

# Kamke differential equations. Mathematica 11 and Maple 2016.1

Nasser M. Abbasi

February 3, 2024

Compiled on February 3, 2024 at 11:23pm

## Contents

<b>1</b>	<b>Introduction and summary of results</b>	<b>49</b>
<b>2</b>	<b>Problems table lookup</b>	<b>54</b>
2.1	ODE No. 1 . . . . .	139
2.2	ODE No. 2 . . . . .	139
2.3	ODE No. 3 . . . . .	140
2.4	ODE No. 4 . . . . .	142
2.5	ODE No. 5 . . . . .	142
2.6	ODE No. 6 . . . . .	143
2.7	ODE No. 7 . . . . .	144
2.8	ODE No. 8 . . . . .	145
2.9	ODE No. 9 . . . . .	146
2.10	ODE No. 10 . . . . .	147
2.11	ODE No. 11 . . . . .	148
2.12	ODE No. 12 . . . . .	149
2.13	ODE No. 13 . . . . .	150
2.14	ODE No. 14 . . . . .	152
2.15	ODE No. 15 . . . . .	154
2.16	ODE No. 16 . . . . .	156
2.17	ODE No. 17 . . . . .	158
2.18	ODE No. 18 . . . . .	159
2.19	ODE No. 19 . . . . .	161
2.20	ODE No. 20 . . . . .	162
2.21	ODE No. 21 . . . . .	163
2.22	ODE No. 22 . . . . .	165
2.23	ODE No. 23 . . . . .	168
2.24	ODE No. 24 . . . . .	169
2.25	ODE No. 25 . . . . .	172
2.26	ODE No. 26 . . . . .	173
2.27	ODE No. 27 . . . . .	175
2.28	ODE No. 28 . . . . .	177

2.29	ODE No. 29 . . . . .	179
2.30	ODE No. 30 . . . . .	180
2.31	ODE No. 31 . . . . .	186
2.32	ODE No. 32 . . . . .	187
2.33	ODE No. 33 . . . . .	189
2.34	ODE No. 34 . . . . .	189
2.35	ODE No. 35 . . . . .	191
2.36	ODE No. 36 . . . . .	193
2.37	ODE No. 37 . . . . .	195
2.38	ODE No. 38 . . . . .	195
2.39	ODE No. 39 . . . . .	197
2.40	ODE No. 40 . . . . .	197
2.41	ODE No. 41 . . . . .	197
2.42	ODE No. 42 . . . . .	198
2.43	ODE No. 43 . . . . .	198
2.44	ODE No. 44 . . . . .	199
2.45	ODE No. 45 . . . . .	200
2.46	ODE No. 46 . . . . .	200
2.47	ODE No. 47 . . . . .	201
2.48	ODE No. 48 . . . . .	201
2.49	ODE No. 49 . . . . .	201
2.50	ODE No. 50 . . . . .	202
2.51	ODE No. 51 . . . . .	202
2.52	ODE No. 52 . . . . .	202
2.53	ODE No. 53 . . . . .	203
2.54	ODE No. 54 . . . . .	203
2.55	ODE No. 55 . . . . .	203
2.56	ODE No. 56 . . . . .	204
2.57	ODE No. 57 . . . . .	204
2.58	ODE No. 58 . . . . .	204
2.59	ODE No. 59 . . . . .	205
2.60	ODE No. 60 . . . . .	205
2.61	ODE No. 61 . . . . .	206
2.62	ODE No. 62 . . . . .	207
2.63	ODE No. 63 . . . . .	209
2.64	ODE No. 64 . . . . .	209
2.65	ODE No. 65 . . . . .	210
2.66	ODE No. 66 . . . . .	210
2.67	ODE No. 67 . . . . .	210
2.68	ODE No. 68 . . . . .	211
2.69	ODE No. 69 . . . . .	211

2.70	ODE No. 70 . . . . .	211
2.71	ODE No. 71 . . . . .	212
2.72	ODE No. 72 . . . . .	212
2.73	ODE No. 73 . . . . .	212
2.74	ODE No. 74 . . . . .	213
2.75	ODE No. 75 . . . . .	213
2.76	ODE No. 76 . . . . .	214
2.77	ODE No. 77 . . . . .	217
2.78	ODE No. 78 . . . . .	218
2.79	ODE No. 79 . . . . .	220
2.80	ODE No. 80 . . . . .	221
2.81	ODE No. 81 . . . . .	221
2.82	ODE No. 82 . . . . .	221
2.83	ODE No. 83 . . . . .	222
2.84	ODE No. 84 . . . . .	222
2.85	ODE No. 85 . . . . .	222
2.86	ODE No. 86 . . . . .	223
2.87	ODE No. 87 . . . . .	223
2.88	ODE No. 88 . . . . .	223
2.89	ODE No. 89 . . . . .	225
2.90	ODE No. 90 . . . . .	227
2.91	ODE No. 91 . . . . .	228
2.92	ODE No. 92 . . . . .	228
2.93	ODE No. 93 . . . . .	229
2.94	ODE No. 94 . . . . .	229
2.95	ODE No. 95 . . . . .	230
2.96	ODE No. 96 . . . . .	232
2.97	ODE No. 97 . . . . .	233
2.98	ODE No. 98 . . . . .	234
2.99	ODE No. 99 . . . . .	234
2.100	ODE No. 100 . . . . .	235
2.101	ODE No. 101 . . . . .	236
2.102	ODE No. 102 . . . . .	238
2.103	ODE No. 103 . . . . .	239
2.104	ODE No. 104 . . . . .	242
2.105	ODE No. 105 . . . . .	243
2.106	ODE No. 106 . . . . .	244
2.107	ODE No. 107 . . . . .	244
2.108	ODE No. 108 . . . . .	244
2.109	ODE No. 109 . . . . .	246
2.110	ODE No. 110 . . . . .	247

2.111	ODE No. 111 . . . . .	249
2.112	ODE No. 112 . . . . .	249
2.113	ODE No. 113 . . . . .	250
2.114	ODE No. 114 . . . . .	252
2.115	ODE No. 115 . . . . .	253
2.116	ODE No. 116 . . . . .	255
2.117	ODE No. 117 . . . . .	255
2.118	ODE No. 118 . . . . .	255
2.119	ODE No. 119 . . . . .	256
2.120	ODE No. 120 . . . . .	256
2.121	ODE No. 121 . . . . .	256
2.122	ODE No. 122 . . . . .	257
2.123	ODE No. 123 . . . . .	257
2.124	ODE No. 124 . . . . .	257
2.125	ODE No. 125 . . . . .	258
2.126	ODE No. 126 . . . . .	258
2.127	ODE No. 127 . . . . .	258
2.128	ODE No. 128 . . . . .	259
2.129	ODE No. 129 . . . . .	259
2.130	ODE No. 130 . . . . .	259
2.131	ODE No. 131 . . . . .	260
2.132	ODE No. 132 . . . . .	260
2.133	ODE No. 133 . . . . .	260
2.134	ODE No. 134 . . . . .	261
2.135	ODE No. 135 . . . . .	261
2.136	ODE No. 136 . . . . .	261
2.137	ODE No. 137 . . . . .	262
2.138	ODE No. 138 . . . . .	262
2.139	ODE No. 139 . . . . .	262
2.140	ODE No. 140 . . . . .	263
2.141	ODE No. 141 . . . . .	263
2.142	ODE No. 142 . . . . .	263
2.143	ODE No. 143 . . . . .	264
2.144	ODE No. 144 . . . . .	264
2.145	ODE No. 145 . . . . .	264
2.146	ODE No. 146 . . . . .	265
2.147	ODE No. 147 . . . . .	265
2.148	ODE No. 148 . . . . .	266
2.149	ODE No. 149 . . . . .	266
2.150	ODE No. 150 . . . . .	266
2.151	ODE No. 151 . . . . .	267

2.152	ODE No. 152 . . . . .	267
2.153	ODE No. 153 . . . . .	267
2.154	ODE No. 154 . . . . .	268
2.155	ODE No. 155 . . . . .	268
2.156	ODE No. 156 . . . . .	268
2.157	ODE No. 157 . . . . .	269
2.158	ODE No. 158 . . . . .	269
2.159	ODE No. 159 . . . . .	269
2.160	ODE No. 160 . . . . .	270
2.161	ODE No. 161 . . . . .	270
2.162	ODE No. 162 . . . . .	270
2.163	ODE No. 163 . . . . .	271
2.164	ODE No. 164 . . . . .	271
2.165	ODE No. 165 . . . . .	271
2.166	ODE No. 166 . . . . .	272
2.167	ODE No. 167 . . . . .	272
2.168	ODE No. 168 . . . . .	272
2.169	ODE No. 169 . . . . .	273
2.170	ODE No. 170 . . . . .	273
2.171	ODE No. 171 . . . . .	273
2.172	ODE No. 172 . . . . .	274
2.173	ODE No. 173 . . . . .	274
2.174	ODE No. 174 . . . . .	274
2.175	ODE No. 175 . . . . .	275
2.176	ODE No. 176 . . . . .	275
2.177	ODE No. 177 . . . . .	275
2.178	ODE No. 178 . . . . .	276
2.179	ODE No. 179 . . . . .	276
2.180	ODE No. 180 . . . . .	276
2.181	ODE No. 181 . . . . .	277
2.182	ODE No. 182 . . . . .	277
2.183	ODE No. 183 . . . . .	278
2.184	ODE No. 184 . . . . .	278
2.185	ODE No. 185 . . . . .	278
2.186	ODE No. 186 . . . . .	279
2.187	ODE No. 187 . . . . .	279
2.188	ODE No. 188 . . . . .	279
2.189	ODE No. 189 . . . . .	280
2.190	ODE No. 190 . . . . .	280
2.191	ODE No. 191 . . . . .	280
2.192	ODE No. 192 . . . . .	281

2.193	ODE No. 193 . . . . .	281
2.194	ODE No. 194 . . . . .	281
2.195	ODE No. 195 . . . . .	282
2.196	ODE No. 196 . . . . .	282
2.197	ODE No. 197 . . . . .	282
2.198	ODE No. 198 . . . . .	283
2.199	ODE No. 199 . . . . .	283
2.200	ODE No. 200 . . . . .	283
2.201	ODE No. 201 . . . . .	284
2.202	ODE No. 202 . . . . .	284
2.203	ODE No. 203 . . . . .	284
2.204	ODE No. 204 . . . . .	284
2.205	ODE No. 205 . . . . .	285
2.206	ODE No. 206 . . . . .	285
2.207	ODE No. 207 . . . . .	285
2.208	ODE No. 208 . . . . .	285
2.209	ODE No. 209 . . . . .	286
2.210	ODE No. 210 . . . . .	286
2.211	ODE No. 211 . . . . .	286
2.212	ODE No. 212 . . . . .	287
2.213	ODE No. 213 . . . . .	287
2.214	ODE No. 214 . . . . .	287
2.215	ODE No. 215 . . . . .	288
2.216	ODE No. 216 . . . . .	288
2.217	ODE No. 217 . . . . .	288
2.218	ODE No. 218 . . . . .	289
2.219	ODE No. 219 . . . . .	289
2.220	ODE No. 220 . . . . .	289
2.221	ODE No. 221 . . . . .	290
2.222	ODE No. 222 . . . . .	290
2.223	ODE No. 223 . . . . .	290
2.224	ODE No. 224 . . . . .	291
2.225	ODE No. 225 . . . . .	291
2.226	ODE No. 226 . . . . .	291
2.227	ODE No. 227 . . . . .	292
2.228	ODE No. 228 . . . . .	292
2.229	ODE No. 229 . . . . .	293
2.230	ODE No. 230 . . . . .	293
2.231	ODE No. 231 . . . . .	293
2.232	ODE No. 232 . . . . .	294
2.233	ODE No. 233 . . . . .	294

2.234	ODE No. 234 . . . . .	294
2.235	ODE No. 235 . . . . .	295
2.236	ODE No. 236 . . . . .	295
2.237	ODE No. 237 . . . . .	295
2.238	ODE No. 238 . . . . .	296
2.239	ODE No. 239 . . . . .	296
2.240	ODE No. 240 . . . . .	296
2.241	ODE No. 241 . . . . .	297
2.242	ODE No. 242 . . . . .	297
2.243	ODE No. 243 . . . . .	297
2.244	ODE No. 244 . . . . .	298
2.245	ODE No. 245 . . . . .	298
2.246	ODE No. 246 . . . . .	298
2.247	ODE No. 247 . . . . .	299
2.248	ODE No. 248 . . . . .	299
2.249	ODE No. 249 . . . . .	299
2.250	ODE No. 250 . . . . .	300
2.251	ODE No. 251 . . . . .	300
2.252	ODE No. 252 . . . . .	300
2.253	ODE No. 253 . . . . .	301
2.254	ODE No. 254 . . . . .	301
2.255	ODE No. 255 . . . . .	301
2.256	ODE No. 256 . . . . .	302
2.257	ODE No. 257 . . . . .	302
2.258	ODE No. 258 . . . . .	302
2.259	ODE No. 259 . . . . .	303
2.260	ODE No. 260 . . . . .	303
2.261	ODE No. 261 . . . . .	303
2.262	ODE No. 262 . . . . .	304
2.263	ODE No. 263 . . . . .	304
2.264	ODE No. 264 . . . . .	304
2.265	ODE No. 265 . . . . .	305
2.266	ODE No. 266 . . . . .	305
2.267	ODE No. 267 . . . . .	305
2.268	ODE No. 268 . . . . .	306
2.269	ODE No. 269 . . . . .	306
2.270	ODE No. 270 . . . . .	306
2.271	ODE No. 271 . . . . .	307
2.272	ODE No. 272 . . . . .	307
2.273	ODE No. 273 . . . . .	307
2.274	ODE No. 274 . . . . .	308

2.275	ODE No. 275 . . . . .	308
2.276	ODE No. 276 . . . . .	308
2.277	ODE No. 277 . . . . .	309
2.278	ODE No. 278 . . . . .	309
2.279	ODE No. 279 . . . . .	309
2.280	ODE No. 280 . . . . .	310
2.281	ODE No. 281 . . . . .	310
2.282	ODE No. 282 . . . . .	310
2.283	ODE No. 283 . . . . .	311
2.284	ODE No. 284 . . . . .	311
2.285	ODE No. 285 . . . . .	312
2.286	ODE No. 286 . . . . .	312
2.287	ODE No. 287 . . . . .	312
2.288	ODE No. 288 . . . . .	313
2.289	ODE No. 289 . . . . .	313
2.290	ODE No. 290 . . . . .	313
2.291	ODE No. 291 . . . . .	314
2.292	ODE No. 292 . . . . .	314
2.293	ODE No. 293 . . . . .	315
2.294	ODE No. 294 . . . . .	315
2.295	ODE No. 295 . . . . .	315
2.296	ODE No. 296 . . . . .	316
2.297	ODE No. 297 . . . . .	316
2.298	ODE No. 298 . . . . .	316
2.299	ODE No. 299 . . . . .	317
2.300	ODE No. 300 . . . . .	317
2.301	ODE No. 301 . . . . .	317
2.302	ODE No. 302 . . . . .	318
2.303	ODE No. 303 . . . . .	318
2.304	ODE No. 304 . . . . .	318
2.305	ODE No. 305 . . . . .	319
2.306	ODE No. 306 . . . . .	319
2.307	ODE No. 307 . . . . .	319
2.308	ODE No. 308 . . . . .	320
2.309	ODE No. 309 . . . . .	320
2.310	ODE No. 310 . . . . .	320
2.311	ODE No. 311 . . . . .	321
2.312	ODE No. 312 . . . . .	321
2.313	ODE No. 313 . . . . .	322
2.314	ODE No. 314 . . . . .	322
2.315	ODE No. 315 . . . . .	322



2.316	ODE No. 316 . . . . .	323
2.317	ODE No. 317 . . . . .	323
2.318	ODE No. 318 . . . . .	324
2.319	ODE No. 319 . . . . .	324
2.320	ODE No. 320 . . . . .	325
2.321	ODE No. 321 . . . . .	325
2.322	ODE No. 322 . . . . .	325
2.323	ODE No. 323 . . . . .	326
2.324	ODE No. 324 . . . . .	326
2.325	ODE No. 325 . . . . .	327
2.326	ODE No. 326 . . . . .	327
2.327	ODE No. 327 . . . . .	328
2.328	ODE No. 328 . . . . .	328
2.329	ODE No. 329 . . . . .	328
2.330	ODE No. 330 . . . . .	329
2.331	ODE No. 331 . . . . .	329
2.332	ODE No. 332 . . . . .	329
2.333	ODE No. 333 . . . . .	330
2.334	ODE No. 334 . . . . .	330
2.335	ODE No. 335 . . . . .	330
2.336	ODE No. 336 . . . . .	331
2.337	ODE No. 337 . . . . .	331
2.338	ODE No. 338 . . . . .	331
2.339	ODE No. 339 . . . . .	332
2.340	ODE No. 340 . . . . .	332
2.341	ODE No. 341 . . . . .	332
2.342	ODE No. 342 . . . . .	333
2.343	ODE No. 343 . . . . .	333
2.344	ODE No. 344 . . . . .	333
2.345	ODE No. 345 . . . . .	334
2.346	ODE No. 346 . . . . .	334
2.347	ODE No. 347 . . . . .	334
2.348	ODE No. 348 . . . . .	335
2.349	ODE No. 349 . . . . .	335
2.350	ODE No. 350 . . . . .	335
2.351	ODE No. 351 . . . . .	336
2.352	ODE No. 352 . . . . .	336
2.353	ODE No. 353 . . . . .	336
2.354	ODE No. 354 . . . . .	337
2.355	ODE No. 355 . . . . .	337
2.356	ODE No. 356 . . . . .	337

2.357	ODE No. 357 . . . . .	338
2.358	ODE No. 358 . . . . .	338
2.359	ODE No. 359 . . . . .	338
2.360	ODE No. 360 . . . . .	339
2.361	ODE No. 361 . . . . .	339
2.362	ODE No. 362 . . . . .	340
2.363	ODE No. 363 . . . . .	340
2.364	ODE No. 364 . . . . .	340
2.365	ODE No. 365 . . . . .	341
2.366	ODE No. 366 . . . . .	341
2.367	ODE No. 367 . . . . .	341
2.368	ODE No. 368 . . . . .	342
2.369	ODE No. 369 . . . . .	342
2.370	ODE No. 370 . . . . .	342
2.371	ODE No. 371 . . . . .	343
2.372	ODE No. 372 . . . . .	343
2.373	ODE No. 373 . . . . .	343
2.374	ODE No. 374 . . . . .	344
2.375	ODE No. 375 . . . . .	344
2.376	ODE No. 376 . . . . .	344
2.377	ODE No. 377 . . . . .	345
2.378	ODE No. 378 . . . . .	345
2.379	ODE No. 379 . . . . .	345
2.380	ODE No. 380 . . . . .	346
2.381	ODE No. 381 . . . . .	346
2.382	ODE No. 382 . . . . .	347
2.383	ODE No. 383 . . . . .	347
2.384	ODE No. 384 . . . . .	347
2.385	ODE No. 385 . . . . .	348
2.386	ODE No. 386 . . . . .	348
2.387	ODE No. 387 . . . . .	348
2.388	ODE No. 388 . . . . .	349
2.389	ODE No. 389 . . . . .	349
2.390	ODE No. 390 . . . . .	349
2.391	ODE No. 391 . . . . .	350
2.392	ODE No. 392 . . . . .	350
2.393	ODE No. 393 . . . . .	351
2.394	ODE No. 394 . . . . .	351
2.395	ODE No. 395 . . . . .	351
2.396	ODE No. 396 . . . . .	352
2.397	ODE No. 397 . . . . .	352

2.398	ODE No. 398 . . . . .	352
2.399	ODE No. 399 . . . . .	353
2.400	ODE No. 400 . . . . .	353
2.401	ODE No. 401 . . . . .	353
2.402	ODE No. 402 . . . . .	354
2.403	ODE No. 403 . . . . .	354
2.404	ODE No. 404 . . . . .	354
2.405	ODE No. 405 . . . . .	355
2.406	ODE No. 406 . . . . .	355
2.407	ODE No. 407 . . . . .	356
2.408	ODE No. 408 . . . . .	356
2.409	ODE No. 409 . . . . .	356
2.410	ODE No. 410 . . . . .	357
2.411	ODE No. 411 . . . . .	357
2.412	ODE No. 412 . . . . .	357
2.413	ODE No. 413 . . . . .	358
2.414	ODE No. 414 . . . . .	358
2.415	ODE No. 415 . . . . .	359
2.416	ODE No. 416 . . . . .	359
2.417	ODE No. 417 . . . . .	359
2.418	ODE No. 418 . . . . .	360
2.419	ODE No. 419 . . . . .	360
2.420	ODE No. 420 . . . . .	361
2.421	ODE No. 421 . . . . .	361
2.422	ODE No. 422 . . . . .	362
2.423	ODE No. 423 . . . . .	362
2.424	ODE No. 424 . . . . .	362
2.425	ODE No. 425 . . . . .	363
2.426	ODE No. 426 . . . . .	363
2.427	ODE No. 427 . . . . .	363
2.428	ODE No. 428 . . . . .	364
2.429	ODE No. 429 . . . . .	364
2.430	ODE No. 430 . . . . .	364
2.431	ODE No. 431 . . . . .	365
2.432	ODE No. 432 . . . . .	365
2.433	ODE No. 433 . . . . .	365
2.434	ODE No. 434 . . . . .	366
2.435	ODE No. 435 . . . . .	366
2.436	ODE No. 436 . . . . .	366
2.437	ODE No. 437 . . . . .	367
2.438	ODE No. 438 . . . . .	367

2.439	ODE No. 439 . . . . .	367
2.440	ODE No. 440 . . . . .	368
2.441	ODE No. 441 . . . . .	368
2.442	ODE No. 442 . . . . .	368
2.443	ODE No. 443 . . . . .	369
2.444	ODE No. 444 . . . . .	369
2.445	ODE No. 445 . . . . .	369
2.446	ODE No. 446 . . . . .	370
2.447	ODE No. 447 . . . . .	370
2.448	ODE No. 448 . . . . .	370
2.449	ODE No. 449 . . . . .	371
2.450	ODE No. 450 . . . . .	371
2.451	ODE No. 451 . . . . .	371
2.452	ODE No. 452 . . . . .	372
2.453	ODE No. 453 . . . . .	372
2.454	ODE No. 454 . . . . .	372
2.455	ODE No. 455 . . . . .	373
2.456	ODE No. 456 . . . . .	373
2.457	ODE No. 457 . . . . .	373
2.458	ODE No. 458 . . . . .	374
2.459	ODE No. 459 . . . . .	374
2.460	ODE No. 460 . . . . .	374
2.461	ODE No. 461 . . . . .	375
2.462	ODE No. 462 . . . . .	375
2.463	ODE No. 463 . . . . .	375
2.464	ODE No. 464 . . . . .	376
2.465	ODE No. 465 . . . . .	376
2.466	ODE No. 466 . . . . .	376
2.467	ODE No. 467 . . . . .	377
2.468	ODE No. 468 . . . . .	377
2.469	ODE No. 469 . . . . .	377
2.470	ODE No. 470 . . . . .	378
2.471	ODE No. 471 . . . . .	378
2.472	ODE No. 472 . . . . .	378
2.473	ODE No. 473 . . . . .	379
2.474	ODE No. 474 . . . . .	379
2.475	ODE No. 475 . . . . .	379
2.476	ODE No. 476 . . . . .	380
2.477	ODE No. 477 . . . . .	380
2.478	ODE No. 478 . . . . .	380
2.479	ODE No. 479 . . . . .	381

2.480	ODE No. 480 . . . . .	381
2.481	ODE No. 481 . . . . .	381
2.482	ODE No. 482 . . . . .	382
2.483	ODE No. 483 . . . . .	382
2.484	ODE No. 484 . . . . .	382
2.485	ODE No. 485 . . . . .	383
2.486	ODE No. 486 . . . . .	383
2.487	ODE No. 487 . . . . .	383
2.488	ODE No. 488 . . . . .	384
2.489	ODE No. 489 . . . . .	384
2.490	ODE No. 490 . . . . .	384
2.491	ODE No. 491 . . . . .	385
2.492	ODE No. 492 . . . . .	385
2.493	ODE No. 493 . . . . .	385
2.494	ODE No. 494 . . . . .	386
2.495	ODE No. 495 . . . . .	386
2.496	ODE No. 496 . . . . .	386
2.497	ODE No. 497 . . . . .	387
2.498	ODE No. 498 . . . . .	387
2.499	ODE No. 499 . . . . .	387
2.500	ODE No. 500 . . . . .	388
2.501	ODE No. 501 . . . . .	388
2.502	ODE No. 502 . . . . .	388
2.503	ODE No. 503 . . . . .	389
2.504	ODE No. 504 . . . . .	389
2.505	ODE No. 505 . . . . .	389
2.506	ODE No. 506 . . . . .	390
2.507	ODE No. 507 . . . . .	390
2.508	ODE No. 508 . . . . .	390
2.509	ODE No. 509 . . . . .	391
2.510	ODE No. 510 . . . . .	391
2.511	ODE No. 511 . . . . .	391
2.512	ODE No. 512 . . . . .	392
2.513	ODE No. 513 . . . . .	392
2.514	ODE No. 514 . . . . .	393
2.515	ODE No. 515 . . . . .	393
2.516	ODE No. 516 . . . . .	393
2.517	ODE No. 517 . . . . .	394
2.518	ODE No. 518 . . . . .	394
2.519	ODE No. 519 . . . . .	395
2.520	ODE No. 520 . . . . .	395

2.521	ODE No. 521 . . . . .	396
2.522	ODE No. 522 . . . . .	396
2.523	ODE No. 523 . . . . .	396
2.524	ODE No. 524 . . . . .	397
2.525	ODE No. 525 . . . . .	397
2.526	ODE No. 526 . . . . .	397
2.527	ODE No. 527 . . . . .	398
2.528	ODE No. 528 . . . . .	398
2.529	ODE No. 529 . . . . .	398
2.530	ODE No. 530 . . . . .	399
2.531	ODE No. 531 . . . . .	399
2.532	ODE No. 532 . . . . .	399
2.533	ODE No. 533 . . . . .	400
2.534	ODE No. 534 . . . . .	400
2.535	ODE No. 535 . . . . .	400
2.536	ODE No. 536 . . . . .	401
2.537	ODE No. 537 . . . . .	401
2.538	ODE No. 538 . . . . .	401
2.539	ODE No. 539 . . . . .	402
2.540	ODE No. 540 . . . . .	402
2.541	ODE No. 541 . . . . .	402
2.542	ODE No. 542 . . . . .	403
2.543	ODE No. 543 . . . . .	403
2.544	ODE No. 544 . . . . .	403
2.545	ODE No. 545 . . . . .	404
2.546	ODE No. 546 . . . . .	404
2.547	ODE No. 547 . . . . .	404
2.548	ODE No. 548 . . . . .	405
2.549	ODE No. 549 . . . . .	405
2.550	ODE No. 550 . . . . .	406
2.551	ODE No. 551 . . . . .	406
2.552	ODE No. 552 . . . . .	406
2.553	ODE No. 553 . . . . .	407
2.554	ODE No. 554 . . . . .	407
2.555	ODE No. 555 . . . . .	407
2.556	ODE No. 556 . . . . .	408
2.557	ODE No. 557 . . . . .	408
2.558	ODE No. 558 . . . . .	408
2.559	ODE No. 559 . . . . .	409
2.560	ODE No. 560 . . . . .	409
2.561	ODE No. 561 . . . . .	410

2.562	ODE No. 562 . . . . .	410
2.563	ODE No. 563 . . . . .	410
2.564	ODE No. 564 . . . . .	411
2.565	ODE No. 565 . . . . .	411
2.566	ODE No. 566 . . . . .	411
2.567	ODE No. 567 . . . . .	412
2.568	ODE No. 568 . . . . .	412
2.569	ODE No. 569 . . . . .	412
2.570	ODE No. 570 . . . . .	413
2.571	ODE No. 571 . . . . .	413
2.572	ODE No. 572 . . . . .	413
2.573	ODE No. 573 . . . . .	414
2.574	ODE No. 574 . . . . .	414
2.575	ODE No. 575 . . . . .	414
2.576	ODE No. 576 . . . . .	415
2.577	ODE No. 577 . . . . .	415
2.578	ODE No. 578 . . . . .	415
2.579	ODE No. 579 . . . . .	416
2.580	ODE No. 580 . . . . .	416
2.581	ODE No. 581 . . . . .	416
2.582	ODE No. 582 . . . . .	417
2.583	ODE No. 583 . . . . .	417
2.584	ODE No. 584 . . . . .	418
2.585	ODE No. 585 . . . . .	418
2.586	ODE No. 586 . . . . .	418
2.587	ODE No. 587 . . . . .	419
2.588	ODE No. 588 . . . . .	419
2.589	ODE No. 589 . . . . .	420
2.590	ODE No. 590 . . . . .	420
2.591	ODE No. 591 . . . . .	421
2.592	ODE No. 592 . . . . .	421
2.593	ODE No. 593 . . . . .	421
2.594	ODE No. 594 . . . . .	422
2.595	ODE No. 595 . . . . .	422
2.596	ODE No. 596 . . . . .	423
2.597	ODE No. 597 . . . . .	423
2.598	ODE No. 598 . . . . .	423
2.599	ODE No. 599 . . . . .	424
2.600	ODE No. 600 . . . . .	424
2.601	ODE No. 601 . . . . .	425
2.602	ODE No. 602 . . . . .	425

2.603	ODE No. 603	426
2.604	ODE No. 604	426
2.605	ODE No. 605	426
2.606	ODE No. 606	427
2.607	ODE No. 607	427
2.608	ODE No. 608	428
2.609	ODE No. 609	428
2.610	ODE No. 610	428
2.611	ODE No. 611	429
2.612	ODE No. 612	429
2.613	ODE No. 613	429
2.614	ODE No. 614	430
2.615	ODE No. 615	430
2.616	ODE No. 616	430
2.617	ODE No. 617	431
2.618	ODE No. 618	431
2.619	ODE No. 619	432
2.620	ODE No. 620	432
2.621	ODE No. 621	432
2.622	ODE No. 622	433
2.623	ODE No. 623	433
2.624	ODE No. 624	433
2.625	ODE No. 625	434
2.626	ODE No. 626	434
2.627	ODE No. 627	434
2.628	ODE No. 628	435
2.629	ODE No. 629	435
2.630	ODE No. 630	435
2.631	ODE No. 631	436
2.632	ODE No. 632	436
2.633	ODE No. 633	437
2.634	ODE No. 634	437
2.635	ODE No. 635	437
2.636	ODE No. 636	438
2.637	ODE No. 637	438
2.638	ODE No. 638	438
2.639	ODE No. 639	439
2.640	ODE No. 640	439
2.641	ODE No. 641	439
2.642	ODE No. 642	440
2.643	ODE No. 643	440



2.644	ODE No. 644 . . . . .	440
2.645	ODE No. 645 . . . . .	441
2.646	ODE No. 646 . . . . .	441
2.647	ODE No. 647 . . . . .	441
2.648	ODE No. 648 . . . . .	442
2.649	ODE No. 649 . . . . .	442
2.650	ODE No. 650 . . . . .	442
2.651	ODE No. 651 . . . . .	443
2.652	ODE No. 652 . . . . .	443
2.653	ODE No. 653 . . . . .	443
2.654	ODE No. 654 . . . . .	444
2.655	ODE No. 655 . . . . .	444
2.656	ODE No. 656 . . . . .	444
2.657	ODE No. 657 . . . . .	445
2.658	ODE No. 658 . . . . .	445
2.659	ODE No. 659 . . . . .	445
2.660	ODE No. 660 . . . . .	446
2.661	ODE No. 661 . . . . .	446
2.662	ODE No. 662 . . . . .	446
2.663	ODE No. 663 . . . . .	447
2.664	ODE No. 664 . . . . .	447
2.665	ODE No. 665 . . . . .	447
2.666	ODE No. 666 . . . . .	448
2.667	ODE No. 667 . . . . .	448
2.668	ODE No. 668 . . . . .	448
2.669	ODE No. 669 . . . . .	449
2.670	ODE No. 670 . . . . .	449
2.671	ODE No. 671 . . . . .	450
2.672	ODE No. 672 . . . . .	450
2.673	ODE No. 673 . . . . .	450
2.674	ODE No. 674 . . . . .	451
2.675	ODE No. 675 . . . . .	451
2.676	ODE No. 676 . . . . .	451
2.677	ODE No. 677 . . . . .	452
2.678	ODE No. 678 . . . . .	452
2.679	ODE No. 679 . . . . .	452
2.680	ODE No. 680 . . . . .	453
2.681	ODE No. 681 . . . . .	453
2.682	ODE No. 682 . . . . .	453
2.683	ODE No. 683 . . . . .	454
2.684	ODE No. 684 . . . . .	454

2.685	ODE No. 685 . . . . .	454
2.686	ODE No. 686 . . . . .	455
2.687	ODE No. 687 . . . . .	455
2.688	ODE No. 688 . . . . .	455
2.689	ODE No. 689 . . . . .	456
2.690	ODE No. 690 . . . . .	456
2.691	ODE No. 691 . . . . .	456
2.692	ODE No. 692 . . . . .	457
2.693	ODE No. 693 . . . . .	457
2.694	ODE No. 694 . . . . .	457
2.695	ODE No. 695 . . . . .	458
2.696	ODE No. 696 . . . . .	458
2.697	ODE No. 697 . . . . .	458
2.698	ODE No. 698 . . . . .	459
2.699	ODE No. 699 . . . . .	459
2.700	ODE No. 700 . . . . .	459
2.701	ODE No. 701 . . . . .	460
2.702	ODE No. 702 . . . . .	460
2.703	ODE No. 703 . . . . .	460
2.704	ODE No. 704 . . . . .	461
2.705	ODE No. 705 . . . . .	461
2.706	ODE No. 706 . . . . .	461
2.707	ODE No. 707 . . . . .	462
2.708	ODE No. 708 . . . . .	462
2.709	ODE No. 709 . . . . .	462
2.710	ODE No. 710 . . . . .	463
2.711	ODE No. 711 . . . . .	463
2.712	ODE No. 712 . . . . .	463
2.713	ODE No. 713 . . . . .	464
2.714	ODE No. 714 . . . . .	464
2.715	ODE No. 715 . . . . .	464
2.716	ODE No. 716 . . . . .	465
2.717	ODE No. 717 . . . . .	465
2.718	ODE No. 718 . . . . .	465
2.719	ODE No. 719 . . . . .	466
2.720	ODE No. 720 . . . . .	466
2.721	ODE No. 721 . . . . .	466
2.722	ODE No. 722 . . . . .	467
2.723	ODE No. 723 . . . . .	467
2.724	ODE No. 724 . . . . .	468
2.725	ODE No. 725 . . . . .	468

2.726	ODE No. 726 . . . . .	468
2.727	ODE No. 727 . . . . .	469
2.728	ODE No. 728 . . . . .	469
2.729	ODE No. 729 . . . . .	470
2.730	ODE No. 730 . . . . .	470
2.731	ODE No. 731 . . . . .	470
2.732	ODE No. 732 . . . . .	471
2.733	ODE No. 733 . . . . .	471
2.734	ODE No. 734 . . . . .	471
2.735	ODE No. 735 . . . . .	472
2.736	ODE No. 736 . . . . .	472
2.737	ODE No. 737 . . . . .	472
2.738	ODE No. 738 . . . . .	473
2.739	ODE No. 739 . . . . .	473
2.740	ODE No. 740 . . . . .	474
2.741	ODE No. 741 . . . . .	474
2.742	ODE No. 742 . . . . .	474
2.743	ODE No. 743 . . . . .	475
2.744	ODE No. 744 . . . . .	475
2.745	ODE No. 745 . . . . .	475
2.746	ODE No. 746 . . . . .	476
2.747	ODE No. 747 . . . . .	476
2.748	ODE No. 748 . . . . .	476
2.749	ODE No. 749 . . . . .	477
2.750	ODE No. 750 . . . . .	477
2.751	ODE No. 751 . . . . .	478
2.752	ODE No. 752 . . . . .	478
2.753	ODE No. 753 . . . . .	478
2.754	ODE No. 754 . . . . .	479
2.755	ODE No. 755 . . . . .	479
2.756	ODE No. 756 . . . . .	479
2.757	ODE No. 757 . . . . .	480
2.758	ODE No. 758 . . . . .	480
2.759	ODE No. 759 . . . . .	480
2.760	ODE No. 760 . . . . .	481
2.761	ODE No. 761 . . . . .	481
2.762	ODE No. 762 . . . . .	481
2.763	ODE No. 763 . . . . .	482
2.764	ODE No. 764 . . . . .	482
2.765	ODE No. 765 . . . . .	482
2.766	ODE No. 766 . . . . .	483

2.767	ODE No. 767	483
2.768	ODE No. 768	483
2.769	ODE No. 769	484
2.770	ODE No. 770	484
2.771	ODE No. 771	484
2.772	ODE No. 772	485
2.773	ODE No. 773	485
2.774	ODE No. 774	486
2.775	ODE No. 775	486
2.776	ODE No. 776	486
2.777	ODE No. 777	487
2.778	ODE No. 778	487
2.779	ODE No. 779	487
2.780	ODE No. 780	488
2.781	ODE No. 781	488
2.782	ODE No. 782	488
2.783	ODE No. 783	489
2.784	ODE No. 784	489
2.785	ODE No. 785	489
2.786	ODE No. 786	490
2.787	ODE No. 787	490
2.788	ODE No. 788	490
2.789	ODE No. 789	491
2.790	ODE No. 790	491
2.791	ODE No. 791	491
2.792	ODE No. 792	492
2.793	ODE No. 793	492
2.794	ODE No. 794	492
2.795	ODE No. 795	493
2.796	ODE No. 796	493
2.797	ODE No. 797	494
2.798	ODE No. 798	494
2.799	ODE No. 799	494
2.800	ODE No. 800	495
2.801	ODE No. 801	495
2.802	ODE No. 802	496
2.803	ODE No. 803	496
2.804	ODE No. 804	497
2.805	ODE No. 805	497
2.806	ODE No. 806	497
2.807	ODE No. 807	498

2.808	ODE No. 808 . . . . .	498
2.809	ODE No. 809 . . . . .	498
2.810	ODE No. 810 . . . . .	499
2.811	ODE No. 811 . . . . .	499
2.812	ODE No. 812 . . . . .	500
2.813	ODE No. 813 . . . . .	500
2.814	ODE No. 814 . . . . .	500
2.815	ODE No. 815 . . . . .	501
2.816	ODE No. 816 . . . . .	501
2.817	ODE No. 817 . . . . .	501
2.818	ODE No. 818 . . . . .	502
2.819	ODE No. 819 . . . . .	502
2.820	ODE No. 820 . . . . .	502
2.821	ODE No. 821 . . . . .	503
2.822	ODE No. 822 . . . . .	503
2.823	ODE No. 823 . . . . .	504
2.824	ODE No. 824 . . . . .	504
2.825	ODE No. 825 . . . . .	504
2.826	ODE No. 826 . . . . .	505
2.827	ODE No. 827 . . . . .	506
2.828	ODE No. 828 . . . . .	506
2.829	ODE No. 829 . . . . .	506
2.830	ODE No. 830 . . . . .	507
2.831	ODE No. 831 . . . . .	507
2.832	ODE No. 832 . . . . .	507
2.833	ODE No. 833 . . . . .	508
2.834	ODE No. 834 . . . . .	508
2.835	ODE No. 835 . . . . .	509
2.836	ODE No. 836 . . . . .	509
2.837	ODE No. 837 . . . . .	509
2.838	ODE No. 838 . . . . .	510
2.839	ODE No. 839 . . . . .	510
2.840	ODE No. 840 . . . . .	510
2.841	ODE No. 841 . . . . .	511
2.842	ODE No. 842 . . . . .	511
2.843	ODE No. 843 . . . . .	511
2.844	ODE No. 844 . . . . .	512
2.845	ODE No. 845 . . . . .	512
2.846	ODE No. 846 . . . . .	513
2.847	ODE No. 847 . . . . .	513
2.848	ODE No. 848 . . . . .	513

2.849	ODE No. 849 . . . . .	514
2.850	ODE No. 850 . . . . .	514
2.851	ODE No. 851 . . . . .	514
2.852	ODE No. 852 . . . . .	515
2.853	ODE No. 853 . . . . .	515
2.854	ODE No. 854 . . . . .	515
2.855	ODE No. 855 . . . . .	516
2.856	ODE No. 856 . . . . .	516
2.857	ODE No. 857 . . . . .	516
2.858	ODE No. 858 . . . . .	517
2.859	ODE No. 859 . . . . .	517
2.860	ODE No. 860 . . . . .	517
2.861	ODE No. 861 . . . . .	518
2.862	ODE No. 862 . . . . .	518
2.863	ODE No. 863 . . . . .	518
2.864	ODE No. 864 . . . . .	519
2.865	ODE No. 865 . . . . .	519
2.866	ODE No. 866 . . . . .	519
2.867	ODE No. 867 . . . . .	520
2.868	ODE No. 868 . . . . .	520
2.869	ODE No. 869 . . . . .	520
2.870	ODE No. 870 . . . . .	521
2.871	ODE No. 871 . . . . .	521
2.872	ODE No. 872 . . . . .	521
2.873	ODE No. 873 . . . . .	522
2.874	ODE No. 874 . . . . .	522
2.875	ODE No. 875 . . . . .	522
2.876	ODE No. 876 . . . . .	523
2.877	ODE No. 877 . . . . .	523
2.878	ODE No. 878 . . . . .	524
2.879	ODE No. 879 . . . . .	524
2.880	ODE No. 880 . . . . .	524
2.881	ODE No. 881 . . . . .	525
2.882	ODE No. 882 . . . . .	525
2.883	ODE No. 883 . . . . .	525
2.884	ODE No. 884 . . . . .	526
2.885	ODE No. 885 . . . . .	526
2.886	ODE No. 886 . . . . .	526
2.887	ODE No. 887 . . . . .	527
2.888	ODE No. 888 . . . . .	527
2.889	ODE No. 889 . . . . .	527

2.890	ODE No. 890 . . . . .	528
2.891	ODE No. 891 . . . . .	528
2.892	ODE No. 892 . . . . .	528
2.893	ODE No. 893 . . . . .	529
2.894	ODE No. 894 . . . . .	529
2.895	ODE No. 895 . . . . .	529
2.896	ODE No. 896 . . . . .	530
2.897	ODE No. 897 . . . . .	530
2.898	ODE No. 898 . . . . .	530
2.899	ODE No. 899 . . . . .	531
2.900	ODE No. 900 . . . . .	531
2.901	ODE No. 901 . . . . .	531
2.902	ODE No. 902 . . . . .	532
2.903	ODE No. 903 . . . . .	532
2.904	ODE No. 904 . . . . .	532
2.905	ODE No. 905 . . . . .	533
2.906	ODE No. 906 . . . . .	533
2.907	ODE No. 907 . . . . .	533
2.908	ODE No. 908 . . . . .	534
2.909	ODE No. 909 . . . . .	534
2.910	ODE No. 910 . . . . .	534
2.911	ODE No. 911 . . . . .	535
2.912	ODE No. 912 . . . . .	535
2.913	ODE No. 913 . . . . .	535
2.914	ODE No. 914 . . . . .	536
2.915	ODE No. 915 . . . . .	536
2.916	ODE No. 916 . . . . .	536
2.917	ODE No. 917 . . . . .	537
2.918	ODE No. 918 . . . . .	537
2.919	ODE No. 919 . . . . .	537
2.920	ODE No. 920 . . . . .	538
2.921	ODE No. 921 . . . . .	538
2.922	ODE No. 922 . . . . .	538
2.923	ODE No. 923 . . . . .	539
2.924	ODE No. 924 . . . . .	539
2.925	ODE No. 925 . . . . .	539
2.926	ODE No. 926 . . . . .	540
2.927	ODE No. 927 . . . . .	540
2.928	ODE No. 928 . . . . .	540
2.929	ODE No. 929 . . . . .	541
2.930	ODE No. 930 . . . . .	541

2.931	ODE No. 931 . . . . .	541
2.932	ODE No. 932 . . . . .	542
2.933	ODE No. 933 . . . . .	542
2.934	ODE No. 934 . . . . .	542
2.935	ODE No. 935 . . . . .	543
2.936	ODE No. 936 . . . . .	543
2.937	ODE No. 937 . . . . .	543
2.938	ODE No. 938 . . . . .	544
2.939	ODE No. 939 . . . . .	544
2.940	ODE No. 940 . . . . .	544
2.941	ODE No. 941 . . . . .	545
2.942	ODE No. 942 . . . . .	545
2.943	ODE No. 943 . . . . .	546
2.944	ODE No. 944 . . . . .	546
2.945	ODE No. 945 . . . . .	546
2.946	ODE No. 946 . . . . .	547
2.947	ODE No. 947 . . . . .	547
2.948	ODE No. 948 . . . . .	547
2.949	ODE No. 949 . . . . .	548
2.950	ODE No. 950 . . . . .	548
2.951	ODE No. 951 . . . . .	548
2.952	ODE No. 952 . . . . .	549
2.953	ODE No. 953 . . . . .	549
2.954	ODE No. 954 . . . . .	549
2.955	ODE No. 955 . . . . .	550
2.956	ODE No. 956 . . . . .	550
2.957	ODE No. 957 . . . . .	551
2.958	ODE No. 958 . . . . .	551
2.959	ODE No. 959 . . . . .	552
2.960	ODE No. 960 . . . . .	552
2.961	ODE No. 961 . . . . .	552
2.962	ODE No. 962 . . . . .	553
2.963	ODE No. 963 . . . . .	553
2.964	ODE No. 964 . . . . .	554
2.965	ODE No. 965 . . . . .	554
2.966	ODE No. 966 . . . . .	554
2.967	ODE No. 967 . . . . .	555
2.968	ODE No. 968 . . . . .	555
2.969	ODE No. 969 . . . . .	556
2.970	ODE No. 970 . . . . .	556
2.971	ODE No. 971 . . . . .	556



2.972	ODE No. 972 . . . . .	557
2.973	ODE No. 973 . . . . .	557
2.974	ODE No. 974 . . . . .	557
2.975	ODE No. 975 . . . . .	558
2.976	ODE No. 976 . . . . .	558
2.977	ODE No. 977 . . . . .	558
2.978	ODE No. 978 . . . . .	559
2.979	ODE No. 979 . . . . .	559
2.980	ODE No. 980 . . . . .	559
2.981	ODE No. 981 . . . . .	560
2.982	ODE No. 982 . . . . .	560
2.983	ODE No. 983 . . . . .	560
2.984	ODE No. 984 . . . . .	561
2.985	ODE No. 985 . . . . .	561
2.986	ODE No. 986 . . . . .	561
2.987	ODE No. 987 . . . . .	562
2.988	ODE No. 988 . . . . .	562
2.989	ODE No. 989 . . . . .	562
2.990	ODE No. 990 . . . . .	563
2.991	ODE No. 991 . . . . .	563
2.992	ODE No. 992 . . . . .	563
2.993	ODE No. 993 . . . . .	564
2.994	ODE No. 994 . . . . .	564
2.995	ODE No. 995 . . . . .	564
2.996	ODE No. 996 . . . . .	565
2.997	ODE No. 997 . . . . .	565
2.998	ODE No. 998 . . . . .	565
2.999	ODE No. 999 . . . . .	566
2.1000	ODE No. 1000 . . . . .	566
2.1001	ODE No. 1001 . . . . .	566
2.1002	ODE No. 1002 . . . . .	567
2.1003	ODE No. 1003 . . . . .	568
2.1004	ODE No. 1004 . . . . .	571
2.1005	ODE No. 1005 . . . . .	573
2.1006	ODE No. 1006 . . . . .	576
2.1007	ODE No. 1007 . . . . .	576
2.1008	ODE No. 1008 . . . . .	578
2.1009	ODE No. 1009 . . . . .	579
2.1010	ODE No. 1010 . . . . .	579
2.1011	ODE No. 1011 . . . . .	583
2.1012	ODE No. 1012 . . . . .	586

2.1013	ODE No. 1013	586
2.1014	ODE No. 1014	587
2.1015	ODE No. 1015	587
2.1016	ODE No. 1016	587
2.1017	ODE No. 1017	588
2.1018	ODE No. 1018	588
2.1019	ODE No. 1019	588
2.1020	ODE No. 1020	589
2.1021	ODE No. 1021	589
2.1022	ODE No. 1022	589
2.1023	ODE No. 1023	590
2.1024	ODE No. 1024	590
2.1025	ODE No. 1025	590
2.1026	ODE No. 1026	591
2.1027	ODE No. 1027	591
2.1028	ODE No. 1028	591
2.1029	ODE No. 1029	592
2.1030	ODE No. 1030	592
2.1031	ODE No. 1031	592
2.1032	ODE No. 1032	593
2.1033	ODE No. 1033	593
2.1034	ODE No. 1034	594
2.1035	ODE No. 1035	596
2.1036	ODE No. 1036	596
2.1037	ODE No. 1037	597
2.1038	ODE No. 1038	597
2.1039	ODE No. 1039	597
2.1040	ODE No. 1040	598
2.1041	ODE No. 1041	598
2.1042	ODE No. 1042	598
2.1043	ODE No. 1043	599
2.1044	ODE No. 1044	601
2.1045	ODE No. 1045	601
2.1046	ODE No. 1046	601
2.1047	ODE No. 1047	602
2.1048	ODE No. 1048	604
2.1049	ODE No. 1049	604
2.1050	ODE No. 1050	605
2.1051	ODE No. 1051	605
2.1052	ODE No. 1052	605
2.1053	ODE No. 1053	606

2.1054	ODE No. 1054	606
2.1055	ODE No. 1055	606
2.1056	ODE No. 1056	607
2.1057	ODE No. 1057	607
2.1058	ODE No. 1058	607
2.1059	ODE No. 1059	608
2.1060	ODE No. 1060	608
2.1061	ODE No. 1061	608
2.1062	ODE No. 1062	609
2.1063	ODE No. 1063	609
2.1064	ODE No. 1064	609
2.1065	ODE No. 1065	610
2.1066	ODE No. 1066	610
2.1067	ODE No. 1067	610
2.1068	ODE No. 1068	611
2.1069	ODE No. 1069	611
2.1070	ODE No. 1070	611
2.1071	ODE No. 1071	612
2.1072	ODE No. 1072	612
2.1073	ODE No. 1073	612
2.1074	ODE No. 1074	613
2.1075	ODE No. 1075	613
2.1076	ODE No. 1076	613
2.1077	ODE No. 1077	614
2.1078	ODE No. 1078	614
2.1079	ODE No. 1079	614
2.1080	ODE No. 1080	615
2.1081	ODE No. 1081	615
2.1082	ODE No. 1082	615
2.1083	ODE No. 1083	616
2.1084	ODE No. 1084	616
2.1085	ODE No. 1085	616
2.1086	ODE No. 1086	617
2.1087	ODE No. 1087	617
2.1088	ODE No. 1088	617
2.1089	ODE No. 1089	618
2.1090	ODE No. 1090	618
2.1091	ODE No. 1091	618
2.1092	ODE No. 1092	619
2.1093	ODE No. 1093	619
2.1094	ODE No. 1094	619

2.1095	ODE No. 1095	620
2.1096	ODE No. 1096	620
2.1097	ODE No. 1097	620
2.1098	ODE No. 1098	621
2.1099	ODE No. 1099	621
2.1100	ODE No. 1100	621
2.1101	ODE No. 1101	627
2.1102	ODE No. 1102	632
2.1103	ODE No. 1103	635
2.1104	ODE No. 1104	635
2.1105	ODE No. 1105	636
2.1106	ODE No. 1106	636
2.1107	ODE No. 1107	636
2.1108	ODE No. 1108	637
2.1109	ODE No. 1109	637
2.1110	ODE No. 1110	637
2.1111	ODE No. 1111	638
2.1112	ODE No. 1112	639
2.1113	ODE No. 1113	640
2.1114	ODE No. 1114	640
2.1115	ODE No. 1115	640
2.1116	ODE No. 1116	641
2.1117	ODE No. 1117	641
2.1118	ODE No. 1118	641
2.1119	ODE No. 1119	642
2.1120	ODE No. 1120	642
2.1121	ODE No. 1121	642
2.1122	ODE No. 1122	643
2.1123	ODE No. 1123	643
2.1124	ODE No. 1124	643
2.1125	ODE No. 1125	644
2.1126	ODE No. 1126	644
2.1127	ODE No. 1127	644
2.1128	ODE No. 1128	645
2.1129	ODE No. 1129	645
2.1130	ODE No. 1130	645
2.1131	ODE No. 1131	646
2.1132	ODE No. 1132	646
2.1133	ODE No. 1133	646
2.1134	ODE No. 1134	647
2.1135	ODE No. 1135	647

2.1136	ODE No. 1136	647
2.1137	ODE No. 1137	648
2.1138	ODE No. 1138	648
2.1139	ODE No. 1139	648
2.1140	ODE No. 1140	649
2.1141	ODE No. 1141	649
2.1142	ODE No. 1142	649
2.1143	ODE No. 1143	650
2.1144	ODE No. 1144	650
2.1145	ODE No. 1145	650
2.1146	ODE No. 1146	651
2.1147	ODE No. 1147	651
2.1148	ODE No. 1148	651
2.1149	ODE No. 1149	652
2.1150	ODE No. 1150	652
2.1151	ODE No. 1151	652
2.1152	ODE No. 1152	653
2.1153	ODE No. 1153	653
2.1154	ODE No. 1154	653
2.1155	ODE No. 1155	654
2.1156	ODE No. 1156	654
2.1157	ODE No. 1157	654
2.1158	ODE No. 1158	655
2.1159	ODE No. 1159	655
2.1160	ODE No. 1160	655
2.1161	ODE No. 1161	656
2.1162	ODE No. 1162	656
2.1163	ODE No. 1163	656
2.1164	ODE No. 1164	657
2.1165	ODE No. 1165	657
2.1166	ODE No. 1166	657
2.1167	ODE No. 1167	658
2.1168	ODE No. 1168	658
2.1169	ODE No. 1169	658
2.1170	ODE No. 1170	659
2.1171	ODE No. 1171	659
2.1172	ODE No. 1172	659
2.1173	ODE No. 1173	660
2.1174	ODE No. 1174	660
2.1175	ODE No. 1175	660
2.1176	ODE No. 1176	661

2.1177	ODE No. 1177	661
2.1178	ODE No. 1178	661
2.1179	ODE No. 1179	662
2.1180	ODE No. 1180	662
2.1181	ODE No. 1181	662
2.1182	ODE No. 1182	663
2.1183	ODE No. 1183	663
2.1184	ODE No. 1184	663
2.1185	ODE No. 1185	664
2.1186	ODE No. 1186	664
2.1187	ODE No. 1187	664
2.1188	ODE No. 1188	665
2.1189	ODE No. 1189	665
2.1190	ODE No. 1190	665
2.1191	ODE No. 1191	666
2.1192	ODE No. 1192	666
2.1193	ODE No. 1193	666
2.1194	ODE No. 1194	667
2.1195	ODE No. 1195	667
2.1196	ODE No. 1196	667
2.1197	ODE No. 1197	668
2.1198	ODE No. 1198	668
2.1199	ODE No. 1199	668
2.1200	ODE No. 1200	669
2.1201	ODE No. 1201	669
2.1202	ODE No. 1202	669
2.1203	ODE No. 1203	670
2.1204	ODE No. 1204	670
2.1205	ODE No. 1205	670
2.1206	ODE No. 1206	671
2.1207	ODE No. 1207	671
2.1208	ODE No. 1208	671
2.1209	ODE No. 1209	672
2.1210	ODE No. 1210	672
2.1211	ODE No. 1211	672
2.1212	ODE No. 1212	673
2.1213	ODE No. 1213	673
2.1214	ODE No. 1214	673
2.1215	ODE No. 1215	674
2.1216	ODE No. 1216	674
2.1217	ODE No. 1217	674

2.1218	ODE No. 1218	675
2.1219	ODE No. 1219	675
2.1220	ODE No. 1220	675
2.1221	ODE No. 1221	676
2.1222	ODE No. 1222	676
2.1223	ODE No. 1223	676
2.1224	ODE No. 1224	677
2.1225	ODE No. 1225	677
2.1226	ODE No. 1226	677
2.1227	ODE No. 1227	678
2.1228	ODE No. 1228	678
2.1229	ODE No. 1229	678
2.1230	ODE No. 1230	679
2.1231	ODE No. 1231	679
2.1232	ODE No. 1232	679
2.1233	ODE No. 1233	680
2.1234	ODE No. 1234	680
2.1235	ODE No. 1235	680
2.1236	ODE No. 1236	681
2.1237	ODE No. 1237	681
2.1238	ODE No. 1238	681
2.1239	ODE No. 1239	682
2.1240	ODE No. 1240	682
2.1241	ODE No. 1241	682
2.1242	ODE No. 1242	683
2.1243	ODE No. 1243	683
2.1244	ODE No. 1244	683
2.1245	ODE No. 1245	684
2.1246	ODE No. 1246	684
2.1247	ODE No. 1247	684
2.1248	ODE No. 1248	685
2.1249	ODE No. 1249	685
2.1250	ODE No. 1250	685
2.1251	ODE No. 1251	686
2.1252	ODE No. 1252	686
2.1253	ODE No. 1253	686
2.1254	ODE No. 1254	687
2.1255	ODE No. 1255	687
2.1256	ODE No. 1256	687
2.1257	ODE No. 1257	688
2.1258	ODE No. 1258	688

2.1259	ODE No. 1259	688
2.1260	ODE No. 1260	689
2.1261	ODE No. 1261	689
2.1262	ODE No. 1262	689
2.1263	ODE No. 1263	690
2.1264	ODE No. 1264	690
2.1265	ODE No. 1265	690
2.1266	ODE No. 1266	691
2.1267	ODE No. 1267	691
2.1268	ODE No. 1268	691
2.1269	ODE No. 1269	692
2.1270	ODE No. 1270	692
2.1271	ODE No. 1271	692
2.1272	ODE No. 1272	693
2.1273	ODE No. 1273	693
2.1274	ODE No. 1274	693
2.1275	ODE No. 1275	694
2.1276	ODE No. 1276	694
2.1277	ODE No. 1277	694
2.1278	ODE No. 1278	695
2.1279	ODE No. 1279	695
2.1280	ODE No. 1280	695
2.1281	ODE No. 1281	696
2.1282	ODE No. 1282	696
2.1283	ODE No. 1283	696
2.1284	ODE No. 1284	697
2.1285	ODE No. 1285	697
2.1286	ODE No. 1286	697
2.1287	ODE No. 1287	698
2.1288	ODE No. 1288	698
2.1289	ODE No. 1289	698
2.1290	ODE No. 1290	699
2.1291	ODE No. 1291	699
2.1292	ODE No. 1292	699
2.1293	ODE No. 1293	700
2.1294	ODE No. 1294	700
2.1295	ODE No. 1295	700
2.1296	ODE No. 1296	701
2.1297	ODE No. 1297	701
2.1298	ODE No. 1298	701
2.1299	ODE No. 1299	702



2.1300	ODE No. 1300	702
2.1301	ODE No. 1301	702
2.1302	ODE No. 1302	703
2.1303	ODE No. 1303	703
2.1304	ODE No. 1304	703
2.1305	ODE No. 1305	704
2.1306	ODE No. 1306	704
2.1307	ODE No. 1307	704
2.1308	ODE No. 1308	705
2.1309	ODE No. 1309	705
2.1310	ODE No. 1310	705
2.1311	ODE No. 1311	706
2.1312	ODE No. 1312	706
2.1313	ODE No. 1313	706
2.1314	ODE No. 1314	707
2.1315	ODE No. 1315	707
2.1316	ODE No. 1316	707
2.1317	ODE No. 1317	708
2.1318	ODE No. 1318	708
2.1319	ODE No. 1319	708
2.1320	ODE No. 1320	709
2.1321	ODE No. 1321	709
2.1322	ODE No. 1322	709
2.1323	ODE No. 1323	710
2.1324	ODE No. 1324	710
2.1325	ODE No. 1325	710
2.1326	ODE No. 1326	711
2.1327	ODE No. 1327	711
2.1328	ODE No. 1328	711
2.1329	ODE No. 1329	712
2.1330	ODE No. 1330	712
2.1331	ODE No. 1331	712
2.1332	ODE No. 1332	713
2.1333	ODE No. 1333	713
2.1334	ODE No. 1334	713
2.1335	ODE No. 1335	714
2.1336	ODE No. 1336	714
2.1337	ODE No. 1337	714
2.1338	ODE No. 1338	715
2.1339	ODE No. 1339	715
2.1340	ODE No. 1340	716

2.1341	ODE No. 1341	716
2.1342	ODE No. 1342	716
2.1343	ODE No. 1343	717
2.1344	ODE No. 1344	717
2.1345	ODE No. 1345	717
2.1346	ODE No. 1346	718
2.1347	ODE No. 1347	718
2.1348	ODE No. 1348	718
2.1349	ODE No. 1349	719
2.1350	ODE No. 1350	719
2.1351	ODE No. 1351	719
2.1352	ODE No. 1352	720
2.1353	ODE No. 1353	720
2.1354	ODE No. 1354	720
2.1355	ODE No. 1355	721
2.1356	ODE No. 1356	721
2.1357	ODE No. 1357	721
2.1358	ODE No. 1358	722
2.1359	ODE No. 1359	722
2.1360	ODE No. 1360	722
2.1361	ODE No. 1361	723
2.1362	ODE No. 1362	723
2.1363	ODE No. 1363	723
2.1364	ODE No. 1364	724
2.1365	ODE No. 1365	724
2.1366	ODE No. 1366	724
2.1367	ODE No. 1367	725
2.1368	ODE No. 1368	725
2.1369	ODE No. 1369	725
2.1370	ODE No. 1370	726
2.1371	ODE No. 1371	726
2.1372	ODE No. 1372	726
2.1373	ODE No. 1373	727
2.1374	ODE No. 1374	727
2.1375	ODE No. 1375	727
2.1376	ODE No. 1376	728
2.1377	ODE No. 1377	728
2.1378	ODE No. 1378	728
2.1379	ODE No. 1379	729
2.1380	ODE No. 1380	729
2.1381	ODE No. 1381	730

2.1382	ODE No. 1382	730
2.1383	ODE No. 1383	731
2.1384	ODE No. 1384	731
2.1385	ODE No. 1385	731
2.1386	ODE No. 1386	732
2.1387	ODE No. 1387	732
2.1388	ODE No. 1388	732
2.1389	ODE No. 1389	733
2.1390	ODE No. 1390	733
2.1391	ODE No. 1391	733
2.1392	ODE No. 1392	734
2.1393	ODE No. 1393	734
2.1394	ODE No. 1394	734
2.1395	ODE No. 1395	735
2.1396	ODE No. 1396	735
2.1397	ODE No. 1397	735
2.1398	ODE No. 1398	736
2.1399	ODE No. 1399	736
2.1400	ODE No. 1400	736
2.1401	ODE No. 1401	737
2.1402	ODE No. 1402	737
2.1403	ODE No. 1403	737
2.1404	ODE No. 1404	738
2.1405	ODE No. 1405	738
2.1406	ODE No. 1406	738
2.1407	ODE No. 1407	739
2.1408	ODE No. 1408	739
2.1409	ODE No. 1409	739
2.1410	ODE No. 1410	740
2.1411	ODE No. 1411	740
2.1412	ODE No. 1412	740
2.1413	ODE No. 1413	741
2.1414	ODE No. 1414	741
2.1415	ODE No. 1415	741
2.1416	ODE No. 1416	742
2.1417	ODE No. 1417	742
2.1418	ODE No. 1418	742
2.1419	ODE No. 1419	743
2.1420	ODE No. 1420	743
2.1421	ODE No. 1421	743
2.1422	ODE No. 1422	744

2.1423	ODE No. 1423	744
2.1424	ODE No. 1424	744
2.1425	ODE No. 1425	745
2.1426	ODE No. 1426	745
2.1427	ODE No. 1427	746
2.1428	ODE No. 1428	746
2.1429	ODE No. 1429	746
2.1430	ODE No. 1430	747
2.1431	ODE No. 1431	747
2.1432	ODE No. 1432	747
2.1433	ODE No. 1433	748
2.1434	ODE No. 1434	748
2.1435	ODE No. 1435	748
2.1436	ODE No. 1436	749
2.1437	ODE No. 1437	749
2.1438	ODE No. 1438	749
2.1439	ODE No. 1439	750
2.1440	ODE No. 1440	750
2.1441	ODE No. 1441	750
2.1442	ODE No. 1442	751
2.1443	ODE No. 1443	751
2.1444	ODE No. 1444	751
2.1445	ODE No. 1445	752
2.1446	ODE No. 1446	752
2.1447	ODE No. 1447	752
2.1448	ODE No. 1448	753
2.1449	ODE No. 1449	753
2.1450	ODE No. 1450	753
2.1451	ODE No. 1451	754
2.1452	ODE No. 1452	754
2.1453	ODE No. 1453	754
2.1454	ODE No. 1454	755
2.1455	ODE No. 1455	755
2.1456	ODE No. 1456	755
2.1457	ODE No. 1457	756
2.1458	ODE No. 1458	756
2.1459	ODE No. 1459	756
2.1460	ODE No. 1460	757
2.1461	ODE No. 1461	757
2.1462	ODE No. 1462	757
2.1463	ODE No. 1463	758

2.1464	ODE No. 1464	758
2.1465	ODE No. 1465	758
2.1466	ODE No. 1466	759
2.1467	ODE No. 1467	759
2.1468	ODE No. 1468	759
2.1469	ODE No. 1469	760
2.1470	ODE No. 1470	760
2.1471	ODE No. 1471	760
2.1472	ODE No. 1472	761
2.1473	ODE No. 1473	761
2.1474	ODE No. 1474	761
2.1475	ODE No. 1475	762
2.1476	ODE No. 1476	762
2.1477	ODE No. 1477	762
2.1478	ODE No. 1478	763
2.1479	ODE No. 1479	763
2.1480	ODE No. 1480	763
2.1481	ODE No. 1481	764
2.1482	ODE No. 1482	764
2.1483	ODE No. 1483	764
2.1484	ODE No. 1484	765
2.1485	ODE No. 1485	765
2.1486	ODE No. 1486	765
2.1487	ODE No. 1487	766
2.1488	ODE No. 1488	766
2.1489	ODE No. 1489	766
2.1490	ODE No. 1490	767
2.1491	ODE No. 1491	767
2.1492	ODE No. 1492	767
2.1493	ODE No. 1493	768
2.1494	ODE No. 1494	768
2.1495	ODE No. 1495	768
2.1496	ODE No. 1496	769
2.1497	ODE No. 1497	769
2.1498	ODE No. 1498	769
2.1499	ODE No. 1499	770
2.1500	ODE No. 1500	770
2.1501	ODE No. 1501	770
2.1502	ODE No. 1502	771
2.1503	ODE No. 1503	771
2.1504	ODE No. 1504	771

2.1505	ODE No. 1505	772
2.1506	ODE No. 1506	772
2.1507	ODE No. 1507	772
2.1508	ODE No. 1508	773
2.1509	ODE No. 1509	773
2.1510	ODE No. 1510	773
2.1511	ODE No. 1511	774
2.1512	ODE No. 1512	774
2.1513	ODE No. 1513	774
2.1514	ODE No. 1514	775
2.1515	ODE No. 1515	775
2.1516	ODE No. 1516	775
2.1517	ODE No. 1517	776
2.1518	ODE No. 1518	776
2.1519	ODE No. 1519	777
2.1520	ODE No. 1520	777
2.1521	ODE No. 1521	777
2.1522	ODE No. 1522	778
2.1523	ODE No. 1523	778
2.1524	ODE No. 1524	778
2.1525	ODE No. 1525	779
2.1526	ODE No. 1526	779
2.1527	ODE No. 1527	779
2.1528	ODE No. 1528	780
2.1529	ODE No. 1529	780
2.1530	ODE No. 1530	780
2.1531	ODE No. 1531	781
2.1532	ODE No. 1532	781
2.1533	ODE No. 1533	781
2.1534	ODE No. 1534	782
2.1535	ODE No. 1535	782
2.1536	ODE No. 1536	782
2.1537	ODE No. 1537	783
2.1538	ODE No. 1538	783
2.1539	ODE No. 1539	783
2.1540	ODE No. 1540	784
2.1541	ODE No. 1541	784
2.1542	ODE No. 1542	784
2.1543	ODE No. 1543	785
2.1544	ODE No. 1544	785
2.1545	ODE No. 1545	785

2.1546	ODE No. 1546	786
2.1547	ODE No. 1547	786
2.1548	ODE No. 1548	786
2.1549	ODE No. 1549	787
2.1550	ODE No. 1550	787
2.1551	ODE No. 1551	787
2.1552	ODE No. 1552	788
2.1553	ODE No. 1553	788
2.1554	ODE No. 1554	788
2.1555	ODE No. 1555	789
2.1556	ODE No. 1556	789
2.1557	ODE No. 1557	789
2.1558	ODE No. 1558	790
2.1559	ODE No. 1559	790
2.1560	ODE No. 1560	790
2.1561	ODE No. 1561	791
2.1562	ODE No. 1562	791
2.1563	ODE No. 1563	791
2.1564	ODE No. 1564	792
2.1565	ODE No. 1565	792
2.1566	ODE No. 1566	792
2.1567	ODE No. 1567	793
2.1568	ODE No. 1568	793
2.1569	ODE No. 1569	793
2.1570	ODE No. 1570	794
2.1571	ODE No. 1571	794
2.1572	ODE No. 1572	794
2.1573	ODE No. 1573	795
2.1574	ODE No. 1574	795
2.1575	ODE No. 1575	795
2.1576	ODE No. 1576	796
2.1577	ODE No. 1577	796
2.1578	ODE No. 1578	796
2.1579	ODE No. 1579	797
2.1580	ODE No. 1580	797
2.1581	ODE No. 1581	797
2.1582	ODE No. 1582	798
2.1583	ODE No. 1583	798
2.1584	ODE No. 1584	798
2.1585	ODE No. 1585	799
2.1586	ODE No. 1586	799

2.1587	ODE No. 1587	799
2.1588	ODE No. 1588	800
2.1589	ODE No. 1589	800
2.1590	ODE No. 1590	800
2.1591	ODE No. 1591	801
2.1592	ODE No. 1592	801
2.1593	ODE No. 1593	801
2.1594	ODE No. 1594	802
2.1595	ODE No. 1595	802
2.1596	ODE No. 1596	802
2.1597	ODE No. 1597	803
2.1598	ODE No. 1598	803
2.1599	ODE No. 1599	803
2.1600	ODE No. 1600	804
2.1601	ODE No. 1601	804
2.1602	ODE No. 1602	804
2.1603	ODE No. 1603	805
2.1604	ODE No. 1604	805
2.1605	ODE No. 1605	805
2.1606	ODE No. 1606	806
2.1607	ODE No. 1607	806
2.1608	ODE No. 1608	806
2.1609	ODE No. 1609	807
2.1610	ODE No. 1610	807
2.1611	ODE No. 1611	807
2.1612	ODE No. 1612	808
2.1613	ODE No. 1613	808
2.1614	ODE No. 1614	808
2.1615	ODE No. 1615	809
2.1616	ODE No. 1616	809
2.1617	ODE No. 1617	809
2.1618	ODE No. 1618	810
2.1619	ODE No. 1619	810
2.1620	ODE No. 1620	810
2.1621	ODE No. 1621	811
2.1622	ODE No. 1622	811
2.1623	ODE No. 1623	811
2.1624	ODE No. 1624	812
2.1625	ODE No. 1625	812
2.1626	ODE No. 1626	812
2.1627	ODE No. 1627	813



2.1628	ODE No. 1628	813
2.1629	ODE No. 1629	813
2.1630	ODE No. 1630	814
2.1631	ODE No. 1631	814
2.1632	ODE No. 1632	814
2.1633	ODE No. 1633	815
2.1634	ODE No. 1634	815
2.1635	ODE No. 1635	815
2.1636	ODE No. 1636	816
2.1637	ODE No. 1637	816
2.1638	ODE No. 1638	816
2.1639	ODE No. 1639	817
2.1640	ODE No. 1640	817
2.1641	ODE No. 1641	817
2.1642	ODE No. 1642	818
2.1643	ODE No. 1643	818
2.1644	ODE No. 1644	818
2.1645	ODE No. 1645	818
2.1646	ODE No. 1646	819
2.1647	ODE No. 1647	819
2.1648	ODE No. 1648	819
2.1649	ODE No. 1649	820
2.1650	ODE No. 1650	820
2.1651	ODE No. 1651	820
2.1652	ODE No. 1652	821
2.1653	ODE No. 1653	821
2.1654	ODE No. 1654	821
2.1655	ODE No. 1655	822
2.1656	ODE No. 1656	822
2.1657	ODE No. 1657	822
2.1658	ODE No. 1658	823
2.1659	ODE No. 1659	823
2.1660	ODE No. 1660	823
2.1661	ODE No. 1661	824
2.1662	ODE No. 1662	824
2.1663	ODE No. 1663	824
2.1664	ODE No. 1664	825
2.1665	ODE No. 1665	825
2.1666	ODE No. 1666	825
2.1667	ODE No. 1667	826
2.1668	ODE No. 1668	826

2.1669	ODE No. 1669	826
2.1670	ODE No. 1670	827
2.1671	ODE No. 1671	827
2.1672	ODE No. 1672	827
2.1673	ODE No. 1673	828
2.1674	ODE No. 1674	828
2.1675	ODE No. 1675	828
2.1676	ODE No. 1676	829
2.1677	ODE No. 1677	829
2.1678	ODE No. 1678	829
2.1679	ODE No. 1679	830
2.1680	ODE No. 1680	830
2.1681	ODE No. 1681	830
2.1682	ODE No. 1682	831
2.1683	ODE No. 1683	831
2.1684	ODE No. 1684	831
2.1685	ODE No. 1685	832
2.1686	ODE No. 1686	832
2.1687	ODE No. 1687	832
2.1688	ODE No. 1688	833
2.1689	ODE No. 1689	833
2.1690	ODE No. 1690	833
2.1691	ODE No. 1691	834
2.1692	ODE No. 1692	834
2.1693	ODE No. 1693	834
2.1694	ODE No. 1694	835
2.1695	ODE No. 1695	835
2.1696	ODE No. 1696	835
2.1697	ODE No. 1697	836
2.1698	ODE No. 1698	836
2.1699	ODE No. 1699	836
2.1700	ODE No. 1700	837
2.1701	ODE No. 1701	837
2.1702	ODE No. 1702	837
2.1703	ODE No. 1703	838
2.1704	ODE No. 1704	838
2.1705	ODE No. 1705	838
2.1706	ODE No. 1706	839
2.1707	ODE No. 1707	839
2.1708	ODE No. 1708	839
2.1709	ODE No. 1709	840

2.1710	ODE No. 1710	840
2.1711	ODE No. 1711	840
2.1712	ODE No. 1712	841
2.1713	ODE No. 1713	841
2.1714	ODE No. 1714	841
2.1715	ODE No. 1715	842
2.1716	ODE No. 1716	842
2.1717	ODE No. 1717	842
2.1718	ODE No. 1718	843
2.1719	ODE No. 1719	843
2.1720	ODE No. 1720	843
2.1721	ODE No. 1721	844
2.1722	ODE No. 1722	844
2.1723	ODE No. 1723	844
2.1724	ODE No. 1724	845
2.1725	ODE No. 1725	845
2.1726	ODE No. 1726	845
2.1727	ODE No. 1727	846
2.1728	ODE No. 1728	846
2.1729	ODE No. 1729	846
2.1730	ODE No. 1730	847
2.1731	ODE No. 1731	847
2.1732	ODE No. 1732	847
2.1733	ODE No. 1733	848
2.1734	ODE No. 1734	848
2.1735	ODE No. 1735	848
2.1736	ODE No. 1736	849
2.1737	ODE No. 1737	849
2.1738	ODE No. 1738	849
2.1739	ODE No. 1739	850
2.1740	ODE No. 1740	850
2.1741	ODE No. 1741	850
2.1742	ODE No. 1742	851
2.1743	ODE No. 1743	851
2.1744	ODE No. 1744	851
2.1745	ODE No. 1745	852
2.1746	ODE No. 1746	852
2.1747	ODE No. 1747	852
2.1748	ODE No. 1748	853
2.1749	ODE No. 1749	853
2.1750	ODE No. 1750	853

2.1751	ODE No. 1751	854
2.1752	ODE No. 1752	854
2.1753	ODE No. 1753	854
2.1754	ODE No. 1754	855
2.1755	ODE No. 1755	855
2.1756	ODE No. 1756	855
2.1757	ODE No. 1757	856
2.1758	ODE No. 1758	856
2.1759	ODE No. 1759	856
2.1760	ODE No. 1760	857
2.1761	ODE No. 1761	857
2.1762	ODE No. 1762	857
2.1763	ODE No. 1763	858
2.1764	ODE No. 1764	858
2.1765	ODE No. 1765	858
2.1766	ODE No. 1766	859
2.1767	ODE No. 1767	859
2.1768	ODE No. 1768	859
2.1769	ODE No. 1769	860
2.1770	ODE No. 1770	860
2.1771	ODE No. 1771	860
2.1772	ODE No. 1772	861
2.1773	ODE No. 1773	861
2.1774	ODE No. 1774	861
2.1775	ODE No. 1775	862
2.1776	ODE No. 1776	862
2.1777	ODE No. 1777	862
2.1778	ODE No. 1778	863
2.1779	ODE No. 1779	863
2.1780	ODE No. 1780	863
2.1781	ODE No. 1781	864
2.1782	ODE No. 1782	864
2.1783	ODE No. 1783	864
2.1784	ODE No. 1784	865
2.1785	ODE No. 1785	865
2.1786	ODE No. 1786	865
2.1787	ODE No. 1787	866
2.1788	ODE No. 1788	866
2.1789	ODE No. 1789	866
2.1790	ODE No. 1790	867
2.1791	ODE No. 1791	867

2.1792	ODE No. 1792	868
2.1793	ODE No. 1793	868
2.1794	ODE No. 1794	868
2.1795	ODE No. 1795	869
2.1796	ODE No. 1796	869
2.1797	ODE No. 1797	870
2.1798	ODE No. 1798	870
2.1799	ODE No. 1799	870
2.1800	ODE No. 1800	871
2.1801	ODE No. 1801	871
2.1802	ODE No. 1802	871
2.1803	ODE No. 1803	871
2.1804	ODE No. 1804	872
2.1805	ODE No. 1805	872
2.1806	ODE No. 1806	873
2.1807	ODE No. 1807	873
2.1808	ODE No. 1808	873
2.1809	ODE No. 1809	874
2.1810	ODE No. 1810	874
2.1811	ODE No. 1811	874
2.1812	ODE No. 1812	875
2.1813	ODE No. 1813	875
2.1814	ODE No. 1814	875
2.1815	ODE No. 1815	876
2.1816	ODE No. 1816	876
2.1817	ODE No. 1817	876
2.1818	ODE No. 1818	877
2.1819	ODE No. 1819	877
2.1820	ODE No. 1820	877
2.1821	ODE No. 1821	878
2.1822	ODE No. 1822	878
2.1823	ODE No. 1823	880
2.1824	ODE No. 1824	880
2.1825	ODE No. 1825	880
2.1826	ODE No. 1826	881
2.1827	ODE No. 1827	881
2.1828	ODE No. 1828	881
2.1829	ODE No. 1829	882
2.1830	ODE No. 1830	882
2.1831	ODE No. 1831	882
2.1832	ODE No. 1832	883

2.1833	ODE No. 1833	883
2.1834	ODE No. 1834	883
2.1835	ODE No. 1835	884
2.1836	ODE No. 1836	884
2.1837	ODE No. 1837	884
2.1838	ODE No. 1838	885
2.1839	ODE No. 1839	885
2.1840	ODE No. 1840	885
2.1841	ODE No. 1841	886
2.1842	ODE No. 1842	886
2.1843	ODE No. 1843	886
2.1844	ODE No. 1844	887
2.1845	ODE No. 1845	887
2.1846	ODE No. 1846	887
2.1847	ODE No. 1847	888
2.1848	ODE No. 1848	888
2.1849	ODE No. 1849	888
2.1850	ODE No. 1850	889
2.1851	ODE No. 1851	889
2.1852	ODE No. 1852	889
2.1853	ODE No. 1853	890
2.1854	ODE No. 1854	890
2.1855	ODE No. 1855	890
2.1856	ODE No. 1856	890
2.1857	ODE No. 1857	891
2.1858	ODE No. 1858	891
2.1859	ODE No. 1859	891
2.1860	ODE No. 1860	892
2.1861	ODE No. 1861	892
2.1862	ODE No. 1862	892
2.1863	ODE No. 1863	893
2.1864	ODE No. 1864	893
2.1865	ODE No. 1865	893
2.1866	ODE No. 1866	894
2.1867	ODE No. 1867	894
2.1868	ODE No. 1868	894
2.1869	ODE No. 1869	895
2.1870	ODE No. 1870	895
2.1871	ODE No. 1871	895
2.1872	ODE No. 1872	896
2.1873	ODE No. 1873	896

2.1874	ODE No. 1874	896
2.1875	ODE No. 1875	897
2.1876	ODE No. 1876	897
2.1877	ODE No. 1877	897
2.1878	ODE No. 1878	898
2.1879	ODE No. 1879	898
2.1880	ODE No. 1880	898
2.1881	ODE No. 1881	899
2.1882	ODE No. 1882	899
2.1883	ODE No. 1883	899
2.1884	ODE No. 1884	900
2.1885	ODE No. 1885	900
2.1886	ODE No. 1886	900
2.1887	ODE No. 1887	901
2.1888	ODE No. 1888	901
2.1889	ODE No. 1889	901
2.1890	ODE No. 1890	902
2.1891	ODE No. 1891	902
2.1892	ODE No. 1892	902
2.1893	ODE No. 1893	903
2.1894	ODE No. 1894	903
2.1895	ODE No. 1895	903
2.1896	ODE No. 1896	904
2.1897	ODE No. 1897	904
2.1898	ODE No. 1898	904
2.1899	ODE No. 1899	905
2.1900	ODE No. 1900	905
2.1901	ODE No. 1901	905
2.1902	ODE No. 1902	906
2.1903	ODE No. 1903	906
2.1904	ODE No. 1904	906
2.1905	ODE No. 1905	907
2.1906	ODE No. 1906	907
2.1907	ODE No. 1907	907
2.1908	ODE No. 1908	908
2.1909	ODE No. 1909	908
2.1910	ODE No. 1910	908
2.1911	ODE No. 1911	909
2.1912	ODE No. 1912	909
2.1913	ODE No. 1913	909
2.1914	ODE No. 1914	910

2.1915	ODE No. 1915	911
2.1916	ODE No. 1916	911
2.1917	ODE No. 1917	911
2.1918	ODE No. 1918	912
2.1919	ODE No. 1919	912
2.1920	ODE No. 1920	912
2.1921	ODE No. 1921	913
2.1922	ODE No. 1922	913
2.1923	ODE No. 1923	913
2.1924	ODE No. 1924	914
2.1925	ODE No. 1925	914
2.1926	ODE No. 1926	914
2.1927	ODE No. 1927	915
2.1928	ODE No. 1928	915
2.1929	ODE No. 1929	915
2.1930	ODE No. 1930	916
2.1931	ODE No. 1931	916
2.1932	ODE No. 1932	916
2.1933	ODE No. 1933	917
2.1934	ODE No. 1934	917
2.1935	ODE No. 1935	917
2.1936	ODE No. 1936	918
2.1937	ODE No. 1937	918
2.1938	ODE No. 1938	918
2.1939	ODE No. 1939	919
2.1940	ODE No. 1940	919

**3 Appendix**

**920**



# 1 Introduction and summary of results

This report gives the result of solving the 1,940 differential equations from Kamke book in Mathematica 11 and Maple 2016.1 on windows 7, 64 bit OS. The PC used is an Intel i7-3930k running at 3.20 GHz with 16 GB memory.

The command `AboluteTiming[]` was used in Mathematica to obtain the CPU time. In Maple the following commands were used for this purpose

```
t0 := time[real]():
timeOut := 5*60;
result_of_solve := timelimit(timeOut,dsolve(ode[i]));
cpu_time := time[real]()-t0:
```

Both Maple and Mathematica had a CPU time limit of 5 minutes to complete each problem else the problem is considered not solved and marked as timed out.

When Mathematica returned `DifferentialRoot` as a solution to an ODE this was counted as not solved. Similarly, when Maple returned `DESol` this was also counted as not solved.

Table 1 below summarizes the performance of each CAS system

system	% solved	mean CPU time (sec)	mean leaf size of result	total CPU (minutes)	total leaf
Mathematica	74.48	4.95	173.13	119.11	25017
Maple	92.01	1.22	207.3	36.3	37002

Table 1: Summary of final results

Table 2 summarizes the Kamke equations used

book chapter	kamke book numbering of equations	Numbering in this doc
Chapter 1, linear first order	1.1—1.576	1—576
Additional non-linear first order	N/A	577—1000
Chapter 2, linear second order	2.1—2.448	1001—1448
Chapter 3, linear third order	3.1—3.85	1449—1533
Chapter 4, linear fourth order	4.1—4.44	1534—1577
Chapter 5, linear fifth and higher order	5.1—5.13	1578—1590
Chapter 6, non-linear second order	6.1—6.246	1591—1836
Chapter 7, non-linear third and higher order	7.1—7.19	1837—1855
Chapter 8, system of ode, first order	8.1—8.57	1856—1912
Chapter 9, system of ode, higher order	9.1—9.28	1913—1940

Table 2: Kamke equation numbering

The following summarizes which equations are solved by each system

**Not solved by Mathematica** 16, 22, 45, 47, 48, 49, 50, 52, 54, 55, 56, 58, 63, 66, 69, 70, 71, 74, 79, 80, 81, 82, 83, 86, 87, 110, 121, 127, 185, 188, 189, 202, 203, 204, 205, 206, 219, 231, 234, 237, 249, 250, 253, 265, 266, 269, 272, 331, 338, 340, 365, 367, 368, 370, 383, 385, 394, 395, 400, 402, 404, 413, 414, 416, 428, 429, 430, 451, 452, 459, 460, 461, 465, 467, 468, 470, 476, 479, 480, 482, 485, 487, 489, 494, 495, 501, 503, 504, 506, 508, 509, 510, 513, 515, 523, 524, 527, 528, 530, 531, 532, 533, 534, 535, 537, 538, 541, 542, 543, 544, 546, 550, 554, 555, 561, 562, 566, 567, 570, 571, 572, 575, 576, 592, 607, 608, 613, 620, 622, 638, 639, 640, 672, 696, 701, 702, 703, 704, 706, 707, 710, 714, 730, 733, 735, 743, 745, 746, 747, 752, 759, 765, 766, 769, 776, 782, 783, 784, 785, 786, 788, 789, 790, 791, 792, 796, 807, 815, 835, 837, 854, 855, 862, 865, 885, 889, 892, 894, 909, 913, 915, 916, 917, 918, 919, 922, 923, 925, 929, 932, 942, 953, 961, 971, 983, 984, 993, 996, 1000, 1015, 1019, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1038, 1072, 1073, 1074, 1075, 1076, 1077, 1080, 1081, 1082, 1083, 1084, 1085, 1099, 1126, 1128, 1156, 1157, 1177, 1205, 1212, 1216, 1219, 1232, 1233, 1236, 1248, 1261, 1263, 1267, 1268, 1270, 1278, 1303, 1306, 1323, 1329, 1330, 1341, 1343, 1348, 1362, 1367, 1372, 1373, 1398, 1402, 1403, 1406, 1407, 1408, 1413, 1418, 1419, 1427, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1450, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1470, 1471, 1472, 1473, 1474, 1476, 1482, 1484, 1487, 1489, 1500, 1505, 1506, 1507, 1510, 1515, 1516, 1520, 1526, 1527, 1529, 1530, 1531, 1540, 1541, 1542, 1543, 1544, 1547, 1552, 1569, 1572, 1573, 1574, 1575, 1576, 1581, 1583, 1586, 1590, 1593, 1595, 1596, 1598, 1599, 1601, 1602, 1603, 1605, 1606, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1631, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1642, 1643, 1644, 1645, 1648, 1649, 1652, 1656, 1658, 1659, 1660, 1662, 1663, 1664, 1665, 1666, 1667, 1672, 1673, 1675, 1677, 1678, 1680, 1681, 1682, 1684, 1685, 1686, 1688, 1690, 1691, 1692, 1693, 1695, 1696, 1702, 1704, 1705, 1706, 1708, 1709, 1710, 1711, 1713, 1719, 1720, 1721, 1729, 1732, 1734, 1735, 1737, 1738, 1739, 1742, 1746, 1751, 1755, 1757, 1760, 1761, 1762, 1776, 1777, 1779, 1780, 1787, 1788, 1789, 1797, 1798, 1801, 1802, 1806, 1807, 1809, 1811, 1813, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1823, 1825, 1827, 1831, 1832, 1833, 1834, 1836, 1837, 1838, 1839, 1840, 1841, 1844, 1845, 1848, 1850, 1851, 1853, 1854, 1855, 1875, 1880, 1885, 1890, 1893, 1894, 1905, 1911, 1912, 1915, 1916, 1917, 1918, 1919, 1920, 1921, 1922, 1925, 1926, 1927, 1928, 1929, 1932, 1933, 1934, 1935, 1936, 1937, 1939, 1940

**Not solved by Maple** 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 110, 121, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 269, 331, 340, 367, 368, 370, 383, 395, 460, 461, 480, 482, 485, 503, 506, 507, 510, 531, 572, 575, 576, 708, 733, 789, 790, 835, 837, 878, 885, 894, 909, 912, 920, 1015, 1019, 1026, 1028, 1030, 1031, 1038, 1072, 1073, 1075, 1076, 1077, 1081, 1157, 1205, 1212, 1216, 1234, 1236, 1278, 1408, 1439, 1440, 1441, 1443, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1473, 1474, 1476, 1484, 1489, 1510, 1515, 1531, 1540, 1541, 1542, 1543, 1581, 1582, 1586, 1593, 1595, 1596, 1598, 1599, 1606, 1608, 1609, 1617, 1619, 1623, 1625, 1628, 1634, 1642, 1643, 1645, 1649, 1675, 1685, 1698, 1702, 1704, 1705, 1706, 1721, 1729, 1732, 1734, 1735, 1737, 1738, 1739, 1751, 1757, 1761, 1788, 1789, 1797, 1801, 1802, 1807, 1835, 1851, 1854, 1855, 1890, 1905, 1921, 1922, 1927, 1928, 1940

**Solved by Mathematica but not by Maple** 507, 708, 878, 912, 920, 1234, 1582, 1698, 1835

**Solved by Maple but not by Mathematica** 16, 22, 45, 52, 54, 58, 63, 66, 69, 70, 71, 80, 81, 83, 86, 127, 185, 188, 189, 204, 231, 249, 266, 272, 338, 365, 385, 394, 400, 402, 404, 413, 414, 416, 428, 429, 430, 451, 452, 459, 465, 467, 468, 470, 476, 479, 487, 489, 494, 495, 501, 504, 508, 509, 513, 515, 523, 524, 527, 528, 530, 532, 533, 534, 535, 537, 538, 541, 542, 543, 544, 546, 550, 554, 555, 561, 562, 566, 567, 570, 571, 592, 607, 608, 613, 620, 622, 638, 639, 640, 672, 696, 701, 702, 703, 704, 706, 707, 710, 714, 730, 735, 743, 745, 746, 747, 752, 759, 765, 766, 769, 776, 782, 783, 784, 785, 786, 788, 791, 792, 796, 807, 815, 854, 855, 862, 865, 889, 892, 913, 915, 916, 917, 918, 919, 922, 923, 925, 929, 932, 942, 953, 961, 971, 983, 984, 993, 996, 1000, 1027, 1029, 1032, 1074, 1080, 1082, 1083, 1084, 1085, 1099, 1126, 1128, 1156, 1177, 1219, 1232, 1233, 1248, 1261, 1263, 1267, 1268, 1270, 1303, 1306, 1323, 1329, 1330, 1341, 1343, 1348, 1362, 1367, 1372, 1373, 1398, 1402, 1403, 1406, 1407, 1413, 1418, 1419, 1427, 1442, 1444, 1445, 1450, 1470, 1471, 1472, 1482, 1487, 1500, 1505, 1506, 1507, 1516, 1520, 1526, 1527, 1529, 1530, 1544, 1547, 1552, 1569, 1572, 1573, 1574, 1575, 1576, 1583, 1590, 1601, 1602, 1603, 1605, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1618, 1620, 1621, 1622, 1624, 1626, 1627, 1629, 1631, 1633, 1635, 1636, 1637, 1638, 1639, 1640, 1644, 1648, 1652, 1656, 1658, 1659, 1660, 1662, 1663, 1664, 1665, 1666, 1667, 1672, 1673, 1677, 1678, 1680, 1681, 1682, 1684, 1686, 1688, 1690, 1691, 1692, 1693, 1695, 1696, 1708, 1709, 1710, 1711, 1713, 1719, 1720, 1742, 1746, 1755, 1760, 1762, 1776, 1777, 1779, 1780, 1787, 1798, 1806, 1809, 1811, 1813, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1823, 1825, 1827, 1831, 1832, 1833, 1834, 1836, 1837, 1838, 1839, 1840, 1841, 1844, 1845, 1848, 1850, 1853, 1875, 1880, 1885, 1893, 1894, 1911, 1912, 1915, 1916, 1917, 1918, 1919, 1920, 1925, 1926, 1929, 1932, 1933, 1934, 1935, 1936, 1937, 1939

**Solved by both Maple and Mathematica** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 51, 53, 57, 59, 60, 61, 62, 64, 65, 67, 68, 72, 73, 75, 76, 77, 78, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 232, 233, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 369, 371,

372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 386, 387, 388, 389, 390, 391, 392, 393, 396, 397, 398, 399, 401, 403, 405, 406, 407, 408, 409, 410, 411, 412, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 453, 454, 455, 456, 457, 458, 462, 463, 464, 466, 469, 471, 472, 473, 474, 475, 477, 478, 481, 483, 484, 486, 488, 490, 491, 492, 493, 496, 497, 498, 499, 500, 502, 505, 511, 512, 514, 516, 517, 518, 519, 520, 521, 522, 525, 526, 529, 536, 539, 540, 545, 547, 548, 549, 551, 552, 553, 556, 557, 558, 559, 560, 563, 564, 565, 568, 569, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 609, 610, 611, 612, 614, 615, 616, 617, 618, 619, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 697, 698, 699, 700, 705, 709, 711, 712, 713, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 731, 732, 734, 736, 737, 738, 739, 740, 741, 742, 744, 748, 749, 750, 751, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 767, 768, 770, 771, 772, 773, 774, 775, 777, 778, 779, 780, 781, 787, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 808, 809, 810, 811, 812, 813, 814, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 836, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 856, 857, 858, 859, 860, 861, 863, 864, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 879, 880, 881, 882, 883, 884, 886, 887, 888, 890, 891, 893, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 910, 911, 914, 921, 924, 926, 927, 928, 930, 931, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 954, 955, 956, 957, 958, 959, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 985, 986, 987, 988, 989, 990, 991, 992, 994, 995, 997, 998, 999, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1020, 1021, 1022, 1023, 1024, 1025, 1033, 1034, 1035, 1036, 1037, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1078, 1079, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1127, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1209, 1210, 1211, 1213, 1214, 1215, 1217, 1218, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1235, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1249, 1250, 1251, 1252, 1253,

1254, 1255, 1256, 1257, 1258, 1259, 1260, 1262, 1264, 1265, 1266, 1269, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1304, 1305, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1324, 1325, 1326, 1327, 1328, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1342, 1344, 1345, 1346, 1347, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1363, 1364, 1365, 1366, 1368, 1369, 1370, 1371, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1399, 1400, 1401, 1404, 1405, 1409, 1410, 1411, 1412, 1414, 1415, 1416, 1417, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1446, 1447, 1448, 1449, 1451, 1452, 1453, 1454, 1455, 1456, 1464, 1465, 1466, 1467, 1468, 1469, 1475, 1477, 1478, 1479, 1480, 1481, 1483, 1485, 1486, 1488, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1501, 1502, 1503, 1504, 1508, 1509, 1511, 1512, 1513, 1514, 1517, 1518, 1519, 1521, 1522, 1523, 1524, 1525, 1528, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1545, 1546, 1548, 1549, 1550, 1551, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1570, 1571, 1577, 1578, 1579, 1580, 1584, 1585, 1587, 1588, 1589, 1591, 1592, 1594, 1597, 1600, 1604, 1607, 1630, 1632, 1641, 1646, 1647, 1650, 1651, 1653, 1654, 1655, 1657, 1661, 1668, 1669, 1670, 1671, 1674, 1676, 1679, 1683, 1687, 1689, 1694, 1697, 1699, 1700, 1701, 1703, 1707, 1712, 1714, 1715, 1716, 1717, 1718, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1730, 1731, 1733, 1736, 1740, 1741, 1743, 1744, 1745, 1747, 1748, 1749, 1750, 1752, 1753, 1754, 1756, 1758, 1759, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1778, 1781, 1782, 1783, 1784, 1785, 1786, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1799, 1800, 1803, 1804, 1805, 1808, 1810, 1812, 1814, 1822, 1824, 1826, 1828, 1829, 1830, 1842, 1843, 1846, 1847, 1849, 1852, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1876, 1877, 1878, 1879, 1881, 1882, 1883, 1884, 1886, 1887, 1888, 1889, 1891, 1892, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1906, 1907, 1908, 1909, 1910, 1913, 1914, 1923, 1924, 1930, 1931, 1938

**Both systems unable to solve** 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 110, 121, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 269, 331, 340, 367, 368, 370, 383, 395, 460, 461, 480, 482, 485, 503, 506, 510, 531, 572, 575, 576, 733, 789, 790, 835, 837, 885, 894, 909, 1015, 1019, 1026, 1028, 1030, 1031, 1038, 1072, 1073, 1075, 1076, 1077, 1081, 1157, 1205, 1212, 1216, 1236, 1278, 1408, 1439, 1440, 1441, 1443, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1473, 1474, 1476, 1484, 1489, 1510, 1515, 1531, 1540, 1541, 1542, 1543, 1581, 1586, 1593, 1595, 1596, 1598, 1599, 1606, 1608, 1609, 1617, 1619, 1623, 1625, 1628, 1634, 1642, 1643, 1645, 1649, 1675, 1685, 1702, 1704, 1705, 1706, 1721, 1729, 1732, 1734, 1735, 1737, 1738, 1739, 1751, 1757, 1761, 1788, 1789, 1797, 1801, 1802, 1807, 1851, 1854, 1855, 1890, 1905, 1921, 1922, 1927, 1928, 1940

## 2 Problems table lookup

Final conclusion table for each equation is given by table 3 below. Clicking on the problem opens a new page that shows the result and links to download each problem as well.

Table 3: Breakdown of results for each Kamke differential equation

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1	✓	0.728	1117	✓	0.022	1089	Linear first order, To Do
Kamke 2	✓	0.014	33	✓	0.013	25	Linear first order, integrating factor $y'(x) + ay(x) = f(x)$
Kamke 3	✓	0.033	40	✓	0.021	37	Linear first order, integrating factor $y'(x) + ay(x) = f(x)$
Kamke 4	✓	0.009	24	✓	0.005	18	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 5	✓	2.681	31	✓	0.087	21	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 6	✓	0.023	18	✓	0.021	15	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 7	✓	0.023	16	✓	0.006	13	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 8	✓	0.023	15	✓	0.011	13	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 9	✓	0.017	17	✓	0.011	14	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Continued on next page							

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 10	✓	0.009	18	✓	0.01	15	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 11	✓	0.462	48	✓	0.017	24	Linear first order, integrating factor $y'(x) + g(x)y(x) = f(x)$
Kamke 12	✓	0.029	34	✓	0.03	8	Non-linear first order, Riccati, separable $y'(x) + y^2(x) = 1$
Kamke 13	✓	0.025	69	✓	0.22	79	Non-linear first order, Riccati, transform to second order Airy ODE using $y = -\frac{u'(x)}{uR(x)}$
Kamke 14	✓	0.013	253	✓	0.109	187	Non-linear first order, Riccati, transform to second order Airy ODE using $y = -\frac{u'(x)}{uR(x)}$
Kamke 15	✓	0.02	25	✓	0.29	35	Non-linear first order, Riccati, transform to separable first order
Kamke 16	✗	0	0	✓	0.146	49	Non-linear first order, Riccati, transform to first order separable using $y = y_p + \frac{1}{u}$
Kamke 17	✓	0.024	30	✓	0.109	24	Non-linear first order, Riccati, transform to second order standard ODE using $y = -\frac{u'(x)}{uR(x)}$

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 18	✓	0.023	49	✓	0.07	39	Non-linear first order, Riccati, transform to first order ODE solved using integrating factor using transformation $y = y_p + \frac{1}{u}$
Kamke 19	✓	0.01	30	✓	0.037	16	Non-linear first order, Riccati, transform to first order separable
Kamke 20	✓	0.731	48	✓	0.082	34	Non-linear first order, Riccati, transform to Bernoulli first order non-linear first order then solve Bernoulli
Kamke 21	✓	6.297	57	✓	0.141	25	Non-linear first order, Riccati, transform to first order ODE solved using integrating factor using transformation $y = y_p + \frac{1}{u}$
Kamke 22	✗	0	0	✓	0.408	128	Non-linear first order, Riccati, transform to first order ODE solved using integrating factor using transformation $y = y_p + \frac{1}{u}$
Kamke 23	✓	0.023	32	✓	0.036	23	Non-linear first order, Riccati, Separable
Kamke 24	✓	0.014	276	✓	0.076	201	Non-linear first order, Riccati, transform to second order Emden-Fowler ODE using $y = -\frac{u'(x)}{uR(x)}$ solution in terms of Bessel functions
Kamke 25	✓	0.23	618	✓	0.309	348	Non-linear first order, Riccati. To do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 26	✓	0.077	56	✓	0.064	45	Non-Linear first order, Riccati, transform to second order standard ODE using $y = -\frac{u'(x)}{uR(x)}$
Kamke 27	✓	0.046	88	✓	0.196	72	Non-linear first order, Riccati, transform to Bernoulli first order non-linear first order then solve Bernoulli
Kamke 28	✓	0.049	63	✓	0.095	51	Non-linear first order, Riccati, transform to Bernoulli first order non-linear first order then solve Bernoulli
Kamke 29	✓	0.022	39	✓	0.012	19	Non-linear first order, Bernoulli
Kamke 30	✓	0.065	197	✓	0.085	54	Non-Linear first order, Riccati, transform to second order Bessel like ODE using $y = -\frac{u'(x)}{uR(x)}$ , solution uses Bessel functions
Kamke 31	✓	0.029	21	✓	0.036	23	Non-Linear first order, Riccati, separable
Kamke 32	✓	0.144	26	✓	0.279	28	Non-Linear first order, Riccati, has particular solution, solution using $y = y_p + \frac{1}{u}$ leads to first order solved using integrating factor
Kamke 33	✓	34.539	114	✓	0.544	58	Non-Linear first order, Riccati. Complicated algebra, will do later
Kamke 34	✓	0.512	51	✓	0.026	28	Non-Linear first order, Bernoulli. Standard method.

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 35	✓	0.058	48	✓	0.042	35	Non-Linear first order, Riccati. Transform to second order ODE using $y(x) = -\frac{u'(x)}{u(x)R(x)}$
Kamke 36	✓	0.299	171	✓	0.098	62	Non-Linear first order of Abel first kind with None constant invariant. Transform to a reverse Riccati then Solve the resulting second order Airy ODE, and transform solution back. Lots of algebra involved. Hardest ODE so far
Kamke 37	✓	0.878	73	✓	0.096	50	To Do
Kamke 38	✓	0.88	99	✓	0.317	34	Non-Linear first order. smart transformation makes it proper Abel first kind
Kamke 39	✓	0.05	1	✓	0.014	30	To Do
Kamke 40	✓	0.397	161	✓	0.083	48	To Do
Kamke 41	✓	5.238	98	✓	0.207	103	To Do
Kamke 42	✓	1.32	136	✓	0.027	40	To Do
Kamke 43	✓	9.395	376	✓	1.825	373	To Do
Kamke 44	✓	0.017	65	✓	0.02	53	Non-Linear first order Bernoulli. Solved using standard method of solving Bernoulli.
Kamke 45	✗	0	0	✓	0.122	123	To Do
Kamke 46	✓	0.248	254	✓	0.095	956	To Do
Kamke 47	✗	0	0	✗	0	0	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 48	✗	0	0	✗	0	0	To Do
Kamke 49	✗	0	0	✗	0	0	To Do
Kamke 50	✗	0	0	✗	0	0	To Do
Kamke 51	✓	16.702	322	✓	0.217	237	To Do
Kamke 52	✗	0	0	✓	0.205	61	To Do
Kamke 53	✓	104.189	95	✓	0.062	281	To Do
Kamke 54	✗	0	0	✓	0.159	38	To Do
Kamke 55	✗	0	0	✗	0	0	To Do
Kamke 56	✗	0	0	✗	0	0	To Do
Kamke 57	✓	100.704	283	✓	0.071	31	To Do
Kamke 58	✗	0	0	✓	0.071	68	To Do
Kamke 59	✓	0.18	96	✓	0.05	26	To Do
Kamke 60	✓	0.049	44	✓	0.016	29	Non-Linear first order, separable.
Kamke 61	✓	0.176	75	✓	0.014	50	To Do
Kamke 62	✓	3.784	36	✓	0.412	34	Non-Linear first order, special transformation makes it exact differential.
Kamke 63	✗	0	0	✓	0.141	35	To Do
Kamke 64	✓	0.175	90	✓	0.082	124	To Do
Kamke 65	✓	1.522	312	✓	0.042	47	To Do
Kamke 66	✗	0	0	✓	0.07	40	To Do
Kamke 67	✓	0.061	14	✓	0.014	51	To Do
Kamke 68	✓	0.852	373	✓	0.063	77	To Do
Kamke 69	✗	0	0	✓	0.146	111	To Do
Kamke 70	✗	0	0	✓	0.163	113	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 71	✗	0	0	✓	0.13	113	To Do
Kamke 72	✓	0.831	84	✓	0.012	64	To Do
Kamke 73	✓	49.986	733	✓	0.349	91	To Do
Kamke 74	✗	0	0	✗	0	0	To Do
Kamke 75	✓	0.02	18	✓	0.132	20	Non-Linear first order, Separable
Kamke 76	✓	0.12	51	✓	0.043	41	Non-Linear first order, Separable, integral requires the tangent half-angle substitution (Weierstrass substitution)
Kamke 77	✓	0.297	58	✓	0.068	54	Non-Linear first order, transform to Separable, becomes same as problem 76 above. Transform back after solution.
Kamke 78	✓	0.861	86	✓	2.176	89	Non-Linear first order, transform to Separable, integral requires the tangent half-angle substitution (Weierstrass substitution). Kamke calls this d'Alembertsche differential equation
Kamke 79	✗	0	0	✗	0	0	To Do
Kamke 80	✗	0	0	✓	1.497	41	To Do
Kamke 81	✗	0	0	✓	1.296	78	To Do
Kamke 82	✗	0	0	✗	0	0	To Do
Kamke 83	✗	0	0	✓	0.449	44	To Do
Kamke 84	✓	10.343	120	✓	0.037	37	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 85	✓	197.699	182	✓	0.468	153	To Do
Kamke 86	✗	0	0	✓	0.499	52	To Do
Kamke 87	✗	0	0	✗	0	0	To Do
Kamke 88	✓	0.284	2479	✓	0.233	256	Non-Linear first order, Riccati, Solved using $y = -\frac{u'}{uR(x)}$ substitution. Convert to second order Bessel ODE
Kamke 89	✓	0.032	46	✓	0.016	56	Linear first order, separable. Integration tricky. requires tangent half-angle substitution
Kamke 90	✓	0.014	19	✓	0.012	17	Linear first order, separable. integrating factor
Kamke 91	✓	0.007	13	✓	0.006	11	To Do
Kamke 92	✓	0.013	14	✓	0.005	12	Linear first order, separable. integrating factor
Kamke 93	✓	0.02	14	✓	0.015	12	To Do
Kamke 94	✓	0.016	25	✓	0.009	23	Linear first order, separable. integrating factor
Kamke 95	✓	0.016	30	✓	0.068	27	Non-Linear first order, Riccati, conversion to second order linear Lienard ODE using $y = \frac{u'}{uR}$ . Solution in terms of Bessel functions
Kamke 96	✓	0.023	33	✓	0.031	11	Non-Linear first order, Riccati, but it is separable, so easy to solve by direct integration

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 97	✓	0.027	36	✓	0.03	25	Non-Linear first order, Riccati, smart substitution transfer it to separable first order, so easy to solve by direct integration
Kamke 98	✓	0.025	123	✓	0.046	38	To Do
Kamke 99	✓	0.019	243	✓	0.119	171	To Do
Kamke 100	✓	0.009	133	✓	0.08	59	Non-Linear first order, Riccati, conversion to second order linear Lienard ODE using $y = \frac{u'}{uR}$ . Solution in terms of Bessel functions
Kamke 101	✓	0.107	18	✓	0.01	16	Non-Linear first order, Bernoulli, standard method of solving Bernoulli
Kamke 102	✓	0.074	30	✓	0.036	22	Non-Linear first order, Riccati, but transformed using smart substitution $y = xv$ to separable first order which is easily solved
Kamke 103	✓	0.135	72	✓	0.033	29	Non-Linear first order, Riccati, conversion to second order linear Sturm-Liouville ODE using $y = \frac{u'}{uR}$ , then smart substitution $t = \frac{x^2}{2}$ is used to solve Sturm-Liouville by converting it to constant coefficients second order ODE
Kamke 104	✓	0.016	43	✓	0.093	63	Non-Linear first order, Riccati, conversion using smart transformation to separable first order

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 105	✓	0.217	301	✓	0.337	844	To Do
Kamke 106	✓	0.035	40	✓	0.039	41	To Do
Kamke 107	✓	0.258	1286	✓	0.239	174	To Do
Kamke 108	✓	0.011	15	✓	0.011	13	Non-Linear first order, Bernoulli, standard method of solving Bernoulli, leads to first order separable
Kamke 109	✓	0.011	17	✓	0.012	15	Non-Linear first order, Bernoulli, standard method of solving Bernoulli, leads to first order separable
Kamke 110	✗	0	0	✗	0	0	Non-Linear first order, Riccati, has known particular solution. Using $y = y_p + \frac{1}{u}$ convert it to first order separable ODE
Kamke 111	✓	0.5	55	✓	0.128	54	To Do
Kamke 112	✓	0.021	13	✓	0.038	27	Non-Linear first order, smart transformation $y(x) = xv(x)$ makes it separable
Kamke 113	✓	0.022	16	✓	0.028	33	Non-Linear first order, very similar to 112. Smart transformation $y(x) = xv(x)$ makes it separable
Kamke 114	✓	0.02	12	✓	2.378	28	Non-Linear first order, very similar to 113. Smart transformation $y(x) = xv(x)$ makes it separable

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 115	✓	0.109	81	✓	0.227	49	Non-Linear first order, Smart transformation $y(x) = xv(x)$ makes it separable
Kamke 116	✓	3.999	142	✓	0.229	86	To Do
Kamke 117	✓	0.03	21	✓	0.119	20	To Do
Kamke 118	✓	0.011	13	✓	0.054	8	To Do
Kamke 119	✓	0.031	17	✓	0.09	14	To Do
Kamke 120	✓	0.045	20	✓	0.178	17	To Do
Kamke 121	✗	0	0	✗	0	0	To Do
Kamke 122	✓	0.085	19	✓	0.457	16	To Do
Kamke 123	✓	0.057	19	✓	0.051	44	To Do
Kamke 124	✓	0.027	16	✓	0.032	12	To Do
Kamke 125	✓	0.036	16	✓	0.062	14	To Do
Kamke 126	✓	17.951	93	✓	0.024	29	To Do
Kamke 127	✗	0	0	✓	0.11	39	To Do
Kamke 128	✓	4.269	39	✓	0.283	33	To Do
Kamke 129	✓	0.027	37	✓	0.027	33	To Do
Kamke 130	✓	0.006	21	✓	0.007	15	To Do
Kamke 131	✓	0.017	20	✓	0.193	31	To Do
Kamke 132	✓	0.012	115	✓	0.028	153	To Do
Kamke 133	✓	0.007	22	✓	0.01	16	To Do
Kamke 134	✓	0.011	21	✓	0.009	17	To Do
Kamke 135	✓	0.007	14	✓	0.006	11	To Do
Kamke 136	✓	0.013	24	✓	0.018	18	To Do
Kamke 137	✓	0.009	16	✓	0.013	14	To Do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 138	✓	0.014	13	✓	0.035	11	To Do
Kamke 139	✓	0.125	397	✓	0.142	219	To Do
Kamke 140	✓	0.01	17	✓	0.045	20	To Do
Kamke 141	✓	0.025	61	✓	0.051	51	To Do
Kamke 142	✓	0.176	109	✓	0.094	52	To Do
Kamke 143	✓	0.009	51	✓	0.049	41	To Do
Kamke 144	✓	0.159	1588	✓	0.118	219	To Do
Kamke 145	✓	0.655	239	✓	0.122	117	To Do
Kamke 146	✓	0.757	73	✓	0.176	84	To Do
Kamke 147	✓	0.908	279	✓	0.215	178	To Do
Kamke 148	✓	0.013	20	✓	0.01	16	To Do
Kamke 149	✓	0.012	27	✓	0.011	20	To Do
Kamke 150	✓	0.008	25	✓	0.006	23	To Do
Kamke 151	✓	0.719	161	✓	0.072	85	To Do
Kamke 152	✓	0.226	39	✓	0.859	25	To Do
Kamke 153	✓	0.016	21	✓	0.014	20	To Do
Kamke 154	✓	0.014	18	✓	0.01	16	To Do
Kamke 155	✓	0.017	47	✓	0.114	14	To Do
Kamke 156	✓	0.015	21	✓	0.017	20	To Do
Kamke 157	✓	0.094	32	✓	0.279	231	To Do
Kamke 158	✓	0.034	31	✓	0.017	22	To Do
Kamke 159	✓	0.016	17	✓	0.125	13	To Do
Kamke 160	✓	0.02	27	✓	0.031	21	To Do
Kamke 161	✓	0.014	34	✓	0.018	27	To Do
Kamke 162	✓	0.26	102	✓	0.199	58	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 163	✓	0.012	43	✓	0.038	26	To Do
Kamke 164	✓	0.079	76	✓	0.217	102	To Do
Kamke 165	✓	0.017	22	✓	0.022	17	To Do
Kamke 166	✓	0.088	63	✓	0.174	97	To Do
Kamke 167	✓	0.022	29	✓	0.035	20	To Do
Kamke 168	✓	0.091	99	✓	0.191	140	To Do
Kamke 169	✓	2.573	110	✓	0.158	153	To Do
Kamke 170	✓	0.023	22	✓	0.018	23	To Do
Kamke 171	✓	0.01	17	✓	0.01	15	To Do
Kamke 172	✓	0.04	27	✓	0.275	26	To Do
Kamke 173	✓	0.015	25	✓	0.046	27	To Do
Kamke 174	✓	0.008	17	✓	0.005	13	To Do
Kamke 175	✓	0.02	23	✓	0.025	20	To Do
Kamke 176	✓	0.144	73	✓	0.103	30	To Do
Kamke 177	✓	0.016	20	✓	0.026	17	To Do
Kamke 178	✓	0.088	52	✓	0.122	63	To Do
Kamke 179	✓	1.656	1619	✓	0.167	112	To Do
Kamke 180	✓	0.122	104	✓	0.052	58	To Do
Kamke 181	✓	0.012	100	✓	0.082	28	To Do
Kamke 182	✓	0.167	24	✓	0.14	18	To Do
Kamke 183	✓	0.014	22	✓	0.013	18	To Do
Kamke 184	✓	1.444	612	✓	0.385	493	To Do
Kamke 185	✗	0	0	✓	0.059	63	To Do
Kamke 186	✓	0.029	19	✓	0.037	17	To Do
Kamke 187	✓	0.069	166	✓	0.053	60	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 188	✗	0	0	✓	0.028	32	To Do
Kamke 189	✗	0	0	✓	0.247	60	To Do
Kamke 190	✓	0.047	44	✓	0.01	29	To Do
Kamke 191	✓	0.029	48	✓	0.017	16	To Do
Kamke 192	✓	0.026	42	✓	0.018	36	To Do
Kamke 193	✓	0.008	16	✓	0.007	14	To Do
Kamke 194	✓	0.073	30	✓	0.024	23	To Do
Kamke 195	✓	0.056	24	✓	0.124	28	To Do
Kamke 196	✓	0.056	40	✓	0.153	29	To Do
Kamke 197	✓	0.043	98	✓	0.082	237	To Do
Kamke 198	✓	0.024	15	✓	0.013	13	To Do
Kamke 199	✓	0.183	15	✓	0.165	102	To Do
Kamke 200	✓	0.048	59	✓	0.062	53	To Do
Kamke 201	✓	0.073	38	✓	0.04	23	To Do
Kamke 202	✗	0	0	✗	0	0	To Do
Kamke 203	✗	0	0	✗	0	0	To Do
Kamke 204	✗	0	0	✓	0.316	91	To Do
Kamke 205	✗	0	0	✗	0	0	To Do
Kamke 206	✗	0	0	✗	0	0	To Do
Kamke 207	✓	0.012	47	✓	0.02	37	To Do
Kamke 208	✓	0.07	120	✓	0.073	106	To Do
Kamke 209	✓	0.022	58	✓	0.011	21	To Do
Kamke 210	✓	0.017	47	✓	0.019	33	To Do
Kamke 211	✓	52.383	38	✓	0.032	31	To Do
Kamke 212	✓	27.708	92	✓	0.135	30	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 213	✓	0.586	71	✓	0.713	66	To Do
Kamke 214	✓	0.492	78	✓	0.161	48	To Do
Kamke 215	✓	0.614	80	✓	0.184	51	To Do
Kamke 216	✓	0.593	82	✓	0.174	51	To Do
Kamke 217	✓	0.02	29	✓	0.03	23	To Do
Kamke 218	✓	0.099	232	✓	0.203	57	To Do
Kamke 219	✗	0	0	✗	0	0	To Do
Kamke 220	✓	0.014	57	✓	0.021	43	To Do
Kamke 221	✓	0.018	30	✓	0.051	21	To Do
Kamke 222	✓	0.117	65	✓	0.052	32	To Do
Kamke 223	✓	0.026	55	✓	0.169	51	To Do
Kamke 224	✓	0.019	29	✓	0.058	35	To Do
Kamke 225	✓	0.017	26	✓	0.046	20	To Do
Kamke 226	✓	0.017	28	✓	0.049	21	To Do
Kamke 227	✓	0.013	71	✓	0.183	33	To Do
Kamke 228	✓	0.287	1677	✓	0.329	271	To Do
Kamke 229	✓	0.013	77	✓	0.174	32	To Do
Kamke 230	✓	0.12	96	✓	0.042	100	To Do
Kamke 231	✗	0	0	✓	0.215	178	To Do
Kamke 232	✓	0.01	46	✓	0.016	39	To Do
Kamke 233	✓	0.023	38	✓	0.023	30	To Do
Kamke 234	✗	0	0	✗	0	0	To Do
Kamke 235	✓	0.078	33	✓	0.051	30	To Do
Kamke 236	✓	0.017	96	✓	0.063	141	To Do
Kamke 237	✗	0	0	✗	0	0	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 238	✓	0.043	176	✓	0.08	91	To Do
Kamke 239	✓	0.028	54	✓	0.176	59	To Do
Kamke 240	✓	0.01	39	✓	0.021	34	To Do
Kamke 241	✓	0.01	37	✓	0.014	33	To Do
Kamke 242	✓	0.014	60	✓	0.015	39	To Do
Kamke 243	✓	14.97	451	✓	0.137	391	To Do
Kamke 244	✓	14.89	457	✓	0.12	391	To Do
Kamke 245	✓	0.401	1453	✓	0.322	31	To Do
Kamke 246	✓	0.032	71	✓	0.168	63	To Do
Kamke 247	✓	14.815	586	✓	0.222	517	To Do
Kamke 248	✓	0.014	83	✓	0.022	75	To Do
Kamke 249	✗	0	0	✓	0.209	232	To Do
Kamke 250	✗	0	0	✗	0	0	To Do
Kamke 251	✓	0.012	57	✓	0.02	51	To Do
Kamke 252	✓	14.742	738	✓	0.873	1338	To Do
Kamke 253	✗	0	0	✗	0	0	To Do
Kamke 254	✓	0.016	81	✓	0.031	59	To Do
Kamke 255	✓	4.807	30	✓	0.226	74	To Do
Kamke 256	✓	0.019	21	✓	0.045	31	To Do
Kamke 257	✓	0.405	38	✓	0.113	98	To Do
Kamke 258	✓	0.013	43	✓	0.021	33	To Do
Kamke 259	✓	0.019	50	✓	0.026	51	To Do
Kamke 260	✓	0.015	74	✓	0.033	59	To Do
Kamke 261	✓	0.893	32	✓	0.156	18	To Do
Kamke 262	✓	0.065	101	✓	0.317	65	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 263	✓	0.039	120	✓	0.148	173	To Do
Kamke 264	✓	0.361	680	✓	0.531	574	To Do
Kamke 265	✗	0	0	✗	0	0	To Do
Kamke 266	✗	0	0	✓	1.716	55	To Do
Kamke 267	✓	0.041	36	✓	0.023	32	To Do
Kamke 268	✓	0.988	140	✓	0.099	118	To Do
Kamke 269	✗	0	0	✗	0	0	To Do
Kamke 270	✓	11.142	326	✓	0.025	319	To Do
Kamke 271	✓	4.285	372	✓	0.185	352	To Do
Kamke 272	✗	0	0	✓	0.151	43	To Do
Kamke 273	✓	0.027	294	✓	0.023	401	To Do
Kamke 274	✓	0.036	396	✓	0.031	657	To Do
Kamke 275	✓	0.365	16	✓	0.104	30	To Do
Kamke 276	✓	0.037	61	✓	0.064	47	To Do
Kamke 277	✓	0.015	53	✓	0.35	41	To Do
Kamke 278	✓	0.11	32	✓	0.058	28	To Do
Kamke 279	✓	0.722	106	✓	0.18	116	To Do
Kamke 280	✓	107.166	20	✓	0.054	24	To Do
Kamke 281	✓	0.073	75	✓	0.069	55	To Do
Kamke 282	✓	0.157	1089	✓	0.235	71	To Do
Kamke 283	✓	0.06	501	✓	0.069	407	To Do
Kamke 284	✓	0.052	59	✓	0.151	21	To Do
Kamke 285	✓	0.035	382	✓	0.177	432	To Do
Kamke 286	✓	0.194	3501	✓	1.803	1337	To Do
Kamke 287	✓	3.276	69	✓	0.074	56	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 288	✓	0.026	518	✓	0.031	579	To Do
Kamke 289	✓	0.022	115	✓	0.034	115	To Do
Kamke 290	✓	0.092	744	✓	0.203	1388	To Do
Kamke 291	✓	0.798	39	✓	0.17	50	To Do
Kamke 292	✓	58.3	1	✓	0.052	115	To Do
Kamke 293	✓	0.103	661	✓	0.373	35	To Do
Kamke 294	✓	0.033	65	✓	0.108	112	To Do
Kamke 295	✓	0.098	30	✓	0.234	29	To Do
Kamke 296	✓	0.571	88	✓	0.779	135	To Do
Kamke 297	✓	0.058	216	✓	0.332	29	To Do
Kamke 298	✓	0.01	72	✓	0.017	73	To Do
Kamke 299	✓	0.024	328	✓	0.183	276	To Do
Kamke 300	✓	0.01	99	✓	0.019	83	To Do
Kamke 301	✓	0.039	64	✓	0.206	25	To Do
Kamke 302	✓	0.018	60	✓	0.153	133	To Do
Kamke 303	✓	0.078	24	✓	0.204	34	To Do
Kamke 304	✓	45.291	45	✓	0.225	44	To Do
Kamke 305	✓	0.098	1211	✓	0.023	21	To Do
Kamke 306	✓	0.05	201	✓	0.382	231	To Do
Kamke 307	✓	0.026	149	✓	0.052	125	To Do
Kamke 308	✓	0.007	48	✓	0.017	37	To Do
Kamke 309	✓	0.013	151	✓	0.04	113	To Do
Kamke 310	✓	0.044	159	✓	0.22	125	To Do
Kamke 311	✓	0.171	2201	✓	0.191	50	To Do
Kamke 312	✓	0.249	190	✓	1.655	240	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 313	✓	0.083	524	✓	0.22	748	To Do
Kamke 314	✓	0.044	164	✓	0.051	158	To Do
Kamke 315	✓	0.106	331	✓	0.089	376	To Do
Kamke 316	✓	0.07	41	✓	0.053	53	To Do
Kamke 317	✓	0.371	23	✓	0.124	29	To Do
Kamke 318	✓	0.145	2353	✓	0.022	28	To Do
Kamke 319	✓	0.023	302	✓	0.036	35	To Do
Kamke 320	✓	0.056	76	✓	0.095	78	To Do
Kamke 321	✓	0.298	42	✓	0.173	42	To Do
Kamke 322	✓	0.2	2097	✓	0.03	29	To Do
Kamke 323	✓	0.045	491	✓	0.135	630	To Do
Kamke 324	✓	0.034	672	✓	0.137	815	To Do
Kamke 325	✓	0.221	133	✓	0.508	124	To Do
Kamke 326	✓	4.869	6861	✓	0.494	160	To Do
Kamke 327	✓	0.39	584	✓	0.175	583	To Do
Kamke 328	✓	27.933	35	✓	0.193	33	To Do
Kamke 329	✓	0.834	97	✓	0.355	71	To Do
Kamke 330	✓	40.05	49	✓	0.03	22	To Do
Kamke 331	✗	0	0	✗	0	0	To Do
Kamke 332	✓	0.2	23	✓	0.014	33	To Do
Kamke 333	✓	0.35	53	✓	0.102	32	To Do
Kamke 334	✓	0.035	39	✓	0.025	19	To Do
Kamke 335	✓	0.176	75	✓	0.011	50	To Do
Kamke 336	✓	0.123	43	✓	0.033	41	To Do
Kamke 337	✓	0.061	52	✓	0.059	28	To Do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 338	✗	0	0	✓	0.744	129	To Do
Kamke 339	✓	0.178	27	✓	0.183	27	To Do
Kamke 340	✗	0	0	✗	0	0	To Do
Kamke 341	✓	0.06	33	✓	0.065	30	To Do
Kamke 342	✓	0.264	163	✓	0.04	17	To Do
Kamke 343	✓	0.078	22	✓	0.047	27	To Do
Kamke 344	✓	0.04	23	✓	0.026	19	To Do
Kamke 345	✓	0.079	23	✓	0.063	36	To Do
Kamke 346	✓	0.081	20	✓	0.276	19	To Do
Kamke 347	✓	0.138	32	✓	0.153	16	To Do
Kamke 348	✓	0.058	17	✓	0.101	15	To Do
Kamke 349	✓	0.037	15	✓	0.033	17	To Do
Kamke 350	✓	0.519	53	✓	1.077	226	To Do
Kamke 351	✓	0.336	61	✓	0.504	55	To Do
Kamke 352	✓	0.165	32	✓	0.302	33	To Do
Kamke 353	✓	0.019	14	✓	0.06	12	To Do
Kamke 354	✓	0.059	145	✓	0.042	108	To Do
Kamke 355	✓	0.058	17	✓	0.1	15	To Do
Kamke 356	✓	0.072	21	✓	0.122	19	To Do
Kamke 357	✓	0.282	35	✓	0.58	13	To Do
Kamke 358	✓	0.042	29	✓	0.072	11	To Do
Kamke 359	✓	0.056	42	✓	0.051	28	To Do
Kamke 360	✓	50.991	6218	✓	0.2	48	To Do
Kamke 361	✓	0.358	23	✓	0.253	22	To Do
Kamke 362	✓	0.287	20	✓	0.231	23	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 363	✓	0.07	28	✓	0.067	35	To Do
Kamke 364	✓	0.139	27	✓	0.106	23	To Do
Kamke 365	✗	0	0	✓	0.304	42	To Do
Kamke 366	✓	247.697	88	✓	0.071	45	To Do
Kamke 367	✗	0	0	✗	0	0	To Do
Kamke 368	✗	0	0	✗	0	0	To Do
Kamke 369	✓	0.049	99	✓	0.728	68	To Do
Kamke 370	✗	0	0	✗	0	0	To Do
Kamke 371	✓	0.024	35	✓	0.676	20	To Do
Kamke 372	✓	0.005	27	✓	0.661	232	To Do
Kamke 373	✓	0.096	71	✓	0.411	49	To Do
Kamke 374	✓	0.067	73	✓	0.659	85	To Do
Kamke 375	✓	0.043	68	✓	0.654	49	To Do
Kamke 376	✓	0.314	110	✓	1.237	219	To Do
Kamke 377	✓	0.005	17	✓	0.641	24	To Do
Kamke 378	✓	0.005	13	✓	0.638	20	To Do
Kamke 379	✓	0.005	15	✓	0.648	22	To Do
Kamke 380	✓	0.412	1445	✓	0.654	619	To Do
Kamke 381	✓	0.428	1445	✓	0.662	579	To Do
Kamke 382	✓	0.245	190	✓	0.661	146	To Do
Kamke 383	✗	0	0	✗	0	0	To Do
Kamke 384	✓	1.995	133	✓	0.025	50	To Do
Kamke 385	✗	0	0	✓	0.274	169	To Do
Kamke 386	✓	0.192	56	✓	0.429	27	To Do
Kamke 387	✓	0.871	133	✓	0.676	115	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 388	✓	0.85	41	✓	0.087	223	To Do
Kamke 389	✓	0.044	55	✓	0.57	71	To Do
Kamke 390	✓	2.812	82	✓	0.262	281	To Do
Kamke 391	✓	0.007	29	✓	0.007	22	To Do
Kamke 392	✓	0.247	25	✓	0.465	50	To Do
Kamke 393	✓	0.039	31	✓	0.119	77	To Do
Kamke 394	✗	0	0	✓	5.143	164	To Do
Kamke 395	✗	0	0	✗	0	0	To Do
Kamke 396	✓	0.01	29	✓	0.011	20	To Do
Kamke 397	✓	0.722	136	✓	0.353	128	To Do
Kamke 398	✓	1.247	175	✓	2.454	138	To Do
Kamke 399	✓	0.005	15	✓	0.019	22	To Do
Kamke 400	✗	0	0	✓	0.19	74	To Do
Kamke 401	✓	0.33	1093	✓	0.032	580	To Do
Kamke 402	✗	0	0	✓	0.177	101	To Do
Kamke 403	✓	0.296	116	✓	0.627	197	To Do
Kamke 404	✗	0	0	✓	0.266	389	To Do
Kamke 405	✓	1.446	40	✓	0.218	378	To Do
Kamke 406	✓	1.122	38	✓	0.079	262	To Do
Kamke 407	✓	0.017	41	✓	0.027	39	To Do
Kamke 408	✓	1.235	163	✓	0.053	73	To Do
Kamke 409	✓	30.808	39	✓	0.055	63	To Do
Kamke 410	✓	31.039	40	✓	0.056	64	To Do
Kamke 411	✓	1.311	180	✓	0.041	65	To Do
Kamke 412	✓	27.564	11973	✓	0.04	146	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 413	✗	0	0	✓	0.193	269	To Do
Kamke 414	✗	0	0	✓	0.199	269	To Do
Kamke 415	✓	0.207	133	✓	0.181	95	To Do
Kamke 416	✗	0	0	✓	0.059	136	To Do
Kamke 417	✓	0.395	183	✓	0.032	35	To Do
Kamke 418	✓	1.261	158	✓	0.038	42	To Do
Kamke 419	✓	1.474	6977	✓	0.04	109	To Do
Kamke 420	✓	1.69	9391	✓	0.038	689	To Do
Kamke 421	✓	0.032	27	✓	0.036	32	To Do
Kamke 422	✓	0.046	29	✓	0.038	30	To Do
Kamke 423	✓	0.077	51	✓	0.042	44	To Do
Kamke 424	✓	1.794	223	✓	0.072	193	To Do
Kamke 425	✓	0.258	57	✓	0.039	45	To Do
Kamke 426	✓	0.397	150	✓	0.038	51	To Do
Kamke 427	✓	0.631	300	✓	0.042	60	To Do
Kamke 428	✗	0	0	✓	0.07	66	To Do
Kamke 429	✗	0	0	✓	0.069	72	To Do
Kamke 430	✗	0	0	✓	1.064	1602	To Do
Kamke 431	✓	0.036	103	✓	0.149	62	To Do
Kamke 432	✓	1.855	54	✓	10.948	242	To Do
Kamke 433	✓	0.501	22	✓	0.235	34	To Do
Kamke 434	✓	0.033	27	✓	0.004	7	To Do
Kamke 435	✓	0.036	55	✓	0.233	22	To Do
Kamke 436	✓	0.033	26	✓	1.738	61	To Do
Kamke 437	✓	0.271	47	✓	0.045	36	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 438	✓	0.007	21	✓	0.011	17	To Do
Kamke 439	✓	0.013	49	✓	0.052	33	To Do
Kamke 440	✓	0.006	19	✓	0.011	15	To Do
Kamke 441	✓	0.068	59	✓	1.188	83	To Do
Kamke 442	✓	0.009	26	✓	0.012	21	To Do
Kamke 443	✓	0.533	1921	✓	1.733	221	To Do
Kamke 444	✓	0.143	73	✓	0.877	120	To Do
Kamke 445	✓	0.009	49	✓	0.015	35	To Do
Kamke 446	✓	0.481	167	✓	0.054	57	To Do
Kamke 447	✓	0.016	41	✓	0.026	33	To Do
Kamke 448	✓	0.084	88	✓	266.143	166	To Do
Kamke 449	✓	0.01	27	✓	0.012	23	To Do
Kamke 450	✓	0.437	26	✓	0.455	51	To Do
Kamke 451	✗	0	0	✓	0.054	78	To Do
Kamke 452	✗	0	0	✓	1.744	37	To Do
Kamke 453	✓	1.227	369	✓	0.742	229	To Do
Kamke 454	✓	0.151	113	✓	0.123	106	To Do
Kamke 455	✓	0.387	123	✓	0.18	66	To Do
Kamke 456	✓	0.118	61	✓	0.392	33	To Do
Kamke 457	✓	1.557	406	✓	0.778	45	To Do
Kamke 458	✓	0.062	139	✓	0.053	90	To Do
Kamke 459	✗	0	0	✓	0.614	65	To Do
Kamke 460	✗	0	0	✗	0	0	To Do
Kamke 461	✗	0	0	✗	0	0	To Do
Kamke 462	✓	0.015	43	✓	0.026	27	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 463	✓	0.017	47	✓	0.075	50	To Do
Kamke 464	✓	0.06	52	✓	0.443	70	To Do
Kamke 465	✗	0	0	✓	0.07	210	To Do
Kamke 466	✓	0.244	119	✓	0.391	71	To Do
Kamke 467	✗	0	0	✓	0.069	148	To Do
Kamke 468	✗	0	0	✓	0.081	181	To Do
Kamke 469	✓	0.632	245	✓	0.092	264	To Do
Kamke 470	✗	0	0	✓	0.29	87	To Do
Kamke 471	✓	0.009	47	✓	0.015	33	To Do
Kamke 472	✓	0.184	121	✓	0.475	121	To Do
Kamke 473	✓	0.378	137	✓	0.641	71	To Do
Kamke 474	✓	0.227	135	✓	1.041	152	To Do
Kamke 475	✓	0.067	57	✓	0.454	67	To Do
Kamke 476	✗	0	0	✓	0.304	87	To Do
Kamke 477	✓	0.304	136	✓	0.517	622	To Do
Kamke 478	✓	0.167	141	✓	0.079	88	To Do
Kamke 479	✗	0	0	✓	0.304	929	To Do
Kamke 480	✗	0	0	✗	0	0	To Do
Kamke 481	✓	0.01	49	✓	0.018	35	To Do
Kamke 482	✗	0	0	✗	0	0	To Do
Kamke 483	✓	0.16	69	✓	0.08	103	To Do
Kamke 484	✓	0.153	79	✓	0.08	115	To Do
Kamke 485	✗	0	0	✗	0	0	To Do
Kamke 486	✓	0.027	89	✓	0.098	54	To Do
Kamke 487	✗	0	0	✓	0.417	100	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 488	✓	0.362	85	✓	0.434	111	To Do
Kamke 489	✗	0	0	✓	2.375	551	To Do
Kamke 490	✓	0.58	63	✓	0.434	145	To Do
Kamke 491	✓	1.01	65	✓	0.75	195	To Do
Kamke 492	✓	0.272	111	✓	0.421	122	To Do
Kamke 493	✓	15.844	393	✓	1.113	111	To Do
Kamke 494	✗	0	0	✓	0.173	161	To Do
Kamke 495	✗	0	0	✓	0.465	61	To Do
Kamke 496	✓	95.165	53	✓	0.241	130	To Do
Kamke 497	✓	0.172	203	✓	0.462	203	To Do
Kamke 498	✓	0.103	107	✓	0.696	99	To Do
Kamke 499	✓	0.287	148	✓	0.19	189	To Do
Kamke 500	✓	1.292	86	✓	0.936	220	To Do
Kamke 501	✗	0	0	✓	4.984	215	To Do
Kamke 502	✓	1.708	71	✓	0.39	195	To Do
Kamke 503	✗	0	0	✗	0	0	To Do
Kamke 504	✗	0	0	✓	0.824	247	To Do
Kamke 505	✓	0.014	73	✓	0.037	52	To Do
Kamke 506	✗	0	0	✗	0	0	To Do
Kamke 507	✓	32.433	408	✗	0	0	To Do
Kamke 508	✗	0	0	✓	1.838	60	To Do
Kamke 509	✗	0	0	✓	1.46	212	To Do
Kamke 510	✗	0	0	✗	0	0	To Do
Kamke 511	✓	1.861	225	✓	5.059	199	To Do
Kamke 512	✓	11.754	725	✓	11.819	135	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 513	✗	0	0	✓	2.854	1134	To Do
Kamke 514	✓	15.197	605	✓	0.228	87	To Do
Kamke 515	✗	0	0	✓	2.286	113	To Do
Kamke 516	✓	3.943	229	✓	1.141	72	To Do
Kamke 517	✓	4.033	253	✓	1.198	155	To Do
Kamke 518	✓	0.782	236	✓	0.195	126	To Do
Kamke 519	✓	1.885	473	✓	0.423	197	To Do
Kamke 520	✓	232.636	3323	✓	0.178	245	To Do
Kamke 521	✓	0.004	14	✓	0.031	33	To Do
Kamke 522	✓	0.004	17	✓	0.042	44	To Do
Kamke 523	✗	0	0	✓	0.057	231	To Do
Kamke 524	✗	0	0	✓	0.058	261	To Do
Kamke 525	✓	0.064	121	✓	0.06	122	To Do
Kamke 526	✓	0.01	43	✓	0.013	32	To Do
Kamke 527	✗	0	0	✓	0.847	43	To Do
Kamke 528	✗	0	0	✓	0.072	86	To Do
Kamke 529	✓	53.649	1496	✓	0.04	1251	To Do
Kamke 530	✗	0	0	✓	0.12	432	To Do
Kamke 531	✗	0	0	✗	0	0	To Do
Kamke 532	✗	0	0	✓	0.157	848	To Do
Kamke 533	✗	0	0	✓	0.035	76	To Do
Kamke 534	✗	0	0	✓	0.074	84	To Do
Kamke 535	✗	0	0	✓	0.056	51	To Do
Kamke 536	✓	0.024	64	✓	0.035	52	To Do
Kamke 537	✗	0	0	✓	598.448	209	To Do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 538	✗	0	0	✓	10.397	1532	To Do
Kamke 539	✓	0.028	45	✓	3.576	32	To Do
Kamke 540	✓	0.021	61	✓	0.35	109	To Do
Kamke 541	✗	0	0	✓	3.656	103	To Do
Kamke 542	✗	0	0	✓	3.376	107	To Do
Kamke 543	✗	0	0	✓	12.393	277	To Do
Kamke 544	✗	0	0	✓	15.838	4201	To Do
Kamke 545	✓	0.727	383	✓	0.926	144	To Do
Kamke 546	✗	0	0	✓	2.064	171	To Do
Kamke 547	✓	2.03	341	✓	3.52	118	To Do
Kamke 548	✓	1.02	569	✓	0.768	246	To Do
Kamke 549	✓	0.241	406	✓	4.315	545	To Do
Kamke 550	✗	0	0	✓	3.802	60	To Do
Kamke 551	✓	0.434	79	✓	1.128	55	To Do
Kamke 552	✓	0.183	39	✓	1.036	43	To Do
Kamke 553	✓	39.766	51	✓	0.308	36	To Do
Kamke 554	✗	0	0	✓	1.642	32	To Do
Kamke 555	✗	0	0	✓	0.137	15	To Do
Kamke 556	✓	6.928	50	✓	1.483	581	To Do
Kamke 557	✓	0.018	37	✓	1.015	74	To Do
Kamke 558	✓	1.256	369	✓	1.734	223	To Do
Kamke 559	✓	0.304	148	✓	1.228	215	To Do
Kamke 560	✓	21.387	86	✓	2.583	1120	To Do
Kamke 561	✗	0	0	✓	5.855	50	To Do
Kamke 562	✗	0	0	✓	1.398	3306	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 563	✓	0.365	57	✓	1.438	66	To Do
Kamke 564	✓	0.047	21	✓	3.761	32	To Do
Kamke 565	✓	0.012	24	✓	4.548	17	To Do
Kamke 566	✗	0	0	✓	0.113	16	To Do
Kamke 567	✗	0	0	✓	0.159	18	To Do
Kamke 568	✓	0.069	27	✓	0.106	32	To Do
Kamke 569	✓	0.044	59	✓	2.899	147	To Do
Kamke 570	✗	0	0	✓	0.303	30	To Do
Kamke 571	✗	0	0	✓	4.12	169	To Do
Kamke 572	✗	0	0	✗	0	0	To Do
Kamke 573	✓	0.021	42	✓	3.723	16	To Do
Kamke 574	✓	0.016	62	✓	0.508	41	To Do
Kamke 575	✗	0	0	✗	0	0	To Do
Kamke 576	✗	0	0	✗	0	0	To Do
Kamke 577	✓	15.259	141	✓	1.804	28	To Do
Kamke 578	✓	20.415	88	✓	6.828	22	To Do
Kamke 579	✓	15.982	184	✓	4.297	35	To Do
Kamke 580	✓	31.813	161	✓	0.504	31	To Do
Kamke 581	✓	48.465	107	✓	0.37	32	To Do
Kamke 582	✓	20.695	103	✓	1.818	30	To Do
Kamke 583	✓	50.861	111	✓	2.022	31	To Do
Kamke 584	✓	23.906	108	✓	5.504	35	To Do
Kamke 585	✓	154.336	141	✓	9.802	122	To Do
Kamke 586	✓	185.373	398	✓	1.078	39	To Do
Kamke 587	✓	257.684	109	✓	0.594	29	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 588	✓	38.249	99	✓	0.773	53	To Do
Kamke 589	✓	23.812	134	✓	1.869	38	To Do
Kamke 590	✓	37.404	91	✓	1.005	28	To Do
Kamke 591	✓	22.34	182	✓	2.249	108	To Do
Kamke 592	✗	0	0	✓	0.31	33	To Do
Kamke 593	✓	44.492	166	✓	0.559	35	To Do
Kamke 594	✓	22.051	188	✓	1.821	67	To Do
Kamke 595	✓	23.325	127	✓	0.37	72	To Do
Kamke 596	✓	237.635	103	✓	1.519	26	To Do
Kamke 597	✓	31.28	112	✓	1.647	37	To Do
Kamke 598	✓	0.117	35	✓	0.781	29	To Do
Kamke 599	✓	27.175	92	✓	0.423	57	To Do
Kamke 600	✓	28.114	135	✓	6.479	38	To Do
Kamke 601	✓	39.651	129	✓	0.335	61	To Do
Kamke 602	✓	234.397	111	✓	2.294	33	To Do
Kamke 603	✓	20.047	102	✓	1.204	27	To Do
Kamke 604	✓	28.152	97	✓	0.512	30	To Do
Kamke 605	✓	211.45	103	✓	0.2	29	To Do
Kamke 606	✓	76.383	180	✓	1.879	34	To Do
Kamke 607	✗	0	0	✓	1.092	22	To Do
Kamke 608	✗	0	0	✓	2.772	40	To Do
Kamke 609	✓	61.083	107	✓	0.859	22	To Do
Kamke 610	✓	0.077	24	✓	0.019	20	To Do
Kamke 611	✓	45.945	116	✓	1.11	28	To Do
Kamke 612	✓	56.152	169	✓	3.195	27	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 613	✗	0	0	✓	0.749	23	To Do
Kamke 614	✓	84.939	144	✓	3.014	60	To Do
Kamke 615	✓	19.007	74	✓	1.848	26	To Do
Kamke 616	✓	55.705	126	✓	1.794	26	To Do
Kamke 617	✓	270.631	302	✓	1.109	47	To Do
Kamke 618	✓	0.097	25	✓	5.764	34	To Do
Kamke 619	✓	257.989	230	✓	2.099	81	To Do
Kamke 620	✗	0	0	✓	5.51	37	To Do
Kamke 621	✓	0.082	445	✓	4.549	59	To Do
Kamke 622	✗	0	0	✓	2.735	77	To Do
Kamke 623	✓	11.562	77	✓	3.777	49	To Do
Kamke 624	✓	48.546	1	✓	6.144	46	To Do
Kamke 625	✓	0.376	67	✓	4.48	55	To Do
Kamke 626	✓	1.364	104	✓	7.711	115	To Do
Kamke 627	✓	0.755	25	✓	14.909	35	To Do
Kamke 628	✓	0.077	32	✓	10.563	23	To Do
Kamke 629	✓	0.666	38	✓	20.097	62	To Do
Kamke 630	✓	0.581	101	✓	19.011	98	To Do
Kamke 631	✓	0.079	31	✓	45.07	23	To Do
Kamke 632	✓	0.367	65	✓	11.1	54	To Do
Kamke 633	✓	0.376	85	✓	14.888	52	To Do
Kamke 634	✓	0.157	31	✓	10.351	26	To Do
Kamke 635	✓	0.103	33	✓	6.954	22	To Do
Kamke 636	✓	0.046	24	✓	4.838	19	To Do
Kamke 637	✓	15.677	53	✓	27.176	84	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 638	✗	0	0	✓	1.461	35	To Do
Kamke 639	✗	0	0	✓	1.004	50	To Do
Kamke 640	✗	0	0	✓	0.566	47	To Do
Kamke 641	✓	0.156	33	✓	2.438	26	To Do
Kamke 642	✓	0.12	95	✓	1.088	286	To Do
Kamke 643	✓	0.105	31	✓	0.189	22	To Do
Kamke 644	✓	0.254	34	✓	0.666	27	To Do
Kamke 645	✓	0.033	20	✓	0.66	14	To Do
Kamke 646	✓	0.155	35	✓	0.467	23	To Do
Kamke 647	✓	0.367	117	✓	0.461	460	To Do
Kamke 648	✓	0.31	96	✓	0.857	41	To Do
Kamke 649	✓	0.159	36	✓	0.29	27	To Do
Kamke 650	✓	0.223	39	✓	0.202	28	To Do
Kamke 651	✓	0.031	15	✓	0.069	13	To Do
Kamke 652	✓	1.845	103	✓	0.181	27	To Do
Kamke 653	✓	0.163	31	✓	0.162	24	To Do
Kamke 654	✓	0.146	37	✓	0.207	23	To Do
Kamke 655	✓	20.416	79	✓	1.842	66	To Do
Kamke 656	✓	0.037	20	✓	0.067	15	To Do
Kamke 657	✓	0.165	37	✓	0.326	26	To Do
Kamke 658	✓	0.222	46	✓	0.279	28	To Do
Kamke 659	✓	0.398	51	✓	0.441	41	To Do
Kamke 660	✓	0.256	42	✓	0.293	29	To Do
Kamke 661	✓	0.384	54	✓	0.214	39	To Do
Kamke 662	✓	0.179	37	✓	0.183	26	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 663	✓	2.15	103	✓	0.17	27	To Do
Kamke 664	✓	0.181	34	✓	0.164	25	To Do
Kamke 665	✓	0.236	39	✓	0.52	28	To Do
Kamke 666	✓	0.066	29	✓	0.146	24	To Do
Kamke 667	✓	1.127	84	✓	0.305	82	To Do
Kamke 668	✓	1.145	59	✓	1.044	58	To Do
Kamke 669	✓	0.788	222	✓	1.24	72	To Do
Kamke 670	✓	0.628	83	✓	1.01	70	To Do
Kamke 671	✓	0.378	162	✓	0.832	231	To Do
Kamke 672	✗	0	0	✓	0.458	36	To Do
Kamke 673	✓	0.122	23	✓	1.068	17	To Do
Kamke 674	✓	0.212	32	✓	0.276	27	To Do
Kamke 675	✓	0.049	45	✓	0.063	37	To Do
Kamke 676	✓	0.287	120	✓	0.626	43	To Do
Kamke 677	✓	0.033	51	✓	0.036	48	To Do
Kamke 678	✓	0.226	90	✓	0.277	37	To Do
Kamke 679	✓	0.03	44	✓	0.063	37	To Do
Kamke 680	✓	0.212	39	✓	0.259	28	To Do
Kamke 681	✓	0.04	54	✓	0.075	45	To Do
Kamke 682	✓	0.118	33	✓	0.395	28	To Do
Kamke 683	✓	0.339	72	✓	0.299	152	To Do
Kamke 684	✓	0.023	18	✓	3.328	30	To Do
Kamke 685	✓	0.036	62	✓	0.116	48	To Do
Kamke 686	✓	15.417	49	✓	3.229	85	To Do
Kamke 687	✓	0.057	111	✓	0.193	39	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 688	✓	0.118	45	✓	0.086	42	To Do
Kamke 689	✓	0.065	55	✓	0.168	25	To Do
Kamke 690	✓	0.281	106	✓	0.346	40	To Do
Kamke 691	✓	0.068	21	✓	0.727	17	To Do
Kamke 692	✓	0.024	18	✓	2.739	30	To Do
Kamke 693	✓	1.022	1	✓	0.216	40	To Do
Kamke 694	✓	0.266	43	✓	0.279	30	To Do
Kamke 695	✓	0.052	34	✓	0.07	39	To Do
Kamke 696	✗	0	0	✓	0.386	32	To Do
Kamke 697	✓	0.36	1	✓	0.152	40	To Do
Kamke 698	✓	0.366	1	✓	0.174	34	To Do
Kamke 699	✓	0.218	90	✓	0.393	36	To Do
Kamke 700	✓	0.062	72	✓	0.175	62	To Do
Kamke 701	✗	0	0	✓	6.756	71	To Do
Kamke 702	✗	0	0	✓	0.247	35	To Do
Kamke 703	✗	0	0	✓	0.602	44	To Do
Kamke 704	✗	0	0	✓	0.048	38	To Do
Kamke 705	✓	0.053	30	✓	0.159	24	To Do
Kamke 706	✗	0	0	✓	0.633	65	To Do
Kamke 707	✗	0	0	✓	0.539	105	To Do
Kamke 708	✓	0.295	1	✗	0	0	To Do
Kamke 709	✓	4.133	143	✓	0.302	39	To Do
Kamke 710	✗	0	0	✓	2.625	31	To Do
Kamke 711	✓	0.065	24	✓	0.139	31	To Do
Kamke 712	✓	0.257	102	✓	0.335	38	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 713	✓	0.116	607	✓	0.447	86	To Do
Kamke 714	✗	0	0	✓	2.222	96	To Do
Kamke 715	✓	0.242	89	✓	0.312	39	To Do
Kamke 716	✓	4.024	133	✓	0.361	37	To Do
Kamke 717	✓	0.303	44	✓	0.381	33	To Do
Kamke 718	✓	1.119	1	✓	0.068	44	To Do
Kamke 719	✓	0.088	44	✓	0.193	57	To Do
Kamke 720	✓	4.36	272	✓	0.244	48	To Do
Kamke 721	✓	0.018	27	✓	0.095	19	To Do
Kamke 722	✓	39.38	493	✓	0.372	70	To Do
Kamke 723	✓	0.07	672	✓	0.085	856	To Do
Kamke 724	✓	61.518	422	✓	0.056	18	To Do
Kamke 725	✓	0.247	19	✓	0.866	25	To Do
Kamke 726	✓	0.079	607	✓	0.305	83	To Do
Kamke 727	✓	0.425	29	✓	0.403	25	To Do
Kamke 728	✓	0.361	72	✓	0.358	50	To Do
Kamke 729	✓	0.299	319	✓	0.108	404	To Do
Kamke 730	✗	0	0	✓	2.161	41	To Do
Kamke 731	✓	0.304	42	✓	0.194	42	To Do
Kamke 732	✓	0.457	105	✓	0.405	43	To Do
Kamke 733	✗	0	0	✗	0	0	To Do
Kamke 734	✓	0.106	37	✓	0.152	39	To Do
Kamke 735	✗	0	0	✓	0.069	78	To Do
Kamke 736	✓	0.097	30	✓	0.2	43	To Do
Kamke 737	✓	0.03	36	✓	0.083	29	To Do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 738	✓	0.473	1200	✓	0.904	1054	To Do
Kamke 739	✓	0.214	34	✓	0.18	35	To Do
Kamke 740	✓	0.062	74	✓	0.104	72	To Do
Kamke 741	✓	3.074	1	✓	1.023	246	To Do
Kamke 742	✓	4.444	221	✓	1.95	239	To Do
Kamke 743	✗	0	0	✓	0.57	301	To Do
Kamke 744	✓	0.046	534	✓	0.243	621	To Do
Kamke 745	✗	0	0	✓	0.071	78	To Do
Kamke 746	✗	0	0	✓	0.489	243	To Do
Kamke 747	✗	0	0	✓	0.358	75	To Do
Kamke 748	✓	0.3	286	✓	0.107	404	To Do
Kamke 749	✓	0.098	102	✓	0.135	192	To Do
Kamke 750	✓	0.321	68	✓	0.265	49	To Do
Kamke 751	✓	0.073	29	✓	0.114	26	To Do
Kamke 752	✗	0	0	✓	1.624	723	To Do
Kamke 753	✓	0.101	41	✓	0.155	38	To Do
Kamke 754	✓	0.038	1	✓	0.019	26	To Do
Kamke 755	✓	0.186	2213	✓	0.165	44	To Do
Kamke 756	✓	0.261	1	✓	0.034	37	To Do
Kamke 757	✓	0.027	33	✓	0.067	26	To Do
Kamke 758	✓	0.958	459	✓	0.247	41	To Do
Kamke 759	✗	0	0	✓	0.704	315	To Do
Kamke 760	✓	1.281	85	✓	1.639	136	To Do
Kamke 761	✓	0.027	26	✓	0.056	18	To Do
Kamke 762	✓	0.059	26	✓	0.112	22	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 763	✓	0.058	21	✓	0.101	14	To Do
Kamke 764	✓	0.096	46	✓	0.126	36	To Do
Kamke 765	✗	0	0	✓	0.208	106	To Do
Kamke 766	✗	0	0	✓	0.256	89	To Do
Kamke 767	✓	0.026	35	✓	0.081	26	To Do
Kamke 768	✓	1.142	66	✓	0.11	26	To Do
Kamke 769	✗	0	0	✓	0.547	251	To Do
Kamke 770	✓	0.133	687	✓	0.132	1105	To Do
Kamke 771	✓	0.033	48	✓	0.111	84	To Do
Kamke 772	✓	0.063	21	✓	0.119	18	To Do
Kamke 773	✓	0.304	59	✓	0.247	48	To Do
Kamke 774	✓	0.03	46	✓	0.089	51	To Do
Kamke 775	✓	0.099	943	✓	0.091	32	To Do
Kamke 776	✗	0	0	✓	0.934	96	To Do
Kamke 777	✓	0.139	34	✓	0.139	51	To Do
Kamke 778	✓	0.214	1	✓	0.03	37	To Do
Kamke 779	✓	0.071	51	✓	0.11	50	To Do
Kamke 780	✓	0.026	15	✓	0.408	27	To Do
Kamke 781	✓	0.536	82	✓	0.29	61	To Do
Kamke 782	✗	0	0	✓	1.546	96	To Do
Kamke 783	✗	0	0	✓	0.238	75	To Do
Kamke 784	✗	0	0	✓	36.043	24	To Do
Kamke 785	✗	0	0	✓	123.082	24	To Do
Kamke 786	✗	0	0	✓	0.063	33	To Do
Kamke 787	✓	41.148	484	✓	0.619	191	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 788	✗	0	0	✓	0.706	108	To Do
Kamke 789	✗	0	0	✗	0	0	To Do
Kamke 790	✗	0	0	✗	0	0	To Do
Kamke 791	✗	0	0	✓	35.476	306	To Do
Kamke 792	✗	0	0	✓	0.852	112	To Do
Kamke 793	✓	23.356	399	✓	0.109	32	To Do
Kamke 794	✓	0.15	66	✓	2.51	32	To Do
Kamke 795	✓	1.175	106	✓	0.042	37	To Do
Kamke 796	✗	0	0	✓	1.451	143	To Do
Kamke 797	✓	2.06	146	✓	0.425	252	To Do
Kamke 798	✓	0.653	25	✓	0.154	30	To Do
Kamke 799	✓	0.302	66	✓	0.43	147	To Do
Kamke 800	✓	1.207	118	✓	0.03	41	To Do
Kamke 801	✓	0.325	1	✓	0.072	63	To Do
Kamke 802	✓	0.083	88	✓	0.147	27	To Do
Kamke 803	✓	0.118	234	✓	0.598	65	To Do
Kamke 804	✓	0.5	43	✓	1.401	38	To Do
Kamke 805	✓	0.033	35	✓	0.68	42	To Do
Kamke 806	✓	0.276	23	✓	0.802	22	To Do
Kamke 807	✗	0	0	✓	0.821	43	To Do
Kamke 808	✓	1.762	149	✓	0.096	45	To Do
Kamke 809	✓	0.981	118	✓	0.029	41	To Do
Kamke 810	✓	0.019	26	✓	0.05	16	To Do
Kamke 811	✓	2.299	29	✓	2.608	32	To Do
Kamke 812	✓	0.329	76	✓	0.483	32	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 813	✓	0.495	64	✓	0.542	40	To Do
Kamke 814	✓	0.116	70	✓	0.042	38	To Do
Kamke 815	✗	0	0	✓	1.298	168	To Do
Kamke 816	✓	0.176	1	✓	1.025	190	To Do
Kamke 817	✓	0.367	59	✓	0.869	27	To Do
Kamke 818	✓	0.086	34	✓	0.155	34	To Do
Kamke 819	✓	0.225	63	✓	0.258	30	To Do
Kamke 820	✓	0.33	59	✓	0.864	27	To Do
Kamke 821	✓	0.157	1993	✓	0.173	25	To Do
Kamke 822	✓	0.042	32	✓	0.144	25	To Do
Kamke 823	✓	0.452	35	✓	0.145	38	To Do
Kamke 824	✓	0.334	66	✓	0.512	61	To Do
Kamke 825	✓	7.463	144	✓	0.119	48	To Do
Kamke 826	✓	0.605	70	✓	0.401	51	To Do
Kamke 827	✓	0.122	93	✓	0.273	49	To Do
Kamke 828	✓	0.384	46	✓	0.353	54	To Do
Kamke 829	✓	0.384	81	✓	0.491	40	To Do
Kamke 830	✓	0.525	34	✓	0.153	38	To Do
Kamke 831	✓	4.461	79	✓	0.376	35	To Do
Kamke 832	✓	3.264	2323	✓	0.214	31	To Do
Kamke 833	✓	0.127	87	✓	0.22	49	To Do
Kamke 834	✓	0.741	90	✓	0.46	60	To Do
Kamke 835	✗	0	0	✗	0	0	To Do
Kamke 836	✓	16.226	379	✓	0.261	73	To Do
Kamke 837	✗	0	0	✗	0	0	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 838	✓	0.028	29	✓	0.119	25	To Do
Kamke 839	✓	0.072	28	✓	0.093	19	To Do
Kamke 840	✓	0.088	28	✓	0.082	19	To Do
Kamke 841	✓	1.356	208	✓	0.378	97	To Do
Kamke 842	✓	0.118	44	✓	0.029	43	To Do
Kamke 843	✓	0.124	44	✓	0.028	43	To Do
Kamke 844	✓	19.409	386	✓	0.286	97	To Do
Kamke 845	✓	4.975	218	✓	0.301	44	To Do
Kamke 846	✓	1.404	174	✓	0.221	40	To Do
Kamke 847	✓	0.357	69	✓	0.323	34	To Do
Kamke 848	✓	0.125	91	✓	0.741	27	To Do
Kamke 849	✓	0.319	73	✓	0.302	33	To Do
Kamke 850	✓	0.253	114	✓	1.39	32	To Do
Kamke 851	✓	0.485	1	✓	0.076	42	To Do
Kamke 852	✓	0.534	1	✓	0.079	42	To Do
Kamke 853	✓	0.019	75	✓	0.044	63	To Do
Kamke 854	✗	0	0	✓	0.277	51	To Do
Kamke 855	✗	0	0	✓	0.271	51	To Do
Kamke 856	✓	0.933	100	✓	0.359	65	To Do
Kamke 857	✓	0.342	77	✓	0.315	32	To Do
Kamke 858	✓	0.509	1	✓	0.075	42	To Do
Kamke 859	✓	1.323	102	✓	0.355	63	To Do
Kamke 860	✓	0.148	35	✓	2.612	29	To Do
Kamke 861	✓	1.841	137	✓	0.205	26	To Do
Kamke 862	✗	0	0	✓	0.253	27	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 863	✓	0.035	26	✓	7.155	38	To Do
Kamke 864	✓	0.043	98	✓	0.12	162	To Do
Kamke 865	✗	0	0	✓	0.276	23	To Do
Kamke 866	✓	0.482	85	✓	0.565	37	To Do
Kamke 867	✓	0.092	1	✓	0.073	30	To Do
Kamke 868	✓	0.076	1	✓	0.075	28	To Do
Kamke 869	✓	0.036	42	✓	0.083	37	To Do
Kamke 870	✓	1.657	32	✓	1.234	30	To Do
Kamke 871	✓	0.022	22	✓	0.089	26	To Do
Kamke 872	✓	0.044	92	✓	0.072	49	To Do
Kamke 873	✓	0.64	48	✓	0.29	50	To Do
Kamke 874	✓	0.199	1	✓	0.064	40	To Do
Kamke 875	✓	0.295	213	✓	0.332	73	To Do
Kamke 876	✓	0.019	89	✓	0.05	41	To Do
Kamke 877	✓	0.017	47	✓	0.052	73	To Do
Kamke 878	✓	0.3	1	✗	0	0	To Do
Kamke 879	✓	0.157	109	✓	0.282	55	To Do
Kamke 880	✓	0.171	10	✓	0.07	41	To Do
Kamke 881	✓	0.019	57	✓	0.054	77	To Do
Kamke 882	✓	0.246	1	✓	0.072	41	To Do
Kamke 883	✓	1.465	1	✓	0.848	352	To Do
Kamke 884	✓	0.563	71	✓	0.423	107	To Do
Kamke 885	✗	0	0	✗	0	0	To Do
Kamke 886	✓	0.099	1	✓	0.042	42	To Do
Kamke 887	✓	0.025	91	✓	0.058	72	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 888	✓	0.019	67	✓	0.059	79	To Do
Kamke 889	✗	0	0	✓	1.447	49	To Do
Kamke 890	✓	0.161	7	✓	1.19	34	To Do
Kamke 891	✓	0.024	86	✓	0.072	56	To Do
Kamke 892	✗	0	0	✓	0.645	40	To Do
Kamke 893	✓	0.098	1	✓	0.039	41	To Do
Kamke 894	✗	0	0	✗	0	0	To Do
Kamke 895	✓	0.024	63	✓	0.064	79	To Do
Kamke 896	✓	0.246	1	✓	0.928	63	To Do
Kamke 897	✓	0.025	65	✓	0.093	87	To Do
Kamke 898	✓	0.021	95	✓	0.056	87	To Do
Kamke 899	✓	0.389	1	✓	0.049	47	To Do
Kamke 900	✓	0.092	381	✓	0.082	48	To Do
Kamke 901	✓	0.221	29	✓	0.675	30	To Do
Kamke 902	✓	0.093	195	✓	0.297	183	To Do
Kamke 903	✓	0.052	19	✓	0.081	48	To Do
Kamke 904	✓	0.05	23	✓	0.056	64	To Do
Kamke 905	✓	0.115	1	✓	0.056	46	To Do
Kamke 906	✓	0.049	326	✓	0.421	37	To Do
Kamke 907	✓	0.05	21	✓	0.178	20	To Do
Kamke 908	✓	1.29	1023	✓	0.484	1742	To Do
Kamke 909	✗	0	0	✗	0	0	To Do
Kamke 910	✓	0.181	1	✓	0.043	42	To Do
Kamke 911	✓	3.558	53	✓	0.681	30	To Do
Kamke 912	✓	1.399	199	✗	0	0	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 913	✗	0	0	✓	3.729	43	To Do
Kamke 914	✓	1.459	401	✓	4.	71	To Do
Kamke 915	✗	0	0	✓	0.072	43	To Do
Kamke 916	✗	0	0	✓	1.471	73	To Do
Kamke 917	✗	0	0	✓	0.398	38	To Do
Kamke 918	✗	0	0	✓	2.477	41	To Do
Kamke 919	✗	0	0	✓	0.316	82	To Do
Kamke 920	✓	0.235	301	✗	0	0	To Do
Kamke 921	✓	2.785	47	✓	0.367	30	To Do
Kamke 922	✗	0	0	✓	0.885	47	To Do
Kamke 923	✗	0	0	✓	0.526	36	To Do
Kamke 924	✓	0.993	53	✓	0.144	46	To Do
Kamke 925	✗	0	0	✓	0.29	38	To Do
Kamke 926	✓	0.026	77	✓	0.116	67	To Do
Kamke 927	✓	0.353	1	✓	0.142	68	To Do
Kamke 928	✓	1.368	22	✓	0.517	20	To Do
Kamke 929	✗	0	0	✓	0.06	42	To Do
Kamke 930	✓	1.575	38	✓	0.679	36	To Do
Kamke 931	✓	0.022	68	✓	0.049	73	To Do
Kamke 932	✗	0	0	✓	0.396	54	To Do
Kamke 933	✓	0.133	93	✓	0.043	39	To Do
Kamke 934	✓	0.152	1	✓	0.082	39	To Do
Kamke 935	✓	20.048	117	✓	0.316	55	To Do
Kamke 936	✓	0.144	1	✓	0.081	39	To Do
Kamke 937	✓	0.027	83	✓	0.066	79	To Do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 938	✓	0.289	101	✓	0.036	39	To Do
Kamke 939	✓	0.683	137	✓	0.184	70	To Do
Kamke 940	✓	0.023	75	✓	0.056	63	To Do
Kamke 941	✓	0.382	1	✓	0.063	35	To Do
Kamke 942	✗	0	0	✓	0.63	43	To Do
Kamke 943	✓	0.428	1	✓	0.057	40	To Do
Kamke 944	✓	1.92	1	✓	0.091	47	To Do
Kamke 945	✓	1.502	1	✓	0.08	41	To Do
Kamke 946	✓	0.091	115	✓	0.187	85	To Do
Kamke 947	✓	0.115	28	✓	0.39	44	To Do
Kamke 948	✓	0.378	39	✓	0.313	68	To Do
Kamke 949	✓	0.024	55	✓	0.048	81	To Do
Kamke 950	✓	0.282	1	✓	0.104	42	To Do
Kamke 951	✓	0.25	1	✓	0.084	41	To Do
Kamke 952	✓	0.169	141	✓	0.67	62	To Do
Kamke 953	✗	0	0	✓	0.454	145	To Do
Kamke 954	✓	0.342	108	✓	0.093	53	To Do
Kamke 955	✓	0.04	73	✓	0.116	101	To Do
Kamke 956	✓	0.23	28	✓	0.113	79	To Do
Kamke 957	✓	0.207	28	✓	0.057	79	To Do
Kamke 958	✓	0.097	80	✓	0.05	40	To Do
Kamke 959	✓	0.05	20	✓	0.09	16	To Do
Kamke 960	✓	0.037	14	✓	0.049	16	To Do
Kamke 961	✗	0	0	✓	0.451	45	To Do
Kamke 962	✓	5.403	1191	✓	1.996	79	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 963	✓	0.58	101	✓	0.26	39	To Do
Kamke 964	✓	4.883	13	✓	3.314	80	To Do
Kamke 965	✓	0.06	29	✓	0.126	25	To Do
Kamke 966	✓	0.737	1	✓	0.829	50	To Do
Kamke 967	✓	101.916	145	✓	0.099	91	To Do
Kamke 968	✓	0.085	30	✓	0.158	29	To Do
Kamke 969	✓	0.058	19	✓	0.155	26	To Do
Kamke 970	✓	0.53	66	✓	1.079	181	To Do
Kamke 971	✗	0	0	✓	0.35	86	To Do
Kamke 972	✓	0.028	32	✓	0.096	27	To Do
Kamke 973	✓	1.039	1	✓	0.451	134	To Do
Kamke 974	✓	0.011	39	✓	0.05	57	To Do
Kamke 975	✓	0.013	47	✓	0.046	59	To Do
Kamke 976	✓	0.226	1	✓	0.872	57	To Do
Kamke 977	✓	1.176	1	✓	0.217	122	To Do
Kamke 978	✓	0.189	58	✓	0.191	71	To Do
Kamke 979	✓	0.013	37	✓	0.046	57	To Do
Kamke 980	✓	0.013	44	✓	0.019	35	To Do
Kamke 981	✓	0.018	49	✓	0.028	41	To Do
Kamke 982	✓	0.646	1	✓	0.548	145	To Do
Kamke 983	✗	0	0	✓	0.409	233	To Do
Kamke 984	✗	0	0	✓	0.322	40	To Do
Kamke 985	✓	0.427	1	✓	0.046	43	To Do
Kamke 986	✓	0.017	41	✓	0.028	36	To Do
Kamke 987	✓	0.099	34	✓	0.075	22	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 988	✓	0.378	77	✓	0.052	29	To Do
Kamke 989	✓	0.103	44	✓	0.038	29	To Do
Kamke 990	✓	0.467	48	✓	0.743	44	To Do
Kamke 991	✓	0.248	75	✓	0.033	29	To Do
Kamke 992	✓	0.108	36	✓	0.038	25	To Do
Kamke 993	✗	0	0	✓	0.033	35	To Do
Kamke 994	✓	0.138	44	✓	0.023	43	To Do
Kamke 995	✓	0.018	17	✓	0.146	14	To Do
Kamke 996	✗	0	0	✓	0.089	15	To Do
Kamke 997	✓	0.03	18	✓	0.066	16	To Do
Kamke 998	✓	0.429	26	✓	0.498	27	To Do
Kamke 999	✓	0.025	24	✓	0.063	36	To Do
Kamke 1000	✗	0	0	✓	0.187	19	To Do
Kamke 1001	✓	0.004	12	✓	0.022	9	Linear second order, homogeneous, constant coefficient. First problem. Direct integration
Kamke 1002	✓	0.005	16	✓	0.005	13	Linear second order, homogeneous, constant coefficient. Direct method
Kamke 1003	✓	0.116	40	✓	0.078	26	Linear second order, Non-homogeneous, constant coefficient. variation of parameters
Kamke 1004	✓	0.1	41	✓	0.049	27	Linear second order, Non-homogeneous, constant coefficient. variation of parameters

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1005	✓	0.528	159	✓	0.123	82	Linear second order, Non-homogeneous, constant coefficient. variation of parameters
Kamke 1006	✓	0.005	20	✓	0.013	15	Linear second order, homogeneous, constant coefficient
Kamke 1007	✓	0.069	36	✓	0.016	26	Linear second order, Non-homogeneous, constant coefficient. variation of parameters
Kamke 1008	✓	0.042	46	✓	0.081	41	To Do
Kamke 1009	✓	0.005	28	✓	0.016	21	To Do
Kamke 1010	✓	0.008	42	✓	0.039	31	Linear second order, homogeneous, variable coefficient. Airy ODE with plus sign, series solution
Kamke 1011	✓	0.008	33	✓	0.032	17	Linear second order, homogeneous, variable coefficient. power series solution
Kamke 1012	✓	0.009	47	✓	0.095	29	To Do
Kamke 1013	✓	0.021	43	✓	0.039	22	To Do
Kamke 1014	✓	0.044	119	✓	0.086	63	To Do
Kamke 1015	✗	0	0	✗	0	0	To Do
Kamke 1016	✓	0.138	225	✓	0.204	91	To Do
Kamke 1017	✓	0.029	46	✓	0.043	17	To Do
Kamke 1018	✓	0.023	55	✓	0.042	39	To Do
Kamke 1019	✗	0	0	✗	0	0	To Do
Kamke 1020	✓	0.686	136	✓	0.186	58	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1021	✓	0.05	40	✓	0.214	39	To Do
Kamke 1022	✓	0.028	28	✓	0.201	21	To Do
Kamke 1023	✓	0.016	40	✓	0.193	29	To Do
Kamke 1024	✓	0.162	54	✓	0.139	30	To Do
Kamke 1025	✓	0.958	158	✓	0.204	102	To Do
Kamke 1026	✗	0	0	✗	0	0	To Do
Kamke 1027	✗	0	0	✓	0.349	69	To Do
Kamke 1028	✗	0	0	✗	0	0	To Do
Kamke 1029	✗	0	0	✓	0.128	22	To Do
Kamke 1030	✗	0	0	✗	0	0	To Do
Kamke 1031	✗	0	0	✗	0	0	To Do
Kamke 1032	✗	0	0	✓	0.112	48	To Do
Kamke 1033	✓	0.018	37	✓	0.01	27	Linear second order, homogeneous, variable coefficient. smart substitution turns it to second order with constant coefficient
Kamke 1034	✓	0.013	20	✓	0.008	15	Linear second order, homogeneous, variable coefficient. smart substitution turns it to second order with constant coefficient
Kamke 1035	✓	0.006	47	✓	0.011	41	To Do
Kamke 1036	✓	0.5	150	✓	0.093	124	To Do
Kamke 1037	✓	0.03	74	✓	0.102	64	To Do
Kamke 1038	✗	0	0	✗	0	0	To Do
Kamke 1039	✓	0.014	41	✓	0.013	25	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1040	✓	0.031	45	✓	0.042	33	To Do
Kamke 1041	✓	0.01	47	✓	0.072	41	To Do
Kamke 1042	✓	0.009	53	✓	0.065	41	To Do
Kamke 1043	✓	0.048	55	✓	0.211	39	Linear second order, homogeneous, variable coefficient. power series solution
Kamke 1044	✓	0.009	39	✓	0.067	35	To Do
Kamke 1045	✓	0.023	39	✓	0.008	21	To Do
Kamke 1046	✓	0.008	31	✓	0.067	31	To Do
Kamke 1047	✓	0.016	20	✓	0.03	16	Linear second order, homogeneous, variable coefficient. power series solution
Kamke 1048	✓	0.012	37	✓	0.069	37	To Do
Kamke 1049	✓	0.07	105	✓	0.167	66	To Do
Kamke 1050	✓	0.014	18	✓	0.028	14	To Do
Kamke 1051	✓	0.038	34	✓	0.037	27	To Do
Kamke 1052	✓	0.022	67	✓	0.082	58	To Do
Kamke 1053	✓	0.032	56	✓	0.046	35	To Do
Kamke 1054	✓	0.051	132	✓	0.049	98	To Do
Kamke 1055	✓	0.157	305	✓	0.233	262	To Do
Kamke 1056	✓	0.039	51	✓	0.108	48	To Do
Kamke 1057	✓	0.877	43	✓	0.132	50	To Do
Kamke 1058	✓	0.894	43	✓	0.117	29	To Do
Kamke 1059	✓	0.056	49	✓	0.088	56	To Do
Kamke 1060	✓	0.038	81	✓	0.155	81	To Do
Kamke 1061	✓	0.096	45	✓	0.088	28	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1062	✓	0.03	30	✓	0.024	19	To Do
Kamke 1063	✓	0.048	28	✓	0.36	61	To Do
Kamke 1064	✓	0.56	502	✓	0.296	125	To Do
Kamke 1065	✓	0.162	83	✓	0.237	60	To Do
Kamke 1066	✓	0.034	18	✓	0.067	15	To Do
Kamke 1067	✓	0.034	21	✓	0.046	17	To Do
Kamke 1068	✓	0.14	20	✓	0.223	45	To Do
Kamke 1069	✓	0.037	19	✓	0.089	15	To Do
Kamke 1070	✓	0.32	129	✓	0.174	60	To Do
Kamke 1071	✓	0.107	58	✓	0.047	24	To Do
Kamke 1072	✗	0	0	✗	0	0	To Do
Kamke 1073	✗	0	0	✗	0	0	To Do
Kamke 1074	✗	0	0	✓	0.017	21	To Do
Kamke 1075	✗	0	0	✗	0	0	To Do
Kamke 1076	✗	0	0	✗	0	0	To Do
Kamke 1077	✗	0	0	✗	0	0	To Do
Kamke 1078	✓	0.072	69	✓	0.031	33	To Do
Kamke 1079	✓	0.246	135	✓	0.018	37	To Do
Kamke 1080	✗	0	0	✓	0.329	74	To Do
Kamke 1081	✗	0	0	✗	0	0	To Do
Kamke 1082	✗	0	0	✓	0.131	74	To Do
Kamke 1083	✗	0	0	✓	0.106	31	To Do
Kamke 1084	✗	0	0	✓	0.083	20	To Do
Kamke 1085	✗	0	0	✓	0.079	24	To Do
Kamke 1086	✓	0.006	42	✓	0.024	29	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1087	✓	0.01	36	✓	0.076	33	To Do
Kamke 1088	✓	0.103	97	✓	0.14	31	To Do
Kamke 1089	✓	0.045	63	✓	0.052	58	To Do
Kamke 1090	✓	0.033	45	✓	0.043	40	To Do
Kamke 1091	✓	0.028	41	✓	0.043	35	To Do
Kamke 1092	✓	0.098	53	✓	0.052	29	To Do
Kamke 1093	✓	0.006	13	✓	0.006	10	To Do
Kamke 1094	✓	0.024	41	✓	0.01	29	To Do
Kamke 1095	✓	0.01	30	✓	0.032	23	To Do
Kamke 1096	✓	0.014	55	✓	0.066	39	To Do
Kamke 1097	✓	0.028	45	✓	0.013	31	To Do
Kamke 1098	✓	0.01	41	✓	0.01	27	To Do
Kamke 1099	✗	0	0	✓	0.044	25	To Do
Kamke 1100	✓	0.03	37	✓	0.039	23	Linear second order, non-homogeneous, variable coefficient. Singularity, Frobenius series solution and also using smart transformation. Variation of parameters.
Kamke 1101	✓	0.028	51	✓	0.033	29	Linear second order, homogeneous, variable coefficient. Singularity, Frobenius series solution and also using smart transformation

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1102	✓	0.007	36	✓	0.033	33	Linear second order, homogeneous, variable coefficient. Use smart transformation to convert to Emden-Fowler then use Power series solution
Kamke 1103	✓	0.028	56	✓	0.01	33	To Do
Kamke 1104	✓	0.037	77	✓	0.013	41	To Do
Kamke 1105	✓	0.023	54	✓	0.04	39	To Do
Kamke 1106	✓	0.053	165	✓	0.081	71	To Do
Kamke 1107	✓	0.031	36	✓	0.074	30	To Do
Kamke 1108	✓	0.037	33	✓	0.072	26	To Do
Kamke 1109	✓	0.056	45	✓	0.028	33	To Do
Kamke 1110	✓	0.042	36	✓	0.043	23	To Do
Kamke 1111	✓	0.019	19	✓	0.021	13	Linear second order, homogeneous, variable coefficient. Solved using Laplace transform instead of series method
Kamke 1112	✓	0.027	30	✓	0.029	22	To Do
Kamke 1113	✓	0.025	24	✓	0.058	17	To Do
Kamke 1114	✓	0.043	39	✓	0.043	34	To Do
Kamke 1115	✓	0.065	63	✓	0.091	47	To Do
Kamke 1116	✓	0.051	38	✓	0.081	31	To Do
Kamke 1117	✓	0.095	87	✓	0.11	82	To Do
Kamke 1118	✓	0.1	46	✓	0.099	39	To Do
Kamke 1119	✓	0.171	75	✓	0.041	20	To Do
Kamke 1120	✓	0.064	135	✓	0.216	109	To Do
Continued on next page							

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1121	✓	10.486	36	✓	0.024	23	To Do
Kamke 1122	✓	10.424	44	✓	0.151	28	To Do
Kamke 1123	✓	0.014	53	✓	0.053	45	To Do
Kamke 1124	✓	0.072	65	✓	0.079	29	To Do
Kamke 1125	✓	0.208	45	✓	0.05	36	To Do
Kamke 1126	✗	0	0	✓	0.04	19	To Do
Kamke 1127	✓	0.041	25	✓	0.021	21	To Do
Kamke 1128	✗	0	0	✓	0.223	32	To Do
Kamke 1129	✓	0.04	42	✓	0.034	30	To Do
Kamke 1130	✓	0.013	46	✓	0.013	31	To Do
Kamke 1131	✓	0.013	48	✓	0.075	33	To Do
Kamke 1132	✓	0.012	44	✓	0.072	29	To Do
Kamke 1133	✓	0.091	78	✓	0.099	37	To Do
Kamke 1134	✓	0.095	44	✓	0.044	21	To Do
Kamke 1135	✓	0.011	27	✓	0.01	17	To Do
Kamke 1136	✓	0.021	23	✓	0.031	16	To Do
Kamke 1137	✓	0.098	48	✓	0.05	25	To Do
Kamke 1138	✓	0.032	32	✓	0.069	26	To Do
Kamke 1139	✓	0.015	59	✓	0.074	37	To Do
Kamke 1140	✓	0.05	120	✓	0.023	66	To Do
Kamke 1141	✓	0.109	62	✓	0.054	55	To Do
Kamke 1142	✓	0.049	89	✓	0.099	53	To Do
Kamke 1143	✓	0.05	70	✓	0.101	57	To Do
Kamke 1144	✓	0.047	69	✓	0.083	60	To Do
Kamke 1145	✓	0.368	301	✓	0.2	248	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1146	✓	0.022	18	✓	0.007	15	To Do
Kamke 1147	✓	0.018	18	✓	0.008	15	To Do
Kamke 1148	✓	0.011	42	✓	0.012	35	To Do
Kamke 1149	✓	0.066	95	✓	0.014	45	To Do
Kamke 1150	✓	0.01	42	✓	0.067	27	To Do
Kamke 1151	✓	0.019	88	✓	0.09	43	To Do
Kamke 1152	✓	0.021	79	✓	0.243	53	To Do
Kamke 1153	✓	0.035	44	✓	0.029	31	To Do
Kamke 1154	✓	0.022	88	✓	0.122	57	To Do
Kamke 1155	✓	0.053	116	✓	0.04	67	To Do
Kamke 1156	✗	0	0	✓	0.136	71	To Do
Kamke 1157	✗	0	0	✗	0	0	To Do
Kamke 1158	✓	14.12	37	✓	0.224	178	To Do
Kamke 1159	✓	0.017	41	✓	0.016	19	To Do
Kamke 1160	✓	0.01	30	✓	0.013	23	To Do
Kamke 1161	✓	0.05	78	✓	0.012	31	To Do
Kamke 1162	✓	0.06	18	✓	0.01	15	To Do
Kamke 1163	✓	0.347	70	✓	0.033	49	To Do
Kamke 1164	✓	0.023	30	✓	0.025	23	To Do
Kamke 1165	✓	0.053	33	✓	0.014	19	To Do
Kamke 1166	✓	0.015	23	✓	0.013	21	To Do
Kamke 1167	✓	0.083	130	✓	0.026	63	To Do
Kamke 1168	✓	0.007	15	✓	0.009	11	To Do
Kamke 1169	✓	0.071	103	✓	0.025	49	To Do
Kamke 1170	✓	0.024	58	✓	0.036	43	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1171	✓	0.056	92	✓	0.122	49	To Do
Kamke 1172	✓	0.058	145	✓	0.03	47	To Do
Kamke 1173	✓	0.063	74	✓	0.076	37	To Do
Kamke 1174	✓	0.023	32	✓	0.021	25	To Do
Kamke 1175	✓	0.175	33	✓	0.052	29	To Do
Kamke 1176	✓	0.019	33	✓	0.029	15	To Do
Kamke 1177	✗	0	0	✓	0.071	34	To Do
Kamke 1178	✓	0.057	63	✓	0.027	23	To Do
Kamke 1179	✓	0.042	53	✓	0.03	19	To Do
Kamke 1180	✓	0.229	66	✓	0.047	49	To Do
Kamke 1181	✓	0.029	27	✓	0.017	25	To Do
Kamke 1182	✓	0.017	20	✓	0.013	20	To Do
Kamke 1183	✓	0.027	27	✓	0.016	22	To Do
Kamke 1184	✓	0.021	30	✓	0.027	25	To Do
Kamke 1185	✓	0.038	65	✓	0.027	33	To Do
Kamke 1186	✓	0.034	37	✓	0.024	36	To Do
Kamke 1187	✓	0.013	57	✓	0.012	53	To Do
Kamke 1188	✓	0.089	243	✓	0.155	114	To Do
Kamke 1189	✓	0.071	168	✓	0.028	79	To Do
Kamke 1190	✓	0.031	95	✓	0.09	38	To Do
Kamke 1191	✓	0.01	72	✓	0.015	23	To Do
Kamke 1192	✓	10.816	34	✓	0.132	51	To Do
Kamke 1193	✓	0.049	42	✓	0.036	38	To Do
Kamke 1194	✓	0.051	66	✓	0.055	48	To Do
Kamke 1195	✓	0.029	63	✓	0.12	93	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1196	✓	0.026	34	✓	0.035	31	To Do
Kamke 1197	✓	0.02	67	✓	0.041	43	To Do
Kamke 1198	✓	0.031	41	✓	0.039	37	To Do
Kamke 1199	✓	0.013	41	✓	0.02	35	To Do
Kamke 1200	✓	0.023	45	✓	0.016	27	To Do
Kamke 1201	✓	0.056	44	✓	0.02	34	To Do
Kamke 1202	✓	0.015	21	✓	0.031	14	To Do
Kamke 1203	✓	0.023	80	✓	0.02	28	To Do
Kamke 1204	✓	0.021	84	✓	0.049	35	To Do
Kamke 1205	✗	0	0	✗	0	0	To Do
Kamke 1206	✓	0.113	102	✓	0.069	76	To Do
Kamke 1207	✓	0.121	223	✓	0.221	110	To Do
Kamke 1208	✓	0.038	49	✓	0.043	35	To Do
Kamke 1209	✓	0.022	59	✓	0.055	41	To Do
Kamke 1210	✓	0.259	231	✓	0.679	81	To Do
Kamke 1211	✓	0.056	60	✓	0.056	36	To Do
Kamke 1212	✗	0	0	✗	0	0	To Do
Kamke 1213	✓	0.075	54	✓	0.053	53	To Do
Kamke 1214	✓	0.29	191	✓	0.62	71	To Do
Kamke 1215	✓	0.158	414	✓	0.176	148	To Do
Kamke 1216	✗	0	0	✗	0	0	To Do
Kamke 1217	✓	0.144	29	✓	0.037	24	To Do
Kamke 1218	✓	0.145	37	✓	0.038	30	To Do
Kamke 1219	✗	0	0	✓	0.082	69	To Do
Kamke 1220	✓	192.315	61	✓	0.02	40	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1221	✓	0.059	30	✓	0.026	35	To Do
Kamke 1222	✓	0.021	30	✓	0.013	23	To Do
Kamke 1223	✓	0.02	25	✓	0.014	39	To Do
Kamke 1224	✓	0.019	30	✓	0.016	23	To Do
Kamke 1225	✓	0.032	29	✓	0.037	23	To Do
Kamke 1226	✓	0.02	30	✓	0.045	25	To Do
Kamke 1227	✓	0.05	21	✓	0.014	16	To Do
Kamke 1228	✓	0.016	66	✓	0.086	53	To Do
Kamke 1229	✓	0.042	33	✓	0.017	31	To Do
Kamke 1230	✓	0.026	68	✓	0.106	36	To Do
Kamke 1231	✓	0.076	56	✓	0.049	52	To Do
Kamke 1232	✗	0	0	✓	0.19	409	To Do
Kamke 1233	✗	0	0	✓	0.154	409	To Do
Kamke 1234	✓	0.03	38	✗	0	0	To Do
Kamke 1235	✓	0.029	50	✓	0.021	45	To Do
Kamke 1236	✗	0	0	✗	0	0	To Do
Kamke 1237	✓	0.011	27	✓	0.019	20	To Do
Kamke 1238	✓	0.019	36	✓	0.018	26	To Do
Kamke 1239	✓	0.015	46	✓	0.049	35	To Do
Kamke 1240	✓	0.019	18	✓	0.04	15	To Do
Kamke 1241	✓	0.017	26	✓	0.048	24	To Do
Kamke 1242	✓	0.077	50	✓	0.077	41	To Do
Kamke 1243	✓	0.032	41	✓	0.04	21	To Do
Kamke 1244	✓	0.032	32	✓	0.071	27	To Do
Kamke 1245	✓	0.024	32	✓	0.053	27	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1246	✓	0.024	32	✓	0.158	28	To Do
Kamke 1247	✓	0.225	87	✓	0.029	27	To Do
Kamke 1248	✗	0	0	✓	0.221	134	To Do
Kamke 1249	✓	0.176	190	✓	0.078	134	To Do
Kamke 1250	✓	0.054	38	✓	0.03	41	To Do
Kamke 1251	✓	0.039	23	✓	0.052	20	To Do
Kamke 1252	✓	0.167	131	✓	0.056	124	To Do
Kamke 1253	✓	0.027	28	✓	0.007	16	To Do
Kamke 1254	✓	0.095	60	✓	0.084	42	To Do
Kamke 1255	✓	0.224	87	✓	0.026	42	To Do
Kamke 1256	✓	0.025	26	✓	0.113	51	To Do
Kamke 1257	✓	0.054	33	✓	0.23	27	To Do
Kamke 1258	✓	0.175	130	✓	0.054	110	To Do
Kamke 1259	✓	0.146	112	✓	0.059	92	To Do
Kamke 1260	✓	0.171	65	✓	0.566	76	To Do
Kamke 1261	✗	0	0	✓	0.234	105	To Do
Kamke 1262	✓	40.198	45	✓	0.263	53	To Do
Kamke 1263	✗	0	0	✓	0.086	52	To Do
Kamke 1264	✓	0.062	23	✓	0.045	19	To Do
Kamke 1265	✓	0.043	57	✓	0.131	93	To Do
Kamke 1266	✓	0.029	22	✓	0.009	19	To Do
Kamke 1267	✗	0	0	✓	0.072	41	To Do
Kamke 1268	✗	0	0	✓	0.111	39	To Do
Kamke 1269	✓	0.088	59	✓	0.095	40	To Do
Kamke 1270	✗	0	0	✓	0.185	46	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1271	✓	0.014	24	✓	0.007	14	To Do
Kamke 1272	✓	0.014	28	✓	0.036	23	To Do
Kamke 1273	✓	0.018	20	✓	0.074	17	To Do
Kamke 1274	✓	0.044	38	✓	0.01	19	To Do
Kamke 1275	✓	0.042	97	✓	0.159	53	To Do
Kamke 1276	✓	0.061	55	✓	0.066	31	To Do
Kamke 1277	✓	0.029	49	✓	0.056	27	To Do
Kamke 1278	✗	0	0	✗	0	0	To Do
Kamke 1279	✓	0.178	45	✓	0.167	32	To Do
Kamke 1280	✓	0.041	52	✓	0.055	40	To Do
Kamke 1281	✓	0.022	21	✓	0.046	15	To Do
Kamke 1282	✓	0.024	32	✓	0.046	21	To Do
Kamke 1283	✓	0.108	89	✓	0.138	48	To Do
Kamke 1284	✓	0.046	41	✓	0.022	41	To Do
Kamke 1285	✓	0.328	119	✓	0.102	52	To Do
Kamke 1286	✓	0.108	80	✓	0.022	32	To Do
Kamke 1287	✓	0.021	57	✓	0.027	27	To Do
Kamke 1288	✓	0.039	43	✓	0.026	21	To Do
Kamke 1289	✓	0.085	51	✓	0.085	33	To Do
Kamke 1290	✓	0.174	103	✓	0.02	47	To Do
Kamke 1291	✓	0.076	82	✓	0.058	62	To Do
Kamke 1292	✓	0.041	53	✓	0.06	31	To Do
Kamke 1293	✓	0.339	44	✓	0.073	33	To Do
Kamke 1294	✓	0.07	44	✓	0.072	33	To Do
Kamke 1295	✓	0.278	229	✓	0.302	106	To Do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1296	✓	0.55	272	✓	0.347	150	To Do
Kamke 1297	✓	0.034	52	✓	0.023	63	To Do
Kamke 1298	✓	0.078	135	✓	0.125	124	To Do
Kamke 1299	✓	0.015	19	✓	0.019	27	To Do
Kamke 1300	✓	0.017	39	✓	0.054	31	To Do
Kamke 1301	✓	0.033	30	✓	0.023	19	To Do
Kamke 1302	✓	0.084	165	✓	0.061	98	To Do
Kamke 1303	✗	0	0	✓	0.225	501	To Do
Kamke 1304	✓	0.035	50	✓	0.036	38	To Do
Kamke 1305	✓	0.074	47	✓	0.055	44	To Do
Kamke 1306	✗	0	0	✓	0.158	69	To Do
Kamke 1307	✓	0.087	44	✓	0.058	36	To Do
Kamke 1308	✓	0.022	41	✓	0.019	40	To Do
Kamke 1309	✓	0.082	83	✓	0.094	85	To Do
Kamke 1310	✓	0.013	27	✓	0.013	20	To Do
Kamke 1311	✓	0.132	61	✓	0.15	52	To Do
Kamke 1312	✓	0.025	26	✓	0.021	19	To Do
Kamke 1313	✓	0.213	75	✓	0.095	35	To Do
Kamke 1314	✓	0.19	75	✓	0.083	33	To Do
Kamke 1315	✓	0.029	44	✓	0.02	45	To Do
Kamke 1316	✓	0.093	38	✓	0.041	18	To Do
Kamke 1317	✓	0.124	38	✓	0.039	13	To Do
Kamke 1318	✓	0.32	146	✓	0.141	122	To Do
Kamke 1319	✓	0.099	100	✓	0.116	31	To Do
Kamke 1320	✓	0.069	21	✓	0.047	17	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1321	✓	0.028	17	✓	0.03	15	To Do
Kamke 1322	✓	0.035	44	✓	0.03	44	To Do
Kamke 1323	✗	0	0	✓	0.02	17	To Do
Kamke 1324	✓	0.031	24	✓	0.032	18	To Do
Kamke 1325	✓	0.267	52	✓	0.137	86	To Do
Kamke 1326	✓	0.027	26	✓	0.03	22	To Do
Kamke 1327	✓	0.172	105	✓	0.127	81	To Do
Kamke 1328	✓	0.025	33	✓	0.03	27	To Do
Kamke 1329	✗	0	0	✓	0.377	64	To Do
Kamke 1330	✗	0	0	✓	1.211	1147	To Do
Kamke 1331	✓	0.045	41	✓	0.026	19	To Do
Kamke 1332	✓	0.026	21	✓	0.021	17	To Do
Kamke 1333	✓	0.115	70	✓	0.078	45	To Do
Kamke 1334	✓	0.205	89	✓	0.1	89	To Do
Kamke 1335	✓	0.315	510	✓	0.055	57	To Do
Kamke 1336	✓	0.055	51	✓	0.054	44	To Do
Kamke 1337	✓	0.085	53	✓	0.035	27	To Do
Kamke 1338	✓	0.071	40	✓	0.037	27	To Do
Kamke 1339	✓	0.272	66	✓	0.142	76	To Do
Kamke 1340	✓	0.038	23	✓	0.033	20	To Do
Kamke 1341	✗	0	0	✓	0.182	201	To Do
Kamke 1342	✓	0.044	51	✓	0.027	31	To Do
Kamke 1343	✗	0	0	✓	0.108	58	To Do
Kamke 1344	✓	0.589	100	✓	0.048	23	To Do
Kamke 1345	✓	0.032	45	✓	0.049	25	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1346	✓	0.346	75	✓	0.054	25	To Do
Kamke 1347	✓	0.088	31	✓	0.029	19	To Do
Kamke 1348	✗	0	0	✓	0.207	73	To Do
Kamke 1349	✓	0.099	73	✓	0.075	85	To Do
Kamke 1350	✓	0.011	25	✓	0.008	21	To Do
Kamke 1351	✓	0.029	44	✓	0.033	24	To Do
Kamke 1352	✓	0.015	51	✓	0.043	43	To Do
Kamke 1353	✓	0.111	79	✓	0.211	66	To Do
Kamke 1354	✓	0.074	81	✓	0.256	33	To Do
Kamke 1355	✓	0.131	57	✓	0.142	30	To Do
Kamke 1356	✓	0.31	78	✓	0.084	29	To Do
Kamke 1357	✓	0.681	264	✓	0.128	97	To Do
Kamke 1358	✓	0.069	45	✓	0.046	20	To Do
Kamke 1359	✓	0.119	84	✓	0.098	57	To Do
Kamke 1360	✓	0.102	68	✓	0.079	47	To Do
Kamke 1361	✓	0.508	36	✓	0.035	33	To Do
Kamke 1362	✗	0	0	✓	0.252	109	To Do
Kamke 1363	✓	0.807	211	✓	0.153	161	To Do
Kamke 1364	✓	0.17	29	✓	0.114	25	To Do
Kamke 1365	✓	0.098	66	✓	0.064	59	To Do
Kamke 1366	✓	0.025	22	✓	0.01	17	To Do
Kamke 1367	✗	0	0	✓	0.26	88	To Do
Kamke 1368	✓	0.029	92	✓	0.078	71	To Do
Kamke 1369	✓	0.107	67	✓	0.055	55	To Do
Kamke 1370	✓	0.031	53	✓	0.011	19	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1371	✓	0.024	48	✓	0.054	37	To Do
Kamke 1372	✗	0	0	✓	0.297	110	To Do
Kamke 1373	✗	0	0	✓	0.244	84	To Do
Kamke 1374	✓	0.038	26	✓	0.058	23	To Do
Kamke 1375	✓	0.056	34	✓	0.081	29	To Do
Kamke 1376	✓	0.104	82	✓	0.031	73	To Do
Kamke 1377	✓	0.245	97	✓	0.132	83	To Do
Kamke 1378	✓	0.059	56	✓	0.052	48	To Do
Kamke 1379	✓	0.083	72	✓	0.069	59	To Do
Kamke 1380	✓	0.312	121	✓	0.089	67	To Do
Kamke 1381	✓	0.75	371	✓	0.177	175	To Do
Kamke 1382	✓	0.72	141	✓	0.121	104	To Do
Kamke 1383	✓	0.144	44	✓	0.052	39	To Do
Kamke 1384	✓	0.036	106	✓	0.27	73	To Do
Kamke 1385	✓	0.021	70	✓	0.066	55	To Do
Kamke 1386	✓	0.096	68	✓	0.072	58	To Do
Kamke 1387	✓	0.042	45	✓	0.032	28	To Do
Kamke 1388	✓	0.308	109	✓	0.083	76	To Do
Kamke 1389	✓	0.394	91	✓	0.081	68	To Do
Kamke 1390	✓	0.043	49	✓	0.045	25	To Do
Kamke 1391	✓	0.062	27	✓	0.028	20	To Do
Kamke 1392	✓	92.707	1	✓	0.237	561	To Do
Kamke 1393	✓	19.122	1	✓	0.161	272	To Do
Kamke 1394	✓	0.059	73	✓	0.12	79	To Do
Kamke 1395	✓	0.169	76	✓	0.06	39	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1396	✓	1.437	199	✓	0.161	178	To Do
Kamke 1397	✓	0.031	38	✓	0.089	27	To Do
Kamke 1398	✗	0	0	✓	0.189	69	To Do
Kamke 1399	✓	0.056	51	✓	0.085	34	To Do
Kamke 1400	✓	0.053	56	✓	0.048	35	To Do
Kamke 1401	✓	0.016	56	✓	0.054	45	To Do
Kamke 1402	✗	0	0	✓	0.305	58	To Do
Kamke 1403	✗	0	0	✓	1.052	298	To Do
Kamke 1404	✓	0.026	25	✓	0.048	19	To Do
Kamke 1405	✓	0.078	70	✓	0.074	42	To Do
Kamke 1406	✗	0	0	✓	0.158	44	To Do
Kamke 1407	✗	0	0	✓	2.766	2597	To Do
Kamke 1408	✗	0	0	✗	0	0	To Do
Kamke 1409	✓	0.026	44	✓	0.016	39	To Do
Kamke 1410	✓	0.137	405	✓	0.296	253	To Do
Kamke 1411	✓	0.337	36	✓	0.021	27	To Do
Kamke 1412	✓	0.019	29	✓	0.009	23	To Do
Kamke 1413	✗	0	0	✓	0.05	12	To Do
Kamke 1414	✓	1.175	127	✓	0.267	97	To Do
Kamke 1415	✓	0.858	145	✓	0.145	36	To Do
Kamke 1416	✓	0.195	35	✓	0.183	26	To Do
Kamke 1417	✓	0.149	45	✓	0.148	31	To Do
Kamke 1418	✗	0	0	✓	15.536	59	To Do
Kamke 1419	✗	0	0	✓	0.239	12	To Do
Kamke 1420	✓	0.453	126	✓	0.325	123	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1421	✓	0.252	62	✓	0.076	27	To Do
Kamke 1422	✓	0.089	54	✓	0.254	50	To Do
Kamke 1423	✓	0.074	61	✓	0.28	132	To Do
Kamke 1424	✓	0.172	65	✓	0.289	120	To Do
Kamke 1425	✓	0.741	194	✓	0.47	91	To Do
Kamke 1426	✓	6.121	1362	✓	0.643	549	To Do
Kamke 1427	✗	0	0	✓	1.68	179	To Do
Kamke 1428	✓	0.428	87	✓	0.323	183	To Do
Kamke 1429	✓	0.059	51	✓	0.036	25	To Do
Kamke 1430	✓	0.467	22	✓	0.366	101	To Do
Kamke 1431	✓	0.201	64	✓	0.233	30	To Do
Kamke 1432	✓	0.1	33	✓	0.045	22	To Do
Kamke 1433	✓	0.242	35	✓	0.116	28	To Do
Kamke 1434	✓	107.774	1	✓	0.68	517	To Do
Kamke 1435	✓	0.15	61	✓	0.171	38	To Do
Kamke 1436	✓	0.548	33	✓	0.281	113	To Do
Kamke 1437	✓	0.285	36	✓	0.177	29	To Do
Kamke 1438	✓	0.977	158	✓	0.181	102	To Do
Kamke 1439	✗	0	0	✗	0	0	To Do
Kamke 1440	✗	0	0	✗	0	0	To Do
Kamke 1441	✗	0	0	✗	0	0	To Do
Kamke 1442	✗	0	0	✓	0.07	30	To Do
Kamke 1443	✗	0	0	✗	0	0	To Do
Kamke 1444	✗	0	0	✓	0.015	37	To Do
Kamke 1445	✗	0	0	✓	0.279	20	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1446	✓	0.026	26	✓	0.046	22	To Do
Kamke 1447	✓	0.023	24	✓	0.06	20	To Do
Kamke 1448	✓	0.339	142	✓	0.103	77	To Do
Kamke 1449	✓	0.02	53	✓	0.013	47	To Do
Kamke 1450	✗	0	0	✓	0.233	1616	To Do
Kamke 1451	✓	0.022	164	✓	0.092	114	To Do
Kamke 1452	✓	0.008	52	✓	0.008	35	To Do
Kamke 1453	✓	0.656	128	✓	0.138	122	To Do
Kamke 1454	✓	0.011	79	✓	0.056	55	To Do
Kamke 1455	✓	0.029	127	✓	0.145	71	To Do
Kamke 1456	✓	0.039	183	✓	0.066	73	To Do
Kamke 1457	✗	0	0	✗	0	0	To Do
Kamke 1458	✗	0	0	✗	0	0	To Do
Kamke 1459	✗	0	0	✗	0	0	To Do
Kamke 1460	✗	0	0	✗	0	0	To Do
Kamke 1461	✗	0	0	✗	0	0	To Do
Kamke 1462	✗	0	0	✗	0	0	To Do
Kamke 1463	✗	0	0	✗	0	0	To Do
Kamke 1464	✓	0.008	34	✓	0.006	27	To Do
Kamke 1465	✓	0.093	52	✓	0.077	214	To Do
Kamke 1466	✓	0.018	34	✓	0.02	27	To Do
Kamke 1467	✓	0.007	84	✓	0.022	590	To Do
Kamke 1468	✓	0.091	57	✓	0.082	59	To Do
Kamke 1469	✓	0.032	68	✓	0.026	37	To Do
Kamke 1470	✗	0	0	✓	0.087	36	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1471	✗	0	0	✓	0.204	36	To Do
Kamke 1472	✗	0	0	✓	0.285	33	To Do
Kamke 1473	✗	0	0	✗	0	0	To Do
Kamke 1474	✗	0	0	✗	0	0	To Do
Kamke 1475	✓	0.028	37	✓	0.021	23	To Do
Kamke 1476	✗	0	0	✗	0	0	To Do
Kamke 1477	✓	0.179	43	✓	0.02	41	To Do
Kamke 1478	✓	0.036	90	✓	0.127	48	To Do
Kamke 1479	✓	0.153	153	✓	0.244	92	To Do
Kamke 1480	✓	0.227	91	✓	0.267	35	To Do
Kamke 1481	✓	1.037	378	✓	0.053	44	To Do
Kamke 1482	✗	0	0	✓	0.328	1616	To Do
Kamke 1483	✓	0.15	105	✓	0.273	37	To Do
Kamke 1484	✗	0	0	✗	0	0	To Do
Kamke 1485	✓	0.123	65	✓	0.379	51	To Do
Kamke 1486	✓	0.174	63	✓	0.25	51	To Do
Kamke 1487	✗	0	0	✓	0.086	38	To Do
Kamke 1488	✓	0.562	97	✓	0.553	132	To Do
Kamke 1489	✗	0	0	✗	0	0	To Do
Kamke 1490	✓	0.02	33	✓	0.064	18	To Do
Kamke 1491	✓	0.046	102	✓	0.09	88	To Do
Kamke 1492	✓	0.414	43	✓	0.114	39	To Do
Kamke 1493	✓	7.336	872	✓	0.323	1033	To Do
Kamke 1494	✓	0.031	43	✓	0.022	32	To Do
Kamke 1495	✓	0.021	24	✓	0.013	16	To Do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1496	✓	0.279	58	✓	0.028	57	To Do
Kamke 1497	✓	0.498	127	✓	0.277	77	To Do
Kamke 1498	✓	8.515	378	✓	0.267	53	To Do
Kamke 1499	✓	0.244	91	✓	0.277	25	To Do
Kamke 1500	✗	0	0	✓	0.227	55	To Do
Kamke 1501	✓	0.192	80	✓	0.246	37	To Do
Kamke 1502	✓	0.057	98	✓	0.49	103	To Do
Kamke 1503	✓	0.116	62	✓	0.029	67	To Do
Kamke 1504	✓	0.109	41	✓	0.216	18	To Do
Kamke 1505	✗	0	0	✓	0.157	79	To Do
Kamke 1506	✗	0	0	✓	0.06	43	To Do
Kamke 1507	✗	0	0	✓	0.651	1210	To Do
Kamke 1508	✓	0.852	136	✓	0.153	81	To Do
Kamke 1509	✓	0.011	33	✓	0.057	29	To Do
Kamke 1510	✗	0	0	✗	0	0	To Do
Kamke 1511	✓	0.038	52	✓	0.033	49	To Do
Kamke 1512	✓	0.039	29	✓	0.013	18	To Do
Kamke 1513	✓	0.078	23	✓	0.226	18	To Do
Kamke 1514	✓	0.746	97	✓	0.546	132	To Do
Kamke 1515	✗	0	0	✗	0	0	To Do
Kamke 1516	✗	0	0	✓	0.477	188	To Do
Kamke 1517	✓	0.405	1656	✓	0.519	866	To Do
Kamke 1518	✓	0.245	96	✓	0.474	60	To Do
Kamke 1519	✓	0.032	58	✓	0.155	19	To Do
Kamke 1520	✗	0	0	✓	0.596	288	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1521	✓	0.079	29	✓	0.454	28	To Do
Kamke 1522	✓	0.021	42	✓	0.043	34	To Do
Kamke 1523	✓	0.121	46	✓	0.469	23	To Do
Kamke 1524	✓	0.153	96	✓	0.536	98	To Do
Kamke 1525	✓	0.448	101	✓	0.502	133	To Do
Kamke 1526	✗	0	0	✓	0.238	19	To Do
Kamke 1527	✗	0	0	✓	0.598	437	To Do
Kamke 1528	✓	0.624	56	✓	0.199	71	To Do
Kamke 1529	✗	0	0	✓	0.087	25	To Do
Kamke 1530	✗	0	0	✓	0.252	113	To Do
Kamke 1531	✗	0	0	✗	0	0	To Do
Kamke 1532	✓	0.017	103	✓	0.102	58	To Do
Kamke 1533	✓	0.019	106	✓	0.099	58	To Do
Kamke 1534	✓	0.005	22	✓	0.009	21	To Do
Kamke 1535	✓	1.312	168	✓	0.019	36	To Do
Kamke 1536	✓	0.006	76	✓	0.013	50	To Do
Kamke 1537	✓	0.933	93	✓	0.147	67	To Do
Kamke 1538	✓	0.26	47	✓	0.523	51	To Do
Kamke 1539	✓	0.008	44	✓	0.027	35	To Do
Kamke 1540	✗	0	0	✗	0	0	To Do
Kamke 1541	✗	0	0	✗	0	0	To Do
Kamke 1542	✗	0	0	✗	0	0	To Do
Kamke 1543	✗	0	0	✗	0	0	To Do
Kamke 1544	✗	0	0	✓	0.017	41	To Do
Kamke 1545	✓	0.178	40	✓	0.137	27	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1546	✓	0.69	148	✓	0.053	73	To Do
Kamke 1547	✗	0	0	✓	0.023	87	To Do
Kamke 1548	✓	0.092	50	✓	0.069	32	To Do
Kamke 1549	✓	0.014	34	✓	0.02	26	To Do
Kamke 1550	✓	5.112	214	✓	0.789	157	To Do
Kamke 1551	✓	0.43	84	✓	0.265	62	To Do
Kamke 1552	✗	0	0	✓	0.076	89	To Do
Kamke 1553	✓	0.026	29	✓	0.011	17	To Do
Kamke 1554	✓	0.025	27	✓	0.013	18	To Do
Kamke 1555	✓	0.067	156	✓	0.181	61	To Do
Kamke 1556	✓	0.026	27	✓	0.014	19	To Do
Kamke 1557	✓	0.07	146	✓	0.142	61	To Do
Kamke 1558	✓	0.165	222	✓	0.194	67	To Do
Kamke 1559	✓	0.292	100	✓	0.185	33	To Do
Kamke 1560	✓	0.027	27	✓	0.011	18	To Do
Kamke 1561	✓	3.907	310	✓	0.285	69	To Do
Kamke 1562	✓	1.099	140	✓	0.408	77	To Do
Kamke 1563	✓	1.862	193	✓	0.283	87	To Do
Kamke 1564	✓	1.353	196	✓	0.263	88	To Do
Kamke 1565	✓	0.559	242	✓	0.437	71	To Do
Kamke 1566	✓	0.654	237	✓	0.399	35	To Do
Kamke 1567	✓	0.026	27	✓	0.017	19	To Do
Kamke 1568	✓	0.012	116	✓	0.03	89	To Do
Kamke 1569	✗	0	0	✓	0.55	63	To Do
Kamke 1570	✓	0.148	213	✓	0.102	49	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1571	✓	0.09	389	✓	0.204	143	To Do
Kamke 1572	✗	0	0	✓	0.464	35	To Do
Kamke 1573	✗	0	0	✓	0.049	41	To Do
Kamke 1574	✗	0	0	✓	0.886	252	To Do
Kamke 1575	✗	0	0	✓	0.5	719	To Do
Kamke 1576	✗	0	0	✓	0.027	67	To Do
Kamke 1577	✓	1.134	39	✓	0.007	21	To Do
Kamke 1578	✓	262.552	128	✓	1.366	89	To Do
Kamke 1579	✓	0.683	80	✓	0.414	69	To Do
Kamke 1580	✓	0.981	111	✓	0.803	147	To Do
Kamke 1581	✗	0	0	✗	0	0	To Do
Kamke 1582	✓	0.637	528	✗	0	0	To Do
Kamke 1583	✗	0	0	✓	0.03	40	To Do
Kamke 1584	✓	2.837	207	✓	0.274	118	To Do
Kamke 1585	✓	0.227	214	✓	0.032	679	To Do
Kamke 1586	✗	0	0	✗	0	0	To Do
Kamke 1587	✓	0.372	492	✓	0.556	174	To Do
Kamke 1588	✓	6.157	103	✓	0.151	90	To Do
Kamke 1589	✓	0.049	659	✓	9.116	4339	To Do
Kamke 1590	✗	0	0	✓	2.786	553	To Do
Kamke 1591	✓	0.053	26	✓	0.017	12	To Do
Kamke 1592	✓	0.029	14	✓	0.013	10	To Do
Kamke 1593	✗	0	0	✗	0	0	To Do
Kamke 1594	✓	0.712	200	✓	0.099	59	To Do
Kamke 1595	✗	0	0	✗	0	0	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1596	✗	0	0	✗	0	0	To Do
Kamke 1597	✓	2.242	131	✓	0.026	21	To Do
Kamke 1598	✗	0	0	✗	0	0	To Do
Kamke 1599	✗	0	0	✗	0	0	To Do
Kamke 1600	✓	3.875	869	✓	0.097	89	To Do
Kamke 1601	✗	0	0	✓	3.115	151	To Do
Kamke 1602	✗	0	0	✓	0.244	73	To Do
Kamke 1603	✗	0	0	✓	71.214	8411	To Do
Kamke 1604	✓	0.062	32	✓	0.353	23	To Do
Kamke 1605	✗	0	0	✓	1.309	104	To Do
Kamke 1606	✗	0	0	✗	0	0	To Do
Kamke 1607	✓	0.107	79	✓	0.122	49	To Do
Kamke 1608	✗	0	0	✗	0	0	To Do
Kamke 1609	✗	0	0	✗	0	0	To Do
Kamke 1610	✗	0	0	✓	0.295	92	To Do
Kamke 1611	✗	0	0	✓	0.529	57	To Do
Kamke 1612	✗	0	0	✓	1.318	57	To Do
Kamke 1613	✗	0	0	✓	0.023	27	To Do
Kamke 1614	✗	0	0	✓	0.066	33	To Do
Kamke 1615	✗	0	0	✓	4.878	91	To Do
Kamke 1616	✗	0	0	✓	1.356	63	To Do
Kamke 1617	✗	0	0	✗	0	0	To Do
Kamke 1618	✗	0	0	✓	1.736	56	To Do
Kamke 1619	✗	0	0	✗	0	0	To Do
Kamke 1620	✗	0	0	✓	0.125	253	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1621	✗	0	0	✓	1.625	1088	To Do
Kamke 1622	✗	0	0	✓	0.527	817	To Do
Kamke 1623	✗	0	0	✗	0	0	To Do
Kamke 1624	✗	0	0	✓	2.	131	To Do
Kamke 1625	✗	0	0	✗	0	0	To Do
Kamke 1626	✗	0	0	✓	0.232	48	To Do
Kamke 1627	✗	0	0	✓	0.961	58	To Do
Kamke 1628	✗	0	0	✗	0	0	To Do
Kamke 1629	✗	0	0	✓	0.039	38	To Do
Kamke 1630	✓	11.192	1670	✓	0.625	783	To Do
Kamke 1631	✗	0	0	✓	0.049	38	To Do
Kamke 1632	✓	0.063	34	✓	0.095	23	To Do
Kamke 1633	✗	0	0	✓	0.279	97	To Do
Kamke 1634	✗	0	0	✗	0	0	To Do
Kamke 1635	✗	0	0	✓	0.178	79	To Do
Kamke 1636	✗	0	0	✓	0.89	59	To Do
Kamke 1637	✗	0	0	✓	0.554	58	To Do
Kamke 1638	✗	0	0	✓	0.192	115	To Do
Kamke 1639	✗	0	0	✓	3.636	56	To Do
Kamke 1640	✗	0	0	✓	0.191	70	To Do
Kamke 1641	✓	2.06	57	✓	0.039	29	To Do
Kamke 1642	✗	0	0	✗	0	0	To Do
Kamke 1643	✗	0	0	✗	0	0	To Do
Kamke 1644	✗	0	0	✓	0.556	56	To Do
Kamke 1645	✗	0	0	✗	0	0	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1646	✓	10.776	262	✓	0.178	94	To Do
Kamke 1647	✓	51.548	59	✓	0.461	60	To Do
Kamke 1648	✗	0	0	✓	2.059	205	To Do
Kamke 1649	✗	0	0	✗	0	0	To Do
Kamke 1650	✓	0.022	19	✓	0.238	16	To Do
Kamke 1651	✓	0.264	414	✓	0.163	31	To Do
Kamke 1652	✗	0	0	✓	0.309	36	To Do
Kamke 1653	✓	0.065	75	✓	0.135	41	To Do
Kamke 1654	✓	0.29	308	✓	0.187	38	To Do
Kamke 1655	✓	0.838	350	✓	0.22	84	To Do
Kamke 1656	✗	0	0	✓	0.78	771	To Do
Kamke 1657	✓	0.152	33	✓	0.268	35	To Do
Kamke 1658	✗	0	0	✓	0.143	115	To Do
Kamke 1659	✗	0	0	✓	0.093	60	To Do
Kamke 1660	✗	0	0	✓	0.944	125	To Do
Kamke 1661	✓	0.029	90	✓	0.085	51	To Do
Kamke 1662	✗	0	0	✓	0.4	56	To Do
Kamke 1663	✗	0	0	✓	0.959	125	To Do
Kamke 1664	✗	0	0	✓	3.272	155	To Do
Kamke 1665	✗	0	0	✓	0.617	84	To Do
Kamke 1666	✗	0	0	✓	0.963	93	To Do
Kamke 1667	✗	0	0	✓	1.548	121	To Do
Kamke 1668	✓	0.071	46	✓	0.162	24	To Do
Kamke 1669	✓	125.481	74	✓	0.119	32	To Do
Kamke 1670	✓	83.924	49	✓	0.398	35	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1671	✓	0.035	59	✓	0.092	35	To Do
Kamke 1672	✗	0	0	✓	1.094	65	To Do
Kamke 1673	✗	0	0	✓	0.813	60	To Do
Kamke 1674	✓	0.061	69	✓	0.069	25	To Do
Kamke 1675	✗	0	0	✗	0	0	To Do
Kamke 1676	✓	54.917	117	✓	0.28	72	To Do
Kamke 1677	✗	0	0	✓	2.267	101	To Do
Kamke 1678	✗	0	0	✓	0.265	60	To Do
Kamke 1679	✓	0.078	33	✓	0.152	27	To Do
Kamke 1680	✗	0	0	✓	0.638	103	To Do
Kamke 1681	✗	0	0	✓	0.056	31	To Do
Kamke 1682	✗	0	0	✓	0.638	94	To Do
Kamke 1683	✓	0.078	25	✓	0.056	23	To Do
Kamke 1684	✗	0	0	✓	1.717	100	To Do
Kamke 1685	✗	0	0	✗	0	0	To Do
Kamke 1686	✗	0	0	✓	1.18	128	To Do
Kamke 1687	✓	0.069	83	✓	0.104	21	To Do
Kamke 1688	✗	0	0	✓	0.113	32	To Do
Kamke 1689	✓	0.648	259	✓	0.162	37	To Do
Kamke 1690	✗	0	0	✓	0.837	99	To Do
Kamke 1691	✗	0	0	✓	1.267	254	To Do
Kamke 1692	✗	0	0	✓	3.955	156	To Do
Kamke 1693	✗	0	0	✓	0.329	68	To Do
Kamke 1694	✓	0.189	111	✓	0.135	54	To Do
Kamke 1695	✗	0	0	✓	0.627	103	To Do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1696	✗	0	0	✓	0.513	100	To Do
Kamke 1697	✓	0.066	68	✓	0.044	39	To Do
Kamke 1698	✓	0.044	63	✗	0	0	To Do
Kamke 1699	✓	0.042	32	✓	0.096	33	To Do
Kamke 1700	✓	0.085	44	✓	0.331	86	To Do
Kamke 1701	✓	0.197	79	✓	0.289	42	To Do
Kamke 1702	✗	0	0	✗	0	0	To Do
Kamke 1703	✓	0.095	63	✓	0.07	25	To Do
Kamke 1704	✗	0	0	✗	0	0	To Do
Kamke 1705	✗	0	0	✗	0	0	To Do
Kamke 1706	✗	0	0	✗	0	0	To Do
Kamke 1707	✓	0.083	28	✓	0.066	39	To Do
Kamke 1708	✗	0	0	✓	1.043	73	To Do
Kamke 1709	✗	0	0	✓	1.977	84	To Do
Kamke 1710	✗	0	0	✓	2.734	91	To Do
Kamke 1711	✗	0	0	✓	0.553	81	To Do
Kamke 1712	✓	11.071	57	✓	0.092	61	To Do
Kamke 1713	✗	0	0	✓	0.333	54	To Do
Kamke 1714	✓	0.069	25	✓	0.063	68	To Do
Kamke 1715	✓	0.035	26	✓	0.045	25	To Do
Kamke 1716	✓	0.679	172	✓	0.216	68	To Do
Kamke 1717	✓	1.686	290	✓	0.272	107	To Do
Kamke 1718	✓	1.56	398	✓	0.276	133	To Do
Kamke 1719	✗	0	0	✓	0.579	70	To Do
Kamke 1720	✗	0	0	✓	0.339	173	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1721	✗	0	0	✗	0	0	To Do
Kamke 1722	✓	1.978	797	✓	0.44	98	To Do
Kamke 1723	✓	0.84	227	✓	0.13	16	To Do
Kamke 1724	✓	0.22	27	✓	0.587	21	To Do
Kamke 1725	✓	0.349	59	✓	0.577	105	To Do
Kamke 1726	✓	0.839	73	✓	0.112	39	To Do
Kamke 1727	✓	0.184	129	✓	0.222	87	To Do
Kamke 1728	✓	0.007	31	✓	0.035	24	To Do
Kamke 1729	✗	0	0	✗	0	0	To Do
Kamke 1730	✓	0.499	77	✓	0.098	53	To Do
Kamke 1731	✓	1.468	359	✓	0.102	61	To Do
Kamke 1732	✗	0	0	✗	0	0	To Do
Kamke 1733	✓	2.61	437	✓	0.101	71	To Do
Kamke 1734	✗	0	0	✗	0	0	To Do
Kamke 1735	✗	0	0	✗	0	0	To Do
Kamke 1736	✓	8.576	285	✓	0.099	49	To Do
Kamke 1737	✗	0	0	✗	0	0	To Do
Kamke 1738	✗	0	0	✗	0	0	To Do
Kamke 1739	✗	0	0	✗	0	0	To Do
Kamke 1740	✓	0.029	16	✓	0.036	13	To Do
Kamke 1741	✓	0.096	17	✓	0.075	34	To Do
Kamke 1742	✗	0	0	✓	0.191	60	To Do
Kamke 1743	✓	42.773	2761	✓	0.111	71	To Do
Kamke 1744	✓	0.963	173	✓	0.225	87	To Do
Kamke 1745	✓	0.306	204	✓	0.386	117	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1746	✗	0	0	✓	0.596	207	To Do
Kamke 1747	✓	0.028	20	✓	0.018	17	To Do
Kamke 1748	✓	0.095	43	✓	0.125	67	To Do
Kamke 1749	✓	0.542	181	✓	0.289	57	To Do
Kamke 1750	✓	59.24	1	✓	0.426	87	To Do
Kamke 1751	✗	0	0	✗	0	0	To Do
Kamke 1752	✓	0.13	26	✓	0.096	33	To Do
Kamke 1753	✓	0.324	43	✓	0.135	147	To Do
Kamke 1754	✓	0.033	17	✓	0.038	15	To Do
Kamke 1755	✗	0	0	✓	0.341	418	To Do
Kamke 1756	✓	0.287	111	✓	0.121	75	To Do
Kamke 1757	✗	0	0	✗	0	0	To Do
Kamke 1758	✓	0.06	31	✓	0.059	42	To Do
Kamke 1759	✓	0.037	18	✓	0.027	31	To Do
Kamke 1760	✗	0	0	✓	0.083	114	To Do
Kamke 1761	✗	0	0	✗	0	0	To Do
Kamke 1762	✗	0	0	✓	0.911	108	To Do
Kamke 1763	✓	0.143	33	✓	0.053	148	To Do
Kamke 1764	✓	0.064	37	✓	0.195	18	To Do
Kamke 1765	✓	0.127	24	✓	0.03	27	To Do
Kamke 1766	✓	0.047	21	✓	0.033	64	To Do
Kamke 1767	✓	0.076	54	✓	0.345	50	To Do
Kamke 1768	✓	0.115	79	✓	0.058	43	To Do
Kamke 1769	✓	0.047	18	✓	0.035	21	To Do
Kamke 1770	✓	0.804	22	✓	0.062	26	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1771	✓	0.082	20	✓	0.162	22	To Do
Kamke 1772	✓	0.934	36	✓	0.151	37	To Do
Kamke 1773	✓	0.183	44	✓	0.043	30	To Do
Kamke 1774	✓	1.364	92	✓	0.253	136	To Do
Kamke 1775	✓	0.131	24	✓	0.1	31	To Do
Kamke 1776	✗	0	0	✓	0.352	49	To Do
Kamke 1777	✗	0	0	✓	0.74	79	To Do
Kamke 1778	✓	0.982	75	✓	0.469	245	To Do
Kamke 1779	✗	0	0	✓	0.543	112	To Do
Kamke 1780	✗	0	0	✓	0.599	160	To Do
Kamke 1781	✓	0.083	14	✓	0.053	11	To Do
Kamke 1782	✓	0.09	93	✓	0.048	33	To Do
Kamke 1783	✓	1.458	24	✓	0.174	23	To Do
Kamke 1784	✓	0.381	72	✓	0.569	82	To Do
Kamke 1785	✓	0.378	95	✓	0.359	83	To Do
Kamke 1786	✓	1.026	87	✓	0.164	42	To Do
Kamke 1787	✗	0	0	✓	0.283	80	To Do
Kamke 1788	✗	0	0	✗	0	0	To Do
Kamke 1789	✗	0	0	✗	0	0	To Do
Kamke 1790	✓	23.341	182	✓	0.255	119	To Do
Kamke 1791	✓	23.172	164	✓	0.364	90	To Do
Kamke 1792	✓	26.665	222	✓	0.905	194	To Do
Kamke 1793	✓	1.364	113	✓	0.069	40	To Do
Kamke 1794	✓	1.289	98	✓	0.093	46	To Do
Kamke 1795	✓	0.418	103	✓	1.037	529	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1796	✓	0.33	65	✓	0.227	51	To Do
Kamke 1797	✗	0	0	✗	0	0	To Do
Kamke 1798	✗	0	0	✓	0.242	166	To Do
Kamke 1799	✓	1.902	58	✓	0.135	46	To Do
Kamke 1800	✓	0.496	78	✓	0.05	60	To Do
Kamke 1801	✗	0	0	✗	0	0	To Do
Kamke 1802	✗	0	0	✗	0	0	To Do
Kamke 1803	✓	86.209	1	✓	2.665	115620	To Do
Kamke 1804	✓	3.922	415	✓	0.034	31	To Do
Kamke 1805	✓	3.298	436	✓	0.046	34	To Do
Kamke 1806	✗	0	0	✓	5.365	718	To Do
Kamke 1807	✗	0	0	✗	0	0	To Do
Kamke 1808	✓	104.936	155	✓	0.141	72	To Do
Kamke 1809	✗	0	0	✓	0.705	336	To Do
Kamke 1810	✓	0.103	1881	✓	0.119	91	To Do
Kamke 1811	✗	0	0	✓	3.85	1862	To Do
Kamke 1812	✓	0.03	29	✓	0.037	19	To Do
Kamke 1813	✗	0	0	✓	0.421	138	To Do
Kamke 1814	✓	13.496	116	✓	0.183	87	To Do
Kamke 1815	✗	0	0	✓	0.999	71	To Do
Kamke 1816	✗	0	0	✓	1.231	46	To Do
Kamke 1817	✗	0	0	✓	0.234	40	To Do
Kamke 1818	✗	0	0	✓	0.4	66	To Do
Kamke 1819	✗	0	0	✓	0.063	42	To Do
Kamke 1820	✗	0	0	✓	1.209	88	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1821	✗	0	0	✓	2.286	54	To Do
Kamke 1822	✓	2.961	369	✓	1.215	291	To Do
Kamke 1823	✗	0	0	✓	0.434	289	To Do
Kamke 1824	✓	0.458	347	✓	0.988	96	To Do
Kamke 1825	✗	0	0	✓	1.053	49	To Do
Kamke 1826	✓	1.86	119	✓	0.375	173	To Do
Kamke 1827	✗	0	0	✓	9.289	81	To Do
Kamke 1828	✓	0.011	32	✓	0.597	59	To Do
Kamke 1829	✓	0.007	24	✓	0.391	32	To Do
Kamke 1830	✓	0.029	24	✓	0.552	304	To Do
Kamke 1831	✗	0	0	✓	1.269	163	To Do
Kamke 1832	✗	0	0	✓	0.976	117	To Do
Kamke 1833	✗	0	0	✓	3.877	145	To Do
Kamke 1834	✗	0	0	✓	0.497	82	To Do
Kamke 1835	✓	0.115	135	✗	0	0	To Do
Kamke 1836	✗	0	0	✓	0.451	100	To Do
Kamke 1837	✗	0	0	✓	0.369	95	To Do
Kamke 1838	✗	0	0	✓	0.657	73	To Do
Kamke 1839	✗	0	0	✓	0.911	116	To Do
Kamke 1840	✗	0	0	✓	0.976	129	To Do
Kamke 1841	✗	0	0	✓	0.625	60	To Do
Kamke 1842	✓	0.171	282	✓	0.655	190	To Do
Kamke 1843	✓	2.981	409	✓	0.316	77	To Do
Kamke 1844	✗	0	0	✓	0.292	17	To Do
Kamke 1845	✗	0	0	✓	0.27	17	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1846	✓	0.046	51	✓	0.011	28	To Do
Kamke 1847	✓	0.131	95	✓	0.27	49	To Do
Kamke 1848	✗	0	0	✓	1.503	789	To Do
Kamke 1849	✓	0.596	415	✓	0.19	197	To Do
Kamke 1850	✗	0	0	✓	1.597	164	To Do
Kamke 1851	✗	0	0	✗	0	0	To Do
Kamke 1852	✓	0.036	28	✓	0.321	30	To Do
Kamke 1853	✗	0	0	✓	0.681	110	To Do
Kamke 1854	✗	0	0	✗	0	0	To Do
Kamke 1855	✗	0	0	✗	0	0	To Do
Kamke 1856	✓	0.006	22	✓	0.033	19	To Do
Kamke 1857	✓	0.055	39	✓	0.048	35	To Do
Kamke 1858	✓	0.019	158	✓	0.053	64	To Do
Kamke 1859	✓	0.006	43	✓	0.039	37	To Do
Kamke 1860	✓	0.047	362	✓	0.084	177	To Do
Kamke 1861	✓	0.014	145	✓	0.109	152	To Do
Kamke 1862	✓	0.075	46	✓	0.04	39	To Do
Kamke 1863	✓	0.008	72	✓	0.038	35	To Do
Kamke 1864	✓	0.014	52	✓	0.041	44	To Do
Kamke 1865	✓	1.204	930	✓	0.136	224	To Do
Kamke 1866	✓	0.032	47	✓	0.04	39	To Do
Kamke 1867	✓	0.088	45	✓	0.036	42	To Do
Kamke 1868	✓	0.048	83	✓	0.073	64	To Do
Kamke 1869	✓	0.116	84	✓	0.06	51	To Do
Kamke 1870	✓	0.132	71	✓	0.113	47	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1871	✓	0.166	83	✓	0.077	62	To Do
Kamke 1872	✓	0.06	81	✓	0.068	65	To Do
Kamke 1873	✓	0.046	112	✓	0.063	52	To Do
Kamke 1874	✓	0.14	105	✓	0.458	57	To Do
Kamke 1875	✗	0	0	✓	1.264	1447	To Do
Kamke 1876	✓	0.107	41	✓	0.139	18	To Do
Kamke 1877	✓	0.006	31	✓	0.036	31	To Do
Kamke 1878	✓	0.013	39	✓	0.058	39	To Do
Kamke 1879	✓	0.091	58	✓	0.056	54	To Do
Kamke 1880	✗	0	0	✓	0.083	23	To Do
Kamke 1881	✓	0.028	44	✓	0.029	48	To Do
Kamke 1882	✓	0.454	199	✓	0.086	99	To Do
Kamke 1883	✓	0.599	186	✓	0.112	80	To Do
Kamke 1884	✓	0.193	116	✓	0.107	69	To Do
Kamke 1885	✗	0	0	✓	0.098	47	To Do
Kamke 1886	✓	0.022	103	✓	0.065	49	To Do
Kamke 1887	✓	0.546	5647	✓	0.14	360	To Do
Kamke 1888	✓	26.089	1	✓	0.269	457	To Do
Kamke 1889	✓	0.111	151	✓	0.062	60	To Do
Kamke 1890	✗	0	0	✗	0	0	To Do
Kamke 1891	✓	0.458	200	✓	0.059	64	To Do
Kamke 1892	✓	0.4	3522	✓	0.155	463	To Do
Kamke 1893	✗	0	0	✓	1.276	1579	To Do
Kamke 1894	✗	0	0	✓	1.103	1056	To Do
Kamke 1895	✓	0.451	6816	✓	0.25	1008	To Do

Continued on next page



Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1896	✓	0.196	252	✓	0.068	67	To Do
Kamke 1897	✓	0.11	123	✓	0.169	86	To Do
Kamke 1898	✓	0.037	263	✓	0.074	71	To Do
Kamke 1899	✓	0.013	93	✓	0.089	52	To Do
Kamke 1900	✓	0.01	88	✓	0.084	50	To Do
Kamke 1901	✓	0.011	93	✓	0.072	43	To Do
Kamke 1902	✓	0.015	109	✓	0.065	51	To Do
Kamke 1903	✓	0.09	741	✓	0.155	299	To Do
Kamke 1904	✓	0.065	1084	✓	0.098	257	To Do
Kamke 1905	✗	0	0	✗	0	0	To Do
Kamke 1906	✓	0.053	177	✓	0.086	120	To Do
Kamke 1907	✓	0.013	157	✓	0.069	66	To Do
Kamke 1908	✓	0.022	551	✓	0.749	1213	To Do
Kamke 1909	✓	0.059	1630	✓	27.579	33085	To Do
Kamke 1910	✓	0.01	39	✓	0.109	37	To Do
Kamke 1911	✗	0	0	✓	0.159	308	To Do
Kamke 1912	✗	0	0	✓	2.798	2956	To Do
Kamke 1913	✓	0.034	52	✓	0.135	57	To Do
Kamke 1914	✓	0.652	198	✓	0.411	92	To Do
Kamke 1915	✗	0	0	✓	11.342	147	To Do
Kamke 1916	✗	0	0	✓	0.579	180	To Do
Kamke 1917	✗	0	0	✓	0.858	109	To Do
Kamke 1918	✗	0	0	✓	2.155	184	To Do
Kamke 1919	✗	0	0	✓	3.442	203	To Do
Kamke 1920	✗	0	0	✓	3.463	205	To Do

Continued on next page

Table 3 – continued from previous page

#	Mathematica			Maple			note on solution
	solved	cpu	leaf	solved	cpu	leaf	
Kamke 1921	✗	0	0	✗	0	0	To Do
Kamke 1922	✗	0	0	✗	0	0	To Do
Kamke 1923	✓	0.014	39	✓	0.04	35	To Do
Kamke 1924	✓	0.071	179	✓	0.331	180	To Do
Kamke 1925	✗	0	0	✓	0.312	194	To Do
Kamke 1926	✗	0	0	✓	0.115	96	To Do
Kamke 1927	✗	0	0	✗	0	0	To Do
Kamke 1928	✗	0	0	✗	0	0	To Do
Kamke 1929	✗	0	0	✓	4.038	116	To Do
Kamke 1930	✓	0.048	135	✓	0.05	45	To Do
Kamke 1931	✓	5.661	1461	✓	0.828	1117	To Do
Kamke 1932	✗	0	0	✓	1.132	383	To Do
Kamke 1933	✗	0	0	✓	2.514	17738	To Do
Kamke 1934	✗	0	0	✓	1.358	377	To Do
Kamke 1935	✗	0	0	✓	2.187	741	To Do
Kamke 1936	✗	0	0	✓	0.778	704	To Do
Kamke 1937	✗	0	0	✓	0.93	242	To Do
Kamke 1938	✓	0.01	137	✓	0.127	101	To Do
Kamke 1939	✗	0	0	✓	1.591	899	To Do
Kamke 1940	✗	0	0	✗	0	0	To Do

## 2.1 ODE No. 1

$$y'(x) - \frac{1}{\sqrt{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}} = 0$$

✓ **Mathematica** : cpu = 0.727546 (sec), leaf count = 1117

$$\left\{ \left\{ y(x) \rightarrow c_1 - \frac{2F\left(\sin^{-1}\left(\sqrt{\frac{(x-\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,1])\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,2]}{(x-\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,2])\text{Root}[a_4\#1^4+a_3\#1^3+a_2\#1^2+a_1\#1+a_0\&,1]}- \right)}{\right. \right.$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 1089

$$\left\{ y(x) = 2 \frac{(-\text{RootOf}(a_4\_Z^4 + a_3\_Z^3 + a_2\_Z^2 + a_1\_Z + a_0, \text{index} = 4) - \text{RootOf}(a_4\_Z^4 + a_3\_Z^3 + a_2\_Z^2 + a_1\_Z + a_0, \text{index} = 1))}{(\text{RootOf}(a_4\_Z^4 + a_3\_Z^3 + a_2\_Z^2 + a_1\_Z + a_0, \text{index} = 4) - \text{RootOf}(a_4\_Z^4 + a_3\_Z^3 + a_2\_Z^2 + a_1\_Z + a_0, \text{index} = 1))} \right.$$

Hand solution

$$y' - \frac{1}{\sqrt{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}} = 0 \quad (1)$$

To Do.

## 2.2 ODE No. 2

$$ay(x) + c(-e^{bx}) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0142307 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-ax}(ce^{x(a+b)} + c_1(a+b))}{a+b} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 25

$$\left\{ y(x) = \left( \frac{ce^{(a+b)x}}{a+b} + \_C1 \right) e^{-ax} \right\}$$

Hand solution

$$\frac{dy}{dx} + ay(x) = ce^{bx} \quad (1)$$

Integrating factor  $\mu = e^{\int adx} = e^{ax}$ . Hence (1) becomes

$$\begin{aligned} \frac{d}{dx}(\mu y(x)) &= \mu ce^{bx} \\ \mu y(x) &= \int \mu ce^{bx} dx + C \end{aligned}$$

Replacing  $\mu$  by  $e^{ax}$

$$\begin{aligned} y(x) &= ce^{-ax} \int e^{(a+b)x} dx + Ce^{-ax} \\ &= ce^{-ax} \frac{e^{(a+b)x}}{a+b} + Ce^{-ax} \\ &= \frac{ce^{(a+b)x-ax}}{a+b} + Ce^{-ax} \end{aligned}$$

Can be reduced to

$$y(x) = c \frac{e^{bx}}{a+b} + Ce^{-ax}$$

### 2.3 ODE No. 3

$$ay(x) - b \sin(cx) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0328369 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \frac{b(a \sin(cx) - c \cos(cx))}{a^2 + c^2} + c_1 e^{-ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 37

$$\left\{ y(x) = e^{-ax} \_C1 + \frac{b(\sin(cx) a - c \cos(cx))}{a^2 + c^2} \right\}$$

<b>Hand solution</b>
----------------------

$$\frac{dy}{dx} + ay(x) = b \sin(cx) \quad (1)$$

Integrating factor  $\mu = e^{\int adx} = e^{ax}$ . Hence (1) becomes

$$\begin{aligned} \frac{d}{dx}(\mu y(x)) &= \mu b \sin(cx) \\ \mu y(x) &= b \int \mu \sin(cx) dx + C \end{aligned}$$

Replacing  $\mu$  by  $e^{ax}$

$$y(x) = be^{-ax} \int e^{ax} \sin(cx) dx + Ce^{-ax} \quad (2)$$

Using  $\sin(cx) = \frac{e^{icx} - e^{-icx}}{2i}$  then

$$\begin{aligned} \int e^{ax} \sin(cx) dx &= \int \frac{e^{(ic+a)x} - e^{(-ic+a)x}}{2i} dx \\ &= \frac{1}{2i} \left( \frac{e^{(ic+a)x}}{ic+a} - \frac{e^{(-ic+a)x}}{-ic+a} \right) \\ &= \frac{1}{2i} e^{ax} \left( \frac{e^{icx}}{ic+a} - \frac{e^{-icx}}{-ic+a} \right) \\ &= \frac{1}{2i} e^{ax} \left( \frac{e^{icx}(-ic+a) - e^{-icx}(ic+a)}{(ic+a)(-ic+a)} \right) \\ &= \frac{1}{2i} e^{ax} \left( \frac{-ice^{icx} + ae^{icx} - ice^{-icx} - ae^{-icx}}{c^2 + a^2} \right) \\ &= \frac{1}{2i} e^{ax} \left( \frac{-ic(e^{icx} + e^{-icx}) + a(e^{icx} - e^{-icx})}{c^2 + a^2} \right) \\ &= \frac{e^{ax}}{(c^2 + a^2)} \left( \frac{-ic(e^{icx} + e^{-icx})}{2i} + \frac{a(e^{icx} - e^{-icx})}{2i} \right) \\ &= \frac{e^{ax}}{(c^2 + a^2)} (-c \cos cx + a \sin cx) \end{aligned}$$

Therefore (2) becomes

$$\begin{aligned} y(x) &= be^{-ax} \left[ \frac{e^{ax}}{(c^2 + a^2)} (-c \cos cx + a \sin cx) \right] + Ce^{-ax} \\ &= \frac{b}{(c^2 + a^2)} (-c \cos cx + a \sin cx) + Ce^{-ax} \end{aligned}$$

## 2.4 ODE No. 4

$$-e^{-x^2} x + y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.00919349 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-x^2} (2c_1 + x^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.005 (sec), leaf count = 18

$$\left\{ y(x) = \left( \frac{x^2}{2} + \_C1 \right) e^{-x^2} \right\}$$

**Hand solution**

$$\frac{dy}{dx} + 2xy(x) = e^{-x^2} x \quad (1)$$

Integrating factor  $\mu = e^{\int 2xdx} = e^{x^2}$ . Hence (1) becomes

$$\begin{aligned} \frac{d}{dx} \left( e^{x^2} y(x) \right) &= e^{x^2} e^{-x^2} x \\ \frac{d}{dx} \left( e^{x^2} y(x) \right) &= x \end{aligned}$$

Integrating both sides

$$\begin{aligned} e^{x^2} y(x) &= \frac{x^2}{2} + C \\ y(x) &= e^{-x^2} \left( \frac{x^2}{2} + C \right) \end{aligned}$$

## 2.5 ODE No. 5

$$y'(x) + y(x) \cos(x) - e^{2x} = 0$$

✓ **Mathematica** : cpu = 2.6808 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow e^{-\sin(x)} \left( \int_1^x e^{2K[1] + \sin(K[1])} dK[1] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.087 (sec), leaf count = 21

$$\left\{ y(x) = e^{-\sin(x)} \left( \int e^{2x + \sin(x)} dx + \_C1 \right) \right\}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) \cos(x) = e^{2x} \quad (1)$$

Integrating factor  $\mu = e^{\int \cos(x) dx} = e^{\sin(x)}$ . Hence (1) becomes

$$\frac{d}{dx} \left( e^{\sin(x)} y(x) \right) = e^{\sin(x)} e^{2x}$$

Integrating both sides

$$\begin{aligned} e^{\sin(x)} y(x) &= \int e^{\sin(x)} e^{2x} + C \\ y(x) &= e^{-\sin(x)} \int e^{2x+\sin(x)} + C e^{-\sin(x)} \end{aligned}$$

## 2.6 ODE No. 6

$$y'(x) + y(x) \cos(x) - \frac{1}{2} \sin(2x) = 0$$

✓ **Mathematica** : cpu = 0.0226681 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-\sin(x)} + \sin(x) - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 15

$$\left\{ y(x) = \sin(x) - 1 + e^{-\sin(x)} \_C1 \right\}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) \cos(x) = \frac{1}{2} \sin(2x) \quad (1)$$

Integrating factor  $\mu = e^{\int \cos dx} = e^{\sin(x)}$ . Therefore (1) becomes

$$\frac{d}{dx} \left( e^{\sin(x)} y(x) \right) = \frac{1}{2} e^{\sin(x)} \sin(2x)$$

Integrating

$$\begin{aligned} e^{\sin(x)} y(x) &= \frac{1}{2} \int e^{\sin(x)} \sin(2x) + C \\ y(x) &= \frac{e^{-\sin(x)}}{2} \int e^{\sin(x)} \sin(2x) + e^{-\sin(x)} C \end{aligned}$$

But  $e^{\sin(x)} \sin(2x)$  can be integrated by parts which gives  $e^{\sin(x)}(-2 + 2 \sin(x))$ . Hence the above becomes

$$\begin{aligned} y(x) &= \frac{e^{-\sin(x)}}{2} \left( e^{\sin(x)}(-2 + 2 \sin(x)) \right) + e^{-\sin(x)}C \\ &= -1 + \sin(x) + e^{-\sin(x)}C \end{aligned}$$

## 2.7 ODE No. 7

$$y'(x) + y(x) \cos(x) - e^{-\sin(x)} = 0$$

✓ **Mathematica** : cpu = 0.0231071 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow (c_1 + x) e^{-\sin(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 13

$$\left\{ y(x) = (x + \_C1) e^{-\sin(x)} \right\}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) \cos(x) = e^{-\sin(x)} \tag{1}$$

Integrating factor  $\mu = e^{\int \cos x dx} = e^{\sin x}$ . Hence (1) becomes

$$\frac{d}{dx}(\mu y(x)) = \mu e^{-\sin(x)}$$

Replacing  $\mu$  by  $e^{\sin x}$  and integrating both sides

$$e^{\sin x} y(x) = \int e^{\sin x} e^{-\sin(x)} dx + C$$

$$e^{\sin x} y(x) = \int dx + C$$

$$e^{\sin x} y(x) = x + C$$

$$y(x) = x e^{-\sin x} + C e^{-\sin(x)}$$



## 2.8 ODE No. 8

$$y'(x) + y(x) \tan(x) - \sin(2x) = 0$$

✓ **Mathematica** : cpu = 0.0234229 (sec), leaf count = 15

$$\{y(x) \rightarrow \cos(x) (c_1 - 2 \cos(x))\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 13

$$\{y(x) = \cos(x) (-2 \cos(x) + \_C1)\}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) \tan(x) = \sin(2x) \quad (1)$$

Integrating factor  $\mu = e^{\int \tan x} = e^{-\ln(\cos(x))} = \frac{1}{\cos(x)}$ . Hence (1) becomes

$$\frac{d}{dx} \left( y(x) \frac{1}{\cos(x)} \right) = \frac{1}{\cos(x)} \sin(2x)$$

Integrating both sides

$$\begin{aligned} y(x) \frac{1}{\cos(x)} &= \int \frac{1}{\cos(x)} \sin(2x) dx + C \\ y(x) &= \cos(x) \int \frac{\sin(2x)}{\cos(x)} dx + C \cos(x) \end{aligned}$$

But  $\sin(2x) = 2 \sin(x) \cos(x)$  hence

$$\begin{aligned} y(x) &= \cos(x) \int \frac{2 \sin(x) \cos(x)}{\cos(x)} dx + C \cos(x) \\ &= 2 \cos(x) \int \sin(x) dx + C \cos(x) \\ &= -2 \cos^2(x) + C \cos(x) \end{aligned}$$

## 2.9 ODE No. 9

$$y'(x) - y(x)(a + \sin(\log(x)) + \cos(\log(x))) = 0$$

✓ **Mathematica** : cpu = 0.0167515 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{x(a + \sin(\log(x)))} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 14

$$\left\{ y(x) = \_C1 e^{x(\sin(\ln(x)) + a)} \right\}$$

**Hand solution**

$$\frac{dy}{dx} - y(x)[a + \sin(\log(x)) + \cos(\log(x))] = 0 \quad (1)$$

Integrating factor  $\mu = e^{-\int a - \sin(\log(x)) - \cos(\log(x)) dx} = e^{-ax} e^{-\int \sin(\log(x)) + \cos(\log(x)) dx}$ . To integrate  $\int \sin(\log(x)) + \cos(\log(x)) dx$ , let  $r = \log(x)$ ,  $\frac{dr}{dx} = \frac{1}{x}$ , then  $dx = x dr$ , But  $x = e^r$ , hence the integral becomes

$$\begin{aligned} \int \sin(\log(x)) + \cos(\log(x)) dx &= \int [\sin(r) + \cos(r)] e^r dr \\ &= \int e^r \sin(r) dr + \int e^r \cos(r) dr \end{aligned} \quad (2)$$

Integrating by parts  $\int e^r \cos(r) dr$ ,  $\int u dv = uv - \int v du$ , Let  $u = e^r \rightarrow du = e^r$  and  $dv = \cos(r) \rightarrow v = \sin(r)$ , hence (2) becomes

$$\begin{aligned} \int e^r \sin(r) dr + \int e^r \cos(r) dr &= \int e^r \sin(r) dr + e^r \sin(r) - \int \sin(r) e^r dr \\ &= e^r \sin(r) \end{aligned}$$

Therefore, substituting back  $r = \log(x)$  gives

$$\begin{aligned} \int \sin(\log(x)) + \cos(\log(x)) dx &= e^{\log(x)} \sin(\log(x)) \\ &= x \sin(\log(x)) \end{aligned}$$

Hence the integration factor is

$$\begin{aligned}\mu &= e^{-ax} e^{-\int \sin(\log(x)) + \cos(\log(x)) dx} \\ &= e^{-ax} e^{-x \sin(\log(x))}\end{aligned}$$

Therefore (1) becomes

$$\frac{d}{dx}(\mu y(x)) = 0$$

Integrating

$$\begin{aligned}y(x) e^{-ax} e^{-x \sin(\log(x))} &= C \\ y(x) &= C e^{ax} e^{x \sin(\log(x))} \\ &= C e^{ax + x \sin(\log(x))} \\ &= C e^{x(a + \sin(\log(x)))}\end{aligned}$$

## 2.10 ODE No. 10

$$y(x)f'(x) - f(x)f'(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.00862264 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-f(x)} + f(x) - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 15

$$\left\{ y(x) = f(x) - 1 + e^{-f(x)} \_ C1 \right\}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) \frac{df}{dx} = f(x) \frac{df}{dx} \tag{1}$$

Integrating factor  $\mu = e^{\int \frac{df}{dx} dx} = e^f$ . Therefore (1) becomes

$$\frac{d}{dx} \left( e^f y(x) \right) = e^f f(x) \frac{df}{dx}$$

Integrating

$$\begin{aligned}e^f y(x) &= \int e^f f(x) \frac{df}{dx} dx + C \\ y(x) &= e^{-f} \int e^f f df + e^{-f} C\end{aligned}$$

But  $\int e^f f df$  is the same as  $\int e^x x dx$  which by integration by parts gives  $e^x(x - 1)$  or in terms of  $f$ , gives  $e^f(f - 1)$ . Hence the above becomes

$$\begin{aligned} y(x) &= e^{-f} \left( e^f (f - 1) \right) + e^{-f} C \\ &= f - 1 + e^{-f} C \end{aligned}$$

## 2.11 ODE No. 11

$$f(x)y(x) - g(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.462212 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow e^{\int_1^x -f(K[1]) dK[1]} \left( \int_1^x g(K[2]) e^{-\int_1^{K[2]} -f(K[1]) dK[1]} dK[2] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 24

$$\left\{ y(x) = \left( \int g(x) e^{\int f(x) dx} dx + \_C1 \right) e^{\int -f(x) dx} \right\}$$

**Hand solution**

$$\frac{dy}{dx} + y(x) f(x) = g(x) \tag{1}$$

Integrating factor  $\mu = e^{\int f(x) dx}$ . Therefore (1) becomes

$$\frac{d}{dx} \left( e^{\int f(x) dx} y(x) \right) = e^{\int f(x) dx} g(x)$$

Integrating

$$\begin{aligned} e^{\int f(x) dx} y(x) &= \int e^{\int f(x) dx} g(x) dx + C \\ y(x) &= e^{-\int f(x) dx} \int e^{\int f(x) dx} g(x) dx + e^{-\int f(x) dx} C \\ &= \left( \int e^{\int f(x) dx} g(x) dx + C \right) e^{-\int f(x) dx} \end{aligned}$$

2.12 ODE No. 12

$$y'(x) + y(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0285497 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{2x} - e^{2c_1}}{e^{2c_1} + e^{2x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 8

$$\{y(x) = \tanh(x + \_C1)\}$$

**Hand solution**

$$\begin{aligned} \frac{dy}{dx} + y^2(x) - 1 &= 0 \\ \frac{dy}{dx} &= 1 - y^2(x) \end{aligned} \tag{1}$$

This is separable. Hence

$$\begin{aligned} \frac{dy}{dx} \frac{1}{1 - y^2(x)} &= 1 \\ \frac{dy}{1 - y^2(x)} &= dx \end{aligned}$$

Integrating

$$\int \frac{dy}{1 - y^2(x)} = x + C$$

Using  $\int \frac{1}{a+by^2} dy = \frac{\sqrt{-\frac{a}{b}} \tanh^{-1}\left(\frac{y}{\sqrt{-\frac{a}{b}}}\right)}{a}$  and since  $a = 1, b = -1$ , then  $\int \frac{dy}{1-y^2(x)} = \tanh^{-1}(y)$  and the above becomes

$$\tanh^{-1}(y) = x + C$$

Therefore

$$y = \tanh(x + C) \tag{2}$$

In terms of exponential, since  $\tanh(u) = \frac{e^u - e^{-u}}{e^u + e^{-u}}$  then (2) can also be written as

$$y = \frac{e^{x+C} - e^{-(x+C)}}{e^{x+C} + e^{-(x+C)}} = \frac{e^x e^C - e^{-x} e^{-C}}{e^x e^C + e^{-x} e^{-C}}$$

Multiplying numerator and denominator by  $e^{-C} e^x$

$$y = \frac{e^{2x} - e^{-2C}}{e^{2x} + e^{-2C}}$$

To get same answer as Mathematica, since  $C$  is constant, let  $C_1 = -C$ , then

$$y = \frac{e^{2x} - e^{2C_1}}{e^{2x} + e^{2C_1}}$$

### 2.13 ODE No. 13

$$-ax - b + y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.024844 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{a} \left( c_1 \text{Ai}'\left(\frac{b+ax}{a^{2/3}}\right) + \text{Bi}'\left(\frac{b+ax}{a^{2/3}}\right) \right)}{c_1 \text{Ai}\left(\frac{b+ax}{a^{2/3}}\right) + \text{Bi}\left(\frac{b+ax}{a^{2/3}}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.22 (sec), leaf count = 79

$$\left\{ y(x) = -i \sqrt[3]{-ia} \left( \text{Ai}^{(1)}\left(- (ax+b) (-ia)^{-\frac{2}{3}}\right) C_1 + \text{Bi}^{(1)}\left(- (ax+b) (-ia)^{-\frac{2}{3}}\right) \right) \left( \text{Ai}\left(- (ax+b) (-ia)^{-\frac{2}{3}}\right) C_1 \right. \right.$$

**Hand solution**

$$\begin{aligned} y'(x) + y^2(x) - ax - b &= 0 \\ y'(x) &= b + ax - y^2(x) \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form

$$y'(x) = P(x) + Q(x)y + R(x)y^2(x) \tag{2}$$

where in this case  $Q(x) = 0, R(x) = -1, P(x) = b + ax$ . We can solve this in two ways. If we know one particular solution  $y_p(x)$  for (1) then we use the substitution  $y = y_p + \frac{1}{u}$  and convert (1) to new associated linear ODE of the form  $u' + (Q(x) + 2R(x))y_p + R(x) = 0$ . If we do not know a particular solution, then we use the standard substitution  $y = \frac{-u'}{uR(x)} = \frac{u'}{u}$  since  $R(x) = -1$  and this is what we will do here.

Since  $u' = yu$  then

$$\begin{aligned}
 u'' &= yu' + y'u \\
 &= y(yu) + (b + ax - y^2)u \\
 &= y^2u + (b + ax)u - y^2u \\
 &= (b + ax)u
 \end{aligned}$$

So we have new second order ODE

$$u'' - (b + ax)u = 0 \quad (3)$$

which we solve for  $u$ . This ODE is of the form  $u'' - q(x)u = 0$  which has solutions in terms of Airy function of first  $Ai(x)$  and second kind  $Bi(x)$ , where

$$\begin{aligned}
 Ai(x) &= \frac{1}{\pi} \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt \\
 Bi(x) &= \frac{1}{\pi} \int_0^\infty \exp\left(-\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right) dt
 \end{aligned}$$

Therefore the solution to (3) is

$$u(x) = c_1 Ai\left(\frac{b + ax}{a^{\frac{2}{3}}}\right) + c_2 Bi\left(\frac{b + ax}{a^{\frac{2}{3}}}\right)$$

We need to find  $u'(x)$  now. Using  $Ai'(x)$ ,  $Bi'(x)$  for derivative of Airy functions of first and second kind, then

$$u'(x) = c_1 Ai'\left(\frac{b + ax}{a^{\frac{2}{3}}}\right) a^{\frac{1}{3}} + c_2 Bi'\left(\frac{b + ax}{a^{\frac{2}{3}}}\right) a^{\frac{1}{3}}$$

Therefore since  $u' = yu$  then

$$\begin{aligned}
 y &= \frac{u'}{u} \\
 &= \frac{c_1 Ai'\left(\frac{b+ax}{a^{\frac{2}{3}}}\right) a^{\frac{1}{3}} + c_2 Bi'\left(\frac{b+ax}{a^{\frac{2}{3}}}\right) a^{\frac{1}{3}}}{c_1 Ai\left(\frac{b+ax}{a^{\frac{2}{3}}}\right) + c_2 Bi\left(\frac{b+ax}{a^{\frac{2}{3}}}\right)}
 \end{aligned}$$

Let  $C_1 = \frac{c_1}{c_2}$  then the above can be written as

$$y = \frac{C_1 Ai'\left(\frac{b+ax}{a^{\frac{2}{3}}}\right) a^{\frac{1}{3}} + Bi'\left(\frac{b+ax}{a^{\frac{2}{3}}}\right) a^{\frac{1}{3}}}{C_1 Ai\left(\frac{b+ax}{a^{\frac{2}{3}}}\right) + Bi\left(\frac{b+ax}{a^{\frac{2}{3}}}\right)}$$

Reference: Airy function

## 2.14 ODE No. 14

$$ax^m + y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0131027 (sec), leaf count = 253

$$\left\{ \left\{ y(x) \rightarrow -\frac{-i\sqrt{-ax}^{\frac{m}{2}+1} \left( c_1 \left( J_{-\frac{m+3}{m+2}} \left( \frac{2i\sqrt{-ax}^{\frac{m}{2}+1}}{m+2} \right) - J_{\frac{m+1}{m+2}} \left( \frac{2i\sqrt{-ax}^{\frac{m}{2}+1}}{m+2} \right) \right) + 2J_{\frac{1}{m+2}-1} \left( \frac{2i\sqrt{-ax}^{\frac{m}{2}+1}}{m+2} \right) \right) - c_1 J_{-\frac{1}{m+2}} \left( \frac{2i\sqrt{-ax}^{\frac{m}{2}+1}}{m+2} \right)}{2x \left( c_1 J_{-\frac{1}{m+2}} \left( \frac{2i\sqrt{-ax}^{\frac{m}{2}+1}}{m+2} \right) + J_{\frac{1}{m+2}} \left( \frac{2i\sqrt{-ax}^{\frac{m}{2}+1}}{m+2} \right) \right)} \right. \right.$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 187

$$\left\{ y(x) = \frac{1}{x} \left( -J_{\frac{3+m}{m+2}} \left( 2 \frac{\sqrt{ax}^{m/2+1}}{m+2} \right) \sqrt{ax}^{\frac{m}{2}+1} - C1 - Y_{\frac{3+m}{m+2}} \left( 2 \frac{\sqrt{ax}^{m/2+1}}{m+2} \right) \sqrt{ax}^{\frac{m}{2}+1} + -C1 J_{(m+2)^{-1}} \left( 2 \frac{\sqrt{ax}^{m/2+1}}{m+2} \right) \right.$$

**Hand solution**

$$\begin{aligned} y'(x) + y^2(x) + ax^m &= 0 \\ y'(x) &= -ax^m - y^2(x) \end{aligned} \quad (1)$$

This is Riccati first order non-linear ODE of the form

$$y'(x) = P(x) + Q(x)y + R(x)y^2(x) \quad (2)$$

where in this case  $Q(x) = 0, R(x) = -1, P(x) = -ax^m$ . We can solve this in two ways. If we know one particular solution  $y_p(x)$  for (1) then we use the substitution  $y = y_p + \frac{1}{u}$  and convert (1) to new associated linear ODE of the form  $u' + (Q(x) + 2R(x))y_p + R(x) = 0$ . If we do not know a particular solution, then we use the standard substitution  $y = \frac{-u'}{uR(x)} = \frac{y'}{u}$  since  $R(x) = -1$  and this is what we will do here.

Since  $u' = yu$  then

$$\begin{aligned} u'' &= yu' + y'u \\ &= y(yu) + (-ax^m - y^2)u \\ &= y^2u - ax^m u - y^2u \\ &= -ax^m u \end{aligned}$$

So we have new second order ODE

$$u'' + ax^m u = 0 \quad (3)$$



which we solve for  $u$ . This is Airy ODE but with a positive sign. Of the form  $u'' + q(x)u = 0$ .

Recall that the solution to  $u'' - axu = 0$  is

$$u = c_1 Ai\left(a^{\frac{1}{3}}x\right) + c_2 Bi\left(a^{\frac{1}{3}}x\right)$$

When  $x$  has power on it (there are restriction on what values the power can take), the solution is written in terms of Bessel functions. The solution to  $u'' - ax^m u = 0$  is

$$u = c_1 \sqrt{x} BesselI\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right) + c_2 \sqrt{x} BesselK\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right)$$

When the sign is positive, the solution to  $u'' + ax^m u = 0$  is

$$u(x) = c_1 \sqrt{x} BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right) + c_2 \sqrt{x} BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right) \quad (4)$$

We need to find  $u'(x)$  now. From (4)

$$\frac{d}{dx} \left[ c_1 \sqrt{x} BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right) \right] = c_1 \frac{BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right) - \sqrt{ax}^{\frac{m+1}{2}} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right)}{\sqrt{x}}$$

And

$$\frac{d}{dx} \left[ c_2 \sqrt{x} BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right) \right] = c_2 \frac{BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right) - \sqrt{ax}^{\frac{m+1}{2}} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right)}{\sqrt{x}}$$

Therefore

$$u'(x) = c_1 \frac{BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right) - \sqrt{ax}^{\frac{m+1}{2}} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right)}{\sqrt{x}} + c_2 \frac{BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right) - \sqrt{ax}^{\frac{m+1}{2}} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax}^{\frac{m+1}{2}}}{m+2}\right)}{\sqrt{x}}$$

Since  $u' = yu$  then

$$\begin{aligned}
y &= \frac{u'}{u} \\
&= \frac{c_1 \frac{BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) - \sqrt{ax} \frac{m+1}{2} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right)}{\sqrt{x}} + c_2 \frac{BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) - \sqrt{ax} \frac{m+1}{2} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right)}{\sqrt{x}}}{c_1 \sqrt{x} BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) + c_2 \sqrt{x} BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right)} \\
&= \frac{c_1 \left[ BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) - \sqrt{ax} \frac{m+1}{2} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) \right] + c_2 \left[ BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) - \sqrt{ax} \frac{m+1}{2} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) \right]}{\sqrt{x} \left[ c_1 \sqrt{x} BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) + c_2 \sqrt{x} BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) \right]} \\
&= \frac{c_1 \left[ BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) - \sqrt{ax} \frac{m+1}{2} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) \right] + c_2 \left[ BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) - \sqrt{ax} \frac{m+1}{2} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) \right]}{c_1 x BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) + c_2 x BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right)}
\end{aligned}$$

Let  $C_1 = \frac{c_1}{c_2}$  then the above can be written as

$$y = \frac{1}{x} \frac{C_1 \left[ BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) - \sqrt{ax} \frac{m+1}{2} BesselJ\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) \right] + BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) - \sqrt{ax} \frac{m+1}{2} BesselY\left(\frac{m+3}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right)}{C_1 BesselJ\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right) + BesselY\left(\frac{1}{m+2}, \frac{2\sqrt{ax} \frac{m+1}{2}}\right)}$$

## 2.15 ODE No. 15

$$x^4 - 2x^2y(x) + y'(x) + y(x)^2 - 2x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0199354 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow -\frac{2}{2c_1 e^{2x} + 1} + x^2 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.29 (sec), leaf count = 35

$$\left\{ y(x) = \frac{x^2(e^x)^2 - x^2\_C1 + (e^x)^2 + \_C1}{(e^x)^2 - \_C1} \right\}$$

**Hand solution**

$$\begin{aligned}
x^4 - 2x^2y(x) + y'(x) + y^2(x) - 2x - 1 &= 0 \\
y'(x) &= -x^4 + 2x + 1 + 2x^2y(x) - y^2(x) \quad (1)
\end{aligned}$$

This is Riccati first order non-linear ODE of the form

$$y'(x) = P(x) + Q(x)y + R(x)y^2(x) \quad (2)$$

where in this case  $Q(x) = 2x^2$ ,  $R(x) = -1$ ,  $P(x) = -x^4 + 2x + 1$ .

Let  $u = y - x^2$  or  $y = u + x^2$  then

$$\begin{aligned}u' &= y' - 2x \\&= (-x^4 + 2x + 1 + 2x^2y - y^2) - 2x \\&= (-x^4 + 2x + 1 + 2x^2(u + x^2) - (u + x^2)^2) - 2x \\&= (-x^4 + 2x + 1 + 2x^2u + 2x^4 - (u^2 + x^4 + 2ux^2)) - 2x \\&= -x^4 + 2x + 1 + 2x^2u + 2x^4 - u^2 - x^4 - 2ux^2 - 2x \\&= 1 - u^2\end{aligned}$$

Hence

$$u' = 1 - u^2$$

This is separable

$$\begin{aligned}\frac{du}{dx} &= 1 - u^2 \\ \frac{du}{1 - u^2} &= dx\end{aligned}$$

Integrating both sides

$$\begin{aligned}\tanh^{-1}(u) &= x + C \\ u(x) &= \tanh(x + C) \\ &= \frac{e^{x+C} - e^{-x-C}}{e^{x+C} + e^{-x-C}} \\ &= \frac{e^x e^C - e^{-x} e^{-C}}{e^x e^C + e^{-x} e^{-C}}\end{aligned}$$

Multiplying numerator and denominator by  $e^{-C}e^x$

$$u(x) = \frac{e^{2x} - e^{-2C}}{e^{2x} + e^{-2C}}$$

Let  $e^{-2C} = C_1$

$$u(x) = \frac{e^{2x} - C_1}{e^{2x} + C_1}$$

Since  $u = y - x^2$  then

$$\begin{aligned}y &= u + x^2 \\ &= \frac{e^{2x} - C_1}{e^{2x} + C_1} + x^2 \\ &= \frac{e^{2x} - C_1 + x^2 e^{2x} + x^2 C_1}{e^{2x} + C_1}\end{aligned}$$

To obtain same solution as Maple, we divide by  $C_1$

$$y = \frac{\frac{1}{C_1}e^{2x} - 1 + \frac{1}{C_1}x^2e^{2x} + x^2}{\frac{1}{C_1}e^{2x} + 1}$$

Let  $\frac{1}{C_1} = -C$  then

$$\begin{aligned} y &= \frac{-Ce^{2x} - 1 - Cx^2e^{2x} + x^2}{-Ce^{2x} + 1} \\ &= \frac{Ce^{2x} + 1 + Cx^2e^{2x} - x^2}{Ce^{2x} - 1} \end{aligned}$$

Which now agrees with the Maple solution form. Mathematica solution also verified to be correct.

## 2.16 ODE No. 16

$$f(x)(xy(x) - 1) + y'(x) + y(x)^2 = 0$$

✗ **Mathematica** : cpu = 20.6974 (sec), leaf count = 0 , could not solve

`DSolve[y[x]^2 + f[x]*(-1 + x*y[x]) + Derivative[1][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.146 (sec), leaf count = 49

$$\left\{ y(x) = 1e^{\int \frac{-x^2 f(x) - 2}{x} dx} \left( -C_1 + \int e^{\int \frac{-x^2 f(x) - 2}{x} dx} dx \right)^{-1} + x^{-1} \right\}$$

**Hand solution**

$$\begin{aligned} y' + y^2 + (xy - 1)f &= 0 \\ y'(x) &= (-xy + 1)f - y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. We can see a particular solution is  $y_p = \frac{1}{x}$ , therefore, we use the substitution

$$\begin{aligned} y(x) &= y_p(x) + \frac{1}{u(x)} \\ &= \frac{1}{x} + \frac{1}{u} \end{aligned}$$

Hence

$$\begin{aligned} y'(x) &= y_p'(x) - \frac{u'(x)}{u^2(x)} \\ &= \frac{-1}{x^2} - \frac{u'(x)}{u^2(x)} \end{aligned} \tag{2}$$

Equating (1) and (2) gives

$$\begin{aligned}
 (-xy + 1)f - y^2 &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 \left(-x\left(\frac{1}{x} + \frac{1}{u}\right) + 1\right)f - \left(\frac{1}{x} + \frac{1}{u}\right)^2 &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 \left(\left(-1 - \frac{x}{u}\right) + 1\right)f - \left(\frac{1}{x^2} + \frac{1}{u^2} + \frac{2}{xu}\right) &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 -\frac{x}{u}f - \left(\frac{1}{x^2} + \frac{1}{u^2} + \frac{2}{xu}\right) &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 -\frac{x}{u}f - \frac{1}{x^2} - \frac{1}{u^2} - \frac{2}{xu} &= \frac{-1}{x^2} - \frac{u'}{u^2} \\
 -xuf - 1 - \frac{2u}{x} &= -u' \\
 u' &= xuf + 1 + \frac{2u}{x}
 \end{aligned}$$

Hence

$$u' - \left(xf + \frac{2}{x}\right)u = 1$$

Integrating factor is  $\mu = e^{\int(xf + \frac{2}{x})dx}$ , hence the solution is

$$d(\mu u) = \mu$$

Integrating both sides

$$\begin{aligned}
 \mu u &= \int \mu dx + C \\
 u &= e^{-\int(xf + \frac{2}{x})dx} \int e^{\int(xf + \frac{2}{x})dx} dx + C e^{-\int(xf + \frac{2}{x})dx} \\
 &= e^{-\int(xf + \frac{2}{x})dx} \left( \int e^{\int(xf + \frac{2}{x})dx} dx + C \right)
 \end{aligned}$$

Hence

$$\begin{aligned}
 y &= y_p + \frac{1}{u} \\
 &= \frac{1}{x} + \frac{1}{e^{-\int(xf + \frac{2}{x})dx} \left( \int e^{\int(xf + \frac{2}{x})dx} dx + C \right)}
 \end{aligned}$$

Hence

$$y(x) = \frac{1}{x} + e^{\int(xf + \frac{2}{x})dx} \left( \int e^{\int(xf + \frac{2}{x})dx} dx + C \right)^{-1}$$

2.17 ODE No. 17

$$y'(x) - y(x)^2 - 3y(x) + 4 = 0$$

✓ **Mathematica** : cpu = 0.0237199 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{-4e^{5(c_1+x)} - 1}{e^{5(c_1+x)} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 24

$$\left\{ y(x) = \frac{-4e^{5x} - C1 - 1}{-1 + e^{5x} - C1} \right\}$$

**Hand solution**

$$\begin{aligned} y' - y^2 - 3y + 4 &= 0 \\ y' &= 3y - 4 + y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. The general form is

$$y' = P(x) + Q(x)y + R(x)y^2$$

Where  $P(x) = -4$ ,  $Q(x) = 3$ ,  $R(x) = 1$ . Using the substitution  $y = -\frac{u'}{uR(x)} = \frac{-u'}{u}$  then

$$\begin{aligned} u' &= -yu \\ u'' &= -yu' - y'u \\ &= -y(-yu) - (3y - 4 + y^2)u \\ &= y^2u - 3\left(-\frac{u'}{u}\right)u + 4u - y^2u \\ &= 3u' + 4u \end{aligned}$$

Hence

$$u'' - 3u' - 4u = 0$$

This is standard second order ODE. The characteristic equation is  $\lambda^2 - 3\lambda - 4 = 0$ , with roots  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} = \{4, -1\}$ , hence

$$u(x) = c_1e^{4x} + c_2e^{-x}$$

And

$$u'(x) = c_14e^{4x} - c_2e^{-x}$$

Since  $y = \frac{-u'}{u}$  then

$$\begin{aligned} y(x) &= \frac{-c_1 4e^{4x} + c_2 e^{-x}}{c_1 e^{4x} + c_2 e^{-x}} \\ &= \frac{-\frac{c_1}{c_2} 4e^{4x} + e^{-x}}{\frac{c_1}{c_2} e^{4x} + e^{-x}} \end{aligned}$$

Let  $\frac{c_1}{c_2} = C_1$  then

$$y(x) = \frac{-4C_1 e^{4x} + e^{-x}}{C_1 e^{4x} + e^{-x}}$$

Dividing by  $e^{-x}$

$$y(x) = \frac{-4C_1 e^{5x} + 1}{C_1 e^{5x} + 1}$$

This is the same result given by CAS. To see it better, let  $C_2 = -C_1$  then the above becomes

$$\begin{aligned} y(x) &= \frac{4C_2 e^{5x} + 1}{-C_2 e^{5x} + 1} \\ &= -\frac{4C_2 e^{5x} + 1}{C_2 e^{5x} - 1} \end{aligned}$$

## 2.18 ODE No. 18

$$y'(x) - y(x)^2 - xy(x) - x + 1 = 0$$

✓ **Mathematica** : cpu = 0.0225362 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{2e^{\frac{1}{2}(x-2)^2}}{2e^2 c_1 - \sqrt{2\pi} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right)} - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 39

$$\left\{ y(x) = -1 + \frac{1}{-C1 + \frac{i}{2}\sqrt{\pi}e^{-2}\sqrt{2}\operatorname{Erf}\left(\frac{i}{2}\sqrt{2}(x-2)\right)} e^{\frac{x(x-4)}{2}} \right\}$$

**Hand solution**

$$\begin{aligned} y' - y^2 - xy - x + 1 &= 0 \\ y' &= x - 1 + xy + y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. The general form is

$$y' = P(x) + Q(x)y + R(x)y^2$$

Where  $P(x) = x - 1, Q(x) = x, R(x) = 1$ . We see that  $y_p = -1$  is a particular solution, therefore we use the substitution  $y = y_p + \frac{1}{u}$ , hence  $y' = -\frac{u'}{u^2}$  and equating this to (1) we obtain

$$\begin{aligned} -\frac{u'}{u^2} &= x - 1 + xy + y^2 \\ &= x - 1 + x\left(-1 + \frac{1}{u}\right) + \left(-1 + \frac{1}{u}\right)^2 \\ &= x - 1 - x + \frac{x}{u} + \left(1 + \frac{1}{u^2} - \frac{2}{u}\right) \\ &= \frac{x}{u} + \frac{1}{u^2} - \frac{2}{u} \end{aligned}$$

Hence

$$\begin{aligned} u' &= -u^2\left(\frac{x}{u} + \frac{1}{u^2} - \frac{2}{u}\right) \\ &= -xu - 1 + 2u \\ u' + xu - 2u &= -1 \\ u' + u(x - 2) &= -1 \end{aligned}$$

Integration factor is  $e^{\int(x-2)dx} = e^{\frac{x^2}{2}-2x} = e^{\frac{1}{2}x(x-4)}$ , therefore

$$d\left(e^{\frac{1}{2}x(x-4)}u\right) = -e^{\frac{1}{2}x(x-4)}$$

Integrating both sides

$$e^{\frac{1}{2}x(x-4)}u = -\int e^{\frac{1}{2}x(x-4)} + C$$

But

$$\int e^{\frac{1}{2}x(x-4)} = \frac{1}{e^2} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right)$$

Hence

$$u(x) = e^{-\frac{1}{2}x(x-4)} \left( \frac{-1}{e^2} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + C \right)$$

Since  $y = y_p + \frac{1}{u}$  then

$$y = -1 + \frac{1}{e^{-\frac{1}{2}x(x-4)} \left( \frac{-1}{e^2} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right) + C \right)}$$

Or

$$y = \frac{e^{\frac{1}{2}x(x-4)}}{C - \frac{1}{e^2} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{x-2}{\sqrt{2}}\right)} - 1$$



2.19 ODE No. 19

$$y'(x) - (y(x) + x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0104082 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 e^{2ix} - \frac{i}{2}} - x - i \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 16

$$\{y(x) = -x - \tan(-x + \_C1)\}$$

**Hand solution**

$$\begin{aligned} y' - (y + x)^2 &= 0 \\ y' &= (y + x)^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form. Let  $u = y + x$ , then  $u' = y' + 1$  and (1) becomes

$$\begin{aligned} u' - 1 &= u^2 \\ u' &= 1 + u^2 \end{aligned}$$

This is separable

$$\begin{aligned} \frac{du}{dx} \frac{1}{1 + u^2} &= 1 \\ \int \frac{du}{1 + u^2} &= \int dx \\ \tan^{-1} u &= x + C \\ u &= \tan(x + C) \end{aligned}$$

Since  $u = y + x$  then

$$y = \tan(x + C) - x$$

2.20 ODE No. 20

$$(x^2 + 1)y(x) + y'(x) - y(x)^2 - 2x = 0$$

✓ **Mathematica** : cpu = 0.73127 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\frac{x^3}{3}+x}}{c_1 - \int_1^x e^{\frac{K[1]^3}{3}+K[1]} dK[1]} + x^2 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 34

$$\left\{ y(x) = x^2 + 1 + 1e^{\frac{x^3}{3}+x} \left( -C1 - \int e^{\frac{x^3}{3}+x} dx \right)^{-1} \right\}$$

**Hand solution**

$$\begin{aligned} (x^2 + 1)y + y' - y^2 - 2x &= 0 \\ y' &= -(x^2 + 1)y + y^2 + 2x \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form of the general form  $y' = P(x) + Q(x)y + R(x)y^2$  where  $P(x) = 2x$ ,  $Q(x) = -(x^2 + 1)$ ,  $R(x) = 1$ . We can convert this to Bernoulli first order ODE in  $u(x)$ , which is little easier to solve by using  $u = y - x^2 - 1$ . The difference between Bernoulli and Riccati is that the term  $P(x) = 0$  in Bernoulli. If  $P(x) \neq 0$  and  $R(x) \neq 0$  then it is called Riccati.

Using  $u = y - x^2 - 1$  gives

$$\begin{aligned} u' &= y' - 2x \\ u' &= [-(x^2 + 1)y + y^2 + 2x] - 2x \\ &= -(x^2 + 1)(u + x^2 + 1) + (u + x^2 + 1)^2 \\ &= (u + x^2 + 1)[(u + x^2 + 1) - (x^2 + 1)] \\ &= (u + x^2 + 1)u \\ &= u^2 + u(1 + x^2) \end{aligned}$$

We see now this is Bernoulli since  $P(x) = 0$ . To solve Bernoulli we always start by dividing by  $u^2$  giving

$$\frac{u'}{u^2} = 1 + \frac{1}{u}(1 + x^2)$$

Next we let  $v = \frac{1}{u}$ , hence  $v' = -\frac{u'}{u^2}$  therefore the above becomes

$$\begin{aligned} -v' &= 1 + v(1 + x^2) \\ v' + v(1 + x^2) &= -1 \end{aligned}$$

Integrating factor is  $e^{\int(1+x^2)dx} = e^{\left(x+\frac{x^3}{2}\right)}$ , therefore

$$d\left(e^{\left(x+\frac{x^3}{2}\right)}v\right) = -e^{\left(x+\frac{x^3}{2}\right)}$$

Integrating

$$\begin{aligned} e^{\left(x+\frac{x^3}{2}\right)}v &= -\int e^{\left(x+\frac{x^3}{2}\right)}dx + C \\ v(x) &= e^{-\left(x+\frac{x^3}{2}\right)}\left(C - \int e^{\left(x+\frac{x^3}{2}\right)}dx\right) \end{aligned}$$

Therefore

$$u = \frac{1}{v} = \frac{e^{\left(x+\frac{x^3}{2}\right)}}{\left(C - \int e^{\left(x+\frac{x^3}{2}\right)}dx\right)}$$

And since  $u = y - x^2 - 1$  then

$$\begin{aligned} y(x) &= u + 1 + x^2 \\ &= \frac{e^{\left(x+\frac{x^3}{2}\right)}}{\left(C - \int e^{\left(x+\frac{x^3}{2}\right)}dx\right)} + 1 + x^2 \end{aligned}$$

## 2.21 ODE No. 21

$$y'(x) - y(x)^2 + y(x) \sin(x) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 6.29656 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \sin(x) \left( \int_1^x e^{-\cos(K[1])} dK[1] \right) + c_1 \left( -e^{-\cos(x)} \right) + \sin(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 25

$$\left\{ y(x) = -\frac{e^{-\cos(x)}}{-C1 + \int e^{-\cos(x)} dx} + \sin(x) \right\}$$

**Hand solution**

$$\begin{aligned} y' - y^2 + y \sin(x) - \cos(x) &= 0 \\ y' &= y^2 - y \sin(x) + \cos(x) \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE of the form of the general form  $y' = P(x) + Q(x)y + R(x)y^2$  where  $P(x) = \cos(x)$ ,  $Q(x) = -\sin(x)$ ,  $R(x) = 1$ . It is best to first try to spot a particular solution  $y_p$  and use the transformation  $y = y_p + \frac{1}{u}$  otherwise we use  $y = -\frac{u'}{yR(x)}$  transformation. For this problem

$$y_p = \sin(x)$$

Therefore

$$y = \sin x + \frac{1}{u}$$

$$y' = \cos x - \frac{u'}{u^2}$$

Equating this to (1) gives

$$y^2 - y \sin(x) + \cos(x) = \cos x - \frac{u'}{u^2}$$

$$\left(\sin x + \frac{1}{u}\right)^2 - \left(\sin x + \frac{1}{u}\right) \sin x + \cos x = \cos x - \frac{u'}{u^2}$$

$$\sin^2 x + \frac{1}{u^2} + \frac{2}{u} \sin x - \sin^2 x - \frac{1}{u} \sin x = -\frac{u'}{u^2}$$

$$\frac{1}{u^2} + \frac{1}{u} \sin x = -\frac{u'}{u^2}$$

$$1 + u \sin x = -u'$$

$$u' + u \sin x = -1$$

Integrating factor is  $e^{\int \sin x} = e^{-\cos x}$ , hence

$$d(e^{-\cos x} u) = -e^{-\cos x}$$

Integrating both sides

$$e^{-\cos x} u = -\int e^{-\cos x} dx + C$$

$$u = e^{\cos x} \left( C - \int e^{-\cos x} dx \right)$$

Since  $y = \sin x + \frac{1}{u}$  then

$$y = \sin x + \frac{e^{-\cos x}}{C - \int e^{-\cos x} dx}$$

Or letting  $C_1 = -C$  to make match Maple form, we obtain

$$y = -\frac{e^{-\cos x}}{C_1 + \int e^{-\cos x} dx} + \sin x$$

## 2.22 ODE No. 22

$$y'(x) - y(x)^2 - y(x) \sin(2x) - \cos(2x) = 0$$

✗ **Mathematica** : cpu = 0 (sec), leaf count = 0 , crash

Kernel Crash

✓ **Maple** : cpu = 0.408 (sec), leaf count = 128

$$\left\{ y(x) = 2 \frac{\sin(2x)}{\sqrt{2 \cos(2x) + 2}} \left( -C1 (\cos(2x) + 1) \operatorname{HeunCPrime} \left( 1, 1/2, -1/2, -1, \frac{7}{8}, 1/2 \cos(2x) + 1/2 \right) + \operatorname{HeunCPrime} \left( 1, 1/2, -1/2, -1, \frac{7}{8}, 1/2 \cos(2x) + 1/2 \right) \right) + \operatorname{HeunCPrime} \left( 1, 1/2, -1/2, -1, \frac{7}{8}, 1/2 \cos(2x) + 1/2 \right) \right\}$$

**Hand solution**

$$\begin{aligned} y' - y^2 - y \sin(2x) - \cos(2x) &= 0 \\ y' &= y^2 + y \sin(2x) + \cos(2x) \end{aligned} \quad (1)$$

This is Riccati first order non-linear ODE of the form of the general form  $y' = P(x) + Q(x)y + R(x)y^2$  where  $P(x) = \cos(2x)$ ,  $Q(x) = \sin(2x)$ ,  $R(x) = 1$ . It is best to first try to spot a particular solution  $y_p$  and use the transformation  $y = y_p + \frac{1}{u}$  otherwise we use  $y = -\frac{u'}{yR(x)}$  transformation. For this problem

$$y_p = \tan(x)$$

To verify, since  $y'_p = \frac{1}{\cos^2 x}$  then plugging this particular in (1) gives

$$\frac{1}{\cos^2 x} - \tan^2(x) - \tan(x) \sin(2x) - \cos(2x) = 0$$

But  $\cos(2x) = \cos^2 x - \sin^2 x$  and  $\sin(2x) = 2 \sin x \cos x$  and  $\tan(x) = \frac{\sin x}{\cos x}$  therefore the above becomes

$$\begin{aligned}
\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - \frac{\sin x}{\cos x}(2 \sin x \cos x) - (\cos^2 x - \sin^2 x) &= 0 \\
\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - 2 \sin^2 x - \cos^2 x + \sin^2 x &= 0 \\
\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - \sin^2 x - \cos^2 x &= 0 \\
\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - 1 &= 0 \\
\frac{1 - \sin^2 x}{\cos^2 x} - 1 &= 0 \\
\frac{\cos^2 x}{\cos^2 x} - 1 &= 0 \\
1 - 1 &= 0 \\
0 &= 0
\end{aligned}$$

Therefore we, we can use  $y = y_p + \frac{1}{u}$

$$\begin{aligned}
y &= \tan x + \frac{1}{u} \\
y' &= \frac{1}{\cos^2 x} - \frac{u'}{u^2}
\end{aligned}$$

Equating this to (1) gives

$$\begin{aligned}
-\frac{u'}{u^2} &= y^2 + y \sin(2x) + \cos(2x) \\
-\frac{u'}{u^2} &= -\frac{1}{\cos^2 x} + \left(\tan x + \frac{1}{u}\right)^2 + \left(\tan x + \frac{1}{u}\right) \sin(2x) + \cos(2x)
\end{aligned}$$

Using  $\sin(2x) = 2 \sin x \cos x$  and  $\cos 2x = \cos^2 x - \sin^2 x$  then above becomes

$$\begin{aligned}
-\frac{u'}{u^2} &= -\frac{1}{\cos^2 x} + \left( \tan^2 x + \frac{1}{u^2} + \frac{2}{u} \tan x \right) + \left( \frac{\sin x}{\cos x} + \frac{1}{u} \right) 2 \sin x \cos x + (\cos^2 x - \sin^2 x) \\
u' &= \frac{u^2}{\cos^2 x} - \left( u^2 \frac{\sin^2 x}{\cos^2 x} + 1 + 2u \frac{\sin x}{\cos x} \right) - \left( u^2 \frac{\sin x}{\cos x} + u \right) 2 \sin x \cos x - u^2 \cos^2 x + u^2 \sin^2 x \\
&= \frac{u^2}{\cos^2 x} - u^2 \frac{\sin^2 x}{\cos^2 x} - 1 - 2u \frac{\sin x}{\cos x} - 2u^2 \frac{\sin x}{\cos x} \sin x \cos x - 2u \sin x \cos x - u^2 \cos^2 x + u^2 \sin^2 x \\
&= \frac{u^2}{\cos^2 x} - u^2 \frac{\sin^2 x}{\cos^2 x} - 1 - 2u \frac{\sin x}{\cos x} - 2u^2 \sin^2 x - 2u \sin x \cos x - u^2 \cos^2 x + u^2 \sin^2 x \\
&= \frac{u^2}{\cos^2 x} - u^2 \frac{\sin^2 x}{\cos^2 x} - 1 - 2u \frac{\sin x}{\cos x} - u^2 \sin^2 x - 2u \sin x \cos x - u^2 \cos^2 x \\
&= u^2 \left( \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} - (\sin^2 x + \cos^2 x) \right) - 1 + u \left( -2 \frac{\sin x}{\cos x} - 2 \sin x \cos x \right) \\
&= u^2 \left( \frac{1 - \sin^2 x}{\cos^2 x} - 1 \right) - 1 + u \left( -2 \frac{\sin x}{\cos x} - 2 \sin x \cos x \right) \\
&= u^2 \left( \frac{\cos^2 x}{\cos^2 x} - 1 \right) - 1 + u \left( -2 \frac{\sin x}{\cos x} - 2 \sin x \cos x \right) \\
&= -1 + 2u \left( -\frac{\sin x}{\cos x} - \sin x \cos x \right)
\end{aligned}$$

Hence

$$u' + 2u(\tan x + \sin x \cos x) = -1$$

Integrating factor is  $e^{2 \int \tan x + \sin x \cos x dx}$ . But

$$\int \tan x dx = -\ln(\cos x)$$

And

$$\int \sin x \cos x dx = \frac{-1}{2} \cos^2 x$$

Hence  $\mu = e^{-2 \ln \cos x} e^{-\cos^2 x} = \frac{1}{\cos^2 x} e^{-\cos^2 x}$ , therefore

$$d \left( \frac{1}{\cos^2 x} e^{-\cos^2 x} u \right) = \frac{-1}{\cos^2 x} e^{-\cos^2 x}$$

Integrating both sides

$$\begin{aligned}
\frac{1}{\cos^2 x} e^{-\cos^2 x} u &= -\int \frac{e^{-\cos^2 x}}{\cos^2 x} dx + C \\
u &= \cos^2 x e^{\cos^2 x} \left( C - \int \frac{e^{-\cos^2 x}}{\cos^2 x} dx \right)
\end{aligned}$$

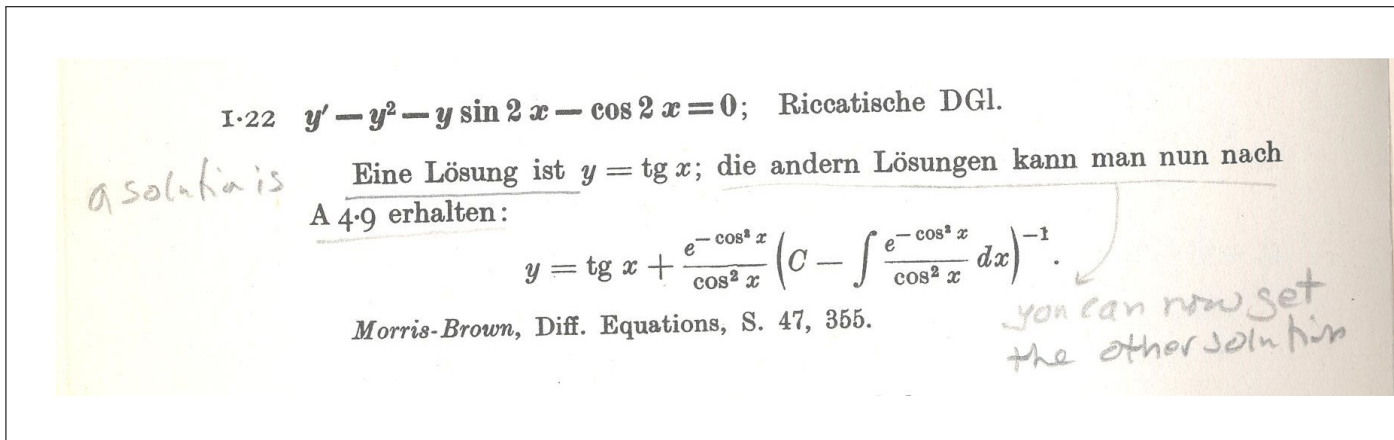
Since  $y = \tan x + \frac{1}{u}$  then

$$y = \tan x + \frac{1}{\cos^2 x e^{\cos^2 x} \left( C - \int \frac{e^{-\cos^2 x}}{\cos^2 x} dx \right)}$$

$$= \tan x + \frac{e^{-\cos^2 x}}{\cos^2 x} \left( C - \int \frac{e^{-\cos^2 x}}{\cos^2 x} dx \right)^{-1}$$

I do not know how Maple came up with the solution involving HeunC functions since  $\int \frac{e^{-\cos^2 x}}{\cos^2 x} dx$  has no closed form solution. I should ask CAS experts about this.

Below is screen shot from Kamke book of the solution it gives, which matches the above result



## 2.23 ODE No. 23

$$ay(x)^2 - b + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0227724 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{b} \tanh(\sqrt{a}\sqrt{b}(c_1 + x))}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 23

$$\left\{ y(x) = \frac{1}{a} \tanh(\sqrt{ab}(x + \_C1)) \sqrt{ab} \right\}$$

**Hand solution**



$$y' + ay^2 - b = 0$$

$$\frac{dy}{dx} = b - ay^2$$

Separable,

$$\frac{dy}{b - ay^2} = dx$$

$$\int \frac{dy}{b - ay^2} = \int dx$$

But

$$\int \frac{dy}{b - ay^2} = \frac{1}{\sqrt{ab}} \tanh^{-1} \left( \sqrt{\frac{a}{b}} y \right)$$

Hence

$$\frac{1}{\sqrt{ab}} \tanh^{-1} \left( \sqrt{\frac{a}{b}} y \right) = x + C$$

$$\tanh^{-1} \left( \sqrt{\frac{a}{b}} y \right) = \sqrt{ab}(x + C)$$

$$\sqrt{\frac{a}{b}} y = \tanh \left( \sqrt{ab}(x + C) \right)$$

$$y = \sqrt{\frac{b}{a}} \tanh \left( \sqrt{ab}(x + C) \right)$$

## 2.24 ODE No. 24

$$ay(x)^2 - bx^\nu + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.013633 (sec), leaf count = 276

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-a}\sqrt{b}x^{\frac{\nu}{2}+1} \left( c_1 \left( J_{\frac{\nu+1}{\nu+2}} \left( \frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2} \right) - J_{-\frac{\nu+3}{\nu+2}} \left( \frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2} \right) \right) - 2J_{\frac{1}{\nu+2}-1} \left( \frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2} \right) \right) - c_1 J_{-\frac{1}{\nu+2}} \left( \frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2} \right)}{2ax \left( c_1 J_{-\frac{1}{\nu+2}} \left( \frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2} \right) + J_{\frac{1}{\nu+2}} \left( \frac{2\sqrt{-a}\sqrt{b}x^{\frac{\nu}{2}+1}}{\nu+2} \right) \right)} \right\} \right.$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 201

$$\left\{ y(x) = \frac{1}{ax} \left( - \left( J_{\frac{3+\nu}{\nu+2}} \left( 2 \frac{\sqrt{-ab}x^{\nu/2+1}}{\nu+2} \right) - C1 + Y_{\frac{3+\nu}{\nu+2}} \left( 2 \frac{\sqrt{-ab}x^{\nu/2+1}}{\nu+2} \right) \right) \sqrt{-ab}x^{\frac{\nu}{2}+1} + -C1 J_{(\nu+2)^{-1}} \left( 2 \frac{\sqrt{-ab}x^{\nu/2+1}}{\nu+2} \right) \right.$$

Hand solution

$$\begin{aligned}
y' + ay^2 - bx^v &= 0 \\
y' &= bx^v - ay^2 \\
&= P(x) + Q(x)y + R(x)y^2
\end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE with  $P(x) = bx^v$ ,  $Q(x) = 0$ ,  $R(x) = -a$ . Using the standard substitution

$$y = -\frac{u'}{uR(x)} = \frac{u'}{au}$$

Hence

$$y' = \frac{u''}{au} - \frac{(u')^2}{au^2}$$

Therefore (1) becomes

$$\begin{aligned}
\frac{u''}{au} - \frac{(u')^2}{au^2} &= bx^v - ay^2 \\
&= bx^v - a\left(\frac{u'}{au}\right)^2 \\
&= bx^v - \frac{(u')^2}{au^2}
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{u''}{au} &= bx^v \\
u'' - abx^v u &= 0
\end{aligned}$$

This is an Emden-Fowler equation, of the general form  $u'' = Ax^n u^m$ , where here  $m = 1$  and  $n = v$  and  $A = ab$ .

For any  $n$ , the solution uses Bessel functions and modified Bessel functions of first and second kind. From Handbook of exact solutions for ODE, page 237, equation 2.1.2.7 we see the solution is given as

$$u = \begin{cases} C_1 \sqrt{x} J_{\frac{1}{2q}}\left(\frac{\sqrt{-ab}}{q} x^q\right) + C_2 \sqrt{x} Y_{\frac{1}{2q}}\left(\frac{\sqrt{-ab}}{q} x^q\right) & ab < 0 \\ C_1 \sqrt{x} I_{\frac{1}{2q}}\left(\frac{\sqrt{ab}}{q} x^q\right) + C_2 \sqrt{x} K_{\frac{1}{2q}}\left(\frac{\sqrt{ab}}{q} x^q\right) & ab > 0 \end{cases}$$

Where  $q = \frac{n+1}{2}$ .  $J$  is Bessel function of first kind and  $Y$  is Bessel function of second kind.  $I$  is modified Bessel function of first kind and  $K$  is modified Bessel function of second kind. To find  $y$  we now use  $y = \frac{u'}{au}$ . Derivative of Bessel functions is given by

$$\begin{aligned} J'_m(x) &= \frac{1}{2}(J_{m-1}(x) - J_{m+1}(x)) \\ Y'_m(x) &= \frac{1}{2}(Y_{m-1}(x) - Y_{m+1}(x)) \\ I'_m(x) &= \frac{1}{2}(I_{m-1}(x) + I_{m+1}(x)) \\ K'_m(x) &= -\frac{1}{2}(K_{m-1}(x) + K_{m+1}(x)) \end{aligned}$$

Using these, then

$$u' = \begin{cases} C_1 \left[ \frac{1}{2\sqrt{x}} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} J'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] + C_2 \left[ \frac{1}{2\sqrt{x}} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} Y'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] & ab < 0 \\ C_1 \left[ \frac{1}{2\sqrt{x}} I_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) + \sqrt{x} I'_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) \right] + C_2 \left[ \frac{1}{2\sqrt{x}} K_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) + \sqrt{x} K'_{\frac{1}{2q}} \left( \frac{\sqrt{ab}}{q} x^q \right) \right] & ab > 0 \end{cases}$$

Hence for  $ab < 0$

$$\begin{aligned} y &= \frac{u'}{au} \\ &= \frac{C_1 \left[ \frac{1}{2\sqrt{x}} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} J'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] + C_2 \left[ \frac{1}{2\sqrt{x}} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \sqrt{x} Y'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right]}{aC_1 \sqrt{x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 \sqrt{x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)} \\ &= \frac{\sqrt{x} C_1 \left[ \frac{1}{2x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + J'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] + \sqrt{x} C_2 \left[ \frac{1}{2x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + Y'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right]}{aC_1 \sqrt{x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 \sqrt{x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)} \\ &= \frac{C_1 \left[ \frac{1}{2x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + J'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right] + C_2 \left[ \frac{1}{2x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + Y'_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right]}{aC_1 J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)} \end{aligned}$$

Using derivatives the above becomes

$$y = \frac{C_1 \left[ \frac{1}{2x} J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \frac{1}{2} \left( J_{\frac{1}{2q}-1} \left( \frac{\sqrt{-ab}}{q} x^q \right) - J_{\frac{1}{2q}+1} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right) \right]}{aC_1 J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)} + \frac{C_2 \left[ \frac{1}{2x} Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + \frac{1}{2} \left( Y_{\frac{1}{2q}-1} \left( \frac{\sqrt{-ab}}{q} x^q \right) - Y_{\frac{1}{2q}+1} \left( \frac{\sqrt{-ab}}{q} x^q \right) \right) \right]}{aC_1 J_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right) + aC_2 Y_{\frac{1}{2q}} \left( \frac{\sqrt{-ab}}{q} x^q \right)}$$

Similar result can be found for  $ab > 0$

## 2.25 ODE No. 25

$$ay(x)^2 - bx^{2\nu} - cx^{\nu-1} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.229701 (sec), leaf count = 618

$$\left\{ \left\{ \begin{array}{l} x^\nu \left( \sqrt{bc_1(\nu+1)} \sqrt{(\nu+1)^2} U \left( \frac{\sqrt{b}\sqrt{(\nu+1)^2\nu+\sqrt{ac}(\nu+1)}}{2\sqrt{b(\nu+1)}\sqrt{(\nu+1)^2}}, \frac{\nu}{\nu+1}, \frac{2\sqrt{a}\sqrt{b}x^{\nu+1}}{\sqrt{(\nu+1)^2}} \right) + c_1 \left( \sqrt{ac}(\nu+1) + \sqrt{b}\sqrt{(\nu+1)^2} \right) \right. \right. \\ \left. \left. y(x) \rightarrow \frac{\sqrt{a}(\nu+1)^2 \left( c_1 \right)}{\sqrt{a}(\nu+1)^2} \right. \right. \end{array} \right.$$

✓ **Maple** : cpu = 0.309 (sec), leaf count = 348

$$\left\{ y(x) = -\frac{1}{2ax} \left( ((-\nu-2)b^{\frac{3}{2}} + \sqrt{abc}) M_{-\frac{1}{2\nu+2}}((-2\nu-2)\sqrt{b+\sqrt{ac}})^{\frac{1}{\sqrt{b}}}, (2\nu+2)^{-1} \left( 2 \frac{\sqrt{a}\sqrt{b}x^{\nu+1}}{\nu+1} \right) + 2b^{3/2} - C_1(\nu+1) \right) \right.$$

**Hand solution**

$$\begin{aligned} y' + ay^2 - bx^{2\nu} - cx^{\nu-1} &= 0 \\ y' &= bx^\nu + cx^{\nu-1} - ay^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE with  $P(x) = bx^\nu + cx^{\nu-1}$ ,  $Q(x) = 0$ ,  $R(x) = -a$ .

Need to do this later.

2.26 ODE No. 26

$$y'(x) - (Ay(x) - a)(By(x) - b) = 0$$

✓ **Mathematica** : cpu = 0.0773237 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow \frac{ae^{Ab(c_1+x)} - be^{aB(c_1+x)}}{Ae^{Ab(c_1+x)} - Be^{aB(c_1+x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.064 (sec), leaf count = 45

$$\left\{ y(x) = \frac{e^{(x+C_1)(Ab-aB)}a - b}{e^{(x+C_1)(Ab-aB)}A - B} \right\}$$

**Hand solution**

$$\begin{aligned} y' - (Ay - a)(By - b) &= 0 \\ y' &= (Ay - a)(By - b) \\ &= ab - y(Ab + Ba) + AB y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE with  $P(x) = ab, Q(x) = -(Ab + Ba), R(x) = AB$ .

Let  $y = -\frac{u'}{uR(x)} = -\frac{u'}{ABu}$ , hence

$$y' = \frac{-u''}{ABu} - \frac{(u')^2}{ABu^2}$$

Comparing to (1) results in

$$\begin{aligned} \frac{-u''}{ABu} - \frac{(u')^2}{ABu^2} &= ab - y(Ab + Ba) + AB y^2 \\ &= ab - \left(-\frac{u'}{ABu}\right)(Ab + Ba) + AB \left(-\frac{u'}{ABu}\right)^2 \\ &= ab + \frac{u'}{ABu}(Ab + Ba) + AB \frac{(u')^2}{(ABu)^2} \\ &= ab + \frac{u'}{ABu}(Ab + Ba) + \frac{(u')^2}{ABu^2} \end{aligned}$$

Hence

$$\begin{aligned} \frac{-u''}{ABu} &= ab + \frac{u'}{ABu}(Ab + Ba) \\ -u'' &= ABabu + u'(Ab + Ba) \\ u'' + u'(Ab + Ba) + u(ABab) &= 0 \end{aligned}$$

This is second order ODE with constant coefficient. Solution is

$$u = c_1 e^{-aBx} + c_2 e^{-Abx}$$

Therefore

$$u' = -aBc_1 e^{-aBx} - c_2 A b e^{-Abx}$$

And therefore the solution is

$$\begin{aligned} y &= -\frac{u'}{ABu} = -\frac{1}{AB} \frac{-aBc_1 e^{-aBx} - c_2 A b e^{-Abx}}{c_1 e^{-aBx} + c_2 e^{-Abx}} \\ &= \frac{aBc_1 e^{-aBx} + c_2 A b e^{-Abx}}{AB(c_1 e^{-aBx} + c_2 e^{-Abx})} \end{aligned}$$

Dividing by  $c_2$  and letting  $c = \frac{c_1}{c_2}$

$$y = \frac{aBc e^{-aBx} + A b e^{-Abx}}{AB(c e^{-aBx} + e^{-Abx})}$$

Verification

```
eq:=diff(y(x),x)-(A*y(x)-a)*(B*y(x)-b) = 0;
sol:=(a*B*_C1*exp(-a*B*x)+A*b*exp(-A*b*x))/(A*B*(C1*exp(-a*B*x)+exp(-A*b*x)));
odetest(y(x)=sol,eq);
0
```

2.27 ODE No. 27

$$ay(x)(y(x) - x) + y'(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0459351 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{2\pi}c_1 x \operatorname{erf}\left(\frac{\sqrt{ax}}{\sqrt{2}}\right) + \frac{2\left(c_1 e^{-\frac{ax^2}{2}} + ax\right)}{\sqrt{a}}}{\sqrt{2\pi}c_1 \operatorname{erf}\left(\frac{\sqrt{ax}}{\sqrt{2}}\right) + 2\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.196 (sec), leaf count = 72

$$\left\{ y(x) = 1 \left( 2\sqrt{a}e^{-1/2ax^2} + x \left( \sqrt{2}\sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{2}x}{2}\sqrt{a}\right) a + 2a^{3/2} - C1 \right) \right) \left( \sqrt{2}\sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{2}x}{2}\sqrt{a}\right) a + 2a^{3/2} - C1 \right) \right\}$$

**Hand solution**

$$\begin{aligned} y' + ay(y - x) - 1 &= 0 \\ y' &= 1 - (ay^2 - ayx) \\ &= 1 + ayx - ay^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE  $y' = P(x) + A(x)y + R(x)y^2$  with  $P(x) = 1, Q(x) = -ax, R(x) = -a$ . We can convert Riccati to Bernoulli which is easier to solve using the substitution  $u = y - x$

$$\begin{aligned} u' &= y' - 1 \\ &= (1 + ayx - ay^2) - 1 \\ &= \left( 1 + a(u + x)x - a(u + x)^2 \right) - 1 \\ &= 1 + aux + ax^2 - a(u^2 + x^2 + 2ux) - 1 \\ &= 1 + aux + ax^2 - au^2 - ax^2 - 2aux - 1 \\ &= -aux - au^2 \\ u' &= -aux - au^2 \end{aligned}$$

This is of the form  $u' = P(x) + Q(x)u + R(x)u^2$  and since  $P(x) = 0$  then it is Bernoulli differential equation. (when  $P(x) \neq 0$  and  $R(x) \neq 0$  it is Riccati). To solve Bernoulli we always start by dividing by  $u^2$

$$\frac{u'}{u^2} = -\frac{ax}{u} - a$$

Then we let  $\zeta = \frac{1}{u}$ , hence  $\zeta' = -\frac{u'}{u^2}$ , therefore the above becomes

$$\begin{aligned} -\zeta' &= -ax\zeta - a \\ \zeta' - ax\zeta &= a \end{aligned}$$

Integrating factor is  $e^{-\int ax dx} = e^{-a\frac{x^2}{2}}$ , hence  $d\left(e^{-a\frac{x^2}{2}}\zeta\right) = ae^{-a\frac{x^2}{2}}$ . Integrating both sides gives

$$e^{-a\frac{x^2}{2}}\zeta = a \int e^{-a\frac{x^2}{2}} dx + C$$

But

$$\int e^{-a\frac{x^2}{2}} dx = \sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right)$$

Therefore

$$\begin{aligned} e^{-a\frac{x^2}{2}}\zeta &= a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C \\ \zeta &= e^{a\frac{x^2}{2}}\left(a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C\right) \end{aligned}$$

Hence

$$\begin{aligned} u &= \frac{1}{\zeta} \\ &= e^{-a\frac{x^2}{2}}\left(a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C\right)^{-1} \end{aligned}$$

Since  $u = y - x$  then

$$\begin{aligned} y &= u + x \\ &= e^{-a\frac{x^2}{2}}\left(a\sqrt{\frac{\pi}{2a}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C\right)^{-1} + x \\ &= \frac{e^{-a\frac{x^2}{2}}}{\sqrt{\frac{a\pi}{2}} \operatorname{erf}\left(\sqrt{\frac{a}{2}}x\right) + C} + x \end{aligned}$$

Verification

```
eq:=diff(y(x),x)+a*y(x)*(y(x)-x)-1 = 0;
sol:=exp(-a*x^2/2)/(sqrt(a*Pi/2)*erf(sqrt(a/2)*x)+_C1)+x;
odetest(y(x)=sol,eq);
0
```



2.28 ODE No. 28

$$x^3(-y(x)) + y'(x) + xy(x)^2 - 2x = 0$$

✓ **Mathematica** : cpu = 0.0486214 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow \frac{2c_1 x^2 + \sqrt{\pi} x^2 \operatorname{erf}\left(\frac{x^2}{2}\right) + 2e^{-\frac{x^4}{4}}}{2c_1 + \sqrt{\pi} \operatorname{erf}\left(\frac{x^2}{2}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1}{\sqrt{\pi}} \left( \operatorname{Erf}\left(\frac{x^2}{2}\right) \sqrt{\pi} - C1 x^2 + x^2 \sqrt{\pi} + 2 e^{-1/4 x^4} - C1 \right) \left( \operatorname{Erf}\left(\frac{x^2}{2}\right) - C1 + 1 \right)^{-1} \right\}$$

**Hand solution**

$$\begin{aligned} y' - yx^3 + xy^2 - 2x &= 0 \\ y' &= 2x + yx^3 - xy^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE with  $P(x) = 2x, Q(x) = x^3, R(x) = -x$ . We can convert Riccati to Bernoulli which is easier to solve using the substitution  $u = x^2 - y$  or  $y = x^2 - u$

$$\begin{aligned} u' &= 2x - y' \\ &= 2x - (2x + yx^3 - xy^2) \\ &= 2x - (2x + (x^2 - u)x^3 - x(x^2 - u)^2) \\ &= 2x - (2x + (x^5 - ux^3) - x(x^4 + u^2 - 2x^2u)) \\ u' &= 2x - (2x + (x^5 - ux^3) - (x^5 + xu^2 - 2x^3u)) \\ &= 2x - 2x - (x^5 - ux^3) + (x^5 + xu^2 - 2x^3u) \\ &= -x^5 + ux^3 + x^5 + xu^2 - 2x^3u \\ &= -ux^3 + xu^2 \end{aligned}$$

This is of the form  $u' = P(x) + Q(x)u + R(x)u^2$  and since  $P(x) = 0$  then it is Bernoulli differential equation. (when  $P(x) \neq 0$  and  $R(x) \neq 0$  it is Riccati). To solve Bernoulli we always start by dividing by  $u^2$

$$\frac{u'}{u^2} = -\frac{1}{u}x^3 + x$$

Then we let  $\zeta = -\frac{1}{u}$ , hence  $\zeta' = \frac{u'}{u^2}$ , therefore the above becomes

$$\begin{aligned}\zeta' &= x^3\zeta + x \\ \zeta' - x^3\zeta &= x\end{aligned}$$

Integrating factor is  $e^{-\int x^3 dx} = e^{-\frac{x^4}{4}}$ , hence

$$d\left(e^{-\frac{x^4}{4}}\zeta\right) = xe^{-\frac{x^4}{4}}$$

Integrating both sides gives

$$e^{-\frac{x^4}{4}}\zeta = \int xe^{-\frac{x^4}{4}} dx + C$$

$\int xe^{-\frac{x^4}{4}} dx = \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right)$ , hence from above

$$\begin{aligned}e^{-\frac{x^4}{4}}\zeta &= \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \\ \zeta &= e^{\frac{x^4}{4}} \left( \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \right)\end{aligned}$$

Since  $\zeta = -\frac{1}{u}$  then

$$u = -e^{-\frac{x^4}{4}} \left( \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \right)^{-1}$$

And since  $y = x^2 - u$  then

$$\begin{aligned}y &= x^2 + e^{-\frac{x^4}{4}} \left( \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C \right)^{-1} \\ &= x^2 + \frac{e^{-\frac{x^4}{4}}}{\frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{x^2}{2}\right) + C}\end{aligned}$$

Verification

```
eq:=diff(y(x),x)+x*y(x)^2-x^3*y(x)-2*x = 0;
sol:=x^2+ exp(-x^4/4)/(_C1+ sqrt(Pi)/2*erf(x^2/2));
odetest(y(x)=sol,eq);
0
```

2.29 ODE No. 29

$$y'(x) - xy(x)^2 - 3xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0222322 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -\frac{3e^{3c_1 + \frac{3x^2}{2}}}{e^{3c_1 + \frac{3x^2}{2}} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 19

$$\left\{ y(x) = 3 \left( -1 + 3e^{-3/2x^2} \_C1 \right)^{-1} \right\}$$

**Hand solution**

$$\begin{aligned} y' - xy^2 - 3xy &= 0 \\ y' &= 3xy + xy^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Bernoulli first order non-linear ODE since  $P(x) = 0$ . To solve Bernoulli we always start by dividing by  $y^2$

$$\frac{y'}{y^2} = \frac{3x}{y} + x$$

Then we let  $u = \frac{1}{y}$ , hence  $u' = \frac{-y'}{y^2}$ , therefore the above becomes

$$\begin{aligned} -u' &= 3xu + x \\ u' + 3ux &= -x \end{aligned}$$

Integrating factor is  $e^{\int 3xdx} = e^{\frac{3x^2}{2}}$ , hence

$$d\left(e^{\frac{3x^2}{2}} u\right) = -xe^{\frac{3x^2}{2}}$$

Integrating both sides gives

$$\begin{aligned} e^{\frac{3x^2}{2}} u &= \int -xe^{\frac{3x^2}{2}} dx + C \\ &= -\frac{1}{3}e^{\frac{3x^2}{2}} + C \end{aligned}$$

Hence from above

$$u = e^{-\frac{3x^2}{2}} \left( -\frac{1}{3}e^{\frac{3x^2}{2}} + C \right)$$

And since  $y = \frac{1}{u}$  then

$$y = \frac{e^{\frac{3x^2}{2}}}{C - \frac{1}{3}e^{\frac{3x^2}{2}}}$$

Verification

```
eq:=diff(y(x),x)-x*y(x)^2-3*x*y(x) = 0;
sol:=exp(3*x^2/2)/(_C1- 1/3*exp(3*x^2/2));
odetest(y(x)=sol,eq);
0
```

### 2.30 ODE No. 30

$$x^{-a-1}y(x)^2 - x^a + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0654721 (sec), leaf count = 197

$$\left\{ \left\{ y(x) \rightarrow \frac{x^a((-1)^a c_1 \sqrt{x} \Gamma(a+1) I_{a-1}(2\sqrt{x}) + (-1)^{a+1} a c_1 \Gamma(a+1) I_a(2\sqrt{x}) + (-1)^a c_1 \sqrt{x} \Gamma(a+1) I_{a+1}(2\sqrt{x}) - \Gamma(1-a))}{2((-1)^a c_1 \Gamma(a+1) I_a(2\sqrt{x}) + \Gamma(1-a))} \right. \right.$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 54

$$\left\{ y(x) = x^{a+1}(-K_{a+1}(2\sqrt{x}) - C1 + I_{a+1}(2\sqrt{x})) \frac{1}{\sqrt{x}} (K_a(2\sqrt{x}) - C1 + I_a(2\sqrt{x}))^{-1} \right\}$$

**Hand solution**

$$\begin{aligned} y' + x^{-a-1}y^2 - x^a &= 0 \\ y' &= x^a - x^{-a-1}y^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Ricatti first order non-linear ODE. Using standard transformation

$$y = -\frac{u'}{uR(x)} = x^{a+1} \frac{u'}{u}$$

Hence

$$y' = (a+1)x^a \frac{u'}{u} + x^{a+1} \frac{u''}{u} - x^{a+1} \frac{(u')^2}{u^2}$$

Comparing to (1) gives

$$\begin{aligned}
x^a - x^{-a-1}y^2 &= (a+1)x^a \frac{u'}{u} + x^{a+1} \frac{u''}{u} - x^{a+1} \frac{(u')^2}{u^2} \\
x^a - x^{-a-1} \left( x^{a+1} \frac{u'}{u} \right)^2 &= (a+1)x^a \frac{u'}{u} + x^{a+1} \frac{u''}{u} - x^{a+1} \frac{(u')^2}{u^2} \\
1 - \frac{x^{-a-1}}{x^a} x^{2a+2} \frac{(u')^2}{u^2} &= (a+1) \frac{u'}{u} + x \frac{u''}{u} - x \frac{(u')^2}{u^2} \\
1 - x \frac{(u')^2}{u^2} &= (a+1) \frac{u'}{u} + x \frac{u''}{u} - x \frac{(u')^2}{u^2} \\
1 &= (a+1) \frac{u'}{u} + x \frac{u''}{u} \\
xu'' + (1+a)u' - u &= 0 \tag{2}
\end{aligned}$$

In standard form  $u'' + \frac{1}{x}(1+a)u' - \frac{1}{x}u = 0$  or  $u'' + p(x)(1+a)u' + q(x)u = 0$ . We see that  $p(x)$  is not analytic at  $x = 0$  (the expansion point). So we can't use power series solution, and will use Forbenius series. Power series, which is  $u = \sum_{n=0}^{\infty} c_n x^n$  is used when the expansion point is not singular point. (i.e.  $p(x)$  and  $q(x)$  are analytic there). Forbenius series  $u = x^r \sum_{n=0}^{\infty} c_n x^n$  is used when there is a removable singular point (called also regular singular point), as in this case. Starting with

$$u = x^r \sum_{n=0}^{\infty} c_n x^n = \sum_{n=0}^{\infty} c_n x^{n+r}$$

Hence

$$\begin{aligned}
u' &= \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} \\
u'' &= \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2}
\end{aligned}$$

Substituting in (2) gives

$$\begin{aligned}
x \sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-2} + (1+a) \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} - \sum_{n=0}^{\infty} c_n x^{n+r} &= 0 \\
\sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n+r-1} + (1+a) \sum_{n=0}^{\infty} (n+r) c_n x^{n+r-1} - \sum_{n=0}^{\infty} c_n x^{n+r} &= 0
\end{aligned}$$

Dividing out  $x^r$

$$\sum_{n=0}^{\infty} (n+r)(n+r-1) c_n x^{n-1} + (1+a) \sum_{n=0}^{\infty} (n+r) c_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n = 0$$

Each term should have  $x^{n-1}$  in it. So we adjust the last term

$$\sum_{n=0}^{\infty} (n+r)(n+r-1)c_n x^{n-1} + (1+a) \sum_{n=0}^{\infty} (n+r)c_n x^{n-1} - \sum_{n=1}^{\infty} c_{n-1} x^{n-1} = 0$$

Expanding the second term

$$\sum_{n=0}^{\infty} (n+r)(n+r-1)c_n x^{n-1} + \sum_{n=0}^{\infty} (n+r)c_n x^{n-1} + \sum_{n=0}^{\infty} a(n+r)c_n x^{n-1} - \sum_{n=1}^{\infty} c_{n-1} x^{n-1} = 0$$

Hence for  $n = 0$

$$\begin{aligned} (n+r)(n+r-1)c_n x^{n-1} + (n+r)c_n x^{n-1} + a(n+r)c_n x^{n-1} &= 0 \\ r(r-1)c_0 + rc_0 + arc_0 &= 0 \end{aligned}$$

Since  $c_0 \neq 0$  then

$$r(r-1) + r + ar = 0$$

Hence  $r = -a$  or  $r = 0$ . Now for  $n \geq 1$

$$\begin{aligned} (n+r)(n+r-1)c_n x^{n-1} + (n+r)c_n x^{n-1} + a(n+r)c_n x^{n-1} - c_{n-1} x^{n-1} &= 0 \\ (n+r)(n+r-1)c_n + (n+r)c_n + a(n+r)c_n - c_{n-1} &= 0 \\ ((n+r)(n+r-1) + (n+r) + a(n+r))c_n &= c_{n-1} \\ c_n &= \frac{c_{n-1}}{(n+r)(n+r-1) + (n+r)} \end{aligned}$$

For  $r = 0$ , we obtain

$$c_n = \frac{c_{n-1}}{n(n-1) + n + an} \quad (3)$$

For  $r = -a$

$$c_n = \frac{c_{n-1}}{(n-a)(n-a-1) + (n-a) + a(n-a)} \quad (4)$$

There are two solutions. Looking at (3) for now, for  $n = 1$

$$c_1 = \frac{c_0}{1+a}$$

For  $n = 2$

$$c_2 = \frac{c_1}{4+2a} = \frac{c_0}{1+a} \frac{1}{2(2+a)}$$

For  $n = 3$

$$c_3 = \frac{c_2}{3(2) + 3 + 3a} = \frac{c_2}{3(3+a)} = \frac{c_0}{1+a} \frac{1}{2(2+a)} \frac{1}{3(3+a)}$$

And so on. Since the solution is assumed to be  $x^r \sum_{n=0}^{\infty} c_n x^n$  and we are looking at case  $r = 0$  then

$$\begin{aligned}
u_{r=0}(x) &= \sum_{n=1}^{\infty} c_n x^n \\
&= c_0 + c_1 x + c_2 x^2 + \dots \\
&= c_0 x^0 + \frac{c_0}{1+a} x + \frac{c_0}{1+a} \frac{1}{2(2+a)} x^2 + \frac{c_0}{1+a} \frac{1}{2(2+a)} \frac{1}{3(3+a)} x^3 + \dots \\
&= c_0 \left( x^0 + \frac{1}{1+a} x + \frac{1}{(1+a)} \frac{1}{2(2+a)} x^2 + \frac{1}{(1+a)} \frac{1}{2(2+a)} \frac{1}{3(3+a)} x^3 + \dots \right)
\end{aligned} \tag{5}$$

Since

$$\Gamma(n) = (n-1)!$$

and

$$a(1+a)(2+a)\dots(n+a) = \frac{\Gamma(a+n+1)}{\Gamma(a)}$$

Then

$$(1+a)(2+a)\dots(n+a) = \frac{\Gamma(a+n+1)}{a\Gamma(a)}$$

And (5) can now be written as

$$y_{r=0}(x) = c_0 \sum_{n=1}^{\infty} \frac{1}{n!} \frac{a\Gamma(a)}{\Gamma(a+n+1)} x^n \tag{6}$$

But modified Bessel function of first kind is

$$\text{BesselI}(a, z) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} \left(\frac{z}{2}\right)^{2n+a}$$

So if we let  $z = 2\sqrt{x}$  we obtain

$$\begin{aligned}
\text{BesselI}(a, 2\sqrt{x}) &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} \left(\frac{2\sqrt{x}}{2}\right)^{2n+a} \\
&= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} (\sqrt{x})^{2n} (\sqrt{x})^a \\
&= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} x^n (\sqrt{x})^a
\end{aligned}$$

Hence

$$\frac{1}{\sqrt{x}^a} \text{BesselI}(a, 2\sqrt{x}) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} x^n \tag{7}$$

If we now compare (6) and (7), we see that if we set  $c_0$ , which is arbitrary, to be  $c_0 = \frac{1}{a\Gamma(a)}$ , then we obtain

$$\begin{aligned} u_{r=0}(x) &= \frac{1}{a\Gamma(a)} \sum_{n=0}^{\infty} \frac{1}{n!} \frac{a\Gamma(a)}{\Gamma(a+n+1)} x^n \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{\Gamma(a+n+1)} x^n \end{aligned}$$

But this is (7). Hence we found the first solution, which is

$$u_{r=0}(x) = \frac{1}{\sqrt{x^a}} \text{Bessell}(a, 2\sqrt{x}) \quad (8)$$

The above was for  $r = 0$ . Now we find the second solution for  $r = -a$ . From (4)

$$c_n = \frac{c_{n-1}}{(n-a)(n-a-1) + (n-a) + a(n-a)}$$

For  $n = 1$

$$c_1 = \frac{c_0}{-a(1-a) + (1-a) + a(1-a)} = \frac{c_0}{(1-a)}$$

For  $n = 2$

$$c_2 = \frac{c_1}{(2-a)(1-a) + (2-a) + a(2-a)} = \frac{c_1}{4-2a} = \frac{c_0}{(1-a)} \frac{1}{2(2-a)}$$

For  $n = 3$

$$c_3 = \frac{c_2}{(3-a)(2-a) + (3-a) + a(3-a)} = \frac{c_2}{3(3-a)} = \frac{c_0}{(1-a)} \frac{1}{2(2-a)} \frac{1}{3(3-a)}$$

And so on. Since the solution is assumed to be  $x^r \sum_{n=0}^{\infty} c_n x^n$  then

$$\begin{aligned} u_{r=-a} &= x^{-a} \sum_{n=0}^{\infty} c_n x^n \\ &= \sum_{n=0}^{\infty} c_n x^{n-a} \\ &= c_0 x^{-a} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{1}{(1-a)} \frac{1}{(2-a)} \frac{1}{(3-a)} \cdots \frac{1}{(n-a)} \right) x^{n-a} \end{aligned}$$



But as we found above, we obtain that  $(1-a)(2-a)\cdots(n-a) = \frac{\Gamma(-a+n+1)}{-a\Gamma(-a)}$ , therefore

$$u_{r=-a} = c_0 \sum_{n=0}^{\infty} \frac{1}{n!} \frac{-a\Gamma(-a)}{\Gamma(-a+n+1)} x^{n-a}$$

Modified Bessel function of second kind is  $\text{BesselK}(a, z) = \frac{\pi}{2} \frac{1}{\sin(a\pi)} (\text{BesselI}(-a, z) - \text{BesselI}(a, z))$ . The above should result in  $\frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})$  for  $z = 2\sqrt{x}$  by setting  $c_0$  to appropriate arbitrary value. I need to work out this final manipulation later. Hence we find  $u_{r=-a}(x) = \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})$ . Therefore, the solution is

$$u = C_1 \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) + C_2 \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})$$

But

$$\begin{aligned} \frac{d}{dx} \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) &= \frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) \\ \frac{d}{dx} \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x}) &= -\frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x}) \end{aligned}$$

Hence

$$u' = C_1 \frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) - C_2 \frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x})$$

And from  $y = x^{a+1} \frac{u'}{u}$

$$y = x^{1+a} \frac{C_1 \frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) - C_2 \frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x})}{C_1 \frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) + C_2 \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})}$$

Let  $C = \frac{C_2}{C_1}$  hence

$$y = x^{1+a} \frac{\frac{1}{\sqrt{x^{1+a}}} \text{BesselI}(1+a, 2\sqrt{x}) - C \frac{1}{\sqrt{x^{1+a}}} \text{BesselK}(1+a, 2\sqrt{x})}{\frac{1}{\sqrt{x^a}} \text{BesselI}(a, 2\sqrt{x}) + C \frac{1}{\sqrt{x^a}} \text{BesselK}(a, 2\sqrt{x})}$$

Or

$$\begin{aligned} y &= x^{1+a} \frac{x^{-\frac{1}{2}} \text{BesselI}(1+a, 2\sqrt{x}) - C x^{-\frac{1}{2}} \text{BesselK}(1+a, 2\sqrt{x})}{\text{BesselI}(a, 2\sqrt{x}) + C \text{BesselK}(a, 2\sqrt{x})} \\ &= \frac{x^{\frac{1}{2}+a} \text{BesselI}(1+a, 2\sqrt{x}) - C x^{\frac{1}{2}+a} \text{BesselK}(1+a, 2\sqrt{x})}{\text{BesselI}(a, 2\sqrt{x}) + C \text{BesselK}(a, 2\sqrt{x})} \end{aligned}$$

Verification

```

eq:=diff(y(x),x)+x^(-a-1)*y(x)^2-x^a = 0;
num:=x^(1/2+a)*BesselI(1+a,2*sqrt(x))-C1*x^(1/2+a)*BesselK(1+a,2*sqrt(x));
den:=BesselI(a,2*sqrt(x))+C1*BesselK(a,2*sqrt(x));
my_sol:=num/den;
odetest(y(x)=my_sol,eq);
0

```

**2.31 ODE No. 31**

$$y'(x) - ax^n(y(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.0293922 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \tan \left( \frac{ax^{n+1}}{n+1} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 23

$$\left\{ y(x) = \tan \left( \frac{(x^{n+1} + (n+1) - C1) a}{n+1} \right) \right\}$$

**Hand solution**

$$\begin{aligned} y' - ax^n(y^2 + 1) &= 0 \\ y' &= ax^n + ax^ny^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Ricatti first order non-linear ODE.  $P(x) = ax^n, Q(x) = 0, R(x) = ax^n$ . But this is separable also. Hence

$$\begin{aligned} \frac{y'}{(y^2 + 1)} &= ax^n \\ \frac{dy}{(y^2 + 1)} &= ax^ndx \end{aligned}$$

Integrating

$$\arctan(y(x)) = a \frac{x^{n+1}}{n+1} + C$$

Or

$$y(x) = \tan \left( a \frac{x^{n+1}}{n+1} + C \right)$$

Verification

```
restart;
eq:=diff(y(x),x)-a*x^n*(y(x)^2+1) = 0;
sol:=tan(a*x^(n+1)/(n+1)+_C1);
odetest(y(x)=sol,eq);
0
```

**2.32 ODE No. 32**

$$y'(x) + y(x)^2 \sin(x) - 2 \tan(x) \sec(x) = 0$$

✓ **Mathematica** : cpu = 0.143819 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \frac{\sec(x) (c_1 - 2 \cos^3(x))}{c_1 + \cos^3(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.279 (sec), leaf count = 28

$$\left\{ y(x) = \frac{-2 (\cos(x))^3 - C1 - 2}{((\cos(x))^3 - C1 - 2) \cos(x)} \right\}$$

**Hand solution**

$$\begin{aligned} y' + y^2 \sin(x) - 2 \frac{\sin x}{\cos^2 x} &= 0 \\ y' &= 2 \frac{\sin x}{\cos^2 x} - y^2 \sin(x) \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Ricatti first order non-linear ODE.  $P(x) = 2 \frac{\sin x}{\cos^2 x}$ ,  $Q(x) = 0$ ,  $R(x) = -\sin(x)$ . A particular solution is  $y_p = \frac{1}{\cos x}$ , therefore the solution is

$$\begin{aligned} y &= y_p + \frac{1}{u} \\ y &= \frac{1}{\cos x} + \frac{1}{u} \end{aligned}$$

Hence

$$y' = \frac{\sin x}{\cos^2 x} - \frac{u'}{u^2}$$

Equating this to RHS of (1) gives

$$\begin{aligned} \frac{\sin x}{\cos^2 x} - \frac{u'}{u^2} &= 2 \frac{\sin x}{\cos^2 x} - y^2 \sin(x) \\ &= 2 \frac{\sin x}{\cos^2 x} - \left( \frac{1}{\cos x} + \frac{1}{u} \right)^2 \sin(x) \\ &= 2 \frac{\sin x}{\cos^2 x} - \left( \frac{1}{\cos^2 x} + \frac{1}{u^2} + \frac{2}{u \cos x} \right) \sin(x) \end{aligned}$$

Hence

$$\begin{aligned} -\frac{u'}{u^2} &= -\frac{\sin x}{\cos^2 x} + 2 \frac{\sin x}{\cos^2 x} - \frac{\sin(x)}{\cos^2 x} - \frac{\sin(x)}{u^2} - \frac{2 \sin(x)}{u \cos x} \\ &= -\frac{\sin(x)}{u^2} - \frac{2 \sin(x)}{u \cos x} \end{aligned}$$

Or

$$u' = \sin(x) + \frac{2u \sin(x)}{\cos x}$$
$$u' - 2u \tan(x) = \sin(x)$$

Integrating factor is  $e^{-2 \int \tan x dx} = e^{2 \ln(\cos x)} = \cos^2(x)$ . Hence the above becomes

$$d(u \cos^2 x) = \cos^2(x) \sin(x)$$

Integrating both sides

$$u \cos^2 x = \int \cos^2(x) \sin(x) dx + C$$
$$= \frac{-1}{3} \cos^3(x) + C$$

Hence

$$u = \frac{-1}{3} \cos(x) + \frac{C}{\cos^2 x}$$

Therefore

$$y = y_p + \frac{1}{u}$$
$$= \frac{1}{\cos x} + \frac{1}{\frac{-1}{3} \cos(x) + \frac{C}{\cos^2 x}}$$
$$= \frac{1}{\cos x} + \frac{3 \cos^2 x}{3C - \cos^3(x)}$$

Let  $3C = C_1$

$$y = \frac{1}{\cos x} + \frac{3 \cos^2 x}{C_1 - \cos^3(x)}$$

Verification

```
restart;
ode:=diff(y(x),x)+y(x)^2*sin(x)-2*sin(x)/cos(x)^2 = 0;
my_sol:=1/cos(x)+ 3*cos(x)^2/(_C1-cos(x)^3);
odetest(y(x)=my_sol,ode);
0
```

**2.33 ODE No. 33**

$$-\frac{y(x)^2 f'(x)}{g(x)} + \frac{g'(x)}{f(x)} + y'(x) = 0$$

✓ **Mathematica** : cpu = 34.5391 (sec), leaf count = 114

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{1}{(f(x)K[2] + g(x))^2} - \int_1^x \frac{2(K[2]f'(K[1]) + g'(K[1]))}{(K[2]f(K[1]) + g(K[1]))^3} dK[1] \right) dK[2] + \int_1^x \frac{g(K[1])g'(K[1])}{f(K[1])g(K[1])} \right]$$

✓ **Maple** : cpu = 0.544 (sec), leaf count = 58

$$\left\{ y(x) = \frac{1}{(f(x))^2} \left( -f(x)g(x) \int \frac{\frac{d}{dx}f(x)}{g(x)(f(x))^2} dx - g(x)f(x) - C1 - 1 \right) \left( -C1 + \int \frac{\frac{d}{dx}f(x)}{g(x)(f(x))^2} dx \right)^{-1} \right\}$$

**Hand solution**

$$\begin{aligned} -\frac{f'}{g}y^2 + \frac{g'}{f} + y' &= 0 \\ y' &= -\frac{g'}{f} + \frac{f'}{g}y^2 \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Ricatti first order non-linear ODE.  $P(x) = -\frac{g'}{f}$ ,  $Q(x) = 0$ ,  $R(x) = \frac{f'}{g}$ .

To do.

**2.34 ODE No. 34**

$$f(x)y(x)^2 + g(x)y(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.512224 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\int_1^x -g(K[1]) dK[1]}}{c_1 - \int_1^x f(K[2]) \left( -e^{\int_1^{K[2]} -g(K[1]) dK[1]} \right) dK[2]} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 28

$$\left\{ y(x) = \frac{e^{\int -g(x) dx}}{\int e^{\int -g(x) dx} f(x) dx + C1} \right\}$$

**Hand solution**

$$\begin{aligned}y^2 f + gy + y' &= 0 \\y' &= -gy - y^2 f \\&= P(x) + Q(x)y + R(x)y^2\end{aligned}\tag{1}$$

This is Bernoulli first order non-linear ODE.  $P(x) = 0, Q(x) = -g, R(x) = f$ . First step is to divide by  $y^2$

$$\frac{y'}{y^2} = -g\frac{1}{y} - f\tag{2}$$

Let  $u = \frac{1}{y}$ , then  $u' = \frac{-y'}{y^2}$  and (2) becomes

$$\begin{aligned}-u' &= -gu - f \\u' - gu &= f\end{aligned}$$

Integrating factor is  $e^{-\int g dx}$  hence

$$\begin{aligned}d\left(e^{-\int g dx} u\right) &= f e^{-\int g dx} \\e^{-\int g dx} u &= \int f e^{-\int g dx} dx + C \\u &= e^{\int g dx} \left( \int f e^{-\int g dx} dx + C \right)\end{aligned}$$

Hence

$$\begin{aligned}y &= \frac{1}{e^{\int g dx} \left( \int f e^{-\int g dx} dx + C \right)} \\&= \frac{e^{-\int g dx}}{\int f e^{-\int g dx} dx + C}\end{aligned}$$

Let  $\beta = e^{-\int g dx}$  then

$$y = \frac{\beta}{\int f \beta dx + C}$$

Verification

```
restart;
eq:=diff(y(x),x)+f(x)*y(x)^2+g(x)*y(x) = 0;
beta:=exp(-Int(g(x),x));
my_sol:=beta/(Int(f(x)*beta,x)+_C1);
odetest(y(x)=my_sol,eq);
0
```

**2.35 ODE No. 35**

$$f(x) (2ay(x) + b + y(x)^2) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0584369 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \sqrt{b - a^2} \tan \left( \sqrt{b - a^2} \left( \int_1^x -f(K[1]) dK[1] + c_1 \right) \right) - a \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 35

$$\left\{ y(x) = \tanh \left( \sqrt{a^2 - b} \left( -C1 + \int f(x) dx \right) \right) \sqrt{a^2 - b} - a \right\}$$

**Hand solution**

$$\begin{aligned} y'(x) + f(x) (2ay(x) + b + y^2(x)) &= 0 \\ y'(x) &= -2af(x)y(x) - bf(x) - f(x)y^2(x) \\ &= P(x) + Q(x)y + R(x)y^2 \end{aligned} \tag{1}$$

This is Riccati first order non-linear ODE.  $P(x) = -bf(x)$ ,  $Q(x) = -2af(x)$ ,  $R(x) = -f(x)$ .

Let

$$y(x) = -\frac{u'(x)}{u(x)R(x)} = \frac{u'(x)}{u(x)f(x)}$$

Hence

$$y'(x) = \frac{u''(x)}{u(x)f(x)} - \frac{(u'(x))^2}{u^2(x)f(x)} - \frac{u'(x)f'(x)}{u(x)f^2(x)}$$

Equating this to RHS of (1) gives

$$\begin{aligned} \frac{u''(x)}{u(x)f(x)} - \frac{(u'(x))^2}{u^2(x)f(x)} - \frac{u'(x)f'(x)}{u(x)f^2(x)} &= -2af(x)y(x) - bf(x) - f(x)y^2(x) \\ &= -2af(x) \left[ \frac{u'(x)}{u(x)f(x)} \right] - bf(x) - f(x) \left[ \frac{u'(x)}{u(x)f(x)} \right]^2 \\ &= -2a \frac{u'(x)}{u(x)} - bf(x) - \frac{u'(x)^2}{u^2(x)f(x)} \end{aligned}$$

Simplifying

$$u''(x) - \frac{(u'(x))^2}{u(x)} - \frac{u'(x)f'(x)}{f(x)} = -2au'(x)f(x) - u(x)bf^2(x) - \frac{u'(x)^2}{u(x)}$$

$$u''(x) - \frac{u'(x)f'(x)}{f(x)} = -2au'(x)f(x) - u(x)bf^2(x)$$

$$u''(x) + u'(x) \left( -\frac{f'(x)}{f(x)} + 2af(x) \right) + u(x)bf^2(x) = 0$$

Second order ODE with variable coefficients. Since coefficients are variables and not constants, a power series method is the standard way to continue. When I tried solving this now pretending the coefficients are constants in time, using the standard auxiliary equation method, the solution did verify OK. I need to look more into this. For now, this is solved using standard method for solving second order ODE with constant coefficients.

$$u(x) = C_1 \exp \left( \frac{\int f(x) \sqrt{-b} dx \left( \sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b} \right)}{b} \right) + C_2 \exp \left( \frac{\int f(x) \sqrt{-b} dx \left( -\sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b} \right)}{b} \right)$$

Hence

$$u'(x) = \frac{C_1 f(x) \sqrt{-b}}{b} \left( \sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) e^{\frac{\int f(x) \sqrt{-b} dx}{b} \left( \sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right)}$$

$$+ \frac{C_2 f(x) \sqrt{-b}}{b} \left( -\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) e^{\frac{\int f(x) \sqrt{-b} dx}{b} \left( -\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right)}$$

Therefore

$$y = \frac{u'(x)}{u(x)f(x)}$$

$$= \frac{C_1 f(x) \sqrt{-b} \left( \sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) e^{\frac{\int f(x) \sqrt{-b} dx}{b} \left( \sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right)} + C_2 f(x) \sqrt{-b} \left( -\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right) e^{\frac{\int f(x) \sqrt{-b} dx}{b} \left( -\sqrt{\frac{-a^2+b}{b}} b + \sqrt{-ba} \right)} }{f(x) \left[ C_1 \exp \left( \frac{\int f(x) \sqrt{-b} dx \left( \sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b} \right)}{b} \right) + C_2 \exp \left( \frac{\int f(x) \sqrt{-b} dx \left( -\sqrt{\frac{b-a^2}{b}} b + a\sqrt{-b} \right)}{b} \right) \right]}$$

Verification

```
restart;
book:=diff(y(x),x)+f(x)*(2*a*y(x)+b+y(x)^2)=0;
eqU:=diff(u(x),x$2)+diff(u(x),x)*(-diff(f(x),x)/f(x)+2*a*f(x))+u(x)*f(x)^2*b=0;
solU:=dsolve(eqU,u(x));
```



```
my_sol:=diff(rhs(solU),x)/(rhs(solU)*f(x));
odetest(y(x)=my_sol,book);
0
```

### 2.36 ODE No. 36

$$axy(x)^2 + y'(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.298568 (sec), leaf count = 171

$$\text{Solve} \left[ \frac{(-1)^{2/3} \sqrt[3]{2} a^{2/3} x \text{Ai} \left( -\frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a} (ax^2 y(x) - 2)}{2y(x)} \right) - 2 \text{Ai}' \left( -\frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a} (ax^2 y(x) - 2)}{2y(x)} \right)}{(-1)^{2/3} \sqrt[3]{2} a^{2/3} x \text{Bi} \left( -\frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a} (ax^2 y(x) - 2)}{2y(x)} \right) - 2 \text{Bi}' \left( -\frac{\sqrt[3]{-\frac{1}{2}} \sqrt[3]{a} (ax^2 y(x) - 2)}{2y(x)} \right)} + c_1 = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 62

$$\left\{ y(x) = 2 \frac{a}{a^2 x^2 + 2 \text{RootOf} \left( \text{Bi}(\_Z) \sqrt[3]{-2 a^2} \_C1 x + \sqrt[3]{-2 a^2} x \text{Ai}(\_Z) + 2 \text{Bi}^{(1)}(\_Z) \_C1 + 2 \text{Ai}^{(1)}(\_Z) \right) \sqrt[3]{-2 a^2}} \right.$$

**Hand solution**

$$y'(x) = -axy^2 - y^3 \tag{1}$$

This is Abel first order non-linear. The general form is of Abel first kind is

$$y'(x) = f_0(x) + f_1(x)y(x) + f_2(x)y^2(x) + f_3(x)y^3(x)$$

In this case,  $f_0(x) = 0$ ,  $f_1(x) = 0$ ,  $f_2(x) = -ax$ ,  $f_3(x) = -1$ . Note  $\left(\frac{f_3}{f_2}\right)' = \left(\frac{1}{ax}\right)' = -\frac{1}{a}$ . While Abel second kind has the form

$$(y + g(x))y'(x) = f_0(x) + f_1(x)y(x) + f_2(x)y^2(x)$$

For  $g(x) \neq 0$ .

Looking at (1) again, using the transformation suggested in Kamke  $u = \frac{1}{y} - \frac{1}{2}ax^2$  or  $y = \frac{1}{u + \frac{1}{2}ax^2}$  Then

$$y' = \frac{-u' - ax}{\left(u + \frac{1}{2}ax^2\right)^2}$$

Equating the above to the RHS of (1) gives

$$\begin{aligned}\frac{-u' - ax}{\left(u + \frac{1}{2}ax^2\right)^2} &= -ax \left(\frac{1}{u + \frac{1}{2}ax^2}\right)^2 - \left(\frac{1}{u + \frac{1}{2}ax^2}\right)^3 \\ -u' - ax &= -ax - \frac{1}{u + \frac{1}{2}ax^2} \\ \frac{du}{dx} &= \frac{1}{u + \frac{1}{2}ax^2}\end{aligned}$$

Writing as

$$\frac{dx}{du} = u + \frac{1}{2}ax^2 \quad (2)$$

This can now be viewed as reverse Riccati in  $x$ . Using the standard transformation

$$x = -\frac{z'}{z\left(\frac{1}{2}a\right)} = -\frac{2z'}{az} \quad (3)$$

Hence

$$\frac{dx}{du} = -\frac{2}{a} \left( \frac{z''}{z} - \frac{(z')^2}{z^2} \right)$$

Equating this to RHS of (2) gives a second order Airy ODE where the dependent variable is  $z$  and the independent variable is  $u$

$$\begin{aligned}-\frac{2}{a} \left( \frac{z''}{z} - \frac{(z')^2}{z^2} \right) &= u + \frac{1}{2}a \left( -\frac{2z'}{az} \right)^2 \\ -\frac{2}{a} \frac{z''}{z} + \frac{2}{a} \frac{(z')^2}{z^2} &= u + \frac{1}{2}a \frac{4(z')^2}{a^2 z^2} \\ -\frac{2}{a} \frac{z''}{z} + \frac{2}{a} \frac{(z')^2}{z^2} &= u + \frac{2}{a} \frac{(z')^2}{z^2} \\ -\frac{2}{a} \frac{z''}{z} &= u \\ z''(u) + \frac{a}{2}uz(u) &= 0\end{aligned}$$

This is Airy ODE whose solution is found using power series method. The solution is

$$z(u) = C_1 \text{AiryAI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) + C_2 \text{AiryBI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) \quad (4)$$

We now go back to (3) and find  $x$

$$x = -\frac{2z'}{az}$$

Since

$$\begin{aligned}\frac{d}{du} \text{AiryAI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) &= -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryAI} \left( 1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) \\ \frac{d}{du} \text{AiryBI} \left( -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right) &= -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}} \text{AiryBI} \left( 1, -\frac{1}{2}2^{\frac{2}{3}}a^{\frac{1}{3}}u \right)\end{aligned}$$

Then

$$x = -\frac{2 - C_1 \frac{1}{2} 2^{\frac{2}{3}} a^{\frac{1}{3}} \text{AiryAI}\left(1, -\frac{1}{2} 2^{\frac{2}{3}} a^{\frac{1}{3}} u\right) - C_2 \frac{1}{2} 2^{\frac{2}{3}} a^{\frac{1}{3}} \text{AiryBI}\left(1, -\frac{1}{2} 2^{\frac{2}{3}} a^{\frac{1}{3}} u\right)}{C_1 \text{AiryAI}\left(-\frac{1}{2} 2^{\frac{2}{3}} a^{\frac{1}{3}} u\right) + C_2 \text{AiryBI}\left(-\frac{1}{2} 2^{\frac{2}{3}} a^{\frac{1}{3}} u\right)}$$

Therefore  $\frac{dx}{du}$  is now found from above. Once we find  $\frac{dx}{du}$  then  $\frac{du}{dx}$  is also found. Using  $\frac{du}{dx} = \frac{1}{u + \frac{1}{2}ax^2}$  now  $u(x)$  is found. Once  $u(x)$  is found then  $y(x)$  is found from the original transformation  $y = \frac{1}{u + \frac{1}{2}ax^2}$ . This is all now just algebra.

### 2.37 ODE No. 37

$$-ae^x y(x)^2 + y'(x) - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.87792 (sec), leaf count = 73

$$\text{Solve}\left[-iae^x = \frac{2e^{-\frac{(ae^x y(x)+1)^2}{2y(x)^2}}}{2c_1 - i\sqrt{2\pi}\text{erf}\left(\frac{ae^x y(x)+1}{\sqrt{2}y(x)}\right)}, y(x)\right]$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 50

$$\left\{-C1 + \frac{1}{e^x a} e^{-\frac{(e^x a + (y(x))^{-1})^2}{2}} + \frac{\sqrt{2}\sqrt{\pi}}{2} \text{Erf}\left(\frac{(e^x a + (y(x))^{-1})\sqrt{2}}{2}\right) = 0\right\}$$

### 2.38 ODE No. 38

$$-ay(x)^3 - \frac{b}{x^{3/2}} + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.880345 (sec), leaf count = 99

$$\text{Solve}\left[-2\text{RootSum}\left[-2\#1^3 + \#1\sqrt[3]{-\frac{1}{ab^2}} - 2\&, \frac{\log\left(y(x)\sqrt[3]{\frac{ax^{3/2}}{b}} - \#1\right)}{\sqrt[3]{-\frac{1}{ab^2}} - 6\#1^2}\&\right] = \frac{ax \log(x)}{\left(\frac{ax^{3/2}}{b}\right)^{2/3}} + c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.317 (sec), leaf count = 34

$$\left\{y(x) = \text{RootOf}\left(-\ln(x) + \_C1 + 2 \int^{-Z} (2a\_a^3 + \_a + 2b)^{-1} d\_a\right) \frac{1}{\sqrt{x}}\right\}$$

Hand solution

$$y'(x) = ay^3 + bx^{-\frac{3}{2}} \quad (1)$$

This can be transformed to Abel first order non-linear ode as follows. Let  $y(x) = x^{-\frac{1}{2}}\eta(\xi)$  where  $\xi = \ln x$  hence

$$\begin{aligned} \frac{dy}{dx} &= -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{1}{2}}\frac{d\eta}{d\xi}\frac{d\xi}{dx} \\ &= -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{1}{2}}\frac{d\eta}{d\xi}\frac{1}{x} \\ &= -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{3}{2}}\frac{d\eta}{d\xi} \end{aligned}$$

Substituting in (1) gives

$$\begin{aligned} -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{3}{2}}\frac{d\eta}{d\xi} &= a\left(x^{-\frac{1}{2}}\eta(\xi)\right)^3 + bx^{-\frac{3}{2}} \\ -\frac{1}{2}x^{-\frac{3}{2}}\eta(\xi) + x^{-\frac{3}{2}}\frac{d\eta}{d\xi} &= ax^{-\frac{3}{2}}\eta^3(\xi) + bx^{-\frac{3}{2}} \\ -\frac{1}{2}\eta + \eta' &= a\eta^3 + b \\ \eta' &= b + \frac{1}{2}\eta + a\eta^3 \end{aligned}$$

This is Abel first kind. In general form it is

$$\eta' = f_0 + f_1\eta + f_2\eta^2 + f_3\eta^3$$

Where in this case  $f_0 = b, f_1 = \frac{1}{2}, f_2 = 0, f_3 = a$ . Using Maple, the solution to the above is (I need to learn how to solve Able by hand more) is implicit, given as

$$\eta = \xi - \int^{\eta(\xi)} \frac{1}{b + \frac{1}{2}z + az^3} dz + C$$

Where  $C$  is constant of integration. Hence, since  $y(x) = x^{-\frac{1}{2}}\eta(\xi)$ , then  $\eta(\xi) = \sqrt{xy}$  and the above becomes

$$\begin{aligned} \sqrt{xy} &= \ln x - \int^{\sqrt{xy}} \frac{1}{b + \frac{1}{2}z + az^3} dz + C \\ y(x) &= \left( \ln x - \int^{\sqrt{xy}} \frac{1}{b + \frac{1}{2}z + az^3} dz + C \right) \frac{1}{\sqrt{x}} \end{aligned}$$

Did not verify. Need to look more into this later.

**2.39 ODE No. 39**

$$-a0 - a1y(x) - a2y(x)^2 - a3y(x)^3 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0497017 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.014 (sec), leaf count = 30

$$\left\{ x - \int^{y(x)} (-a^3 a3 + -a^2 a2 + -a a1 + a0)^{-1} d_a + -C1 = 0 \right\}$$

**2.40 ODE No. 40**

$$3ay(x)^3 + 6axy(x)^2 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.39657 (sec), leaf count = 161

$$\text{Solve} \left[ \frac{\sqrt[3]{-3} \sqrt[3]{ax} \text{Ai} \left( \frac{(-1)^{2/3} (3ax^2 y(x) - 1)}{\sqrt[3]{3} \sqrt[3]{ay(x)}} \right) + \text{Ai}' \left( \frac{(-1)^{2/3} (3ax^2 y(x) - 1)}{\sqrt[3]{3} \sqrt[3]{ay(x)}} \right)}{\sqrt[3]{-3} \sqrt[3]{ax} \text{Bi} \left( \frac{(-1)^{2/3} (3ax^2 y(x) - 1)}{\sqrt[3]{3} \sqrt[3]{ay(x)}} \right) + \text{Bi}' \left( \frac{(-1)^{2/3} (3ax^2 y(x) - 1)}{\sqrt[3]{3} \sqrt[3]{ay(x)}} \right)} + c_1 = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 48

$$\left\{ y(x) = \left( 3ax^2 + \text{RootOf} \left( \text{Bi}(\_Z) \sqrt[3]{-3a} \_C1 x + \sqrt[3]{-3ax} \text{Ai}(\_Z) + \text{Bi}^{(1)}(\_Z) \_C1 + \text{Ai}^{(1)}(\_Z) \right) \sqrt[3]{-3a} \right)^{-1} \right\}$$

**2.41 ODE No. 41**

$$axy(x)^3 + by(x)^2 + y'(x) = 0$$

✓ **Mathematica** : cpu = 5.23807 (sec), leaf count = 98

$$\text{Solve} \left[ \frac{b^2 \log(x)}{a} = \frac{b^2 \left( \frac{2 \tan^{-1} \left( \frac{-2axy(x) - b}{b \sqrt{-\frac{4a}{b^2} - 1}} \right)}{\sqrt{-\frac{4a}{b^2} - 1}} - \log \left( \frac{ax^2 y(x)^2 + bxy(x) - 1}{ax^2 y(x)^2} \right) \right)}{2a} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 103

$$\left\{ y(x) = \frac{1}{x} e^{\text{RootOf}\left(2\sqrt{b^2+4ab}\text{Artanh}\left(\frac{2ae^{-Z}+b}{\sqrt{b^2+4a}}\right) - \ln(x^2(ae^{-Z}+be^{-Z}-1))b^2+2-C1\right) b^2+2-Zb^2-4\ln(x^2(ae^{-Z}+be^{-Z}-1))a+8-C1} \right.$$

## 2.42 ODE No. 42

$$y'(x) - x(x+2)y(x)^3 - (x+3)y(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.32011 (sec), leaf count = 136

$$\text{Solve} \left[ c_1 = -\frac{i\left(x \sinh\left(\frac{1}{2}\sqrt{\frac{2}{y(x)}+x(x+2)}\right) + \sqrt{\frac{2}{y(x)}+x(x+2)} \cosh\left(\frac{1}{2}\sqrt{\frac{2}{y(x)}+x(x+2)}\right)\right)}{\sqrt{\frac{2}{y(x)}+x(x+2)} \sinh\left(\frac{1}{2}\sqrt{\frac{2}{y(x)}+x(x+2)}\right) + x \cosh\left(\frac{1}{2}\sqrt{\frac{2}{y(x)}+x(x+2)}\right)}, y(x) \right]$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 40

$$\left\{ -C1 + \text{Artanh}\left(x\sqrt{y(x)}\frac{1}{\sqrt{x(x+2)y(x)+2}}\right) + \frac{1}{2}\sqrt{x(x+2)y(x)+2}\frac{1}{\sqrt{y(x)}} = 0 \right\}$$

## 2.43 ODE No. 43

$$y(x)^3(4a^2x+3ax^2+b) + y'(x) + 3xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 9.39528 (sec), leaf count = 376

$$\text{Solve} \left[ c_1 = \frac{\left(3x - a\left(\sqrt{4 - \frac{3b}{a^3}} - 2\right)\right) J_{\frac{1}{2}\sqrt{4 - \frac{3b}{a^3}}}\left(-\frac{1}{2}i\sqrt{3}\sqrt{\frac{(b+ax(4a+3x))y(x)-2a}{a^3y(x)}}\right) - i\sqrt{3}a\sqrt{\frac{y(x)(ax(4a+3x)+b)-2a}{a^3y(x)}} J_{\frac{1}{2}}}{\left(a\left(\sqrt{4 - \frac{3b}{a^3}} - 2\right) - 3x\right) Y_{\frac{1}{2}\sqrt{4 - \frac{3b}{a^3}}}\left(-\frac{1}{2}i\sqrt{3}\sqrt{\frac{(b+ax(4a+3x))y(x)-2a}{a^3y(x)}}\right) + i\sqrt{3}a\sqrt{\frac{y(x)(ax(4a+3x)+b)-2a}{a^3y(x)}} Y_{\frac{1}{2}}}$$

✓ **Maple** : cpu = 1.825 (sec), leaf count = 373

$$\left\{ -C1 + 1\left(-K_{\frac{1}{2}\sqrt{\frac{4a^3-3b}{a^3}+1}}\left(-\frac{\sqrt{3}}{2}\sqrt{\frac{4y(x)a^2x+3ax^2y(x)+by(x)-2a}{a^3y(x)}}\right)\sqrt{3}\sqrt{\frac{4y(x)a^2x+3ax^2y(x)+by(x)}{a^3y(x)}}\right) \right.$$

2.44 ODE No. 44

$$2ax^3y(x)^3 + y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0170798 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{\sqrt{c_1 e^{2x^2} - \frac{1}{2}a(2x^2 + 1)}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 e^{2x^2} - \frac{1}{2}a(2x^2 + 1)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 53

$$\left\{ y(x) = -2 \frac{1}{\sqrt{-4ax^2 + 4e^{2x^2} - C1 - 2a}}, y(x) = 2 \frac{1}{\sqrt{-4ax^2 + 4e^{2x^2} - C1 - 2a}} \right\}$$

**Hand solution**

$$y' = -2xy - 2ax^3y^3 \tag{1}$$

This is of the form  $y' = f_0 + f_1y + f_2y^2 + f_3y^3$  where  $f_0 = 0, f_2 = 0$ . Hence this is Bernoulli first order non-linear ODE. We start by dividing by  $y^3$

$$\frac{y'}{y^3} = -2x \frac{1}{y^2} - 2ax^3$$

Let  $u = \frac{1}{y^2}$ , hence  $u' = -2\frac{y'}{y^3}$  and the above becomes

$$\begin{aligned} -\frac{1}{2}u' &= -2xu - 2ax^3 \\ u' - 4xu &= 4ax^3 \end{aligned}$$

Integrating factor is  $e^{-4 \int x dx} = e^{-2x^2}$  hence

$$\frac{d}{dx} (e^{-2x^2} u) = 4ax^3 e^{-2x^2}$$

Integrating

$$\begin{aligned} e^{-2x^2} u &= 4a \int x^3 e^{-2x^2} dx + C \\ &= 4a \left( \frac{-1}{8} (2x^2 + 1) e^{-2x^2} \right) + C \end{aligned}$$

Therefore

$$u = -\frac{1}{2}a(2x^2 + 1) + Ce^{2x^2}$$

Hence

$$y^2 = \frac{1}{u} = \frac{1}{-\frac{1}{2}a(2x^2 + 1) + Ce^{2x^2}}$$

Or

$$y = \pm \frac{\sqrt{2}}{\sqrt{-a(2x^2 + 1) + Ce^{2x^2}}}$$

Verification

```
ode:=2*a*x^3*y(x)^3+diff(y(x),x)+2*x*y(x)=0;
my_sol:=sqrt(2)/sqrt(-a*(2*x^2+1)+_C1*exp(2*x^2));
odetest(y(x)=my_sol,ode);
0
my_sol:=-sqrt(2)/sqrt(-a*(2*x^2+1)+_C1*exp(2*x^2));
odetest(y(x)=my_sol,ode);
0
```

### 2.45 ODE No. 45

$$2y(x)^3 (a^2x^3 - b^2x) + 3by(x)^2 + y'(x) = 0$$

✗ **Mathematica** : cpu = 300.494 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.122 (sec), leaf count = 123

$$\left\{ -C_1 + 1 \sqrt[4]{\left(\frac{ax}{b} + \left(\frac{b^2y(x)}{a} - \frac{b}{ax}\right)^{-1}\right)^2 - 1 \left(\frac{b^2y(x)}{a} - \frac{b}{ax}\right)^{-1} \frac{1}{\sqrt{\frac{ax}{b} + \left(\frac{b^2y(x)}{a} - \frac{b}{ax}\right)^{-1}}} - \int^{\frac{ax^2y(x)}{bxy(x)^{-1}}} 1 \sqrt[4]{-a^2} \right.$$

### 2.46 ODE No. 46

$$-x^{-a}y(x) + ax^{-a-1} - x^{-2a} - x^a y(x)^3 + y'(x) + 3y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.247645 (sec), leaf count = 254

$$\left\{ \left\{ y(x) \rightarrow x^{-a} - \frac{e^{\frac{2x^{1-a}}{a-1}}}{\sqrt[4]{c_1 - \frac{2x \left( \frac{\frac{a+1}{4^{\frac{a-1}{a-1}}} x \left( \frac{x^{1-a}}{1-a} \right)^{\frac{2}{a-1}} \Gamma\left(-\frac{2}{a-1}, -\frac{4x^{1-a}}{a-1}\right)}{a-1} + e^{\frac{4x^{1-a}}{a-1}} x^a \right)}}{a+1}} \right\}, \left\{ y(x) \rightarrow \frac{e^{\frac{2x}{a}}}{\sqrt[4]{c_1 - \frac{2x \left( \frac{\frac{a+1}{4^{\frac{a-1}{a-1}}} x \left( \frac{x^{1-a}}{1-a} \right)^{\frac{2}{a-1}} \right)}}{a-1}}}{a-1}} \right\} \right.$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 956



$$\left\{ \begin{array}{l} y(x) = -1e^{2\frac{x}{(a-1)x^a}} \\ \sqrt{-C1 - 2\frac{1}{1-a}2^{-2\frac{a}{1-a}-2(1-a)^{-1}} \left( (1-a)^{-1} \right)^{-\frac{a}{1-a}-(1-a)^{-1}} \left( -\frac{(a-1)(1-a)}{(a+1)(-3+a)} 2^{-3+2\frac{a}{1-a}+2(1-a)^{-1}} \right)} \end{array} \right.$$

**2.47 ODE No. 47**

$$-a(x^n - x)y(x)^3 + y'(x) - y(x)^2 = 0$$

✗ **Mathematica** : cpu = 33.2356 (sec), leaf count = 0 , could not solve

`DSolve[-y[x]^2 - a*(-x + x^n)*y[x]^3 + Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)-a*(x^n-x)*y(x)^3-y(x)^2 = 0,y(x))`

**2.48 ODE No. 48**

$$y(x)^3(-ax^n + bx) - cy(x)^2 + y'(x) = 0$$

✗ **Mathematica** : cpu = 36.5586 (sec), leaf count = 0 , could not solve

`DSolve[-(c*y[x]^2) - (b*x + a*x^n)*y[x]^3 + Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)-(a*x^n+b*x)*y(x)^3-c*y(x)^2 = 0,y(x))`

**2.49 ODE No. 49**

$$ay(x)^3\phi'(x) + \frac{(2a+1)y(x)\phi''(x)}{\phi'(x)} + 6a\phi(x)y(x)^2 + 2a + y'(x) + 2 = 0$$

✗ **Mathematica** : cpu = 28.9583 (sec), leaf count = 0 , could not solve

`DSolve[2 + 2*a + 6*a*phi[x]*y[x]^2 + a*y[x]^3*Derivative[1][phi][x] + Derivative[1][y][x] +`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)+a*diff(phi(x),x)*y(x)^3+6*a*phi(x)*y(x)^2+(2*a+1)*y(x)*diff(diff(phi(x),`

**2.50 ODE No. 50**

$$-f_0(x) - f_1(x)y(x) - f_2(x)y(x)^2 - f_3(x)y(x)^3 + y'(x) = 0$$

✗ **Mathematica** : cpu = 130.264 (sec), leaf count = 0 , could not solve

`DSolve[-f0[x] - f1[x]*y[x] - f2[x]*y[x]^2 - f3[x]*y[x]^3 + Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)-f3(x)*y(x)^3-f2(x)*y(x)^2-f1(x)*y(x)-f0(x) = 0,y(x))`

**2.51 ODE No. 51**

$$-h(x)(y(x)-f(x))(y(x)-g(x)) \left( y(x) - \frac{af(x) + bg(x)}{a + b} \right) - \frac{f'(x)(y(x) - g(x))}{f(x) - g(x)} - \frac{(y(x) - f(x))g'(x)}{g(x) - f(x)} + y'(x) = 0$$

✓ **Mathematica** : cpu = 16.7016 (sec), leaf count = 322

$$\text{Solve} \left[ \begin{array}{l} -\frac{1}{3}(a-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3} \text{RootSum} \left[ \#1^3(a-b)^{2/3}(2a+b)^{2/3}(a+2b)^{2/3} - 3\#1a^2 - 3\#1ab - 3\#1b^3 \right] \end{array} \right]$$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 237

$$\left\{ y(x) = \frac{1}{9a^3 + 18a^2b + 18ab^2 + 9b^3} \left( 2(f(x) - g(x))(b/2 + a)(a + 2b)(a - b) \text{RootOf} \left( -27 \int^{-Z} \frac{1}{(2_a a^2 - \dots} \right) \right) \right.$$

**2.52 ODE No. 52**

$$-ay(x)^n - bx^{\frac{n}{1-n}} + y'(x) = 0$$

✗ **Mathematica** : cpu = 300.022 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.205 (sec), leaf count = 61

$$\left\{ - \int_{-b}^{y(x)} 1x^{\frac{n}{n-1}} \left( (ax(n-1) - a^n + a)x^{\frac{n}{n-1}} + b(n-1)x \right)^{-1} d_a(n-1) + \ln(x) - C1 = 0 \right\}$$

**2.53 ODE No. 53**

$$f(x)^{1-n}g'(x)y(x)^n(-ag(x)+b)^{-n} - \frac{y(x)f'(x)}{f(x)} - f(x)g'(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 104.189 (sec), leaf count = 95

$$\text{Solve} \left[ \int_1^{y(x)(f(x)^{-n}(ag(x)+b)^{-n})^{\frac{1}{n}}} \frac{1}{-(a^n)^{\frac{1}{n}} K[1] + K[1]^n + 1} dK[1] = \frac{f(x)(ag(x)+b) \log(ag(x)+b) (f(x)^{-n}(ag(x)+b)^{-n})^{\frac{1}{n}}}{a} \right]$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 281

$$\left\{ y(x) = \frac{f(x)(ag(x)+b)}{a} \text{RootOf} \left( - \int^{-Z} \frac{1}{-a \left( (f(x))^{1-n} \left( \frac{d}{dx}g(x) \right) (ag(x)+b)^{-n} \right)^{-n-1} (f(x) \frac{d}{dx}g(x))^{-2n+1}} \right) \right\}$$

**2.54 ODE No. 54**

$$-a^n f(x)^{1-n}g'(x)y(x)^n - \frac{y(x)f'(x)}{f(x)} - f(x)g'(x) + y'(x) = 0$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.159 (sec), leaf count = 38

$$\left\{ \frac{ay(x)}{nf(x)} \text{LerchPhi} \left( - \left( \frac{ay(x)}{f(x)} \right)^n, 1, n^{-1} \right) - ag(x) + \_C1 = 0 \right\}$$

**2.55 ODE No. 55**

$$-f(x)y(x)^n - g(x)y(x) - h(x) + y'(x) = 0$$

✗ **Mathematica** : cpu = 4.07045 (sec), leaf count = 0 , could not solve

`DSolve[-h[x] - g[x]*y[x] - f[x]*y[x]^n + Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)-f(x)*y(x)^n-g(x)*y(x)-h(x) = 0,y(x))`

**2.56 ODE No. 56**

$$-f(x)y(x)^a - g(x)y(x)^b + y'(x) = 0$$

✗ **Mathematica** : cpu = 2.31721 (sec), leaf count = 0 , could not solve

`DSolve[-(f[x]*y[x]^a) - g[x]*y[x]^b + Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)-f(x)*y(x)^a-g(x)*y(x)^b = 0,y(x))`

**2.57 ODE No. 57**

$$y'(x) - \sqrt{|y(x)|} = 0$$

✓ **Mathematica** : cpu = 100.704 (sec), leaf count = 283

$$\left\{ \left\{ \begin{array}{l} y(x) \rightarrow \text{InverseFunction} \left[ -\frac{2 \cdot 2^{3/4} (1 - \#1) \sqrt[4]{|\Im(\#1)| + i(1 - \Re(\#1))} (i|\Im(\#1)| - \Re(\#1) + 1) {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{7}{4}; \frac{2\Im(\#1)}{|\Im(\#1)| + i(1 - \Re(\#1))}\right)}{3 \sqrt[4]{|\Im(\#1)|} (\Im(\#1)^2 + (1 - \Re(\#1))^2)} \right] \end{array} \right. \right.$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 31

$$\left\{ x - \begin{cases} -2 \sqrt{-y(x)} & y(x) \leq 0 \\ 2 \sqrt{y(x)} & 0 < y(x) \end{cases} + C1 = 0 \right\}$$

**2.58 ODE No. 58**

$$a(-\sqrt{y(x)}) - bx + y'(x) = 0$$

✗ **Mathematica** : cpu = 301.207 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.071 (sec), leaf count = 68

$$\left\{ -\frac{1}{2} \ln(\sqrt{y(x)}ax + bx^2 - 2y(x)) + a\sqrt{y(x)} \text{Artanh}\left(1(a\sqrt{y(x)} + 2bx) \frac{1}{\sqrt{y(x)}(a^2 + 8b)}\right) \frac{1}{\sqrt{y(x)}(a^2 + 8b)} \right\}$$

2.59 ODE No. 59

$$a\left(-\sqrt{y(x)^2 + 1}\right) - b + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.179906 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{b \tan^{-1} \left( \frac{\#1b}{\sqrt{\#1^2 + 1} \sqrt{a^2 - b^2}} \right) - \frac{b \tan^{-1} \left( \frac{\#1a}{\sqrt{a^2 - b^2}} \right) + \sinh^{-1}(\#1)}{a} \right] \& [c_1 + x] \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 26

$$\left\{ x - \int^{y(x)} \left( a \sqrt{-a^2 + 1} + b \right)^{-1} d\_a + \_C1 = 0 \right\}$$

2.60 ODE No. 60

$$y'(x) - \frac{\sqrt{y(x)^2 - 1}}{\sqrt{x^2 - 1}} = 0$$

✓ **Mathematica** : cpu = 0.048519 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-c_1} \left( (e^{2c_1} - 1) \sqrt{x^2 - 1} + (e^{2c_1} + 1) x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 29

$$\left\{ \ln \left( x + \sqrt{x^2 - 1} \right) - \ln \left( y(x) + \sqrt{(y(x))^2 - 1} \right) + \_C1 = 0 \right\}$$

**Hand solution**

$$y' = \pm \frac{\sqrt{y^2 - 1}}{\sqrt{x^2 - 1}} \tag{1}$$

Separable. For the positive case

$$\begin{aligned} \frac{dy}{dx} \frac{1}{\sqrt{y^2 - 1}} &= \frac{1}{\sqrt{x^2 - 1}} \\ \frac{dy}{(y^2 - 1)^{\frac{1}{2}}} &= \frac{dx}{(x^2 - 1)^{\frac{1}{2}}} \end{aligned}$$

Integrating

$$\int \frac{dy}{(y^2 - 1)^{\frac{1}{2}}} = \int \frac{dx}{(x^2 - 1)^{\frac{1}{2}}} + C$$

But  $\int \frac{dy}{(y^2 - 1)^{\frac{1}{2}}} = \tanh^{-1} \frac{y}{(y^2 - 1)^{\frac{1}{2}}} = \ln(y + \sqrt{y^2 - 1})$ , hence

$$\ln(y + \sqrt{y^2 - 1}) = \ln(x + \sqrt{x^2 - 1}) + C$$

For the negative case

$$\begin{aligned} \frac{dy}{dx} \frac{1}{\sqrt{y^2 - 1}} &= -\frac{1}{\sqrt{x^2 - 1}} \\ \frac{dy}{(y^2 - 1)^{\frac{1}{2}}} &= -\frac{dx}{(x^2 - 1)^{\frac{1}{2}}} \end{aligned}$$

Integrating

$$\int \frac{dy}{(y^2 - 1)^{\frac{1}{2}}} = -\int \frac{dx}{(x^2 - 1)^{\frac{1}{2}}} + C$$

But  $\int \frac{dy}{(y^2 - 1)^{\frac{1}{2}}} = \tanh^{-1} \frac{y}{(y^2 - 1)^{\frac{1}{2}}} = \ln(y + \sqrt{y^2 - 1})$ , hence

$$\ln(y + \sqrt{y^2 - 1}) = -\ln(x + \sqrt{x^2 - 1}) + C$$

Therefore

$$\ln(y + \sqrt{y^2 - 1}) = \pm \ln(x + \sqrt{x^2 - 1}) + C$$

## 2.61 ODE No. 61

$$y'(x) - \frac{\sqrt{x^2 - 1}}{\sqrt{y(x)^2 - 1}} = 0$$

✓ **Mathematica** : cpu = 0.176101 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{1}{2} \#1 \sqrt{\#1^2 - 1} - \frac{1}{2} \log \left( \sqrt{\#1^2 - 1} + \#1 \right) \right] \& \right\} \left[ c_1 + \frac{1}{2} \sqrt{x^2 - 1} x - \frac{1}{2} \log \left( \sqrt{x^2 - 1} \right) \right] \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 50

$$\left\{ -C1 + x\sqrt{x^2 - 1} - \ln(x + \sqrt{x^2 - 1}) - y(x)\sqrt{(y(x))^2 - 1} + \ln(y(x) + \sqrt{(y(x))^2 - 1}) = 0 \right\}$$

2.62 ODE No. 62

$$y'(x) - \frac{y(x) - x^2\sqrt{x^2 - y(x)^2}}{xy(x)\sqrt{x^2 - y(x)^2} + x} = 0$$

✓ **Mathematica** : cpu = 3.78422 (sec), leaf count = 36

$$\text{Solve} \left[ 2 \tan^{-1} \left( \frac{y(x)}{\sqrt{x^2 - y(x)^2}} \right) + x^2 + y(x)^2 = 2c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.412 (sec), leaf count = 34

$$\left\{ \frac{(y(x))^2}{2} + \arctan \left( y(x) \frac{1}{\sqrt{x^2 - (y(x))^2}} \right) + \frac{x^2}{2} - C1 = 0 \right\}$$

**Hand solution**

$$y' = \frac{y - x^2\sqrt{x^2 - y^2}}{xy\sqrt{x^2 - y^2} + x} \tag{1}$$

Let  $y = ux$  then  $y' = u + xu'$  therefore

$$\begin{aligned} u + xu' &= \frac{y - x^2\sqrt{x^2 - y^2}}{xy\sqrt{x^2 - y^2} + x} \\ &= \frac{ux - x^2\sqrt{x^2 - (ux)^2}}{x(ux)\sqrt{x^2 - (ux)^2} + x} \\ &= \frac{ux - x^3\sqrt{1 - u^2}}{x^3u\sqrt{1 - u^2} + x} \\ &= \frac{u - x^2\sqrt{1 - u^2}}{x^2u\sqrt{1 - u^2} + 1} \end{aligned}$$

Hence

$$\begin{aligned} u(x^2u\sqrt{1 - u^2} + 1) + xu'(x^2u\sqrt{1 - u^2} + 1) &= u - x^2\sqrt{1 - u^2} \\ x^2u^2\sqrt{1 - u^2} + u + u'(x^3u\sqrt{1 - u^2} + x) &= u - x^2\sqrt{1 - u^2} \\ x^2u^2\sqrt{1 - u^2} + u' (x^3u\sqrt{1 - u^2} + x) &= -x^2\sqrt{1 - u^2} \\ xu^2\sqrt{1 - u^2} + u' (x^2u\sqrt{1 - u^2} + 1) &= -x\sqrt{1 - u^2} \\ xu^2 + u' \left( x^2u + \frac{1}{\sqrt{1 - u^2}} \right) &= -x \\ x(1 + u^2) + u' \left( x^2u + \frac{1}{\sqrt{1 - u^2}} \right) &= 0 \end{aligned}$$

Hence

$$x(1+u^2) dx + \left(x^2u + \frac{1}{\sqrt{1-u^2}}\right) du = 0 \quad (2)$$

Let  $M = x(1+u^2)$ ,  $N = \left(x^2u + \frac{1}{\sqrt{1-u^2}}\right)$ .

$$\begin{aligned} \frac{\partial M}{\partial u} &= 2xu \\ \frac{\partial N}{\partial x} &= 2xu \end{aligned}$$

Therefore (2) is exact. Let

$$x(1+u^2) dx + \left(x^2u + \frac{1}{\sqrt{1-u^2}}\right) du = dU$$

Since  $dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial u} du$ . Comparing with the above, we see that

$$\frac{\partial U}{\partial x} = x(1+u^2) \quad (3)$$

$$\frac{\partial U}{\partial u} = x^2u + \frac{1}{\sqrt{1-u^2}} \quad (4)$$

From (3)

$$\begin{aligned} U &= \int x(1+u^2) dx \\ &= \frac{x^2}{2}(1+u^2) + f(u) \end{aligned} \quad (5)$$

From (4)

$$\begin{aligned} \frac{d}{du} \left( \frac{x^2}{2}(1+u^2) + f(u) \right) &= x^2u + \frac{1}{\sqrt{1-u^2}} \\ x^2u + f'(u) &= x^2u + \frac{1}{\sqrt{1-u^2}} \\ f'(u) &= \frac{1}{\sqrt{1-u^2}} \end{aligned}$$

Therefore

$$f(u) = \arcsin(u)$$

From (5) we find

$$U(x, u) = \frac{x^2}{2}(1+u^2) + \arcsin(u)$$

Since  $dU = 0$  then

$$\begin{aligned} \frac{x^2}{2}(1+u^2) + \arcsin(u) &= C \\ \frac{x^2}{2}(1+u^2) + \arcsin(u) - C &= 0 \end{aligned}$$



Since  $y = ux$  then the above can be written as

$$\begin{aligned}\frac{x^2}{2} \left( 1 + \left( \frac{y}{x} \right)^2 \right) + \arcsin \left( \frac{y}{x} \right) - C &= 0 \\ \frac{x^2}{2} \left( \frac{x^2 + y^2}{x^2} \right) + \arcsin \left( \frac{y}{x} \right) - C &= 0 \\ \frac{1}{2}(x^2 + y^2) + \arcsin \left( \frac{y}{x} \right) - C &= 0 \\ \arcsin \left( \frac{y}{x} \right) &= C - \frac{1}{2}(x^2 + y^2)\end{aligned}$$

Hence

$$\begin{aligned}\frac{y}{x} &= \sin \left( C - \frac{1}{2}(x^2 + y^2) \right) \\ y(x) &= x \sin \left( C - \frac{1}{2}(x^2 + y^2) \right)\end{aligned}$$

### 2.63 ODE No. 63

$$y'(x) - \frac{y(x)^2 + 1}{(x+1)^{3/2} |y(x) + \sqrt{y(x)+1}|} = 0$$

✗ **Mathematica** : cpu = 300.007 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.141 (sec), leaf count = 35

$$\left\{ -2 \frac{1}{\sqrt{1+x}} - \int^{y(x)} \frac{1}{-a^2+1} | -a + \sqrt{-a+1} | d_a + _C1 = 0 \right\}$$

### 2.64 ODE No. 64

$$y'(x) - \sqrt{\frac{ay(x)^2 + by(x) + c}{ax^2 + bx + c}} = 0$$

✓ **Mathematica** : cpu = 0.175228 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\sqrt{ac_1}} \left( 2\sqrt{a} \left( e^{2\sqrt{ac_1}} - 1 \right) \sqrt{x(ax+b) + c} + b \left( e^{\sqrt{ac_1}} - 1 \right)^2 + 2ax \left( e^{2\sqrt{ac_1}} + 1 \right) \right)}{4a} \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 124

$$\left\{ -1 \sqrt{\frac{a(y(x))^2 + by(x) + c}{ax^2 + bx + c}} \sqrt{ax^2 + bx + c} \ln \left( \frac{1}{2} \left( 2 \sqrt{ax^2 + bx + c} \sqrt{a} + 2ax + b \right) \frac{1}{\sqrt{a}} \right) \frac{1}{\sqrt{a(y(x))^2 + by(x) + c}} \right\}$$

2.65 ODE No. 65

$$y'(x) - \sqrt{\frac{y(x)^3 + 1}{x^3 + 1}} = 0$$

✓ **Mathematica** : cpu = 1.52161 (sec), leaf count = 312

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{i(\#1 + 1) \sqrt{1 + \frac{6i}{(\sqrt{3}-3i)(\#1+1)}} \sqrt{\frac{2}{3} - \frac{4i}{(\sqrt{3}+3i)(\#1+1)}} F \left( i \sinh^{-1} \left( \frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{\#1+1}} \right) \right) \frac{3i+\sqrt{3}}{3i-\sqrt{3}}}{\sqrt{-\frac{i}{\sqrt{3}+3i}} \sqrt{\#1^2 - \#1 + 1}} \right] \right\} \right.$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 47

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^3 + 1}} da + \int^x -1 \sqrt{\frac{(y(x))^3 + 1}{-a^3 + 1}} \frac{1}{\sqrt{(y(x))^3 + 1}} da + C1 = 0 \right\}$$

2.66 ODE No. 66

$$y'(x) - \frac{\sqrt{|(1-y(x))y(x)(1-ay(x))|}}{\sqrt{|(1-x)x(1-ax)|}} = 0$$

✗ **Mathematica** : cpu = 300.005 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.07 (sec), leaf count = 40

$$\left\{ \int \frac{1}{\sqrt{|x(x-1)(ax-1)|}} dx - \int^{y(x)} \frac{1}{\sqrt{|-a(-a-1)(-aa-1)|}} da + C1 = 0 \right\}$$

2.67 ODE No. 67

$$y'(x) - \frac{\sqrt{1-y(x)^4}}{\sqrt{1-x^4}} = 0$$

✓ **Mathematica** : cpu = 0.0605338 (sec), leaf count = 14

$$\{ \{ y(x) \rightarrow \text{sn}(c_1 + F(\sin^{-1}(x) | -1) | -1) \} \}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 51

$$\left\{ \text{EllipticF}(x, i) \sqrt{-x^2 + 1} \sqrt{x^2 + 1} \frac{1}{\sqrt{-x^4 + 1}} - \int^{y(x)} \frac{1}{\sqrt{-a^4 + 1}} da + C1 = 0 \right\}$$

2.68 ODE No. 68

$$y'(x) - \sqrt{\frac{ay(x)^4 + by(x)^2 + 1}{ax^4 + bx^2 + 1}} = 0$$

✓ **Mathematica** : cpu = 0.851989 (sec), leaf count = 373

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{i \sqrt{\frac{2\#1^2 a + \sqrt{b^2 - 4a + b}}{\sqrt{b^2 - 4a + b}}} \sqrt{\frac{2\#1^2 a}{b - \sqrt{b^2 - 4a}}} + 1 F\left(i \sinh^{-1}\left(\sqrt{2} \sqrt{\frac{a}{b + \sqrt{b^2 - 4a}}}\#1\right) \middle| \frac{b + \sqrt{b^2 - 4a}}{b - \sqrt{b^2 - 4a}}\right)}{\sqrt{2} \sqrt{\frac{a}{\sqrt{b^2 - 4a + b}}} \sqrt{\#1^4 a + \#1^2 b + 1}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 77

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^4 a + -a^2 b + 1}} d_a + \int^x -1 \sqrt{\frac{a(y(x))^4 + b(y(x))^2 + 1}{-a^4 a + -a^2 b + 1}} \frac{1}{\sqrt{a(y(x))^4 + b(y(x))^2 + 1}} d_a + -C1 = 0 \right\}$$

2.69 ODE No. 69

$$y'(x) - \sqrt{(a0 + a1x + a2x^2 + a3x^3 + a4x^4)(b0 + b1y(x) + b2y(x)^2 + b3y(x)^3 + b4y(x)^4)} = 0$$

✗ **Mathematica** : cpu = 300.666 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.146 (sec), leaf count = 111

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^4 b4 + -a^3 b3 + -a^2 b2 + -a b1 + b0}} d_a + \int^x -1 \sqrt{(b4(y(x))^4 + b3(y(x))^3 + b2(y(x))^2 + b1 y(x) + b0)} d_a + -C1 = 0 \right\}$$

2.70 ODE No. 70

$$y'(x) - \sqrt{\frac{a0 + a1x + a2x^2 + a3x^3 + a4x^4}{b0 + b1y(x) + b2y(x)^2 + b3y(x)^3 + b4y(x)^4}} = 0$$

✗ **Mathematica** : cpu = 300.062 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.163 (sec), leaf count = 113

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^4 b4 + -a^3 b3 + -a^2 b2 + -a b1 + b0}} d_a + \int^x -\sqrt{\frac{-a^4 a4 + -a^3 a3 + -a^2 a2 + -a a1 + a0}{b4(y(x))^4 + b3(y(x))^3 + b2(y(x))^2 + b1 y(x) + b0}} d_a + -C1 = 0 \right\}$$

2.71 ODE No. 71

$$y'(x) - \sqrt{\frac{b_0 + b_1 y(x) + b_2 y(x)^2 + b_3 y(x)^3 + b_4 y(x)^4}{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4}} = 0$$

✗ **Mathematica** : cpu = 303.658 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.13 (sec), leaf count = 113

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^4 b_4 + -a^3 b_3 + -a^2 b_2 + -a b_1 + b_0}} d_a + \int^x -1 \sqrt{\frac{b_4 (y(x))^4 + b_3 (y(x))^3 + b_2 (y(x))^2 + b_1 y(x) + b_0}{-a^4 a_4 + -a^3 a_3 + -a^2 a_2 + -a a_1 + a_0}} dx \right.$$

2.72 ODE No. 72

$$y'(x) - R1(x, \sqrt{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4}) R2(y(x), \sqrt{b_0 + b_1 y(x) + b_2 y(x)^2 + b_3 y(x)^3 + b_4 y(x)^4}) = 0$$

✓ **Mathematica** : cpu = 0.831465 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} \frac{1}{R2(K[1], \sqrt{b_1 K[1] + b_2 K[1]^2 + b_3 K[1]^3 + b_4 K[1]^4 + b_0})} dK[1] \& \right] \left[ \int_1^x R1 \right. \right. \right.$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 64

$$\left\{ \int R1(x, \sqrt{a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0}) dx - \int^{y(x)} \left( R2(-a, \sqrt{-a^4 b_4 + -a^3 b_3 + -a^2 b_2 + -a b_1 + b_0}) \right) dy \right.$$

2.73 ODE No. 73

$$y'(x) - \left( \frac{a_0 + a_1 x + a_2 x^2 + a_3 x^3}{a_0 + a_1 y(x) + a_2 y(x)^2 + a_3 y(x)^3} \right)^{2/3} = 0$$

✓ **Mathematica** : cpu = 49.9862 (sec), leaf count = 733

$$\text{Solve} \left[ \frac{3(a_0 + y(x)(a_1 + y(x)(a_2 + a_3 y(x))))^{2/3} (y(x) - \text{Root}[\#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 1]) F_1\left(\frac{5}{3}; -\frac{2}{3}, -\frac{2}{3}; \frac{y(x) - \text{Root}[\#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 2]}{\text{Root}[\#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 1]} - \text{Root}[\#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 1]}\right)}{5 \left( \frac{y(x) - \text{Root}[\#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 2]}{\text{Root}[\#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 1]} - \text{Root}[\#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 1]}{\text{Root}[\#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 1]} - \text{Root}[\#1^3 a_3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 1]} \right)^{2/3}} \right]$$

✓ **Maple** : cpu = 0.349 (sec), leaf count = 91

$$\left\{ \int^{y(x)} (-a^3 a_3 + -a^2 a_2 + -a a_1 + a_0)^{2/3} d_a + \int^x - \left( \frac{-a^3 a_3 + -a^2 a_2 + -a a_1 + a_0}{a_3 (y(x))^3 + a_2 (y(x))^2 + a_1 y(x) + a_0} \right)^{2/3} (a_3 (y(x)))^{2/3} dy \right.$$

2.74 ODE No. 74

$$y'(x) - f(x)(y(x) - g(x))\sqrt{(y(x) - a)(y(x) - b)} = 0$$

✗ **Mathematica** : cpu = 2.99035 (sec), leaf count = 0 , could not solve

DSolve[-(f[x]\*Sqrt[(-a + y[x])\*(-b + y[x])]\*(-g[x] + y[x])) + Derivative[1][y][x] == 0, y[x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(y(x),x)-f(x)\*(y(x)-g(x))\*((y(x)-a)\*(y(x)-b))^(1/2) = 0,y(x))

2.75 ODE No. 75

$$y'(x) - e^{x-y(x)} + e^x = 0$$

✓ **Mathematica** : cpu = 0.0195488 (sec), leaf count = 18

$$\{ \{ y(x) \rightarrow \log(e^{C1 - e^x} + 1) \} \}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 20

$$\{ y(x) = -e^x + \ln(-1 + e^{e^x + C1}) - C1 \}$$

**Hand solution**

$$y' = e^{x-y} - e^x$$

$$y' = e^x(e^{-y} - 1)$$

$$\frac{1}{e^{-y} - 1} dy = e^x dx \tag{1}$$

Integrating both sides.  $\int \frac{1}{e^{-y}-1} dy$ . Let  $e^{-y} = u$ , then  $\frac{du}{dy} = -e^{-y} = -u$ . Hence  $dy = -\frac{du}{u}$ , therefore the integral becomes

$$\int \frac{1}{u-1} \left( -\frac{du}{u} \right) = - \int \frac{1}{u(u-1)} du$$

But  $\frac{1}{u(u-1)} = -\left(\frac{1}{u} - \frac{1}{u-1}\right)$ , hence

$$\begin{aligned} - \int \frac{1}{u(u-1)} du &= \int \left( \frac{1}{u} - \frac{1}{u-1} \right) du \\ &= \ln u - \ln(u-1) \\ &= \ln e^{-y} - \ln(e^{-y} - 1) \\ &= -(\ln(e^{-y} - 1) - \ln e^{-y}) \end{aligned}$$

But  $\ln x - \ln y = \ln\left(\frac{x}{y}\right)$  and the above becomes

$$\int \frac{1}{e^{-y} - 1} dy = - \left[ \ln\left(\frac{e^{-y} - 1}{e^{-y}}\right) \right] \\ = - \ln(1 - e^y)$$

Back to (1), when we integrate both sides, and since  $\int e^x dx = e^x + C$

$$- \ln(1 - e^y) = e^x + C \\ \ln(1 - e^y) = -e^x + C_1$$

Hence

$$1 - e^y = \exp(-e^x + C_1) \\ e^y = 1 - \exp(-e^x + C_1)$$

Taking logs

$$y = \ln(1 - \exp(-e^x + C_1))$$

Let  $e^{C_1} = C_2$  then

$$y = \ln(1 - C_2 e^{-e^x})$$

Verification

```
ode:=diff(y(x),x)=exp(x-y(x))-exp(x);
my_sol:=log(1-C1*exp(-exp(x)));
odetest(y(x)=my_sol,ode);
0
```

## 2.76 ODE No. 76

$$-a \cos(y(x)) + b + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.119814 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow 2 \tan^{-1} \left( \frac{(a-b) \tanh\left(\frac{1}{2}\sqrt{a^2-b^2}(x-c_1)\right)}{\sqrt{a^2-b^2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 41

$$\left\{ y(x) = 2 \arctan \left( \frac{\tanh\left(\frac{1}{2}\sqrt{a^2-b^2}(x+_C1)\right) \sqrt{a^2-b^2}}{a+b} \right) \right\}$$

**Hand solution**

$$y' = a \cos y + b$$

This is separable.

$$\begin{aligned} \frac{dy}{a \cos y + b} &= dx \\ \int \frac{dy}{a \cos y + b} &= x + C \end{aligned} \quad (1)$$

Using standard Tangent half-angle substitution, let  $t = \tan \frac{y}{2}$ ,  $\cos y = \frac{1-t^2}{1+t^2}$ ,  $dy = \frac{2}{1+t^2} dt$ , then the integral becomes

$$\begin{aligned} \int \frac{dy}{a \cos y + b} &= \int \frac{2}{1+t^2} \frac{1}{\left(a \frac{1-t^2}{1+t^2} + b\right)} dt \\ &= 2 \int \frac{1+t^2}{(1+t^2)(a(1-t^2) + b(1+t^2))} dt \\ &= 2 \int \frac{dt}{a - at^2 + b + bt^2} \\ &= 2 \int \frac{dt}{(a+b) + t^2(b-a)} \\ &= 2 \int \frac{dt}{(a+b) \left(1 + \frac{t^2(b-a)}{(a+b)}\right)} \\ &= \frac{2}{a+b} \int \frac{dt}{\left(1 + \frac{t^2(b-a)}{(a+b)}\right)} \end{aligned}$$

Let  $z^2 = \frac{t^2(b-a)}{(a+b)}$ , or  $z = \frac{t\sqrt{b-a}}{\sqrt{a+b}}$ , then  $\frac{dz}{dt} = \frac{\sqrt{b-a}}{\sqrt{a+b}}$  and the above integral becomes

$$\begin{aligned} \frac{2}{a+b} \int \frac{dt}{\left(1 + \frac{t^2(b-a)}{(a+b)}\right)} &= \frac{2}{a+b} \int \frac{\sqrt{a+b}}{\sqrt{b-a}} \frac{dz}{(1+z^2)} \\ &= \frac{2}{a+b} \frac{\sqrt{a+b}}{\sqrt{b-a}} \int \frac{dz}{(1+z^2)} \\ &= \frac{2}{\sqrt{a+b}} \frac{1}{\sqrt{b-a}} \int \frac{dz}{(1+z^2)} \\ &= \frac{2}{\sqrt{(a+b)(b-a)}} \int \frac{dz}{(1+z^2)} \\ &= \frac{2}{\sqrt{b^2 - a^2}} \int \frac{dz}{(1+z^2)} \end{aligned}$$

Now,  $\int \frac{dz}{(1+z^2)} = \arctan(z)$ , hence

$$\begin{aligned} \frac{2}{\sqrt{b^2 - a^2}} \int \frac{dz}{(1+z^2)} &= \frac{2}{\sqrt{b^2 - a^2}} \arctan(z) \\ &= \frac{2}{\sqrt{b^2 - a^2}} \arctan\left(\frac{t\sqrt{b-a}}{\sqrt{a+b}}\right) \end{aligned}$$

But  $t = \tan \frac{y}{2}$  therefore

$$\frac{2}{\sqrt{b^2 - a^2}} \arctan\left(\frac{t\sqrt{b-a}}{\sqrt{a+b}}\right) = \frac{2}{\sqrt{b^2 - a^2}} \arctan\left(\frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}}\right)$$

Going back to (1)

$$\begin{aligned} \int \frac{dy}{a \cos y + b} &= x + C \\ \frac{2}{\sqrt{b^2 - a^2}} \arctan\left(\frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}}\right) &= x + C \\ \arctan\left(\frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}}\right) &= \frac{1}{2}\sqrt{b^2 - a^2}(x + C) \\ \frac{\tan\left(\frac{y}{2}\right)\sqrt{b-a}}{\sqrt{a+b}} &= \tan\left(\frac{1}{2}\sqrt{b^2 - a^2}(x + C)\right) \\ \tan\left(\frac{y}{2}\right) &= \frac{\sqrt{a+b}}{\sqrt{b-a}} \tan\left(\frac{1}{2}\sqrt{b^2 - a^2}(x + C)\right) \\ \frac{y}{2} &= \arctan\left(\frac{(a+b)}{\sqrt{(a+b)(b-a)}} \tan\left(\frac{1}{2}\sqrt{b^2 - a^2}(x + C)\right)\right) \\ &= \arctan\left(\frac{(a+b)}{\sqrt{b^2 - a^2}} \tan\left(\frac{1}{2}\sqrt{b^2 - a^2}(x + C)\right)\right) \\ y &= 2 \arctan\left(\frac{a+b}{\sqrt{b^2 - a^2}} \tan\left(\frac{1}{2}\sqrt{b^2 - a^2}(x + C)\right)\right) \end{aligned}$$

Verification

```
ode:=diff(y(x),x)=a*cos(y(x))+b;
my_sol:=2*arctan((a+b)/sqrt(b^2-a^2) * tan(1/2*sqrt(b^2-a^2)*(x+C1)));
odetest(y(x)=my_sol,ode);
0
```



2.77 ODE No. 77

$$y'(x) - \cos(ay(x) + bx) = 0$$

✓ **Mathematica** : cpu = 0.296727 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow \frac{2 \tan^{-1} \left( \frac{(a+b) \tanh \left( \frac{1}{2} \sqrt{a^2 - b^2} (x - c_1) \right)}{\sqrt{a^2 - b^2}} \right) - bx}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 54

$$\left\{ y(x) = \frac{1}{a} \left( -bx + 2 \arctan \left( \frac{\tanh \left( 1/2 \sqrt{a^2 - b^2} (x - C1) \right) \sqrt{a^2 - b^2}}{a - b} \right) \right) \right\}$$

**Hand solution**

$$y' = \cos(ay + bx)$$

This is separable after transformation of  $u = ay + bx$ , hence  $u' = ay' + b$  or  $y' = \frac{1}{a}(u' - b)$ . Therefore the above becomes

$$\begin{aligned} \frac{1}{a}(u' - b) &= \cos(u) \\ u' &= a \cos u + b \\ \frac{du}{a \cos u + b} &= dx \end{aligned}$$

This is the same as Kamke 76 (the problem before this), which we solved using half angle tan transformation, and the answer is

$$u = 2 \arctan \left( \frac{a + b}{\sqrt{b^2 - a^2}} \tan \left( \frac{1}{2} \sqrt{b^2 - a^2} (x + C) \right) \right)$$

Since  $u = ay + bx$  then  $y = \frac{u - bx}{a}$ , hence

$$y = \frac{1}{a} \left( 2 \arctan \left( \frac{a + b}{\sqrt{b^2 - a^2}} \tan \left( \frac{1}{2} \sqrt{b^2 - a^2} (x + C) \right) \right) - bx \right)$$

Verification

```
ode:=diff(y(x),x)=cos(a*y(x)+b*x);
my_sol:=(1/a)*(2*arctan((a+b)/sqrt(b^2-a^2))*tan(1/2*sqrt(b^2-a^2)*(x+C1)))-b*x;
odetest(y(x)=my_sol,ode);
0
```

## 2.78 ODE No. 78

$$a \sin(\alpha y(x) + \beta x) + b + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.860759 (sec), leaf count = 86

$$\left\{ \left\{ y(x) \rightarrow \frac{2 \tan^{-1} \left( \frac{\sqrt{(\beta - \alpha b)^2 - a^2 \alpha^2} \tan \left( \frac{1}{2} (c_1 - x) \sqrt{(\beta - \alpha b)^2 - a^2 \alpha^2} \right) - a \alpha}{\alpha b - \beta} \right) - \beta x}{\alpha} \right\} \right\}$$

✓ **Maple** : cpu = 2.176 (sec), leaf count = 89

$$\left\{ y(x) = \frac{1}{\alpha} \left( -\beta x + 2 \arctan \left( \frac{-\tan \left( \frac{1}{2} \sqrt{(-a^2 + b^2) \alpha^2 - 2 \alpha b \beta + \beta^2} (x - C_1) \right) \sqrt{(-a^2 + b^2) \alpha^2 - 2 \alpha b \beta}}{b \alpha - \beta} \right) \right) \right\}$$

**Hand solution**

$$y' = -a \sin(\alpha y + \beta x) - b$$

This is separable after transformation of  $u = \alpha y + \beta x$ , hence  $u' = \alpha y' + \beta$  or  $y' = \frac{1}{\alpha}(u' - \beta)$ .  
Therefore the above becomes

$$\begin{aligned} \frac{1}{\alpha}(u' - \beta) &= -a \sin(u) - b \\ u' &= -\alpha(a \sin(u) + b) + \beta \\ \frac{du}{\beta - \alpha(a \sin(u) + b)} &= dx \end{aligned} \tag{1}$$

Using half angle tan transformation where  $\tan\left(\frac{u}{2}\right) = t$ ,  $\sin(u) = \frac{2t}{t^2+1}$ ,  $du = \frac{2}{1+t^2} dt$  then

$$\begin{aligned}
\int \frac{du}{\beta - \alpha(a \sin(u) + b)} &= \int \frac{2}{1+t^2} \frac{dt}{\beta - \alpha\left(a \frac{2t}{t^2+1} + b\right)} \\
&= 2 \int \frac{dt}{\beta(t^2+1) - \alpha(a2t + b(t^2+1))} \\
&= 2 \int \frac{dt}{t\beta^2 + \beta - (\alpha a 2t + t^2 \alpha b + \alpha b)} \\
&= 2 \int \frac{dt}{(\beta - \alpha b) \left( \frac{t\beta^2 - \alpha a 2t - t^2 \alpha b}{(\beta - \alpha b)} + 1 \right)} \\
&= \frac{2}{(\beta - \alpha b)} \int \frac{dt}{\frac{t\beta^2 - \alpha a 2t - t^2 \alpha b}{(\beta - \alpha b)} + 1} \\
&= \frac{2}{(\beta - \alpha b)} \frac{-(\alpha b - \beta)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}} \tanh^{-1} \left( \frac{\left(t + \frac{\alpha a}{b\alpha - \beta}\right) (b\alpha - \beta)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}} \right) \\
&= \frac{2}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}} \tanh^{-1} \left( \frac{t(b\alpha - \beta) + \alpha a}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}} \right)
\end{aligned}$$

But  $t = \tan\left(\frac{u}{2}\right)$  therefore

$$\int \frac{du}{\beta - \alpha(a \sin(u) + b)} = \frac{2}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}} \tanh^{-1} \left( \frac{\tan\left(\frac{u}{2}\right) (b\alpha - \beta) + \alpha a}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}} \right)$$

But  $u = \alpha y + \beta x$ , and the above becomes

$$\int \frac{du}{\beta - \alpha(a \sin(u) + b)} = \frac{2 \tanh^{-1} \left( \frac{\tan\left(\frac{\alpha y + \beta x}{2}\right) (b\alpha - \beta) + \alpha a}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}} \right)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}}$$

Back to (1), therefore after integrating both sides

$$\frac{2 \tanh^{-1} \left( \frac{\tan\left(\frac{\alpha y + \beta x}{2}\right) (b\alpha - \beta) + \alpha a}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}} \right)}{\sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}} = x + C$$

Let

$$A = \sqrt{\alpha^2 a^2 - (\alpha^2 b^2 + \beta^2 - 2\alpha b \beta)}$$

Then

$$\begin{aligned} \tanh^{-1} \left( \frac{\tan \left( \frac{\alpha y + \beta x}{2} \right) (b\alpha - \beta) + a\alpha}{A} \right) &= \frac{1}{2} A(x + C) \\ \frac{\tan \left( \frac{\alpha y + \beta x}{2} \right) (b\alpha - \beta) + a\alpha}{A} &= \tanh \left( \frac{1}{2} A(x + C) \right) \\ \tan \left( \frac{\alpha y + \beta x}{2} \right) (b\alpha - \beta) + a\alpha &= A \tanh \left( \frac{1}{2} A(x + C) \right) \\ \tan \left( \frac{\alpha y + \beta x}{2} \right) &= \frac{A}{(b\alpha - \beta)} \tanh \left( \frac{1}{2} A(x + C) \right) - \frac{a\alpha}{(b\alpha - \beta)} \\ \frac{\alpha y + \beta x}{2} &= \arctan \left( \frac{A}{(b\alpha - \beta)} \tanh \left( \frac{1}{2} A(x + C) \right) - \frac{a\alpha}{(b\alpha - \beta)} \right) \\ y &= \frac{2}{\alpha} \arctan \left( \frac{A}{(b\alpha - \beta)} \tanh \left( \frac{1}{2} A(x + C) \right) - \frac{a\alpha}{(b\alpha - \beta)} \right) - \frac{\beta x}{\alpha} \end{aligned}$$

Verification

```
ode:=diff(y(x),x)=-a*sin(alpha*y(x)+beta*x)-b;
A0:=sqrt(alpha^2*a^2-(alpha^2*b^2+beta^2-2*alpha*b*beta));
B0:=(alpha*b-beta);
my_sol:=2/alpha*arctan(A0/B0*tanh((1/2)*A0*(x+C1))-a*alpha/(B0))-beta*x/alpha;
odetest(y(x)=my_sol,ode);
0
```

## 2.79 ODE No. 79

$$f(x) \cos(ay(x)) + g(x) \sin(ay(x)) + h(x) + y'(x) = 0$$

✗ **Mathematica** : cpu = 25.2598 (sec), leaf count = 0 , could not solve

```
DSolve[Cos[a*y[x]]*f[x] + h[x] + g[x]*Sin[a*y[x]] + Derivative[1][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(diff(y(x),x)+f(x)*cos(a*y(x))+g(x)*sin(a*y(x))+h(x) = 0,y(x))
```

**2.80 ODE No. 80**

$$(1 - f'(x)) \cos(y(x)) - f'(x) + f(x) \sin(y(x)) + y'(x) - 1 = 0$$

✗ **Mathematica** : cpu = 23.7792 (sec), leaf count = 0 , could not solve

`DSolve[-1 + f[x]*Sin[y[x]] + Cos[y[x]]*(1 - Derivative[1][f][x]) - Derivative[1][f][x] + Derivative[1][y][x] - 1 == 0, y[x], x]`

✓ **Maple** : cpu = 1.497 (sec), leaf count = 41

$$\left\{ y(x) = 2 \arctan \left( \frac{-e^{\int f(x) dx} + \int e^{\int f(x) dx} dx f(x) + f(x) - C1}{-C1 + \int e^{\int f(x) dx} dx} \right) \right\}$$

**2.81 ODE No. 81**

$$y'(x) + 2 \tan(x) \tan(y(x)) - 1 = 0$$

✗ **Mathematica** : cpu = 43.3971 (sec), leaf count = 0 , could not solve

`DSolve[-1 + 2*Tan[x]*Tan[y[x]] + Derivative[1][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.296 (sec), leaf count = 78

$$\left\{ -C1 + \tan(x) \frac{1}{\sqrt[4]{\frac{(1+\tan(y(x)))^2(1+\tan(x))^2}{(\tan(y(x))\tan(x)-1)^2}}} + \frac{\tan(y(x)) + \tan(x)}{2 \tan(y(x)) \tan(x) - 2} {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{3}{2}; -\frac{(\tan(y(x)) + \tan(x))^2}{(\tan(y(x)) \tan(x) - 1)}\right) \right\}$$

**2.82 ODE No. 82**

$$-a(\tan^2(y(x)) + 1) + y'(x) + \tan(x) \tan(y(x)) = 0$$

✗ **Mathematica** : cpu = 49.7927 (sec), leaf count = 0 , could not solve

`DSolve[Tan[x]*Tan[y[x]] - a*(1 + Tan[y[x]]^2) + Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x), x) - a*(1+tan(y(x))^2) + tan(y(x))*tan(x) = 0, y(x))`

2.83 ODE No. 83

$$y'(x) - \tan(xy(x)) = 0$$

✗ **Mathematica** : cpu = 40.3737 (sec), leaf count = 0 , could not solve

DSolve[-Tan[x\*y[x]] + Derivative[1][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.449 (sec), leaf count = 44

$$\left\{ y(x) = -i\text{RootOf}\left(-\text{Erf}\left(\frac{(-x + \_Z)\sqrt{2}}{2}\right)\sqrt{\pi} - \sqrt{\pi}\text{Erf}\left(\frac{\sqrt{2}(x + \_Z)}{2}\right) + \sqrt{2}\_C1\right) \right\}$$

2.84 ODE No. 84

$$y'(x) - f(ax + by(x)) = 0$$

✓ **Mathematica** : cpu = 10.3426 (sec), leaf count = 120

$$\text{Solve}\left[ c_1 = \int_1^{y(x)} \frac{(bf(bK[2] + ax) + a) \int_1^x \frac{ab^2 f'(aK[1] + bK[2])}{(bf(aK[1] + bK[2]) + a)^2} dK[1] + b}{bf(bK[2] + ax) + a} dK[2] + \int_1^x \frac{bf(aK[1] + by(x))}{bf(aK[1] + by(x)) + a} dK[1] \right]$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 37

$$\left\{ y(x) = \frac{\text{RootOf}\left(\int^{-Z}(f(\_a b) b + a)^{-1} d\_ab - x + \_C1\right) b - ax}{b} \right\}$$

2.85 ODE No. 85

$$y'(x) - x^{a-1}y(x)^{1-b}f\left(\frac{x^a}{a} + \frac{y(x)^b}{b}\right) = 0$$

✓ **Mathematica** : cpu = 197.699 (sec), leaf count = 182

$$\text{Solve}\left[ c_1 = \int_1^{y(x)} \frac{K[2] \left(f\left(\frac{K[2]^b}{b} + \frac{x^a}{a}\right) + 1\right) \left(\int_1^x \frac{K[1]^{a-1} K[2]^{b-1} f'\left(\frac{K[1]^a}{a} + \frac{K[2]^b}{b}\right)}{\left(f\left(\frac{K[1]^a}{a} + \frac{K[2]^b}{b}\right) + 1\right)^2} dK[1]\right) + K[2]^b}{K[2] \left(f\left(\frac{K[2]^b}{b} + \frac{x^a}{a}\right) + 1\right)} dK[2] + \int_1^x \frac{K[1]^{a-1} K[2]^{b-1} f'\left(\frac{K[1]^a}{a} + \frac{K[2]^b}{b}\right)}{\left(f\left(\frac{K[1]^a}{a} + \frac{K[2]^b}{b}\right) + 1\right)^2} dK[1] \right]$$

✓ **Maple** : cpu = 0.468 (sec), leaf count = 153

$$\left\{ y(x) = \sqrt[b]{-\frac{1}{a} \left( -\text{RootOf} \left( \int^{-Z} \left( (\sqrt[b]{-b+_-a})^{-b} (\sqrt[a]{a})^a f \left( \frac{(\sqrt[a]{a})^a b + (\sqrt[b]{-b+_-a})^b a}{ab} \right) -a - (\sqrt[b]{-b+_-a}) \right) \right)} \right.$$

**2.86 ODE No. 86**

$$y'(x) - \frac{y(x) - x f(ay(x)^2 + x^2)}{ay(x) f(ay(x)^2 + x^2) + x} = 0$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.499 (sec), leaf count = 52

$$\left\{ 1 \arctan \left( x \sqrt{a} \frac{1}{\sqrt{a^2 (y(x))^2}} \right) \frac{1}{\sqrt{a}} - \frac{1}{2} \int^{(y(x))^2 + \frac{x^2}{a}} \frac{f(-a a)}{-a} d_-a - C1 = 0 \right\}$$

**2.87 ODE No. 87**

$$y'(x) - \frac{cx^a y(x)^b + ay(x) f(x^c y(x))}{bx f(x^c y(x)) - x^a y(x)^b} = 0$$

✗ **Mathematica** : cpu = 15.2498 (sec), leaf count = 0 , could not solve

`DSolve[-((a*f[x^c*y[x]]*y[x] + c*x^a*y[x]^b)/(b*x*f[x^c*y[x]] - x^a*y[x]^b)) + Derivative[1]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)-(y(x)*a*f(x^c*y(x))+c*x^a*y(x)^b)/(x*b*f(x^c*y(x))-x^a*y(x)^b) = 0,y(x))`

**2.88 ODE No. 88**

$$-ce^{-2ax} - 4ay(x) - b + 2y'(x) - 3y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.283671 (sec), leaf count = 2479

$$\left\{ \left\{ y(x) \rightarrow \frac{3^{\frac{a+\sqrt{4a^2-3b}}{2a}} 4^{\frac{\sqrt{4a^4-3a^2b}}{a^2}} a^{\frac{\sqrt{4a^4-3a^2b}}{a^2}+1} b^{\frac{a+\sqrt{4a^2-3b}}{2a}} c^{\frac{a+\sqrt{4a^2-3b}}{2a}} J_{\frac{\sqrt{4a^4-3a^2b}}{2a^2}-1} \left( \frac{\sqrt{3}\sqrt{b}\sqrt{c}\sqrt{\frac{e^{-2ax}}{b}}}{2a} \right) \Gamma \left( \frac{\sqrt{4a^4-3a^2b}}{2a^2} + \right)}{\right. \right.$$

✓ **Maple** : cpu = 0.233 (sec), leaf count = 256

$$\left\{ y(x) = 1 \left( -e^{-ax} \sqrt{3} \left( Y_{-\frac{1}{2a}}(\sqrt{4a^2-3b-2a}) \left( \frac{e^{-ax} \sqrt{3}}{2a} \sqrt{c} \right) - C_1 + J_{-\frac{1}{2a}}(\sqrt{4a^2-3b-2a}) \left( \frac{e^{-ax} \sqrt{3}}{2a} \sqrt{c} \right) \right) \sqrt{c} - \left( \sqrt{4} \right) \right. \right.$$

**Hand solution**

$$y' = \frac{1}{2}b + \frac{1}{2}ce^{-2ax} + 2ay + \frac{3}{2}y^2$$

This is of the form  $y' = f_0 + f_1y + f_2y^2$  with  $f_0 = \frac{1}{2}b + \frac{1}{2}ce^{-2ax}$ ,  $f_1 = 2a$ ,  $f_2 = \frac{3}{2}$ . Hence it is Riccati non-linear first order. Transforming to second order ODE using

$$\begin{aligned} y &= -\frac{u'}{uf_2} \\ &= \frac{-2u'}{3u} \end{aligned}$$

Hence  $y' = \frac{-2}{3} \left( \frac{u''}{u} - \frac{(u')^2}{u^2} \right)$  and equating this to RHS of the ODE gives

$$\begin{aligned} \frac{-2}{3} \left( \frac{u''}{u} - \frac{(u')^2}{u^2} \right) &= \frac{1}{2}b + \frac{1}{2}ce^{-2ax} + 2a \left( \frac{-2u'}{3u} \right) + \frac{3}{2} \left( \frac{-2u'}{3u} \right)^2 \\ \frac{-2u''}{3u} + \frac{2(u')^2}{3u^2} &= \frac{1}{2}b + \frac{1}{2}ce^{-2ax} - \frac{4}{3}a \frac{u'}{u} + \frac{2(u')^2}{3u^2} \\ \frac{-2u''}{3u} &= \frac{1}{2}b + \frac{1}{2}ce^{-2ax} - \frac{4}{3}a \frac{u'}{u} \\ \frac{u''}{u} &= -\frac{3}{4}b - \frac{3}{4}ce^{-2ax} + 2a \frac{u'}{u} \\ u'' &= -\left( \frac{3}{4}b + \frac{3}{4}ce^{-2ax} \right) u + 2au' \end{aligned}$$

$$u'' - 2au' + \frac{3}{4}(b + ce^{-2ax})u = 0$$

This is second order linear ODE with varying coefficient. Solved using power series method giving solutions using special functions (Bessel functions). Let  $A = \frac{\sqrt{4a^2-3b}}{a}$ ,  $B = \frac{\sqrt{3ce^{-ax}}}{a}$  then

$$u(x) = C_1 e^{ax} \text{BesselJ} \left( -\frac{1}{2} \frac{\sqrt{4a^2-3b}}{a}, \frac{1}{2} \frac{\sqrt{3ce^{-ax}}}{a} \right) + C_2 e^{ax} \text{BesselY} \left( -\frac{1}{2} \frac{\sqrt{4a^2-3b}}{a}, \frac{1}{2} \frac{\sqrt{3ce^{-ax}}}{a} \right)$$

But



$$\begin{aligned}
u'(x) &= C_1 a \exp(ax) \operatorname{BesselJ}\left(-1/2 \frac{\sqrt{4a^2 - 3b}}{a}, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) \\
&- 1/2 C_1 \exp(ax) \left( -\operatorname{BesselJ}\left(-1/2 \frac{\sqrt{4a^2 - 3b}}{a} + 1, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) - 1/3 \frac{\sqrt{3}\sqrt{4a^2 - 3b}}{\sqrt{c} \exp(-ax)} \operatorname{BesselJ}\left(-1/2 \frac{\sqrt{4a^2 - 3b}}{a}, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) \right. \\
&\quad \left. + C_2 a \exp(ax) \operatorname{BesselY}\left(-1/2 \frac{\sqrt{4a^2 - 3b}}{a}, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) \right) \\
&- 1/2 C_1 \exp(ax) \left( -\operatorname{BesselY}\left(-1/2 \frac{\sqrt{4a^2 - 3b}}{a} + 1, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) - 1/3 \frac{\sqrt{3}\sqrt{4a^2 - 3b}}{\sqrt{c} \exp(-ax)} \operatorname{BesselY}\left(-1/2 \frac{\sqrt{4a^2 - 3b}}{a}, 1/2 \frac{\sqrt{3}\sqrt{c} \exp(-ax)}{a}\right) \right)
\end{aligned}$$

Hence from  $y = \frac{-2}{3} \frac{u'}{u}$  the solution is now found.

Verification

```

ode:=2*diff(y(x),x)-3*y(x)^2-4*a*y(x)=b+c*exp(-2*a*x);
uode:=diff(u(x),x^2)-2*a*diff(u(x),x)+3/4*(b+c*(exp(-2*a*x)))*u(x)=0;
uSol:=dsolve(uode,u(x));
my_sol:=(-2/3)*diff(rhs(uSol),x)/rhs(uSol);
odetest(y(x)=my_sol,ode);
0

```

### 2.89 ODE No. 89

$$xy'(x) - \sqrt{a^2 - x^2} = 0$$

✓ **Mathematica** : cpu = 0.0317198 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \sqrt{a^2 - x^2} - a \log\left(a\left(\sqrt{a^2 - x^2} + a\right)\right) + a \log(x) + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 56

$$\left\{ y(x) = \sqrt{a^2 - x^2} - a^2 \ln\left(\frac{1}{x}\left(2a^2 + 2\sqrt{a^2}\sqrt{a^2 - x^2}\right)\right) \frac{1}{\sqrt{a^2}} + -C1 \right\}$$

**Hand solution**

$$xy' = \pm \sqrt{a^2 - x^2}$$

This is separable.  $y' = \frac{\pm \sqrt{a^2 - x^2}}{x}$  or  $dy = \frac{\pm \sqrt{a^2 - x^2}}{x} dx$ . Hence

$$y = \pm \int \frac{\sqrt{a^2 - x^2}}{x} dx + C$$

Let  $x = a \sin u$ , then  $dx = a \cos(u) du$  and the integral becomes

$$\begin{aligned}
\int \frac{\sqrt{a^2 - x^2}}{x} dx &= \int \frac{\sqrt{a^2 - a^2 \sin^2 u}}{a \sin u} a \cos(u) du \\
&= \int \frac{a\sqrt{1 - \sin^2 u}}{a \sin u} a \cos(u) du \\
&= a \int \frac{\cos u}{\sin u} \cos(u) du \\
&= a \int \frac{\cos^2 u}{\sin u} du \\
&= a \int \frac{1 - \sin^2 u}{\sin u} du \\
&= a \left( \int \frac{1}{\sin u} du - \int \sin u du \right) \\
&= a \left( \int \frac{1}{\sin u} du + \cos u \right) \tag{1}
\end{aligned}$$

For  $\int \frac{1}{\sin u} du$ , using half tan angle, let  $t = \tan\left(\frac{u}{2}\right)$ ,  $du = \frac{2}{1+t^2} dt$ ,  $\sin u = \frac{2t}{1+t^2}$ , therefore

$$\begin{aligned}
\int \frac{1}{\sin u} du &= \int \frac{1+t^2}{2t} \frac{2}{1+t^2} dt \\
&= \int \frac{1}{t} dt \\
&= \ln(t)
\end{aligned}$$

Hence  $\int \frac{1}{\sin u} du = \ln\left(\tan\left(\frac{u}{2}\right)\right)$  and from (1)

$$\begin{aligned}
\int \frac{\sqrt{a^2 - x^2}}{x} dx &= a \left( \int \frac{1}{\sin u} du + \cos u \right) \\
&= a \left( \ln\left(\tan\left(\frac{u}{2}\right)\right) + \cos u \right)
\end{aligned}$$

But  $x = a \sin u$ , hence  $u = \arcsin\left(\frac{x}{a}\right)$  and the integral becomes

$$\int \frac{\sqrt{a^2 - x^2}}{x} dx = a \left[ \ln\left(\tan\left(\frac{\arcsin\left(\frac{x}{a}\right)}{2}\right)\right) + \cos\left(\arcsin\left(\frac{x}{a}\right)\right) \right]$$

Hence the solution is

$$y = \pm a \left[ \ln\left(\tan\left(\frac{\arcsin\left(\frac{x}{a}\right)}{2}\right)\right) + \cos\left(\arcsin\left(\frac{x}{a}\right)\right) \right] + C$$

Maple do not verify the above, but I do not see what is wrong with the solution. Will investigate more later.

2.90 ODE No. 90

$$xy'(x) + y(x) - x \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.0142992 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 + \sin(x) - x \cos(x)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 17

$$\left\{ y(x) = \frac{\sin(x) - \cos(x)x + \_C1}{x} \right\}$$

**Hand solution**

$$xy' + y = x \sin x$$

Linear first order, exact, separable.  $y' + \frac{y}{x} = \sin x$ , integrating factor  $\mu = e^{\int \frac{1}{x} dx} = x$ , hence

$$\begin{aligned} d(\mu y) &= \mu \sin x \\ xy &= \int x \sin x dx + C \end{aligned}$$

Using integration by parts.  $\int u dv = uv - \int v du$ . Let  $u = x, dv = \sin x$ , hence  $du = 1, v = -\cos x$ , therefore

$$\begin{aligned} \int x \sin x dx &= -x \cos x + \int \cos x \\ &= -x \cos x + \sin x \end{aligned}$$

Hence

$$\begin{aligned} xy &= -x \cos x + \sin x + C \\ y &= \frac{\sin x}{x} - \cos x + \frac{C}{x} \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+y(x)=x*sin(x);
my_sol:=sin(x)/x-cos(x)+_C1/x;
odetest(y(x)=my_sol,ode);
0
```

**2.91 ODE No. 91**

$$xy'(x) - y(x) - \frac{x}{\log(x)} = 0$$

✓ **Mathematica** : cpu = 0.00705183 (sec), leaf count = 13

$$\{\{y(x) \rightarrow x(c_1 + \log(\log(x)))\}\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 11

$$\{y(x) = (\ln(\ln(x)) + \_C1)x\}$$

**2.92 ODE No. 92**

$$x^2(-\sin(x)) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0130106 (sec), leaf count = 14

$$\{\{y(x) \rightarrow x(c_1 - \cos(x))\}\}$$

✓ **Maple** : cpu = 0.005 (sec), leaf count = 12

$$\{y(x) = x(\_C1 - \cos(x))\}$$

**Hand solution**

$$xy' - y = x^2 \sin x$$

Linear first order, exact, separable.  $y' - \frac{y}{x} = x \sin x$ , integrating factor  $\mu = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$ , hence

$$\begin{aligned} d(\mu y) &= \mu \sin x \\ \frac{1}{x} y &= \int \sin x dx + C \\ y &= x(C - \cos x) \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)-y(x)=x^2*sin(x);
my_sol:=x*(\_C1-cos(x));
odetest(y(x)=my_sol,ode);
0
```

**2.93 ODE No. 93**

$$xy'(x) - y(x) - \frac{x \cos(\log(\log(x)))}{\log(x)} = 0$$

✓ **Mathematica** : cpu = 0.0201479 (sec), leaf count = 14

$$\{\{y(x) \rightarrow x(c_1 + \sin(\log(\log(x))))\}\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 12

$$\{y(x) = x(\sin(\ln(\ln(x))) + \_C1)\}$$

**2.94 ODE No. 94**

$$ay(x) + bx^n + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.016094 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{-a} - \frac{bx^n}{a+n} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 23

$$\left\{ y(x) = -\frac{bx^n}{n+a} + x^{-a} \_C1 \right\}$$

**Hand solution**

$$xy' + ay + bx^n = 0$$

Linear first order, exact, separable.  $y' + \frac{ay}{x} = -bx^{n-1}$ , integrating factor  $\mu = e^{\int \frac{a}{x} dx} = e^{a \ln x} = x^a$ , hence

$$\begin{aligned} d(\mu y) &= -\mu bx^{n-1} \\ x^a y &= -\int bx^{a+n-1} + C \end{aligned}$$

If  $a = -n$  then

$$\begin{aligned} x^a y &= -\int bx^{-1} + C \\ y &= -x^{-a} b \ln(x) + x^{-a} C \\ &= x^{-a} (C - b \ln x) \end{aligned}$$

If  $a \neq -n$  then

$$x^a y = -\frac{bx^{a+n}}{a+n} + C$$

$$y = -b\frac{x^n}{a+n} + Cx^{-a}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+a*y(x)+b*x^n=0;
s1:=x^(-a)*(_C1-b*ln(x));
s2:=-b*(x^n/(a+n))+_C1*x^(-a);
odetest(y(x)=s2,ode);
0
```

## 2.95 ODE No. 95

$$x^2 + xy'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0157967 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow -\frac{x(c_1 J_1(x) + Y_1(x))}{c_1 J_0(x) + Y_0(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 27

$$\left\{ y(x) = -\frac{(-C1 Y_1(x) + J_1(x)) x}{-C1 Y_0(x) + J_0(x)} \right\}$$

**Hand solution**

$$xy' + y^2 + x^2 = 0$$

This is Riccati first order non-linear. Writing it in standard form and for  $x \neq 0$

$$y' = -x - \frac{1}{x}y^2 \tag{1}$$

$$= f_0 + f_1 y + f_2 y^2$$

Where  $f_0 = -x$ ,  $f_1 = 0$ ,  $f_2 = -\frac{1}{x}$ . Using standard substitution  $y = \frac{-u'}{uf_2}$  changes the ODE to second order linear ODE

$$y = \frac{xu'}{u} \tag{2}$$

Hence

$$y' = \frac{u'}{u} + x \frac{u''}{u} - \frac{x(u')^2}{u^2}$$

Equating this to RHS of (1) gives

$$\begin{aligned} \frac{u'}{u} + x \frac{u''}{u} - \frac{x(u')^2}{u^2} &= -x - \frac{1}{x} \left( \frac{xu'}{u} \right)^2 \\ \frac{u'}{u} + x \frac{u''}{u} &= -x \\ u'' + \frac{1}{x}u' + u &= 0 \end{aligned}$$

This is Lienard ODE. Since it is not constant coefficient ODE, the solution will be in Bessel functions, using Power series method. The solution is

$$u = C_1 \text{BesselJ}(0, x) + C_2 \text{BesselY}(0, x)$$

But  $\frac{d}{dx} \text{BesselJ}(0, x) = -\text{BesselJ}(1, x)$  and  $\frac{d}{dx} \text{BesselY}(0, x) = -\text{BesselY}(1, x)$ , hence

$$u'(x) = -C_1 \text{BesselJ}(1, x) - C_2 \text{BesselY}(1, x)$$

And from (2) the solution is

$$\begin{aligned} y &= \frac{xu'}{u} \\ &= x \frac{[-C_1 \text{BesselJ}(1, x) - C_2 \text{BesselY}(1, x)]}{C_1 \text{BesselJ}(0, x) + C_2 \text{BesselY}(0, x)} \\ &= -x \frac{C_1 \text{BesselJ}(1, x) + C_2 \text{BesselY}(1, x)}{C_1 \text{BesselJ}(0, x) + C_2 \text{BesselY}(0, x)} \end{aligned}$$

Let  $C = \frac{C_1}{C_2}$  then

$$y = -x \frac{C \text{BesselJ}(1, x) + \text{BesselY}(1, x)}{C \text{BesselJ}(0, x) + \text{BesselY}(0, x)}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+y(x)^2+x^2=0;
my_sol:=-x*(C1*BesselJ(1, x)+BesselY(1,x))/(C1*BesselJ(0, x)+BesselY(0,x));
odetest(y(x)=my_sol,ode);
0
```

2.96 ODE No. 96

$$xy'(x) - y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.0229439 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{1 - e^{2c_1 x^2}}{e^{2c_1 x^2} + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 11

$$\{y(x) = -\tanh(\ln(x) + \_C1)\}$$

**Hand solution**

$$xy' - y^2 + 1 = 0$$

This is Riccati first order non-linear. But it is separable. Hence

$$y' = \frac{y^2 - 1}{x} \tag{1}$$

Hence

$$\begin{aligned} \frac{dy}{dx} &= \frac{y^2 - 1}{x} \\ \frac{dy}{y^2 - 1} &= \frac{dx}{x} \end{aligned}$$

Integrating

$$\begin{aligned} -\tanh^{-1}(y) &= \ln x + C \\ y &= -\tanh(\ln x + C) \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)-y(x)^2+1=0;
my_sol:=-tanh(ln(x)+_C1);
odetest(y(x)=my_sol,ode);
0
```



2.97 ODE No. 97

$$ay(x)^2 + bx^2 + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0271706 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{b}x \tan\left(\sqrt{a}\sqrt{b}(x - c_1)\right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 25

$$\left\{ y(x) = -\frac{x}{a} \tan\left(\sqrt{ab}(x + \_C1)\right) \sqrt{ab} \right\}$$

**Hand solution**

$$ay^2 + bx^2 + xy' - y = 0$$

This is Riccati first order non-linear. Let  $y = ux$ , hence the above becomes

$$\begin{aligned} au^2x^2 + bx^2 + x(u'x + u) - ux &= 0 \\ au^2x + bx + u'x &= 0 \\ au^2 + b + u' &= 0 \\ u' &= -au^2 - b \end{aligned}$$

Which is separable, Hence

$$\frac{du}{au^2 + b} = -dx$$

Integrating

$$\begin{aligned} \frac{1}{\sqrt{ab}} \arctan\left(\frac{au}{\sqrt{ab}}\right) &= -x + C \\ \frac{au}{\sqrt{ab}} &= \tan\left(\sqrt{ab}(-x + C)\right) \\ u &= \frac{\sqrt{ab}}{a} \tan\left(\sqrt{ab}(-x + C)\right) \end{aligned}$$

Therefore

$$\begin{aligned}
 y &= ux \\
 &= x \frac{\sqrt{ab}}{a} \tan(\sqrt{ab}(-x + C))
 \end{aligned}$$

Verification

```

restart;
ode:=a*y(x)^2+b*x^2+x*diff(y(x),x)-y(x)=0;
my_sol:=x*sqrt(a*b)/a*tan(sqrt(a*b)*(-x+_C1));
odetest(y(x)=my_sol,ode);
0

```

## 2.98 ODE No. 98

$$ay(x)^2 + cx^{2b} - by(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0250049 (sec), leaf count = 123

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{-cx^b} \left( c_1 \sin\left(\frac{\sqrt{-a}\sqrt{-cx^b}}{b}\right) - \cos\left(\frac{\sqrt{-a}\sqrt{-cx^b}}{b}\right) \right)}{\sqrt{-a} \left( \sin\left(\frac{\sqrt{-a}\sqrt{-cx^b}}{b}\right) + c_1 \cos\left(\frac{\sqrt{-a}\sqrt{-cx^b}}{b}\right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 38

$$\left\{ y(x) = -\frac{1}{x^{-b}} \tan\left(\frac{1}{b} \left( x^b \sqrt{c} \sqrt{a} + \_C1 b \right)\right) \sqrt{c} \frac{1}{\sqrt{a}} \right\}$$

## 2.99 ODE No. 99

$$ay(x)^2 - by(x) - cx^\beta + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.018554 (sec), leaf count = 243

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-a}\sqrt{cx^{\beta/2}} \left( c_1 \left( J_{1-\frac{b}{\beta}}\left(\frac{2\sqrt{-a}\sqrt{cx^{\beta/2}}}{\beta}\right) - J_{-\frac{b+\beta}{\beta}}\left(\frac{2\sqrt{-a}\sqrt{cx^{\beta/2}}}{\beta}\right) \right) - 2J_{\frac{b}{\beta}-1}\left(\frac{2\sqrt{-a}\sqrt{cx^{\beta/2}}}{\beta}\right) \right) - bc_1 J_{-\frac{b}{\beta}}\left(\frac{2\sqrt{-a}\sqrt{cx^{\beta/2}}}{\beta}\right)}{2a \left( J_{\frac{b}{\beta}}\left(\frac{2\sqrt{-a}\sqrt{cx^{\beta/2}}}{\beta}\right) + c_1 J_{-\frac{b}{\beta}}\left(\frac{2\sqrt{-a}\sqrt{cx^{\beta/2}}}{\beta}\right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 171

$$\left\{ y(x) = \frac{1}{a} \left( -\left( Y_{\frac{b+\beta}{\beta}}\left(2\frac{\sqrt{-acx^{\beta/2}}}{\beta}\right) - C1 + J_{\frac{b+\beta}{\beta}}\left(2\frac{\sqrt{-acx^{\beta/2}}}{\beta}\right) \right) \sqrt{-acx^{\frac{\beta}{2}}} + b \left( Y_{\frac{b}{\beta}}\left(2\frac{\sqrt{-acx^{\beta/2}}}{\beta}\right) - C1 + J_{\frac{b}{\beta}}\left(2\frac{\sqrt{-acx^{\beta/2}}}{\beta}\right) \right) \right)$$

**2.100 ODE No. 100**

$$a + xy'(x) + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.00877463 (sec), leaf count = 133

$$\left\{ \left\{ y(x) \rightarrow \frac{i\sqrt{-a}(c_1 - 2)\sqrt{x}J_0(2i\sqrt{-a}\sqrt{x}) + c_1(J_1(2i\sqrt{-a}\sqrt{x}) - i\sqrt{-a}\sqrt{x}J_2(2i\sqrt{-a}\sqrt{x}))}{2(c_1 - 1)xJ_1(2i\sqrt{-a}\sqrt{x})} \right\} \right\}$$

✓ **Maple** : cpu = 0.08 (sec), leaf count = 59

$$\left\{ y(x) = 1\sqrt{a}(J_0(2\sqrt{a}\sqrt{x})_C1 + Y_0(2\sqrt{a}\sqrt{x})) \frac{1}{\sqrt{x}} (-C1 J_1(2\sqrt{a}\sqrt{x}) + Y_1(2\sqrt{a}\sqrt{x}))^{-1} \right\}$$

**Hand solution**

$$xy' + xy^2 + a = 0$$

$$y' = -\frac{a}{x} - y^2$$

This is Riccati first order non-linear. Let  $y = -\frac{u'}{uR} = \frac{u'}{u}$ , hence  $y' = \frac{u''}{u} - \frac{(u')^2}{u^2}$ . Equating this to RHS of the above gives

$$\frac{u''}{u} - \frac{(u')^2}{u^2} = -\frac{a}{x} - \left(\frac{u'}{u}\right)^2$$

$$\frac{u''}{u} = -\frac{a}{x}$$

$$u'' + \frac{a}{x}u = 0$$

This is linear second order, an Emden Fowler ODE, with removal singularity. Solved using power series method. The solution is

$$u = C_1\sqrt{x} \text{BesselJ}(1, 2\sqrt{ax}) + C_2\sqrt{x} \text{BesselY}(1, 2\sqrt{ax})$$

But

$$\frac{d}{dx} \text{BesselJ}(1, 2\sqrt{ax}) = \frac{\sqrt{a}}{\sqrt{x}} \left( \text{BesselJ}(0, 2\sqrt{ax}) - \frac{1}{2} \frac{1}{\sqrt{ax}} \text{BesselJ}(1, 2\sqrt{ax}) \right)$$

And

$$\frac{d}{dx} \text{BesselY}(1, 2\sqrt{ax}) = \frac{\sqrt{a}}{\sqrt{x}} \left( \text{BesselY}(0, 2\sqrt{ax}) - \frac{1}{2} \frac{1}{\sqrt{ax}} \text{BesselY}(1, 2\sqrt{ax}) \right)$$

Therefore,

$$u' = C_1 \left( \frac{1}{2\sqrt{x}} \text{BesselJ}(1, 2\sqrt{a}\sqrt{x}) + \sqrt{x} \frac{\sqrt{a}}{\sqrt{x}} \left( \text{BesselJ}(0, 2\sqrt{a}\sqrt{x}) - \frac{1}{2} \frac{1}{\sqrt{ax}} \text{BesselJ}(1, 2\sqrt{a}\sqrt{x}) \right) \right) \\ + C_2 \left( \frac{1}{2\sqrt{x}} \text{BesselY}(1, 2\sqrt{a}\sqrt{x}) + \sqrt{x} \frac{\sqrt{a}}{\sqrt{x}} \left( \text{BesselY}(0, 2\sqrt{a}\sqrt{x}) - \frac{1}{2} \frac{1}{\sqrt{ax}} \text{BesselY}(1, 2\sqrt{a}\sqrt{x}) \right) \right)$$

Which is simplified to

$$u' = C_1 \sqrt{a} \text{BesselJ}(0, 2\sqrt{a}\sqrt{x}) + C_2 \sqrt{a} \text{BesselY}(0, 2\sqrt{a}\sqrt{x})$$

Therefore, from  $y = \frac{u'}{u}$ , the solution is

$$y = \frac{C_1 \sqrt{a} \text{BesselJ}(0, 2\sqrt{a}\sqrt{x}) + C_2 \sqrt{a} \text{BesselY}(0, 2\sqrt{a}\sqrt{x})}{C_1 \sqrt{x} \text{BesselJ}(1, 2\sqrt{a}\sqrt{x}) + C_2 \sqrt{x} \text{BesselY}(1, 2\sqrt{a}\sqrt{x})}$$

Let  $C = \frac{C_1}{C_2}$ , hence

$$y = \frac{C \sqrt{a} \text{BesselJ}(0, 2\sqrt{a}\sqrt{x}) + \sqrt{a} \text{BesselY}(0, 2\sqrt{a}\sqrt{x})}{C \sqrt{x} \text{BesselJ}(1, 2\sqrt{a}\sqrt{x}) + \sqrt{x} \text{BesselY}(1, 2\sqrt{a}\sqrt{x})}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+x*y(x)^2+a=0;
num:=-C1*sqrt(a)*BesselJ(0,2*sqrt(a)*sqrt(x))+sqrt(a)*BesselY(0,2*sqrt(a)*sqrt(x));
den:=-C1*sqrt(x)*BesselJ(1,2*sqrt(a)*sqrt(x))+sqrt(x)*BesselY(1,2*sqrt(a)*sqrt(x));
my_solution:=num/den;
odetest(y(x)=my_solution,ode);
0
```

## 2.101 ODE No. 101

$$xy'(x) + xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.107177 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow \frac{2x}{2c_1 + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 16

$$\left\{ y(x) = 2 \frac{x}{x^2 + 2\_C1} \right\}$$

**Hand solution**

$$\begin{aligned}
 xy' + xy^2 - y &= 0 \\
 y' &= \frac{1}{x}y - y^2
 \end{aligned} \tag{1}$$

This is of the form  $y' = f_0 + f_1y + f_2y^2$  with  $f_0 = 0, f_1 = \frac{1}{x}, f_2 = -1$ . Since  $f_0 = 0$  this is Bernoulli differential equation. We always start by dividing by  $y^2$

$$\frac{y'}{y^2} = \frac{1}{x} \frac{1}{y} - 1$$

Then  $u = \frac{1}{y}$  or  $y = \frac{1}{u}$ , therefore  $y' = -\frac{u'}{u^2}$ . Equating this to RHS of (1) gives

$$\begin{aligned}
 -\frac{u'}{u^2}u^2 &= \frac{1}{x}u - 1 \\
 -u' &= \frac{u}{x} - 1 \\
 u' + \frac{u}{x} &= 1
 \end{aligned}$$

Integrating factor is  $e^{\int \frac{1}{x} dx} = x$  and the above becomes

$$d(xu) = x$$

Integrating

$$\begin{aligned}
 xu &= \frac{x^2}{2} + C \\
 u &= \frac{x}{2} + \frac{C}{x} \\
 &= \frac{x^2 + 2C}{2x}
 \end{aligned}$$

Hence

$$\begin{aligned}
 y &= \frac{1}{u} \\
 &= \frac{2x}{x^2 + 2C}
 \end{aligned}$$

Verification

```

restart;
ode:=x*diff(y(x),x)+x*y(x)^2-y=0;
my_solution:=2*x/(x^2+2*_C1);
odetest(y(x)=my_solution,ode);
0

```

2.102 ODE No. 102

$$-ax^3 + xy'(x) + xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0737486 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \sqrt{ax} \tanh \left( \frac{1}{2} \sqrt{a} (2c_1 + x^2) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 22

$$\left\{ y(x) = \tanh \left( \frac{x^2 + 2\_C1}{2} \sqrt{a} \right) x \sqrt{a} \right\}$$

**Hand solution**

$$xy' + xy^2 - y - ax^3 = 0$$

This is Riccati non-linear first order. But using the transformation  $y = xv$  it is transformed to easily solved ODE

$$y' = v + xv'$$

Therefore the ODE becomes

$$\begin{aligned} x(v + xv') + x(xv)^2 - xv - ax^3 &= 0 \\ xv + x^2v' + x^3v^2 - xv - ax^3 &= 0 \\ x^2v' + x^3v^2 - ax^3 &= 0 \\ v' + xv^2 - ax &= 0 \\ \frac{dv}{dx} &= x(a - v^2) \\ \frac{dv}{a - v^2} &= x dx \end{aligned}$$

Integrating

$$\begin{aligned} \frac{1}{\sqrt{a}} \tanh^{-1} \left( \frac{v}{\sqrt{a}} \right) &= \frac{x^2}{2} + C \\ \tanh^{-1} \left( \frac{v}{\sqrt{a}} \right) &= \sqrt{a} \left( \frac{x^2}{2} + C \right) \\ \frac{v}{\sqrt{a}} &= \tanh \left( \sqrt{a} \left( \frac{x^2}{2} + C \right) \right) \\ v &= \sqrt{a} \tanh \left( \sqrt{a} \left( \frac{x^2}{2} + C \right) \right) \end{aligned}$$

Therefore

$$y = xv$$

$$= x\sqrt{a} \tanh\left(\sqrt{a}\left(\frac{x^2}{2} + C\right)\right)$$

Verification

```
restart;
ode:=x*diff(y(x),x)+x*y(x)^2-y(x)-a*x^3=0;
my_solution:=x*sqrt(a)*tanh(sqrt(a)*(x^2/2+_C1));
odetest(y(x)=my_solution,ode);
0
```

### 2.103 ODE No. 103

$$-x^3 - (2x^2 + 1)y(x) + xy'(x) + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.134539 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( (1 + \sqrt{2}) e^{\sqrt{2}x^2} - (\sqrt{2} - 1) e^{2\sqrt{2}c_1} \right)}{e^{2\sqrt{2}c_1} + e^{\sqrt{2}x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 29

$$\left\{ y(x) = \frac{\sqrt{2}x}{2} \left( \sqrt{2} + 2 \tanh\left(\frac{1}{2}(x^2 + 2\_C1)\sqrt{2}\right) \right) \right\}$$

**Hand solution**

$$xy' - xy^2 - (2x^2 + 1)y - x^3 = 0$$

This is Riccati non-linear first order. Converting it to standard form

$$y' = x^2 + \frac{(2x^2 + 1)}{x}y + y^2 \tag{1}$$

$$= f_0 + f_1y + f_2y^2$$

Using standard transformation  $y = -\frac{u'}{uf_2} = -\frac{u'}{u}$ , therefore

$$y' = -\frac{u''}{u} + \frac{(u')^2}{u^2}$$

Equating the above to RHS of (1) gives

$$\begin{aligned}
 -\frac{u''}{u} + \frac{(u')^2}{u^2} &= x^2 - \frac{(2x^2 + 1)u'}{xu} + \left(-\frac{u'}{u}\right)^2 \\
 -\frac{u''}{u} &= x^2 - \frac{(2x^2 + 1)u'}{xu} \\
 -u'' &= ux^2 - \frac{(2x^2 + 1)u'}{x} \\
 -u'' + \frac{(2x^2 + 1)u'}{x} - ux^2 &= 0 \\
 xu'' - (2x^2 + 1)u' + ux^3 &= 0 \tag{2}
 \end{aligned}$$

This is second order linear ODE (Sturm-Liouville). Using the transformation  $t = \frac{x^2}{2}$ , then  $\frac{dt}{dx} = x$  and

$$\frac{du}{dx} = \frac{du}{dt} \frac{dt}{dx} = x \frac{du}{dt} = \sqrt{2t} \frac{du}{dt}$$

And

$$\begin{aligned}
 \frac{d^2u}{dx^2} &= \frac{d}{dx} \left( \frac{du}{dx} \right) \\
 &= \frac{d}{dx} \left( x \frac{du}{dt} \right) \\
 &= \frac{du}{dt} + x \frac{d^2u}{dt^2} \frac{dt}{dx} \\
 &= \frac{du}{dt} + x \frac{d^2u}{dt^2} x \\
 &= \frac{du}{dt} + x^2 \frac{d^2u}{dt^2} \\
 &= \frac{du}{dt} + 2t \frac{d^2u}{dt^2}
 \end{aligned}$$

Hence (2) can be written as

$$\begin{aligned}
 \sqrt{2t} \left( \frac{du}{dt} + 2t \frac{d^2u}{dt^2} \right) - (2(2t) + 1) \sqrt{2t} \frac{du}{dt} + u(2t)^{\frac{3}{2}} &= 0 \\
 \sqrt{2t} \frac{du}{dt} + \sqrt{2t} 2t \frac{d^2u}{dt^2} - (4t + 1) \sqrt{2t} \frac{du}{dt} + u(\sqrt{2t})^3 &= 0 \\
 \frac{du}{dt} + 2t \frac{d^2u}{dt^2} - (4t + 1) \frac{du}{dt} + 2tu &= 0 \\
 \frac{du}{dt} + 2t \frac{d^2u}{dt^2} - 4t \frac{du}{dt} - \frac{du}{dt} + 2tu &= 0 \\
 2t \frac{d^2u}{dt^2} - 4t \frac{du}{dt} + 2tu &= 0 \\
 2t \left( \frac{d^2u}{dt^2} - 2 \frac{du}{dt} + u \right) &= 0
 \end{aligned}$$



Hence

$$\frac{d^2u}{dt^2} - 2\frac{du}{dt} + u = 0$$

This is linear second order with constant coefficients. The indicial equation is  $\lambda^2 - 2\lambda + 1 = 0$  with roots  $\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4}}{2} = 1$  double root. Hence

$$u(t) = Ae^t + tBe^t$$

Since  $t = \frac{x^2}{2}$  then

$$u(x) = Ae^{\frac{x^2}{2}} + \frac{x^2}{2}Be^{\frac{x^2}{2}}$$

But  $y = -\frac{u'}{u}$  therefore

$$\begin{aligned} u' &= Axe^{\frac{x^2}{2}} + \left( xBe^{\frac{x^2}{2}} + \frac{x^2}{2}xBe^{\frac{x^2}{2}} \right) \\ &= Axe^{\frac{x^2}{2}} + xBe^{\frac{x^2}{2}} + \frac{x^3}{2}Be^{\frac{x^2}{2}} \end{aligned}$$

Hence

$$y = -\frac{Axe^{\frac{x^2}{2}} + xBe^{\frac{x^2}{2}} + \frac{x^3}{2}Be^{\frac{x^2}{2}}}{Ae^{\frac{x^2}{2}} + \frac{x^2}{2}Be^{\frac{x^2}{2}}} = -\frac{Axe^{\frac{x^2}{2}} + B\left(xe^{\frac{x^2}{2}} + \frac{x^3}{2}e^{\frac{x^2}{2}}\right)}{Ae^{\frac{x^2}{2}} + \frac{x^2}{2}Be^{\frac{x^2}{2}}}$$

Let  $C = \frac{A}{B}$

$$\begin{aligned} y &= -\frac{xe^{\frac{x^2}{2}}\left(C + 1 + \frac{x^2}{2}\right)}{e^{\frac{x^2}{2}}\left(C + \frac{x^2}{2}\right)} \\ &= -\frac{x\left(C + 1 + \frac{x^2}{2}\right)}{C + \frac{x^2}{2}} \\ &= -\frac{x(2C + 2 + x^2)}{2C + x^2} \\ &= -\frac{x(C_1 + 2 + x^2)}{C_1 + x^2} \end{aligned}$$

Verification

```
restart;
ode:=x*diff(y(x),x)-x*y(x)^2-(2*x^2+1)*y(x)-x^3;
my_solution:=- (x*(C1+2+x^2))/(C1+x^2);
odetest(y(x)=my_solution,ode);
0
```

2.104 ODE No. 104

$$axy(x)^2 + bx + xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0160872 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{\frac{b}{a}} \tan \left( ax\sqrt{\frac{b}{a}} - c_1 \right) - \frac{1}{ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 63

$$\left\{ y(x) = -\frac{1}{a} \left( -\frac{1}{x} \left( i\sqrt{a}\sqrt{bx} - 1 \right) + 1e^{-2ix\sqrt{a}\sqrt{b}} \left( -C1 - \frac{i}{2} e^{-2ix\sqrt{a}\sqrt{b}} \frac{1}{\sqrt{a}} \frac{1}{\sqrt{b}} \right)^{-1} \right) \right\}$$

**Hand solution**

$xy' + axy^2 + 2y + bx = 0$  This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned} y' &= -b - \frac{2}{x}y - ay^2 \\ &= f_0 + f_1y + f_2y^2 \end{aligned} \tag{1}$$

Using transformation suggested by Kamke  $y = u(x) - \frac{1}{ax}$  then  $y' = u' + \frac{1}{ax^2}$ . Equating this to RHS of (1) gives

$$\begin{aligned} u' + \frac{1}{ax^2} &= -b - \frac{2}{x} \left( u - \frac{1}{ax} \right) - a \left( u - \frac{1}{ax} \right)^2 \\ &= -b - \frac{2}{x}u + \frac{2}{ax^2} - a \left( u^2 + \frac{1}{a^2x^2} - \frac{2u}{ax} \right) \\ &= -b - \frac{2}{x}u + \frac{2}{ax^2} - au^2 - \frac{1}{ax^2} + \frac{2u}{x} \end{aligned}$$

Hence

$$\begin{aligned} u' &= -b - au^2 \\ \frac{du}{dx} &= -b - au^2 \end{aligned}$$

This is separable

$$\frac{du}{b + au^2} = -dx$$

Integrating

$$\int \frac{du}{b+au^2} = -x + C$$

$$\frac{1}{\sqrt{ba}} \arctan\left(\frac{au}{\sqrt{ba}}\right) = -x + C$$

$$\arctan\left(\frac{au}{\sqrt{ba}}\right) = -\sqrt{ba}x + C$$

$$u = \frac{\sqrt{ba}}{a} \tan(-\sqrt{ba}x + C)$$

Hence

$$y = u - \frac{1}{ax}$$

$$= \frac{\sqrt{ba}}{a} \tan(-\sqrt{ba}x + C) - \frac{1}{ax}$$

Verification

```
restart;
ode:=x*diff(y(x),x)+a*x*y(x)^2+2*y(x)+b*x = 0;
my_solution:=sqrt(b*a)/a*tan(-sqrt(b*a)*x+_C1)-1/(a*x);
odetest(y(x)=my_solution,ode);
0
```

## 2.105 ODE No. 105

$$axy(x)^2 + by(x) + cx + d + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.216978 (sec), leaf count = 301

$$\left\{ \left\{ y(x) \rightarrow -\frac{i\left(\sqrt{c}c_1 U\left(\frac{1}{2}\left(b + \frac{i\sqrt{ad}}{\sqrt{c}}\right), b, 2i\sqrt{a}\sqrt{cx}\right) + c_1(b\sqrt{c} + i\sqrt{ad}) U\left(\frac{1}{2}\left(b + \frac{i\sqrt{ad}}{\sqrt{c}} + 2\right), b + 1, 2i\sqrt{a}\sqrt{cx}\right) + \sqrt{a}\left(c_1 U\left(\frac{1}{2}\left(b + \frac{i\sqrt{ad}}{\sqrt{c}}\right), b, 2i\sqrt{a}\sqrt{cx}\right) + L_{-\frac{b}{2} - \frac{i\sqrt{ad}}{2\sqrt{c}}}^{b-1}\right)}{\right. \right.$$

✓ **Maple** : cpu = 0.337 (sec), leaf count = 844

$$\left\{ y(x) = -4c^2 \left( -1/4\_C1 \left( a^3c^2d^2 + a^2b^2c^3 - 2(-ac)^{3/2}abcd - 2(-ac)^{5/2}bd \right) U\left( 1/2 \frac{(-ac)^{3/2}d + c(2\sqrt{-acd}}{c^2a} \right. \right.$$

**2.106 ODE No. 106**

$$\frac{1}{2}(a-b)y(x) + x^a y(x)^2 + x^b + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0351584 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow -x^{\frac{b-a}{2}} \tan \left( \frac{2x^{\frac{a+b}{2}}}{a+b} - c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 41

$$\left\{ y(x) = -1 \tan \left( \frac{1}{a+b} \left( 2x^{a/2+b/2} + \_C1 (a+b) \right) \right) \left( x^{\frac{a}{2}-\frac{b}{2}} \right)^{-1} \right\}$$

**2.107 ODE No. 107**

$$ax^\alpha y(x)^2 + by(x) - cx^\beta + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.258438 (sec), leaf count = 1286

$$\left\{ \left\{ y(x) \rightarrow \frac{x^{-\alpha} \left( (-1)^{\frac{b}{\alpha+\beta}} \sqrt{a} \alpha (\alpha + \beta)^{\frac{2b}{\alpha+\beta}} \sqrt{cx^{\alpha+\beta}} I_{\frac{b+\beta}{\alpha+\beta}} \left( \frac{2\sqrt{a}\sqrt{c}\sqrt{x^{\alpha+\beta}}}{\sqrt{(\alpha+\beta)^2}} \right) c_1 \Gamma \left( \frac{b+\beta}{\alpha+\beta} \right) ((\alpha + \beta)^2)^{\frac{\alpha}{\alpha+\beta}} + (-1)^{\frac{b}{\alpha+\beta}} \sqrt{a} \beta \right)}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.239 (sec), leaf count = 174

$$\left\{ y(x) = -\frac{x^{1-\alpha}}{ax} \left( Y_{\frac{b+\beta}{\alpha+\beta}} \left( 2 \frac{\sqrt{-ac} x^{\alpha/2+\beta/2}}{\alpha + \beta} \right) \_C1 + J_{\frac{b+\beta}{\alpha+\beta}} \left( 2 \frac{\sqrt{-ac} x^{\alpha/2+\beta/2}}{\alpha + \beta} \right) \right) x^{\frac{\alpha}{2}+\frac{\beta}{2}} \sqrt{-ac} \left( Y_{\frac{b-\alpha}{\alpha+\beta}} \left( 2 \frac{\sqrt{-ac} x^{\alpha/2+\beta/2}}{\alpha + \beta} \right) \right) \right\}$$

**2.108 ODE No. 108**

$$xy'(x) + y(x) + y(x)^2(-\log(x)) = 0$$

✓ **Mathematica** : cpu = 0.0105073 (sec), leaf count = 15

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 x + \log(x) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 13

$$\left\{ y(x) = (1 + \_C1 x + \ln(x))^{-1} \right\}$$

**Hand solution**

$xy' + axy^2 + 2y + bx = 0$  This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned}xy' - y^2 \ln x + y &= 0 \\y' &= -\frac{1}{x}y + y^2 \frac{\ln x}{x} \\&= f_0 + f_1 y + f_2 y^2\end{aligned}\tag{1}$$

This is Bernoulli non-linear first order ODE since  $f_0 = 0$ . Dividing by  $y^2$  gives

$$\frac{y'}{y^2} = -\frac{1}{x} \frac{1}{y} + \frac{\ln x}{x}$$

Let  $u = \frac{1}{y}$ , hence  $u' = -\frac{y'}{y^2}$ , and the above becomes

$$\begin{aligned}-u' &= -\frac{1}{x}u + \frac{\ln x}{x} \\u' - \frac{1}{x}u &= -\frac{\ln x}{x}\end{aligned}$$

Integrating factor is  $\mu = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$ , hence

$$d(\mu u) = -\mu \frac{\ln x}{x}$$

Integrating

$$\begin{aligned}\frac{1}{x}u &= -\int \frac{1}{x^2} \ln x dx + C \\&= -\left(-\frac{\ln x}{x} - \frac{1}{x}\right) + C\end{aligned}$$

Therefore

$$u = \ln x + 1 + Cx$$

Since  $u = \frac{1}{y}$  then

$$y = \frac{1}{\ln x + 1 + Cx}$$

Verification

```

restart;
ode:=x*diff(y(x),x)-y(x)^2*ln(x)+y(x)=0;
my_solution:=1/(ln(x)+1+_C1*x);
odetest(y(x)=my_solution,ode);
0

```

## 2.109 ODE No. 109

$$xy'(x) - y(x)(2y(x) \log(x) - 1) = 0$$

✓ **Mathematica** : cpu = 0.0109896 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 x + 2 \log(x) + 2} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 15

$$\left\{ y(x) = (2 + \_C1 x + 2 \ln(x))^{-1} \right\}$$

**Hand solution**

$xy' + axy^2 + 2y + bx = 0$  This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned}
 xy' - y(2y \ln x - 1) &= 0 \\
 xy' &= y(2y \ln x - 1) \\
 y' &= -\frac{1}{x}y + y^2 \frac{2}{x} \ln x \\
 y' &= f_0 + f_1 y + f_2 y^2
 \end{aligned} \tag{1}$$

This is Bernoulli non-linear first order ODE since  $f_0 = 0$ . Dividing by  $y^2$  gives

$$\frac{y'}{y^2} = -\frac{1}{x} \frac{1}{y} + \frac{2}{x} \ln x$$

Putting  $u = \frac{1}{y}$ , hence  $u' = -\frac{y'}{y^2}$ , and the above becomes

$$\begin{aligned}
 -u' &= -\frac{1}{x}u + 2\frac{\ln x}{x} \\
 -u' + \frac{1}{x}u &= 2\frac{\ln x}{x} \\
 u' - \frac{1}{x}u &= -2\frac{\ln x}{x}
 \end{aligned}$$

Integrating factor is  $\mu = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$ , hence

$$\begin{aligned} d(\mu u) &= -2\mu \frac{\ln x}{x} \\ d\left(\frac{1}{x}u\right) &= -2\frac{\ln x}{x^2} \end{aligned}$$

Integrating

$$\begin{aligned} \frac{1}{x}u &= -2 \int \frac{1}{x^2} \ln x dx + C \\ &= -2 \left( -\frac{\ln x}{x} - \frac{1}{x} \right) + C \end{aligned}$$

Therefore

$$\begin{aligned} u &= -2x \left( -\frac{\ln x}{x} - \frac{1}{x} \right) + Cx \\ &= 2(\ln x + 1) + Cx \end{aligned}$$

Since  $u = \frac{1}{y}$  then

$$y = \frac{1}{2(\ln x + 1) + Cx}$$

Verification

```
restart;
ode:=x*diff(y(x),x)-y(x)*(2*y(x)*ln(x)-1)=0;
my_solution:=1/(2*(ln(x)+1)+_C1*x);
odetest(y(x)=my_solution,ode);
0
```

## 2.110 ODE No. 110

$$f(x) (y(x)^2 - x^2) + xy'(x) = 0$$

✗ **Mathematica** : cpu = 15.5816 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*(-x^2 + y[x]^2) + x*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*diff(y(x),x)+f(x)*(y(x)^2-x^2) = 0,y(x))`

**Hand solution**

$xy' + axy^2 + 2y + bx = 0$  This is Riccati non-linear first order. Converting it to standard form

$$\begin{aligned} xy' + f(x) (y^2 - x^2) - y &= 0 \\ xy' &= -f(y^2 - x^2) + y \\ y' &= -\frac{f}{x}y^2 + fx + \frac{1}{x}y \end{aligned} \tag{1}$$

This is Riccati non-linear first order order. There are two particular solutions  $y_p = \pm x$ . Using  $y_p = x$ , then using the transformation  $y = y_p + \frac{1}{u}$ , gives  $y' = 1 - \frac{u'}{u^2}$  and (1) becomes

$$\begin{aligned} 1 - \frac{u'}{u^2} &= -\frac{f}{x} \left(x + \frac{1}{u}\right)^2 + fx + \frac{1}{x} \left(x + \frac{1}{u}\right) \\ &= -\frac{f}{x} \left(x^2 + \frac{1}{u^2} + 2\frac{x}{u}\right) + fx + \left(1 + \frac{1}{ux}\right) \\ &= -fx - \frac{f}{x} \frac{1}{u^2} - 2\frac{f}{u} + fx + 1 + \frac{1}{ux} \\ &= -\frac{f}{x} \frac{1}{u^2} - 2\frac{f}{u} + 1 + \frac{1}{ux} \end{aligned}$$

Hence

$$\begin{aligned} u^2 - u' &= -\frac{f}{x} - 2fu + u^2 + \frac{u}{x} \\ -u' &= -\frac{f}{x} - 2fu + \frac{u}{x} \\ -u' - \frac{u}{x} + 2fu &= \frac{-f}{x} \\ u' + \frac{u}{x} - 2fu &= \frac{-f}{x} \\ u' + u \left(\frac{1}{x} - 2f\right) &= \frac{-f}{x} \end{aligned}$$

Integrating factor is  $\mu = e^{\int \frac{1}{x} - 2f} = e^{\ln x} e^{-2 \int f dx} = x e^{-2 \int f dx}$ , hence

$$\begin{aligned} d(\mu u) &= -\mu \frac{f}{x} \\ d\left(x e^{-2 \int f dx} u\right) &= -\left(x e^{-2 \int f dx}\right) \frac{f}{x} \\ d\left(x e^{-2 \int f dx} u\right) &= -f \left(e^{-2 \int f dx}\right) \end{aligned}$$

Integrating

$$\begin{aligned} x e^{-2 \int f dx} u &= -\int f \left(e^{-2 \int f dx}\right) + C \\ u &= -\frac{1}{x} e^{2 \int f dx} \int f \left(e^{-2 \int f dx}\right) + C \frac{1}{x} e^{2 \int f dx} \end{aligned}$$

Since  $u = \frac{1}{y}$  then

$$\begin{aligned} y &= \frac{1}{-\frac{1}{x} e^{2 \int f dx} \int f \left(e^{-2 \int f dx}\right) + C \frac{1}{x} e^{2 \int f dx}} \\ &= \frac{x e^{-2 \int f dx}}{-\int f e^{-2 \int f dx} dx + C} \end{aligned}$$

Verification (Maple does not verify it, need to look more into this)



```

ode:=x*diff(y(x),x)+f(x)*(y(x)^2-x^2) =0;
dsolve(ode,y(x));
fint:=Int(f(x),x);
my_solution:=x*exp(-2*fint)/(-Int(f*exp(-2*fint),x)+_C1);
odetest(y(x)=my_solution,ode);
not zero

```

### 2.111 ODE No. 111

$$xy'(x) + y(x)^3 + 3xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.500033 (sec), leaf count = 55

$$\text{Solve} \left[ -3x = \frac{2e^{\frac{1}{2} \left( \frac{1}{y(x)} - 3x \right)^2}}{2c_1 + \sqrt{2\pi} \operatorname{erfi} \left( \frac{\frac{1}{y(x)} - 3x}{\sqrt{2}} \right)}, y(x) \right]$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 54

$$\left\{ -C1 - \frac{i}{x} e^{\frac{(3xy(x)-1)^2}{2(y(x))^2}} + \frac{\sqrt{2}\sqrt{\pi}}{2} \operatorname{Erf} \left( \frac{(-i + 3iy(x)x)\sqrt{2}}{2y(x)} \right) = 0 \right\}$$

### 2.112 ODE No. 112

$$-\sqrt{x^2 + y(x)^2} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0211027 (sec), leaf count = 13

$$\{\{y(x) \rightarrow x \sinh(c_1 + \log(x))\}\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 27

$$\left\{ \frac{1}{x^2} \sqrt{(y(x))^2 + x^2} + \frac{y(x)}{x^2} - C1 = 0 \right\}$$

**Hand solution**

$$xy' = \sqrt{x^2 + y^2} + y$$

Let  $y = xv$ , then  $y' = v + xv'$  and the above becomes

$$\begin{aligned}
x(v + xv') &= \sqrt{x^2 + (xv)^2} + xv \\
x(v + xv') &= x\sqrt{1 + v^2} + xv \\
(v + xv') &= \sqrt{1 + v^2} + v \\
xv' &= \sqrt{1 + v^2}
\end{aligned}$$

Separable.

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{1}{x} dx$$

Integrating

$$\begin{aligned}
\operatorname{arcsinh}(v) &= \ln x + C \\
v &= \sinh(\ln x + C)
\end{aligned}$$

Since  $y = xv$  then

$$y = x \sinh(\ln x + C)$$

Verification

```

ode:=x*diff(y(x),x)=sqrt(x^2+y(x)^2)+y(x);
y0:=x*sinh(ln(x)+_C1);
odetest(y(x)=y0,ode) assuming x>= 0;
0

```

### 2.113 ODE No. 113

$$a\sqrt{x^2 + y(x)^2} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0220716 (sec), leaf count = 16

$$\{ \{ y(x) \rightarrow x \sinh(c_1 - a \log(x)) \} \}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 33

$$\left\{ \frac{x^a}{x} \sqrt{(y(x))^2 + x^2} + \frac{x^a y(x)}{x} - C1 = 0 \right\}$$

**Hand solution**

$$xy' = -a\sqrt{x^2 + y^2} + y$$

Let  $y = xv$ , then  $y' = v + xv'$  and the above becomes

$$x(v + xv') = -a\sqrt{x^2 + (xv)^2} + xv$$

$$x(v + xv') = -ax\sqrt{1 + v^2} + xv$$

$$(v + xv') = -a\sqrt{1 + v^2} + v$$

$$xv' = -a\sqrt{1 + v^2}$$

Separable.

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{-a}{x} dx$$

Integrating

$$\operatorname{arcsinh}(v) = -a \ln x + C$$

$$v = \sinh(C - a \ln x)$$

Since  $y = xv$  then

$$y = x \sinh(C - a \ln x)$$

Verification

```
ode:=x*diff(y(x),x)=-a*sqrt(x^2+y(x)^2)+y(x);
y0:=x*sinh(_C1-a*ln(x));
odetest(y(x)=y0,ode) assuming x >=0;
0
```

2.114 ODE No. 114

$$-x\sqrt{x^2 + y(x)^2} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.01981 (sec), leaf count = 12

$$\{\{y(x) \rightarrow x \sinh(c_1 + x)\}\}$$

✓ **Maple** : cpu = 2.378 (sec), leaf count = 28

$$\left\{ \ln \left( \sqrt{(y(x))^2 + x^2} + y(x) \right) - x - \ln(x) - \_C1 = 0 \right\}$$

**Hand solution**

$$xy' = x\sqrt{x^2 + y^2} + y$$

Let  $y = xv$ , then  $y' = v + xv'$  and the above becomes

$$x(v + xv') = x\sqrt{x^2 + (xv)^2} + xv$$

$$(v + xv') = x\sqrt{1 + v^2} + v$$

$$xv' = x\sqrt{1 + v^2}$$

$$v' = \sqrt{1 + v^2}$$

Separable.

$$\frac{dv}{\sqrt{1 + v^2}} = dx$$

Integrating

$$\operatorname{arcsinh}(v) = x + C$$

$$v = \sinh(x + C)$$

Since  $y = xv$  then

$$y = x \sinh(x + C)$$

Verification

```
ode:=x*diff(y(x),x)=x*sqrt(x^2+y(x)^2)+y(x);
y0:=x*sinh(x+_C1);
odetest(y(x)=y0,ode) assuming x>0;
0
```

## 2.115 ODE No. 115

$$-x(y(x) - x)\sqrt{x^2 + y(x)^2} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.109111 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( -2e^{\frac{2c_1+x^2}{\sqrt{2}}} + e^{\sqrt{2}(2c_1+x^2)} - 1 \right)}{2e^{\frac{2c_1+x^2}{\sqrt{2}}} + e^{\sqrt{2}(2c_1+x^2)} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 49

$$\left\{ \ln \left( 2 \frac{\left( \sqrt{2(y(x))^2 + 2x^2 + y(x) + x} \right) x}{y(x) - x} \right) + \frac{\sqrt{2}x^2}{2} - \ln(x) - \_C1 = 0 \right\}$$

**Hand solution**

$$xy' = x(y - x)\sqrt{y^2 - x^2} + y$$

Let  $y = xu$ , then  $y' = u + xu'$  and the above becomes

$$x(u + xu') = x(xu - x)\sqrt{(xu)^2 - x^2} + xu$$

$$(u + xu') = (xu - x)\sqrt{(xu)^2 - x^2} + u$$

$$xu' = (xu - x)x\sqrt{u^2 - 1}$$

$$u' = x(u - 1)\sqrt{u^2 - 1}$$

Separable.

$$\frac{du}{(u-1)\sqrt{u^2-1}} = xdx$$

$$\frac{-u-1}{\sqrt{u^2-1}} = \frac{x^2}{2} + C$$

But  $y = xu$ , hence

$$\frac{-\frac{y}{x} - 1}{\sqrt{\left(\frac{y}{x}\right)^2 - 1}} = \frac{x^2}{2} + C$$

Let  $\frac{y}{x} = z$

$$\frac{-z - 1}{\sqrt{z^2 - 1}} = \frac{x^2}{2} + C$$

$$-z - 1 = \sqrt{z^2 - 1} \left( \frac{x^2}{2} + C \right)$$

$$(-z - 1)^2 = (z^2 - 1) \left( \frac{x^2}{2} + C \right)^2$$

$$z^2 + 1 + 2z = z^2 \left( \frac{x^2}{2} + C \right)^2 - \left( \frac{x^2}{2} + C \right)^2$$

$$z^2 \left( 1 - \left( \frac{x^2}{2} + C \right)^2 \right) + 2z + 1 + \left( \frac{x^2}{2} + C \right)^2 = 0$$

Solving for  $z$  (quadratic formula, some conditions apply), one of the solutions is

$$z = \frac{4Cx^2 + 4C^2 + x^4 + 4}{4Cx^2 + 4C^2 + x^4 - 4}$$

Hence

$$y = x \frac{4Cx^2 + 4C^2 + x^4 + 4}{4Cx^2 + 4C^2 + x^4 - 4}$$

Need to work on verification. Kamke gives the final solution as

$$y = x \frac{-2Cx^2 + C^2 + x^4 + 4}{-2Cx^2 + C^2 + x^4 - 4}$$

I am not sure where my error now is. Need to look at this again.

**2.116 ODE No. 116**

$$-x\sqrt{(y(x)^2 - 4x^2)(y(x)^2 - x^2)} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 3.99901 (sec), leaf count = 142

$$\text{Solve} \left[ \frac{2\sqrt{\frac{y(x)+x}{2x-y(x)}} \sqrt{\frac{y(x)+2x}{2x-y(x)}} \left(\frac{y(x)}{x} - 2\right)^{3/2} \sqrt{\frac{1}{\frac{y(x)}{x} - 2}} + 1F\left(\sin^{-1}\left(\sqrt{\frac{x+y(x)}{4x-2y(x)}}\right) \mid -8\right)}{\sqrt{\frac{y(x)+x}{x}} \sqrt{\frac{y(x)}{x} - 1} \sqrt{\frac{y(x)}{x} + 2}} = c_1 + \frac{x^2}{2}, y(x) \right]$$

✓ **Maple** : cpu = 0.229 (sec), leaf count = 86

$$\left\{ \int_{-b}^x 1 \left( -a \sqrt{4 - a^4 - 5 - a^2 (y(x))^2 + (y(x))^4} + y(x) \right) \frac{1}{\sqrt{4 - a^4 - 5 - a^2 (y(x))^2 + (y(x))^4}} d_a + \int^{y(x)} -_b \frac{1}{\sqrt{4 - a^4 - 5 - a^2 (y(x))^2 + (y(x))^4}} d_a \right\}$$

**2.117 ODE No. 117**

$$xy'(x) + x\left(-e^{\frac{y(x)}{x}}\right) - y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0298949 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow -x \log\left(\frac{e^{-c_1}}{x} - 1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 20

$$\left\{ y(x) = \left( \ln\left(-\frac{x}{-1 + xe^{-C_1}}\right) + -C_1 \right) x \right\}$$

**2.118 ODE No. 118**

$$xy'(x) - y(x) \log(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0107921 (sec), leaf count = 13

$$\left\{ \left\{ y(x) \rightarrow e^{e^{c_1} x} \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 8

$$\left\{ y(x) = e^{-C_1 x} \right\}$$

**2.119 ODE No. 119**

$$xy'(x) - y(x)(\log(xy(x)) - 1) = 0$$

✓ **Mathematica** : cpu = 0.0314593 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{e^{c_1 x}}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 14

$$\left\{ y(x) = \frac{1}{x} e^{-\frac{x}{c_1}} \right\}$$

**2.120 ODE No. 120**

$$xy'(x) - y(x) \left( x \log \left( \frac{x^2}{y(x)} \right) + 2 \right) = 0$$

✓ **Mathematica** : cpu = 0.0453985 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow x^2 e^{-2c_1 e^{-x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.178 (sec), leaf count = 17

$$\left\{ y(x) = x^2 \left( e^{\frac{c_1}{e^x}} \right)^{-1} \right\}$$

**2.121 ODE No. 121**

$$xy'(x) - \sin(x - y(x)) = 0$$

✗ **Mathematica** : cpu = 2.78423 (sec), leaf count = 0 , could not solve

`DSolve[-Sin[x - y[x]] + x*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*diff(y(x),x)-sin(x-y(x)) = 0,y(x))`



**2.122 ODE No. 122**

$$\cos(y(x)) (\sin(y(x)) - 3x^2 \cos(y(x))) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0852591 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{c_1}{2x} + x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.457 (sec), leaf count = 16

$$\left\{ y(x) = \arctan \left( \frac{x^3 + 2\_C1}{x} \right) \right\}$$

**2.123 ODE No. 123**

$$xy'(x) - y(x) - x \sin \left( \frac{y(x)}{x} \right) = 0$$

✓ **Mathematica** : cpu = 0.0566239 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow 2x \cot^{-1} \left( \frac{e^{-c_1}}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 44

$$\left\{ y(x) = \arctan \left( 2 \frac{\_C1 x}{\_C1^2 x^2 + 1}, \frac{-\_C1^2 x^2 + 1}{\_C1^2 x^2 + 1} \right) x \right\}$$

**2.124 ODE No. 124**

$$xy'(x) - y(x) + x \cos \left( \frac{y(x)}{x} \right) + x = 0$$

✓ **Mathematica** : cpu = 0.0272515 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow 2x \tan^{-1} (c_1 - \log(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 12

$$\{y(x) = -2 \arctan (\ln (x) + \_C1) x\}$$

**2.125 ODE No. 125**

$$xy'(x) - y(x) + x \tan\left(\frac{y(x)}{x}\right) = 0$$

✓ **Mathematica** : cpu = 0.0364949 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow x \sin^{-1}\left(\frac{e^{c_1}}{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 14

$$\left\{ y(x) = \arcsin\left(\frac{1}{-C1 x}\right) x \right\}$$

**2.126 ODE No. 126**

$$xy'(x) - y(x)f(xy(x)) = 0$$

✓ **Mathematica** : cpu = 17.9509 (sec), leaf count = 93

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} -\frac{K[2](f(xK[2]) + 1) \left( \int_1^x \frac{f'(K[1]K[2])}{(f(K[1]K[2]) + 1)^2} dK[1] \right) + 1}{K[2](f(xK[2]) + 1)} dK[2] + \int_1^x \frac{f(y(x)K[1])}{K[1]f(y(x)K[1]) + K[1]} dK[1] \right]$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 29

$$\left\{ y(x) = \frac{1}{x} \text{RootOf}\left(-\ln(x) + -C1 + \int^{-Z} \frac{1}{-a(1+f(-a))} d_{-a}\right) \right\}$$

**2.127 ODE No. 127**

$$xy'(x) - y(x)f(x^a y(x)^b) = 0$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.11 (sec), leaf count = 39

$$\left\{ \int_{-b}^{y(x)} \frac{1}{(f(x^a - a^b) b + a) - a} d_{-a} - \frac{\ln(x)}{b} - -C1 = 0 \right\}$$

**2.128 ODE No. 128**

$$-f(x)g(x^a y(x)) + ay(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 4.26887 (sec), leaf count = 39

$$\text{Solve} \left[ \int_1^x K[2]^{a-1} f(K[2]) dK[2] + c_1 = \int_1^{x^a y(x)} \frac{1}{g(K[1])} dK[1], y(x) \right]$$

✓ **Maple** : cpu = 0.283 (sec), leaf count = 33

$$\left\{ y(x) = \frac{\text{RootOf} \left( - \int f(x) x^{a-1} dx + \int^{-Z} (g(-a))^{-1} d_a + \_C1 \right)}{x^a} \right\}$$

**2.129 ODE No. 129**

$$(x + 1)y'(x) + y(x)(y(x) - x) = 0$$

✓ **Mathematica** : cpu = 0.0274262 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow - \frac{e^{x+1}}{e(e^x - c_1(x+1)) - (x+1)\text{Ei}(x+1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 33

$$\left\{ y(x) = \frac{e^x}{-e^{-1}(1+x)\text{Ei}(1, -1-x) - e^x + \_C1(1+x)} \right\}$$

**2.130 ODE No. 130**

$$-2x^3 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.00647746 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow c_1 \sqrt{x} + \frac{2x^3}{5} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 15

$$\left\{ y(x) = \frac{2x^3}{5} + \sqrt{x} \_C1 \right\}$$

**2.131 ODE No. 131**

$$(2x + 1)y'(x) - 4e^{-y(x)} + 2 = 0$$

✓ **Mathematica** : cpu = 0.0169886 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow \log \left( \frac{e^{c_1}}{2x + 1} + 2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 31

$$\left\{ y(x) = -\ln \left( \frac{2x + 1}{-1 + (4x + 2)e^{2-C1}} \right) - 2 - C1 \right\}$$

**2.132 ODE No. 132**

$$3xy'(x) - y(x) - 3xy(x)^4 \log(x) = 0$$

✓ **Mathematica** : cpu = 0.0118807 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow \frac{(-2)^{2/3} \sqrt[3]{x}}{\sqrt[3]{4c_1 + 3x^2 - 6x^2 \log(x)}} \right\}, \left\{ y(x) \rightarrow \frac{2^{2/3} \sqrt[3]{x}}{\sqrt[3]{4c_1 + 3x^2 - 6x^2 \log(x)}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-12}^{2/3} \sqrt[3]{x}}{\sqrt[3]{4c_1 + 3x^2 - 6x^2 \log(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 153

$$\left\{ y(x) = \frac{1}{6x^2 \ln(x) - 3x^2 - 4 - C1} \sqrt[3]{-4x(6x^2 \ln(x) - 3x^2 - 4 - C1)^2}, y(x) = \frac{i\sqrt{3} - 1}{12x^2 \ln(x) - 6x^2 - 8 - C1} \sqrt[3]{-} \right\}$$

**2.133 ODE No. 133**

$$x^2 y'(x) + y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.00705503 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{1}{x}} \left( c_1 - \text{Ei} \left( -\frac{1}{x} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 16

$$\left\{ y(x) = (\text{Ei}(1, x^{-1}) + -C1) e^{x^{-1}} \right\}$$

**2.134 ODE No. 134**

$$x^2 y'(x) + e^{x-\frac{1}{x}} x^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0111796 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow e^{-1/x} (c_1 - e^x) \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 17

$$\left\{ y(x) = (-e^x + \_C1) e^{-x^{-1}} \right\}$$

**2.135 ODE No. 135**

$$x^2 y'(x) - (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.00708575 (sec), leaf count = 14

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{x}} x \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 11

$$\left\{ y(x) = \_C1 x e^{x^{-1}} \right\}$$

**2.136 ODE No. 136**

$$x^2 y'(x) + x^2 + xy(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.012529 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{x(-c_1 + \log(x) - 1)}{c_1 - \log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 18

$$\left\{ y(x) = -\frac{x(\ln(x) + \_C1 - 1)}{\ln(x) + \_C1} \right\}$$

**2.137 ODE No. 137**

$$x^2 y'(x) - xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.00878679 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{c_1 - \log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 14

$$\left\{ y(x) = \frac{x}{-\ln(x) + \_C1} \right\}$$

**2.138 ODE No. 138**

$$x^2 y'(x) - x^2 - xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0135795 (sec), leaf count = 13

$$\{ \{ y(x) \rightarrow x \tan(c_1 + \log(x)) \} \}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 11

$$\{ y(x) = \tan(\ln(x) + \_C1) x \}$$

**2.139 ODE No. 139**

$$ax^k - (b-1)b + x^2(y'(x) + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.125317 (sec), leaf count = 397

$$\left\{ \left\{ y(x) \rightarrow \frac{-\sqrt{a}c_1 x^k \Gamma\left(\frac{-2b+k+1}{k}\right) J_{-\frac{2b+k+1}{k}}\left(\frac{2\sqrt{a}\sqrt{x^k}}{k}\right) + \sqrt{a}c_1 x^k \Gamma\left(\frac{-2b+k+1}{k}\right) J_{-\frac{2b+k-1}{k}}\left(\frac{2\sqrt{a}\sqrt{x^k}}{k}\right) + c_1 \sqrt{x^k} \Gamma\left(\frac{-2b+k+1}{k}\right)}{2x\sqrt{x^k} \left( c_1 \Gamma\left(\frac{-2b+k+1}{k}\right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 219

$$\left\{ y(x) = \frac{1}{2x} \left( -2 J_{\frac{\sqrt{(-1+2b)^2+k}}{k}} \left( 2 \frac{\sqrt{ax^{k/2}}}{k} \right) \sqrt{ax^{k/2}} - 2 \sqrt{ax^{k/2}} Y_{\frac{\sqrt{(-1+2b)^2+k}}{k}} \left( 2 \frac{\sqrt{ax^{k/2}}}{k} \right) - C1 + 2(1/2 + (b-1)/2) \right) \right\}$$

**2.140 ODE No. 140**

$$x^2(y'(x) + y(x)^2) + 4xy(x) + 2 = 0$$

✓ **Mathematica** : cpu = 0.0098277 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 + x} - \frac{2}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 20

$$\left\{ y(x) = \frac{-2\_C1 + x}{x(-x + \_C1)} \right\}$$

**2.141 ODE No. 141**

$$axy(x) + b + x^2(y'(x) + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.0254922 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{a^2 - 2a - 4b + 1} \left( 1 - \frac{2c_1}{x\sqrt{a^2 - 2a - 4b + 1} + c_1} \right) - a + 1}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1}{2x} \left( -\tanh \left( \frac{-\ln(x) + \_C1}{2} \sqrt{a^2 - 2a - 4b + 1} \right) \sqrt{a^2 - 2a - 4b + 1} - a + 1 \right) \right\}$$

**2.142 ODE No. 142**

$$-ax^2y(x) + ax + x^2(y'(x) - y(x)^2) + 2 = 0$$

✓ **Mathematica** : cpu = 0.176258 (sec), leaf count = 109

$$\left\{ \left\{ y(x) \rightarrow \frac{a^6 c_1 x^3 e^{ax} - a^5 c_1 x^2 e^{ax} + 2a^4 c_1 x e^{ax} - 2a^3 c_1 e^{ax} + 1}{a^5 c_1 x^3 (-e^{ax}) + 2a^4 c_1 x^2 e^{ax} - 2a^3 c_1 x e^{ax} + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 52

$$\left\{ y(x) = \frac{-(ax - 1)(a^2 x^2 + 2)e^{ax} + \_C1}{((a^2 x^2 - 2ax + 2)e^{ax} + \_C1)x} \right\}$$

**2.143 ODE No. 143**

$$x^2(ay(x)^2 + y'(x)) - b = 0$$

✓ **Mathematica** : cpu = 0.00916341 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{4ab+1} \left( \frac{2c_1}{x\sqrt{4ab+1}+c_1} - 1 \right) - 1}{2ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 41

$$\left\{ y(x) = \frac{1}{2ax} \left( -\tanh \left( \frac{-\ln(x) + -C1}{2} \sqrt{4ab+1} \right) \sqrt{4ab+1} + 1 \right) \right\}$$

**2.144 ODE No. 144**

$$x^2(ay(x)^2 + y'(x)) + bx^\alpha + c = 0$$

✓ **Mathematica** : cpu = 0.159374 (sec), leaf count = 1588

$$\left\{ \left\{ y(x) \rightarrow \frac{a \frac{\sqrt{\alpha^2(1-4ac)}}{\alpha^2} \alpha^{\frac{2i\sqrt{4ac-1}}{\alpha}+1} (x^\alpha)^{\frac{\sqrt{\alpha^2(1-4ac)}}{\alpha^2} + \frac{1}{2}} J_{-\frac{\sqrt{\alpha^2(1-4ac)}}{\alpha^2}} \left( \frac{2\sqrt{a}\sqrt{b}\sqrt{x^\alpha}}{\alpha} \right) c_1 \Gamma \left( 1 - \frac{\sqrt{1-4ac}}{\alpha} \right) b \frac{\sqrt{\alpha^2(1-4ac)}}{\alpha^2} + a \frac{\sqrt{\alpha^2(1-4ac)}}{\alpha^2}}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.118 (sec), leaf count = 219

$$\left\{ y(x) = \frac{1}{2ax} \left( -2\sqrt{ab} \left( Y_{\frac{\sqrt{-4ac+1}+\alpha}} \left( 2 \frac{\sqrt{ab}x^{\alpha/2}}{\alpha} \right) - C1 + J_{\frac{\sqrt{-4ac+1}+\alpha}} \left( 2 \frac{\sqrt{ab}x^{\alpha/2}}{\alpha} \right) \right) x^{\alpha/2} + (\sqrt{-4ac+1} + 1) \right) \right\}$$

**2.145 ODE No. 145**

$$-ax^2y(x)^2 + ay(x)^3 + x^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.654849 (sec), leaf count = 239

$$\text{Solve} \left[ \frac{\text{Ai}' \left( \frac{2ay(x)x^2+x+a(ax^3+2)y(x)^2}{2\sqrt[3]{2a^4/3}xy(x)^2} \right) - \frac{(axy(x)+1)\text{Ai} \left( \frac{2ay(x)x^2+x+a(ax^3+2)y(x)^2}{2\sqrt[3]{2a^4/3}xy(x)^2} \right)}{2^{2/3}a^{2/3}y(x)}}{\text{Bi}' \left( \frac{2ay(x)x^2+x+a(ax^3+2)y(x)^2}{2\sqrt[3]{2a^4/3}xy(x)^2} \right) - \frac{(axy(x)+1)\text{Bi} \left( \frac{2ay(x)x^2+x+a(ax^3+2)y(x)^2}{2\sqrt[3]{2a^4/3}xy(x)^2} \right)}{2^{2/3}a^{2/3}y(x)}} + c_1 = 0, y(x) \right]$$



✓ **Maple** : cpu = 0.122 (sec), leaf count = 117

$$\left\{ y(x) = - \left( ax + (-2a)^{\frac{2}{3}} \operatorname{RootOf} \left( \operatorname{Bi} \left( \frac{1}{x} (-Z^2 \sqrt[3]{-2ax} - 1) \frac{1}{\sqrt[3]{-2a}} \right) - C1 - Z + -Z \operatorname{Ai} \left( \frac{1}{x} (-Z^2 \sqrt[3]{-2ax} - 1) \frac{1}{\sqrt[3]{-2a}} \right) \right) \right.$$

**2.146 ODE No. 146**

$$ay(x)^2 + x^2 y'(x) + xy(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.757026 (sec), leaf count = 73

$$\operatorname{Solve} \left[ -\frac{ia}{x} = \frac{2e^{-\frac{(ay(x)+x)^2}{2x^2y(x)^2}}}{2c_1 - i\sqrt{2\pi}\operatorname{erf} \left( \frac{ay(x)+x}{\sqrt{2xy(x)}} \right)}, y(x) \right]$$

✓ **Maple** : cpu = 0.176 (sec), leaf count = 84

$$\left\{ \frac{1}{2} \left( a\sqrt{\pi}\sqrt{2}\operatorname{Erf} \left( \frac{\sqrt{2}(ay(x)+x)}{2xy(x)} \right) e^{\frac{(ay(x)+x)^2}{2x^2(y(x))^2}} + 2x \right) e^{-\frac{((a-x)y(x)+x)((x+a)y(x)+x)}{2x^2(y(x))^2}} + -C1 = 0 \right\}$$

**2.147 ODE No. 147**

$$ax^2y(x)^3 + by(x)^2 + x^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.907638 (sec), leaf count = 279

$$\operatorname{Solve} \left[ \frac{(by(x)+x)\operatorname{Ai} \left( \frac{x^2+2by(x)x+(b^2-2ax^3)y(x)^2}{2\sqrt[3]{2a^2/3}b^{2/3}x^2y(x)^2} \right)}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}xy(x)} + \operatorname{Ai}' \left( \frac{x^2+2by(x)x+(b^2-2ax^3)y(x)^2}{2\sqrt[3]{2a^2/3}b^{2/3}x^2y(x)^2} \right)}{\frac{(by(x)+x)\operatorname{Bi} \left( \frac{x^2+2by(x)x+(b^2-2ax^3)y(x)^2}{2\sqrt[3]{2a^2/3}b^{2/3}x^2y(x)^2} \right)}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}xy(x)} + \operatorname{Bi}' \left( \frac{x^2+2by(x)x+(b^2-2ax^3)y(x)^2}{2\sqrt[3]{2a^2/3}b^{2/3}x^2y(x)^2} \right)} + c_1 = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.215 (sec), leaf count = 178

$$\left\{ y(x) = -\sqrt[3]{2}abx \left( \sqrt[3]{2}ab^2 - 2(a^2b^2)^{2/3} \operatorname{RootOf} \left( \operatorname{Bi} \left( -1/2 \frac{a2^{2/3}x - 2-Z^2\sqrt[3]{a^2b^2}}{\sqrt[3]{a^2b^2}} \right) - C1 - Z + -Z \operatorname{Ai} \left( -1/2 \frac{a2^{2/3}x - 2-Z^2\sqrt[3]{a^2b^2}}{\sqrt[3]{a^2b^2}} \right) \right) \right.$$

2.148 ODE No. 148

$$(x^2 + 1) y'(x) + xy(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0126685 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 + \sinh^{-1}(x)}{\sqrt{x^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 16

$$\left\{ y(x) = (\text{Arcsinh}(x) + \_C1) \frac{1}{\sqrt{x^2 + 1}} \right\}$$

2.149 ODE No. 149

$$(x^2 + 1) y'(x) - x(x^2 + 1) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0117758 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{\sqrt{x^2 + 1}} + \frac{1}{3}(x^2 + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 20

$$\left\{ y(x) = \frac{x^2}{3} + \frac{1}{3} + \_C1 \frac{1}{\sqrt{x^2 + 1}} \right\}$$

2.150 ODE No. 150

$$(x^2 + 1) y'(x) - 2x^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.00799195 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow \frac{3c_1 + 2x^3}{3x^2 + 3} \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 23

$$\left\{ y(x) = \frac{2x^3 + 3\_C1}{3x^2 + 3} \right\}$$

**2.151 ODE No. 151**

$$(x^2 + 1) y'(x) + (2xy(x) - 1) (y(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.719283 (sec), leaf count = 161

$$\text{Solve} \left[ c_1 = \frac{i \left( x \left( \sqrt[4]{\frac{(x^2+1)(y(x)^2+1)}{(xy(x)-1)^2}} \right) {}_2F_1 \left( \frac{1}{2}, \frac{5}{4}; \frac{3}{2}; -\frac{(x+y(x))^2}{(xy(x)-1)^2} \right) - 2 \right) + y(x) \left( \sqrt[4]{\frac{(x^2+1)(y(x)^2+1)}{(xy(x)-1)^2}} \right) {}_2F_1 \left( \frac{1}{2}, \frac{5}{4}; \frac{3}{2}; -\frac{(x+y(x))^2}{(xy(x)-1)^2} \right)}{2(xy(x) - 1) \sqrt[4]{-\frac{(x^2+1)(y(x)^2+1)}{(xy(x)-1)^2}}} \right]$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 85

$$\left\{ -C1 + x \frac{1}{\sqrt[4]{\left( x^{-1} + x^2 \left( \frac{y(x)x^4}{x^2+1} - \frac{x^3}{x^2+1} \right)^{-1} \right)^2} + 1} + \frac{y(x) + x}{2xy(x) - 2} {}_2F_1 \left( \frac{1}{2}, \frac{5}{4}; \frac{3}{2}; -\frac{(y(x) + x)^2}{(xy(x) - 1)^2} \right) = 0 \right\}$$

**2.152 ODE No. 152**

$$(x^2 + 1) y'(x) - x(x^2 + 1) \cos^2(y(x)) + x \sin(y(x)) \cos(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.225988 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{-6c_1 \sqrt{x^2 + 1} + x^4 + 2x^2 + 1}{3x^2 + 3} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.859 (sec), leaf count = 25

$$\left\{ y(x) = \arctan \left( \frac{1}{3} \left( (x^2 + 1)^{\frac{3}{2}} + 3 - C1 \right) \frac{1}{\sqrt{x^2 + 1}} \right) \right\}$$

**2.153 ODE No. 153**

$$a + (x^2 - 1) y'(x) - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0158703 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow ax + c_1 \sqrt{x^2 - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 20

$$\left\{ y(x) = \sqrt{1 + x} \sqrt{x - 1} - C1 + ax \right\}$$

**2.154 ODE No. 154**

$$(x^2 - 1) y'(x) + 2xy(x) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0139773 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 + \sin(x)}{x^2 - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 16

$$\left\{ y(x) = \frac{\sin(x) + \_C1}{x^2 - 1} \right\}$$

**2.155 ODE No. 155**

$$(x^2 - 1) y'(x) + y(x)^2 - 2xy(x) + 1 = 0$$

✓ **Mathematica** : cpu = 0.0170661 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{2c_1x + x \log(1 - x) - x \log(x + 1) + 2}{2c_1 + \log(1 - x) - \log(x + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 14

$$\left\{ y(x) = x + (\_C1 - \text{Arctanh}(x))^{-1} \right\}$$

**2.156 ODE No. 156**

$$(x^2 - 1) y'(x) - y(x)(y(x) - x) = 0$$

✓ **Mathematica** : cpu = 0.0151967 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 \sqrt{x^2 - 1} + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 20

$$\left\{ y(x) = \left( \sqrt{x - 1} \sqrt{1 + x} \_C1 + x \right)^{-1} \right\}$$

**2.157 ODE No. 157**

$$a(y(x)^2 - 2xy(x) + 1) + (x^2 - 1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0941492 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 P_a(x) + Q_a(x)}{c_1 P_{a-1}(x) + Q_{a-1}(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.279 (sec), leaf count = 231

$$\left\{ y(x) = \frac{1}{(4 + 4x)a} \left( 8((a - 1/2)x - a/2 + 1/2)(1 + x) \_C1 \text{HeunC} \left( 0, -2a + 1, 0, 0, a^2 - a + 1/2, 2(1 + x)^{-1} \right) \right) \right\}$$

**2.158 ODE No. 158**

$$axy(x)^2 + (x^2 - 1)y'(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0341872 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{c_1}}{ae^{c_1} - \sqrt{x^2 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 22

$$\left\{ y(x) = \left( \sqrt{x - 1} \sqrt{1 + x} \_C1 - a \right)^{-1} \right\}$$

**2.159 ODE No. 159**

$$(x^2 - 1)y'(x) - 2xy(x) \log(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0161058 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow e^{e^{c_1}(x^2 - 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 13

$$\left\{ y(x) = e^{-C1(x-1)(1+x)} \right\}$$

**2.160 ODE No. 160**

$$(x^2 - 4) y'(x) + (x + 2)y(x)^2 - 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0197447 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{2 - x}{(x + 2)(c_1 - \log(x + 2))} \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 21

$$\left\{ y(x) = \frac{x - 2}{(x + 2)(\ln(x + 2) + C1)} \right\}$$

**2.161 ODE No. 161**

$$(x^2 - 5x + 6) y'(x) + x^2 + 3xy(x) - 8y(x) = 0$$

✓ **Mathematica** : cpu = 0.0135984 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{-12c_1 - 3x^4 + 8x^3}{12(x - 3)(x - 2)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 27

$$\left\{ y(x) = \frac{1}{(x - 2)^2(x - 3)} \left( -\frac{x^4}{4} + \frac{2x^3}{3} + C1 \right) \right\}$$

**2.162 ODE No. 162**

$$k(-a + y(x) + x)(-b + y(x) + x) + (x - a)(x - b)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.259849 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow \frac{(k + 1) \sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}} \tan \left( \frac{(k+1) \sqrt{-\frac{k^2(a-b)^2}{(k+1)^2}} (\log(x-b) - \log(x-a))}{2(a-b)} + c_1 \right) + k(a + b - 2x)}{2(k + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.199 (sec), leaf count = 58

$$\left\{ y(x) = \frac{(-C1(a - x)(a - x)^k + (b - x)^k(b - x))k}{(k + 1)(-C1(a - x)^k + (b - x)^k)} \right\}$$

**2.163 ODE No. 163**

$$2a^2x + 2x^2y'(x) - 2y(x)^2 - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0124506 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-a^2}\sqrt{x} \tan\left(\frac{2\sqrt{-a^2}}{\sqrt{x}} - c_1\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 26

$$\left\{ y(x) = i \tan\left(1(-C1 \sqrt{x} - 2ia) \frac{1}{\sqrt{x}}\right) \sqrt{xa} \right\}$$

**2.164 ODE No. 164**

$$2a^2x + 2x^2y'(x) - 2y(x)^2 - 3xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0791822 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow \frac{4a^2c_1\sqrt{x} + 2ac_1x + 2a\sqrt{x}e^{\frac{4a}{\sqrt{x}}} - xe^{\frac{4a}{\sqrt{x}}}}{2e^{\frac{4a}{\sqrt{x}}} - 4ac_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.217 (sec), leaf count = 102

$$\left\{ y(x) = 1 \left( \left( -2x_{-C1} \sqrt{-\frac{a^2}{x}} - x \right) \sin\left(2\sqrt{-\frac{a^2}{x}}\right) - x \left( -C1 - 2\sqrt{-\frac{a^2}{x}} \right) \cos\left(2\sqrt{-\frac{a^2}{x}}\right) \right) \left( 2 \cos\left(2\sqrt{-\frac{a^2}{x}}\right) \right) \right\}$$

**2.165 ODE No. 165**

$$x(2x - 1)y'(x) + y(x)^2 - (4x + 1)y(x) + 4x = 0$$

✓ **Mathematica** : cpu = 0.0171774 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{x(2x - 1)}{x - c_1} + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 17

$$\left\{ y(x) = \frac{2x^2 + _C1}{x + _C1} \right\}$$

**2.166 ODE No. 166**

$$2(x-1)xy'(x) + (x-1)y(x)^2 - x = 0$$

✓ **Mathematica** : cpu = 0.0877914 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow - \frac{2 \left( \pi G_{2,2}^{2,0} \left( x \left| \begin{array}{l} \frac{1}{2}, \frac{3}{2} \\ 0, 1 \end{array} \right. \right) + c_1(K(x) - E(x)) \right)}{\pi G_{2,2}^{2,0} \left( x \left| \begin{array}{l} \frac{1}{2}, \frac{3}{2} \\ 0, 0 \end{array} \right. \right) + 2c_1 E(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 97

$$\left\{ y(x) = \frac{x}{2x-2} \left( LegendreQ \left( -\frac{1}{2}, 1, \frac{2-x}{x} \right) - C1 - LegendreQ \left( \frac{1}{2}, 1, \frac{2-x}{x} \right) - C1 + LegendreP \left( -\frac{1}{2}, 1, \frac{2-x}{x} \right) \right) \right\}$$

**2.167 ODE No. 167**

$$3x^2y'(x) - x^2 - 3xy(x) - 7y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0220684 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan \left( \frac{1}{3} \sqrt{7} (3c_1 + \log(x)) \right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 20

$$\left\{ y(x) = \frac{x\sqrt{7}}{7} \tan \left( \frac{(\ln(x) + C1)\sqrt{7}}{3} \right) \right\}$$

**2.168 ODE No. 168**

$$3(x^2-4)y'(x) + y(x)^2 - xy(x) - 3 = 0$$

✓ **Mathematica** : cpu = 0.0906268 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow \frac{-2c_1 x P_{-\frac{1}{6}}^{\frac{1}{3}} \left( \frac{x}{2} \right) + 3c_1 P_{\frac{5}{6}}^{\frac{1}{3}} \left( \frac{x}{2} \right) - 2x Q_{-\frac{1}{6}}^{\frac{1}{3}} \left( \frac{x}{2} \right) + 3Q_{\frac{5}{6}}^{\frac{1}{3}} \left( \frac{x}{2} \right)}{c_1 P_{-\frac{1}{6}}^{\frac{1}{3}} \left( \frac{x}{2} \right) + Q_{-\frac{1}{6}}^{\frac{1}{3}} \left( \frac{x}{2} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.191 (sec), leaf count = 140

$$\left\{ y(x) = -3(x+2) \left( HeunC \left( 0, 4/3, -1/3, 0, \frac{25}{36}, 4(x+2)^{-1} \right) - C1 - 1/3(-x/4 - 1/2)^{4/3} HeunC \left( 0, -4/3, -1 \right) \right) \right\}$$



**2.169 ODE No. 169**

$$(ax + b)^2 y'(x) + y(x)^3 (ax + b) + cy(x)^2 = 0$$

✓ **Mathematica** : cpu = 2.57346 (sec), leaf count = 110

$$\text{Solve} \left[ -\frac{c}{\sqrt{-a(ax+b)^2}} = \frac{2 \exp\left(-\frac{a(ax+b)+cy(x)^2}{2ay(x)^2(ax+b)^2}\right)}{2c_1 - \sqrt{2\pi} \operatorname{erfi}\left(\frac{a(ax+b)+cy(x)}{\sqrt{2y(x)}\sqrt{-a(ax+b)^2}}\right)}, y(x) \right]$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 153

$$\left\{ \frac{1}{2} \left( \left( \sqrt{2} \sqrt{\pi} \operatorname{Erf} \left( \frac{(cy(x) + a(ax+b)) \sqrt{2}}{2(ax+b)y(x)} \right) \frac{1}{\sqrt{a}} \right) e^{\frac{(cy(x)+a(ax+b))^2}{2(y(x))^2(ax+b)^2 a}} ac + 2(ax+b)a^{3/2} \right) e^{-\frac{((ax+b+c)y(x)+a(ax+b))((-ax-b)+a)}{2(y(x))^2(ax+b)^2 a}} \right\}$$

**2.170 ODE No. 170**

$$-x^4 + x^3 y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0230476 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2(c_1 + \log(x) - 1)}{c_1 + \log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 23

$$\left\{ y(x) = \frac{(\ln(x) - \_C1 - 1)x^2}{\ln(x) - \_C1} \right\}$$

**2.171 ODE No. 171**

$$x^3 y'(x) - x^2 y(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0102898 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2}{c_1 x + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 15

$$\left\{ y(x) = \frac{x^2}{\_C1 x + 1} \right\}$$

**2.172 ODE No. 172**

$$x^4(-y(x)^2) + x^3y'(x) + x^2y(x) + 20 = 0$$

✓ **Mathematica** : cpu = 0.0399386 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{4 - 5c_1x^9}{c_1x^{11} + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.275 (sec), leaf count = 26

$$\left\{ y(x) = \frac{5x^9 + 4\_C1}{(-x^9 + \_C1)x^2} \right\}$$

**2.173 ODE No. 173**

$$x^6(-y(x)^2) + x^3y'(x) - (2x - 3)x^2y(x) + 3 = 0$$

✓ **Mathematica** : cpu = 0.0153097 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow \frac{\frac{1}{c_1e^{4x+\frac{1}{4}} - 3}}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 27

$$\left\{ y(x) = \frac{-3(e^x)^4\_C1 - 3}{x^3((e^x)^4\_C1 - 3)} \right\}$$

**2.174 ODE No. 174**

$$(x^2 + 1)xy'(x) + x^2y(x) = 0$$

✓ **Mathematica** : cpu = 0.00753693 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{\sqrt{x^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.005 (sec), leaf count = 13

$$\left\{ y(x) = \_C1 \frac{1}{\sqrt{x^2 + 1}} \right\}$$

**2.175 ODE No. 175**

$$ax^3 + (x^2 - 1)xy'(x) - (2x^2 - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0195034 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow x \left( a + c_1 \sqrt{1 - x^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 20

$$\left\{ y(x) = x \left( \sqrt{x - 1} \sqrt{1 + x} \_C1 + a \right) \right\}$$

**2.176 ODE No. 176**

$$(x^2 - 1)xy'(x) + (x^2 - 1)y(x)^2 - x^2 = 0$$

✓ **Mathematica** : cpu = 0.144153 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow - \frac{2 \left( \pi G_{2,2}^{2,0} \left( x^2 \mid \begin{matrix} \frac{1}{2}, \frac{3}{2} \\ 0, 1 \end{matrix} \right) + c_1 (K(x^2) - E(x^2)) \right)}{\pi G_{2,2}^{2,0} \left( x^2 \mid \begin{matrix} \frac{1}{2}, \frac{3}{2} \\ 0, 0 \end{matrix} \right) + 2c_1 E(x^2)} \right\} \right\}$$

✓ **Maple** : cpu = 0.103 (sec), leaf count = 30

$$\left\{ y(x) = \frac{\_C1 \text{EllipticCE}(x) + \text{EllipticE}(x) - \text{EllipticK}(x)}{\_C1 \text{EllipticCE}(x) - \_C1 \text{EllipticCK}(x) + \text{EllipticE}(x)} \right\}$$

**2.177 ODE No. 177**

$$(x - 1)x^2y'(x) - (x - 2)xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.016476 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2}{c_1(-x) + c_1 + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 17

$$\left\{ y(x) = \frac{x^2}{1 + \_C1(x - 1)} \right\}$$

**2.178 ODE No. 178**

$$2(x^2 - 1)xy'(x) + 2(x^2 - 1)y(x)^2 - (3x^2 - 5)y(x) + x^2 - 3 = 0$$

✓ **Mathematica** : cpu = 0.0882532 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{x}}{c_1 \sqrt{1-x^2} - 2\sqrt{1-\frac{1}{x^2}} x F\left(\sin^{-1}\left(\frac{1}{\sqrt{x}}\right) \middle| -1\right)} + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 63

$$\left\{ y(x) = 1 - 2 \frac{\sqrt{x}}{\sqrt{x-1}\sqrt{1+x}} \left( -C1 - 2 \frac{\text{EllipticF}(\sqrt{1+x}, 1/2\sqrt{2}) \sqrt{-x}\sqrt{-2x+2\sqrt{2}}}{\sqrt{2x-2}\sqrt{x}} \right)^{-1} \right\}$$

**2.179 ODE No. 179**

$$3x(x^2 - 1)y'(x) - (x^2 + 1)y(x) + xy(x)^2 - 3x = 0$$

✓ **Mathematica** : cpu = 1.6559 (sec), leaf count = 1619

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-2 \int_1^x \text{Root}[125K[1]^8 - 164K[1]^6 + 70K[1]^4 - 20K[1]^2 + (1296K[1]^{12} - 5184K[1]^{10} + 7776K[1]^8 - 5184K[1]^6 + 1296K[1]^4) \#1^4 + (-3} \right\} \right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 112

$$\left\{ y(x) = 35 \frac{1}{\sqrt[3]{x} (8x^{2/3} {}_2F_1(5/6, 7/6; 4/3; x^2) - C1 + 8 {}_2F_1(1/2, 5/6; 2/3; x^2))} \left( -C1 \left( \frac{8x^2}{7} - \frac{16}{35} \right) {}_2F_1(5/6, 7/6; \right. \right.$$

**2.180 ODE No. 180**

$$(xy'(x) - y(x))(ax^2 + bx + c) + x^2 - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.122081 (sec), leaf count = 104

$$\left\{ \left\{ y(x) \rightarrow - \frac{x \left( \exp \left( \frac{4 \tan^{-1} \left( \frac{2ax+b}{\sqrt{4ac-b^2}} \right) + 2c_1}{\sqrt{4ac-b^2}} \right) - 1 \right)}{\exp \left( \frac{4 \tan^{-1} \left( \frac{2ax+b}{\sqrt{4ac-b^2}} \right) + 2c_1}{\sqrt{4ac-b^2}} \right) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 58

$$\left\{ y(x) = -\tanh \left( 1 \left( -C1 \sqrt{4ac - b^2} + 2 \arctan \left( \frac{2ax + b}{\sqrt{4ac - b^2}} \right) \right) \frac{1}{\sqrt{4ac - b^2}} \right) x \right\}$$

**2.181 ODE No. 181**

$$a + x^4(y'(x) + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.0123108 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow \frac{(x + i\sqrt{-a}c_1) \cosh \left( \frac{\sqrt{-a}}{x} \right) - (\sqrt{-a} + ic_1x) \sinh \left( \frac{\sqrt{-a}}{x} \right)}{x^2 \left( \cosh \left( \frac{\sqrt{-a}}{x} \right) - ic_1 \sinh \left( \frac{\sqrt{-a}}{x} \right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 28

$$\left\{ y(x) = \frac{1}{x^2} \left( -\tan \left( \frac{-C1x - 1}{x} \sqrt{a} \right) \sqrt{a} + x \right) \right\}$$

**2.182 ODE No. 182**

$$(x^3 - 1)xy'(x) + x^2 - 2xy(x)^2 + y(x) = 0$$

✓ **Mathematica** : cpu = 0.167045 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{x(2c_1x + 1)}{2c_1 + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.14 (sec), leaf count = 18

$$\left\{ y(x) = \frac{x(x + \frac{-C1}{x^2})}{-C1x^2 + 1} \right\}$$

**2.183 ODE No. 183**

$$(2x^4 - x) y'(x) - 2(x^3 - 1) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0144835 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^2}{\sqrt[3]{1 - 2x^3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 18

$$\left\{ y(x) = -C1 x^2 \frac{1}{\sqrt[3]{2x^3 - 1}} \right\}$$

**2.184 ODE No. 184**

$$(y'(x) + y(x)^2) (ax^2 + bx + c)^2 + A = 0$$

✓ **Mathematica** : cpu = 1.44383 (sec), leaf count = 612

$$\left\{ \left\{ y(x) \rightarrow \frac{b^2 c_1 \left( -\exp \left( \frac{2\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}} \tan^{-1} \left( \frac{2ax+b}{\sqrt{4ac-b^2}} \right) \right)}{\sqrt{b^2-4ac}} \right) + bc_1 \sqrt{b^2-4ac} \sqrt{1-\frac{4A}{b^2-4ac}} \exp \left( \frac{2\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}}}{\sqrt{b^2-4ac}} \right)}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.385 (sec), leaf count = 493

$$\left\{ y(x) = 2 \frac{a}{\sqrt{-4ac+b^2} (2ax+b+i\sqrt{4ac-b^2}) (i\sqrt{4ac-b^2}-2ax-b)} \left( \left( i\sqrt{\frac{-4ac+b^2-4A}{a^2}} a\sqrt{4ac-b^2} \right) \right) \right\}$$

**2.185 ODE No. 185**

$$x^7 y'(x) + 5x^3 y(x)^2 + 2(x^2 + 1) y(x)^3 = 0$$

✗ **Mathematica** : cpu = 300.184 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.059 (sec), leaf count = 63

$$\left\{ -C1 + x \frac{1}{\sqrt[4]{\left(x^{-1} + \frac{x^2}{y(x)}\right)^2 + 1}} + \frac{x^3 + y(x)}{2xy(x)} {}_2F_1\left(\frac{1}{2}, \frac{5}{4}; \frac{3}{2}; -\frac{(x^3 + y(x))^2}{x^2 (y(x))^2}\right) = 0 \right\}$$

**2.186 ODE No. 186**

$$-(n-1)x^{n-1}y(x) + x^{2n-2} + x^n y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.029254 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow x^{n-1} \tan(c_1 - \log(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 17

$$\{y(x) = \tan(-\ln(x) + \_C1) x^{n-1}\}$$

**2.187 ODE No. 187**

$$-ay(x)^2 - bx^{2n-2} + x^n y'(x) = 0$$

✓ **Mathematica** : cpu = 0.068649 (sec), leaf count = 166

$$\left\{ \left\{ y(x) \rightarrow \frac{x^{n-1} \left( c_1 \left( \sqrt{a} \sqrt{b} \sqrt{\frac{(n-1)^2 - 4ab}{ab}} + n - 1 \right) + \left( -\sqrt{a} \sqrt{b} \sqrt{\frac{(n-1)^2 - 4ab}{ab}} + n - 1 \right) x^{\sqrt{a} \sqrt{b} \sqrt{\frac{(n-1)^2 - 4ab}{ab}}} \right)}{2a \left( x^{\sqrt{a} \sqrt{b} \sqrt{\frac{(n-1)^2 - 4ab}{ab}}} + c_1 \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 60

$$\left\{ y(x) = \frac{x^{n-1}}{2a} \left( -\sqrt{4ab - n^2 + 2n - 1} \tan\left(\frac{-\ln(x) + \_C1}{2} \sqrt{4ab - n^2 + 2n - 1}\right) + n - 1 \right) \right\}$$

**2.188 ODE No. 188**

$$-ay(x)^3 - bx^3 + x^{2n+1}y'(x) = 0$$

✗ **Mathematica** : cpu = 19.9668 (sec), leaf count = 0 , could not solve

`DSolve[-(b*n*x^3) - a*y[x]^3 + x^(1 + 2*n)*Derivative[1][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.028 (sec), leaf count = 32

$$\left\{ y(x) = \text{RootOf}\left(-\ln(x) + \_C1 + \int^{-Z} (\_a^3 a - n\_a + b)^{-1} d\_a\right) x^n \right\}$$

**2.189 ODE No. 189**

$$-ay(x)^n - bx^{(m+1)n} + x^{m(n-1)+n}y'(x) = 0$$

✗ **Mathematica** : cpu = 300.016 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.247 (sec), leaf count = 60

$$\left\{ \int_{-b}^{y(x)} -\frac{x^{mn}x^n}{(bx^m x - (m+1)_a)x^n x^{mn} + a_a^n x^m x} d_a + \ln(x) - _C1 = 0 \right\}$$

**2.190 ODE No. 190**

$$\sqrt{x^2 - 1}y'(x) - \sqrt{y(x)^2 - 1} = 0$$

✓ **Mathematica** : cpu = 0.0468695 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}e^{-c_1} \left( (e^{2c_1} - 1) \sqrt{x^2 - 1} + (e^{2c_1} + 1) x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 29

$$\left\{ \ln \left( x + \sqrt{x^2 - 1} \right) - \ln \left( y(x) + \sqrt{(y(x))^2 - 1} \right) + _C1 = 0 \right\}$$

**2.191 ODE No. 191**

$$\sqrt{1 - x^2}y'(x) - y(x)\sqrt{y(x)^2 - 1} = 0$$

✓ **Mathematica** : cpu = 0.0292108 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow -\cot \left( c_1 + \sin^{-1}(x) \right) \sqrt{\sec^2 \left( c_1 + \sin^{-1}(x) \right)} \right\}, \left\{ y(x) \rightarrow \cot \left( c_1 + \sin^{-1}(x) \right) \sqrt{\sec^2 \left( c_1 + \sin^{-1}(x) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 16

$$\left\{ \arcsin(x) + \arctan \left( \frac{1}{\sqrt{(y(x))^2 - 1}} \right) + _C1 = 0 \right\}$$



**2.192 ODE No. 192**

$$\sqrt{a^2 + x^2}y'(x) - \sqrt{a^2 + x^2} + y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.026449 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{a^2 \log(\sqrt{a^2 + x^2} + x) + c_1}{\sqrt{a^2 + x^2} + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 36

$$\left\{ y(x) = 1 \left( a^2 \ln(x + \sqrt{a^2 + x^2}) + \_C1 \right) \left( x + \sqrt{a^2 + x^2} \right)^{-1} \right\}$$

**2.193 ODE No. 193**

$$-ax(\log(x) + 1) + x \log(x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.00815962 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow ax + \frac{c_1}{\log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 14

$$\left\{ y(x) = ax + \frac{C1}{\ln(x)} \right\}$$

**2.194 ODE No. 194**

$$x \log(x)y'(x) - y(x) (2 \log^2(x) + 1) - y(x)^2 \log(x) - \log^3(x) = 0$$

✓ **Mathematica** : cpu = 0.0725777 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow -\frac{\log(x) (2c_1 + \log^2(x) + 2)}{2c_1 + \log^2(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 23

$$\left\{ y(x) = -\frac{\ln(x) \left( (\ln(x))^2 + \_C1 + 2 \right)}{(\ln(x))^2 + \_C1} \right\}$$

**2.195 ODE No. 195**

$$\sin(x)y'(x) + y(x)^2(-\sin^2(x)) + y(x)(\cos(x) - 3\sin(x)) + 4 = 0$$

✓ **Mathematica** : cpu = 0.0564182 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \left( \frac{1}{c_1 e^{5x} + \frac{1}{5}} - 4 \right) \csc(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 28

$$\left\{ y(x) = \frac{-4(e^x)^5\_C1 - 4}{\sin(x) \left( (e^x)^5\_C1 - 4 \right)} \right\}$$

**2.196 ODE No. 196**

$$\cos(x)y'(x) + y(x) + (\sin(x) + 1)\cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0564329 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow e^{-2 \tanh^{-1}(\tan(\frac{x}{2}))} \left( c_1 + \sin(x) + 4 \log \left( \cos \left( \frac{x}{2} \right) - \sin \left( \frac{x}{2} \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 29

$$\left\{ y(x) = \frac{\sin(x) + 2 \ln(\cos(x)) - 2 \ln(\sec(x) + \tan(x)) +\_C1}{\sec(x) + \tan(x)} \right\}$$

**2.197 ODE No. 197**

$$\cos(x)y'(x) - y(x)^4 - y(x)\sin(x) = 0$$

✓ **Mathematica** : cpu = 0.0430447 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\sqrt[3]{c_1 \cos^3(x) - \sin(x) - 2 \sin(x) \cos^2(x)}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1}}{\sqrt[3]{c_1 \cos^3(x) - \sin(x) - 2 \sin(x) \cos^2(x)}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt[3]{-1}}{\sqrt[3]{c_1 \cos^3(x) - \sin(x) - 2 \sin(x) \cos^2(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 237

$$\left\{ y(x) = \frac{1}{\_C1 (\sin(x))^4 + 2 \cos(x) (\sin(x))^3 - 2\_C1 (\sin(x))^2 - 3 \cos(x) \sin(x) +\_C1 \sqrt[3]{\cos(x) \left( -C1 (\sin(x))^3 + \sin(x) \right)}} \right\}$$

**2.198 ODE No. 198**

$$\sin(x) \cos(x) y'(x) - y(x) - \sin^3(x) = 0$$

✓ **Mathematica** : cpu = 0.0237215 (sec), leaf count = 15

$$\{ \{ y(x) \rightarrow c_1 \tan(x) - \sin(x) \} \}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 13

$$\{ y(x) = \tan(x) (-\cos(x) + \_C1) \}$$

**2.199 ODE No. 199**

$$\sin(2x) y'(x) + \sin(2y(x)) = 0$$

✓ **Mathematica** : cpu = 0.183384 (sec), leaf count = 15

$$\{ \{ y(x) \rightarrow \cot^{-1}(e^{-2c_1} \tan(x)) \} \}$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 102

$$\left\{ y(x) = \frac{1}{2} \arctan \left( 2 \frac{\_C1 (2 \sin(2x) + \sin(4x))}{-\_C1^2 \cos(4x) + \_C1^2 + 4 \cos(2x) + \cos(4x) + 3}, \frac{\_C1^2 \cos(4x) - \_C1^2 + 4 \cos(2x)}{-\_C1^2 \cos(4x) - \_C1^2 - 4 \cos(2x)} \right) \right\}$$

**2.200 ODE No. 200**

$$Ax(a \sin^2(x) + c) + y'(x)(a \sin^2(x) + b) + ay(x) \sin(2x) = 0$$

✓ **Mathematica** : cpu = 0.048288 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow \frac{2aAx^2 - 2aAx \sin(2x) - aA \cos(2x) + 4Acx^2 + 4c_1}{4a \cos(2x) - 4(a + 2b)} \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 53

$$\left\{ y(x) = \frac{-A \cos(2x) a - 2A \sin(2x) ax + 2x^2(a + 2c) A - 8\_C1}{4a \cos(2x) - 4a - 8b} \right\}$$

2.201 ODE No. 201

$$-y(x)f'(x) + 2f(x)y'(x) + 2f(x)y(x)^2 - 2f(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0730679 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow i\sqrt{f(x)} \tan \left( c_1 + i \int_1^x -\sqrt{f(K[1])} dK[1] \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 23

$$\left\{ y(x) = i \tan \left( -i \int \sqrt{f(x)} dx + \_C1 \right) \sqrt{f(x)} \right\}$$

2.202 ODE No. 202

$$f(x)y'(x) + g(x)\text{tg}(y(x)) + h(x) = 0$$

✗ **Mathematica** : cpu = 20.5291 (sec), leaf count = 0 , could not solve

`DSolve[h[x] + g[x]*tg[y[x]] + f[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(f(x)*diff(y(x),x)+g(x)*tg(y(x))+h(x) = 0,y(x))`

2.203 ODE No. 203

$$x^3 + y(x)y'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 3.20427 (sec), leaf count = 0 , could not solve

`DSolve[x^3 + y[x] + y[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(y(x)*diff(y(x),x)+y(x)+x^3 = 0,y(x))`

2.204 ODE No. 204

$$ay(x) + y(x)y'(x) + x = 0$$

✗ **Mathematica** : cpu = 300.505 (sec), leaf count = 0 , timed out

`$Aborted`

✓ **Maple** : cpu = 0.316 (sec), leaf count = 91

$$\left\{ y(x) = \text{RootOf} \left( \_Z^2 - e^{\text{RootOf} \left( x^2 \left( -\left( \tanh \left( \frac{2\_C1 + \_Z + 2 \ln(x)}{2a} \sqrt{(a-2)(a+2)} \right) \right)^2 a^2 + 4 \left( \tanh \left( 1/2 \frac{\sqrt{(a-2)(a+2)}(2\_C1 + \_Z + 2 \ln(x))}{a} \right) \right) \right)} \right) \right\}$$

2.205 ODE No. 205

$$\frac{1}{4}(a^2 - 1)x + ay(x) + bx^n + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 24.3198 (sec), leaf count = 0 , could not solve

DSolve[((-1 + a^2)\*x)/4 + b\*x^n + a\*y[x] + y[x]\*Derivative[1][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(y(x)\*diff(y(x),x)+a\*y(x)+1/4\*(a^2-1)\*x+b\*x^n = 0,y(x))

2.206 ODE No. 206

$$ay(x) - 2a + be^x + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 29.7803 (sec), leaf count = 0 , could not solve

DSolve[-2\*a + b\*E^x + a\*y[x] + y[x]\*Derivative[1][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(y(x)\*diff(y(x),x)+a\*y(x)+b\*exp(x)-2\*a = 0,y(x))

2.207 ODE No. 207

$$y(x)y'(x) + y(x)^2 + 4x(x + 1) = 0$$

✓ **Mathematica** : cpu = 0.0123076 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 e^{-2x} - 4x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 e^{-2x} - 4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 37

$$\left\{ y(x) = \sqrt{e^{-2x} C1 - 4x^2}, y(x) = -\sqrt{e^{-2x} C1 - 4x^2} \right\}$$

2.208 ODE No. 208

$$ay(x)^2 - b \cos(c + x) + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0704223 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{e^{-2ax} (4a^2 c_1 + 2be^{2ax} \sin(c + x) + c_1) + 4ab \cos(c + x)}}{\sqrt{4a^2 + 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{e^{-2ax} (4a^2 c_1 + 2be^{2ax} \sin(c + x) + c_1) + 4ab \cos(c + x)}}{\sqrt{4a^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 106

$$\left\{ y(x) = \frac{1}{4a^2 + 1} \sqrt{16 (a^2 + 1/4)^2 C1 e^{-2ax} + 16 (\cos(x + c) a + 1/2 \sin(x + c)) (a^2 + 1/4) b}, y(x) = -\frac{1}{4a^2 + 1} \sqrt{16 (a^2 + 1/4)^2 C1 e^{-2ax} + 16 (\cos(x + c) a + 1/2 \sin(x + c)) (a^2 + 1/4) b} \right\}$$

**2.209 ODE No. 209**

$$y(x)y'(x) - \sqrt{ay(x)^2 + b} = 0$$

✓ **Mathematica** : cpu = 0.0220095 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^2(c_1 + x)^2 - b}}{\sqrt{a}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^2(c_1 + x)^2 - b}}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 21

$$\left\{ x - \frac{1}{a} \sqrt{a(y(x))^2 + b} + \_C1 = 0 \right\}$$

**2.210 ODE No. 210**

$$y(x)y'(x) + xy(x)^2 - 4x = 0$$

✓ **Mathematica** : cpu = 0.0167611 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{e^{2c_1 - x^2} + 4} \right\}, \left\{ y(x) \rightarrow \sqrt{e^{2c_1 - x^2} + 4} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{e^{-x^2} \_C1 + 4}, y(x) = -\sqrt{e^{-x^2} \_C1 + 4} \right\}$$

**2.211 ODE No. 211**

$$y(x)y'(x) - xe^{\frac{x}{y(x)}} = 0$$

✓ **Mathematica** : cpu = 52.3828 (sec), leaf count = 38

$$\text{Solve} \left[ c_1 = \int_1^{\frac{y(x)}{x}} \frac{K[1]}{K[1]^2 - e^{\frac{1}{K[1]}}} dK[1] + \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 31

$$\left\{ y(x) = \text{RootOf} \left( - \int^{-Z} \frac{-a}{-a^2 + e^{-a^{-1}}} d\_a + \ln(x) + \_C1 \right) x \right\}$$

**2.212 ODE No. 212**

$$g(x)f(x^2 + y(x)^2) + y(x)y'(x) + x = 0$$

✓ **Mathematica** : cpu = 27.708 (sec), leaf count = 92

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{K[2]}{f(K[2]^2 + x^2)} - \int_1^x -\frac{2K[1]K[2]f'(K[1]^2 + K[2]^2)}{f(K[1]^2 + K[2]^2)^2} dK[1] \right) dK[2] + \int_1^x \left( \frac{K[1]}{f(K[1]^2 + y(x)^2)} \right) \right]$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 30

$$\left\{ \int_{-b}^{y(x)} \frac{-a}{f(-a^2 + x^2)} d_a + \int g(x) dx - \_C1 = 0 \right\}$$

**2.213 ODE No. 213**

$$(y(x) + 1)y'(x) - y(x) - x = 0$$

✓ **Mathematica** : cpu = 0.585608 (sec), leaf count = 71

$$\text{Solve} \left[ \frac{1}{2} \log \left( \frac{x^2 - y(x)^2 + (x - 3)y(x) - x - 1}{(x - 1)^2} \right) + \log(1 - x) = c_1 + \frac{\tanh^{-1} \left( \frac{y(x) + 2x - 1}{\sqrt{5}(y(x) + 1)} \right)}{\sqrt{5}}, y(x) \right]$$

✓ **Maple** : cpu = 0.713 (sec), leaf count = 66

$$\left\{ -\frac{1}{2} \ln \left( \frac{(y(x))^2 + (-x + 3)y(x) - x^2 + x + 1}{(x - 1)^2} \right) - \frac{\sqrt{5}}{5} \text{Artanh} \left( \frac{(-2y(x) - 3 + x)\sqrt{5}}{5x - 5} \right) - \ln(x - 1) - \_C1 = 0 \right\}$$

**2.214 ODE No. 214**

$$(y(x) + x - 1)y'(x) - y(x) + 2x + 3 = 0$$

✓ **Mathematica** : cpu = 0.49238 (sec), leaf count = 78

$$\text{Solve} \left[ 2\sqrt{2} \tan^{-1} \left( \frac{-y(x) + 2x + 3}{\sqrt{2}(y(x) + x - 1)} \right) = 3c_1 + 2 \log \left( \frac{6x^2 + 3y(x)^2 - 10y(x) + 8x + 11}{(3x + 2)^2} \right) + 4 \log(3x + 2), y(x) \right]$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 48

$$\left\{ y(x) = \frac{5}{3} + \frac{(-3x - 2)\sqrt{2} \tan \left( \text{RootOf} \left( \sqrt{2} \ln \left( 2 \left( (\tan(\_Z))^2 + 1 \right) (3x + 2)^2 \right) + 2\sqrt{2}\_C1 - 2\_Z \right) \right)}{3} \right\}$$

**2.215 ODE No. 215**

$$(y(x) + 2x - 2)y'(x) - y(x) + x + 1 = 0$$

✓ **Mathematica** : cpu = 0.614392 (sec), leaf count = 80

$$\text{Solve} \left[ 6\sqrt{3} \tan^{-1} \left( \frac{4 - 3y(x)}{\sqrt{3}(y(x) + 2x - 2)} \right) = 2c_1 + 3 \log \left( \frac{3x^2 + 3y(x)^2 + 3(x - 3)y(x) - 6x + 7}{(1 - 3x)^2} \right) + 6 \log(3x - 1), \right.$$

✓ **Maple** : cpu = 0.184 (sec), leaf count = 51

$$\left\{ y(x) = \frac{3}{2} - \frac{x}{2} + \frac{\sqrt{3}(3x - 1)}{6} \tan \left( \text{RootOf} \left( \sqrt{3} \ln \left( \frac{(3(\tan(_Z))^2 + 3)(3x - 1)^2}{4} \right) + 2\sqrt{3}_C1 + 6_Z \right) \right) \right\}$$

**2.216 ODE No. 216**

$$(y(x) - 2x + 1)y'(x) + y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.593147 (sec), leaf count = 82

$$\text{Solve} \left[ 6\sqrt{3} \tan^{-1} \left( \frac{3y(x) + 1}{\sqrt{3}(-y(x) + 2x - 1)} \right) = 2c_1 + 3 \log \left( \frac{3x^2 + 3y(x)^2 - 3(x - 1)y(x) - 3x + 1}{(1 - 3x)^2} \right) + 6 \log(3x - 1), \right.$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 51

$$\left\{ y(x) = \frac{(-3x + 1)\sqrt{3}}{6} \tan \left( \text{RootOf} \left( \sqrt{3} \ln \left( \frac{(3(\tan(_Z))^2 + 3)(3x - 1)^2}{4} \right) + 2\sqrt{3}_C1 + 6_Z \right) \right) + \frac{x}{2} - \right.$$

**2.217 ODE No. 217**

$$(y(x) - x^2)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0199568 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( W \left( -e^{c_1 - 2x^2 - 1} \right) + 1 \right) + x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 23

$$\left\{ y(x) = x^2 + \frac{\text{lambertW} \left( -4_C1 e^{-2x^2 - 1} \right)}{2} + \frac{1}{2} \right\}$$



**2.218 ODE No. 218**

$$(y(x) - x^2) y'(x) + 4xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0985292 (sec), leaf count = 232

$$\left\{ \left\{ y(x) \rightarrow x^2 \left( 1 + \frac{2 - 2i}{\frac{i\sqrt{2}}{\sqrt{x^2 \sinh\left(\frac{2c_1}{9}\right) + x^2 \cosh\left(\frac{2c_1}{9}\right) - i}} - (1 - i)} \right) \right\}, \left\{ y(x) \rightarrow x^2 \left( 1 + \frac{2 - 2i}{(-1 + i) - \frac{i\sqrt{2}}{\sqrt{x^2 \sinh\left(\frac{2c_1}{9}\right) + x^2 \cosh\left(\frac{2c_1}{9}\right)}}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.203 (sec), leaf count = 57

$$\left\{ y(x) = -\frac{C1}{2} \sqrt{-C1^2 - 4x^2} + \frac{C1^2}{2} - x^2, y(x) = \frac{C1}{2} \sqrt{-C1^2 - 4x^2} + \frac{C1^2}{2} - x^2 \right\}$$

**2.219 ODE No. 219**

$$-f_0(x) - f_1(x)y(x) - f_2(x)y(x)^2 + (g(x) + y(x))y'(x) = 0$$

✗ **Mathematica** : cpu = 300.947 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((y(x)+g(x))*diff(y(x),x)-f2(x)*y(x)^2-f1(x)*y(x)-f0(x) = 0,y(x))`

**2.220 ODE No. 220**

$$-x^3 + 2y(x)y'(x) - xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0143251 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 e^{\frac{x^2}{2}} - x^2 - 2} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 e^{\frac{x^2}{2}} - x^2 - 2} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 43

$$\left\{ y(x) = \sqrt{e^{\frac{x^2}{2}} - C1 - x^2 - 2}, y(x) = -\sqrt{e^{\frac{x^2}{2}} - C1 - x^2 - 2} \right\}$$

**2.221 ODE No. 221**

$$(2y(x) + x + 1)y'(x) - 2y(x) - x + 1 = 0$$

✓ **Mathematica** : cpu = 0.0183278 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} \left( 4W \left( -e^{c_1 + \frac{9x}{4}} - 1 \right) - 3x + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 21

$$\left\{ y(x) = -\frac{x}{2} + \frac{2}{3} \text{lambertW} \left( \frac{e^{-\frac{1}{4}} C1}{4} e^{\frac{9x}{4}} \right) + \frac{1}{6} \right\}$$

**2.222 ODE No. 222**

$$(2y(x) + x + 7)y'(x) - y(x) + 2x + 4 = 0$$

✓ **Mathematica** : cpu = 0.116907 (sec), leaf count = 65

$$\text{Solve} \left[ 5c_1 + 2 \log \left( \frac{4(x^2 + y(x)^2 + 4y(x) + 6x + 13)}{5(x+3)^2} \right) + 2 \tan^{-1} \left( \frac{y(x) - 2(x+2)}{2y(x) + x + 7} \right) + 4 \log(x+3) = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 32

$$\left\{ y(x) = -2 + (-x - 3) \tan \left( \text{RootOf} \left( -_Z + \ln \left( (\cos(_Z))^{-2} \right) + 2 \ln(x+3) + 2\_C1 \right) \right) \right\}$$

**2.223 ODE No. 223**

$$(2y(x) - x)y'(x) - y(x) - 2x = 0$$

✓ **Mathematica** : cpu = 0.0257185 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( x - \sqrt{5x^2 - 4e^{c_1}} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{5x^2 - 4e^{c_1}} + x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.169 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1}{2\_C1} \left( -C1 x - \sqrt{5\_C1^2 x^2 + 4} \right), y(x) = \frac{1}{2\_C1} \left( -C1 x + \sqrt{5\_C1^2 x^2 + 4} \right) \right\}$$

**2.224 ODE No. 224**

$$(2y(x) - 6x)y'(x) - y(x) + 3x + 2 = 0$$

✓ **Mathematica** : cpu = 0.0187114 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow 3x - \frac{2}{5} \left( W \left( -e^{c_1 + \frac{25x}{4}} - 1 \right) + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 35

$$\left\{ y(x) = \frac{1}{5} e^{-\text{lambertW} \left( -\frac{e^{-1}}{2} e^{\frac{25x}{4}} e^{-\frac{25}{4} C1} \right) + \frac{25x}{4} - 1 - \frac{25}{4} C1} + 3x - \frac{2}{5} \right\}$$

**2.225 ODE No. 225**

$$(4y(x) + 2x + 3)y'(x) - 2y(x) - x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0169784 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} \left( W \left( -e^{c_1 + 8x - 1} \right) - 4x - 5 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 20

$$\left\{ y(x) = -\frac{x}{2} + \frac{\text{lambertW} \left( e^5 (e^x)^8 - C1 \right)}{8} - \frac{5}{8} \right\}$$

**2.226 ODE No. 226**

$$(4y(x) - 2x - 3)y'(x) + 2y(x) - x - 1 = 0$$

✓ **Mathematica** : cpu = 0.017131 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} \left( -W \left( -e^{c_1 + 8x - 1} \right) + 4x + 5 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 21

$$\left\{ y(x) = \frac{x}{2} - \frac{\text{lambertW} \left( -e^5 (e^x)^8 - C1 \right)}{8} + \frac{5}{8} \right\}$$

**2.227 ODE No. 227**

$$(4y(x) - 3x - 5)y'(x) - 3y(x) + 7x + 2 = 0$$

✓ **Mathematica** : cpu = 0.012626 (sec), leaf count = 71

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} \left( -i \sqrt{-16c_1 + 19x^2 - 14x - 25} + 3x + 5 \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{4} \left( i \sqrt{-16c_1 + 19x^2 - 14x - 25} + 3x + 5 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 33

$$\left\{ y(x) = \frac{1}{76 - C1} \left( -\sqrt{4 - 6859 \left( x - \frac{7}{19} \right)^2 - C1^2 + (57x + 95) - C1} \right) \right\}$$

**2.228 ODE No. 228**

$$(4y(x) + 11x - 11)y'(x) - 25y(x) - 8x + 62 = 0$$

✓ **Mathematica** : cpu = 0.287119 (sec), leaf count = 1677

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} \left( \sqrt[3]{-6561 \cosh\left(\frac{3c_1}{4}\right)x^4 - 6561 \sinh\left(\frac{3c_1}{4}\right)x^4 + 2916 \cosh\left(\frac{3c_1}{4}\right)x^3 + 2916 \sinh\left(\frac{3c_1}{4}\right)x^3 + 162 \cosh\left(\frac{3c_1}{8}\right)x^2 - 486 \cosh\left(\frac{3c_1}{4}\right)x^2 + \dots} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.329 (sec), leaf count = 271

$$\left\{ y(x) = 1 \left( (-76x + 28) \sqrt[3]{64 - 8748(9x - 1)^2 - C1} + 108 \sqrt{43046721} \sqrt{-C1(x - 1/9)^2} \left( -\frac{32}{177147} + (x - 1/9) \right) \right) \right\}$$

**2.229 ODE No. 229**

$$(12y(x) - 5x - 8)y'(x) - 5y(x) + 2x + 3 = 0$$

✓ **Mathematica** : cpu = 0.012704 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12} \left( -i \sqrt{-16(9c_1 + 4) - x^2 - 8x + 5x + 8} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{12} \left( i \sqrt{-16(9c_1 + 4) - x^2 - 8x + 5x + 8} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{12\_C1} \left( -\sqrt{(x+4)^2 - C1^2 + 24} + (5x+8) - C1 \right) \right\}$$

**2.230 ODE No. 230**

$$ay(x)y'(x) + by(x)^2 + f(x) = 0$$

✓ **Mathematica** : cpu = 0.120341 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow -e^{-\frac{bx}{a}} \sqrt{2 \int_1^x -\frac{f(K[1])e^{\frac{2bK[1]}}{a}} dK[1] + c_1} \right\}, \left\{ y(x) \rightarrow e^{-\frac{bx}{a}} \sqrt{2 \int_1^x -\frac{f(K[1])e^{\frac{2bK[1]}}{a}} dK[1] + c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 100

$$\left\{ y(x) = \frac{1}{a} \sqrt{e^{\frac{bx}{a}} a \left( -C1 a - 2 \int \left( e^{\frac{bx}{a}} \right)^2 f(x) dx \right)} \left( e^{\frac{bx}{a}} \right)^{-1}, y(x) = -\frac{1}{a} \sqrt{e^{\frac{bx}{a}} a \left( -C1 a - 2 \int \left( e^{\frac{bx}{a}} \right)^2 f(x) dx \right)} \right\}$$

**2.231 ODE No. 231**

$$y'(x)(ay(x) + bx + c) + \alpha y(x) + \beta x + \gamma = 0$$

✗ **Mathematica** : cpu = 300.189 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.215 (sec), leaf count = 178

$$\left\{ y(x) = \frac{1}{-a\beta + b\alpha} \left( -b\gamma + \beta c + \frac{x(a\beta - b\alpha) + a\gamma - \alpha c}{2a} \left( \sqrt{4a\beta - \alpha^2 - 2b\alpha - b^2} \tan \left( \text{RootOf} \left( \sqrt{4a\beta - \alpha^2} - \right. \right. \right) \right) \right)$$

**2.232 ODE No. 232**

$$x^2 + xy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0100015 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{c_1 - \frac{x^4}{2}}}{x} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{c_1 - \frac{x^4}{2}}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 39

$$\left\{ y(x) = -\frac{1}{2x} \sqrt{-2x^4 + 4\_C1}, y(x) = \frac{1}{2x} \sqrt{-2x^4 + 4\_C1} \right\}$$

**2.233 ODE No. 233**

$$ax^3 \cos(x) + xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0229631 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow -x \sqrt{c_1 - 2a \sin(x)} \right\}, \left\{ y(x) \rightarrow x \sqrt{c_1 - 2a \sin(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 30

$$\left\{ y(x) = \sqrt{-2a \sin(x) + \_C1} x, y(x) = -\sqrt{-2a \sin(x) + \_C1} x \right\}$$

**2.234 ODE No. 234**

$$x^3 - 2x^2 + xy(x)y'(x) + xy(x) - y(x)^2 = 0$$

✗ **Mathematica** : cpu = 28.8078 (sec), leaf count = 0 , could not solve

`DSolve[-2*x^2 + x^3 + x*y[x] - y[x]^2 + x*y[x]*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*y(x)*diff(y(x),x)-y(x)^2+x*y(x)+x^3-2*x^2 = 0,y(x))`

**2.235 ODE No. 235**

$$(a + xy(x))y'(x) + by(x) = 0$$

✓ **Mathematica** : cpu = 0.0775916 (sec), leaf count = 33

$$\text{Solve} \left[ x = \frac{e^{-\frac{y(x)}{b}} \left( bc_1 - a \text{Ei} \left( \frac{y(x)}{b} \right) \right)}{b}, y(x) \right]$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 30

$$\left\{ -C1 + \left( -e^{\frac{y(x)}{b}} bx + a \text{Ei} \left( 1, -\frac{y(x)}{b} \right) \right)^{-1} = 0 \right\}$$

**2.236 ODE No. 236**

$$x(y(x) + 4)y'(x) - y(x)^2 - 2y(x) - 2x = 0$$

✓ **Mathematica** : cpu = 0.0167394 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\frac{1}{x+4} - \frac{\left(\frac{x}{x+4}\right)^{3/2}}{x\sqrt{c_1(x+4)-4}}} - 4 \right\}, \left\{ y(x) \rightarrow \frac{1}{\frac{\left(\frac{x}{x+4}\right)^{3/2}}{x\sqrt{c_1(x+4)-4}} + \frac{1}{x+4}} - 4 \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 141

$$\left\{ y(x) = 1 \left( -(x+4)^{\frac{3}{2}} \sqrt{\frac{-C1(x+4)-4}{x+4}} x - 16\sqrt{x} - 4x^{3/2} \right) \left( -(x+4)^{\frac{3}{2}} \sqrt{\frac{-C1(x+4)-4}{x+4}} + 4\sqrt{x} + x^{\frac{3}{2}} \right) \right\}$$

**2.237 ODE No. 237**

$$x(a + y(x))y'(x) + by(x) + cx = 0$$

✗ **Mathematica** : cpu = 8.02614 (sec), leaf count = 0 , could not solve

`DSolve[c*x + b*y[x] + x*(a + y[x])*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*(y(x)+a)*diff(y(x),x)+b*y(x)+c*x = 0,y(x))`

**2.238 ODE No. 238**

$$(a + x(y(x) + x))y'(x) - b - y(x)(y(x) + x) = 0$$

✓ **Mathematica** : cpu = 0.0431231 (sec), leaf count = 176

$$\left\{ \left\{ y(x) \rightarrow -\frac{-\frac{a}{a^2+ax^2+bx^2} - \frac{1}{(a^2+ax^2+bx^2)^{3/2}} \frac{x}{\sqrt{c_1 - \frac{1}{(a+b)(a^2+ax^2+bx^2)}}} + a + x^2}{x} \right\}, \left\{ y(x) \rightarrow -\frac{-\frac{1}{(a^2+ax^2+bx^2)^{3/2}} \frac{x}{\sqrt{c_1 - \frac{1}{(a+b)(a^2+ax^2+bx^2)}}}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.08 (sec), leaf count = 91

$$\left\{ y(x) = \frac{1}{-C1 a^2 - 1} \left( -C1 abx + x - \sqrt{(-1 + (ax^2 + bx^2 + a^2) - C1) (a + b)} \right), y(x) = \frac{1}{-C1 a^2 - 1} \left( -C1 abx + \dots \right) \right\}$$

**2.239 ODE No. 239**

$$(xy(x) - x^2)y'(x) - 2x^2 - 3xy(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0283072 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow x - \frac{\sqrt{e^{2c_1} + 2x^4}}{x} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{e^{2c_1} + 2x^4}}{x} + x \right\} \right\}$$

✓ **Maple** : cpu = 0.176 (sec), leaf count = 59

$$\left\{ y(x) = \frac{1}{-C1 x} \left( x^2 - C1 - \sqrt{2x^4 - C1^2 + 1} \right), y(x) = \frac{1}{-C1 x} \left( x^2 - C1 + \sqrt{2x^4 - C1^2 + 1} \right) \right\}$$

**2.240 ODE No. 240**

$$ax + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.00965267 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{x(c_1 - a \log(x))} \right\}, \left\{ y(x) \rightarrow \sqrt{x(c_1 - a \log(x))} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 34

$$\left\{ y(x) = \sqrt{-ax \ln(x) + -C1 x}, y(x) = -\sqrt{-x(a \ln(x) - -C1)} \right\}$$



**2.241 ODE No. 241**

$$ax^2 + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0097477 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{x(c_1 - ax)} \right\}, \left\{ y(x) \rightarrow \sqrt{x(c_1 - ax)} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{-ax^2 + \_C1 x}, y(x) = -\sqrt{-ax^2 + \_C1 x} \right\}$$

**2.242 ODE No. 242**

$$2xy(x)y'(x) + 2y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.0144028 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{e^{4c_1} - x^2}}{\sqrt{2x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{e^{4c_1} - x^2}}{\sqrt{2x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 39

$$\left\{ y(x) = -\frac{1}{2x} \sqrt{-2x^2 + 4\_C1}, y(x) = \frac{1}{2x} \sqrt{-2x^2 + 4\_C1} \right\}$$

**2.243 ODE No. 243**

$$x(2y(x) + x - 1)y'(x) - y(x)(y(x) + 2x + 1) = 0$$

✓ **Mathematica** : cpu = 14.9697 (sec), leaf count = 451

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{2x}}{\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2 + 27c_1^2x}}} + \frac{\sqrt[3]{-27c_1^2x^2 + \sqrt{108c_1^3x^3 + (27c_1^2x - 27c_1^2x^2)^2 + 27c_1^2x}}}{3\sqrt[3]{2c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.137 (sec), leaf count = 391

$$\left\{ y(x) = \frac{1}{80\_C1} \left( 3 \left( x \left( \sqrt{5} \sqrt{\frac{80(x-1)^2\_C1 - x}{\_C1}} + 20x - 20 \right) \_C1^2 \right)^{2/3} (i\sqrt{3} - 1) \sqrt[3]{5} - 3\_C1 \left( (1 + i\sqrt{3}) \right)^{2/3} \right)$$

**2.244 ODE No. 244**

$$x(2y(x) - x - 1)y'(x) + (-y(x) + 2x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 14.8898 (sec), leaf count = 457

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{2x}}{\sqrt[3]{27c_1^2x^2 + \sqrt{(27c_1^2x^2 + 27c_1^2x)^2 - 108c_1^3x} + 27c_1^2x}} - \frac{\sqrt[3]{27c_1^2x^2 + \sqrt{(27c_1^2x^2 + 27c_1^2x)^2 - 108c_1^3x}}}{3\sqrt[3]{2}c_1} \right. \right.$$

✓ **Maple** : cpu = 0.12 (sec), leaf count = 391

$$\left\{ y(x) = \frac{3}{80\_C1} \left( \left( x \left( \sqrt{5} \sqrt{\frac{80(1+x)^2\_C1 - x}{\_C1}} - 20x - 20 \right) - C1^2 \right)^{\frac{2}{3}} (i\sqrt{3} - 1) \sqrt[3]{5} - \left( (1 + i\sqrt{3}) x 5^{\frac{2}{3}} + \right. \right.$$

**2.245 ODE No. 245**

$$(4x^3 + 2xy(x))y'(x) + 112x^2y(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.401064 (sec), leaf count = 1453

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[ -1521681143169024\#1x^{22} - 697437190619136\#1^2x^{20} - 145299414712320\#1^3x^{18} - 18162426 \right. \right.$$

✓ **Maple** : cpu = 0.322 (sec), leaf count = 31

$$\left\{ y(x) = \frac{\_C1}{x^{28} (\text{RootOf} (x^{30} \_Z^{360} - 24 x^{30} \_Z^{330} - \_C1))^{330}} \right\}$$

**2.246 ODE No. 246**

$$x(3y(x) + 2x)y'(x) + 3(y(x) + x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0323422 (sec), leaf count = 71

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{6e^{4c_1} - 2x^4 + 4x^2}}{6x} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{6e^{4c_1} - 2x^4 - 4x^2}}{6x} \right\} \right\}$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 63

$$\left\{ y(x) = \frac{1}{6\_C1 x} \left( -4x^2\_C1 - \sqrt{-2x^4\_C1^2 + 6} \right), y(x) = \frac{1}{6\_C1 x} \left( -4x^2\_C1 + \sqrt{-2x^4\_C1^2 + 6} \right) \right\}$$

**2.247 ODE No. 247**

$$-7x^2 + (3x + 2)(y(x) - 2x - 1)y'(x) + xy(x) - y(x)^2 - 9x - 3 = 0$$

✓ **Mathematica** : cpu = 14.8155 (sec), leaf count = 586

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( \sqrt[3]{-2e^{2c_1}(3x+2) + 2\sqrt{e^{2c_1}(3x+2)^2(e^{2c_1} - (3x+2)^2)} + 27x^3 + 54x^2 + 36x + 8 + 12} \right) + (-2e^{2c_1} - 2\sqrt{e^{2c_1}(3x+2)^2(e^{2c_1} - (3x+2)^2)})}{2\sqrt[3]{-2e^{2c_1}(3x+2) + 2\sqrt{e^{2c_1}(3x+2)^2(e^{2c_1} - (3x+2)^2)}}} \right\} \right.$$

✓ **Maple** : cpu = 0.222 (sec), leaf count = 517

$$\left\{ y(x) = -\frac{1}{3} + \frac{3x+2}{6} \left( 7 \left( -1/4 \sqrt[3]{2(3x+2)C1 - 27(3x+2)^3 - C1^3} + 2\sqrt{-27(3x+2)^4 - C1^4} + (3x+2) \right) \right)$$

**2.248 ODE No. 248**

$$(x^2 + 6xy(x) + 3)y'(x) + 3y(x)^2 + 2xy(x) + 2x = 0$$

✓ **Mathematica** : cpu = 0.0141296 (sec), leaf count = 83

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{36c_1x + x^4 - 12x^3 + 6x^2 + 9} + x^2 + 3}{6x} \right\}, \left\{ y(x) \rightarrow -\frac{-\sqrt{36c_1x + x^4 - 12x^3 + 6x^2 + 9} + x^2 + 3}{6x} \right\} \right.$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 75

$$\left\{ y(x) = \frac{1}{6x} \left( -x^2 - 3 - \sqrt{x^4 - 12x^3 - 12C1x + 6x^2 + 9} \right), y(x) = \frac{1}{6x} \left( -x^2 - 3 + \sqrt{x^4 - 12x^3 - 12C1x + 6x^2 + 9} \right) \right.$$

**2.249 ODE No. 249**

$$y'(x)(axy(x) + bx^n) + \alpha y(x)^3 + \beta y(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.135 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.209 (sec), leaf count = 232

$$\left\{ y(x) = \beta \left( eval \left( RootOf \left( -x^{1-n} Z^{\frac{\alpha(n-1)}{\beta}} a^2 \beta n + C1 a^2 b n^2 + x^{1-n} Z^{\frac{\alpha(n-1)}{\beta}} a^2 \beta - x^{1-n} Z^{\frac{\alpha(n-1)}{\beta}} a \beta^2 - Z^{\alpha n} \right) \right) \right)$$

**2.250 ODE No. 250**

$$y'(x)(ax + Ax^2 + by(x) + Bxy(x) + c) + Axy(x) + \alpha x - By(x)^2 + \beta y(x) + \gamma = 0$$

✗ **Mathematica** : cpu = 301.413 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((B*x*y(x)+A*x^2+a*x+b*y(x)+c)*diff(y(x),x)-B*g(x)^2+A*x*y(x)+alpha*x+beta*y(x)+gamma`

**2.251 ODE No. 251**

$$(x^2y(x) - 1)y'(x) + xy(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0121921 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow \frac{1 - \sqrt{c_1x^2 + 2x^3 + 1}}{x^2} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{c_1x^2 + 2x^3 + 1} + 1}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1}{x^2} \left( 1 - \sqrt{-2x^2 - C1 + 2x^3 + 1} \right), y(x) = \frac{1}{x^2} \left( 1 + \sqrt{-2x^2 - C1 + 2x^3 + 1} \right) \right\}$$

**2.252 ODE No. 252**

$$(x^2y(x) - 1)y'(x) - xy(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 14.742 (sec), leaf count = 738

$$\left\{ \left\{ y(x) \rightarrow \frac{(1 - 6c_1)x^2 + (6c_1 - 1)x \sqrt{-(1 - 6c_1)^2x^3 + \sqrt{(6c_1 - 1)^3(6c_1x^6 + (2 - 12c_1)x^3 + 6c_1 - 1) + 36c_1^2}}}{(6c_1 - 1) \sqrt[3]{-(1 - 6c_1)^2x^3 + \sqrt{(6c_1 - 1)^3(6c_1x^6 + (2 - 12c_1)x^3 + 6c_1 - 1) + 36c_1^2}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.873 (sec), leaf count = 1338

$$\left\{ y(x) = 1 \left( ((-C1 + 80)x^7 - 160x^4 + 80x) \sqrt[3]{4} \sqrt[3]{(-80 + (-C1 - 80)x^6 + 160x^3)^2 - C1} \left( -\frac{1}{4} + \sqrt{\frac{-}{-80 + (}} \right) \right) \right\}$$

**2.253 ODE No. 253**

$$(x^2y(x) - 1)y'(x) + 8xy(x)^2 - 8 = 0$$

✗ **Mathematica** : cpu = 18.7908 (sec), leaf count = 0 , could not solve

`DSolve[-8 + 8*x*y[x]^2 + (-1 + x^2*y[x])*Derivative[1][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((x^2*y(x)-1)*diff(y(x),x)+8*x*y(x)^2-8 = 0,y(x))`

**2.254 ODE No. 254**

$$x^2y(x)^3 + x(xy(x) - 2)y'(x) + xy(x)^2 - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0160879 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow \frac{2}{\sqrt{-\frac{1}{x^3}x^2\sqrt{-x(4c_1 - 4\log(x) + 1)} + x}} \right\}, \left\{ y(x) \rightarrow \frac{2}{\left(-\frac{1}{x^3}\right)^{3/2}x^5\sqrt{-x(4c_1 - 4\log(x) + 1)} + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 59

$$\left\{ y(x) = \frac{1}{(-2\ln(x) + 2\_C1)x} \left(-1 + \sqrt{1 - 4\ln(x) + 4\_C1}\right), y(x) = \frac{1}{(2\ln(x) - 2\_C1)x} \left(1 + \sqrt{1 - 4\ln(x)}\right) \right\}$$

**2.255 ODE No. 255**

$$x(xy(x) - 3)y'(x) + xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 4.80664 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow -\frac{3W\left(e^{\frac{9c_1}{2^{2/3}}-1}x^{2/3}\right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.226 (sec), leaf count = 74

$$\left\{ y(x) = -3\frac{\text{lambertW}\left(2/3\sqrt[3]{-1/8x^2-C1}\right)}{x}, y(x) = -3\frac{\text{lambertW}\left(-1/3\sqrt[3]{-1/8x^2-C1}(1+i\sqrt{3})\right)}{x}, y(x) = -3\frac{\text{lambertW}\left(-1/3\sqrt[3]{-1/8x^2-C1}(1-i\sqrt{3})\right)}{x} \right\}$$

**2.256 ODE No. 256**

$$x^2(y(x) - 1)y'(x) + (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0191802 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow -W\left(x\left(-e^{\frac{1}{x}-c_1}\right)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 31

$$\left\{ y(x) = e^{\frac{-C1 x + x \ln(x) - \text{lambertW}\left(-x e^{-C1 + x^{-1}}\right)_{x+1}}{x}} \right\}$$

**2.257 ODE No. 257**

$$x(x^4 + xy(x) - 1)y'(x) - y(x)(-x^4 + xy(x) - 1) = 0$$

✓ **Mathematica** : cpu = 0.404905 (sec), leaf count = 38

$$\text{Solve}\left[\frac{c_1 + 2xy(x) - 2 \log\left(\frac{1}{1-xy(x)}\right) - 2}{x^2y(x)^2} + \frac{1}{x^4} = 0, y(x)\right]$$

✓ **Maple** : cpu = 0.113 (sec), leaf count = 98

$$\left\{ y(x) = \frac{-C1 + e^{\text{RootOf}\left(-2_Z x^4 (e^{-Z})^2 + 2x^4 (e^{-Z})^2 - 2e^{-Z} - C1 x^4 + (e^{-Z})^2 - 2e^{-Z} - C1 + C1^2\right)}}{x e^{\text{RootOf}\left(-2_Z x^4 (e^{-Z})^2 + 2x^4 (e^{-Z})^2 - 2e^{-Z} - C1 x^4 + (e^{-Z})^2 - 2e^{-Z} - C1 + C1^2\right)}} \right\}$$

**2.258 ODE No. 258**

$$-2x^3 + 2x^2y(x)y'(x) - x^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0131559 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 e^{\frac{1}{x}} + x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 e^{\frac{1}{x}} + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{e^{x^{-1}} - C1 + x^2}, y(x) = -\sqrt{e^{x^{-1}} - C1 + x^2} \right\}$$

**2.259 ODE No. 259**

$$2x^2y(x)y'(x) - e^{x-\frac{1}{x}}x^2 - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0188996 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow -e^{-\frac{1}{2}/x} \sqrt{c_1 + e^x} \right\}, \left\{ y(x) \rightarrow e^{-\frac{1}{2}/x} \sqrt{c_1 + e^x} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 51

$$\left\{ y(x) = \sqrt{e^{-x^{-1}} C1 + e^{\frac{x^2-1}{x}}}, y(x) = -\sqrt{e^{-x^{-1}} C1 + e^{\frac{x^2-1}{x}}} \right\}$$

**2.260 ODE No. 260**

$$(2x^2y(x) + x) y'(x) - x^2y(x)^3 + 2xy(x)^2 + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0149123 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{\frac{\sqrt{x(c_1-2\log(x)+4)}}{\sqrt{\frac{1}{x^3}}} - 2x^2} \right\}, \left\{ y(x) \rightarrow -\frac{x}{\frac{\sqrt{x(c_1-2\log(x)+4)}}{\sqrt{\frac{1}{x^3}}} + 2x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 59

$$\left\{ y(x) = \frac{1}{(2 \ln(x) - 2 C1) x} \left( -2 + \sqrt{4 - 2 \ln(x) + 2 C1} \right), y(x) = \frac{1}{(-2 \ln(x) + 2 C1) x} \left( 2 + \sqrt{4 - 2 \ln(x)} \right) \right\}$$

**2.261 ODE No. 261**

$$(2x^2y(x) - x) y'(x) - 2xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.893384 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2xW\left(\frac{e^{\frac{9c_1}{2^{2/3}}-1}}{x^2}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.156 (sec), leaf count = 18

$$\left\{ y(x) = -\frac{1}{2x} \left( \text{lambertW}\left(-\frac{C1}{2x^2}\right) \right)^{-1} \right\}$$

**2.262 ODE No. 262**

$$2x^3 + (2x^2y(x) - x^3) y'(x) - 4xy(x)^2 + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0651598 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow \frac{2x^3 - \sqrt{e^{2c_1}x^2(e^{2c_1} - 3x^2)}}{e^{2c_1} + x^2} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{e^{2c_1}x^2(e^{2c_1} - 3x^2)} + 2x^3}{e^{2c_1} + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.317 (sec), leaf count = 65

$$\left\{ y(x) = \frac{x}{-C1 x^2 - 1} \left( 2 - C1 x^2 - \sqrt{3 - C1 x^2 + 1} \right), y(x) = \frac{x}{-C1 x^2 - 1} \left( 2 - C1 x^2 + \sqrt{3 - C1 x^2 + 1} \right) \right\}$$

**2.263 ODE No. 263**

$$2x^3 + 3x^2y(x)^2 + y(x)y'(x) + 7 = 0$$

✓ **Mathematica** : cpu = 0.0394964 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{3} \sqrt{9c_1 e^{-2x^3} + \frac{20 \cdot 2^{2/3} e^{-2x^3} x \Gamma\left(\frac{1}{3}, -2x^3\right)}{\sqrt[3]{-x^3}} - 6x} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 e^{-2x^3} + \frac{20 \cdot 2^{2/3} e^{-2x^3} x \Gamma\left(\frac{1}{3}, -2x^3\right)}{9 \sqrt[3]{-x^3}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 173

$$\left\{ y(x) = -\frac{2^{\frac{2}{3}}}{18 \Gamma(2/3)} \sqrt{-240 \Gamma(2/3) \left( \frac{\left( -\frac{27 e^{-2x^3} - C1}{2} + 9x \right) \Gamma(2/3) \sqrt[3]{2} \sqrt[3]{-x^3}}{40} + e^{-2x^3} x \left( \pi \sqrt{3} - 3/2 \Gamma(1/3, -2x^3) \right) \right)} \right\}$$

**2.264 ODE No. 264**

$$2x(x^3y(x) + 1) y'(x) + y(x) (3x^3y(x) - 1) = 0$$

✓ **Mathematica** : cpu = 0.36064 (sec), leaf count = 680

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[ 81 \#1^7 e^{\frac{21c_1}{2}} x^{12} + 756 \#1^6 e^{\frac{21c_1}{2}} x^9 + 2646 \#1^5 e^{\frac{21c_1}{2}} x^6 + 4116 \#1^4 e^{\frac{21c_1}{2}} x^3 + 2401 \#1^3 e^{\frac{21c_1}{2}} - x^{3/2} \& \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.531 (sec), leaf count = 574

$$\left\{ y(x) = \frac{-40353607 \left( \text{RootOf} \left( 9x^7 - Z^{98} - 49 - C1 - Z^{42} + 14 - C1 - Z^{21} - C1 \right) \right)^{91} - C1 + 756315 \left( \text{RootOf} \left( 9x^7 - Z^{98} - 49 - C1 - Z^{42} + 14 - C1 - Z^{21} - C1 \right) \right)^7 \left( 5764801 - C1 \left( \text{RootOf} \left( 9x^7 - Z^{98} - 49 - C1 - Z^{42} + 14 - C1 - Z^{21} - C1 \right) \right)^7 \right)}{3x^3 \left( \text{RootOf} \left( 9x^7 - Z^{98} - 49 - C1 - Z^{42} + 14 - C1 - Z^{21} - C1 \right) \right)^7 \left( 5764801 - C1 \left( \text{RootOf} \left( 9x^7 - Z^{98} - 49 - C1 - Z^{42} + 14 - C1 - Z^{21} - C1 \right) \right)^7 \right)}$$



2.265 ODE No. 265

$$2(n+1)^2 x^{n-1} (x^{n^2} y(x)^2 - 1) + (x^{n(n+1)} y(x) - 1) y'(x) = 0$$

✗ **Mathematica** : cpu = 300.029 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((x^(n\*(n+1))\*y(x)-1)\*diff(y(x),x)+2\*(n+1)^2\*x^(n-1)\*(x^(n^2)\*y(x)^2-1) = 0,y(x))

2.266 ODE No. 266

$$\sqrt{x^2 + 1}(y(x) - x)y'(x) - a\sqrt{(y(x)^2 + 1)^3} = 0$$

✗ **Mathematica** : cpu = 300.043 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.716 (sec), leaf count = 55

$$\left\{ y(x) = \tan \left( \text{RootOf} \left( -\arctan(x) + \int^{-\arctan(x)+_Z} -\frac{1}{2a^2 + \cos(2\_a) - 1} \left( \cos(2\_a) - 1 + \sqrt{-2a^2(\cos(2\_a) - 1)} \right) \right) \right) \right\}$$

2.267 ODE No. 267

$$y(x) \sin^2(x)y'(x) + y(x)^2 \sin(x) \cos(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0407511 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{c_1 + 2x} \csc(x) \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 + 2x} \csc(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{\sin(x)} \sqrt{2x + \_C1}, y(x) = -\frac{1}{\sin(x)} \sqrt{2x + \_C1} \right\}$$

**2.268 ODE No. 268**

$$f(x)y(x)y'(x) + g(x)y(x)^2 + h(x) = 0$$

✓ **Mathematica** : cpu = 0.988086 (sec), leaf count = 140

$$\left\{ \left\{ y(x) \rightarrow -e^{\int_1^x -\frac{g(K[1])}{f(K[1])} dK[1]} \sqrt{2 \int_1^x -\frac{h(K[2]) \exp\left(-2 \int_1^{K[2]} -\frac{g(K[1])}{f(K[1])} dK[1]\right)}{f(K[2])} dK[2] + c_1} \right\}, \left\{ y(x) \rightarrow e^{\int_1^x -\frac{g(K[1])}{f(K[1])} dK[1]} \right\} \right.$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 118

$$\left\{ y(x) = 1 \sqrt{e^{2 \int \frac{g(x)}{f(x)} dx} \left( -2 \int \frac{h(x)}{f(x)} \left( e^{\int \frac{g(x)}{f(x)} dx} \right)^2 dx + _C1 \right) \left( e^{2 \int \frac{g(x)}{f(x)} dx} \right)^{-1}}, y(x) = -1 \sqrt{e^{2 \int \frac{g(x)}{f(x)} dx} \left( -2 \int \frac{h(x)}{f(x)} \left( e^{\int \frac{g(x)}{f(x)} dx} \right)^2 dx + _C1 \right) \left( e^{2 \int \frac{g(x)}{f(x)} dx} \right)^{-1}} \right.$$

**2.269 ODE No. 269**

$$-f_0(x) - f_1(x)y(x) - f_2(x)y(x)^2 - f_3(x)y(x)^3 + y'(x)(g_0(x) + g_1(x)y(x)) = 0$$

✗ **Mathematica** : cpu = 399.683 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((g1(x)*y(x)+g0(x))*diff(y(x),x)-f1(x)*y(x)-f2(x)*y(x)^2-f3(x)*y(x)^3-f0(x) = 0,y(x))`

**2.270 ODE No. 270**

$$x^2 + (y(x)^2 - x) y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 11.1423 (sec), leaf count = 326

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{2} \left( \sqrt{(6c_1 - 4)x^3 + 9c_1^2 + x^6 + 3c_1 + x^3} \right)^{2/3} + 2x}{2^{2/3} \sqrt[3]{\sqrt{(6c_1 - 4)x^3 + 9c_1^2 + x^6 + 3c_1 + x^3}}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt[3]{2}(1 - i\sqrt{3}) \left( \sqrt{(6c_1 - 4)x^3 + 9c_1^2 + x^6 + 3c_1 + x^3} \right)^{2/3} + 2x}{2^{2/3} \sqrt[3]{\sqrt{(6c_1 - 4)x^3 + 9c_1^2 + x^6 + 3c_1 + x^3}}} \right\} \right.$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 319

$$\left\{ y(x) = \frac{1}{2} \left( \left( -4x^3 - 12\_C1 + 4 \sqrt{x^6 + (6\_C1 - 4)x^3 + 9\_C1^2} \right)^{2/3} + 4x \right) \frac{1}{\sqrt[3]{-4x^3 - 12\_C1 + 4 \sqrt{x^6 + (6\_C1 - 4)x^3 + 9\_C1^2}}} \right.$$

**2.271 ODE No. 271**

$$(x^2 + y(x)^2) y'(x) + 2x(y(x) + 2x) = 0$$

✓ **Mathematica** : cpu = 4.28525 (sec), leaf count = 372

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{\sqrt{-8e^{3c_1}x^3 + e^{6c_1} + 20x^6 + e^{3c_1} - 4x^3}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}x^2}{\sqrt[3]{\sqrt{-8e^{3c_1}x^3 + e^{6c_1} + 20x^6 + e^{3c_1} - 4x^3}}} \right\}, \left\{ y(x) \rightarrow \dots \right. \right.$$

✓ **Maple** : cpu = 0.185 (sec), leaf count = 352

$$\left\{ y(x) = 1 \left( \frac{1}{2} \sqrt[3]{4 - 16x^3 - C1^{3/2} + 4\sqrt{20 - C1^3x^6 - 8x^3 - C1^{3/2} + 1}} - 2 \frac{-C1x^2}{\sqrt[3]{4 - 16x^3 - C1^{3/2} + 4\sqrt{20 - C1^3x^6 - 8x^3 - C1^{3/2} + 1}}} \right) \right.$$

**2.272 ODE No. 272**

$$(x^2 + y(x)^2) y'(x) - y(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.151 (sec), leaf count = 43

$$\left\{ y(x) = e^{\frac{2\sqrt{3}}{3} \text{RootOf}(-\sqrt{3}xe^{-C1} + 3 \tan(_Z)xe^{-C1} + 2\sqrt{3}e^{2/3\sqrt{3}-Z}) - C1} \right\}$$

**2.273 ODE No. 273**

$$(a + x^2 + y(x)^2) y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0265402 (sec), leaf count = 294

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{2} \left( \sqrt{4(a+x^2)^3 + 9c_1^2 + 3c_1} \right)^{2/3} - 2a - 2x^2}{2^{2/3} \sqrt[3]{\sqrt{4(a+x^2)^3 + 9c_1^2 + 3c_1}}} \right\}, \left\{ y(x) \rightarrow \frac{(1+i\sqrt{3})(a+x^2)}{2^{2/3} \sqrt[3]{\sqrt{4(a+x^2)^3 + 9c_1^2 + 3c_1}}} + \frac{i(\sqrt{3} \dots)}{\dots} \right. \right.$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 401

$$\left\{ y(x) = \frac{1}{4} \left( \left( i \left( -12 - C1 + 4 \sqrt{4x^6 + 12ax^4 + 12a^2x^2 + 4a^3 + 9 - C1^2} \right)^{\frac{2}{3}} + 4ix^2 + 4ia \right) \sqrt{3} - (-12 - C1 + 4 \dots) \right)$$

**2.274 ODE No. 274**

$$(a + x^2 + y(x)^2) y'(x) + b + x^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0359814 (sec), leaf count = 396

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{2} \left( \sqrt{4(a+x^2)^3 + (3bx - 3c_1 + x^3)^2} - 3bx + 3c_1 - x^3 \right)^{2/3} - 2a - 2x^2}{2^{2/3} \sqrt[3]{\sqrt{4(a+x^2)^3 + (3bx - 3c_1 + x^3)^2} - 3bx + 3c_1 - x^3}} \right\}, \left\{ y(x) \rightarrow \frac{1}{2^{2/3} \sqrt[3]{\sqrt{4(a+x^2)^3 + (3bx - 3c_1 + x^3)^2} - 3bx + 3c_1 - x^3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 657

$$\left\{ y(x) = \frac{1}{2} \left( \left( -4x^3 - 12bx - 12\_C1 + 4\sqrt{5x^6 + (12a + 6b)x^4 + 6x^3\_C1 + (12a^2 + 9b^2)x^2 + 18bx\_C1 + 4\_C1^2} \right)^{1/3} - 2a - 2x^2 \right) \right\}$$

**2.275 ODE No. 275**

$$(x^2 + y(x)^2 + x) y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.365142 (sec), leaf count = 16

$$\text{Solve} \left[ c_1 + \tan^{-1} \left( \frac{x}{y(x)} \right) = y(x), y(x) \right]$$

✓ **Maple** : cpu = 0.104 (sec), leaf count = 30

$$\left\{ -C1 + \frac{(ix + y(x)) e^{-2iy(x)}}{2iy(x) + 2x} = 0 \right\}$$

**2.276 ODE No. 276**

$$(y(x)^2 - x^2) y'(x) + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0374162 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( e^{c_1} - \sqrt{e^{2c_1} - 4x^2} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{e^{2c_1} - 4x^2} + e^{c_1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.064 (sec), leaf count = 47

$$\left\{ y(x) = \frac{1}{2\_C1} \left( 1 + \sqrt{-4\_C1^2 x^2 + 1} \right), y(x) = \frac{1}{2\_C1} \left( 1 - \sqrt{-4\_C1^2 x^2 + 1} \right) \right\}$$

**2.277 ODE No. 277**

$$(x^4 + y(x)^2) y'(x) - 4x^3 y(x) = 0$$

✓ **Mathematica** : cpu = 0.0153401 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( c_1 - \sqrt{c_1^2 + 4x^4} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{c_1^2 + 4x^4} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.35 (sec), leaf count = 41

$$\left\{ y(x) = -\frac{1}{2} \sqrt{4x^4 + \_C1^2} + \frac{\_C1}{2}, y(x) = \frac{1}{2} \sqrt{4x^4 + \_C1^2} + \frac{\_C1}{2} \right\}$$

**2.278 ODE No. 278**

$$y'(x) (y(x)^2 + 4 \sin(x)) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.109515 (sec), leaf count = 32

$$\text{Solve} \left[ -\frac{1}{32} e^{-4y(x)} (8y(x)^2 + 4y(x) + 32 \sin(x) + 1) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 28

$$\left\{ \frac{\left( -8(y(x))^2 - 4y(x) - 32 \sin(x) - 1 \right) e^{-4y(x)}}{32} + \_C1 = 0 \right\}$$

**2.279 ODE No. 279**

$$(y(x)^2 + 2y(x) + x) y'(x) + y(x)^2 (y(x) + x)^2 + y(x)(y(x) + 1) = 0$$

✓ **Mathematica** : cpu = 0.722104 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{(c_1 x - x^2 + 1)^2 + 4(x - c_1)} - c_1 x + x^2 - 1}{2(x - c_1)} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{(c_1 x - x^2 + 1)^2 + 4(x - c_1)} + c_1 x - 1}{2(x - c_1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 116

$$\left\{ y(x) = \frac{1}{-2\_C1 + 4x} \left( -2x^2 + \_C1 x + \sqrt{4x^4 - 4x^3 \_C1 + (\_C1^2 - 8)x^2 + (4\_C1 + 16)x - 8\_C1 + 4} \right) \right\}$$

**2.280 ODE No. 280**

$$(y(x) + x)^2 y'(x) - a^2 = 0$$

✓ **Mathematica** : cpu = 107.166 (sec), leaf count = 20

$$\text{Solve} \left[ a \tan^{-1} \left( \frac{y(x) + x}{a} \right) + c_1 = y(x), y(x) \right]$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 24

$$\{y(x) = a \text{RootOf}(\tan(\_Z) a - \_Z a + \_C1 - x) - \_C1\}$$

**2.281 ODE No. 281**

$$(-x^2 + 2xy(x) + y(x)^2) y'(x) + x^2 + 2xy(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0726545 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( e^{c_1} - \sqrt{4e^{c_1} x + e^{2c_1} - 4x^2} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{4e^{c_1} x + e^{2c_1} - 4x^2} + e^{c_1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 55

$$\left\{ y(x) = \frac{1}{2\_C1} \left( 1 + \sqrt{-4\_C1^2 x^2 + 4\_C1 x + 1} \right), y(x) = \frac{1}{2\_C1} \left( 1 - \sqrt{-4\_C1^2 x^2 + 4\_C1 x + 1} \right) \right\}$$

**2.282 ODE No. 282**

$$(y(x) + 3x - 1)^2 y'(x) - (2y(x) - 1)(4y(x) + 6x - 3) = 0$$

✓ **Mathematica** : cpu = 0.157449 (sec), leaf count = 1089

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} \left( 12x + 4e^{c_1} - \sqrt{36x^2 - 12x + 16e^{2c_1} + 16e^{c_1}(6x - 1) + 3 \cdot 2^{2/3} \sqrt[3]{-e^{c_1}(6x - 1)^4 (6x + e^{c_1} - 1)} + 1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.235 (sec), leaf count = 71

$$\left\{ -\ln \left( \frac{-6y(x) + 4 - 6x}{6x - 1} \right) + 3 \ln \left( \frac{-6y(x) + 3}{6x - 1} \right) - 3 \ln \left( \frac{-6y(x) + 18x}{6x - 1} \right) - \ln(6x - 1) - \_C1 = 0 \right\}$$

**2.283 ODE No. 283**

$$3(y(x)^2 - x^2)y'(x) + 2y(x)^3 - 6x(x+1)y(x) - 3e^x = 0$$

✓ **Mathematica** : cpu = 0.0597815 (sec), leaf count = 501

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{-2x} \sqrt[3]{\sqrt{e^{8x}(-2c_1 e^{3x} + c_1^2 - 4e^{4x}x^6 + e^{6x})} + c_1 e^{4x} - e^{7x}}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2} e^{2x} x^2}{\sqrt[3]{\sqrt{e^{8x}(-2c_1 e^{3x} + c_1^2 - 4e^{4x}x^6 + e^{6x})}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 407

$$\left\{ y(x) = \frac{1}{2e^{2x}} \left( 4x^2 (e^{2x})^2 + \left( \left( 4e^{3x} - 4\_C1 + 4\sqrt{-4x^6 (e^{2x})^2 + (e^{3x})^2 - 2e^{3x}\_C1 + \_C1^2} \right) (e^{2x})^2 \right)^{\frac{2}{3}} \right) \right\}$$

**2.284 ODE No. 284**

$$(x^2 + 4y(x)^2)y'(x) - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0517688 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow -\frac{x}{2\sqrt{W\left(\frac{1}{4}e^{-\frac{c_1}{2}}x^2\right)}} \right\}, \left\{ y(x) \rightarrow \frac{x}{2\sqrt{W\left(\frac{1}{4}e^{-\frac{c_1}{2}}x^2\right)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 21

$$\left\{ y(x) = e^{\frac{1}{2} \text{lambertW}\left(\frac{(e^{-C1})^2 x^2}{4}\right) - C1} \right\}$$

**2.285 ODE No. 285**

$$(3x^2 + 2xy(x) + 4y(x)^2) y'(x) + 2x^2 + 6xy(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0353497 (sec), leaf count = 382

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} \left( \sqrt[3]{2\sqrt{4e^{3c_1}x^3 + 16e^{6c_1} + 333x^6 + 8e^{3c_1} + x^3}} - \frac{11x^2}{\sqrt[3]{2\sqrt{4e^{3c_1}x^3 + 16e^{6c_1} + 333x^6 + 8e^{3c_1} + x^3}}} - x \right) \right. \right.$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 432

$$\left\{ y(x) = \frac{1}{-C1} \left( \frac{1}{4} \sqrt[3]{x^3 - C1^3 + 8 + 2\sqrt{333 - C1^6 x^6 + 4x^3 - C1^3 + 16}} - \frac{11 - C1^2 x^2}{4} \sqrt[3]{x^3 - C1^3 + 8 + 2\sqrt{333 - C1^6 x^6 + 4x^3 - C1^3 + 16}} \right) \right.$$

**2.286 ODE No. 286**

$$(2y(x) - 3x + 1)^2 y'(x) - (3y(x) - 2x - 4)^2 = 0$$

✓ **Mathematica** : cpu = 0.193603 (sec), leaf count = 3501

✓ **Maple** : cpu = 1.803 (sec), leaf count = 1337

$$\left\{ y(x) = \frac{(5x + 3) (\text{RootOf}((115330078125 - C1 x^9 - 2283535546875 - C1 x^8 + 20095112812500 - C1 x^7 - 1031))}{5 (\text{RootOf}((115330078125 - C1 x^9 - 2283535546875 - C1 x^8 + 20095112812500 - C1 x^7 - 1031))} \right.$$

**2.287 ODE No. 287**

$$(2y(x) - 4x + 1)^2 y'(x) - (y(x) - 2x)^2 = 0$$

✓ **Mathematica** : cpu = 3.27605 (sec), leaf count = 69

Solve  $\left[ \frac{1}{196} (112y(x) + (9\sqrt{2} - 8) \log(-7y(x) + 14x + \sqrt{2} - 4) - (8 + 9\sqrt{2}) \log(7y(x) - 14x + \sqrt{2} + 4) - 28 \right]$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 56

$$\left\{ -\frac{x}{7} + \frac{4y(x)}{7} - \frac{2 \ln(7(y(x) - 2x)^2 + 8y(x) - 16x + 2)}{49} - \frac{9\sqrt{2}}{98} \text{Artanh}\left(\frac{(7y(x) - 14x + 4)\sqrt{2}}{2}\right) - C1 = \right.$$



**2.288 ODE No. 288**

$$(-3x^2y(x) + 6y(x)^2 + 1)y'(x) - 3xy(x)^2 + x = 0$$

✓ **Mathematica** : cpu = 0.0260503 (sec), leaf count = 518

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36} \left( -3\sqrt[3]{3}\sqrt[3]{4\sqrt{3}\sqrt{-54c_1x^6 + 648c_1x^2 + 432c_1^2 - 27x^8 + 207x^4 + 32} + 144c_1 - 9x^6 + 108x^2} - \sqrt[3]{4\sqrt{3}\sqrt{-54c_1x^6 + 648c_1x^2 + 432c_1^2 - 27x^8 + 207x^4 + 32}} \right) \right. \right.$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 579

$$\left\{ y(x) = -\frac{1}{24} \left( -6x^2\sqrt[3]{-324x^2 - 432\_C1 + 27x^6 + 12\sqrt{-81x^8 - 162\_C1x^6 + 621x^4 + 1944\_C1x^2 + 1296}} \right) \right.$$

**2.289 ODE No. 289**

$$a + (6y(x) - x)^2y'(x) - 6y(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0218595 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} \left( \sqrt[3]{-18ax + 18c_1 - x^3} + x \right) \right\}, \left\{ y(x) \rightarrow \frac{x}{6} + \frac{1}{12}i(\sqrt{3} + i)\sqrt[3]{-18ax + 18c_1 - x^3} \right\}, \left\{ y(x) \rightarrow \frac{x}{6} - \frac{1}{12}i(\sqrt{3} + i)\sqrt[3]{-18ax + 18c_1 - x^3} \right\} \right.$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 115

$$\left\{ y(x) = \frac{1}{6}\sqrt[3]{-x^3 - 18ax - 18\_C1} + \frac{x}{6}, y(x) = -\frac{1}{12}\sqrt[3]{-x^3 - 18ax - 18\_C1} - \frac{i}{12}\sqrt{3}\sqrt[3]{-x^3 - 18ax - 18\_C1} \right.$$

**2.290 ODE No. 290**

$$y'(x)(ay(x)^2 + 2bxy(x) + cx^2) + by(x)^2 + 2cxy(x) + dx^2 = 0$$

✓ **Mathematica** : cpu = 0.0915474 (sec), leaf count = 744

$$\left\{ \left\{ y(x) \rightarrow \frac{2^{2/3}\sqrt[3]{\sqrt{(a^2(e^{3c_1} - dx^3) + 3abcx^3 - 2b^3x^3)^2 - 4x^6(b^2 - ac)^3} + a^2e^{3c_1} - a^2dx^3 + 3abcx^3 - 2b^3x^3}}{2a} \right. \right.$$

✓ **Maple** : cpu = 0.203 (sec), leaf count = 1388

$$\left\{ y(x) = \frac{1}{-C1} \left( \frac{1}{2a} \sqrt[3]{-4\_C1^3 a^2 dx^3 + 12 cx^3\_C1^3 ba - 8 b^3 x^3\_C1^3 + 4 \sqrt{-C1^6 a^2 d^2 x^6 - 6\_C1^6 abcdx^6 + 4\_C1^6 a^2 d^2 x^6}} \right) \right.$$

**2.291 ODE No. 291**

$$y'(x) (b(\alpha x + \beta y(x))^2 - \beta(ax + by(x))) - \alpha(ax + by(x)) + a(\alpha x + \beta y(x))^2 = 0$$

✓ **Mathematica** : cpu = 0.798025 (sec), leaf count = 39

$$\text{Solve} \left[ \frac{a\beta \left( \log(ax + by(x)) + \frac{1}{\alpha x + \beta y(x)} \right)}{a\beta - \alpha b} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.17 (sec), leaf count = 50

$$\left\{ y(x) = \frac{-ax + e^{\text{RootOf}(-C1 a\beta x - C1 \alpha bx - Z a\beta x + Z \alpha bx - C1 \beta e^{-Z} + e^{-Z} Z \beta + b)}}{b} \right\}$$

**2.292 ODE No. 292**

$$y'(x)(ay(x) + bx + c)^2 + (\alpha y(x) + \beta x + \gamma)^2 = 0$$

✓ **Mathematica** : cpu = 58.3004 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.052 (sec), leaf count = 115

$$\left\{ y(x) = \frac{1}{a\beta - b\alpha} \left( ((bx + c)\alpha - a(\beta x + \gamma)) \text{RootOf} \left( \int^{-Z} \frac{(-a a - b)^2}{-a^3 a^2 - 2\_a^2 ab -\_a^2 \alpha^2 + 2\_a \alpha \beta +\_a b^2 - \beta^2} d \right) \right) \right.$$

**2.293 ODE No. 293**

$$x(y(x)^2 - 3x) y'(x) + 2y(x)^3 - 5xy(x) = 0$$

✓ **Mathematica** : cpu = 0.102911 (sec), leaf count = 661

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[ -\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}} \&, 1 \right] \right\}, \left\{ y(x) \rightarrow \text{Root} \left[ -\#1^{15} - \frac{25\#1^2 e^{\frac{65c_1}{2}}}{x^{26}} + \frac{65e^{\frac{65c_1}{2}}}{x^{25}} \&, 2 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.373 (sec), leaf count = 35

$$\left\{ \ln(x) - C1 - \frac{2}{65} \ln \left( \frac{5(y(x))^2 - 13x}{x} \right) + \frac{6}{13} \ln \left( y(x) \frac{1}{\sqrt{x}} \right) = 0 \right\}$$

**2.294 ODE No. 294**

$$x(-a + x^2 + y(x)^2) y'(x) - y(x) (a + x^2 + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.0325358 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( c_1 x - \sqrt{(c_1^2 + 4) x^2 - 4a} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{(c_1^2 + 4) x^2 - 4a} + c_1 x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 112

$$\left\{ \left( (y(x))^{-2} - (-x^2 + a)^{-1} \right)^{-1} = -x \sqrt{x^2 - a} \frac{1}{\sqrt{-C1 + 4 \frac{a}{x^2 - a}}} + \frac{x^2}{2} - \frac{a}{2}, \left( (y(x))^{-2} - (-x^2 + a)^{-1} \right)^{-1} = x \sqrt{x^2 - a} \right\}$$

**2.295 ODE No. 295**

$$x(-x^2 + xy(x) + y(x)^2) y'(x) + x^2 y(x) - y(x)^3 + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.098372 (sec), leaf count = 30

$$\text{Solve} \left[ c_1 = \frac{x}{y(x)} + \frac{y(x)}{x} + \log \left( \frac{y(x)}{x} \right) + 2 \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.234 (sec), leaf count = 29

$$\left\{ y(x) = e^{\text{RootOf} \left( (e^{-Z})^2 + 2e^{-Z} \ln(x) + 2e^{-Z} C1 + Z e^{-Z} + 1 \right)} x \right\}$$

**2.296 ODE No. 296**

$$x^4 + x(x^2y(x) + x^2 + y(x)^2)y'(x) - 2x^2y(x)^2 - 2y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.57085 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow -e^{-c_1} \left( \sqrt{x^2(-e^{c_1}x^2 + e^{2c_1} + x^2)} + x^2 \right) \right\}, \left\{ y(x) \rightarrow e^{-c_1} \left( \sqrt{x^2(-e^{c_1}x^2 + e^{2c_1} + x^2)} - x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.779 (sec), leaf count = 135

$$\left\{ y(x) = -x \left( -x^3 + \_C1 x + x^2 + \sqrt{-\_C1 x^4 + \_C1^2 x^2 + x^4} \right) \left( \_C1 x - x^2 + \sqrt{-\_C1 x^4 + \_C1^2 x^2 + x^4} \right) \right\}$$

**2.297 ODE No. 297**

$$2x(5x^2 + y(x)^2)y'(x) - x^2y(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0576747 (sec), leaf count = 216

$$\left\{ \left\{ y(x) \rightarrow \text{Root} \left[ -\#1^5 + \frac{\#1^2 e^{3c_1}}{x^{3/2}} + 3e^{3c_1} \sqrt{x} \&, 1 \right] \right\}, \left\{ y(x) \rightarrow \text{Root} \left[ -\#1^5 + \frac{\#1^2 e^{3c_1}}{x^{3/2}} + 3e^{3c_1} \sqrt{x} \&, 2 \right] \right\}, \left\{ y(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.332 (sec), leaf count = 29

$$\left\{ y(x) = \left( \text{RootOf}(x^9 \_C1 \_Z^{45} - \_Z^{18} - 6 \_Z^9 - 9) \right)^{\frac{9}{2}} x \right\}$$

**2.298 ODE No. 298**

$$3xy(x)^2y'(x) + y(x)^3 - 2x = 0$$

✓ **Mathematica** : cpu = 0.00956467 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{c_1 + x^2}}{\sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1} \sqrt[3]{c_1 + x^2}}{\sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow \frac{(-1)^{2/3} \sqrt[3]{c_1 + x^2}}{\sqrt[3]{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 73

$$\left\{ y(x) = \frac{1}{x} \sqrt[3]{(x^2 + \_C1) x^2}, y(x) = -\frac{1 + i\sqrt{3}}{2x} \sqrt[3]{(x^2 + \_C1) x^2}, y(x) = \frac{i\sqrt{3} - 1}{2x} \sqrt[3]{(x^2 + \_C1) x^2} \right\}$$

**2.299 ODE No. 299**

$$(3xy(x)^2 - x^2) y'(x) + y(x)^3 - 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0243934 (sec), leaf count = 328

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{2} \left( 9c_1 x^2 + \sqrt{81c_1^2 x^4 - 12x^9} \right)^{2/3} + 2\sqrt[3]{3} x^3}{6^{2/3} x \sqrt[3]{9c_1 x^2 + \sqrt{81c_1^2 x^4 - 12x^9}}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt[3]{3} (1 - i\sqrt{3}) \left( 18c_1 x^2 + 2\sqrt{81c_1^2 x^4 - 12x^9} \right)}{12x \sqrt[3]{9c_1 x^2 + \sqrt{81c_1^2 x^4 - 12x^9}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 276

$$\left\{ y(x) = -\frac{12^{\frac{2}{3}}}{144x} \left( \left( -12ix^3 + i \left( \left( 12\sqrt{-12x^5 + 81\_C1^2} + 108\_C1 \right) x^2 \right)^{\frac{2}{3}} \right) \sqrt{3} + 12x^3 + \left( \left( 12\sqrt{-12x^5 + 81\_C1^2} + 108\_C1 \right) x^2 \right)^{\frac{2}{3}} \right) \right\}$$

**2.300 ODE No. 300**

$$6xy(x)^2 y'(x) + 2y(x)^3 + x = 0$$

✓ **Mathematica** : cpu = 0.00980018 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{4c_1 - x^2}}{2^{2/3} \sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1} \sqrt[3]{4c_1 - x^2}}{2^{2/3} \sqrt[3]{x}} \right\}, \left\{ y(x) \rightarrow \frac{(-1)^{2/3} \sqrt[3]{4c_1 - x^2}}{2^{2/3} \sqrt[3]{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 83

$$\left\{ y(x) = \frac{1}{2x} \sqrt[3]{-2(x^2 - 4\_C1)x^2}, y(x) = -\frac{1 + i\sqrt{3}}{4x} \sqrt[3]{-2(x^2 - 4\_C1)x^2}, y(x) = \frac{i\sqrt{3} - 1}{4x} \sqrt[3]{-2(x^2 - 4\_C1)x^2} \right\}$$

**2.301 ODE No. 301**

$$(x^2 + 6xy(x)^2) y'(x) - y(x) (3y(x)^2 - x) = 0$$

✓ **Mathematica** : cpu = 0.0391927 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x} \sqrt{W\left(\frac{6e^{3c_1}}{x^3}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.206 (sec), leaf count = 25

$$\left\{ y(x) = \frac{1}{x} e^{-\frac{1}{2} \text{lambert} W\left(6 \frac{e^3 - C1}{x^3}\right) + \frac{3}{2} C1} \right\}$$

**2.302 ODE No. 302**

$$(x^2y(x)^2 + x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.018058 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( c_1 - \frac{\sqrt{c_1^2 x + 4}}{\sqrt{x}} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \frac{\sqrt{c_1^2 x + 4}}{\sqrt{x}} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 133

$$\left\{ y(x) = -\frac{1}{2\_C1 x} \sqrt{-2x\_C1 (-2\_C1 - x + \sqrt{x(4\_C1 + x)})}, y(x) = \frac{1}{2\_C1 x} \sqrt{-2x\_C1 (-2\_C1 - x - \sqrt{x(4\_C1 + x)})} \right\}$$

**2.303 ODE No. 303**

$$y(x)(x^2y(x)^2 + 1) + x(xy(x) - 1)^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0781378 (sec), leaf count = 24

$$\text{Solve} \left[ c_1 + \frac{1}{xy(x)} + 2 \log(y(x)) = xy(x), y(x) \right]$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 34

$$\left\{ y(x) = \frac{e^{\text{RootOf}(-e^{2-Z} - 2e^{-Z} \ln(x) + 2e^{-Z} C1 + 2_Z e^{-Z} + 1)}}{x} \right\}$$

**2.304 ODE No. 304**

$$5x^2y(x)^3 + (10x^3y(x)^2 + x^2y(x) + 2x)y'(x) + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 45.2911 (sec), leaf count = 45

$$\text{Solve} \left[ c_1 + \frac{1}{2} \log(5x^2y(x)^2 + 2) + \log(y(x)) + \frac{\tan^{-1} \left( \sqrt{\frac{5}{2}} xy(x) \right)}{\sqrt{10}} = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.225 (sec), leaf count = 44

$$\left\{ y(x) = \frac{\sqrt{10}}{5x} \tan \left( \text{RootOf} \left( \sqrt{10} \ln \left( \frac{4 \left( (\tan(\_Z))^2 + 1 \right) (\tan(\_Z))^2}{5x^2} \right) + 2\sqrt{10}\_C1 + 2\_Z \right) \right) \right\}$$

**2.305 ODE No. 305**

$$x^2 + (y(x)^3 - 3x)y'(x) - 3y(x) = 0$$

✓ **Mathematica** : cpu = 0.0979621 (sec), leaf count = 1211

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{\frac{4x^3+12c_1+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}{\sqrt{6}} - \frac{1}{2} \sqrt{\frac{12\sqrt{6}x}{\sqrt{\frac{4x^3+12c_1+\left(243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}\right)^{2/3}}{\sqrt[3]{243x^2-\frac{1}{432}\sqrt{11019960576x^4-4(144x^3+432c_1)^3}}}}}}}} \right\} \right.$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 21

$$\left\{ \frac{x^3}{3} - 3xy(x) + \frac{(y(x))^4}{4} + \_C1 = 0 \right\}$$

**2.306 ODE No. 306**

$$(y(x)^3 - x^3)y'(x) - x^2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0499333 (sec), leaf count = 201

$$\left\{ \left\{ y(x) \rightarrow \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}} \right\}, \left\{ y(x) \rightarrow -\sqrt[3]{-1} \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}} \right\}, \left\{ y(x) \rightarrow (-1)^{2/3} \sqrt[3]{x^3 - \sqrt{x^6 - e^{6c_1}}} \right\} \right.,$$

✓ **Maple** : cpu = 0.382 (sec), leaf count = 231

$$\left\{ y(x) = x \frac{1}{\sqrt[3]{-(x^3 - C1 - \sqrt{-C1^2 x^6 + 1}) x^3 - C1}}, y(x) = x \frac{1}{\sqrt[3]{-(x^3 - C1 + \sqrt{-C1^2 x^6 + 1}) x^3 - C1}}, y(x) = 4 \dots \right.$$

**2.307 ODE No. 307**

$$y(x)(a + x^2 + y(x)^2)y'(x) + x(-a + x^2 + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.0259569 (sec), leaf count = 149

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{-\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \right\}, \left\{ y(x) \rightarrow -\sqrt{\sqrt{a^2 + 4ax^2 + 4c_1} - a - x^2} \right\} \right.,$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 125

$$\left\{ y(x) = \sqrt{-x^2 - a - \sqrt{4ax^2 + a^2 - 4\_C1}}, y(x) = \sqrt{-x^2 - a + \sqrt{4ax^2 + a^2 - 4\_C1}}, y(x) = -\sqrt{-x^2 - a - \sqrt{4ax^2 + a^2 - 4\_C1}} \right.$$

**2.308 ODE No. 308**

$$2y(x)^3 y'(x) + xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.00724318 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow 0 \right\}, \left\{ y(x) \rightarrow -\sqrt{2c_1 - \frac{x^2}{2}} \right\}, \left\{ y(x) \rightarrow \sqrt{2c_1 - \frac{x^2}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 37

$$\left\{ y(x) = 0, y(x) = -\frac{1}{2}\sqrt{-2x^2 + 4\_C1}, y(x) = \frac{1}{2}\sqrt{-2x^2 + 4\_C1} \right\}$$

**2.309 ODE No. 309**

$$-2x^3 + (2y(x)^3 + y(x)) y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0129898 (sec), leaf count = 151

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-\sqrt{8c_1 + 4x^4 + 4x^2 + 1} - 1}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-\sqrt{8c_1 + 4x^4 + 4x^2 + 1} - 1}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt{\sqrt{8c_1 + 4x^4 + 4x^2 + 1} + 1}}{\sqrt{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 113

$$\left\{ y(x) = -\frac{1}{2}\sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8\_C1 + 1}}, y(x) = \frac{1}{2}\sqrt{-2 - 2\sqrt{4x^4 + 4x^2 + 8\_C1 + 1}}, y(x) = -\frac{1}{2}\sqrt{-2 + 2\sqrt{4x^4 + 4x^2 + 8\_C1 + 1}} \right\}$$

**2.310 ODE No. 310**

$$x^3 + (5x^2 y(x) + 2y(x)^3) y'(x) + 5xy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0435922 (sec), leaf count = 159

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-\sqrt{2e^{4c_1} + 23x^4 - 5x^2}}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-\sqrt{2e^{4c_1} + 23x^4 - 5x^2}}}{\sqrt{2}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt{\sqrt{2e^{4c_1} + 23x^4 - 5x^2}}}{\sqrt{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.22 (sec), leaf count = 125

$$\left\{ y(x) = -\frac{1}{2}\sqrt{-10\_C1 x^2 - 2\sqrt{23x^4 - C1^2} + 2}\frac{1}{\sqrt{-C1}}, y(x) = \frac{1}{2}\sqrt{-10\_C1 x^2 - 2\sqrt{23x^4 - C1^2} + 2}\frac{1}{\sqrt{-C1}}, y(x) = -\frac{1}{2}\sqrt{-10\_C1 x^2 - 2\sqrt{23x^4 - C1^2} - 2}\frac{1}{\sqrt{-C1}} \right\}$$



**2.311 ODE No. 311**

$$4x^3 + 9x^2y(x) + (3x^3 + 6x^2y(x) - 3xy(x)^2 + 20y(x)^3) y'(x) + 6xy(x)^2 - y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.171012 (sec), leaf count = 2201

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{20} + \frac{1}{2} \sqrt{-\frac{39x^2}{100} + \frac{\sqrt[3]{99x^6 + 351e^{c_1}x^2 + \sqrt{3}\sqrt{-67037x^{12} + 185406e^{c_1}x^8 - 83733e^{2c_1}x^4 + 32000e^{3c_1}}}}{5\sqrt[3]{23^{2/3}}}} \right. \right.$$

✓ **Maple** : cpu = 0.191 (sec), leaf count = 50

$$\left\{ y(x) = \frac{\text{RootOf}(x^4 - C1^4 + 3x^3 - C1^3 - Z + 3 - C1^2 - Z^2x^2 - C1 - Z^3x + 5 - Z^4 - 1)}{-C1} \right\}$$

**2.312 ODE No. 312**

$$(y(x)y'(x) + x) \left( \frac{x^2}{a} + \frac{y(x)^2}{b} \right) + \frac{(a-b)(y(x)y'(x) - x)}{a+b} = 0$$

✓ **Mathematica** : cpu = 0.248978 (sec), leaf count = 190

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{b} \sqrt{2a^2 W\left(\frac{c_1(a+b)e^{-\frac{a^2(b+x^2)+ab^2-b^2x^2}}{2a^2b}}\right) + (a+b)(a-x^2)}}{\sqrt{a}\sqrt{a+b}} \right. \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{b} \sqrt{2a^2 W\left(\frac{c_1(a+b)e^{-\frac{a^2(b+x^2)+ab^2-b^2x^2}}{2a^2b}}\right) + (a+b)(a-x^2)}}{\sqrt{a}\sqrt{a+b}} \right\}$$

✓ **Maple** : cpu = 1.655 (sec), leaf count = 240

$$\left\{ y(x) = \frac{1}{a} \sqrt{\left( e^{\frac{1}{2a^2b}} \left( -2 \text{lambertW}\left( \frac{1}{2} \frac{(a+b)e^{-1/2} e^{-1/2} \frac{x^2}{b} e^{1/2} \frac{bx^2}{a^2} e^{-1/2} \frac{b}{a} \left( e^{-\frac{C1}{ab}} \right)^{-1} \right) a^2b + (-x^2-b)a^2 + (-b^2-2-C1)a+b^2x^2 \right) \right)} \right\} +$$

**2.313 ODE No. 313**

$$y'(x) (3axy(x)^2 + 2ay(x)^3 - bx^3 + cx^2) - ay(x)^3 + 2bx^3 + 3bx^2y(x) + cy(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0834291 (sec), leaf count = 524

$$\left\{ \left\{ y(x) \rightarrow \frac{-\sqrt[3]{2} \left( \sqrt{3} \sqrt{a^3 (27ax^2 (bx^2 + c_1)^2 + 4 (cx + c_1)^3) + 9a^2bx^3 + 9a^2c_1x} \right)^{2/3} + 2\sqrt[3]{3}acx + 2\sqrt[3]{3}ac_1}{6^{2/3}a\sqrt[3]{\sqrt{3}\sqrt{a^3 (27ax^2 (bx^2 + c_1)^2 + 4 (cx + c_1)^3) + 9a^2bx^3 + 9a^2c_1x}}} \right\}, \left\{ \right.$$

✓ **Maple** : cpu = 0.22 (sec), leaf count = 748

$$\left\{ y(x) = -\frac{1}{12a} \left( \left( (12icx - 12i\_C1) a + i \left( \left( -108bx^3 + 108\_C1x + 12\sqrt{\frac{81ab^2x^6 - 162\_C1abx^4 + 12c^3x^2}{\dots}} \right) \right) \right) \right)$$

**2.314 ODE No. 314**

$$xy(x)^3y'(x) + y(x)^4 - x \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.0439039 (sec), leaf count = 164

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[4]{c_1 + 16x(x^2 - 6) \sin(x) - 4(x^4 - 12x^2 + 24) \cos(x)}}{x} \right\}, \left\{ y(x) \rightarrow -\frac{i\sqrt[4]{c_1 + 16x(x^2 - 6) \sin(x) - 4(x^4 - 12x^2 + 24) \cos(x)}}{x} \right\} \right.$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 158

$$\left\{ y(x) = \frac{1}{x} \sqrt[4]{(-4x^4 + 48x^2 - 96) \cos(x) + (16x^3 - 96x) \sin(x) + \_C1}, y(x) = \frac{-i}{x} \sqrt[4]{(-4x^4 + 48x^2 - 96) \cos(x) + (16x^3 - 96x) \sin(x) + \_C1} \right.$$

**2.315 ODE No. 315**

$$(2xy(x)^3 - x^4) y'(x) + 2x^3y(x) - y(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.106468 (sec), leaf count = 331

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{2} \left( \sqrt{81x^6 - 12e^{3c_1}x^3 - 9x^3} \right)^{2/3} + 2\sqrt[3]{3}e^{c_1}x}{6^{2/3}\sqrt[3]{\sqrt{81x^6 - 12e^{3c_1}x^3 - 9x^3}}} \right\}, \left\{ y(x) \rightarrow \frac{i\sqrt[3]{2}\sqrt[3]{3}(\sqrt{3} + i) \left( \sqrt{81x^6 - 12e^{3c_1}x^3 - 9x^3} \right)^{2/3} + 2\sqrt[3]{3}e^{c_1}x}{2 \cdot 2^{2/3}3^{5/6}\sqrt[3]{\sqrt{81x^6 - 12e^{3c_1}x^3 - 9x^3}}} \right\} \right.$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 376

$$\left\{ y(x) = \frac{\sqrt[3]{12}}{6\_C1} \left( x \sqrt[3]{12}\_C1 + \left( x \left( -9\_C1 x^2 + \sqrt{3} \sqrt{\frac{27\_C1^3 x^4 - 4x}{\_C1}} \right) - C1^2 \right)^{\frac{2}{3}} \right) \frac{1}{\sqrt[3]{x \left( -9\_C1 x^2 + \sqrt{3} \sqrt{\frac{27\_C1^3 x^4 - 4x}{\_C1}} \right)}} \right\}$$

**2.316 ODE No. 316**

$$(2xy(x)^3 + y(x)) y'(x) + 2y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0697356 (sec), leaf count = 41

$$\left\{ \{y(x) \rightarrow 0\}, \text{Solve} \left[ x = \frac{1}{4} e^{-\frac{1}{2}y(x)^2} \left( 4c_1 - \text{Ei} \left( \frac{y(x)^2}{2} \right) \right), y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 53

$$\left\{ y(x) = 0, y(x) = \sqrt{-2 \text{RootOf}(e^{-Z} \text{Ei}(1, -Z) + 4e^{-Z}\_C1 - 4x)}, y(x) = -\sqrt{-2 \text{RootOf}(e^{-Z} \text{Ei}(1, -Z) + 4e^{-Z}\_C1 - 4x)} \right\}$$

**2.317 ODE No. 317**

$$(x^2 + 2xy(x)^3 + xy(x)) y'(x) + y(x)^2 - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.370949 (sec), leaf count = 23

$$\text{Solve} \left[ c_1 + \frac{x}{y(x)} = y(x)^2 + \log(y(x)) + \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.124 (sec), leaf count = 29

$$\left\{ y(x) = e^{\text{RootOf} \left( -(e^{-Z})^3 - e^{-Z} \ln(x) + e^{-Z}\_C1 -\_Z e^{-Z} + x \right)} \right\}$$

**2.318 ODE No. 318**

$$(3xy(x)^3 - 4xy(x) + y(x)) y'(x) + (y(x)^2 - 2) y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.144994 (sec), leaf count = 2353

$$\left\{ \begin{array}{l} \{y(x) \rightarrow 0\}, \\ y(x) \rightarrow -\sqrt{\frac{8\sqrt[3]{2}x^4 + 8\sqrt[3]{2}x^3 + 2\left(2\sqrt[3]{16x^6 + 24x^5 - 3(9c_1^2 - 4)x^4 + 2x^3 + 3\sqrt{3}\sqrt{-x^7 c_1^2(32x^3 + 48x^2 - 27c_1^2 x + 24x + 4)} + \sqrt[3]{2}\right)}{x^2\sqrt[3]{16x^6}}}} \end{array} \right.$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 28

$$\left\{ x + (y(x))^{-2} - \frac{-C1}{(y(x))^2} \frac{1}{\sqrt{(y(x))^2 - 2}} = 0, y(x) = 0 \right\}$$

**2.319 ODE No. 319**

$$(7xy(x)^3 + y(x) - 5x) y'(x) + y(x)^4 - 5y(x) = 0$$

✓ **Mathematica** : cpu = 0.0229439 (sec), leaf count = 302

$$\left\{ \{y(x) \rightarrow \text{Root}[10\#1^7 x + 2\#1^5 - 100\#1^4 x - 25\#1^2 + 250\#1 x - 10c_1 \&, 1]\}, \{y(x) \rightarrow \text{Root}[10\#1^7 x + 2\#1^5 - \dots]\} \right.$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 35

$$\left\{ x + \frac{2(y(x))^5 - 25(y(x))^2 - 10-C1}{10y(x)((y(x))^3 - 5)^2} = 0 \right\}$$

**2.320 ODE No. 320**

$$(x^2y(x)^3 + xy(x))y'(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0555107 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-1}\right) + 2x - 1}}{\sqrt{x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2xW\left(c_1e^{\frac{1}{2x}-1}\right) + 2x - 1}}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 78

$$\left\{ y(x) = \frac{1}{x} \sqrt{2x^2 \text{lambertW}\left(1/2\_C1 e^{-1/2 \frac{2x-1}{x}}\right) + 2x^2 - x}, y(x) = -\frac{1}{x} \sqrt{2x^2 \text{lambertW}\left(1/2\_C1 e^{-1/2 \frac{2x-1}{x}}\right) + 2x^2 - x} \right\}$$

**2.321 ODE No. 321**

$$(2x^2y(x)^3 + x^2y(x)^2 - 2x)y'(x) - 2y(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.297609 (sec), leaf count = 42

$$\text{Solve}\left[\frac{1}{64}\left(-4y(x)^2 + 4y(x) - \frac{16}{2xy(x) + x} - 2\log(8y(x) + 4) + 3\right) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.173 (sec), leaf count = 42

$$\left\{ y(x) = \frac{e^{\text{RootOf}\left(x(e^{-z})^3 - 4x(e^{-z})^2 + 8\_C1xe^{-z} + 2\_Zxe^{-z} + 3xe^{-z} + 16\right)}}{2} - \frac{1}{2} \right\}$$

**2.322 ODE No. 322**

$$(10x^2y(x)^3 - 3y(x)^2 - 2)y'(x) + 5xy(x)^4 + x = 0$$

✓ **Mathematica** : cpu = 0.199764 (sec), leaf count = 2097

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{3} \sqrt{\frac{5 \sqrt[3]{6} \sqrt[3]{189x^2 - 18c_1 + \sqrt{3} \sqrt{27(21x^2 - 2c_1)^2 - 16(5x^4 - 10c_1x^2 - 2)^3} x^2 + \frac{10 \cdot 6^{2/3} (5x^4 - 10c_1x^2 - 2) x^2}{\sqrt[3]{189x^2 - 18c_1 + \sqrt{3} \sqrt{27(21x^2 - 2c_1)^2 - 16(5x^4 - 10c_1x^2 - 2)^3}}}}}{x^4}} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 29

$$\left\{ \frac{5x^2(y(x))^4}{2} - (y(x))^3 + \frac{x^2}{2} - 2y(x) + \_C1 = 0 \right\}$$

**2.323 ODE No. 323**

$$xy'(x)(axy(x)^3 + c) + y(x)(bx^3y(x) + c) = 0$$

✓ **Mathematica** : cpu = 0.0451371 (sec), leaf count = 491

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{3} \left( \sqrt{3} \sqrt{a^3 x^4 (27ac^2 + x^2 (bx^2 - 2c_1)^3)} + 9a^2 cx^2 \right)^{2/3} + 3^{2/3} ax^2 (2c_1 - bx^2)}{3ax \sqrt[3]{\sqrt{3} \sqrt{a^3 x^4 (27ac^2 + x^2 (bx^2 - 2c_1)^3)} + 9a^2 cx^2}} \right\}, \left\{ y(x) \rightarrow \frac{i \sqrt[6]{3} (\sqrt{3} \dots)}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 630

$$\left\{ y(x) = -\frac{3^{\frac{2}{3}}}{18ax} \left( \left( 3i(bx^2 - 2\_C1)x^2a + i \left( \left( 27c + 3 \sqrt{\frac{3b^3x^8 - 18\_C1b^2x^6 + 36\_C1^2bx^4 - 24\_C1^3x^2 + \dots}{a}} \right) \right) \right) \right)$$

**2.324 ODE No. 324**

$$(2x^3y(x)^3 - x)y'(x) + 2x^3y(x)^3 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0335472 (sec), leaf count = 672

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1x^2 + \frac{x^4(c_1-2x)^2}{\sqrt[3]{12c_1x^8 - 6c_1^2x^7 + c_1^3x^6 + 3\sqrt{3}\sqrt{x^8(-24c_1x^4 + 12c_1^2x^3 - 2c_1^3x^2 + 16x^5 + 27)} - 8x^9 - 27x^4}} + \sqrt[3]{12c_1x^8 - 6c_1^2x^7 + c_1^3x^6}}{6x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.137 (sec), leaf count = 815

$$\left\{ y(x) = \frac{1}{12x} \left( (2\_C1x - 4x^2) \sqrt[3]{(-C1^3x^2 - 6\_C1^2x^3 + 12\_C1x^4 - 8x^5 + 3\sqrt{-6\_C1^3x^2 + 36\_C1^2x^3 - \dots}} \right) \right)$$

**2.325 ODE No. 325**

$$y(x) (y(x)^3 - 2x^3) y'(x) + x(2y(x)^3 - x^3) = 0$$

✓ **Mathematica** : cpu = 0.221447 (sec), leaf count = 133

$$\text{Solve} \left[ 7c_1 + \log \left( 1 - \frac{y(x)}{x} \right) = \text{RootSum} \left[ \#1^4 + \#1^3 + 3\#1^2 + \#1 + 1 \&, \frac{8\#1^3 \log \left( \frac{y(x)}{x} - \#1 \right) + 9\#1^2 \log \left( \frac{y(x)}{x} - \#1 \right)}{4\#1^4} \right] \right]$$

✓ **Maple** : cpu = 0.508 (sec), leaf count = 124

$$\left\{ \frac{1}{7} \ln \left( \frac{y(x) - x}{x} \right) - \frac{2}{7} \ln \left( \frac{4x^4 + 4x^3y(x) + 12x^2(y(x))^2 + 4x(y(x))^3 + 4(y(x))^4}{x^4} \right) - \frac{2\sqrt{3}}{7} \arctan \left( \frac{(x + 2y(x))^2}{3x} \right) \right\}$$

**2.326 ODE No. 326**

$$y(x)y'(x) ((ay(x) + bx)^3 + bx^3) + x((ay(x) + bx)^3 + ay(x)^3) = 0$$

✓ **Mathematica** : cpu = 4.86889 (sec), leaf count = 6861

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{3} \sqrt{(x^2 - e^{2c_1})a^2 + 3b^2x^2 + (b^2 + 1)x^2 + 3(a^2(e^{2c_1} - x^2) - (b^2 + 1)x^2) + \sqrt[3]{-(e^{2c_1} - x^2)^3a^6 + 3(b^2 + 1)x^2(e^{2c_1} - x^2)^2a^4 - 3(b^4 - 16b^2x^2 + 3x^4)(e^{2c_1} - x^2)^2a^2}}{(e^{2c_1} - x^2)^3a^6 + 3(b^2 + 1)x^2(e^{2c_1} - x^2)^2a^4 - 3(b^4 - 16b^2x^2 + 3x^4)(e^{2c_1} - x^2)^2a^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.494 (sec), leaf count = 160

$$\left\{ y(x) = \frac{x(-C1x - b\text{RootOf}(b^2_Z^4 - 2bx_C1_Z^3 + (a^2x^2_C1^2 + b^2x^2_C1^2 + C1^2x^2 - a^2)_Z^2 - 2bx^3_C1^3)}{a\text{RootOf}(b^2_Z^4 - 2bx_C1_Z^3 + (a^2x^2_C1^2 + b^2x^2_C1^2 + C1^2x^2 - a^2)_Z^2 - 2bx^3_C1^3)} \right\}$$

**2.327 ODE No. 327**

$$(2x^2y(x)^3 + xy(x)^4 + 2y(x) + x)y'(x) + y(x)^5 + y(x) = 0$$

✓ **Mathematica** : cpu = 0.38988 (sec), leaf count = 584

$$\left\{ \left\{ y(x) \rightarrow \frac{\frac{2\sqrt[3]{2}c_1(c_1+3x^2)}{\sqrt[3]{9(c_1^2+3)x^2+3\sqrt{3}\sqrt{-4c_1^3x^6+(-c_1^4+18c_1^2+27)x^4+4c_1^3x^2+2c_1^3}} + 2^{2/3}\sqrt[3]{9(c_1^2+3)x^2+3\sqrt{3}\sqrt{-4c_1^3x^6+(-c_1^4+18c_1^2+27)x^4+4c_1^3x^2+2c_1^3}}}{6x} \right. \right.$$

✓ **Maple** : cpu = 0.175 (sec), leaf count = 583

$$\left\{ y(x) = \frac{1}{12\_C1 x} \left( \left( -12ix^2\_C1 - i \left( 108\_C1^3x^2 + 12\sqrt{3}\sqrt{27\_C1^4x^2 + 18\_C1^2x^2 + (4x^4 - 4)\_C1 - x^2} \right) \right) \right.$$

**2.328 ODE No. 328**

$$ax^2y(x)^ny'(x) - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 27.9327 (sec), leaf count = 35

$$\text{Solve} \left[ \frac{n(-\log(-axy(x)^n + n + 2) - 2\log(y(x)) + \log(x))}{n + 2} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 33

$$\left\{ \frac{((y(x))^n ax - n - 2)^n ((y(x))^n)^2}{x^n} - \_C1 = 0 \right\}$$

**2.329 ODE No. 329**

$$x^ny(x)^m (axy'(x) + by(x)) + \alpha xy'(x) + \beta y(x) = 0$$

✓ **Mathematica** : cpu = 0.834377 (sec), leaf count = 97

$$\text{Solve} \left[ \frac{m((\alpha\beta - \alpha b) \log(x^n y(x)^m (bm - an) - \alpha n + \beta m) + \alpha(bm - an) \log(y(x)(\beta m - \alpha n)) + \beta \log(x)(bm - an))}{(bm - an)(\beta m - \alpha n)} \right]$$

✓ **Maple** : cpu = 0.355 (sec), leaf count = 71

$$\left\{ (x^n(an - bm)(y(x))^m - \beta m + \alpha n)^{-\alpha\beta m + \alpha bm} ((y(x))^m)^\alpha (an - bm) x^{\beta m(an - bm)} - \_C1 = 0 \right\}$$



**2.330 ODE No. 330**

$$(f(y(x) + x) + 1)y'(x) + f(y(x) + x) = 0$$

✓ **Mathematica** : cpu = 40.0502 (sec), leaf count = 49

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x f'(K[1] + K[2]) dK[1] + f(K[2] + x) + 1 \right) dK[2] + \int_1^x f(K[1] + y(x)) dK[1], y(x) \right]$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 22

$$\left\{ y(x) = -x + \text{RootOf} \left( -x + \int^{-Z} 1 + f(_a) d_a + _C1 \right) \right\}$$

**2.331 ODE No. 331**

$$y'(x) \left( \sum_{\nu=1}^p y(x)^\nu f(\nu)(x) \right) - \sum_{\nu=1}^q y(x)^\nu g(\nu)(x) = 0$$

✗ **Mathematica** : cpu = 54.8205 (sec), leaf count = 0 , could not solve

`DSolve[-Sum[y[x]^nu*g[nu][x], {nu, 1, q}] + Sum[y[x]^nu*f[nu][x], {nu, 1, p}]*Derivative[1][y[x]]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.332 ODE No. 332**

$$x \left( \sqrt{xy(x)} - 1 \right) y'(x) - y(x) \left( \sqrt{xy(x)} + 1 \right) = 0$$

✓ **Mathematica** : cpu = 0.199599 (sec), leaf count = 23

$$\text{Solve} \left[ c_1 + \log(x) = \frac{2}{\sqrt{xy(x)}} + \log(y(x)), y(x) \right]$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 33

$$\left\{ -1 \left( 1 + \left( -C1 - \ln(x) + \frac{\ln(xy(x))}{2} \right) \sqrt{xy(x)} \right) \frac{1}{\sqrt{xy(x)}} = 0 \right\}$$

**2.333 ODE No. 333**

$$-x^{3/2}y(x)^{5/2} + \left(2x^{5/2}y(x)^{3/2} + x^2y(x) - x\right)y'(x) + xy(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.350317 (sec), leaf count = 53

$$\text{Solve} \left[ \frac{\sqrt{xy(x)}(-3x^{3/2}y(x)^{3/2}(\log(x) - 2\log(y(x))) - 6xy(x) + 2)}{3x^2y(x)^2} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 32

$$\left\{ \ln(y(x)) - 1 \frac{1}{\sqrt{x}} \frac{1}{\sqrt{y(x)}} + \frac{1}{3}(y(x))^{-\frac{3}{2}} x^{-\frac{3}{2}} - \frac{\ln(x)}{2} - \_C1 = 0 \right\}$$

**2.334 ODE No. 334**

$$\left(\sqrt{y(x)} + x + 1\right)y'(x) + 1 = 0$$

✓ **Mathematica** : cpu = 0.0346758 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -2\sqrt{c_1 + x + 1} + c_1 + 2 \right\}, \left\{ y(x) \rightarrow 2\sqrt{c_1 + x + 1} + c_1 + 2 \right\} \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 19

$$\left\{ -y(x) - 2\sqrt{y(x)} + x - \_C1 = 0 \right\}$$

**2.335 ODE No. 335**

$$\sqrt{y(x)^2 - 1}y'(x) - \sqrt{x^2 - 1} = 0$$

✓ **Mathematica** : cpu = 0.175713 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{1}{2} \#1 \sqrt{\#1^2 - 1} - \frac{1}{2} \log \left( \sqrt{\#1^2 - 1} + \#1 \right) \right] \& \right\} \left[ c_1 + \frac{1}{2} \sqrt{x^2 - 1} x - \frac{1}{2} \log \left( \sqrt{x^2 - 1} + x \right) \right] \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 50

$$\left\{ \_C1 + x\sqrt{x^2 - 1} - \ln \left( x + \sqrt{x^2 - 1} \right) - y(x) \sqrt{(y(x))^2 - 1} + \ln \left( y(x) + \sqrt{(y(x))^2 - 1} \right) = 0 \right\}$$

**2.336 ODE No. 336**

$$(ax + \sqrt{y(x)^2 + 1}) y'(x) + ay(x) + \sqrt{x^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.12316 (sec), leaf count = 43

$$\text{Solve} \left[ y(x) \left( 2ax + \sqrt{y(x)^2 + 1} \right) + \sqrt{x^2 + 1}x + \sinh^{-1}(y(x)) + \sinh^{-1}(x) = 2c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 41

$$\left\{ \frac{x}{2} \sqrt{x^2 + 1} + \frac{\text{Arcsinh}(x)}{2} + axy(x) + \frac{y(x)}{2} \sqrt{(y(x))^2 + 1} + \frac{\text{Arcsinh}(y(x))}{2} + \_C1 = 0 \right\}$$

**2.337 ODE No. 337**

$$\left( \sqrt{x^2 + y(x)^2} + x \right) y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.060938 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow -e^{\frac{c_1}{2}} \sqrt{e^{c_1} + 2x} \right\}, \left\{ y(x) \rightarrow e^{\frac{c_1}{2}} \sqrt{e^{c_1} + 2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 28

$$\left\{ \_C1 + \frac{1}{(y(x))^2} \sqrt{(y(x))^2 + x^2} + \frac{x}{(y(x))^2} = 0 \right\}$$

**2.338 ODE No. 338**

$$y'(x) \left( \sin(\alpha) (y(x)^2 - x^2) - 2x \cos(\alpha) y(x) + \sqrt{x^2 + y(x)^2} y(x) \right) + \cos(\alpha) (y(x)^2 - x^2) + 2x \sin(\alpha) y(x) + x \sqrt{x^2 + y(x)^2} = 0$$

✗ **Mathematica** : cpu = 0 (sec), leaf count = 0 , crash

Kernel Crash

✓ **Maple** : cpu = 0.744 (sec), leaf count = 129

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \int^{-Z} \frac{1}{(\_a^2 + 1) (\cos(2\alpha) \_a^2 + 2\_a \sin(2\alpha) + \_a^2 - \cos(2\alpha) + 1)} \right) \left( -\cos(2\alpha) \_a \right) \right\}$$

**2.339 ODE No. 339**

$$\left(x\sqrt{x^2 + y(x)^2 + 1} - y(x)(x^2 + y(x)^2)\right) y'(x) - \sqrt{x^2 + y(x)^2 + 1} y(x) - x(x^2 + y(x)^2) = 0$$

✓ **Mathematica** : cpu = 0.178129 (sec), leaf count = 27

$$\text{Solve}\left[\sqrt{x^2 + y(x)^2 + 1} + \tan^{-1}\left(\frac{x}{y(x)}\right) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 27

$$\left\{\arctan\left(\frac{y(x)}{x}\right) - \sqrt{x^2 + (y(x))^2 + 1} - \_C1 = 0\right\}$$

**2.340 ODE No. 340**

$$y'(x) \left( \frac{e_1(a+x)}{((a+x)^2 + y(x)^2)^{3/2}} + \frac{e_2(x-a)}{((x-a)^2 + y(x)^2)^{3/2}} \right) - y(x) \left( \frac{e_1}{((a+x)^2 + y(x)^2)^{3/2}} + \frac{e_2}{((x-a)^2 + y(x)^2)^{3/2}} \right)$$

✗ **Mathematica** : cpu = 0 (sec), leaf count = 0 , crash

Kernel Crash

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.341 ODE No. 341**

$$(xe^{y(x)} + e^x) y'(x) + e^x y(x) + e^{y(x)} = 0$$

✓ **Mathematica** : cpu = 0.0601693 (sec), leaf count = 33

$$\left\{\left\{y(x) \rightarrow c_1 e^{-x} - W\left(x e^{c_1 e^{-x} - x}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 30

$$\left\{y(x) = -\text{lambertW}\left(\frac{x}{e^x} \left(e^{\frac{C1}{e^x}}\right)^{-1}\right) - \frac{C1}{e^x}\right\}$$

**2.342 ODE No. 342**

$$x(2e^{-xy(x)} + 3e^{xy(x)})(xy'(x) + y(x)) + 1 = 0$$

✓ **Mathematica** : cpu = 0.264116 (sec), leaf count = 163

$$\left\{ \left\{ y(x) \rightarrow -\frac{\cosh^{-1}\left(\frac{1}{24}\left(-5\sqrt{\log^2\left(\frac{c_1}{x}\right) + 24} - \log\left(\frac{c_1}{x}\right)\right)\right)}{x} \right\} \right\}, \left\{ \left\{ y(x) \rightarrow \frac{\cosh^{-1}\left(\frac{1}{24}\left(-5\sqrt{\log^2\left(\frac{c_1}{x}\right) + 24} - \log\left(\frac{c_1}{x}\right)\right)\right)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 17

$$\left\{ y(x) = \frac{1}{x} \ln\left(-\frac{\ln(x)}{5} + \frac{C1}{5}\right) \right\}$$

**2.343 ODE No. 343**

$$y'(x)(\log(y(x)) + x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0784293 (sec), leaf count = 22

$$\text{Solve}\left[\log(y(x)) + x = e^{y(x)}(c_1 + \text{Ei}(-y(x))), y(x)\right]$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 27

$$\left\{ y(x) = e^{\text{RootOf}\left(-_Z - x - e^{-Z} \text{Ei}(1, e^{-Z}) + _C1 e^{e^{-Z}}\right)} \right\}$$

**2.344 ODE No. 344**

$$y'(x)(\log(y(x)) + 2x - 1) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0403015 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow -\frac{W(-2c_1 e^{-2x})}{2c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 19

$$\left\{ y(x) = e^{-\text{lambertW}(-2e^{-2x} - C1) - 2x} \right\}$$

**2.345 ODE No. 345**

$$xy'(x) (2x^2y(x) \log(y(x)) + 1) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0794142 (sec), leaf count = 23

$$\text{Solve}\left[y(x) \left(\frac{1}{x^2} + y(x) \left(\log(y(x)) - \frac{1}{2}\right)\right) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 36

$$\left\{y(x) = e^{\text{RootOf}\left(2\_Z x^2 (e^{-Z})^2 - x^2 (e^{-Z})^2 + 2\_C1 x^2 + 2 e^{-Z}\right)}\right\}$$

**2.346 ODE No. 346**

$$xy'(x)(-ax + y(x) + y(x) \log(xy(x))) - y(x)(ax \log(xy(x)) + ax - y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0813728 (sec), leaf count = 20

$$\text{Solve}[(ax - y(x)) \log(xy(x)) = c_1, y(x)]$$

✓ **Maple** : cpu = 0.276 (sec), leaf count = 19

$$\left\{(xy(x))^{-ax+y(x)} - \_C1 = 0\right\}$$

**2.347 ODE No. 347**

$$(\sin(x) + 1)y'(x) \sin(y(x)) + \cos(x)(\cos(y(x)) - 1) = 0$$

✓ **Mathematica** : cpu = 0.137987 (sec), leaf count = 32

$$\left\{\left\{y(x) \rightarrow 0\right\}, \left\{y(x) \rightarrow 2 \sin^{-1} \left(\frac{1}{4} c_1 \left(\sin \left(\frac{x}{2}\right) + \cos \left(\frac{x}{2}\right)\right)\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 16

$$\{y(x) = \pi - \arccos(\sin(x) \_C1 + \_C1 - 1)\}$$

**2.348 ODE No. 348**

$$y'(x)(x \cos(y(x)) + \sin(x)) + \sin(y(x)) + y(x) \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0584059 (sec), leaf count = 17

$$\text{Solve}[c_1 = x \sin(y(x)) + y(x) \sin(x), y(x)]$$

✓ **Maple** : cpu = 0.101 (sec), leaf count = 15

$$\{y(x) \sin(x) + x \sin(y(x)) + \_C1 = 0\}$$

**2.349 ODE No. 349**

$$xy'(x) \cot\left(\frac{y(x)}{x}\right) + 2x \sin\left(\frac{y(x)}{x}\right) - y(x) \cot\left(\frac{y(x)}{x}\right) = 0$$

✓ **Mathematica** : cpu = 0.0373771 (sec), leaf count = 15

$$\{\{y(x) \rightarrow x \csc^{-1}(2(c_1 + \log(x)))\}\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 17

$$\{y(x) = \arcsin\left(\left(2 \ln(x) + 2 \_C1\right)^{-1}\right) x\}$$

**2.350 ODE No. 350**

$$y'(x) \cos(y(x)) - \sin(y(x)) - \cos(x) \sin^2(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.518928 (sec), leaf count = 53

$$\left\{\left\{y(x) \rightarrow \csc^{-1}\left(\frac{1}{2}(-2c_1 e^{-x} - \sin(x) - \cos(x))\right)\right\}, \left\{y(x) \rightarrow -\csc^{-1}\left(\frac{1}{2}(2c_1 e^{-x} + \sin(x) + \cos(x))\right)\right\}\right\}$$

✓ **Maple** : cpu = 1.077 (sec), leaf count = 226

$$\left\{y(x) = \arctan\left(-2 \frac{e^x}{e^x (\cos(x) + \sin(x)) + 2 \_C1}, \frac{\sqrt{16}}{4 \_C1^2 + 4 e^x (\cos(x) + \sin(x)) \_C1 + (e^x)^2 (2 \cos(x) \sin(x))}\right)\right\}$$

**2.351 ODE No. 351**

$$y'(x) \cos(y(x)) - \sin^3(y(x)) + x \sin(y(x)) \cos^2(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.336463 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow -\cot^{-1} \left( \sqrt{e^{x^2} (4c_1 - \sqrt{\pi} \operatorname{erf}(x))} \right) \right\}, \left\{ y(x) \rightarrow \cot^{-1} \left( \sqrt{e^{x^2} (4c_1 - \sqrt{\pi} \operatorname{erf}(x))} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.504 (sec), leaf count = 55

$$\left\{ y(x) = -\arcsin \left( \frac{1}{\sqrt{1 - \sqrt{\pi} \operatorname{Erf}(x) e^{x^2} - 2\_C1 e^{x^2}}} \right), y(x) = \arcsin \left( \frac{1}{\sqrt{1 - \sqrt{\pi} \operatorname{Erf}(x) e^{x^2} - 2\_C1 e^{x^2}}} \right) \right\}$$

**2.352 ODE No. 352**

$$y'(x) \cos(y(x)) (\cos(y(x)) - \sin(\alpha) \sin(x)) + \cos(x) (\cos(x) - \sin(\alpha) \sin(y(x))) = 0$$

✓ **Mathematica** : cpu = 0.165351 (sec), leaf count = 32

$$\operatorname{Solve}[c_1 + 2y(x) + \sin(2y(x)) + 2x + \sin(2x) = 4 \sin(\alpha) \sin(x) \sin(y(x)), y(x)]$$

✓ **Maple** : cpu = 0.302 (sec), leaf count = 33

$$\left\{ \frac{(-2 \sin(\alpha) \sin(x) + \cos(y(x))) \sin(y(x))}{2} + \frac{\cos(x) \sin(x)}{2} + \frac{x}{2} + \_C1 + \frac{y(x)}{2} = 0 \right\}$$

**2.353 ODE No. 353**

$$xy'(x) \cos(y(x)) + \sin(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0194951 (sec), leaf count = 14

$$\left\{ \left\{ y(x) \rightarrow \sin^{-1} \left( \frac{e^{c_1}}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 12

$$\left\{ y(x) = \arcsin \left( \frac{1}{\_C1 x} \right) \right\}$$



**2.354 ODE No. 354**

$$y'(x)(x \sin(y(x)) - 1) + \cos(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0593614 (sec), leaf count = 145

$$\left\{ \left\{ y(x) \rightarrow -\cos^{-1} \left( \frac{c_1 x - \sqrt{c_1^2 - x^2 + 1}}{c_1^2 + 1} \right) \right\}, \left\{ y(x) \rightarrow \cos^{-1} \left( \frac{c_1 x - \sqrt{c_1^2 - x^2 + 1}}{c_1^2 + 1} \right) \right\}, \left\{ y(x) \rightarrow -\cos^{-1} \left( \frac{\sqrt{c_1^2 - x^2 + 1}}{c_1^2 + 1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 108

$$\left\{ y(x) = \arctan \left( \frac{1}{-C1^2 + 1} \left( -C1 \sqrt{-C1^2 - x^2 + 1} + x \right), \frac{1}{-C1^2 + 1} \left( -C1 x + \sqrt{-C1^2 - x^2 + 1} \right) \right), y(x) = \arctan \left( \frac{1}{-C1^2 + 1} \left( -C1 \sqrt{-C1^2 - x^2 + 1} + x \right), \frac{1}{-C1^2 + 1} \left( -C1 x + \sqrt{-C1^2 - x^2 + 1} \right) \right) \right\}$$

**2.355 ODE No. 355**

$$y'(x)(x \cos(y(x)) + \cos(x)) + \sin(y(x)) - y(x) \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.0577579 (sec), leaf count = 17

$$\text{Solve}[c_1 = x \sin(y(x)) + y(x) \cos(x), y(x)]$$

✓ **Maple** : cpu = 0.1 (sec), leaf count = 15

$$\{y(x) \cos(x) + x \sin(y(x)) + \_C1 = 0\}$$

**2.356 ODE No. 356**

$$y'(x) (x^2 \cos(y(x)) + 2y(x) \sin(x)) + 2x \sin(y(x)) + y(x)^2 \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0715877 (sec), leaf count = 21

$$\text{Solve}[c_1 = x^2 \sin(y(x)) + y(x)^2 \sin(x), y(x)]$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 19

$$\{(y(x))^2 \sin(x) + x^2 \sin(y(x)) + \_C1 = 0\}$$

**2.357 ODE No. 357**

$$x \log(x)y'(x) \sin(y(x)) + \cos(y(x))(1 - x \cos(y(x))) = 0$$

✓ **Mathematica** : cpu = 0.282095 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1} \left( \frac{x - c_1}{\log(x)} \right) \right\}, \left\{ y(x) \rightarrow \sec^{-1} \left( \frac{x - c_1}{\log(x)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.58 (sec), leaf count = 13

$$\left\{ y(x) = \arccos \left( \frac{\ln(x)}{x + \_C1} \right) \right\}$$

**2.358 ODE No. 358**

$$\cos(x)y'(x) \sin(y(x)) + \sin(x) \cos(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0420563 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow -\cos^{-1} \left( \frac{1}{2} c_1 \sec(x) \right) \right\}, \left\{ y(x) \rightarrow \cos^{-1} \left( \frac{1}{2} c_1 \sec(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 11

$$\left\{ y(x) = \arccos \left( \frac{\_C1}{\cos(x)} \right) \right\}$$

**2.359 ODE No. 359**

$$3 \sin(x)y'(x) \sin(y(x)) + 5y(x) \cos^4(x) = 0$$

✓ **Mathematica** : cpu = 0.0556384 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \text{SinIntegral}^{(-1)} \left( c_1 - \frac{5}{36} \left( 15 \cos(x) + \cos(3x) + 12 \left( \log \left( \sin \left( \frac{x}{2} \right) \right) - \log \left( \cos \left( \frac{x}{2} \right) \right) \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.051 (sec), leaf count = 28

$$\left\{ \frac{\cos(3x)}{12} + \frac{5 \cos(x)}{4} + \ln(\csc(x) - \cot(x)) + \frac{3 \text{Si}(y(x))}{5} + \_C1 = 0 \right\}$$

**2.360 ODE No. 360**

$$y'(x) \cos(ay(x)) - b(1 - c \cos(ay(x))) \sqrt{c \cos(ay(x)) + \cos^2(ay(x)) - 1} = 0$$

✓ **Mathematica** : cpu = 50.9914 (sec), leaf count = 6218

$$\left\{ \left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\sqrt{2}(\cos(a\#1) + 1) \sqrt{\frac{2c \cos(a\#1) + \cos(2a\#1) - 1}{(\cos(a\#1) + 1)^2}}}{\left( \sqrt{-\frac{\sqrt{c^2+4}}{c} - \frac{2}{c}} + \sqrt{\frac{\sqrt{c^2+4}}{c} - \frac{2}{c}} \right) \left( \frac{\sqrt{c-1}}{\sqrt{c+1}} + \sqrt{\dots} \right)} \right. \right. \right. \right.$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 48

$$\left\{ x + \int^{y(x)} 2 \frac{\cos(\_a a)}{b(c \cos(\_a a) - 1) \sqrt{2 \cos(2\_a a) - 2 + 4 c \cos(\_a a)}} d\_a + \_C1 = 0 \right\}$$

**2.361 ODE No. 361**

$$y'(x)(-\sin(y(x)) + x \sin(xy(x)) + \cos(y(x) + x)) + y(x) \sin(xy(x)) + \cos(y(x) + x) + \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.357535 (sec), leaf count = 23

$$\text{Solve}[c_1 + \cos(xy(x)) = \sin(y(x) + x) + \cos(y(x)) + \sin(x), y(x)]$$

✓ **Maple** : cpu = 0.253 (sec), leaf count = 22

$$\{-\cos(xy(x)) + \sin(y(x) + x) + \sin(x) + \cos(y(x)) + \_C1 = 0\}$$

**2.362 ODE No. 362**

$$y'(x) (x^2 y(x) \sin(xy(x)) - 4x) - y(x) + xy(x)^2 \sin(xy(x)) = 0$$

✓ **Mathematica** : cpu = 0.286668 (sec), leaf count = 20

$$\text{Solve}[c_1 + 4 \log(y(x)) + \cos(xy(x)) + \log(x) = 0, y(x)]$$

✓ **Maple** : cpu = 0.231 (sec), leaf count = 23

$$\left\{ y(x) = \frac{1}{x} \text{RootOf} \left( -Z + e^{-\frac{\cos(-Z)}{4}} - C1 x^{\frac{3}{4}} \right) \right\}$$

**2.363 ODE No. 363**

$$(xy'(x) - y(x)) \cos^2 \left( \frac{y(x)}{x} \right) + x = 0$$

✓ **Mathematica** : cpu = 0.0699583 (sec), leaf count = 28

$$\text{Solve} \left[ 4c_1 = \frac{2y(x)}{x} + \sin \left( \frac{2y(x)}{x} \right) + 4 \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 35

$$\left\{ -\frac{1}{2x} \left( \cos \left( \frac{y(x)}{x} \right) \sin \left( \frac{y(x)}{x} \right) x + y(x) \right) - \ln(x) - C1 = 0 \right\}$$

**2.364 ODE No. 364**

$$xy'(x) \left( y(x) \sin \left( \frac{y(x)}{x} \right) - x \cos \left( \frac{y(x)}{x} \right) \right) - y(x) \left( y(x) \sin \left( \frac{y(x)}{x} \right) + x \cos \left( \frac{y(x)}{x} \right) \right) = 0$$

✓ **Mathematica** : cpu = 0.139455 (sec), leaf count = 27

$$\text{Solve} \left[ c_1 + \log \left( \frac{y(x)}{x} \right) + \log \left( \cos \left( \frac{y(x)}{x} \right) \right) + 2 \log(x) = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.106 (sec), leaf count = 23

$$\left\{ y(x) = \frac{-C1}{\cos(\text{RootOf}(-Z x^2 \cos(-Z) + C1)) x} \right\}$$

**2.365 ODE No. 365**

$$(y(x)f(x^2 + y(x)^2) - x)y'(x) + xf(x^2 + y(x)^2) + y(x) = 0$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.304 (sec), leaf count = 42

$$\left\{ y(x) = x \left( \tan \left( \text{RootOf} \left( -2\_Z - \int \frac{x^2((\tan(\_Z))^2+1)}{(\tan(\_Z))^2} \frac{f(\_a)}{\_a} d\_a + 2\_C1 \right) \right) \right)^{-1} \right\}$$

**2.366 ODE No. 366**

$$f(ay(x)^2 + x^2) (ay(x)y'(x) + x) - xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 247.697 (sec), leaf count = 88

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x (1 - 2aK[1]K[2]f'(aK[2]^2 + K[1]^2)) dK[1] - aK[2]f(aK[2]^2 + x^2) + x \right) dK[2] + \int_1^x \right]$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 45

$$\left\{ -ax(y(x))^2 \frac{1}{\sqrt{a^2(y(x))^2}} - \int^{-\frac{a(y(x))^2}{2} - \frac{x^2}{2}} f(-2\_a) d\_a + \_C1 = 0 \right\}$$

**2.367 ODE No. 367**

$$f(x^c y(x)) (bxy'(x) - a) - x^a y(x)^b (cy(x) + xy'(x)) = 0$$

✗ **Mathematica** : cpu = 13.3047 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[-(x^a*y[x]^b*(c*y[x] + x*Derivative[1][y][x])) + f[x^c*y[x]]*(-a + b*x*Derivative[1][y][x])]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(f(x^c*y(x))*(b*x*diff(y(x),x)-a)-x^a*y(x)^b*(x*diff(y(x),x)+c*y(x)) = 0,y(x))$$

**2.368 ODE No. 368**

$$ay(x) + bx^2 + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.369 ODE No. 369**

$$-a^2 + y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0487004 (sec), leaf count = 99

$$\left\{ \left\{ y(x) \rightarrow -\frac{a \tan(x - c_1)}{\sqrt{\sec^2(x - c_1)}} \right\}, \left\{ y(x) \rightarrow \frac{a \tan(x - c_1)}{\sqrt{\sec^2(x - c_1)}} \right\}, \left\{ y(x) \rightarrow -\frac{a \tan(c_1 + x)}{\sqrt{\sec^2(c_1 + x)}} \right\}, \left\{ y(x) \rightarrow \frac{a \tan(c_1 + x)}{\sqrt{\sec^2(c_1 + x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.728 (sec), leaf count = 68

$$\left\{ y(x) = a, y(x) = \tan(-x + \_C1) \sqrt{\frac{a^2}{(\tan(-x + \_C1))^2 + 1}}, y(x) = -a, y(x) = -\tan(-x + \_C1) \sqrt{\frac{a^2}{(\tan(-x + \_C1))^2 + 1}} \right\}$$

**2.370 ODE No. 370**

$$-f(x)^2 + y'(x)^2 + y(x)^2 = 0$$

✗ **Mathematica** : cpu = 17.1967 (sec), leaf count = 0 , could not solve

`DSolve[-f[x]^2 + y[x]^2 + Derivative[1][y][x]^2 == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x), x)^2 + y(x)^2 - f(x)^2 = 0, y(x))`

**2.371 ODE No. 371**

$$y'(x)^2 - y(x)^3 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0242757 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \sec^2 \left( \frac{1}{2}(x - c_1) \right) \right\}, \left\{ y(x) \rightarrow \tan^2 \left( \frac{1}{2}(c_1 + x) \right) + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.676 (sec), leaf count = 20

$$\left\{ y(x) = 1, y(x) = \left( \tan \left( -\frac{x}{2} + \frac{C1}{2} \right) \right)^2 + 1 \right\}$$

**2.372 ODE No. 372**

$$ay(x) + b + y'(x)^2 - 4y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.00482505 (sec), leaf count = 27

$$\{ \{ y(x) \rightarrow \wp(x - c_1; a, b) \}, \{ y(x) \rightarrow \wp(x + c_1; a, b) \} \}$$

✓ **Maple** : cpu = 0.661 (sec), leaf count = 232

$$\left\{ y(x) = -\frac{1}{12} \left( \left( i \left( 27b + 3\sqrt{-3a^3 + 81b^2} \right)^{\frac{2}{3}} - 3ia \right) \sqrt{3} + \left( 27b + 3\sqrt{-3a^3 + 81b^2} \right)^{\frac{2}{3}} + 3a \right) \frac{1}{\sqrt[3]{27b + 3\sqrt{-3a^3 + 81b^2}}} \right\}$$

**2.373 ODE No. 373**

$$a^2 y(x)^2 (\log^2(y(x)) - 1) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0961571 (sec), leaf count = 71

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{1}{2}(e^{-c_1+iax}+e^{c_1-iax})} \right\}, \left\{ y(x) \rightarrow \exp \left( \frac{1}{2}(e^{-c_1-iax} + e^{c_1+iax}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.411 (sec), leaf count = 49

$$\left\{ y(x) = \left( e^{-\sin(a(x-C1))} \right)^{-1}, y(x) = e^{-\sin(a(x-C1))}, y(x) = e^{\text{RootOf}(a^2(e^{-Z})^2(-Z^2-1))} \right\}$$

**2.374 ODE No. 374**

$$y'(x)^2 - 2y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0667006 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{\#1^2 + 1}}{\#1} - \frac{1}{\#1} + \sinh^{-1}(\#1) \& \right] [c_1 - x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{\#1^2 + 1}}{\#1} \right] [c_1 - x] \right\} \right\}$$

✓ **Maple** : cpu = 0.659 (sec), leaf count = 85

$$\left\{ x - (y(x))^{-1} - \frac{1}{y(x)} \left( (y(x))^2 + 1 \right)^{\frac{3}{2}} + y(x) \sqrt{(y(x))^2 + 1} + \text{Arcsinh}(y(x)) - \_C1 = 0, x - (y(x))^{-1} + \frac{1}{y(x)} \left( (y(x))^2 + 1 \right)^{\frac{3}{2}} - \text{Arcsinh}(y(x)) + \_C1 = 0 \right\}$$

**2.375 ODE No. 375**

$$ay'(x) + bx + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0430597 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow c_1 - \frac{(a^2 - 4bx)^{3/2} + 6abx}{12b} \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \frac{(a^2 - 4bx)^{3/2}}{6b} - ax \right) + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.654 (sec), leaf count = 49

$$\left\{ y(x) = -\frac{ax}{2} - \frac{1}{12b} (a^2 - 4bx)^{\frac{3}{2}} + \_C1, y(x) = -\frac{ax}{2} + \frac{1}{12b} (a^2 - 4bx)^{\frac{3}{2}} + \_C1 \right\}$$

**2.376 ODE No. 376**

$$ay'(x) + by(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.313729 (sec), leaf count = 110

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{a^2 - 4\#1b} + a \log \left( \sqrt{a^2 - 4\#1b} - a \right)}{2b} \& \right] \left[ c_1 + \frac{x}{2} \right] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{a^2 - 4\#1b}}{2b} \& \right] \left[ c_1 + \frac{x}{2} \right] \right\} \right\}$$

✓ **Maple** : cpu = 1.237 (sec), leaf count = 219

$$\left\{ y(x) = -\frac{1}{4b} e^{\frac{1}{2a}} \left( -2 a \text{lambertW} \left( 2 \frac{e^{-1}}{a} e^{-\frac{C1 b}{a}} \frac{1}{\sqrt{-b-1}} \left( e^{\frac{bx}{a}} \right)^{-1} \right) - a \ln \left( -\frac{1}{4b} \right) - 2a + (-2x + 2\_C1)b \right) \left( e^{\frac{1}{2a}} \left( -2 a \text{lambertW} \left( 2 \frac{e^{-1}}{a} e^{-\frac{C1 b}{a}} \frac{1}{\sqrt{-b-1}} \left( e^{\frac{bx}{a}} \right)^{-1} \right) - a \ln \left( -\frac{1}{4b} \right) - 2a + (-2x + 2\_C1)b \right) \right)^{-1} \right.$$



**2.377 ODE No. 377**

$$y'(x)^2 + (x - 2)y'(x) - y(x) + 1 = 0$$

✓ **Mathematica** : cpu = 0.00489705 (sec), leaf count = 17

$$\{ \{ y(x) \rightarrow c_1(x - 2) + c_1^2 + 1 \} \}$$

✓ **Maple** : cpu = 0.641 (sec), leaf count = 24

$$\left\{ y(x) = -\frac{x^2}{4} + x, y(x) = 1 + \_C1^2 + (x - 2)\_C1 \right\}$$

**2.378 ODE No. 378**

$$(a + x)y'(x) + y'(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0051076 (sec), leaf count = 13

$$\{ \{ y(x) \rightarrow c_1(a + c_1 + x) \} \}$$

✓ **Maple** : cpu = 0.638 (sec), leaf count = 20

$$\left\{ y(x) = \_C1(\_C1 + a + x), y(x) = -\frac{(x + a)^2}{4} \right\}$$

**2.379 ODE No. 379**

$$y'(x)^2 - (x + 1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0051428 (sec), leaf count = 15

$$\{ \{ y(x) \rightarrow c_1(-c_1 + x + 1) \} \}$$

✓ **Maple** : cpu = 0.648 (sec), leaf count = 22

$$\left\{ y(x) = \_C1(-\_C1 + x + 1), y(x) = \frac{(1 + x)^2}{4} \right\}$$

**2.380 ODE No. 380**

$$y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.411793 (sec), leaf count = 1445

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36} \left( -9x^2 - \frac{9(x^3 + 8 \cosh(3c_1) + 8 \sinh(3c_1))}{\sqrt[3]{x^6 - 20 \cosh(3c_1)x^3 - 20 \sinh(3c_1)x^3 - 8 \cosh(6c_1) - 8 \sinh(6c_1) + 8\sqrt{-(x^3 - 1)}}} \right) \right. \right.$$

✓ **Maple** : cpu = 0.654 (sec), leaf count = 619

$$\left\{ y(x) = \frac{1}{16} \left( i\sqrt{3} \left( 6\_C1 - x^3 + 2\sqrt{-3x^3\_C1 + 9\_C1^2} \right)^{\frac{2}{3}} - i\sqrt{3}x^2 - \left( 6\_C1 - x^3 + 2\sqrt{-3x^3\_C1 + 9\_C1^2} \right) \right.$$

**2.381 ODE No. 381**

$$y'(x)^2 - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.427669 (sec), leaf count = 1445

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36} \left( 9x^2 + \frac{9(x^3 + 8 \cosh(3c_1) + 8 \sinh(3c_1))}{\sqrt[3]{x^6 - 20 \cosh(3c_1)x^3 - 20 \sinh(3c_1)x^3 - 8 \cosh(6c_1) - 8 \sinh(6c_1) + 8\sqrt{-(x^3 - 1)}}} \right) \right. \right.$$

✓ **Maple** : cpu = 0.662 (sec), leaf count = 579

$$\left\{ y(x) = -\frac{1}{16} \left( i\sqrt{3} \left( -6\_C1 + x^3 + 2\sqrt{-3x^3\_C1 + 9\_C1^2} \right)^{\frac{2}{3}} - i\sqrt{3}x^2 - \left( -6\_C1 + x^3 + 2\sqrt{-3x^3\_C1 + 9\_C1^2} \right) \right.$$

**2.382 ODE No. 382**

$$axy'(x) - bx^2 - c + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.245327 (sec), leaf count = 190

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}x\sqrt{x^2(a^2 + 4b) + 4c} + \frac{c \log\left(\sqrt{a^2 + 4b}\sqrt{x^2(a^2 + 4b) + 4c} + a^2x + 4bx\right)}{\sqrt{a^2 + 4b}} - \frac{ax^2}{4} + c_1 \right\}, \left\{ y(x) \rightarrow \dots \right\} \right.$$

✓ **Maple** : cpu = 0.661 (sec), leaf count = 146

$$\left\{ y(x) = -\frac{ax^2}{4} - \frac{x}{4}\sqrt{(a^2 + 4b)x^2 + 4c} - c \ln\left(\sqrt{a^2 + 4b}x + \sqrt{(a^2 + 4b)x^2 + 4c}\right) \frac{1}{\sqrt{a^2 + 4b}} + C_1, y(x) = \dots \right.$$

**2.383 ODE No. 383**

$$axy'(x) + by(x) + cx^2 + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.384 ODE No. 384**

$$(ax + b)y'(x) - ay(x) + c + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.99463 (sec), leaf count = 133

$$\left\{ \left\{ y(x) \rightarrow -\frac{2\sqrt{-a^4e^{2c_1}(x+1)^2} + a^3(-(2x+1)) + 2a^2bx + a(b^2 + e^{2c_1} - 4c)}{4a^2} \right\}, \left\{ y(x) \rightarrow \frac{2\sqrt{-a^4e^{2c_1}(x+1)^2}}{4a^2} \right\} \right.$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 50

$$\left\{ y(x) = \frac{-C_1^2 + (ax + b)C_1 + c}{a}, y(x) = \frac{-a^2x^2 - 2abx - b^2 + 4c}{4a} \right\}$$

**2.385 ODE No. 385**

$$-2x^2y'(x) + y'(x)^2 + 2xy(x) = 0$$

✗ **Mathematica** : cpu = 300.074 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.274 (sec), leaf count = 169

$$\left\{ y(x) = \frac{x^4 - (\text{RootOf}(x^{16} - 12\_Z^2x^{12} - 16\_Z^3x^{10} + 30\_Z^4x^8 + 96\_Z^5x^6 + 100\_Z^6x^4 + 48\_Z^7x^2 + 9\_Z^8))}{2x} \right\}$$

**2.386 ODE No. 386**

$$ax^3y'(x) - 2ax^2y(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.192215 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow 2ae^{2c_1}(4e^{2c_1} + x^2) \right\}, \left\{ y(x) \rightarrow \frac{e^{4c_1} - 2ae^{2c_1}x^2}{8a} \right\} \right\}$$

✓ **Maple** : cpu = 0.429 (sec), leaf count = 27

$$\left\{ y(x) = -\frac{ax^4}{8}, y(x) = \_C1 x^2 + 2\frac{C1^2}{a} \right\}$$

**2.387 ODE No. 387**

$$y'(x)^2 + e^x(y'(x) - y(x)) = 0$$

✓ **Mathematica** : cpu = 0.870862 (sec), leaf count = 133

$$\left\{ \text{Solve} \left[ \frac{e^{x/2} \sqrt{4y(x) + e^x} + 4y(x) \log(\sqrt{4y(x) + e^x} + e^{x/2}) - e^x}{2y(x)} = c_1, y(x) \right], \text{Solve} \left[ 2 \log(y(x)) = c_1 + \frac{e^{x/2} \sqrt{4y(x) + e^x}}{2} \right] \right\}$$

✓ **Maple** : cpu = 0.676 (sec), leaf count = 115

$$\left\{ -\frac{e^x}{2y(x)} + \ln(y(x)) + 2 \text{Artanh}\left(\sqrt{e^{2x} + 4y(x)e^x}\right) + \frac{1}{2y(x)} \sqrt{e^{2x} + 4y(x)e^x} - \_C1 = 0, -2 \text{Artanh}\left(\sqrt{e^{2x} + 4y(x)e^x}\right) \right\}$$

**2.388 ODE No. 388**

$$y'(x)^2 - 2y(x)y'(x) - 2x = 0$$

✓ **Mathematica** : cpu = 0.84966 (sec), leaf count = 41

$$\text{Solve} \left[ \left\{ x = \frac{\text{K\$162581}(2c_1 + \sinh^{-1}(\text{K\$162581}))}{2\sqrt{\text{K\$162581}^2 + 1}}, \text{K\$162581} = 2 \left( \frac{x}{\text{K\$162581}} + y(x) \right) \right\}, \{y(x), \text{K\$162581}\} \right]$$

✓ **Maple** : cpu = 0.087 (sec), leaf count = 223

$$\left\{ 1 \left( \left( -\frac{y(x)}{2} - \frac{1}{2} \sqrt{(y(x))^2 + 2x} \right) \text{Arcsinh} \left( y(x) + \sqrt{(y(x))^2 + 2x} \right) + x \sqrt{2(y(x))^2 + 2x + 2y(x) \sqrt{(y(x))^2 + 2x}} \right) \right\}$$

**2.389 ODE No. 389**

$$y'(x)^2 - (4y(x) + 1)y'(x) + y(x)(4y(x) + 1) = 0$$

✓ **Mathematica** : cpu = 0.0440891 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} e^{x-4c_1} (e^x - 2e^{2c_1}) \right\}, \left\{ y(x) \rightarrow \frac{1}{4} e^{2c_1+x} (e^{2c_1+x} - 2) \right\} \right\}$$

✓ **Maple** : cpu = 0.57 (sec), leaf count = 71

$$\left\{ y(x) = -\frac{1}{4}, y(x) = \frac{1}{-C1} \left( -(e^x)^2 \sqrt{\frac{-C1}{(e^x)^2} + -C1} \right) \frac{1}{\sqrt{\frac{-C1}{(e^x)^2} + -C1}}, y(x) = -\frac{1}{-C1} \left( (e^x)^2 \sqrt{\frac{-C1}{(e^x)^2} + -C1} \right) \frac{1}{\sqrt{\frac{-C1}{(e^x)^2} + -C1}} \right\}$$

**2.390 ODE No. 390**

$$ay(x)y'(x) - bx - c + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 2.81242 (sec), leaf count = 82

$$\text{Solve} \left[ \left\{ \frac{c}{b} + x = \frac{\text{K\$163371} \left( \tan^{-1} \left( \frac{\sqrt{a}\text{K\$163371}}{\sqrt{b-a}\text{K\$163371}^2} \right) + \sqrt{ac_1} \right)}{\sqrt{a}\sqrt{b-a}\text{K\$163371}^2}, y(x) = \frac{bx + c - \text{K\$163371}^2}{a\text{K\$163371}} \right\}, \{y(x), \text{K\$163371}\} \right]$$

✓ **Maple** : cpu = 0.262 (sec), leaf count = 281

$$\left\{ y(x) = 2 \frac{\left( -1/4 \left( e^{2 \operatorname{RootOf}(\sqrt{a} - C1 b e^{2-z} - a e^{2-z} b x + \sqrt{a} - C1 b^2 - e^{2-z} - z b - a e^{2-z} c + a b^2 x - z b^2 + a b c)} + b \right)^2 e^{-2 \operatorname{RootOf}(\sqrt{a} - C1 b e^{2-z} - a e^{2-z} b x + \sqrt{a} - C1 b^2 - e^{2-z} - z b - a e^{2-z} c + a b^2 x - z b^2 + a b c)}}{a^{3/2} \left( e^{2 \operatorname{RootOf}(\sqrt{a} - C1 b e^{2-z} - a e^{2-z} b x + \sqrt{a} - C1 b^2 - e^{2-z} - z b - a e^{2-z} c + a b^2 x - z b^2 + a b c)} + b \right)} \right\}$$

**2.391 ODE No. 391**

$$y'(x)(ay(x) + bx) + abxy(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.00666689 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-ax} \right\}, \left\{ y(x) \rightarrow c_1 - \frac{bx^2}{2} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 22

$$\left\{ y(x) = -C1 e^{-ax}, y(x) = -\frac{bx^2}{2} + -C1 \right\}$$

**2.392 ODE No. 392**

$$y(x)^2 \log(ay(x)) - xy(x)y'(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.246927 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\frac{1}{4}c_1(2x-c_1)}}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.465 (sec), leaf count = 50

$$\left\{ y(x) = \frac{1}{a} e^{\frac{x^2}{4}}, y(x) = \frac{1}{e^{-C1^2} e^{-C1 x a}}, y(x) = \frac{e^{-C1 x}}{e^{-C1^2 a}} \right\}$$

**2.393 ODE No. 393**

$$y'(x)^2 + 2y(x) \cot(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0388865 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow c_1 \csc^2 \left( \frac{x}{2} \right) \right\}, \left\{ y(x) \rightarrow c_1 \sec^2 \left( \frac{x}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 77

$$\left\{ y(x) = \frac{-C1}{\tan(x)} \left( 1 + \sqrt{(\tan(x))^2 + 1} \right) \frac{1}{\sqrt{\frac{(\tan(x))^2}{(\tan(x))^2 + 1}}}, y(x) = \frac{-C1 \left( (\tan(x))^2 + 1 \right)}{\tan(x)} \sqrt{\frac{(\tan(x))^2}{(\tan(x))^2 + 1}} \left( 1 + \sqrt{(\tan(x))^2 + 1} \right) \right.$$

**2.394 ODE No. 394**

$$-(g(x) - f(x)^2) e^{-2 \int_a^x f(xp) dxp} + 2f(x)y(x)y'(x) + g(x)y(x)^2 + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 53.4131 (sec), leaf count = 0 , could not solve

`DSolve[-((-f[x]^2 + g[x])/E^(2*Integrate[f[xp], {xp, a, x}])) + g[x]*y[x]^2 + 2*f[x]*y[x]*Derivative[1][y][x] + Derivative[1][y][x]^2 == 0, y[x]]`

✓ **Maple** : cpu = 5.143 (sec), leaf count = 164

$$\left\{ y(x) = -\tan \left( \frac{1}{2 \cos(2) + 2} \left( \sqrt{2} \int \left( e^{\int_a^x f(xp) dxp} \right)^2 \sqrt{-\frac{(4 \cos(2) + \cos(4) + 3) \left( (f(x))^2 - g(x) \right)}{\left( e^{\int_a^x f(xp) dxp} \right)^4}} dx - 2 \right) \right.$$

**2.395 ODE No. 395**

$$2f(x)y(x)y'(x) + g(x)y(x)^2 + h(x) + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 32.0098 (sec), leaf count = 0 , could not solve

`DSolve[h[x] + g[x]*y[x]^2 + 2*f[x]*y[x]*Derivative[1][y][x] + Derivative[1][y][x]^2 == 0, y[x]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x), x)^2 + 2*f(x)*y(x)*diff(y(x), x) + g(x)*y(x)^2 + h(x) = 0, y(x))`

**2.396 ODE No. 396**

$$(y(x) - x)y(x)y'(x) + y'(x)^2 - xy(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0101554 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{x - c_1} \right\}, \left\{ y(x) \rightarrow c_1 e^{\frac{x^2}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 20

$$\left\{ y(x) = (x + \_C1)^{-1}, y(x) = e^{\frac{x^2}{2}} \_C1 \right\}$$

**2.397 ODE No. 397**

$$-2x^3y(x)^2y'(x) - 4x^2y(x)^3 + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.722035 (sec), leaf count = 136

$$\left\{ \text{Solve} \left[ 4c_1 + \frac{2x\sqrt{x^4y(x) + 4y(x)^{3/2}} \sinh^{-1} \left( \frac{1}{2}x^2\sqrt{y(x)} \right)}{\sqrt{x^2y(x)^3(x^4y(x) + 4)}} + \log(y(x)) = 0, y(x) \right], \text{Solve} \left[ 4c_1 + \log(y(x)) = \frac{2xy(x)}{x^4 - 4} \right] \right\}$$

✓ **Maple** : cpu = 0.353 (sec), leaf count = 128

$$\left\{ y(x) = \frac{-2\sqrt{2}x^2 - 2\_C1}{2\_C1x^4 - \_C1^3}, y(x) = \frac{2\sqrt{2}x^2 - 2\_C1}{2\_C1x^4 - \_C1^3}, y(x) = \frac{(\sqrt{2}x^2 - \_C1 - 2)\_C1^2}{2\_C1^2x^4 - 4}, y(x) = -4x^{-4}, y(x) = -\frac{2xy(x)}{x^4 - 4} \right\}$$

**2.398 ODE No. 398**

$$y'(x)^2 - 3xy(x)^{2/3}y'(x) + 9y(x)^{5/3} = 0$$

✓ **Mathematica** : cpu = 1.24727 (sec), leaf count = 175

$$\left\{ \text{Solve} \left[ \frac{1}{6} \left( -6c_1 + \frac{(x^2 - 4\sqrt[3]{y(x)})^{3/2} y(x)^2 \left( 6 \log \left( \sqrt{x^2 - 4\sqrt[3]{y(x)}} + x \right) - \log(y(x)) \right)}{\left( (x^2 - 4\sqrt[3]{y(x)}) y(x)^{4/3} \right)^{3/2}} + \log(y(x)) \right) = 0, y(x) \right] \right\}$$

✓ **Maple** : cpu = 2.454 (sec), leaf count = 138

$$\left\{ \ln(x) + \frac{1}{6} \ln \left( \frac{y(x)}{x^6} \right) - \frac{1}{6} \ln \left( 4\sqrt[3]{\frac{y(x)}{x^6}} - 1 \right) - 1\sqrt{-4\left(\frac{y(x)}{x^6}\right)^{5/3} + \left(\frac{y(x)}{x^6}\right)^{4/3}} \text{Artanh} \left( \sqrt{-4\sqrt[3]{\frac{y(x)}{x^6}} + 1} \right) \right\}$$



**2.399 ODE No. 399**

$$2y'(x)^2 + (x - 1)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.00486121 (sec), leaf count = 15

$$\{\{y(x) \rightarrow c_1(2c_1 + x - 1)\}\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 22

$$\left\{ y(x) = \_C1(2\_C1 + x - 1), y(x) = -\frac{(x - 1)^2}{8} \right\}$$

**2.400 ODE No. 400**

$$-2x^2y'(x) + 2y'(x)^2 + 3xy(x) = 0$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.19 (sec), leaf count = 74

$$\left\{ y(x) = \frac{x^3}{6}, y(x) = \frac{1}{3\_C1} \left( -\sqrt{-6\_C1}xx + 3 \right), y(x) = \frac{1}{3\_C1} \left( \sqrt{-6\_C1}xx + 3 \right), y(x) = -\frac{x}{3}\sqrt{-6\_C1}x + \dots \right\}$$

**2.401 ODE No. 401**

$$3y'(x)^2 - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.329671 (sec), leaf count = 1093

$$\{\{y(x) \rightarrow \text{Root}[-16e^{6c_1}x^6 + 3\#1^4x^4 + 144e^{6c_1}\#1x^4 - 24\#1^5x^2 - 378e^{6c_1}\#1^2x^2 + 243e^{12c_1} + 48\#1^6 + 216e^{6c_1}]\}\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 580

$$\left\{ y(x) = -\frac{1}{48} \left( i\sqrt{3} \left( -54\_C1 + x^3 + 6\sqrt{-3x^3\_C1 + 81\_C1^2} \right)^{\frac{2}{3}} - i\sqrt{3}x^2 - \left( -54\_C1 + x^3 + 6\sqrt{-3x^3\_C1} \right)^{\frac{2}{3}} \right) \right\}$$

**2.402 ODE No. 402**

$$x^2 + 4xy'(x) + 3y'(x)^2 - y(x) = 0$$

✗ **Mathematica** : cpu = 300.007 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.177 (sec), leaf count = 101

$$\left\{ y(x) = -\frac{x^2}{3}, y(x) = \frac{-3 - C1^2 x^2 - 2\sqrt{3} - C1 x + 3}{12 - C1^2}, y(x) = \frac{-3 - C1^2 x^2 + 2\sqrt{3} - C1 x + 3}{12 - C1^2}, y(x) = -\frac{\sqrt{3} - C1 x}{6} \right.$$

**2.403 ODE No. 403**

$$ay'(x)^2 + by'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.296377 (sec), leaf count = 116

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\sqrt{4\#1a + b^2} + b \log \left( \sqrt{4\#1a + b^2} - b \right)}{2a} \right] \& \left[ \frac{x}{2a} + c_1 \right] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \right.$$

✓ **Maple** : cpu = 0.627 (sec), leaf count = 197

$$\left\{ y(x) = \frac{1}{4a} e^{-\frac{1}{2b} \left( 2 \text{blambertW} \left( 2 \frac{e^{-1}}{b\sqrt{a-1}} e^{\frac{x}{b}} \left( e^{-\frac{C1}{b}} \right)^{-1} \right) + b \ln \left( \frac{1}{4a} \right) + 2 - C1 + 2b - 2x \right)} \left( e^{-\frac{1}{2b} \left( 2 \text{blambertW} \left( 2 \frac{e^{-1}}{b\sqrt{a-1}} e^{\frac{x}{b}} \left( e^{-\frac{C1}{b}} \right)^{-1} \right)} \right)} \right.$$

**2.404 ODE No. 404**

$$ay'(x)^2 + bx^2y'(x) + cxy(x) = 0$$

✗ **Mathematica** : cpu = 300.002 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.266 (sec), leaf count = 389

$$\left\{ \int_{-b}^x 1 \left( -b - a^2 - \sqrt{-a^4 b^2 - 4 - a c y(x)} \right) \left( b - a^3 + \sqrt{-a^4 b^2 - 4 - a c y(x)} - a + 6 a y(x) \right)^{-1} d - a + \int^{y(x)} -2 \frac{1}{b x^3} \right.$$

**2.405 ODE No. 405**

$$ay'(x)^2 + y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 1.44613 (sec), leaf count = 40

$$\text{Solve} \left[ \left\{ x = \frac{\text{K\$222473}(a \sin^{-1}(\text{K\$222473}) + c_1)}{\sqrt{1 - \text{K\$222473}^2}}, a\text{K\$222473} + y(x) = \frac{x}{\text{K\$222473}} \right\}, \{y(x), \text{K\$222473}\} \right]$$

✓ **Maple** : cpu = 0.218 (sec), leaf count = 378

$$\left\{ -C1 \left( y(x) - \sqrt{4ax + (y(x))^2} \right) \frac{1}{\sqrt{\frac{1}{a} \left( -y(x) + \sqrt{4ax + (y(x))^2} + 2a \right)}} \frac{1}{\sqrt{\frac{1}{a} \left( -y(x) + \sqrt{4ax + (y(x))^2} - 2a \right)}} \right\}$$

**2.406 ODE No. 406**

$$ay'(x)^2 - y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 1.12206 (sec), leaf count = 38

$$\text{Solve} \left[ \left\{ x = \frac{\text{K\$223140}(a \sinh^{-1}(\text{K\$223140}) + c_1)}{\sqrt{\text{K\$223140}^2 + 1}}, a\text{K\$223140} = \frac{x}{\text{K\$223140}} + y(x) \right\}, \{y(x), \text{K\$223140}\} \right]$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 262

$$\left\{ 1 \left( -\frac{\sqrt{2}}{2} \left( y(x) + \sqrt{4ax + (y(x))^2} \right) \text{Arcsinh} \left( \frac{1}{2a} \left( y(x) + \sqrt{4ax + (y(x))^2} \right) \right) + x \sqrt{\frac{1}{a^2} \left( y(x) \sqrt{4ax + (y(x))^2} \right)} \right) \right\}$$

**2.407 ODE No. 407**

$$xy'(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0170014 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}(c_1 - 2\sqrt{x})^2 \right\}, \left\{ y(x) \rightarrow \frac{1}{4}(c_1 + 2\sqrt{x})^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 39

$$\left\{ y(x) = 0, y(x) = \frac{1}{x}(-x + \sqrt{-C1 x})^2, y(x) = \frac{1}{x}(x + \sqrt{-C1 x})^2 \right\}$$

**2.408 ODE No. 408**

$$xy'(x)^2 - 2y(x) + x = 0$$

✓ **Mathematica** : cpu = 1.23538 (sec), leaf count = 163

$$\left\{ \text{Solve} \left[ \frac{\left( \sqrt{\frac{2y(x)}{x} - 1} - 1 \right) \left( \left( \sqrt{\frac{2y(x)}{x} - 1} - 1 \right) \log \left( \sqrt{\frac{2y(x)}{x} - 1} - 1 \right) - 1 \right)}{\sqrt{\frac{2y(x)}{x} - 1} - \frac{y(x)}{x}} = c_1 + \log(x), y(x) \right], \text{Solve} \left[ \frac{x \left( \sqrt{\frac{2y(x)}{x} - 1} - 1 \right)}{\sqrt{\frac{2y(x)}{x} - 1} - \frac{y(x)}{x}} = c_1 + \log(x), y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 73

$$\left\{ y(x) = \left( \frac{1}{2} \left( \text{lambertW} \left( \frac{1}{-C1} \sqrt{-C1 x} \right) + 1 \right) \right)^2 \left( \text{lambertW} \left( \frac{1}{-C1} \sqrt{-C1 x} \right) \right)^{-2} + \frac{1}{2} \right\} x, y(x) = \left( \frac{1}{2} \left( \text{lambertW} \left( \frac{1}{-C1} \sqrt{-C1 x} \right) + 1 \right) \right)^2 \left( \text{lambertW} \left( \frac{1}{-C1} \sqrt{-C1 x} \right) \right)^{-2} + \frac{1}{2} \right\}$$

**2.409 ODE No. 409**

$$xy'(x)^2 - 2y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 30.8079 (sec), leaf count = 39

$$\text{Solve} \left[ \left\{ K\$224706x = \frac{y(K\$224706)}{K\$224706} + 2, y(x) = \frac{K\$224706(c_1 K\$224706 - 2K\$224706 \log(K\$224706) - 2)}{(K\$224706 - 1)^2} \right\}, \{y(x) = \dots\} \right]$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 63

$$\left\{ y(x) = xe^{2 \text{RootOf}(-xe^{2-Z} + 2xe^{-Z} + 2e^{-Z} + C1 - 2_Z - x)} - 2e^{\text{RootOf}(-xe^{2-Z} + 2xe^{-Z} + 2e^{-Z} + C1 - 2_Z - x)} \right\}$$

2.410 ODE No. 410

$$xy'(x)^2 + 4y'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 31.0387 (sec), leaf count = 40

$$\text{Solve} \left[ \left\{ \text{K\$225417}x + 4 = \frac{2y(\text{K\$225417})}{\text{K\$225417}}, y(x) = \frac{\text{K\$225417}(c_1\text{K\$225417} + 4\text{K\$225417} \log(\text{K\$225417}) + 8)}{(\text{K\$225417} - 2)^2} \right\}, \{x\} \right]$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 64

$$\left\{ y(x) = \frac{x e^{2 \text{RootOf}(-x e^{2-Z} + 4 x e^{-Z} - 4 e^{-Z} + C1 + 8 - Z - 4 x)}}{2} + 2 e^{\text{RootOf}(-x e^{2-Z} + 4 x e^{-Z} - 4 e^{-Z} + C1 + 8 - Z - 4 x)} \right\}$$

2.411 ODE No. 411

$$xy'(x)^2 + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 1.31124 (sec), leaf count = 180

$$\left\{ \text{Solve} \left[ \frac{x \left( \sqrt{\frac{4y(x)}{x} + 1} - 1 \right) \left( \left( \sqrt{\frac{4y(x)}{x} + 1} - 1 \right) \log \left( \sqrt{\frac{4y(x)}{x} + 1} - 1 \right) - 1 \right)}{2x \left( \sqrt{\frac{4y(x)}{x} + 1} - 1 \right) - 4y(x)} = c_1 + \frac{\log(x)}{2}, y(x) \right], \text{Solve} \left[ \frac{x \left( \sqrt{\frac{4y(x)}{x} + 1} - 1 \right)}{2x \left( \sqrt{\frac{4y(x)}{x} + 1} - 1 \right) - 4y(x)} = c_1 + \frac{\log(x)}{2}, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 65

$$\left\{ y(x) = \frac{x}{4} \left( 1 + 2 \text{lambertW} \left( -1/2 \frac{1}{\sqrt{-C1/x}} \right) \right) \left( \text{lambertW} \left( -\frac{1}{2} \frac{1}{\sqrt{-C1/x}} \right) \right)^{-2}, y(x) = \frac{x}{4} \left( 1 + 2 \text{lambertW} \left( 1/\sqrt{-C1/x} \right) \right) \right\}$$

2.412 ODE No. 412

$$a + xy'(x)^2 + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 27.5637 (sec), leaf count = 11973

✓ **Maple** : cpu = 0.04 (sec), leaf count = 146

$$\left\{ -C1 x^2 \left( \frac{1}{x} \left( -y(x) + \sqrt{(y(x))^2 - 4ax} \right) \right)^{\frac{3}{2}} \left( -y(x) + \sqrt{(y(x))^2 - 4ax} \right)^{-2} + x + \frac{4ax^2}{3} \left( -y(x) + \sqrt{(y(x))^2 - 4ax} \right)^2 \right\}$$

**2.413 ODE No. 413**

$$-x^2 + xy'(x)^2 + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 300.002 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.193 (sec), leaf count = 269

$$\left\{ \int_{-b-a}^x \frac{1}{-y(x) - \sqrt{4a^3 + (y(x))^2}} \left( \sqrt{4a^3 + (y(x))^2} + 4y(x) \right)^{-1} da + \int^{y(x)} 1 \left( -2 + \left( -48f - 12\sqrt{f^2 - 4a^3} \right) \right) dy \right\}$$

**2.414 ODE No. 414**

$$x^3 + xy'(x)^2 + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 0 (sec), leaf count = 0 , crash

Kernel Crash

✓ **Maple** : cpu = 0.199 (sec), leaf count = 269

$$\left\{ \int_{-b-a}^x \frac{1}{-y(x) - \sqrt{-4a^4 + (y(x))^2}} \left( \sqrt{-4a^4 + (y(x))^2} + 5y(x) \right)^{-1} da + \int^{y(x)} 1 \left( -2 + \left( 80f + 16\sqrt{f^2 - 4a^4} \right) \right) dy \right\}$$

**2.415 ODE No. 415**

$$y(x)y'(x) + xy'(x)^2 - y(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.207273 (sec), leaf count = 133

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{\tanh^2\left(\frac{1}{2}(c_1 - \log(x))\right) - 1}}{2\sqrt{x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{\tanh^2\left(\frac{1}{2}(c_1 - \log(x))\right) - 1}}{2\sqrt{x}} \right\}, \left\{ y(x) \rightarrow -\sqrt{\tanh^2\left(\frac{1}{2}(c_1 - \log(x))\right) - 1} \right\} \right.$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 95

$$\left\{ y(x) = -\frac{1}{2} \frac{1}{\sqrt{-x}}, y(x) = \frac{1}{2} \frac{1}{\sqrt{-x}}, y(x) = -\frac{1}{2x} \sqrt{-\left(\tanh\left(-\frac{\ln(x)}{2} + \frac{-C1}{2}\right)\right)^2 x + x \left(\tanh\left(-\frac{\ln(x)}{2} + \frac{-C1}{2}\right)\right)^2} \right.$$

**2.416 ODE No. 416**

$$xy'(x)^2 + (y(x) - 3x)y'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 301.967 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.059 (sec), leaf count = 136

$$\left\{ -\frac{C1}{x} \left( 5x - y(x) + \sqrt{9x^2 - 10xy(x) + (y(x))^2} \right) \left( \frac{1}{x} \left( 3x - y(x) + \sqrt{9x^2 - 10xy(x) + (y(x))^2} \right) \right)^{-\frac{3}{2}} + x \right.$$

**2.417 ODE No. 417**

$$a + xy'(x)^2 - y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.39547 (sec), leaf count = 183

$$\left\{ \left\{ y(x) \rightarrow -\frac{-8a^2 - \sqrt{a(\sinh(2c_1) + \cosh(2c_1))((-4a + x - 1)\sinh\left(\frac{c_1}{2}\right) + (4a - x - 1)\cosh\left(\frac{c_1}{2}\right))^2 + 2a\sinh(2c_1)}}{-4a + \sinh(c_1) + \cosh(c_1)} \right\} \right.$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 35

$$\left\{ y(x) = \frac{x - C1^2 + a}{-C1}, y(x) = -2\sqrt{ax}, y(x) = 2\sqrt{ax} \right\}$$

**2.418 ODE No. 418**

$$ay(x) + xy'(x)^2 - y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 1.26093 (sec), leaf count = 158

$$\left\{ \text{Solve} \left[ \frac{y(x)}{ax} + \frac{\sqrt{\frac{y(x)}{x}} \sqrt{\frac{y(x)}{x} - 4a}}{a} + 4c_1 + 2 \log(x) = 4 \log \left( \sqrt{\frac{y(x)}{x} - 4a} + \sqrt{\frac{y(x)}{x}} \right), y(x) \right], \text{Solve} \left[ \frac{\sqrt{\frac{y(x)}{x}} \sqrt{\frac{y(x)}{x} - 4a}}{a} \right. \right.$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 42

$$\left\{ y(x) = 0, y(x) = -ax \left( \text{lambertW} \left( -\frac{xe}{-C1 a} \right) - 1 \right)^2 \left( \text{lambertW} \left( -\frac{xe}{-C1 a} \right) \right)^{-1} \right\}$$

**2.419 ODE No. 419**

$$xy'(x)^2 + 2y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 1.47432 (sec), leaf count = 6977

$$\left\{ y(x) \rightarrow -\sqrt{\frac{36x^6 + \sqrt{36 \cdot 2^{2/3} (2x^6 - \cosh(6c_1) - \sinh(6c_1)) x^4}}{\sqrt[3]{32x^{12} + 40 \cosh(6c_1)x^6 + 40 \sinh(6c_1)x^6 - \cosh(12c_1) - \sinh(12c_1)} + \sqrt{((16x^6 + 1) \cosh(3c_1) + (1 - 16x^6) \sinh(3c_1))^3 (\cosh(15c_1) + \sinh(15c_1))}}}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 109

$$\left\{ x + \frac{C1}{x} \left( y(x) - \sqrt{(y(x))^2 + x^2} \right) \left( \frac{1}{x^2} \left( 2x^2 + 6(y(x))^2 - 6y(x) \sqrt{(y(x))^2 + x^2} \right) \right)^{-\frac{2}{3}} = 0, \frac{C1}{x} \left( \sqrt{(y(x))^2 + x^2} \right) \right.$$



**2.420 ODE No. 420**

$$a + xy'(x)^2 - 2y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 1.69031 (sec), leaf count = 9391

✓ **Maple** : cpu = 0.038 (sec), leaf count = 689

$$\left\{ y(x) = \frac{x}{12\_C1} \left( 4 \frac{x^2}{\sqrt[3]{-36 a\_C1^2 + 8 x^3 + 12 \sqrt{a (9 a\_C1^2 - 4 x^3)}\_C1}} + 2 x + \sqrt[3]{-36 a\_C1^2 + 8 x^3 + 12 \sqrt{a (9 a\_C1^2 - 4 x^3)}\_C1} \right) \right.$$

**2.421 ODE No. 421**

$$xy'(x)^2 - 2y(x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0319847 (sec), leaf count = 27

$$\{\{y(x) \rightarrow x \sinh(c_1 - \log(x))\}, \{y(x) \rightarrow x \sinh(c_1 + \log(x))\}\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 32

$$\left\{ y(x) = -ix, y(x) = ix, y(x) = -\frac{C1^2 - x^2}{2\_C1} \right\}$$

**2.422 ODE No. 422**

$$xy'(x)^2 - 2y(x)y'(x) + 4x = 0$$

✓ **Mathematica** : cpu = 0.0463495 (sec), leaf count = 29

$$\{\{y(x) \rightarrow 2x \cosh(c_1 - \log(x))\}, \{y(x) \rightarrow 2x \cosh(c_1 + \log(x))\}\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 30

$$\left\{ \left\{ y(x) = -2x, y(x) = 2x, y(x) = \frac{4\_C1^2 + x^2}{2\_C1} \right\} \right\}$$

**2.423 ODE No. 423**

$$xy'(x)^2 - 2y(x)y'(x) + 2y(x) + x = 0$$

✓ **Mathematica** : cpu = 0.0767615 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow -e^{-c_1}x^2 - \frac{e^{c_1}}{2} + x \right\}, \left\{ y(x) \rightarrow -\frac{1}{2}e^{c_1}x^2 - e^{-c_1} + x \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 44

$$\left\{ y(x) = (1 - \sqrt{2})x, y(x) = (1 + \sqrt{2})x, y(x) = \frac{2\_C1^2 + 2\_C1x + x^2}{2\_C1} \right\}$$

**2.424 ODE No. 424**

$$ay(x)y'(x) + bx + xy'(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.7937 (sec), leaf count = 223

$$\left\{ \text{Solve} \left[ \frac{-2a \tan^{-1} \left( \frac{ay(x)}{x\sqrt{4b - \frac{a^2y(x)^2}{x^2}}} \right) + (a+2) \left( 2 \tan^{-1} \left( \frac{(a+2)y(x)}{x\sqrt{4b - \frac{a^2y(x)^2}{x^2}}} \right) - i \log \left( \frac{(a+1)y(x)^2}{x^2} + b \right) \right)}{8(a+1)} = c_1 + \frac{1}{2}i \log \right. \right.$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 193

$$\left\{ \frac{1}{x} \left( -\_C1 \left( ay(x) - \sqrt{a^2(y(x))^2 - 4bx^2} \right) \left( \frac{a}{2x^2} \left( -y(x)(a+1) \sqrt{a^2(y(x))^2 - 4bx^2} + (a^2+a)(y(x))^2 - 2bx \right) \right) \right. \right.$$

**2.425 ODE No. 425**

$$(x+1)y'(x)^2 - (y(x)+x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.257634 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{c_1}(e^{c_1} - 2x)}{2(e^{c_1} + 2)} \right\}, \left\{ y(x) \rightarrow \frac{2e^{c_1}(x - 2e^{c_1})}{2e^{c_1} + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 45

$$\left\{ y(x) = \frac{-C1(-C1x + -C1 - x)}{-C1 - 1}, y(x) = x + 2 - 2\sqrt{1+x}, y(x) = x + 2 + 2\sqrt{1+x} \right\}$$

**2.426 ODE No. 426**

$$(3x+1)y'(x)^2 - 3(y(x)+2)y'(x) + 9 = 0$$

✓ **Mathematica** : cpu = 0.397118 (sec), leaf count = 150

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{(\sinh(2c_1) + \cosh(2c_1)) \left( (3x - 34) \cosh\left(\frac{c_1}{2}\right) - 3(x - 12) \sinh\left(\frac{c_1}{2}\right) \right)^2 + 8 \sinh(c_1) + 8 \cosh(c_1)}}{\sinh(c_1) + \cosh(c_1) - 36} \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 51

$$\left\{ y(x) = \frac{9 + (3x+1)C1^2 - 6C1}{3C1}, y(x) = -2 - 2\sqrt{3x+1}, y(x) = -2 + 2\sqrt{3x+1} \right\}$$

**2.427 ODE No. 427**

$$(3x+5)y'(x)^2 - (3y(x)+x)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.631388 (sec), leaf count = 300

$$\left\{ \left\{ y(x) \rightarrow -\frac{-3e^{\frac{4c_1}{3}}(2x+5) + \sqrt{5}\sqrt{-e^{\frac{4c_1}{3}}\left(e^{\frac{4c_1}{3}} - 12x - 15\right)^2 + 30x + 25}}{18\left(e^{\frac{4c_1}{3}} + 5\right)} \right\}, \left\{ y(x) \rightarrow \frac{3e^{\frac{4c_1}{3}}(2x+5) + \sqrt{5}\sqrt{\dots}}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 60

$$\left\{ y(x) = \frac{(3x+5)C1^2 - C1x}{3C1 - 1}, y(x) = \frac{x}{3} + \frac{10}{9} - \frac{2}{9}\sqrt{15x+25}, y(x) = \frac{x}{3} + \frac{10}{9} + \frac{2}{9}\sqrt{15x+25} \right\}$$

2.428 ODE No. 428

$$y'(x)(-ay(x) + bx + c) + axy'(x)^2 - by(x) = 0$$

✗ **Mathematica** : cpu = 300.033 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.07 (sec), leaf count = 66

$$\left\{ y(x) = \frac{1}{a}(-bx + c - 2\sqrt{-bcx}), y(x) = \frac{1}{a}(-bx + c + 2\sqrt{-bcx}), y(x) = \frac{-C1(-C1ax + bx + c)}{-C1a + b} \right\}$$

2.429 ODE No. 429

$$-y'(x)(ay(x) - a + bx - b) + axy'(x)^2 + by(x) = 0$$

✗ **Mathematica** : cpu = 300.106 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.069 (sec), leaf count = 72

$$\left\{ y(x) = \frac{1}{a}(bx + a + b - 2\sqrt{bx(a+b)}), y(x) = \frac{1}{a}(bx + a + b + 2\sqrt{bx(a+b)}), y(x) = \frac{-C1(-C1ax - bx + a)}{-C1a - b} \right\}$$

2.430 ODE No. 430

$$a0x + y'(x)(a1x + b1y(x) + c1) + (a2x + c2)y'(x)^2 + b0y(x) + c0 = 0$$

✗ **Mathematica** : cpu = 300.006 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.064 (sec), leaf count = 1602

$$\left\{ \frac{1}{2a2x + 2c2} \left( -2 \left( -C1 - 1/2 \int \frac{-a1x - b1y(x) - c1 + \sqrt{b1^2(y(x))^2 + ((2b1a1 - 4a2b0)x - 4b0c2 + 2b1c1)y(x) + (-4a0a2 + a1^2)x^2 + (-4a0c2 + a1^2)}}{2a2x + 2c2} \right) \right) \right\}$$

**2.431 ODE No. 431**

$$x^2 y'(x)^2 - y(x)^4 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0359298 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow \sqrt{\sec^2(c_1 - \log(x))} (-\cot(c_1 - \log(x))) \right\}, \left\{ y(x) \rightarrow \sqrt{\sec^2(c_1 - \log(x))} \cot(c_1 - \log(x)) \right\}, \left\{ y(x) \rightarrow \right.$$

✓ **Maple** : cpu = 0.149 (sec), leaf count = 62

$$\left\{ y(x) = -1, y(x) = 1, y(x) = \frac{1}{\tan(-\ln(x) + \_C1)} \sqrt{(\tan(-\ln(x) + \_C1))^2 + 1}, y(x) = -\frac{1}{\tan(-\ln(x) + \_C1)} \right.$$

**2.432 ODE No. 432**

$$(a + xy'(x))^2 - 2ay(x) + x^2 = 0$$

✓ **Mathematica** : cpu = 1.85483 (sec), leaf count = 54

$$\text{Solve} \left[ \left\{ y(x) = \frac{a^2 + 2a\text{K\$741630}x + (\text{K\$741630}^2 + 1)x^2}{2a}, x = \frac{c_1 - a \sinh^{-1}(\text{K\$741630})}{\sqrt{\text{K\$741630}^2 + 1}} \right\}, \{y(x), \text{K\$741630}\} \right]$$

✓ **Maple** : cpu = 10.948 (sec), leaf count = 242

$$\left\{ y(x) = \frac{1}{2a \left( \left( \text{RootOf} \left( (\text{Arcsinh}(\_Z))^2 a^2 - \_Z^2 x^2 - 2 \text{Arcsinh}(\_Z) \_C1 a + \_C1^2 - x^2 \right) \right)^2 + 1 \right)} \left( -2 a \text{Ro}$$

**2.433 ODE No. 433**

$$-4a - 4x^2 + (xy'(x) + y(x) + 2x)^2 - 4xy(x) = 0$$

✓ **Mathematica** : cpu = 0.50141 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1(c_1 - 2x) - a}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.235 (sec), leaf count = 34

$$\left\{ y(x) = \frac{-x^2 - a}{x}, y(x) = \_C1 + \frac{\_C1^2 - 4a}{4x} \right\}$$

**2.434 ODE No. 434**

$$x^2 y'(x)^2 - x^2 - 2xy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0325646 (sec), leaf count = 27

$$\{ \{y(x) \rightarrow x \sinh(c_1 - \log(x))\}, \{y(x) \rightarrow x \sinh(c_1 + \log(x))\} \}$$

✓ **Maple** : cpu = 0.004 (sec), leaf count = 7

$$\{y(x) = x + \_C1\}$$

**2.435 ODE No. 435**

$$x^2 y'(x)^2 - 2xy(x)y'(x) + y(x)(y(x) + 1) - x = 0$$

✓ **Mathematica** : cpu = 0.0363125 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1^2 x}{4} - ic_1 \sqrt{x} + x - 1 \right\}, \left\{ y(x) \rightarrow \frac{c_1^2 x}{4} + ic_1 \sqrt{x} + x - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.233 (sec), leaf count = 22

$$\left\{ y(x) = x, y(x) = \_C1 \sqrt{x} - \frac{x \_C1^2}{4} + x - 1 \right\}$$

**2.436 ODE No. 436**

$$-x^4 + x^2 y'(x)^2 + (1 - x^2) y(x)^2 - 2xy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0330548 (sec), leaf count = 26

$$\{ \{y(x) \rightarrow -x \sinh(x - c_1)\}, \{y(x) \rightarrow x \sinh(c_1 + x)\} \}$$

✓ **Maple** : cpu = 1.738 (sec), leaf count = 61

$$\left\{ y(x) = -ix, y(x) = ix, y(x) = -\frac{x((e^x)^2 - \_C1^2)}{2e^x \_C1}, y(x) = \frac{x((e^x)^2 \_C1^2 - 1)}{2e^x \_C1} \right\}$$

**2.437 ODE No. 437**

$$-(a + 2xy(x))y'(x) + x^2y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.2708 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{x - 2\sqrt{ac_1}}{4c_1^2} \right\}, \left\{ y(x) \rightarrow \frac{2\sqrt{ac_1} + x}{4c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 36

$$\left\{ y(x) = -\frac{a}{4x}, y(x) = -C1 x - \sqrt{-C1 a}, y(x) = -C1 x + \sqrt{-C1 a} \right\}$$

**2.438 ODE No. 438**

$$x^2y'(x)^2 + 3xy(x)y'(x) + 2y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.00667521 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x^2} \right\}, \left\{ y(x) \rightarrow \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 17

$$\left\{ y(x) = \frac{C1}{x^2}, y(x) = \frac{C1}{x} \right\}$$

**2.439 ODE No. 439**

$$x^2y'(x)^2 + 3xy(x)y'(x) + 3y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0134663 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{-\frac{3}{2} - \frac{i\sqrt{3}}{2}} \right\}, \left\{ y(x) \rightarrow c_1 x^{\frac{1}{2}i(\sqrt{3}+3i)} \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 33

$$\left\{ y(x) = -C1 x^{-\frac{i}{2}\sqrt{3}} x^{-\frac{3}{2}}, y(x) = -C1 x^{\frac{i}{2}\sqrt{3}} x^{-\frac{3}{2}} \right\}$$

**2.440 ODE No. 440**

$$x^2 y'(x)^2 + 4xy(x)y'(x) - 5y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.00649762 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x^5} \right\}, \left\{ y(x) \rightarrow c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 15

$$\left\{ y(x) = \frac{C1}{x^5}, y(x) = -C1 x \right\}$$

**2.441 ODE No. 441**

$$x^2 y'(x)^2 - 4x(y(x) + 2)y'(x) + 4y(x)(y(x) + 2) = 0$$

✓ **Mathematica** : cpu = 0.0683744 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow e^{-c_1 x} \left( x - 2\sqrt{2} e^{\frac{c_1}{2}} \right) \right\}, \left\{ y(x) \rightarrow e^{c_1 x^2} - 2\sqrt{2} e^{\frac{c_1}{2}} x \right\} \right\}$$

✓ **Maple** : cpu = 1.188 (sec), leaf count = 83

$$\left\{ y(x) = -2, y(x) = \frac{1}{-C1} \left( x^2 - 2\sqrt{2}\sqrt{-C1 x^2} \right), y(x) = \frac{1}{-C1} \left( 2\sqrt{2}\sqrt{-C1 x^2} + x^2 \right), y(x) = \frac{x(-2\sqrt{2}-C1 + x)}{-C1^2} \right\}$$

**2.442 ODE No. 442**

$$x^2 y'(x)^2 + (1-x)(y(x)^2 - x^2 y(x)) + (x^3 + x^2 y(x) - 2xy(x)) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.00851512 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-x} \right\}, \left\{ y(x) \rightarrow x(c_1 - x) \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 21

$$\left\{ y(x) = (-x + -C1) x, y(x) = -C1 e^{-x} \right\}$$



**2.443 ODE No. 443**

$$x(xy'(x) - y(x))^2 - y'(x) = 0$$

✓ **Mathematica** : cpu = 0.533008 (sec), leaf count = 1921

$$\{ \{ y(x) \rightarrow \text{Root}[1024x^{12} - 576e^{12c_1}\#1^4x^8 - 2176e^{12c_1}\#1^3x^6 + 81e^{24c_1}\#1^8x^4 - 1536e^{12c_1}\#1^2x^4 + 36e^{24c_1}\#1^7x^2$$

✓ **Maple** : cpu = 1.733 (sec), leaf count = 221

$$\left\{ y(x) = -\frac{2}{9x^2}, y(x) = \frac{(\text{RootOf}(-729\_C1x^{12} + \_Z^8 - 12\_Z^7 + 60\_Z^6 - 160\_Z^5 + 240\_Z^4 - 192\_Z^3 + \dots$$

**2.444 ODE No. 444**

$$x^2y'(x)^2 - (y(x) - 2x)y(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.142518 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow \frac{\sinh(2c_1) - \cosh(2c_1)}{x \sinh(2c_1) + x \cosh(2c_1) - 1} \right\}, \left\{ y(x) \rightarrow \frac{\sinh(2c_1) - \cosh(2c_1)}{x \sinh(2c_1) + x \cosh(2c_1) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.877 (sec), leaf count = 120

$$\left\{ y(x) = \frac{C1^3\sqrt{2} - 2x\_C1^2}{-2\_C1^2 + 4x^2}, y(x) = \frac{-C1^2(\sqrt{2}\_C1 + 2x)}{2\_C1^2 - 4x^2}, y(x) = 4x, y(x) = -2\frac{-C1^2(-\sqrt{2}\_C1 + x)}{-2\_C1^2 + x^2}, y(x) \right\}$$

**2.445 ODE No. 445**

$$y'(x)(ax^2y(x)^3 + b) + aby(x)^3 + x^2y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0090415 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{\sqrt{2ax - 2c_1}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{2ax - 2c_1}} \right\}, \left\{ y(x) \rightarrow \frac{b}{x} + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 35

$$\left\{ y(x) = \frac{1}{\sqrt{2ax + \_C1}}, y(x) = -\frac{1}{\sqrt{2ax + \_C1}}, y(x) = \frac{b}{x} + \_C1 \right\}$$

**2.446 ODE No. 446**

$$(x^2 + 1) y'(x)^2 - 2xy(x)y'(x) + y(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.480779 (sec), leaf count = 167

$$\left\{ \left\{ y(x) \rightarrow -\frac{-e^{2c_1}x + 2e^{c_1} + x}{e^{2c_1} + 1} \right\}, \left\{ y(x) \rightarrow \frac{e^{2c_1}x + 2e^{c_1} - x}{e^{2c_1} + 1} \right\}, \left\{ y(x) \rightarrow \frac{(e^{4c_1} - 1)x + 2\sqrt{-e^{2c_1}(e^{2c_1} - 1)^2}}{(e^{2c_1} - 1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 57

$$\left\{ y(x) = \sqrt{x^2 + 1}, y(x) = -\sqrt{x^2 + 1}, y(x) = \_C1 x - \sqrt{-\_C1^2 + 1}, y(x) = \_C1 x + \sqrt{-\_C1^2 + 1} \right\}$$

**2.447 ODE No. 447**

$$(x^2 - 1) y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0156761 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 - \log(\sqrt{x^2 - 1} + x) \right\}, \left\{ y(x) \rightarrow c_1 + \log(\sqrt{x^2 - 1} + x) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 33

$$\left\{ y(x) = -\ln(x + \sqrt{x^2 - 1}) + \_C1, y(x) = \ln(x + \sqrt{x^2 - 1}) + \_C1 \right\}$$

**2.448 ODE No. 448**

$$(x^2 - 1) y'(x)^2 - y(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.0843593 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}e^{-c_1} \left( (e^{2c_1} + 1)x - (e^{2c_1} - 1)\sqrt{x^2 - 1} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2}e^{-c_1} \left( (e^{2c_1} - 1)\sqrt{x^2 - 1} + (e^{2c_1} + 1)x \right) \right\} \right\}$$

✓ **Maple** : cpu = 266.143 (sec), leaf count = 166

$$\left\{ 1\sqrt{(-1 + y(x))(1 + y(x))} \ln\left(y(x) + \sqrt{(y(x))^2 - 1}\right) \frac{1}{\sqrt{-1 + y(x)}} \frac{1}{\sqrt{1 + y(x)}} + \int^x \frac{1}{-a^2 - 1} \sqrt{(-a^2 - 1)} \left( (y(x) \right) \right. \right.$$

**2.449 ODE No. 449**

$$(x^2 - a^2) y'(x)^2 + 2xy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0103637 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{a-x} \right\}, \left\{ y(x) \rightarrow \frac{c_1}{a+x} \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 23

$$\left\{ y(x) = \frac{-C1}{a-x}, y(x) = \frac{-C1}{x+a} \right\}$$

**2.450 ODE No. 450**

$$(x^2 - a^2) y'(x)^2 - x^2 - 2xy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.43678 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \frac{a^2 + c_1^2 - x^2}{2c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.455 (sec), leaf count = 51

$$\left\{ y(x) = \sqrt{a^2 - x^2}, y(x) = -\sqrt{a^2 - x^2}, y(x) = \_C1 x^2 - \_C1 a^2 - \frac{1}{4\_C1} \right\}$$

**2.451 ODE No. 451**

$$(a + x^2) y'(x)^2 + b - 2xy(x)y'(x) + y(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.391 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.054 (sec), leaf count = 78

$$\left\{ y(x) = \frac{1}{a} \sqrt{-ab(x^2 + a)}, y(x) = -\frac{1}{a} \sqrt{-ab(x^2 + a)}, y(x) = \_C1 x - \sqrt{-a\_C1^2 - b}, y(x) = \_C1 x + \sqrt{-a\_C1^2 - b} \right\}$$

**2.452 ODE No. 452**

$$(2x^2 + 1) y'(x)^2 + (x^2 + 2xy(x) + y(x)^2 + 2) y'(x) + 2y(x)^2 + 1 = 0$$

✗ **Mathematica** : cpu = 300.737 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.744 (sec), leaf count = 37

$$\left\{ y(x) = -3x - 2\sqrt{2x^2 + 1}, y(x) = -3x + 2\sqrt{2x^2 + 1} \right\}$$

**2.453 ODE No. 453**

$$(a^2 - 1) x^2 y'(x)^2 + a^2 x^2 + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.22687 (sec), leaf count = 369

$$\left\{ \text{Solve} \left[ \frac{a \left( 2 \log(x - a^2 x) - \log \left( \frac{(a^2 - 1) \left( y(x) + ix \left( a \sqrt{a^2 - \frac{y(x)^2}{x^2} - 1} + a^2 - 1 \right) \right)}{a^3 (x + iy(x))} \right) \right) + \log \left( \frac{i(a^2 - 1) \left( x \left( a \sqrt{a^2 - \frac{y(x)^2}{x^2} - 1} + a^2 - 1 \right) \right)}{a^3 (x - iy(x))} \right)}{2(a^2 - 1)} \right. \right.$$

✓ **Maple** : cpu = 0.742 (sec), leaf count = 229

$$\left\{ \frac{1}{2a} \left( -2\_C1 a + 2a \ln(x) + \ln \left( \frac{(y(x))^2 + x^2}{x^2} \right) a - 2\sqrt{-a^2} \arctan \left( \frac{a^2 y(x)}{\sqrt{-a^2} x} \frac{1}{\sqrt{\frac{(y(x))^2 + (-a^2 + 1)x^2}{x^2}}} \right) + 2 \ln \left( \frac{1}{x} \right) \right. \right.$$

**2.454 ODE No. 454**

$$ax^2 y'(x)^2 - (a - 1)ax^2 - 2axy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.150596 (sec), leaf count = 113

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-c_1} x^{1 - \sqrt{\frac{a-1}{a}}} \left( e^{2c_1} - ax^2 \sqrt{\frac{a-1}{a}} \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} e^{c_1} x^{\sqrt{\frac{a-1}{a}} + 1} - \frac{1}{2} a e^{-c_1} x^{1 - \sqrt{\frac{a-1}{a}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.123 (sec), leaf count = 106

$$\left\{ y(x) = \sqrt{-ax}, y(x) = \text{RootOf} \left( -\ln(x) - \int^{-Z} \frac{1}{(a-1)(-a^2+a)} \sqrt{(a-1)(-a^2+a)} ad\_a + \_C1 \right) x, y(x) = \dots \right.$$

**2.455 ODE No. 455**

$$a + x^3 y'(x)^2 + x^2 y(x) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.386812 (sec), leaf count = 123

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{-\frac{c_1}{2}}(2ae^{c_1} + x)}{\sqrt{2x}} \right\}, \left\{ y(x) \rightarrow \frac{e^{-\frac{c_1}{2}}(2ae^{c_1} + x)}{\sqrt{2x}} \right\}, \left\{ y(x) \rightarrow -\frac{e^{-\frac{c_1}{2}}(2ax + e^{c_1})}{\sqrt{2x}} \right\}, \left\{ y(x) \rightarrow \frac{e^{-\frac{c_1}{2}}(2ax + e^{c_1})}{\sqrt{2x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.18 (sec), leaf count = 66

$$\left\{ y(x) = -2 \frac{\sqrt{ax}}{x}, y(x) = 2 \frac{\sqrt{ax}}{x}, y(x) = \frac{C1^2 + 4ax}{2\_C1 x}, y(x) = \frac{x\_C1^2 + 4a}{2\_C1 x} \right\}$$

**2.456 ODE No. 456**

$$2(1 - x^2) y(x) y'(x) + x(x^2 - 1) y'(x)^2 + xy(x)^2 - x = 0$$

✓ **Mathematica** : cpu = 0.117533 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow x \cos \left( 2 \tan^{-1} \left( \sqrt{\frac{x-1}{x+1}} \right) + ic_1 \right) \right\}, \left\{ y(x) \rightarrow x \cos \left( 2 \tan^{-1} \left( \sqrt{\frac{x-1}{x+1}} \right) - ic_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.392 (sec), leaf count = 33

$$\left\{ y(x) = x, y(x) = -x, y(x) = \sqrt{-\_C1^2 + 1} + \sqrt{x^2 - 1}\_C1 \right\}$$

**2.457 ODE No. 457**

$$x^4 y'(x)^2 - xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 1.55681 (sec), leaf count = 406

$$\left\{ \text{Solve} \left[ \frac{2x\sqrt{4x^2y(x)+1}\log(x) + x\sqrt{4x^2y(x)+1}\log(y(x)) - x\sqrt{4x^2y(x)+1}\log(4x^2y(x)+1) - 2x\sqrt{4x^2y(x)+1}}{4x^2y(x)+1} \right] \right\}$$

✓ **Maple** : cpu = 0.778 (sec), leaf count = 45

$$\left\{ y(x) = \frac{i\_C1 - x}{x\_C1^2}, y(x) = \frac{-i\_C1 - x}{x\_C1^2}, y(x) = -\frac{1}{4x^2} \right\}$$

2.458 ODE No. 458

$$x^2(x^2 - a^2)y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0617069 (sec), leaf count = 139

$$\left\{ \left\{ y(x) \rightarrow c_1 - \frac{ix\sqrt{x^2 - a^2} \log\left(\frac{2(\sqrt{x^2 - a^2} - ia)}{x}\right)}{a\sqrt{x^4 - a^2x^2}} \right\}, \left\{ y(x) \rightarrow c_1 + \frac{ix\sqrt{x^2 - a^2} \log\left(\frac{2(\sqrt{x^2 - a^2} - ia)}{x}\right)}{a\sqrt{x^4 - a^2x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 90

$$\left\{ y(x) = -1 \ln\left(\frac{1}{x}(-2a^2 + 2\sqrt{-a^2}\sqrt{-a^2 + x^2})\right) \frac{1}{\sqrt{-a^2}} + _C1, y(x) = 1 \ln\left(\frac{1}{x}(-2a^2 + 2\sqrt{-a^2}\sqrt{-a^2 + x^2})\right) \right\}$$

2.459 ODE No. 459

$$-(y'(x) - 1)^2 + e^{-2x}y'(x)^2 + e^{-2y(x)} = 0$$

✗ **Mathematica** : cpu = 300.86 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.614 (sec), leaf count = 65

$$\left\{ y(x) = x + \ln\left(\frac{1}{e^x}\left(-1 - \sqrt{(e^x)^2 - \frac{(e^x)^2}{(e^{-C1})^2}}\right)\right) + _C1, y(x) = x + \ln\left(\frac{1}{e^x}\left(-1 + \sqrt{(e^x)^2 - \frac{(e^x)^2}{(e^{-C1})^2}}\right)\right) + _C1 \right\}$$

2.460 ODE No. 460

$$\cos^4(x)(y'(x)^2 + y(x)^2) - a^2 = 0$$

✗ **Mathematica** : cpu = 55.6 (sec), leaf count = 0 , could not solve

`DSolve[-a^2 + Cos[x]^4*(y[x]^2 + Derivative[1][y][x]^2) == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((diff(y(x), x)^2 + y(x)^2)*cos(x)^4 - a^2 = 0, y(x))`

**2.461 ODE No. 461**

$$a(x)y'(x)^2 + 2b(x)y(x)y'(x) + c(x)y(x)^2 + 2d(x)y'(x) + 2e(x)y(x) + f(x) = 0$$

✗ **Mathematica** : cpu = 300.093 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(d0(x)\*diff(y(x),x)^2+2\*b0(x)\*y(x)\*diff(y(x),x)+c0(x)\*y(x)^2+2\*d0(x)\*diff(y(x),x)+2\*e0(x)\*y(x)+f0(x)=0)

**2.462 ODE No. 462**

$$y(x)y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.0154201 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \left( \frac{3}{2} \right)^{2/3} (c_1 - x)^{2/3} \right\}, \left\{ y(x) \rightarrow \left( \frac{3}{2} \right)^{2/3} (c_1 + x)^{2/3} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 27

$$\left\{ x - \frac{2}{3}(y(x))^{\frac{3}{2}} - \_C1 = 0, x + \frac{2}{3}(y(x))^{\frac{3}{2}} - \_C1 = 0 \right\}$$

**2.463 ODE No. 463**

$$y(x)y'(x)^2 - e^{2x} = 0$$

✓ **Mathematica** : cpu = 0.0173387 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \left( \frac{3}{2} \right)^{2/3} (c_1 - e^x)^{2/3} \right\}, \left\{ y(x) \rightarrow \left( \frac{3}{2} \right)^{2/3} (c_1 + e^x)^{2/3} \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 50

$$\left\{ -1\sqrt{y(x)(e^x)^2} \frac{1}{\sqrt{y(x)}} + \frac{2}{3}(y(x))^{\frac{3}{2}} + \_C1 = 0, 1\sqrt{y(x)(e^x)^2} \frac{1}{\sqrt{y(x)}} + \frac{2}{3}(y(x))^{\frac{3}{2}} + \_C1 = 0 \right\}$$

**2.464 ODE No. 464**

$$y(x)y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0600474 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow -e^{\frac{c_1}{2}} \sqrt{e^{c_1} - 2x} \right\}, \left\{ y(x) \rightarrow e^{\frac{c_1}{2}} \sqrt{e^{c_1} - 2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.443 (sec), leaf count = 70

$$\left\{ y(x) = \sqrt{-C1^2 - 2\_C1 x}, y(x) = \sqrt{-C1^2 + 2\_C1 x}, y(x) = -ix, y(x) = ix, y(x) = -\sqrt{-C1 (2x + \_C1)}, y(x) = \sqrt{-C1 (2x + \_C1)} \right\}$$

**2.465 ODE No. 465**

$$y(x)y'(x)^2 + 2xy'(x) - 9y(x) = 0$$

✗ **Mathematica** : cpu = 300.65 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.07 (sec), leaf count = 210

$$\left\{ -C1 x \left( x + \sqrt{x^2 + 9 (y(x))^2} \right) \left( \frac{1}{y(x)} \left( -x - \sqrt{x^2 + 9 (y(x))^2} \right) \right)^{\frac{2}{7}} \left( x \sqrt{x^2 + 9 (y(x))^2} + x^2 + (y(x))^2 \right)^{-1} \right\}$$

**2.466 ODE No. 466**

$$y(x)y'(x)^2 - 2xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.243837 (sec), leaf count = 119

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-(\sinh(c_1) + \cosh(c_1)) (\sinh(c_1) + \cosh(c_1) + 2x)} \right\}, \left\{ y(x) \rightarrow \sqrt{-(\sinh(c_1) + \cosh(c_1)) (\sinh(c_1) + \cosh(c_1) + 2x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.391 (sec), leaf count = 71

$$\left\{ y(x) = x, y(x) = \sqrt{-C1^2 - 2ix\_C1}, y(x) = \sqrt{-C1^2 + 2ix\_C1}, y(x) = -x, y(x) = -\sqrt{-C1^2 - 2ix\_C1}, y(x) = \sqrt{-C1^2 - 2ix\_C1} \right\}$$



2.467 ODE No. 467

$$y(x)y'(x)^2 - 4xy'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 300.11 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.069 (sec), leaf count = 148

$$\left\{ -\frac{C1 x}{y(x)} \frac{1}{\sqrt[3]{\frac{1}{(y(x))^2} \left( 8x^2 - 4(y(x))^2 - 4x\sqrt{4x^2 - (y(x))^2} \right)}} \frac{1}{\sqrt[3]{\frac{1}{y(x)} \left( 2x - \sqrt{4x^2 - (y(x))^2} \right)}} + x = 0, -\frac{C1}{y(x)} \right.$$

2.468 ODE No. 468

$$-4a^2xy'(x) + a^2y(x) + y(x)y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.238 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.081 (sec), leaf count = 181

$$\left\{ -\frac{C1 x}{ay(x)} \frac{1}{\sqrt[3]{\frac{a}{y(x)} \left( 2ax + \sqrt{4a^2x^2 - (y(x))^2} \right)}} \frac{1}{\sqrt[3]{\frac{a^2}{(y(x))^2} \left( 2a^2x^2 + \sqrt{4a^2x^2 - (y(x))^2}ax - (y(x))^2 \right)}} + x = 0, -\frac{C1}{ay(x)} \right.$$

2.469 ODE No. 469

$$axy'(x) + by(x) + y(x)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.632101 (sec), leaf count = 245

$$\left\{ \text{Solve} \left[ \frac{1}{8} \left( \frac{2a \tanh^{-1} \left( \frac{\sqrt{a^2 - \frac{4by(x)^2}{x^2}}}{a} \right) - 2(a+2b) \tanh^{-1} \left( \frac{\sqrt{a^2 - \frac{4by(x)^2}{x^2}}}{a+2b} \right) + a \log \left( a + b + \frac{y(x)^2}{x^2} \right) + 2b \log \left( a + b + \frac{y(x)^2}{x^2} \right)}{a+b} \right) \right]$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 264

$$\left\{ \frac{x}{(y(x))^2} \left( -C1 \left( -\frac{1}{2y(x)} \left( ax + \sqrt{a^2x^2 - 4b(y(x))^2} \right) \right) \right)^{-\frac{a}{a+b}} \left( ax + \sqrt{a^2x^2 - 4b(y(x))^2} \right) \left( \frac{a}{2(y(x))^2} \left( ax^2 + \dots \right) \right) \right.$$

2.470 ODE No. 470

$$x^3 y'(x) - x^2 y(x) + y(x) y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.29 (sec), leaf count = 87

$$\left\{ y(x) = -\frac{i}{2}x^2, y(x) = \frac{i}{2}x^2, y(x) = -\frac{1}{4}\sqrt{-4\_C1 x^2 + \_C1^2}, y(x) = \frac{1}{4}\sqrt{-4\_C1 x^2 + \_C1^2}, y(x) = -2\frac{\sqrt{-C1}}{\_C1} \right\}$$

2.471 ODE No. 471

$$y(x)y'(x)^2 - (y(x) - x)y'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.00931508 (sec), leaf count = 47

$$\left\{ \{y(x) \rightarrow c_1 + x\}, \{y(x) \rightarrow -\sqrt{2c_1 - x^2}\}, \{y(x) \rightarrow \sqrt{2c_1 - x^2}\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{-x^2 + \_C1}, y(x) = -\sqrt{-x^2 + \_C1}, y(x) = x + \_C1 \right\}$$

2.472 ODE No. 472

$$(y(x) + x)y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.184077 (sec), leaf count = 121

$$\left\{ \left\{ y(x) \rightarrow -\frac{2}{3}\sqrt{e^{c_1}(e^{c_1} - 3x)} - \frac{e^{c_1}}{3} \right\}, \left\{ y(x) \rightarrow \frac{2}{3}\sqrt{e^{c_1}(e^{c_1} - 3x)} - \frac{e^{c_1}}{3} \right\}, \left\{ y(x) \rightarrow e^{c_1} - 2\sqrt{e^{c_1}(e^{c_1} + x)} \right\}, \left\{ y(x) \rightarrow e^{c_1} + 2\sqrt{e^{c_1}(e^{c_1} + x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.475 (sec), leaf count = 121

$$\left\{ \ln(x) - \operatorname{Artanh}\left(\frac{y(x) + 2x}{2x} \frac{1}{\sqrt{\frac{(y(x))^2 + xy(x) + x^2}{x^2}}}\right) + \ln\left(\frac{y(x)}{x}\right) - \_C1 = 0, \ln(x) + \operatorname{Artanh}\left(\frac{y(x) + 2x}{2x} \frac{1}{\sqrt{\frac{(y(x))^2 + xy(x) + x^2}{x^2}}}\right) + \ln\left(\frac{y(x)}{x}\right) - \_C1 = 0 \right\}$$

**2.473 ODE No. 473**

$$(y(x) - 2x)y'(x)^2 - 2(x - 1)y'(x) + y(x) - 2 = 0$$

✓ **Mathematica** : cpu = 0.378289 (sec), leaf count = 137

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}\sqrt{-e^{c_1}(e^{c_1} + 4x - 4)} - \frac{e^{c_1}}{2} + 2 \right\}, \left\{ y(x) \rightarrow \frac{1}{2}\left(\sqrt{-e^{c_1}(e^{c_1} + 4x - 4)} - e^{c_1} + 4\right) \right\}, \left\{ y(x) \rightarrow -\sqrt{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.641 (sec), leaf count = 71

$$\left\{ y(x) = 2 + \_C1 - \sqrt{\_C1(-\_C1 + 2x - 2)}, y(x) = 2 + \frac{\_C1}{2} - \frac{1}{2}\sqrt{\_C1(-\_C1 + 4x - 4)}, y(x) = (x - 1) \right\}$$

**2.474 ODE No. 474**

$$2y(x)y'(x)^2 - (4x - 5)y'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.226992 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow -i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{8e^{c_1} + 4x - 5} \right\}, \left\{ y(x) \rightarrow i\sqrt{2}e^{\frac{c_1}{2}}\sqrt{8e^{c_1} + 4x - 5} \right\}, \left\{ y(x) \rightarrow -\frac{1}{4}ie^{\frac{c_1}{2}}\sqrt{e^{c_1} + 8x - 10} \right\}, \left\{ y(x) \rightarrow \dots \right\} \right\}$$

✓ **Maple** : cpu = 1.041 (sec), leaf count = 152

$$\left\{ \ln\left(x - \frac{5}{4}\right) + \frac{1}{2}\ln\left(16\frac{(y(x))^2}{(4x - 5)^2} - 1\right) + \sqrt{-16\frac{(y(x))^2}{(4x - 5)^2} + 1} - \text{Artanh}\left(\frac{1}{\sqrt{-16\frac{(y(x))^2}{(4x - 5)^2} + 1}}\right) - \frac{\sqrt{4}}{2}\sqrt{\frac{1}{(4x - 5)^2}} \right\}$$

**2.475 ODE No. 475**

$$4y(x)y'(x)^2 + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0668289 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}e^{2c_1}\sqrt{e^{4c_1} - 2x} \right\}, \left\{ y(x) \rightarrow \frac{1}{2}e^{2c_1}\sqrt{e^{4c_1} - 2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.454 (sec), leaf count = 67

$$\left\{ y(x) = \sqrt{\_C1^2 - \_C1 x}, y(x) = \sqrt{\_C1^2 + \_C1 x}, y(x) = -\frac{i}{2}x, y(x) = \frac{i}{2}x, y(x) = -\sqrt{\_C1(x + \_C1)}, y(x) = \dots \right\}$$

2.476 ODE No. 476

$$4x^3y'(x) - 4x^2y(x) + 9y(x)y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.304 (sec), leaf count = 87

$$\left\{ y(x) = -\frac{i}{3}x^2, y(x) = \frac{i}{3}x^2, y(x) = -\frac{1}{6}\sqrt{-4\_C1 x^2 + \_C1^2}, y(x) = \frac{1}{6}\sqrt{-4\_C1 x^2 + \_C1^2}, y(x) = -2\frac{\sqrt{-C1}}{\_C1} \right\}$$

2.477 ODE No. 477

$$ay(x)y'(x)^2 + (2x - b)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.304121 (sec), leaf count = 136

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{\frac{c_1}{2}}\sqrt{2b + e^{c_1} - 4x}}{2\sqrt{a}} \right\}, \left\{ y(x) \rightarrow \frac{e^{\frac{c_1}{2}}\sqrt{2b + e^{c_1} - 4x}}{2\sqrt{a}} \right\}, \left\{ y(x) \rightarrow -e^{\frac{c_1}{2}}\sqrt{4(ae^{c_1} + x) - 2b} \right\}, \left\{ y(x) \rightarrow e^{\frac{c_1}{2}}\sqrt{4(ae^{c_1} + x) - 2b} \right\} \right\}$$

✓ **Maple** : cpu = 0.517 (sec), leaf count = 622

$$\left\{ \int_{-b}^x 1 \left( -4\_a + 2b - 2\sqrt{4a(y(x))^2 + (b - 2\_a)^2} \right) \left( (2\_a - b)\sqrt{4a(y(x))^2 + (b - 2\_a)^2} + 4a(y(x))^2 + (b - 2\_a)^2 \right) dx \right\}$$

2.478 ODE No. 478

$$(y'(x)^2 + 1)(ay(x) + b) - c = 0$$

✓ **Mathematica** : cpu = 0.167065 (sec), leaf count = 141

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{c \tan^{-1} \left( \frac{\sqrt{\#1a+b}}{\sqrt{-\#1a-b+c}} \right) - \sqrt{\#1a+b}\sqrt{-\#1a-b+c}}{a} \& \right] [c_1 - x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{c \tan^{-1} \left( \frac{\sqrt{\#1a+b}}{\sqrt{-\#1a-b+c}} \right) + \sqrt{\#1a+b}\sqrt{-\#1a-b+c}}{a} \& \right] [c_1 - x] \right\} \right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 88

$$\left\{ x - \int^{y(x)} (\_a a + b) \frac{1}{\sqrt{-(\_a a + b)(\_a a + b - c)}} d\_a - \_C1 = 0, x - \int^{y(x)} -(\_a a + b) \frac{1}{\sqrt{-(\_a a + b)(\_a a + b - c)}} d\_a - \_C1 = 0 \right\}$$

**2.479 ODE No. 479**

$$a_0x + y'(x)(a_1x + b_1y(x) + c_1) + y'(x)^2(a_2x + b_2y(x) + c_2) + b_0y(x) + c_0 = 0$$

✗ **Mathematica** : cpu = 300.031 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.304 (sec), leaf count = 929

$$\left\{ x - e^{\int \frac{1}{2b_2y(x)+2a_2x+2c_2} \left( -a_1x - b_1y(x) - c_1 + \sqrt{-4a_0a_2x^2 - 4a_0b_2xy(x) + a_1^2x^2 + 2a_1b_1xy(x) - 4a_2b_0xy(x) - 4b_0b_2(y(x))^2 + b_1^2(y(x))^2 - 4a_0c_2x + c_1^2} \right) dx} \right.$$

**2.480 ODE No. 480**

$$(ay(x) - x^2) y'(x)^2 + 2xy(x)y'(x)^2 - y(x)^2 = 0$$

✗ **Mathematica** : cpu = 30.5828 (sec), leaf count = 0 , could not solve

`DSolve[-y[x]^2 + 2*x*y[x]*Derivative[1][y][x]^2 + (-x^2 + a*y[x])*Derivative[1][y][x]^2 == 0`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((a*y(x)-x^2)*diff(y(x),x)^2+2*x*y(x)*diff(y(x),x)^2-y(x)^2 = 0,y(x))`

**2.481 ODE No. 481**

$$(x^2 + y(x)^2) y'(x) + xy(x)y'(x)^2 + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.00980946 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x} \right\}, \left\{ y(x) \rightarrow -\sqrt{2c_1 - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{2c_1 - x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 35

$$\left\{ y(x) = \sqrt{-x^2 + \_C1}, y(x) = \frac{-C1}{x}, y(x) = -\sqrt{-x^2 + \_C1} \right\}$$

**2.482 ODE No. 482**

$$(a + x^{22} - y(x)^2) y'(x) + xy(x)y'(x)^2 - xy(x) = 0$$

✗ **Mathematica** : cpu = 63.1457 (sec), leaf count = 0 , could not solve

`DSolve[-(x*y[x]) + (a + x^22 - y[x]^2)*Derivative[1][y][x] + x*y[x]*Derivative[1][y][x]^2 ==`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x*y(x)*diff(y(x),x)^2+(x^22-y(x)^2+a)*diff(y(x),x)-x*y(x) = 0,y(x))`

**2.483 ODE No. 483**

$$(2xy(x) - x^2) y'(x)^2 + 2xy(x)y'(x) - y(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.159806 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{c_1}{2}} - \sqrt{x \left( 2e^{\frac{c_1}{2}} - x \right)} \right\}, \left\{ y(x) \rightarrow \sqrt{x \left( 2e^{\frac{c_1}{2}} - x \right)} + e^{\frac{c_1}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.08 (sec), leaf count = 103

$$\left\{ y(x) = 0, y(x) = \text{RootOf} \left( -2 \ln(x) + \int^{-Z} \frac{1}{-a(-a^2+1)} \left( -2-a^2 + \sqrt{2} \sqrt{-a(-a-1)^2} \right) d_a + 2_C1 \right) x, y \right\}$$

**2.484 ODE No. 484**

$$(2xy(x) - x^2) y'(x)^2 - 6xy(x)y'(x) - y(x)^2 + 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.153447 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{x \left( 3x - 2e^{\frac{c_1}{2}} \right)} - e^{\frac{c_1}{2}} + 2x \right\}, \left\{ y(x) \rightarrow \sqrt{x \left( 3x - 2e^{\frac{c_1}{2}} \right)} - e^{\frac{c_1}{2}} + 2x \right\} \right\}$$

✓ **Maple** : cpu = 0.08 (sec), leaf count = 115

$$\left\{ y(x) = 0, y(x) = \text{RootOf} \left( -2 \ln(x) + \int^{-Z} \frac{1}{-a(-a^2-4_a+1)} \left( -2-a^2 + \sqrt{2} \sqrt{-a(-a+1)^2} + 4_a \right) d_a \right) \right\}$$

**2.485 ODE No. 485**

$$-y'(x) (ay(x)^2 + bx^2 + c) + axy(x)y'(x)^2 + bxy(x) = 0$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.486 ODE No. 486**

$$-a^2 + y(x)^2 y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0265232 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{a^2 - (c_1 + x)^2} \right\}, \left\{ y(x) \rightarrow \sqrt{a^2 - (c_1 + x)^2} \right\}, \left\{ y(x) \rightarrow -\sqrt{a^2 - (x - c_1)^2} \right\}, \left\{ y(x) \rightarrow \sqrt{a^2 - (x - c_1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 54

$$\left\{ y(x) = a, y(x) = \sqrt{-\_C1^2 + 2\_C1 x + a^2 - x^2}, y(x) = -a, y(x) = -\sqrt{(a - \_C1 + x)(a + \_C1 - x)} \right\}$$

**2.487 ODE No. 487**

$$-6x^3 y'(x) + 4x^2 y(x) + y(x)^2 y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.417 (sec), leaf count = 100

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \int^{-Z} -\frac{3}{4\_a(4\_a^3 - 9)} \left( 4\_a^3 + 3\sqrt{-4\_a^3 + 9} - 9 \right) d\_a + \_C1 \right) x^{\frac{4}{3}}, y(x) = \frac{\sqrt[3]{1}}{2} \right\}$$

2.488 ODE No. 488

$$4a^2 - 4ay(x)y'(x) - 4ax + y(x)^2y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.362329 (sec), leaf count = 85

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1^2}}{2a} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{16a^3x - 4a^2x^2 - 4ac_1x - c_1^2}}{2a} \right\} \right\}$$

✓ **Maple** : cpu = 0.434 (sec), leaf count = 111

$$\left\{ y(x) = -2\sqrt{ax}, y(x) = 2\sqrt{ax}, y(x) = -\frac{1}{4a}\sqrt{-16a^4 + 32a^3x + (-16x^2 + 8\_C1)a^2 + 8\_C1ax - \_C1^2}, y(x) = \frac{1}{4a}\sqrt{-16a^4 + 32a^3x + (-16x^2 + 8\_C1)a^2 + 8\_C1ax - \_C1^2} \right\}$$

2.489 ODE No. 489

$$ay(x)^2 + bx + c + y(x)^2y'(x)^2 + 2xy(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 304.237 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.375 (sec), leaf count = 551

$$\left\{ y(x) = -\frac{\sqrt{16}}{2a(a+1)}\sqrt{\left((a+1)^2\left(ax - \frac{b}{2} + x\right)^2 aRootOf\left(-b\ln(2ax - b + 2x) + 2\int^{-Z} -1/4 \frac{1}{(a+1)\_a(4}\right)\right)} \right\}$$

2.490 ODE No. 490

$$a - x^2 - 2xy(x)y'(x) + y(x)^2y'(x)^2 + 2y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.579728 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-\frac{a}{2} + 4c_1x - 2c_1^2 - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{-\frac{a}{2} + 4c_1x - 2c_1^2 - x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.434 (sec), leaf count = 145

$$\left\{ y(x) = \sqrt{-2\sqrt{a + 2\_C1x - \_C1} - x^2 - a}, y(x) = \sqrt{2\sqrt{a + 2\_C1x - \_C1} - x^2 - a}, y(x) = -\sqrt{-2\sqrt{a + 2\_C1x - \_C1} - x^2 - a} \right\}$$



**2.491 ODE No. 491**

$$(a - 1)b + ax^2 + 2axy(x)y'(x) + (1 - a)y(x)^2 + y(x)^2y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.01032 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-2(a-1)c_1x + (a-1)c_1^2 + b - x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{-2(a-1)c_1x + (a-1)c_1^2 + b - x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.75 (sec), leaf count = 195

$$\left\{ y(x) = \sqrt{-ax^2 + b}, y(x) = \frac{1}{a} \sqrt{\left(-2x\sqrt{-a(b - C1)(a-1)} + (-x^2 + b)a - b + C1\right)}, a, y(x) = \frac{1}{a} \sqrt{\left(2x\sqrt{-a(b - C1)(a-1)} + (-x^2 + b)a - b + C1\right)} \right\}$$

**2.492 ODE No. 492**

$$(y(x)^2 - a^2)y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.271795 (sec), leaf count = 111

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction}\left[\sqrt{a^2 - \#1^2} - a \log\left(a\sqrt{a^2 - \#1^2} + a^2\right) + a \log(\#1)\right] [c_1 - x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction}\left[\sqrt{a^2 - \#1^2} + a \log\left(a\sqrt{a^2 - \#1^2} + a^2\right) + a \log(\#1)\right] [c_1 - x] \right\} \right\}$$

✓ **Maple** : cpu = 0.421 (sec), leaf count = 122

$$\left\{ x - \sqrt{a^2 - (y(x))^2} + a^2 \ln\left(\frac{1}{y(x)}\left(2a^2 + 2\sqrt{a^2}\sqrt{a^2 - (y(x))^2}\right)\right) \frac{1}{\sqrt{a^2}} - C1 = 0, x + \sqrt{a^2 - (y(x))^2} - a^2 \ln\left(\frac{1}{y(x)}\left(2a^2 + 2\sqrt{a^2}\sqrt{a^2 - (y(x))^2}\right)\right) \frac{1}{\sqrt{a^2}} - C1 = 0 \right\}$$

**2.493 ODE No. 493**

$$(a^2 - 2ax + y(x)^2)y'(x)^2 + 2ay(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 15.8438 (sec), leaf count = 393

$$\left\{ \text{Solve}\left[\left\{ y(x) = -\frac{\sqrt{-aK\$779473^2\left(aK\$779473^2 - 2\left(K\$779473^2 + 1\right)x\right) + aK\$779473}}{K\$779473^2 + 1}, x = \frac{a\left(c_1^2K\$779473^2 - a^2\right)}{K\$779473^2 + 1} \right\} \right] \right\}$$

✓ **Maple** : cpu = 1.113 (sec), leaf count = 111

$$\left\{ [x(-T) = \frac{1}{2a} \left( \left( \text{Artanh}\left(\frac{1}{\sqrt{-T^2 + 1}}\right) \right)^2 \sqrt{-T^2 + 1} a^2 + (-2a - C1) \sqrt{-T^2 + 1} - 2a^2 \right) \text{Artanh}\left(\frac{1}{\sqrt{-T^2 + 1}}\right) \right\}$$

**2.494 ODE No. 494**

$$(y(x)^2 - a^2 x^2) y'(x)^2 + (1 - a^2) x^2 + 2xy(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 301.079 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.173 (sec), leaf count = 161

$$\left\{ y(x) = \sqrt{a^2 - 1}x, y(x) = \text{RootOf}\left(-\ln(x) + \int^{-Z} \frac{1}{(\_a^2 + 1)(\_a^2 - a^2 + 1)} \left(-\_a^3 + \_a a^2 + \sqrt{\_a^2 a^2 - a^4 + \dots}\right) \right) \right.$$

**2.495 ODE No. 495**

$$((1 - a)x^2 + y(x)^2) y'(x)^2 + 2axy(x)y'(x) + (1 - a)y(x)^2 + x^2 = 0$$

✗ **Mathematica** : cpu = 300.388 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.465 (sec), leaf count = 61

$$\left\{ y(x) = \tan\left(\text{RootOf}\left(-2\_Z \sqrt{a - 1} - \ln\left(\frac{x^2}{(\cos(\_Z))^2}\right) + 2\_C1\right)\right) x, y(x) = \tan\left(\text{RootOf}\left(2\_Z \sqrt{a - 1} - \dots\right)\right) \right.$$

**2.496 ODE No. 496**

$$(y(x) - x)^2 (y'(x)^2 + 1) - a^2 (y'(x) + 1)^2 = 0$$

✓ **Mathematica** : cpu = 95.1652 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 - \sqrt{a^2 - (x - c_1)^2} \right\}, \left\{ y(x) \rightarrow \sqrt{a^2 - (x - c_1)^2} + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.241 (sec), leaf count = 130

$$\left\{ y(x) = x - \sqrt{2}a, y(x) = x + \sqrt{2}a, y(x) = x + \text{RootOf}\left(-x + \int^{-Z} -\frac{1}{2\_a^2 - 4a^2} \left(-a^2 - 2a^2 + \sqrt{\_a^4 + 2\_a^2 - \dots}\right) \right) \right.$$

**2.497 ODE No. 497**

$$-x^2 - 2xy(x)y'(x) + 3y(x)^2y'(x)^2 + 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.171613 (sec), leaf count = 203

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-4ix \sinh(3c_1) - 4ix \cosh(3c_1) + \sinh(6c_1) + \cosh(6c_1) - 3x^2}}{\sqrt{3}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-4ix \sinh(3c_1) - 4ix \cosh(3c_1) + \sinh(6c_1) + \cosh(6c_1) - 3x^2}}{\sqrt{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.462 (sec), leaf count = 203

$$\left\{ \ln(x) - \frac{1}{2} \sqrt{\frac{x^2 - 3(y(x))^2}{x^2}} + \operatorname{Artanh}\left(\frac{1}{2} \sqrt{\frac{x^2 - 3(y(x))^2}{x^2}}\right) + \frac{\sqrt{3}}{6} \sqrt{\frac{(\sqrt{3}x - 3y(x))(\sqrt{3}x + 3y(x))}{x^2}} + \frac{1}{2} \ln\left(\frac{\sqrt{3}x - 3y(x)}{\sqrt{3}x + 3y(x)}\right) \right\}$$

**2.498 ODE No. 498**

$$(3y(x) - 2)y'(x)^2 + 4y(x) - 4 = 0$$

✓ **Mathematica** : cpu = 0.103357 (sec), leaf count = 107

$$\left\{ \left\{ y(x) \rightarrow \operatorname{InverseFunction}\left[-\sqrt{1 - \#1} \sqrt{3\#1 - 2} - \frac{\sin^{-1}(\sqrt{3 - 3\#1})}{\sqrt{3}}\right] [c_1 - 2x] \right\}, \left\{ y(x) \rightarrow \operatorname{InverseFunction}\left[\sqrt{1 - \#1} \sqrt{3\#1 - 2} - \frac{\sin^{-1}(\sqrt{3 - 3\#1})}{\sqrt{3}}\right] [c_1 - 2x] \right\} \right\}$$

✓ **Maple** : cpu = 0.696 (sec), leaf count = 99

$$\left\{ y(x) = 1, y(x) = \frac{\sin\left(\operatorname{RootOf}\left(-8\sqrt{3}C1_Z + 8\sqrt{3}x_Z - (\cos(_Z))^2 + 48C1^2 - 96C1x + 48x^2 + _Z\right)\right)}{6} \right\}$$

**2.499 ODE No. 499**

$$a^2(-x^2) - 2a^2xy(x)y'(x) + (1 - a^2)y(x)^2y'(x)^2 + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.286948 (sec), leaf count = 148

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{e^{-2c_1} \left( (a^2 - 1)^3 (-e^{2c_1}) x^2 + 2(a^2 - 1) x e^{(a^2+1)c_1} + e^{2a^2c_1} \right)}}{\sqrt{(a^2 - 1)^3}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{e^{-2c_1} \left( (a^2 - 1)^3 (-e^{2c_1}) x^2 + 2(a^2 - 1) x e^{(a^2+1)c_1} + e^{2a^2c_1} \right)}}{\sqrt{(a^2 - 1)^3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.19 (sec), leaf count = 189

$$\left\{ y(x) = \operatorname{RootOf}\left(-\ln(x) + \int^{-Z} \frac{-a}{(a^2 + 1)(a^2 a^2 - a^2 + a^2)} \left(-a^2 a^2 + a^2 - a^2 + \sqrt{-a^2 a^2 - a^2 + a^2}\right) dZ\right) \right\}$$

**2.500 ODE No. 500**

$$(a - b)y(x)^2 y'(x)^2 - ab + ay(x)^2 - bx^2 - 2bxy(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 1.29245 (sec), leaf count = 86

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a((x - c_1)^2 - b) + b(b - x^2)}}{\sqrt{b - a}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a((x - c_1)^2 - b) + b(b - x^2)}}{\sqrt{b - a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.936 (sec), leaf count = 220

$$\left\{ y(x) = \frac{1}{b} \sqrt{b(2x\sqrt{-ab(b - C1)} + (-x^2 + C1 + a)b - C1a)}, y(x) = \frac{1}{b} \sqrt{(-2x\sqrt{-ab(b - C1)} + (-x^2 + C1 + a)b - C1a)} \right\}$$

**2.501 ODE No. 501**

$$y'(x)^2 (ay(x)^2 + bx + c) - by(x)y'(x) + dy(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.489 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 4.984 (sec), leaf count = 215

$$\left\{ [x(-T) = -\frac{1}{4bd} \left( \sqrt{-T^2a + d} \left( \ln \left( \frac{1}{-T} \left( \sqrt{d} \sqrt{-T^2a + d} + d \right) \right) \right)^2 b^2 + \left( (2 \ln(2) b^2 + 4 \sqrt{d} C1 b) \sqrt{-T^2a + d} \right) \right] \right\}$$

**2.502 ODE No. 502**

$$(ay(x) - bx)^2 (a^2 y'(x)^2 + b^2) - c^2 (ay'(x) + b)^2 = 0$$

✓ **Mathematica** : cpu = 1.70754 (sec), leaf count = 71

$$\left\{ \left\{ y(x) \rightarrow \frac{bc_1 - \sqrt{c^2 - b^2(x - c_1)^2}}{a} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{c^2 - b^2(x - c_1)^2} + bc_1}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.39 (sec), leaf count = 195

$$\left\{ y(x) = \frac{bx - \sqrt{2}c}{a}, y(x) = \frac{bx + \sqrt{2}c}{a}, y(x) = \frac{1}{a} \left( \text{RootOf} \left( -x + \int^{-Z} \frac{a}{(2 - a^2 a^2 - 4 c^2) b} (-a^2 a^2 + 2 c^2 + \sqrt{-a^2}) \right) \right) \right\}$$

**2.503 ODE No. 503**

$$a_0 + y'(x)(a_1x + b_1y(x) + c_1) + y'(x)^2(a_2x + b_2y(x) + c_2)^2 + b_0y(x) + c_0 = 0$$

✗ **Mathematica** : cpu = 300.041 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((b2\*y(x)+a2\*x+c2)^2\*diff(y(x),x)^2+(a1\*x+b1\*y(x)+c1)\*diff(y(x),x)+b0\*y(x)+a0+c0=0,y(x))

**2.504 ODE No. 504**

$$-(-a + x^3 + y(x)^3) y'(x) + x^2 y(x) + x y(x)^2 y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.824 (sec), leaf count = 247

$$\left\{ \int_{-b}^{y(x)} -a^2 \frac{1}{\sqrt{x^6 + (-2a^3 - 2a)x^3 + (-a^3 + a)^2}} dx - \frac{\ln(x)}{2} - C1 = 0, \int_{-b}^{y(x)} -a^2 \frac{1}{\sqrt{x^6 + (-2a^3 - 2a)x^3 + (-a^3 + a)^2}} dx - \frac{\ln(x)}{2} - C1 = 0 \right\}$$

**2.505 ODE No. 505**

$$-x^3 + x y(x)^2 y'(x)^2 - 2 y(x)^3 y'(x) + 2 x y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0144896 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{2c_1 + x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{2c_1 + x^2} \right\}, \left\{ y(x) \rightarrow -\sqrt{c_1 x^4 + x^2} \right\}, \left\{ y(x) \rightarrow \sqrt{c_1 x^4 + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 52

$$\left\{ y(x) = \sqrt{x^2 + C1}, y(x) = \sqrt{-C1 x^2 + 1x}, y(x) = -\sqrt{x^2 + C1}, y(x) = -\sqrt{-C1 x^2 + 1x} \right\}$$

**2.506 ODE No. 506**

$$2x^2(y(x) - x)y(x)^2y'(x) + x^2(xy(x)^2 - 1)y'(x)^2 - (x^2y(x) - 1)y(x)^2 = 0$$

✗ **Mathematica** : cpu = 63.0111 (sec), leaf count = 0 , could not solve

DSolve[-(y[x]^2\*(-1 + x^2\*y[x])) + 2\*x^2\*y[x]^2\*(-x + y[x])\*Derivative[1][y][x] + x^2\*(-1 + x\*y[x]^2)\*Derivative[1][y][x]^2 == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(x^2\*(x\*y(x)^2-1)\*diff(y(x),x)^2+2\*x^2\*y(x)^2\*(y(x)-x)\*diff(y(x),x)-y(x)^2\*(x^2\*y(x)-1)=0,y(x))

**2.507 ODE No. 507**

$$(y(x)^4 - a^2x^2)y'(x)^2 + 2a^2xy(x)y'(x) + y(x)^2(y(x)^2 - a^2) = 0$$

✓ **Mathematica** : cpu = 32.4331 (sec), leaf count = 408

$$\left\{ \text{Solve} \left[ \left[ x = \frac{K\$931688y(K\$931688) - \sqrt{a^2(K\$931688^4 + K\$931688^2)y(K\$931688)^4}}{K\$931688^2}, ac_1\sqrt{K\$931688} + \frac{2a\sqrt[4]{\frac{1}{K\$931688^2}}}{\sqrt[4]{K\$931688^2}} \right] \right. \right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception  
time expired

**2.508 ODE No. 508**

$$(x^2y(x)^2 - x^2 + y(x)^4)y'(x)^2 + 2xy(x)y'(x) - y(x)^2 = 0$$

✗ **Mathematica** : cpu = 55.151 (sec), leaf count = 0 , could not solve

DSolve[-y[x]^2 + 2\*x\*y[x]\*Derivative[1][y][x] + (-x^2 + x^2\*y[x]^2 + y[x]^4)\*Derivative[1][y][x]^2 == 0, y[x], x]

✓ **Maple** : cpu = 1.838 (sec), leaf count = 60

$$\left\{ y(x) = -ix, y(x) = ix, y(x) = -\text{Artanh}\left(\text{RootOf}\left(\left(\text{Artanh}(\_Z)\right)^2 \_Z^2 - 2 \text{Artanh}(\_Z) \_C1 \_Z^2 + \_C1^2 \_Z^2\right)\right) \right.$$

**2.509 ODE No. 509**

$$9(x^2 - 1)y(x)^4 y'(x)^2 - 4x^2 - 6xy(x)^5 y'(x) = 0$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.46 (sec), leaf count = 212

$$\left\{ y(x) = \sqrt[6]{-4x^2 + 4}, y(x) = -\sqrt[6]{-4x^2 + 4}, y(x) = -\frac{1 + i\sqrt{3}}{2} \sqrt[6]{-4x^2 + 4}, y(x) = \frac{1 + i\sqrt{3}}{2} \sqrt[6]{-4x^2 + 4}, y(x) = \dots \right.$$

**2.510 ODE No. 510**

$$-(x^4 y(x)^2 - 1)y(x)^2 + x^2(x^2 y(x)^4 - 1)y'(x)^2 + 2x^3(y(x)^2 - x^2)y(x)^3 y'(x) = 0$$

✗ **Mathematica** : cpu = 60.5607 (sec), leaf count = 0 , could not solve

DSolve[-(y[x]^2\*(-1 + x^4\*y[x]^2)) + 2\*x^3\*y[x]^3\*(-x^2 + y[x]^2)\*Derivative[1][y][x] + x^2\*1 + x^2\*y[x]^4)\*Derivative[1][y][x]^2 == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(x^2\*(x^2\*y(x)^4-1)\*diff(y(x),x)^2+2\*x^3\*y(x)^3\*(y(x)^2-x^2)\*diff(y(x),x)-y(x)^2\*(x^4\*y(x)^2-1)=0,y(x))

**2.511 ODE No. 511**

$$(a^2 \sqrt{x^2 + y(x)^2} - x^2) y'(x)^2 + a^2 \sqrt{x^2 + y(x)^2} + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.8612 (sec), leaf count = 225

$$\left\{ \text{Solve} \left[ \tan^{-1} \left( \frac{x}{y(x)} \right) = \frac{2a\sqrt{x^2 + y(x)^2} \sqrt{\sqrt{x^2 + y(x)^2} - a^2} \tan^{-1} \left( \frac{\sqrt{\sqrt{x^2 + y(x)^2} - a^2}}{a} \right)}{\sqrt{a^2(x^2 + y(x)^2) (\sqrt{x^2 + y(x)^2} - a^2)}} + c_1, y(x) \right], \text{Solve} \left[ \dots \right. \right.$$

✓ **Maple** : cpu = 5.059 (sec), leaf count = 199

$$\left\{ \arctan \left( \frac{x}{y(x)} \right) - 2 \frac{\sqrt{a^2((y(x))^2 + x^2) (-a^2 + \sqrt{(y(x))^2 + x^2})}}{a\sqrt{(y(x))^2 + x^2} \sqrt{-a^2 + \sqrt{(y(x))^2 + x^2}}} \arctan \left( \frac{\sqrt{-a^2 + \sqrt{(y(x))^2 + x^2}}}{a} \right) - \dots \right.$$

**2.512 ODE No. 512**

$$(a(x^2 + y(x)^2)^{3/2} - x^2) y'(x)^2 + a(x^2 + y(x)^2)^{3/2} + 2xy(x)y'(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 11.7543 (sec), leaf count = 725

$$\left\{ \text{Solve} \left[ \tan^{-1} \left( \frac{x}{y(x)} \right) - \frac{i\sqrt{a}(x^2 + y(x)^2) \sqrt{\sqrt{x^2 + y(x)^2} - a(x^2 + y(x)^2)} \left( \sqrt{2} \left( \log \left( \frac{a^{3/2} (3i\sqrt{2}a\sqrt{x^2 + y(x)^2} + 4\sqrt{a}}{4a\sqrt{x}} \right) \right) \right)}{\right. \right.$$

✓ **Maple** : cpu = 11.819 (sec), leaf count = 135

$$\left\{ y(x) = x \left( \tan \left( \text{RootOf} \left( -Z + \int \frac{x^2((\tan(Z))^2 + 1)}{(\tan(Z))^2} - \frac{1}{(2_a a^2 - 2)_a^2} (\sqrt{-aa} + 1) \sqrt{-a^{\frac{5}{2}} a (\sqrt{-aa} - 1) d_a} \right) \right) \right)$$

**2.513 ODE No. 513**

$$y'(x)^2 \sin(y(x)) + 2xy'(x) \cos^3(y(x)) - \sin(y(x)) \cos^4(y(x)) = 0$$

✗ **Mathematica** : cpu = 300.032 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.854 (sec), leaf count = 1134

$$\left\{ x(-T) = \frac{1}{2_{-T}} \left( \cos \left( \frac{1}{2} \arctan \left( 1 \left( -C1^2_{-T} - 2_{-T}_{-C1} \sqrt[3]{-C1^3_{-T} + 54_{-T}_{-C1} + 6\sqrt{3}\sqrt{-C1^2_{-T}}} \right) \right) \right)$$



**2.514 ODE No. 514**

$$y'(x)^2(a \cos(y(x)) + b) - c \cos(y(x)) + d = 0$$

✓ **Mathematica** : cpu = 15.1971 (sec), leaf count = 605

$$\left\{ \left\{ \begin{array}{l} y(x) \rightarrow \text{InverseFunction} \left[ \frac{4 \sin^2\left(\frac{\#1}{2}\right) \csc(\#1) \sqrt{a \cos(\#1) + b} \sqrt{\frac{\cot^2\left(\frac{\#1}{2}\right)(c-d)}{c+d}} \sqrt{\frac{\csc^2\left(\frac{\#1}{2}\right)(a+b)(d-c \cos(\#1))}{ad+bc}} \left( c(a \cos(\#1) + b) - c \cos(\#1) + d \right)}{c(a \cos(\#1) + b) - c \cos(\#1) + d} \right] \end{array} \right. \right.$$

✓ **Maple** : cpu = 0.228 (sec), leaf count = 87

$$\left\{ x - \int^{y(x)} (a \cos(\_a) + b) \frac{1}{\sqrt{(a \cos(\_a) + b)(c \cos(\_a) - d)}} d\_a - \_C1 = 0, x - \int^{y(x)} -(a \cos(\_a) + b) \frac{1}{\sqrt{(a \cos(\_a) + b)(c \cos(\_a) - d)}} d\_a - \_C1 = 0 \right.$$

**2.515 ODE No. 515**

$$f(x^2 + y(x)^2) (y'(x)^2 + 1) - (xy'(x) - y(x))^2 = 0$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.286 (sec), leaf count = 113

$$\left\{ y(x) = x \left( \tan \left( \text{RootOf} \left( -\_Z + \int \frac{x^2 ((\tan(\_Z))^2 + 1)}{(\tan(\_Z))^2} - \frac{1}{2\_a (f(\_a) - \_a)} \sqrt{-(f(\_a) - \_a) f(\_a) d\_a + \_C1} \right) \right) \right)$$

**2.516 ODE No. 516**

$$(x^2 + y(x)^2) f\left(\frac{x}{\sqrt{x^2 + y(x)^2}}\right) (y'(x)^2 + 1) - (xy'(x) - y(x))^2 = 0$$

✓ **Mathematica** : cpu = 3.94272 (sec), leaf count = 229

$$\left\{ \text{Solve} \left[ c_1 = \log(x) + \int_1^{\frac{y(x)}{x}} \frac{(K[1]^2 + 1) f\left(\frac{1}{\sqrt{K[1]^2 + 1}}\right) - 1}{(K[1] - i)(K[1] + i) \sqrt{f\left(\frac{1}{\sqrt{K[1]^2 + 1}}\right)} \left( K[1] \sqrt{f\left(\frac{1}{\sqrt{K[1]^2 + 1}}\right)} + i \sqrt{f\left(\frac{1}{\sqrt{K[1]^2 + 1}}\right)} - 1 \right)} \right]$$

✓ **Maple** : cpu = 1.141 (sec), leaf count = 72

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \int^{-Z} -\frac{1}{-a^2+1} \left( -a f \left( \frac{1}{\sqrt{-a^2+1}} \right) - \sqrt{-\left( f \left( \frac{1}{\sqrt{-a^2+1}} \right) \right)^2 + f \left( \frac{1}{\sqrt{-a^2+1}} \right)} \right) \right.$$

**2.517 ODE No. 517**

$$(x^2 + y(x)^2) f \left( \frac{y(x)}{\sqrt{x^2 + y(x)^2}} \right) (y'(x)^2 + 1) - (xy'(x) - y(x))^2 = 0$$

✓ **Mathematica** : cpu = 4.0333 (sec), leaf count = 253

$$\left\{ \text{Solve} \left[ c_1 = \log(x) + \int_1^{\frac{y(x)}{x}} \frac{(K[1]^2 + 1) f \left( \frac{K[1]}{\sqrt{K[1]^2 + 1}} \right) - 1}{(K[1] - i)(K[1] + i) \sqrt{f \left( \frac{K[1]}{\sqrt{K[1]^2 + 1}} \right)} \left( K[1] \sqrt{f \left( \frac{K[1]}{\sqrt{K[1]^2 + 1}} \right)} + i \sqrt{f \left( \frac{K[1]}{\sqrt{K[1]^2 + 1}} \right)} - 1 \right)} \right.$$

✓ **Maple** : cpu = 1.198 (sec), leaf count = 155

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \int^{-Z} \frac{1}{-a^2+1} \left( -a f \left( -a \frac{1}{\sqrt{-a^2+1}} \right) + \sqrt{-\left( f \left( -a \frac{1}{\sqrt{-a^2+1}} \right) \right)^2 + f \left( -a \frac{1}{\sqrt{-a^2+1}} \right)} \right) \right.$$

**2.518 ODE No. 518**

$$y'(x)^3 - (y(x) - a)^2(y(x) - b)^2 = 0$$

✓ **Mathematica** : cpu = 0.781937 (sec), leaf count = 236

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{3 \sqrt[3]{a - \#1} \left( \frac{\#1 - b}{a - b} \right)^{2/3} {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; \frac{a - \#1}{a - b} \right)}{(b - \#1)^{2/3}} \right] \& [c_1 + x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \right.$$

✓ **Maple** : cpu = 0.195 (sec), leaf count = 126

$$\left\{ x - \int^{y(x)} \frac{1}{\sqrt[3]{(-a-a)^2(-b-a)^2}} d_{-a} - C1 = 0, x - \int^{y(x)} -2 \frac{1}{(1+i\sqrt{3}) \sqrt[3]{(-a+a)^2(b-a)^2}} d_{-a} - C2 = 0 \right.$$

**2.519 ODE No. 519**

$$y'(x)^3 - f(x) (ay(x)^2 + by(x) + c)^2 = 0$$

✓ **Mathematica** : cpu = 1.88514 (sec), leaf count = 473

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{3 \left( 2\#1a - \sqrt{b^2 - 4ac} + b \right) \left( \frac{2\#1a + \sqrt{b^2 - 4ac} + b}{\sqrt{b^2 - 4ac}} \right)^{2/3} {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{-b - 2a\#1 + \sqrt{b^2 - 4ac}}{2\sqrt{b^2 - 4ac}} \right)}{2 \cdot 2^{2/3} a (\#1(\#1a + b) + c)^{2/3}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.423 (sec), leaf count = 197

$$\left\{ \int^{y(x)} (\_a^2 a + \_a b + c)^{-\frac{2}{3}} d\_a + \int^x -1 \sqrt[3]{f(\_a) (a(y(x))^2 + by(x) + c)^2 (a(y(x))^2 + by(x) + c)^{-\frac{2}{3}}} d\_a + \dots \right\}$$

**2.520 ODE No. 520**

$$y'(x)^3 + y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 232.636 (sec), leaf count = 3323

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\left( 243\#1^2 - 27\sqrt{81\#1^2 + 12}\#1 - 24\sqrt[3]{2}\sqrt[6]{3} \tan^{-1} \left( \frac{1}{\sqrt{3}} - \left( \frac{2}{3} \right)^{2/3} \sqrt[3]{\sqrt{81\#1^2 + 12}} \right)}{\dots} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.178 (sec), leaf count = 245

$$\left\{ x - \int^{y(x)} 6 \frac{\sqrt[3]{108\_a + 12\sqrt{81\_a^2 + 12}}}{(108\_a + 12\sqrt{81\_a^2 + 12})^{2/3} - 12} d\_a - \_C1 = 0, x - \int^{y(x)} -12 \frac{\dots}{(1 + i\sqrt{3}) \left( \sqrt[3]{108\_a + 12\sqrt{81\_a^2 + 12}} \right)} d\_a \dots \right\}$$

**2.521 ODE No. 521**

$$y'(x)^3 + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.00432108 (sec), leaf count = 14

$$\{\{y(x) \rightarrow c_1(c_1^2 + x)\}\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 33

$$\left\{y(x) = -C1(-C1^2 + x), y(x) = -\frac{2x}{9}\sqrt{-3x}, y(x) = \frac{2x}{9}\sqrt{-3x}\right\}$$

**2.522 ODE No. 522**

$$y'(x)^3 - (x + 5)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.00434508 (sec), leaf count = 17

$$\{\{y(x) \rightarrow c_1(-c_1^2 + x + 5)\}\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 44

$$\left\{y(x) = -C1(-C1^2 + x + 5), y(x) = -\frac{2x + 10}{9}\sqrt{3x + 15}, y(x) = \frac{2x + 10}{9}\sqrt{3x + 15}\right\}$$

**2.523 ODE No. 523**

$$-axy'(x) + x^3 + y'(x)^3 = 0$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.057 (sec), leaf count = 231

$$\left\{y(x) = \int i \left( \left( \frac{i}{12} - \frac{\sqrt{3}}{12} \right) \left( -108x^3 + 12\sqrt{-12a^3x^3 + 81x^6} \right)^{\frac{2}{3}} + (\sqrt{3} + i)xa \right) \frac{1}{\sqrt[3]{-108x^3 + 12\sqrt{-12a^3x^3 + 81x^6}}} dx \right\}$$

**2.524 ODE No. 524**

$$y'(x)^3 - 2y(x)y'(x) + y(x)^2 = 0$$

✗ **Mathematica** : cpu = 0 (sec), leaf count = 0 , crash

Kernel Crash

✓ **Maple** : cpu = 0.058 (sec), leaf count = 261

$$\left\{ x - \int^{y(x)} 6 \frac{\sqrt[3]{-108_a^2 + 12\sqrt{3}\sqrt{27_a^4 - 32_a^3}}}{(-108_a^2 + 12\sqrt{3}\sqrt{27_a^4 - 32_a^3})^{2/3} + 24_a} d_a - C1 = 0, x - \int^{y(x)} 24 \frac{1}{(i\sqrt{3} - 1) \left( (i\sqrt{3} \right)} \right.$$

**2.525 ODE No. 525**

$$-axy(x)y'(x) + 2ay(x)^2 + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0637903 (sec), leaf count = 121

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{\frac{1}{4}(ax^2 + \sqrt{ax}\sqrt{ax^2-8})}}{(\sqrt{a}\sqrt{ax^2-8} + ax)^2} \right\}, \left\{ y(x) \rightarrow ac_1 e^{\frac{1}{4}(ax^2 - \sqrt{ax}\sqrt{ax^2-8})} (\sqrt{ax^2-8} + \sqrt{ax})^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 122

$$\left\{ y(x) = C1 \left( a^2 x \frac{1}{\sqrt{a^2}} + \sqrt{a^2 x^2 - 8a} \right)^{-2 \frac{a}{\sqrt{a^2}}} e^{\frac{x}{4} (ax + \sqrt{a^2 x^2 - 8a})}, y(x) = C1 \left( a^2 x \frac{1}{\sqrt{a^2}} + \sqrt{a^2 x^2 - 8a} \right)^2 \frac{a}{\sqrt{a^2}} \right\}$$

**2.526 ODE No. 526**

$$-x^3y(x)^3 - (x^2 + xy(x) + y(x)^2) y'(x)^2 + (x^3y(x) + x^2y(x)^2 + xy(x)^3) y'(x) + y'(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.0100629 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{c_1 + x} \right\}, \left\{ y(x) \rightarrow c_1 e^{\frac{x^2}{2}} \right\}, \left\{ y(x) \rightarrow c_1 + \frac{x^3}{3} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 32

$$\left\{ y(x) = (-x + C1)^{-1}, y(x) = e^{\frac{x^2}{2}} C1, y(x) = \frac{x^3}{3} + C1 \right\}$$

2.527 ODE No. 527

$$-xy(x)^4y'(x) + y'(x)^3 - y(x)^5 = 0$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.847 (sec), leaf count = 43

$$\left\{ y(x) = \sqrt{\frac{-C1^{10}}{(-C1^4x - 1)^2} - C1}, y(x) = -\frac{3\sqrt{3}}{2}x^{-\frac{3}{2}}, y(x) = \frac{3\sqrt{3}}{2}x^{-\frac{3}{2}} \right\}$$

2.528 ODE No. 528

$$abx + ay'(x)^2 + by(x) + y'(x)^3 = 0$$

✗ **Mathematica** : cpu = 300.008 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.072 (sec), leaf count = 86

$$\left\{ y(x) = -ax - \frac{\left( e^{\text{RootOf}(-2a^2_Z - 3e^{-Z} + 8ae^{-Z} + 2_C1 b - 5a^2 - 2bx)} - a \right)^2 e^{\text{RootOf}(-2a^2_Z - 3e^{-Z} + 8ae^{-Z} + 2_C1 b - 5a^2 - 2bx)}}{b} \right\}$$

2.529 ODE No. 529

$$y'(x)^3 + xy'(x)^2 - y(x) = 0$$

✓ **Mathematica** : cpu = 53.6486 (sec), leaf count = 1496

$$\left\{ \left\{ y(x) \rightarrow \frac{-16x^4 + 8 \left( \sqrt[3]{-8x^3 - 36x^2 - 54x + 108c_1 + 6\sqrt{6}\sqrt{(2c_1 + 1)(-4x^3 - 18x^2 - 27x + 27c_1)} + 27 - 12 \right)}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 1251

$$\left\{ y(x) = 0, y(x) = 1 \left( (-8x - 6) \sqrt[3]{-36x^2 - 54x + 108_C1 - 8x^3 + 27 + 6\sqrt{-6(1 + 2_C1)(4x^3 + 18x^2 - 27x + 27c_1)}} \right) \right\}$$

**2.530 ODE No. 530**

$$y'(x)^3 - y(x)y'(x)^2 + y(x)^2 = 0$$

✗ **Mathematica** : cpu = 0 (sec), leaf count = 0 , crash

Kernel Crash

✓ **Maple** : cpu = 0.12 (sec), leaf count = 432

$$\left\{ x - \int^{y(x)} 6 \frac{\sqrt[3]{-108 a^2 + 8 a^3 + 12 \sqrt{-12 a^5 + 81 a^4}}}{4 a^2 + 2 a \sqrt[3]{-108 a^2 + 8 a^3 + 12 \sqrt{-12 a^5 + 81 a^4}} + (-108 a^2 + 8 a^3 + 12 \sqrt{-12 a^5 + 81 a^4}} \right.$$

**2.531 ODE No. 531**

$$-x^3y(x)^6 - (x^2 + y(x)^4 + xy(x)^2) y'(x)^2 + (x^3y(x)^2 + x^2y(x)^4 + xy(x)^6) y'(x) + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 70.3251 (sec), leaf count = 0 , could not solve

`DSolve[-(x^3*y[x]^6) + (x^3*y[x]^2 + x^2*y[x]^4 + x*y[x]^6)*Derivative[1][y][x] + Derivative`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)^2-(y(x)^4+x*y(x)^2+x^2)*diff(y(x),x)^2+(x*y(x)^6+x^2*y(x)^4+x^3*y(x)^2)*x^3*y(x)^6=0,y(x))`

**2.532 ODE No. 532**

$$ay'(x)^3 + by'(x)^2 + cy'(x) - d - y(x) = 0$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.157 (sec), leaf count = 848

$$\left\{ x - \int^{y(x)} 6 \frac{a \sqrt[3]{12 \sqrt{3} \sqrt{27 (d + a)^2 a^2}}}{\left( 12 \sqrt{3} \sqrt{27 (d + a)^2 a^2} + 18 ((d + a) b + 2/9 c^2) ca + (-4 d - 4 a) b^3 - b^2 c^2 a + (108 d + 108 a) \right)} \right.$$

**2.533 ODE No. 533**

$$a + xy'(x)^3 - y(x)y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.078 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.035 (sec), leaf count = 76

$$\left\{ y(x) = \frac{-C1^3x + a}{-C1^2}, y(x) = \frac{3\sqrt[3]{2}}{2}\sqrt[3]{ax^2}, y(x) = -\frac{3\sqrt[3]{2}(1 + i\sqrt{3})}{4}\sqrt[3]{ax^2}, y(x) = \frac{3\sqrt[3]{2}(i\sqrt{3} - 1)}{4}\sqrt[3]{ax^2} \right\}$$

**2.534 ODE No. 534**

$$4xy'(x)^3 - 6y(x)y'(x)^2 + 3y(x) - x = 0$$

✗ **Mathematica** : cpu = 300.019 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.074 (sec), leaf count = 84

$$\left\{ y(x) = x, y(x) = -\frac{(1 + \sqrt{3})x}{2}, y(x) = \frac{(\sqrt{3} - 1)x}{2}, y(x) = \frac{1}{3-C1} \left( -\sqrt{2}(x + -C1) \sqrt{-C1(x + -C1)} - -C1 \right) \right\}$$

**2.535 ODE No. 535**

$$8xy'(x)^3 - 12y(x)y'(x)^2 + 9y(x) = 0$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.056 (sec), leaf count = 51

$$\left\{ y(x) = 0, y(x) = -\frac{3x}{2}, y(x) = \frac{3x}{2}, y(x) = -\frac{1}{3-C1^2}(-C1(3-C1+x))^{\frac{3}{2}}, y(x) = \frac{1}{3-C1^2}(-C1(3-C1+x))^{\frac{3}{2}} \right\}$$



**2.536 ODE No. 536**

$$bx(x^2 - a^2)y'(x)^2 + (x^2 - a^2)y'(x)^3 + bx + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0239762 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow c_1 - \frac{bx^2}{2} \right\}, \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{x}{\sqrt{a^2 - x^2}} \right) + c_1 \right\}, \left\{ y(x) \rightarrow c_1 - \tan^{-1} \left( \frac{x}{\sqrt{a^2 - x^2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 52

$$\left\{ y(x) = -\frac{bx^2}{2} + \_C1, y(x) = -\arctan \left( x \frac{1}{\sqrt{a^2 - x^2}} \right) + \_C1, y(x) = \arctan \left( x \frac{1}{\sqrt{a^2 - x^2}} \right) + \_C1 \right\}$$

**2.537 ODE No. 537**

$$(x^6 + 3xy(x)^2)y'(x) - 2x^5y(x) + x^3y'(x)^3 - 3x^2y(x)y'(x)^2 - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 598.448 (sec), leaf count = 209

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \int^{-Z} -\frac{1}{6\_a} \left( 3\sqrt{81\_a^2 + 12\_a} 4^{2/3} \left( \frac{3\sqrt{81\_a^2 + 12\_a} - 27\_a^2 - 4}{(27\_a^2 + 4)^2} \right)^{2/3} + 27 \right) \right) \right\}$$

**2.538 ODE No. 538**

$$2(xy'(x) + y(x))^3 - y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 10.397 (sec), leaf count = 1532

$$\left\{ \int_{-b}^x 1 \left( -6^{\frac{2}{3}} \left( -9\_a^2 \left( -1/9\sqrt{3} \sqrt{\frac{27\_a(y(x))^2 - 2y(x)}{-a}} + y(x) \right) y(x) \right)^{\frac{2}{3}} + 6\_a y(x) \left( \sqrt[3]{6}^3 \sqrt{-9\_a^2 \left( -1/9 \right)} \right) \right) \right\}$$

**2.539 ODE No. 539**

$$\sin(x)y'(x)^3 - y'(x)^2 (y(x) \sin(x) - \cos^2(x)) - y'(x) (y(x) \cos^2(x) + \sin(x)) + y(x) \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.0283843 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow c_1 e^x \right\}, \left\{ y(x) \rightarrow c_1 - \cos(x) \right\}, \left\{ y(x) \rightarrow c_1 - \log \left( \sin \left( \frac{x}{2} \right) \right) + \log \left( \cos \left( \frac{x}{2} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 3.576 (sec), leaf count = 32

$$\{y(x) = \_C1 e^x, y(x) = -\cos(x) + \_C1, y(x) = -\ln(\csc(x) - \cot(x)) + \_C1\}$$

**2.540 ODE No. 540**

$$2y(x)y'(x)^3 - y(x)y'(x)^2 + 2xy'(x) - x = 0$$

✓ **Mathematica** : cpu = 0.0209184 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow c_1 + \frac{x}{2} \right\}, \left\{ y(x) \rightarrow \left( \frac{3c_1}{2} - ix^{3/2} \right)^{2/3} \right\}, \left\{ y(x) \rightarrow \left( \frac{3c_1}{2} + ix^{3/2} \right)^{2/3} \right\} \right\}$$

✓ **Maple** : cpu = 0.35 (sec), leaf count = 109

$$\left\{ x + \frac{C1 x}{y(x)} \left( \frac{1}{y(x)} \left( -\sqrt{-xy(x)} + y(x) \right) \right)^{-\frac{2}{3}} \left( \frac{1}{y(x)} \left( -x + \sqrt{-xy(x)} + y(x) \right) \right)^{-\frac{2}{3}} = 0, x + \frac{C1 x}{y(x)} \left( \frac{1}{y(x)} \left( \sqrt{-xy(x)} + y(x) \right) \right)^{-\frac{2}{3}} \left( \frac{1}{y(x)} \left( -x + \sqrt{-xy(x)} + y(x) \right) \right)^{-\frac{2}{3}} = 0 \right\}$$

**2.541 ODE No. 541**

$$y(x)^2 y'(x)^3 + 2xy'(x) - y(x) = 0$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 3.656 (sec), leaf count = 103

$$\left\{ y(x) = \sqrt{\_C1^3 + 2\_C1 x}, y(x) = -\frac{2i}{3} \sqrt[4]{2} \sqrt[4]{3} \sqrt[4]{-x^3}, y(x) = \frac{2i}{3} \sqrt[4]{2} \sqrt[4]{3} \sqrt[4]{-x^3}, y(x) = -\sqrt{\_C1^3 + 2\_C1 x}, y(x) = \sqrt{\_C1^3 + 2\_C1 x} \right\}$$

2.542 ODE No. 542

$$16y(x)^2y'(x)^3 + 2xy'(x) - y(x) = 0$$

✗ **Mathematica** : cpu = 300.002 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 3.376 (sec), leaf count = 107

$$\left\{ y(x) = \sqrt{16\_C1^3 + 2\_C1 x}, y(x) = -\frac{i}{3}\sqrt[4]{2}\sqrt[4]{3}\sqrt[4]{-x^3}, y(x) = \frac{i}{3}\sqrt[4]{2}\sqrt[4]{3}\sqrt[4]{-x^3}, y(x) = -\sqrt{16\_C1^3 + 2\_C1 x} \right.$$

2.543 ODE No. 543

$$x(x^2 + 1)y'(x) - x^2y(x) + y(x)^3(-y'(x)^2) + xy(x)^2y'(x)^3 = 0$$

✗ **Mathematica** : cpu = 300.006 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 12.393 (sec), leaf count = 277

$$\left\{ y(x) = -\frac{i}{2}\sqrt[4]{-16x^4 + 40x^2 + 2 - 2\sqrt{-(8x^2 - 1)^3}}, y(x) = -\frac{i}{2}\sqrt[4]{-16x^4 + 40x^2 + 2 + 2\sqrt{-(8x^2 - 1)^3}}, y(x) \right.$$

2.544 ODE No. 544

$$x^7y(x)^2y'(x)^3 - (3x^6y(x)^3 - 1)y'(x)^2 + 3x^5y(x)^4y'(x) - x^4y(x)^5 = 0$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 15.838 (sec), leaf count = 4201

$$\left\{ \int_{-b-a}^x \frac{1}{-a} \left( (1 - i\sqrt{3}) \left( -108(y(x))^6 - a^{12} + 12\sqrt{3}\sqrt{\frac{27 - a^6(y(x))^3 - 4}{y(x)}}(y(x))^5 - a^9 + 72 - a^6(y(x))^3 - 8 \right) \right)^{\frac{2}{3}} + \right.$$

**2.545 ODE No. 545**

$$y'(x)^4 - (y(x) - a)^3(y(x) - b)^2 = 0$$

✓ **Mathematica** : cpu = 0.727267 (sec), leaf count = 383

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt[4]{a - \#1} \sqrt{\frac{\#1 - b}{a - b}} B_{\frac{a - \#1}{a - b}} \left( \frac{1}{4}, \frac{1}{2} \right)}{\sqrt{b - \#1} \sqrt[4]{\frac{a - \#1}{a - b}}} \right] \& \right] [c_1 - \sqrt[4]{-1}x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{1}{\sqrt[4]{-(-a + a)^3 (b - a)^2}} \right] [c_1 - \sqrt[4]{-1}x] \right\} \right.$$

✓ **Maple** : cpu = 0.926 (sec), leaf count = 144

$$\left\{ x - \int^{y(x)} \frac{1}{\sqrt[4]{(-a - a)^3 (-b + a)^2}} d_a - C1 = 0, x - \int^{y(x)} -i \frac{1}{\sqrt[4]{-(-a + a)^3 (b - a)^2}} d_a - C1 = 0, x \right.$$

**2.546 ODE No. 546**

$$y'(x)^4 + 3(x - 1)y'(x)^2 - 3(2y(x) - 1)y'(x) + 3x = 0$$

✗ **Mathematica** : cpu = 300.007 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.064 (sec), leaf count = 171

$$\left\{ y(x) = 1 \left( (-6 + C1^3 + (6x - 6)C1) \sqrt{-C1^2 + 4x} - 2C1^4 + (-14x + 6)C1^2 + ((-C1^2 + 4x)^{\frac{3}{2}} + 6) \right) \right.$$

**2.547 ODE No. 547**

$$y'(x)^4 - 4y(x)(xy'(x) - 2y(x))^2 = 0$$

✓ **Mathematica** : cpu = 2.03011 (sec), leaf count = 341

$$\left\{ \text{Solve} \left[ \frac{1}{4} \left( -4c_1 + \frac{\sqrt{x^2 - 4\sqrt{y(x)}\sqrt{y(x)}} \left( 4 \log \left( \sqrt{x^2 - 4\sqrt{y(x)} + x} \right) - \log(y(x)) \right)}{\sqrt{(x^2 - 4\sqrt{y(x)})y(x)}} + \log(y(x)) \right) = 0, y(x) \right] \right.$$

✓ **Maple** : cpu = 3.52 (sec), leaf count = 118

$$\left\{ 1 \left( \sqrt{x^2 - 4\sqrt{y(x)}} - x \right)^{1\sqrt{x^2y(x) - 4(y(x))^{3/2}} \frac{1}{\sqrt{x^2 - 4\sqrt{y(x)}} \sqrt{y(x)}}} \frac{1}{\sqrt{y(x)}} \left( \left( \sqrt{x^2 - 4\sqrt{y(x)}} + x \right)^{1\sqrt{x^2y(x) - 4(y(x))^{3/2}} \frac{1}{\sqrt{x^2 - 4\sqrt{y(x)}} \sqrt{y(x)}}} \right) \right.$$

2.548 ODE No. 548

$$y'(x)^6 - (y(x) - a)^4(y(x) - b)^3 = 0$$

✓ **Mathematica** : cpu = 1.02001 (sec), leaf count = 569

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt[3]{a - \#1} \sqrt{\frac{\#1 - b}{a - b}} B_{\frac{a - \#1}{a - b}} \left( \frac{1}{3}, \frac{1}{2} \right)}{\sqrt{b - \#1} \sqrt[3]{\frac{a - \#1}{a - b}}} \right] \& [c_1 - ix] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt[3]{a - \#1} \sqrt{\frac{\#1 - b}{a - b}} B_{\frac{a - \#1}{a - b}} \left( \frac{1}{3}, \frac{1}{2} \right)}{\sqrt{b - \#1} \sqrt[3]{\frac{a - \#1}{a - b}}} \right] \& [c_1 - ix] \right\} \right.$$

✓ **Maple** : cpu = 0.768 (sec), leaf count = 246

$$\left\{ x - \int^{y(x)} \frac{1}{\sqrt[6]{(-a - a)^4 (-b + -a)^3}} d_{-a} - _C1 = 0, x - \int^{y(x)} \frac{-2i}{-i + \sqrt{3}} \frac{1}{\sqrt[6]{-(-a + a)^4 (b - -a)^3}} d_{-a} - _C1 \right.$$

2.549 ODE No. 549

$$x^2(y'(x)^2 + 1)^3 - a^2 = 0$$

✓ **Mathematica** : cpu = 0.240722 (sec), leaf count = 406

$$\left\{ \left\{ y(x) \rightarrow c_1 - \frac{\sqrt[3]{x} (2x^{2/3} + (1 + i\sqrt{3}) a^{2/3}) \sqrt{\frac{-2x^{2/3} + (-1 - i\sqrt{3}) a^{2/3}}{x^{2/3}}}}{2\sqrt{2}} \right\}, \left\{ y(x) \rightarrow c_1 + \frac{\sqrt[3]{x} \sqrt{\frac{-2x^{2/3} + (-1 - i\sqrt{3}) a^{2/3}}{x^{2/3}}}}{2\sqrt{2}} \right\} \right.$$

✓ **Maple** : cpu = 4.315 (sec), leaf count = 545

$$\left\{ y(x) = 1 \sqrt{-\frac{1}{a^4} (a^2 x)^{\frac{4}{3}} \left( (a^2 x)^{\frac{2}{3}} - a^2 \right) \left( a^2 - (a^2 x)^{\frac{2}{3}} \right) (a^2 x)^{-\frac{2}{3}} + _C1}, y(x) = 1 \sqrt{-\frac{1}{a^4} (a^2 x)^{\frac{4}{3}} \left( (a^2 x)^{\frac{2}{3}} - a^2 \right) \left( a^2 - (a^2 x)^{\frac{2}{3}} \right) (a^2 x)^{-\frac{2}{3}} + _C1} \right.$$

**2.550 ODE No. 550**

$$-ay(x)^s - bx^{\frac{rs}{r-s}} + y'(x)^r = 0$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 3.802 (sec), leaf count = 60

$$\left\{ (-r + s) \int_{-b}^{y(x)} \left( x(r - s) \sqrt[r]{a_{-}a^s + bx^{\frac{rs}{r-s}} - r_{-}a} \right)^{-1} d_{-}a + \ln(x) - \_C1 = 0 \right\}$$

**2.551 ODE No. 551**

$$y'(x)^n - f(x)^n(y(x) - a)^{n+1}(y(x) - b)^{n-1} = 0$$

✓ **Mathematica** : cpu = 0.434267 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow \frac{a(a-b)^n \left( \int_1^x (-1)^{\frac{1}{n}+1} f(K[1]) dK[1] + c_1 \right)^n + bn^n}{(a-b)^n \left( \int_1^x (-1)^{\frac{1}{n}+1} f(K[1]) dK[1] + c_1 \right)^n + n^n} \right\} \right\}$$

✓ **Maple** : cpu = 1.128 (sec), leaf count = 55

$$\left\{ y(x) = 1 \left( b \left( -\frac{n}{(a-b) \left( \int f(x) dx + \_C1 \right)} \right)^n - a \right) \left( -1 + \left( -\frac{n}{(a-b) \left( \int f(x) dx + \_C1 \right)} \right)^n \right)^{-1} \right\}$$

**2.552 ODE No. 552**

$$y'(x)^n - f(x)g(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.18269 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} g(K[1])^{-1/n} dK[1] \& \right] \left[ \int_1^x f(K[2])^{\frac{1}{n}} dK[2] + c_1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 1.036 (sec), leaf count = 43

$$\left\{ \int^{y(x)} (g(\_a))^{-n-1} d_{-}a + \int^x -\frac{\sqrt[n]{f(\_a)g(y(x))}}{\sqrt[n]{g(y(x))}} d_{-}a + \_C1 = 0 \right\}$$

**2.553 ODE No. 553**

$$ay'(x)^m + by'(x)^n - y(x) = 0$$

✓ **Mathematica** : cpu = 39.7662 (sec), leaf count = 51

$$\text{Solve} \left[ \left\{ x = \frac{amK\$434891^{m-1}}{m-1} + \frac{bnK\$434891^{n-1}}{n-1} + c_1, aK\$434891^m + bK\$434891^n = y(x) \right\}, \{y(x), K\$434891\} \right]$$

✓ **Maple** : cpu = 0.308 (sec), leaf count = 36

$$\left\{ x - \int^{y(x)} (\text{RootOf}(-a\_Z^m - b\_Z^n + \_a))^{-1} d\_a - \_C1 = 0, y(x) = 0 \right\}$$

**2.554 ODE No. 554**

$$x^{n-1}y'(x)^n - nxy'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 300.139 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.642 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{\_C1} \left( \_C1^2 \sqrt[n]{\frac{x}{\_C1}} n - (\_C1^{-1})^{-n} \right) \right\}$$

**2.555 ODE No. 555**

$$xy'(x) + \sqrt{y'(x)^2 + 1} - y(x) = 0$$

✗ **Mathematica** : cpu = 303.153 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.137 (sec), leaf count = 15

$$\left\{ y(x) = \sqrt{\_C1^2 + 1} + \_C1 x \right\}$$

**2.556 ODE No. 556**

$$xy'(x)^2 + \sqrt{y'(x)^2 + 1} + y(x) = 0$$

✓ **Mathematica** : cpu = 6.92788 (sec), leaf count = 50

$$\text{Solve} \left[ \left\{ x = -\frac{-c_1 + \sqrt{K\$529059^2 + 1} + \sinh^{-1}(K\$529059)}{(K\$529059 + 1)^2}, K\$529059^2 x + \sqrt{K\$529059^2 + 1} + y(x) = 0 \right\}, \{y(x)\} \right]$$

✓ **Maple** : cpu = 1.483 (sec), leaf count = 581

$$\left\{ -C1 x^2 \left( \sqrt{-4xy(x) + 2 + 2\sqrt{4x^2 - 4xy(x) + 1}} - 2x \right)^{-2} + x + 2 \frac{x^2}{\left( \sqrt{-4xy(x) + 2 + 2\sqrt{4x^2 - 4xy(x) + 1}} \right)^2} \right\}$$

**2.557 ODE No. 557**

$$x(y'(x) + \sqrt{y'(x)^2 + 1}) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.017548 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{-x(x - c_1)} \right\}, \left\{ y(x) \rightarrow \sqrt{-x(x - c_1)} \right\} \right\}$$

✓ **Maple** : cpu = 1.015 (sec), leaf count = 74

$$\left\{ 2xy(x) - C1 \frac{1}{\sqrt{\frac{(y(x))^2 + x^2}{x^2(y(x))^2}}} \left( \sqrt{\frac{x^4 + 2x^2(y(x))^2 + (y(x))^4}{x^2(y(x))^2}} y(x)x + (y(x))^2 - x^2 \right)^{-1} + x = 0 \right\}$$

**2.558 ODE No. 558**

$$ax\sqrt{y'(x)^2 + 1} + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 1.25621 (sec), leaf count = 369

$$\text{Solve} \left[ \frac{a \left( 2 \log(x - a^2 x) - \log \left( \frac{(a^2 - 1) \left( y(x) + ix \left( a \sqrt{a^2 - \frac{y(x)^2}{x^2}} - 1 + a^2 - 1 \right) \right)}{a^3(x + iy(x))} \right) \right) + \log \left( \frac{i(a^2 - 1) \left( x \left( a \sqrt{a^2 - \frac{y(x)^2}{x^2}} - 1 + a^2 - 1 \right) \right)}{a^3(x - iy(x))} \right)}{2(a^2 - 1)} \right]$$



✓ **Maple** : cpu = 1.734 (sec), leaf count = 223

$$\left\{ x - \frac{1}{a} \operatorname{Arcsinh} \left( \frac{1}{(a^2-1)x} \left( \sqrt{-a^2x^2+x^2+(y(x))^2} a + y(x) \right) \right) \right\} \frac{1}{\sqrt{\frac{1}{(a^2-1)^2x^2} \left( -a^2x^2 + a^2(y(x))^2 + 2\sqrt{-a^2x^2+x^2+(y(x))^2} \right)}}$$

**2.559 ODE No. 559**

$$-ay(x)y'(x) - ax + y(x)\sqrt{y'(x)^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.30371 (sec), leaf count = 148

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{e^{-2c_1} \left( (a^2-1)^3 (-e^{2c_1}) x^2 + 2(a^2-1) x e^{(a^2+1)c_1} + e^{2a^2c_1} \right)}}{\sqrt{(a^2-1)^3}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{e^{-2c_1} \left( (a^2-1)^3 (-e^{2c_1}) x^2 + 2(a^2-1) x e^{(a^2+1)c_1} + e^{2a^2c_1} \right)}}{\sqrt{(a^2-1)^3}} \right\} \right\}$$

✓ **Maple** : cpu = 1.228 (sec), leaf count = 215

$$\left\{ -e^{\int \frac{1}{(a^2-1)y(x)} \left( -a^2x - \sqrt{(a^2-1)(y(x))^2 + a^2x^2} \right) dx} a \left( a\sqrt{-a^2+1} - a \right) \frac{1}{\sqrt{-a^2+1}} \left( -a a - \sqrt{-a^2+1} \right)^{-1} \left( -a^2 a - \sqrt{-a^2+1} a + a \right)^{-1} d_a C1 + x \right\}$$

**2.560 ODE No. 560**

$$ay(x)\sqrt{y'(x)^2 + 1} - x^2 - 2xy(x)y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 21.3867 (sec), leaf count = 86

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{4x^2 - a^2(c_1x + 2)^2}}{\sqrt{a^2c_1^2 - 4}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{4x^2 - a^2(c_1x + 2)^2}}{\sqrt{a^2c_1^2 - 4}} \right\} \right\}$$

✓ **Maple** : cpu = 2.583 (sec), leaf count = 1120

$$\left\{ \int_{-b}^x 1 \left( 2a^3 - 2a(y(x))^2 + \sqrt{a^2 \left( -a^4 + 2a^2(y(x))^2 - a^2(y(x))^2 + (y(x))^4 \right)} \right) \left( -2a^2a(y(x))^2 + 2a^5 \right) dx \right\}$$

**2.561 ODE No. 561**

$$f(x^2 + y(x)^2) \sqrt{y'(x)^2 + 1} - xy'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 300.005 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 5.855 (sec), leaf count = 50

$$\left\{ y(x) = x \left( \tan \left( \text{RootOf} \left( -2\_Z + \int \frac{x^2((\tan(-Z))^2+1)}{(\tan(-Z))^2} \frac{f(-a)}{-a} \frac{1}{\sqrt{-(f(-a))^2 + -a}} d\_a + 2\_C1 \right) \right) \right) \right\}^{-1}$$

**2.562 ODE No. 562**

$$a \sqrt[3]{y'(x)^3 + 1} + bxy'(x) - y(x) = 0$$

✗ **Mathematica** : cpu = 300.252 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.398 (sec), leaf count = 3306

$$\left\{ \left( - \int \frac{1}{2b^3x^3+2a^3} \left( 2b^2x^2y(x) \sqrt[3]{-4b^6x^6-8a^3b^3x^3-4b^3x^3(y(x))^3+4\sqrt{b^6x^6+2a^3b^3x^3+2b^3x^3(y(x))^3+a^6-2(y(x))^3a^3+(y(x))^6b^3x^3-4a^6}} \right) \right) \right\}$$

**2.563 ODE No. 563**

$$ay(x) + b + xy'(x) + \log(y'(x)) = 0$$

✓ **Mathematica** : cpu = 0.364966 (sec), leaf count = 57

$$\text{Solve} \left[ \frac{a^2y(x) + aW(xe^{-ay(x)-b}) + (a+1) \log(1 - aW(xe^{-ay(x)-b}))}{a} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 1.438 (sec), leaf count = 66

$$\left\{ - \left( e^{-ay(x)-\text{lambertW}(xe^{-ay(x)-b})-b} \right)^{-(a+1)^{-1}} \_C1 + x - \frac{e^{ay(x)+\text{lambertW}(xe^{-ay(x)-b})+b}}{a} = 0 \right\}$$

**2.564 ODE No. 564**

$$a(xy'(x) - y(x)) + \log(y'(x)) = 0$$

✓ **Mathematica** : cpu = 0.0469476 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow e^{-c_1 x} - \frac{c_1}{a} \right\} \right\}$$

✓ **Maple** : cpu = 3.761 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{a} \left( \ln \left( -\frac{1}{ax} \right) - 1 \right), y(x) = -C1 x + \frac{\ln(-C1)}{a} \right\}$$

**2.565 ODE No. 565**

$$y'(x) + y(x) \log(y'(x)) - xy(x) - y(x) \log(y(x)) = 0$$

✓ **Mathematica** : cpu = 0.0119614 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\frac{1}{2} W(e^x)(W(e^x)+2)} \right\} \right\}$$

✓ **Maple** : cpu = 4.548 (sec), leaf count = 17

$$\left\{ y(x) = -C1 e^{\frac{\text{lambertW}(e^x)(\text{lambertW}(e^x)+2)}{2}} \right\}$$

**2.566 ODE No. 566**

$$y'(x) + \sin(y'(x)) - x = 0$$

✗ **Mathematica** : cpu = 0.0112574 (sec), leaf count = 0 , could not solve

`DSolve[-x + Sin[Derivative[1][y][x]] + Derivative[1][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.113 (sec), leaf count = 16

$$\left\{ y(x) = \int \text{RootOf}(-Z + \sin(-Z) - x) dx + -C1 \right\}$$

**2.567 ODE No. 567**

$$a \cos(y'(x)) + by'(x) + x = 0$$

✗ **Mathematica** : cpu = 0.0121905 (sec), leaf count = 0 , could not solve

`DSolve[x + a*Cos[Derivative[1][y][x]] + b*Derivative[1][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.159 (sec), leaf count = 18

$$\left\{ y(x) = \int \text{RootOf}(a \cos(_Z) + _Z b + x) dx + _C1 \right\}$$

**2.568 ODE No. 568**

$$y'(x)^2 \sin(y'(x)) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0685818 (sec), leaf count = 27

`Solve[{cos(K$572988) + x = c1 + K$572988 sin(K$572988), K$572988^2 sin(K$572988) = y(x)}, {y(x), K$572988}]`

✓ **Maple** : cpu = 0.106 (sec), leaf count = 32

$$\left\{ x - \int^{y(x)} (\text{RootOf}(\sin(_Z) _Z^2 - _a))^{-1} d_a - _C1 = 0, y(x) = 0 \right\}$$

**2.569 ODE No. 569**

$$(y'(x)^2 + 1) \sin^2(y(x) - xy'(x)) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0444267 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow c_1 x - \frac{1}{2} \cos^{-1} \left( \frac{c_1^2 - 1}{c_1^2 + 1} \right) \right\}, \left\{ y(x) \rightarrow c_1 x + \frac{1}{2} \cos^{-1} \left( \frac{c_1^2 - 1}{c_1^2 + 1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 2.899 (sec), leaf count = 147

$$\left\{ y(x) = _C1 x - \arcsin \left( \frac{1}{\sqrt{-C1^2 + 1}} \right), y(x) = _C1 x + \arcsin \left( \frac{1}{\sqrt{-C1^2 + 1}} \right), y(x) = -\sqrt{1-x} \sqrt{x^{-1}x} - a \right\}$$

**2.570 ODE No. 570**

$$(y'(x)^2 + 1)(ax + \tan^{-1}(y'(x))) + y'(x) = 0$$

✗ **Mathematica** : cpu = 0.10703 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] + (a*x + ArcTan[Derivative[1][y][x]])*(1 + Derivative[1][y][x]^2)`

✓ **Maple** : cpu = 0.303 (sec), leaf count = 30

$$\left\{ y(x) = \int \tan \left( \text{RootOf} \left( ax(\tan(\_Z))^2 + (\tan(\_Z))^2\_Z + ax + \tan(\_Z) + \_Z \right) \right) dx + \_C1 \right\}$$

**2.571 ODE No. 571**

$$ax^n f(y'(x)) + xy'(x) - y(x) = 0$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

`$Aborted`

✓ **Maple** : cpu = 4.12 (sec), leaf count = 169

$$\left\{ [y(\_T) = a \left( \left( \frac{1}{f(\_T)na} \left( (1-n) \int (f(\_T))^{-n-1} d\_T + \_C1 \right) \right)^{(n-1)^{-1}} (f(\_T))^{\frac{1}{n(n-1)}} \right)^n f(\_T) + \_T \right\}$$

**2.572 ODE No. 572**

$$f(y'(x))(xy'(x) - y(x))^n + y(x)g(y'(x)) + xh(y'(x)) = 0$$

✗ **Mathematica** : cpu = 0.0319639 (sec), leaf count = 0 , could not solve

`DSolve[x*h[Derivative[1][y][x]] + g[Derivative[1][y][x]]*y[x] + f[Derivative[1][y][x]]*(-y[x] + x*Derivative[1][y][x])^n == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve((x*diff(y(x),x)-y(x))^n*f(diff(y(x),x))+y(x)*g(diff(y(x),x))+x*h(diff(y(x),x))=0,y(x)`

**2.573 ODE No. 573**

$$f(xy'(x)^2) + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0213676 (sec), leaf count = 42

$$\{ \{y(x) \rightarrow f(c_1) - 2\sqrt{c_1}\sqrt{x}, y(x) \rightarrow f(c_1) + 2\sqrt{c_1}\sqrt{x}\} \}$$

✓ **Maple** : cpu = 3.723 (sec), leaf count = 16

$$\left\{ y(x) = f\left(\frac{-C1^2}{4}\right) + \_C1 \sqrt{x} \right\}$$

**2.574 ODE No. 574**

$$f\left(x - \frac{3}{2}y'(x)^2\right) + y'(x)^3 - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0161279 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{9}\left(9f(c_1) + 2\sqrt{6}(x - c_1)^{3/2}\right), y(x) \rightarrow \frac{1}{9}\left(9f(c_1) - 2\sqrt{6}(x - c_1)^{3/2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.508 (sec), leaf count = 41

$$\left\{ y(x) = f(\_C1) - \frac{2\sqrt{6}}{9}\sqrt{(x - \_C1)^3}, y(x) = f(\_C1) + \frac{2\sqrt{6}}{9}\sqrt{(x - \_C1)^3} \right\}$$

**2.575 ODE No. 575**

$$y'(x)f(xy(x)y'(x) - y(x)^2) + x^2(-y'(x)) + xy(x) = 0$$

✗ **Mathematica** : cpu = 0.0254695 (sec), leaf count = 0 , could not solve

`DSolve[x*y[x] - x^2*Derivative[1][y][x] + f[-y[x]^2 + x*y[x]*Derivative[1][y][x]]*Derivative`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x)*f(x*y(x)*diff(y(x),x)-y(x)^2)-x^2*diff(y(x),x)+x*y(x)=0,y(x))`

2.576 ODE No. 576

$$\phi(f(x, y(x), y'(x)), g(x, y(x), y'(x))) = 0$$

✗ **Mathematica** : cpu = 0.00766076 (sec), leaf count = 0 , could not solve

DSolve[phi[f[x, y[x], Derivative[1][y][x]], g[x, y[x], Derivative[1][y][x]]] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(phi(f(x,y(x),diff(y(x),x)),g(x,y(x),diff(y(x),x)))=0,y(x))

2.577 ODE No. 577

$$y'(x) = F\left(\frac{y(x)}{a+x}\right)$$

✓ **Mathematica** : cpu = 15.2591 (sec), leaf count = 141

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{1}{K[2] - (a+x)F\left(\frac{K[2]}{a+x}\right)} - \int_1^x \frac{(K[1]+a)F\left(\frac{K[2]}{K[1]+a}\right) - K[2]F'\left(\frac{K[2]}{K[1]+a}\right)}{(K[1]+a)\left(K[2] - (K[1]+a)F\left(\frac{K[2]}{K[1]+a}\right)\right)^2} dK[1] \right) dK[2] + \dots \right]$$

✓ **Maple** : cpu = 1.804 (sec), leaf count = 28

$$\left\{ y(x) = -\text{RootOf} \left( \int^{-Z} (F(-_a) + _a)^{-1} d_a + \ln(x+a) + _C1 \right) (x+a) \right\}$$

2.578 ODE No. 578

$$y'(x) = F(y(x) - x^2) + 2x$$

✓ **Mathematica** : cpu = 20.4146 (sec), leaf count = 88

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x - \frac{2K[1]F'(K[2] - K[1]^2)}{F(K[2] - K[1]^2)^2} dK[1] - \frac{1}{F(K[2] - x^2)} \right) dK[2] + \int_1^x \left( \frac{2K[1]}{F(y(x) - K[1]^2)} + \dots \right) \right]$$

✓ **Maple** : cpu = 6.828 (sec), leaf count = 22

$$\left\{ y(x) = x^2 + \text{RootOf} \left( -x + \int^{-Z} (F(_a))^{-1} d_a + _C1 \right) \right\}$$

**2.579 ODE No. 579**

$$y'(x) = F\left(\frac{ax^2}{4} + \frac{bx}{2} + y(x)\right) - \frac{ax}{2}$$

✓ **Mathematica** : cpu = 15.9821 (sec), leaf count = 184

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} - \frac{\left(2F\left(K[2] + \frac{ax^2}{4} + \frac{bx}{2}\right) + b\right) \int_1^x \frac{2(aK[1]+b)F'\left(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2]\right) dK[1] + 2}{\left(2F\left(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2]\right) + b\right)^2} dK[1] + 2}{2F\left(K[2] + \frac{ax^2}{4} + \frac{bx}{2}\right) + b} dK[2] + \int_1^x \frac{2F\left(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2]\right)}{2F\left(\frac{1}{4}aK[1]^2 + \frac{1}{2}bK[1]+K[2]\right) + b} dK[1] \right]$$

✓ **Maple** : cpu = 4.297 (sec), leaf count = 35

$$\left\{ y(x) = -\frac{ax^2}{4} - \frac{bx}{2} + \text{RootOf}\left(-x + 2 \int^{-Z} (2F(\_a) + b)^{-1} d\_a + \_C1\right) \right\}$$

**2.580 ODE No. 580**

$$y'(x) = e^{bx} F(e^{-bx} y(x))$$

✓ **Mathematica** : cpu = 31.8126 (sec), leaf count = 161

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{1}{bK[2] - e^{bx} F(e^{-bx} K[2])} - \int_1^x \frac{b(e^{bK[1]} F(K[2]e^{-bK[1]}) - K[2]F'(K[2]e^{-bK[1]}))}{(e^{bK[1]} F(K[2]e^{-bK[1]}) - bK[2])^2} dK[1] \right) dK[2] \right]$$

✓ **Maple** : cpu = 0.504 (sec), leaf count = 31

$$\left\{ y(x) = \frac{\text{RootOf}\left(-x + \int^{-Z} (F(\_a) - \_a b)^{-1} d\_a + \_C1\right)}{e^{-bx}} \right\}$$

**2.581 ODE No. 581**

$$y'(x) = \frac{x F\left(\frac{x^2 y(x) + \frac{1}{4}}{x^2}\right) + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 48.465 (sec), leaf count = 107

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{F'\left(K[2] + \frac{1}{4K[1]^2}\right)}{2K[1]^3 F\left(K[2] + \frac{1}{4K[1]^2}\right)^2} dK[1] - \frac{1}{F\left(K[2] + \frac{1}{4x^2}\right)} \right) dK[2] + \int_1^x \frac{1}{F\left(\frac{1}{4K[1]^2} + y(x)\right) + \frac{1}{2}} \frac{1}{2K[1]^3} dK[1] \right]$$



✓ **Maple** : cpu = 0.37 (sec), leaf count = 32

$$\left\{ y(x) = \frac{4 \operatorname{RootOf}\left(\int^{-Z}(F(_a))^{-1} d_a x + _C1 x + 1\right) x^2 - 1}{4 x^2} \right\}$$

**2.582 ODE No. 582**

$$y'(x) = \frac{ax^2 F\left(\frac{axy(x)+1}{ax}\right) + 1}{ax^2}$$

✓ **Mathematica** : cpu = 20.6945 (sec), leaf count = 103

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{1}{F(K[2] + \frac{1}{ax})} - \int_1^x \frac{F'\left(\frac{1}{aK[1]} + K[2]\right)}{aK[1]^2 F\left(\frac{1}{aK[1]} + K[2]\right)^2} dK[1] \right) dK[2] + \int_1^x \left( -\frac{1}{aK[1]^2 F\left(\frac{1}{aK[1]} + K[2]\right)} \right) dK[2] \right]$$

✓ **Maple** : cpu = 1.818 (sec), leaf count = 30

$$\left\{ y(x) = \frac{\operatorname{RootOf}\left(-x + \int^{-Z}(F(_a))^{-1} d_a + _C1\right) ax - 1}{ax} \right\}$$

**2.583 ODE No. 583**

$$y'(x) = -\frac{1}{2}x \left( ax^2 - 2F\left(\frac{ax^4}{8} + y(x)\right) \right)$$

✓ **Mathematica** : cpu = 50.8612 (sec), leaf count = 111

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( -\int_1^x \frac{aK[1]^3 F'\left(\frac{1}{8}aK[1]^4 + K[2]\right)}{2F\left(\frac{1}{8}aK[1]^4 + K[2]\right)^2} dK[1] - \frac{1}{F\left(K[2] + \frac{ax^4}{8}\right)} \right) dK[2] + \int_1^x \left( K[1] - \frac{a}{2F\left(\frac{1}{8}aK[1]^4 + K[2]\right)} \right) dK[2] \right]$$

✓ **Maple** : cpu = 2.022 (sec), leaf count = 31

$$\left\{ y(x) = -\frac{ax^4}{8} + \operatorname{RootOf}\left(-x^2 + 2 \int^{-Z}(F(_a))^{-1} d_a + 2_C1\right) \right\}$$

**2.584 ODE No. 584**

$$y'(x) = \frac{2a}{2aF(y(x)^2 - 4ax) + y(x)}$$

✓ **Mathematica** : cpu = 23.9055 (sec), leaf count = 108

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \frac{1}{4} \left( \frac{\frac{K[2]}{F(K[2]^2 - 4ax)} + 2a}{a^2} - 4 \int_1^x \frac{K[2]F'(K[2]^2 - 4aK[1])}{aF(K[2]^2 - 4aK[1])^2} dK[1] \right) dK[2] + \int_1^x -\frac{1}{2aF(y(x)^2 - 4ax)} dx \right]$$

✓ **Maple** : cpu = 5.504 (sec), leaf count = 35

$$\left\{ \frac{y(x)}{2a} + \frac{\int^{(y(x))^2 - 4ax} (F(-a))^{-1} d_a}{8a^2} - C1 = 0 \right\}$$

**2.585 ODE No. 585**

$$y'(x) = y(x)F(\log(\log(y(x))) - \log(x))$$

✓ **Mathematica** : cpu = 154.336 (sec), leaf count = 141

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{1}{xK[2]F(\log(\log(K[2])) - \log(x)) - K[2]\log(K[2])} - \int_1^x \frac{F'(\log(\log(K[2])) - \log(K[1])) - F(\log(\log(K[2])) - \log(K[1]))}{K[2](\log(K[2]) - K[1]F(\log(\log(K[2])) - \log(K[1])))} dK[1] \right) dK[2] \right]$$

✓ **Maple** : cpu = 9.802 (sec), leaf count = 122

$$\left\{ \int_{-b}^x \frac{F(\ln(\ln(y(x))) - \ln(-a))}{aF(\ln(\ln(y(x))) - \ln(-a)) - \ln(y(x))} d_a + \int^{y(x)} -\frac{1}{-f(xF(\ln(\ln(-f)) - \ln(x)) - \ln(-f))} - \int_{-b}^x \frac{F(\ln(\ln(-f)) - \ln(x))}{-f(xF(\ln(\ln(-f)) - \ln(x)) - \ln(-f))} dx \right\}$$

**2.586 ODE No. 586**

$$y'(x) = \frac{x F\left(\frac{y(x)}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}}$$

✓ **Mathematica** : cpu = 185.373 (sec), leaf count = 398

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \frac{-\left( (x^2 + 1) F\left(\frac{K[2]}{\sqrt{x^2+1}}\right)^2 - K[2]^2 \right) \left( \int_1^x -\frac{K[1] \left( -(K[1]^2+1)^{3/2} K[2] F\left(\frac{K[2]}{\sqrt{K[1]^2+1}}\right) \right)^2 \left( F'\left(\frac{K[2]}{\sqrt{K[1]^2+1}}\right) - 2 \right)}{\dots} \right)}{\dots} \right]$$

✓ **Maple** : cpu = 1.078 (sec), leaf count = 39

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x^2 + 1) + 2 \int^{-Z} (F(_a) - _a)^{-1} d_a + 2\_C1 \right) \sqrt{x^2 + 1} \right\}$$

**2.587 ODE No. 587**

$$y'(x) = \frac{1}{2} \sqrt{x} \left( 2F\left(y(x) - \frac{x^3}{6}\right) + x^{3/2} \right)$$

✓ **Mathematica** : cpu = 257.684 (sec), leaf count = 109

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( -\int_1^x -\frac{K[1]^2 F'\left(K[2] - \frac{K[1]^3}{6}\right)}{2F\left(K[2] - \frac{K[1]^3}{6}\right)^2} dK[1] - \frac{1}{F\left(K[2] - \frac{x^3}{6}\right)} \right) dK[2] + \int_1^x \left( \frac{K[1]^2}{2F\left(y(x) - \frac{K[1]^3}{6}\right)} \right) \right]$$

✓ **Maple** : cpu = 0.594 (sec), leaf count = 29

$$\left\{ \int_{-b}^{y(x)} \left( F\left(-a - \frac{x^3}{6}\right) \right)^{-1} d_a - \frac{2}{3} x^{\frac{3}{2}} - \_C1 = 0 \right\}$$

**2.588 ODE No. 588**

$$y'(x) = \frac{F(-(x - y(x))(y(x) + x)) + x}{y(x)}$$

✓ **Mathematica** : cpu = 38.2493 (sec), leaf count = 99

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( -\int_1^x -\frac{2K[1]K[2]F'(K[2]^2 - K[1]^2)}{F(K[2]^2 - K[1]^2)^2} dK[1] - \frac{K[2]}{F(K[2]^2 - x^2)} \right) dK[2] + \int_1^x \left( \frac{K[1]}{F(y(x)^2 - K[1]^2)} \right) \right]$$

✓ **Maple** : cpu = 0.773 (sec), leaf count = 53

$$\left\{ y(x) = \sqrt{x^2 + \text{RootOf}\left(-2x + \int^{-Z} (F(-a))^{-1} d_a + 2\_C1\right)}, y(x) = -\sqrt{x^2 + \text{RootOf}\left(-2x + \int^{-Z} (F(-a))^{-1} d_a + 2\_C1\right)} \right.$$

**2.589 ODE No. 589**

$$y'(x) = \frac{y(x)^2 F\left(\frac{1-y(x)\log(x)}{y(x)}\right)}{x}$$

✓ **Mathematica** : cpu = 23.8117 (sec), leaf count = 134

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} -\frac{K[2]^2 \left(F\left(\frac{1}{K[2]} - \log(x)\right) + 1\right) \left(\int_1^x -\frac{F'\left(\frac{1}{K[2]} - \log(K[1])\right)}{K[1]K[2]^2 \left(F\left(\frac{1}{K[2]} - \log(K[1])\right) + 1\right)^2 dK[1]\right) + 1}{K[2]^2 \left(F\left(\frac{1}{K[2]} - \log(x)\right) + 1\right)} dK[2] + \int_1^x \right.$$

✓ **Maple** : cpu = 1.869 (sec), leaf count = 38

$$\left\{ \int_{-b}^{y(x)} \frac{1}{-a^2} \left(F\left(\frac{1-a \ln(x)}{-a}\right) + 1\right)^{-1} d_a - \ln(x) - \_C1 = 0 \right\}$$

**2.590 ODE No. 590**

$$y'(x) = \frac{x}{F(x^2 + y(x)^2) - y(x)}$$

✓ **Mathematica** : cpu = 37.4044 (sec), leaf count = 91

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left(-\int_1^x \frac{2K[1]K[2]F'(K[1]^2 + K[2]^2)}{F(K[1]^2 + K[2]^2)^2} dK[1] - \frac{K[2]}{F(K[2]^2 + x^2)} + 1\right) dK[2] + \int_1^x -\frac{K[1]}{F(K[1]^2 + x^2)} dK[1] \right.$$

✓ **Maple** : cpu = 1.005 (sec), leaf count = 28

$$\left\{ -y(x) + \frac{\int^{(y(x))^2+x^2} (F(-a))^{-1} d_a}{2} - \_C1 = 0 \right\}$$

**2.591 ODE No. 591**

$$y'(x) = \frac{x F\left(\frac{ay(x)^2 + bx^2}{a}\right)}{\sqrt{ay(x)}}$$

✓ **Mathematica** : cpu = 22.3397 (sec), leaf count = 182

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \frac{\left(\sqrt{a} F\left(K[2]^2 + \frac{bx^2}{a}\right) + b\right) \int_1^x \frac{2b^2 K[1] K[2] F'\left(\frac{bK[1]^2}{a} + K[2]^2\right)}{\sqrt{a} \left(\sqrt{a} F\left(\frac{bK[1]^2}{a} + K[2]^2\right) + b\right)^2} dK[1] + bK[2]}{\sqrt{a} F\left(K[2]^2 + \frac{bx^2}{a}\right) + b} dK[2] + \int_1^x \frac{bK[1] F\left(\frac{bK[1]^2}{a}\right)}{a F\left(\frac{bK[1]^2}{a}\right)} dK[1] \right]$$

✓ **Maple** : cpu = 2.249 (sec), leaf count = 108

$$\left\{ y(x) = \frac{1}{a} \sqrt{a \left(-bx^2 + \text{RootOf}\left(\int^{-Z} (b\sqrt{a} + F(\_a) a)^{-1} d\_aba^{\frac{3}{2}} - bx^2 + 2\_C1 a\right) a\right)}, y(x) = -\frac{1}{a} \sqrt{a \left(-bx^2 + \dots\right)} \right\}$$

**2.592 ODE No. 592**

$$y'(x) = \frac{F\left(-\frac{2x^3}{5} + y(x) - 2\sqrt{x}\right) + \frac{6x^3}{5} + \sqrt{x}}{x}$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.31 (sec), leaf count = 33

$$\left\{ \int_{-b}^{y(x)} \left(F\left(-a - \frac{2x^3}{5} - 2\sqrt{x}\right)\right)^{-1} d\_a - \ln(x) - \_C1 = 0 \right\}$$

**2.593 ODE No. 593**

$$y'(x) = \frac{e^x F\left(y(x)^{3/2} - \frac{3e^x}{2}\right)}{\sqrt{y(x)}}$$

✓ **Mathematica** : cpu = 44.4923 (sec), leaf count = 166

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \frac{\sqrt{K[2]} - \left(F\left(K[2]^{3/2} - \frac{3e^x}{2}\right) - 1\right) \int_1^x \frac{3e^{K[1]} \sqrt{K[2]} F'\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right)}{2 \left(F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right) - 1\right)^2} dK[1]}{F\left(K[2]^{3/2} - \frac{3e^x}{2}\right) - 1} dK[2] + \int_1^x \frac{e^{K[1]} F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right)}{F\left(K[2]^{3/2} - \frac{3e^{K[1]}}{2}\right)} dK[1] \right]$$

✓ **Maple** : cpu = 0.559 (sec), leaf count = 35

$$\left\{ \int_{-b}^{y(x)} 1\sqrt{-a} \left( F\left(-a^{\frac{3}{2}} - \frac{3e^x}{2}\right) - 1 \right)^{-1} d_a - e^x - C1 = 0 \right\}$$

**2.594 ODE No. 594**

$$y'(x) = \frac{x F\left(\frac{y(x)^2 - b}{x^2}\right)}{y(x)}$$

✓ **Mathematica** : cpu = 22.0513 (sec), leaf count = 188

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{K[2]}{x^2 F\left(\frac{K[2]^2 - b}{x^2}\right) - K[2]^2 + b} - \int_1^x -\frac{2K[2] \left( (b - K[2]^2) F'\left(\frac{K[2]^2 - b}{K[1]^2}\right) + K[1]^2 F\left(\frac{K[2]^2 - b}{K[1]^2}\right) \right)}{K[1] \left( K[1]^2 F\left(\frac{K[2]^2 - b}{K[1]^2}\right) - K[2]^2 + b \right)^2} dx \right) dx \right]$$

✓ **Maple** : cpu = 1.821 (sec), leaf count = 67

$$\left\{ y(x) = \sqrt{\text{RootOf}\left(-2 \ln(x) + \int^{-Z} (F(-a) - a)^{-1} d_a + 2 C1\right) x^2 + b}, y(x) = -\sqrt{\text{RootOf}\left(-2 \ln(x) + \int^{-Z} (F(-a) - a)^{-1} d_a + 2 C1\right) x^2 + b} \right\}$$

**2.595 ODE No. 595**

$$y'(x) = \frac{F\left(\frac{xy(x)^2 + 1}{x}\right)}{x^2 y(x)}$$

✓ **Mathematica** : cpu = 23.3251 (sec), leaf count = 127

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \frac{(1 - 2F(K[2]^2 + \frac{1}{x})) \int_1^x \frac{2K[2]F'(K[2]^2 + \frac{1}{K[1]})}{K[1]^2(1 - 2F(K[2]^2 + \frac{1}{K[1]}))^2} dK[1] + K[2]}{2F(K[2]^2 + \frac{1}{x}) - 1} dK[2] + \int_1^x -\frac{F\left(\frac{1}{K[1]} + \frac{1}{x}\right)}{K[1]^2 \left( 2F\left(\frac{1}{K[1]} + \frac{1}{x}\right) - 1 \right)} dx \right]$$

✓ **Maple** : cpu = 0.37 (sec), leaf count = 72

$$\left\{ y(x) = \frac{1}{x} \sqrt{x \left( \text{RootOf}\left(\int^{-Z} (-1 + 2F(-a))^{-1} d_a x + C1 x + 1\right) x - 1 \right)}, y(x) = -\frac{1}{x} \sqrt{x \left( \text{RootOf}\left(\int^{-Z} (-1 + 2F(-a))^{-1} d_a x + C1 x + 1\right) x - 1 \right)} \right\}$$

**2.596 ODE No. 596**

$$y'(x) = \frac{F(x^2 + y(x) - x) - 2x^2 + x}{x}$$

✓ **Mathematica** : cpu = 237.635 (sec), leaf count = 103

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{(2K[1] - 1)F'(K[1]^2 - K[1] + K[2])}{F(K[1]^2 - K[1] + K[2])^2} dK[1] - \frac{1}{F(K[2] + x^2 - x)} \right) dK[2] + \int_1^x \left( \frac{1}{F(K[2] + x^2 - x)} \right) dK[2] \right]$$

✓ **Maple** : cpu = 1.519 (sec), leaf count = 26

$$\left\{ y(x) = -x^2 + \text{RootOf} \left( -\ln(x) + \int^{-Z} (F(_a))^{-1} d_a + \_C1 \right) + x \right\}$$

**2.597 ODE No. 597**

$$y'(x) = \frac{2a}{x^2 \left( 2aF \left( \frac{xy(x)^2 - 4a}{x} \right) - y(x) \right)}$$

✓ **Mathematica** : cpu = 31.2804 (sec), leaf count = 112

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{2K[2]F'(K[2]^2 - \frac{4a}{K[1]})}{K[1]^2 F(K[2]^2 - \frac{4a}{K[1]})^2} dK[1] - \frac{K[2]}{2aF(K[2]^2 - \frac{4a}{x})} + 1 \right) dK[2] + \int_1^x -\frac{1}{K[1]^2 F(y(x))} dK[2] \right]$$

✓ **Maple** : cpu = 1.647 (sec), leaf count = 37

$$\left\{ -\frac{y(x)}{2a} + \frac{1}{8a^2} \int^{(y(x))^2 - 4\frac{a}{x}} (F(_a))^{-1} d_a - \_C1 = 0 \right\}$$

**2.598 ODE No. 598**

$$y'(x) = \frac{F\left(\frac{y(x)}{x}\right) + y(x)}{x - 1}$$

✓ **Mathematica** : cpu = 0.117319 (sec), leaf count = 35

$$\text{Solve} \left[ c_1 + \log(1 - x) = \int_1^{\frac{y(x)}{x}} \frac{1}{F(K[1]) + K[1]} dK[1] + \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.781 (sec), leaf count = 29

$$\left\{ y(x) = \text{RootOf} \left( - \int^{-Z} (F(_a) + _a)^{-1} d_a + \ln(x - 1) - \ln(x) + \_C1 \right) x \right\}$$

**2.599 ODE No. 599**

$$y'(x) = \frac{F(x^2 + y(x)^2) - x}{y(x)}$$

✓ **Mathematica** : cpu = 27.1751 (sec), leaf count = 92

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{2K[1]K[2]F'(K[1]^2 + K[2]^2)}{F(K[1]^2 + K[2]^2)^2} dK[1] - \frac{K[2]}{F(K[2]^2 + x^2)} \right) dK[2] + \int_1^x \left( 1 - \frac{K[1]}{F(K[1]^2 + x^2)} \right) dx \right]$$

✓ **Maple** : cpu = 0.423 (sec), leaf count = 57

$$\left\{ y(x) = \sqrt{-x^2 + \text{RootOf} \left( -2x + \int^{-Z} (F(_a))^{-1} d_a + 2\_C1 \right)}, y(x) = -\sqrt{-x^2 + \text{RootOf} \left( -2x + \int^{-Z} (F(_a))^{-1} d_a + 2\_C1 \right)} \right\}$$

**2.600 ODE No. 600**

$$y'(x) = \frac{y(x)^2 F\left(\frac{1-2y(x)\log(x)}{y(x)}\right)}{x}$$

✓ **Mathematica** : cpu = 28.1145 (sec), leaf count = 135

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \frac{K[2]^2 \left( F\left(\frac{1}{K[2]} - 2\log(x)\right) + 2 \right) \left( \int_1^x - \frac{4F'\left(\frac{1}{K[2]} - 2\log(K[1])\right)}{K[1]K[2]^2 \left( F\left(\frac{1}{K[2]} - 2\log(K[1])\right) + 2 \right)^2} dK[1] \right) + 2}{K[2]^2 \left( F\left(\frac{1}{K[2]} - 2\log(x)\right) + 2 \right)} dK[2] + \dots \right]$$

✓ **Maple** : cpu = 6.479 (sec), leaf count = 38

$$\left\{ \int_{-b}^{y(x)} \frac{1}{-a^2} \left( F\left(\frac{-2\_a \ln(x) + 1}{-a}\right) + 2 \right)^{-1} d\_a - \ln(x) - \_C1 = 0 \right\}$$



**2.601 ODE No. 601**

$$y'(x) = \frac{x F(-(x - y(x))(y(x) + x))}{y(x)}$$

✓ **Mathematica** : cpu = 39.651 (sec), leaf count = 129

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \frac{K[2] - (F(K[2]^2 - x^2) - 1) \int_1^x \frac{2K[1]K[2]F'(K[2]^2 - K[1]^2)}{(F(K[2]^2 - K[1]^2) - 1)^2} dK[1]}{F(K[2]^2 - x^2) - 1} dK[2] + \int_1^x \frac{K[1]F(y(x)^2 - K[1]^2)}{1 - F(y(x)^2 - K[1]^2)} dK[1] \right]$$

✓ **Maple** : cpu = 0.335 (sec), leaf count = 61

$$\left\{ y(x) = \sqrt{x^2 + \text{RootOf} \left( -x^2 + \int^{-Z} (F(_a) - 1)^{-1} d_a + 2\_C1 \right)}, y(x) = -\sqrt{x^2 + \text{RootOf} \left( -x^2 + \int^{-Z} (F(_a) - 1)^{-1} d_a + 2\_C1 \right)} \right\}$$

**2.602 ODE No. 602**

$$y'(x) = \frac{y(x)^2 \left( x^2 F\left(\frac{x^2 - y(x)}{x^2 y(x)}\right) + 2 \right)}{x^3}$$

✓ **Mathematica** : cpu = 234.397 (sec), leaf count = 111

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{2F'\left(\frac{1}{K[2]} - \frac{1}{K[1]^2}\right)}{K[1]^3 K[2]^2 F\left(\frac{1}{K[2]} - \frac{1}{K[1]^2}\right)^2} dK[1] - \frac{1}{K[2]^2 F\left(\frac{1}{K[2]} - \frac{1}{x^2}\right)} \right) dK[2] + \int_1^x \left( \frac{1}{K[1]^3 F\left(\frac{1}{K[2]} - \frac{1}{K[1]^2}\right)} \right) dK[1] \right]$$

✓ **Maple** : cpu = 2.294 (sec), leaf count = 33

$$\left\{ y(x) = \frac{x^2}{\text{RootOf} \left( -\ln(x) - \int^{-Z} (F(_a))^{-1} d_a + \_C1 \right) x^2 + 1} \right\}$$

**2.603 ODE No. 603**

$$y'(x) = \frac{2xF(y(x) + \log(2x + 1)) + F(y(x) + \log(2x + 1)) - 2}{2x + 1}$$

✓ **Mathematica** : cpu = 20.0471 (sec), leaf count = 102

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{1}{F(K[2] + \log(2x + 1))} - \int_1^x -\frac{2F'(K[2] + \log(2K[1] + 1))}{(2K[1] + 1)F(K[2] + \log(2K[1] + 1))^2} dK[1] \right) dK[2] + \int_1^x \left( \right. \right.$$

✓ **Maple** : cpu = 1.204 (sec), leaf count = 27

$$\left\{ y(x) = -\ln(2x + 1) + \text{RootOf} \left( -x + \int^{-Z} (F(\_a))^{-1} d\_a + \_C1 \right) \right\}$$

**2.604 ODE No. 604**

$$y'(x) = \frac{2y(x)^3}{2y(x)F\left(\frac{4xy(x)^2+1}{y(x)^2}\right) + 1}$$

✓ **Mathematica** : cpu = 28.1519 (sec), leaf count = 97

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{\frac{1}{F\left(\frac{1}{K[2]^2} + 4x\right)} + 2K[2]}{2K[2]^3} - \int_1^x -\frac{2F'\left(4K[1] + \frac{1}{K[2]^2}\right)}{K[2]^3 F\left(4K[1] + \frac{1}{K[2]^2}\right)^2} dK[1] \right) dK[2] + \int_1^x -\frac{1}{F\left(4K[1] + \frac{1}{K[2]^2}\right)} \right.$$

✓ **Maple** : cpu = 0.512 (sec), leaf count = 30

$$\left\{ -\_C1 - (y(x))^{-1} - \frac{\int^{4x+(y(x))^{-2}} (F(\_a))^{-1} d\_a}{4} = 0 \right\}$$

**2.605 ODE No. 605**

$$y'(x) = -\frac{y(x)^2 \left( 2x - F\left(\frac{1-\frac{1}{2}xy(x)}{y(x)}\right) \right)}{4x}$$

✓ **Mathematica** : cpu = 211.45 (sec), leaf count = 103

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( -\int_1^x -\frac{2F'\left(\frac{1}{K[2]} - \frac{K[1]}{2}\right)}{K[2]^2 F\left(\frac{1}{K[2]} - \frac{K[1]}{2}\right)^2} dK[1] - \frac{4}{K[2]^2 F\left(\frac{1}{K[2]} - \frac{x}{2}\right)} \right) dK[2] + \int_1^x \left( \frac{1}{K[1]} - \frac{1}{F\left(\frac{1}{y(x)}\right)} \right.$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 29

$$\left\{ y(x) = 2 \left( 2 \operatorname{RootOf} \left( -\ln(x) - 4 \int^{-Z} (F(\_a))^{-1} d\_a + \_C1 \right) + x \right)^{-1} \right\}$$

**2.606 ODE No. 606**

$$y'(x) = -x \left( -F \left( y(x) - \frac{1}{2} e^{-x^2} x^2 \right) + e^{-x^2} x^2 - e^{-x^2} \right)$$

✓ **Mathematica** : cpu = 76.3829 (sec), leaf count = 180

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{e^{-K[1]^2} K[1] (K[1]^2 - 1) F'(K[2] - \frac{1}{2} e^{-K[1]^2} K[1]^2)}{F(K[2] - \frac{1}{2} e^{-K[1]^2} K[1]^2)^2} dK[1] - \frac{1}{F(K[2] - \frac{1}{2} e^{-x^2} x^2)} \right) dL \right]$$

✓ **Maple** : cpu = 1.879 (sec), leaf count = 34

$$\left\{ y(x) = \frac{x^2 e^{-x^2}}{2} + \operatorname{RootOf} \left( x^2 - 2 \int^{-Z} (F(\_a))^{-1} d\_a + 2 \_C1 \right) \right\}$$

**2.607 ODE No. 607**

$$y'(x) = \frac{x^3 F \left( \frac{y(x)}{x^2} \right) + 2y(x)}{x}$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.092 (sec), leaf count = 22

$$\left\{ y(x) = \operatorname{RootOf} \left( -x + \int^{-Z} (F(\_a))^{-1} d\_a + \_C1 \right) x^2 \right\}$$

2.608 ODE No. 608

$$y'(x) = \frac{\sqrt{y(x)}}{F\left(\frac{x-y(x)}{\sqrt{y(x)}}\right) + \sqrt{y(x)}}$$

✗ **Mathematica** : cpu = 300.005 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.772 (sec), leaf count = 40

$$\left\{ \frac{\ln(y(x))}{2} - \int^{x \frac{1}{\sqrt{y(x)}} - \sqrt{y(x)}} (2F(_a) - _a)^{-1} d_a - _C1 = 0 \right\}$$

2.609 ODE No. 609

$$y'(x) = \frac{F(x^3 y(x)) - 3x^2 y(x)}{x^3}$$

✓ **Mathematica** : cpu = 61.0835 (sec), leaf count = 107

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{3K[1]^5 K[2] F'(K[1]^3 K[2]) - 3K[1]^2 F(K[1]^3 K[2])}{F(K[1]^3 K[2])^2} dK[1] - \frac{x^3}{F(x^3 K[2])} \right) dK[2] + \int_1^x \right.$$

✓ **Maple** : cpu = 0.859 (sec), leaf count = 22

$$\left\{ y(x) = \frac{\text{RootOf}\left(x - \int^{-Z} (F(_a))^{-1} d_a + _C1\right)}{x^3} \right\}$$

2.610 ODE No. 610

$$y'(x) = \frac{x^2 F\left(\frac{y(x)}{x}\right) + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.0770719 (sec), leaf count = 24

$$\text{Solve} \left[ c_1 + x = \int_1^{\frac{y(x)}{x}} \frac{1}{F(K[1])} dK[1], y(x) \right]$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 20

$$\left\{ y(x) = \text{RootOf}\left(x - \int^{-Z} (F(_a))^{-1} d_a + _C1\right) x \right\}$$

**2.611 ODE No. 611**

$$y'(x) = \frac{F(x(y(x) + x)) - y(x) - 2x}{x}$$

✓ **Mathematica** : cpu = 45.9447 (sec), leaf count = 116

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{K[1](2K[1] + K[2])F'(K[1](K[1] + K[2])) - F(K[1](K[1] + K[2]))}{F(K[1](K[1] + K[2]))^2} dK[1] - \frac{x}{F(x(K[2] \right.$$

✓ **Maple** : cpu = 1.11 (sec), leaf count = 28

$$\left\{ y(x) = \frac{-x^2 + \text{RootOf}\left(-x + \int^{-Z} (F(_a))^{-1} d_a + \_C1\right)}{x} \right\}$$

**2.612 ODE No. 612**

$$y'(x) = \frac{1}{2} e^{\frac{x^2}{4}} \left( 2F\left(e^{-\frac{x^2}{4}} y(x)\right) + e^{-\frac{x^2}{4}} xy(x) \right)$$

✓ **Mathematica** : cpu = 56.1524 (sec), leaf count = 169

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{e^{-\frac{1}{2}K[1]^2} K[1] \left( e^{\frac{K[1]^2}{4}} F\left(e^{-\frac{1}{4}K[1]^2} K[2]\right) - K[2]F'\left(e^{-\frac{1}{4}K[1]^2} K[2]\right) \right)}{2F\left(e^{-\frac{1}{4}K[1]^2} K[2]\right)^2} dK[1] - \frac{e^{-\frac{x^2}{4}}}{F\left(e^{-\frac{x^2}{4}} K \right. \right.$$

✓ **Maple** : cpu = 3.195 (sec), leaf count = 27

$$\left\{ y(x) = \text{RootOf}\left(-x + \int^{-Z} (F(_a))^{-1} d_a + \_C1\right) \left(e^{-\frac{x^2}{4}}\right)^{-1} \right\}$$

**2.613 ODE No. 613**

$$y'(x) = \frac{x^2 F\left(\frac{y(x)-x \log(x)}{x}\right) + y(x) + x}{x}$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.749 (sec), leaf count = 23

$$\left\{ y(x) = \left( \ln(x) + \text{RootOf}\left(-x + \int^{-Z} (F(_a))^{-1} d_a + \_C1\right) \right) x \right\}$$

**2.614 ODE No. 614**

$$y'(x) = \frac{(a-1)(a+1)x}{a^2 F\left(-\frac{1}{2}a^2x^2 + \frac{x^2}{2} + \frac{y(x)^2}{2}\right) - F\left(-\frac{1}{2}a^2x^2 + \frac{x^2}{2} + \frac{y(x)^2}{2}\right) + y(x)}$$

✓ **Mathematica** : cpu = 84.9394 (sec), leaf count = 144

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{K[1]K[2]F'\left(\frac{1}{2}(K[2]^2 - (a^2 - 1)K[1]^2)\right)}{F\left(\frac{1}{2}(K[2]^2 - (a^2 - 1)K[1]^2)\right)^2} dK[1] + \frac{K[2]}{(a-1)(a+1)F\left(\frac{1}{2}(K[2]^2 - a^2x^2 + \dots)}\right)} \right) \right]$$

✓ **Maple** : cpu = 3.014 (sec), leaf count = 60

$$\left\{ \frac{y(x)}{(a-1)(a+1)} + \frac{1}{2a^4 - 4a^2 + 2} \int^{-a^2x^2 + x^2 + (y(x))^2} \left( F\left(\frac{-a}{2}\right) \right)^{-1} d_a - C1 = 0 \right\}$$

**2.615 ODE No. 615**

$$y'(x) = \frac{y(x)}{x(y(x)F(xy(x)) - 1)}$$

✓ **Mathematica** : cpu = 19.0071 (sec), leaf count = 74

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{F'(K[1]K[2])}{F(K[1]K[2])^2} dK[1] - \frac{1}{K[2]F(xK[2])} + 1 \right) dK[2] + \int_1^x - \frac{1}{K[1]F(y(x)K[1])} dK[1], y(x) \right]$$

✓ **Maple** : cpu = 1.848 (sec), leaf count = 26

$$\left\{ -y(x) + \int^{xy(x)} \frac{1}{F(-a)_a} d_a - C1 = 0 \right\}$$

**2.616 ODE No. 616**

$$y'(x) = \frac{F(xxy(x) - 1) - 2x^3y(x) + x^2}{x^4}$$

✓ **Mathematica** : cpu = 55.7055 (sec), leaf count = 126

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{K[1](K[1](2K[1]K[2] - 1)F'(K[1](K[1]K[2] - 1)) - 2F(K[1](K[1]K[2] - 1)))}{F(K[1](K[1]K[2] - 1))^2} dK[1] \right) \right]$$

✓ **Maple** : cpu = 1.794 (sec), leaf count = 26

$$\left\{ y(x) = \frac{\text{RootOf}\left(\int^{-Z}(F(_a))^{-1} d_a x + _C1 x + 1\right) + x}{x^2} \right\}$$

**2.617 ODE No. 617**

$$y'(x) = \frac{1}{9} e^{-\frac{3x^2}{2}} xy(x)^2 F\left(\frac{e^{\frac{3x^2}{2}}(y(x)+3)}{3y(x)}\right)$$

✓ **Mathematica** : cpu = 270.631 (sec), leaf count = 302

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{9e^{\frac{3K[1]^2}{2}} K[1] \left( e^{\frac{3K[1]^2}{2}} (K[2]+3) F' \left( \frac{e^{\frac{3K[1]^2}{2}} (K[2]+3)}{3K[2]} \right) - 3K[2] F \left( \frac{e^{\frac{3K[1]^2}{2}} (K[2]+3)}{3K[2]} \right) \right)}{K[2] \left( K[2] F \left( \frac{e^{\frac{3K[1]^2}{2}} (K[2]+3)}{3K[2]} \right) - 9e^{\frac{3K[1]^2}{2}} (K[2]+3) \right)^2} dx \right) dy(x) \right]$$

✓ **Maple** : cpu = 1.109 (sec), leaf count = 47

$$\left\{ y(x) = -3 \frac{e^{3/2 x^2}}{e^{3/2 x^2} - 3 \text{RootOf}\left(-x^2 - 18 \int^{-Z}(F(_a) - 27_a)^{-1} d_a + 2_C1\right)} \right\}$$

**2.618 ODE No. 618**

$$y'(x) = \frac{(y(x)+1)(x(y(x)) - \log(y(x)+1) - \log(x)) + 1}{xy(x)}$$

✓ **Mathematica** : cpu = 0.0973369 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow -W\left(-\frac{e^{c_1 e^x - 1}}{x}\right) - 1 \right\} \right\}$$

✓ **Maple** : cpu = 5.764 (sec), leaf count = 34

$$\left\{ y(x) = \frac{1}{x} \left( e^{-\text{lambertW}\left(-\frac{e^{e^x - C1 - 1}}{x}\right) + e^x - C1 - 1} - x \right) \right\}$$

**2.619 ODE No. 619**

$$y'(x) = \frac{6y(x)}{-F\left(-\frac{1}{3}y(x)^4 - \frac{y(x)^3}{2} - y(x)^2 - y(x) + x\right) + 8y(x)^4 + 9y(x)^3 + 12y(x)^2 + 6y(x)}$$

✓ **Mathematica** : cpu = 257.989 (sec), leaf count = 230

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \frac{F\left(x - \frac{1}{6}K[2] (2K[2]^3 + 3K[2]^2 + 6K[2] + 6)\right) \left(1 - K[2]\right) \left(\int_1^x \frac{(8K[2]^3 + 9K[2]^2 + 12K[2] + 6)F'(K[1] - \frac{1}{6}K[2])}{F(K[1] - \frac{1}{6}K[2](2K[2]^3 + 3K[2]^2 + 6K[2] + 6))} dx\right)}{K[2]F\left(x - \frac{1}{6}K[2] (2K[2]^3 + 3K[2]^2 + 6K[2] + 6)\right)} \right]$$

✓ **Maple** : cpu = 2.099 (sec), leaf count = 81

$$\left\{ \int_{-b}^{y(x)} \frac{1}{-a} \left( -8_a^4 - 9_a^3 - 12_a^2 + F\left(-\frac{a^4}{3} - \frac{a^3}{2} - a^2 - a + x\right) - 6_a \right) \left( F\left(-\frac{a^4}{3} - \frac{a^3}{2} - a^2 - a + x\right) \right) dx \right\}$$

**2.620 ODE No. 620**

$$y'(x) = \frac{e^{2F(-(x-y(x))(y(x)+x))} + x^2 + 2xy(x) + y(x)^2}{-e^{2F(-(x-y(x))(y(x)+x))} + x^2 + 2xy(x) + y(x)^2}$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 5.51 (sec), leaf count = 37

$$\left\{ y(x) = e^{\text{RootOf}\left(-_Z + \int (e^{-Z})^2 - 2xe^{-Z} (e^{2F(-a)} + a)^{-1} d_a + C1\right)} - x \right\}$$

**2.621 ODE No. 621**

$$y'(x) = \frac{1}{y(x) + \sqrt{x}}$$

✓ **Mathematica** : cpu = 0.0822352 (sec), leaf count = 445

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\text{Root}\left[\#1^6 (16e^{12c_1} + 16x^3) - 24\#1^4 x^2 + 8\#1^3 x^{3/2} + 9\#1^2 x - 6\#1\sqrt{x} + 1\&, 1\right]} - \sqrt{x} \right\}, \left\{ y(x) \rightarrow \dots \right\} \right\}$$

✓ **Maple** : cpu = 4.549 (sec), leaf count = 59

$$\left\{ y(x) = 1 \left( \sqrt{x} (\text{RootOf}(_Z^{18} C1 - 9x_Z^6 - 6\sqrt{x}_Z^3 - 1))^3 + 1 \right) (\text{RootOf}(_Z^{18} C1 - 9x_Z^6 - 6\sqrt{x}_Z^3 - 1))^3 \right\}$$



**2.622 ODE No. 622**

$$y'(x) = \frac{1}{y(x) + \sqrt{3x+1} + 2}$$

✗ **Mathematica** : cpu = 300.029 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.735 (sec), leaf count = 77

$$\left\{ \ln \left( (3y(x) + 6) \sqrt{3x+1} + 3(y(x))^2 - 6x + 12y(x) + 10 \right) - 6 \frac{\sqrt{3x+1}}{\sqrt{99x+33}} \operatorname{Artanh} \left( \frac{3\sqrt{3x+1} + 6y(x) + 12}{\sqrt{99x+33}} \right) \right\}$$

**2.623 ODE No. 623**

$$y'(x) = \frac{x^2}{x^{3/2} + y(x)}$$

✓ **Mathematica** : cpu = 11.5619 (sec), leaf count = 77

$$\text{Solve} \left[ 44c_1 + 6\sqrt{33} \tanh^{-1} \left( \frac{7x^{3/2} + 3y(x)}{\sqrt{33}(x^{3/2} + y(x))} \right) = 33 \left( \log \left( -\frac{3y(x)}{2x^{3/2}} - \frac{3y(x)^2}{2x^3} + 1 \right) + 3 \log(x) \right), y(x) \right]$$

✓ **Maple** : cpu = 3.777 (sec), leaf count = 49

$$\left\{ \ln \left( 3x^{3/2}y(x) - 2x^3 + 3(y(x))^2 \right) - \frac{2\sqrt{33}}{11} \operatorname{Artanh} \left( \frac{\sqrt{33}}{11} \left( x^{3/2} + 2y(x) \right) x^{-3/2} \right) - C1 = 0 \right\}$$

**2.624 ODE No. 624**

$$y'(x) = \frac{x^{5/3}}{x^{4/3} + y(x)}$$

✓ **Mathematica** : cpu = 48.5463 (sec), leaf count = 1

\$Aborted

✓ **Maple** : cpu = 6.144 (sec), leaf count = 46

$$\left\{ y(x) = \frac{1}{2} \left( \operatorname{RootOf} \left( -Z^{192} + 12x^{4/3} Z^{176} + 48x^{8/3} Z^{160} + 64x^4 Z^{144} - C1 \right) \right)^{16} + \frac{1}{2} x^{4/3} \right\}$$

**2.625 ODE No. 625**

$$y'(x) = \frac{1}{2}ix^2(-2\sqrt{6y(x) - x^3} + i)$$

✓ **Mathematica** : cpu = 0.375532 (sec), leaf count = 67

Solve  $\left[12ic_1 + 2\sqrt{6y(x) - x^3} + i \log(-x^3 + 6y(x) + 1) + 2ix^3 = 2 \tan^{-1}(\sqrt{6y(x) - x^3}), y(x)\right]$

✓ **Maple** : cpu = 4.48 (sec), leaf count = 55

$$\left\{-2i\sqrt{-x^3 + 6y(x)} + 2i \arctan(\sqrt{-x^3 + 6y(x)}) + \ln(-x^3 + 6y(x) + 1) + 2x^3 - \_C1 = 0\right\}$$

**2.626 ODE No. 626**

$$y'(x) = \frac{x}{\sqrt{x^2 + 1} + y(x)}$$

✓ **Mathematica** : cpu = 1.36352 (sec), leaf count = 104

Solve  $\left[\frac{1}{2}\left(\log\left(\frac{-\sqrt{x^2 + 1}y(x)^2 - (x^2 + 1)y(x) + (x^2 + 1)^{3/2}}{(x^2 + 1)^{3/2}}\right) + \log(x^2 + 1)\right) = c_1 + \frac{\tanh^{-1}\left(\frac{3\sqrt{x^2 + 1} + y(x)}{\sqrt{5}(\sqrt{x^2 + 1} + y(x))}\right)}{\sqrt{5}}\right]$

✓ **Maple** : cpu = 7.711 (sec), leaf count = 115

$$\left\{\frac{2}{3}\ln\left(-\frac{1296}{11}(y(x)\sqrt{x^2 + 1} - x^2 + (y(x))^2 - 1)(y(x) + \sqrt{x^2 + 1})^{-2}\right) - \frac{4\sqrt{5}}{15}\text{Artanh}\left(\sqrt{5}(3\sqrt{x^2 + 1} + y(x))\right)\right\}$$

**2.627 ODE No. 627**

$$y'(x) = \frac{(y(x)\log(x) - 1)^2}{x}$$

✓ **Mathematica** : cpu = 0.754884 (sec), leaf count = 25

$$\left\{\left\{y(x) \rightarrow \frac{\tan(c_1 + \log(x))}{\log(x)\tan(c_1 + \log(x)) + 1}\right\}\right\}$$

✓ **Maple** : cpu = 14.909 (sec), leaf count = 35

$$\left\{y(x) = \frac{\sin(\ln(x))\_C1 + \cos(\ln(x))}{(\ln(x) + \_C1)\cos(\ln(x)) + \sin(\ln(x))(\_C1 \ln(x) - 1)}\right\}$$

**2.628 ODE No. 628**

$$y'(x) = \frac{1}{3}x(3\sqrt{x^2 + 3y(x)} - 2)$$

✓ **Mathematica** : cpu = 0.0767801 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{48}(-2(27c_1 + 8)x^2 + 81c_1^2 + 9x^4) \right\} \right\}$$

✓ **Maple** : cpu = 10.563 (sec), leaf count = 23

$$\left\{ -C1 + \frac{3x^2}{4} + \frac{2}{3} - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

**2.629 ODE No. 629**

$$y'(x) = \frac{(2y(x)\log(x) - 1)^2}{x}$$

✓ **Mathematica** : cpu = 0.665909 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\sqrt{2} \left( \sqrt{2} \log(x) - \tan \left( \frac{c_1 + 2 \log(x)}{\sqrt{2}} \right) \right)} \right\} \right\}$$

✓ **Maple** : cpu = 20.097 (sec), leaf count = 62

$$\left\{ y(x) = \frac{\sin(\sqrt{2} \ln(x)) - C1 - \cos(\sqrt{2} \ln(x))}{\sin(\sqrt{2} \ln(x)) (2 - C1 \ln(x) + \sqrt{2}) + (\sqrt{2} - C1 - 2 \ln(x)) \cos(\sqrt{2} \ln(x))} \right\}$$

**2.630 ODE No. 630**

$$y'(x) = \frac{e^{bx}}{e^{-bx}y(x) + 1}$$

✓ **Mathematica** : cpu = 0.581247 (sec), leaf count = 101

$$\text{Solve} \left[ \frac{1}{2}b \left( \log \left( -be^{-2bx}y(x)^2 - be^{-bx}y(x) + 1 \right) + 2bx \right) = \frac{b \tan^{-1} \left( \frac{(b+2)(-e^{bx}) - by(x)}{b\sqrt{-\frac{b+4}{b}}(e^{bx} + y(x))} \right)}{\sqrt{-\frac{b+4}{b}}} + c_1, y(x) \right]$$

✓ **Maple** : cpu = 19.011 (sec), leaf count = 98

$$\left\{ y(x) = \frac{1}{e^{-bx}} \text{RootOf} \left( -e^{\text{RootOf} \left( \left( \tanh \left( \frac{2-C1 b-2bx-Z \sqrt{b^2+4b}}{2b} \right) \right)^2 b+4 \left( \tanh \left( \frac{1/2 \sqrt{b^2+4b}(2-C1 b-2bx-Z)}{b} \right) \right)^2 -4 e^{-Z-b-4} \right)} \right. \right.$$

**2.631 ODE No. 631**

$$y'(x) = \frac{1}{2} x^2 (2\sqrt{x^3 - 6y(x)} + 1)$$

✓ **Mathematica** : cpu = 0.0790744 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} ((1 - 12c_1) x^3 - 36c_1^2 - x^6) \right\} \right\}$$

✓ **Maple** : cpu = 45.07 (sec), leaf count = 23

$$\left\{ -C1 - x^3 - \frac{1}{2} - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

**2.632 ODE No. 632**

$$y'(x) = \frac{e^x}{e^{-x}y(x) + 1}$$

✓ **Mathematica** : cpu = 0.366834 (sec), leaf count = 65

$$\text{Solve} \left[ \frac{1}{2} \log(-e^{-2x}y(x)^2 - e^{-x}y(x) + 1) + x = c_1 + \frac{\tanh^{-1} \left( \frac{y(x)+3e^x}{\sqrt{5}(y(x)+e^x)} \right)}{\sqrt{5}}, y(x) \right]$$

✓ **Maple** : cpu = 11.1 (sec), leaf count = 54

$$\left\{ x + \frac{\ln \left( (y(x))^2 (e^{-x})^2 + y(x) e^{-x} - 1 \right)}{2} - \frac{\sqrt{5}}{5} \text{Artanh} \left( \frac{2y(x) \sqrt{5} e^{-x}}{5} + \frac{\sqrt{5}}{5} \right) - C1 = 0 \right\}$$

**2.633 ODE No. 633**

$$y'(x) = \frac{e^{2x/3}}{e^{-2x/3}y(x) + 1}$$

✓ **Mathematica** : cpu = 0.375931 (sec), leaf count = 85

$$\text{Solve} \left[ 7 \left( -9c_1 + 3 \log \left( -\frac{2}{3} e^{-4x/3} y(x)^2 - \frac{2}{3} e^{-2x/3} y(x) + 1 \right) + 4x \right) = 6\sqrt{7} \tanh^{-1} \left( \frac{y(x) + 4e^{2x/3}}{\sqrt{7}(y(x) + e^{2x/3})} \right), y(x) \right]$$

✓ **Maple** : cpu = 14.888 (sec), leaf count = 52

$$\left\{ y(x) = 1 \text{RootOf} \left( -e^{\text{RootOf} \left( 343 - 343 \left( \tanh \left( 1/6 (4\_C1 - 4x - 3\_Z) \sqrt{7} \right) \right)^2 + 98 e^{-Z} \right) - 3 + 2\_Z + 2\_Z^2} \right) \left( e^{-\frac{2x}{3}} \right)^{-1} \right\}$$

**2.634 ODE No. 634**

$$y'(x) = \frac{x^5 \sqrt{4x^2 y(x) + 1} + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.156921 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{16} \left( -8c_1 x^4 + 16c_1^2 + x^8 - \frac{4}{x^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 10.351 (sec), leaf count = 26

$$\left\{ -C1 - \frac{1}{x} \sqrt{4x^2 y(x) + 1} + \frac{x^4}{2} = 0 \right\}$$

**2.635 ODE No. 635**

$$y'(x) = \frac{1}{2} x \left( 2\sqrt{x^3 - 6y(x)} + x \right)$$

✓ **Mathematica** : cpu = 0.103267 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{24} (36c_1 x^2 - 36c_1^2 - 9x^4 + 4x^3) \right\} \right\}$$

✓ **Maple** : cpu = 6.954 (sec), leaf count = 22

$$\left\{ -C1 - \frac{3x^2}{2} - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

**2.636 ODE No. 636**

$$y'(x) = y(x) (x^2 - \log(y(x)))$$

✓ **Mathematica** : cpu = 0.0456087 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow e^{-2c_1 e^{-x} + x^2 - 2x + 2} \right\} \right\}$$

✓ **Maple** : cpu = 4.838 (sec), leaf count = 19

$$\left\{ y(x) = e^{\frac{C1}{e^x} + x^2 - 2x + 2} \right\}$$

**2.637 ODE No. 637**

$$y'(x) = \frac{e^{-x^2} x}{e^{x^2} y(x) + 1}$$

✓ **Mathematica** : cpu = 15.6765 (sec), leaf count = 53

$$\text{Solve} \left[ 2x^2 = 4c_1 + \log \left( 2e^{2x^2} y(x)^2 + 2e^{x^2} y(x) + 1 \right) + 2 \tan^{-1} \left( 2e^{x^2} y(x) + 1 \right), y(x) \right]$$

✓ **Maple** : cpu = 27.176 (sec), leaf count = 84

$$\left\{ y(x) = -\frac{1}{e^{x^2}} \tan \left( \text{RootOf} \left( 2x^2 - \ln \left( \frac{81 (\tan(\_Z))^2}{10} + \frac{81}{10} \right) + 2 \ln(9/2 \tan(\_Z) - 9/2) + 6\_C1 - 2\_Z \right) \right) \right\}$$

**2.638 ODE No. 638**

$$y'(x) = y(x) (-(\log(x) - \log(\log(y(x))))))$$

✗ **Mathematica** : cpu = 2.36564 (sec), leaf count = 0 , could not solve

$$\text{DSolve}[\text{Derivative}[1][y][x] == -((\text{Log}[x] - \text{Log}[\text{Log}[y[x]]])*y[x]), y[x], x]$$

✓ **Maple** : cpu = 1.461 (sec), leaf count = 35

$$\left\{ \int_{\_b}^{y(x)} \frac{1}{\_a (x \ln(x) - \ln(\ln(\_a)) x + \ln(\_a))} d\_a + \ln(x) - \_C1 = 0 \right\}$$

**2.639 ODE No. 639**

$$y'(x) = y(x)(\log(x) - \log(\log(y(x))))^2$$

✗ **Mathematica** : cpu = 0.293853 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == (Log[x] - Log[Log[y[x]]])^2\*y[x], y[x], x]

✓ **Maple** : cpu = 1.004 (sec), leaf count = 50

$$\left\{ \int_{-b}^{y(x)} \frac{1}{-a \left( x (\ln(x))^2 - 2 \ln(x) \ln(\ln(-a)) x + (\ln(\ln(-a)))^2 x - \ln(-a) \right)} d_{-a} - \ln(x) - _C1 = 0 \right\}$$

**2.640 ODE No. 640**

$$y'(x) = \frac{y(x)}{\log(\log(y(x))) - \log(x) + 1}$$

✗ **Mathematica** : cpu = 3.25119 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == y[x]/(1 - Log[x] + Log[Log[y[x]]]), y[x], x]

✓ **Maple** : cpu = 0.566 (sec), leaf count = 47

$$\left\{ \int_{-b}^{y(x)} \frac{-\ln(\ln(-a)) + \ln(x) - 1}{(\ln(-a) \ln(x) - \ln(-a) \ln(\ln(-a)) - \ln(-a) + x)_{-a}} d_{-a} - _C1 = 0 \right\}$$

**2.641 ODE No. 641**

$$y'(x) = \frac{x^4 \sqrt{4x^2 y(x) + 1} + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.155894 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow -\frac{2c_1 x^3}{3} + c_1^2 + \frac{x^6}{9} - \frac{1}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 2.438 (sec), leaf count = 26

$$\left\{ -C1 - \frac{1}{x} \sqrt{4x^2 y(x) + 1} + \frac{2x^3}{3} = 0 \right\}$$

**2.642 ODE No. 642**

$$y'(x) = \frac{(4ax - y(x)^2)^2}{y(x)}$$

✓ **Mathematica** : cpu = 0.12027 (sec), leaf count = 95

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{4ax - \sqrt{2}\sqrt{a} \tanh\left(\frac{\sqrt{2}(2ax - c_1)}{\sqrt{a}}\right)} \right\}, \left\{ y(x) \rightarrow \sqrt{4ax - \sqrt{2}\sqrt{a} \tanh\left(\frac{\sqrt{2}(2ax - c_1)}{\sqrt{a}}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 1.088 (sec), leaf count = 286

$$\left\{ y(x) = \sqrt{4} \sqrt{\left( -C1 \left( ax - \frac{\sqrt{2}}{4} \sqrt{a} \right) e^{2x(\sqrt{2}\sqrt{a}-2ax)} + e^{-2x(\sqrt{2}\sqrt{a}+2ax)} \left( ax + \frac{\sqrt{2}}{4} \sqrt{a} \right) \right) \left( -C1 e^{2x(\sqrt{2}\sqrt{a}-2ax)} \right)} \right\}$$

**2.643 ODE No. 643**

$$y'(x) = \frac{1}{3}x(3x\sqrt{x^2 + 3y(x)} - 2)$$

✓ **Mathematica** : cpu = 0.10456 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12}(-6c_1x^3 + 9c_1^2 + x^6 - 4x^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.189 (sec), leaf count = 22

$$\left\{ -C1 + \frac{x^3}{2} - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

**2.644 ODE No. 644**

$$y'(x) = -\frac{1}{2}x^2(ax - 2\sqrt{a(ax^4 + 8y(x))})$$

✓ **Mathematica** : cpu = 0.253832 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72}a(-96c_1x^3 + 144c_1^2 + 16x^6 - 9x^4) \right\} \right\}$$

✓ **Maple** : cpu = 0.666 (sec), leaf count = 27

$$\left\{ -C1 + \frac{4ax^3}{3} - \sqrt{a(ax^4 + 8y(x))} = 0 \right\}$$



**2.645 ODE No. 645**

$$y'(x) = y(x)(x - \log(y(x)))$$

✓ **Mathematica** : cpu = 0.0326839 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow e^{-e^{c_1 - x + x - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.66 (sec), leaf count = 14

$$\left\{ y(x) = e^{\frac{C_1}{e^x} - 1 + x} \right\}$$

**2.646 ODE No. 646**

$$y'(x) = \frac{\sqrt{x^3 - 6y(x)} + \frac{x^3}{2} + \frac{x^2}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.155395 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} (18c_1 \log(x + 1) - 9c_1^2 + x^3 - 9 \log^2(x + 1)) \right\} \right\}$$

✓ **Maple** : cpu = 0.467 (sec), leaf count = 23

$$\left\{ -C_1 - 3 \ln(1 + x) - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

**2.647 ODE No. 647**

$$y'(x) = \frac{x(ay(x)^2 + bx^2)^2}{a^{5/2}y(x)}$$

✓ **Mathematica** : cpu = 0.367267 (sec), leaf count = 117

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{\frac{a^{3/4}\sqrt{b} \tan\left(\frac{a^{3/2}bx^2 + 2c_1}{a^{9/4}\sqrt{b}}\right) - bx^2}{a}} \right\}, \left\{ y(x) \rightarrow \sqrt{\frac{a^{3/4}\sqrt{b} \tan\left(\frac{a^{3/2}bx^2 + 2c_1}{a^{9/4}\sqrt{b}}\right) - bx^2}{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.461 (sec), leaf count = 460

$$\left\{ y(x) = \frac{1}{a} \sqrt{-\left(-C_1 e^{\frac{x^2}{2}} \left(2 \sqrt{-\frac{b}{a^{3/2}}} a^{3/2} + bx^2\right) a^{-\frac{3}{2}} + e^{\frac{x^2}{2}} \left(-2 \sqrt{-\frac{b}{a^{3/2}}} a^{3/2} + bx^2\right) a^{-\frac{3}{2}}\right)} a \left( \left( bx^2 - \sqrt{-ba^{-\frac{3}{2}} a^{\frac{3}{2}}} \right) e^{\frac{x^2}{2}} \left(-2 \sqrt{-\frac{b}{a^{3/2}}} a^{3/2} + bx^2\right) a^{-\frac{3}{2}} \right)} \right\}$$

**2.648 ODE No. 648**

$$y'(x) = -\frac{\sqrt{ax^3}(-2\sqrt{ax^4 + 8y(x)} + \sqrt{ax} + \sqrt{a})}{2(x+1)}$$

✓ **Mathematica** : cpu = 0.309911 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72}a(-96c_1x^3 + 72(2c_1 - 1)x^2 - 48(-6c_1 + 2x^3 - 3x^2 + 6x + 9)\log(x+1) - 144(2c_1 - 3)x + 36(3) \right\} \right\}$$

✓ **Maple** : cpu = 0.857 (sec), leaf count = 41

$$\left\{ \frac{1}{4}\sqrt{ax^4 + 8y(x)}\frac{1}{\sqrt{a}} - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln(1+x) - \_C1 = 0 \right\}$$

**2.649 ODE No. 649**

$$y'(x) = x\sqrt{x^2 + 8y(x) - 2x + 1} - \frac{x}{4} + \frac{1}{4}$$

✓ **Mathematica** : cpu = 0.158918 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8}(-(16c_1 + 1)x^2 + 16c_1^2 + 4x^4 + 2x - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.29 (sec), leaf count = 27

$$\left\{ \_C1 + 2x^2 + \frac{1}{4} - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0 \right\}$$

**2.650 ODE No. 650**

$$y'(x) = x\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{a}{2} - \frac{x}{2}$$

✓ **Mathematica** : cpu = 0.222808 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}(-a^2 - 2ax - (4c_1 + 1)x^2 + 4c_1^2 + x^4) \right\} \right\}$$

✓ **Maple** : cpu = 0.202 (sec), leaf count = 28

$$\left\{ \_C1 + x^2 + \frac{1}{2} - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0 \right\}$$

2.651 ODE No. 651

$$y'(x) = \frac{y(x) (x^2 + \log(y(x)))}{x}$$

✓ **Mathematica** : cpu = 0.0308514 (sec), leaf count = 15

$$\left\{ \left\{ y(x) \rightarrow e^{x(2c_1+x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 13

$$\left\{ y(x) = e^{x-C1} e^{x^2} \right\}$$

2.652 ODE No. 652

$$y'(x) = \frac{x\sqrt{4ax - y(x)^2} + 2a}{y(x)}$$

✓ **Mathematica** : cpu = 1.84488 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{256a^4x(16a-x^3) + 32a^2e^{c_1}x^2 - e^{2c_1}}}{32a^2} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{256a^4x(16a-x^3) + 32a^2e^{c_1}x^2 - e^{2c_1}}}{32a^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 27

$$\left\{ -\sqrt{-(y(x))^2 + 4ax} - \frac{x^2}{2} - C1 = 0 \right\}$$

2.653 ODE No. 653

$$y'(x) = x\sqrt{x^2 + 4y(x) - 4x} - \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 0.162937 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{4}(4c_1 + 1)x^2 + c_1^2 + \frac{x^4}{4} + x \right\} \right\}$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 24

$$\left\{ -C1 + x^2 + \frac{1}{2} - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$

**2.654 ODE No. 654**

$$y'(x) = \frac{\sqrt{x^2 + 3y(x)} - \frac{2x^2}{3} - \frac{2x}{3}}{x + 1}$$

✓ **Mathematica** : cpu = 0.145788 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{12}(-18c_1 \log(x + 1) + 9c_1^2 - 4x^2 + 9 \log^2(x + 1)) \right\} \right\}$$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 23

$$\left\{ -C1 + \frac{3 \ln(1 + x)}{2} - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

**2.655 ODE No. 655**

$$y'(x) = \frac{e^{-4x/3}y(x)^3}{e^{-2x/3}y(x) + 1}$$

✓ **Mathematica** : cpu = 20.4162 (sec), leaf count = 79

$$\text{Solve} \left[ \frac{3}{2} \log(y(x)) - \frac{3}{4} \log(-3y(x)^2 + 2e^{2x/3}y(x) + 2e^{4x/3}) + \frac{3 \tanh^{-1}\left(\frac{y(x) + 2e^{2x/3}}{\sqrt{7}y(x)}\right)}{2\sqrt{7}} + x = c_1, y(x) \right]$$

✓ **Maple** : cpu = 1.842 (sec), leaf count = 66

$$\left\{ x + \frac{3}{2} \ln(y(x) e^{-\frac{2x}{3}}) - \frac{3}{4} \ln\left(3(y(x))^2 (e^{-2/3x})^2 - 2y(x) e^{-2/3x} - 2\right) + \frac{3\sqrt{7}}{14} \text{Artanh}\left(\frac{3y(x)\sqrt{7}}{7} e^{-\frac{2x}{3}} - \frac{\sqrt{7}}{7}\right) \right\}$$

**2.656 ODE No. 656**

$$y'(x) = \frac{y(x)(x^3 + \log(y(x)))}{x}$$

✓ **Mathematica** : cpu = 0.0366767 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow e^{3c_1x + \frac{x^3}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 15

$$\left\{ y(x) = e^{\frac{x^3}{2}} e^{x-C1} \right\}$$

**2.657 ODE No. 657**

$$y'(x) = x^2 \sqrt{x^2 + 8y(x) - 2x + 1} - \frac{x}{4} + \frac{1}{4}$$

✓ **Mathematica** : cpu = 0.165406 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72} (-96c_1x^3 + 144c_1^2 + 16x^6 - 9x^2 + 18x - 9) \right\} \right\}$$

✓ **Maple** : cpu = 0.326 (sec), leaf count = 26

$$\left\{ -C1 + \frac{4x^3}{3} - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0 \right\}$$

**2.658 ODE No. 658**

$$y'(x) = \frac{\sqrt{x^2 + 8y(x) - 2x + 1} - \frac{x^2}{4} + \frac{1}{4}}{x + 1}$$

✓ **Mathematica** : cpu = 0.221547 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} (16c_1^2 - x^2 + 2x - 1) - 4c_1 \log(4(x + 1)) + 2 \log^2(4(x + 1)) \right\} \right\}$$

✓ **Maple** : cpu = 0.279 (sec), leaf count = 28

$$\left\{ -C1 + 4 \ln(1 + x) - \frac{1}{4} - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0 \right\}$$

**2.659 ODE No. 659**

$$y'(x) = x \sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c} - \frac{ax}{2} - \frac{b}{2}$$

✓ **Mathematica** : cpu = 0.398067 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{a^2(-4c_1 + 1)x^2 + 4c_1^2 + x^4 - 2abx - b^2 + 4c}{4a} \right\} \right\}$$

✓ **Maple** : cpu = 0.441 (sec), leaf count = 41

$$\left\{ -C1 + ax^2 + \frac{a}{2} - \sqrt{a^2x^2 + 2abx + b^2 + 4ay(x) - 4c} = 0 \right\}$$

**2.660 ODE No. 660**

$$y'(x) = x^2 \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{a}{2} - \frac{x}{2}$$

✓ **Mathematica** : cpu = 0.256166 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36}(-9a^2 - 18ax - 24c_1x^3 + 36c_1^2 + 4x^6 - 9x^2) \right\} \right\}$$

✓ **Maple** : cpu = 0.293 (sec), leaf count = 29

$$\left\{ -C1 + \frac{2x^3}{3} - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0 \right\}$$

**2.661 ODE No. 661**

$$y'(x) = x^2 \sqrt{a^2x^2 + 2abx + 4ay(x) + b^2 - 4c} - \frac{ax}{2} - \frac{b}{2}$$

✓ **Mathematica** : cpu = 0.383816 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow \frac{a^2(-24c_1x^3 + 36c_1^2 + 4x^6 - 9x^2) - 18abx - 9b^2 + 36c}{36a} \right\} \right\}$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 39

$$\left\{ -C1 + \frac{2ax^3}{3} - \sqrt{a^2x^2 + 2abx + b^2 + 4ay(x) - 4c} = 0 \right\}$$

**2.662 ODE No. 662**

$$y'(x) = x^2 \sqrt{x^2 - 4y(x) + 2x + 1} + \frac{x}{2} + \frac{1}{2}$$

✓ **Mathematica** : cpu = 0.179378 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36}(24c_1x^3 - 36c_1^2 - 4x^6 + 9x^2 + 18x + 9) \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 26

$$\left\{ -C1 - \frac{2x^3}{3} - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0 \right\}$$

**2.663 ODE No. 663**

$$y'(x) = \frac{x^2 \sqrt{4ax - y(x)^2} + 2a}{y(x)}$$

✓ **Mathematica** : cpu = 2.14988 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{4096a^6x(36a-x^5) + 128a^3e^{c_1}x^3 - e^{2c_1}}}{192a^3} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{4096a^6x(36a-x^5) + 128a^3e^{c_1}x^3 - e^{2c_1}}}{192a^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.17 (sec), leaf count = 27

$$\left\{ -\sqrt{-(y(x))^2 + 4ax} - \frac{x^3}{3} - \_C1 = 0 \right\}$$

**2.664 ODE No. 664**

$$y'(x) = x^2 \sqrt{x^2 + 4y(x)} - 4x - \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 0.181336 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow -\frac{2c_1x^3}{3} + c_1^2 + \frac{x^6}{9} - \frac{x^2}{4} + x \right\} \right\}$$

✓ **Maple** : cpu = 0.164 (sec), leaf count = 25

$$\left\{ -C1 + \frac{2x^3}{3} - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$

**2.665 ODE No. 665**

$$y'(x) = -\frac{\sqrt{a}(-2\sqrt{ax^4 + 8y(x)} + \sqrt{ax^4} + \sqrt{ax^3})}{2(x+1)}$$

✓ **Mathematica** : cpu = 0.235641 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow -4ac_1 \log(x+1) + 2ac_1^2 - \frac{ax^4}{8} + 2a \log^2(x+1) \right\} \right\}$$

✓ **Maple** : cpu = 0.52 (sec), leaf count = 28

$$\left\{ -\frac{1}{4} \sqrt{ax^4 + 8y(x)} \frac{1}{\sqrt{a}} + \ln(1+x) - \_C1 = 0 \right\}$$

**2.666 ODE No. 666**

$$y'(x) = y(x) (x^3 + x^2 - \log(y(x)) + 1)$$

✓ **Mathematica** : cpu = 0.0660065 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow e^{-c_1 e^{-x} + x^3 - 2x^2 + 4x - 3} \right\} \right\}$$

✓ **Maple** : cpu = 0.146 (sec), leaf count = 24

$$\left\{ y(x) = e^{\frac{C_1}{e^x} + x^3 - 2x^2 + 4x - 3} \right\}$$

**2.667 ODE No. 667**

$$y'(x) = \frac{e^{-2bx} y(x)^3}{e^{-bx} y(x) + 1}$$

✓ **Mathematica** : cpu = 1.12704 (sec), leaf count = 84

$$\text{Solve} \left[ \frac{\log(y(x))}{b} - \frac{\log(y(x)^2 - b e^{bx} (e^{bx} + y(x)))}{2b} + \frac{\tanh^{-1} \left( \frac{\sqrt{\frac{b}{b+4}} (2e^{bx} + y(x))}{y(x)} \right)}{\sqrt{b} \sqrt{b+4}} + x = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.305 (sec), leaf count = 82

$$\left\{ bx - b \operatorname{Artanh} \left( (-2y(x) e^{-bx} + b) \frac{1}{\sqrt{b^2 + 4b}} \right) \frac{1}{\sqrt{b^2 + 4b}} + \ln(y(x) e^{-bx}) - \frac{\ln(-by(x) e^{-bx} + (y(x))^2 (e^{-bx})^2)}{2} \right\}$$

**2.668 ODE No. 668**

$$y'(x) = \frac{e^{-2x} y(x)^3}{e^{-x} y(x) + 1}$$

✓ **Mathematica** : cpu = 1.14488 (sec), leaf count = 59

$$\text{Solve} \left[ \log(y(x)) - \frac{1}{2} \log(-y(x)^2 + e^x y(x) + e^{2x}) + \frac{\tanh^{-1} \left( \frac{y(x) + 2e^x}{\sqrt{5} y(x)} \right)}{\sqrt{5}} + x = c_1, y(x) \right]$$



✓ **Maple** : cpu = 1.044 (sec), leaf count = 58

$$\left\{ y(x) = e^{\text{RootOf}\left(2\sqrt{5}\text{Artanh}\left(1/5\frac{(-2e^{-Z}+e^x)\sqrt{5}}{e^x}\right)+5\ln(-(e^x)^2-e^{-Z+x}+(e^{-Z})^2)+10\_C1-10\_Z-10x\right)} \right\}$$

**2.669 ODE No. 669**

$$y'(x) = \frac{e^x(3e^x - 2y(x)^{3/2})^2}{4\sqrt{y(x)}}$$

✓ **Mathematica** : cpu = 0.788413 (sec), leaf count = 222

$$\left\{ \left\{ y(x) \rightarrow \frac{(\frac{3}{2}e^{3c_1+x} + e^{3c_1} - e^{3e^x} + \frac{3}{2}e^{x+3e^x})^{2/3}}{\sqrt[3]{(e^{3c_1} + e^{3e^x})^2}} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1}(\frac{3}{2}e^{3c_1+x} + e^{3c_1} - e^{3e^x} + \frac{3}{2}e^{x+3e^x})^{2/3}}{\sqrt[3]{(e^{3c_1} + e^{3e^x})^2}} \right\}, \left\{ \right. \right\}$$

✓ **Maple** : cpu = 1.24 (sec), leaf count = 72

$$\left\{ -C1 + 1e^{-\frac{3e^x}{2} - \frac{9e^{2x}}{8}} \left(2(y(x))^{3/2} e^x - 2e^x - 3e^{2x}\right) \left(e^{\frac{3e^x}{2} - \frac{9e^{2x}}{8}}\right)^{-1} \left(2(y(x))^{3/2} e^x + 2e^x - 3e^{2x}\right)^{-1} = 0 \right\}$$

**2.670 ODE No. 670**

$$y'(x) = \frac{1}{2}ixy(x) \left(-2\sqrt{4\log(a) - x^2 + 4\log(y(x))} + i\right)$$

✓ **Mathematica** : cpu = 0.627842 (sec), leaf count = 83

$$\text{Solve}\left[2i\sqrt{4\log(a) - x^2 + 4\log(y(x))} - 2i\tan^{-1}\left(\sqrt{4\log(a) - x^2 + 4\log(y(x))}\right) + 8\log(a) = \log(4\log(a) - x^2 + 4\log(y(x)))\right]$$

✓ **Maple** : cpu = 1.01 (sec), leaf count = 70

$$\left\{ -\frac{1}{2}\sqrt{-x^2 + 4\ln(a) + 4\ln(y(x))} + \frac{1}{2}\arctan\left(\sqrt{-x^2 + 4\ln(a) + 4\ln(y(x))}\right) - \frac{i}{4}\ln(x^2 - 4\ln(a) - 4\ln(y(x))) \right\}$$

**2.671 ODE No. 671**

$$y'(x) = \frac{(xy(x)^2 + 1)^2}{x^4 y(x)}$$

✓ **Mathematica** : cpu = 0.377582 (sec), leaf count = 162

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{\frac{(\sqrt{2x-2})e^{\frac{2\sqrt{2}(c_1x+1)}{x}} - \sqrt{2x-2}}{x}}}{\sqrt{2}\sqrt{e^{\frac{2\sqrt{2}(c_1x+1)}{x}} + 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{\frac{(\sqrt{2x-2})e^{\frac{2\sqrt{2}(c_1x+1)}{x}} - \sqrt{2x-2}}{x}}}{\sqrt{2}\sqrt{e^{\frac{2\sqrt{2}(c_1x+1)}{x}} + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.832 (sec), leaf count = 231

$$\left\{ y(x) = -\frac{1}{2x} \sqrt{-2x \left( -C1 e^{\frac{-1-\sqrt{2x}}{x^2}} + e^{\frac{-1+\sqrt{2x}}{x^2}} \right) \left( -C1 (\sqrt{2x} + 2) e^{\frac{-1-\sqrt{2x}}{x^2}} + (2 - \sqrt{2x}) e^{\frac{-1+\sqrt{2x}}{x^2}} \right) \left( -C1 e^{\frac{-1-\sqrt{2x}}{x^2}} \right)} \right\}$$

**2.672 ODE No. 672**

$$y'(x) = \frac{x^2 \left( \sqrt{4y(x)^3 - 9x^4} + 3x \right)}{y(x)^2}$$

✗ **Mathematica** : cpu = 300.063 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.458 (sec), leaf count = 36

$$\left\{ \int_{-b}^{y(x)} -a^2 \frac{1}{\sqrt{-9x^4 + 4a^3}} dx - \frac{x^3}{3} - C1 = 0 \right\}$$

**2.673 ODE No. 673**

$$y'(x) = \frac{\frac{1}{2}x^2 \cos(2y(x)) + \frac{x^2}{2} - \frac{1}{2} \sin(2y(x))}{x}$$

✓ **Mathematica** : cpu = 0.121779 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{3c_1 + 2x^3}{6x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.068 (sec), leaf count = 17

$$\left\{ y(x) = \arctan \left( \frac{x^3 + 6C1}{3x} \right) \right\}$$

**2.674 ODE No. 674**

$$y'(x) = \frac{\sqrt{x^2 + 4y(x) - 4x - \frac{x^2}{2} + \frac{x}{2} + 1}}{x + 1}$$

✓ **Mathematica** : cpu = 0.212119 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow -2c_1 \log(x + 1) + c_1^2 - \frac{x^2}{4} + x + \log^2(x + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.276 (sec), leaf count = 27

$$\left\{ -C1 + 2 \ln(1 + x) - 1 - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$

**2.675 ODE No. 675**

$$y'(x) = \frac{ax^4 + ae^x x^3 + ax^3 - x^2 y(x)^2 - e^x x y(x)^2 - x y(x)^2 + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.0493513 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \sqrt{ax} \tanh \left( \frac{1}{6} \sqrt{a} (6c_1 + 2x^3 + 3x^2 + 6e^x(x - 1)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 37

$$\left\{ y(x) = \tanh \left( \frac{(6x - 6)e^x + 2x^3 + 3x^2 + 6 - C1}{6} \sqrt{a} \right) x \sqrt{a} \right\}$$

**2.676 ODE No. 676**

$$y'(x) = \frac{x^6 \sqrt{4x^2 y(x) + 1 + \frac{x}{2} + \frac{1}{2}}}{x^3(x + 1)}$$

✓ **Mathematica** : cpu = 0.286888 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{12} (6c_1 - 11) x^4 + \left( \frac{2c_1}{3} - 1 \right) x^3 - (c_1 - 1) x^2 + \left( -2c_1 + \frac{x^4}{2} - \frac{2x^3}{3} + x^2 - 2x \right) \log(x + 1) + 2c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.626 (sec), leaf count = 43

$$\left\{ -C1 + 2 \ln(1 + x) - \frac{1}{x} \sqrt{4x^2 y(x) + 1} - 2x + x^2 - \frac{2x^3}{3} + \frac{x^4}{2} = 0 \right\}$$

**2.677 ODE No. 677**

$$y'(x) = \frac{ax^4 + ax^3 + ax^3 \log(x+1) - x^2 y(x)^2 - xy(x)^2 + y(x) - xy(x)^2 \log(x+1)}{x}$$

✓ **Mathematica** : cpu = 0.03253 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \sqrt{ax} \tanh \left( \frac{1}{12} \sqrt{a} (12c_1 + 4x^3 + 3x^2 + 6(x^2 - 1) \log(x+1) + 6x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 48

$$\left\{ y(x) = \tanh \left( \frac{6 \ln(1+x)x^2 + 4x^3 + 3x^2 - 6 \ln(1+x) + 12\_C1 + 6x + 9}{12} \sqrt{a} \right) x \sqrt{a} \right\}$$

**2.678 ODE No. 678**

$$y'(x) = \frac{x^2 (2x\sqrt{x^3 - 6y(x)} + x + 1)}{2(x+1)}$$

✓ **Mathematica** : cpu = 0.226 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow \left( -3c_1 + x^3 - \frac{3x^2}{2} + 3x \right) \log(x+1) + \frac{1}{24} (8(3c_1 + 5)x^3 - 36(c_1 + 1)x^2 + 72c_1x - 36c_1^2 - 4x^6 + 12x^5) \right\} \right\}$$

✓ **Maple** : cpu = 0.277 (sec), leaf count = 37

$$\left\{ -C1 - x^3 + \frac{3x^2}{2} - 3x + 3 \ln(1+x) - \frac{1}{2} - \sqrt{x^3 - 6y(x)} = 0 \right\}$$

**2.679 ODE No. 679**

$$y'(x) = \frac{x^4 + x^3 + x^3 \log(x) + 7x^2 y(x)^2 + 7xy(x)^2 + y(x) + 7xy(x)^2 \log(x)}{x}$$

✓ **Mathematica** : cpu = 0.0298885 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan \left( \frac{1}{12} \sqrt{7} (12c_1 + 4x^3 + 3x^2 + 6x^2 \log(x)) \right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 37

$$\left\{ y(x) = \frac{x\sqrt{7}}{7} \tan \left( \frac{(6x^2 \ln(x) + 4x^3 + 3x^2 + 12\_C1) \sqrt{7}}{12} \right) \right\}$$

**2.680 ODE No. 680**

$$y'(x) = \frac{\sqrt{x^2 - 4y(x) + 2x + 1} + \frac{x^2}{2} + x + \frac{1}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.211896 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} (8c_1 \log(x + 1) - 4c_1^2 + x^2 + 2x - 4 \log^2(x + 1) + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.259 (sec), leaf count = 28

$$\left\{ -C1 - 2 \ln(1 + x) - \frac{1}{2} - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0 \right\}$$

**2.681 ODE No. 681**

$$y'(x) = \frac{ax^2y(x)^2 + axy(x)^2 + axy(x)^2 \log\left(\frac{1}{x}\right) + bx^4 + bx^3 + bx^3 \log\left(\frac{1}{x}\right) + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.0399783 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{bx} \tan\left(\frac{1}{12} \sqrt{a} \sqrt{b} (12c_1 + 4x^3 + 9x^2 - 6x^2 \log(x))\right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 45

$$\left\{ y(x) = \frac{x}{a} \tan\left(\frac{4x^3 + 6x^2 \ln(x^{-1}) + 9x^2 + 12 - C1}{12} \sqrt{ab}\right) \sqrt{ab} \right\}$$

**2.682 ODE No. 682**

$$y'(x) = \frac{2a}{x(-8a^2 + 2axy(x)^2 - xy(x))}$$

✓ **Mathematica** : cpu = 0.117546 (sec), leaf count = 33

$$\text{Solve}\left[\frac{e^{-4ay(x)}(xy(x)^2 - 4a)}{8ax} = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.395 (sec), leaf count = 28

$$\left\{ -C1 + \frac{-x(y(x))^2 + 4a}{e^{4ay(x)}x} = 0 \right\}$$

**2.683 ODE No. 683**

$$y'(x) = \frac{y(x) (x^4 y(x) \log(x(x+1)) - x^3 \log(x(x+1)) - 1)}{x}$$

✓ **Mathematica** : cpu = 0.338565 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\frac{1}{9}x(2x^2+3)}}{x \left( c_1 e^{\frac{x^2}{6}} \sqrt[3]{x+1} (x(x+1))^{\frac{x^3}{3}} + e^{\frac{1}{9}x(2x^2+3)} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.299 (sec), leaf count = 152

$$\left\{ y(x) = \frac{1}{x} (x(1+x))^{-\frac{x^3}{3}} e^{\frac{2x^3}{9}} e^{-\frac{x^2}{6}} e^{\frac{x}{3}} \left( x^{-\frac{x^3}{3}} (1+x)^{-\frac{x^3}{3}} e^{\frac{x}{6}} (ix^2 \pi (\operatorname{csgn}(ix(1+x)))^3 - ix^2 (\operatorname{csgn}(ix) + \operatorname{csgn}(i+ix)) \pi (\operatorname{csgn}(ix(1+x)))) \right) \right\}$$

**2.684 ODE No. 684**

$$y'(x) = \frac{x^2 \sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.022502 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left( c_1 + \frac{x^2}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 3.328 (sec), leaf count = 30

$$\left\{ \ln \left( \sqrt{(y(x))^2 + x^2} + y(x) \right) - \frac{x^2}{2} - \ln(x) - \_C1 = 0 \right\}$$

**2.685 ODE No. 685**

$$y'(x) = \frac{x^3 \log((x-1)(x+1)) + y(x) + 7xy(x)^2 \log((x-1)(x+1))}{x}$$

✓ **Mathematica** : cpu = 0.0364041 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan \left( \frac{1}{2} \sqrt{7} (2c_1 - x^2 + x^2 \log(x-1) + x^2 \log(x+1) - \log(1-x) - \log(x+1)) \right)}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 48

$$\left\{ y(x) = \frac{x\sqrt{7}}{7} \tan \left( \frac{(x^2 \ln((1+x)(x-1)) - x^2 - \ln((1+x)(x-1)) + 2\_C1 + 1) \sqrt{7}}{2} \right) \right\}$$

**2.686 ODE No. 686**

$$y'(x) = \frac{e^{2x^2} xy(x)^3}{e^{x^2} y(x) + 1}$$

✓ **Mathematica** : cpu = 15.4173 (sec), leaf count = 49

$$\text{Solve} \left[ -\frac{1}{2} \log \left( e^{2x^2} y(x)^2 + 2e^{x^2} y(x) + 2 \right) + \tan^{-1} \left( e^{x^2} y(x) + 1 \right) + \log(y(x)) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 3.229 (sec), leaf count = 85

$$\left\{ y(x) = \frac{1}{e^{x^2}} \left( 1 - \tan \left( \text{RootOf} \left( -2x^2 - \ln \left( \frac{81(\tan(_Z))^2}{10} + \frac{81}{10} \right) + 2 \ln(9/2 \tan(_Z) - 9/2) + 6\_C1 - 2 \right) \right) \right)$$

**2.687 ODE No. 687**

$$y'(x) = \frac{x^3 \left( -\log \left( \frac{x+1}{x-1} \right) \right) + y(x) + xy(x)^2 \log \left( \frac{x+1}{x-1} \right)}{x}$$

✓ **Mathematica** : cpu = 0.0569657 (sec), leaf count = 111

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( -(x+1)^{x^2} e^{2(c_1+x)} + x \left( (x+1)^{x^2} e^{2(c_1+x)} + (x-1)^{x^2} \right) + (x-1)^{x^2} \right)}{(x+1)^{x^2} e^{2(c_1+x)} + x \left( (x-1)^{x^2} - (x+1)^{x^2} e^{2(c_1+x)} \right) + (x-1)^{x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 39

$$\left\{ y(x) = -\tanh \left( \frac{x^2}{2} \ln \left( \frac{1+x}{x-1} \right) - \frac{1}{2} \ln \left( \frac{1+x}{x-1} \right) + \_C1 + x - 1 \right) x \right\}$$

**2.688 ODE No. 688**

$$y'(x) = \frac{e^{\frac{x+1}{x-1}} x^3 + e^{\frac{x+1}{x-1}} xy(x)^2 + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.117954 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow x \tan \left( c_1 - 4e \text{Ei} \left( \frac{2}{x-1} \right) + \frac{1}{2} e^{\frac{x+1}{x-1}} (x^2 + 2x - 3) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 42

$$\left\{ y(x) = \tan \left( \frac{x^2 + 2x - 3}{2} e^{\frac{1+x}{x-1}} + 4e \text{Ei} \left( 1, -2(x-1)^{-1} \right) + \_C1 \right) x \right\}$$

**2.689 ODE No. 689**

$$y'(x) = \frac{-e^{x+1}x^3 + e^{x+1}xy(x)^2 + xy(x) - y(x)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.065307 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{x - xe^{2(c_1 + e^2 \text{Ei}(x-1) + e^{x+1})}}{e^{2(c_1 + e^2 \text{Ei}(x-1) + e^{x+1})} + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.168 (sec), leaf count = 25

$$\{y(x) = -\tanh(e^{1+x} - e^2 \text{Ei}(1, 1-x) + \_C1) x\}$$

**2.690 ODE No. 690**

$$y'(x) = \frac{-\frac{x^2}{4} + x^3 \sqrt{x^2 + 8y(x) - 2x + 1} + \frac{1}{4}}{x + 1}$$

✓ **Mathematica** : cpu = 0.281456 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow -\frac{2}{3}(-6c_1 + 2x^3 - 3x^2 + 6x + 11) \log(4(x+1)) + \frac{1}{72}((32 - 96c_1)x^3 + 3(48c_1 - 43)x^2 - 6(48c_1 - 91) \right.$$

✓ **Maple** : cpu = 0.346 (sec), leaf count = 40

$$\left. \left\{ -C1 + \frac{4x^3}{3} - 2x^2 + 4x - 4 \ln(1+x) - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0 \right\} \right\}$$

**2.691 ODE No. 691**

$$y'(x) = \frac{\frac{1}{2}x^3 \cos(2y(x)) + \frac{x^3}{2} - \frac{1}{2} \sin(2y(x))}{x}$$

✓ **Mathematica** : cpu = 0.0676791 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{2c_1 + x^4}{4x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.727 (sec), leaf count = 17

$$\left\{ y(x) = \arctan \left( \frac{x^4 + 8\_C1}{4x} \right) \right\}$$



**2.692 ODE No. 692**

$$y'(x) = \frac{x^3 \sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.0235355 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left( c_1 + \frac{x^3}{3} \right) \right\} \right\}$$

✓ **Maple** : cpu = 2.739 (sec), leaf count = 30

$$\left\{ \ln \left( \sqrt{(y(x))^2 + x^2} + y(x) \right) - \frac{x^3}{3} - \ln(x) - \_C1 = 0 \right\}$$

**2.693 ODE No. 693**

$$y'(x) = e^{bx} \left( e^{-3bx} y(x)^3 + e^{-2bx} y(x)^2 + 1 \right)$$

✓ **Mathematica** : cpu = 1.02161 (sec), leaf count = 1

$$\{ \}$$

✓ **Maple** : cpu = 0.216 (sec), leaf count = 40

$$\left\{ y(x) = \frac{1}{e^{-bx}} \text{RootOf} \left( -x - \int^{-Z} -(\_a^3 + \_a^2 - \_a b + 1)^{-1} d\_a + \_C1 \right) \right\}$$

**2.694 ODE No. 694**

$$y'(x) = \frac{x^3 \sqrt{4x^2 y(x) + 1} + \frac{x}{2} + \frac{1}{2}}{x^3(x+1)}$$

✓ **Mathematica** : cpu = 0.265644 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow -2c_1 x - 2(x - c_1) \log(x + 1) + c_1^2 + x^2 - \frac{1}{4x^2} + \log^2(x + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.279 (sec), leaf count = 30

$$\left\{ -2 \ln(1 + x) - \frac{1}{x} \sqrt{4x^2 y(x) + 1} + 2x + \_C1 = 0 \right\}$$

**2.695 ODE No. 695**

$$y'(x) = \frac{x^4 + x^3 + x^2 y(x)^2 + xy(x)^2 + y(x) \log(x-1)}{x \log(x-1)}$$

✓ **Mathematica** : cpu = 0.0519688 (sec), leaf count = 34

$$\{\{y(x) \rightarrow x \tan (c_1 + 2\text{Ei}(\log(x-1)) + 3\text{Ei}(2 \log(x-1)) + \text{Ei}(3 \log(x-1)))\}\}$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 39

$$\{y(x) = \tan (-\text{Ei}(1, -3 \ln (x-1)) - 3 \text{Ei}(1, -2 \ln (x-1)) - 2 \text{Ei}(1, -\ln (x-1)) + \_C1) x\}$$

**2.696 ODE No. 696**

$$y'(x) = \frac{e^{x+1} x^3 + 7e^{x+1} x y(x)^2 + y(x) \log(x-1)}{x \log(x-1)}$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.386 (sec), leaf count = 32

$$\left\{ y(x) = \frac{x\sqrt{7}}{7} \tan \left( \left( e \int \frac{x e^x}{\ln(x-1)} dx + \_C1 \right) \sqrt{7} \right) \right\}$$

**2.697 ODE No. 697**

$$y'(x) = e^{2x/3} (e^{-2x} y(x)^3 + e^{-4x/3} y(x)^2 + 1)$$

✓ **Mathematica** : cpu = 0.359558 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.152 (sec), leaf count = 40

$$\left\{ y(x) = \text{RootOf} \left( -x + 3 \int^{-Z} (3 \_a^3 + 3 \_a^2 - 2 \_a + 3)^{-1} d\_a + \_C1 \right) \left( e^{-\frac{2x}{3}} \right)^{-1} \right\}$$

**2.698 ODE No. 698**

$$y'(x) = e^x (e^{-3x} y(x)^3 + e^{-2x} y(x)^2 + 1)$$

✓ **Mathematica** : cpu = 0.365757 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.174 (sec), leaf count = 34

$$\left\{ y(x) = \frac{\text{RootOf}\left(-x + \int^{-Z} (-a^3 + a^2 - a + 1)^{-1} da + C1\right)}{e^{-x}} \right\}$$

**2.699 ODE No. 699**

$$y'(x) = \frac{x(3x^2 \sqrt{x^2 + 3y(x)} - 2x - 2)}{3(x+1)}$$

✓ **Mathematica** : cpu = 0.218162 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{48} (-12(2c_1 + 3)x^3 + 4(9c_1 + 5)x^2 - 12(-6c_1 + 2x^3 - 3x^2 + 6x) \log(x+1) - 72c_1x + 36c_1^2 + 4x^6 - \dots \right. \right.$$

✓ **Maple** : cpu = 0.393 (sec), leaf count = 36

$$\left\{ -C1 + \frac{x^3}{2} - \frac{3x^2}{4} + \frac{3x}{2} - \frac{3 \ln(1+x)}{2} - \sqrt{x^2 + 3y(x)} = 0 \right\}$$

**2.700 ODE No. 700**

$$y'(x) = \frac{1}{xy(x)(xy(x)^2 + x + 1)}$$

✓ **Mathematica** : cpu = 0.0621526 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2xW\left(c_1 e^{\frac{1}{2}\left(\frac{1}{x}-1\right)}\right)} + x - 1}{\sqrt{x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2xW\left(c_1 e^{\frac{1}{2}\left(\frac{1}{x}-1\right)}\right)} + x - 1}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.175 (sec), leaf count = 62

$$\left\{ y(x) = \frac{1}{x} \sqrt{x \left( 2 \text{lambertW} \left( 1/2 \_C1 e^{-1/2 \frac{x-1}{x}} \right) x + x - 1 \right)}, y(x) = -\frac{1}{x} \sqrt{x \left( 2 \text{lambertW} \left( 1/2 \_C1 e^{-1/2 \frac{x-1}{x}} \right) \right)} \right\}$$

**2.701 ODE No. 701**

$$y'(x) = \frac{x^4 + x^4 \log(x) - 2x^2 y(x) - 2x^2 y(x) \log(x) + y(x)^2 + y(x)^2 \log(x) + 2e^x x - 2x - \log(x) - 1}{e^x - 1}$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 6.756 (sec), leaf count = 71

$$\left\{ y(x) = 1 \left( -x^2 \left( e^{\int \frac{\ln(x)+1}{e^x-1} dx} \right)^2 + x^2 \_C1 + \left( e^{\int \frac{\ln(x)+1}{e^x-1} dx} \right)^2 + \_C1 \right) \left( - \left( e^{\int \frac{\ln(x)+1}{e^x-1} dx} \right)^2 + \_C1 \right)^{-1} \right\}$$

**2.702 ODE No. 702**

$$y'(x) = \frac{-x^3 + x^3(-\log(x)) - xy(x)^2 + xy(x) - e^x y(x) - xy(x)^2 \log(x)}{x(x - e^x)}$$

✗ **Mathematica** : cpu = 300.032 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.247 (sec), leaf count = 35

$$\left\{ y(x) = \tan \left( \int \frac{x \ln(x)}{e^x - x} dx + \int \frac{x}{e^x - x} dx + \_C1 \right) x \right\}$$

**2.703 ODE No. 703**

$$y'(x) = \frac{y(x) (x^3 y(x) + x^2 y(x) \log(x) - x^2 - x - x \log(x) + 1)}{(x - 1)x}$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.602 (sec), leaf count = 44

$$\left\{ y(x) = \frac{e^{dilog(x)}}{x e^x (x - 1)} \left( \int -\frac{e^{dilog(x)} (x + \ln(x))}{e^x (x - 1)^2} dx + \_C1 \right)^{-1} \right\}$$

2.704 ODE No. 704

$$y'(x) = \frac{2ax^3y(x)^2 + 2bx^5 - y(x) + xy(x)\log(x)}{x(x\log(x) - 1)}$$

✗ **Mathematica** : cpu = 300.024 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.048 (sec), leaf count = 38

$$\left\{ y(x) = \frac{x}{a} \tan \left( 2\sqrt{ab} \left( -C1 + \int \frac{x^3}{x \ln(x) - 1} dx \right) \right) \sqrt{ab} \right\}$$

2.705 ODE No. 705

$$y'(x) = \frac{y(x)(x^4 + x^3 + \log(y(x)) + x)}{x}$$

✓ **Mathematica** : cpu = 0.0534148 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow x^x e^{c_1 x + \frac{x^4}{3} + \frac{x^3}{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 24

$$\left\{ y(x) = e^{\frac{x^4}{3}} e^{\frac{x^3}{2}} e^{x-C1} x^x \right\}$$

2.706 ODE No. 706

$$y'(x) = -\frac{1}{8}x(y(x) + 1)^2(-\log(y(x) - 1) + \log(y(x) + 1) + 2\log(x))$$

✗ **Mathematica** : cpu = 300.069 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.633 (sec), leaf count = 65

$$\left\{ \int_{-b}^{y(x)} \frac{1}{2+2-a} \left( -\frac{x^2(1+a)\ln(a-1)}{2} + \frac{x^2(1+a)\ln(1+a)}{2} + x^2(1+a)\ln(x) + 4a-4 \right)^{-1} da + \right.$$

**2.707 ODE No. 707**

$$y'(x) = \frac{1}{16}x(y(x) + 1)^2(-\log(y(x) - 1) + \log(y(x) + 1) + 2\log(x))^2$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.539 (sec), leaf count = 105

$$\left\{ \int_{-b}^{y(x)} \frac{1}{4+4-a} \left( \frac{x^2(1+a)(\ln(1+a))^2}{4} + x^2(1+a) \left( -\frac{\ln(a-1)}{2} + \ln(x) \right) \ln(1+a) + \frac{x^2(1+a)}{4} \right) \right.$$

**2.708 ODE No. 708**

$$y'(x) = \frac{(4ax - y(x)^2)^3}{y(x)(4ax - y(x)^2 - 1)}$$

✓ **Mathematica** : cpu = 0.294938 (sec), leaf count = 1

{}

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.709 ODE No. 709**

$$y'(x) = \frac{x^3\sqrt{4ax - y(x)^2} + 2ax + 2a}{(x+1)y(x)}$$

✓ **Mathematica** : cpu = 4.13278 (sec), leaf count = 143

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{6}\sqrt{144ax - (6c_1 + 2x^3 - 3x^2 + 6x)^2 + 12(6c_1 + 2x^3 - 3x^2 + 6x)\log(x+1) - 36\log^2(x+1)} \right\}, \left\{ y \right. \right.$$

✓ **Maple** : cpu = 0.302 (sec), leaf count = 39

$$\left\{ -\sqrt{-(y(x))^2 + 4ax} - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln(1+x) - C1 = 0 \right\}$$

2.710 ODE No. 710

$$y'(x) = \frac{2x^3 + 4x^2y(x) + 2xy(x)^2 + 2x + e^{\frac{1}{x}} - \log(x)}{\log(x) - e^{\frac{1}{x}}}$$

✗ **Mathematica** : cpu = 300.03 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.625 (sec), leaf count = 31

$$\left\{ y(x) = -x + \tan \left( 2_{-}C1 - 2 \int -\frac{x}{\ln(x) - e^{x^{-1}}} dx \right) \right\}$$

2.711 ODE No. 711

$$y'(x) = -\frac{y(x)(x \log(y(x)) + \log(y(x)) - 1)}{x + 1}$$

✓ **Mathematica** : cpu = 0.0646671 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow e^{e^{-x-1}(ec_1 + \text{Ei}(x+1))} \right\} \right\}$$

✓ **Maple** : cpu = 0.139 (sec), leaf count = 31

$$\left\{ y(x) = 1e^{-\frac{C1}{e^x}} \left( e^{\frac{\text{Ei}(1, -1-x)}{e^x e}} \right)^{-1} \right\}$$

2.712 ODE No. 712

$$y'(x) = \frac{\frac{x^2}{2} + x^3 \sqrt{x^2 - 4y(x) + 2x + 1} + x + \frac{1}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.256642 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{36} (8(3c_1 - 1)x^3 + (39 - 36c_1)x^2 + 12(-6c_1 + 2x^3 - 3x^2 + 6x + 11) \log(x + 1) + 6(12c_1 - 19)x - 4) \right\} \right\}$$

✓ **Maple** : cpu = 0.335 (sec), leaf count = 38

$$\left\{ -C1 - \frac{2x^3}{3} + x^2 - 2x + 2 \ln(1 + x) - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0 \right\}$$

**2.713 ODE No. 713**

$$y'(x) = \frac{-a^2 - aby(x) - ab\sqrt{x} + ab + b^2x + b^2}{a(a(-y(x)) - a\sqrt{x} + a + bx + b)}$$

✓ **Mathematica** : cpu = 0.116247 (sec), leaf count = 607

$$\left\{ \left\{ y(x) \rightarrow \frac{\frac{1}{\text{Root}\left[\#1^6(16e^{12c_1}+16x^3) - \frac{24\#1^4x^2}{a^4} + \frac{8\#1^3x^{3/2}}{a^6} + \frac{9\#1^2x}{a^8} - \frac{6\#1\sqrt{x}}{a^{10}} + \frac{1}{a^{12}}\&,1\right]} + a(a(-\sqrt{x}) + a + bx + b)}{a^2} \right\}, \left\{ y(x) \right.$$

✓ **Maple** : cpu = 0.447 (sec), leaf count = 86

$$\left\{ y(x) = \frac{1}{2a} \left( 3 \tanh \left( \text{RootOf} \left( 729 x^3 (\tanh(\_Z))^6 a^6 - 2187 x^3 (\tanh(\_Z))^4 a^6 + 2187 x^3 (\tanh(\_Z))^2 a^6 - 729 \right) \right) \right)$$

**2.714 ODE No. 714**

$$y'(x) = -\frac{y(x)(x^3y(x) + x^2y(x)\log(x) - x^2 + e^x - x\log(x) - \log(\frac{1}{x}))}{x(e^x - \log(\frac{1}{x}))}$$

✗ **Mathematica** : cpu = 300.039 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.222 (sec), leaf count = 96

$$\left\{ y(x) = 1e^{\int \frac{x \ln(x) + x^2 + \ln(x^{-1}) - e^x}{(-\ln(x^{-1}) + e^x)x} dx} \left( \int \frac{x(x + \ln(x))}{-\ln(x^{-1}) + e^x} e^{\int \frac{x \ln(x) + x^2 + \ln(x^{-1}) - e^x}{(-\ln(x^{-1}) + e^x)x} dx} dx + \_C1 \right)^{-1} \right\}$$

**2.715 ODE No. 715**

$$y'(x) = \frac{-\frac{x^2}{2} + x^3\sqrt{x^2 + 4y(x)} - 4x + \frac{x}{2} + 1}{x + 1}$$

✓ **Mathematica** : cpu = 0.242357 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{3}(2c_1 + 3)x^3 + \left(c_1 + \frac{3}{4}\right)x^2 + \left(2c_1 - \frac{2x^3}{3} + x^2 - 2x\right)\log(x + 1) - 2c_1x + c_1^2 + \frac{x^6}{9} - \frac{x^5}{3} + \frac{11x^4}{12} \right.$$

✓ **Maple** : cpu = 0.312 (sec), leaf count = 39

$$\left\{ \_C1 + \frac{2x^3}{3} - x^2 + 2x - 2\ln(1 + x) - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$



**2.716 ODE No. 716**

$$y'(x) = \frac{\sqrt{9x^4 - 4y(x)^3} + 3x^4 + 3x^3}{(x+1)y(x)^2}$$

✓ **Mathematica** : cpu = 4.02357 (sec), leaf count = 133

$$\left\{ \left\{ y(x) \rightarrow \left(-\frac{3}{2}\right)^{2/3} \sqrt[3]{8c_1 \log(x+1) - 4c_1^2 + x^4 - 4\log^2(x+1)} \right\}, \left\{ y(x) \rightarrow \left(\frac{3}{2}\right)^{2/3} \sqrt[3]{8c_1 \log(x+1) - 4c_1^2 + x^4 - 4\log^2(x+1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.361 (sec), leaf count = 37

$$\left\{ \int_{-b}^{y(x)} -a^2 \frac{1}{\sqrt{9x^4 - 4a^3}} d_a - \ln(1+x) - C1 = 0 \right\}$$

**2.717 ODE No. 717**

$$y'(x) = \frac{\sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{ax}{2} - \frac{a}{2} - \frac{x^2}{2} - \frac{x}{2}}{x+1}$$

✓ **Mathematica** : cpu = 0.302686 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow -\frac{a^2}{4} - \frac{ax}{2} - 2c_1 \log(x+1) + c_1^2 - \frac{x^2}{4} + \log^2(x+1) \right\} \right\}$$

✓ **Maple** : cpu = 0.381 (sec), leaf count = 33

$$\left\{ -C1 + \frac{a}{2} + 2 \ln(1+x) - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0 \right\}$$

**2.718 ODE No. 718**

$$y'(x) = e^{-x^2} x \left( e^{3x^2} y(x)^3 + e^{2x^2} y(x)^2 + 1 \right)$$

✓ **Mathematica** : cpu = 1.1192 (sec), leaf count = 1

$$\{ \}$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 44

$$\left\{ y(x) = \frac{-11 \text{RootOf}\left(-5x^2 + 20250 \int^{-Z} (121 a^3 + 3375 a - 3375) d_a + 6 C1\right) - 15}{45 e^{x^2}} \right\}$$

**2.719 ODE No. 719**

$$y'(x) = \frac{e^{-x}y(x)(x^2y(x)\log(2x) - e^x - x\log(2x))}{x}$$

✓ **Mathematica** : cpu = 0.0876951 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{2e^{-x}}{x(c_1x^{-e^{-x}}e^{Ei(-x)} + 2e^{-x})} \right\} \right\}$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 57

$$\left\{ y(x) = -\frac{x^{e^{-x}}2^{e^{-x}}e^{Ei(1,x)}}{x\left(\int x^{e^{-x}}2^{e^{-x}}e^{Ei(1,x)}e^{-x}(\ln(2) + \ln(x))dx + C1\right)} \right\}$$

**2.720 ODE No. 720**

$$y'(x) = \frac{x^3\left(\sqrt{9x^4 - 4y(x)^3} + 3x + 3\right)}{(x+1)y(x)^2}$$

✓ **Mathematica** : cpu = 4.36012 (sec), leaf count = 272

$$\left\{ \left\{ y(x) \rightarrow \sqrt[3]{(6c_1 + 9)x^3 - 9(c_1 + 1)x^2 + 3(-6c_1 + 2x^3 - 3x^2 + 6x)\log(x+1) + 18c_1x - 9c_1^2 - x^6 + 3x^5 - 6x^4} \right\} \right\}$$

✓ **Maple** : cpu = 0.244 (sec), leaf count = 48

$$\left\{ \int_{-b}^{y(x)} -a^2 \frac{1}{\sqrt{9x^4 - 4a^3}} da - \frac{x^3}{3} + \frac{x^2}{2} - x + \ln(1+x) - C1 = 0 \right\}$$

**2.721 ODE No. 721**

$$y'(x) = \frac{1}{36}\sqrt{x}\left(18x^{3/2} + x^6 - 12x^3y(x) + 36y(x)^2\right)$$

✓ **Mathematica** : cpu = 0.0180446 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \frac{2x^{3/2}}{3}} + \frac{x^3}{6} \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 19

$$\left\{ y(x) = \frac{x^3}{6} + \left(-C1 - \frac{2}{3}x^{3/2}\right)^{-1} \right\}$$

**2.722 ODE No. 722**

$$y'(x) = -\frac{y(x)^3}{x(-y(x) + 2y(x)\log(x) - 1)}$$

✓ **Mathematica** : cpu = 39.3801 (sec), leaf count = 493

$$\text{Solve} \left[ -\frac{\sqrt[3]{-2} \left( (-2)^{2/3} - \frac{(1-2\log(x))^2 \left( -\frac{1}{(2\log(x)-1)^3} \right)^{2/3} (y(x)(5-4\log(x))+2)}{2\sqrt[3]{2}(y(x)(2\log(x)-1)-1)} \right)}{\left( \frac{y(x)(4\log(x)-5)-2}{\sqrt[3]{2} \sqrt[3]{-\frac{1}{(2\log(x)-1)^3} (2\log(x)-1)(y(x)(2\log(x)-1)-1)}} \right)} \right]$$

✓ **Maple** : cpu = 0.372 (sec), leaf count = 70

$$\left\{ y(x) = 1e^{\text{RootOf}\left(-e^{-Z} \ln\left(\frac{e^{-Z}+2}{2x^4}\right)+3\_C1 e^{-Z}+_Z e^{-Z}+2\right)} \left( 1 + (2 \ln(x) - 1) e^{\text{RootOf}\left(-e^{-Z} \ln\left(\frac{e^{-Z}+2}{2x^4}\right)+3\_C1 e^{-Z}+_Z e^{-Z}+2\right)} \right) \right.$$

**2.723 ODE No. 723**

$$y'(x) = \frac{2a}{32a^3x^2 - 16a^2xy(x)^2 + 2ay(x)^4 + y(x)}$$

✓ **Mathematica** : cpu = 0.0702015 (sec), leaf count = 672

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt[3]{-1024a^6c_1^3 + 9216a^5c_1x - 432a^2 + 16\sqrt{a^4 \left( (64a^4c_1^3 - 576a^3c_1x + 27)^2 - 4096a^5 (ac_1^2 + 3x)^3 \right)}}}{12\sqrt[3]{2}a} \right. \right.$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 856

$$\left\{ y(x) = -\frac{1}{12a} \left( -8\_C1 a^2 \sqrt[3]{\left( 64\_C1^3 a^4 - 576\_C1 a^3 x + 3 \sqrt{-12288\_C1^4 a^7 x + 24576\_C1^2 a^6 x^2 - 12288} \right)} \right) \right.$$

2.724 ODE No. 724

$$y'(x) = -\frac{y(x)^3}{x(-y(x) + y(x) \log(x) - 1)}$$

✓ **Mathematica** : cpu = 61.5178 (sec), leaf count = 422

Solve  $\left[ \frac{\sqrt[3]{-2} \left( \frac{1-y(x)(\log(x)-4)}{\sqrt[3]{2} \sqrt[3]{-\frac{1}{(\log(x)-1)^3} (\log(x)-1)(y(x)(\log(x)-1)-1)}} + (-2)^{2/3} \right)}{\sqrt[3]{-\frac{1}{(\log(x)-1)^3} (\log(x)-1)(y(x)(\log(x)-1)-1)}} + (-2)^{2/3} \right) \left( \frac{2^{2/3}(y(x)(\log(x)-4)-1)}{\sqrt[3]{-\frac{1}{(\log(x)-1)^3} (\log(x)-1)(y(x)(\log(x)-1)-1)}} + (-2)^{2/3} \right)}{\sqrt[3]{-\frac{1}{(\log(x)-1)^3} (\log(x)-1)(y(x)(\log(x)-1)-1)}} + (-2)^{2/3} \right)} + (-2)^{2/3} \right]$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 18

$$\{y(x) = (-\text{lambertW}(-C1 e^{-2x}) + \ln(x) - 2)^{-1}\}$$

2.725 ODE No. 725

$$y'(x) = \frac{x^2 \log(2x) + 2xy(x) \log(2x) + y(x)^2 \log(2x) - \log(x) + \log(2x)}{\log(x)}$$

✓ **Mathematica** : cpu = 0.247218 (sec), leaf count = 19

$$\{\{y(x) \rightarrow \tan(c_1 + \log(2)\text{li}(x) + x) - x\}\}$$

✓ **Maple** : cpu = 0.866 (sec), leaf count = 25

$$\{y(x) = -x - \tan(\ln(2) Ei(1, -\ln(x)) + C1 - x)\}$$

2.726 ODE No. 726

$$y'(x) = \frac{a^2 - aby(x) - ab\sqrt{x} - b^2x + bc}{a(ay(x) + a\sqrt{x} + bx - c)}$$

✓ **Mathematica** : cpu = 0.078635 (sec), leaf count = 607

$$\left\{ \left\{ y(x) \rightarrow \frac{\text{Root}\left[\#1^6(16e^{12c_1} + 16x^3) - \frac{24\#1^4x^2}{a^4} + \frac{8\#1^3x^{3/2}}{a^6} + \frac{9\#1^2x}{a^8} - \frac{6\#1\sqrt{x}}{a^{10}} + \frac{1}{a^{12}} \&, 1\right] + a(a(-\sqrt{x}) - bx + c)}{a^2} \right\}, \left\{ y(x) \rightarrow \dots \right\} \right\}$$

✓ **Maple** : cpu = 0.305 (sec), leaf count = 83

$$\left\{ y(x) = \frac{1}{2a} \left( 3 \tanh \left( \text{RootOf} \left( -729 x^3 (\tanh (_Z))^6 a^6 + 2187 x^3 (\tanh (_Z))^4 a^6 - 2187 x^3 (\tanh (_Z))^2 a^6 + 7 \right) \right) \right) \right\}$$

**2.727 ODE No. 727**

$$y'(x) = \frac{y(x)(y(x) + 2x + 2)}{(x + 1)(\log(y(x)) + 2x - 1)}$$

✓ **Mathematica** : cpu = 0.424875 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{W(e^{-2x}(c_1 + \log(x + 1)))}{c_1 + \log(x + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.403 (sec), leaf count = 25

$$\left\{ y(x) = e^{-\text{lambertW}((\ln(1+x) - C1)e^{-2x}) - 2x} \right\}$$

**2.728 ODE No. 728**

$$y'(x) = \frac{y(x)(x^3 + 3y(x)^2)}{x(6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.360757 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W\left(\frac{6e^{2c_1+x^2}}{x}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x} \sqrt{W\left(\frac{6e^{2c_1+x^2}}{x}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.358 (sec), leaf count = 50

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\text{RootOf} \left( x^2 e^{-Z} - e^{-Z} \ln \left( \frac{(e^{-Z} + 9)x}{2} \right) + 3C1 e^{-Z} + Z e^{-Z} + 9 \right)} + 9 \right) \right\}$$

**2.729 ODE No. 729**

$$y'(x) = \frac{(x - y(x))y(x)}{x(x - y(x)^3)}$$

✓ **Mathematica** : cpu = 0.298813 (sec), leaf count = 319

$$\left\{ \left\{ y(x) \rightarrow \frac{12\sqrt[3]{2}(c_1 - \log(x)) - 2^{2/3} \left( 2\sqrt{(6c_1 - 6\log(x))^3 + 729x^2 + 54x} \right)^{2/3}}{6\sqrt[3]{2}\sqrt{(6c_1 - 6\log(x))^3 + 729x^2 + 54x}} \right\}, \left\{ y(x) \rightarrow \frac{2^{2/3}(1 + i\sqrt{3}) \left( 2\sqrt{(6c_1 - 6\log(x))^3 + 729x^2 + 54x} \right)^{2/3}}{6\sqrt[3]{2}\sqrt{(6c_1 - 6\log(x))^3 + 729x^2 + 54x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 404

$$\left\{ y(x) = \frac{1}{3} \left( \left( -27x + 3\sqrt{24_{-}C1^3 - 72_{-}C1^2 \ln(x) + 72_{-}C1 (\ln(x))^2 - 24 (\ln(x))^3 + 81x^2} \right)^{\frac{2}{3}} + 6 \ln(x) - 6 \right) \right\}$$

**2.730 ODE No. 730**

$$y'(x) = \frac{e^x(2y(x)^{3/2} - 3e^x)^3}{4\sqrt{y(x)}(2y(x)^{3/2} - 3e^x + 2)}$$

✗ **Mathematica** : cpu = 49.0156 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == (E^x*(-3*E^x + 2*y[x]^(3/2)))^3/(4*sqrt[y[x]]*(2 - 3*E^x + 2*y[x]^(3/2))), y[x], x]`

✓ **Maple** : cpu = 2.161 (sec), leaf count = 41

$$\left\{ e^x - \int^{(y(x))^{\frac{3}{2}} - \frac{3e^x}{2}} \frac{2 + 2_{-}a}{3_{-}a^3 - 3_{-}a - 3} d_{-}a -_{-}C1 = 0 \right\}$$

**2.731 ODE No. 731**

$$y'(x) = \frac{2y(x) + 1}{x(2xy(x)^3 + xy(x)^2 - 2)}$$

✓ **Mathematica** : cpu = 0.304187 (sec), leaf count = 42

$$\text{Solve} \left[ \frac{1}{64} \left( -4y(x)^2 + 4y(x) - \frac{16}{2xy(x) + x} - 2 \log(8y(x) + 4) + 3 \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.194 (sec), leaf count = 42

$$\left\{ y(x) = \frac{e^{\text{RootOf}(x(e^{-z})^3 - 4x(e^{-z})^2 + 8x_{-}C1 e^{-z} + 2_{-}Z x e^{-z} + 3x e^{-z} + 16)}}{2} - \frac{1}{2} \right\}$$

**2.732 ODE No. 732**

$$y'(x) = \frac{x^3 \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{ax}{2} - \frac{a}{2} - \frac{x^2}{2} - \frac{x}{2}}{x + 1}$$

✓ **Mathematica** : cpu = 0.457121 (sec), leaf count = 105

$$\left\{ \left\{ y(x) \rightarrow -\frac{a^2}{4} - \frac{ax}{2} - \frac{2c_1x^3}{3} + c_1x^2 + \left( 2c_1 - \frac{2x^3}{3} + x^2 - 2x \right) \log(x + 1) - 2c_1x + c_1^2 + \frac{x^6}{9} - \frac{x^5}{3} + \frac{11x^4}{12} - x^3 \right. \right.$$

✓ **Maple** : cpu = 0.405 (sec), leaf count = 43

$$\left\{ -C1 + \frac{2x^3}{3} - x^2 + 2x - 2 \ln(1 + x) - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0 \right\}$$

**2.733 ODE No. 733**

$$y'(x) = \csc(x) (x^4 \log(2x) - 2x^2 y(x) \log(2x) + y(x)^2 \log(2x) - \log(2x) + 2x \sin(x))$$

✗ **Mathematica** : cpu = 300.004 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(y(x),x) = (2\*x\*sin(x)-ln(2\*x)+ln(2\*x)\*x^4-2\*ln(2\*x)\*x^2\*y(x)+ln(2\*x)\*y(x)^2)/sin

**2.734 ODE No. 734**

$$y'(x) = \frac{y(x) (x^3 - x \log(y(x)) - \log(y(x)))}{x + 1}$$

✓ **Mathematica** : cpu = 0.105668 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \exp(-c_1 e^{-x} - e^{-x-1} \text{Ei}(x + 1) + x^2 - 3x + 4) \right\} \right\}$$

✓ **Maple** : cpu = 0.152 (sec), leaf count = 39

$$\left\{ y(x) = \frac{e^{x^2} e^4}{(e^x)^3} e^{-\frac{C1}{e^x}} e^{\frac{\text{Ei}(1, -1-x)}{e^x e}} \right\}$$

**2.735 ODE No. 735**

$$y'(x) = \frac{(2y(x) \log(x) - 1)^3}{x(-y(x) + 2y(x) \log(x) - 1)}$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.069 (sec), leaf count = 78

$$\left\{ y(x) = \frac{71 \operatorname{RootOf}\left(-82944 \int^{-Z} (5041 \_a^3 - 27648 \_a + 27648)^{-1} d\_a - 16 \ln(x) + 3 \_C1\right) - 1}{(142 \ln(x) - 71) \operatorname{RootOf}\left(-82944 \int^{-Z} (5041 \_a^3 - 27648 \_a + 27648)^{-1} d\_a - 16 \ln(x) + 3 \_C1\right) - 1} \right\}$$

**2.736 ODE No. 736**

$$y'(x) = \frac{x^4 - 2x^2y(x) + 2x^2 + y(x)^2 + 2x - 1}{x + 1}$$

✓ **Mathematica** : cpu = 0.0969318 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow -\frac{2(x+1)^2}{-2c_1 + x^2 + 2x} + x^2 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 43

$$\left\{ y(x) = \frac{-C1(x^4 + 2x^3 - x^2 - 2x - 2) + x^2 + 1}{1 + (x^2 + 2x) \_C1} \right\}$$

**2.737 ODE No. 737**

$$y'(x) = \frac{x(2x^3 - 2xy(x) + x - 1)}{x^2 - y(x)}$$

✓ **Mathematica** : cpu = 0.0300024 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( W\left(-e^{c_1 + \frac{4x^3}{3} - 2x^2 - 1}\right) + 1 \right) + x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 29

$$\left\{ y(x) = x^2 + \frac{1}{2} \operatorname{lambertW}\left(-2 \frac{e^{4/3 x^3} \_C1 e^{-1}}{(e^{x^2})^2}\right) + \frac{1}{2} \right\}$$





2.740 ODE No. 740

$$y'(x) = \frac{x^4 - 2x^2y(x)^2 + y(x)^4 + x}{y(x)}$$

✓ **Mathematica** : cpu = 0.0620416 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{2c_1x^2 + 2x^3 - 1}}{\sqrt{2}\sqrt{c_1 + x}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{2c_1x^2 + 2x^3 - 1}}{\sqrt{2}\sqrt{c_1 + x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.104 (sec), leaf count = 72

$$\left\{ y(x) = \frac{\sqrt{2}}{2_{-C1} + 2x} \sqrt{(-C1 + x)(2x^2_{-C1} + 2x^3 - 1)}, y(x) = -\frac{\sqrt{2}}{2_{-C1} + 2x} \sqrt{(-C1 + x)(2x^2_{-C1} + 2x^3 - 1)} \right\}$$

2.741 ODE No. 741

$$y'(x) = \frac{x(ay(x)^2 + bx^2)^3}{a^{5/2}y(x)(ay(x)^2 + a + bx^2)}$$

✓ **Mathematica** : cpu = 3.0744 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 1.023 (sec), leaf count = 246

$$\left\{ \int_{-b}^x \frac{(-a^2b + a(y(x))^2)^3 - a}{a^3} \left( b((y(x))^2 + 1) a^{5/2} + a^{3/2}b^2 - a^2 + (-a^2b + a(y(x))^2)^3 \right)^{-1} d_a + \int^{y(x)} 1 \left( ((-f^2) \right) \right.$$

2.742 ODE No. 742

$$y'(x) = -\frac{(-\cos(y(x)) + x + 1) \cos(y(x))}{(x + 1)(x \sin(y(x)) - 1)}$$

✓ **Mathematica** : cpu = 4.4438 (sec), leaf count = 221

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1} \left( \frac{-\sqrt{2c_1 \log(x+1) + c_1^2 - x^2 + \log^2(x+1) + 1 + c_1x + x \log(x+1)}}{x^2 - 1} \right) \right\}, \left\{ y(x) \rightarrow \sec^{-1} \left( \frac{-\sqrt{2c_1 \log(x+1) + c_1^2 - x^2 + \log^2(x+1) + 1 + c_1x + x \log(x+1)}}{x^2 - 1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.95 (sec), leaf count = 239

$$\left\{ y(x) = \arctan \left( \frac{1}{-C1^2 - 2_{-C1} \ln(1+x) + (\ln(1+x))^2 + 1} \left( (-\ln(1+x) +_{-C1}) \sqrt{(\ln(1+x))^2 - 2_{-C1} \ln(1+x) + C1^2} \right) \right) \right\}$$

2.743 ODE No. 743

$$y'(x) = -\frac{i(x^4 + 8x^2y(x)^2 + 16y(x)^4 + 8ix)}{32y(x)}$$

✗ **Mathematica** : cpu = 46.9439 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == ((-I/32)\*((8\*I)\*x + x^4 + 8\*x^2\*y[x]^2 + 16\*y[x]^4))/y[x], y[x]

✓ **Maple** : cpu = 0.57 (sec), leaf count = 301

$$\left\{ y(x) = -i \sqrt{-i \left( -2 \left( -\sqrt{3} + i \right) {}_0F_1 \left( 1/2 \left( -\sqrt{3} + i \right) x \right) + \left( 2\sqrt{3} - 2i \right) \text{Bi}^{(1)} \left( \frac{\left( -\sqrt{3} + i \right) x}{2} \right) + ix^2 \right)} \right.$$

2.744 ODE No. 744

$$y'(x) = \frac{x}{x^4 + 2x^2y(x)^2 + y(x)^4 - y(x)}$$

✓ **Mathematica** : cpu = 0.0456241 (sec), leaf count = 534

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{144c_1x^2 + 2\sqrt{(12x^2 - 4c_1^2)^3 + 4(36c_1x^2 + 4c_1^3 - 27)^2 + 16c_1^3 - 108}}}{6\sqrt[3]{2}} + \frac{\sqrt[3]{36c_1x^2 + 3\sqrt{3}\sqrt{32c_1^2x^4}}}{3\sqrt[3]{36c_1x^2 + 3\sqrt{3}\sqrt{32c_1^2x^4}}} \right. \right.$$

✓ **Maple** : cpu = 0.243 (sec), leaf count = 621

$$\left\{ y(x) = \frac{1}{12} \left( -2 {}_0F_1 \sqrt[3]{-36x^2 {}_0F_1 - 54 - {}_0F_1^3 + 6\sqrt{48x^6 + 24x^4 {}_0F_1^2 + (3 {}_0F_1^4 + 108 {}_0F_1)x^2 + 3 {}_0F_1}} \right) \right.$$

2.745 ODE No. 745

$$y'(x) = \frac{(y(x) \log(x) - 1)^3}{x(-y(x) + y(x) \log(x) - 1)}$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.071 (sec), leaf count = 78

$$\left\{ y(x) = \frac{47 \text{RootOf} \left( -27783 \int^{-Z} (2209 {}_a^3 - 9261 {}_a + 9261)^{-1} d{}_a - 7 \ln(x) + 3 {}_0F_1 \right) - 84}{(47 \ln(x) - 47) \text{RootOf} \left( -27783 \int^{-Z} (2209 {}_a^3 - 9261 {}_a + 9261)^{-1} d{}_a - 7 \ln(x) + 3 {}_0F_1 \right) - 84 \ln(x)} \right.$$

2.746 ODE No. 746

$$y'(x) = -\frac{i(x^4 + 2x^2y(x)^2 + y(x)^4 + ix)}{y(x)}$$

✗ **Mathematica** : cpu = 45.7178 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == ((-I)*(I*x + x^4 + 2*x^2*y[x]^2 + y[x]^4))/y[x], y[x], x]`

✓ **Maple** : cpu = 0.489 (sec), leaf count = 243

$$\left\{ y(x) = \frac{-i\sqrt{2}}{2 \operatorname{Ai}(-\sqrt[3]{-8ix})_C1 + 2 \operatorname{Bi}(-\sqrt[3]{-8ix})} \sqrt{-2i \left( \operatorname{Ai}(-\sqrt[3]{-8ix})_C1 + \operatorname{Bi}(-\sqrt[3]{-8ix}) \right)} \left( -\frac{C1(-\sqrt[3]{-8ix})}{\dots} \right) \right\}$$

2.747 ODE No. 747

$$y'(x) = -\frac{y(x) \cot(x) (x^2y(x)(-\log(2x)) + x \log(2x) + \tan(x))}{x}$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.358 (sec), leaf count = 75

$$\left\{ y(x) = 1e^{\int \frac{-x \ln(x) - x \ln(2) - \tan(x)}{x \tan(x)} dx} \left( \int -\frac{x(\ln(2) + \ln(x))}{\tan(x)} e^{\int \frac{-x \ln(x) - x \ln(2) - \tan(x)}{x \tan(x)} dx} dx + C1 \right)^{-1} \right\}$$

2.748 ODE No. 748

$$y'(x) = \frac{y(x)(y(x) + x)}{x(y(x)^3 + x)}$$

✓ **Mathematica** : cpu = 0.299796 (sec), leaf count = 286

$$\left\{ \left\{ y(x) \rightarrow \frac{2\sqrt[3]{2}(c_1 + \log(x))}{\sqrt[3]{\sqrt{2916x^2 - 864(c_1 + \log(x))^3 + 54x}}} + \frac{\sqrt[3]{\frac{1}{6}\sqrt{2916x^2 - 864(c_1 + \log(x))^3 + 9x}}}{3^{2/3}} \right\}, \left\{ y(x) \rightarrow \frac{(-1)}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.107 (sec), leaf count = 404

$$\left\{ y(x) = \frac{1}{6} \left( \left( i \left( 27x + 3 \sqrt{-24_C1^3 - 72_C1^2 \ln(x) - 72_C1 (\ln(x))^2 - 24 (\ln(x))^3 + 81x^2} \right) \right)^{\frac{2}{3}} - 6i_C1 \right) \right\}$$

**2.749 ODE No. 749**

$$y'(x) = \frac{x(x - y(x))^2(y(x) + x)^2}{y(x)}$$

✓ **Mathematica** : cpu = 0.0983858 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{(x^2 - 1)e^{4c_1 + 2x^2} + x^2 + 1}}{\sqrt{e^{4c_1 + 2x^2} + 1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{(x^2 - 1)e^{4c_1 + 2x^2} + x^2 + 1}}{\sqrt{e^{4c_1 + 2x^2} + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 192

$$\left\{ y(x) = 1 \sqrt{\left( -C1 e^{-\frac{x^2(x^2-2)}{2}} + e^{-\frac{x^2(x^2+2)}{2}} \right) \left( (x^2 + 1) e^{-\frac{x^2(x^2+2)}{2}} + -C1 (x^2 - 1) e^{-\frac{x^2(x^2-2)}{2}} \right) \left( -C1 e^{-\frac{x^2(x^2-2)}{2}} \right)} \right\}$$

**2.750 ODE No. 750**

$$y'(x) = \frac{y(x)(x^2 + 3y(x)^2)}{x(6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.320823 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W\left(\frac{6e^{2(c_1+x)}}{x}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x} \sqrt{W\left(\frac{6e^{2(c_1+x)}}{x}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.265 (sec), leaf count = 49

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\text{RootOf}\left(-e^{-Z} \ln\left(\frac{(e^{-Z}+9)x}{2}\right) + 3 - C1 e^{-Z} + -Z e^{-Z} + 2xe^{-Z} + 9\right)} + 9 \right) \right\}$$

2.751 ODE No. 751

$$y'(x) = \frac{y(x) (x^4 + x \log(y(x)) + \log(y(x)))}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.073266 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow (x+1)^x e^{\frac{1}{2}x(2c_1+x^2-2x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 26

$$\left\{ y(x) = \frac{(1+x)^x e^{x-C1}}{e^{x^2}} e^{\frac{x^3}{2}} \right\}$$

2.752 ODE No. 752

$$y'(x) = \frac{\cos(y(x)) (x^3 \cos(y(x)) - x - 1)}{(x+1)(x \sin(y(x)) - 1)}$$

✗ **Mathematica** : cpu = 31.5955 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == (Cos[y[x]]\*(-1 - x + x^3\*Cos[y[x]]))/((1 + x)\*(-1 + x\*Sin[y[x]])), y[x], x]

✓ **Maple** : cpu = 1.624 (sec), leaf count = 723

$$\left\{ y(x) = \arctan \left( \frac{1}{36 (\ln(1+x))^2 + (-24x^3 + 36x^2 - 72\_C1 - 72x) \ln(1+x) + 4x^6 - 12x^5 + 33x^4 + (24\_C1 - 72x)} \right) \right\}$$

2.753 ODE No. 753

$$y'(x) = \frac{y(x) \log(y(x)) (x^4 \log(y(x)) + x + 1)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.100631 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \exp \left( \frac{12x}{12c_1 - 3x^4 + 4x^3 - 6x^2 + 12x - 12 \log(x+1)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 38

$$\left\{ y(x) = e^{-12 \frac{x}{3x^4 - 4x^3 + 6x^2 + 12 \ln(1+x) - 12\_C1 - 12x}} \right\}$$

**2.754 ODE No. 754**

$$y'(x) = \frac{x^3 + xy(x)^2 + xy(x) + y(x)^3}{x^2}$$

✓ **Mathematica** : cpu = 0.0384468 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.019 (sec), leaf count = 26

$$\left\{ y(x) = \text{RootOf} \left( - \int^{-Z} (_a^3 + _a^2 + 1)^{-1} d_a + x + _C1 \right) x \right\}$$

**2.755 ODE No. 755**

$$y'(x) = \frac{y(x)^{3/2}}{x^2 - 2xy(x) + y(x)^2 + y(x)^{3/2}}$$

✓ **Mathematica** : cpu = 0.186236 (sec), leaf count = 2213

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{3} \left( 2(x + e^{c_1} + 2e^{2c_1}) - \sqrt[3]{x^3 + 3e^{c_1}x^2 - 3e^{2c_1}(4x - 1)x - 96e^{5c_1} - 64e^{6c_1} + 6e^{4c_1}(8x - 5) + e^{3c_1}(12} \right) \right. \right.$$

✓ **Maple** : cpu = 0.165 (sec), leaf count = 44

$$\left\{ 2 \frac{\sqrt{y(x)}}{y(x) - x} + (y(x) - x)^{-1} - 2 \frac{x}{\sqrt{y(x)}(y(x) - x)} - _C1 = 0 \right\}$$

**2.756 ODE No. 756**

$$y'(x) = \frac{x^6 + 2x^3y(x) + x^2y(x)^2 + y(x)^3}{x^4}$$

✓ **Mathematica** : cpu = 0.261215 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.034 (sec), leaf count = 37

$$\left\{ y(x) = \frac{\left( -3 + 29 \text{RootOf} \left( -81 \int^{-Z} (841 _a^3 - 27 _a + 27)^{-1} d_a + x + 3 _C1 \right) \right) x^2}{9} \right\}$$

2.757 ODE No. 757

$$y'(x) = \frac{x^3 + 2x^2 - 4xy(x) - 4x - 8}{2x^2 - 8y(x) + 4x - 8}$$

✓ **Mathematica** : cpu = 0.0265405 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} \left( 8W \left( -e^{c_1 - \frac{x}{4}} - 1 \right) + x^2 + 2x + 4 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 26

$$\left\{ y(x) = \frac{x^2}{4} + 2 \operatorname{lambertW} \left( \frac{1}{2} C_1 e^{-x/4} e^{-1/2} \right) + \frac{x}{2} + 1 \right\}$$

2.758 ODE No. 758

$$y'(x) = \frac{y(x) (x^3 y(x) + 2x + 2)}{(x + 1)(\log(y(x)) + 2x - 1)}$$

✓ **Mathematica** : cpu = 0.958157 (sec), leaf count = 459

$$\left\{ \left\{ y(x) \rightarrow \frac{6W \left( -\frac{1}{6} \sqrt[6]{e^{-12x} (6c_1 + 2x^3 - 3x^2 + 6x - 6 \log(x + 1))} \right)^6}{6c_1 + 2x^3 - 3x^2 + 6x - 6 \log(x + 1)} \right\}, \left\{ y(x) \rightarrow \frac{6W \left( \frac{1}{6} \sqrt[6]{e^{-12x} (6c_1 + 2x^3 - 3x^2 + 6x - 6 \log(x + 1))} \right)^6}{6c_1 + 2x^3 - 3x^2 + 6x - 6 \log(x + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.247 (sec), leaf count = 41

$$\left\{ y(x) = e^{-\operatorname{lambertW} \left( -\frac{(-2x^3 + 3x^2 + 6 \ln(1+x) + 6 - C_1 - 6x)e^{-2x}}{6} \right) - 2x} \right\}$$

2.759 ODE No. 759

$$y'(x) = -\frac{ix(x^8 + 18x^4 y(x)^2 + 54ix^2 + 81y(x)^4)}{243y(x)}$$

✗ **Mathematica** : cpu = 40.7893 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == ((-I/243)*x*((54*I)*x^2 + x^8 + 18*x^4*y[x]^2 + 81*y[x]^4))/y[x], y[x], x]`

✓ **Maple** : cpu = 0.704 (sec), leaf count = 315

$$\left\{ y(x) = \frac{-\frac{1}{6} - \frac{i}{6}}{x} \sqrt{(1-i) \left( J_{\frac{1}{3}} \left( \left( \frac{2}{27} - \frac{2i}{27} \right) \sqrt{6} x^3 \right) - C_1 + Y_{\frac{1}{3}} \left( \left( \frac{2}{27} - \frac{2i}{27} \right) \sqrt{6} x^3 \right) \right) \left( (-27 - 27i - (1-i)x^6) \right)} \right\}$$



**2.760 ODE No. 760**

$$y'(x) = \frac{(xy(x)^2 + 1)^3}{x^4 y(x) (xy(x)^2 + x + 1)}$$

✓ **Mathematica** : cpu = 1.28116 (sec), leaf count = 85

$$\text{Solve} \left[ 5 \left( c_1 + \frac{1}{x} \right) + 2 \log(xy(x)^2 - x + 1) + \tan^{-1}(2xy(x)^4 + 2(x+1)y(x)^2 + x + 1) = \log(2x^2 y(x)^4 + x^2 + 2) \right]$$

✓ **Maple** : cpu = 1.639 (sec), leaf count = 136

$$\left\{ -\frac{(-1 + y(x)) \left( 2(y(x))^4 + 2(y(x))^2 + 1 \right) \left( \ln \left( 2x^2(y(x))^4 + (2x^2 + 4x)(y(x))^2 + x^2 + 2x + 2 \right) x - \arctan \left( 2 \right) \right)}{5x \left( 2 \right)} \right\}$$

**2.761 ODE No. 761**

$$y'(x) = \frac{-x^3 + 4x^2 - 4xy(x) - 4x + 8}{2x^2 + 8y(x) - 8x + 8}$$

✓ **Mathematica** : cpu = 0.0266788 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow W(-e^{c_1 - x - 1}) - \frac{x^2}{4} + x \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 18

$$\left\{ y(x) = -\frac{x^2}{4} + \text{lambertW} \left( \frac{-C1}{e^x} \right) + x \right\}$$

**2.762 ODE No. 762**

$$y'(x) = -\frac{y(x)(x \log(y(x)) + \log(y(x)) - x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.0593319 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow (x+1)^{-1/x} e^{1 - \frac{c_1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 22

$$\left\{ y(x) = \frac{e}{\sqrt{x+1}} e^{-\frac{C1}{x}} \right\}$$

**2.763 ODE No. 763**

$$y'(x) = \frac{y(x)(x \log(y(x)) + \log(y(x)) + x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.0577186 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \left( \frac{x}{x+1} \right)^x e^{c_1 x} \right\} \right\}$$

✓ **Maple** : cpu = 0.101 (sec), leaf count = 14

$$\left\{ y(x) = \left( \frac{x - C1}{1+x} \right)^x \right\}$$

**2.764 ODE No. 764**

$$y'(x) = \frac{y(x)(x^4 - x \log(y(x)) - \log(y(x)))}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.0959562 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow (x+1)^{\frac{1}{x}} \exp \left( -\frac{12c_1 - 3x^4 + 4x^3 - 6x^2 + 12x + 25}{12x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.126 (sec), leaf count = 36

$$\left\{ y(x) = e^{\frac{x^3}{4}} e^{-\frac{x^2}{3}} e^{\frac{x}{2}} \sqrt{1+x} e^{-\frac{C1}{x}} e^{-1} \right\}$$

**2.765 ODE No. 765**

$$y'(x) = \frac{y(x) \left( xy(x) \log \left( \frac{(x-1)(x+1)}{x} \right) - \log \left( \frac{(x-1)(x+1)}{x} \right) - 1 \right)}{x}$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.208 (sec), leaf count = 106

$$\left\{ y(x) = \frac{e^{dilog(1+x)} x^{\ln(1+x)}}{e^{dilog(x)} x} e^{-\frac{(\ln(x))^2}{2}} \left( \int -\frac{e^{dilog(1+x)} x^{\ln(1+x)}}{e^{dilog(x)} x} e^{-\frac{(\ln(x))^2}{2}} \ln \left( \frac{(1+x)(x-1)}{x} \right) \left( x^{\ln \left( \frac{(1+x)(x-1)}{x} \right)} \right)^{-1} dx \right) \right\}$$

**2.766 ODE No. 766**

$$y'(x) = \frac{y(x) \left( x^2 y(x) \log \left( \frac{(x-1)(x+1)}{x} \right) - x \log \left( \frac{(x-1)(x+1)}{x} \right) - \log(x) \right)}{x \log(x)}$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.256 (sec), leaf count = 89

$$\left\{ y(x) = 1 e^{\int \frac{1}{x \ln(x)} \left( -x \ln \left( \frac{(1+x)(x-1)}{x} \right) - \ln(x) \right) dx} \left( \int -\frac{x}{\ln(x)} e^{\int \frac{1}{x \ln(x)} \left( -x \ln \left( \frac{(1+x)(x-1)}{x} \right) - \ln(x) \right) dx} \ln \left( \frac{(1+x)(x-1)}{x} \right) dx \right) \right\}$$

**2.767 ODE No. 767**

$$y'(x) = \frac{-x^3 + 2x^2 - 8xy(x) - 8x + 32}{4x^2 + 32y(x) - 8x + 32}$$

✓ **Mathematica** : cpu = 0.0261665 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow 4W \left( -e^{c_1 - \frac{x}{16} - 1} \right) - \frac{x^2}{8} + \frac{x}{4} + 3 \right\} \right\}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 26

$$\left\{ y(x) = -\frac{x^2}{8} + 4 \text{lambertW} \left( 1/4\_C1 e^{-x/16} e^{-3/4} \right) + \frac{x}{4} + 3 \right\}$$

**2.768 ODE No. 768**

$$y'(x) = \frac{y(x)(y(x) + 1)}{x(xy(x) - y(x) - 1)}$$

✓ **Mathematica** : cpu = 1.14209 (sec), leaf count = 66

$$\text{Solve} \left[ \frac{2^{2/3} \left( xy(x) \left( -\log \left( \frac{xy(x)}{(x-1)y(x)-1} \right) + \log \left( \frac{y(x)+1}{-xy(x)+y(x)+1} \right) + \log(x) + 1 \right) - 1 \right)}{9xy(x)} = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 26

$$\left\{ y(x) = - \left( x \text{lambertW} \left( \frac{1}{x e^{x-1} - C1} \right) + 1 \right)^{-1} \right\}$$

2.769 ODE No. 769

$$y'(x) = -\frac{ix(x^8 + 8x^4y(x)^2 + 16ix^2 + 16y(x)^4)}{32y(x)}$$

✗ **Mathematica** : cpu = 42.6012 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == ((-I/32)\*x\*((16\*I)\*x^2 + x^8 + 8\*x^4\*y[x]^2 + 16\*y[x]^4))/y[x]

✓ **Maple** : cpu = 0.547 (sec), leaf count = 251

$$\left\{ y(x) = -\frac{\sqrt{4}}{2x} \sqrt{\left(-2\left(\frac{1}{8}x^6 + i\right)_{-C1} J_{1/3}\left(\left(\frac{1}{3} - \frac{i}{3}\right)x^3\right) + \left(-\frac{x^6}{4} - 2i\right) Y_{1/3}\left(\left(\frac{1}{3} - \frac{i}{3}\right)x^3\right) + (1+i)x^3\left(J_{4/3}\right)}\right.}$$

2.770 ODE No. 770

$$y'(x) = \frac{2y(x)^6}{32x^2y(x)^4 + y(x)^3 + 16xy(x)^2 + 2}$$

✓ **Mathematica** : cpu = 0.133224 (sec), leaf count = 687

$$\left\{ \left\{ y(x) \rightarrow \frac{2^3 \sqrt{2}^3 \sqrt{4608c_1^2 x^2 + 3\sqrt{3}\sqrt{(1-16c_1x)^2(2048c_1^2 x^2 + 64c_1(4c_1^3 - 9)x - 16c_1^3 + 4096x^3 + 27)} - 720c_1x}}{6(1-16c_1x)} \right\} \right.$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 1105

$$\left\{ y(x) = \frac{1}{96x + 6_{-C1}} \left( 32x_{-C1} \sqrt[3]{96(-C1/16 + x)\sqrt{3}\sqrt{(4096x^3 + 27)_{-C1}^4 + 576x_{-C1}^3 + 2048_{-C1}^2 x^2}} \right) \right.$$

2.771 ODE No. 771

$$y'(x) = \frac{-a^2x^3 - 2abx^2 - 4axy(x) - 4ax + 8}{2ax^2 + 4bx + 8y(x) + 8}$$

✓ **Mathematica** : cpu = 0.032786 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow -\frac{abx^2 + 8W\left(-e^{-\frac{b^2x}{4} + c_1 - 1}\right) + 2b^2x + 4b + 8}{4b} \right\} \right.$$

✓ **Maple** : cpu = 0.111 (sec), leaf count = 84

$$\left\{ y(x) = \frac{1}{4b} \left( -ax^2b - 2b^2x - 4b + 4e^{1/4 \frac{1}{a}} \left( -4 \operatorname{lambertW} \left( -1/2 e^{-1/4 b^2 x} e^{-1/2 - \frac{C1 b^2}{a}} e^{-b/2} e^{-1} \right) \right)_{a+(-b^2x-2b-4)a-2-C1 b^2} \right) \right\}$$

**2.772 ODE No. 772**

$$y'(x) = \frac{y(x) \log(y(x))(x \log(y(x)) + x + 1)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.0633986 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{x}{c_1 - x + \log(x+1)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 18

$$\left\{ y(x) = e^{\frac{x}{\ln(1+x) + C1 - x}} \right\}$$

**2.773 ODE No. 773**

$$y'(x) = \frac{y(x)^2 + xy(x) + x}{(x - 1)(y(x) + x)}$$

✓ **Mathematica** : cpu = 0.304325 (sec), leaf count = 59

$$\text{Solve} \left[ \frac{1}{2} \log \left( \frac{x^2 + xy(x) + y(x)^2}{x^2} \right) + \frac{\tan^{-1} \left( \frac{2y(x)+x}{\sqrt{3x}} \right)}{\sqrt{3}} + \log(x) = c_1 + \log(1 - x), y(x) \right]$$

✓ **Maple** : cpu = 0.247 (sec), leaf count = 48

$$\left\{ y(x) = \frac{\sqrt{3}x}{2} \tan \left( \operatorname{RootOf} \left( -\sqrt{3} \ln \left( \frac{3x^2 \left( (\tan(\_Z))^2 + 1 \right)}{4(x-1)^2} \right) + 2\sqrt{3} C1 - 2\_Z \right) \right) - \frac{x}{2} \right\}$$

2.774 ODE No. 774

$$y'(x) = \frac{-2ax^2 - x^3 - 4xy(x) - 4x + 8}{4ax + 2x^2 + 8y(x) + 8}$$

✓ **Mathematica** : cpu = 0.0295103 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow -\frac{8W\left(-e^{-\frac{a^2x}{4}+c_1-1}\right) + 2a^2x + a(x^2 + 4) + 8}{4a} \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1}{4a} \left( -2a^2x - ax^2 - 8 \operatorname{lambertW}\left(-1/2 e^{-1/4 a^2 x} e^{-a/2} e^{-1} e^{1/4 - C1 a^2}\right) - 4a - 8 \right) \right\}$$

2.775 ODE No. 775

$$y'(x) = \frac{-y(x) + \sqrt{y(x)} + x}{-y(x) + \sqrt{y(x)} + x + 1}$$

✓ **Mathematica** : cpu = 0.0985144 (sec), leaf count = 943

$$\left\{ \left\{ y(x) \rightarrow \operatorname{Root}\left[x^6 - 2e^{3c_1}x^3 + e^{6c_1} + \#1^6 + (-6x - 6)\#1^5 + (15x^2 + 24x + 9)\#1^4 + (-20x^3 - 36x^2 - 18x + \dots\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.091 (sec), leaf count = 32

$$\left\{ \sqrt{y(x) - 2\sqrt{y(x)}} - x(y(x) + \sqrt{y(x)} - x) - C1 = 0 \right\}$$

2.776 ODE No. 776

$$y'(x) = \frac{y(x) \left( x^2 y(x) \log\left(\frac{x^2+1}{x}\right) - x \log\left(\frac{x^2+1}{x}\right) - \log\left(\frac{1}{x}\right) \right)}{x \log\left(\frac{1}{x}\right)}$$

✗ **Mathematica** : cpu = 300.028 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.934 (sec), leaf count = 96

$$\left\{ y(x) = 1e^{\int \frac{1}{x \ln(x^{-1})} (-\ln\left(\frac{x^2+1}{x}\right) x - \ln(x^{-1})) dx} \left( \int -\frac{x}{\ln(x^{-1})} e^{\int \frac{1}{x \ln(x^{-1})} (-\ln\left(\frac{x^2+1}{x}\right) x - \ln(x^{-1})) dx} \ln\left(\frac{x^2+1}{x}\right) dx + \dots \right) \right\}$$

2.777 ODE No. 777

$$y'(x) = \frac{y(x)(y(x) + 1)}{x(xy(x)^4 - y(x) - 1)}$$

✓ **Mathematica** : cpu = 0.139007 (sec), leaf count = 34

$$\text{Solve} \left[ y(x) + \frac{3}{2} = c_1 + \frac{y(x)^2}{2} + \frac{1}{xy(x)} + \log(y(x) + 1), y(x) \right]$$

✓ **Maple** : cpu = 0.139 (sec), leaf count = 51

$$\left\{ y(x) = e^{\text{RootOf}(x(e^{-z})^3 - 5x(e^{-z})^2 + 2x\_C1 e^{-z} + 2\_Z x e^{-z} + 7x e^{-z} - 2x\_C1 - 2\_Z x - 3x + 2)} - 1 \right\}$$

2.778 ODE No. 778

$$y'(x) = \frac{x^9 y(x)^3 + x^6 y(x)^2 - 3x^2 y(x) + 1}{x^3}$$

✓ **Mathematica** : cpu = 0.213563 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.03 (sec), leaf count = 37

$$\left\{ y(x) = \frac{-3 + 29 \text{RootOf}(-81 \int^{-z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + x + 3\_C1)}{9x^3} \right\}$$

2.779 ODE No. 779

$$y'(x) = \frac{x^3 y(x) + x^3 + xy(x)^2 + y(x)^3}{(x-1)x^3}$$

✓ **Mathematica** : cpu = 0.0712789 (sec), leaf count = 51

$$\text{Solve} \left[ 4c_1 + \log\left(\frac{y(x)^2}{x^2} + 1\right) + 4 \log(1-x) = 2\left(\log\left(\frac{y(x)+x}{x}\right) + \tan^{-1}\left(\frac{y(x)}{x}\right) + 2 \log(x)\right), y(x) \right]$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 50

$$\left\{ -\frac{1}{4} \ln\left(\frac{(y(x))^2 + x^2}{x^2}\right) + \frac{1}{2} \arctan\left(\frac{y(x)}{x}\right) + \frac{1}{2} \ln\left(\frac{y(x)+x}{x}\right) - \ln(x-1) + \ln(x) - \_C1 = 0 \right\}$$

2.780 ODE No. 780

$$y'(x) = \frac{x\sqrt{x^2 + y(x)^2} + xy(x) + y(x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.0259316 (sec), leaf count = 15

$$\{\{y(x) \rightarrow x \sinh(c_1 + \log(x+1))\}\}$$

✓ **Maple** : cpu = 0.408 (sec), leaf count = 27

$$\left\{ -C1 + \frac{1}{x(1+x)} \left( \sqrt{(y(x))^2 + x^2} + y(x) \right) = 0 \right\}$$

2.781 ODE No. 781

$$y'(x) = \frac{y(x)(x^4 + x^3 + 3y(x)^2 + x)}{x(6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.535517 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x}\sqrt{W\left(6xe^{2c_1 + \frac{2x^3}{3} + x^2}\right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x}\sqrt{W\left(6xe^{2c_1 + \frac{2x^3}{3} + x^2}\right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.29 (sec), leaf count = 61

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\text{RootOf}\left(2x^3e^{-Z} + 3x^2e^{-Z} - 3e^{-Z}\ln\left(1/2\frac{e^{-Z}+9}{x}\right) + 9 - C1e^{-Z} + 3Ze^{-Z} + 27\right)} + 9 \right) \right\}$$

2.782 ODE No. 782

$$y'(x) = \frac{y(x) \coth\left(\frac{1}{x}\right) \left( x^2 y(x) \log\left(\frac{x^2+1}{x}\right) - x \log\left(\frac{x^2+1}{x}\right) - \tanh\left(\frac{1}{x}\right) \right)}{x}$$

✗ **Mathematica** : cpu = 300.019 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.546 (sec), leaf count = 96

$$\left\{ y(x) = 1e^{\int \frac{1}{x \tanh(x^{-1})} \left( -\ln\left(\frac{x^2+1}{x}\right) x - \tanh(x^{-1}) \right) dx} \left( \int -\frac{x}{\tanh(x^{-1})} e^{\int \frac{1}{x \tanh(x^{-1})} \left( -\ln\left(\frac{x^2+1}{x}\right) x - \tanh(x^{-1}) \right) dx} \ln\left(\frac{x^2+1}{x}\right) dx \right) \right\}$$



**2.783 ODE No. 783**

$$y'(x) = -\frac{y(x) \coth(x) (x^2 y(x) (-\log(2x)) + x \log(2x) + \tanh(x))}{x}$$

✗ **Mathematica** : cpu = 300.034 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.238 (sec), leaf count = 75

$$\left\{ y(x) = 1e^{\int \frac{-x \ln(x) - x \ln(2) - \tanh(x)}{x \tanh(x)} dx} \left( \int -\frac{x(\ln(2) + \ln(x))}{\tanh(x)} e^{\int \frac{-x \ln(x) - x \ln(2) - \tanh(x)}{x \tanh(x)} dx} dx + _C1 \right)^{-1} \right\}$$

**2.784 ODE No. 784**

$$y'(x) = \operatorname{csch}(x) (x^2 \log(x) + 2xy(x) \log(x) + y(x)^2 \log(x) + \log(x) - \sinh(x))$$

✗ **Mathematica** : cpu = 300.063 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 36.043 (sec), leaf count = 24

$$\left\{ y(x) = -x - \tan \left( -C1 - \int \frac{\ln(x)}{\sinh(x)} dx \right) \right\}$$

**2.785 ODE No. 785**

$$y'(x) = \frac{x^2 \sinh(x) + 2xy(x) \sinh(x) + y(x)^2 \sinh(x) - \log(x) + \sinh(x)}{\log(x)}$$

✗ **Mathematica** : cpu = 300.056 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 123.082 (sec), leaf count = 24

$$\left\{ y(x) = -x - \tan \left( -C1 - \int \frac{\sinh(x)}{\ln(x)} dx \right) \right\}$$

2.786 ODE No. 786

$$y'(x) = \frac{axy(x)^2 \cosh(x) + bx^3 \cosh(x) + y(x) \log(x)}{x \log(x)}$$

✗ **Mathematica** : cpu = 300.042 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.063 (sec), leaf count = 33

$$\left\{ y(x) = \frac{x}{a} \tan \left( \sqrt{ab} \left( -C1 + \int \frac{x \cosh(x)}{\ln(x)} dx \right) \right) \sqrt{ab} \right\}$$

2.787 ODE No. 787

$$y'(x) = \frac{x(2x^4 - 2x^2y(x) + x^2 - x - 1)}{(x + 1)(x^2 - y(x))}$$

✓ **Mathematica** : cpu = 41.148 (sec), leaf count = 484

Solve

$$\left[ 2^{2/3} \left( 2 - \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} \right) \left( \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} + 4 \right) \left( \left( 1 - \frac{x(x^2-x-1)(2x^2-2y(x)+3)}{2\sqrt[3]{x^3(x^2-x-1)^3(x^2-y(x))}} \right) \log \left( \dots \right) \right) \right]$$

$$9 \left( -\frac{(2x^2-2y(x)+3)^3}{(x^2-y(x))^3} + \frac{12x(x^2-x-1)}{\sqrt[3]{x^3(x^2-x-1)^3}} \right)$$

✓ **Maple** : cpu = 0.619 (sec), leaf count = 191

$$\left\{ y(x) = 1 \left( 4x^2 e^{\text{RootOf} \left( 8x^3 e^{-Z} - 24x^2 e^{-Z} - 36x^3 + 6 \ln \left( \frac{2e^{-Z}-9}{(1+x)^4} \right) e^{-Z} + 18 - C1 e^{-Z} - 6 - Z e^{-Z} + 24x e^{-Z} + 108x^2 - 27 \ln \left( \frac{2e^{-Z}-9}{(1+x)^4} \right) \right)} \right) \right\}$$

2.788 ODE No. 788

$$y'(x) = -\frac{y(x) (x^2 y(x) (-\coth(x+1)) + \log(x-1) + x \coth(x+1))}{x \log(x-1)}$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.706 (sec), leaf count = 108

$$\left\{ y(x) = 1 \left( e^{-\int \frac{-\ln(x-1) \sinh(1+x) - x \cosh(1+x)}{\sinh(1+x)x \ln(x-1)} dx} \right)^{-1} \left( -C1 + \int -\frac{x \cosh(1+x)}{\ln(x-1) \sinh(1+x)} e^{\int \frac{-\ln(x-1) \sinh(1+x) - x \cosh(1+x)}{\sinh(1+x)x \ln(x-1)} dx} \right)$$

**2.789 ODE No. 789**

$$y'(x) = \frac{x^2 \coth(x+1) + 2xy(x) \coth(x+1) + y(x)^2 \coth(x+1) - \log(x-1) + \coth(x+1)}{\log(x-1)}$$

✗ **Mathematica** : cpu = 300.004 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.790 ODE No. 790**

$$y'(x) = \frac{x^4 \coth\left(\frac{x+1}{x-1}\right) - 2x^2 y(x) \coth\left(\frac{x+1}{x-1}\right) + y(x)^2 \coth\left(\frac{x+1}{x-1}\right) + 2x \log\left(\frac{1}{x-1}\right) - \coth\left(\frac{x+1}{x-1}\right)}{\log\left(\frac{1}{x-1}\right)}$$

✗ **Mathematica** : cpu = 300.005 (sec), leaf count = 0 , timed out

\$Aborted

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(y(x),x) = (2\*x\*ln(1/(x-1))-coth((1+x)/(x-1))+coth((1+x)/(x-1))\*y(x)^2-2\*coth((1+x)/(x-1))\*x^2\*y(x)+coth((1+x)/(x-1))\*x^4)/ln(1/(x-1)),y(x))

**2.791 ODE No. 791**

$$y'(x) = \frac{\operatorname{sech}\left(\frac{1}{x-1}\right) \left(x^5 + x^4 - 2x^3 y(x) - 2x^2 y(x) + 2x^2 \cosh\left(\frac{1}{x-1}\right) + xy(x)^2 + y(x)^2 - x - 2x \cosh\left(\frac{1}{x-1}\right) - 1\right)}{x-1}$$

✗ **Mathematica** : cpu = 300.263 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 35.476 (sec), leaf count = 306

$$\left\{ y(x) = 1 \left( (-x^2 + 1) \left( e^{\frac{1}{\left(\frac{e^{(x-1)^{-1}}}{2} + 1\right)^2} \int \frac{e^{(x-1)^{-1}(1+x)}}{\left(\frac{e^{(x-1)^{-1}}}{2} + 1\right)(x-1)} dx} \right)^4 \left( e^{\frac{1}{\left(\frac{e^{(x-1)^{-1}}}{2} + 1\right)^2} \int \frac{e^{(x-1)^{-1}(1+x)}}{\left(\frac{e^{(x-1)^{-1}}}{2} + 1\right)(x-1)} dx e^{2(x-1)^{-1}} \right) \right.$$

**2.792 ODE No. 792**

$$y'(x) = \frac{y(x)\operatorname{sech}\left(\frac{1}{x+1}\right) \left(x^3y(x) + x^2y(x) - x^2 - x - x \cosh\left(\frac{1}{x+1}\right) + \cosh\left(\frac{1}{x+1}\right)\right)}{(x-1)x}$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.852 (sec), leaf count = 112

$$\left\{ y(x) = 1e^{\int \frac{(1-x) \cosh((1+x)^{-1}) - x^2 - x}{x(x-1) \cosh((1+x)^{-1})} dx} \left( \int -\frac{x(1+x)}{(x-1) \cosh((1+x)^{-1})} e^{\int \frac{(1-x) \cosh((1+x)^{-1}) - x^2 - x}{x(x-1) \cosh((1+x)^{-1})} dx} dx + \_C1 \right)^{-1} \right\}$$

**2.793 ODE No. 793**

$$y'(x) = -\frac{y(x)(xy(x) + 1)}{x(xy(x) - y(x) + 1)}$$

✓ **Mathematica** : cpu = 23.3557 (sec), leaf count = 399

Solve  $\left[ \frac{\sqrt[3]{-2} \left( \frac{2^{2/3}((x-1)y(x)-2)}{\sqrt[3]{-\frac{1}{(x-1)^3}(x-1)((x-1)y(x)+1)}} + (-2)^{2/3} \right) \left( \frac{-xy(x)+y(x)+2}{\sqrt[3]{2} \sqrt[3]{-\frac{1}{(x-1)^3}(x-1)((x-1)y(x)+1)}} + (-2)^{2/3} \right) \left( \left( \frac{\sqrt[3]{-1}(-x)}{\sqrt[3]{-\frac{1}{(x-1)^3}(x-1)((x-1)y(x)+1)}} \right)^{-1} \right)}{\dots} \right]$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 32

$$\left\{ y(x) = -2 \frac{1}{x} e^{-\operatorname{lambertW}\left(-2 \frac{(x-1)(e^{-C1})^3 e^{-1}}{x}\right) + 3\_C1 - 1} \right\}$$

**2.794 ODE No. 794**

$$y'(x) = \frac{y(x)}{x(x^3y(x)^4 + x^2y(x)^3 + y(x) - 1)}$$

✓ **Mathematica** : cpu = 0.149541 (sec), leaf count = 66

Solve  $\left[ c_1 + \log(x) = \operatorname{RootSum}\left[\#1^3y(x)^3 + \#1^2y(x)^2 + 1\&, \frac{\#1y(x) \log(x - \#1) + \log(x - \#1)}{3\#1y(x) + 2}\right] \& \right] + y(x), y(x)$

✓ **Maple** : cpu = 2.51 (sec), leaf count = 32

$$\left\{ -y(x) + \int^{xy(x)} \frac{1}{-a(-a^3 - a^2 + 1)} da - C1 = 0 \right\}$$

**2.795 ODE No. 795**

$$y'(x) = \frac{a^3 + 3a^2x + 3ax^2 + ay(x)^2 + x^3 + y(x)^3 + xy(x)^2}{(a+x)^3}$$

✓ **Mathematica** : cpu = 1.17545 (sec), leaf count = 106

$$\text{Solve} \left[ 57\text{RootSum} \left[ -19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log \left( \frac{a+3y(x)+x}{\sqrt[3]{38} \sqrt[3]{\frac{1}{(a+x)^6} (a+x)^3}} - \#1 \right)}{2\sqrt[3]{38} - 19\#1^2} \& \right] + 38^{2/3} \left( \frac{1}{(a+x)^6} \right)^{2/3} (a+x) \right]$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 37

$$\left\{ y(x) = -\text{RootOf} \left( - \int^{-Z} (-a^3 - a^2 - a - 1)^{-1} da + \ln(x+a) + C1 \right) (x+a) \right\}$$

**2.796 ODE No. 796**

$$y'(x) = \frac{e^{-\frac{3x^2}{2}} xy(x)^3}{3 \left( e^{\frac{3x^2}{2}} y(x) + 3e^{\frac{3x^2}{2}} + 3y(x) \right)}$$

✗ **Mathematica** : cpu = 301.119 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.451 (sec), leaf count = 143

$$\left\{ y(x) = \text{RootOf} \left( \left( 7 e^{3x^2 + \text{RootOf} \left( \left( e^{3/2 x^2} \right)^2 \left( 42 \sqrt{93} \tanh \left( \frac{(-C1 - 5Z)\sqrt{93}}{90} \right) e^{3x^2 - Z} + 217 \left( \tanh \left( \frac{(-C1 - 5Z)\sqrt{93}}{90} \right) \right)^2 e^{3x^2 - Z} + 189 \right) \right) \right) \right)$$

**2.797 ODE No. 797**

$$y'(x) = \frac{y(x) \left( x^3 y(x) \cosh\left(\frac{x+1}{x-1}\right) + x^2 y(x) \cosh\left(\frac{x+1}{x-1}\right) - x^2 \cosh\left(\frac{x+1}{x-1}\right) - x \cosh\left(\frac{x+1}{x-1}\right) - 1 \right)}{x}$$

✓ **Mathematica** : cpu = 2.06017 (sec), leaf count = 146

$$\left\{ \left\{ y(x) \rightarrow \frac{\exp\left(\frac{(3e^2-1)\text{Chi}\left(\frac{2}{x-1}\right) + (1+3e^2)\text{Shi}\left(\frac{2}{x-1}\right)}{e}\right)}{x \left( c_1 \exp\left(\frac{(x-1)\left((-x+e^2(x+5)-1)\sinh\left(\frac{2}{x-1}\right) + (x+e^2(x+5)+1)\cosh\left(\frac{2}{x-1}\right)\right)}{4e}\right) + \exp\left(\frac{(3e^2-1)\text{Chi}\left(\frac{2}{x-1}\right) + (1+3e^2)\text{Shi}\left(\frac{2}{x-1}\right)}{e}\right)} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.425 (sec), leaf count = 252

$$\left\{ y(x) = \frac{1}{x} e^{\frac{5e}{4} e^{2(x-1)^{-1}}} e^{-\frac{ex^2}{4} e^{2(x-1)^{-1}}} e^{\frac{e^{-1}}{4} e^{-2(x-1)^{-1}}} e^{-1} \text{Ei}\left(1, 2(x-1)^{-1}\right) e^{-\frac{e^{-1}x^2}{4} e^{-2(x-1)^{-1}}} \left( e^{e^{2(x-1)^{-1}} x} \right)^{-1} \left( e^{e \text{Ei}\left(1, \dots\right)} \right)$$

**2.798 ODE No. 798**

$$y'(x) = \frac{y(x)(y(x) + x + 1)}{(x + 1)(2y(x)^3 + y(x) + x)}$$

✓ **Mathematica** : cpu = 0.652985 (sec), leaf count = 25

$$\text{Solve}\left[ c_1 + \frac{x}{y(x)} + \log(x + 1) = y(x)^2 + \log(y(x)), y(x) \right]$$

✓ **Maple** : cpu = 0.154 (sec), leaf count = 30

$$\left\{ y(x) = e^{\text{RootOf}\left(-\left(e^{-Z}\right)^3 + \ln(1+x)e^{-Z} + \_C1 e^{-Z} - \_Z e^{-Z} + x\right)} \right\}$$

**2.799 ODE No. 799**

$$y'(x) = \frac{y(x) \left( e^{\frac{x+1}{x-1}} x^3 y(x) + e^{\frac{x+1}{x-1}} x^2 y(x) - e^{\frac{x+1}{x-1}} x^2 - e^{\frac{x+1}{x-1}} x - 1 \right)}{x}$$

✓ **Mathematica** : cpu = 0.301862 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{6e \text{Ei}\left(\frac{2}{x-1}\right)}}{x \left( c_1 e^{\frac{1}{2} e^{\frac{x+1}{x-1}} (x^2 + 4x - 5)} + e^{6e \text{Ei}\left(\frac{2}{x-1}\right)} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.43 (sec), leaf count = 147

$$\left\{ y(x) = \frac{1}{x} e^{\frac{5}{2} e^{\frac{1+x}{x-1}}} e^{-\frac{x^2}{2} e^{\frac{1+x}{x-1}}} \left( e^{Ei(1, -2(x-1)^{-1})} \right)^{-6} \left( e^{x e^{\frac{1+x}{x-1}}} \right)^{-2} \left( \int -(1+x) e^{\frac{1+x}{x-1}} e^{\frac{5}{2} e^{\frac{1+x}{x-1}}} e^{-\frac{x^2}{2} e^{\frac{1+x}{x-1}}} \left( e^{Ei(1, -2(x-1)^{-1})} \right) \right)$$

**2.800 ODE No. 800**

$$y'(x) = \frac{-b^3 + 6b^2x - 12bx^2 - 4by(x)^2 + 8x^3 + 8y(x)^3 + 8xy(x)^2}{(2x - b)^3}$$

✓ **Mathematica** : cpu = 1.20698 (sec), leaf count = 118

$$\text{Solve} \left[ 57\text{RootSum} \left[ -19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log \left( \frac{b-6y(x)-2x}{\sqrt[3]{38} \sqrt[3]{\frac{1}{(b-2x)^6} (b-2x)^3}} - \#1 \right)}{2\sqrt[3]{38} - 19\#1^2} \& \right] + 38^{2/3} \left( \frac{1}{(b-2x)^6} \right)^{2/3} (b -$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 41

$$\left\{ y(x) = \frac{\text{RootOf} \left( -\int^{-Z} (\_a^3 - \_a^2 - \_a - 1)^{-1} d\_a + \ln(-2x + b) + \_C1 \right) (-2x + b)}{2} \right\}$$

**2.801 ODE No. 801**

$$y'(x) = \frac{1}{2} e^{\frac{x^2}{4}} \left( 2e^{-\frac{3x^2}{4}} y(x)^3 + 2e^{-\frac{x^2}{2}} y(x)^2 + e^{-\frac{x^2}{4}} xy(x) + 2 \right)$$

✓ **Mathematica** : cpu = 0.325384 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.072 (sec), leaf count = 63

$$\left\{ y(x) = \frac{1}{9} \left( -3e^{-1/4x^2} e^{1/4x^2} + 29 \text{RootOf} \left( -81 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + x + 3\_C1 \right) \right) \left( e^{\frac{x^2}{4}} \right)^{-1}$$

**2.802 ODE No. 802**

$$y'(x) = \frac{-F1(y(x) + \frac{1}{x}) + \frac{1}{x}}{x}$$

✓ **Mathematica** : cpu = 0.0829548 (sec), leaf count = 88

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x - \frac{-F1'(K[2] + \frac{1}{K[1]})}{K[1]^2 (-F1(K[2] + \frac{1}{K[1]}))^2} dK[1] - \frac{1}{-F1(K[2] + \frac{1}{x})} \right) dK[2] + \int_1^x \frac{\frac{1}{K[1]} + y(x)}{K[1]^2} \right]$$

✓ **Maple** : cpu = 0.147 (sec), leaf count = 27

$$\left\{ y(x) = \frac{\text{RootOf} \left( -\ln(x) + \int^{-Z} (-F1(\_a))^{-1} d\_a + \_C1 \right) x - 1}{x} \right\}$$

**2.803 ODE No. 803**

$$y'(x) = \frac{-F1(y(x)^2 - 2 \log(x))}{x \sqrt{y(x)^2}}$$

✓ **Mathematica** : cpu = 0.118493 (sec), leaf count = 234

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \frac{-((\_F1(K[2]^2 - 2 \log(x)))^2 - 1) \left( \int_1^x \frac{2(\sqrt{K[2]^2} (\_F1(K[2]^2 - 2 \log(K[1])))^2 + 2K[2] \_F1(K[2]^2 - 2 \log(K[1])))^2 - 2K[2] (\_F1(K[2]^2 - 2 \log(K[1])))^2}{K[1]((\_F1(K[2]^2 - 2 \log(K[1])))^2 - 1)} dx \right)}{(\_F1(K[2]^2 - 2 \log(x)))^2 - 1} \right]$$

✓ **Maple** : cpu = 0.598 (sec), leaf count = 65

$$\left\{ y(x) = \sqrt{2 \ln(x) + 2 \text{RootOf} \left( \ln(x) - \int^{-Z} (-F1(2\_a) - 1)^{-1} d\_a + \_C1 \right)}, y(x) = -\sqrt{2 \ln(x) + 2 \text{RootOf} \left( \ln(x) - \int^{-Z} (-F1(2\_a) - 1)^{-1} d\_a + \_C1 \right)} \right\}$$



**2.804 ODE No. 804**

$$y'(x) = \frac{\frac{1}{2}x^4 \cos(2y(x)) + \frac{x^4}{2} - \frac{1}{2}x \sin(2y(x)) - \frac{1}{2} \sin(2y(x))}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.500022 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{-12c_1 + 3x^4 - 4x^3 + 6x^2 - 12x + 12 \log(x+1) - 25}{12x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.401 (sec), leaf count = 38

$$\left\{ y(x) = \arctan \left( \frac{3x^4 - 4x^3 + 6x^2 + 12 \ln(1+x) - 12\_C1 - 12x}{12x} \right) \right\}$$

**2.805 ODE No. 805**

$$y'(x) = \frac{x^4 \sqrt{x^2 + y(x)^2} + xy(x) + y(x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.033338 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left( c_1 + \frac{x^3}{3} - \frac{x^2}{2} + x - \log(x+1) + \frac{11}{6} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.68 (sec), leaf count = 42

$$\left\{ \ln \left( \sqrt{(y(x))^2 + x^2} + y(x) \right) - \frac{x^3}{3} + \frac{x^2}{2} - x - \ln(x) + \ln(1+x) - \_C1 = 0 \right\}$$

**2.806 ODE No. 806**

$$y'(x) = \frac{-\frac{1}{2}x \sin(2y(x)) - \frac{1}{2} \sin(2y(x)) + \frac{1}{2}x \cos(2y(x)) + \frac{x}{2}}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.276378 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{-c_1 + x - \log(x+1) + 1}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.802 (sec), leaf count = 22

$$\left\{ y(x) = -\arctan \left( \frac{-x + \ln(1+x) - \_C1}{x} \right) \right\}$$

2.807 ODE No. 807

$$y'(x) = -\frac{1}{-e^{y(x)}y(x) - \text{F1}(y(x) - \log(x)) - x}$$

✗ **Mathematica** : cpu = 1.80062 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == -(-x - E^y[x]\*y[x]\*\_F1[-Log[x] + y[x]])^(-1), y[x], x]

✓ **Maple** : cpu = 0.821 (sec), leaf count = 43

$$\left\{ \frac{(\ln(x))^2}{2} - y(x) \ln(x) - \int^{y(x) - \ln(x)} \frac{\text{F1}(-a) - a + e^{-a}}{-\text{F1}(-a)} da + C1 = 0 \right\}$$

2.808 ODE No. 808

$$y'(x) = \frac{(y(x) + 1)(2y(x) + 1)}{x(2xy(x) - 2y(x) + x - 2)}$$

✓ **Mathematica** : cpu = 1.76159 (sec), leaf count = 149

$$\text{Solve} \left[ \frac{2^{2/3} \left( x \log \left( -\frac{6 \cdot 2^{2/3}(y(x)+1)}{2(x-1)y(x)+x-2} \right) - x \log \left( \frac{3 \cdot 2^{2/3}(2xy(x)+x)}{2(x-1)y(x)+x-2} \right) + 2xy(x) \left( \log \left( -\frac{6 \cdot 2^{2/3}(y(x)+1)}{2(x-1)y(x)+x-2} \right) - \log \left( \frac{3 \cdot 2^{2/3}(2xy(x)+x)}{2(x-1)y(x)+x-2} \right) \right) \right)}{9(2xy(x) + x)} \right]$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 45

$$\left\{ y(x) = 1 \left( -x \text{lambertW} \left( \frac{1}{x e^{x-1} C1} \right) - 2 \right) \left( 2 x \text{lambertW} \left( \frac{1}{x e^{x-1} C1} \right) + 2 \right)^{-1} \right\}$$

2.809 ODE No. 809

$$y'(x) = \frac{64x^3 - 240x^2 + 64xy(x)^2 + 64y(x)^3 - 80y(x)^2 + 300x - 125}{(4x - 5)^3}$$

✓ **Mathematica** : cpu = 0.980723 (sec), leaf count = 118

$$\text{Solve} \left[ 57\text{RootSum} \left[ -19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log \left( \frac{12y(x)+4x-5}{\sqrt[3]{38} \sqrt[3]{\frac{1}{(5-4x)^6} (4x-5)^3} - \#1} \right)}{2\sqrt[3]{38} - 19\#1^2} \& \right] + 9c_1 + 38^{2/3} \left( \frac{1}{(5-4x)^6} \right)^2 \right]$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 41

$$\left\{ y(x) = -\frac{\text{RootOf}\left(-\int^{-Z}(-a^3 - a^2 - a - 1)^{-1} da + \ln(4x - 5) + C1\right)(4x - 5)}{4} \right\}$$

**2.810 ODE No. 810**

$$y'(x) = \frac{x^2 \log^2(x) + y(x)^2 + y(x) - 2xy(x) \log(x) + x}{x}$$

✓ **Mathematica** : cpu = 0.018715 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \frac{x((x - c_1) \log(x) - 1)}{x - c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 16

$$\left\{ y(x) = \left( \ln(x) + (C1 - x)^{-1} \right) x \right\}$$

**2.811 ODE No. 811**

$$y'(x) = \frac{x^4 + x^3 e^{y(x)} + xy(x) + e^{y(x)} y(x) - x \log(e^{y(x)} + x) - e^{y(x)} \log(e^{y(x)} + x) + x}{x^2}$$

✓ **Mathematica** : cpu = 2.29885 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow -\log\left(\frac{e^{-\frac{1}{2}x(2c_1+x^2)} - 1}{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 2.608 (sec), leaf count = 32

$$\left\{ y(x) = \frac{x^3}{2} + x C1 + \ln\left(-x\left(-1 + e^{\frac{x^3}{2}} e^{x-C1}\right)^{-1}\right) \right\}$$

**2.812 ODE No. 812**

$$y'(x) = x^3 \sqrt{x^3 - 6y(x)} + \sqrt{x^3 - 6y(x)} + \frac{x^2}{2} + x^2 \sqrt{x^3 - 6y(x)}$$

✓ **Mathematica** : cpu = 0.328669 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow \left( \frac{3c_1}{4} - 1 \right) x^4 + \left( c_1 + \frac{1}{6} \right) x^3 + 3c_1 x - \frac{3c_1^2}{2} - \frac{3x^8}{32} - \frac{x^7}{4} - \frac{x^6}{6} - \frac{3x^5}{4} - \frac{3x^2}{2} \right\} \right\}$$

✓ **Maple** : cpu = 0.483 (sec), leaf count = 32

$$\left\{ -\frac{1}{3} \sqrt{x^3 - 6y(x)} - \frac{x^4}{4} - \frac{x^3}{3} - x - \_C1 = 0 \right\}$$

**2.813 ODE No. 813**

$$y'(x) = \frac{1}{2} \sqrt{a} \left( 2\sqrt{ax^4 + 8y(x)} - \sqrt{a}x^3 + 2x^3 \sqrt{ax^4 + 8y(x)} + 2x^2 \sqrt{ax^4 + 8y(x)} \right)$$

✓ **Mathematica** : cpu = 0.495116 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{72} a \left( (87 - 72c_1) x^4 - 96c_1 x^3 - 288c_1 x + 144c_1^2 + 9x^8 + 24x^7 + 16x^6 + 72x^5 + 144x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.542 (sec), leaf count = 40

$$\left\{ \frac{1}{4} \sqrt{ax^4 + 8y(x)} + \frac{-3x^4 - 4x^3 - 12x}{12} \sqrt{a} - \_C1 = 0 \right\}$$

**2.814 ODE No. 814**

$$y'(x) = \frac{y(x) (x^7 y(x)^2 - 3x^3 y(x) - 3)}{x (x^3 y(x) + 1)}$$

✓ **Mathematica** : cpu = 0.115929 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow \frac{x}{\frac{\sqrt{x(c_1 - 2x + 1)}}{\sqrt{\frac{1}{x^7}}} - x^4} \right\}, \left\{ y(x) \rightarrow -\frac{x}{\frac{\sqrt{x(c_1 - 2x + 1)}}{\sqrt{\frac{1}{x^7}}} + x^4} \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 38

$$\left\{ y(x) = \frac{1}{x^3} \left( \sqrt{-C1 - 2x} - 1 \right)^{-1}, y(x) = -\frac{1}{x^3} \left( \sqrt{-C1 - 2x} + 1 \right)^{-1} \right\}$$

2.815 ODE No. 815

$$y'(x) = \frac{e^{3x^2} x(y(x) + 3)^3}{81 \left( e^{\frac{3x^2}{2}} y(x) + 3e^{\frac{3x^2}{2}} + 3y(x) \right)}$$

✗ **Mathematica** : cpu = 301.469 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.298 (sec), leaf count = 168

$$\left\{ -10 \ln \left( \frac{10 e^{3/2 x^2} (3 + y(x))}{27 e^{3/2 x^2} + 9 e^{3/2 x^2} y(x) + 27 y(x)} \right) + 5 \ln \left( \frac{100 (3 + y(x))^2 (e^{3/2 x^2})^2 + (-8100 (y(x))^2 - 24300 y(x))}{189 (e^{3/2 x^2} (3 + y(x)) + 3 y(x))} \right) \right\}$$

2.816 ODE No. 816

$$y'(x) = \frac{x(x - y(x))^3 (y(x) + x)^3}{y(x) (x^2 - y(x)^2 - 1)}$$

✓ **Mathematica** : cpu = 0.176433 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 1.025 (sec), leaf count = 190

$$\left\{ \int_{-b}^x \frac{(-a - y(x))^3 (-a + y(x))^3 - a}{-b - a^6 - 3 - a^4 (y(x))^2 + 3 - a^2 (y(x))^4 - (y(x))^6 - a^2 + (y(x))^2 + 1} d_a + \int^{y(x)} \frac{(-f^2 + 1)}{-f^6 + 3 - f^4 x^2 - 3} d_f \right\}$$

2.817 ODE No. 817

$$y'(x) = \frac{\csc(y(x)) \left( \frac{1}{2} x^3 \log(x) \cos(2y(x)) + \frac{1}{2} x^3 \log(x) - \cos(y(x)) \right)}{x \log(x)}$$

✓ **Mathematica** : cpu = 0.367238 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow -\sec^{-1} \left( -\frac{9c_1 + x^3 - 3x^3 \log(x)}{9 \log(x)} \right) \right\}, \left\{ y(x) \rightarrow \sec^{-1} \left( -\frac{9c_1 + x^3 - 3x^3 \log(x)}{9 \log(x)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.869 (sec), leaf count = 27

$$\left\{ y(x) = \arccos \left( 9 \frac{\ln(x)}{3x^3 \ln(x) - x^3 + 9 - C1} \right) \right\}$$

**2.818 ODE No. 818**

$$y'(x) = \frac{y(x)}{x(xy(x)^4 + xy(x)^3 + xy(x) - 1)}$$

✓ **Mathematica** : cpu = 0.0861051 (sec), leaf count = 34

$$\text{Solve}\left[\frac{y(x)^3}{3} + \frac{y(x)^2}{2} + \frac{1}{xy(x)} + \log(y(x)) = c_1, y(x)\right]$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 34

$$\left\{y(x) = e^{\text{RootOf}\left(-2x(e^{-Z})^4 - 3x(e^{-Z})^3 + 6x_{C1}e^{-Z} - 6_Zxe^{-Z} - 6\right)}\right\}$$

**2.819 ODE No. 819**

$$y'(x) = x^2\sqrt{x^2 + 3y(x)} + \sqrt{x^2 + 3y(x)} + x^3\sqrt{x^2 + 3y(x)} - \frac{2x}{3}$$

✓ **Mathematica** : cpu = 0.225381 (sec), leaf count = 63

$$\left\{\left\{y(x) \rightarrow \frac{1}{192}\left((96 - 72c_1)x^4 - 96c_1x^3 - 288c_1x + 144c_1^2 + 9x^8 + 24x^7 + 16x^6 + 72x^5 + 80x^2\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.258 (sec), leaf count = 30

$$\left\{-C1 + \frac{3x^4}{8} + \frac{x^3}{2} + \frac{3x}{2} - \sqrt{x^2 + 3y(x)} = 0\right\}$$

**2.820 ODE No. 820**

$$y'(x) = \frac{\csc(y(x))\left(\frac{1}{2}x^2\log(x)\cos(2y(x)) + \frac{1}{2}x^2\log(x) - \cos(y(x))\right)}{x\log(x)}$$

✓ **Mathematica** : cpu = 0.329933 (sec), leaf count = 59

$$\left\{\left\{y(x) \rightarrow -\sec^{-1}\left(-\frac{4c_1 + x^2 - 2x^2\log(x)}{4\log(x)}\right)\right\}, \left\{y(x) \rightarrow \sec^{-1}\left(-\frac{4c_1 + x^2 - 2x^2\log(x)}{4\log(x)}\right)\right\}\right\}$$

✓ **Maple** : cpu = 0.864 (sec), leaf count = 27

$$\left\{y(x) = \arccos\left(4\frac{\ln(x)}{2x^2\ln(x) - x^2 + 4_{C1}}\right)\right\}$$

**2.821 ODE No. 821**

$$y'(x) = \frac{y(x)(xy(x) + 1)}{x(x^3y(x)^4 - xy(x) - 1)}$$

✓ **Mathematica** : cpu = 0.156574 (sec), leaf count = 1993

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{4} - \frac{1}{2} \sqrt{\frac{c_1^2}{4} + \frac{\sqrt[3]{36c_1^2x^6 + 27x^5 + \sqrt{x^9(216x^3(6c_1 - 1)c_1^3 + 216x^2c_1^2 - 9x(512c_1 - 81) - 4096)}}{6x^3}}}{4} \right\} \right.$$

✓ **Maple** : cpu = 0.173 (sec), leaf count = 25

$$\left\{ \frac{1}{2x^2(y(x))^2} + \frac{1}{3x^3(y(x))^3} + y(x) + \_C1 = 0 \right\}$$

**2.822 ODE No. 822**

$$y'(x) = \frac{1}{4}x(-4e^{-x^2}x^2y(x) - 4e^{-x^2}x^2 + 4e^{-x^2} + e^{-2x^2}x^4 + 4y(x)^2)$$

✓ **Mathematica** : cpu = 0.0416643 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \frac{x^2}{2}} + \frac{1}{2}e^{-x^2}x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.144 (sec), leaf count = 25

$$\left\{ y(x) = \frac{x^2e^{-x^2}}{2} + \left(-C1 - \frac{x^2}{2}\right)^{-1} \right\}$$

**2.823 ODE No. 823**

$$y'(x) = \frac{y(x)(y(x) + x)}{x(y(x)^4 + y(x)^3 + y(x) + x)}$$

✓ **Mathematica** : cpu = 0.452093 (sec), leaf count = 35

$$\text{Solve} \left[ \frac{y(x)^3}{3} + \frac{y(x)^2}{2} + \log(y(x)) = c_1 + \frac{x}{y(x)} + \log(x), y(x) \right]$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 38

$$\left\{ y(x) = e^{\text{RootOf}(-2(e^{-Z})^4 - 3(e^{-Z})^3 + 6e^{-Z} \ln(x) + 6\_C1 e^{-Z} - 6\_Z e^{-Z} + 6x)} \right\}$$

**2.824 ODE No. 824**

$$y'(x) = \frac{y(x)(x^3 + x^2y(x) + y(x)^2)}{(x-1)x^2(y(x) + x)}$$

✓ **Mathematica** : cpu = 0.333558 (sec), leaf count = 66

$$\text{Solve} \left[ \log\left(\frac{y(x)}{x}\right) + \frac{\tan^{-1}\left(\frac{2y(x)+x}{\sqrt{3}x}\right)}{\sqrt{3}} + \log(x) = c_1 + \frac{1}{2} \log\left(\frac{x^2 + xy(x) + y(x)^2}{x^2}\right) + \log(1-x), y(x) \right]$$

✓ **Maple** : cpu = 0.512 (sec), leaf count = 61

$$\left\{ -\frac{1}{2} \ln\left(\frac{(y(x))^2 + xy(x) + x^2}{x^2}\right) + \frac{\sqrt{3}}{3} \arctan\left(\frac{(x + 2y(x))\sqrt{3}}{3x}\right) + \ln\left(\frac{y(x)}{x}\right) - \ln(x-1) + \ln(x) - \_C1 = 0 \right\}$$

**2.825 ODE No. 825**

$$y'(x) = \frac{x(x^2y(x)^3 + (x^2 + 1)^{3/2}y(x)^2 + x^2(x^2 + 1)^{3/2} + (x^2 + 1)^{3/2} + y(x)^3)}{(x^2 + 1)^3}$$

✓ **Mathematica** : cpu = 7.46348 (sec), leaf count = 144



$$\text{Solve} \left[ -\frac{19}{3} \text{RootSum} \left[ -19\#1^3 + 6\sqrt[3]{38}\#1 - 19\&, \frac{\log \left( \frac{x \left( \frac{3y(x)}{(x^2+1)^2} + \frac{1}{(x^2+1)^{3/2}} \right) - \#1}{\sqrt[3]{38} \sqrt[3]{\frac{x^3}{(x^2+1)^{9/2}}}} \right)}{2\sqrt[3]{38} - 19\#1^2} \& \right] = \frac{1}{18} \left( 18c_1 + \frac{38^{2/3}}{\sqrt{x^2+1}} \right) \right]$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 48

$$\left\{ y(x) = \frac{19 \text{RootOf} \left( -1296 \int^{-Z} (361\_a^3 - 432\_a + 432)^{-1} d\_a + 2 \ln(x^2 + 1) + 3\_C1 \right) - 6}{18} \sqrt{x^2 + 1} \right\}$$

## 2.826 ODE No. 826

$$y'(x) = \frac{y(x) (3xy(x)^2 + 3y(x)^2 + x)}{x(x+1) (6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.605153 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W \left( \frac{6e^{2c_1} x}{(x+1)^2} \right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x} \sqrt{W \left( \frac{6e^{2c_1} x}{(x+1)^2} \right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.401 (sec), leaf count = 51

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\text{RootOf} \left( -e^{-Z} \ln \left( \frac{(1+x)^2 (e^{-Z} + 9)}{2x} \right) + 3\_C1 e^{-Z} + \_Z e^{-Z} + 9 \right)} + 9 \right) \right\}$$

**2.827 ODE No. 827**

$$y'(x) = \frac{x^2 y(x) \sqrt{x^2 + y(x)^2} + x^3 \left( -\sqrt{x^2 + y(x)^2} \right) + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.121895 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( -2e^{\frac{1}{3}\sqrt{2}(3c_1+x^3)} + e^{\frac{2}{3}\sqrt{2}(3c_1+x^3)} - 1 \right)}{2e^{\frac{1}{3}\sqrt{2}(3c_1+x^3)} + e^{\frac{2}{3}\sqrt{2}(3c_1+x^3)} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 49

$$\left\{ \ln \left( 2 \frac{x \left( \sqrt{2(y(x))^2 + 2x^2 + y(x) + x} \right)}{y(x) - x} \right) + \frac{\sqrt{2}x^3}{3} - \ln(x) - C1 = 0 \right\}$$

**2.828 ODE No. 828**

$$y'(x) = \frac{(y(x) + 1)(2y(x) + 1)}{x(2xy(x)^4 + xy(x)^3 - 2y(x) - 2)}$$

✓ **Mathematica** : cpu = 0.384254 (sec), leaf count = 46

$$\text{Solve} \left[ \frac{1}{16} \left( -2y(x)^2 + 6y(x) - \frac{8}{2xy(x) + x} - 8 \log(y(x) + 1) + \log(2y(x) + 1) \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.353 (sec), leaf count = 54

$$\left\{ y(x) = \frac{1}{2} e^{\text{RootOf}(x(e^{-Z})^3 - 8x(e^{-Z})^2 + 16 \ln(1/2 e^{-Z} + 1/2) x e^{-Z} + 8x - C1 e^{-Z} - 2 - Z x e^{-Z} + 7 x e^{-Z} + 16)} - \frac{1}{2} \right\}$$

**2.829 ODE No. 829**

$$y'(x) = \frac{x^6 \sqrt{4x^2 y(x) + 1} + x^5 \sqrt{4x^2 y(x) + 1} + x^3 \sqrt{4x^2 y(x) + 1} + \frac{1}{2}}{x^3}$$

✓ **Mathematica** : cpu = 0.383536 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow -\frac{2c_1 x^5}{5} - \frac{1}{4}(2c_1 - 1)x^4 - c_1 x^2 + c_1^2 + \frac{x^{10}}{25} + \frac{x^9}{10} + \frac{x^8}{16} + \frac{x^7}{5} + \frac{x^6}{4} - \frac{1}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.491 (sec), leaf count = 40

$$\left\{ -C1 + \frac{1}{x} \left( -4x^6 - 5x^5 - 10x^3 + 10 \sqrt{4x^2 y(x) + 1} \right) = 0 \right\}$$

**2.830 ODE No. 830**

$$y'(x) = \frac{(x - y(x))y(x)}{x(-y(x)^4 - y(x)^3 - y(x) + x)}$$

✓ **Mathematica** : cpu = 0.52528 (sec), leaf count = 34

$$\text{Solve} \left[ \log(x) = c_1 + \frac{y(x)^3}{3} + \frac{y(x)^2}{2} + \frac{x}{y(x)} + \log(y(x)), y(x) \right]$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 38

$$\left\{ y(x) = e^{\text{RootOf}(2(e^{-Z})^4 + 3(e^{-Z})^3 - 6e^{-Z} \ln(x) + 6_{-C1}e^{-Z} + 6_{-Z}e^{-Z} + 6x)} \right\}$$

**2.831 ODE No. 831**

$$y'(x) = \frac{x^3 \sqrt{4ax - y(x)^2} + x^2 \sqrt{4ax - y(x)^2} + \sqrt{4ax - y(x)^2} + 2a}{y(x)}$$

✓ **Mathematica** : cpu = 4.46135 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{12} \sqrt{576ax - (12c_1 + 3x^4 + 4x^3 + 12x)^2} \right\}, \left\{ y(x) \rightarrow \frac{1}{12} \sqrt{576ax - (12c_1 + 3x^4 + 4x^3 + 12x)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.376 (sec), leaf count = 35

$$\left\{ -\sqrt{-(y(x))^2 + 4ax} - \frac{x^4}{4} - \frac{x^3}{3} - x - _{C1} = 0 \right\}$$

**2.832 ODE No. 832**

$$y'(x) = \frac{y(x)(y(x) + x + 1)}{(x + 1)(y(x)^4 + y(x)^3 + y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 3.26419 (sec), leaf count = 2323

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} \left( -\sqrt{\frac{16(-8x + 3c_1 + 3 \log(x + 1) + \sqrt{36(c_1 + \log(x + 1))^2 + 18(c_1 + \log(x + 1)) + 69x + \sqrt{36(c_1 + \log(x + 1))^2 + 18(c_1 + \log(x + 1))})}{\sqrt{36(c_1 + \log(x + 1))^2 + 18(c_1 + \log(x + 1)) + 69x + \sqrt{36(c_1 + \log(x + 1))^2 + 18(c_1 + \log(x + 1))})}} \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 31

$$\left\{ \ln(1 + x) + \frac{x}{y(x)} - \frac{(y(x))^3}{3} - \frac{(y(x))^2}{2} - y(x) + _{C1} = 0 \right\}$$

**2.833 ODE No. 833**

$$y'(x) = \frac{x^4 \left( -\sqrt{x^2 + y(x)^2} \right) + x^3 y(x) \sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.127492 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( -2e^{\frac{4c_1+x^4}{2\sqrt{2}}} + e^{\frac{4c_1+x^4}{\sqrt{2}}} - 1 \right)}{2e^{\frac{4c_1+x^4}{2\sqrt{2}}} + e^{\frac{4c_1+x^4}{\sqrt{2}}} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.22 (sec), leaf count = 49

$$\left\{ \ln \left( 2 \frac{x \left( \sqrt{2} (y(x))^2 + 2x^2 + y(x) + x \right)}{y(x) - x} \right) + \frac{\sqrt{2}x^4}{4} - \ln(x) - C1 = 0 \right\}$$

**2.834 ODE No. 834**

$$y'(x) = \frac{y(x) (x^4 + 3xy(x)^2 + 3y(x)^2)}{x(x+1) (6y(x)^2 + x)}$$

✓ **Mathematica** : cpu = 0.741366 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{x} \sqrt{W \left( \frac{6(x+1)^2 e^{2c_1+x^2-2x-3}}{x} \right)}}{\sqrt{6}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{x} \sqrt{W \left( \frac{6(x+1)^2 e^{2c_1+x^2-2x-3}}{x} \right)}}{\sqrt{6}} \right\} \right\}$$

✓ **Maple** : cpu = 0.46 (sec), leaf count = 60

$$\left\{ \left( (y(x))^{-2} + 6x^{-1} \right)^{-1} = \frac{x}{54} \left( e^{\text{RootOf} \left( x^2 e^{-Z} - e^{-Z} \ln \left( \frac{(e^{-Z}+9)x}{2(1+x)^2} \right) + 3 - C1 e^{-Z} + Z e^{-Z} - 2x e^{-Z} + 9 \right)} + 9 \right) \right\}$$

**2.835 ODE No. 835**

$$y'(x) = -\frac{1}{x \left( -\sqrt[3]{y(x)^3} \right) \_F1(y(x)^3 - 3 \log(x)) - x (y(x)^3)^{2/3}}$$

✗ **Mathematica** : cpu = 2.59853 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == -(-(x*(y[x]^3)^(2/3)) - x*(y[x]^3)^(1/3)*_F1[-3*Log[x] + y[x]^3]), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x), x) = -1/(-(y(x)^3)^(2/3)*x-_F1(y(x)^3-3*ln(x))*(y(x)^3)^(1/3)*x), y(x))`

**2.836 ODE No. 836**

$$y'(x) = \frac{(x - y(x))y(x)(y(x) + 1)}{x(xy(x) - y(x) + x)}$$

✓ **Mathematica** : cpu = 16.2258 (sec), leaf count = 379

$$\text{Solve} \left[ \frac{1}{9} 2^{2/3} \left( \frac{\left( 1 - \frac{(x-1)^2 \left( \frac{x^6}{(x-1)^3} \right)^{2/3} ((x+2)y(x)+x)}{x^4((x-1)y(x)+x)} \right) \left( \frac{\left( \frac{x^6}{(x-1)^3} \right)^{2/3} (x-1)^2 ((x+2)y(x)+x)}{x^4((x-1)y(x)+x)} + 2 \right) \left( \left( 1 - \frac{(x-1)^2 \left( \frac{x^6}{(x-1)^3} \right)^{2/3} ((x+2)y(x)+x)}{x^4((x-1)y(x)+x)} \right) \right)}{\right. \right]$$

✓ **Maple** : cpu = 0.261 (sec), leaf count = 73

$$\left\{ y(x) = -xe^{\text{RootOf}\left(-\ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right)e^{-Z} + 3\_C1 e^{-Z} +\_Z e^{-Z} - xe^{-Z} + 9\right)} \left( -9 + (x-1) e^{\text{RootOf}\left(-\ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right)e^{-Z} + 3\_C1 e^{-Z} +\_Z e^{-Z} - xe^{-Z} + 9\right)} \right) \right.$$

**2.837 ODE No. 837**

$$y'(x) = -\frac{1}{-\sqrt[3]{y(x)^3} \log(x) \_F1(3\text{Ei}(-\log(x)) + y(x)^3) - (y(x)^3)^{2/3} \log(x)}$$

✗ **Mathematica** : cpu = 3.16435 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == -(-(Log[x]*(y[x]^3)^(2/3)) - Log[x]*(y[x]^3)^(1/3)*_F1[3*ExpIntegralEi[-Log[x]] + y[x]^3])^(-1), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x), x) = -1/(-ln(x)*(y(x)^3)^(2/3)-_F1(y(x)^3+3*Ei(1,-ln(x)))*ln(x)*(y(x)^3)^(1/3)), y(x))`

**2.838 ODE No. 838**

$$y'(x) = \frac{\frac{8x^{7/2}}{5} + \frac{4x^6}{25} - \frac{4}{5}x^3y(x) + \frac{6x^3}{5} - 4\sqrt{x}y(x) + y(x)^2 + 4x + \sqrt{x}}{x}$$

✓ **Mathematica** : cpu = 0.0284624 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \log(x)} + \frac{2x^3}{5} + 2\sqrt{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 25

$$\left\{ y(x) = \frac{2x}{5} \left( x^2 + 5 \frac{1}{\sqrt{x}} \right) + (_C1 - \ln(x))^{-1} \right\}$$

**2.839 ODE No. 839**

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left( x^2 + xe^{-\frac{y(x)}{x}} + e^{-\frac{y(x)}{x}} y(x) \right)}{x}$$

✓ **Mathematica** : cpu = 0.0719576 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow -x \log \left( \frac{e^{2c_1} - x^2}{2x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 19

$$\left\{ y(x) = \ln \left( 2 \frac{x}{-x^2 + _C1} \right) x \right\}$$

**2.840 ODE No. 840**

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left( x^3 + xe^{-\frac{y(x)}{x}} + e^{-\frac{y(x)}{x}} y(x) \right)}{x}$$

✓ **Mathematica** : cpu = 0.0879489 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow -x \log \left( \frac{e^{3c_1} - x^3}{3x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 19

$$\left\{ y(x) = \ln \left( 3 \frac{x}{-x^3 + _C1} \right) x \right\}$$

**2.841 ODE No. 841**

$$y'(x) = \frac{-2a^{3/2}bx^2y(x)^2 + 2a^{3/2}cy(x)^2 + a^{5/2}y(x)^4 + \sqrt{ab^2x^4 - 2\sqrt{abc}x^2 + \sqrt{ac^2 + bx^3}}}{ax^2y(x)}$$

✓ **Mathematica** : cpu = 1.35593 (sec), leaf count = 208

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-2a^{5/2}(c - bx^2) + 4a^3bx(bx^2 - c) + a^2x + 4\sqrt{abc}c_1(bx^2 - c) + 2bc_1x}}{\sqrt{2}\sqrt{2a^{3/2}bc_1 + a^{7/2} + 2a^4bx}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-2a^{5/2}(c - bx^2) + 4a^3bx(bx^2 - c) + a^2x + 4\sqrt{abc}c_1(bx^2 - c) + 2bc_1x}}{\sqrt{2}\sqrt{2a^{3/2}bc_1 + a^{7/2} + 2a^4bx}} \right\} \right\}$$

✓ **Maple** : cpu = 0.378 (sec), leaf count = 97

$$\left\{ y(x) = \frac{1}{x\_C1 + 1} \sqrt{(x\_C1 + 1) \left( (x\_C1 + 1) (bx^2 - c) \sqrt{a + \frac{x}{2}} \right) a^{\frac{3}{2}} a^{-\frac{3}{2}}}, y(x) = -2 \frac{\sqrt{(x\_C1 + 1) \left( (x\_C1 + 1) (bx^2 - c) \sqrt{a + \frac{x}{2}} \right) a^{\frac{3}{2}} a^{-\frac{3}{2}}}}{a^{3/2} (2x + 1)} \right\}$$

**2.842 ODE No. 842**

$$y'(x) = \frac{2x^2y(x) \log^2(x) + x^2y(x)^2 \log(x) + x^2 \log^3(x) + y(x)}{x \log(x)}$$

✓ **Mathematica** : cpu = 0.118365 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{\log(x) (-4(c_1 + 1) + x^2 - 2x^2 \log(x))}{4c_1 - x^2 + 2x^2 \log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 43

$$\left\{ y(x) = -\frac{\ln(x) (2x^2 \ln(x) - x^2 + 2\_C1 + 4)}{2x^2 \ln(x) - x^2 + 2\_C1} \right\}$$

**2.843 ODE No. 843**

$$y'(x) = \frac{2x^3y(x) \log^2(x) + x^3y(x)^2 \log(x) + x^3 \log^3(x) + y(x)}{x \log(x)}$$

✓ **Mathematica** : cpu = 0.124244 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{\log(x) (-9(c_1 + 1) + x^3 - 3x^3 \log(x))}{9c_1 - x^3 + 3x^3 \log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 43

$$\left\{ y(x) = -\frac{\ln(x) (6x^3 \ln(x) - 2x^3 + 9\_C1 + 18)}{6x^3 \ln(x) - 2x^3 + 9\_C1} \right\}$$

2.844 ODE No. 844

$$y'(x) = \frac{y(x)(y(x) + 1)(y(x) + x)}{x(xy(x) + y(x) + x)}$$

✓ **Mathematica** : cpu = 19.4085 (sec), leaf count = 386

$$\text{Solve} \left[ \frac{2^{2/3} \left( 1 - \frac{\left(\frac{x^6}{(x+1)^3}\right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4((x+1)y(x)+x)} \right) \left( \frac{\left(\frac{x^6}{(x+1)^3}\right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4((x+1)y(x)+x)} + 2 \right) \left( \left( 1 - \frac{\left(\frac{x^6}{(x+1)^3}\right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4((x+1)y(x)+x)} \right)^{3/2} \right)}{9 \left( \frac{3 \left( \left( 1 - \frac{\left(\frac{x^6}{(x+1)^3}\right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4((x+1)y(x)+x)} \right)^{3/2} \right)^{2/3}}{\left( 1 - \frac{\left(\frac{x^6}{(x+1)^3}\right)^{2/3} (x+1)^2 ((x-2)y(x)+x)}{x^4((x+1)y(x)+x)} \right)} \right)^{2/3}} \right]$$

✓ **Maple** : cpu = 0.286 (sec), leaf count = 97

$$\left\{ y(x) = -x e^{\text{RootOf}\left(-\ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) e^{-Z} + 3 - C1 e^{-Z} - Z e^{-Z} + x e^{-Z} + 9\right)} \left( e^{\text{RootOf}\left(-\ln\left(\frac{e^{-Z}}{2} + \frac{9}{2}\right) e^{-Z} + 3 - C1 e^{-Z} - Z e^{-Z} + x e^{-Z} + 9\right)} \right) \right\}$$

2.845 ODE No. 845

$$y'(x) = \frac{\sqrt{4y(x)^3 - 9x^4} + 3x^3 + x^3 \sqrt{4y(x)^3 - 9x^4} + x^2 \sqrt{4y(x)^3 - 9x^4}}{y(x)^2}$$

✓ **Mathematica** : cpu = 4.97511 (sec), leaf count = 218

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt[3]{-\frac{1}{2} \sqrt[3]{12(6c_1 + 11)x^4 + 96c_1x^3 + 288c_1x + 144c_1^2 + 9x^8 + 24x^7 + 16x^6 + 72x^5 + 144x^2}} \right\}, \left\{ y(x) \rightarrow \dots \right\} \right\}$$

✓ **Maple** : cpu = 0.301 (sec), leaf count = 44

$$\left\{ \int_{-b}^{y(x)} -a^2 \frac{1}{\sqrt{-9x^4 + 4a^3}} dx - \frac{x^4}{4} - \frac{x^3}{3} - x - C1 = 0 \right\}$$



**2.846 ODE No. 846**

$$y'(x) = \frac{1}{x^2 \left( -\left(\frac{1}{y(x)} + 1\right) \right) \_F1 \left( x \left( \frac{1}{y(x)} + 1 \right) \right) + x^2 \_F1 \left( x \left( \frac{1}{y(x)} + 1 \right) \right) + x \left( \frac{1}{y(x)} + 1 \right) - x}$$

✓ **Mathematica** : cpu = 1.40407 (sec), leaf count = 174

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{x \_F1 \left( x \left( \frac{1}{K[2]} + 1 \right) \right) - 1}{x(K[2] + 1) \_F1 \left( x \left( \frac{1}{K[2]} + 1 \right) \right) - K[2]} - \int_1^x \frac{K[1](K[2] + 1) \_F1' \left( K[1] \left( \frac{1}{K[2]} + 1 \right) \right) + K[2]}{K[2] \left( K[2] - K[1](K[2] + 1) \_F1 \left( K[1] \left( \frac{1}{K[2]} + 1 \right) \right) \right)} dK[1] \right) dK[2] + \dots \right]$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 40

$$\left\{ y(x) = e^{\text{RootOf} \left( -Z - \int \frac{x e^{-Z}}{e^{-Z} - 1} \frac{1}{(\_F1(\_a) \_a - 1) \_a} d\_a + \_C1 \right) - 1} \right\}$$

**2.847 ODE No. 847**

$$y'(x) = x^2 \sqrt{x^2 - 4y(x) + 2x + 1} + \sqrt{x^2 - 4y(x) + 2x + 1} + x^3 \sqrt{x^2 - 4y(x) + 2x + 1} + \frac{x}{2} + \frac{1}{2}$$

✓ **Mathematica** : cpu = 0.357435 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{144} (24(3c_1 - 4)x^4 + 96c_1x^3 + 72(4c_1 + 1)x - 144c_1^2 - 9x^8 - 24x^7 - 16x^6 - 72x^5 - 108x^2 + 36) \right\} \right\}$$

✓ **Maple** : cpu = 0.323 (sec), leaf count = 34

$$\left\{ \_C1 - \frac{x^4}{2} - \frac{2x^3}{3} - 2x - \sqrt{x^2 + 2x + 1 - 4y(x)} = 0 \right\}$$

**2.848 ODE No. 848**

$$y'(x) = \_F1(y(x) - \log(\sinh(x))) + \coth(x)$$

✓ **Mathematica** : cpu = 0.124532 (sec), leaf count = 91

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{1}{\_F1(K[2] - \log(\sinh(x)))} - \int_1^x \frac{\coth(K[1]) \_F1'(K[2] - \log(\sinh(K[1])))}{(\_F1(K[2] - \log(\sinh(K[1])))^2} dK[1] \right) dK[2] + \dots \right]$$

✓ **Maple** : cpu = 0.741 (sec), leaf count = 27

$$\left\{ \int_{\_b}^{y(x)} (\_F1(\_a - \ln(\sinh(x))))^{-1} d\_a - x - \_C1 = 0 \right\}$$

**2.849 ODE No. 849**

$$y'(x) = x^2 \sqrt{x^2 + 4y(x) - 4x} + \sqrt{x^2 + 4y(x) - 4x} + x^3 \sqrt{x^2 + 4y(x) - 4x} - \frac{x}{2} + 1$$

✓ **Mathematica** : cpu = 0.31874 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{6}(3c_1 - 4)x^4 - \frac{2c_1x^3}{3} - 2c_1x + c_1^2 + \frac{x^8}{16} + \frac{x^7}{6} + \frac{x^6}{9} + \frac{x^5}{2} + \frac{3x^2}{4} + x \right\} \right\}$$

✓ **Maple** : cpu = 0.302 (sec), leaf count = 33

$$\left\{ -C1 + \frac{x^4}{2} + \frac{2x^3}{3} + 2x - \sqrt{x^2 - 4x + 4y(x)} = 0 \right\}$$

**2.850 ODE No. 850**

$$y'(x) = \_F1(y(x) - \log(\sin(x)) + \log(\cos(x) + 1)) + \csc(x)$$

✓ **Mathematica** : cpu = 0.252632 (sec), leaf count = 114

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{1}{\_F1(K[2] - \log(\sin(x)) + \log(\cos(x) + 1))} - \int_1^x \frac{\csc(K[1])\_F1'(K[2] - \log(\sin(K[1])) + \log(\cos(K[1]))}{(\_F1(K[2] - \log(\sin(K[1])) + \log(\cos(K[1])))} dx \right) dx \right]$$

✓ **Maple** : cpu = 1.39 (sec), leaf count = 32

$$\left\{ \int_{-b}^{y(x)} (\_F1(\_a - \ln(\sin(x)) + \ln(\cos(x) + 1)))^{-1} d\_a - x - \_C1 = 0 \right\}$$

**2.851 ODE No. 851**

$$y'(x) = \frac{a^3x^3 + 3a^2bx^2y(x) + a^2bx^2 + 3ab^2xy(x)^2 + 2ab^2xy(x) + b^3y(x)^3 + b^3y(x)^2 + b^3}{b^3}$$

✓ **Mathematica** : cpu = 0.485403 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.076 (sec), leaf count = 42

$$\left\{ y(x) = \frac{\text{RootOf}\left(\int^{-Z} (\_a^3b + \_a^2b + a + b)^{-1} d\_ab - x + \_C1\right) b - ax}{b} \right\}$$

**2.852 ODE No. 852**

$$y'(x) = \frac{\alpha^3 y(x)^3 + \alpha^3 y(x)^2 + \alpha^3 + 3\alpha^2 \beta x y(x)^2 + 2\alpha^2 \beta x y(x) + 3\alpha \beta^2 x^2 y(x) + \alpha \beta^2 x^2 + \beta^3 x^3}{\alpha^3}$$

✓ **Mathematica** : cpu = 0.534372 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.079 (sec), leaf count = 42

$$\left\{ y(x) = \frac{\text{RootOf}\left(\int^{-Z} (_a^3 \alpha + _a^2 \alpha + \alpha + \beta)^{-1} d_a \alpha - x + _C1\right) \alpha - \beta x}{\alpha} \right\}$$

**2.853 ODE No. 853**

$$y'(x) = \frac{x^3 y(x)^3 + 6x^2 y(x)^2 + 14x y(x) + 2x + 12}{x^2 (x y(x) + x + 2)}$$

✓ **Mathematica** : cpu = 0.0186209 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{-2\sqrt{c_1 - 2x} + x + 2}{x(\sqrt{c_1 - 2x} - 1)} \right\}, \left\{ y(x) \rightarrow -\frac{2\sqrt{c_1 - 2x} + x + 2}{x\sqrt{c_1 - 2x} + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 63

$$\left\{ y(x) = \frac{1}{x} \left( -2\sqrt{-C1 - 2x} - x - 2 \right) \left( \sqrt{-C1 - 2x} + 1 \right)^{-1}, y(x) = \frac{1}{x} \left( -2\sqrt{-C1 - 2x} + x + 2 \right) \left( \sqrt{-C1 - 2x} \right)^{-1} \right\}$$

**2.854 ODE No. 854**

$$y'(x) = \frac{y(x) (x^2 \log^2(y(x)) + 2x^2 \log(x) \log(y(x)) + x^2 \log^2(x) + \log(y(x)) + \log(x) - 1)}{x}$$

✗ **Mathematica** : cpu = 0.636945 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == ((-1 + Log[x] + x^2\*Log[x]^2 + Log[y[x]] + 2\*x^2\*Log[x]\*Log[y[x]]

✓ **Maple** : cpu = 0.277 (sec), leaf count = 51

$$\left\{ y(x) = 1 \left( x^{\frac{x^3}{x^3+3-C1}} \right)^{-1} \left( x^{\frac{-C1}{x^3+3-C1}} \right)^{-3} \left( e^{\frac{x}{x^3+3-C1}} \right)^{-3} \right\}$$

**2.855 ODE No. 855**

$$y'(x) = \frac{y(x) (x^3 \log^2(y(x)) + 2x^3 \log(x) \log(y(x)) + x^3 \log^2(x) + \log(y(x)) + \log(x) - 1)}{x}$$

✗ **Mathematica** : cpu = 0.684774 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == ((-1 + Log[x] + x^3\*Log[x]^2 + Log[y[x]] + 2\*x^3\*Log[x]\*Log[y[x]]

✓ **Maple** : cpu = 0.271 (sec), leaf count = 51

$$\left\{ y(x) = 1 \left( x^{\frac{x^4}{x^4+4}-C1} \right)^{-1} \left( x^{\frac{-C1}{x^4+4}-C1} \right)^{-4} \left( e^{\frac{x}{x^4+4}-C1} \right)^{-4} \right\}$$

**2.856 ODE No. 856**

$$y'(x) = -\frac{x(-F1(y(x)^2 - 2x) - \frac{1}{x})}{\sqrt{y(x)^2}}$$

✓ **Mathematica** : cpu = 0.933042 (sec), leaf count = 100

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{\sqrt{K[2]^2}}{-F1(K[2]^2 - 2x)} - \int_1^x \frac{2K[2]_F1'(K[2]^2 - 2K[1])}{(-F1(K[2]^2 - 2K[1]))^2} dK[1] \right) dK[2] + \int_1^x \left( -\frac{1}{-F1(y(x)^2 - 2x)} \right) dx \right]$$

✓ **Maple** : cpu = 0.359 (sec), leaf count = 65

$$\left\{ y(x) = \sqrt{2 \text{RootOf} \left( x^2 - 2 \int^{-Z} (-F1(2_a))^{-1} d_a + 4_C1 \right) + 2x}, y(x) = -\sqrt{2 \text{RootOf} \left( x^2 - 2 \int^{-Z} (-F1(2_a))^{-1} d_a + 4_C1 \right) + 2x} \right\}$$

**2.857 ODE No. 857**

$$y'(x) = x^2 \sqrt{x^2 + 8y(x) - 2x + 1} + \sqrt{x^2 + 8y(x) - 2x + 1} + x^3 \sqrt{x^2 + 8y(x) - 2x + 1} - \frac{x}{4} + \frac{1}{4}$$

✓ **Mathematica** : cpu = 0.341675 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow \left( \frac{4}{3} - c_1 \right) x^4 - \frac{4c_1 x^3}{3} + \left( \frac{1}{4} - 4c_1 \right) x + 2c_1^2 + \frac{x^8}{8} + \frac{x^7}{3} + \frac{2x^6}{9} + x^5 + \frac{15x^2}{8} - \frac{1}{8} \right\} \right\}$$

✓ **Maple** : cpu = 0.315 (sec), leaf count = 32

$$\left\{ -C1 + x^4 + \frac{4x^3}{3} + 4x - \sqrt{x^2 - 2x + 1 + 8y(x)} = 0 \right\}$$

**2.858 ODE No. 858**

$$y'(x) = \frac{a^3 y(x)^3 + a^3 y(x)^2 + a^3 + 3a^2 b x y(x)^2 + 2a^2 b x y(x) + 3ab^2 x^2 y(x) + ab^2 x^2 + b^3 x^3}{a^3}$$

✓ **Mathematica** : cpu = 0.509249 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.075 (sec), leaf count = 42

$$\left\{ y(x) = \frac{\text{RootOf}\left(\int^{-Z} (a\_a^3 + a\_a^2 + a + b)^{-1} d\_a a - x + \_C1\right) a - b x}{a} \right\}$$

**2.859 ODE No. 859**

$$y'(x) = \frac{-F1(y(x)^2 - 2x) + x}{x \sqrt{y(x)^2}}$$

✓ **Mathematica** : cpu = 1.32325 (sec), leaf count = 102

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( \frac{\sqrt{K[2]^2}}{-F1(K[2]^2 - 2x)} - \int_1^x \frac{2K[2]_F1'(K[2]^2 - 2K[1])}{(-F1(K[2]^2 - 2K[1]))^2} dK[1] \right) dK[2] + \int_1^x \left( -\frac{1}{-F1(y(x)^2 - 2x)} \right) dx \right]$$

✓ **Maple** : cpu = 0.355 (sec), leaf count = 63

$$\left\{ y(x) = \sqrt{2 \text{RootOf}\left(\ln(x) - \int^{-Z} (-F1(2\_a))^{-1} d\_a + 2\_C1\right)} + 2x, y(x) = -\sqrt{2 \text{RootOf}\left(\ln(x) - \int^{-Z} (-F1(2\_a))^{-1} d\_a + 2\_C1\right)} - 2x \right\}$$

**2.860 ODE No. 860**

$$y'(x) = \frac{\frac{1}{2}x^4 \cos(2y(x)) + \frac{x^4}{2} + \frac{1}{2}x^3 \cos(2y(x)) + \frac{x^3}{2} - \frac{1}{2} \sin(2y(x)) + \frac{1}{2}x \cos(2y(x)) + \frac{x}{2}}{x}$$

✓ **Mathematica** : cpu = 0.147787 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \tan^{-1} \left( \frac{c_1}{2x} + \frac{x^4}{5} + \frac{x^3}{4} + \frac{x}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 2.612 (sec), leaf count = 29

$$\left\{ y(x) = \arctan \left( \frac{4x^5 + 5x^4 + 10x^2 + 40\_C1}{20x} \right) \right\}$$

**2.861 ODE No. 861**

$$y'(x) = -\frac{e^{-1/x} \left( -\text{F1} \left( e^{\frac{1}{x}} y(x) \right) - \frac{e^{\frac{1}{x}} y(x)}{x} \right)}{x}$$

✓ **Mathematica** : cpu = 1.84134 (sec), leaf count = 137

$$\text{Solve} \left[ c_1 = \int_1^{y(x)} \left( - \int_1^x \frac{e^{\frac{1}{K[1]}} \left( -\text{F1} \left( e^{\frac{1}{K[1]}} K[2] \right) - e^{\frac{1}{K[1]}} K[2] \text{F1}' \left( e^{\frac{1}{K[1]}} K[2] \right) \right)}{K[1]^2 \left( -\text{F1} \left( e^{\frac{1}{K[1]}} K[2] \right) \right)^2} dK[1] - \frac{e^{\frac{1}{x}}}{-\text{F1} \left( e^{\frac{1}{x}} K[2] \right)} \right) dK[2] \right]$$

✓ **Maple** : cpu = 0.205 (sec), leaf count = 26

$$\left\{ y(x) = \frac{\text{RootOf} \left( -\ln(x) + \int^{-Z} (-\text{F1}(\_a))^{-1} d\_a + \_C1 \right)}{e^{x^{-1}}} \right\}$$

**2.862 ODE No. 862**

$$y'(x) = -\log(y(x) - 1) \left( \frac{\text{Ei}(-\log(y(x) - 1))}{x} - \text{F1}(x) \right)$$

✗ **Mathematica** : cpu = 1.13558 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == -(Log[-1 + y[x]]*(ExpIntegralEi[-Log[-1 + y[x]]])/x - F1[x]),`

✓ **Maple** : cpu = 0.253 (sec), leaf count = 27

$$\left\{ y(x) = e^{\text{RootOf} \left( \int \frac{-\text{F1}(x)}{x} dx + x\_C1 + \text{Ei}(1, -Z) \right)} + 1 \right\}$$

**2.863 ODE No. 863**

$$y'(x) = \frac{x\sqrt{x^2 + y(x)^2} + x^4\sqrt{x^2 + y(x)^2} + x^3\sqrt{x^2 + y(x)^2} + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.0347075 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow x \sinh \left( c_1 + \frac{x^4}{4} + \frac{x^3}{3} + x \right) \right\} \right\}$$

✓ **Maple** : cpu = 7.155 (sec), leaf count = 38

$$\left\{ \ln \left( \sqrt{(y(x))^2 + x^2} + y(x) \right) - \frac{x^4}{4} - \frac{x^3}{3} - x - \ln(x) - \_C1 = 0 \right\}$$

**2.864 ODE No. 864**

$$y'(x) = \frac{e^{\frac{x^2}{4}} y(x) \left( 2e^{-\frac{3x^2}{4}} y(x)^2 + e^{-\frac{x^2}{2}} xy(x) + e^{-\frac{x^2}{4}} x \right)}{2e^{-\frac{x^2}{4}} y(x) + 2}$$

✓ **Mathematica** : cpu = 0.0431205 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\frac{x^2}{2}}}{\sqrt{e^{\frac{x^2}{2}} (c_1 - 2x + 1) - e^{\frac{x^2}{4}}}} \right\}, \left\{ y(x) \rightarrow -\frac{e^{\frac{x^2}{2}}}{\sqrt{e^{\frac{x^2}{2}} (c_1 - 2x + 1) + e^{\frac{x^2}{4}}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.12 (sec), leaf count = 162

$$\left\{ y(x) = 1 \left( e^{\frac{x^2}{2}} \left( \sqrt{-C1 - 2x} - 1 \right) e^{-\frac{x^2}{4}} - e^{\frac{x^2}{4}} \sqrt{-C1 - 2x} \right) \left( e^{-\frac{x^2}{4}} \right)^{-1} \left( e^{-\frac{x^2}{4}} e^{\frac{x^2}{2}} + e^{\frac{x^2}{4}} \sqrt{-C1 - 2x} \right)^{-1}, y(x) \right\}$$

**2.865 ODE No. 865**

$$y'(x) = (1 - y(x)) \left( -f(x) + \frac{y(x) \log(y(x) - 1)}{x(1 - y(x)) \log(x)} - \frac{\log(y(x) - 1)}{x(1 - y(x)) \log(x)} \right)$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.276 (sec), leaf count = 23

$$\left\{ y(x) = e^{\int \frac{f(x)}{\ln(x)} dx \ln(x)} x^{-C1} + 1 \right\}$$

**2.866 ODE No. 866**

$$y'(x) = x^2 \sqrt{a^2 + 2ax + x^2 + 4y(x)} + \sqrt{a^2 + 2ax + x^2 + 4y(x)} + x^3 \sqrt{a^2 + 2ax + x^2 + 4y(x)} - \frac{a}{2} - \frac{x}{2}$$

✓ **Mathematica** : cpu = 0.482286 (sec), leaf count = 85

$$\left\{ \left\{ y(x) \rightarrow -\frac{a^2}{4} - \frac{ax}{2} - \frac{1}{6}(3c_1 - 4)x^4 - \frac{2c_1 x^3}{3} - 2c_1 x + c_1^2 + \frac{x^8}{16} + \frac{x^7}{6} + \frac{x^6}{9} + \frac{x^5}{2} + \frac{3x^2}{4} \right\} \right\}$$

✓ **Maple** : cpu = 0.565 (sec), leaf count = 37

$$\left\{ -C1 + \frac{x^4}{2} + \frac{2x^3}{3} + 2x - \sqrt{x^2 + 2ax + a^2 + 4y(x)} = 0 \right\}$$

**2.867 ODE No. 867**

$$y'(x) = \frac{x^6}{27} + \frac{1}{3}x^4y(x) + \frac{x^4}{9} + x^2y(x)^2 + \frac{2}{3}x^2y(x) + y(x)^3 + y(x)^2 - \frac{2x}{3} + 1$$

✓ **Mathematica** : cpu = 0.0920309 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.073 (sec), leaf count = 30

$$\left\{ y(x) = -\frac{x^2}{3} + \text{RootOf}\left(-x + \int^{-Z} (_a^3 + _a^2 + 1)^{-1} d_a + _C1\right) \right\}$$

**2.868 ODE No. 868**

$$y'(x) = -x^6 + 3x^4y(x) + x^4 - 3x^2y(x)^2 - 2x^2y(x) + y(x)^3 + y(x)^2 + 2x + 1$$

✓ **Mathematica** : cpu = 0.0757485 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.075 (sec), leaf count = 28

$$\left\{ y(x) = x^2 + \text{RootOf}\left(-x + \int^{-Z} (_a^3 + _a^2 + 1)^{-1} d_a + _C1\right) \right\}$$

**2.869 ODE No. 869**

$$y'(x) = \frac{2x^5 + 2x^4 - 2x^3y(x) + x^3 - 2x^2y(x) + 3x^2 - 2y(x) - x + 1}{x^2 - y(x)}$$

✓ **Mathematica** : cpu = 0.0356223 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( W\left(-e^{c_1+x^4+\frac{4x^3}{3}-2x^2+4x-1}\right) + 1\right) + x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 37

$$\left\{ y(x) = x^2 + \frac{1}{2} \text{lambertW}\left(-2 \frac{e^{x^4} e^{4/3 x^3} - C1 (e^x)^4 e^{-1}}{(e^{x^2})^2}\right) + \frac{1}{2} \right\}$$



**2.870 ODE No. 870**

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left( x^4 + x^3 + x e^{-\frac{y(x)}{x}} + e^{-\frac{y(x)}{x}} y(x) + x \right)}{x}$$

✓ **Mathematica** : cpu = 1.65651 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow -x \log \left( -\frac{c_1}{x} - \frac{x^3}{4} - \frac{x^2}{3} - 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.234 (sec), leaf count = 30

$$\left\{ y(x) = -\ln \left( -\frac{3x^4 + 4x^3 + 12\_C1 + 12x}{12x} \right) x \right\}$$

**2.871 ODE No. 871**

$$y'(x) = \frac{2xy(x)^2 + y(x)^2 + 4xy(x) \log(2x + 1) + 2y(x) \log(2x + 1) + 2x \log^2(2x + 1) + \log^2(2x + 1) - 2}{2x + 1}$$

✓ **Mathematica** : cpu = 0.0224972 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - x} - \log(2x + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 26

$$\left\{ y(x) = \frac{-1 + (\_C1 - x) \ln(2x + 1)}{-\_C1 + x} \right\}$$

**2.872 ODE No. 872**

$$y'(x) = \frac{14x^{7/2} + \frac{12x^6}{5} - 6x^3 y(x) - 6x^3 - 5\sqrt{x} y(x) + 10x - 5\sqrt{x} - 5}{x(2x^3 - 5y(x) + 10\sqrt{x} - 5)}$$

✓ **Mathematica** : cpu = 0.0443797 (sec), leaf count = 92

$$\left\{ \left\{ y(x) \rightarrow \sqrt{-\frac{1}{x}} \sqrt{-x(c_1 + 2 \log(x) + 1)} + \frac{2x^3}{5} + 2\sqrt{x} - 1 \right\}, \left\{ y(x) \rightarrow \left( -\frac{1}{x} \right)^{3/2} x \sqrt{-x(c_1 + 2 \log(x) + 1)} + \right. \right.$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 49

$$\left\{ y(x) = \frac{2x^3}{5} + 2\sqrt{x} - \sqrt{-C1 + 2 \ln(x)} - 1, y(x) = \frac{2x^3}{5} + 2\sqrt{x} + \sqrt{-C1 + 2 \ln(x)} - 1 \right\}$$

**2.873 ODE No. 873**

$$y'(x) = \frac{2y(x) + 1}{x(2xy(x)^4 + 3xy(x)^3 + xy(x)^2 + 2xy(x) + x - 2)}$$

✓ **Mathematica** : cpu = 0.640179 (sec), leaf count = 48

$$\text{Solve} \left[ \frac{1}{192} \left( -16y(x)^3 - 12y(x)^2 + 12y(x) - \frac{96}{2xy(x) + x} - 54 \log(4y(x) + 2) + 7 \right) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.29 (sec), leaf count = 50

$$\left\{ y(x) = \frac{e^{\text{RootOf}(2x(e^{-Z})^4 - 3x(e^{-Z})^3 - 6x(e^{-Z})^2 + 48x\_C1 e^{-Z} + 54\_Z x e^{-Z} + 7x e^{-Z} + 96)}}{2} - \frac{1}{2} \right\}$$

**2.874 ODE No. 874**

$$y'(x) = \frac{1}{512} x(a^3 x^{12} + 24a^2 x^8 y(x) + 8a^2 x^8 + 192ax^4 y(x)^2 + 128ax^4 y(x) - 256ax^2 + 512y(x)^3 + 512y(x)^2 + 512y(x) + 512)$$

✓ **Mathematica** : cpu = 0.199203 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.064 (sec), leaf count = 40

$$\left\{ y(x) = -\frac{ax^4}{8} - \frac{1}{3} + \frac{29 \text{RootOf}(x^2 - 162 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + 6\_C1)}{9} \right\}$$

**2.875 ODE No. 875**

$$y'(x) = \frac{x^5 \left( -\sqrt{x^2 + y(x)^2} \right) + x^4 y(x) \sqrt{x^2 + y(x)^2} + xy(x) + y(x)}{x(x + 1)}$$

✓ **Mathematica** : cpu = 0.294532 (sec), leaf count = 213

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( 2(x + 1)^{\sqrt{2}} \exp \left( \frac{12c_1 + 3x^4 + 6x^2 + 4(x^2 + 3)x + 25}{6\sqrt{2}} \right) + (x + 1)^{2\sqrt{2}} \left( -e^{\frac{4c_1 + x^4 + 2x^2}{\sqrt{2}}} \right) + e^{\frac{4x^3 + 12x + 25}{3\sqrt{2}}} \right)}{-2(x + 1)^{\sqrt{2}} \exp \left( \frac{12c_1 + 3x^4 + 6x^2 + 4(x^2 + 3)x + 25}{6\sqrt{2}} \right) + (x + 1)^{2\sqrt{2}} \left( -e^{\frac{4c_1 + x^4 + 2x^2}{\sqrt{2}}} \right) + e^{\frac{4x^3 + 12x + 25}{3\sqrt{2}}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.332 (sec), leaf count = 73

$$\left\{ \ln \left( 2 \frac{x \left( \sqrt{2 (y(x))^2 + 2x^2 + y(x) + x} \right)}{y(x) - x} \right) + \sqrt{2} \ln(1+x) + \frac{(3x^4 - 4x^3 + 6x^2 - 12x)\sqrt{2}}{12} - \_C1 - \ln(x) \right\}$$

**2.876 ODE No. 876**

$$y'(x) = -\frac{y(x)^2 (x^2 y(x) - 2xy(x) + y(x) - 2x)}{2x(xy(x) - 2y(x) - 2)}$$

✓ **Mathematica** : cpu = 0.0190893 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow \frac{2}{\sqrt{2} \sqrt{-\frac{1}{x} \sqrt{-x(2c_1 - \log(x) + 2)} + x - 2}} \right\}, \left\{ y(x) \rightarrow -\frac{2}{\sqrt{2} \sqrt{-\frac{1}{x} \sqrt{-x(2c_1 - \log(x) + 2)} - x + 2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 41

$$\left\{ y(x) = -4 \left( \sqrt{-C1 - 8 \ln(x)} - 2x + 4 \right)^{-1}, y(x) = 4 \left( \sqrt{-C1 - 8 \ln(x)} + 2x - 4 \right)^{-1} \right\}$$

**2.877 ODE No. 877**

$$y'(x) = \frac{x^6 - 3x^4 y(x) + 2x^3 + 3x^2 y(x)^2 - 2xy(x) - y(x)^3 - 2x}{x^2 - y(x) - 1}$$

✓ **Mathematica** : cpu = 0.0167199 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2x} - 1} + x^2 \right\}, \left\{ y(x) \rightarrow x^2 - \frac{1}{\sqrt{c_1 - 2x} + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 73

$$\left\{ y(x) = \frac{1}{2x - 2\_C1} \left( -2x^2\_C1 + 2x^3 + \sqrt{2\_C1 - 2x + 1} - 1 \right), y(x) = \frac{1}{-2x + 2\_C1} \left( 2x^2\_C1 - 2x^3 + \sqrt{2\_C1 - 2x + 1} - 1 \right) \right\}$$

**2.878 ODE No. 878**

$$y'(x) = \frac{-64a^3x^3 + 48a^2x^2y(x)^2 + 16a^2x^2 - 12axy(x)^4 - 8axy(x)^2 + y(x)^6 + y(x)^4 + 1}{y(x)}$$

✓ **Mathematica** : cpu = 0.300102 (sec), leaf count = 1

{}

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x),x) = (1+y(x)^4-8*a*x*y(x)^2+16*a^2*x^2+y(x)^6-12*y(x)^4*a*x+48*y(x)^2*a^2*x+64*a^3*x^3)/y(x),y(x))`

**2.879 ODE No. 879**

$$y'(x) = \frac{x^2(-\sqrt{x^2+y(x)^2}) + xy(x)\sqrt{x^2+y(x)^2} + xy(x) + y(x)}{x(x+1)}$$

✓ **Mathematica** : cpu = 0.15672 (sec), leaf count = 109

$$\left\{ \left\{ y(x) \rightarrow \frac{x(-2(x+1)^{\sqrt{2}}e^{\sqrt{2}(c_1+x)} + e^{2\sqrt{2}(c_1+x)} - (x+1)^{2\sqrt{2}})}{2(x+1)^{\sqrt{2}}e^{\sqrt{2}(c_1+x)} + e^{2\sqrt{2}(c_1+x)} - (x+1)^{2\sqrt{2}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.282 (sec), leaf count = 55

$$\left\{ \ln \left( 2 \frac{x(\sqrt{2(y(x))^2 + 2x^2 + y(x) + x})}{y(x) - x} \right) + \sqrt{2}x - \ln(x) - \sqrt{2}\ln(1+x) - C1 = 0 \right\}$$

**2.880 ODE No. 880**

$$y'(x) = -\frac{2a}{128a^4x^3 - 96a^3x^2y(x)^2 - 32a^3x^2 + 24a^2xy(x)^4 + 16a^2xy(x)^2 - 2ay(x)^6 - 2ay(x)^4 - 2a - y(x)}$$

✓ **Mathematica** : cpu = 0.171101 (sec), leaf count = 10

{{y(x) → 2ac<sub>1</sub>}}

✓ **Maple** : cpu = 0.07 (sec), leaf count = 41

$$\left\{ \frac{y(x)}{2a} + \frac{\int^{(y(x))^2-4ax} (-a^3 + a^2 + 1)^{-1} d_a}{8a^2} - C1 = 0 \right\}$$

**2.881 ODE No. 881**

$$y'(x) = \frac{x^6 + 9x^4y(x) - 6x^3 + 27x^2y(x)^2 - 18xy(x) + 27y(x)^3 - 18x}{9x^2 + 27y(x) + 27}$$

✓ **Mathematica** : cpu = 0.0185486 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow \frac{27}{\sqrt{c_1 - 1458x} - 27} - \frac{x^2}{3} \right\}, \left\{ y(x) \rightarrow -\frac{27}{\sqrt{c_1 - 1458x} + 27} - \frac{x^2}{3} \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 77

$$\left\{ y(x) = \frac{1}{-6x + 6\_C1} \left( -2x^2\_C1 + 2x^3 - 3\sqrt{2\_C1 - 2x + 1} + 3 \right), y(x) = \frac{1}{-6x + 6\_C1} \left( -2x^2\_C1 + 2x^3 \right) \right\}$$

**2.882 ODE No. 882**

$$y'(x) = -\frac{1}{216}\sqrt{x} \left( -108x^{3/2} + x^9 - 18x^6y(x) - 6x^6 + 108x^3y(x)^2 + 72x^3y(x) - 216y(x)^3 - 216y(x)^2 - 216 \right)$$

✓ **Mathematica** : cpu = 0.245541 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.072 (sec), leaf count = 41

$$\left\{ y(x) = \frac{x^3}{6} - \frac{1}{3} + \frac{29}{9} \text{RootOf} \left( 2x^{3/2} - 243 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + 9\_C1 \right) \right\}$$

**2.883 ODE No. 883**

$$y'(x) = \frac{x(a^3y(x)^6 + a^3y(x)^4 + a^3 + 3a^2bx^2y(x)^4 + 2a^2bx^2y(x)^2 + 3ab^2x^4y(x)^2 + ab^2x^4 + b^3x^6)}{a^{7/2}y(x)}$$

✓ **Mathematica** : cpu = 1.46485 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.848 (sec), leaf count = 352

$$\left\{ \int_{-b}^x \left( b^3\_a^6 + 3(y(x))^2 ab^2\_a^4 + 3(y(x))^4 a^2b\_a^2 + (y(x))^6 a^3 + a\_a^4b^2 + 2(y(x))^2 a^2b\_a^2 + (y(x))^4 a^3 + a^3 \right) \right\}$$

**2.884 ODE No. 884**

$$y'(x) = -\frac{x(x^6 - 3x^4y(x)^2 - x^4 + 3x^2y(x)^4 + 2x^2y(x)^2 - y(x)^6 - y(x)^4 - 1)}{y(x)}$$

✓ **Mathematica** : cpu = 0.563111 (sec), leaf count = 71

$$\text{Solve}\left[\frac{1}{4}\left(2\log(-x^2 + y(x)^2 + 1) - 2x^2 - \frac{1}{y(x)(y(x) + x)} + \frac{1}{xy(x) - y(x)^2} - 2\log(x - y(x)) - 2\log(y(x) + x)\right)\right]$$

✓ **Maple** : cpu = 0.423 (sec), leaf count = 107

$$\left\{ y(x) = e^{\text{RootOf}\left(-3x^2(e^{-Z})^2 + 6x^3e^{-Z} + 3(e^{-Z})^2 \ln\left(\frac{(e^{-Z})^2 - 2xe^{-Z} + 1}{e^{-Z} - 2x}\right) - 2\_C1(e^{-Z})^2 - 3\_Z(e^{-Z})^2 - 6e^{-Z} \ln\left(\frac{(e^{-Z})^2 - 2xe^{-Z} + 1}{e^{-Z} - 2x}\right)\right)} x \right.$$

**2.885 ODE No. 885**

$$y'(x) = -\frac{i(x^6 + 12x^4y(x)^2 + 4x^4 + 48x^2y(x)^4 + 32x^2y(x)^2 + 64y(x)^6 + 64y(x)^4 + 32ix + 64)}{128y(x)}$$

✗ **Mathematica** : cpu = 40.7424 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == ((-I/128)*(64 + (32*I)*x + 4*x^4 + x^6 + 32*x^2*y[x]^2 + 12*x^`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x), x) = -1/128*I*(32*I*x+64+64*y(x)^4+32*x^2*y(x)^2+4*x^4+64*y(x)^6+48*x^2*y(x)`

**2.886 ODE No. 886**

$$y'(x) = \frac{x^6y(x)^3 - 3x^5y(x)^2 + x^4y(x)^2 + 3x^4y(x) - 4x^3y(x) - x^3 + 2x^2 + 1}{x^4}$$

✓ **Mathematica** : cpu = 0.0986971 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.042 (sec), leaf count = 42

$$\left\{ y(x) = \frac{9x - 3 + 29 \text{RootOf}\left(-81 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_ax + 3x\_C1 - 1\right)}{9x^2} \right\}$$

**2.887 ODE No. 887**

$$y'(x) = \frac{a^3 x^3 y(x)^3 + 3a^2 x^2 y(x)^2 + a^2 x y(x) + a^2 x + 3a x y(x) + a + 1}{a^2 x^2 (a x y(x) + a x + 1)}$$

✓ **Mathematica** : cpu = 0.0254922 (sec), leaf count = 91

$$\left\{ \left\{ y(x) \rightarrow \frac{a^3}{\sqrt{c_1 - 2a^6 x} - a^3} - \frac{1}{ax} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt{c_1 - 2a^6 x} + a^4 x + a^3}{ax \sqrt{c_1 - 2a^6 x} + a^4 x} \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 72

$$\left\{ y(x) = \frac{1}{ax} \left( -ax - \sqrt{-C1 - 2x} - 1 \right) \left( \sqrt{-C1 - 2x} + 1 \right)^{-1}, y(x) = \frac{1}{ax} \left( ax - \sqrt{-C1 - 2x} + 1 \right) \left( \sqrt{-C1 - 2x} \right) \right\}$$

**2.888 ODE No. 888**

$$y'(x) = \frac{x^4 y(x)^3 - 5x^3 y(x)^2 + 6x^2 y(x) - 2x y(x) - 2x + 1}{x^2 (x^2 y(x) - x + 1)}$$

✓ **Mathematica** : cpu = 0.0188414 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{x^4 \left( \frac{1}{x^2} - \frac{1}{x^2 \sqrt{c_1 + \frac{2}{x}}} \right)} + \frac{x-1}{x^2} \right\}, \left\{ y(x) \rightarrow \frac{\frac{1}{\sqrt{c_1 + \frac{2}{x}} + 1} + x - 1}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 79

$$\left\{ y(x) = \frac{1}{x^2} \left( \sqrt{\frac{x-C1+2}{x}} x - x + 1 \right) \left( \sqrt{\frac{x-C1+2}{x}} - 1 \right)^{-1}, y(x) = \frac{1}{x^2} \left( \sqrt{\frac{x-C1+2}{x}} x + x - 1 \right) \left( \sqrt{\frac{x-C1-2}{x}} \right) \right\}$$

**2.889 ODE No. 889**

$$y'(x) = -\frac{e^x (-8y(x)^{9/2} + 36e^x y(x)^3 - 8y(x)^3 + 24e^x y(x)^{3/2} - 54e^{2x} y(x)^{3/2} - 18e^{2x} + 27e^{3x} - 8)}{8\sqrt{y(x)}}$$

✗ **Mathematica** : cpu = 300.214 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.447 (sec), leaf count = 49

$$\left\{ e^x + \frac{2}{3} \ln \left( (y(x))^{\frac{3}{2}} - \frac{3e^x}{2} \right) - 4 \left( -6(y(x))^{3/2} + 9e^x \right)^{-1} - \frac{2}{3} \ln \left( (y(x))^{\frac{3}{2}} - \frac{3e^x}{2} + 1 \right) - C1 = 0 \right\}$$

**2.890 ODE No. 890**

$$y'(x) = \frac{x}{x^6 + 3x^4y(x)^2 + x^4 + 3x^2y(x)^4 + 2x^2y(x)^2 + y(x)^6 + y(x)^4 - y(x) + 1}$$

✓ **Mathematica** : cpu = 0.161322 (sec), leaf count = 7

$$\{\{y(x) \rightarrow c_1\}\}$$

✓ **Maple** : cpu = 1.19 (sec), leaf count = 34

$$\left\{ -y(x) + \frac{\int^{(y(x))^2+x^2} (-a^3 + a^2 + 1)^{-1} da}{2} - C1 = 0 \right\}$$

**2.891 ODE No. 891**

$$y'(x) = \frac{y(x)^2 (x^4y(x) + 2x^2y(x) + 2x^2 - 2y(x))}{x^3 (x^2y(x) + x^2 - y(x))}$$

✓ **Mathematica** : cpu = 0.0242318 (sec), leaf count = 86

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2}{x^2 \left( \sqrt{\frac{1}{x^5}} \sqrt{x^5 (c_1 - 2 \log(x) + 1)} - 1 \right) + 1} \right\}, \left\{ y(x) \rightarrow -\frac{x^2}{x^2 \left( \sqrt{\frac{1}{x^5}} \sqrt{x^5 (c_1 - 2 \log(x) + 1)} + 1 \right) - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 56

$$\left\{ y(x) = x^2 \left( \sqrt{-C1 - 2 \ln(x)x^2 - x^2 + 1} \right)^{-1}, y(x) = -x^2 \left( \sqrt{-C1 - 2 \ln(x)x^2 + x^2 - 1} \right)^{-1} \right\}$$

**2.892 ODE No. 892**

$$y'(x) = \frac{e^{-\frac{2}{x^2-y(x)^2-1}} + x^2 + 2xy(x) + y(x)^2}{-e^{-\frac{2}{x^2-y(x)^2-1}} + x^2 + 2xy(x) + y(x)^2}$$

✗ **Mathematica** : cpu = 300.007 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.645 (sec), leaf count = 40

$$\left\{ y(x) = e^{\text{RootOf}\left(-Z + \int^{(e^{-Z})^2 - 2xe^{-Z}} (e^{2(1+a)^{-1}} + a)^{-1} da + C1\right)} - x \right\}$$



**2.893 ODE No. 893**

$$y'(x) = \frac{x^3 y(x)^3 + x^3 y(x)^2 + x^3 + 6x^2 y(x)^2 + 4x^2 y(x) + 12xy(x) + 6x + 8}{x^3}$$

✓ **Mathematica** : cpu = 0.0975865 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.039 (sec), leaf count = 41

$$\left\{ y(x) = \frac{29 \operatorname{RootOf}\left(-81 \int^{-Z} (841 \_a^3 - 27 \_a + 27)^{-1} d\_a + x + 3 \_C1\right) x - 3x - 18}{9x} \right\}$$

**2.894 ODE No. 894**

$$y'(x) = -\frac{i(x^6 + 3x^4 y(x)^2 + x^4 + 3x^2 y(x)^4 + 2x^2 y(x)^2 + y(x)^6 + y(x)^4 + ix + 1)}{y(x)}$$

✗ **Mathematica** : cpu = 40.6733 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == ((-I)*(1 + I*x + x^4 + x^6 + 2*x^2*y[x]^2 + 3*x^4*y[x]^2 + y[x]^6 + y[x]^4 + ix + 1)/y[x]), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x), x) = -I*(I*x+1+x^4+2*x^2*y(x)^2+y(x)^4+x^6+3*x^4*y(x)^2+3*x^2*y(x)^4+y(x)^6+y(x)^4+ix+1)/y(x), y(x))`

**2.895 ODE No. 895**

$$y'(x) = \frac{x(a^3 x^{12} + 24a^2 x^8 y(x) - 32a^2 x^6 + 192ax^4 y(x)^2 - 256ax^2 y(x) - 256ax^2 + 512y(x)^3)}{64ax^4 + 512y(x) + 512}$$

✓ **Mathematica** : cpu = 0.0243419 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow \frac{512}{\sqrt{c_1 - 262144x^2} - 512} - \frac{ax^4}{8} \right\}, \left\{ y(x) \rightarrow -\frac{ax^4}{8} - \frac{512}{\sqrt{c_1 - 262144x^2} + 512} \right\} \right\}$$

✓ **Maple** : cpu = 0.064 (sec), leaf count = 79

$$\left\{ y(x) = 1\left(-8 + \left(-\sqrt{-x^2 + \_C1} - 1\right) ax^4\right) \left(8 + 8\sqrt{-x^2 + \_C1}\right)^{-1}, y(x) = 1\left(8 + \left(-\sqrt{-x^2 + \_C1} + 1\right) ax^4\right) \left(8 + 8\sqrt{-x^2 + \_C1}\right)^{-1} \right\}$$

**2.896 ODE No. 896**

$$y'(x) = \frac{-x^6 + 3x^4y(x)^2 + x^4 - 3x^2y(x)^4 - 2x^2y(x)^2 + y(x)^6 + y(x)^4 + x + 1}{y(x)}$$

✓ **Mathematica** : cpu = 0.246037 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.928 (sec), leaf count = 63

$$\left\{ \int_{-b}^{y(x)} \frac{-a}{-a^6 + 3a^4x^2 - 3a^2x^4 + x^6 - a^4 + 2a^2x^2 - x^4 - 1} da + x - C1 = 0 \right\}$$

**2.897 ODE No. 897**

$$y'(x) = \frac{\sqrt{x}(-108x^{3/2}y(x) + 18x^{9/2} - 108x^{3/2} + x^9 - 18x^6y(x) + 108x^3y(x)^2 - 216y(x)^3)}{36x^3 - 216y(x) - 216}$$

✓ **Mathematica** : cpu = 0.0251383 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{x^3}{6} - \frac{216}{\sqrt{c_1 - 62208x^{3/2} + 216}} \right\}, \left\{ y(x) \rightarrow \frac{216}{\sqrt{c_1 - 62208x^{3/2} - 216}} + \frac{x^3}{6} \right\} \right\}$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 87

$$\left\{ y(x) = 1 \left( \sqrt{9 - C1 - 12x^{3/2}x^3} - 3x^3 + 18 \right) \left( -18 + 6\sqrt{9 - C1 - 12x^{3/2}} \right)^{-1}, y(x) = 1 \left( \sqrt{9 - C1 - 12x^{3/2}x^3} + \dots \right) \right\}$$

**2.898 ODE No. 898**

$$y'(x) = \frac{4x^6y(x)^3 + 2x^5y(x) + 2x^5 + 3x^4y(x)^2 + \frac{x^3}{2} + \frac{3}{4}x^2y(x) + \frac{1}{16}}{x^6(4x^2y(x) + 4x^2 + 1)}$$

✓ **Mathematica** : cpu = 0.0213216 (sec), leaf count = 95

$$\left\{ \left\{ y(x) \rightarrow \frac{-\sqrt{c_1 + \frac{8192}{x}} + 256x^2 + 64}{4x^2 \left( \sqrt{c_1 + \frac{8192}{x}} - 64 \right)} \right\}, \left\{ y(x) \rightarrow -\frac{\sqrt{c_1 + \frac{8192}{x}} + 256x^2 + 64}{4x^2 \left( \sqrt{c_1 + \frac{8192}{x}} + 64 \right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 87

$$\left\{ y(x) = \frac{1}{4x^2} \left( -4x^2 - \sqrt{\frac{x - C1 + 2}{x}} - 1 \right) \left( \sqrt{\frac{x - C1 + 2}{x}} + 1 \right)^{-1}, y(x) = \frac{1}{4x^2} \left( 4x^2 - \sqrt{\frac{x - C1 + 2}{x}} + 1 \right) \left( \sqrt{\dots} \right) \right\}$$

**2.899 ODE No. 899**

$$y'(x) = \frac{x^6 y(x)^3 + x^6 y(x)^2 + x^6 + \frac{x^5}{2} + \frac{3}{4} x^4 y(x)^2 + \frac{1}{2} x^4 y(x) + \frac{3}{16} x^2 y(x) + \frac{x^2}{16} + \frac{1}{64}}{x^8}$$

✓ **Mathematica** : cpu = 0.389486 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.049 (sec), leaf count = 47

$$\left\{ y(x) = \frac{116 \operatorname{RootOf}\left(-81 \int^{-Z} (841 \_a^3 - 27 \_a + 27)^{-1} d\_ax + 3 x \_C1 - 1\right) x^2 - 12 x^2 - 9}{36 x^2} \right\}$$

**2.900 ODE No. 900**

$$y'(x) = \frac{2a(4ax - y(x)^2 - 1)}{128a^4x^3 - 96a^3x^2y(x)^2 + 24a^2xy(x)^4 - 2ay(x)^6 + 4axy(x) - y(x)^3 - y(x)}$$

✓ **Mathematica** : cpu = 0.0922565 (sec), leaf count = 381

$$\left\{ \{y(x) \rightarrow \operatorname{Root}[8\#1^5 a - 16\#1^4 a^2 c_1 - 64\#1^3 a^2 x + \#1^2 (128a^3 c_1 x - 2) + 128\#1 a^3 x^2 - 256a^4 c_1 x^2 + 8ax - 1\&, \right.$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 48

$$\left\{ \frac{y(x)}{2a} + \left(32a^3x - 8a^2(y(x))^2\right)^{-1} - \frac{1}{16a^2 \left((y(x))^2 - 4ax\right)^2} - \_C1 = 0 \right\}$$

**2.901 ODE No. 901**

$$y'(x) = \frac{y(x) (-ax \log(y(x)) + x^2 + y(x))}{x(ax - y(x) - y(x) \log(x) - y(x) \log(y(x)))}$$

✓ **Mathematica** : cpu = 0.220843 (sec), leaf count = 29

$$\operatorname{Solve}[2c_1 + x^2 + 2y(x)(\log(y(x)) + \log(x)) = 2ax \log(y(x)), y(x)]$$

✓ **Maple** : cpu = 0.675 (sec), leaf count = 30

$$\left\{ y(x) = e^{\operatorname{RootOf}(-2 \_Z ax + 2 e^{-Z} \ln(x) + 2 \_Z e^{-Z} + 2 \_C1 a + x^2)} \right\}$$

**2.902 ODE No. 902**

$$y'(x) = \frac{x^6 - 3x^4 y(x)^2 + x^3 + 3x^2 y(x)^4 - xy(x)^2 - y(x)^6 - x}{y(x)(x^2 - y(x)^2 - 1)}$$

✓ **Mathematica** : cpu = 0.0932561 (sec), leaf count = 195

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2} \sqrt{-\frac{4c_1 x^2 + \sqrt{4c_1 - 4x + 1} - 4x^3 + 1}{x - c_1}} \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \sqrt{-\frac{4c_1 x^2 + \sqrt{4c_1 - 4x + 1} - 4x^3 + 1}{x - c_1}} \right\}, \left\{ \right.$$

✓ **Maple** : cpu = 0.297 (sec), leaf count = 183

$$\left\{ y(x) = \frac{1}{2\_C1 + 6x} \sqrt{(-C1 + 3x)(4x^2\_C1 + 12x^3 - \sqrt{-12\_C1 - 36x + 9} - 3)}, y(x) = \frac{1}{2\_C1 + 6x} \sqrt{(-$$

**2.903 ODE No. 903**

$$y'(x) = \frac{\sin\left(\frac{y(x)}{x}\right) \csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \left(2x^2 \sin\left(\frac{y(x)}{2x}\right) \cos\left(\frac{y(x)}{2x}\right) + y(x)\right)}{2x}$$

✓ **Mathematica** : cpu = 0.0517637 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow 2x \cot^{-1}\left(e^{-c_1 - x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 48

$$\left\{ y(x) = \arctan\left(2 \frac{-C1 e^x}{-C1^2 (e^x)^2 + 1}, \frac{-C1^2 (e^x)^2 + 1}{-C1^2 (e^x)^2 + 1}\right) x \right\}$$

**2.904 ODE No. 904**

$$y'(x) = \frac{\sin\left(\frac{y(x)}{x}\right) \csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \left(2x^3 \sin\left(\frac{y(x)}{2x}\right) \cos\left(\frac{y(x)}{2x}\right) + y(x)\right)}{2x}$$

✓ **Mathematica** : cpu = 0.0498038 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow 2x \cot^{-1}\left(e^{-c_1 - \frac{x^2}{2}}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 64

$$\left\{ y(x) = \arctan\left(2 \frac{e^{1/2 x^2} - C1}{(e^{1/2 x^2})^2 - C1^2 + 1}, 1 \left(-\left(e^{\frac{x^2}{2}}\right)^2 - C1^2 + 1\right) \left(\left(e^{\frac{x^2}{2}}\right)^2 - C1^2 + 1\right)^{-1}\right) x \right\}$$

**2.905 ODE No. 905**

$$y'(x) = \frac{a^3 x^3 y(x)^3 + a^3 x^3 y(x)^2 + a^3 x^3 + 3a^2 x^2 y(x)^2 + 2a^2 x^2 y(x) + a^2 x + 3axy(x) + ax + 1}{a^3 x^3}$$

✓ **Mathematica** : cpu = 0.114729 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.056 (sec), leaf count = 46

$$\left\{ y(x) = \frac{29 \operatorname{RootOf}\left(-81 \int^{-Z} (841 \_a^3 - 27 \_a + 27)^{-1} d\_a + x + 3 \_C1\right) ax - 3 ax - 9}{9 ax} \right\}$$

**2.906 ODE No. 906**

$$y'(x) = \frac{x(x^2 + y(x)^2 + 1)}{x^6 + 3x^4 y(x)^2 + 3x^2 y(x)^4 - x^2 y(x) + y(x)^6 - y(x)^3 - y(x)}$$

✓ **Mathematica** : cpu = 0.0491267 (sec), leaf count = 326

{ {y(x) → Root[4#1<sup>5</sup> - 4#1<sup>4</sup>c<sub>1</sub> + 8#1<sup>3</sup>x<sup>2</sup> + #1<sup>2</sup>(2 - 8c<sub>1</sub>x<sup>2</sup>) + 4#1x<sup>4</sup> - 4c<sub>1</sub>x<sup>4</sup> + 2x<sup>2</sup> + 1&, 1] } , {y(x) → Root[

✓ **Maple** : cpu = 0.421 (sec), leaf count = 37

$$\left\{ -\left(2x^2 + 2(y(x))^2\right)^{-1} - \frac{1}{4\left((y(x))^2 + x^2\right)^2} - y(x) + \_C1 = 0 \right\}$$

**2.907 ODE No. 907**

$$y'(x) = \frac{\frac{3x^2}{2} + x^2 \sin(x) - 2x^2 \cos(x) + \frac{1}{2}x^2 \cos(2x) - 2xy(x) + y(x)^2 + 2xy(x) \cos(x) + x - x \cos(x)}{x}$$

✓ **Mathematica** : cpu = 0.0502402 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \log(x)} + x + x(-\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.178 (sec), leaf count = 20

$$\left\{ y(x) = -(\cos(x) - 1)x + (\_C1 - \ln(x))^{-1} \right\}$$

**2.908 ODE No. 908**

$$y'(x) = \frac{4(a-1)(a+1)x}{a^6x^4 - 3a^4x^4 - 2a^4x^2y(x)^2 + 3a^2x^4 + 4a^2x^2y(x)^2 + a^2y(x)^4 - x^4 - 2x^2y(x)^2 - y(x)^4 + 4y(x)}$$

✓ **Mathematica** : cpu = 1.29047 (sec), leaf count = 1023

$$\left\{ \left\{ \begin{array}{l} -c_1 + \sqrt[3]{9x^2c_1a^6 - 27x^2c_1a^4 - 27a^4 + 27x^2c_1a^2 + 54a^2 - c_1^3 - 9x^2c_1 + \frac{1}{2}\sqrt{4(-9x^2c_1a^6 + 27(c_1x^2 + \dots)}} \end{array} \right. \right. \\ \left. \left. y(x) \rightarrow \frac{\dots}{\dots} \right. \right.$$

✓ **Maple** : cpu = 0.484 (sec), leaf count = 1742

$$\left\{ y(x) = \frac{9^{\frac{2}{3}}}{27a^2 - 27} \left( (-C_1 a^2 + C_1) \sqrt[3]{9} \sqrt[3]{\left(\frac{1}{3}\sqrt{-3(a-1)^5(a+1)^5x^6 + 6C_1^2(a-1)^4(a+1)^4x^4 - 3C_1^3}\right)} \right) \right.$$

**2.909 ODE No. 909**

$$y'(x) = \frac{x^3y(x)^6 + x^3y(x)^4 + x^3 + 3x^2y(x)^4 + 2x^2y(x)^2 + 3xy(x)^2 + x + 1}{x^5y(x)}$$

✗ **Mathematica** : cpu = 40.6348 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == (1 + x + x^3 + 3*x*y[x]^2 + 2*x^2*y[x]^2 + 3*x^2*y[x]^4 + x^3*y[x]^6 + x^3 + 3*x^2*y[x]^4 + 2*x^2*y[x]^2 + 3*x*y[x]^2 + x + 1)/x^5, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(y(x), x) = (x^3+y(x)^4*x^3+2*x^2*y(x)^2+x+x^3*y(x)^6+3*x^2*y(x)^4+3*x*y(x)^2+1)/x^5, y(x))`

**2.910 ODE No. 910**

$$y'(x) = \frac{x^6 + 3x^5y(x) + 3x^4y(x)^2 + x^4 + x^3y(x)^3 + 2x^3y(x) + x^2y(x)^2 - y(x) - 2x + 1}{x}$$

✓ **Mathematica** : cpu = 0.181165 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.043 (sec), leaf count = 42

$$\left\{ y(x) = \frac{-9x^2 + 29 \text{RootOf}\left(-81 \int^{-Z} (841_a^3 - 27_a + 27)^{-1} d_a + x + 3_C1\right) - 3}{9x} \right\}$$

**2.911 ODE No. 911**

$$y'(x) = -y(x) \left( -_F1(x) - \frac{\log(y(x))}{x} + \cot(x) \log(y(x)) \right)$$

✓ **Mathematica** : cpu = 3.5578 (sec), leaf count = 53

$$\text{Solve} \left[ c_1 + 2 \sin(1) \log(y(x)) = \int_1^x \frac{2K[1] \sin(K[1]) \_F1(K[1]) + 2 \log(y(x)) (\sin(K[1]) - K[1] \cos(K[1]))}{K[1]^2} dK[1], y \right]$$

✓ **Maple** : cpu = 0.681 (sec), leaf count = 30

$$\left\{ y(x) = e^{\frac{x \_C1}{\sin(x)}} e^{\frac{x}{\sin(x)} \int \frac{\_F1(x) \sin(x)}{x} dx} \right\}$$

**2.912 ODE No. 912**

$$y'(x) = \frac{2ax}{-128a^4 + 96a^3xy(x)^2 + 32a^3x - 24a^2x^2y(x)^4 - 16a^2x^2y(x)^2 + 2ax^3y(x)^6 + 2ax^3y(x)^4 + 2ax^3 - x^3y(x)}$$

✓ **Mathematica** : cpu = 1.39892 (sec), leaf count = 199

$$\text{Solve} \left[ y(x) = \text{RootSum} \left[ -\#1^3y(x)^6 - \#1^3y(x)^4 - \#1^3 + 12\#1^2ay(x)^4 + 8\#1^2ay(x)^2 - 48\#1a^2y(x)^2 - 16\#1a^2 \right] \right]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.913 ODE No. 913**

$$y'(x) = \frac{y(x)^3 + y(x) + y(x)^3 (-\log^3(x)) + y(x)^3 \log^2(x) + 3y(x)^2 \log^2(x) - 2y(x)^2 \log(x) - 3y(x) \log(x) + 1}{xy(x)}$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 3.729 (sec), leaf count = 43

$$\left\{ y(x) = 9 \left( 9 \ln(x) + 56 \text{RootOf} \left( -81 \int^{-Z} (3136\_a^3 - 27\_a + 27)^{-1} d\_a - \ln(x) + 3\_C1 \right) - 3 \right)^{-1} \right\}$$

**2.914 ODE No. 914**

$$y'(x) = \frac{2a(-4a + xy(x)^2 + x)}{-128a^4 + 96a^3xy(x)^2 - 24a^2x^2y(x)^4 + 2ax^3y(x)^6 + 4ax^2y(x) - x^3y(x)^3 - x^3y(x)}$$

✓ **Mathematica** : cpu = 1.45864 (sec), leaf count = 401

$$\{ \{y(x) \rightarrow \text{Root}[8\#1^5ax^2 - 8\#1^4ac_1x^2 - 64\#1^3a^2x + \#1^2(64a^2c_1x + 2x^2) + 128\#1a^3 - 128a^3c_1 - 8ax + x^2\&, \dots]$$

✓ **Maple** : cpu = 4. (sec), leaf count = 71

$$\left\{ \frac{x(y(x))^4 + (-4a + x)(y(x))^2 - 2a}{2a(y(x))^4(-x(y(x))^2 + 4a)^2} + \frac{8a(y(x))^5 + 2(y(x))^2 + 1}{16a^2(y(x))^4} + \_C1 = 0 \right\}$$

**2.915 ODE No. 915**

$$y'(x) = \frac{y(x)^3 + y(x) - 8y(x)^3 \log^3(x) + 4y(x)^3 \log^2(x) + 12y(x)^2 \log^2(x) - 4y(x)^2 \log(x) - 6y(x) \log(x) + 1}{xy(x)}$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.072 (sec), leaf count = 43

$$\left\{ y(x) = 9 \left( 18 \ln(x) + 83 \text{RootOf} \left( -81 \int^{-Z} (6889\_a^3 - 27\_a + 27)^{-1} d\_a - \ln(x) + 3\_C1 \right) - 3 \right)^{-1} \right\}$$

**2.916 ODE No. 916**

$$y'(x) = \frac{y(x)(x^4 \log^2(y(x)) + 2x^4 \log(x) \log(y(x)) + x^4 \log^2(x) + x \log(y(x)) + \log(y(x)) - x + x \log(x) + \log(x))}{x(x+1)}$$

✗ **Mathematica** : cpu = 2.26876 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == ((-1 - x + Log[x] + x\*Log[x] + x^4\*Log[x]^2 + Log[y[x]] + x\*Lo

✓ **Maple** : cpu = 1.471 (sec), leaf count = 73

$$\left\{ y(x) = e^{\frac{-12 \ln(x) \ln(1+x) + (-3x^4 + 4x^3 - 6x^2 + 12\_C1 + 12x) \ln(x) - 12x}{3x^4 - 4x^3 + 6x^2 + 12 \ln(1+x) - 12\_C1 - 12x}} \right\}$$



**2.917 ODE No. 917**

$$y'(x) = \frac{y(x) (x \log^2(y(x)) + 2x \log(x) \log(y(x)) + x \log(y(x)) + \log(y(x)) - x + x \log^2(x) + x \log(x) + \log(x) - 1)}{x(x+1)}$$

✗ **Mathematica** : cpu = 1.16325 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x] == ((-1 - x + Log[x] + x\*Log[x] + x\*Log[x]^2 + Log[y[x]] + x\*Log[

✓ **Maple** : cpu = 0.398 (sec), leaf count = 38

$$\left\{ y(x) = e^{\frac{\ln(x) \ln(1+x) + (-x + C1) \ln(x) - x}{-\ln(1+x) - C1 + x}} \right\}$$

**2.918 ODE No. 918**

$$y'(x) = \frac{2y(x)^8}{128x^3y(x)^6 + 32x^2y(x)^6 + 96x^2y(x)^4 + 2y(x)^6 + y(x)^5 + 16xy(x)^4 + 24xy(x)^2 + 2y(x)^2 + 2}$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.477 (sec), leaf count = 41

$$\left\{ x - \text{RootOf} \left( \int^{-Z} (64\_a^3 + 16\_a^2 + 1)^{-1} d\_a y(x) + \_C1 y(x) + 1 \right) + \frac{1}{4 (y(x))^2} = 0 \right\}$$

**2.919 ODE No. 919**

$$y'(x) = \frac{(-y(x) + \sqrt{y(x)} + x) y(x)^{3/2}}{x^3 - 3x^2y(x) + 3xy(x)^2 + xy(x)^{3/2} - y(x)^3 - y(x)^{5/2} + y(x)^2}$$

✗ **Mathematica** : cpu = 300.386 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.316 (sec), leaf count = 82

$$\left\{ -\frac{1}{(x-y(x))^3} \left( (-C1 x^3 - 6x - 1) (y(x))^{\frac{5}{2}} + (-3x^2 - C1 + 3) (y(x))^{\frac{7}{2}} - (y(x))^{\frac{11}{2}} - C1 + 3(y(x))^{3/2} x^2 + 3(y(x)) \right) \right\}$$

**2.920 ODE No. 920**

$$y'(x) = \frac{2y(x)^6 (4xy(x)^2 + y(x)^2 + 1)}{128x^3y(x)^6 + 96x^2y(x)^4 + 4xy(x)^5 + y(x)^5 + y(x)^3 + 24xy(x)^2 + 2}$$

✓ **Mathematica** : cpu = 0.234568 (sec), leaf count = 301

$$\{ \{y(x) \rightarrow \text{Root}[\#1^5(128c_1x^2 - 8x - 1) + 128\#1^4x^2 + \#1^3(64c_1x - 2) + 64\#1^2x + 8\#1c_1 + 8\&, 1] \}, \{y(x) \rightarrow \dots\} \}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.921 ODE No. 921**

$$y'(x) = -y(x) \left( -\text{F1}(x) - \frac{\log(y(x))}{x} + \frac{\log(y(x))}{x \log(x)} \right)$$

✓ **Mathematica** : cpu = 2.78472 (sec), leaf count = 47

$$\text{Solve} \left[ \text{ConditionalExpression} \left[ c_1 = \int_1^x -\frac{K[1] \log(K[1]) \text{F1}(K[1]) + \log(y(x))(\log(K[1]) - 1)}{K[1]^2} dK[1], \Re(x) > 0 \right] \right]$$

✓ **Maple** : cpu = 0.367 (sec), leaf count = 30

$$\left\{ y(x) = e^{\frac{C1 x}{\ln(x)}} e^{\frac{x}{\ln(x)} \int \frac{\text{F1}(x) \ln(x)}{x} dx} \right\}$$

**2.922 ODE No. 922**

$$y'(x) = \frac{y(x)^2}{x^3 - 3x^2y(x) + x^2\sqrt{y(x)} + 3xy(x)^2 - 2xy(x)^{3/2} - y(x)^3 + y(x)^{5/2} + y(x)^2 + y(x)^{3/2}}$$

✗ **Mathematica** : cpu = 300.001 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.885 (sec), leaf count = 47

$$\left\{ \frac{\ln(y(x))}{2} - \int^x \frac{1}{\sqrt{y(x)} - \sqrt{y(x)}} (2\_a^3 + 2\_a^2 - \_a + 2)^{-1} d\_a - \_C1 = 0 \right\}$$

**2.923 ODE No. 923**

$$y'(x) = \frac{x^2 + 2xy(x) + e^{-2(x-y(x))(y(x)+x)} + y(x)^2}{x^2 + 2xy(x) - e^{-2(x-y(x))(y(x)+x)} + y(x)^2}$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.526 (sec), leaf count = 36

$$\left\{ y(x) = e^{\text{RootOf}\left(-Z + \int (e^{-Z})^2 - 2e^{-Z}x(e^2 - a + a)^{-1} d_a + C1\right) - x} \right\}$$

**2.924 ODE No. 924**

$$y'(x) = -\frac{y(x) \left( -F1(x) - \frac{\log^2(y(x))}{2x} \right)}{\log(y(x))}$$

✓ **Mathematica** : cpu = 0.992868 (sec), leaf count = 53

Solve [ConditionalExpression [2c1 = 2 ∫<sub>1</sub><sup>x</sup> - $\frac{2K[1]_F1(K[1]) + \log^2(y(x))}{2K[1]^2} dx + C1$ ,  $\Re(x) > 0 \vee x \notin \mathbb{R}$ ]

✓ **Maple** : cpu = 0.144 (sec), leaf count = 46

$$\left\{ y(x) = e^{\sqrt{2 \int \frac{F1(x)}{x} dx + 2 - C1} x}, y(x) = e^{-\sqrt{2} \sqrt{x \left( \int \frac{F1(x)}{x} dx + C1 \right)}} \right\}$$

**2.925 ODE No. 925**

$$y'(x) = \frac{x^2 + 2xy(x) + e^{2(x-y(x))^2(y(x)+x)^2} + y(x)^2}{x^2 + 2xy(x) - e^{2(x-y(x))^2(y(x)+x)^2} + y(x)^2}$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.29 (sec), leaf count = 38

$$\left\{ y(x) = e^{\text{RootOf}\left(-Z + \int (e^{-Z})^2 - 2e^{-Z}x(e^2 - a^2 + a)^{-1} d_a + C1\right) - x} \right\}$$

**2.926 ODE No. 926**

$$y'(x) = \frac{\frac{1}{16}x^3y(x)^3 - \frac{1}{2}x^2y(x)^3 - \frac{3}{8}x^2y(x)^2 + xy(x)^3 + xy(x)^2 + \frac{3}{4}xy(x) - \frac{1}{2}}{xxy(x) - 2y(x) - 2}$$

✓ **Mathematica** : cpu = 0.0260068 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow \frac{2\left(\sqrt{c_1 + 2048 \log(x)} - 64\right)}{x\left(\sqrt{c_1 + 2048 \log(x)} - 64\right) + 128} \right\}, \left\{ y(x) \rightarrow \frac{2\left(\sqrt{c_1 + 2048 \log(x)} + 64\right)}{x\left(\sqrt{c_1 + 2048 \log(x)} + 64\right) - 128} \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 67

$$\left\{ y(x) = 1\left(2\sqrt{-C1 + 8 \ln(x)} - 8\right) \left(x\sqrt{-C1 + 8 \ln(x)} - 4x + 8\right)^{-1}, y(x) = 1\left(2\sqrt{-C1 + 8 \ln(x)} + 8\right) \left(x\sqrt{-C1 + 8 \ln(x)} - 4x + 8\right)^{-1} \right\}$$

**2.927 ODE No. 927**

$$y'(x) = -\frac{1}{8}x\left(12e^{-x^2}x^2y(x)^2 + 8e^{-x^2}x^2y(x) + 8e^{-x^2}x^2 - 8e^{-x^2} + e^{-3x^2}x^6 - 6e^{-2x^2}x^4y(x) - 2e^{-2x^2}x^4 - 8y(x)^3\right)$$

✓ **Mathematica** : cpu = 0.352624 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.142 (sec), leaf count = 68

$$\left\{ y(x) = \frac{58 \text{RootOf}\left(x^2 - 162 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + 6\_C1\right) + \left(9x^2 - 6e^{x^2}\right) e^{-x^2}}{18e^{-x^2}e^{x^2}} \right\}$$

**2.928 ODE No. 928**

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left(x^2 e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} y(x) + e^{-\frac{y(x)}{x}} y(x) + x\right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 1.36814 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow -x \log\left(\frac{c_1 - \log(x+1)}{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.517 (sec), leaf count = 20

$$\left\{ y(x) = -\ln\left(\frac{-\ln(1+x) + \_C1}{x}\right) x \right\}$$

**2.929 ODE No. 929**

$$y'(x) = \frac{-\frac{1}{32}x^3y(x)^3 + \frac{1}{16}x^2y(x)^3 + \frac{3}{16}x^2y(x)^2 - \frac{1}{2}xy(x)^3 + \frac{y(x)^3}{4} - \frac{1}{4}xy(x)^2 - \frac{3}{8}xy(x) + \frac{y(x)}{4} + \frac{1}{4}}{xy(x)}$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.06 (sec), leaf count = 42

$$\left\{ y(x) = 18 \left( 58 \operatorname{RootOf} \left( -324 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a - \ln(x) + 12\_C1 \right) + 9x - 6 \right)^{-1} \right\}$$

**2.930 ODE No. 930**

$$y'(x) = \frac{e^{\frac{y(x)}{x}} \left( x^4 + x^2 e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} + x e^{-\frac{y(x)}{x}} y(x) + e^{-\frac{y(x)}{x}} y(x) \right)}{x(x+1)}$$

✓ **Mathematica** : cpu = 1.57469 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow -x \log \left( -\frac{c_1 + \frac{x^3}{3} - \frac{x^2}{2} + x - \log(x+1)}{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.679 (sec), leaf count = 36

$$\left\{ y(x) = -\ln \left( \frac{-2x^3 + 3x^2 + 6 \ln(1+x) - 6\_C1 - 6x}{6x} \right) x \right\}$$

**2.931 ODE No. 931**

$$y'(x) = \frac{x^6 + 3x^5y(x) + 3x^4y(x)^2 + x^3y(x)^3 - 2x^3 - 3x^2y(x) - xy(x)^2 - y(x) - 2x}{x(x^2 + xy(x) + 1)}$$

✓ **Mathematica** : cpu = 0.0222134 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow \frac{1 - x^2(\sqrt{c_1 - 2x} - 1)}{x(\sqrt{c_1 - 2x} - 1)} \right\}, \left\{ y(x) \rightarrow -\frac{1}{x\sqrt{c_1 - 2x} + x} - x \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 73

$$\left\{ y(x) = \frac{1}{x} \left( -\sqrt{-C1 - 2xx^2 - x^2 - 1} \right) \left( \sqrt{-C1 - 2x + 1} \right)^{-1}, y(x) = \frac{1}{x} \left( -\sqrt{-C1 - 2xx^2 + x^2 + 1} \right) \left( \sqrt{-C1 - 2x + 1} \right)^{-1} \right\}$$

**2.932 ODE No. 932**

$$y'(x) = \frac{e^{-\frac{3x^2}{2}} x \left( 3e^{3x^2} y(x)^3 + e^{\frac{9x^2}{2}} y(x)^3 + 18e^{3x^2} y(x)^2 + 9e^{\frac{9x^2}{2}} y(x)^2 + 27e^{3x^2} y(x) + 27e^{\frac{9x^2}{2}} y(x) + 27e^{\frac{9x^2}{2}} + 27y(x) \right)}{243y(x)}$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.396 (sec), leaf count = 54

$$\left\{ y(x) = -369 \frac{e^{3/2 x^2}}{123 + 123 e^{3/2 x^2} - 136 \operatorname{RootOf} \left( -41 x^2 - 50243409 \int^{-Z} (9248 \_a^3 - 1860867 \_a + 1860867)^{-1} d\_a \right)} \right\}$$

**2.933 ODE No. 933**

$$y'(x) = \frac{x^3 + x^3(-\log^3(x)) + x^3 \log^2(x) + 3x^2 y(x) \log^2(x) - 2x^2 y(x) \log(x) + x^2 + xy(x)^2 + xy(x) + y(x)^3 - 3xy(x)}{x^2}$$

✓ **Mathematica** : cpu = 0.13299 (sec), leaf count = 93

$$\operatorname{Solve} \left[ 87 \operatorname{RootSum} \left[ -29 \#1^3 + 3 \sqrt[3]{29} \#1 - 29 \&, \frac{\log \left( \frac{3y(x)+x-3x \log(x)}{\sqrt[3]{29} \sqrt[3]{\frac{1}{x^3} x^2}} - \#1 \right)}{\sqrt[3]{29} - 29 \#1^2} \& \right] + 9c_1 + \frac{29^{2/3}}{\sqrt[3]{\frac{1}{x^3}}} = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 39

$$\left\{ y(x) = \frac{x \left( 9 \ln(x) - 3 + 29 \operatorname{RootOf} \left( -81 \int^{-Z} (841 \_a^3 - 27 \_a + 27)^{-1} d\_a + x + 3 \_C1 \right) \right)}{9} \right\}$$

**2.934 ODE No. 934**

$$y'(x) = -\frac{x^6}{64} - \frac{3x^5}{32} + \frac{3}{16} x^4 y(x) - \frac{x^4}{8} + \frac{3}{4} x^3 y(x) + \frac{x^3}{8} - \frac{3}{4} x^2 y(x)^2 + \frac{1}{4} x^2 y(x) + \frac{x^2}{4} - \frac{3}{2} xy(x)^2 - xy(x) + y(x)^3 + y(x)^2 + \frac{x}{2}$$

✓ **Mathematica** : cpu = 0.15233 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.082 (sec), leaf count = 39

$$\left\{ y(x) = \frac{x^2}{4} + \frac{x}{2} + \operatorname{RootOf} \left( -x + 2 \int^{-Z} (2 \_a^3 + 2 \_a^2 + 1)^{-1} d\_a + \_C1 \right) \right\}$$

**2.935 ODE No. 935**

$$y'(x) = \frac{x^6}{64} - \frac{3x^5}{16} + \frac{3}{16}x^4y(x) + \frac{13x^4}{16} - \frac{3}{2}x^3y(x) - \frac{3x^3}{2} + \frac{3}{4}x^2y(x)^2 + \frac{7}{2}x^2y(x) + x^2 - 3xy(x)^2 - 2xy(x) + y(x)^3 + y(x)^2 - \frac{3}{2}$$

✓ **Mathematica** : cpu = 20.0475 (sec), leaf count = 117

$$\text{Solve} \left[ -\frac{2^{2/3}(x^2(-\log(4y(x) + (x-2)^2)) + (x-4)x \log(-4y(x) - (x-4)x) + 4x \log(4y(x) + (x-2)^2) + 4y(x)^3 + y(x)^2 - \frac{3}{2})}{9(4y(x) + (x-4)x)} \right]$$

✓ **Maple** : cpu = 0.316 (sec), leaf count = 55

$$\left\{ y(x) = \frac{e^{\text{RootOf}(\ln(e^{-Z}-4)e^{-Z} + \_C1 e^{-Z} - \_Z e^{-Z} + e^{-Z}x - 4 \ln(e^{-Z}-4) - 4 \_C1 + 4 \_Z - 4x + 4)}}{4} - 1 - \frac{x^2}{4} + x \right\}$$

**2.936 ODE No. 936**

$$y'(x) = \frac{x^6}{512} - \frac{3x^5}{256} + \frac{3}{64}x^4y(x) + \frac{5x^4}{128} - \frac{3}{16}x^3y(x) - \frac{5x^3}{64} + \frac{3}{8}x^2y(x)^2 + \frac{7}{16}x^2y(x) + \frac{x^2}{16} - \frac{3}{4}xy(x)^2 - \frac{1}{2}xy(x) + y(x)^3 + y(x)$$

✓ **Mathematica** : cpu = 0.143892 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.081 (sec), leaf count = 39

$$\left\{ y(x) = -\frac{x^2}{8} + \frac{x}{4} + \text{RootOf} \left( -x + 4 \int^{-Z} (4\_a^3 + 4\_a^2 + 3)^{-1} d\_a + \_C1 \right) \right\}$$

**2.937 ODE No. 937**

$$y'(x) = \frac{2xy(x)^3 + y(x)^3 - 2y(x) + 6xy(x) \log^2(2x+1) + 3y(x) \log^2(2x+1) + 6xy(x)^2 \log(2x+1) + 3y(x)^2 \log(2x+1)}{(2x+1)(y(x) + \log(2x+1) + 1)}$$

✓ **Mathematica** : cpu = 0.0274723 (sec), leaf count = 83

$$\left\{ \left\{ y(x) \rightarrow \frac{1 - (\sqrt{c_1 - 2x} - 1) \log(2x + 1)}{\sqrt{c_1 - 2x} - 1} \right\}, \left\{ y(x) \rightarrow -\frac{(\sqrt{c_1 - 2x} + 1) \log(2x + 1) + 1}{\sqrt{c_1 - 2x} + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 79

$$\left\{ y(x) = 1 \left( -\sqrt{\_C1 - 2x} \ln(2x + 1) - \ln(2x + 1) - 1 \right) \left( \sqrt{\_C1 - 2x} + 1 \right)^{-1}, y(x) = 1 \left( -\sqrt{\_C1 - 2x} \ln(2x + 1) - \ln(2x + 1) - 1 \right) \left( \sqrt{\_C1 - 2x} - 1 \right)^{-1} \right\}$$

**2.938 ODE No. 938**

$$y'(x) = \frac{x^6 - 3x^5 + 3x^4y(x) + 4x^4 - 6x^3y(x) - 3x^3 + 3x^2y(x)^2 + 5x^2y(x) - x^2 - 3xy(x)^2 - 2xy(x) + y(x)^3 + y(x)}{x}$$

✓ **Mathematica** : cpu = 0.28895 (sec), leaf count = 101

$$\text{Solve} \left[ 87\text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{3x^2 + 3y(x) - 3x + 1}{\sqrt[3]{29} \sqrt[3]{\frac{1}{x^3} x}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] + 9c_1 + 29^{2/3} \left( \frac{1}{x^3} \right)^{2/3} x^2 \log(x) \right]$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 39

$$\left\{ y(x) = -x^2 + x - \frac{1}{3} + \frac{29 \text{RootOf} \left( -81 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + \ln(x) + 3\_C1 \right)}{9} \right\}$$

**2.939 ODE No. 939**

$$y'(x) = \frac{x^6 + 6x^5 - 12x^4y(x) + 12x^4 - 48x^3y(x) + 16x^3 + 48x^2y(x)^2 - 48x^2y(x) + 16x^2 + 96xy(x)^2 - 32xy(x) - 16x^2 - 64y(x) + 32x - 64}{16x^2 - 64y(x) + 32x - 64}$$

✓ **Mathematica** : cpu = 0.683326 (sec), leaf count = 137

$$\text{Solve} \left[ 2\text{RootSum} \left[ \#1^4 + 4\#1^3 - 8\#1^2y(x) - 16\#1y(x) - 8\#1 + 16y(x)^2 + 16y(x) + 8\&, \frac{\#1^2(-\log(x - \#1))}{\#1^2(-\log(x - \#1)) + 8\&} \right] \right]$$

✓ **Maple** : cpu = 0.184 (sec), leaf count = 70

$$\left\{ x - \frac{4}{5} \ln \left( y(x) - \frac{x^2}{4} - \frac{x}{2} - 1 \right) + \frac{2}{5} \ln \left( 2 \left( y(x) - \frac{1}{4}x^2 - \frac{x}{2} \right)^2 + 2y(x) - \frac{x^2}{2} - x + 1 \right) - \frac{2}{5} \arctan \left( -2y(x) \right) \right\}$$

**2.940 ODE No. 940**

$$y'(x) = \frac{x^3 \log^3(x) - 3x^2y(x) \log^2(x) - x^2 + x^2 \log(x) - y(x)^3 - y(x)^2 - 2xy(x) + 3xy(x)^2 \log(x) + xy(x) \log(x)}{x(-y(x) - x + x \log(x))}$$

✓ **Mathematica** : cpu = 0.0225705 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{x((\sqrt{c_1 - 2x} + 1) \log(x) - 1)}{\sqrt{c_1 - 2x} + 1} \right\}, \left\{ y(x) \rightarrow \frac{x((\sqrt{c_1 - 2x} - 1) \log(x) + 1)}{\sqrt{c_1 - 2x} - 1} \right\} \right\}$$



✓ **Maple** : cpu = 0.056 (sec), leaf count = 63

$$\left\{ y(x) = x \left( \sqrt{-C1 - 2x \ln(x) - \ln(x) + 1} \right) \left( \sqrt{-C1 - 2x - 1} \right)^{-1}, y(x) = x \left( \sqrt{-C1 - 2x \ln(x) + \ln(x) - 1} \right) \right\}$$

**2.941 ODE No. 941**

$$y'(x) = \frac{x^6 - 12x^5 + 12x^4 y(x) + 48x^4 - 96x^3 y(x) - 72x^3 + 48x^2 y(x)^2 + 192x^2 y(x) + 32x^2 - 192xy(x)^2 - 32xy(x)}{16x^2 + 64y(x) - 64x + 64}$$

✓ **Mathematica** : cpu = 0.382097 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.063 (sec), leaf count = 35

$$\left\{ y(x) = -\frac{x^2}{4} + x + \text{RootOf} \left( -x + \int^{-Z} \frac{-a + 1}{-a^3 - a - 1} d_a + -C1 \right) \right\}$$

**2.942 ODE No. 942**

$$y'(x) = \frac{-\exp\left(\frac{2(x-y(x))^3(y(x)+x)^3}{x^2-y(x)^2-1}\right) - x^2 - 2xy(x) - y(x)^2}{\exp\left(\frac{2(x-y(x))^3(y(x)+x)^3}{x^2-y(x)^2-1}\right) - x^2 - 2xy(x) - y(x)^2}$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.63 (sec), leaf count = 43

$$\left\{ y(x) = e^{\text{RootOf} \left( -Z + \int^{(e^{-Z})^2 - 2e^{-Z}x} \left( e^2 \frac{a^3}{-a+1} + a \right)^{-1} d_a + -C1 \right)} - x \right\}$$

**2.943 ODE No. 943**

$$y'(x) = \frac{x^6 - 6x^5 + 24x^4y(x) + 12x^4 - 96x^3y(x) - 24x^3 + 192x^2y(x)^2 + 96x^2y(x) + 32x^2 - 384xy(x)^2 - 128xy(x)}{64x^2 + 512y(x) - 128x + 512}$$

✓ **Mathematica** : cpu = 0.42759 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.057 (sec), leaf count = 40

$$\left\{ y(x) = -\frac{x^2}{8} + \frac{x}{4} + \text{RootOf}\left(-x + \int^{-Z} 4 \frac{-a + 1}{4 - a^3 - a - 1} d_a + -C1\right) \right\}$$

**2.944 ODE No. 944**

$$y'(x) = \frac{a^3x^6 + 6a^2bx^5 + 12a^2x^4y(x) - 8a^2x^3 + 12ab^2x^4 + 48abx^3y(x) - 16abx^2 + 48ax^2y(x)^2 - 32axy(x) - 32a}{16ax^2 + 32bx + 64y(x) + 64}$$

✓ **Mathematica** : cpu = 1.91953 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.091 (sec), leaf count = 47

$$\left\{ y(x) = -\frac{ax^2}{4} - \frac{bx}{2} + \text{RootOf}\left(bx + 2 \int^{-Z} -\frac{b(-a + 1)}{2 - a^3 + -a b + b} d_a + 2 - C1\right) \right\}$$

**2.945 ODE No. 945**

$$y'(x) = \frac{8a^3x^3 + 12a^2x^4 + 48a^2x^2y(x) + 6ax^5 + 48ax^3y(x) - 16ax^2 + 96axy(x)^2 + x^6 + 12x^4y(x) - 8x^3 + 48x^2y(x)}{32ax + 16x^2 + 64y(x) + 64}$$

✓ **Mathematica** : cpu = 1.50167 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.08 (sec), leaf count = 41

$$\left\{ y(x) = -\frac{x^2}{4} - \frac{ax}{2} + \text{RootOf}\left(-x + \int^{-Z} 2 \frac{-a + 1}{2 - a^3 + -a a + a} d_a + -C1\right) \right\}$$

**2.946 ODE No. 946**

$$y'(x) = \frac{x \left( 12e^{-x^2} x^2 y(x)^2 + 8e^{-x^2} x^2 y(x) - 8e^{-x^2} y(x) + 4e^{-2x^2} x^2 + 8e^{-x^2} x^2 - 8e^{-x^2} + e^{-3x^2} x^6 - 6e^{-2x^2} x^4 y(x) \right)}{4e^{-x^2} x^2 - 8y(x) - 8}$$

✓ **Mathematica** : cpu = 0.0905033 (sec), leaf count = 115

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-x^2} \left( x^2 (\sqrt{c_1 - 64x^2} - 8) + 16e^{x^2} \right)}{2 (\sqrt{c_1 - 64x^2} - 8)} \right\}, \left\{ y(x) \rightarrow \frac{e^{-x^2} \left( x^2 (\sqrt{c_1 - 64x^2} + 8) - 16e^{x^2} \right)}{2 (\sqrt{c_1 - 64x^2} + 8)} \right\} \right\}$$

✓ **Maple** : cpu = 0.187 (sec), leaf count = 85

$$\left\{ y(x) = 1 \left( -2 + x^2 \left( 1 + \sqrt{-x^2 + \_C1} \right) e^{-x^2} \right) \left( 2 \sqrt{-x^2 + \_C1} + 2 \right)^{-1}, y(x) = 1 \left( 2 + x^2 \left( -1 + \sqrt{-x^2 + \_C1} \right) \right) \right\}$$

**2.947 ODE No. 947**

$$y'(x) = \frac{x^3 \sin(x) + x^2 y(x)^2 + 2x^2 y(x) \cos(x) + \frac{x^2}{2} + x^2 \cos(x) + \frac{1}{2} x^2 \cos(2x) + 2xy(x) - 2xy(x) \sin(x) + x - x \sin(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.114814 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \log(x)} + \frac{\sin(x) - x \cos(x) - 1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.39 (sec), leaf count = 44

$$\left\{ y(x) = \frac{(\cos(x) x - \sin(x) + 1) \ln(x) - \cos(x) \_C1 x + \sin(x) \_C1 + x - \_C1}{x (\_C1 - \ln(x))} \right\}$$

**2.948 ODE No. 948**

$$y'(x) = - \frac{216y(x)}{36x^2 + 4y(x)^8 + 12y(x)^7 + 33y(x)^6 + 60y(x)^5 - 24xy(x)^4 - 216y(x)^4 - 36xy(x)^3 - 252y(x)^3 - 72xy(x)^2}$$

✓ **Mathematica** : cpu = 0.377973 (sec), leaf count = 39

$$\text{Solve} \left[ \frac{36}{y(x) (2y(x)^3 + 3y(x)^2 + 6y(x) + 6) - 6x} + \log(y(x)) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 0.313 (sec), leaf count = 68

$$\left\{ y(x) = e^{\text{RootOf}(-12 \_C1 (e^{-Z})^4 - 2 (e^{-Z})^4 \_Z - 18 \_C1 (e^{-Z})^3 - 3 (e^{-Z})^3 \_Z - 36 \_C1 (e^{-Z})^2 - 6 (e^{-Z})^2 \_Z - 36 \_C1 e^{-Z} - 6 \_Z e^{-Z} + 36)}$$

**2.949 ODE No. 949**

$$y'(x) = \frac{x^6 - 3x^5 + 3x^4y(x) + x^4 - 6x^3y(x) + 2x^3 + 3x^2y(x)^2 + x^2y(x) - 3x^2 - 3xy(x)^2 + xy(x) + y(x)^3 + x}{x(x^2 + y(x) - x + 1)}$$

✓ **Mathematica** : cpu = 0.0244424 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2 \log(x)} - 1} - x^2 + x \right\}, \left\{ y(x) \rightarrow -\frac{1}{\sqrt{c_1 - 2 \log(x)} + 1} - x^2 + x \right\} \right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 81

$$\left\{ y(x) = 1 \left( (-x^2 + x) \sqrt{-C1 - 2 \ln(x)} - x^2 + x - 1 \right) \left( 1 + \sqrt{-C1 - 2 \ln(x)} \right)^{-1}, y(x) = 1 \left( (-x^2 + x) \sqrt{-C1 - 2 \ln(x)} - x^2 + x + 1 \right) \left( 1 - \sqrt{-C1 - 2 \ln(x)} \right)^{-1} \right\}$$

**2.950 ODE No. 950**

$$y'(x) = \frac{a^3x^6}{64} + \frac{3}{32}a^2bx^5 + \frac{3}{16}a^2x^4y(x) + \frac{a^2x^4}{16} + \frac{3}{16}ab^2x^4 + \frac{3}{4}abx^3y(x) + \frac{1}{4}abx^3 + \frac{3}{4}ax^2y(x)^2 + \frac{1}{2}ax^2y(x) - \frac{ax}{2} + \frac{b^3x^3}{8} + \dots$$

✓ **Mathematica** : cpu = 0.282194 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.104 (sec), leaf count = 42

$$\left\{ y(x) = -\frac{ax^2}{4} - \frac{bx}{2} + \text{RootOf} \left( -x + 2 \int^{-Z} (2\_a^3 + 2\_a^2 + b + 2)^{-1} d\_a + \_C1 \right) \right\}$$

**2.951 ODE No. 951**

$$y'(x) = \frac{a^3x^3}{8} + \frac{3a^2x^4}{16} + \frac{3}{4}a^2x^2y(x) + \frac{a^2x^2}{4} + \frac{3ax^5}{32} + \frac{3}{4}ax^3y(x) + \frac{ax^3}{4} + \frac{3}{2}axy(x)^2 + axy(x) + \frac{x^6}{64} + \frac{3}{16}x^4y(x) + \frac{x^4}{16} + \frac{3}{4}x^2y(x)^2 + \dots$$

✓ **Mathematica** : cpu = 0.250089 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.084 (sec), leaf count = 41

$$\left\{ y(x) = -\frac{x^2}{4} - \frac{ax}{2} + \text{RootOf} \left( -x + 2 \int^{-Z} (2\_a^3 + 2\_a^2 + a + 2)^{-1} d\_a + \_C1 \right) \right\}$$

**2.952 ODE No. 952**

$$y'(x) = \frac{-x^2\sqrt{x^2+y(x)^2} + xy(x)\sqrt{x^2+y(x)^2} + x^5\left(-\sqrt{x^2+y(x)^2}\right) + x^4y(x)\sqrt{x^2+y(x)^2} - x^4\sqrt{x^2+y(x)^2} + x^2}{x}$$

✓ **Mathematica** : cpu = 0.168824 (sec), leaf count = 141

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( -2e^{\frac{20c_1+4x^5+5x^4+10x^2}{10\sqrt{2}}} + e^{\frac{20c_1+4x^5+5x^4+10x^2}{5\sqrt{2}}} - 1 \right)}{2e^{\frac{20c_1+4x^5+5x^4+10x^2}{10\sqrt{2}}} + e^{\frac{20c_1+4x^5+5x^4+10x^2}{5\sqrt{2}}} - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.67 (sec), leaf count = 62

$$\left\{ \ln \left( 2 \frac{\left( \sqrt{2} (y(x))^2 + 2x^2 + y(x) + x \right) x}{y(x) - x} \right) + \frac{(4x^5 + 5x^4 + 10x^2)\sqrt{2}}{20} - C1 - \ln(x) = 0 \right\}$$

**2.953 ODE No. 953**

$$y'(x) = \frac{y(x) (x^4 \log^2(y(x)) + 2x^4 \log(x) \log(y(x)) + x^4 \log^2(x) + x^3 \log^2(y(x)) + 2x^3 \log(x) \log(y(x)) + x^3 \log^2(x))}{x}$$

✗ **Mathematica** : cpu = 1.63457 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == ((-1 + Log[x] + x*Log[x]^2 + x^3*Log[x]^2 + x^4*Log[x]^2 + Log`

✓ **Maple** : cpu = 0.454 (sec), leaf count = 145

$$\left\{ y(x) = 1 \left( x^{\frac{x^5}{4x^5+5x^4+10x^2+20-C1}} \right)^{-4} \left( x^{\frac{x^4}{4x^5+5x^4+10x^2+20-C1}} \right)^{-5} \left( x^{\frac{x^2}{4x^5+5x^4+10x^2+20-C1}} \right)^{-10} \left( x^{\frac{-C1}{4x^5+5x^4+10x^2+20-C1}} \right)$$

**2.954 ODE No. 954**

$$y'(x) = \frac{\frac{24}{5}x^{7/2}y(x) - \frac{24x^{13/2}}{25} + \frac{8x^{7/2}}{5} - 8x^{3/2} - \frac{8x^9}{125} + \frac{12}{25}x^6y(x) + \frac{4x^6}{25} - \frac{24x^4}{5} - \frac{6}{5}x^3y(x)^2 - \frac{4}{5}x^3y(x) + \frac{6x^3}{5} + 12xy}{x}$$

✓ **Mathematica** : cpu = 0.341599 (sec), leaf count = 108

$$\text{Solve} \left[ 87\text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{-6x^3+15y(x)-30\sqrt{x}+5}{5\sqrt[3]{29}\sqrt[3]{\frac{1}{x^3}x}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] + 9c_1 + 29^{2/3} \left( \frac{1}{x^3} \right)^{2/3} x^2 \log \right]$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 53

$$\left\{ y(x) = \frac{1}{45} \left( 18 x^{7/2} + 145 \operatorname{RootOf} \left( -81 \int^{-Z} (841 \_a^3 - 27 \_a + 27)^{-1} d\_a + \ln(x) + 3 \_C1 \right) \sqrt{x} - 15 \sqrt{x} + \right. \right.$$

### 2.955 ODE No. 955

$$y'(x) = \frac{-24x^{7/2}y(x) + \frac{24x^{13/2}}{5} + 14x^{7/2} + 40x^{3/2} + \frac{8x^9}{25} - \frac{12}{5}x^6y(x) + \frac{12x^6}{5} + 24x^4 + 6x^3y(x)^2 - 6x^3y(x) - 6x^3 -}{x(2x^3 - 5y(x) + 10\sqrt{x} - 5)}$$

✓ **Mathematica** : cpu = 0.0397453 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow -\frac{125}{\sqrt{c_1 - 31250 \log(x)} + 125} + \frac{2x^3}{5} + 2\sqrt{x} \right\}, \left\{ y(x) \rightarrow \frac{125}{\sqrt{c_1 - 31250 \log(x)} - 125} + \frac{2x^3}{5} + 2\sqrt{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 101

$$\left\{ y(x) = 1 \left( (2x^3 + 10\sqrt{x}) \sqrt{-C1 - 2 \ln(x) - 2x^3 - 10\sqrt{x} + 5} \left( 5 \sqrt{-C1 - 2 \ln(x) - 5} \right)^{-1}, y(x) = 1 \left( (2x^3 + \right. \right.$$

### 2.956 ODE No. 956

$$y'(x) = \frac{y(x) \left( y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} x^{\frac{2}{\log(x)+1}+2} + y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} \log^2(x) x^{\frac{2}{\log(x)+1}+2} + 2y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} \log(x) x^{\frac{2}{\log(x)+1}+2} - e^{\frac{2 \log^2(x)}{\log(x)+1}} \right)}{x(\log(x) + 1)}$$

✓ **Mathematica** : cpu = 0.229734 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\left( c_1 e^{\frac{x^4}{4}} + 1 \right) (\log(x) + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.113 (sec), leaf count = 79

$$\left\{ y(x) = \frac{1}{\ln(x) + 1} e^{-\frac{x^4}{4}} \left( x^{-2 \frac{\ln(x)}{\ln(x)+1}} (\ln(x) + 1) e^{\frac{(-4 \ln(x) - 4) \ln(\ln(x)+1) - x^4 \ln(x) - x^4 + 8 (\ln(x))^2}{4 \ln(x) + 4}} + \_C1 \right)^{-1} \right\}$$

**2.957 ODE No. 957**

$$y'(x) = \frac{y(x) \left( y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} x^{\frac{2}{\log(x)+1}+3} + y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} \log^2(x) x^{\frac{2}{\log(x)+1}+3} + 2y(x) e^{\frac{2 \log^2(x)}{\log(x)+1}} \log(x) x^{\frac{2}{\log(x)+1}+3} - e^{\frac{2 \log^2(x)}{\log(x)+1}} \right)}{x(\log(x) + 1)}$$

✓ **Mathematica** : cpu = 0.206588 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{\left( c_1 e^{\frac{x^5}{5}} + 1 \right) (\log(x) + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.057 (sec), leaf count = 79

$$\left\{ y(x) = \frac{1}{\ln(x) + 1} e^{-\frac{x^5}{5}} \left( x^{-2 \frac{\ln(x)}{\ln(x)+1}} (\ln(x) + 1) e^{\frac{(-5 \ln(x)-5) \ln(\ln(x)+1) - x^5 \ln(x) - x^5 + 10 (\ln(x))^2}{5 \ln(x)+5}} + -C1 \right)^{-1} \right\}$$

**2.958 ODE No. 958**

$$y'(x) = \frac{2xy(x)^3 + y(x)^3 + 2xy(x)^2 + y(x)^2 + 6xy(x) \log^2(2x + 1) + 3y(x) \log^2(2x + 1) + 6xy(x)^2 \log(2x + 1) + 3y(x) \log(2x + 1)}{x^2}$$

✓ **Mathematica** : cpu = 0.0968764 (sec), leaf count = 80

$$\text{Solve} \left[ 87\text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{3y(x)+3\log(2x+1)+1}{\sqrt[3]{29}} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \& \right] + 9c_1 + 29^{2/3}x = 0, y(x) \right]$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 40

$$\left\{ y(x) = -\ln(2x + 1) - \frac{1}{3} + \frac{29 \text{RootOf} \left( -81 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + x + 3\_C1 \right)}{9} \right\}$$

**2.959 ODE No. 959**

$$y'(x) = \frac{\csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{x}\right) \left(x^3 \sin\left(\frac{y(x)}{2x}\right) \sin\left(\frac{y(x)}{x}\right) \cos\left(\frac{y(x)}{2x}\right) - \frac{1}{2}y(x) \sin\left(\frac{y(x)}{x}\right) + \frac{1}{2}y(x) \sin\left(\frac{y(x)}{2x}\right)\right)}{x}$$

✓ **Mathematica** : cpu = 0.0496633 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow x \sin^{-1} \left( e^{C1 + \frac{x^2}{2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 16

$$\left\{ y(x) = \frac{\arccos\left(e^{x^2} - C1 + 1\right) x}{2} \right\}$$

**2.960 ODE No. 960**

$$y'(x) = \frac{\csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{x}\right) \left(x^2 \sin\left(\frac{y(x)}{2x}\right) \sin\left(\frac{y(x)}{x}\right) \cos\left(\frac{y(x)}{2x}\right) - \frac{1}{2}y(x) \sin\left(\frac{y(x)}{x}\right) + \frac{1}{2}y(x) \sin\left(\frac{y(x)}{2x}\right)\right)}{x}$$

✓ **Mathematica** : cpu = 0.0371157 (sec), leaf count = 14

$$\left\{ \left\{ y(x) \rightarrow x \sin^{-1} \left( e^{C1 + x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 16

$$\left\{ y(x) = \frac{\arccos\left((e^x)^2 - C1 + 1\right) x}{2} \right\}$$

**2.961 ODE No. 961**

$$y'(x) = \frac{\exp(-2x^6 + 6x^4y(x)^2 + 2x^4 - 6x^2y(x)^4 - 4x^2y(x)^2 + 2y(x)^6 + 2y(x)^4 + 2) + x^2 + 2xy(x) + y(x)^2}{-\exp(-2x^6 + 6x^4y(x)^2 + 2x^4 - 6x^2y(x)^4 - 4x^2y(x)^2 + 2y(x)^6 + 2y(x)^4 + 2) + x^2 + 2xy(x) + y(x)^2}$$

✗ **Mathematica** : cpu = 300.003 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.451 (sec), leaf count = 45

$$\left\{ y(x) = e^{\text{RootOf}\left(-_Z + \int (e^{-Z})^2 - 2e^{-Z} x (e^2 - a^3 + 2 - a^2 + 2 + \_a)^{-1} d\_a + \_C1\right)} - x \right\}$$





**2.964 ODE No. 964**

$$y'(x) = -\frac{a^8x^6 - 4a^6x^6 - 3a^6x^4y(x)^2 - 2a^6x^4 + 6a^4x^6 + 9a^4x^4y(x)^2 + 6a^4x^4 + 3a^4x^2y(x)^4 + 4a^4x^2y(x)^2 - 4a^2x^2}{a^8x^6 - 4a^6x^6 - 3a^6x^4y(x)^2 - 2a^6x^4 + 6a^4x^6 + 9a^4x^4y(x)^2 + 6a^4x^4 + 3a^4x^2y(x)^4 + 4a^4x^2y(x)^2 - 4a^2x^2}$$

✓ **Mathematica** : cpu = 4.88308 (sec), leaf count = 13

$$\{ \{ y(x) \rightarrow (a^2 - 1) c_1 \} \}$$

✓ **Maple** : cpu = 3.314 (sec), leaf count = 80

$$\left\{ \frac{y(x)}{(a-1)(a+1)} + 4 \frac{1}{a^4 - 2a^2 + 1} \sum_{R=RootOf(-Z^3+2-Z^2+8)} \frac{\ln(-a^2x^2 + (y(x))^2 + x^2 - R)}{3R^2 + 4R} - C1 = 0 \right\}$$

**2.965 ODE No. 965**

$$y'(x) = \frac{\csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{x}\right) \left(x^4 \sin\left(\frac{y(x)}{2x}\right) \sin\left(\frac{y(x)}{x}\right) \cos\left(\frac{y(x)}{2x}\right) + x^3 \sin\left(\frac{y(x)}{2x}\right) \sin\left(\frac{y(x)}{x}\right) \cos\left(\frac{y(x)}{2x}\right)\right)}{a^8x^6 - 4a^6x^6 - 3a^6x^4y(x)^2 - 2a^6x^4 + 6a^4x^6 + 9a^4x^4y(x)^2 + 6a^4x^4 + 3a^4x^2y(x)^4 + 4a^4x^2y(x)^2 - 4a^2x^2}$$

✓ **Mathematica** : cpu = 0.0598215 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow x \sin^{-1} \left( x e^{c_1 + \frac{x^3}{3} + \frac{x^2}{2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.126 (sec), leaf count = 25

$$\left\{ y(x) = \frac{x}{2} \arccos \left( e^{\frac{2x^3}{3}} e^{x^2} C1 x^2 + 1 \right) \right\}$$

**2.966 ODE No. 966**

$$y'(x) = -\frac{216x^3 - 216x^2y(x)^4 - 324x^2y(x)^3 - 648x^2y(x)^2 - 648x^2y(x) + 216x^2 - 8y(x)^{12} - 36y(x)^{11} - 126y(x)^{10}}{216x^3 - 216x^2y(x)^4 - 324x^2y(x)^3 - 648x^2y(x)^2 - 648x^2y(x) + 216x^2 - 8y(x)^{12} - 36y(x)^{11} - 126y(x)^{10}}$$

✓ **Mathematica** : cpu = 0.73691 (sec), leaf count = 1

\$Aborted

✓ **Maple** : cpu = 0.829 (sec), leaf count = 50

$$\left\{ y(x) = e^{RootOf(-Z-6 \int^{-1/3} (e^{-Z})^4 - 1/2 (e^{-Z})^3 - (e^{-Z})^2 + x - e^{-Z} (-a^3 + a^2 + 1)^{-1} d_a + C1)} \right\}$$

**2.967 ODE No. 967**

$$y'(x) = -\frac{x(64x^9 - 288x^8y(x) - 96x^8 + 432x^7y(x)^2 + 288x^7y(x) - 144x^7 - 216x^6y(x)^3 - 216x^6y(x)^2 - 288x^6y(x))}{(x^2+1)^3}$$

✓ **Mathematica** : cpu = 101.916 (sec), leaf count = 145

$$\text{Solve} \left[ -\frac{29}{3} \text{RootSum} \left[ -29\#1^3 + 3\sqrt[3]{29}\#1 - 29\&, \frac{\log \left( \frac{\left( \frac{x^3}{(x^2+1)^3} \right)^{2/3} (x^2+1)(-4x^3+6x^2y(x)+2x^2+6y(x)+5)}{2\sqrt[3]{29}x^2} - \#1 \right)}{\sqrt[3]{29} - 29\#1^2} \right] \& \right]$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 91

$$\left\{ y(x) = \frac{58 \text{RootOf} \left( -162 \int^{-Z} (841\_a^3 - 27\_a + 27)^{-1} d\_a + \ln(x^2 + 1) + 6\_C1 \right) x^2 + 12x^3 - 6x^2 + 58R}{18x^2 + 18} \right\}$$

**2.968 ODE No. 968**

$$y'(x) = \frac{\csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{x}\right) \left(x^4 \sin\left(\frac{y(x)}{2x}\right) \sin\left(\frac{y(x)}{x}\right) \cos\left(\frac{y(x)}{2x}\right) - \frac{1}{2}xy(x) \sin\left(\frac{y(x)}{x}\right) - \frac{1}{2}y(x) \sin\left(\frac{y(x)}{x}\right)\right)}{(x^2+1)^3}$$

✓ **Mathematica** : cpu = 0.0854174 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow x \sin^{-1} \left( (x+1)e^{c_1 + \frac{x^2}{2} - x - \frac{3}{2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 29

$$\left\{ y(x) = \frac{x}{2} \arccos \left( \frac{-C1(1+x)^2 e^{x^2} + (e^x)^2}{(e^x)^2} \right) \right\}$$

**2.969 ODE No. 969**

$$y'(x) = \frac{\csc\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{2x}\right) \sec\left(\frac{y(x)}{x}\right) \left(-\frac{1}{2}xy(x) \sin\left(\frac{y(x)}{x}\right) - \frac{1}{2}y(x) \sin\left(\frac{y(x)}{x}\right) + x \sin\left(\frac{y(x)}{2x}\right) \sin\left(\frac{y(x)}{x}\right) \cos\left(\frac{y(x)}{2x}\right)\right)}{\dots}$$

✓ **Mathematica** : cpu = 0.0583077 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow x \sin^{-1} \left( \frac{e^{c_1 x}}{x+1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 26

$$\left\{ y(x) = \frac{x}{2} \arccos \left( \frac{1 + (-C_1 + 1)x^2 + 2x}{(1+x)^2} \right) \right\}$$

**2.970 ODE No. 970**

$$y'(x) = -\frac{216x^3 - 216x^2y(x)^4 - 324x^2y(x)^3 - 648x^2y(x)^2 - 648x^2y(x) - 8y(x)^{12} - 36y(x)^{11} - 126y(x)^{10} - 315y(x)^9 - 126y(x)^8 - 36y(x)^7 - 6y(x)^6 - 6y(x)^5 - 6y(x)^4 - 6y(x)^3 - 6y(x)^2 - 6y(x) - 6}{\dots}$$

✓ **Mathematica** : cpu = 0.529505 (sec), leaf count = 66

$$\text{Solve} \left[ \frac{36(2y(x)^4 + 3y(x)^3 + 6y(x)^2 + 6y(x) - 6x - 3)}{(y(x)(2y(x)^3 + 3y(x)^2 + 6y(x) + 6) - 6x)^2} + \log(y(x)) = c_1, y(x) \right]$$

✓ **Maple** : cpu = 1.079 (sec), leaf count = 181

$$\left\{ \frac{1}{6-C_1 - 6 \ln(y(x))} \left( -6 \sqrt{3 \ln(y(x)) - 3-C_1 + 9} + \left( 2(y(x))^4 + 3(y(x))^3 + 6(y(x))^2 - 6x + 6y(x) \right) \ln(y(x)) \right) \right\}$$

**2.971 ODE No. 971**

$$y'(x) = \frac{(xy(x) + 1)^3}{x^5}$$

✗ **Mathematica** : cpu = 300.46 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.35 (sec), leaf count = 86

$$\left\{ y(x) = \frac{\sqrt{3}}{6x} \left( 3 \tan \left( \text{RootOf} \left( -18x^3(-x^{-6})^{2/3} - 6-Z\sqrt{3} - \ln \left( \frac{(\sqrt{3} + \tan(-Z))^6}{(\tan(-Z))^2 + 1} \right) + 18-C_1 \right) \right) \right) x^3 \sqrt{\dots} \right\}$$

**2.972 ODE No. 972**

$$y'(x) = \frac{x(-2x^4 + 2x^2y(x) - x^2 + 1)}{y(x) - x^2}$$

✓ **Mathematica** : cpu = 0.0277161 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( W \left( -e^{c_1 + x^4 - 2x^2 - 1} \right) + 1 \right) + x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 27

$$\left\{ y(x) = x^2 + \frac{1}{2} \text{lambertW} \left( -2 \frac{e^{x^4} - C_1 e^{-1}}{(e^{x^2})^2} \right) + \frac{1}{2} \right\}$$

**2.973 ODE No. 973**

$$y'(x) = e^{-2bx}y(x) \left( e^{bx}y(x) + e^{2bx} + y(x)^2 \right)$$

✓ **Mathematica** : cpu = 1.03914 (sec), leaf count = 1

$$\{ \}$$

✓ **Maple** : cpu = 0.451 (sec), leaf count = 134

$$\left\{ y(x) = -\frac{1}{2} \tan \left( \text{RootOf} \left( -\ln \left( (4(\tan(\_Z))^2 b - 3(\tan(\_Z))^2 + 4b - 3) \left( \tan(\_Z) \sqrt{-(e^{bx})^2 (4b - 3) + e^{bx}} \right) \right) \right) \right) \right\}$$

**2.974 ODE No. 974**

$$y'(x) = -x^6 + 3x^4y(x) - 3x^2y(x)^2 + y(x)^3 + 2x$$

✓ **Mathematica** : cpu = 0.0111819 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow x^2 - \frac{1}{\sqrt{c_1 - 2x}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2x}} + x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 57

$$\left\{ y(x) = 1 \left( x^2 \sqrt{2\_C1 - 2x} - 1 \right) \frac{1}{\sqrt{2\_C1 - 2x}}, y(x) = 1 \left( x^2 \sqrt{2\_C1 - 2x} + 1 \right) \frac{1}{\sqrt{2\_C1 - 2x}} \right\}$$

**2.975 ODE No. 975**

$$y'(x) = \frac{x^6}{27} + \frac{1}{3}x^4y(x) + x^2y(x)^2 + y(x)^3 - \frac{2x}{3}$$

✓ **Mathematica** : cpu = 0.0127396 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{\sqrt{c_1 - 2x}} - \frac{x^2}{3} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2x}} - \frac{x^2}{3} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 59

$$\left\{ y(x) = -\frac{1}{3} \left( x^2 \sqrt{-54\_C1 - 2x} - 3 \right) \frac{1}{\sqrt{-54\_C1 - 2x}}, y(x) = -\frac{1}{3} \left( x^2 \sqrt{-54\_C1 - 2x} + 3 \right) \frac{1}{\sqrt{-54\_C1 - 2x}} \right\}$$

**2.976 ODE No. 976**

$$y'(x) = \frac{y(x) (x^7y(x)^2 + x^4y(x) + x - 3)}{x}$$

✓ **Mathematica** : cpu = 0.226211 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.872 (sec), leaf count = 57

$$\left\{ y(x) = \frac{1}{2x^3} \left( \sqrt{3} \tan \left( \text{RootOf} \left( -\sqrt{3} \ln \left( \frac{9 (\tan(\_Z))^2 + 9}{7 (\sqrt{3} - 3 \tan(\_Z))^2} \right) + 3 \sqrt{3} \_C1 - 2 \sqrt{3}x - 2 \_Z \right) \right) - 1 \right) \right\}$$

**2.977 ODE No. 977**

$$y'(x) = e^{2x^2} xy(x) (e^{-x^2} y(x) + e^{-2x^2} + y(x)^2)$$

✓ **Mathematica** : cpu = 1.17609 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.217 (sec), leaf count = 122

$$\left\{ y(x) = \frac{1}{2e^{x^2}} \left( \sqrt{11} \tan \left( \text{RootOf} \left( -4 \sqrt{11}x^2 + 8 \ln \left( -\frac{36 \sqrt{11}}{11} + 36 \tan(\_Z) \right) \right) \sqrt{11} - 4 \ln \left( \frac{2592 \sqrt{11} (e^{x^2})^2}{25} \right) \right) \right\}$$

**2.978 ODE No. 978**

$$y'(x) = \frac{y(x)(x^2 + xy(x) + y(x)^2 + x)}{x^2}$$

✓ **Mathematica** : cpu = 0.188581 (sec), leaf count = 58

$$\text{Solve} \left[ \log \left( \frac{y(x)}{x} \right) = c_1 + \frac{1}{2} \log \left( \frac{x^2 + xy(x) + y(x)^2}{x^2} \right) + \frac{\tan^{-1} \left( \frac{2y(x)+x}{\sqrt{3}x} \right)}{\sqrt{3}} + x, y(x) \right]$$

✓ **Maple** : cpu = 0.191 (sec), leaf count = 71

$$\left\{ y(x) = -\frac{x}{2} + \frac{\sqrt{3}x}{2} \tan \left( \text{RootOf} \left( -\sqrt{3} \ln(3) - \sqrt{3} \ln \left( \frac{4}{3 + 3(\tan(_Z))^2} \right) - 2\sqrt{3} \ln \left( -1/6\sqrt{3} + 1/2 \tan(_Z) \right) \right) \right.$$

**2.979 ODE No. 979**

$$y'(x) = \frac{-x^3 + 3x^2y(x) - 3xy(x)^2 + y(x)^3 + x}{x}$$

✓ **Mathematica** : cpu = 0.0127904 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow x - \frac{1}{\sqrt{c_1 - 2 \log(x)}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2 \log(x)}} + x \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 57

$$\left\{ y(x) = 1 \left( x \sqrt{2\_C1 - 2 \ln(x)} - 1 \right) \frac{1}{\sqrt{2\_C1 - 2 \ln(x)}}, y(x) = 1 \left( x \sqrt{2\_C1 - 2 \ln(x)} + 1 \right) \frac{1}{\sqrt{2\_C1 - 2 \ln(x)}} \right\}$$

**2.980 ODE No. 980**

$$y'(x) = \frac{x^3y(x)^3 + 6x^2y(x)^2 + 12xy(x) + 2x + 8}{x^3}$$

✓ **Mathematica** : cpu = 0.0132951 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow -\frac{\frac{x}{\sqrt{c_1 - 2x}} + 2}{x} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2x}} - \frac{2}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 35

$$\left\{ y(x) = -\frac{1}{\sqrt{-C1 - 2x}} - 2x^{-1}, y(x) = \frac{1}{\sqrt{-C1 - 2x}} - 2x^{-1} \right\}$$

**2.981 ODE No. 981**

$$y'(x) = \frac{a^3 x^3 y(x)^3 + 3a^2 x^2 y(x)^2 + a^2 x + 3axy(x) + 1}{a^3 x^3}$$

✓ **Mathematica** : cpu = 0.0178718 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{ax} - \frac{1}{\sqrt{c_1 - 2x}} \right\}, \left\{ y(x) \rightarrow \frac{1}{\sqrt{c_1 - 2x}} - \frac{1}{ax} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 41

$$\left\{ y(x) = -\frac{1}{\sqrt{-C1 - 2x}} - \frac{1}{ax}, y(x) = \frac{1}{\sqrt{-C1 - 2x}} - \frac{1}{ax} \right\}$$

**2.982 ODE No. 982**

$$y'(x) = \frac{1}{2} e^{-\frac{x^2}{2}} y(x) \left( 2e^{\frac{x^2}{4}} y(x) + 2e^{\frac{x^2}{2}} + e^{\frac{x^2}{2}} x + 2y(x)^2 \right)$$

✓ **Mathematica** : cpu = 0.646207 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.548 (sec), leaf count = 145

$$\left\{ -\frac{2}{3} \ln \left( -6 + \left( 18 y(x) e^{-1/2 x^2} + 6 e^{-1/4 x^2} \right) e^{\frac{x^2}{4}} \right) + \frac{1}{3} \ln \left( 36 + \frac{324}{7} \left( y(x) e^{-\frac{x^2}{2}} + \frac{1}{3} e^{-\frac{x^2}{4}} \right)^2 \left( e^{\frac{x^2}{4}} \right)^2 + \frac{1}{7} (108 y(x) e^{-\frac{x^2}{2}} + 36 e^{-\frac{x^2}{4}}) \right) \right\}$$

**2.983 ODE No. 983**

$$y'(x) = \frac{-x^3 + 3x^2 y(x) + x^2 - 3xy(x)^2 + y(x)^3}{(x-1)(x+1)}$$

✗ **Mathematica** : cpu = 300.002 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.409 (sec), leaf count = 233

$$\left\{ y(x) = \frac{\sqrt{3}}{2} \left( \frac{x^2 - 1}{3} \left( 3 \tan \left( \text{RootOf} \left( -18 \ln(1+x) \left( \frac{1}{(1+x)^3 (x-1)^3} \right)^{2/3} x^4 + 18 \ln(x-1) \left( \frac{1}{(1+x)^3 (x-1)^3} \right)^{2/3} \right) \right) \right)^{2/3} \right\}$$



2.984 ODE No. 984

$$y'(x) = \frac{e^{-2x}(x-1)y(x)(x^2y(x)^2 + e^xxy(x) + e^{2x})}{x}$$

✗ **Mathematica** : cpu = 300.986 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.322 (sec), leaf count = 40

$$\left\{ y(x) = \frac{1}{9x} e^{\text{RootOf}\left(-e^{-Z} \ln\left(\frac{x(e^{-Z}+9)}{2}\right) + 3\_C1 e^{-Z} + \_Z e^{-Z} + e^{-Z}x+9\right) + x} \right\}$$

2.985 ODE No. 985

$$y'(x) = \frac{(xy(x) + 1)(x^2y(x)^2 + x^2y(x) + x^2 + 2xy(x) + x + 1)}{x^5}$$

✓ **Mathematica** : cpu = 0.426885 (sec), leaf count = 1

{}

✓ **Maple** : cpu = 0.046 (sec), leaf count = 43

$$\left\{ y(x) = \frac{17 \text{RootOf}\left(162 \int^{-Z} (289\_a^3 + 54\_a - 54)^{-1} d\_ax + 3\_C1 x + 2\right) x - 3x - 9}{9x} \right\}$$

2.986 ODE No. 986

$$y'(x) = \frac{-x^3 \log^3(x) + 3x^2y(x) \log^2(x) + x^2 + y(x)^3 + xy(x) - 3xy(x)^2 \log(x)}{x^2}$$

✓ **Mathematica** : cpu = 0.0166328 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow x \left( \log(x) - \frac{1}{\sqrt{c_1 - 2x}} \right) \right\}, \left\{ y(x) \rightarrow x \left( \frac{1}{\sqrt{c_1 - 2x}} + \log(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 36

$$\left\{ y(x) = -x \frac{1}{\sqrt{-C1 - 2x}} + x \ln(x), y(x) = x \frac{1}{\sqrt{-C1 - 2x}} + x \ln(x) \right\}$$

**2.987 ODE No. 987**

$$y'(x) = \frac{y(x)}{x} - F(x) (y(x)^2 - ax^2)$$

✓ **Mathematica** : cpu = 0.0994344 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \sqrt{ax} \tanh \left( \sqrt{a} \left( \int_1^x K[1] F(K[1]) dK[1] + c_1 \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 22

$$\left\{ y(x) = \tanh \left( \sqrt{a} \left( -C1 + \int F(x) x dx \right) \right) x \sqrt{a} \right\}$$

**2.988 ODE No. 988**

$$y'(x) = \frac{y(x)}{x} - F(x) (-x^2 - 2xy(x) + y(x)^2)$$

✓ **Mathematica** : cpu = 0.378247 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow \frac{x(-(\sqrt{2}-1) \exp(2\sqrt{2}(\int_1^x K[1](-F(K[1])) dK[1] + c_1)) + 1 + \sqrt{2})}{\exp(2\sqrt{2}(\int_1^x K[1](-F(K[1])) dK[1] + c_1)) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 29

$$\left\{ y(x) = \frac{x(\sqrt{2} + 2 \tanh((\_C1 + \int F(x) x dx) \sqrt{2})) \sqrt{2}}{2} \right\}$$

**2.989 ODE No. 989**

$$y'(x) = \frac{y(x)}{x} - F(x) (-ay(x)^2 - bx^2)$$

✓ **Mathematica** : cpu = 0.102763 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{bx} \tan \left( \sqrt{a}\sqrt{b} \left( \int_1^x K[1] F(K[1]) dK[1] + c_1 \right) \right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 29

$$\left\{ y(x) = \frac{x}{a} \tan \left( \sqrt{ab} \left( -C1 + \int F(x) x dx \right) \right) \sqrt{ab} \right\}$$

**2.990 ODE No. 990**

$$y'(x) = 2x - F(x) (-x^4 + 2x^2y(x) - y(x)^2 + 1)$$

✓ **Mathematica** : cpu = 0.466894 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow -\frac{2e^{\int_1^x 2F(K[5]) dK[5]}}{e^{\text{Integrate}[2F(K[5]),\{K[5],1,x\},\text{Assumptions}\rightarrow\text{True}] - 2c_1}} + x^2 + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.743 (sec), leaf count = 44

$$\left\{ y(x) = \frac{-x^2 \left( e^{\int F(x) dx} \right)^2 + \_C1 x^2 + \left( e^{\int F(x) dx} \right)^2 + \_C1}{-\left( e^{\int F(x) dx} \right)^2 + \_C1} \right\}$$

**2.991 ODE No. 991**

$$y'(x) = \frac{y(x)}{x} - F(x) (x^2 + 2xy(x) - y(x)^2)$$

✓ **Mathematica** : cpu = 0.24759 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{x(-(\sqrt{2}-1) \exp(2\sqrt{2}(\int_1^x K[1]F(K[1]) dK[1] + c_1)) + 1 + \sqrt{2})}{\exp(2\sqrt{2}(\int_1^x K[1]F(K[1]) dK[1] + c_1)) + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 29

$$\left\{ y(x) = \frac{x(\sqrt{2} - 2 \tanh((\_C1 + \int F(x) x dx) \sqrt{2})) \sqrt{2}}{2} \right\}$$

**2.992 ODE No. 992**

$$y'(x) = \frac{y(x)}{x} - F(x) (-x^3 - 7xy(x)^2)$$

✓ **Mathematica** : cpu = 0.108445 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{x \tan(\sqrt{7}(\int_1^x K[1]^2 F(K[1]) dK[1] + c_1))}{\sqrt{7}} \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 25

$$\left\{ y(x) = \frac{\tan((\int F(x) x^2 dx + \_C1) \sqrt{7}) x \sqrt{7}}{7} \right\}$$

**2.993 ODE No. 993**

$$y'(x) = \frac{y(x)}{x \log(x)} - F(x) (-y(x)^2 - 2y(x) \log(x) - \log^2(x))$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.033 (sec), leaf count = 35

$$\left\{ y(x) = -\frac{\ln(x) \left( \int -2 \ln(x) F(x) dx - \_C1 - 2 \right)}{\int -2 \ln(x) F(x) dx - \_C1} \right\}$$

**2.994 ODE No. 994**

$$y'(x) = \frac{y(x)}{x \log(x)} - x^3 (-y(x)^2 - 2y(x) \log(x) - \log^2(x))$$

✓ **Mathematica** : cpu = 0.138185 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{\log(x) (-16(c_1 + 1) + x^4 - 4x^4 \log(x))}{16c_1 - x^4 + 4x^4 \log(x)} \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 43

$$\left\{ y(x) = -\frac{\ln(x) (4x^4 \ln(x) - x^4 + 8\_C1 + 16)}{4x^4 \ln(x) - x^4 + 8\_C1} \right\}$$

**2.995 ODE No. 995**

$$y'(x) = (y(x) - e^x)^2 + e^x$$

✓ **Mathematica** : cpu = 0.0176696 (sec), leaf count = 17

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - x} + e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.146 (sec), leaf count = 14

$$\{ y(x) = e^x + (\_C1 - x)^{-1} \}$$

**2.996 ODE No. 996**

$$y'(x) = \frac{(y(x) - \text{Si}(x))^2 + \sin(x)}{x}$$

✗ **Mathematica** : cpu = 59.2191 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] == (Sin[x] + (-SinIntegral[x] + y[x])^2)/x, y[x], x]`

✓ **Maple** : cpu = 0.089 (sec), leaf count = 15

$$\left\{ y(x) = \text{Si}(x) + (_C1 - \ln(x))^{-1} \right\}$$

**2.997 ODE No. 997**

$$y'(x) = (y(x) + \cos(x))^2 + \sin(x)$$

✓ **Mathematica** : cpu = 0.0300991 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - x} - \cos(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 16

$$\left\{ y(x) = -\cos(x) + (_C1 - x)^{-1} \right\}$$

**2.998 ODE No. 998**

$$y'(x) = \frac{(-\text{Ci}(x) + y(x) - \log(x))^2 + \cos(x)}{x}$$

✓ **Mathematica** : cpu = 0.42943 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow -\frac{2x^2}{x^2 - 2c_1} + \text{Ci}(x) + \log(x) + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.498 (sec), leaf count = 27

$$\left\{ y(x) = \ln(x) + \text{Ci}(x) + \frac{-_C1 x^2 + 1}{-_C1 x^2 + 1} \right\}$$

**2.999 ODE No. 999**

$$y'(x) = \frac{(y(x) - x + \log(x + 1))^2 + x}{x + 1}$$

✓ **Mathematica** : cpu = 0.0247035 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_1 - \log(x + 1)} + x - \log(x + 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 36

$$\left\{ y(x) = \frac{-(\ln(1 + x))^2 + (x - \_C1) \ln(1 + x) + \_C1 x - 1}{\ln(1 + x) + \_C1} \right\}$$

**2.1000 ODE No. 1000**

$$y'(x) = \frac{x^3 + 2x^2y(x) - xy(x) - y(x)^2 + xy(x) \log(x)}{x^2(x + \log(x))}$$

✗ **Mathematica** : cpu = 300.04 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.187 (sec), leaf count = 19

$$\left\{ y(x) = \frac{x(\_C1 x - 1)}{\_C1 \ln(x) + 1} \right\}$$

**2.1001 ODE No. 1001**

$$y''(x) = 0$$

✓ **Mathematica** : cpu = 0.00409005 (sec), leaf count = 12

$$\{\{y(x) \rightarrow c_2x + c_1\}\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 9

$$\{y(x) = \_C1 x + \_C2\}$$

**Hand solution**

$$y'' = 0$$

Integration twice gives

$$y(x) = c_1x + c_2$$

**2.1002 ODE No. 1002**

$$y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.00499081 (sec), leaf count = 16

$$\{\{y(x) \rightarrow c_2 \sin(x) + c_1 \cos(x)\}\}$$

✓ **Maple** : cpu = 0.005 (sec), leaf count = 13

$$\{y(x) = \_C1 \sin(x) + \_C2 \cos(x)\}$$

**Hand solution**

$$y'' + y = 0$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\lambda^2 e^{\lambda x} + e^{\lambda x} = 0$$

$$\lambda^2 + 1 = 0$$

Hence  $\lambda = \pm i$ , therefore the solution is

$$\begin{aligned} y &= Ae^{ix} + Be^{-ix} \\ &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\ &= \cos x(A + B) + \sin x(Ai - iB) \\ &= \cos x(A + B) + \sin x(i(A - B)) \end{aligned}$$

Let  $A + B = c_1, i(A - B) = c_2$  hence

$$y = c_1 \cos x + c_2 \sin x$$

**2.1003 ODE No. 1003**

$$-\sin(nx) + y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.115598 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2(n^2 - 1) \sin(x) + c_1(n^2 - 1) \cos(x) - \sin(nx)}{n^2 - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 26

$$\left\{ y(x) = \sin(x) \_C2 + \cos(x) \_C1 - \frac{\sin(nx)}{n^2 - 1} \right\}$$

**Hand solution**

$$y'' + y = \sin nx \tag{1}$$

We start by solving the homogeneous equation

$$y'' + y = 0$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\begin{aligned} \lambda^2 e^{\lambda x} + e^{\lambda x} &= 0 \\ \lambda^2 + 1 &= 0 \end{aligned}$$

Hence  $\lambda = \pm i$ , therefore the solution is

$$\begin{aligned} y_h &= Ae^{ix} + Be^{-ix} \\ &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\ &= \cos x(A + B) + \sin x(Ai - iB) \\ &= \cos x(A + B) + \sin x(i(A - B)) \end{aligned}$$

Let  $A + B = c_1, i(A - B) = c_2$  hence

$$y_h = c_1 \cos x + c_2 \sin x$$

Now we solve for the particular solution using variation of parameters. Let

$$\begin{aligned} y_p &= u_1(x) \cos x + u_2(x) \sin x \\ y'_p &= u'_1 \cos x - u_1 \sin x + u'_2 \sin x + u_2 \cos x \\ &= u_2 \cos x - u_1 \sin x + u'_1 \cos x + u'_2 \sin x \end{aligned}$$

Let first condition be

$$u'_1 \cos x + u'_2 \sin x = 0 \tag{2}$$



Hence

$$\begin{aligned}y_p' &= u_2 \cos x - u_1 \sin x \\y_p'' &= u_2' \cos x - u_2 \sin x - u_1' \sin x - u_1 \cos x\end{aligned}$$

Substituting in (1) gives

$$\begin{aligned}y_p'' + y_p &= \sin nx \\u_2' \cos x - u_2 \sin x - u_1' \sin x - u_1 \cos x + u_1 \cos x + u_2 \sin x &= \sin nx \\u_2' \cos x - u_1' \sin x &= \sin nx\end{aligned}\tag{3}$$

So we have two equations (1)(2) to solve for  $u_1, u_2$

$$\begin{aligned}u_1' \cos x + u_2' \sin x &= 0 \\u_2' \cos x - u_1' \sin x &= \sin nx\end{aligned}$$

From the first equation

$$u_1' = -u_2' \frac{\sin x}{\cos x}\tag{4}$$

Substituting in the second equation

$$\begin{aligned}u_2' \cos x - \left(-u_2' \frac{\sin x}{\cos x}\right) \sin x &= \sin nx \\u_2' \left(\cos x + \frac{\sin x}{\cos x} \sin x\right) &= \sin nx \\u_2' \left(\frac{\cos^2 x + \sin^2 x}{\cos x}\right) &= \sin nx \\u_2' &= \cos x \sin nx\end{aligned}$$

Hence

$$\begin{aligned}u_2 &= \int \cos x \sin (nx) dx \\&= \frac{-n \cos x \cos (nx) - \sin x \sin (nx)}{n^2 - 1}\end{aligned}$$

From (4)

$$\begin{aligned}u_1' &= -\cos x \sin nx \frac{\sin x}{\cos x} \\u_1 &= -\int \sin (nx) \sin x dx \\&= \frac{n \cos (nx) \sin x - \cos x \sin (nx)}{n^2 - 1}\end{aligned}$$

Since  $y_p = u_1(x) \cos x + u_2(x) \sin x$  then

$$\begin{aligned} y_p &= \left( \frac{n \cos(nx) \sin x - \cos x \sin(nx)}{n^2 - 1} \right) \cos x + \left( \frac{-n \cos x \cos(nx) - \sin x \sin(nx)}{n^2 - 1} \right) \sin x \\ &= \frac{n \cos(nx) \cos x \sin x - \cos^2 x \sin(nx) - n \cos x \sin x \cos(nx) - \sin^2 x \sin(nx)}{n^2 - 1} \\ &= \frac{-\sin(nx) (\cos^2 x + \sin^2 x)}{n^2 - 1} \\ &= \frac{\sin(nx)}{1 - n^2} \end{aligned}$$

Therefore, the full solution is (for  $n^2 \neq 1$ )

$$\begin{aligned} y &= y_h + y_p \\ &= c_1 \cos x + c_2 \sin x + \frac{\sin(nx)}{1 - n^2} \end{aligned}$$

Solution using undetermined coefficients: Since RHS is  $\sin nx$  we guess  $y_p = A \cos(nx) + B \sin(nx)$ , therefore

$$\begin{aligned} y_p' &= -An \sin(nx) + Bn \cos(nx) \\ y_p'' &= -An^2 \cos(nx) - Bn^2 \sin(nx) \end{aligned}$$

Plug into the ODE gives

$$\begin{aligned} y_p'' + y_p &= \sin nx \\ -An^2 \cos(nx) - Bn^2 \sin(nx) + A \cos(nx) + B \sin(nx) &= \sin nx \\ \cos(nx) (-An^2 + A) + \sin(nx) (-Bn^2 + B) &= \sin(nx) \end{aligned}$$

Hence  $-Bn^2 - B = 1$  and  $-An^2 + A = 0$ . Therefore  $A = 0$  and from the first equation

$$\begin{aligned} B(n^2 + 1) &= -1 \\ B &= \frac{-1}{n^2 + 1} \end{aligned}$$

Hence

$$\begin{aligned} y_p &= A \cos(nx) + B \sin(nx) \\ &= \frac{\sin(nx)}{1 - n^2} \end{aligned}$$

Which is the same as variation of parameters method.

Note: Full solution should also really consider the case for  $n = 1$ . Will update later.

```

restart;
ode:=diff(diff(y(x),x),x)+y(x)-sin(n*x)=0;
y0:=-C1*cos(x)+_C2*sin(x)+sin(n*x)/(1-n^2);
odetest(y(x)=y0,ode);
0

```

## 2.1004 ODE No. 1004

$$-a \cos(bx) + y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.100485 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{-a \cos(bx) + (b^2 - 1) c_2 \sin(x) + (b^2 - 1) c_1 \cos(x)}{b^2 - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 27

$$\left\{ y(x) = \sin(x) \_C2 + \cos(x) \_C1 - \frac{a \cos(bx)}{b^2 - 1} \right\}$$

**Hand solution**

$$y'' + y = a \cos bx \tag{1}$$

We start by solving the homogeneous equation

$$y'' + y = 0$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\lambda^2 e^{\lambda x} + e^{\lambda x} = 0$$

$$\lambda^2 + 1 = 0$$

Hence  $\lambda = \pm i$ , therefore the solution is

$$\begin{aligned}
 y_h &= Ae^{ix} + Be^{-ix} \\
 &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\
 &= \cos x(A + B) + \sin x(Ai - iB) \\
 &= \cos x(A + B) + \sin x(i(A - B))
 \end{aligned}$$

Let  $A + B = c_1, i(A - B) = c_2$  hence

$$y_h = c_1 \cos x + c_2 \sin x$$

Now we solve for the particular solution using variation of parameters. Let

$$\begin{aligned}y_p &= u_1(x) \cos x + u_2(x) \sin x \\y_p' &= u_1' \cos x - u_1 \sin x + u_2' \sin x + u_2 \cos x \\&= u_2 \cos x - u_1 \sin x + u_1' \cos x + u_2' \sin x\end{aligned}$$

Let first condition be

$$u_1' \cos x + u_2' \sin x = 0 \quad (2)$$

Hence

$$\begin{aligned}y_p' &= u_2 \cos x - u_1 \sin x \\y_p'' &= u_2' \cos x - u_2 \sin x - u_1' \sin x - u_1 \cos x\end{aligned}$$

Substituting in (1) gives

$$\begin{aligned}y_p'' + y_p &= a \cos bx \\u_2' \cos x - u_2 \sin x - u_1' \sin x - u_1 \cos x + u_1 \cos x + u_2 \sin x &= a \cos bx \\u_2' \cos x - u_1' \sin x &= a \cos bx\end{aligned} \quad (3)$$

So we have two equations (1)(2) to solve for  $u_1, u_2$

$$\begin{aligned}u_1' \cos x + u_2' \sin x &= 0 \\u_2' \cos x - u_1' \sin x &= a \cos bx\end{aligned}$$

From the first equation

$$u_1' = -u_2' \frac{\sin x}{\cos x} \quad (4)$$

Substituting in the second equation

$$\begin{aligned}u_2' \cos x - \left(-u_2' \frac{\sin x}{\cos x}\right) \sin x &= a \cos bx \\u_2' \left(\cos x + \frac{\sin x}{\cos x} \sin x\right) &= a \cos bx \\u_2' \left(\frac{\cos^2 x + \sin^2 x}{\cos x}\right) &= a \cos bx \\u_2' &= a \cos x \cos bx\end{aligned}$$

Hence

$$\begin{aligned}u_2 &= a \int \cos x \cos (bx) dx \\&= a \frac{-\cos (bx) \sin x + b \cos x \sin (bx)}{b^2 - 1}\end{aligned}$$

From (4)

$$\begin{aligned} u_1' &= -a \cos(bx) \sin x \\ u_1 &= -a \int \cos(bx) \sin x dx \\ &= -a \frac{\cos(bx) \cos x + b \sin x \sin(bx)}{b^2 - 1} \end{aligned}$$

Since  $y_p = u_1(x) \cos x + u_2(x) \sin x$  then

$$\begin{aligned} y_p &= \left( -a \frac{\cos(bx) \cos x + b \sin x \sin(bx)}{b^2 - 1} \right) \cos x + \left( a \frac{-\cos(bx) \sin x + b \cos x \sin(bx)}{b^2 - 1} \right) \sin x \\ &= \frac{-a \cos(bx) \cos^2 x - ab \cos x \sin x \sin(bx) - a \cos(bx) \sin^2 x + ab \sin x \cos x \sin(bx)}{b^2 - 1} \\ &= \frac{-a \cos(bx) \cos^2 x - a \cos(bx) \sin^2 x}{b^2 - 1} \\ &= \frac{-a \cos(bx) (\cos^2 x + \sin^2 x)}{b^2 - 1} \\ &= \frac{-a \cos(bx)}{b^2 - 1} \\ &= \frac{a \cos(bx)}{1 - b^2} \end{aligned}$$

Therefore, the full solution is (for  $b^2 \neq 1$ )

$$\begin{aligned} y &= y_h + y_p \\ &= c_1 \cos x + c_2 \sin x + \frac{a \cos(bx)}{1 - b^2} \end{aligned}$$

```
restart;
ode:=diff(diff(y(x),x),x)+y(x)-a*cos(b*x)=0;
y0:=-C1*cos(x)+_C2*sin(x)+a*cos(b*x)/(1-b^2);
odetest(y(x)=y0,ode);
0
```

## 2.1005 ODE No. 1005

$$-\sin(ax) \sin(bx) + y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.52799 (sec), leaf count = 159

$$\left\{ \left\{ y(x) \rightarrow \frac{a^4 c_2 \sin(x) - 2a^2 b^2 c_2 \sin(x) - a^2 \sin(ax) \sin(bx) - 2a^2 c_2 \sin(x) + c_1 (a^4 - 2a^2(b^2 + 1) + (b^2 - 1)^2) c_2}{(a - b - 1)(a - b + 1)} \right. \right.$$

✓ **Maple** : cpu = 0.123 (sec), leaf count = 82

$$\left\{ y(x) = \sin(x) \_C2 + \cos(x) \_C1 + \frac{-(a+b+1)(a+b-1)\cos(x(a-b)) + \cos((a+b)x)(a-b+1)(a-b)}{2a^4 + (-4b^2 - 4)a^2 + 2b^4 - 4b^2 + 2} \right.$$

**Hand solution**

$$y'' + y = \sin ax \sin bx \tag{1}$$

We start by solving the homogeneous equation

$$y'' + y = 0$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\lambda^2 e^{\lambda x} + e^{\lambda x} = 0$$

$$\lambda^2 + 1 = 0$$

Hence  $\lambda = \pm i$ , therefore the solution is

$$\begin{aligned} y_h &= Ae^{ix} + Be^{-ix} \\ &= A(\cos x + i \sin x) + B(\cos x - i \sin x) \\ &= \cos x(A + B) + \sin x(Ai - iB) \\ &= \cos x(A + B) + \sin x(i(A - B)) \end{aligned}$$

Let  $A + B = c_1, i(A - B) = c_2$  hence

$$\begin{aligned} y_h &= c_1 \cos x + c_2 \sin x \\ &= y_h = c_1 y_1 + c_2 y_2 \end{aligned}$$

Now we solve for the particular solution using variation of parameters.

$$y_p = u_1 y_1 + u_2 y_2$$

Wonskian is

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

Hence, using  $f = \sin ax \sin bx$ , which is the RHS of the ODE, and noting that  $a_0$  is the coefficient of  $y''$  which is one here, then

$$\begin{aligned}
u_1 &= \int \frac{-y_2}{W} \frac{f}{a_0} dx = - \int \sin x \sin(ax) \sin(bx) dx \\
&= -\frac{1}{4} \left( \frac{\cos((a-b-1)x)}{a-b-1} - \frac{\cos((a-b+1)x)}{a-b+1} - \frac{\cos((a+b-1)x)}{a+b-1} + \frac{\cos((a+b+1)x)}{a+b+1} \right)
\end{aligned}$$

And

$$\begin{aligned}
u_2 &= \int \frac{y_1}{W} \frac{f}{a_0} dx = - \int \cos x \sin(ax) \sin(bx) dx \\
&= \frac{1}{4} \left( \frac{\sin((a-b-1)x)}{a-b-1} + \frac{\sin((a-b+1)x)}{a-b+1} - \frac{\sin((a+b-1)x)}{a+b-1} - \frac{\sin((a+b+1)x)}{a+b+1} \right)
\end{aligned}$$

Since  $y_p = u_1(x) \cos x + u_2(x) \sin x$  then

$$\begin{aligned}
y_p &= -\frac{1}{4} \left( \frac{\cos((a-b-1)x)}{a-b-1} - \frac{\cos((a-b+1)x)}{a-b+1} - \frac{\cos((a+b-1)x)}{a+b-1} + \frac{\cos((a+b+1)x)}{a+b+1} \right) \cos x \\
&\quad + \frac{1}{4} \left( \frac{\sin((a-b-1)x)}{a-b-1} + \frac{\sin((a-b+1)x)}{a-b+1} - \frac{\sin((a+b-1)x)}{a+b-1} - \frac{\sin((a+b+1)x)}{a+b+1} \right) \sin x \\
&= \frac{1}{4} \left( -\frac{\cos^2((a-b-1)x)}{a-b-1} + \frac{\sin^2((a-b-1)x)}{a-b-1} \right) + \frac{1}{4} \left( \frac{\cos^2((a-b+1)x)}{a-b+1} + \frac{\sin^2((a-b+1)x)}{a-b+1} \right) \\
&\quad + \frac{1}{4} \left( \frac{\cos^2((a+b-1)x)}{a+b-1} - \frac{\sin^2((a+b-1)x)}{a+b-1} \right) + \frac{1}{4} \left( -\frac{\cos^2((a+b+1)x)}{a+b+1} - \frac{\sin^2((a+b+1)x)}{a+b+1} \right) \\
&= \frac{1}{4} \left( -\frac{\cos^2((a-b-1)x)}{a-b-1} + \frac{\sin^2((a-b-1)x)}{a-b-1} \right) + \frac{1}{4} \left( \frac{1}{a-b+1} \right) \\
&\quad + \frac{1}{4} \left( \frac{\cos^2((a+b-1)x)}{a+b-1} - \frac{\sin^2((a+b-1)x)}{a+b-1} \right) - \frac{1}{4} \left( \frac{1}{a+b+1} \right)
\end{aligned}$$

Let  $a-b-1 = \alpha, a+b-1 = \beta$  then

$$\begin{aligned}
y_p &= \frac{1}{4} \left( \frac{\sin^2(\alpha x) - \cos^2(\alpha x)}{\alpha} \right) + \frac{1}{4} \left( \frac{\cos^2(\beta x) - \sin^2(\beta x)}{\beta} \right) + \frac{1}{4} \left( \frac{1}{\alpha+2} \right) - \frac{1}{4} \left( \frac{1}{\beta+2} \right) \\
&= \frac{1}{4} \left( \frac{\sin^2(\alpha x) - \cos^2(\alpha x)}{\alpha} + \frac{\cos^2(\beta x) - \sin^2(\beta x)}{\beta} + \frac{1}{\alpha+2} - \frac{1}{\beta+2} \right)
\end{aligned}$$

Therefore, the full solution is

$$\begin{aligned}
y &= y_h + y_p \\
&= c_1 \cos x + c_2 \sin x + \frac{1}{4} \left( \frac{\sin^2(\alpha x) - \cos^2(\alpha x)}{\alpha} + \frac{\cos^2(\beta x) - \sin^2(\beta x)}{\beta} + \frac{1}{\alpha+2} - \frac{1}{\beta+2} \right)
\end{aligned}$$

I made mistake. Need to go over it again. I do not see it now. Maple does not verify.

```

restart;
ode:=diff(diff(y(x),x),x)+y(x)-sin(a*x)*sin(b*x)=0;
alpha:=a-b-1;
beta:=a+b-1;
yp:=1/4*(1/alpha*(sin(alpha*x)^2-cos(alpha*x)^2)+1/beta*(cos(beta*x)^2-sin(beta*x)^2))+1/4*(1/(a
y0:=yp+_C1*sin(x)+_C2*cos(x);
not zero

```

## 2.1006 ODE No. 1006

$$y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.00487625 (sec), leaf count = 20

$$\{ \{ y(x) \rightarrow c_1 e^x + c_2 e^{-x} \} \}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 15

$$\{ y(x) = \_C1 e^x + \_C2 e^{-x} \}$$

**Hand solution**

$$y'' - y = 0 \tag{1}$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\lambda^2 e^{\lambda x} - e^{\lambda x} = 0$$

$$\lambda^2 - 1 = 0$$

Hence  $\lambda = \pm 1$ , therefore the solution is

$$y_h = A e^x + B e^{-x}$$

## 2.1007 ODE No. 1007

$$-4e^{x^2} x^2 + y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0694246 (sec), leaf count = 36

$$\{ \{ y(x) \rightarrow c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x} + e^{x^2} \} \}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 26

$$\{ y(x) = e^{\sqrt{2}x} \_C2 + e^{-\sqrt{2}x} \_C1 + e^{x^2} \}$$



**Hand solution**

$$y'' - 2y = 4x^2 e^{x^2} \tag{1}$$

We start by solving the homogeneous equation

$$y'' - 2y = 0$$

Let  $y = e^{\lambda x}$ , substitution in above gives

$$\begin{aligned} \lambda^2 e^{\lambda x} - 2e^{\lambda x} &= 0 \\ \lambda^2 - 2 &= 0 \end{aligned}$$

Hence  $\lambda = \pm\sqrt{2}$ , therefore the solution is

$$\begin{aligned} y_h &= Ae^{\sqrt{2}x} + Be^{-\sqrt{2}x} \\ &= Ay_1 + By_2 \end{aligned}$$

Now we solve for the particular solution using variation of parameters.

$$y_p = u_1 y_1 + u_2 y_2$$

wronskian is

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{\sqrt{2}x} & e^{-\sqrt{2}x} \\ \sqrt{2}e^{\sqrt{2}x} & -\sqrt{2}e^{-\sqrt{2}x} \end{vmatrix} = -\sqrt{2} - \sqrt{2} = -2\sqrt{2}$$

Hence, using  $f = 4x^2 e^{x^2}$ , which is the RHS of the ODE, and noting that  $a_0$  is the coefficient of  $y''$  which is one here, then

$$\begin{aligned} u_1 &= \int \frac{-y_2}{W} \frac{f}{a_0} dx = - \int \frac{e^{-\sqrt{2}x}}{-2\sqrt{2}} 4x^2 e^{x^2} dx \\ &= \frac{2}{\sqrt{2}} \int x^2 e^{x^2 - \sqrt{2}x} dx \\ &= \frac{2}{\sqrt{2}} \left( \frac{1}{4} e^{x(x-\sqrt{2})} (\sqrt{2} + 2x) \right) \end{aligned}$$

And

$$\begin{aligned}
u_2 &= \int \frac{y_1}{W} \frac{f}{a_0} dx = \int \frac{e^{\sqrt{2}x}}{-2\sqrt{2}} 4x^2 e^{x^2} dx \\
&= \frac{-2}{\sqrt{2}} \int x^2 e^{x^2 + \sqrt{2}x} dx \\
&= \frac{-2}{\sqrt{2}} \left( -\frac{1}{4} e^{x(x+\sqrt{2})} (\sqrt{2} - 2x) \right)
\end{aligned}$$

Since  $y_p = u_1 e^{\sqrt{2}x} + u_2 e^{-\sqrt{2}x}$  then

$$\begin{aligned}
y_p &= \frac{2}{\sqrt{2}} \left( \frac{1}{4} e^{x(x-\sqrt{2})} (\sqrt{2} + 2x) \right) e^{\sqrt{2}x} - \frac{2}{\sqrt{2}} \left( -\frac{1}{4} e^{x(x+\sqrt{2})} (\sqrt{2} - 2x) \right) e^{-\sqrt{2}x} \\
&= \frac{2}{\sqrt{2}} \frac{1}{4} e^{x^2} (\sqrt{2} + 2x) + \frac{2}{\sqrt{2}} \frac{1}{4} e^{x^2} (\sqrt{2} - 2x) \\
&= \frac{1}{2} e^{x^2} + \frac{1}{2} e^{x^2} \\
&= e^{x^2}
\end{aligned}$$

Therefore, the full solution is

$$\begin{aligned}
y &= y_h + y_p \\
&= A e^{\sqrt{2}x} + B e^{-\sqrt{2}x} + e^{x^2}
\end{aligned}$$

```

restart;
ode:=diff(y(x),x$2)-2*y(x)=4*x^2*exp(x^2);
y0:=-C1*exp(sqrt(2)*x)+_C2*exp(-sqrt(2)*x)+exp(x^2);
odetest(y(x)=y0,ode);
0

```

## 2.1008 ODE No. 1008

$$a^2 y(x) - \cot(ax) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0416339 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \frac{\sin(ax) \left( a^2 c_2 + \log \left( \sin \left( \frac{ax}{2} \right) \right) - \log \left( \cos \left( \frac{ax}{2} \right) \right) \right)}{a^2} + c_1 \cos(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 41

$$\left\{ y(x) = \sin(ax) \_C2 + \cos(ax) \_C1 + \frac{\sin(ax)}{a^2} \ln \left( \frac{1 - \cos(ax)}{\sin(ax)} \right) \right\}$$

**2.1009 ODE No. 1009**

$$ly(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.00533351 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin(\sqrt{l}x) + c_1 \cos(\sqrt{l}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 21

$$\{y(x) = \_C1 \sin(\sqrt{l}x) + \_C2 \cos(\sqrt{l}x)\}$$

**2.1010 ODE No. 1010**

$$y(x)(ax + b) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.00776316 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{Ai}\left(-\frac{b+ax}{(-a)^{2/3}}\right) + c_2 \text{Bi}\left(-\frac{b+ax}{(-a)^{2/3}}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 31

$$\{y(x) = \_C1 \text{Ai}\left(-(ax+b)a^{-\frac{2}{3}}\right) + \_C2 \text{Bi}\left(-(ax+b)a^{-\frac{2}{3}}\right)\}$$

**Hand solution**

$$y'' + (ax + b)y = 0 \tag{1}$$

For  $a \neq 0$ . Let  $y = \eta(\xi)$ ,  $\xi = ax + b$ , hence

$$\begin{aligned} \frac{dy}{dx} &= \frac{d\eta}{d\xi} \frac{d\xi}{dx} \\ &= \frac{d\eta}{d\xi} a \end{aligned}$$

And

$$\begin{aligned}
\frac{d^2 y}{dx^2} &= \frac{d}{dx} \left( \frac{d\eta}{d\xi} a \right) \\
&= a \frac{d}{dx} \left( \frac{d\eta}{d\xi} \right) \\
&= a \left( \frac{d^2 \eta}{d\xi^2} \frac{d\xi}{dx} \right) \\
&= a^2 \frac{d^2 \eta}{d\xi^2}
\end{aligned}$$

Therefore (1) becomes

$$\begin{aligned}
a^2 \frac{d^2 \eta}{d\xi^2} + \xi \eta(\xi) &= 0 \\
a^2 \eta'' + \xi \eta &= 0
\end{aligned} \tag{2}$$

This is Airy ODE but with plus sign instead of the normal Airy  $\eta'' - \xi \eta = 0$ . Let

$$\begin{aligned}
\eta &= \sum_{n=0}^{\infty} c_n \xi^n \\
\eta' &= \sum_{n=0}^{\infty} n c_n \xi^{n-1} = \sum_{n=1}^{\infty} n c_n \xi^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} \xi^n \\
\eta'' &= \sum_{n=0}^{\infty} n(n+1) c_{n+1} \xi^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} \xi^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} \xi^n
\end{aligned}$$

Hence (2) becomes

$$\begin{aligned}
a^2 \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} \xi^n + \xi \sum_{n=0}^{\infty} c_n \xi^n &= 0 \\
\sum_{n=0}^{\infty} a^2 (n+1)(n+2) c_{n+2} \xi^n + \sum_{n=0}^{\infty} c_n \xi^{n+1} &= 0 \\
\sum_{n=0}^{\infty} a^2 (n+1)(n+2) c_{n+2} \xi^n + \sum_{n=1}^{\infty} c_{n-1} \xi^n &= 0 \\
2a^2 c_2 + \sum_{n=1}^{\infty} (a^2 (n+1)(n+2) c_{n+2} + c_{n-1}) \xi^n &= 0
\end{aligned}$$

Hence

$$2a^2 c_2 = 0 \tag{3}$$

$$a^2(n+1)(n+2)c_{n+2} + c_{n-1} = 0 \quad n \geq 1 \tag{4}$$

From (3) and since  $a \neq 0$

$$c_2 = 0$$

From (4)

$$c_{n+2} = \frac{-c_{n-1}}{a^2(n+1)(n+2)}$$

Hence for  $n = 3$ , we see from the above recurrence equation and because  $c_2 = 0$  that

$$c_5 = \frac{-c_2}{a^2(4)(6)} = 0$$

Similarly, for  $n = 6$

$$c_8 = \frac{-c_5}{a^2(7)(8)} = 0$$

Similarly, for  $n = 9$

$$c_{11} = \frac{-c_8}{a^2(10)(11)} = 0$$

And so on. Hence we found so far that for  $n = 3, 6, 9, 12, \dots$  all terms generated which are  $c_5, c_8, c_{11}, \dots$  are zero.

Now, for  $n = 1$ , the recurrence equation gives

$$c_3 = \frac{-c_0}{a^2} \frac{1}{2 \cdot 3}$$

For  $n = 2$

$$c_4 = \frac{-c_1}{a^2} \frac{1}{3 \cdot 4}$$

For  $n = 4$

$$c_6 = \frac{-c_3}{a^2(5)(6)} = c_0 \left( \frac{1}{a^2} \frac{1}{5 \cdot 6} \right) \left( \frac{1}{a^2} \frac{1}{2 \cdot 3} \right)$$

For  $n = 5$

$$c_7 = \frac{-c_4}{a^2(6)(7)} = c_1 \left( \frac{1}{a^2} \frac{1}{6 \cdot 7} \right) \left( \frac{1}{a^2} \frac{1}{3 \cdot 4} \right)$$

For  $n = 7$

$$c_9 = \frac{-c_6}{a^2(8)(9)} = -c_0 \left( \frac{1}{a^2} \frac{1}{8 \cdot 9} \right) \left( \frac{1}{a^2} \frac{1}{5 \cdot 6} \right) \left( \frac{1}{a^2} \frac{1}{2 \cdot 3} \right)$$

For  $n = 8$

$$c_{10} = \frac{-c_7}{a^2(9)(10)} = -c_1 \left( \frac{1}{a^2} \frac{1}{9 \cdot 10} \right) \left( \frac{1}{a^2} \frac{1}{6 \cdot 7} \right) \left( \frac{1}{a^2} \frac{1}{3 \cdot 4} \right)$$

Therefore, in summary, this is what we have so far. For  $n = 3, 6, 9, 12, \dots$  all terms are zero. For  $n = 1, 4, 7, \dots$  all terms are expressed using  $c_0$  and for  $n = 2, 5, 8, \dots$  all terms are expressed using  $c_1$ . So there are two arbitrary constants  $c_0, c_1$ .

In other words,  $c_2, c_5, c_8, c_{11}, \dots = 0$  and  $c_3, c_6, c_9, c_{12}, \dots = f(c_0)$  and  $c_4, c_7, c_{10}, c_{13}, \dots = f(c_1)$ .

$$\begin{aligned} \eta &= \sum_{n=0}^{\infty} c_n \xi^n \\ &= c_0 + c_1 \xi^1 + c_2 \xi^2 + c_3 \xi^3 + \dots \\ &= c_0 + (c_1 \xi^1) + 0 - \left( c_0 \frac{\xi^3}{a^2} \frac{1}{2 \cdot 3} \right) - \left( c_1 \frac{\xi^4}{a^2} \frac{1}{3 \cdot 4} \right) + 0 + c_0 \frac{\xi^6}{a^4} \frac{1}{2 \cdot 3 \cdot 5 \cdot 6} + c_1 \frac{\xi^7}{a^4} \frac{1}{3 \cdot 4 \cdot 6 \cdot 7} + 0 \\ &= c_0 \left( 1 - \frac{\xi^3}{a^2} \frac{1}{2 \cdot 3} + \frac{\xi^6}{a^4} \frac{1}{2 \cdot 3 \cdot 5 \cdot 6} - \dots \right) + c_1 \left( \xi - \frac{\xi^4}{a^2} \frac{1}{3 \cdot 4} + \frac{\xi^7}{a^4} \frac{1}{3 \cdot 4 \cdot 6 \cdot 7} - \dots \right) \\ &= c_0 \left( 1 - \frac{1}{a^2} \frac{\xi^3}{3!} + \frac{1 \cdot 4}{a^4} \frac{\xi^6}{6!} - \frac{1 \cdot 4 \cdot 7}{a^6} \frac{\xi^9}{9!} + \dots \right) + c_1 \left( \xi - \frac{\xi^4}{a^2} \frac{2}{4!} + \frac{\xi^7}{a^4} \frac{2 \cdot 5}{7!} - \frac{\xi^{10}}{a^6} \frac{2 \cdot 5 \cdot 8}{10!} + \dots \right) \end{aligned}$$

Hence

$$\eta = c_0 \left( \sum_{n=0}^{\infty} 3^n \binom{1}{3}_n \frac{(-1)^n \xi^{3n}}{a^{2n} (3n)!} \right) + c_1 \left( \sum_{n=0}^{\infty} 3^n \binom{2}{3}_n \frac{(-1)^n \xi^{3n+1}}{a^{2n} (3n+1)!} \right) \quad (5)$$

Where

$$\begin{aligned} 3^n \binom{1}{3}_n &= (1) \cdot (4) \cdot (7) \dots (3n - 2) \\ 3^n \binom{2}{3}_n &= (2) \cdot (5) \cdot (8) \dots (3n - 1) \end{aligned}$$

And

$$\left(\frac{1}{3}\right)_0 = \left(\frac{2}{3}\right)_0 = 1$$

Equation (5) can be simplified more by moving  $\frac{(-1)^n}{a^{2n}}$  into  $\xi$  as follows

$$\eta = c_0 \left( \sum_{n=0}^{\infty} 3^n \binom{1}{3}_n \frac{1}{(3n)!} \left(\frac{-\xi}{a^{\frac{2}{3}}}\right)^{3n} \right) + ac_1 \left( \sum_{n=0}^{\infty} 3^n \binom{2}{3}_n \frac{1}{(3n+1)!} \left(\frac{-\xi}{a^{\frac{2}{3}}}\right)^{3n+1} \right)$$

Let  $\left(\frac{-\xi}{a^{\frac{2}{3}}}\right) = z$  then the above is

$$\eta = c_0 \left( \sum_{n=0}^{\infty} 3^n \binom{1}{3}_n \frac{z^{3n}}{(3n)!} \right) + ac_1 \left( \sum_{n=0}^{\infty} 3^n \binom{2}{3}_n \frac{z^{3n+1}}{(3n+1)!} \right)$$

Let

$$f(\xi) = \sum_{n=0}^{\infty} 3^n \binom{1}{3}_n \frac{\xi^{3n}}{(3n)!}$$

$$g(\xi) = \sum_{n=0}^{\infty} 3^n \binom{2}{3}_n \frac{\xi^{3n+1}}{(3n+1)!}$$

Now looking at definition of AiryAI( $z$ ) we see

$$\text{AiryAI}(z) = r_1 f(z) - r_2 g(z)$$

$$\text{AiryBI}(z) = \sqrt{3}(r_1 f(z) + r_2 g(z))$$

These are Airy functions AiryAI and AiryBI with appropriate choice of  $c_0, c_1$ . See definition of these special functions. Converting back to  $x$  using  $\xi = ax + b$  should result in solution given by CAS. Need to write these final details to make sure. Will finish later.

## 2.1011 ODE No. 1011

$$y''(x) - (x^2 + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.00805754 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{-1}(\sqrt{2}x) + c_2 D_0(i\sqrt{2}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 17

$$\left\{ y(x) = e^{\frac{x^2}{2}} (Erf(x) \_C2 + \_C1) \right\}$$

**Hand solution**

$$y'' - (x^2 + 1)y = 0 \tag{1}$$

Second order with varying coefficient. Using power series, let  $y = \sum_{n=0}^{\infty} c_n x^n$ , hence

$$y' = \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n$$

$$y'' = \sum_{n=0}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n$$

Substituting back in the original ODE gives

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - (x^2 + 1) \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - x^2 \sum_{n=0}^{\infty} c_n x^n - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=0}^{\infty} c_n x^{n+2} - \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=2}^{\infty} c_{n-2} x^n - \sum_{n=0}^{\infty} c_n x^n = 0$$

For  $n = 0$

$$(n+1)(n+2) c_{n+2} - c_n = 0$$

$$2c_2 - c_0 = 0$$

$$c_2 = \frac{c_0}{2}$$

For  $n = 1$

$$(n+1)(n+2) c_{n+2} - c_n = 0$$

$$(2)(3) c_3 - c_1 = 0$$

$$c_3 = \frac{c_1}{6}$$

For  $n \geq 2$

$$(n+1)(n+2) c_{n+2} - c_{n-2} - c_n = 0$$

$$c_{n+2} = \frac{c_{n-2} + c_n}{(n+1)(n+2)}$$

Hence for  $n = 2$

$$c_4 = \frac{c_0 + c_2}{(3)(4)} = \frac{c_0 + \frac{c_0}{2}}{(3)(4)} = \frac{2c_0 + c_0}{(2)(3)(4)} = c_0 \frac{3}{(2)(3)(4)}$$



For  $n = 3$

$$c_5 = \frac{c_1 + c_3}{(4)(5)} = \frac{c_1 + \frac{c_1}{6}}{(4)(5)} = \frac{6c_1 + c_1}{(4)(5)(6)} = c_1 \frac{7}{(4)(5)(6)}$$

For  $n = 4$

$$c_6 = \frac{c_2 + c_4}{(5)(6)} = \frac{\frac{c_0}{2} + c_0 \frac{3}{(2)(3)(4)}}{(5)(6)} = \frac{c_0(3)(4) + 3c_0}{(2)(3)(4)(5)(6)} = c_0 \frac{15}{(2)(3)(4)(5)(6)}$$

For  $n = 5$

$$\begin{aligned} c_7 &= \frac{c_3 + c_5}{(6)(7)} \\ &= \frac{\frac{c_1}{6} + c_1 \frac{7}{(4)(5)(6)}}{(6)(7)} = \frac{3}{560} c_1 \end{aligned}$$

And so on. Hence the series is

$$\begin{aligned} y &= \sum_{n=0}^{\infty} c_n x^n \\ &= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + c_1 x + \frac{c_0}{2} x^2 + \frac{c_1}{6} x^3 + c_0 \frac{3}{(2)(3)(4)} x^4 + c_1 \frac{7}{(4)(5)(6)} x^5 + c_0 \frac{15}{(2)(3)(4)(5)(6)} x^6 + c_1 \frac{3}{560} x^7 + \dots \\ &= c_0 + c_1 x + \frac{c_0}{2} x^2 + \frac{c_1}{6} x^3 + c_0 \frac{1}{8} x^4 + c_1 \frac{7}{120} x^5 + c_0 \frac{1}{48} x^6 + c_1 \frac{3}{560} x^7 + \dots \\ &= c_0 \left( 1 + \frac{x^2}{2} + \frac{1}{8} x^4 + \frac{1}{48} x^6 + \dots \right) + c_1 \left( x + \frac{1}{6} x^3 + \frac{7}{120} x^5 + \frac{3}{560} x^7 + \dots \right) \end{aligned}$$

Now the power series for  $e^{\frac{x^2}{2}} = 1 + \frac{x^2}{2} + \frac{x^4}{8} + \frac{x^6}{48} + \dots$ , so we can convert the first term above (the expression for  $c_0$  to be  $e^{\frac{x^2}{2}}$ . Hence

$$c_0 \left( 1 + \frac{x^2}{2} + \frac{1}{8} x^4 + \frac{1}{48} x^6 + \dots \right) = c_0 e^{\frac{x^2}{2}}$$

So now we have to work on the second term (the expression for  $c_1$ )

$$c_1 \left( x + \frac{1}{6} x^3 + \frac{7}{120} x^5 + \frac{3}{560} x^7 + \dots \right) = ?$$

Recall that series for error function is

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{48} + \dots \right)$$

Multiplying  $e^{\frac{x^2}{2}}$  by  $\operatorname{erf}(x)$  gives

$$\begin{aligned} e^{\frac{x^2}{2}} \operatorname{erf}(x) &= \frac{2}{\sqrt{\pi}} \left( 1 + \frac{x^2}{2} + \frac{x^4}{(2)(4)} + \frac{x^6}{(2)(4)(6)} + \dots \right) \left( x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{48} + \dots \right) \\ &= \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{48} + \dots + \frac{x^3}{2} - \frac{x^5}{6} + \frac{x^7}{20} - \frac{x^{15}}{96} + \dots \right) \\ &= \frac{2}{\sqrt{\pi}} \left( x + \frac{x^3}{6} + \frac{7x^5}{120} + \frac{3}{560} x^7 + \dots \right) \end{aligned}$$

Comparing the above to the term next to  $c_1$  above, we see they are the same with a multiplier  $\frac{2}{\sqrt{\pi}}$ , which can be absorbed into the constant  $c_1$ , Hence

$$\begin{aligned} y &= c_0 \left( 1 + \frac{x^2}{2} + \frac{1}{8}x^4 + \frac{1}{48}x^6 + \dots \right) + c_1 \left( x + \frac{1}{6}x^3 + \frac{7}{120}x^5 + \frac{3}{560}x^7 + \dots \right) \\ &= c_0 e^{\frac{x^2}{2}} + c_1 \left( e^{\frac{x^2}{2}} \operatorname{erf}(x) \right) \end{aligned}$$

Hence final solution is

$$y = c_0 e^{\frac{x^2}{2}} + c_2 \left( e^{\frac{x^2}{2}} \operatorname{erf}(x) \right)$$

Verification

```
restart;
ode:=diff(diff(y(x),x),x)-(x^2+1)*y(x)=0;
y0:=-C1*exp(x^2/2)+_C2*exp(x^2/2)*erf(x);
odetest(y(x)=y0,ode);
0
```

## 2.1012 ODE No. 1012

$$y''(x) - (a + x^2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.00873271 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{\frac{1}{2}(-a-1)}(\sqrt{2}x) + c_2 D_{\frac{a-1}{2}}(i\sqrt{2}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 29

$$\left\{ y(x) = 1 \left( -C2 W_{-\frac{a}{4}, \frac{1}{4}}(x^2) + -C1 M_{-\frac{a}{4}, \frac{1}{4}}(x^2) \right) \frac{1}{\sqrt{x}} \right\}$$

## 2.1013 ODE No. 1013

$$y''(x) - (a^2 x^2 + a)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0211046 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{-1}(\sqrt{2}\sqrt{ax}) + c_2 D_0(i\sqrt{2}\sqrt{ax}) \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 22

$$\left\{ y(x) = e^{\frac{ax^2}{2}} (Erf(\sqrt{ax}) - C2 + -C1) \right\}$$

**2.1014 ODE No. 1014**

$$y''(x) - cx^a y(x) = 0$$

✓ **Mathematica** : cpu = 0.0435852 (sec), leaf count = 119

$$\left\{ \left\{ y(x) \rightarrow (a+2)^{-\frac{1}{a+2}} \sqrt{x} c^{\frac{1}{2a+4}} \left( c_1 \Gamma\left(\frac{a+1}{a+2}\right) I_{-\frac{1}{a+2}}\left(\frac{2\sqrt{cx}^{\frac{a}{2}+1}}{a+2}\right) + (-1)^{\frac{1}{a+2}} c_2 \Gamma\left(1 + \frac{1}{a+2}\right) I_{\frac{1}{a+2}}\left(\frac{2\sqrt{cx}^{\frac{a}{2}+1}}{a+2}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 63

$$\left\{ y(x) = \sqrt{x} \left( Y_{(a+2)^{-1}} \left( 2 \frac{\sqrt{-cx}^{a/2+1}}{a+2} \right) - C2 + J_{(a+2)^{-1}} \left( 2 \frac{\sqrt{-cx}^{a/2+1}}{a+2} \right) - C1 \right) \right\}$$

**2.1015 ODE No. 1015**

$$y''(x) - y(x) (a^2 x^{2n} - 1) = 0$$

✗ **Mathematica** : cpu = 0.335244 (sec), leaf count = 0 , could not solve

`DSolve[-((-1 + a^2*x^(2*n))*y[x]) + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} - Y(x) + (-a^2 x^{2n} + 1) - Y(x) \right\}, \{ -Y(x) \} \right) \right\}$$

**2.1016 ODE No. 1016**

$$y(x) (ax^{2c} + bx^{c-1}) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.137968 (sec), leaf count = 225

$$\left\{ \left\{ y(x) \rightarrow 2^{\frac{c}{2c+2}} x^{-c/2} (x^{c+1})^{\frac{c}{2c+2}} e^{-\frac{\sqrt{ax}^{c+1}}{\sqrt{-(c+1)^2}}} \left( c_1 U \left( -\frac{(c+1)(cb+b+\sqrt{ac}\sqrt{-(c+1)^2})}{2\sqrt{a}(-(c+1)^2)^{3/2}}, \frac{c}{c+1}, \frac{2\sqrt{ax}^{c+1}}{\sqrt{-(c+1)^2}} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 91

$$\left\{ y(x) = x^{-\frac{c}{2}} \left( M_{\frac{-ib}{2c+2}, \frac{1}{\sqrt{a}}, (2c+2)^{-1}} \left( \frac{2ix^{c+1}}{c+1} \sqrt{a} \right) - C1 + W_{\frac{-ib}{2c+2}, \frac{1}{\sqrt{a}}, (2c+2)^{-1}} \left( \frac{2ix^{c+1}}{c+1} \sqrt{a} \right) - C2 \right) \right\}$$

**2.1017 ODE No. 1017**

$$(e^{2x} - v^2) y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0286726 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 \Gamma(1-v) J_{-v}(\sqrt{e^{2x}}) + c_2 \Gamma(v+1) J_v(\sqrt{e^{2x}}) \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 17

$$\{y(x) = \_C1 J_v(e^x) + \_C2 Y_v(e^x)\}$$

**2.1018 ODE No. 1018**

$$ae^{bx} y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.022533 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow c_1 J_0\left(\frac{2\sqrt{a}\sqrt{e^{bx}}}{b}\right) + 2c_2 Y_0\left(\frac{2\sqrt{a}\sqrt{e^{bx}}}{b}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 39

$$\left\{ y(x) = \_C1 J_0\left(2 \frac{\sqrt{a}e^{1/2bx}}{b}\right) + \_C2 Y_0\left(2 \frac{\sqrt{a}e^{1/2bx}}{b}\right) \right\}$$

**2.1019 ODE No. 1019**

$$y''(x) - y(x) (4a^2 b^2 x^2 e^{2bx^2} - 1) = 0$$

✗ **Mathematica** : cpu = 0.789006 (sec), leaf count = 0 , could not solve

`DSolve[-((-1 + 4*a^2*b^2*E^(2*b*x^2))*x^2)*y[x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{\frac{d^2}{dx^2} - Y(x) + (-4a^2 b^2 x^2 e^{2bx^2} + 1) - Y(x)\right\}, \{-Y(x)\}\right) \right\}$$

**2.1020 ODE No. 1020**

$$y(x) (ae^{2x} + be^x + c) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.686136 (sec), leaf count = 136

$$\left\{ \left\{ y(x) \rightarrow e^{-i\sqrt{a}e^x} (e^x)^{i\sqrt{c}} \left( c_1 U \left( \frac{ib}{2\sqrt{a}} + i\sqrt{c} + \frac{1}{2}, 2i\sqrt{c} + 1, 2i\sqrt{a}e^x \right) + c_2 L_{-\frac{ib}{2\sqrt{a}} - i\sqrt{c} - \frac{1}{2}}^{2i\sqrt{c}} (2i\sqrt{a}e^x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.186 (sec), leaf count = 58

$$\left\{ y(x) = e^{-\frac{x}{2}} \left( W_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, i\sqrt{c}}(2i\sqrt{a}e^x) \_C2 + M_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, i\sqrt{c}}(2i\sqrt{a}e^x) \_C1 \right) \right\}$$

**2.1021 ODE No. 1021**

$$y(x) (a \cos^2(x) + b) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0504648 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{MathieuC} \left[ \frac{a}{2} + b, -\frac{a}{4}, x \right] + c_2 \text{MathieuS} \left[ \frac{a}{2} + b, -\frac{a}{4}, x \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.214 (sec), leaf count = 39

$$\left\{ y(x) = \_C1 \text{MathieuC} \left( -\frac{a}{2} - b, \frac{a}{4}, ix \right) + \_C2 \text{MathieuS} \left( -\frac{a}{2} - b, \frac{a}{4}, ix \right) \right\}$$

**2.1022 ODE No. 1022**

$$y(x)(a \cos(2x) + b) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0282233 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{MathieuC} \left[ b, -\frac{a}{2}, x \right] + c_2 \text{MathieuS} \left[ b, -\frac{a}{2}, x \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.201 (sec), leaf count = 21

$$\left\{ y(x) = \_C1 \text{MathieuC} \left( b, -\frac{a}{2}, x \right) + \_C2 \text{MathieuS} \left( b, -\frac{a}{2}, x \right) \right\}$$

**2.1023 ODE No. 1023**

$$y(x) (a \cos^2(x) + b) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0155017 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{MathieuC} \left[ \frac{a}{2} + b, -\frac{a}{4}, x \right] + c_2 \text{MathieuS} \left[ \frac{a}{2} + b, -\frac{a}{4}, x \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.193 (sec), leaf count = 29

$$\left\{ y(x) = \_C1 \text{MathieuC} \left( \frac{a}{2} + b, -\frac{a}{4}, x \right) + \_C2 \text{MathieuS} \left( \frac{a}{2} + b, -\frac{a}{4}, x \right) \right\}$$

**2.1024 ODE No. 1024**

$$y''(x) - y(x) (2 \tan^2(x) + 1) = 0$$

✓ **Mathematica** : cpu = 0.161789 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[4]{\sin^2(x)} \left( -c_2 \sqrt{\sin^2(x)} + 2c_1 \sec(x) + c_2 \sec(x) \sin^{-1}(\cos(x)) \right)}{2 \sqrt[4]{-\sin^2(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.139 (sec), leaf count = 30

$$\left\{ y(x) = \frac{i \cos(x) \sin(x) \_C2 + \ln(\cos(x) + i \sin(x)) \_C2 + \_C1}{\cos(x)} \right\}$$

**2.1025 ODE No. 1025**

$$y''(x) - y(x) (a + (m - 1)m \sec^2(x) + (n - 1)n \csc^2(x)) = 0$$

✓ **Mathematica** : cpu = 0.958222 (sec), leaf count = 158

$$\left\{ \left\{ y(x) \rightarrow \frac{(-1)^{-m} \cos^2(x)^{-\frac{m}{2} - \frac{1}{4}} (-\sin^2(x))^{n/2} \left( c_1 (-1)^m \cos^2(x)^{m+\frac{1}{2}} {}_2F_1 \left( \frac{1}{2}(m+n-\sqrt{-a}), \frac{1}{2}(m+n+\sqrt{-a}) \right) \right)}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 102

$$\left\{ y(x) = (\sin(x))^n \left( {}_2F_1 \left( \frac{n}{2} - \frac{m}{2} + \frac{i}{2}\sqrt{a} + \frac{1}{2}, \frac{n}{2} - \frac{m}{2} - \frac{i}{2}\sqrt{a} + \frac{1}{2}; \frac{3}{2} - m; (\cos(x))^2 \right) (\cos(x))^{-m+1} \_C2 + (\cos(x)) \right) \right\}$$

**2.1026 ODE No. 1026**

$$y''(x) - y(x)(B + n(n + 1)\wp(x; g2, g3)) = 0$$

✗ **Mathematica** : cpu = 0.183217 (sec), leaf count = 0 , could not solve

DSolve[-((B + n\*(1 + n)\*WeierstrassP[x, {g2, g3}])\*y[x]) + Derivative[2][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{\frac{d^2}{dx^2}Y(x) + (-n(n + 1) WeierstrassP(x, g2, g3) - B)Y(x)\right\}, \{Y(x)\}\right)\right\}$$

**2.1027 ODE No. 1027**

$$y(x) (as\,n(x|k)^2 + b) + y''(x) = 0$$

✗ **Mathematica** : cpu = 1.29035 (sec), leaf count = 0 , could not solve

DSolve[(b + a\*JacobiSN[x, k]^2)\*y[x] + Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.349 (sec), leaf count = 69

$$\left\{ y(x) = \_C1 HeunG\left(k^{-2}, \frac{b}{4k^2}, -\frac{n}{2}, \frac{n}{2} + \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, (JacobiSN(x, k))^2\right) + \_C2 HeunG\left(k^{-2}, \frac{k^2 + b + 1}{4k^2}, \frac{n}{2} + 1, \dots\right)\right\}$$

**2.1028 ODE No. 1028**

$$y''(x) - y(x) \left( ap(x) + b + \frac{p^4(x)}{30} + \frac{7p''(x)}{3} \right) = 0$$

✗ **Mathematica** : cpu = 0.255097 (sec), leaf count = 0 , could not solve

DSolve[-(y[x]\*(b + a\*p[x] + (p^4)[x]/30 + (7\*Derivative[2][p][x])/3)) + Derivative[2][y][x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{\frac{d^2}{dx^2}Y(x) + \left(-\frac{d^4 p(x)}{30} - \frac{7 \frac{d^2 p(x)}{dx^2}}{3} - ap(x) - b\right)Y(x)\right\}, \{Y(x)\}\right)\right\}$$

**2.1029 ODE No. 1029**

$$y''(x) - y(x) (f'(x) + f(x)^2) = 0$$

✗ **Mathematica** : cpu = 0.126126 (sec), leaf count = 0 , could not solve

DSolve[-(y[x]\*(f[x]^2 + Derivative[1][f][x])) + Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.128 (sec), leaf count = 22

$$\left\{ y(x) = \left( \int e^{\int -2f(x) dx} dx + \_C1 \right) e^{\int f(x) dx} \_C2 \right\}$$

**2.1030 ODE No. 1030**

$$y(x)(l + P(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.131358 (sec), leaf count = 0 , could not solve

DSolve[(1 + P[x])\*y[x] + Derivative[2][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ (P(x) + l) \_Y(x) + \frac{d^2}{dx^2} \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

**2.1031 ODE No. 1031**

$$y''(x) - f(x)y(x) = 0$$

✗ **Mathematica** : cpu = 0.101372 (sec), leaf count = 0 , could not solve

DSolve[-(f[x]\*y[x]) + Derivative[2][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ -f(x) \_Y(x) + \frac{d^2}{dx^2} \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$



**2.1032 ODE No. 1032**

$$y(x) \left( \frac{(\frac{1}{4} - v^2) g'(x)^2}{g(x)} + g'(x)^2 + \frac{g^3(x)}{2g'(x)} - \frac{3g''(x)^2}{4g'(x)^2} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.46679 (sec), leaf count = 0 , could not solve

DSolve[y[x]\*((g^3)[x]/(2\*Derivative[1][g][x]) + Derivative[1][g][x]^2 + ((1/4 - v^2)\*Derivat

✓ **Maple** : cpu = 0.112 (sec), leaf count = 48

$$\left\{ y(x) = 1 \left( -C2 W_{\frac{i}{2}v^2 - \frac{i}{8}, \frac{1}{2}}(2ig(x)) + -C1 M_{\frac{i}{2}v^2 - \frac{i}{8}, \frac{1}{2}}(2ig(x)) \right) \frac{1}{\sqrt{\frac{d}{dx}g(x)}} \right\}$$

**2.1033 ODE No. 1033**

$$ae^{-2x}y(x) + y''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0181377 (sec), leaf count = 37

$$\{ \{ y(x) \rightarrow c_1 \cos(\sqrt{a}e^{-x}) - c_2 \sin(\sqrt{a}e^{-x}) \} \}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 27

$$\{ y(x) = -C1 \sin(e^{-x}\sqrt{a}) + -C2 \cos(e^{-x}\sqrt{a}) \}$$

**Hand solution**

$$y'' + y' + ae^{-2x}y = 0$$

Let  $y(x) = \eta(\xi)$  where  $\xi = e^{-x}$ , hence

$$\begin{aligned} \frac{dy}{dx} &= \frac{d\eta}{d\xi} \frac{d\xi}{dx} \\ &= \frac{d\eta}{d\xi} (-e^{-x}) \end{aligned}$$

And

$$\begin{aligned}
\frac{d^2y}{dx^2} &= \frac{d}{dx} \left( \frac{d\eta}{d\xi} (-e^{-x}) \right) \\
&= \frac{d^2\eta}{d\xi^2} \frac{d\xi}{dx} (-e^{-x}) + \frac{d\eta}{d\xi} (e^{-x}) \\
&= \frac{d^2\eta}{d\xi^2} (-e^{-x}) (-e^{-x}) + \frac{d\eta}{d\xi} (e^{-x}) \\
&= \frac{d^2\eta}{d\xi^2} (e^{-2x}) + \frac{d\eta}{d\xi} (e^{-x})
\end{aligned}$$

Hence the original ODE becomes

$$\begin{aligned}
\frac{d^2\eta}{d\xi^2} (e^{-2x}) + \frac{d\eta}{d\xi} (e^{-x}) + \frac{d\eta}{d\xi} (-e^{-x}) + ae^{-2x}\eta(\xi) &= 0 \\
\eta'' + a\eta &= 0
\end{aligned}$$

This is standard second order with constant coefficients. The solution is

$$\eta = c_1 \cos(\sqrt{a}\xi) + c_2 \sin(\sqrt{a}\xi)$$

Substituting back

$$y(x) = c_1 \cos(\sqrt{a}e^{-x}) + c_2 \sin(\sqrt{a}e^{-x})$$

Verification

```

restart;
ode:=diff(diff(y(x),x),x)+diff(y(x),x)+a*exp(-2*x)*y(x)=0;
ys:=-C1*cos(sqrt(a)*exp(-x))+_C2*sin(sqrt(a)*exp(-x));
odetest(y(x)=ys,ode);
0

```

## 2.1034 ODE No. 1034

$$y''(x) - y'(x) + e^{2x}y(x) = 0$$

✓ **Mathematica** : cpu = 0.0125617 (sec), leaf count = 20

$$\{\{y(x) \rightarrow c_2 \sin(e^x) + c_1 \cos(e^x)\}\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 15

$$\{y(x) = \_C1 \sin(e^x) + \_C2 \cos(e^x)\}$$

**Hand solution**

$$y'' - y' + e^{2x}y = 0$$

Let  $y(x) = \eta(\xi)$  where  $\xi = e^x$ , hence

$$\begin{aligned}\frac{dy}{dx} &= \frac{d\eta}{d\xi} \frac{d\xi}{dx} \\ &= \frac{d\eta}{d\xi} e^x\end{aligned}$$

And

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d}{dx} \left( \frac{d\eta}{d\xi} e^x \right) \\ &= \frac{d^2\eta}{d\xi^2} \frac{d\xi}{dx} (e^x) + \frac{d\eta}{d\xi} (e^x) \\ &= \frac{d^2\eta}{d\xi^2} (e^x) (e^x) + \frac{d\eta}{d\xi} (e^x) \\ &= \frac{d^2\eta}{d\xi^2} (e^{2x}) + \frac{d\eta}{d\xi} (e^x)\end{aligned}$$

Hence the original ODE becomes

$$\begin{aligned}\frac{d^2\eta}{d\xi^2} (e^{2x}) + \frac{d\eta}{d\xi} (e^x) - \frac{d\eta}{d\xi} (e^x) + e^{2x}\eta &= 0 \\ \eta'' + \eta &= 0\end{aligned}$$

This is standard second order with constant coefficients. The solution is

$$\eta = c_1 \cos(\xi) + c_2 \sin(\xi)$$

Substituting back

$$y(x) = c_1 \cos(e^x) + c_2 \sin(e^x)$$

Verification

```

restart;
ode:=diff(diff(y(x),x),x)-diff(y(x),x)+exp(2*x)*y(x)=0;
ys:=-C1*cos(exp(x))+_C2*sin(exp(x));
odetest(y(x)=ys,ode);
0

```

## 2.1035 ODE No. 1035

$$ay'(x) + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.00623299 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{1}{2}x(\sqrt{a^2-4b}+a)} \left( c_2 e^{x\sqrt{a^2-4b}} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 41

$$\left\{ y(x) = \_C1 e^{-\frac{x}{2}(a-\sqrt{a^2-4b})} + \_C2 e^{-\frac{x}{2}(a+\sqrt{a^2-4b})} \right\}$$

## 2.1036 ODE No. 1036

$$ay'(x) + by(x) - f(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.500289 (sec), leaf count = 150

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{1}{2}x(\sqrt{a^2-4b}+a)} \left( \int_1^x \frac{f(K[1]) e^{\frac{1}{2}(\sqrt{a^2-4b}+a)K[1]}}{\sqrt{a^2-4b}} dK[1] + e^{x\sqrt{a^2-4b}} \int_1^x \frac{f(K[2]) e^{\frac{1}{2}(a-\sqrt{a^2-4b})K[2]}}{\sqrt{a^2-4b}} dK[2] \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 124

$$\left\{ y(x) = e^{-\frac{x}{2}(a-\sqrt{a^2-4b})} \_C2 + e^{-\frac{x}{2}(a+\sqrt{a^2-4b})} \_C1 + 1 \left( \int f(x) e^{-\frac{x}{2}(-a+\sqrt{a^2-4b})} dx e^{x\sqrt{a^2-4b}} - \int f(x) e^{\frac{x}{2}(a+\sqrt{a^2-4b})} dx \right) \right\}$$

**2.1037 ODE No. 1037**

$$ay'(x) + y(x) \left(- (b^2x^2 + c)\right) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0300844 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{1}{2}x(a+bx)} \left( c_1 H_{-\frac{a^2+4(b+c)}{8b}}(\sqrt{bx}) + c_2 {}_1F_1\left(\frac{a^2+4(b+c)}{16b}; \frac{1}{2}; bx^2\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 64

$$\left\{ y(x) = e^{-\frac{x(bx+a)}{2}} x \left( M\left(\frac{a^2+12b+4c}{16b}, \frac{3}{2}, bx^2\right) - C1 + U\left(\frac{a^2+12b+4c}{16b}, \frac{3}{2}, bx^2\right) - C2 \right) \right\}$$

**2.1038 ODE No. 1038**

$$2ay'(x) + f(x)y(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.275084 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + 2*a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{ f(x) - Y(x) + 2a \frac{d}{dx} - Y(x) + \frac{d^2}{dx^2} - Y(x) \right\}, \{ -Y(x) \} \right) \right\}$$

**2.1039 ODE No. 1039**

$$y''(x) + xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0136982 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-\frac{x^2}{2}} \left( \sqrt{2\pi} c_1 \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) + 2c_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 25

$$\left\{ y(x) = \left( \operatorname{Erf}\left(\frac{i}{2}\sqrt{2}x\right) - C1 + -C2 \right) \left( e^{\frac{x^2}{2}} \right)^{-1} \right\}$$

**2.1040 ODE No. 1040**

$$y''(x) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0312561 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{\frac{\pi}{2}} c_2 x \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) - c_2 e^{-\frac{x^2}{2}} + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.042 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{\pi} \sqrt{2} e^{-\frac{x^2}{2}} C_2 + x \left( \pi C_2 \operatorname{Erf}\left(\frac{\sqrt{2}x}{2}\right) + C_1 \right) \right\}$$

**2.1041 ODE No. 1041**

$$(n+1)y(x) + y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.00998801 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{x^2}{2}} \left( c_1 H_n\left(\frac{x}{\sqrt{2}}\right) + c_2 {}_1F_1\left(-\frac{n}{2}; \frac{1}{2}; \frac{x^2}{2}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 41

$$\left\{ y(x) = e^{-\frac{x^2}{2}} x \left( M\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, \frac{x^2}{2}\right) C_1 + U\left(-\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, \frac{x^2}{2}\right) C_2 \right) \right\}$$

**2.1042 ODE No. 1042**

$$-ny(x) + y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.00874967 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{x^2}{2}} \left( c_1 H_{-n-1}\left(\frac{x}{\sqrt{2}}\right) + c_2 {}_1F_1\left(\frac{n+1}{2}; \frac{1}{2}; \frac{x^2}{2}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 41

$$\left\{ y(x) = e^{-\frac{x^2}{2}} x \left( U\left(\frac{n}{2} + 1, \frac{3}{2}, \frac{x^2}{2}\right) C_2 + M\left(\frac{n}{2} + 1, \frac{3}{2}, \frac{x^2}{2}\right) C_1 \right) \right\}$$

**2.1043 ODE No. 1043**

$$y''(x) - xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0475565 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow c_1(x^2 - 1) - \frac{1}{4}c_2 \left( 2e^{\frac{x^2}{2}} x - \sqrt{2\pi}(x^2 - 1) \operatorname{erfi}\left(\frac{x}{\sqrt{2}}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.211 (sec), leaf count = 39

$$\left\{ y(x) = -2e^{1/2x^2} - C1 x + (x - 1)(1 + x) \left( \sqrt{\pi}\sqrt{2}\operatorname{erfi}\left(\frac{\sqrt{2}x}{2}\right) - C1 + -C2 \right) \right\}$$

**Hand solution**

$$y'' - xy' + 2y = 0 \tag{1}$$

Second order with varying coefficient. Using power series, let  $y = \sum_{n=0}^{\infty} c_n x^n$ , hence

$$y' = \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n$$

$$y'' = \sum_{n=0}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n$$

Substituting back in the original ODE gives

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - x \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n + 2 \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=0}^{\infty} (n+1) c_{n+1} x^{n+1} + \sum_{n=0}^{\infty} 2c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n - \sum_{n=1}^{\infty} n c_n x^n + \sum_{n=0}^{\infty} 2c_n x^n = 0$$

For  $n = 0$

$$(n+1)(n+2) c_{n+2} + 2c_n = 0$$

$$(1)(2) c_2 + 2c_0 = 0$$

$$c_2 = -c_0$$

For  $n \geq 1$

$$(n+1)(n+2) c_{n+2} - n c_n + 2c_n = 0$$

$$c_{n+2} = \frac{c_n(n-2)}{(n+1)(n+2)}$$

Hence for  $n = 1$

$$c_3 = \frac{-c_1}{(2)(3)}$$

For  $n = 2$

$$c_4 = \frac{c_2(2-2)}{(3)(4)} = 0$$

For  $n = 3$

$$c_5 = \frac{c_3}{(4)(5)} = \frac{-c_1}{(2)(3)(4)(5)}$$

For  $n = 4$  and since  $c_4 = 0$  then

$$c_6 = \frac{c_4(n-2)}{(n+1)(n+2)} = 0$$

For  $n = 5$

$$c_7 = \frac{3c_5}{(6)(7)} = -\frac{3c_1}{(2)(3)(4)(5)(6)(7)}$$

For  $n = 6$  and since  $c_6 = 0$  then

$$c_8 = \frac{c_6(n-2)}{(n+1)(n+2)} = 0$$

For  $n = 7$

$$c_9 = \frac{5c_7}{(8)(9)} = -\frac{(3)(5)c_1}{(2)(3)(4)(5)(6)(7)(8)(9)}$$

And so on. Hence

$$\begin{aligned} y &= \sum_{n=0}^{\infty} c_n x^n \\ &= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + c_1 x - c_0 x^2 - \frac{c_1}{(2)(3)} x^3 - \frac{c_1}{(2)(3)(4)(5)} x^5 - \frac{3c_1}{(2)(3)(4)(5)(6)(7)} x^7 - \frac{(3)(5)c_1}{(2)(3)(4)(5)(6)(7)(8)(9)} x^9 - \dots \\ &= c_0(1 - x^2) + c_1 \left( x - \frac{1}{(2)(3)} x^3 - \frac{1}{(2)(3)(4)(5)} x^5 - \frac{3}{(2)(3)(4)(5)(6)(7)} x^7 - \frac{(3)(5)}{(2)(3)(4)(5)(6)(7)(8)(9)} x^9 - \dots \right) \\ &= c_0(1 - x^2) + c_1 \left( x - \frac{1}{3!} x^3 - \frac{1}{5!} x^5 - \frac{3}{7!} x^7 - \frac{15}{9!} x^9 - \dots \right) \end{aligned}$$

Hence

$$y(x) = c_0(1 - x^2) + c_1 \left( x - \frac{1}{6} x^3 - \frac{1}{120} x^5 - \frac{1}{1680} x^7 - \frac{1}{24192} x^9 - \dots \right)$$

Verification

```
restart;
Order:=10:
sol:=dsolve(ode,y(x),series):
subs({y(0)=c0,D(y)(0)=c1},rhs(sol)):
```



```
sol:=convert(%,polynom):
```

```
sol:=collect(sol,{c0,c1});
```

```
sol := (-x^2+1)*c0+(x-(1/6)*x^3-(1/120)*x^5-(1/1680)*x^7-(1/24192)*x^9)*c1
```

## 2.1044 ODE No. 1044

$$-ay(x) + y''(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0094738 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow c_1 H_{-a} \left( \frac{x}{\sqrt{2}} \right) + c_2 {}_1F_1 \left( \frac{a}{2}; \frac{1}{2}; \frac{x^2}{2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 35

$$\left\{ y(x) = x \left( U \left( \frac{1}{2} + \frac{a}{2}, \frac{3}{2}, \frac{x^2}{2} \right) - C2 + M \left( \frac{1}{2} + \frac{a}{2}, \frac{3}{2}, \frac{x^2}{2} \right) - C1 \right) \right\}$$

## 2.1045 ODE No. 1045

$$y''(x) - xy'(x) + (x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0229103 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow \sqrt{\frac{\pi}{2}} c_2 e^{x-2} \operatorname{erfi} \left( \frac{x-2}{\sqrt{2}} \right) + c_1 e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 21

$$\left\{ y(x) = e^x \left( \operatorname{Erf} \left( \frac{i}{2} \sqrt{2} (x-2) \right) - C1 + -C2 \right) \right\}$$

## 2.1046 ODE No. 1046

$$ay(x) + y''(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.00799483 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow c_1 H_{\frac{a}{2}}(x) + c_2 {}_1F_1 \left( -\frac{a}{4}; \frac{1}{2}; x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 31

$$\left\{ y(x) = x \left( M \left( \frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2 \right) - C1 + U \left( \frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2 \right) - C2 \right) \right\}$$

2.1047 ODE No. 1047

$$(4x^2 + 2)y(x) + y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0157842 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow e^{-x^2} (c_2 x + c_1) \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 16

$$\{y(x) = e^{-x^2} (_C2 x + _C1)\}$$

**Hand solution**

$$y'' + 4xy' + (4x^2 + 2)y = 0 \tag{1}$$

Second order with varying coefficient. Using power series, let  $y = \sum_{n=0}^{\infty} c_n x^n$ , hence

$$y' = \sum_{n=0}^{\infty} n c_n x^{n-1} = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n$$

$$y'' = \sum_{n=0}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1}^{\infty} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n$$

Substituting back in the original ODE gives

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n + 4x \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n + (4x^2 + 2) \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n + \sum_{n=0}^{\infty} 4(n+1) c_{n+1} x^{n+1} + \sum_{n=0}^{\infty} 4c_n x^{n+2} + \sum_{n=0}^{\infty} 2c_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2) c_{n+2} x^n + \sum_{n=1}^{\infty} 4n c_n x^n + \sum_{n=2}^{\infty} 4c_{n-2} x^n + \sum_{n=0}^{\infty} 2c_n x^n = 0$$

For  $n = 0$

$$(n+1)(n+2) c_{n+2} + 2c_n = 0$$

$$(1) (2) c_2 + 2c_0 = 0$$

$$c_2 = -c_0$$

For  $n = 1$

$$(n+1)(n+2) c_{n+2} + 4n c_n + 2c_n = 0$$

$$(2) (3) c_3 + 4c_1 + 2c_1 = 0$$

$$c_3 = -c_1$$

For  $n \geq 2$

$$(n+1)(n+2)c_{n+2} + 4nc_n + 4c_{n-2} + 2c_n = 0$$

$$c_{n+2} = \frac{(-4n-2)c_n - 4c_{n-2}}{(n+1)(n+2)}$$

Hence for  $n = 2$

$$c_4 = \frac{(-8-2)c_2 - 4c_0}{(3)(4)} = \frac{(-8-2)(-c_0) - 4c_0}{(3)(4)} = c_0 \frac{6}{(3)(4)}$$

For  $n = 3$

$$c_5 = \frac{(-12-2)c_3 - 4c_1}{(4)(5)} = \frac{(-12-2)(-c_1) - 4c_1}{(4)(5)} = c_1 \frac{10}{(4)(5)}$$

For  $n = 4$

$$c_6 = \frac{(-16-2)c_4 - 4c_2}{(5)(6)} = \frac{(-16-2)\left(c_0 \frac{6}{(3)(4)}\right) - 4(-c_0)}{(5)(6)} = c_0 \frac{-5}{(5)(6)}$$

For  $n = 5$

$$c_7 = \frac{(-20-2)c_5 - 4c_3}{(6)(7)} = \frac{(-20-2)\left(c_1 \frac{10}{(4)(5)}\right) - 4(-c_1)}{(6)(7)} = c_1 \frac{-7}{(6)(7)}$$

For  $n = 6$

$$c_8 = \frac{(-24-2)c_6 - 4c_4}{(7)(8)} = \frac{(-24-2)\left(c_0 \frac{-5}{(5)(6)}\right) - 4\left(c_0 \frac{6}{(3)(4)}\right)}{(7)(8)} = c_0 \frac{7}{3} \frac{1}{(7)(8)}$$

And so on. Hence

$$y = \sum_{n=0}^{\infty} c_n x^n$$

$$= c_0 + c_1 x + c_2 x^2 + \dots$$

$$= c_0 + c_1 x - c_0 x^2 - c_1 x^3 + c_0 \frac{6}{(3)(4)} x^4 + c_1 \frac{10}{(4)(5)} x^5 - c_0 \frac{5}{(5)(7)} x^6 - c_1 \frac{7}{(6)(7)} x^7 + c_0 \frac{12}{7} \frac{1}{(7)(8)} x^8 + \dots$$

$$= c_0 \left( 1 - x^2 + \frac{6}{(3)(4)} x^4 - \frac{5}{(5)(6)} x^6 + \frac{7}{3} \frac{1}{(7)(8)} x^8 + \dots \right) + c_1 \left( x - x^3 + \frac{10}{(4)(5)} x^5 - \frac{7}{(6)(7)} x^7 + \dots \right)$$

$$= c_0 \left( 1 - x^2 + \frac{1}{2} x^4 - \frac{1}{6} x^6 + \frac{1}{24} x^8 + \dots \right) + c_1 \left( x - x^3 + \frac{1}{2} x^5 - \frac{1}{6} x^7 + \dots \right)$$

But Taylor series for  $e^{-x^2} = 1 - x^2 + \frac{1}{2} x^4 - \frac{x^6}{4} + \dots$ , therefore the above becomes

$$y = c_0 e^{-x^2} + c_1 x e^{-x^2}$$

Verification

```

restart;
restart;
ode:=diff(diff(y(x),x),x)+4*x*diff(y(x),x)+(4*x^2+2)*y(x)=0;
y0:=-C0*exp(-x^2)+C1*x*exp(-x^2);
odetest(y(x)=y0,ode);
0

```

## 2.1048 ODE No. 1048

$$(2n + 3x^2 - 1)y(x) + y''(x) - 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0117684 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{x^2}{2}} \left( c_1 H_n(x) + c_2 {}_1F_1 \left( -\frac{n}{2}; \frac{1}{2}; x^2 \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 37

$$\left\{ y(x) = e^{\frac{x^2}{2}} x \left( U \left( -\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2 \right) - C2 + M \left( -\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, x^2 \right) - C1 \right) \right\}$$

## 2.1049 ODE No. 1049

$$(4x^2 - 1)y(x) + y''(x) - 4xy'(x) - e^x = 0$$

✓ **Mathematica** : cpu = 0.069975 (sec), leaf count = 105

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} e^{x(x-i)-\frac{i}{2}} \left( 2e^{\frac{i}{2}} (2c_1 - ic_2 e^{2ix}) - ie^i \sqrt{\pi} \operatorname{erf} \left( -x + \left( \frac{1}{2} + \frac{i}{2} \right) \right) + \sqrt{\pi} e^{2ix} \operatorname{erfi} \left( \left( \frac{1}{2} + \frac{i}{2} \right) - ix \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 66

$$\left\{ y(x) = \frac{\left( (i \cos(x) + \sin(x)) \sqrt{\pi} e^{\frac{i}{2}} \operatorname{Erf} \left( x - \frac{1}{2} - \frac{i}{2} \right) - (i \cos(x) - \sin(x)) \sqrt{\pi} e^{-\frac{i}{2}} \operatorname{Erf} \left( x - \frac{1}{2} + \frac{i}{2} \right) + 4 \sin(x) - C \right)}{4} \right\}$$

**2.1050 ODE No. 1050**

$$(4x^2 - 2)y(x) + y''(x) - 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0143497 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow e^{x^2} (c_2 x + c_1) \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 14

$$\left\{ y(x) = e^{x^2} (_{C2} x + _{C1}) \right\}$$

**2.1051 ODE No. 1051**

$$(4x^2 - 3)y(x) - e^{x^2} + y''(x) - 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0384542 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{(x-1)x} (c_2 e^{2x} + 2c_1 - 2e^x) \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 27

$$\left\{ y(x) = e^{x(1+x)} _{C2} + e^{x(x-1)} _{C1} - e^{x^2} \right\}$$

**2.1052 ODE No. 1052**

$$axy'(x) + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0224562 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{ax^2}{2}} \left( c_1 H_{\frac{b}{a}-1} \left( \frac{\sqrt{ax}}{\sqrt{2}} \right) + c_2 {}_1F_1 \left( \frac{a-b}{2a}; \frac{1}{2}; \frac{ax^2}{2} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 58

$$\left\{ y(x) = e^{-\frac{ax^2}{2}} x \left( M \left( \frac{2a-b}{2a}, \frac{3}{2}, \frac{ax^2}{2} \right) _{C1} + U \left( \frac{2a-b}{2a}, \frac{3}{2}, \frac{ax^2}{2} \right) _{C2} \right) \right\}$$

**2.1053 ODE No. 1053**

$$a^2x^2y(x) + 2axy'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0315633 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\frac{ax^2}{2} - \sqrt{ax}} (c_2 e^{2\sqrt{ax}} + 2\sqrt{ac_1})}{2\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 35

$$\left\{ y(x) = \_C1 e^{-\frac{x}{2}(ax-2\sqrt{a})} + \_C2 e^{-\frac{x}{2}(ax+2\sqrt{a})} \right\}$$

**2.1054 ODE No. 1054**

$$(ax + b)y'(x) + y(x)(cx + d) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.051213 (sec), leaf count = 132

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{cx}{a} - \frac{ax^2}{2} - bx} \left( c_2 {}_1F_1 \left( \frac{a^3 - da^2 + bca - c^2}{2a^3}; \frac{1}{2}; \frac{(xa^2 + ba - 2c)^2}{2a^3} \right) + c_1 H_{-\frac{a^3 + a^2d - abc + c^2}{a^3}} \left( \frac{a^2x + ab - 2c}{\sqrt{2}a^{3/2}} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 98

$$\left\{ y(x) = e^{-\frac{cx}{a}} \left( U \left( \frac{da^2 - abc + c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x + ab - 2c)^2}{2a^3} \right) - C2 + M \left( \frac{da^2 - abc + c^2}{2a^3}, \frac{1}{2}, -\frac{(a^2x + ab - 2c)^2}{2a^3} \right) \right) \right\}$$

**2.1055 ODE No. 1055**

$$(ax + b)y'(x) + y(x)(a_1x^2 + b_1x + c_1) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.156809 (sec), leaf count = 305

$$\left\{ \left\{ y(x) \rightarrow \exp \left( -\frac{x(a(x\sqrt{a^2 - 4a_1} + 2b) + 2b\sqrt{a^2 - 4a_1} + a^2x - 4(a_1x + b_1))}{4\sqrt{a^2 - 4a_1}} \right) \left( c_1 H_{-\frac{a^3 + 2(a_1(2\sqrt{a^2 - 4a_1} + b^2 - 4c_1))}{a^3}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.233 (sec), leaf count = 262

$$\left\{ y(x) = \left( -C2 (a^2x + ab - 4a_1x - 2b_1) {}_1F_1 \left( \frac{1}{4} \left( 3(a^2 - 4a_1)^{3/2} + a^3 - 2a^2c_1 + (2b_1b - 4a_1)a + (-2b^2 + \dots \right) \right) \right) \right\}$$

**2.1056 ODE No. 1056**

$$x^2(-y'(x)) + y''(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0392222 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow -c_2 e^{\frac{x^3}{3}} + \frac{c_2 \sqrt[3]{-x^3} \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right)}{\sqrt[3]{3}} + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 48

$$\left\{ y(x) = \frac{1}{x^2} \left( -e^{\frac{x^3}{3}} (-x^3)^{\frac{2}{3}} \sqrt[3]{3} C_2 + x^3 \left( -C_2 \Gamma\left(\frac{2}{3}\right) - C_2 \Gamma\left(\frac{2}{3}, -\frac{x^3}{3}\right) + C_1 \right) \right) \right\}$$

**2.1057 ODE No. 1057**

$$x^2(-y'(x)) + y''(x) - (x+1)^2 y(x) = 0$$

✓ **Mathematica** : cpu = 0.87748 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{x^3}{3}+x} \left( c_2 \int_1^x e^{-\frac{1}{3}K[1](K[1]^2+6)} dK[1] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 50

$$\left\{ y(x) = C_1 \operatorname{HeunT}\left(0, -3, 2\sqrt[3]{3}, \frac{3^{\frac{2}{3}}x}{3}\right) e^{-x} + C_2 \operatorname{HeunT}\left(0, 3, 2\sqrt[3]{3}, -\frac{3^{\frac{2}{3}}x}{3}\right) e^{\frac{x(x^2+3)}{3}} \right\}$$

**2.1058 ODE No. 1058**

$$(x^4 - 2)xy(x) - (x+1)x^2y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.89359 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{x^3}{3}} \left( c_2 \int_1^x e^{\frac{1}{12}K[1]^3(3K[1]-4)} dK[1] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.117 (sec), leaf count = 29

$$\left\{ y(x) = e^{\frac{x^3}{3}} \left( \int e^{\frac{x^4}{4} - \frac{x^3}{3}} dx C_2 + C_1 \right) \right\}$$

**2.1059 ODE No. 1059**

$$x^4 y'(x) - x^3 y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0564086 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow -c_2 e^{-\frac{x^5}{5}} + \frac{c_2 \sqrt[5]{x^5} \Gamma\left(\frac{4}{5}, \frac{x^5}{5}\right)}{\sqrt[5]{5}} + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.088 (sec), leaf count = 56

$$\left\{ y(x) = \frac{1}{x^7} \left( 9 - C_2 e^{-1/10 x^5} (x^5 + 4) M_{7/5, \frac{9}{10}}(1/5 x^5) + x^8 \left( x^2 - C_2 e^{-\frac{x^5}{10}} M_{\frac{2}{5}, \frac{9}{10}}\left(\frac{x^5}{5}\right) + -C_1 \right) \right) \right\}$$

**2.1060 ODE No. 1060**

$$ax^{q-1}y'(x) + bx^{q-2}y(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0376037 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow c_2 q^{-1/q} a^{\frac{1}{q}} (x^q)^{\frac{1}{q}} {}_1F_1\left(\frac{a+b}{aq}; 1 + \frac{1}{q}; -\frac{ax^q}{q}\right) + c_1 {}_1F_1\left(\frac{b}{aq}; \frac{q-1}{q}; -\frac{ax^q}{q}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 81

$$\left\{ y(x) = e^{-\frac{ax^q}{q}} x \left( M\left(\frac{aq-b}{aq}, \frac{q+1}{q}, \frac{ax^q}{q}\right) - C_1 + U\left(\frac{aq-b}{aq}, \frac{q+1}{q}, \frac{ax^q}{q}\right) - C_2 \right) \right\}$$

**2.1061 ODE No. 1061**

$$-e^{-\frac{x^{3/2}}{3}} x + y''(x) + \sqrt{x} y'(x) + \left(\frac{x}{4} + \frac{1}{4\sqrt{x}} - 9\right) y(x) = 0$$

✓ **Mathematica** : cpu = 0.095721 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{18} e^{-\frac{1}{3}(\sqrt{x}+9)x} (3c_2 e^{6x} + 18c_1 - 2e^{3x}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.088 (sec), leaf count = 28

$$\left\{ y(x) = -\frac{-9 \cosh(3x) - C_1 - 9 \sinh(3x) - C_2 + x e^{-\frac{1}{3}x^{\frac{3}{2}}}}{9} \right\}$$



**2.1062 ODE No. 1062**

$$\frac{(x + \sqrt{x} - 8) y(x)}{4x^2} + y''(x) - \frac{y'(x)}{\sqrt{x}} = 0$$

✓ **Mathematica** : cpu = 0.0296905 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\sqrt{x}}(c_2 x^3 + 3c_1)}{3x} \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 19

$$\left\{ y(x) = \frac{-C2 x^3 + -C1}{x} e^{\sqrt{x}} \right\}$$

**2.1063 ODE No. 1063**

$$y''(x) - (2e^x + 1) y'(x) + e^{2x} y(x) - e^{3x} = 0$$

✓ **Mathematica** : cpu = 0.0484224 (sec), leaf count = 28

$$\{ \{ y(x) \rightarrow c_1 e^{e^x} + c_2 e^{e^x + e^x} + e^x + 2 \} \}$$

✓ **Maple** : cpu = 0.36 (sec), leaf count = 61

$$\left\{ y(x) = \left( -C1 \cosh\left(\frac{x}{2}\right) + -C2 \sinh\left(\frac{x}{2}\right) \right) e^{e^x + \frac{x}{2}} + e^{\frac{x}{2}} \left( (e^{2x} + e^x + 1) \cosh\left(\frac{x}{2}\right) - 3(e^x + 1/3 e^{2x} + 1) \sinh(x) \right) \right\}$$

**2.1064 ODE No. 1064**

$$ay'(x) + by(x) + y''(x) + \tan(x) = 0$$

✓ **Mathematica** : cpu = 0.559657 (sec), leaf count = 502

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\frac{1}{2}x(\sqrt{a^2-4b}+a)} \left( (2a - i(b-4)) \left( 2ib\sqrt{a^2-4b} (c_2 e^{x\sqrt{a^2-4b}} + c_1) \right) + (\sqrt{a^2-4b} + a) e^{\frac{1}{2}x(\sqrt{a^2-4b}+a)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.296 (sec), leaf count = 125

$$\left\{ y(x) = e^{-\frac{x}{2}(a-\sqrt{a^2-4b})} -C2 + e^{-\frac{x}{2}(a+\sqrt{a^2-4b})} -C1 - 1 \left( \int \tan(x) e^{-\frac{x}{2}(-a+\sqrt{a^2-4b})} dx e^{x\sqrt{a^2-4b}} - \int \tan(x) e^{\frac{x}{2}} \right) \right\}$$

**2.1065 ODE No. 1065**

$$(n^2 - a^2)y(x) + 2n \cot(x)y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.161643 (sec), leaf count = 83

$$\left\{ \left\{ y(x) \rightarrow (-\sin^2(x))^{\frac{1}{4} - \frac{n}{2}} \left( c_1 P_{\sqrt{2n^2 - a^2} - \frac{1}{2}}^{n - \frac{1}{2}}(\cos(x)) + c_2 Q_{\sqrt{2n^2 - a^2} - \frac{1}{2}}^{n - \frac{1}{2}}(\cos(x)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.237 (sec), leaf count = 60

$$\left\{ y(x) = (\sin(x))^{-n + \frac{1}{2}} \left( LegendreQ\left(-\frac{1}{2} + \sqrt{-a^2 + 2n^2}, n - \frac{1}{2}, \cos(x)\right) \_C2 + LegendreP\left(-\frac{1}{2} + \sqrt{-a^2 + 2n^2}, n - \frac{1}{2}, \cos(x)\right) \_C1 \right) \right\}$$

**2.1066 ODE No. 1066**

$$y''(x) + \tan(x)y'(x) + y(x) \cos^2(x) = 0$$

✓ **Mathematica** : cpu = 0.034097 (sec), leaf count = 18

$$\{\{y(x) \rightarrow c_2 \sin(\sin(x)) + c_1 \cos(\sin(x))\}\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 15

$$\{y(x) = \_C1 \sin(\sin(x)) + \_C2 \cos(\sin(x))\}$$

**2.1067 ODE No. 1067**

$$y''(x) + \tan(x)y'(x) - y(x) \cos^2(x) = 0$$

✓ **Mathematica** : cpu = 0.0340813 (sec), leaf count = 21

$$\{\{y(x) \rightarrow c_1 \cosh(\sin(x)) + ic_2 \sinh(\sin(x))\}\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 17

$$\{y(x) = \_C1 e^{\sin(x)} + \_C2 e^{-\sin(x)}\}$$

**2.1068 ODE No. 1068**

$$v(v+1)y(x) + y''(x) + \cot(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.140015 (sec), leaf count = 20

$$\{y(x) \rightarrow c_1 P_v(\cos(x)) + c_2 Q_v(\cos(x))\}$$

✓ **Maple** : cpu = 0.223 (sec), leaf count = 45

$$\left\{ y(x) = \_C1 {}_2F_1\left(-\frac{v}{2}, \frac{1}{2} + \frac{v}{2}; \frac{1}{2}; (\cos(x))^2\right) + \_C2 \cos(x) {}_2F_1\left(1 + \frac{v}{2}, \frac{1}{2} - \frac{v}{2}; \frac{3}{2}; (\cos(x))^2\right) \right\}$$

**2.1069 ODE No. 1069**

$$y''(x) - \cot(x)y'(x) + y(x) \sin^2(x) = 0$$

✓ **Mathematica** : cpu = 0.0366466 (sec), leaf count = 19

$$\{y(x) \rightarrow c_1 \cos(\cos(x)) - c_2 \sin(\cos(x))\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 15

$$\{y(x) = \_C1 \sin(\cos(x)) + \_C2 \cos(\cos(x))\}$$

**2.1070 ODE No. 1070**

$$a \tan(x)y'(x) + by(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.320398 (sec), leaf count = 129

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(\frac{1}{4}(-a - \sqrt{a^2 + 4b}), \frac{1}{4}(\sqrt{a^2 + 4b} - a); \frac{1-a}{2}; \cos^2(x)\right) + i^{a+1} c_2 \cos^{a+1}(x) {}_2F_1\left(\frac{1}{4}(a - \sqrt{a^2 + 4b}), \frac{1}{4}(\sqrt{a^2 + 4b} + a); \frac{1+a}{2}; \cos^2(x)\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 60

$$\left\{ y(x) = (\cos(x))^{\frac{1}{2} + \frac{a}{2}} \left( LegendreQ\left(\frac{1}{2}\sqrt{a^2 + 4b} - \frac{1}{2}, \frac{1}{2} + \frac{a}{2}, \sin(x)\right) \_C2 + LegendreP\left(\frac{1}{2}\sqrt{a^2 + 4b} - \frac{1}{2}, \frac{1}{2} + \frac{a}{2}, \sin(x)\right) \_C1 \right) \right\}$$

**2.1071 ODE No. 1071**

$$(b^2 - a^2) y(x) + 2a \cot(ax) y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.107464 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-\sqrt{-b^2}x} \csc(ax) \left( \frac{c_2 e^{2\sqrt{-b^2}x}}{\sqrt{-b^2}} + 2c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 24

$$\left\{ y(x) = \frac{-C2 \cos(bx) + -C1 \sin(bx)}{\sin(ax)} \right\}$$

**2.1072 ODE No. 1072**

$$y(x) (-4anp(x)^2 + a + bp(x)) + ap''(x)y'(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.280173 (sec), leaf count = 0 , could not solve

`DSolve[(a + b*p[x] - 4*a*n*p[x]^2)*y[x] + a*Derivative[1][y][x]*Derivative[2][p][x] + Deriva`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + a \left( \frac{d^2}{dx^2} p(x) \right) \frac{d}{dx} Y(x) + \left( a + bp(x) - 4na(p(x))^2 \right) Y(x) \right\}, \{ -Y(x) \} \right) \right\}$$

**2.1073 ODE No. 1073**

$$\frac{y'(x) (-\wp(x; a, b)\wp'(x; a, b) + \wp(x; a, b)^3 - 6\wp(x; a, b)^2 + \frac{a}{2})}{\wp'(x; a, b) - \wp(x; a, b)^2} + \frac{y(x) (\wp(x; a, b)^2(-\wp'(x; a, b)) - (6\wp(x; a, b)^2 - \frac{a}{2})\wp'(x; a, b))}{\wp(x; a, b)^2 + \wp'(x; a, b)}$$

✗ **Mathematica** : cpu = 1.32698 (sec), leaf count = 0 , could not solve

`DSolve[(((WeierstrassP[x, {a, b}]*(-a/2 + 6*WeierstrassP[x, {a, b}]^2)) - WeierstrassP[x, {a, b}]^2 + WeierstrassPPrime[x, {a, b}]) + Derivative[2][y][x] == 0, y[x],`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + \frac{\frac{d}{dx} Y(x)}{WeierstrassPPrime(x, a, b) + (WeierstrassP(x, a, b))^2} \right\} \left( 11 WeierstrassP(x, a, b) \right) \right) \right\}$$

**2.1074 ODE No. 1074**

$$\frac{k^2 \operatorname{cn}(x|k) \operatorname{sn}(x|k) y'(x)}{\operatorname{dn}(x|k)} + n^2 y(x) \operatorname{dn}(x|k)^2 + y''(x) = 0$$

✗ **Mathematica** : cpu = 48.3257 (sec), leaf count = 0 , could not solve

`DSolve[n^2*JacobiDN[x, k]^2*y[x] + (k^2*JacobiCN[x, k]*JacobiSN[x, k]*Derivative[1][y][x])/J`

✓ **Maple** : cpu = 0.017 (sec), leaf count = 21

$$\{y(x) = \_C1 \sin(n \operatorname{JacobiAM}(x, k)) + \_C2 \cos(n \operatorname{JacobiAM}(x, k))\}$$

**2.1075 ODE No. 1075**

$$f(x)y'(x) + g(x)y(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.182781 (sec), leaf count = 0 , could not solve

`DSolve[g[x]*y[x] + f[x]*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \operatorname{DESol} \left( \left\{ g(x) \_Y(x) + f(x) \frac{d}{dx} \_Y(x) + \frac{d^2}{dx^2} \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

**2.1076 ODE No. 1076**

$$y(x) (a + f'(x)) + f(x)y'(x) - g(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.204745 (sec), leaf count = 0 , could not solve

`DSolve[-g[x] + y[x]*(a + Derivative[1][f][x]) + f[x]*Derivative[1][y][x] + Derivative[2][y][x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \operatorname{DESol} \left( \left\{ \frac{d^2}{dx^2} \_Y(x) + f(x) \frac{d}{dx} \_Y(x) + \left( \frac{d}{dx} f(x) + a \right) \_Y(x) - g(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

**2.1077 ODE No. 1077**

$$y'(x)(af(x) + b) + y(x)(cf(x) + d) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.310607 (sec), leaf count = 0 , could not solve

`DSolve[(d + c*f[x])*y[x] + (b + a*f[x])*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + (af(x) + b) \frac{d}{dx} Y(x) + (cf(x) + d) Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

**2.1078 ODE No. 1078**

$$y(x) \left( a + \frac{f'(x)}{2} + \frac{f(x)^2}{4} \right) + f(x)y'(x) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0724459 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow \frac{\left( c_2 e^{2\sqrt{-ax}} + 2\sqrt{-ac_1} \right) \exp \left( -\frac{1}{2} \int_1^x f(K[1]) dK[1] - \sqrt{-ax} \right)}{2\sqrt{-a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 33

$$\left\{ y(x) = e^{-\frac{\int f(x) dx}{2}} (\sinh(\sqrt{-ax}) C1 + \cosh(\sqrt{-ax}) C2) \right\}$$

**2.1079 ODE No. 1079**

$$by(x)f(x)^{2a} - \frac{af'(x)y'(x)}{f(x)} + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.246015 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( e^{c_2 + \int_1^x -i\sqrt{b}f(K[1])^a dK[1]} - 2c_1 \exp \left( -c_2 - \int_1^x -i\sqrt{b}f(K[1])^a dK[1] \right) \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( e^{c_2 + \int_1^x i\sqrt{b}f(K[1])^a dK[1]} + 2c_1 \exp \left( -c_2 - \int_1^x -i\sqrt{b}f(K[1])^a dK[1] \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 37

$$\left\{ y(x) = C1 e^{\int i(f(x))^a \sqrt{b} dx} + C2 e^{-\int i(f(x))^a \sqrt{b} dx} \right\}$$

**2.1080 ODE No. 1080**

$$y(x) \left( a^2 + \frac{af'(x)}{f(x)} - b^2 f(x)^2 \right) - y'(x) \left( 2a + \frac{f'(x)}{f(x)} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.305302 (sec), leaf count = 0 , could not solve

DSolve[y[x]\*(a^2 - b^2\*f[x]^2 + (a\*Derivative[1][f][x])/f[x]) - (2\*a + Derivative[1][f][x])/f[x] + Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.329 (sec), leaf count = 74

$$\left\{ y(x) = e^{\int^{-1} \left( \frac{f(x)(e^{-C_1 b})^2 b}{(e^{\int f(x) dx b})^2} + b f(x) - \frac{(e^{-C_1 b})^2 a}{(e^{\int f(x) dx b})^2} + a \right) \left( \frac{(e^{-C_1 b})^2}{(e^{\int f(x) dx b})^2} - 1 \right)^{-1} dx - C_2} \right\}$$

**2.1081 ODE No. 1081**

$$-\frac{a^2 y(x) f'(x)^2}{b^2 + f(x)^2} + \frac{f(x) f^3(x) y'(x)}{b^2 + f(x)^2} + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.822484 (sec), leaf count = 0 , could not solve

DSolve[-((a^2\*y[x]\*Derivative[1][f][x]^2)/(b^2 + f[x]^2)) + (f[x]\*(f^3)[x]\*Derivative[1][y][x])/(b^2 + f[x]^2) + Derivative[2][y][x] == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + \frac{f(x) \left( \frac{d^3}{dx^3} f(x) \right) \frac{d}{dx} Y(x)}{(f(x))^2 + b^2} - \frac{\left( \frac{d}{dx} f(x) \right)^2 a^2 Y(x)}{(f(x))^2 + b^2} \right\}, \{ -Y(x) \} \right) \right\}$$

**2.1082 ODE No. 1082**

$$y(x) \left( \frac{(m^2 - v^2) g'(x)^2}{g(x)} + g'(x)^2 \right) - y'(x) \left( \frac{(2m - 1)g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.539612 (sec), leaf count = 0 , could not solve

DSolve[y[x]\*(Derivative[1][g][x]^2 + ((m^2 - v^2)\*Derivative[1][g][x]^2)/g[x]) - Derivative[1][y][x]\*((2\*m - 1)\*Derivative[1][g][x])/g[x] + Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.131 (sec), leaf count = 74

$$\left\{ y(x) = (g(x))^{2m} e^{-ig(x)} \left( U \left( \frac{i}{2} m^2 - \frac{i}{2} v^2 + m + \frac{1}{2}, 2m + 1, 2ig(x) \right) - C_2 + M \left( \frac{i}{2} m^2 - \frac{i}{2} v^2 + m + \frac{1}{2}, 2m + 1, 2ig(x) \right) \right) \right\}$$

**2.1083 ODE No. 1083**

$$-\frac{f'(x)y'(x)}{f(x)} + y(x) \left( -\frac{f''(x)}{2f(x)} + \frac{3f'(x)^2}{4f(x)^2} + \frac{(\frac{1}{4} - v^2)g'(x)^2}{g(x)^2} + g'(x)^2 + \frac{g^3(x)}{2g'(x)} - \frac{3g''(x)^2}{4g'(x)^2} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.811735 (sec), leaf count = 0 , could not solve

DSolve[-((Derivative[1][f][x]\*Derivative[1][y][x])/f[x]) + y[x]\*((3\*Derivative[1][f][x]^2)/

✓ **Maple** : cpu = 0.106 (sec), leaf count = 31

$$\left\{ y(x) = \sqrt{\frac{f(x)g(x)}{\frac{d}{dx}g(x)}} (J_v(g(x))_C1 + Y_v(g(x))_C2) \right\}$$

**2.1084 ODE No. 1084**

$$-y'(x) \left( \frac{2f'(x)}{f(x)} - \frac{g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} \right) + y(x) \left( -\frac{f''(x)}{f(x)} + \frac{f'(x) \left( \frac{2f'(x)}{f(x)} - \frac{g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} \right)}{f(x)} - \frac{v^2 g'(x)^2}{g(x)^2} + g'(x)^2 \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.861561 (sec), leaf count = 0 , could not solve

DSolve[-(Derivative[1][y][x]\*((2\*Derivative[1][f][x])/f[x] - Derivative[1][g][x]/g[x] + Deri

✓ **Maple** : cpu = 0.083 (sec), leaf count = 20

$$\{y(x) = f(x) (J_v(g(x))_C1 + Y_v(g(x))_C2)\}$$

**2.1085 ODE No. 1085**

$$-y'(x) \left( \frac{(2v-1)g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} + \frac{2h'(x)}{h(x)} \right) + y(x) \left( g'(x)^2 + \frac{h'(x) \left( \frac{(2v-1)g'(x)}{g(x)} + \frac{g''(x)}{g'(x)} + \frac{2h'(x)}{h(x)} \right)}{h(x)} - \frac{h''(x)}{h(x)} \right) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.917306 (sec), leaf count = 0 , could not solve

DSolve[-(Derivative[1][y][x]\*((( -1 + 2\*v)\*Derivative[1][g][x])/g[x] + (2\*Derivative[1][h][x]

✓ **Maple** : cpu = 0.079 (sec), leaf count = 24

$$\{y(x) = h(x) (g(x))^v (J_v(g(x))_C1 + Y_v(g(x))_C2)\}$$



**2.1086 ODE No. 1086**

$$4y''(x) + 9xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0060298 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{Ai} \left( \sqrt[3]{-1} \left( \frac{3}{2} \right)^{2/3} x \right) + c_2 \text{Bi} \left( \sqrt[3]{-1} \left( \frac{3}{2} \right)^{2/3} x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 29

$$\left\{ y(x) = \_C1 \text{Ai} \left( -\frac{3^{2/3} \sqrt[3]{2} x}{2} \right) + \_C2 \text{Bi} \left( -\frac{3^{2/3} \sqrt[3]{2} x}{2} \right) \right\}$$

**2.1087 ODE No. 1087**

$$4y''(x) - (a + x^2) y(x) = 0$$

✓ **Mathematica** : cpu = 0.00957747 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow c_1 D_{\frac{1}{4}(-a-2)}(x) + c_2 D_{\frac{a-2}{4}}(ix) \right\} \right\}$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 33

$$\left\{ y(x) = 1 \left( \_C2 W_{-\frac{a}{8}, \frac{1}{4}} \left( \frac{x^2}{2} \right) + \_C1 M_{-\frac{a}{8}, \frac{1}{4}} \left( \frac{x^2}{2} \right) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1088 ODE No. 1088**

$$4y''(x) + 4 \tan(x)y'(x) + y(x) (-5 \tan^2(x) + 2) = 0$$

✓ **Mathematica** : cpu = 0.102603 (sec), leaf count = 97

$$\left\{ \left\{ y(x) \rightarrow \frac{3 \sqrt[8]{-1} c_2 \sqrt[4]{-\cos^4(x)} \sqrt{1 + i \sqrt{-\cos^4(x)}} + 3(-1)^{7/8} c_2 \sinh^{-1} \left( \frac{(1+i) \sqrt[4]{-\cos^4(x)}}{\sqrt{2}} \right) - 2(-1)^{7/8} c_1}{2 \sqrt[8]{-\cos^4(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.14 (sec), leaf count = 31

$$\left\{ y(x) = (i \cos(x) \sin(x) \_C2 - \ln(\sin(x) + i \cos(x)) \_C2 + \_C1) \frac{1}{\sqrt{\cos(x)}} \right\}$$

**2.1089 ODE No. 1089**

$$-y'(x)(ab + c + x) + ay''(x) + y(x)(b(c + x) + d) = 0$$

✓ **Mathematica** : cpu = 0.0445358 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow e^{bx} \left( c_1 H_d \left( \frac{-ab + c + x}{\sqrt{2}\sqrt{a}} \right) + c_2 {}_1F_1 \left( -\frac{d}{2}; \frac{1}{2}; \frac{(-ab + c + x)^2}{2a} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 58

$$\left\{ y(x) = e^{bx} \left( U \left( -\frac{d}{2}, \frac{1}{2}, \frac{(ab - c - x)^2}{2a} \right) - C2 + M \left( -\frac{d}{2}, \frac{1}{2}, \frac{(ab - c - x)^2}{2a} \right) - C1 \right) \right\}$$

**2.1090 ODE No. 1090**

$$a(a^2 - 2be^{-ax})y'(x) + a^2y''(x) + b^2e^{-2ax}y(x) = 0$$

✓ **Mathematica** : cpu = 0.0331313 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\frac{be^{-ax}}{a^2} - ax} (a^2 c_1 e^{ax} - bc_2)}{a^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 40

$$\left\{ y(x) = e^{-\frac{a^3x + 2be^{-ax}}{2a^2}} \left( \sinh \left( \frac{ax}{2} \right) - C1 + \cosh \left( \frac{ax}{2} \right) - C2 \right) \right\}$$

**2.1091 ODE No. 1091**

$$x(y''(x) + y(x)) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0279811 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin(x) + c_1 \cos(x) + \frac{1}{2} (Ci(2x) \sin(x) - Si(2x) \cos(x) + \log(x) \sin(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 35

$$\left\{ y(x) = \frac{\sin(x) Ci(2x)}{2} - \frac{Si(2x) \cos(x)}{2} + \frac{(2 - C2 + \ln(x)) \sin(x)}{2} + \cos(x) - C1 \right\}$$

**2.1092 ODE No. 1092**

$$(a + x)y(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0983749 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow e^{-ix} x \left( c_2 {}_1F_1 \left( \frac{ia}{2} + 1; 2; 2ix \right) + c_1 U \left( \frac{ia}{2} + 1, 2, 2ix \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 29

$$\{y(x) = \_C1 M_{-\frac{i}{2}a, \frac{1}{2}}(2ix) + \_C2 W_{-\frac{i}{2}a, \frac{1}{2}}(2ix)\}$$

**2.1093 ODE No. 1093**

$$xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.00555654 (sec), leaf count = 13

$$\{\{y(x) \rightarrow c_1 \log(x) + c_2\}\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 10

$$\{y(x) = \_C2 \ln(x) + \_C1\}$$

**2.1094 ODE No. 1094**

$$ay(x) + xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0243928 (sec), leaf count = 41

$$\{\{y(x) \rightarrow c_1 J_0(2\sqrt{a}\sqrt{x}) + 2c_2 Y_0(2\sqrt{a}\sqrt{x})\}\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 29

$$\{y(x) = \_C1 J_0(2\sqrt{a}\sqrt{x}) + \_C2 Y_0(2\sqrt{a}\sqrt{x})\}$$

**2.1095 ODE No. 1095**

$$lxy(x) + xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.00973875 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 J_0(\sqrt{l}x) + c_2 Y_0(\sqrt{l}x) \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 23

$$\left\{ y(x) = \_C1 J_0(\sqrt{l}x) + \_C2 Y_0(\sqrt{l}x) \right\}$$

**2.1096 ODE No. 1096**

$$(a + x)y(x) + xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0139961 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow e^{-ix} \left( c_1 U\left(\frac{ia}{2} + \frac{1}{2}, 1, 2ix\right) + c_2 L_{-\frac{1}{2}i(a-i)}(2ix) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 39

$$\left\{ y(x) = e^{-ix} \left( U\left(\frac{1}{2} + \frac{i}{2}a, 1, 2ix\right) \_C2 + M\left(\frac{1}{2} + \frac{i}{2}a, 1, 2ix\right) \_C1 \right) \right\}$$

**2.1097 ODE No. 1097**

$$ay(x) + xy''(x) - y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0280096 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow 2ax(c_1 J_2(2\sqrt{a}\sqrt{x}) - c_2 Y_2(2\sqrt{a}\sqrt{x})) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 31

$$\left\{ y(x) = x(J_2(2\sqrt{a}\sqrt{x}) \_C1 + Y_2(2\sqrt{a}\sqrt{x}) \_C2) \right\}$$

**2.1098 ODE No. 1098**

$$-ax^3y(x) + xy''(x) - y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0102834 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh\left(\frac{\sqrt{a}x^2}{2}\right) + ic_2 \sinh\left(\frac{\sqrt{a}x^2}{2}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 27

$$\left\{ y(x) = -C1 \sinh\left(\frac{x^2}{2}\sqrt{a}\right) + -C2 \cosh\left(\frac{x^2}{2}\sqrt{a}\right) \right\}$$

**2.1099 ODE No. 1099**

$$x^3(e^{x^3} - v^2)y(x) + xy''(x) - y'(x) = 0$$

✗ **Mathematica** : cpu = 1.04739 (sec), leaf count = 0 , could not solve

DSolve[(E^x^3 - v^2)\*x^3\*y[x] - Derivative[1][y][x] + x\*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.044 (sec), leaf count = 25

$$\left\{ y(x) = -C1 J_v\left(e^{\frac{x^2}{2}}\right) + -C2 Y_v\left(e^{\frac{x^2}{2}}\right) \right\}$$

**2.1100 ODE No. 1100**

$$xy''(x) + 2y'(x) - xy(x) - e^x = 0$$

✓ **Mathematica** : cpu = 0.0302767 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-x}(e^{2x}(2c_2 + 2x - 1) + 4c_1)}{4x} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 23

$$\left\{ y(x) = \frac{\sinh(x) - C2}{x} + \frac{\cosh(x) - C1}{x} + \frac{e^x}{2} \right\}$$

**Hand solution**

$$xy'' + 2y' - xy = e^x \quad (1)$$

First method, much shorter, using transformation. Let  $y_h = \frac{u(x)}{x}$ , hence (now we are solving only the homogeneous part).

$$y' = \frac{u'}{x} - \frac{u}{x^2}$$

$$y'' = \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3}$$

And (1) becomes

$$x\left(\frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3}\right) + 2\left(\frac{u'}{x} - \frac{u}{x^2}\right) - x\left(\frac{u}{x}\right) = 0$$

$$u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \frac{2u'}{x} - \frac{2u}{x^2} - u = 0$$

$$u'' - u = 0$$

Hence the roots of the characteristic equation are  $\pm 1$  and the solution is

$$u = Ae^x + Be^{-x}$$

Hence

$$y_h = \frac{1}{x}(Ae^x + Be^{-x})$$

The particular solution is found below, and given in the second method. The transformation method is much simpler.

Second method, much longer, using series method. This is used if a transformation is now known or can not be found. There is singularity at  $x = 0$ . We need to check if it regular or not. Writing in standard form  $y'' + p(x)y' + q(x)y = 0$  gives (we are looking at the homogeneous part now only)

$$y'' + \frac{2}{x}y' - y = 0$$

Hence  $\lim_{x \rightarrow 0} xp(x) = \lim_{x \rightarrow 0} x \frac{2}{x} = 2$  which is analytic at  $x = 0$ . And  $\lim_{x \rightarrow 0} x^2q(x) = \lim_{x \rightarrow 0} -x^2 = 0$  which is analytic. Hence the singularity is regular (removable). Using Frobenius series, assume that

$$y = \sum_{n=-\infty}^{\infty} c_n x^{n+r}$$

Where  $c_n = 0$  for  $n < 0$ . Hence

$$y' = \sum (n+r) c_n x^{n+r-1}$$

$$y'' = \sum (n+r)(n+r-1) c_n x^{n+r-2}$$

Substituting back in the original ODE gives

$$\sum (n+r)(n+r-1) c_n x^{n+r-1} + \sum 2(n+r) c_n x^{n+r-1} - \sum c_n x^{n+r+1} = 0$$

Adjusting so that all have same power  $x^{n+r}$  gives

$$\sum (n+r+1)(n+r) c_{n+1} x^{n+r} + \sum 2(n+r+1) c_{n+1} x^{n+r} - \sum c_{n-1} x^{n+r} = 0$$

Hence

$$(n+r+1)(n+r) c_{n+1} + 2(n+r+1) c_{n+1} - c_{n-1} = 0$$

$$(n+r+1)(2+(n+r)) c_{n+1} - c_{n-1} = 0 \quad (2)$$

We want equation with  $c_0$  in it. Hence let  $n = -1$

$$(-1+r+1)(2+(-1+r)) c_0 - c_{-2} = 0$$

But  $c_n = 0$  for all  $n < 0$  hence

$$(-1+r+1)(2+(-1+r)) c_0 = 0$$

But  $c_0 \neq 0$ , as this is the basis for this method. Therefore, we obtain the indicial equation for  $r$

$$(-1+r+1)(2+(-1+r)) = 0$$

$$r(r+1) = 0$$

Hence  $r = 0$  or  $r = -1$  are the roots. Now for each  $r$  we find a solution. Using  $r = 0$ , we go back the recurrence equation (2)

$$(n+1)(2+n) c_{n+1} - c_{n-1} = 0$$

$$c_{n+1} = \frac{c_{n-1}}{(n+1)(2+n)}$$

For  $n = 0$

$$c_1 = \frac{c_{-1}}{(n+1)(2+n)} = 0$$

For  $n = 1$

$$c_2 = \frac{c_0}{(2)(3)}$$

For  $n = 2$

$$c_3 = \frac{c_1}{(n+1)(2+n)} = 0$$

For  $n = 3$

$$c_4 = \frac{c_2}{(4)(5)} = \frac{c_0}{(2)(3)(4)(5)}$$

And so on. Hence, for  $r = 0$  we have

$$\begin{aligned} y_{r=0} &= \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + \frac{c_0}{6} x^2 + \frac{c_0}{120} x^4 + \dots \\ &= A \left( 1 + \frac{1}{6} x^2 + \frac{1}{120} x^4 + \dots \right) \end{aligned} \quad (3)$$

Where  $A$  is used as arbitrary constant instead of  $a_0$ . Now we find the solution for  $r = -1$ . we go back the recurrence equation (2)

$$\begin{aligned} (n-1+1)(2+(n-1))c_{n+1} - c_{n-1} &= 0 \\ n(1+n)c_{n+1} - c_{n-1} &= 0 \\ c_{n+1} &= \frac{c_{n-1}}{n(1+n)} \end{aligned}$$

For  $n = 0$

$$c_1 = \frac{c_{-1}}{n(1+n)} = 0$$

For  $n = 1$

$$c_2 = \frac{c_0}{2}$$

For  $n = 2$

$$c_3 = \frac{c_1}{n(1+n)} = 0$$

For  $n = 3$

$$c_4 = \frac{c_2}{(3)(4)} = \frac{c_0}{(2)(3)(4)}$$

For  $n = 4$

$$c_5 = \frac{c_3}{n(1+n)} = 0$$

For  $n = 5$

$$c_6 = \frac{c_4}{(5)(6)} = \frac{c_0}{(2)(3)(4)(5)(6)}$$

And so on. Hence solution is

$$\begin{aligned} y_{r=-1} &= \frac{1}{x} \sum_{n=0}^{\infty} c_n x^n = \frac{1}{x} \left( c_0 + \frac{c_0}{2} x^2 + \frac{c_0}{(2)(3)(4)} x^4 + \frac{c_0}{(2)(3)(4)(5)(6)} x^6 + \dots \right) \\ &= \frac{B}{x} \left( 1 + \frac{1}{2} x^2 + \frac{1}{24} x^4 + \frac{1}{720} x^6 + \dots \right) \end{aligned}$$



Where  $B$  is used as arbitrary constant instead of  $a_0$ . Therefore, the homogeneous solution found is

$$\begin{aligned} y_h &= y_{r=0} + y_{r=-1} \\ &= A \left( 1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \dots \right) + \frac{B}{x} \left( 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 + \dots \right) \end{aligned}$$

But

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \dots \quad (3)$$

And

$$e^{-x} = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \dots \quad (4)$$

Hence adding (3)+(4) gives

$$\begin{aligned} e^x + e^{-x} &= 2 + 2\frac{1}{2}x^2 + 2\frac{1}{24}x^4 + \dots \\ &= 2 \left( 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 \dots \right) \end{aligned}$$

But  $y_{r=-1} = \frac{B}{x} \left( 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 + \dots \right)$ , therefore comparing the result we found above, we see that we can write  $y_{r=-1}$  as

$$y_{r=-1} = \frac{B}{x} \left( \frac{e^x + e^{-x}}{2} \right)$$

Similarly, we obtain  $y_{r=0}$  expression

$$\begin{aligned} \frac{1}{x}e^x &= \frac{1}{x} \left( 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \dots \right) \\ &= \frac{1}{x} + 1 + \frac{1}{2}x + \frac{1}{6}x^2 + \frac{1}{24}x^3 + \frac{1}{120}x^4 + \dots \end{aligned} \quad (3A)$$

And

$$\begin{aligned} \frac{1}{x}e^{-x} &= \frac{1}{x} \left( 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5 + \dots \right) \\ &= \frac{1}{x} - 1 + \frac{1}{2}x - \frac{1}{6}x^2 + \frac{1}{24}x^3 - \frac{1}{120}x^4 + \dots \end{aligned} \quad (4A)$$

Now (3A)-(4A) gives

$$\begin{aligned} \frac{1}{x}e^x - \frac{1}{x}e^{-x} &= \left( \frac{1}{x} + 1 + \frac{1}{2}x + \frac{1}{6}x^2 + \frac{1}{24}x^3 + \frac{1}{120}x^4 + \dots \right) - \left( \frac{1}{x} - 1 + \frac{1}{2}x - \frac{1}{6}x^2 + \frac{1}{24}x^3 - \frac{1}{120}x^4 + \dots \right) \\ &= 2 + 2\frac{1}{6}x^2 + 2\frac{1}{120}x^4 + \dots \\ &= 2 \left( 1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \dots \right) \end{aligned}$$

Hence

$$\left(1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \dots\right) = \frac{1}{2x}(e^x - e^{-x})$$

But  $y_{r=0} = A\left(1 + \frac{1}{6}x^2 + \frac{1}{120}x^4 + \dots\right)$ , therefore comparing the result we found above, we see that we can write  $y_{r=0}$  as

$$\begin{aligned} y_{r=0} &= A\left(\frac{1}{2x}(e^x - e^{-x})\right) \\ &= \frac{A}{2x}(e^x - e^{-x}) \end{aligned}$$

Therefore

$$\begin{aligned} y_h &= y_{r=0} + y_{r=-1} \\ &= \frac{A}{2x}(e^x - e^{-x}) + \frac{B}{2x}(e^x + e^{-x}) \\ &= \frac{1}{x}\left(\frac{A}{2}e^x - \frac{A}{2}e^{-x} + \frac{B}{2}e^x + \frac{B}{2}e^{-x}\right) \\ &= \frac{1}{x}\left(e^x\left(\frac{A}{2} + \frac{B}{2}\right) + e^{-x}\left(-\frac{A}{2} + \frac{B}{2}\right)\right) \end{aligned}$$

Let  $\frac{A+B}{2} = A_0$ ,  $\frac{B-A}{2} = B_0$  hence the above becomes

$$y_h = \frac{1}{x}(A_0e^x + B_0e^{-x})$$

We see this is the same solution using the transformation method given above. We now need to find particular solution. Let  $y_1 = \frac{e^x}{x}$ ,  $y_2 = \frac{e^{-x}}{x}$ , hence  $y_1' = \frac{e^x}{x} - \frac{e^x}{x^2}$  and  $y_2' = \frac{-e^{-x}}{x^2} - \frac{e^{-x}}{x}$ , hence the Wronskian is

$$\begin{aligned} W &= \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \\ &= \begin{vmatrix} \frac{e^x}{x} & \frac{e^{-x}}{x} \\ \frac{e^x}{x} - \frac{e^x}{x^2} & -\frac{e^{-x}}{x^2} - \frac{e^{-x}}{x} \end{vmatrix} \\ &= \frac{e^x}{x} \left(\frac{-e^{-x}}{x^2} - \frac{e^{-x}}{x}\right) - \frac{e^{-x}}{x} \left(\frac{e^x}{x} - \frac{e^x}{x^2}\right) \\ &= \left(\frac{-1}{x^3} - \frac{1}{x^2}\right) - \left(\frac{1}{x^2} - \frac{1}{x^3}\right) \\ &= -\frac{2}{x^2} \end{aligned}$$

Therefore, let  $y_p = u_1y_1 + u_2y_2$  and hence

$$u_1 = - \int \frac{y_2}{aW} e^x dx$$

Where  $a = x$  since the original ODE is  $xy'' + 2y' - xy = e^x$ , and  $a$  is the coefficient of  $y''$  always. Hence the above becomes

$$u_1 = - \int \frac{\frac{e^{-x}}{x}}{x \left(-\frac{2}{x^2}\right)} e^x dx = \int \frac{e^{-x}}{2} e^x dx = \int \frac{1}{2} dx = \frac{x}{2}$$

And

$$u_2 = \int \frac{\frac{e^x}{x}}{x \left(-\frac{2}{x^2}\right)} e^x dx = - \int \frac{\frac{e^x}{x}}{\frac{2}{x}} e^x dx = -\frac{1}{2} \int e^{2x} dx = -\frac{1}{2} \frac{e^{2x}}{2} = -\frac{1}{4} e^{2x}$$

Hence

$$\begin{aligned} y_p &= u_1 y_1 + u_2 y_2 \\ &= \frac{x}{2} \frac{e^x}{x} - \frac{1}{4} e^{2x} \frac{e^{-x}}{x} \\ &= \frac{1}{2} e^x - \frac{1}{4x} e^x \end{aligned}$$

Therefore

$$y_p = e^x \left( \frac{1}{2} - \frac{1}{4x} \right)$$

Hence the general solution is

$$\begin{aligned} y &= y_h + y_p \\ &= \frac{1}{x} (A_0 e^x + B_0 e^{-x}) + e^x \left( \frac{1}{2} - \frac{1}{4x} \right) \end{aligned}$$

Verification

```
restart;
ode:=x*difff(difff(y(x),x),x)+2*difff(y(x),x)-x*y(x)=exp(x);
y0:=1/x*( _C1* exp(x)+ _C2*exp(-x))+ (1/2-1/(4*x))*exp(x);
odetest(y(x)=y0,ode);
0
```

## 2.1101 ODE No. 1101

$$axy(x) + xy''(x) + 2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0277609 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\sqrt{-ax}} \left( \frac{c_2 e^{2\sqrt{-ax}}}{\sqrt{-a}} + 2c_1 \right)}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 29

$$\left\{ y(x) = \frac{1}{x} ( _C2 \cosh(\sqrt{-ax}) + _C1 \sinh(\sqrt{-ax})) \right\}$$

**Hand solution**

$$xy'' + 2y' + axy = 0 \tag{1}$$

First method much shorter, using transformation suggested by Kamke. Let  $y = \frac{u(x)}{x}$ , hence

$$y' = \frac{u'}{x} - \frac{u}{x^2}$$

$$y'' = \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3}$$

And (1) becomes

$$x \left( \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \right) + 2 \left( \frac{u'}{x} - \frac{u}{x^2} \right) + ax \left( \frac{u}{x} \right) = 0$$

$$u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \frac{2u'}{x} - \frac{2u}{x^2} + au = 0$$

$$u'' + au = 0$$

Hence the roots of the characteristic equation are  $\pm\sqrt{-a}$  and the solution is

$$u = A \cos(\sqrt{ax}) + B \sin(\sqrt{ax})$$

Hence

$$y = \frac{1}{x} (A \cos(\sqrt{ax}) + B \sin(\sqrt{ax}))$$

Second method Using series method.

There is singularity at  $x = 0$ . We need to check if it regular or not. Writing in standard form  $y'' + p(x)y' + q(x)y = 0$  gives (we are looking at the homogeneous part now only)

$$y'' + \frac{2}{x}y' + ay = 0$$

Hence  $\lim_{x \rightarrow 0} xp(x) = \lim_{x \rightarrow 0} x \frac{2}{x} = 2$  which is analytic at  $x = 0$ . And  $\lim_{x \rightarrow 0} x^2q(x) = \lim_{x \rightarrow 0} -ax^2 = 0$  which is analytic. Hence the singularity is regular (removable). Using Frobenius series, assume that

$$y = \sum_{n=-\infty}^{\infty} c_n x^{n+r}$$

Where  $c_n = 0$  for  $n < 0$ . Hence

$$y' = \sum (n+r) c_n x^{n+r-1}$$

$$y'' = \sum (n+r)(n+r-1) c_n x^{n+r-2}$$

Substituting back in the original ODE gives

$$\sum (n+r)(n+r-1) c_n x^{n+r-1} + \sum 2(n+r) c_n x^{n+r-1} + \sum a c_n x^{n+r+1} = 0$$

Adjusting so that all have same power  $x^{n+r}$  gives

$$\sum (n+r+1)(n+r) c_{n+1} x^{n+r} + \sum 2(n+r+1) c_{n+1} x^{n+r} + \sum a c_{n-1} x^{n+r} = 0$$

Hence

$$(n+r+1)(n+r) c_{n+1} + 2(n+r+1) c_{n+1} + a c_{n-1} = 0$$

$$(n+r+1)(2+(n+r)) c_{n+1} + a c_{n-1} = 0 \quad (2)$$

We want equation with  $c_0$  in it. Hence let  $n = -1$

$$(-1+r+1)(2+(-1+r)) c_0 + a c_{-2} = 0$$

But  $c_n = 0$  for all  $n < 0$  hence

$$(-1+r+1)(2+(-1+r)) c_0 = 0$$

But  $c_0 \neq 0$ , as this is the basis for this method. Therefore, we obtain the indicial equation for  $r$

$$(-1+r+1)(2+(-1+r)) = 0$$

$$r(r+1) = 0$$

Hence  $r = 0$  or  $r = -1$  are the roots. Now for each  $r$  we find a solution. Using  $r = 0$ , we go back the recurrence equation (2)

$$(n+1)(2+n) c_{n+1} + a c_{n-1} = 0$$

$$c_{n+1} = \frac{-a c_{n-1}}{(n+1)(2+n)}$$

For  $n = 0$

$$c_1 = \frac{-a c_{-1}}{(n+1)(2+n)} = 0$$

For  $n = 1$

$$c_2 = \frac{-a c_0}{(2)(3)}$$

For  $n = 2$

$$c_3 = \frac{-a c_1}{(n+1)(2+n)} = 0$$

For  $n = 3$

$$c_4 = \frac{-ac_2}{(4)(5)} = \frac{-a(-ac_0)}{(2)(3)(4)(5)} = \frac{a^2c_0}{(2)(3)(4)(5)}$$

And so on. Hence, for  $r = 0$  we have

$$\begin{aligned} y_{r=0} &= \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 - a \frac{c_0}{6} x^2 + a^2 \frac{c_0}{120} x^4 + \dots \\ &= A \left( 1 - a \frac{1}{6} x^2 + a^2 \frac{1}{120} x^4 + \dots \right) \end{aligned} \quad (3)$$

Where  $A$  is used as arbitrary constant instead of  $a_0$ . Now we find the solution for  $r = -1$ . we go back the recurrence equation (2)

$$\begin{aligned} (n-1+1)(2+(n-1))c_{n+1} + ac_{n-1} &= 0 \\ n(1+n)c_{n+1} + ac_{n-1} &= 0 \\ c_{n+1} &= \frac{-ac_{n-1}}{n(1+n)} \end{aligned}$$

For  $n = 0$

$$c_1 = \frac{-ac_{-1}}{n(1+n)} = 0$$

For  $n = 1$

$$c_2 = \frac{-ac_0}{2}$$

For  $n = 2$

$$c_3 = \frac{-ac_1}{n(1+n)} = 0$$

For  $n = 3$

$$c_4 = \frac{-ac_2}{(3)(4)} = \frac{-a(-ac_0)}{(2)(3)(4)} = \frac{a^2c_0}{(2)(3)(4)}$$

For  $n = 4$

$$c_5 = \frac{-ac_3}{n(1+n)} = 0$$

For  $n = 5$

$$c_6 = \frac{-ac_4}{(5)(6)} = \frac{-a(a^2c_0)}{(2)(3)(4)(5)(6)} = \frac{-a^3c_0}{(2)(3)(4)(5)(6)}$$

And so on. Hence solution is

$$\begin{aligned} y_{r=-1} &= \frac{1}{x} \sum_{n=0}^{\infty} c_n x^n = \frac{1}{x} \left( c_0 - \frac{ac_0}{2} x^2 + \frac{a^2c_0}{(2)(3)(4)} x^4 - \frac{a^3c_0}{(2)(3)(4)(5)(6)} x^6 + \dots \right) \\ &= \frac{B}{x} \left( 1 - \frac{a}{2} x^2 + \frac{a^2}{24} x^4 - \frac{a^3}{720} x^6 + \dots \right) \end{aligned}$$

Where  $B$  is used as arbitrary constant instead of  $a_0$ . Therefore, the homogeneous solution found is

$$\begin{aligned} y_h &= y_{r=0} + y_{r=-1} \\ &= A \left( 1 - a \frac{1}{6} x^2 + a^2 \frac{1}{120} x^4 + \dots \right) + \frac{B}{x} \left( 1 - \frac{a}{2} x^2 + \frac{a^2}{24} x^4 - \frac{a^3}{720} x^6 + \dots \right) \\ &= A \left( 1 - \frac{1}{6} (\sqrt{ax})^2 + \frac{1}{120} (\sqrt{ax})^4 + \dots \right) + \frac{B}{x} \left( 1 - \frac{1}{2} (\sqrt{ax})^2 + \frac{1}{24} (\sqrt{ax})^4 - \frac{1}{720} (\sqrt{ax})^6 + \dots \right) \end{aligned}$$

But

$$\sin x = x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \dots \quad (3)$$

And

$$\cos x = 1 - \frac{1}{2} x^2 + \frac{1}{24} x^4 - \frac{1}{720} x^6 + \dots \quad (4)$$

Therefore

$$\begin{aligned} \sin(\sqrt{ax}) &= \sqrt{ax} - \frac{1}{6} (\sqrt{ax})^3 + \frac{1}{120} (\sqrt{ax})^5 - \dots \\ \cos(\sqrt{ax}) &= 1 - \frac{1}{2} (\sqrt{ax})^2 + \frac{1}{24} (\sqrt{ax})^4 - \frac{1}{720} (\sqrt{ax})^6 + \dots \end{aligned}$$

Therefore, using the above, we can write  $y_h$  as

$$\begin{aligned} y_h &= \frac{A}{\sqrt{ax}} \sqrt{ax} \left( 1 - \frac{1}{6} (\sqrt{ax})^2 + \frac{1}{120} (\sqrt{ax})^4 + \dots \right) + \frac{B}{x} \cos(\sqrt{ax}) \\ &= \frac{A}{\sqrt{ax}} \left( \sqrt{ax} - \frac{1}{6} (\sqrt{ax})^3 + \frac{1}{120} (\sqrt{ax})^5 + \dots \right) + \frac{B}{x} \cos(\sqrt{ax}) \\ &= \frac{A}{\sqrt{ax}} \sin(\sqrt{ax}) + \frac{B}{x} \cos(\sqrt{ax}) \end{aligned}$$

Let  $A_0 = \frac{A}{\sqrt{a}}$ , hence the above becomes the same solution found using the transformation method

$$y(x) = \frac{1}{x} (A_0 \sin(\sqrt{ax}) + B \cos(\sqrt{ax}))$$

Clearly, the transformation method is much faster and better. But the trick is to see the correct transformation needed and this is not always easy.

Third method Using Laplace transform. Using property  $\mathcal{L}xf(x) = -\frac{d}{ds}F(s)$  then the Laplace transform of the ODE becomes

$$\begin{aligned} \mathcal{L}(xy'' + 2y' + axy) &= 0 \\ -\frac{d}{ds}(\mathcal{L}y'') + 2\left(-\frac{d}{ds}(\mathcal{L}y')\right) + a\left(-\frac{d}{ds}Y\right) &= 0 \\ -\frac{d}{ds}(s^2Y - sA - B) + 2\left(-\frac{d}{ds}(sY - A)\right) + a(-Y') &= 0 \end{aligned}$$

Where  $A = y(0)$ ,  $B = y'(0)$ . Simplifying gives

$$\begin{aligned} -(2sY + s^2Y' - A) - 2(Y + sY') - aY' &= 0 \\ Y'(-s^2 - 2s - a) + Y(-2s - 2) + A &= 0 \\ Y' + Y \frac{2s + 2}{s^2 + 2s + a} &= \frac{A}{s^2 + 2s + a} \end{aligned}$$

This is first order ODE, which is solved easily using an integrating factor. Solving for  $Y(s)$  gives

$$Y(s) = \frac{As + c_1}{s^2 + a + 2s}$$

Where  $c_1$  is constant of integration. The inverse Laplace of the above is

$$y(x) = e^{-x} \left( A \cos(x\sqrt{1-a}) + \frac{(-A + c_1)}{\sqrt{1-a}} \sinh(x\sqrt{1-a}) \right)$$

I need to find why the above does not verify. May be I made a mistake somewhere. Verification of the result of the first two methods is below.

```
restart;
ode:=x*diff(diff(y(x),x),x)+2*diff(y(x),x)+a*x*y(x)=0;
y0:=1/x*( _C1* cos(sqrt(a)*x)+ _C2*sin(sqrt(a)*x));
odetest(y(x)=y0,ode);
0
```

## 2.1102 ODE No. 1102

$$ax^2y(x) + xy''(x) + 2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.00725182 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \text{Ai}(\sqrt[3]{-ax}) + c_2 \text{Bi}(\sqrt[3]{-ax})}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 33

$$\left\{ y(x) = 1 \left( -C_2 Y_{\frac{1}{3}} \left( \frac{2}{3} \sqrt{ax^{\frac{3}{2}}} \right) + -C_1 J_{\frac{1}{3}} \left( \frac{2}{3} \sqrt{ax^{\frac{3}{2}}} \right) \right) \frac{1}{\sqrt{x}} \right\}$$

**Hand solution**

$$xy'' + 2y' + ax^2y = 0 \tag{1}$$



Since there is a term  $2y$ , we can use  $y = \frac{u(x)}{x}$ , hence

$$y' = \frac{u'}{x} - \frac{u}{x^2}$$

$$y'' = \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3}$$

And (1) becomes

$$x \left( \frac{u''}{x} - \frac{u'}{x^2} - \frac{u'}{x^2} + 2\frac{u}{x^3} \right) + 2 \left( \frac{u'}{x} - \frac{u}{x^2} \right) + ax^2 \left( \frac{u}{x} \right) = 0$$

$$u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \frac{2u'}{x} - \frac{2u}{x^2} + axu = 0$$

$$u'' + axu = 0 \tag{2}$$

This is Emdon-Fowler. (form is  $u'' + x^n u = 0$ ) with  $n = 1$ . Assume that

$$u = \sum_{n=0}^{\infty} c_n x^n$$

Hence

$$u' = \sum_{n=0} n c_n x^{n-1} = \sum_{n=1} n c_n x^{n-1} = \sum_{n=0} (n+1) c_{n+1} x^n$$

$$u'' = \sum_{n=0} n(n+1) c_{n+1} x^{n-1} = \sum_{n=1} n(n+1) c_{n+1} x^{n-1} = \sum_{n=0} (n+1)(n+2) c_{n+2} x^n$$

Substituting back in (2) gives

$$\sum_{n=0} (n+1)(n+2) c_{n+2} x^n + \sum_{n=0} a c_n x^{n+1} = 0$$

$$\sum_{n=0} (n+1)(n+2) c_{n+2} x^n + \sum_{n=1} a c_{n-1} x^n = 0$$

For  $n = 0$

$$(1)(2) c_2 = 0$$

Hence  $c_2 = 0$ . For  $n \geq 1$

$$(n+1)(n+2) c_{n+2} + a c_{n-1} = 0$$

$$c_{n+2} = \frac{-a c_{n-1}}{(n+1)(n+2)} \tag{3}$$

For  $n = 1$ , from (3)

$$c_3 = \frac{-a c_0}{(2)(3)}$$

For  $n = 2$ , from (3)

$$c_4 = \frac{-a c_1}{(3)(4)}$$

For  $n = 3$ , from (3)

$$c_5 = \frac{-ac_2}{(4)(5)} = 0$$

For  $n = 4$ , from (3)

$$c_6 = \frac{-ac_3}{(5)(6)} = \frac{-a}{(5)(6)} \left( \frac{-ac_0}{(2)(3)} \right) = \frac{a^2 c_0}{(2)(3)(5)(6)}$$

For  $n = 5$ , from (3)

$$c_7 = \frac{-ac_4}{(6)(7)} = \frac{-a}{(6)(7)} \left( \frac{-ac_1}{(3)(4)} \right) = \frac{a^2 c_1}{(3)(4)(6)(7)}$$

For  $n = 6$ , from (3)

$$c_8 = \frac{-ac_5}{(7)(8)} = 0$$

For  $n = 7$ , from (3)

$$c_9 = \frac{-ac_6}{(8)(9)} = \frac{-a}{(8)(9)} \left( \frac{a^2 c_0}{(2)(3)(5)(6)} \right) = \frac{-a^3 c_0}{(2)(3)(5)(6)(8)(9)}$$

For  $n = 8$ , from (3)

$$c_{10} = \frac{-ac_7}{(9)(10)} = \frac{-a}{(9)(10)} \left( \frac{a^2 c_1}{(3)(4)(6)(7)} \right) = \frac{-a^3 c_1}{(3)(4)(6)(7)(9)(10)}$$

And so on. Hence,

$$\begin{aligned} u &= \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3 + \dots \\ &= c_0 + c_1 x + 0 + c_3 x^3 + c_4 x^4 + 0 + c_6 x^6 + c_7 x^7 + 0 + c_9 x^9 + \dots \\ &= c_0 + c_1 x - \frac{ac_0}{(2)(3)} x^3 - \frac{ac_1}{(3)(4)} x^4 + \frac{a^2 c_0}{(2)(3)(5)(6)} x^6 + \frac{a^2 c_1}{(3)(4)(6)(7)} x^7 - \frac{a^3 c_0}{(2)(3)(5)(6)(8)(9)} x^9 - \frac{a^3 c_1}{(3)(4)(6)(7)(9)(10)} x^{10} + \dots \\ &= c_0 \left( 1 - \frac{a}{6} x^3 + \frac{a^2}{180} x^6 - \frac{a^3}{12960} x^9 + \dots \right) + x c_1 \left( 1 - \frac{a}{12} x^3 + \frac{a^2}{504} x^6 - \frac{a^3}{45360} x^9 + \dots \right) \\ &= c_0 \left( 1 - \frac{1}{6} \left( a^{\frac{1}{3}} x \right)^3 + \frac{1}{180} \left( a^{\frac{1}{3}} x \right)^6 - \frac{1}{12960} \left( a^{\frac{1}{3}} x \right)^9 + \dots \right) + x c_1 \left( 1 - \frac{1}{12} \left( a^{\frac{1}{3}} x \right)^3 + \frac{1}{504} \left( a^{\frac{1}{3}} x \right)^6 - \frac{1}{45360} \left( a^{\frac{1}{3}} x \right)^9 + \dots \right) \end{aligned}$$

Comparing the above to the series expansion of Airy functions, we see that

$$u = c_0 \text{AiryAI} \left( -a^{\frac{1}{3}} x \right) + c_1 \text{AiryBI} \left( -a^{\frac{1}{3}} x \right)$$

And since  $y = \frac{u(x)}{x}$  then

$$y = \frac{1}{x} \left( c_0 \text{AiryAI} \left( -a^{\frac{1}{3}} x \right) + c_1 \text{AiryBI} \left( -a^{\frac{1}{3}} x \right) \right)$$

Verification

```

restart;
#for series solution of u''+axu=0, use this:
Order:=10;
sol:=dsolve(ode,u(x),series);
sol:=convert(sol,polynomial);
sol:=subs({u(0)=c0,D(u)(0)=c1},rhs(sol));
collect(sol,{c0,c1});
(1-(1/6)*a*x^3+(1/180)*a^2*x^6-(1/12960)*a^3*x^9)*c0+(x-(1/12)*a*x^4+(1/504)*a^2*x^7)*c1
#to verify final solution use this:
ode:=x*diff(y(x),x$2)+2*diff(y(x),x)+a*x^2*y(x)=0;
y0:=(1/x)*(_C0*AiryAi(-a^(1/3)*x)+_C1*AiryBi(-a^(1/3)*x));
odetest(y(x)=y0,ode);
0

```

### 2.1103 ODE No. 1103

$$ay(x) + xy''(x) - 2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0284124 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow 2a^{3/2}x^{3/2}(3c_1J_3(2\sqrt{a}\sqrt{x}) - ic_2Y_3(2\sqrt{a}\sqrt{x})) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 33

$$\left\{ y(x) = x^{3/2} (J_3(2\sqrt{a}\sqrt{x})_C1 + Y_3(2\sqrt{a}\sqrt{x})_C2) \right\}$$

### 2.1104 ODE No. 1104

$$ay(x) + vy'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0366258 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow a^{1/2-v}x^{1/2-v} (c_2\Gamma(2-v)J_{1-v}(2\sqrt{a}\sqrt{x}) + c_1\Gamma(v)J_{v-1}(2\sqrt{a}\sqrt{x})) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 41

$$\left\{ y(x) = x^{1/2-v} (Y_{v-1}(2\sqrt{a}\sqrt{x})_C2 + J_{v-1}(2\sqrt{a}\sqrt{x})_C1) \right\}$$

**2.1105 ODE No. 1105**

$$ay'(x) + bxy(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0227935 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow x^{\frac{1}{2} - \frac{a}{2}} \left( c_1 J_{\frac{a-1}{2}}(\sqrt{bx}) + c_2 Y_{\frac{a-1}{2}}(\sqrt{bx}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 39

$$\left\{ y(x) = x^{-\frac{a}{2} + \frac{1}{2}} \left( Y_{\frac{a}{2} - \frac{1}{2}}(\sqrt{bx}) - C2 + J_{\frac{a}{2} - \frac{1}{2}}(\sqrt{bx}) - C1 \right) \right\}$$

**2.1106 ODE No. 1106**

$$ay'(x) + bx^{a1}y(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0525812 (sec), leaf count = 165

$$\left\{ \left\{ y(x) \rightarrow \left( \frac{1}{a1} + 1 \right)^{\frac{a-1}{a1+1}} a1^{\frac{a-1}{a1+1}} b^{\frac{1-a}{2a1+2}} (x^{a1})^{-\frac{a-1}{2a1}} \left( c_2 \Gamma\left(\frac{-a+a1+2}{a1+1}\right) J_{\frac{1-a}{a1+1}}\left(\frac{2\sqrt{b}(x^{a1})^{\frac{a1+1}{2a1}}}{a1+1}\right) + c_1 \Gamma\left(\frac{a+a1}{a1+1}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 71

$$\left\{ y(x) = x^{-\frac{a}{2} + \frac{1}{2}} \left( J_{\frac{a-1}{a1+1}}\left(2 \frac{\sqrt{bx}^{a1/2+1/2}}{a1+1}\right) - C1 + Y_{\frac{a-1}{a1+1}}\left(2 \frac{\sqrt{bx}^{a1/2+1/2}}{a1+1}\right) - C2 \right) \right\}$$

**2.1107 ODE No. 1107**

$$ay(x) + (b+x)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0313736 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow e^{-x} \left( c_1 U(b-a, b, x) + c_2 L_{a-b}^{b-1}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 30

$$\left\{ y(x) = e^{-x} (U(-a+b, b, x) - C2 + M(-a+b, b, x) - C1) \right\}$$

**2.1108 ODE No. 1108**

$$(a + b + x)y'(x) + ay(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0366261 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow e^{-x} \left( c_1 U(b, a + b, x) + c_2 L_{-b}^{a+b-1}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 26

$$\{y(x) = e^{-x}(M(b, a + b, x)_C1 + U(b, a + b, x)_C2)\}$$

**2.1109 ODE No. 1109**

$$xy''(x) - xy'(x) - y(x) - e^x x(x + 1) = 0$$

✓ **Mathematica** : cpu = 0.0558041 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow -c_2(e^x x \text{Ei}(-x) + 1) + c_1 e^x x + e^x(x^2 + x - x \log(-x) - 1) \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 33

$$\{y(x) = e^x(-_C1 \text{Ei}(1, x) x + x^2 + x_C2 - x \ln(x) + e^{-x}_C1 - 1)\}$$

**2.1110 ODE No. 1110**

$$-ay(x) + xy''(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0421462 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left( -x \left| \begin{matrix} 1-a \\ 0,1 \end{matrix} \right. \right) + c_1 x {}_1F_1(a+1; 2; x) \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 23

$$\{y(x) = x(M(a + 1, 2, x)_C1 + U(a + 1, 2, x)_C2)\}$$

2.1111 ODE No. 1111

$$xy''(x) - (x + 1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0194391 (sec), leaf count = 19

$$\{\{y(x) \rightarrow c_1 e^x - c_2(x + 1)\}\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 13

$$\{y(x) = \_C2 e^x + \_C1 x + \_C1\}$$

**Hand solution**

$$xy'' - (x + 1)y' + y = 0 \tag{1}$$

Taking Laplace transform of each term and using property of  $\mathcal{L}(xf(x)) = -\frac{d}{ds}F(s)$  where  $F(s) = \mathcal{L}f(x)$ , then

$$\mathcal{L}(xy'') = -\frac{d}{ds}(\mathcal{L}y'')$$

Let  $\mathcal{L}y(x) = Y(s) \equiv Y$ . Now  $\mathcal{L}y'' = s^2Y - sy(0) - y'(0)$ . Assuming  $y(0) = A, y'(0) = B$  then

$$\begin{aligned} \mathcal{L}(xy'') &= -\frac{d}{ds}(s^2Y - As - B) \\ &= -(2sY + s^2Y' - A) \end{aligned}$$

And

$$\begin{aligned} \mathcal{L}((x + 1)y') &= \mathcal{L}(xy' + y') \\ &= -\frac{d}{ds}(\mathcal{L}y') + \mathcal{L}y' \\ &= -\frac{d}{ds}(sY - y(0)) + (sY - y(0)) \\ &= -\frac{d}{ds}(sY - A) + (sY - y(0)) \\ &= -(Y + sY') + (sY - A) \\ &= -Y - sY' + sY - A \end{aligned}$$

Hence Laplace transform of the ODE becomes

$$\begin{aligned} -(2sY + s^2Y' - A) - (-Y - sY' + sY - A) + Y &= 0 \\ -2sY - s^2Y' + A + Y + sY' - sY + A + Y &= 0 \\ Y'(s - s^2) + Y(-2s + 1 - s + 1) &= -2A \\ Y'(s^2 - s) + Y(3s - 2) &= 2A \\ Y' + \frac{(3s - 2)}{s(s - 1)}Y &= \frac{2A}{s(s - 1)} \end{aligned}$$

The integrating factor is  $\mu = e^{\int \frac{(3s-2)}{s(s-1)} ds} = e^{\ln(s-1)+2\ln(s)} = (s-1)s^2$ , hence

$$\begin{aligned} d((s-1)s^2Y) &= (s-1)s^2 \frac{2A}{s(s-1)} \\ (s-1)s^2Y &= 2A \int s ds + c_1 \\ (s-1)s^2Y &= 2A \frac{s^2}{2} + c_1 \\ Y &= \frac{As^2 + c_1}{(s-1)s^2} \end{aligned}$$

Inverse Laplace transform gives

$$\begin{aligned} y(x) &= -c_1 + (A + c_1)e^x - c_1x \\ &= -c_1(1+x) + (A + c_1)e^x \end{aligned}$$

Let  $-c_1 = A_0, A + c_1 = B_0$ , hence

$$y(x) = A_0(1+x) + B_0e^x$$

Verification

```
rrestart;
ode:=x*diff(diff(y(x),x),x)-(1+x)*diff(y(x),x)+y(x)=0;
y0:=-C0*(1+x)+_C1*exp(x);
odetest(y(x)=y0,ode);
0
```

## 2.1112 ODE No. 1112

$$xy''(x) - (x+1)y'(x) - 2(x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0268887 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{2x} - \frac{1}{9} c_2 e^{-x} (3x+1) \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 22

$$\{y(x) = \_C1 e^{2x} + \_C2 e^{-x} (3x+1)\}$$

**2.1113 ODE No. 1113**

$$-ay(x) + (b-x)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0247243 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow c_1 U(a, b, x) + c_2 L_{-a}^{b-1}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 17

$$\{y(x) = \_C1 M(a, b, x) + \_C2 U(a, b, x)\}$$

**2.1114 ODE No. 1114**

$$xy''(x) - 2(x-1)y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0434092 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left( -2x \left| \begin{array}{c} \frac{1}{2} \\ -1, 0 \end{array} \right. \right) + c_1 e^x (I_0(x) - I_1(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 34

$$\{y(x) = e^x (\_C2 K_1(-x) - \_C2 K_0(-x) + \_C1 (I_0(x) - I_1(x)))\}$$

**2.1115 ODE No. 1115**

$$xy''(x) - (3x-2)y'(x) - (2x-3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0648472 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{1}{2}(\sqrt{17}-3)x} \left( c_2 {}_1F_1 \left( 1 - \frac{6}{\sqrt{17}}; 2; \sqrt{17}x \right) + c_1 U \left( 1 - \frac{6}{\sqrt{17}}, 2, \sqrt{17}x \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.091 (sec), leaf count = 47

$$\left\{ y(x) = e^{-\frac{x(-3+\sqrt{17})}{2}} \left( U \left( 1 - \frac{6\sqrt{17}}{17}, 2, \sqrt{17}x \right) \_C2 + M \left( 1 - \frac{6\sqrt{17}}{17}, 2, \sqrt{17}x \right) \_C1 \right) \right\}$$



**2.1116 ODE No. 1116**

$$y'(x)(ax + b + n) + any(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.051276 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow e^{-ax} \left( c_1 U(b, b + n, ax) + c_2 L_{-b}^{b+n-1}(ax) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 31

$$\{y(x) = e^{-ax}(U(b, b + n, ax)_C2 + M(b, b + n, ax)_C1)\}$$

**2.1117 ODE No. 1117**

$$-(x + 1)(a + b)y'(x) + abxy(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0950784 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow e^{bx} x^{a+b+1} \left( c_1 U\left(\frac{a^2 + ba + a - b}{a - b}, a + b + 2, (a - b)x\right) + c_2 L_{-\frac{a^2 + ab + a - b}{a - b}}^{a+b+1}(x(a - b)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.11 (sec), leaf count = 82

$$\left\{ y(x) = x^{a+b+1} e^{bx} \left( U\left(\frac{a^2 + ab + a - b}{a - b}, a + b + 2, x(a - b)\right)_C2 + M\left(\frac{a^2 + ab + a - b}{a - b}, a + b + 2, x(a - b)\right)_C1 \right) \right\}$$

**2.1118 ODE No. 1118**

$$y'(x)(x(a + b) + m + n) + y(x)(abx + an + bm) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.10049 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow e^{-ax} \left( c_1 U(m, m + n, (a - b)x) + c_2 L_{-m}^{m+n-1}(x(a - b)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 39

$$\{y(x) = e^{-ax}(U(m, m + n, x(a - b))_C2 + M(m, m + n, x(a - b))_C1)\}$$

**2.1119 ODE No. 1119**

$$y(x) (a^2x + 2ab) - 2(ax + b)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.170837 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{ax} x^{b-\frac{1}{2}\sqrt{(2b+1)^2+\frac{1}{2}} \left( c_2 x^{\sqrt{(2b+1)^2} + \sqrt{(2b+1)^2} c_1 \right)}{\sqrt{(2b+1)^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 20

$$\left\{ y(x) = e^{ax} \left( x^{2b+1} \_C2 + \_C1 \right) \right\}$$

**2.1120 ODE No. 1120**

$$(ax + b)y'(x) + y(x)(cx + d) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0642911 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{1}{2}x(\sqrt{a^2-4c}+a)} \left( c_1 U \left( \frac{ab + \sqrt{a^2-4c}b - 2d}{2\sqrt{a^2-4c}}, b, \sqrt{a^2-4c}x \right) + c_2 L^{\frac{b-1}{-b\sqrt{a^2-4c}+ab-2d}} \left( x\sqrt{a^2-4c} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.216 (sec), leaf count = 109

$$\left\{ y(x) = e^{-\frac{x}{2}(a+\sqrt{a^2-4c})} \left( U \left( \frac{1}{2}(b\sqrt{a^2-4c} + ab - 2d) \frac{1}{\sqrt{a^2-4c}}, b, \sqrt{a^2-4c}x \right) \_C2 + M \left( \frac{1}{2}(b\sqrt{a^2-4c} + \right) \right) \right\}$$

**2.1121 ODE No. 1121**

$$-(x^2 - x) y'(x) + xy''(x) + (x - 1)y(x) = 0$$

✓ **Mathematica** : cpu = 10.4856 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow x \left( c_2 \int_1^x \frac{e^{\frac{1}{2}(K[1]-2)K[1]}}{K[1]^2} dK[1] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 23

$$\left\{ y(x) = \left( \int \frac{1}{x^2} e^{\frac{x(x-2)}{2}} dx \_C1 + \_C2 \right) x \right\}$$

**2.1122 ODE No. 1122**

$$-(x^2 - x - 2)y'(x) + xy''(x) - x(x + 3)y(x) = 0$$

✓ **Mathematica** : cpu = 10.4244 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{x^2}{2}} \left( c_2 \int_1^x \frac{e^{-\frac{1}{2}K[1](K[1]+2)}}{K[1]^2} dK[1] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 28

$$\left\{ y(x) = e^{\frac{x^2}{2}} \left( \int \frac{1}{x^2} e^{-\frac{x(x+2)}{2}} dx \_C2 + \_C1 \right) \right\}$$

**2.1123 ODE No. 1123**

$$-(2ax^2 + 1)y'(x) + bx^3y(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0141401 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{1}{2}x^2(\sqrt{a^2-b}-a)} \left( c_2 e^{x^2\sqrt{a^2-b}} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 45

$$\left\{ y(x) = \_C1 e^{\frac{x^2}{2}(\sqrt{a^2-b}+a)} + \_C2 e^{-\frac{x^2}{2}(\sqrt{a^2-b}-a)} \right\}$$

**2.1124 ODE No. 1124**

$$-2(x^2 - a)y'(x) + 2nxy(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0720885 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_1F_1\left(-\frac{n}{2}; a + \frac{1}{2}; x^2\right) + i^{1-2a} c_2 x^{1-2a} {}_1F_1\left(-a - \frac{n}{2} + \frac{1}{2}; \frac{3}{2} - a; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 29

$$\left\{ y(x) = \_C1 M\left(-\frac{n}{2}, \frac{1}{2} + a, x^2\right) + \_C2 U\left(-\frac{n}{2}, \frac{1}{2} + a, x^2\right) \right\}$$

**2.1125 ODE No. 1125**

$$-4x^5 - 4x^3y(x) + (4x^2 - 1)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.208333 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{(\sqrt{2}-1)x^2} + c_2 e^{-(1+\sqrt{2})x^2} - x^2 - 2 \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 36

$$\left\{ y(x) = e^{x^2(\sqrt{2}-1)} \_C2 + e^{-x^2(1+\sqrt{2})} \_C1 - x^2 - 2 \right\}$$

**2.1126 ODE No. 1126**

$$(a^2x^3 + a)y(x) + (2ax^3 - 1)y'(x) + xy''(x) = 0$$

✗ **Mathematica** : cpu = 1.17064 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(a^2x^3 + a)y(x) + (2x^3a - 1)y'(x) + xy''(x) = 0, y(1) = c_1, y'(1) = c_2\}) \right\} \right\} (x)$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 19

$$\left\{ y(x) = e^{-\frac{ax^3}{3}} (\_C2 x^2 + \_C1) \right\}$$

**2.1127 ODE No. 1127**

$$y(x) (a^2x \log^2(x) + a \log(x) + a) + (2ax \log(x) + 1)y'(x) + xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0414915 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow e^{ax} x^{-ax} (c_2 \log(x) + c_1) \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 21

$$\left\{ y(x) = x^{-ax} e^{ax} (\ln(x) \_C2 + \_C1) \right\}$$

**2.1128 ODE No. 1128**

$$(xf(x) + 2)y'(x) + f(x)y(x) + xy''(x) = 0$$

✗ **Mathematica** : cpu = 0.243244 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + (2 + x*f[x])*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.223 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{x} \left( -C2 \int e^{\int \frac{-xf(x)-2}{x} dx} x^2 dx + -C1 \right) \right\}$$

**2.1129 ODE No. 1129**

$$(x - 3)y''(x) - (4x - 9)y'(x) + (3x - 6)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0398148 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} c_2 e^{3x-9} (4x^3 - 42x^2 + 150x - 183) + c_1 e^{x-3} \right\} \right\}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 30

$$\{ y(x) = -C1 e^x + -C2 e^{3x} (4x^3 - 42x^2 + 150x - 183) \}$$

**2.1130 ODE No. 1130**

$$ay(x) + 2xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0126954 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \sqrt{2} \sqrt{a} \sqrt{x} \right) + c_1 \cos \left( \sqrt{2} \sqrt{a} \sqrt{x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 31

$$\{ y(x) = -C1 \sin \left( \sqrt{x} \sqrt{2} \sqrt{a} \right) + -C2 \cos \left( \sqrt{x} \sqrt{2} \sqrt{a} \right) \}$$

**2.1131 ODE No. 1131**

$$ay(x) + 2xy''(x) - (x-1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0129031 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \sqrt{x} \left( c_1 U \left( \frac{1}{2} - a, \frac{3}{2}, \frac{x}{2} \right) + c_2 L_{a-\frac{1}{2}}^{\frac{1}{2}} \left( \frac{x}{2} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{x} \left( U \left( -a + \frac{1}{2}, \frac{3}{2}, \frac{x}{2} \right) - C2 + M \left( -a + \frac{1}{2}, \frac{3}{2}, \frac{x}{2} \right) - C1 \right) \right\}$$

**2.1132 ODE No. 1132**

$$ay(x) + 2xy''(x) - (2x-1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0120772 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \sqrt{x} \left( c_1 U \left( \frac{1-a}{2}, \frac{3}{2}, x \right) + c_2 L_{\frac{a-1}{2}}^{\frac{1}{2}}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 29

$$\left\{ y(x) = \sqrt{x} \left( U \left( -\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, x \right) - C2 + M \left( -\frac{a}{2} + \frac{1}{2}, \frac{3}{2}, x \right) - C1 \right) \right\}$$

**2.1133 ODE No. 1133**

$$(2x-1)y''(x) - (3x-4)y'(x) + (x-3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0906598 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow 2^{5/8} c_1 e^{x-\frac{1}{2}} - \frac{2^{3/8} c_2 e^{\frac{x}{2}-\frac{1}{4}}}{\sqrt[4]{2x-1}} + \frac{c_2 e^{x-\frac{1}{2}} \Gamma\left(\frac{3}{4}, \frac{1}{4}(2x-1)\right)}{\sqrt[8]{2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 37

$$\left\{ y(x) = 1e^{\frac{x}{2}} \left( U \left( 1, \frac{3}{4}, \frac{x}{2} - \frac{1}{4} \right) - C2 + M \left( 1, \frac{3}{4}, \frac{x}{2} - \frac{1}{4} \right) - C1 \right) \frac{1}{\sqrt[4]{2x-1}} \right\}$$

**2.1134 ODE No. 1134**

$$4xy''(x) - (a + x)y(x) = 0$$

✓ **Mathematica** : cpu = 0.095232 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} e^{-x/2} x \left( c_2 {}_1F_1 \left( \frac{a}{4} + 1; 2; x \right) + c_1 U \left( \frac{a}{4} + 1, 2, x \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 21

$$\left\{ y(x) = \_C1 M_{-\frac{a}{4}, \frac{1}{2}}(x) + \_C2 W_{-\frac{a}{4}, \frac{1}{2}}(x) \right\}$$

**2.1135 ODE No. 1135**

$$4xy''(x) + 2y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0107323 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh(\sqrt{x}) + ic_2 \sinh(\sqrt{x}) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 17

$$\left\{ y(x) = \_C1 \sinh(\sqrt{x}) + \_C2 \cosh(\sqrt{x}) \right\}$$

**2.1136 ODE No. 1136**

$$4xy''(x) + 4y'(x) - (x + 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0212604 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow e^{x/2} (c_2 \text{Ei}(-x) + c_1) \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 16

$$\left\{ y(x) = e^{\frac{x}{2}} (\text{Ei}(1, x) \_C2 + \_C1) \right\}$$

**2.1137 ODE No. 1137**

$$ly(x) + 4xy''(x) - (x + 2)y(x) + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0984645 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} e^{-x/2} x \left( c_2 {}_1F_1 \left( \frac{1}{2} - \frac{l}{4}; 2; x \right) + c_1 U \left( \frac{1}{2} - \frac{l}{4}, 2, x \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 25

$$\left\{ y(x) = -C1 M_{\frac{l}{4} + \frac{1}{2}, \frac{1}{2}}(x) + -C2 W_{\frac{l}{4} + \frac{1}{2}, \frac{1}{2}}(x) \right\}$$

**2.1138 ODE No. 1138**

$$y(x)(-(-2m - 4n + x)) + 4my'(x) + 4xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0317822 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow e^{-x/2} (c_1 U(-n, m, x) + c_2 L_n^{m-1}(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 26

$$\left\{ y(x) = e^{-\frac{x}{2}} (U(-n, m, x) - C2 + M(-n, m, x) - C1) \right\}$$

**2.1139 ODE No. 1139**

$$-(a + x)y(x) + 16xy''(x) + 8y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0146467 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow e^{-x/4} \sqrt{x} \left( c_1 U \left( \frac{a+6}{8}, \frac{3}{2}, \frac{x}{2} \right) + c_2 L_{\frac{1}{8}(-a-6)} \left( \frac{x}{2} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 37

$$\left\{ y(x) = \sqrt{x} e^{-\frac{x}{4}} \left( U \left( \frac{a}{8} + \frac{3}{4}, \frac{3}{2}, \frac{x}{2} \right) - C2 + M \left( \frac{a}{8} + \frac{3}{4}, \frac{3}{2}, \frac{x}{2} \right) - C1 \right) \right\}$$



**2.1140 ODE No. 1140**

$$axy''(x) + by'(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0502732 (sec), leaf count = 120

$$\left\{ \left\{ y(x) \rightarrow a^{\frac{1}{2} \left( \frac{b}{a} - 1 \right)} c^{\frac{a-b}{2a}} x^{\frac{a-b}{2a}} \left( c_1 \Gamma \left( \frac{b}{a} \right) J_{\frac{b}{a}-1} \left( \frac{2\sqrt{c}\sqrt{x}}{\sqrt{a}} \right) + c_2 \Gamma \left( 2 - \frac{b}{a} \right) J_{1-\frac{b}{a}} \left( \frac{2\sqrt{c}\sqrt{x}}{\sqrt{a}} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 66

$$\left\{ y(x) = x^{\frac{a-b}{2a}} \left( Y_{-\frac{a+b}{a}} \left( 2 \sqrt{\frac{c}{a}} \sqrt{x} \right) - C2 + J_{-\frac{a+b}{a}} \left( 2 \sqrt{\frac{c}{a}} \sqrt{x} \right) - C1 \right) \right\}$$

**2.1141 ODE No. 1141**

$$(3a + bx)y'(x) + axy''(x) + 3by(x) = 0$$

✓ **Mathematica** : cpu = 0.10938 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-\frac{bx}{a}} \left( \frac{b^2 c_2 \text{Ei} \left( \frac{bx}{a} \right)}{a^2} - \frac{c_2 e^{\frac{bx}{a}} (a + bx)}{ax^2} + 2c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 55

$$\left\{ y(x) = \frac{1}{x^2} \left( \text{Ei} \left( 1, -\frac{bx}{a} \right) e^{-\frac{bx}{a}} - C2 b^2 x^2 + C1 e^{-\frac{bx}{a}} x^2 + a C2 (bx + a) \right) \right\}$$

**2.1142 ODE No. 1142**

$$cy(x) \sqrt[5]{ax+b} + 5(ax+b)y''(x) + 8ay'(x) = 0$$

✓ **Mathematica** : cpu = 0.0485283 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow \frac{3a \left( c_2 \sin \left( \frac{\sqrt{5}\sqrt{c}(ax+b)^{3/5}}{3a} \right) + 2c_1 \cos \left( \frac{\sqrt{5}\sqrt{c}(ax+b)^{3/5}}{3a} \right) \right)}{\sqrt{5}\sqrt{c}(ax+b)^{3/5}} \right\} \right\}$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 53

$$\left\{ y(x) = 1 \left( -C2 \cosh \left( \frac{1}{3a} (ax+b)^{\frac{3}{5}} \sqrt{-5c} \right) + C1 \sinh \left( \frac{1}{3a} (ax+b)^{\frac{3}{5}} \sqrt{-5c} \right) \right) (ax+b)^{-\frac{3}{5}} \right\}$$

**2.1143 ODE No. 1143**

$$(a + bx)y'(x) + 2axy''(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0499673 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow \sqrt{x} e^{-\frac{bx}{2a}} \left( c_1 U \left( 1 - \frac{c}{b}, \frac{3}{2}, \frac{bx}{2a} \right) + c_2 L_{\frac{c}{b}-1}^{\frac{1}{2}} \left( \frac{bx}{2a} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.101 (sec), leaf count = 57

$$\left\{ y(x) = \sqrt{x} e^{-\frac{bx}{2a}} \left( U \left( \frac{b-c}{b}, \frac{3}{2}, \frac{bx}{2a} \right) - C2 + M \left( \frac{b-c}{b}, \frac{3}{2}, \frac{bx}{2a} \right) - C1 \right) \right\}$$

**2.1144 ODE No. 1144**

$$(3a + bx)y'(x) + 2axy''(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0466609 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{bx}{2a}} \left( c_1 U \left( \frac{3}{2} - \frac{c}{b}, \frac{3}{2}, \frac{bx}{2a} \right) + c_2 L_{\frac{c}{b}-\frac{3}{2}}^{\frac{1}{2}} \left( \frac{bx}{2a} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 60

$$\left\{ y(x) = e^{-\frac{bx}{2a}} \left( U \left( \frac{3b-2c}{2b}, \frac{3}{2}, \frac{bx}{2a} \right) - C2 + M \left( \frac{3b-2c}{2b}, \frac{3}{2}, \frac{bx}{2a} \right) - C1 \right) \right\}$$

**2.1145 ODE No. 1145**

$$y(x)(a_0x + b_0) + (a_1x + b_1)y'(x) + (a_2x + b_2)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.367614 (sec), leaf count = 301

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{x(\sqrt{a_1^2 - 4a_0a_2} + a_1)}{2a_2}} (a_2x + b_2)^{\frac{a_1b_2 + a_2^2 - a_2b_1}{a_2^2}} \left( c_1 U \left( \frac{2(\sqrt{a_1^2 - 4a_0a_2} - b_0)a_2^2 + (a_1b_1 - \sqrt{a_1^2 - 4a_0a_2})a_2}{2a_2^2\sqrt{a_1^2 - 4a_0a_2}} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.2 (sec), leaf count = 248

$$\left\{ y(x) = (a_2x + b_2)^{\frac{a_1b_2 + a_2^2 - a_2b_1}{a_2^2}} e^{-\frac{x}{2a_2}(\sqrt{-4a_0a_2 + a_1^2} + a_1)} \left( U \left( \frac{1}{2a_2^2} \left( (a_1b_2 + 2a_2^2 - a_2b_1) \sqrt{-4a_0a_2 + a_1^2} \right) \right) \right) \right\}$$

**2.1146 ODE No. 1146**

$$x^2 y''(x) - 6y(x) = 0$$

✓ **Mathematica** : cpu = 0.0221519 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^5 + c_2}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 15

$$\left\{ y(x) = \frac{-C2 x^5 + -C1}{x^2} \right\}$$

**2.1147 ODE No. 1147**

$$x^2 y''(x) - 12y(x) = 0$$

✓ **Mathematica** : cpu = 0.0184177 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^7 + c_2}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 15

$$\left\{ y(x) = \frac{-C1 x^7 + -C2}{x^3} \right\}$$

**2.1148 ODE No. 1148**

$$ay(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0114753 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow x^{\frac{1}{2} - \frac{1}{2}\sqrt{1-4a}} \left( c_2 x^{\sqrt{1-4a}} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 35

$$\left\{ y(x) = -C1 x^{\frac{1}{2} + \frac{1}{2}\sqrt{1-4a}} + -C2 x^{\frac{1}{2} - \frac{1}{2}\sqrt{1-4a}} \right\}$$

**2.1149 ODE No. 1149**

$$y(x)(ax + b) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0659617 (sec), leaf count = 95

$$\left\{ \left\{ y(x) \rightarrow \sqrt{a}\sqrt{x} \left( c_1 \Gamma(1 - \sqrt{1-4b}) J_{-\sqrt{1-4b}}(2\sqrt{a}\sqrt{x}) + c_2 \Gamma(\sqrt{1-4b} + 1) J_{\sqrt{1-4b}}(2\sqrt{a}\sqrt{x}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 45

$$\left\{ y(x) = \sqrt{x} \left( Y_{\sqrt{1-4b}}(2\sqrt{a}\sqrt{x}) \_C2 + J_{\sqrt{1-4b}}(2\sqrt{a}\sqrt{x}) \_C1 \right) \right\}$$

**2.1150 ODE No. 1150**

$$x^2 y''(x) + (x^2 - 2) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0102824 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{\frac{2}{\pi}}((c_2 x - c_1) \sin(x) + (c_1 x + c_2) \cos(x))}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.067 (sec), leaf count = 27

$$\left\{ y(x) = \frac{(\_C1 x + \_C2) \cos(x) + \sin(x) (\_C2 x - \_C1)}{x} \right\}$$

**2.1151 ODE No. 1151**

$$x^2 y''(x) - (ax^2 + 2) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0193034 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow -\frac{i\sqrt{\frac{2}{\pi}}\sqrt{x}((c_1 + i\sqrt{a}c_2x) \sinh(\sqrt{a}x) - (\sqrt{a}c_1x + ic_2) \cosh(\sqrt{a}x))}{(-i\sqrt{a}x)^{3/2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 43

$$\left\{ y(x) = \frac{1}{x} \left( \_C2 (ax + \sqrt{a}) e^{-\sqrt{a}x} - \_C1 e^{\sqrt{a}x} (ax - \sqrt{a}) \right) \right\}$$

**2.1152 ODE No. 1152**

$$(a^2x^2 - 6)y(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0208397 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{\frac{2}{\pi}}\sqrt{x}((c_1(a^2x^2 - 3) + 3ac_2x) \sin(ax) + (-a^2c_2x^2 + 3ac_1x + 3c_2) \cos(ax))}{(ax)^{5/2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.243 (sec), leaf count = 53

$$\left\{ y(x) = \frac{(-C1 a^2x^2 + 3 C2 ax - 3 C1) \cos(ax) + \sin(ax) (-C2 a^2x^2 - 3 C1 ax - 3 C2)}{x^2} \right\}$$

**2.1153 ODE No. 1153**

$$y(x) (ax^2 - (v - 1)v) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0347533 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \sqrt{x} \left( c_1 J_{v-\frac{1}{2}}(\sqrt{ax}) + c_2 Y_{v-\frac{1}{2}}(\sqrt{ax}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 31

$$\left\{ y(x) = \sqrt{x} \left( Y_{v-\frac{1}{2}}(\sqrt{ax}) C2 + J_{v-\frac{1}{2}}(\sqrt{ax}) C1 \right) \right\}$$

**2.1154 ODE No. 1154**

$$y(x) (ax^2 + bx + c) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0218038 (sec), leaf count = 88

$$\left\{ \left\{ y(x) \rightarrow c_1 M_{-\frac{ib}{2\sqrt{a}}, -\frac{1}{2}i\sqrt{4c-1}}(2i\sqrt{ax}) + c_2 W_{-\frac{ib}{2\sqrt{a}}, -\frac{1}{2}i\sqrt{4c-1}}(2i\sqrt{ax}) \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 57

$$\left\{ y(x) = C1 M_{-\frac{ib}{2\sqrt{a}}, \frac{1}{2}\sqrt{1-4c}}(2i\sqrt{ax}) + C2 W_{-\frac{ib}{2\sqrt{a}}, \frac{1}{2}\sqrt{1-4c}}(2i\sqrt{ax}) \right\}$$

**2.1155 ODE No. 1155**

$$y(x) \left( ax^k - (b-1)b \right) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0532439 (sec), leaf count = 116

$$\left\{ \left\{ y(x) \rightarrow k^{-1/k} a^{1/2/k} (x^k)^{1/2/k} \left( c_1 \Gamma\left(\frac{-2b+k+1}{k}\right) J_{\frac{1-2b}{k}}\left(\frac{2\sqrt{a}\sqrt{x^k}}{k}\right) + c_2 \Gamma\left(\frac{2b+k-1}{k}\right) J_{\frac{2b-1}{k}}\left(\frac{2\sqrt{a}\sqrt{x^k}}{k}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 67

$$\left\{ y(x) = \sqrt{x} \left( Y_{\frac{1}{k}\sqrt{(2b-1)^2}} \left( 2 \frac{\sqrt{ax^{k/2}}}{k} \right) - C2 + J_{\frac{1}{k}\sqrt{(2b-1)^2}} \left( 2 \frac{\sqrt{ax^{k/2}}}{k} \right) - C1 \right) \right\}$$

**2.1156 ODE No. 1156**

$$x^2 y''(x) + \frac{y(x)}{\log(x)} - e^x x(x \log(x) + 2) = 0$$

✗ **Mathematica** : cpu = 0.208473 (sec), leaf count = 0 , could not solve

`DSolve[-(E^x*x*(2 + x*Log[x])) + y[x]/Log[x] + x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.136 (sec), leaf count = 71

$$\left\{ y(x) = \ln(x) - C2 - (Ei(1, -\ln(x)) \ln(x) + x) - C1 - \ln(x) \left( - \int \frac{(Ei(1, -\ln(x)) \ln(x) + x) e^x (2 + x \ln(x))}{x} dx \right) \right\}$$

**2.1157 ODE No. 1157**

$$ay'(x) + x^2 y''(x) - xy(x) = 0$$

✗ **Mathematica** : cpu = 0.497075 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{y''(x)x^2 - y(x)x + ay'(x) = 0, y(1) = c_1, y'(1) = c_2\}) (x) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ \frac{d^2}{dx^2} Y(x) + \frac{a \frac{d}{dx} Y(x)}{x^2} - \frac{Y(x)}{x} \right\}, \{Y(x)\} \right) \right\}$$

**2.1158 ODE No. 1158**

$$-y(x)(ab + b^2x^2) + ay'(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 14.1198 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow e^{bx} \left( c_2 \int_1^x e^{\frac{a}{K[1]} - 2bK[1]} dK[1] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.224 (sec), leaf count = 178

$$\left\{ y(x) = \sqrt{x} \left( e^{bx} \operatorname{HeunD} \left( -4\sqrt{2}\sqrt{ab}, -1 - 4\sqrt{2}\sqrt{ab}, 8\sqrt{2}\sqrt{ab}, -4\sqrt{2}\sqrt{ab} + 1, 1 \left( \sqrt{2}\sqrt{ab}x - a \right) \left( \sqrt{2}\sqrt{ab}x + a \right) \right) \right) \right\}$$

**2.1159 ODE No. 1159**

$$-ax^2 + x^2y''(x) + xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0174398 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{2ax^3 + 3(c_1(x^2 + 1) + ic_2(x^2 - 1))}{6x} \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 19

$$\left\{ y(x) = x\_C2 + \frac{ax^2}{3} + \frac{C1}{x} \right\}$$

**2.1160 ODE No. 1160**

$$ay(x) + x^2y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.010253 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin(\sqrt{a} \log(x)) + c_1 \cos(\sqrt{a} \log(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 23

$$\left\{ y(x) = \_C1 \sin(\sqrt{a} \ln(x)) + \_C2 \cos(\sqrt{a} \ln(x)) \right\}$$

**2.1161 ODE No. 1161**

$$-(a+x)y(x) + x^2y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0497196 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow (-1)^{-\sqrt{a}} c_1 \Gamma(1 - 2\sqrt{a}) I_{-2\sqrt{a}}(2\sqrt{x}) + (-1)^{\sqrt{a}} c_2 \Gamma(2\sqrt{a} + 1) I_{2\sqrt{a}}(2\sqrt{x}) \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 31

$$\left\{ y(x) = \_C1 I_{2\sqrt{a}}(2\sqrt{x}) + \_C2 K_{2\sqrt{a}}(2\sqrt{x}) \right\}$$

**2.1162 ODE No. 1162**

$$(x^2 - v^2) y(x) + x^2 y''(x) + x y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0600414 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_1 J_v(x) + c_2 Y_v(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 15

$$\left\{ y(x) = \_C1 J_v(x) + \_C2 Y_v(x) \right\}$$

**2.1163 ODE No. 1163**

$$-f(x) + (x^2 - v^2) y(x) + x^2 y''(x) + x y'(x) = 0$$

✓ **Mathematica** : cpu = 0.347494 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow J_v(x) \int_1^x -\frac{\pi f(K[1]) Y_v(K[1])}{2K[1]} dK[1] + Y_v(x) \int_1^x \frac{\pi f(K[2]) J_v(K[2])}{2K[2]} dK[2] + c_1 J_v(x) + c_2 Y_v(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 49

$$\left\{ y(x) = -\frac{J_v(x)\pi}{2} \int \frac{Y_v(x)f(x)}{x} dx + \frac{Y_v(x)\pi}{2} \int \frac{J_v(x)f(x)}{x} dx + Y_v(x)\_C1 + J_v(x)\_C2 \right\}$$



**2.1164 ODE No. 1164**

$$y(x)(lx^2 - v^2) + x^2y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0229999 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_1 J_v(\sqrt{lx}) + c_2 Y_v(\sqrt{lx}) \right\} \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 23

$$\left\{ y(x) = \_C1 J_v(\sqrt{lx}) + \_C2 Y_v(\sqrt{lx}) \right\}$$

**2.1165 ODE No. 1165**

$$(a + x)y'(x) + x^2y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.053298 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1(a + x) - \frac{c_2 x e^{a/x}}{a^2}}{a + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 19

$$\left\{ y(x) = (x + a) \_C1 + \_C2 x e^{\frac{a}{x}} \right\}$$

**2.1166 ODE No. 1166**

$$-3x^3 + x^2y''(x) - xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.01483 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow c_1 x + c_2 x \log(x) + \frac{3x^3}{4} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 21

$$\left\{ y(x) = \frac{x(4 \ln(x) \_C1 + 3x^2 + 4 \_C2)}{4} \right\}$$

**2.1167 ODE No. 1167**

$$y(x)(ax^m + b) + x^2y''(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0829769 (sec), leaf count = 130

$$\left\{ \left\{ y(x) \rightarrow m^{-2/m} a^{\frac{1}{m}} (x^m)^{\frac{1}{m}} \left( c_1 \Gamma \left( 1 - \frac{2i\sqrt{b-1}}{m} \right) J_{-\frac{2i\sqrt{b-1}}{m}} \left( \frac{2\sqrt{a}\sqrt{x^m}}{m} \right) + c_2 \Gamma \left( \frac{2i\sqrt{b-1}}{m} + 1 \right) J_{\frac{2i\sqrt{b-1}}{m}} \left( \frac{2\sqrt{a}\sqrt{x^m}}{m} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 63

$$\left\{ y(x) = x \left( Y_{2 \frac{\sqrt{1-b}}{m}} \left( 2 \frac{\sqrt{ax^{m/2}}}{m} \right) - C2 + J_{2 \frac{\sqrt{1-b}}{m}} \left( 2 \frac{\sqrt{ax^{m/2}}}{m} \right) - C1 \right) \right\}$$

**2.1168 ODE No. 1168**

$$x^2y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.00671073 (sec), leaf count = 15

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 11

$$\left\{ y(x) = -C1 + \frac{C2}{x} \right\}$$

**2.1169 ODE No. 1169**

$$y(x)(ax - b^2) + x^2y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0714459 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \Gamma \left( 1 - \sqrt{4b^2 + 1} \right) J_{-\sqrt{4b^2 + 1}} \left( 2\sqrt{a}\sqrt{x} \right) + c_2 \Gamma \left( \sqrt{4b^2 + 1} + 1 \right) J_{\sqrt{4b^2 + 1}} \left( 2\sqrt{a}\sqrt{x} \right)}{\sqrt{a}\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.025 (sec), leaf count = 49

$$\left\{ y(x) = 1 \left( -C2 Y_{\sqrt{4b^2 + 1}} \left( 2\sqrt{a}\sqrt{x} \right) + -C1 J_{\sqrt{4b^2 + 1}} \left( 2\sqrt{a}\sqrt{x} \right) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1170 ODE No. 1170**

$$y(x)(ax^2 + b) + x^2y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0241659 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow c_1 j_{\frac{1}{2}(\sqrt{1-4b}-1)}(\sqrt{ax}) + c_2 y_{\frac{1}{2}(\sqrt{1-4b}-1)}(\sqrt{ax}) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 43

$$\left\{ y(x) = 1 \left( -C2 Y_{\frac{1}{2}\sqrt{1-4b}}(\sqrt{ax}) + -C1 J_{\frac{1}{2}\sqrt{1-4b}}(\sqrt{ax}) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1171 ODE No. 1171**

$$y(x)(ax + lx^2 - n(n+1)) + x^2y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.056143 (sec), leaf count = 92

$$\left\{ \left\{ y(x) \rightarrow e^{-i\sqrt{l}x} x^n \left( c_1 U \left( \frac{ia}{2\sqrt{l}} + n + 1, 2n + 2, 2i\sqrt{l}x \right) + c_2 L_{-\frac{ia}{2\sqrt{l}}-n-1} \left( 2i\sqrt{l}x \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 49

$$\left\{ y(x) = \frac{1}{x} \left( -C1 M_{-\frac{i}{2}a\frac{1}{\sqrt{l}}, n+\frac{1}{2}} \left( 2i\sqrt{l}x \right) + -C2 W_{-\frac{i}{2}a\frac{1}{\sqrt{l}}, n+\frac{1}{2}} \left( 2i\sqrt{l}x \right) \right) \right\}$$

**2.1172 ODE No. 1172**

$$ay(x) + x^2y''(x) + 2(x-1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0580178 (sec), leaf count = 145

$$\left\{ \left\{ y(x) \rightarrow 2^{\frac{1}{2}-\frac{1}{2}\sqrt{1-4a}} \left( \frac{1}{x} \right)^{\frac{1}{2}-\frac{1}{2}\sqrt{1-4a}} \left( 2^{\sqrt{1-4a}} c_2 \left( \frac{1}{x} \right)^{\sqrt{1-4a}} {}_1F_1 \left( \frac{1}{2}(\sqrt{1-4a}+1); \sqrt{1-4a}+1; -\frac{2}{x} \right) + c_1 {}_1F_1 \left( \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 47

$$\left\{ y(x) = e^{-x^{-1}\sqrt{x-1}} \left( K_{\frac{1}{2}\sqrt{1-4a}}(x^{-1}) - C2 + I_{\frac{1}{2}\sqrt{1-4a}}(x^{-1}) - C1 \right) \right\}$$

**2.1173 ODE No. 1173**

$$2(a+x)y'(x) - (b-1)by(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.063279 (sec), leaf count = 74

$$\left\{ \left\{ y(x) \rightarrow (-2)^{1-b} c_1 a^{1-b} \left(\frac{1}{x}\right)^{1-b} {}_1F_1\left(1-b; 2-2b; \frac{2a}{x}\right) + (-2)^b c_2 a^b \left(\frac{1}{x}\right)^b {}_1F_1\left(b; 2b; \frac{2a}{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 37

$$\left\{ y(x) = 1e^{\frac{a}{x}} \left( K_{b-\frac{1}{2}}\left(\frac{a}{x}\right) - C2 + I_{b-\frac{1}{2}}\left(\frac{a}{x}\right) - C1 \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1174 ODE No. 1174**

$$x^5(-\log(x)) + x^2y''(x) - 2xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0234703 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow c_2 x^2 + c_1 x - \frac{7x^5}{144} + \frac{1}{12} x^5 \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 25

$$\left\{ y(x) = \frac{x^5 \ln(x)}{12} - \frac{7x^5}{144} + x^2 - C1 + x - C2 \right\}$$

**2.1175 ODE No. 1175**

$$-(ax^2 + 12a + 4) \cos(x) + x^2y''(x) - 2xy'(x) - 4y(x) - x \sin(x) = 0$$

✓ **Mathematica** : cpu = 0.175151 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{-(2a+1) \sin(x) - ax \cos(x) + c_2 x^5 + c_1}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 29

$$\left\{ y(x) = \frac{(-2a-1) \sin(x) + x^5 - C2 - ax \cos(x) - C1}{x} \right\}$$

**2.1176 ODE No. 1176**

$$x^2 y''(x) + (x^2 + 2) y(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0186801 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{-ix} x - \frac{1}{2} i c_2 e^{ix} x \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 15

$$\{y(x) = x(\cos(x) \_C2 + \sin(x) \_C1)\}$$

**2.1177 ODE No. 1177**

$$x^2 y''(x) + (x^2 + 2) y'(x) + x^2(-\sec(x)) - 2xy'(x) = 0$$

✗ **Mathematica** : cpu = 300.002 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.071 (sec), leaf count = 34

$$\left\{ y(x) = \left( -\cos(x) \int \frac{\sin(x)}{\cos(x)x} dx + \cos(x) \_C1 + \sin(x) (\_C2 + \ln(x)) \right) x \right\}$$

**2.1178 ODE No. 1178**

$$x^3(-\sec(x)) + x^2 y''(x) + (x^2 + 2) y(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0572242 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-ix} x (-i c_2 e^{2ix} + 2c_1 + e^{2ix} \log(1 + e^{-2ix}) + \log(1 + e^{2ix})) \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 23

$$\{y(x) = (\cos(x) \ln(\cos(x)) + \cos(x) \_C1 + \sin(x) (x + \_C2)) x\}$$

**2.1179 ODE No. 1179**

$$(a^2x^2 + 2)y(x) + x^2y''(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0420812 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 x e^{-\sqrt{-a^2}x} + \frac{c_2 x e^{\sqrt{-a^2}x}}{2\sqrt{-a^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 19

$$\{y(x) = x(\cos(ax) \_C2 + \sin(ax) \_C1)\}$$

**2.1180 ODE No. 1180**

$$-f(x) + (-v^2 + x^2 + 1)y(x) + x^2y''(x) + 3xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.228619 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{J_v(x) \int_1^x -\frac{1}{2}\pi f(K[1])Y_v(K[1]) dK[1] + Y_v(x) \int_1^x \frac{1}{2}\pi f(K[2])J_v(K[2]) dK[2] + c_1 J_v(x) + c_2 Y_v(x)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 49

$$\left\{ y(x) = \frac{-J_v(x)\pi \int Y_v(x)f(x) dx + Y_v(x)\pi \int J_v(x)f(x) dx + 2Y_v(x) \_C1 + 2J_v(x) \_C2}{2x} \right\}$$

**2.1181 ODE No. 1181**

$$x^2y''(x) + (3x - 1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0288892 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-1/x}(c_1 - c_2 \text{Ei}(\frac{1}{x}))}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 25

$$\left\{ y(x) = \frac{1}{xe^{x^{-1}}} (\_C1 \text{Ei}(1, -x^{-1}) + \_C2) \right\}$$

**2.1182 ODE No. 1182**

$$x^2 y''(x) - 3xy'(x) + 4y(x) - 5x = 0$$

✓ **Mathematica** : cpu = 0.0172178 (sec), leaf count = 20

$$\{ \{ y(x) \rightarrow x(c_1 x + 2c_2 x \log(x) + 5) \} \}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 20

$$\{ y(x) = x^2 \_C2 + x^2 \ln(x) \_C1 + 5x \}$$

**2.1183 ODE No. 1183**

$$x^2 y''(x) + x^2(-\log(x)) - 3xy'(x) - 5y(x) = 0$$

✓ **Mathematica** : cpu = 0.0266554 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1 x^5 + \frac{c_2}{x} - \frac{1}{9} x^2 \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 22

$$\left\{ y(x) = x^5 \_C2 + \frac{\_C1}{x} - \frac{x^2 \ln(x)}{9} \right\}$$

**2.1184 ODE No. 1184**

$$-x^4 + x^2 y''(x) + x^2 - 4xy'(x) + 6y(x) = 0$$

✓ **Mathematica** : cpu = 0.0212288 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} x^2 (2c_2 x + 2c_1 + x^2 + 2 \log(x) + 2) \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 25

$$\left\{ y(x) = \frac{x^2 (2x \_C1 + x^2 + 2 \ln(x) + 2 \_C2 + 2)}{2} \right\}$$

**2.1185 ODE No. 1185**

$$-(2x^3 - 4)y(x) + x^2y''(x) + 5xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0384065 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{6\sqrt[3]{3}c_2K_0\left(\frac{2}{3}\sqrt{2}x^{3/2}\right) - 3\sqrt[3]{-3}c_1I_0\left(\frac{2}{3}\sqrt{2}x^{3/2}\right)}{2^{2/3}x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 33

$$\left\{ y(x) = \frac{1}{x^2} \left( -C_2 K_0 \left( \frac{2\sqrt{2}}{3} x^{3/2} \right) + -C_1 I_0 \left( \frac{2\sqrt{2}}{3} x^{3/2} \right) \right) \right\}$$

**2.1186 ODE No. 1186**

$$x^3(-\sin(x)) + x^2y''(x) - 5xy'(x) + 8y(x) = 0$$

✓ **Mathematica** : cpu = 0.0341904 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}x^2(2c_2x^2 + 2c_1 + x^2\text{Ci}(x) - x\sin(x) + \cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.024 (sec), leaf count = 36

$$\left\{ y(x) = \frac{x^4\text{Ci}(x)}{2} - \frac{\sin(x)x^3}{2} + \frac{x^2(2\_C1x^2 + 2\_C2 + \cos(x))}{2} \right\}$$

**2.1187 ODE No. 1187**

$$axy'(x) + by(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0131562 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow x^{\frac{1}{2}(-\sqrt{a^2-2a-4b+1}-a+1)} \left( c_2x^{\sqrt{a^2-2a-4b+1}} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.012 (sec), leaf count = 53

$$\left\{ y(x) = -C_1 x^{-\frac{a}{2} + \frac{1}{2} + \frac{1}{2}\sqrt{a^2-2a-4b+1}} + -C_2 x^{-\frac{a}{2} + \frac{1}{2} - \frac{1}{2}\sqrt{a^2-2a-4b+1}} \right\}$$



**2.1188 ODE No. 1188**

$$(ax + b)y'(x) + cy(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0887319 (sec), leaf count = 243

$$\left\{ \left\{ y(x) \rightarrow -i^{-\sqrt{a^2-2a-4c+1}+a+1} b^{\frac{1}{2}(-\sqrt{a^2-2a-4c+1}+a-1)} \left(\frac{1}{x}\right)^{\frac{1}{2}(-\sqrt{a^2-2a-4c+1}+a-1)} \left(c_1 {}_1F_1\left(\frac{1}{2}(a - \sqrt{a^2 - 2a - 4c + 1}), \frac{1}{2}(a + \sqrt{a^2 - 2a - 4c + 1}), -\frac{b}{x}\right) + c_2 \Gamma\left(\frac{1}{2}(a - \sqrt{a^2 - 2a - 4c + 1})\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 114

$$\left\{ y(x) = x^{-\frac{1}{2}\sqrt{a^2-2a-4c+1}-\frac{a}{2}+\frac{1}{2}} \left( U\left(-\frac{1}{2} + \frac{1}{2}\sqrt{a^2-2a-4c+1} + \frac{a}{2}, 1 + \sqrt{a^2-2a-4c+1}, \frac{b}{x}\right) - C_2 + M\left(-\frac{1}{2}\sqrt{a^2-2a-4c+1} - \frac{a}{2} + \frac{1}{2}, 1 + \sqrt{a^2-2a-4c+1}, \frac{b}{x}\right) - C_1 \right) \right\}$$

**2.1189 ODE No. 1189**

$$axy'(x) + y(x)(bx^m + c) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0710866 (sec), leaf count = 168

$$\left\{ \left\{ y(x) \rightarrow m^{\frac{a-1}{m}} b^{-\frac{a-1}{2m}} (x^m)^{-\frac{a-1}{2m}} \left( c_1 \Gamma\left(1 - \frac{\sqrt{a^2-2a-4c+1}}{m}\right) J_{-\frac{\sqrt{a^2-2a-4c+1}}{m}}\left(\frac{2\sqrt{b}x^{m/2}}{m}\right) + c_2 \Gamma\left(\frac{m + \sqrt{a^2-2a-4c+1}}{m}\right) J_{\frac{\sqrt{a^2-2a-4c+1}}{m}}\left(\frac{2\sqrt{b}x^{m/2}}{m}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 79

$$\left\{ y(x) = x^{-\frac{a}{2}+\frac{1}{2}} \left( Y_{\frac{1}{m}\sqrt{a^2-2a-4c+1}}\left(2\frac{\sqrt{b}x^{m/2}}{m}\right) - C_2 + J_{\frac{1}{m}\sqrt{a^2-2a-4c+1}}\left(2\frac{\sqrt{b}x^{m/2}}{m}\right) - C_1 \right) \right\}$$

**2.1190 ODE No. 1190**

$$y(x)(ax + b) + x^2y''(x) + x^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0311144 (sec), leaf count = 95

$$\left\{ \left\{ y(x) \rightarrow e^{-x} x^{\frac{1}{2}(\sqrt{1-4b}+1)} \left( c_1 U\left(\frac{1}{2}(-2a + \sqrt{1-4b} + 1), \sqrt{1-4b} + 1, x\right) + c_2 L_{a-\frac{1}{2}\sqrt{1-4b}-\frac{1}{2}}^{\sqrt{1-4b}}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 38

$$\left\{ y(x) = e^{-\frac{x}{2}} \left( W_{a, \frac{1}{2}\sqrt{1-4b}}(x) - C_2 + M_{a, \frac{1}{2}\sqrt{1-4b}}(x) - C_1 \right) \right\}$$

**2.1191 ODE No. 1191**

$$x^2 y''(x) + x^2 y'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0104053 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-x/2} (2(2c_1 + ic_2x) \sinh(\frac{x}{2}) - 2(c_1x + 2ic_2) \cosh(\frac{x}{2}))}{\sqrt{\pi} \sqrt{-ix} \sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.015 (sec), leaf count = 23

$$\left\{ y(x) = \frac{-C2(x+2)e^{-x} + -C1(x-2)}{x} \right\}$$

**2.1192 ODE No. 1192**

$$x^2 y''(x) + (x^2 - 1) y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 10.8161 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow e^{-x} \left( c_2 \int_1^x e^{K[1] - \frac{1}{K[1]}} dK[1] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 51

$$\left\{ y(x) = \sqrt{x} \left( HeunD\left(-4, 3, -8, 5, \frac{x-1}{1+x}\right) e^{-x^{-1}} -C2 + e^{-x} HeunD\left(4, 3, -8, 5, \frac{x-1}{1+x}\right) -C1 \right) \right\}$$

**2.1193 ODE No. 1193**

$$x^2 y''(x) + (x+1)xy'(x) + (x-9)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0490895 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1((x-8)x+20) - c_2 e^{-x}(x^3+9x^2+36x+60)}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 38

$$\left\{ y(x) = \frac{-C2(x^3+9x^2+36x+60)e^{-x} + -C1(x^2-8x+20)}{x^3} \right\}$$

**2.1194 ODE No. 1194**

$$x^2 y''(x) + (x+1)xy'(x) + (3x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0510296 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-x}(c_2(x-3)x^2 \text{Ei}(x) + 6c_1x^3 - x^2(c_2e^x + 18c_1) + 2c_2e^xx + c_2e^x)}{6x} \right\} \right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 48

$$\left\{ y(x) = \frac{x^2 \_C2 e^{-x}(x-3) \text{Ei}(1, -x) + \_C1 x^2(x-3) e^{-x} + \_C2 (x^2 - 2x - 1)}{x} \right\}$$

**2.1195 ODE No. 1195**

$$x^2 y''(x) + (x+3)xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0285939 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow e^{-x} x^{\sqrt{2}-1} \left( c_1 U\left(2 + \sqrt{2}, 1 + 2\sqrt{2}, x\right) + c_2 L_{-2-\sqrt{2}}^{2\sqrt{2}}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.12 (sec), leaf count = 93

$$\left\{ y(x) = -1 \left( \_C1 \left( \sqrt{2} + x + 1 \right) I_{-\frac{1}{2} + \sqrt{2}} \left( \frac{x}{2} \right) - \_C1 \left( x - \sqrt{2} + 1 \right) I_{\frac{1}{2} + \sqrt{2}} \left( \frac{x}{2} \right) + \left( -\sqrt{2} - x - 1 \right) K_{-\frac{1}{2} + \sqrt{2}} \left( \frac{x}{2} \right) \right) \right\}$$

**2.1196 ODE No. 1196**

$$x^2 y''(x) - (x-1)xy'(x) + (x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0264356 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow c_1 x - \frac{c_2 (e^x(x+1) - x^2 \text{Ei}(x))}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 31

$$\left\{ y(x) = \frac{\text{Ei}(1, -x) \_C2 x^2 + \_C2 (1+x) e^x + \_C1 x^2}{x} \right\}$$

**2.1197 ODE No. 1197**

$$-(a+x)y(x) + x^2y''(x) - (x^2 - 2x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0197664 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{x/2} \left( c_1 J_{\frac{1}{2}\sqrt{4a+1}} \left( -\frac{ix}{2} \right) + c_2 Y_{\frac{1}{2}\sqrt{4a+1}} \left( -\frac{ix}{2} \right) \right)}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 43

$$\left\{ y(x) = 1e^{\frac{x}{2}} \left( K_{\frac{1}{2}\sqrt{4a+1}} \left( \frac{x}{2} \right) - C2 + I_{\frac{1}{2}\sqrt{4a+1}} \left( \frac{x}{2} \right) - C1 \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1198 ODE No. 1198**

$$x^2y''(x) - (x^2 - 2x)y'(x) - (3x + 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0307109 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 e^x x - \frac{c_2 (e^x x^3 \text{Ei}(-x) + x^2 - x + 2)}{6x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 37

$$\left\{ y(x) = \frac{e^x \text{Ei}(1, x) - C2 x^3 + -C1 x^3 e^x - -C2 (x^2 - x + 2)}{x^2} \right\}$$

**2.1199 ODE No. 1199**

$$x^2y''(x) - (x + 4)xy'(x) + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0134595 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_2 e^x x^4 - \frac{1}{6} c_1 x (e^x x^3 \text{Ei}(-x) + x^2 - x + 2) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 35

$$\{ y(x) = (e^x \text{Ei}(1, x) - C2 x^3 + -C1 x^3 e^x - -C2 (x^2 - x + 2)) x \}$$

**2.1200 ODE No. 1200**

$$-(v-1)vy(x) + x^2y''(x) + 2x^2y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0232991 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow e^{-x} \sqrt{x} \left( c_1 J_{v-\frac{1}{2}}(-ix) + c_2 Y_{v-\frac{1}{2}}(-ix) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 27

$$\left\{ y(x) = \sqrt{x} e^{-x} \left( K_{v-\frac{1}{2}}(x) C_2 + I_{v-\frac{1}{2}}(x) C_1 \right) \right\}$$

**2.1201 ODE No. 1201**

$$x^2y''(x) + (2x+1)xy'(x) - 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0559455 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-2x} (c_2 e^{2x} (2x^2 - 4x + 3) + c_1 (4x + 6))}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 34

$$\left\{ y(x) = \frac{-C_1 (2x^2 - 4x + 3)}{x^2} + \frac{-C_2 e^{-2x} (2x + 3)}{x^2} \right\}$$

**2.1202 ODE No. 1202**

$$x^2y''(x) - 2(x+1)xy'(x) + 2(x+1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0154194 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow x \left( \frac{1}{2} c_2 e^{2x} + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 14

$$\{ y(x) = x(e^{2x} C_2 + C_1) \}$$

**2.1203 ODE No. 1203**

$$ax^2y'(x) + x^2y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0226914 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow -\frac{ax^{3/2}e^{-\frac{ax}{2}}(2(2c_1 + iac_2x) \sinh(\frac{ax}{2}) - 2(ac_1x + 2ic_2) \cosh(\frac{ax}{2})))}{\sqrt{\pi}(-iax)^{5/2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 28

$$\left\{ y(x) = \frac{-C2(ax + 2)e^{-ax} + -C1(ax - 2)}{x} \right\}$$

**2.1204 ODE No. 1204**

$$x^2(a + 2b)y'(x) + y(x)(bx^2(a + b) - 2) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0214201 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow -\frac{ax^{3/2}e^{-\frac{1}{2}x(a+2b)}(2(2c_1 + iac_2x) \sinh(\frac{ax}{2}) - 2(ac_1x + 2ic_2) \cosh(\frac{ax}{2})))}{\sqrt{\pi}(-iax)^{5/2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 35

$$\left\{ y(x) = \frac{-C2(ax + 2)e^{-(a+b)x} + -C1e^{-bx}(ax - 2)}{x} \right\}$$

**2.1205 ODE No. 1205**

$$ax^2y'(x) + f(x)y(x) + x^2y''(x) = 0$$

✗ **Mathematica** : cpu = 0.202008 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + a*x^2*Derivative[1][y][x] + x^2*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol\left(\left\{ \frac{f(x)-Y(x)}{x^2} + a\frac{d}{dx}Y(x) + \frac{d^2}{dx^2}Y(x) \right\}, \{-Y(x)\}\right) \right\}$$

**2.1206 ODE No. 1206**

$$y(x) (abx + cx^2 + d) + x(2ax + b)y'(x) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.112513 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow e^{-ax} x^{\frac{1}{2} - \frac{b}{2}} \left( c_1 J_{\frac{1}{2}\sqrt{b^2 - 2b - 4d + 1}}(-i\sqrt{a^2 - cx}) + c_2 Y_{\frac{1}{2}\sqrt{b^2 - 2b - 4d + 1}}(-i\sqrt{a^2 - cx}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 76

$$\left\{ y(x) = e^{-ax} x^{-\frac{b}{2} + \frac{1}{2}} \left( Y_{\frac{1}{2}\sqrt{b^2 - 2b - 4d + 1}}(\sqrt{-a^2 + cx}) - C2 + J_{\frac{1}{2}\sqrt{b^2 - 2b - 4d + 1}}(\sqrt{-a^2 + cx}) - C1 \right) \right\}$$

**2.1207 ODE No. 1207**

$$x(ax + b)y'(x) + y(x) (a1x^2 + b1x + c1) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.120957 (sec), leaf count = 223

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{1}{2}x(\sqrt{a^2 - 4a1} + a)} x^{\frac{1}{2}(\sqrt{b^2 - 2b - 4c1 + 1} - b + 1)} \left( c_1 U \left( \frac{ab - 2b1 + \sqrt{a^2 - 4a1}(\sqrt{b^2 - 2b - 4c1 + 1} + 1)}{2\sqrt{a^2 - 4a1}}, \sqrt{b^2 - 2b - 4c1 + 1} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 110

$$\left\{ y(x) = e^{-\frac{ax}{2}} x^{-\frac{b}{2}} \left( W_{-\frac{ab - 2b1}{2}, \frac{1}{\sqrt{a^2 - 4a1}}, \frac{1}{2}\sqrt{b^2 - 2b - 4c1 + 1}}(\sqrt{a^2 - 4a1}x) - C2 + M_{-\frac{ab - 2b1}{2}, \frac{1}{\sqrt{a^2 - 4a1}}, \frac{1}{2}\sqrt{b^2 - 2b - 4c1 + 1}}(\sqrt{a^2 - 4a1}x) - C1 \right) \right\}$$

**2.1208 ODE No. 1208**

$$x^3y'(x) + x^2y''(x) + (x^2 - 2)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0384849 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{2\pi}c_2 \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) - 2c_2 e^{-\frac{x^2}{2}} x + 2c_1}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 35

$$\left\{ y(x) = \frac{1}{x} \left( \sqrt{\pi} \sqrt{2} \operatorname{Erf}\left(\frac{\sqrt{2}x}{2}\right) - C2 - 2e^{-1/2x^2} - C2 x + -C1 \right) \right\}$$

**2.1209 ODE No. 1209**

$$x^2 y''(x) + (x^2 + 2) x y'(x) + (x^2 - 2) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0218425 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\frac{x^2}{2}} \left( 2 \left( c_1 e^{\frac{x^2}{2}} x + c_2 \right) - \sqrt{2\pi} c_1 \operatorname{erfi} \left( \frac{x}{\sqrt{2}} \right) \right)}{2x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 41

$$\left\{ y(x) = \frac{1}{x^2} \left( \left( -\operatorname{Erf} \left( \frac{i}{2} \sqrt{2} x \right) \pi - C2 + C1 \right) e^{-\frac{x^2}{2}} + i \sqrt{\pi} \sqrt{2} - C2 x \right) \right\}$$

**2.1210 ODE No. 1210**

$$y(x) (a((-1)^n - 1) + 2nx^2) - 2x(x^2 - a) y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.259223 (sec), leaf count = 231

$$\left\{ \left\{ y(x) \rightarrow i^{-a} (-1)^{\frac{1}{4} (1 - \sqrt{4a^2 - 4a(-1)^{n+1}})} x^{\frac{1}{2} (-\sqrt{4a^2 - 4a(-1)^{n+1}} - 2a + 1)} \left( c_1 {}_1F_1 \left( \frac{1}{4} (-2a - 2n - \sqrt{4a^2 - 4(-1)^n a + 1}) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.679 (sec), leaf count = 81

$$\left\{ y(x) = e^{\frac{x^2}{2}} x^{-\frac{1}{2} - a} \left( M_{\frac{n}{2} + \frac{a}{2} + \frac{1}{4}, \frac{1}{4} \sqrt{1 - 4(-1)^n a + 4a^2}}(x^2) - C1 + W_{\frac{n}{2} + \frac{a}{2} + \frac{1}{4}, \frac{1}{4} \sqrt{1 - 4(-1)^n a + 4a^2}}(x^2) - C2 \right) \right\}$$

**2.1211 ODE No. 1211**

$$4x^3 y'(x) + x^2 y''(x) + (4x^4 + 2x^2 + 1) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0558947 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{3} e^{-x^2} x^{\frac{1}{2} - \frac{i\sqrt{3}}{2}} \left( 3c_1 - i\sqrt{3}c_2 x^{i\sqrt{3}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 36

$$\left\{ y(x) = e^{-x^2} \left( x^{\frac{1}{2} - \frac{i\sqrt{3}}{2}} - C2 + x^{\frac{1}{2} + \frac{i\sqrt{3}}{2}} - C1 \right) \right\}$$



**2.1212 ODE No. 1212**

$$x(ax^2 + b)y'(x) + f(x)y(x) + x^2y''(x) = 0$$

✗ **Mathematica** : cpu = 0.372942 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + x*(b + a*x^2)*Derivative[1][y][x] + x^2*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ \frac{f(x)Y(x)}{x^2} + \frac{(ax^2 + b) \frac{d}{dx} Y(x)}{x} + \frac{d^2}{dx^2} Y(x) \right\}, \{Y(x)\} \right) \right\}$$

**2.1213 ODE No. 1213**

$$(x^3 + 1)xy'(x) + x^2y''(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0750474 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[3]{3}c_1 {}_1F_1\left(-\frac{1}{3}; \frac{1}{3}; -\frac{x^3}{3}\right)}{x} + \frac{c_2 x {}_1F_1\left(\frac{1}{3}; \frac{5}{3}; -\frac{x^3}{3}\right)}{\sqrt[3]{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 53

$$\left\{ y(x) = x^{\frac{3}{2}} e^{-\frac{x^3}{6}} \left( -C1 I_{-\frac{1}{6}}\left(\frac{x^3}{6}\right) - C1 I_{\frac{5}{6}}\left(\frac{x^3}{6}\right) - C2 \left( K_{\frac{1}{6}}\left(\frac{x^3}{6}\right) - K_{\frac{5}{6}}\left(\frac{x^3}{6}\right) \right) \right) \right\}$$

**2.1214 ODE No. 1214**

$$y(x) (-a^2 + x^2(2a + 2n + 1) + a(-1)^n - x^4) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.290119 (sec), leaf count = 191

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\frac{x^2}{2}} 2^{\frac{1}{4}} (\sqrt{4a^2 - 4a(-1)^n + 1} + 2) (x^2)^{\frac{1}{4}} (\sqrt{4a^2 - 4a(-1)^n + 1} + 2) \left( c_1 U\left(\frac{1}{4}(-2a - 2n + \sqrt{4a^2 - 4(-1)^n a + 1} + 1) \right) \right)}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.62 (sec), leaf count = 71

$$\left\{ y(x) = 1 \left( -C1 M_{\frac{n}{2} + \frac{a}{2} + \frac{1}{4}, \frac{1}{4} \sqrt{1 - 4(-1)^n a + 4a^2}}(x^2) + -C2 W_{\frac{n}{2} + \frac{a}{2} + \frac{1}{4}, \frac{1}{4} \sqrt{1 - 4(-1)^n a + 4a^2}}(x^2) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1215 ODE No. 1215**

$$xy'(x)(ax^n + b) + y(x)(a1x^{2n} + b1x^n + c1) + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.15753 (sec), leaf count = 414

$$\left\{ \left\{ y(x) \rightarrow 2^{\frac{1}{2}} \left( \frac{\sqrt{n^2(b^2-2b-4c1+1)}}{n^2} + 1 \right) x^{\frac{1}{2}(-b-n+1)} e^{-\frac{(\sqrt{a^2-4a1+a})x^n}{2n}} (x^n)^{\frac{1}{2} \left( \frac{\sqrt{n^2(b^2-2b-4c1+1)}}{n^2} + 1 \right)} \left( c_1 U \left( \frac{(n^2 + \sqrt{(b^2 - \dots}}{\dots}} \right) \right) \right. \right.$$

✓ **Maple** : cpu = 0.176 (sec), leaf count = 148

$$\left\{ y(x) = e^{-\frac{ax^n}{2n}} x^{-\frac{b}{2} - \frac{n}{2} + \frac{1}{2}} \left( W_{-\frac{(b+n-1)a-2b1}{2n}, \frac{1}{\sqrt{a^2-4a1}}, \frac{1}{2n}\sqrt{b^2-2b-4c1+1}} \left( \frac{x^n}{n} \sqrt{a^2-4a1} \right) - C2 + M_{-\frac{(b+n-1)a-2b1}{2n}, \frac{1}{\sqrt{a^2-4a1}}, \frac{1}{2n}\sqrt{b^2-2b-4c1+1}} \left( \frac{x^n}{n} \sqrt{a^2-4a1} \right) \right) \right.$$

**2.1216 ODE No. 1216**

$$xy'(x)(ax^{a1} + b) + y(x)(Ax^{2a1} + Bx^{a1} + Cx^{b1} + DD) + x^2y''(x) = 0$$

✗ **Mathematica** : cpu = 1.44343 (sec), leaf count = 0 , could not solve

`DSolve[(DD + B*x^a1 + A*x^(2*a1) + C*x^b1)*y[x] + x*(b + a*x^a1)*Derivative[1][y][x] + x^2*D`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + \frac{(ax^{a1} + b) \frac{d}{dx} Y(x)}{x} + \frac{(Ax^{2a1} + Bx^{a1} + Cx^{b1} + DD) Y(x)}{x^2} \right\}, \{ Y(x) \} \right) \right.$$

**2.1217 ODE No. 1217**

$$-y(x)(a + x \tan(x)) + x^2y''(x) - (2x^2 \tan(x) - x) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.143896 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \sec(x) \left( c_1 J_{\sqrt{a}}(x) + c_2 Y_{\sqrt{a}}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 24

$$\left\{ y(x) = \frac{1}{\cos(x)} \left( -C1 J_{\sqrt{a}}(x) + -C2 Y_{\sqrt{a}}(x) \right) \right\}$$

**2.1218 ODE No. 1218**

$$y(x)(a + x \cot(x)) + x^2 y''(x) + (2x^2 \cot(x) + x) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.144591 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \csc(x) \left( c_1 J_{i\sqrt{a}}(x) + c_2 Y_{i\sqrt{a}}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 30

$$\left\{ y(x) = \frac{1}{\sin(x)} \left( -C2 Y_{i\sqrt{a}}(x) + -C1 J_{i\sqrt{a}}(x) \right) \right\}$$

**2.1219 ODE No. 1219**

$$y(x) (ax^2 + bx + c + x f'(x) + f(x)^2 - f(x)) + 2x f(x) y'(x) + x^2 y''(x) = 0$$

✗ **Mathematica** : cpu = 300.034 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.082 (sec), leaf count = 69

$$\left\{ y(x) = e^{-\int \frac{f(x)}{x} dx} \left( W_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, \frac{1}{2}\sqrt{1-4c}}(2i\sqrt{ax}) - C2 + M_{-\frac{i}{2}b\frac{1}{\sqrt{a}}, \frac{1}{2}\sqrt{1-4c}}(2i\sqrt{ax}) - C1 \right) \right\}$$

**2.1220 ODE No. 1220**

$$y(x) (x^2 (a + f'(x) + f(x)^2) - (v - 1)v) + 2x^2 f(x) y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 192.315 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \left( c_1 J_{v-\frac{1}{2}}(\sqrt{ax}) + c_2 Y_{v-\frac{1}{2}}(\sqrt{ax}) \right) e^{\int_1^x \left( \frac{1}{2K[1]} - f(K[1]) \right) dK[1]} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 40

$$\left\{ y(x) = \sqrt{x} e^{-\frac{\int 2f(x) dx}{2}} \left( Y_{v-\frac{1}{2}}(\sqrt{ax}) - C2 + J_{v-\frac{1}{2}}(\sqrt{ax}) - C1 \right) \right\}$$

**2.1221 ODE No. 1221**

$$y(x) (x^2 (-f'(x) + f(x)^2 + 1) - xf(x) - v^2) + (x - 2x^2 f(x)) y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0590964 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow (c_1 J_v(x) + c_2 Y_v(x)) e^{\int_1^x f(K[1]) dK[1]} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 35

$$\left\{ y(x) = \sqrt{x} e^{-\frac{1}{2} \int \frac{-2xf(x)+1}{x} dx} (Y_v(x) \_C2 + J_v(x) \_C1) \right\}$$

**2.1222 ODE No. 1222**

$$(x^2 + 1) y''(x) + xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0206646 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \sqrt{2} \sinh^{-1}(x) \right) + c_1 \cos \left( \sqrt{2} \sinh^{-1}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 23

$$\left\{ y(x) = \_C1 \sin \left( \sqrt{2} \operatorname{Arcsinh}(x) \right) + \_C2 \cos \left( \sqrt{2} \operatorname{Arcsinh}(x) \right) \right\}$$

**2.1223 ODE No. 1223**

$$(x^2 + 1) y''(x) + xy'(x) - 9y(x) = 0$$

✓ **Mathematica** : cpu = 0.0199354 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh \left( 3 \sinh^{-1}(x) \right) + ic_2 \sinh \left( 3 \sinh^{-1}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 39

$$\left\{ y(x) = \_C1 \sin \left( 3 \arctan \left( \frac{x}{\sqrt{-x^2 - 1}} \right) \right) + \_C2 \cos \left( 3 \arctan \left( \frac{x}{\sqrt{-x^2 - 1}} \right) \right) \right\}$$

**2.1224 ODE No. 1224**

$$ay(x) + (x^2 + 1)y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0185966 (sec), leaf count = 30

$$\{ \{ y(x) \rightarrow c_2 \sin(\sqrt{a} \sinh^{-1}(x)) + c_1 \cos(\sqrt{a} \sinh^{-1}(x)) \} \}$$

✓ **Maple** : cpu = 0.016 (sec), leaf count = 23

$$\{ y(x) = \_C1 \sin(\sqrt{a} \operatorname{Arcsinh}(x)) + \_C2 \cos(\sqrt{a} \operatorname{Arcsinh}(x)) \}$$

**2.1225 ODE No. 1225**

$$(x^2 + 1)y''(x) - xy'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0316529 (sec), leaf count = 29

$$\{ \{ y(x) \rightarrow -c_2 \sqrt{x^2 + 1} + c_1 x + c_2 x \sinh^{-1}(x) \} \}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 23

$$\{ y(x) = -\sqrt{x^2 + 1} \_C2 + x(\_C2 \operatorname{Arcsinh}(x) + \_C1) \}$$

**2.1226 ODE No. 1226**

$$-(v - 1)vy(x) + (x^2 + 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0196413 (sec), leaf count = 30

$$\{ \{ y(x) \rightarrow c_1 P_{v-1}(ix) + c_2 Q_{v-1}(ix) \} \}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 25

$$\{ y(x) = \_C1 \operatorname{LegendreP}(v - 1, ix) + \_C2 \operatorname{LegendreQ}(v - 1, ix) \}$$

**2.1227 ODE No. 1227**

$$(x^2 + 1) y''(x) - 2xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.049622 (sec), leaf count = 21

$$\{ \{ y(x) \rightarrow c_2 x - c_1 (x - i)^2 \} \}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 16

$$\{ y(x) = \_C2 x^2 + \_C1 x - \_C2 \}$$

**2.1228 ODE No. 1228**

$$ay(x) + (x^2 + 1) y''(x) + 3xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0156607 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 P_{\sqrt{1-a}-\frac{1}{2}}^{\frac{1}{2}}(ix) + c_2 Q_{\sqrt{1-a}-\frac{1}{2}}^{\frac{1}{2}}(ix)}{\sqrt[4]{x^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 53

$$\left\{ y(x) = 1 \left( \_C2 \left( x + \sqrt{x^2 + 1} \right)^{-\sqrt{1-a}} + \_C1 \left( x + \sqrt{x^2 + 1} \right)^{\sqrt{1-a}} \right) \frac{1}{\sqrt{x^2 + 1}} \right\}$$

**2.1229 ODE No. 1229**

$$(x^2 + 1) y''(x) + 4xy'(x) + 2y(x) + 2x - 2 \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0424668 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow -\frac{-3c_2 x - 3c_1 + x^3 + 6 \cos(x)}{3x^2 + 3} \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 31

$$\left\{ y(x) = \frac{-x^3 + 3 \_C1 x - 6 \cos(x) + 3 \_C2}{3x^2 + 3} \right\}$$

**2.1230 ODE No. 1230**

$$axy'(x) + (a - 2)y(x) + (x^2 + 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0260058 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow (x^2 + 1)^{\frac{1}{2} - \frac{a}{4}} \left( c_1 P_{\frac{a-2}{2}}^{\frac{a-2}{2}}(ix) + c_2 Q_{\frac{a-2}{2}}^{\frac{a-2}{2}}(ix) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.106 (sec), leaf count = 36

$$\left\{ y(x) = \_C1 (x^2 + 1)^{1 - \frac{a}{2}} + \_C2 {}_2F_1\left(1, \frac{a}{2} - \frac{1}{2}; \frac{3}{2}; -x^2\right)x \right\}$$

**2.1231 ODE No. 1231**

$$(x^2 - 1)y''(x) - v(v + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0758467 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(-\frac{v}{2} - \frac{1}{2}, \frac{v}{2}; \frac{1}{2}; x^2\right) + ic_2 x {}_2F_1\left(-\frac{v}{2}, \frac{v+1}{2}; \frac{3}{2}; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 52

$$\left\{ y(x) = -(x - 1)(1 + x) \left( {}_2F_1\left(1 - \frac{v}{2}, \frac{3}{2} + \frac{v}{2}; \frac{3}{2}; x^2\right) \_C2 x + \_C1 {}_2F_1\left(1 + \frac{v}{2}, \frac{1}{2} - \frac{v}{2}; \frac{1}{2}; x^2\right) \right) \right\}$$

**2.1232 ODE No. 1232**

$$\frac{nxP_n(x) - nP_{n-1}(x)}{x^2 - 1} - n(n + 1)y(x) + (x^2 - 1)y''(x) = 0$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.19 (sec), leaf count = 409

$$\left\{ y(x) = 3(1 + x) \left( -{}_2F_1(n/2 + 1, -n/2 + 1/2; 1/2; x^2)(n + 1) \int \frac{1}{3} \frac{1}{(1 + x)^3 (x - 1)^3 (({}_2F_1(n/2 + 1, -n/2 + 1/2; 1/2; x^2))} \right) \right\}$$

**2.1233 ODE No. 1233**

$$\frac{nxQ_n(x) - nQ_{n-1}(x)}{x^2 - 1} - n(n+1)y(x) + (x^2 - 1)y''(x) = 0$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.154 (sec), leaf count = 409

$$\left\{ y(x) = 3(1+x) \left( -{}_2F_1(n/2+1, -n/2+1/2; 1/2; x^2)(n+1) \int 1/3 \frac{1}{(1+x)^3 ({}_2F_1(n/2+1, -n/2+1/2; 1/2; x^2))} \right) \right\}$$

**2.1234 ODE No. 1234**

$$(x^2 - 1)y''(x) + xy'(x) + 2 = 0$$

✓ **Mathematica** : cpu = 0.0296834 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \log(\sqrt{x^2 - 1} + x) \left( c_1 - \log(\sqrt{x^2 - 1} + x) \right) + c_2 \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.1235 ODE No. 1235**

$$ay(x) + (x^2 - 1)y''(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0290575 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin(\sqrt{a} \log(\sqrt{x^2 - 1} + x)) + c_1 \cos(\sqrt{a} \log(\sqrt{x^2 - 1} + x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 45

$$\left\{ y(x) = 1 \left( -C1 \left( (x + \sqrt{x^2 - 1})^{i\sqrt{a}} \right)^2 + -C2 \right) \left( (x + \sqrt{x^2 - 1})^{i\sqrt{a}} \right)^{-1} \right\}$$



**2.1236 ODE No. 1236**

$$f(x)y(x) + (x^2 - 1)y''(x) + xy'(x) = 0$$

✗ **Mathematica** : cpu = 0.363797 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + x*Derivative[1][y][x] + (-1 + x^2)*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ \frac{f(x) - Y(x)}{x^2 - 1} + \frac{x \frac{d}{dx} Y(x)}{x^2 - 1} + \frac{d^2}{dx^2} Y(x) \right\}, \{ -Y(x) \} \right) \right\}$$

**2.1237 ODE No. 1237**

$$(x^2 - 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0114497 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}c_1(\log(1 - x) - \log(x + 1)) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 20

$$\left\{ y(x) = -C1 - \frac{(\ln(1 + x) - \ln(x - 1)) - C2}{2} \right\}$$

**2.1238 ODE No. 1238**

$$-a + (x^2 - 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0191837 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(a + c_1)\log(1 - x) + \frac{1}{2}(a - c_1)\log(x + 1) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 26

$$\left\{ y(x) = \frac{(a - C1)\ln(1 + x)}{2} + \frac{(a + C1)\ln(x - 1)}{2} + C2 \right\}$$

**2.1239 ODE No. 1239**

$$-ly(x) + (x^2 - 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0149078 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow c_1 P_{\frac{1}{2}(\sqrt{4l+1}-1)}(x) + c_2 Q_{\frac{1}{2}(\sqrt{4l+1}-1)}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 35

$$\left\{ y(x) = \_C1 \text{LegendreP}\left(\frac{1}{2}\sqrt{1+4l} - \frac{1}{2}, x\right) + \_C2 \text{LegendreQ}\left(\frac{1}{2}\sqrt{1+4l} - \frac{1}{2}, x\right) \right\}$$

**2.1240 ODE No. 1240**

$$-v(v+1)y(x) + (x^2 - 1)y''(x) + 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0191962 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_1 P_v(x) + c_2 Q_v(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 15

$$\{y(x) = \_C1 \text{LegendreP}(v, x) + \_C2 \text{LegendreQ}(v, x)\}$$

**2.1241 ODE No. 1241**

$$-(v-1)(v+2)y(x) + (x^2 - 1)y''(x) - 2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0166607 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow (x^2 - 1) (c_1 P_v^2(x) + c_2 Q_v^2(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 24

$$\{y(x) = (1+x)(x-1)(\_C2 \text{LegendreQ}(v, 2, x) + \_C1 \text{LegendreP}(v, 2, x))\}$$

**2.1242 ODE No. 1242**

$$(x^2 - 1)y''(x) - (x^2 - x)y(x) - (3x + 1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0768598 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow e^{-x-2} (c_2(x+1)^2 \text{Ei}(2(x+1)) + e^2 (c_1(x+1)^2 - 2c_2 e^{2x})) \right\} \right\}$$

✓ **Maple** : cpu = 0.077 (sec), leaf count = 41

$$\left\{ y(x) = \_C2 e^{-x-2} (1+x)^2 \text{Ei}(1, -2x-2) + \_C1 e^{-x} (1+x)^2 + 2e^x \_C2 \right\}$$

**2.1243 ODE No. 1243**

$$(x^2 - 1)y''(x) + (x^2 + 1)y(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.031859 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-ix} (2c_1 - ic_2 e^{2ix})}{2(x^2 - 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 21

$$\left\{ y(x) = \frac{\_C2 \cos(x) + \_C1 \sin(x)}{x^2 - 1} \right\}$$

**2.1244 ODE No. 1244**

$$-(v - n)(n + v + 1)y(x) + 2(n + 1)xy'(x) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0316753 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow (x^2 - 1)^{-n/2} (c_1 P_v^n(x) + c_2 Q_v^n(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.071 (sec), leaf count = 27

$$\left\{ y(x) = (x^2 - 1)^{-\frac{n}{2}} (\text{LegendreP}(v, n, x) \_C1 + \text{LegendreQ}(v, n, x) \_C2) \right\}$$

**2.1245 ODE No. 1245**

$$-(-n + v + 1)(n + v)y(x) - 2(n - 1)xy'(x) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0238485 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow (x^2 - 1)^{n/2} (c_1 P_v^n(x) + c_2 Q_v^n(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 27

$$\left\{ y(x) = (x^2 - 1)^{\frac{n}{2}} (\text{LegendreP}(v, n, x) \_C1 + \text{LegendreQ}(v, n, x) \_C2) \right\}$$

**2.1246 ODE No. 1246**

$$-2(v - 1)xy'(x) - 2vy(x) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0237051 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow (x^2 - 1)^{v/2} (c_1 P_v^v(x) + c_2 Q_v^v(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 28

$$\left\{ y(x) = (x^2 - 1)^v \left( {}_2F_1\left(\frac{1}{2}, v + 1; \frac{3}{2}; x^2\right) \_C2 x + \_C1 \right) \right\}$$

**2.1247 ODE No. 1247**

$$2axy'(x) + (a - 1)ay(x) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.224844 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{1 - x^2} (x^2 - 1)^{-a/2} e^{-\sqrt{(a-1)^2} \tanh^{-1}(x)} \left( c_2 e^{2\sqrt{(a-1)^2} \tanh^{-1}(x)} + 2\sqrt{(a-1)^2} c_1 \right)}{2\sqrt{(a-1)^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 27

$$\left\{ y(x) = \_C1 (1 + x)^{1-a} + \_C2 (x - 1)^{1-a} \right\}$$

**2.1248 ODE No. 1248**

$$axy'(x) + y(x)(bx^2 + cx + d) + (x^2 - 1)y''(x) = 0$$

✗ **Mathematica** : cpu = 2.19498 (sec), leaf count = 0 , DifferentialRoot result

$$\{ \{y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(bx^2 + cx + d)y(x) + xay'(x) + (x^2 - 1)y''(x) = 0, y(0) = c_1, y'(0) = c_2\}) \}$$

✓ **Maple** : cpu = 0.221 (sec), leaf count = 134

$$\left\{ y(x) = e^{\sqrt{-b}x} (x^2 - 1)^{-\frac{a}{4}} \left( \left( \frac{1}{2} + \frac{x}{2} \right)^{1-\frac{a}{4}} \left( -\frac{1}{2} + \frac{x}{2} \right)^{\frac{a}{4}} \text{HeunC} \left( 4\sqrt{-b}, 1 - \frac{a}{2}, \frac{a}{2} - 1, 2c, d - c - \frac{a^2}{8} + b + \frac{1}{2}, \frac{1}{2} \right) \right) \right\}$$

**2.1249 ODE No. 1249**

$$(ax + b)y'(x) + cy(x) + (x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.176327 (sec), leaf count = 190

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(x-1)^{\frac{1}{2}(-a-b)} \left( 2c_1(x-1)^{\frac{a+b}{2}} {}_2F_1 \left( \frac{1}{2} \left( a - \sqrt{a^2 - 2a - 4c + 1} - 1 \right), \frac{1}{2} \left( a + \sqrt{a^2 - 2a - 4c + 1} - 1 \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 134

$$\left\{ y(x) = {}_2F_1 \left( -\frac{1}{2} - \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1} + \frac{a}{2}, -\frac{1}{2} + \frac{1}{2}\sqrt{a^2 - 2a - 4c + 1} + \frac{a}{2}; \frac{a}{2} - \frac{b}{2}; \frac{1}{2} + \frac{x}{2} \right) + {}_2F_1 \left( \frac{1}{2} \left( a - \sqrt{a^2 - 2a - 4c + 1} - 1 \right), \frac{1}{2} \left( a + \sqrt{a^2 - 2a - 4c + 1} - 1 \right); \frac{1}{2} \left( a + \sqrt{a^2 - 2a - 4c + 1} - 1 \right); \frac{1}{2} + \frac{x}{2} \right) \right\}$$

**2.1250 ODE No. 1250**

$$(x^2 - a^2)y''(x) + 8xy'(x) + 12y(x) = 0$$

✓ **Mathematica** : cpu = 0.0535079 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \frac{a^2 + 3x^2}{(a-x)^3} + 3c_1}{3(a+x)^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 41

$$\left\{ y(x) = \frac{3 {}_2F_1 \left( \frac{1}{2} \left( a - \sqrt{a^2 - 2a - 4c + 1} - 1 \right), \frac{1}{2} \left( a + \sqrt{a^2 - 2a - 4c + 1} - 1 \right); \frac{1}{2} \left( a + \sqrt{a^2 - 2a - 4c + 1} - 1 \right); \frac{1}{2} + \frac{x}{2} \right) + 3 {}_2F_1 \left( \frac{1}{2} \left( a - \sqrt{a^2 - 2a - 4c + 1} - 1 \right), \frac{1}{2} \left( a + \sqrt{a^2 - 2a - 4c + 1} - 1 \right); \frac{1}{2} \left( a + \sqrt{a^2 - 2a - 4c + 1} - 1 \right); \frac{1}{2} + \frac{x}{2} \right)}{(a-x)^3 (x+a)^3} \right\}$$

**2.1251 ODE No. 1251**

$$x(x+1)y''(x) - (x-1)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0392478 (sec), leaf count = 23

$$\{ \{ y(x) \rightarrow c_1(x-1) + c_2((x-1)\log(x) - 4) \} \}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 20

$$\{ y(x) = \_C2(x-1)\ln(x) - 4\_C2 + \_C1(x-1) \}$$

**2.1252 ODE No. 1252**

$$(ax+b)y'(x) + cy(x) + x(x+1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.167176 (sec), leaf count = 131

$$\left\{ \left\{ y(x) \rightarrow c_2 x^{1-b} {}_2F_1 \left( \frac{1}{2} (a-2b - \sqrt{a^2 - 2a - 4c + 1} + 1), \frac{1}{2} (a-2b + \sqrt{a^2 - 2a - 4c + 1} + 1); 2-b; -x \right) + \dots \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 124

$$\left\{ y(x) = \_C1 {}_2F_1 \left( -\frac{1}{2} - \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{a}{2}, -\frac{1}{2} + \frac{1}{2} \sqrt{a^2 - 2a - 4c + 1} + \frac{a}{2}; a-b; 1+x \right) + \_C2(1+x) \right\}$$

**2.1253 ODE No. 1253**

$$x(x+1)y''(x) + (3x+2)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0269149 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \log(2(x+1)) + 2c_1}{\sqrt{2x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 16

$$\left\{ y(x) = \frac{\ln(1+x)\_C1 + \_C2}{x} \right\}$$

**2.1254 ODE No. 1254**

$$(x^2 + x - 2)y''(x) + (x^2 - x)y'(x) - (6x^2 + 7x)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0946372 (sec), leaf count = 60

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{5} e^{-3x-5} (195c_2 e^{5x} (x-1) \text{Ei}(5-5x) - e^5 (5c_1 e^{5x} (x-1) - c_2 (x+44))) \right\} \right\}$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 42

$$\{y(x) = -195\_C2 e^{2x-5} (x-1) \text{Ei}(1, 5x-5) + \_C2 (x+44) e^{-3x} + \_C1 e^{2x} (x-1)\}$$

**2.1255 ODE No. 1255**

$$ay'(x) + (x-1)xy''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.224424 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow \frac{(a^2 + a(2x-1) + 2(x-1)x) \left( \frac{c_2 x^{a+1} (1-x)^{1-a}}{(a-1)a(a+1)(a^2+a(2x-1)+2(x-1)x)} + c_1 \right)}{a^2 + 3a + 4} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 42

$$\left\{ y(x) = (a^2 + a(2x-1) + 2x^2 - 2x) \_C1 + \frac{\_C2 x^a x (x-1)}{(x-1)^a} \right\}$$

**2.1256 ODE No. 1256**

$$-v(v+1)y(x) + (x-1)xy''(x) + (2x-1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0252603 (sec), leaf count = 26

$$\{\{y(x) \rightarrow c_1 P_v(2x-1) + c_2 Q_v(2x-1)\}\}$$

✓ **Maple** : cpu = 0.113 (sec), leaf count = 51

$$\{y(x) = \_C1 {}_2F_1(-v, -v; -2v; x^{-1})x^v + \_C2 {}_2F_1(v+1, v+1; 2+2v; x^{-1})x^{-v-1}\}$$

**2.1257 ODE No. 1257**

$$((a+1)x+b)y'(x) + (x-1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.054001 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^{b+1} {}_2F_1(b+1, a+b+1; b+2; x)}{b+1} + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.23 (sec), leaf count = 27

$$\left\{ y(x) = {}_2F_1(b+1, a+b+1; b+2; x) x^{b+1} \right\}$$

**2.1258 ODE No. 1258**

$$(ax+b)y'(x) + cy(x) + (x-1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.174921 (sec), leaf count = 130

$$\left\{ \left\{ y(x) \rightarrow (-1)^{b+1} c_2 x^{b+1} {}_2F_1\left(\frac{1}{2}\left(a+2b-\sqrt{a^2-2a-4c+1}+1\right), \frac{1}{2}\left(a+2b+\sqrt{a^2-2a-4c+1}+1\right); b+2; x\right) + c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 110

$$\left\{ y(x) = {}_2F_1\left(-\frac{1}{2}-\frac{1}{2}\sqrt{a^2-2a-4c+1}+\frac{a}{2}, -\frac{1}{2}+\frac{1}{2}\sqrt{a^2-2a-4c+1}+\frac{a}{2}; -b; x\right) + {}_2F_1\left(\frac{1}{2}\left(a+2b-\sqrt{a^2-2a-4c+1}+1\right), \frac{1}{2}\left(a+2b+\sqrt{a^2-2a-4c+1}+1\right); b+2; x\right) x^{b+1} \right\}$$

**2.1259 ODE No. 1259**

$$((a+1)x+b)y'(x) - ly(x) + (x-1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.146432 (sec), leaf count = 112

$$\left\{ \left\{ y(x) \rightarrow (-1)^{b+1} c_2 x^{b+1} {}_2F_1\left(\frac{1}{2}\left(a+2b-\sqrt{a^2+4l}+2\right), \frac{1}{2}\left(a+2b+\sqrt{a^2+4l}+2\right); b+2; x\right) + c_1 {}_2F_1\left(\frac{1}{2}\left(a+2b-\sqrt{a^2+4l}+2\right), \frac{1}{2}\left(a+2b+\sqrt{a^2+4l}+2\right); b+2; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 92

$$\left\{ y(x) = {}_2F_1\left(\frac{a}{2}-\frac{1}{2}\sqrt{a^2+4l}, \frac{a}{2}+\frac{1}{2}\sqrt{a^2+4l}; -b; x\right) + {}_2F_1\left(\frac{a}{2}-\frac{1}{2}\sqrt{a^2+4l}+b+1, \frac{a}{2}+\frac{1}{2}\sqrt{a^2+4l}+b+1; b+2; x\right) x^{b+1} \right\}$$



**2.1260 ODE No. 1260**

$$y'(x)(x(a1 + b1 + 1) - d1) + a1b1d1 + (x - 1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.171369 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow a1b1x\Gamma(d1 + 1) {}_3\tilde{F}_2(1, a1 + b1 + 1, 1; d1 + 1, 2; x) - \frac{c_1x^{1-d1} {}_2F_1(1 - d1, a1 + b1 - d1 + 1; 2 - d1; x)}{d1 - 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.566 (sec), leaf count = 76

$$\left\{ y(x) = \int \left( -(\text{signum}(x - 1))^{a1+b1-d1} (-\text{signum}(x - 1))^{-a1-b1+d1} a1 b1 {}_2F_1(d1, -a1 - b1 + d1; 1 + d1; x) \right) dx \right\}$$

**2.1261 ODE No. 1261**

$$y(x)(2lx(-n + p - 1) + 2lp + m) + 2(x(-2l + n + 1) - lx^2 + n + 1)y'(x) + x(x + 2)y''(x) = 0$$

✗ **Mathematica** : cpu = 2.72711 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(-2xl - 2xnl + 2xpl + 2pl + m)y(x) + 2(-lx^2 - 2lx + nx + x + n + 1)y'(x) + x(x + 2)y''(x)\}) \right\} \right\}$$

✓ **Maple** : cpu = 0.234 (sec), leaf count = 105

$$\left\{ y(x) = (x + 2)^{-\frac{n}{2} - \frac{1}{2}} \left( -\frac{x}{2} - 1 \right)^{\frac{n}{2} + \frac{1}{2}} \left( x^{-n} \text{HeunC} \left( 4l, -n, n, -4pl, \frac{(4n + 4p + 4)l}{2} - \frac{n^2}{2} + m - n, -\frac{x}{2} \right) - C2 \right) \right\}$$

**2.1262 ODE No. 1262**

$$(x^2 + x - 1)y'(x) + (x + 1)^2y''(x) - (x + 2)y(x) = 0$$

✓ **Mathematica** : cpu = 40.1978 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow e^{-x} \left( c_2 \int_1^x e^{\frac{K[1]^2 + K[1] - 1}{K[1] + 1}} (K[1] + 1) dK[1] + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.263 (sec), leaf count = 53

$$\left\{ y(x) = (1 + x) \left( -C1 e^{-x} \text{HeunD} \left( 4, 4, -8, 12, \frac{x}{x + 2} \right) + -C2 \text{HeunD} \left( -4, 4, -8, 12, \frac{x}{x + 2} \right) e^{\frac{x-1}{2x+2}} \right) \right\}$$

**2.1263 ODE No. 1263**

$$-(20x + 30)(x^2 + 3x)^{7/3} + x(x + 3)y''(x) + (3x - 1)y'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.086 (sec), leaf count = 52

$$\left\{ y(x) = 1 \left( -C2 + \int \frac{1}{x^2 + 3x} \left( -C1 + 3(x^2 + 3x)^{7/3} x(x + 3) \right) (x + 3)^{7/3} x^{-4/3} dx \right) x^{4/3} (x + 3)^{-7/3} \right\}$$

**2.1264 ODE No. 1264**

$$(x^2 + 3x + 4)y''(x) + (x^2 + x + 1)y'(x) - (2x + 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0620275 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow c_2(x^2 + x + 3) + c_1 e^{-x} \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 19

$$\left\{ y(x) = \_C1 e^{-x} + \_C2 (x^2 + x + 3) \right\}$$

**2.1265 ODE No. 1265**

$$(x - 2)(x - 1)y''(x) - (2x - 3)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0433852 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow (x^2 - 3x + 2) \left( c_1 P_{\frac{1}{2}}^2(-1 + \sqrt{5})(2x - 3) + c_2 Q_{\frac{1}{2}}^2(-1 + \sqrt{5})(2x - 3) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.131 (sec), leaf count = 93

$$\left\{ y(x) = (x - 2)^2 \left( -C1 {}_2F_1\left(\frac{1}{2} - \frac{\sqrt{5}}{2}, \frac{5}{2} - \frac{\sqrt{5}}{2}; -\sqrt{5} + 1; (x - 1)^{-1}\right) (x - 1)^{\frac{\sqrt{5}}{2} - \frac{1}{2}} + -C2 {}_2F_1\left(\frac{1}{2} + \frac{\sqrt{5}}{2}, \frac{5}{2} + \frac{\sqrt{5}}{2}; \right) \right) \right\}$$

**2.1266 ODE No. 1266**

$$(x-2)^2 y''(x) - (x-2)y'(x) - 3y(x) = 0$$

✓ **Mathematica** : cpu = 0.0293308 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow c_1(x-2)^3 + \frac{c_2}{x-2} \right\} \right\}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 19

$$\left\{ y(x) = \frac{-C1 + -C2(x-2)^4}{x-2} \right\}$$

**2.1267 ODE No. 1267**

$$-(l+2x^2-5x)y'(x) + 2x^2y''(x) - (4x-1)y(x) = 0$$

✗ **Mathematica** : cpu = 300.101 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.072 (sec), leaf count = 41

$$\left\{ y(x) = e^x \left( -C1 \int \frac{1}{2e^x} e^{\frac{l}{2x}} x^{-\frac{3}{2}} dx + -C2 \right) \frac{1}{\sqrt{x}} \left( e^{\frac{l}{2x}} \right)^{-1} \right\}$$

**2.1268 ODE No. 1268**

$$y(x)(ax+b) + 2(x-1)xy''(x) + (2x-1)y'(x) = 0$$

✗ **Mathematica** : cpu = 1.43729 (sec), leaf count = 0 , DifferentialRoot result

{ { y(x) → DifferentialRoot({ y, x }, { (xa + b)y(x) + (2x - 1)y'(x) + 2(x - 1)xy''(x) = 0, y(2) = c1, y'(2) = c2 }) }

✓ **Maple** : cpu = 0.111 (sec), leaf count = 39

$$\left\{ y(x) = -C1 \operatorname{MathieuC}\left(-a - 2b, \frac{a}{2}, \arccos(\sqrt{x})\right) + -C2 \operatorname{MathieuS}\left(-a - 2b, \frac{a}{2}, \arccos(\sqrt{x})\right) \right\}$$

**2.1269 ODE No. 1269**

$$((2v + 5)x - 2v - 3)y'(x) + (v + 1)y(x) + 2(x - 1)xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0875847 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(\frac{1}{2}, v + 1; v + \frac{3}{2}; x\right) - ic_2 i^{-2v} x^{-v-\frac{1}{2}} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 40

$$\left\{ y(x) = \_C1 {}_2F_1\left(\frac{1}{2}, v + 1; \frac{3}{2} + v; x\right) + \_C2 x^{-\frac{1}{2}-v} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x\right) \right\}$$

**2.1270 ODE No. 1270**

$$(2x^2 + 6x + 4)y''(x) + (10x^2 + 21x + 8)y'(x) + (12x^2 + 17x + 8)y(x) = 0$$

✗ **Mathematica** : cpu = 300.009 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.185 (sec), leaf count = 46

$$\left\{ y(x) = e^{-2x}(x + 2)^4 \left( \_C2 (1 + x)^{\frac{5}{2}} HeunC\left(-1, \frac{5}{2}, 4, -\frac{7}{4}, \frac{7}{2}, -1 - x\right) + \_C1 HeunC\left(-1, -\frac{5}{2}, 4, -\frac{7}{4}, \frac{7}{2}, -1 - x\right) \right) \right\}$$

**2.1271 ODE No. 1271**

$$4x^2y''(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0135552 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}\sqrt{x}(c_2 \log(x) + 2c_1) \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 14

$$\{y(x) = \sqrt{x}(\ln(x) \_C2 + \_C1)\}$$

**2.1272 ODE No. 1272**

$$(4a^2x^2 + 1)y(x) + 4x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.013633 (sec), leaf count = 28

$$\{\{y(x) \rightarrow \sqrt{x}(c_1J_0(ax) + c_2Y_0(ax))\}\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 23

$$\{y(x) = \sqrt{x}(Y_0(ax)_C2 + J_0(ax)_C1)\}$$

**2.1273 ODE No. 1273**

$$4x^2y''(x) - y(x)(-4kx + 4m^2 + x^2 - 1) = 0$$

✓ **Mathematica** : cpu = 0.0180475 (sec), leaf count = 20

$$\{\{y(x) \rightarrow c_1M_{k,m}(x) + c_2W_{k,m}(x)\}\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 17

$$\{y(x) = _C1 M_{k,m}(x) + _C2 W_{k,m}(x)\}$$

**2.1274 ODE No. 1274**

$$(x - v^2)y(x) + 4x^2y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0436968 (sec), leaf count = 38

$$\{\{y(x) \rightarrow c_1\Gamma(1-v)J_{-v}(\sqrt{x}) + c_2\Gamma(v+1)J_v(\sqrt{x})\}\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 19

$$\{y(x) = _C1 J_v(\sqrt{x}) + _C2 Y_v(\sqrt{x})\}$$

**2.1275 ODE No. 1275**

$$y(x) (2x(2l - m + 1) - m^2 - x^2 + 1) + 4x^2 y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0415424 (sec), leaf count = 97

$$\left\{ \left\{ y(x) \rightarrow e^{-x/2} x^{\frac{\sqrt{m^2-1}}{2}} \left( c_1 U \left( \frac{1}{2} \left( -2l + m + \sqrt{m^2-1} \right), \sqrt{m^2-1} + 1, x \right) + c_2 L_{l - \frac{\sqrt{m^2-1}}{2} - \frac{m}{2}}(x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.159 (sec), leaf count = 53

$$\left\{ y(x) = 1 \left( -C2 W_{l - \frac{m}{2} + \frac{1}{2}, \frac{1}{2} \sqrt{m-1} \sqrt{m+1}}(x) + -C1 M_{l - \frac{m}{2} + \frac{1}{2}, \frac{1}{2} \sqrt{m-1} \sqrt{m+1}}(x) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1276 ODE No. 1276**

$$-4e^x \sqrt{x^3} + 4x^2 y''(x) - (4x^2 + 1) y(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0614771 (sec), leaf count = 55

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-x}}{\sqrt{x}} + \frac{c_2 e^x}{2\sqrt{x}} + \frac{e^x \sqrt{x^3} (2x - 1)}{4x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 31

$$\left\{ y(x) = \sinh(x) - C2 \frac{1}{\sqrt{x}} + \cosh(x) - C1 \frac{1}{\sqrt{x}} + \frac{e^x}{2x} \sqrt{x^3} \right\}$$

**2.1277 ODE No. 1277**

$$-(ax^2 + 1) y(x) + 4x^2 y''(x) + 4xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0291308 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\frac{\sqrt{ax}}{2}} \left( c_2 e^{\sqrt{ax}} + \sqrt{ac_1} \right)}{\sqrt{a} \sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 27

$$\left\{ y(x) = 1 \left( -C2 \cosh \left( \frac{x}{2} \sqrt{a} \right) + -C1 \sinh \left( \frac{x}{2} \sqrt{a} \right) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1278 ODE No. 1278**

$$f(x)y(x) + 4x^2y''(x) + 4xy'(x) = 0$$

✗ **Mathematica** : cpu = 0.263434 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + 4*x*Derivative[1][y][x] + 4*x^2*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ \frac{f(x) - Y(x)}{4x^2} + \frac{\frac{d}{dx} - Y(x)}{x} + \frac{d^2}{dx^2} - Y(x) \right\}, \{ -Y(x) \} \right) \right\}$$

**2.1279 ODE No. 1279**

$$4x^2y''(x) + 5xy'(x) - y(x) - \log(x) = 0$$

✓ **Mathematica** : cpu = 0.178292 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{\frac{1}{8}(\sqrt{17}-1)} + c_2 x^{-\frac{1}{8}-\frac{\sqrt{17}}{8}} - \log(x) - 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.167 (sec), leaf count = 32

$$\left\{ y(x) = x^{-\frac{1}{8}+\frac{\sqrt{17}}{8}} C_2 + x^{-\frac{1}{8}-\frac{\sqrt{17}}{8}} C_1 - \ln(x) - 1 \right\}$$

**2.1280 ODE No. 1280**

$$4x^2y''(x) - (4x^2 + 12x + 3)y(x) + 8xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0409331 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 e^{-x} (4e^{2x} x^2 \text{Ei}(-2x) + 2x - 1)}{2x^{3/2}} + c_1 e^x \sqrt{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 40

$$\left\{ y(x) = (4 e^x \text{Ei}(1, 2x) C_2 x^2 + (-2x + 1) C_2 e^{-x} + C_1 x^2 e^x) x^{-\frac{3}{2}} \right\}$$

**2.1281 ODE No. 1281**

$$4x^2y''(x) + (4x^2 - 4x - 1)y(x) - 4(2x - 1)xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0222828 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \frac{e^x(c_2x + c_1)}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 15

$$\left\{ y(x) = e^x(x\_C2 + \_C1) \frac{1}{\sqrt{x}} \right\}$$

**2.1282 ODE No. 1282**

$$4x^3y'(x) + 4x^2y''(x) + (x^2 - 4)(x^2 + 6)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0244552 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\frac{x^2}{4}}(c_2x^5 + 5c_1)}{5x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 21

$$\left\{ y(x) = \frac{\_C2 x^5 + \_C1}{x^2} e^{-\frac{x^2}{4}} \right\}$$

**2.1283 ODE No. 1283**

$$4x^2y''(x) + 4x^2\log(x)y'(x) + y(x)(x^2\log^2(x) + 2x - 8) - 4\sqrt{e^xx^{-x}x^2} = 0$$

✓ **Mathematica** : cpu = 0.108074 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{x/2} x^{-\frac{x}{2}-1} + \frac{1}{3} c_2 e^{x/2} x^{2-\frac{x}{2}} - \frac{1}{9} \sqrt{e^xx^{-x}x^2} + \frac{1}{3} \sqrt{e^xx^{-x}x^2} \log(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 48

$$\left\{ y(x) = \frac{x^2}{3} \left( \ln(x) - \frac{1}{3} \right) \sqrt{x^{-x}e^x} + e^{\frac{x}{2}} \left( \_C1 x^{-\frac{x}{2}+2} + \_C2 x^{-\frac{x}{2}-1} \right) \right\}$$



**2.1284 ODE No. 1284**

$$(2x + 1)^2 y''(x) - 2(2x + 1)y'(x) - 12y(x) - 3x - 1 = 0$$

✓ **Mathematica** : cpu = 0.0464829 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{192c_1(2x + 1)^4 + 192c_2 - 72x^2 - 56x - 7}{192(2x + 1)} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 41

$$\left\{ y(x) = \frac{-C1}{2x + 1} + (2x + 1)^3 - C2 + \frac{-72x^2 - 56x - 7}{384x + 192} \right\}$$

**2.1285 ODE No. 1285**

$$((4a + 2)x - a)y'(x) + (a - 1)ay(x) + x(4x - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.328189 (sec), leaf count = 119

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[4]{4x - 1} x^{\frac{1}{2} - \frac{a}{2}} e^{-\sqrt{-(a-1)^2 \tan^{-1}(\sqrt{4x-1})}} \left( 4\sqrt{-(a-1)^2} c_1 e^{2\sqrt{-(a-1)^2 \tan^{-1}(\sqrt{4x-1})}} - c_2 \right)}{2\sqrt{-(a-1)^2} \sqrt[4]{1 - 4x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 52

$$\left\{ y(x) = -C1 {}_2F_1\left(\frac{a}{2}, \frac{a}{2} - \frac{1}{2}; a; 4x\right) + -C2 x^{1-a} {}_2F_1\left(1 - \frac{a}{2}, -\frac{a}{2} + \frac{1}{2}; -a + 2; 4x\right) \right\}$$

**2.1286 ODE No. 1286**

$$(3x - 1)^2 y''(x) + 3(3x - 1)y'(x) - 9y(x) - \log^2(3x - 1) = 0$$

✓ **Mathematica** : cpu = 0.10815 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow \frac{-81c_1x^2 - 81ic_2x^2 + 54c_1x + 54ic_2x - 18c_1 - 12x + (2 - 6x) \log^2(3x - 1) - 2 \log(1 - 3x) + 2 \log(3x - 1)}{54x - 18} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 32

$$\left\{ y(x) = \frac{-C1}{3x - 1} + (3x - 1) - C2 - \frac{(\ln(3x - 1))^2}{9} - \frac{2}{9} \right\}$$

**2.1287 ODE No. 1287**

$$9(x-1)xy''(x) + 3(2x-1)y'(x) - 20y(x) = 0$$

✓ **Mathematica** : cpu = 0.0213971 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow \frac{3c_2 \sqrt[3]{-(x-1)x} \Gamma\left(\frac{4}{3}\right) Q_1^{\frac{2}{3}}(2x-1) + c_1 x^{2/3} (6x-5)}{3\Gamma\left(\frac{4}{3}\right)} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 27

$$\left\{ y(x) = \_C1 (6x-5) x^{\frac{2}{3}} + \_C2 (6x-1) (x-1)^{\frac{2}{3}} \right\}$$

**2.1288 ODE No. 1288**

$$16x^2y''(x) + (4x+3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0391195 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow e^{-i\sqrt{x}} \sqrt[4]{x} \left( c_1 e^{2i\sqrt{x}} + ic_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 21

$$\left\{ y(x) = \sqrt[4]{x} (\cos(\sqrt{x}) \_C2 + \sin(\sqrt{x}) \_C1) \right\}$$

**2.1289 ODE No. 1289**

$$16x^2y''(x) + 32xy'(x) - (4x+5)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0845864 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\sqrt{x}} \left( c_2 (\sqrt{x}+1) - c_1 e^{2\sqrt{x}} (\sqrt{x}-1) \right)}{x^{5/4}} \right\} \right\}$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 33

$$\left\{ y(x) = 1 \left( \_C2 (\sqrt{x}+1) e^{-\sqrt{x}} + \_C1 e^{\sqrt{x}} (\sqrt{x}-1) \right) x^{-\frac{5}{4}} \right\}$$

**2.1290 ODE No. 1290**

$$(27x^2 + 4)y''(x) + 27xy'(x) - 3y(x) = 0$$

✓ **Mathematica** : cpu = 0.17446 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh \left( \frac{\sqrt{-27x^2 - 4} \tan^{-1} \left( \frac{3x}{\sqrt{-9x^2 - \frac{4}{3}}} \right)}{3\sqrt{27x^2 + 4}} \right) + ic_2 \sinh \left( \frac{\sqrt{-27x^2 - 4} \tan^{-1} \left( \frac{3x}{\sqrt{-9x^2 - \frac{4}{3}}} \right)}{3\sqrt{27x^2 + 4}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 47

$$\left\{ y(x) = -C1 \sin \left( \frac{1}{3} \arctan \left( 3 \frac{\sqrt{3}x}{\sqrt{-27x^2 - 4}} \right) \right) + -C2 \cos \left( \frac{1}{3} \arctan \left( 3 \frac{\sqrt{3}x}{\sqrt{-27x^2 - 4}} \right) \right) \right\}$$

**2.1291 ODE No. 1291**

$$48(x - 1)xy''(x) + (152x - 40)y'(x) + 53y(x) = 0$$

✓ **Mathematica** : cpu = 0.0761593 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow \sqrt[6]{-1}c_2 \sqrt[6]{x} {}_2F_1 \left( \frac{5}{4} - \frac{\sqrt{\frac{5}{2}}}{6}, \frac{1}{12} (15 + \sqrt{10}); \frac{7}{6}; x \right) + c_1 {}_2F_1 \left( \frac{1}{12} (13 - \sqrt{10}), \frac{1}{12} (13 + \sqrt{10}); \frac{5}{6}; x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 62

$$\left\{ y(x) = -C1 {}_2F_1 \left( \frac{13}{12} - \frac{\sqrt{5}\sqrt{2}}{12}, \frac{13}{12} + \frac{\sqrt{5}\sqrt{2}}{12}; \frac{5}{6}; x \right) + -C2 \sqrt[6]{x} {}_2F_1 \left( \frac{5}{4} - \frac{\sqrt{5}\sqrt{2}}{12}, \frac{5}{4} + \frac{\sqrt{5}\sqrt{2}}{12}; \frac{7}{6}; x \right) \right\}$$

**2.1292 ODE No. 1292**

$$50(x - 1)xy''(x) + 25(2x - 1)y'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0412586 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh \left( \frac{2}{5} \log (\sqrt{x - 1} + \sqrt{x}) \right) + ic_2 \sinh \left( \frac{2}{5} \log (\sqrt{x - 1} + \sqrt{x}) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 31

$$\left\{ y(x) = 1 \left( -C1 (\sqrt{x} + \sqrt{x - 1})^{\frac{4}{5}} + -C2 \right) (\sqrt{x} + \sqrt{x - 1})^{-\frac{2}{5}} \right\}$$

**2.1293 ODE No. 1293**

$$144(x-1)xy''(x) + (120x-48)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.339412 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow (-1)^{2/3} c_2 x^{2/3} {}_2F_1\left(\frac{7}{12}, \frac{7}{12}; \frac{5}{3}; x\right) + c_1 {}_2F_1\left(-\frac{1}{12}, -\frac{1}{12}; \frac{1}{3}; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt[3]{x} \left( LegendreP\left(-\frac{1}{2}, \frac{2}{3}, \sqrt{1-x}\right) - C1 + LegendreQ\left(-\frac{1}{2}, \frac{2}{3}, \sqrt{1-x}\right) - C2 \right) \right\}$$

**2.1294 ODE No. 1294**

$$144(x-1)xy''(x) + (168x-96)y'(x) + y(x) = 0$$

✓ **Mathematica** : cpu = 0.0696003 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(\frac{1}{12}, \frac{1}{12}; \frac{2}{3}; x\right) + \sqrt[3]{-1} c_2 \sqrt[3]{x} {}_2F_1\left(\frac{5}{12}, \frac{5}{12}; \frac{4}{3}; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt[6]{x} \left( LegendreP\left(-\frac{1}{2}, \frac{1}{3}, \sqrt{1-x}\right) - C1 + LegendreQ\left(-\frac{1}{2}, \frac{1}{3}, \sqrt{1-x}\right) - C2 \right) \right\}$$

**2.1295 ODE No. 1295**

$$ax^2y''(x) + bxy'(x) + y(x)(cx^2 + dx + f) = 0$$

✓ **Mathematica** : cpu = 0.278479 (sec), leaf count = 229

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{i\sqrt{c}x}{\sqrt{a}}} x^{\frac{\sqrt{a^2-2a(b+2f)+b^2+a-b}}{2a}} \left( c_1 U\left(\frac{a + \frac{id\sqrt{a}}{\sqrt{c}} + \sqrt{a^2-2(b+2f)a+b^2}}{2a}, \frac{a + \sqrt{a^2-2(b+2f)a+b^2}}{a}, 2\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.302 (sec), leaf count = 106

$$\left\{ y(x) = x^{-\frac{b}{2a}} \left( M_{-\frac{i}{2}d\frac{1}{\sqrt{c}}\frac{1}{\sqrt{a}}, \frac{1}{2a}\sqrt{a^2+(-2b-4f)a+b^2}}\left(2ix\sqrt{c}\frac{1}{\sqrt{a}}\right) - C1 + W_{-\frac{i}{2}d\frac{1}{\sqrt{c}}\frac{1}{\sqrt{a}}, \frac{1}{2a}\sqrt{a^2+(-2b-4f)a+b^2}}\left(2ix\sqrt{c}\frac{1}{\sqrt{a}}\right) \right) \right\}$$

**2.1296 ODE No. 1296**

$$y(x) (a_0x^2 + b_0x + c_0) + (a_1x^2 + b_1x) y'(x) + a_2x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.550259 (sec), leaf count = 272

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{x(\sqrt{a_1^2 - 4a_0a_2} + a_1)}{2a_2}} x^{\frac{\sqrt{a_2^2 - 2a_2(b_1 + 2c_0) + b_1^2} + a_2 - b_1}{2a_2}} \left( c_1 U \left( \frac{-\frac{2b_0a_2}{\sqrt{a_1^2 - 4a_0a_2}} + a_2 + \frac{a_1b_1}{\sqrt{a_1^2 - 4a_0a_2}} + \sqrt{a_2^2 - 2(b_1 + 2c_0) + b_1^2}}{2a_2} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.347 (sec), leaf count = 150

$$\left\{ y(x) = x^{-\frac{b_1}{2a_2}} e^{-\frac{a_1x}{2a_2}} \left( M_{-\frac{a_1b_1 - 2a_2b_0}{2a_2}, \frac{1}{\sqrt{-4a_0a_2 + a_1^2}}, \frac{1}{2a_2} \sqrt{a_2^2 + (-2b_1 - 4c_0)a_2 + b_1^2}} \left( \frac{x}{a_2} \sqrt{-4a_0a_2 + a_1^2} \right) - C_1 + W_{-\frac{a_1b_1 - 2a_2b_0}{2a_2}, \frac{1}{\sqrt{-4a_0a_2 + a_1^2}}, \frac{1}{2a_2} \sqrt{a_2^2 + (-2b_1 - 4c_0)a_2 + b_1^2}} \left( \frac{x}{a_2} \sqrt{-4a_0a_2 + a_1^2} \right) \right) \right\}$$

**2.1297 ODE No. 1297**

$$(ax^2 + 1) y''(x) + axy'(x) + by(x) = 0$$

✓ **Mathematica** : cpu = 0.0340672 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \frac{\sqrt{b} \sinh^{-1}(\sqrt{ax})}{\sqrt{a}} \right) + c_1 \cos \left( \frac{\sqrt{b} \sinh^{-1}(\sqrt{ax})}{\sqrt{a}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 63

$$\left\{ y(x) = 1 \left( -C_1 \left( (\sqrt{ax} + \sqrt{ax^2 + 1})^{i\sqrt{b}\frac{1}{\sqrt{a}}} \right)^2 + -C_2 \right) \left( (\sqrt{ax} + \sqrt{ax^2 + 1})^{i\sqrt{b}\frac{1}{\sqrt{a}}} \right)^{-1} \right\}$$

**2.1298 ODE No. 1298**

$$(ax^2 + 1) y''(x) + bxy'(x) + cy(x) = 0$$

✓ **Mathematica** : cpu = 0.0784597 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow (ax^2 + 1)^{\frac{1}{2} - \frac{b}{4a}} \left( c_1 P_{\frac{b}{2a} - 1}^{\frac{b}{2a} - 1} \left( \frac{i\sqrt{ax}}{\sqrt{a^2 - 2(b+2c)a + b^2 - a}} \right) + c_2 Q_{\frac{b}{2a} - 1}^{\frac{b}{2a} - 1} \left( \frac{i\sqrt{ax}}{\sqrt{a^2 - 2(b+2c)a + b^2 - a}} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 124

$$\left\{ y(x) = (ax^2 + 1)^{\frac{2a-b}{4a}} \left( LegendreP \left( \frac{1}{2a} \left( \sqrt{a^2 + (-2b - 4c)a + b^2} - a \right), \frac{2a-b}{2a}, \sqrt{-ax} \right) - C_1 + LegendreQ \left( \frac{1}{2a} \left( \sqrt{a^2 + (-2b - 4c)a + b^2} - a \right), \frac{2a-b}{2a}, \sqrt{-ax} \right) \right) \right\}$$

**2.1299 ODE No. 1299**

$$(a^2x^2 - 1)y''(x) + 2a^2xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0152076 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{c_1 \tanh^{-1}(ax)}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 27

$$\left\{ y(x) = \_C1 - \frac{(\ln(ax + 1) - \ln(ax - 1))\_C2}{2a} \right\}$$

**2.1300 ODE No. 1300**

$$(a^2x^2 - 1)y''(x) + 2a^2xy'(x) - 2a^2y(x) = 0$$

✓ **Mathematica** : cpu = 0.017027 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow ac_1x - \frac{1}{2}c_2(ax \log(1 - ax) - ax \log(ax + 1) + 2) \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 31

$$\left\{ y(x) = -\frac{\_C2 a \ln(ax + 1) x}{2} + \frac{\_C2 a \ln(ax - 1) x}{2} + \_C1 x + \_C2 \right\}$$

**2.1301 ODE No. 1301**

$$(ax^2 + bx)y''(x) - 2ay(x) + 2by'(x) = 0$$

✓ **Mathematica** : cpu = 0.0329597 (sec), leaf count = 30

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2(ax+b)^3}{3x} + 3c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.023 (sec), leaf count = 19

$$\left\{ y(x) = \frac{\_C1 + \_C2 (ax + b)^3}{x} \right\}$$

**2.1302 ODE No. 1302**

$$A_0 y(x)(ax + b) + A_1(ax + b)y'(x) + A_2(ax + b)^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0835107 (sec), leaf count = 165

$$\left\{ \left\{ y(x) \rightarrow (-1)^{-\frac{A_1}{aA_2}} \left( \frac{b}{a} + x \right)^{\frac{A_1}{2aA_2}} (A_2(ax + b))^{-\frac{A_1}{2aA_2}} \left( -\frac{A_0(ax + b)}{a^2 A_2} \right)^{\frac{1}{2} - \frac{A_1}{2aA_2}} \left( c_1 (-1)^{\frac{A_1}{aA_2}} I_{\frac{A_1}{aA_2} - 1} \left( 2\sqrt{-\frac{A_0(b + ax)}{a^2 A_2}} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.061 (sec), leaf count = 98

$$\left\{ y(x) = (ax + b)^{-\frac{-aA_2 + A_1}{2aA_2}} \left( J_{\frac{aA_2 - A_1}{aA_2}} \left( 2\sqrt{A_0} \sqrt{\frac{ax + b}{a^2 A_2}} \right) - C_1 + Y_{\frac{aA_2 - A_1}{aA_2}} \left( 2\sqrt{A_0} \sqrt{\frac{ax + b}{a^2 A_2}} \right) - C_2 \right) \right\}$$

**2.1303 ODE No. 1303**

$$y''(x)(ax^2 + bx + c) + (dx + f)y'(x) + gy(x) = 0$$

✗ **Mathematica** : cpu = 14.1984 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{gy(x) + (xd + f)y'(x) + (ax^2 + bx + c)y''(x) = 0, y(0) = c_1, y'(0) = c_2\}) \right\} \right\} (x)$$

✓ **Maple** : cpu = 0.225 (sec), leaf count = 501

$$\left\{ y(x) = -C_1 {}_2F_1\left(-\frac{1}{2a}\left(a - d + \sqrt{a^2 + (-2d - 4g)a + d^2}\right), \frac{1}{2a}\left(-a + d + \sqrt{a^2 + (-2d - 4g)a + d^2}\right); \frac{1}{2a^2}\left(\dots\right)\right) \right\}$$

**2.1304 ODE No. 1304**

$$x^3 y''(x) + xy'(x) - (2x + 3)y(x) = 0$$

✓ **Mathematica** : cpu = 0.034511 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 e^{\frac{1}{x}} \text{Ei}\left(-\frac{1}{x}\right) + c_2 x(2x^2 - x + 1) + 6c_1 e^{\frac{1}{x}}}{6x} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 38

$$\left\{ y(x) = \frac{1}{x} \left( \text{Ei}(1, x^{-1}) e^{x^{-1}} - C_2 + -C_1 e^{x^{-1}} - 2(x^2 - x/2 + 1/2)x - C_2 \right) \right\}$$

**2.1305 ODE No. 1305**

$$x^3 y''(x) + 2xy'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.0741633 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left( -\frac{2}{x} \middle| \begin{matrix} \frac{1}{2} \\ -1, 0 \end{matrix} \right) + c_1 e^{\frac{1}{x}} \left( I_0 \left( \frac{1}{x} \right) - I_1 \left( \frac{1}{x} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 44

$$\left\{ y(x) = e^{x^{-1}} (-\_C2 K_1(-x^{-1}) + \_C2 K_0(-x^{-1}) + \_C1 (I_0(x^{-1}) - I_1(x^{-1}))) \right\}$$

**2.1306 ODE No. 1306**

$$y(x) (ax^2 + a + bx) + x^3 y''(x) + x^2 y'(x) = 0$$

✗ **Mathematica** : cpu = 0.975969 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{y''(x)x^3 + y'(x)x^2 + (ax^2 + bx + a)y(x) = 0, y(1) = c_1, y'(1) = c_2\}) (x) \right\} \right\}$$

✓ **Maple** : cpu = 0.158 (sec), leaf count = 69

$$\left\{ y(x) = \text{HeunD} \left( 0, 8a + 4b, 0, 8a - 4b, \frac{1+x}{x-1} \right) \left( \int \frac{1}{x} \left( \text{HeunD} \left( 0, 8a + 4b, 0, 8a - 4b, \frac{1+x}{x-1} \right) \right)^{-2} dx \_C2 + \_C1 \right) \right\}$$

**2.1307 ODE No. 1307**

$$x^3 y''(x) + (x+1)xy'(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.087026 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{-c_2 e^{\frac{1}{x}} (x+1) \text{Ei} \left( -\frac{1}{x} \right) + c_1 e^{\frac{1}{x}} (x+1) - c_2 x}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 36

$$\left\{ y(x) = \frac{\_C2 e^{x^{-1}} (1+x) \text{Ei}(1, x^{-1}) + \_C1 (1+x) e^{x^{-1}} - \_C2 x}{x} \right\}$$



**2.1308 ODE No. 1308**

$$x^3 y''(x) - x^2 y'(x) + xy(x) - \log^3(x) = 0$$

✓ **Mathematica** : cpu = 0.0219897 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow c_1 x + c_2 x \log(x) + \frac{2 \log^3(x) + 6 \log^2(x) + 9 \log(x) + 6}{8x} \right\} \right\}$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 40

$$\left\{ y(x) = \frac{2 (\ln(x))^3 + 6 (\ln(x))^2 + (8\_C1 x^2 + 9) \ln(x) + 8\_C2 x^2 + 6}{8x} \right\}$$

**2.1309 ODE No. 1309**

$$x^3 y''(x) - (x^2 - 1) y'(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.082328 (sec), leaf count = 83

$$\left\{ \left\{ y(x) \rightarrow \frac{2c_2 x G_{1,2}^{2,0} \left( -\frac{1}{2x^2} \middle| \begin{matrix} 1 \\ -\frac{1}{2}, -\frac{1}{2} \end{matrix} \right) + \sqrt{2} c_1 e^{\frac{1}{4x^2}} \left( (2x^2 - 1) I_0\left(\frac{1}{4x^2}\right) + I_1\left(\frac{1}{4x^2}\right) \right)}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.094 (sec), leaf count = 85

$$\left\{ y(x) = \frac{C1}{x} e^{\frac{1}{4x^2}} \left( 2x^2 I_0(1/4x^{-2}) + I_1\left(\frac{1}{4x^2}\right) - I_0\left(\frac{1}{4x^2}\right) \right) + \frac{C2}{x} e^{\frac{1}{4x^2}} \left( 2K_0(-1/4x^{-2})x^2 + K_1\left(-\frac{1}{4x^2}\right) \right) \right\}$$

**2.1310 ODE No. 1310**

$$x^3 y''(x) + 3x^2 y'(x) + xy(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0133402 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{2c_2 \log(x) + 2c_1 + \log^2(x)}{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 20

$$\left\{ y(x) = \frac{1}{x} \left( \frac{(\ln(x))^2}{2} + \_C1 \ln(x) + \_C2 \right) \right\}$$

**2.1311 ODE No. 1311**

$$-v(v+1)xy(x) + x(x^2+1)y''(x) + (2x^2+1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.132321 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{2,2}^{2,0} \left( -x^2 \middle| \begin{matrix} \frac{1-v}{2}, \frac{v+2}{2} \\ 0, 0 \end{matrix} \right) + c_1 {}_2F_1 \left( -\frac{v}{2}, \frac{v+1}{2}; 1; -x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.15 (sec), leaf count = 52

$$\left\{ y(x) = -C1 {}_2F_1 \left( -\frac{v}{2}, \frac{1}{2} + \frac{v}{2}; \frac{1}{2}; x^2 + 1 \right) + -C2 \sqrt{x^2 + 1} {}_2F_1 \left( 1 + \frac{v}{2}, \frac{1}{2} - \frac{v}{2}; \frac{3}{2}; x^2 + 1 \right) \right\}$$

**2.1312 ODE No. 1312**

$$x(x^2+1)y''(x) + 2(x^2-1)y'(x) - 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0249963 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x^3 + 3c_1}{3x^2 + 3} \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 19

$$\left\{ y(x) = \frac{-C2 x^3 + -C1}{x^2 + 1} \right\}$$

**2.1313 ODE No. 1313**

$$x(-v-n)(n+v+1)y(x) + (2(n+1)x^2 + 2n+1)y'(x) + x(x^2+1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.213037 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left( \frac{n-v}{2}, \frac{1}{2}(n+v+1); n+1; -x^2 \right) + c_2 x^{-2n} {}_2F_1 \left( \frac{1}{2}(-n-v), \frac{1}{2}(-n+v+1); 1-n; -x^2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 35

$$\left\{ y(x) = x^{-n} \left( LegendreQ(v, n, \sqrt{x^2 + 1}) - C2 + LegendreP(v, n, \sqrt{x^2 + 1}) - C1 \right) \right\}$$

**2.1314 ODE No. 1314**

$$x(n-v-1)(n+v)y(x) - (2(n-1)x^2 + 2n-1)y'(x) + x(x^2+1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.190212 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1\left(\frac{1}{2}(-n-v), \frac{1}{2}(-n+v+1); 1-n; -x^2\right) + c_2 x^{2n} {}_2F_1\left(\frac{n-v}{2}, \frac{1}{2}(n+v+1); n+1; -x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 33

$$\left\{ y(x) = x^n \left( LegendreQ(v, n, \sqrt{x^2+1}) \_C2 + LegendreP(v, n, \sqrt{x^2+1}) \_C1 \right) \right\}$$

**2.1315 ODE No. 1315**

$$ax^3y(x) + (x^2-1)xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0285702 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin\left(\sqrt{a}\sqrt{x^2-1}\right) + c_1 \cos\left(\sqrt{a}\sqrt{x^2-1}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 45

$$\left\{ y(x) = \_C1 \sin\left((1+x)(x-1)\sqrt{a}\frac{1}{\sqrt{x^2-1}}\right) + \_C2 \cos\left((1+x)(x-1)\sqrt{a}\frac{1}{\sqrt{x^2-1}}\right) \right\}$$

**2.1316 ODE No. 1316**

$$x(x^2-1)y''(x) + (x^2-1)y'(x) - xy(x) = 0$$

✓ **Mathematica** : cpu = 0.0931275 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{2,2}^{2,0}\left(x^2 \middle| \begin{matrix} \frac{1}{2}, \frac{3}{2} \\ 0, 0 \end{matrix} \right) + \frac{2c_1 E(x^2)}{\pi} \right\} \right\}$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 18

$$\left\{ y(x) = \_C1 EllipticE(x) + \_C2 (EllipticCE(x) - EllipticCK(x)) \right\}$$

**2.1317 ODE No. 1317**

$$x(x^2 - 1)y''(x) + (3x^2 - 1)y'(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.124127 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{2,2}^{2,0} \left( x^2 \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix} \right) + \frac{2c_1 K(x^2)}{\pi} \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 13

$$\{y(x) = \_C1 \text{EllipticK}(x) + \_C2 \text{EllipticCK}(x)\}$$

**2.1318 ODE No. 1318**

$$(ax^2 + b)y'(x) + cxy(x) + x(x^2 - 1)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.319698 (sec), leaf count = 146

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_2F_1 \left( \frac{1}{4} \left( a - \sqrt{a^2 - 2a - 4c + 1} - 1 \right), \frac{1}{4} \left( a + \sqrt{a^2 - 2a - 4c + 1} - 1 \right); \frac{1-b}{2}; x^2 \right) + i^{b+1} c_2 x^{b+1} {}_2F_1 \right. \right.$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 122

$$\left. \left\{ y(x) = \_C1 {}_2F_1 \left( -\frac{1}{4} + \frac{a}{4} + \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1}, -\frac{1}{4} + \frac{a}{4} - \frac{1}{4} \sqrt{a^2 - 2a - 4c + 1}; -\frac{b}{2} + \frac{1}{2}; x^2 \right) + \_C2 x^{b+1} \right. \right.$$

**2.1319 ODE No. 1319**

$$x(x^2 + 2)y''(x) - y'(x) - 6xy(x) = 0$$

✓ **Mathematica** : cpu = 0.099123 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow \frac{-\sqrt[4]{2}c_2(x^2 + 2)x^2 {}_2F_1 \left( \frac{1}{4}, \frac{3}{4}, \frac{5}{4}; -\frac{x^2}{2} \right) + 3c_1 x^{7/2} + 6c_1 x^{3/2} - c_2 \sqrt[4]{x^2 + 2}x^2 - c_2 \sqrt[4]{x^2 + 2}}{3\sqrt[4]{x^2 + 2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 31

$$\left\{ y(x) = (x^2 + 2)^{\frac{3}{4}} \left( x^{\frac{3}{2}} \_C1 + {}_2F_1 \left( -\frac{3}{4}, \frac{7}{4}; \frac{1}{4}; -\frac{x^2}{2} \right) \_C2 \right) \right\}$$

**2.1320 ODE No. 1320**

$$x(x^2 - 2)y''(x) + (x^2 + 4x + 2)y(x) - (x^3 + 3x^2 - 2x - 2)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0686406 (sec), leaf count = 21

$$\{\{y(x) \rightarrow c_1 e^x x^2 + c_2(x - 1)\}\}$$

✓ **Maple** : cpu = 0.047 (sec), leaf count = 17

$$\{y(x) = \_C1(x - 1) + \_C2 e^x x^2\}$$

**2.1321 ODE No. 1321**

$$(x + 1)x^2 y''(x) - (2x + 1)xy'(x) + (2x + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0281779 (sec), leaf count = 17

$$\{\{y(x) \rightarrow x(c_2(x + \log(x)) + c_1)\}\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 15

$$\{y(x) = x(\_C2 \ln(x) + \_C2 x + \_C1)\}$$

**2.1322 ODE No. 1322**

$$(x + 1)x^2 y''(x) + 2(3x + 2)xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0347721 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1 \left( -\frac{1}{3x^3} + \frac{1}{x^2} - \frac{3}{x} - \frac{1}{x+1} - 4 \log(x) + 4 \log(x+1) \right) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 44

$$\left\{ y(x) = \_C1 + \left( -4 \ln(x) + 4 \ln(1+x) - \frac{12x^3 + 6x^2 - 2x + 1}{3x^3(1+x)} \right) \_C2 \right\}$$

**2.1323 ODE No. 1323**

$$y''(x) = \frac{2(x+1)y(x)}{(x-1)x} - \frac{2(x-2)y'(x)}{(x-1)x}$$

✗ **Mathematica** : cpu = 0.747675 (sec), leaf count = 0 , DifferentialRoot result

$$\{ \{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(-2x-2)y(x) + (2x-4)y'(x) + (x-1)xy''(x) = 0, y(2) = c_1, y'(2) = c_2\}) \}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 17

$$\left\{ y(x) = \frac{-C1 + -C2(x-1)^3}{x^2} \right\}$$

**2.1324 ODE No. 1324**

$$y''(x) = \frac{(5x-4)y'(x)}{(x-1)x} - \frac{(9x-6)y(x)}{(x-1)x^2}$$

✓ **Mathematica** : cpu = 0.0311803 (sec), leaf count = 24

$$\{ \{ y(x) \rightarrow x^2(c_1x - c_2(x \log(x) + 1)) \} \}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 18

$$\{ y(x) = x^2(\ln(x) - C2x + -C1x + -C2) \}$$

**2.1325 ODE No. 1325**

$$y''(x) = -\frac{y(x)(abx - \alpha\beta)}{(x-1)x^2} - \frac{y'(x)(x(a+b+1) + \alpha + \beta - 1)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.267324 (sec), leaf count = 52

$$\{ \{ y(x) \rightarrow (-1)^\alpha c_1 x^\alpha {}_2F_1(a + \alpha, \alpha + b; \alpha - \beta + 1; x) + (-1)^\beta c_2 x^\beta {}_2F_1(a + \beta, b + \beta; -\alpha + \beta + 1; x) \} \}$$

✓ **Maple** : cpu = 0.137 (sec), leaf count = 86

$$\{ y(x) = (x-1)^{1-a-\alpha-b-\beta} \left( {}_2F_1(1-\alpha-b, 1-a-\alpha; 1+\beta-\alpha; x)x^\beta - C2 + {}_2F_1(1-b-\beta, 1-a-\beta; 1-\beta + \right.$$

**2.1326 ODE No. 1326**

$$y''(x) = -\frac{y'(x)}{x+1} - \frac{y(x)}{x(x+1)^2}$$

✓ **Mathematica** : cpu = 0.0265799 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x + c_2 x \log(x) - c_2}{x+1} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 22

$$\left\{ y(x) = \frac{\ln(x) - C2 x + -C1 x - C2}{1+x} \right\}$$

**2.1327 ODE No. 1327**

$$y''(x) = \frac{2y'(x)}{(x-2)x} - \frac{y(x)}{(x-2)x^2}$$

✓ **Mathematica** : cpu = 0.171772 (sec), leaf count = 105

$$\left\{ \left\{ y(x) \rightarrow \left(-\frac{1}{2}\right)^{-\frac{1}{\sqrt{2}}} x^{-\frac{1}{\sqrt{2}}} \left( \left(-\frac{1}{2}\right)^{\sqrt{2}} c_2 x^{\sqrt{2}} {}_2F_1\left(\frac{1}{\sqrt{2}}, -1 + \frac{1}{\sqrt{2}}; 1 + \sqrt{2}; \frac{x}{2}\right) + c_1 {}_2F_1\left(-\frac{1}{\sqrt{2}}, -1 - \frac{1}{\sqrt{2}}; 1 - \sqrt{2}; \frac{x}{2}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.127 (sec), leaf count = 81

$$\left\{ y(x) = (x-2)^2 \left( -C1 {}_2F_1\left(2 - \frac{\sqrt{2}}{2}, 1 - \frac{\sqrt{2}}{2}; 1 - \sqrt{2}; \frac{x}{2}\right) x^{-\frac{\sqrt{2}}{2}} + -C2 {}_2F_1\left(2 + \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}; 1 + \sqrt{2}; \frac{x}{2}\right) x^{\frac{\sqrt{2}}{2}} \right) \right\}$$

**2.1328 ODE No. 1328**

$$y''(x) = \frac{2y(x)}{(x-1)^2 x}$$

✓ **Mathematica** : cpu = 0.0249531 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{-c_2 x^2 - c_1 x + 2c_2 x \log(x) + c_2}{x-1} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 27

$$\left\{ y(x) = \frac{2 \ln(x) - C2 x - C2 x^2 + -C1 x + -C2}{x-1} \right\}$$

**2.1329 ODE No. 1329**

$$y''(x) = -\frac{y'(x)(-x(a(\delta + \text{gamma}1) + \alpha + \beta - \delta + 1) + a\text{gamma}1 + x^2(\alpha + \beta + 1))}{(x-1)x(x-a)} - \frac{y(x)(\alpha\beta x - q)}{(x-1)x(x-a)}$$

✗ **Mathematica** : cpu = 6.54095 (sec), leaf count = 0 , DifferentialRoot result

$$\{ \{y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(x\alpha\beta - q)y(x) + (\alpha x^2 + \beta x^2 + x^2 - \alpha x - \beta x - a\delta x + \delta x - a\text{gamma}1x - x$$

✓ **Maple** : cpu = 0.377 (sec), leaf count = 64

$$\{y(x) = \_C1 \text{HeunG}(a, q, \alpha, \beta, \gamma1, \delta, x) + \_C2 x^{1-\gamma1} \text{HeunG}(a, q - (-1 + \gamma1)(\delta(a-1) + \alpha + \beta - \gamma1 + 1), \beta +$$

**2.1330 ODE No. 1330**

$$y''(x) = -\frac{y'(x)(Ax^2 + Bx + C)}{(x-a)(x-b)(x-c)} - \frac{(DDx + e)y(x)}{(x-a)(x-b)(x-c)}$$

✗ **Mathematica** : cpu = 157.104 (sec), leaf count = 0 , DifferentialRoot result

$$\{ \{y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(xDD + e)y(x) + (Ax^2 + Bx + C)y'(x) - (a-x)(b-x)(c-x)y''(x) = 0, y$$

✓ **Maple** : cpu = 1.211 (sec), leaf count = 1147

$$\{y(x) = \_C1 \text{HeunG}\left(\frac{a-c}{a-b}, \frac{DDa + E}{a-b}, \frac{A}{2} - \frac{1}{2} + \frac{1}{2}\sqrt{A^2 - 2A - 4DD + 1}, 1\left((A(b-c)a - Abc - Bc - C)\sqrt{A}\right.\right.$$

**2.1331 ODE No. 1331**

$$y''(x) = \frac{(x-4)y'(x)}{2(x-2)x} - \frac{(x-3)y(x)}{2(x-2)x^2}$$

✓ **Mathematica** : cpu = 0.0450225 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt[4]{x-2}\sqrt{x}(2c_2\sqrt{x-2} + c_1)}{\sqrt[4]{2-x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 19

$$\{y(x) = \_C1 \sqrt{x} + \_C2 \sqrt{x(x-2)}\}$$



**2.1332 ODE No. 1332**

$$y''(x) = \frac{y'(x)}{x+1} - \frac{(3x+1)y(x)}{4x^2(x+1)}$$

✓ **Mathematica** : cpu = 0.0263018 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \sqrt{x}(c_2(x + \log(x)) + c_1) \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 17

$$\{y(x) = \sqrt{x}(_{C2} \ln(x) + _{C2} x + _{C1})\}$$

**2.1333 ODE No. 1333**

$$y''(x) = \frac{v(v+1)y(x)}{4x^2} - \frac{(3x-1)y'(x)}{2(x-1)x}$$

✓ **Mathematica** : cpu = 0.115185 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{-v} x^{-v/2} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x\right) + c_2 i^{v+1} x^{\frac{v+1}{2}} {}_2F_1\left(\frac{1}{2}, v+1; v + \frac{3}{2}; x\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 45

$$\left\{ y(x) = _{C1} x^{-\frac{v}{2}} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x\right) + _{C2} x^{\frac{1}{2} + \frac{v}{2}} {}_2F_1\left(\frac{1}{2}, v+1; \frac{3}{2} + v; x\right) \right\}$$

**2.1334 ODE No. 1334**

$$y''(x) = -\frac{y(x)(x(a^2 - b^2) + c^2)}{4(x-1)x^2} - \frac{((a+1)x-1)y'(x)}{(x-1)x}$$

✓ **Mathematica** : cpu = 0.204612 (sec), leaf count = 89

$$\left\{ \left\{ y(x) \rightarrow i^{-c} x^{-c/2} \left( c_1 {}_2F_1\left(\frac{1}{2}(a-b-c), \frac{1}{2}(a+b-c); 1-c; x\right) + i^{2c} c_2 x^c {}_2F_1\left(\frac{1}{2}(a-b+c), \frac{1}{2}(a+b+c); c+1; x\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.1 (sec), leaf count = 89

$$\left\{ y(x) = (x-1)^{1-a} \left( x^{-\frac{c}{2}} {}_2F_1\left(-\frac{a}{2} - \frac{b}{2} - \frac{c}{2} + 1, -\frac{a}{2} + \frac{b}{2} - \frac{c}{2} + 1; 1-c; x\right) _{C2} + x^{\frac{c}{2}} {}_2F_1\left(-\frac{a}{2} - \frac{b}{2} + \frac{c}{2} + 1, -\frac{a}{2} - \frac{b}{2} + \frac{c}{2} + 1; c+1; x\right) _{C1} \right) \right\}$$

**2.1335 ODE No. 1335**

$$y''(x) = -\frac{y(x)(ax+b)}{4(x-1)^2x} - \frac{(3x-1)y'(x)}{2(x-1)x}$$

✓ **Mathematica** : cpu = 0.315371 (sec), leaf count = 510

$$\left\{ \left\{ y(x) \rightarrow \frac{(x-1)^{\frac{2a\sqrt{-4\sqrt{(4a-1)(a+b)}-8a-4b+1+2b}(\sqrt{-4\sqrt{(4a-1)(a+b)}-8a-4b+1+2})-\sqrt{(4a-1)(a+b)}\sqrt{-4\sqrt{(4a-1)(a+b)}-8a-4b+1+1}}{8b+2}}}{C_1} \right\} \right.$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 57

$$\left\{ y(x) = \_C1 \text{LegendreP}\left(\frac{1}{2}\sqrt{1-4a} - \frac{1}{2}, \sqrt{-a-b}, \sqrt{x}\right) + \_C2 \text{LegendreQ}\left(\frac{1}{2}\sqrt{1-4a} - \frac{1}{2}, \sqrt{-a-b}, \sqrt{x}\right) \right\}$$

**2.1336 ODE No. 1336**

$$y''(x) = -\frac{(1-3x)y(x)}{(x-1)(2x-1)^2}$$

✓ **Mathematica** : cpu = 0.0551158 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{1-2x}(c_1x + 2c_2(x-1)\log(x-1) - 2c_2(x-1)\log(2x-1) - c_1 + c_2) \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 44

$$\{y(x) = (2\_C2(x-1)\ln(2x-1) - 2\_C2(x-1)\ln(x-1) + \_C1x - \_C1 - \_C2)\sqrt{2x-1}\}$$

**2.1337 ODE No. 1337**

$$y''(x) = -\frac{(a+2b+3x)y'(x)}{2(a+x)(b+x)} - \frac{(a-b)y(x)}{4(a+x)^2(b+x)}$$

✓ **Mathematica** : cpu = 0.0851042 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1\sqrt{a-b} + c_2\sqrt{b+x}}{\sqrt{a-b}\sqrt{\frac{a+x}{a-b}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 27

$$\left\{ y(x) = 1 \left( \sqrt{x+b} \_C1 + \_C2 \right) \frac{1}{\sqrt{\frac{x+a}{a-b}}} \right\}$$

**2.1338 ODE No. 1338**

$$y''(x) = \frac{y(x)}{3(x-2)x^2} + \frac{(6x-1)y'(x)}{3(x-2)x}$$

✓ **Mathematica** : cpu = 0.0708338 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow \frac{3}{935} c_2 x (18x^2 - 102x + 187) + c_1 \sqrt[6]{x} (2-x)^{17/6} \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 27

$$\left\{ y(x) = \_C2 \sqrt[6]{x} (x-2)^{\frac{17}{6}} + 18 \_C1 x \left( x^2 - \frac{17x}{3} + \frac{187}{18} \right) \right\}$$

**2.1339 ODE No. 1339**

$$y''(x) = -\frac{y'(x)(a(b+2)x^2 + x(c-d+1))}{x^2(ax+1)} - \frac{y(x)(abx-cd)}{x^2(ax+1)}$$

✓ **Mathematica** : cpu = 0.272089 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow c_1 a^{-c} x^{-c} {}_2F_1(1-c, b-c; -c-d+1; -ax) + c_2 a^d x^d {}_2F_1(d+1, b+d; c+d+1; -ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 76

$$\left\{ y(x) = (ax+1)^{-b+c-d} \left( x^{-c} {}_2F_1(-d, 1-b-d; 1-d-c; -ax) \_C2 + x^d {}_2F_1(c, 1-b+c; 1+d+c; -ax) \_C1 \right) \right\}$$

**2.1340 ODE No. 1340**

$$y''(x) = \frac{2(ax + 2b)y'(x)}{x(ax + b)} - \frac{y(x)(2ax + 6b)}{x^2(ax + b)}$$

✓ **Mathematica** : cpu = 0.0378897 (sec), leaf count = 23

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2(c_2x + c_1)}{ax + b} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 20

$$\left\{ y(x) = \frac{x^2(-C2x + -C1)}{ax + b} \right\}$$

**2.1341 ODE No. 1341**

$$y''(x) = -\frac{y(x)(avx - b)}{x^2(ax + b)} - \frac{(2ax + b)y'(x)}{x(ax + b)} + Ax$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.182 (sec), leaf count = 201

$$\left\{ y(x) = \frac{1}{a^2(v+6)(v+2)(v+12)} \left( x^{-\frac{1}{2} + \frac{1}{2}\sqrt{1-4v}} a^2 - C2(v+6)(v+2)(v+12) {}_2F_1\left(-\frac{1}{2} - \frac{1}{2}\sqrt{1-4v}, \frac{3}{2} - \frac{1}{2}\sqrt{1-4v}\right) \right) \right\}$$

**2.1342 ODE No. 1342**

$$y''(x) = -\frac{ay(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.0436904 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow c_1 x e^{\frac{\sqrt{-a}}{x}} + \frac{c_2 x e^{-\frac{\sqrt{-a}}{x}}}{2\sqrt{-a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 31

$$\left\{ y(x) = x \left( \cosh\left(\frac{1}{x}\sqrt{-a}\right) - C2 + \sinh\left(\frac{1}{x}\sqrt{-a}\right) - C1 \right) \right\}$$

**2.1343 ODE No. 1343**

$$y''(x) = -\frac{y(x) ((1-a)ax^2 - b(b+x))}{x^4}$$

✗ **Mathematica** : cpu = 0.697167 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{y''(x)x^4 + (-a^2x^2 + ax^2 - bx - b^2)y(x) = 0, y(1) = c_1, y'(1) = c_2\}) (x) \right\} \right\}$$

✓ **Maple** : cpu = 0.108 (sec), leaf count = 58

$$\left\{ y(x) = I_{a+1}\left(\frac{b}{x}\right) - C1 b - K_{a+1}\left(\frac{b}{x}\right) - C2 b + 2 \left( -C1 I_a\left(\frac{b}{x}\right) + -C2 K_a\left(\frac{b}{x}\right) \right) (ax + b/2) \right\}$$

**2.1344 ODE No. 1344**

$$y''(x) = -\frac{(e^{2/x} - v^2) y(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.588664 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow \frac{(-1)^{-v} 2^{\frac{3v}{2} + \frac{1}{2}} (-e^{2/x})^{-v/2} (e^{2/x})^{v/2} \left( c_1 (-1)^v I_v(\sqrt{-e^{2/x}}) + c_2 K_v(\sqrt{-e^{2/x}}) \right)}{\log(e^{2/x})} \right\} \right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 23

$$\left\{ y(x) = x \left( Y_v(e^{x^{-1}}) - C2 + J_v(e^{x^{-1}}) - C1 \right) \right\}$$

**2.1345 ODE No. 1345**

$$y''(x) = \frac{2y(x)}{x^4} - \frac{y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0317188 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{\frac{1}{2x^2}} x \left( 2c_1 - \sqrt{2\pi} c_2 \operatorname{erf}\left(\frac{1}{\sqrt{2}x}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.049 (sec), leaf count = 25

$$\left\{ y(x) = x e^{\frac{1}{2x^2}} \left( \operatorname{Erf}\left(\frac{\sqrt{2}}{2x}\right) - C2 + -C1 \right) \right\}$$

**2.1346 ODE No. 1346**

$$y''(x) = \frac{(a+b)y'(x)}{x^2} - \frac{y(x)(x(a+b) + ab)}{x^4}$$

✓ **Mathematica** : cpu = 0.345831 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow \frac{x e^{-\frac{\sqrt{(a-b)^2 + a + b}}{2x}} \left( c_1 \sqrt{(a-b)^2} e^{\frac{\sqrt{(a-b)^2}}{x}} + c_2 \right)}{\sqrt{(a-b)^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 25

$$\left\{ y(x) = x \left( e^{-\frac{a}{x}} C1 + e^{-\frac{b}{x}} C2 \right) \right\}$$

**2.1347 ODE No. 1347**

$$y''(x) = -\frac{y(x)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0878916 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow c_2 J_0\left(\frac{1}{x}\right) + \frac{c_1 K_0\left(\frac{i}{x}\right)}{\sqrt{\pi}} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 19

$$\{ y(x) = C1 J_0(x^{-1}) + C2 Y_0(x^{-1}) \}$$

**2.1348 ODE No. 1348**

$$y''(x) = -\frac{y(x)(a(x^4 + 1) + bx^2)}{x^4} - \frac{y'(x)}{x}$$

✗ **Mathematica** : cpu = 1.32496 (sec), leaf count = 0 , DifferentialRoot result

$$\{ \{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{y''(x)x^4 + y'(x)x^3 + (ax^4 + bx^2 + a)y(x) = 0, y(1) = c_1, y'(1) = c_2\}) (x) \} \}$$

✓ **Maple** : cpu = 0.207 (sec), leaf count = 73

$$\left\{ y(x) = \text{HeunD}\left(0, 2a + b, 0, 2a - b, \frac{x^2 + 1}{x^2 - 1}\right) \left( \int \frac{1}{x} \left( \text{HeunD}\left(0, 2a + b, 0, 2a - b, \frac{x^2 + 1}{x^2 - 1}\right) \right)^{-2} dx C2 + C1 \right) \right\}$$

**2.1349 ODE No. 1349**

$$y''(x) = -\frac{y(x)}{x^4} - \frac{(x^2 + 1)y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0991669 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow c_2 G_{1,2}^{2,0} \left( -\frac{1}{2x^2} \middle| \frac{3}{2}, 0, 0 \right) + \frac{c_1 e^{\frac{1}{4x^2}} \left( (2x^2 - 1) I_0\left(\frac{1}{4x^2}\right) + I_1\left(\frac{1}{4x^2}\right) \right)}{2x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 85

$$\left\{ y(x) = \frac{C1}{x^2} e^{\frac{1}{4x^2}} \left( 2x^2 I_0(1/4x^{-2}) + I_1\left(\frac{1}{4x^2}\right) - I_0\left(\frac{1}{4x^2}\right) \right) + \frac{C2}{x^2} e^{\frac{1}{4x^2}} \left( 2K_0(-1/4x^{-2})x^2 + K_1\left(-\frac{1}{4x^2}\right) \right) \right\}$$

**2.1350 ODE No. 1350**

$$y''(x) = -\frac{a^2 y(x)}{x^4} - \frac{2y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0114462 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow c_1 \cos\left(\frac{a}{x}\right) - c_2 \sin\left(\frac{a}{x}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 21

$$\left\{ y(x) = \_C1 \sin\left(\frac{a}{x}\right) + \_C2 \cos\left(\frac{a}{x}\right) \right\}$$

**2.1351 ODE No. 1351**

$$y''(x) = \frac{y(x)}{x^4} - \frac{(2x^2 + 1)y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0291263 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{\frac{1}{2x^2}} \left( 2c_1 - \sqrt{2\pi} c_2 \operatorname{erf}\left(\frac{1}{\sqrt{2x}}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 24

$$\left\{ y(x) = e^{\frac{1}{2x^2}} \left( \operatorname{Erf}\left(\frac{\sqrt{2}}{2x}\right) \_C2 + \_C1 \right) \right\}$$

**2.1352 ODE No. 1352**

$$y''(x) = -\frac{2(a+x)y'(x)}{x^2} - \frac{by(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.0153945 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{a-\sqrt{a^2-b}}{x}} \left( c_1 e^{\frac{2\sqrt{a^2-b}}{x}} + c_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 43

$$\left\{ y(x) = -C1 e^{\frac{1}{x}(a-\sqrt{a^2-b})} + -C2 e^{\frac{1}{x}(\sqrt{a^2-b}+a)} \right\}$$

**2.1353 ODE No. 1353**

$$y''(x) = \frac{(2x^2-1)y'(x)}{x^3} - \frac{y(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.111213 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow \frac{2\left(c_2 e^{\frac{1}{2x^2}} x(x^2-1) + 8c_1(x^4+2x^2-1)\right) - \sqrt{2\pi}c_2(x^4+2x^2-1) \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right)}{16x} \right\} \right\}$$

✓ **Maple** : cpu = 0.211 (sec), leaf count = 66

$$\left\{ y(x) = \frac{1}{x} \left( -C1 \sqrt{2}\sqrt{\pi}(x^4+2x^2-1) \operatorname{erfi}\left(\frac{\sqrt{2}}{2x}\right) + (-2-C1 x^3 + 2-C1 x) e^{\frac{1}{2x^2}} + -C2 (x^4+2x^2-1) \right) \right\}$$

**2.1354 ODE No. 1354**

$$y''(x) = \frac{(2x^2-1)y'(x)}{x^3} - \frac{2y(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.0738161 (sec), leaf count = 81

$$\left\{ \left\{ y(x) \rightarrow \frac{5\sqrt{2\pi}c_2(1-5x^2) \operatorname{erfi}\left(\frac{1}{\sqrt{2}x}\right) + 2\left(6c_1(5x^2-1) + 5c_2 e^{\frac{1}{2x^2}} x(2x^4+4x^2-1)\right)}{60x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.256 (sec), leaf count = 33

$$\left\{ y(x) = \frac{1}{x^2} \left( -C2 {}_1F_1\left(-\frac{5}{2}; -\frac{1}{2}; \frac{1}{2x^2}\right)x^5 + 5-C1 x^2 - -C1 \right) \right\}$$



**2.1355 ODE No. 1355**

$$y''(x) = \frac{xy(x)}{x^3+1} - \frac{(x^3-1)y'(x)}{x(x^3+1)}$$

✓ **Mathematica** : cpu = 0.13121 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}c_2 \sqrt[3]{x^3+1} x^2 {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{5}{3}; -x^3\right) + c_1 \sqrt[3]{x^3+1} + c_2 x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 30

$$\left\{ y(x) = \sqrt[3]{x^3+1} \left( {}_2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{5}{3}; -x^3\right) - C1 x^2 + -C2 \right) \right\}$$

**2.1356 ODE No. 1356**

$$y''(x) = -\frac{y(x)(-n^2 - v(v+1)x^2)}{x^2(x^2+1)} - \frac{(2x^2+1)y'(x)}{x(x^2+1)}$$

✓ **Mathematica** : cpu = 0.310373 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{-n} {}_2F_1\left(\frac{1}{2}(-n-v), \frac{1}{2}(-n+v+1); 1-n; -x^2\right) + c_2 x^n {}_2F_1\left(\frac{n-v}{2}, \frac{1}{2}(n+v+1); n+1; -x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 29

$$\left\{ y(x) = -C1 \text{LegendreP}(v, n, \sqrt{x^2+1}) + -C2 \text{LegendreQ}(v, n, \sqrt{x^2+1}) \right\}$$

**2.1357 ODE No. 1357**

$$y''(x) = -\frac{(ax^2+a-1)y'(x)}{x(x^2+1)} - \frac{y(x)(bx^2+c)}{x^2(x^2+1)}$$

✓ **Mathematica** : cpu = 0.680887 (sec), leaf count = 264

$$\left\{ \left\{ y(x) \rightarrow x^{-\frac{1}{2}\sqrt{a^2-4a-4c+4}-\frac{a}{2}+1} \left( c_1 {}_2F_1\left(\frac{1}{4}\left(-\sqrt{a^2-2a-4b+1}-\sqrt{a^2-4a-4c+4}+1\right), \frac{1}{4}\left(\sqrt{a^2-2a-4b+1}+\sqrt{a^2-4a-4c+4}+1\right); \frac{5}{4}; -x^2\right) + c_2 \sqrt{x^2+1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.128 (sec), leaf count = 97

$$\left\{ y(x) = x^{1-\frac{a}{2}} \left( \text{LegendreP}\left(-\frac{1}{2} + \frac{1}{2}\sqrt{a^2-2a-4b+1}, \frac{1}{2}\sqrt{a^2-4a-4c+4}, \sqrt{x^2+1}\right) - C1 + \text{LegendreQ}\left(-\frac{1}{2} + \frac{1}{2}\sqrt{a^2-2a-4b+1}, \frac{1}{2}\sqrt{a^2-4a-4c+4}, \sqrt{x^2+1}\right) - C2 \right) \right\}$$

**2.1358 ODE No. 1358**

$$y''(x) = \frac{(x^2 - 2)y'(x)}{x(x^2 - 1)} - \frac{(x^2 - 2)y(x)}{x^2(x^2 - 1)}$$

✓ **Mathematica** : cpu = 0.0689789 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{x\sqrt[4]{x^2-1} \left( c_2 \log(\sqrt{x^2-1} + x) + c_1 \right)}{\sqrt[4]{1-x^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 20

$$\left\{ y(x) = x \left( \ln(x + \sqrt{x^2 - 1}) - C_2 + -C_1 \right) \right\}$$

**2.1359 ODE No. 1359**

$$y''(x) = -\frac{v(v+1)y(x)}{x^2(x^2-1)} - \frac{2xy'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.119152 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{-v} x^{-v} {}_2F_1\left(\frac{1}{2} - \frac{v}{2}, -\frac{v}{2}; \frac{1}{2} - v; x^2\right) + c_2 i^{v+1} x^{v+1} {}_2F_1\left(\frac{v+1}{2}, \frac{v+2}{2}; v + \frac{3}{2}; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 57

$$\left\{ y(x) = -C_1 {}_2F_1\left(-\frac{v}{2}, \frac{1}{2} - \frac{v}{2}; \frac{1}{2} - v; x^2\right) x^{-v} + -C_2 {}_2F_1\left(1 + \frac{v}{2}, \frac{1}{2} + \frac{v}{2}; \frac{3}{2} + v; x^2\right) x^{v+1} \right\}$$

**2.1360 ODE No. 1360**

$$y''(x) = \frac{v(v+1)y(x)}{x^2} - \frac{2xy'(x)}{x^2-1}$$

✓ **Mathematica** : cpu = 0.101772 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{-v} x^{-v} {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x^2\right) + c_2 i^{v+1} x^{v+1} {}_2F_1\left(\frac{1}{2}, v+1; v + \frac{3}{2}; x^2\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.079 (sec), leaf count = 47

$$\left\{ y(x) = -C_1 {}_2F_1\left(\frac{1}{2}, -v; \frac{1}{2} - v; x^2\right) x^{-v} + -C_2 {}_2F_1\left(\frac{1}{2}, v+1; \frac{3}{2} + v; x^2\right) x^{v+1} \right\}$$

**2.1361 ODE No. 1361**

$$y''(x) = \frac{2xy'(x)}{x^2 - 1} - \frac{(a(a+1) - a(a+3)x^2)y(x)}{x^2(x^2 - 1)}$$

✓ **Mathematica** : cpu = 0.507966 (sec), leaf count = 36

$$\{ \{ y(x) \rightarrow c_1 x^{-a} - c_2 x^{a+1} (2a(x^2 - 1) + x^2 - 3) \} \}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 33

$$\{ y(x) = \_C1 x^{-a} + \_C2 x^{a+1} (2ax^2 + x^2 - 2a - 3) \}$$

**2.1362 ODE No. 1362**

$$y''(x) = \frac{2xy'(x)}{x^2 - 1} - \frac{y(x) ((x^2 - 1)x^2(a - n)(a + n + 1) + 2ax^2 + n(n + 1)(x^2 - 1))}{x^2(x^2 - 1)}$$

✗ **Mathematica** : cpu = 14.183 (sec), leaf count = 0 , DifferentialRoot result

$$\{ \{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{-2y'(x)x^3 + (a^2x^4 - n^2x^4 + ax^4 - nx^4 - a^2x^2 + 2n^2x^2 + ax^2 + 2nx^2 - n^2)\}) \} \}$$

✓ **Maple** : cpu = 0.252 (sec), leaf count = 109

$$\{ y(x) = \_C1 \text{HeunC}\left(0, -n - \frac{1}{2}, -2, -\frac{a^2}{4} + \frac{n^2}{4} - \frac{a}{4} + \frac{n}{4}, -\frac{n^2}{4} - \frac{n}{4} + \frac{3}{4} + \frac{a^2}{4} - \frac{a}{4}, x^2\right) x^{-n} + \_C2 \text{HeunC}\left(0, \dots\right) \}$$

**2.1363 ODE No. 1363**

$$y''(x) = -\frac{(ax^2 + a - 2)y'(x)}{x(x^2 - 1)} - \frac{by(x)}{x^2}$$

✓ **Mathematica** : cpu = 0.80738 (sec), leaf count = 211

$$\{ \{ y(x) \rightarrow (-1)^{\frac{1}{4}(-\sqrt{a^2 - 2a - 4b + 1} + a + 7)} x^{\frac{1}{2}(-\sqrt{a^2 - 2a - 4b + 1} + a - 1)} \left( c_1 {}_2F_1\left(\frac{a-1}{2}, \frac{1}{2}(a - \sqrt{a^2 - 2a - 4b + 1} - 1)\right); 1 - \dots \right) \} \}$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 161

$$\{ y(x) = (x^2 - 1)^{-a+2} \left( {}_2F_1\left(-\frac{a}{2} + \frac{3}{2}, -\frac{a}{2} + \frac{3}{2} + \frac{1}{2}\sqrt{a^2 - 2a - 4b + 1}; 1 + \frac{1}{2}\sqrt{a^2 - 2a - 4b + 1}; x^2\right) x^{\frac{a}{2} - \frac{1}{2} + \frac{1}{2}\sqrt{a^2 - 2a - 4b + 1}} \right) \}$$

**2.1364 ODE No. 1364**

$$y''(x) = \frac{y'(x) (2(a-1)x^2 - 2a + 2bc(x^2 - 1)x^c) - y(x) (bc(2a - c - 1)x^{c+2} - bc(2a - c + 1)x^c + x^2((a-1)a - bc))}{x(x^2 - 1)^2}$$

✓ **Mathematica** : cpu = 0.169674 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow x^a e^{bx^c} (c_1 P_v(x) + c_2 Q_v(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 25

$$\left\{ y(x) = x^a e^{bx^c} (\text{LegendreQ}(v, x) \_C2 + \text{LegendreP}(v, x) \_C1) \right\}$$

**2.1365 ODE No. 1365**

$$y''(x) = -\frac{ay(x)}{(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.098444 (sec), leaf count = 66

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \sqrt{x^2 + 1} e^{-i\sqrt{a+1} \tan^{-1}(x)} \left( 2c_1 e^{2i\sqrt{a+1} \tan^{-1}(x)} + \frac{ic_2}{\sqrt{a+1}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.064 (sec), leaf count = 59

$$\left\{ y(x) = \sqrt{x^2 + 1} \left( \left( \frac{x+i}{-x+i} \right)^{-\frac{1}{2}\sqrt{a+1}} \_C2 + \left( \frac{x+i}{-x+i} \right)^{\frac{1}{2}\sqrt{a+1}} \_C1 \right) \right\}$$

**2.1366 ODE No. 1366**

$$y''(x) = -\frac{2xy'(x)}{x^2 + 1} - \frac{y(x)}{(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.024826 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x + c_1}{\sqrt{x^2 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.01 (sec), leaf count = 17

$$\left\{ y(x) = (\_C1 x + \_C2) \frac{1}{\sqrt{x^2 + 1}} \right\}$$

**2.1367 ODE No. 1367**

$$y''(x) = -\frac{y(x) \left( a^2(x^2 + 1)^2 + m^2 - n(n + 1)(x^2 + 1) \right)}{(x^2 + 1)^2} - \frac{2xy'(x)}{x^2 + 1}$$

✗ **Mathematica** : cpu = 2.37435 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ y''(x) (x^2 + 1)^2 + 2xy'(x) (x^2 + 1) + (a^2x^4 + 2a^2x^2 - n^2x^2 - nx^2 + a^2 + m^2) \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.26 (sec), leaf count = 88

$$\left\{ y(x) = (x^2 + 1)^{\frac{m}{2}} \left( \text{HeunC} \left( 0, \frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \frac{a^2}{4} + \frac{m^2}{4} - \frac{n^2}{4} - \frac{n}{4}, -x^2 \right) - C_2 x + \text{HeunC} \left( 0, -\frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} \right) \right) \right\}$$

**2.1368 ODE No. 1368**

$$y''(x) = -\frac{axy'(x)}{x^2 + 1} - \frac{by(x)}{(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.0293621 (sec), leaf count = 92

$$\left\{ \left\{ y(x) \rightarrow (x^2 + 1)^{\frac{1}{2} - \frac{a}{4}} \left( c_1 P_{\frac{a-2}{2}}^{\frac{1}{2}\sqrt{a^2-4a+4b+4}}(ix) + c_2 Q_{\frac{a-2}{2}}^{\frac{1}{2}\sqrt{a^2-4a+4b+4}}(ix) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.078 (sec), leaf count = 71

$$\left\{ y(x) = (x^2 + 1)^{\frac{1}{2} - \frac{a}{4}} \left( \text{LegendreQ} \left( \frac{a}{2} - 1, \frac{1}{2}\sqrt{a^2 - 4a + 4b + 4}, ix \right) - C_2 + \text{LegendreP} \left( \frac{a}{2} - 1, \frac{1}{2}\sqrt{a^2 - 4a + 4b + 4}, ix \right) \right) \right\}$$

**2.1369 ODE No. 1369**

$$y''(x) = -\frac{ay(x)}{(x^2 - 1)^2}$$

✓ **Mathematica** : cpu = 0.107106 (sec), leaf count = 67

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}\sqrt{1-x^2}e^{-\sqrt{1-a}\tanh^{-1}(x)} \left( \frac{c_2 e^{2\sqrt{1-a}\tanh^{-1}(x)}}{\sqrt{1-a}} + 2c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.055 (sec), leaf count = 55

$$\left\{ y(x) = \sqrt{x^2 - 1} \left( \left( \frac{x-1}{1+x} \right)^{-\frac{1}{2}\sqrt{1-a}} - C_2 + \left( \frac{x-1}{1+x} \right)^{\frac{1}{2}\sqrt{1-a}} - C_1 \right) \right\}$$

**2.1370 ODE No. 1370**

$$y''(x) = \frac{a^2 y(x)}{(x^2 - 1)^2} - \frac{2xy'(x)}{x^2 - 1}$$

✓ **Mathematica** : cpu = 0.0311137 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh \left( \frac{1}{2} a (\log(1-x) - \log(x+1)) \right) + ic_2 \sinh \left( \frac{1}{2} a (\log(1-x) - \log(x+1)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 19

$$\{y(x) = \_C1 \sinh(a \operatorname{Artanh}(x)) + \_C2 \cosh(a \operatorname{Artanh}(x))\}$$

**2.1371 ODE No. 1371**

$$y''(x) = -\frac{y(x)(-a^2 - \lambda(x^2 - 1))}{(x^2 - 1)^2} - \frac{2xy'(x)}{x^2 - 1}$$

✓ **Mathematica** : cpu = 0.024163 (sec), leaf count = 48

$$\left\{ \left\{ y(x) \rightarrow c_1 P_{\frac{1}{2}(\sqrt{4\lambda+1}-1)}^a(x) + c_2 Q_{\frac{1}{2}(\sqrt{4\lambda+1}-1)}^a(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 37

$$\left\{ y(x) = \_C1 \operatorname{LegendreP} \left( \frac{1}{2} \sqrt{1+4\lambda} - \frac{1}{2}, a, x \right) + \_C2 \operatorname{LegendreQ} \left( \frac{1}{2} \sqrt{1+4\lambda} - \frac{1}{2}, a, x \right) \right\}$$

**2.1372 ODE No. 1372**

$$y''(x) = -\frac{y(x)((x^2 - 1)(ax^2 + bx + c) - k^2)}{(x^2 - 1)^2} - \frac{2xy'(x)}{x^2 - 1}$$

✗ **Mathematica** : cpu = 4.09161 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \operatorname{DifferentialRoot}(\{y, x\}, \{(ax^4 + bx^3 - ax^2 + cx^2 - bx - k^2 - c)y(x) + (2x^3 - 2x)y'(x) + (x^4 - 2x^2)\}) \right\} \right\}$$

✓ **Maple** : cpu = 0.297 (sec), leaf count = 110

$$\left\{ y(x) = e^{\sqrt{-a}x} \left( \operatorname{HeunC} \left( 4\sqrt{-a}, -k, k, 2b, \frac{k^2}{2} + a - b + c, \frac{1}{2} + \frac{x}{2} \right) \sqrt{2x-2}(1+x)^{-\frac{k}{2}}(x-1)^{\frac{k}{2}-\frac{1}{2}} \_C2 + \operatorname{HeunC} \left( 4\sqrt{-a}, -k, k, 2b, \frac{k^2}{2} + a - b + c, \frac{1}{2} + \frac{x}{2} \right) \sqrt{2x-2}(1+x)^{-\frac{k}{2}}(x-1)^{\frac{k}{2}-\frac{1}{2}} \_C1 \right) \right\}$$

**2.1373 ODE No. 1373**

$$y''(x) = -\frac{y(x) \left( -a^2(x^2 - 1)^2 - m^2 - n(n+1)(x^2 - 1) \right)}{(x^2 - 1)^2} - \frac{2xy'(x)}{x^2 - 1}$$

✗ **Mathematica** : cpu = 2.33153 (sec), leaf count = 0 , DifferentialRoot result

$$\{ \{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{(-a^2x^4 + 2a^2x^2 - n^2x^2 - nx^2 - a^2 - m^2 + n^2 + n)y(x) + (2x^3 - 2x)y'(x)\}) \}$$

✓ **Maple** : cpu = 0.244 (sec), leaf count = 84

$$\left\{ y(x) = (x^2 - 1)^{\frac{m}{2}} \left( \text{HeunC} \left( 0, \frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \frac{a^2}{4} + \frac{m^2}{4} - \frac{n^2}{4} - \frac{n}{4}, x^2 \right) - C2 x + \text{HeunC} \left( 0, -\frac{1}{2}, m, -\frac{a^2}{4}, \frac{1}{4} + \right. \right.$$

**2.1374 ODE No. 1374**

$$y''(x) = \frac{2(2a - 1)xy'(x)}{x^2 - 1} - \frac{y(x) (x^2(2a(2a - 1) - v(v + 1)) + 2a + v(v + 1))}{(x^2 - 1)^2}$$

✓ **Mathematica** : cpu = 0.0383716 (sec), leaf count = 26

$$\{ \{ y(x) \rightarrow (x^2 - 1)^a (c_1 P_v(x) + c_2 Q_v(x)) \} \}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 23

$$\{ y(x) = (x^2 - 1)^a (\text{LegendreQ}(v, x) - C2 + \text{LegendreP}(v, x) - C1) \}$$

**2.1375 ODE No. 1375**

$$y''(x) = -\frac{y(x) (4ax^2(a - n) - (x^2 - 1) (2a + (v - n)(n + v + 1)))}{(x^2 - 1)^2} - \frac{2x(-2a + n + 1)y'(x)}{x^2 - 1}$$

✓ **Mathematica** : cpu = 0.0559353 (sec), leaf count = 34

$$\{ \{ y(x) \rightarrow (x^2 - 1)^{a - \frac{n}{2}} (c_1 P_v^n(x) + c_2 Q_v^n(x)) \} \}$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 29

$$\{ y(x) = (x^2 - 1)^{a - \frac{n}{2}} (\text{LegendreQ}(v, n, x) - C2 + \text{LegendreP}(v, n, x) - C1) \}$$

**2.1376 ODE No. 1376**

$$y''(x) = -\frac{by(x)}{x^2(a+x^2)} - \frac{(a+2x^2)y'(x)}{x(a+x^2)}$$

✓ **Mathematica** : cpu = 0.104061 (sec), leaf count = 82

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin \left( \frac{\sqrt{b}(\log(x) - \log(\sqrt{a}\sqrt{a+x^2} + a))}{\sqrt{a}} \right) + c_1 \cos \left( \frac{\sqrt{b}(\log(x) - \log(\sqrt{a}\sqrt{a+x^2} + a))}{\sqrt{a}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.031 (sec), leaf count = 73

$$\left\{ y(x) = 1 \left( -C2 \left( \left( \frac{1}{x} (2a + 2\sqrt{a}\sqrt{x^2+a}) \right)^{i\sqrt{b}\frac{1}{\sqrt{a}}} + -C1 \right) \left( \left( \frac{1}{x} (2a + 2\sqrt{a}\sqrt{x^2+a}) \right)^{i\sqrt{b}\frac{1}{\sqrt{a}}} \right)^{-1} \right) \right\}$$

**2.1377 ODE No. 1377**

$$y''(x) = -\frac{b^2y(x)}{(a^2+x^2)^2}$$

✓ **Mathematica** : cpu = 0.244704 (sec), leaf count = 97

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \sqrt{a^2+x^2} e^{-i\sqrt{\frac{b^2}{a^2}+1} \tan^{-1}\left(\frac{x}{a}\right)} \left( 2c_1 e^{2i\sqrt{\frac{b^2}{a^2}+1} \tan^{-1}\left(\frac{x}{a}\right)} + \frac{ic_2}{a\sqrt{\frac{b^2}{a^2}+1}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.132 (sec), leaf count = 83

$$\left\{ y(x) = \sqrt{a^2+x^2} \left( \left( \frac{ix-a}{ix+a} \right)^{\frac{1}{2a}\sqrt{a^2+b^2}} - C1 + \left( \frac{ix-a}{ix+a} \right)^{-\frac{1}{2a}\sqrt{a^2+b^2}} - C2 \right) \right\}$$

**2.1378 ODE No. 1378**

$$y''(x) = -\frac{2(x^2-1)y'(x)}{(x-1)^2x} - \frac{(-2x^2+2x+2)y(x)}{(x-1)^2x^2}$$

✓ **Mathematica** : cpu = 0.0585061 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow -\frac{x(c_1x^2 - c_1x - 2c_2x - 2c_2(x-1)x \log(1-x) + 2c_2(x-1)x \log(x) + c_2)}{(x-1)^2} \right\} \right\}$$



✓ **Maple** : cpu = 0.052 (sec), leaf count = 48

$$\left\{ y(x) = \frac{x}{(x-1)^2} \left( -C2 x(x-1) \ln(x-1) + C2 x(x-1) \ln(x) + C1 x^2 + (-C1 - C2) x + \frac{C2}{2} \right) \right\}$$

**2.1379 ODE No. 1379**

$$y''(x) = \frac{12y(x)}{(x+1)^2(x^2+2x+3)}$$

✓ **Mathematica** : cpu = 0.0827081 (sec), leaf count = 72

$$\left\{ \left\{ y(x) \rightarrow \frac{2(c_1(x^2+2x+3) + c_2(x^3+2x^2+4x+1)) - 3\sqrt{2}c_2(x^2+2x+3) \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{2(x+1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 59

$$\left\{ y(x) = \frac{1}{(1+x)^2} \left( -3 C2 (x^2+2x+3) \arctan\left(\frac{1}{2}(1+x)\sqrt{2}\right) + C2 (x^3+2x^2+4x+1) \sqrt{2} + C1 (x^2 - \dots) \right) \right\}$$

**2.1380 ODE No. 1380**

$$y''(x) = -\frac{by(x)}{x^2(x-a)^2}$$

✓ **Mathematica** : cpu = 0.311979 (sec), leaf count = 121

$$\left\{ \left\{ y(x) \rightarrow \frac{x^{\frac{1}{2}-\frac{1}{2}} \sqrt{1-\frac{4b}{a^2}} (x-a)^{\frac{1}{2}-\frac{1}{2}} \sqrt{1-\frac{4b}{a^2}} \left( ac_1 \sqrt{1-\frac{4b}{a^2}} x^{\sqrt{1-\frac{4b}{a^2}}} + c_2 (x-a)^{\sqrt{1-\frac{4b}{a^2}}} \right)}{a \sqrt{1-\frac{4b}{a^2}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 67

$$\left\{ y(x) = \sqrt{x(a-x)} \left( \left( \frac{x}{a-x} \right)^{\frac{1}{2a} \sqrt{a^2-4b}} -C2 + \left( \frac{a-x}{x} \right)^{\frac{1}{2a} \sqrt{a^2-4b}} -C1 \right) \right\}$$

**2.1381 ODE No. 1381**

$$y''(x) = c - \frac{by(x)}{x^2(x-a)^2}$$

✓ **Mathematica** : cpu = 0.750222 (sec), leaf count = 371

$$\left\{ \left\{ y(x) \rightarrow \frac{acx^2(a-x) \left(1 - \frac{x}{a}\right)^{-\frac{1}{2}} \sqrt{1 - \frac{4b}{a^2}} - \frac{1}{2} \left( \left( \sqrt{1 - \frac{4b}{a^2}} - 3 \right) \left(1 - \frac{x}{a}\right) \sqrt{1 - \frac{4b}{a^2}} \right) {}_2F_1 \left( \frac{1}{2} \sqrt{1 - \frac{4b}{a^2}} - \frac{1}{2}, \frac{1}{2} \sqrt{1 - \frac{4b}{a^2}} + \frac{3}{2} \right)}{2(2a^2 + b)} \right. \right.$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 175

$$\left\{ y(x) = 1\sqrt{x(a-x)} \left( \left( -C2 \sqrt{a^2 - 4b} - \int \sqrt{x(a-x)} \left( \frac{a-x}{x} \right)^{-\frac{1}{2a}\sqrt{a^2-4b}} dx \right) \left( \frac{a-x}{x} \right)^{\frac{1}{2a}\sqrt{a^2-4b}} + \left( \int \sqrt{x(a-x)} \right) \right)$$

**2.1382 ODE No. 1382**

$$y''(x) = \frac{cy(x)}{(x-a)^2(x-b)^2}$$

✓ **Mathematica** : cpu = 0.719563 (sec), leaf count = 141

$$\left\{ \left\{ y(x) \rightarrow (x-a)^{\frac{1}{2}} \left( 1 - \sqrt{\frac{4c}{(a-b)^2} + 1} \right) (x-b)^{\frac{1}{2}} \left( 1 - \sqrt{\frac{4c}{(a-b)^2} + 1} \right) \left( c_1(x-a) \sqrt{\frac{4c}{(a-b)^2} + 1} - \frac{c_2(x-b) \sqrt{\frac{4c}{(a-b)^2} + 1}}{(a-b) \sqrt{\frac{4c}{(a-b)^2} + 1}} \right) \right. \right\}$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 104

$$\left\{ y(x) = \sqrt{(a-x)(b-x)} \left( \left( \frac{a-x}{b-x} \right)^{\frac{1}{2a-2b}\sqrt{a^2-2ab+b^2+4c}} - C1 + \left( \frac{a-x}{b-x} \right)^{-\frac{1}{2a-2b}\sqrt{a^2-2ab+b^2+4c}} - C2 \right) \right\}$$

**2.1383 ODE No. 1383**

$$y''(x) = -\frac{y'(x) \left( (x-a)^2(\alpha + \beta + 1)(x-b) + (x-a)(-\alpha - \beta + 1)(x-b)^2 \right)}{(x-a)^2(x-b)^2} - \frac{\alpha\beta(a-b)^2 y(x)}{(x-a)^2(x-b)^2}$$

✓ **Mathematica** : cpu = 0.144299 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_1(x-a)^\alpha(x-b)^{-\alpha} + c_2(x-a)^\beta(x-b)^{-\beta} \right\} \right\}$$

✓ **Maple** : cpu = 0.052 (sec), leaf count = 39

$$\left\{ y(x) = -C1 \left( \frac{a-x}{b-x} \right)^\beta + -C2 \left( \frac{a-x}{b-x} \right)^\alpha \right\}$$

**2.1384 ODE No. 1384**

$$y''(x) = -\frac{y(x) \left( -(a^2 - 1)x^2 + 2(a + 3)bx - b^2 \right)}{4x^2}$$

✓ **Mathematica** : cpu = 0.0358169 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow c_1 M_{\frac{(a+3)b}{2\sqrt{a^2-1}}, \frac{\sqrt{b^3+b}}{2\sqrt{b}}} \left( \sqrt{a^2-1}x \right) + c_2 W_{\frac{(a+3)b}{2\sqrt{a^2-1}}, \frac{\sqrt{b^3+b}}{2\sqrt{b}}} \left( \sqrt{a^2-1}x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.27 (sec), leaf count = 73

$$\left\{ y(x) = -C1 M_{\frac{b(a+3)}{2}, \frac{1}{\sqrt{a^2-1}}, \frac{1}{2}\sqrt{b^2+1}} \left( \sqrt{a^2-1}x \right) + -C2 W_{\frac{b(a+3)}{2}, \frac{1}{\sqrt{a^2-1}}, \frac{1}{2}\sqrt{b^2+1}} \left( \sqrt{a^2-1}x \right) \right\}$$

**2.1385 ODE No. 1385**

$$y''(x) = -\frac{(ax^2 + a - 3)y(x)}{4(x^2 + 1)^2}$$

✓ **Mathematica** : cpu = 0.021055 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow \sqrt{x^2+1} \left( c_1 P_{\frac{1}{2}}^{\frac{1}{2}}(\sqrt{1-a-1})(ix) + c_2 Q_{\frac{1}{2}}^{\frac{1}{2}}(\sqrt{1-a-1})(ix) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 55

$$\left\{ y(x) = \sqrt[4]{x^2+1} \left( \left( x + \sqrt{x^2+1} \right)^{-\frac{1}{2}\sqrt{1-a}} -C2 + \left( x + \sqrt{x^2+1} \right)^{\frac{1}{2}\sqrt{1-a}} -C1 \right) \right\}$$

**2.1386 ODE No. 1386**

$$y''(x) = \frac{18y(x)}{(2x+1)^2(x^2+x+1)}$$

✓ **Mathematica** : cpu = 0.0960154 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1(x^2+x+1) - 12\sqrt{3}c_2(x^2+x+1)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c_2(16x^3+24x^2+30x+11)}{(2x+1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 58

$$\left\{ y(x) = \frac{1}{(2x+1)^2} \left( 36\_C2 (x^2+x+1) \arctan\left(\frac{1}{3}(2x+1)\sqrt{3}\right) - 16\left(x^3+x^2+\frac{11x}{8}+3/16\right) - C2\sqrt{3} + \dots \right) \right\}$$

**2.1387 ODE No. 1387**

$$y''(x) = \frac{3y(x)}{4(x^2+x+1)^2}$$

✓ **Mathematica** : cpu = 0.041608 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{3}\sqrt{x^2+x+1} \left( 2\sqrt{3}c_2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + 3c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 28

$$\left\{ y(x) = \sqrt{x^2+x+1} \left( \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right) - C2 + \dots - C1 \right) \right\}$$

**2.1388 ODE No. 1388**

$$y''(x) = -\frac{y(x)(v(v+1)(x-1)-a^2x)}{4(x-1)^2x^2} - \frac{(3x-1)y'(x)}{2(x-1)x}$$

✓ **Mathematica** : cpu = 0.307618 (sec), leaf count = 109

$$\left\{ \left\{ y(x) \rightarrow \frac{(-1)^{-v}(x-1)^{\frac{a+1}{2}}x^{-v/2} \left( c_1(-1)^v x^{v+1/2} {}_2F_1\left(\frac{1}{2}(a+v+1), \frac{1}{2}(a+v+2); v+\frac{3}{2}; x\right) - ic_2 {}_2F_1\left(\frac{a-v}{2}, \frac{1}{2}(a-v) \dots \right) \right)}{\sqrt{1-x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.083 (sec), leaf count = 76

$$\left\{ y(x) = (x-1)^{-\frac{a}{2}} \left( x^{-\frac{v}{2}} {}_2F_1\left(-\frac{v}{2}-\frac{a}{2}, \frac{1}{2}-\frac{v}{2}-\frac{a}{2}; \frac{1}{2}-v; x\right) - C1 + x^{\frac{1}{2}+\frac{v}{2}} {}_2F_1\left(1+\frac{v}{2}-\frac{a}{2}, \frac{1}{2}+\frac{v}{2}-\frac{a}{2}; \frac{3}{2}+v; x\right) \dots \right) \right\}$$

**2.1389 ODE No. 1389**

$$y''(x) = -\frac{y(x)(-4n^2x - v(v+1)(x-1)^2)}{4(x-1)^2x^2} - \frac{(3x-1)y'(x)}{2(x-1)x}$$

✓ **Mathematica** : cpu = 0.394426 (sec), leaf count = 91

$$\left\{ \left\{ y(x) \rightarrow \frac{(-1)^{-v}(x-1)^{n+\frac{1}{2}}x^{-v/2} \left( c_1(-1)^v x^{v+\frac{1}{2}} {}_2F_1\left(n+\frac{1}{2}, n+v+1; v+\frac{3}{2}; x\right) - ic_2 {}_2F_1\left(n+\frac{1}{2}, n-v; \frac{1}{2}-v; x\right) \right)}{\sqrt{1-x}} \right. \right.$$

✓ **Maple** : cpu = 0.081 (sec), leaf count = 68

$$\left\{ y(x) = (x-1)^{-n} \left( x^{-\frac{v}{2}} {}_2F_1\left(-v-n, -n+\frac{1}{2}; \frac{1}{2}-v; x\right) C1 + x^{\frac{1}{2}+\frac{v}{2}} {}_2F_1\left(v-n+1, -n+\frac{1}{2}; \frac{3}{2}+v; x\right) C2 \right) \right\}$$

**2.1390 ODE No. 1390**

$$y''(x) = -\frac{3y(x)}{16(x-1)^2x^2}$$

✓ **Mathematica** : cpu = 0.0434111 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow \frac{(1-x)^{3/4} \sqrt[4]{x} \left( c_1 \sqrt{-(x-1)x} + 2c_2x \right)}{\sqrt{-(x-1)x}} \right. \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 25

$$\left\{ y(x) = C1 x^{\frac{3}{4}} \sqrt[4]{x-1} + C2 \sqrt[4]{x} (x-1)^{\frac{3}{4}} \right\}$$

**2.1391 ODE No. 1391**

$$y''(x) = \frac{(7ax^2 + 5)y'(x)}{x(ax^2 + 1)} - \frac{(15ax^2 + 5)y(x)}{x^2(ax^2 + 1)}$$

✓ **Mathematica** : cpu = 0.0615562 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow c_1x^5 - \frac{1}{4}c_2x(2ax^2 + 1) \right. \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 20

$$\left\{ y(x) = C1 x^5 + 2 C2 ax^3 + C2 x \right\}$$

**2.1392 ODE No. 1392**

$$y''(x) = -\frac{bxy'(x)}{a(x^2-1)} - \frac{y(x)(cx^2+dx+e)}{a(x^2-1)^2}$$

✓ **Mathematica** : cpu = 92.707 (sec), leaf count = 1

\$Aborted

✓ **Maple** : cpu = 0.237 (sec), leaf count = 561

$$\left\{ y(x) = \left( -\frac{1}{2} + \frac{x}{2} \right)^{\frac{1}{4a} \left( 2a + \sqrt{4a^2 + (-4b - 4c - 4d - 4e)a + b^2} \right)} (x^2 - 1)^{-\frac{b}{4a}} \left( {}_2F_1 \left( -\frac{1}{4a} \left( -\sqrt{4a^2 + (-4b - 4c - 4d - 4e)a + b^2} \right), \frac{1}{2} \right) \right) \right\}$$

**2.1393 ODE No. 1393**

$$y''(x) = -\frac{y(x)(bx^2+cx+d)}{a(x-1)^2x^2}$$

✓ **Mathematica** : cpu = 19.1225 (sec), leaf count = 1

\$Aborted

✓ **Maple** : cpu = 0.161 (sec), leaf count = 272

$$\left\{ y(x) = (x-1)^{-\frac{1}{2}(\sqrt{a-4b-4c-4d}-\sqrt{a})} \frac{1}{\sqrt{a}} \left( {}_2F_1 \left( -\frac{1}{2} \left( \sqrt{a-4b-4c-4d}-\sqrt{a}-\sqrt{a-4d}+\sqrt{a-4b} \right), \frac{1}{2} \right) \right) \right\}$$

**2.1394 ODE No. 1394**

$$y''(x) = -\frac{cy(x)}{x^2(ax+b)^2} - \frac{2y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0593806 (sec), leaf count = 73

$$\left\{ \left\{ y(x) \rightarrow \left( c_2 e^{\frac{\sqrt{b^2-4c}(\log(x)-\log(ax+b))}{b}} + c_1 \right) \exp \left( -\frac{(\sqrt{b^2-4c}+b)(\log(x)-\log(ax+b))}{2b} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.12 (sec), leaf count = 79

$$\left\{ y(x) = \sqrt{\frac{ax+b}{x}} \left( \left( \frac{x}{ax+b} \right)^{-\frac{a}{2b} \sqrt{\frac{b^2-4c}{a^2}}} - C_2 + \left( \frac{x}{ax+b} \right)^{\frac{a}{2b} \sqrt{\frac{b^2-4c}{a^2}}} - C_1 \right) \right\}$$

**2.1395 ODE No. 1395**

$$y''(x) = -\frac{y(x)}{(ax+b)^4}$$

✓ **Mathematica** : cpu = 0.168937 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-\frac{\sqrt{-\frac{1}{4}a}}{ax+b}} (ax+b) \left( 2c_1 e^{\frac{2\sqrt{-\frac{1}{4}a}}{ax+b}} + \left(-\frac{1}{a^4}\right)^{3/2} a^6 c_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 39

$$\left\{ y(x) = (ax+b) \left( -C2 \cos\left(\frac{1}{a(ax+b)}\right) + -C1 \sin\left(\frac{1}{a(ax+b)}\right) \right) \right\}$$

**2.1396 ODE No. 1396**

$$y''(x) = -\frac{Ay(x)}{(ax^2+bx+c)^2}$$

✓ **Mathematica** : cpu = 1.43711 (sec), leaf count = 199

$$\left\{ \left\{ y(x) \rightarrow \sqrt{x(ax+b)+c} \exp\left(-\frac{\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{b^2-4ac}}\right) \left( c_1 \exp\left(\frac{2\sqrt{4ac-b^2} \sqrt{1-\frac{4A}{b^2-4ac}}}{\sqrt{b^2-4ac}}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.161 (sec), leaf count = 178

$$\left\{ y(x) = \sqrt{ax^2+bx+c} \left( \left( 1 \left( i\sqrt{4ca-b^2} - 2ax - b \right) \left( 2ax + b + i\sqrt{4ca-b^2} \right)^{-1} \right)^{-\frac{a}{2} \sqrt{\frac{-4ca+b^2-4A}{a^2}} \frac{1}{\sqrt{-4ca+b^2}}} - C \right) \right\}$$

**2.1397 ODE No. 1397**

$$y''(x) = \frac{y(x)}{x^5} - \frac{y'(x)}{x^4}$$

✓ **Mathematica** : cpu = 0.0307451 (sec), leaf count = 38

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \Gamma\left(\frac{1}{3}, -\frac{1}{3x^3}\right)}{3^{2/3} \sqrt[3]{-\frac{1}{x^3}}} + c_1 x \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 27

$$\left\{ y(x) = x \left( -\sqrt{3} \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{1}{3}, -\frac{1}{3x^3}\right) - C2 + 2\pi - C2 + -C1 \right) \right\}$$

**2.1398 ODE No. 1398**

$$y''(x) = -\frac{(-(2v+1)^2 + x^2 - 1)y(x)}{(x^2 - 1)^2} - \frac{(3x^2 - 1)y'(x)}{x(x^2 - 1)}$$

✗ **Mathematica** : cpu = 1.48124 (sec), leaf count = 0 , DifferentialRoot result

$$\{ \{y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{x(x^2 - 4v^2 - 4v - 2)y(x) + (3x^4 - 4x^2 + 1)y'(x) + (x^5 - 2x^3 + x)y''(x) = 0\})\} \}$$

✓ **Maple** : cpu = 0.189 (sec), leaf count = 69

$$\{y(x) = \_C1 (x^2 - 1)^{-v-\frac{1}{2}} {}_2F_1(-v, -v; -2v; -x^2 + 1) + \_C2 (x^2 - 1)^{v+\frac{1}{2}} {}_2F_1(v + 1, v + 1; 2 + 2v; -x^2 + 1)\}$$

**2.1399 ODE No. 1399**

$$y''(x) = \frac{(3x+1)y'(x)}{(x-1)(x+1)} - \frac{36(x+1)^2y(x)}{(x-1)^2(3x+5)^2}$$

✓ **Mathematica** : cpu = 0.0564908 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}(1-x)^{3/2}\sqrt{3x+5}(3c_2 \log(1-x) + c_2 \log(3x+5) + 2c_1) \right\} \right\}$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 34

$$\{y(x) = (x-1)^{\frac{3}{2}}\sqrt{3x+5}(\_C2 \ln(3x+5) + 3\_C2 \ln(x-1) + \_C1)\}$$

**2.1400 ODE No. 1400**

$$y''(x) = \frac{y'(x)}{x} - \frac{ay(x)}{x^6}$$

✓ **Mathematica** : cpu = 0.0530903 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}x^2 e^{-\frac{\sqrt{-a}}{2x^2}} \left( 2c_1 e^{\frac{\sqrt{-a}}{x^2}} + \frac{c_2}{\sqrt{-a}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 35

$$\left\{ y(x) = x^2 \left( \cosh \left( \frac{1}{2x^2} \sqrt{-a} \right) \_C2 + \sinh \left( \frac{1}{2x^2} \sqrt{-a} \right) \_C1 \right) \right\}$$



**2.1401 ODE No. 1401**

$$y''(x) = -\frac{(a + 3x^2)y'(x)}{x^3} - \frac{by(x)}{x^6}$$

✓ **Mathematica** : cpu = 0.0157529 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{a - \sqrt{a^2 - 4b}}{4x^2}} \left( c_1 e^{\frac{\sqrt{a^2 - 4b}}{2x^2}} + c_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.054 (sec), leaf count = 45

$$\left\{ y(x) = \_C1 e^{-\frac{1}{4x^2}(-a + \sqrt{a^2 - 4b})} + \_C2 e^{\frac{1}{4x^2}(a + \sqrt{a^2 - 4b})} \right\}$$

**2.1402 ODE No. 1402**

$$y''(x) = -\frac{y(x) \left( 4a(a+1)x^4 - 2a(x^2-1)x^2 + (x^2-1)^2(x^2-v^2) \right)}{x^2(x^2-1)^2} - \frac{((1-4a)x^2-1)y'(x)}{x(x^2-1)}$$

✗ **Mathematica** : cpu = 4.54662 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{ (x^6 + 4a^2x^4 - v^2x^4 + 2ax^4 - 2x^4 + 2v^2x^2 + 2ax^2 + x^2 - v^2) y(x) + (-4ax^5 \right.$$

✓ **Maple** : cpu = 0.305 (sec), leaf count = 58

$$\left\{ y(x) = (x^2 - 1)^a (x^2 - 1) \left( \_C1 x^v \text{HeunC} \left( 0, v, 1, \frac{1}{4}, \frac{a}{2} + \frac{1}{4}, x^2 \right) + \_C2 x^{-v} \text{HeunC} \left( 0, -v, 1, \frac{1}{4}, \frac{a}{2} + \frac{1}{4}, x^2 \right) \right) \right\}$$

**2.1403 ODE No. 1403**

$$y''(x) = -y'(x) \left( \frac{-a_1 - b_1 + 1}{x - c_1} + \frac{-a_2 - b_2 + 1}{x - c_2} + \frac{-a_3 - b_3 + 1}{x - c_3} \right) - \frac{y(x) \left( \frac{a_1 b_1 (c_1 - c_2)(c_1 - c_3)}{x - c_1} + \frac{a_2 b_2 (c_2 - c_1)(c_2 - c_3)}{x - c_2} \right)}{(x - c_1)(x - c_2)(x - c_3)}$$

✗ **Mathematica** : cpu = 96.3648 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}(\{y, x\}, \{ (c_1 - x)^2 (c_2 - x)^2 y''(x) (c_3 - x)^2 + (c_1 - x)(c_2 - x) (a_1 x^2 + a_2 x^2 + a_3 x^2 + \right.$$

✓ **Maple** : cpu = 1.052 (sec), leaf count = 298

$$\left\{ y(x) = (x - c_2)^{a_2} (x - c_3)^{b_3} \left( \text{HeunG} \left( \frac{c_1 - c_3}{c_1 - c_2}, \frac{((-2a_1 - a_3 - b_2 + 2)c_1 + (a_1 + a_3 - 1)c_2 + c_3)(a_1 + b_1)}{(c_1 - c_2)(c_1 - c_3)} \right) \right) \right\}$$

2.1404 ODE No. 1404

$$y''(x) = -\frac{(1-2x^2)y(x)}{4x^6} - \frac{(2x^2+1)y'(x)}{x^3}$$

✓ **Mathematica** : cpu = 0.0256542 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{\frac{1}{4x^2}}(c_2x + c_1)}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 19

$$\left\{ y(x) = \frac{-C1x + -C2e^{\frac{1}{4x^2}}}{x} \right\}$$

2.1405 ODE No. 1405

$$y''(x) = \frac{(2x^2+1)y'(x)}{x^3} - \frac{(ax^4+10x^2+1)y(x)}{4x^6}$$

✓ **Mathematica** : cpu = 0.0784283 (sec), leaf count = 70

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\frac{1}{4x^2}}x^{\frac{3}{2}-\frac{\sqrt{9-a}}{2}}(c_2x^{\sqrt{9-a}} + \sqrt{9-a}c_1)}{\sqrt{9-a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 42

$$\left\{ y(x) = e^{-\frac{1}{4x^2}} \left( x^{\frac{3}{2}-\frac{1}{2}\sqrt{-a+9}} \_C2 + x^{\frac{3}{2}+\frac{1}{2}\sqrt{-a+9}} \_C1 \right) \right\}$$

2.1406 ODE No. 1406

$$y''(x) = -\frac{27xy(x)}{16(x^3-1)^2}$$

✗ **Mathematica** : cpu = 299.999 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.158 (sec), leaf count = 44

$$\left\{ y(x) = \sqrt{x} \sqrt[4]{x^3-1} \left( LegendreQ\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{-x^3+1}\right) \_C2 + LegendreP\left(-\frac{1}{6}, \frac{1}{3}, \sqrt{-x^3+1}\right) \_C1 \right) \right\}$$



**2.1410 ODE No. 1410**

$$y''(x) = -\frac{y'(x)(apx^b + q)}{x(ax^b - 1)} - \frac{y(x)(arx^b + s)}{x^2(ax^b - 1)}$$

✓ **Mathematica** : cpu = 0.136735 (sec), leaf count = 405

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{-\frac{\sqrt{q^2+2q+4s+1}+q+1}{b}} a^{-\frac{\sqrt{q^2+2q+4s+1}+q+1}{2b}} (x^b)^{-\frac{\sqrt{q^2+2q+4s+1}+q+1}{2b}} {}_2F_1\left(\frac{p+q-\sqrt{p^2-2p-4r+1}-\sqrt{q^2+2q+4s+1}}{2b}\right) \right. \right.$$

✓ **Maple** : cpu = 0.296 (sec), leaf count = 253

$$\left\{ y(x) = -C1 {}_2F_1\left(\frac{1}{2b}(p+q+\sqrt{q^2+2q+4s+1}+\sqrt{p^2-2p-4r+1}), \frac{1}{2b}(p+q+\sqrt{q^2+2q+4s+1}-\sqrt{p^2-2p-4r+1})\right) \right.$$

**2.1411 ODE No. 1411**

$$y''(x) = \frac{y(x)}{e^x + 1}$$

✓ **Mathematica** : cpu = 0.336909 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow e^{-x}(c_1(e^x + 1) + c_2(e^x + 1) \log(e^x + 1) + c_2) \right\} \right.$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 27

$$\left\{ y(x) = \frac{-C1(e^x + 1) \ln(e^x + 1) + -C2 e^x + -C1 + -C2}{e^x} \right\}$$

**2.1412 ODE No. 1412**

$$y''(x) = \frac{y'(x)}{x \log(x)} + y(x) \log^2(x)$$

✓ **Mathematica** : cpu = 0.019106 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh(x(\log(x) - 1)) + i c_2 \sinh(x(\log(x) - 1)) \right\} \right.$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 23

$$\left\{ y(x) = -C1 \sinh(x(\ln(x) - 1)) + -C2 \cosh(x(\ln(x) - 1)) \right\}$$

**2.1413 ODE No. 1413**

$$y''(x) = \frac{y'(x)}{x(\log(x) - 1)} - \frac{y(x)}{x^2(\log(x) - 1)}$$

✗ **Mathematica** : cpu = 0.351785 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -(y[x]/(x^2*(-1 + Log[x]))) + Derivative[1][y][x]/(x*(-1 + Log[x])), y[x], x]`

✓ **Maple** : cpu = 0.05 (sec), leaf count = 12

$$\{y(x) = \_C1 x + \_C2 \ln(x)\}$$

**2.1414 ODE No. 1414**

$$y''(x) = y(x) (-\operatorname{csch}^2(x)) (-a^2 \sinh^2(x) - (n - 1)n)$$

✓ **Mathematica** : cpu = 1.17538 (sec), leaf count = 127

$$\left\{ \left\{ y(x) \rightarrow \frac{(-1)^{-n} (-\operatorname{sech}^2(x))^{a/2} \tanh^2(x)^{-\frac{n}{2} - \frac{1}{4}} \left( c_1 (-1)^n \tanh^2(x)^{n + \frac{1}{2}} {}_2F_1\left(\frac{a+n}{2}, \frac{1}{2}(a+n+1); n + \frac{1}{2}; \tanh^2(x)\right) \right)}{\sqrt{\tanh(x)}} \right. \right.$$

✓ **Maple** : cpu = 0.267 (sec), leaf count = 97

$$\left\{ y(x) = \_C1 (\sinh(x))^n {}_2F_1\left(-\frac{a}{2} + \frac{n}{2}, \frac{a}{2} + \frac{n}{2}; \frac{1}{2}; \frac{\cosh(2x)}{2} + \frac{1}{2}\right) + \_C2 (\sinh(x))^n (2 \cosh(2x) + 2)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{2} + \frac{n}{2}, \frac{1}{2}(a+n+1); n + \frac{1}{2}; \tanh^2(x)\right) \right.$$

**2.1415 ODE No. 1415**

$$y''(x) = -(n^2 - a^2) y(x) - 2n \operatorname{coth}(x) y'(x)$$

✓ **Mathematica** : cpu = 0.857674 (sec), leaf count = 145

$$\left\{ \left\{ y(x) \rightarrow (-1)^{-n} (-\operatorname{sech}^2(x))^{\frac{a+1}{2}} \tanh^{-n - \frac{1}{2}}(x) \tanh^2(x)^{-\frac{n}{2} - \frac{1}{4}} \operatorname{sech}^2(x)^{\frac{n-1}{2}} \left( c_1 (-1)^n \tanh^2(x)^{n + \frac{1}{2}} {}_2F_1\left(\frac{a+n}{2}, \frac{1}{2}(a+n+1); n + \frac{1}{2}; \tanh^2(x)\right) \right) \right. \right.$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 36

$$\left\{ y(x) = (\sinh(x))^{-n + \frac{1}{2}} \left( \operatorname{LegendreQ}\left(a - \frac{1}{2}, n - \frac{1}{2}, \cosh(x)\right) \_C2 + \operatorname{LegendreP}\left(a - \frac{1}{2}, n - \frac{1}{2}, \cosh(x)\right) \_C1 \right) \right.$$

**2.1416 ODE No. 1416**

$$y''(x) = -(v - n)(n + v + 1)y(x) - (2n + 1) \cot(x)y'(x)$$

✓ **Mathematica** : cpu = 0.19508 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow (-\sin^2(x))^{-n/2} (c_1 P_v^n(\cos(x)) + c_2 Q_v^n(\cos(x))) \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 26

$$\{y(x) = (\sin(x))^{-n} (\text{LegendreP}(v, n, \cos(x))\_C1 + \text{LegendreQ}(v, n, \cos(x))\_C2)\}$$

**2.1417 ODE No. 1417**

$$y''(x) = -\csc(x)y'(x) (\sin^2(x) - \cos(x)) - y(x) \sin^2(x)$$

✓ **Mathematica** : cpu = 0.148977 (sec), leaf count = 45

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{\cos(x)}{2}} \left( c_1 \cos\left(\frac{1}{2}\sqrt{3}\cos(x)\right) + c_2 \sin\left(\frac{1}{2}\sqrt{3}\cos(x)\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.148 (sec), leaf count = 31

$$\left\{ y(x) = e^{\frac{\cos(x)}{2}} \left( \sin\left(\frac{\sqrt{3}\cos(x)}{2}\right) - C1 + \cos\left(\frac{\sqrt{3}\cos(x)}{2}\right) - C2 \right) \right\}$$

**2.1418 ODE No. 1418**

$$y''(x) = \frac{y(x) \sin(x)}{x \cos(x) - \sin(x)} - \frac{x \sin(x) y'(x)}{x \cos(x) - \sin(x)}$$

✗ **Mathematica** : cpu = 1.31387 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == (Sin[x]*y[x])/(x*Cos[x] - Sin[x]) - (x*Sin[x]*Derivative[1][y][x])`

✓ **Maple** : cpu = 15.536 (sec), leaf count = 59

$$\left\{ y(x) = \sin(x) \left( \int e^{\int \frac{-2(\cos(x))^3 x + 3(\cos(x))^2 \sin(x) - \sin(x)}{\cos(x)(\cos(x)x - \sin(x)) \sin(x)} dx} \cos(x) dx - C1 \right) \right\}$$

**2.1419 ODE No. 1419**

$$y''(x) = -\frac{\sec(x)y'(x)(x^2 \sin(x) - 2x \cos(x))}{x^2} - \frac{y(x) \sec(x)(2x \cos(x) - x \sin(x))}{x^2}$$

✗ **Mathematica** : cpu = 1.17378 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -(Sec[x]*(2*x*Cos[x] - x*Sin[x])*y[x])/x^2 - (Sec[x]*(-2*x*Cos[x] + x^2*Sin[x])*Derivative[1][y][x])/x^2, y[x], x]`

✓ **Maple** : cpu = 0.239 (sec), leaf count = 12

$$\{y(x) = x(\sin(x) \_C2 + \_C1)\}$$

**2.1420 ODE No. 1420**

$$\cos^2(x)y''(x) - y(x)(a \cos^2(x) + (n-1)n) = 0$$

✓ **Mathematica** : cpu = 0.452775 (sec), leaf count = 126

$$\left\{ \left\{ y(x) \rightarrow c_1 i^{1-n} \cos^{1-n}(x) {}_2F_1\left(\frac{1}{2}(-n - i\sqrt{a} + 1), \frac{1}{2}(-n + i\sqrt{a} + 1); \frac{3}{2} - n; \cos^2(x)\right) + c_2 i^n \cos^n(x) {}_2F_1\left(\frac{1}{2}(n - i\sqrt{a} + 1), \frac{1}{2}(n + i\sqrt{a} + 1); \frac{3}{2} - n; \cos^2(x)\right) \right\} \right.$$

✓ **Maple** : cpu = 0.325 (sec), leaf count = 123

$$\left\{ y(x) = \_C1 \sin(2x) (\cos(x))^{-n} {}_2F_1\left(1 + \frac{i}{2}\sqrt{a} - \frac{n}{2}, 1 - \frac{i}{2}\sqrt{a} - \frac{n}{2}; \frac{3}{2} - n; \frac{\cos(2x)}{2} + \frac{1}{2}\right) + \_C2 (\cos(x))^n (-2)^n \right.$$

**2.1421 ODE No. 1421**

$$y''(x) = -a^2 n y(x) \sec^2(ax) ((n-1) \sin^2(ax) + \cos^2(ax)) - a(n-1) \sin(2ax) \sec^2(ax) y'(x)$$

✓ **Mathematica** : cpu = 0.252065 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-\sqrt{-a^2}x} \left( \frac{c_2 e^{2\sqrt{-a^2}x}}{\sqrt{-a^2}} + 2c_1 \right) \cos^{n-1}(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.076 (sec), leaf count = 27

$$\{y(x) = \_C1 (\cos(ax))^n + \_C2 \sin(ax) (\cos(ax))^{n-1}\}$$

**2.1422 ODE No. 1422**

$$y''(x) = 2y(x) \csc^2(x)$$

✓ **Mathematica** : cpu = 0.0885472 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow \frac{\cos(x) \left( c_2 \log \left( \sqrt{-\sin^2(x)} + \cos(x) \right) + c_1 \right) - c_2 \sqrt{-\sin^2(x)}}{\sqrt{-\sin^2(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.254 (sec), leaf count = 50

$$\left\{ y(x) = \frac{-i \ln(\cos(2x) + i \sin(2x)) \sin(2x) - C2 + C1 \sin(2x) + 2 C2 (\cos(2x) - 1)}{\cos(2x) - 1} \right\}$$

**2.1423 ODE No. 1423**

$$y''(x) = -ay(x) \csc^2(x)$$

✓ **Mathematica** : cpu = 0.0737422 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \sqrt[4]{-\sin^2(x)} \left( c_1 P_{-\frac{1}{2}}^{\frac{1}{2}\sqrt{1-4a}}(\cos(x)) + c_2 Q_{-\frac{1}{2}}^{\frac{1}{2}\sqrt{1-4a}}(\cos(x)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.28 (sec), leaf count = 132

$$\left\{ y(x) = 1 \sqrt[4]{2 \cos(2x) + 2} \left( \frac{\cos(2x)}{2} - \frac{1}{2} \right)^{\frac{1}{4}\sqrt{1-4a}} \sqrt{-2 \cos(2x) + 2} \left( {}_2F_1\left(\frac{1}{4}\sqrt{1-4a} + \frac{3}{4}, \frac{1}{4}\sqrt{1-4a} + \frac{3}{4}; \frac{3}{2}; \frac{\cos(2x)}{2} - \frac{1}{2}\right) + c_2 Q_{-\frac{1}{2}}^{\frac{1}{2}\sqrt{1-4a}}(\cos(x)) \right) \right\}$$

**2.1424 ODE No. 1424**

$$\sin^2(x)y''(x) - y(x) (a \sin^2(x) + (n-1)n) = 0$$

✓ **Mathematica** : cpu = 0.171567 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \sqrt[4]{-\sin^2(x)} \left( c_1 P_{i\sqrt{a}-\frac{1}{2}}^{n-\frac{1}{2}}(\cos(x)) + c_2 Q_{i\sqrt{a}-\frac{1}{2}}^{n-\frac{1}{2}}(\cos(x)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.289 (sec), leaf count = 120

$$\left\{ y(x) = 1 \left( \frac{\cos(2x)}{2} - \frac{1}{2} \right)^{\frac{n}{2}} \left( {}_2F_1\left(\frac{1}{2} + \frac{i}{2}\sqrt{a} + \frac{n}{2}, \frac{1}{2} - \frac{i}{2}\sqrt{a} + \frac{n}{2}; \frac{3}{2}; \frac{\cos(2x)}{2} - \frac{1}{2}\right) (2 \cos(2x) + 2)^{\frac{3}{4}} \sqrt[4]{-2 \cos(2x) + 2} \right) \right\}$$



**2.1425 ODE No. 1425**

$$y''(x) = y(x) \csc^2(x) (-(-a^2 \cos^2(x) - (3 - 2a) \cos(x) + 3a - 3))$$

✓ **Mathematica** : cpu = 0.741325 (sec), leaf count = 194

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 \sin^2(x)^{a/2} (-2a \cos(x) + \cos(x) + 2)}{1 - \cos(x)} - \frac{c_2 \sin^2(x)^{-a} (1 - \cos(x))^{\frac{a-1}{2}} (\cos(x) + 1)^{\frac{a+1}{2}} \left( \frac{(2a-1)(\cos(x)-1)}{(2a-1)\cos(x)-2} \right)}{1 - \cos(x)} \right. \right.$$

✓ **Maple** : cpu = 0.47 (sec), leaf count = 91

$$\left\{ y(x) = 1 \left( -C_2 {}_2F_1 \left( a - \frac{1}{2}, -\frac{1}{2} - a; \frac{3}{2} - a; \frac{\cos(x)}{2} + \frac{1}{2} \right) (\cos(x) + 1)^{-\frac{1}{4} - \frac{a}{2}} \sqrt{2 \cos(x) + 2} (\cos(x) - 1)^{\frac{a}{2} - \frac{1}{4}} + 2 \right) \right.$$

**2.1426 ODE No. 1426**

$$\sin^2(x)y''(x) - y(x) \left( a^2 \cos^2(x) + \frac{b^2}{(2a - 3)^2} + 3a + b \cos(x) + 2 \right) = 0$$

✓ **Mathematica** : cpu = 6.12124 (sec), leaf count = 1362

$$\left\{ \left\{ y(x) \rightarrow \frac{(-1)^{-\frac{(2a+3)^2}{(3-2a)^2}} 2^{-\frac{\sqrt{(3-2a)^2(16a^4+8(2b-9)a^2-48ba+(2b+9)^2)}}{2(3-2a)^2}} (1 - \cos(x))^{\frac{-4a^2-9}{(3-2a)^2}} (\cos(x) - 1)^{-\frac{-8a^2+24a+\sqrt{(3-2a)^2(16a^4+8(2b-9)a^2-48ba+(2b+9)^2)}}{4(3-2a)^2}}}{1 - \cos(x)} \right. \right.$$

✓ **Maple** : cpu = 0.643 (sec), leaf count = 549

$$\left\{ y(x) = 1 \left( \frac{\cos(x)}{2} - \frac{1}{2} \right)^{\frac{1}{8a-12}} \left( 4a-6+\sqrt{4b^2+16(a-3/2)^2b+16a^4-72a^2+81} \right) \left( {}_2F_1 \left( \frac{1}{8a-12} \left( 8a^2 - \sqrt{4b^2 - 16(a-3/2)^2b+16a^4-72a^2+81} \right) \right) \right)$$

**2.1427 ODE No. 1427**

$$y''(x) = y(x) (-\csc^2(x)) (-(a^2b^2 - (a+1)^2) \sin^2(x) - a(a+1)b \sin(2x) - (a-1)a)$$

✗ **Mathematica** : cpu = 200.964 (sec), leaf count = 0 , could not solve

DSolve[Derivative[2][y][x] == -(Csc[x]^2\*((-1+a)\*a) - ((-1+a)^2 + a^2\*b^2)\*Sin[x]^2 -

✓ **Maple** : cpu = 1.68 (sec), leaf count = 179

$$\left\{ y(x) = 1e^{\int \frac{1}{\sin(2x)(b \sin(2x) + \cos(2x) + 1)} (2b((a+1) \cos(2x) + a + 1/2) \sin(2x) - ((ab^2 - a - 2) \cos(2x) - ab^2 - a + 1)(\cos(2x) + 1)) dx} \left( \int -2e^{-\dots} \right) \right\}$$

**2.1428 ODE No. 1428**

$$y''(x) = y(x) (-\csc^2(x)) (a \cos^2(x) + b \sin^2(x) + c)$$

✓ **Mathematica** : cpu = 0.427681 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow \sqrt[4]{-\sin^2(x)} \left( c_1 P^{\frac{1}{2}\sqrt{-4a-4c+1}}(\cos(x)) + c_2 Q^{\frac{1}{2}\sqrt{-4a-4c+1}}(\cos(x)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.323 (sec), leaf count = 183

$$\left\{ y(x) = 1 \sqrt[4]{2 \cos(2x) + 2} \left( \frac{\cos(2x)}{2} - \frac{1}{2} \right)^{\frac{1}{4}\sqrt{-4a+1-4c}} \sqrt{-2 \cos(2x) + 2} \left( {}_2F_1\left(\frac{1}{4}\sqrt{-4a+1-4c} + \frac{1}{2}\sqrt{-a+b}\right) \right) \right\}$$

**2.1429 ODE No. 1429**

$$y''(x) = y(x) \csc^2(x) - \cot(x)y'(x)$$

✓ **Mathematica** : cpu = 0.0586197 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow c_1 \cosh \left( \log \left( \cos \left( \frac{x}{2} \right) \right) - \log \left( \sin \left( \frac{x}{2} \right) \right) \right) - ic_2 \sinh \left( \log \left( \cos \left( \frac{x}{2} \right) \right) - \log \left( \sin \left( \frac{x}{2} \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 25

$$\left\{ y(x) = \frac{\sin(x) - C1}{\cos(x) - 1} + \frac{(\cos(x) - 1) - C2}{\sin(x)} \right\}$$

**2.1430 ODE No. 1430**

$$y''(x) = y(x) \csc^2(x) (-(v(v+1) \sin^2(x) - n^2)) - \cot(x)y'(x)$$

✓ **Mathematica** : cpu = 0.467489 (sec), leaf count = 22

$$\{ \{ y(x) \rightarrow c_1 P_v^n(\cos(x)) + c_2 Q_v^n(\cos(x)) \} \}$$

✓ **Maple** : cpu = 0.366 (sec), leaf count = 101

$$\left\{ y(x) = 1 \left( \frac{\cos(2x)}{2} - \frac{1}{2} \right)^{\frac{n}{2}} \left( \sin(2x) {}_2F_1 \left( 1 + \frac{v}{2} + \frac{n}{2}, \frac{1}{2} - \frac{v}{2} + \frac{n}{2}; \frac{3}{2}; \frac{\cos(2x)}{2} + \frac{1}{2} \right) {}_2F_1 \left( -\frac{v}{2} + \frac{n}{2}, \frac{1}{2} + \frac{v}{2} \right) \right) {}_2F_1 \left( -\frac{v}{2} + \frac{n}{2}, \frac{1}{2} + \frac{v}{2} \right) \right\}$$

**2.1431 ODE No. 1431**

$$y''(x) = \cot(2x)y'(x) - 2y(x)$$

✓ **Mathematica** : cpu = 0.200745 (sec), leaf count = 64

$$\left\{ \left\{ y(x) \rightarrow -\frac{2}{3} c_2 \cos(2x) \cos^{\frac{3}{2}}(x) {}_2F_1 \left( \frac{1}{4}, \frac{3}{4}; \frac{7}{4}; \cos^2(x) \right) + \frac{1}{2} c_1 \cos(2x) - 2c_2 \sin^2(x)^{3/4} \cos^{\frac{3}{2}}(x) \right\} \right\}$$

✓ **Maple** : cpu = 0.233 (sec), leaf count = 30

$$\left\{ y(x) = (\sin(2x))^{\frac{3}{4}} \left( LegendreQ \left( \frac{1}{4}, \frac{3}{4}, \cos(2x) \right) {}_2F_1 \left( \frac{1}{4}, \frac{3}{4}; \frac{7}{4}; \cos^2(x) \right) + LegendreP \left( \frac{1}{4}, \frac{3}{4}, \cos(2x) \right) \right) \right\}$$

**2.1432 ODE No. 1432**

$$y''(x) = -\cot(x)y'(x) - \frac{1}{4}y(x) (-17 \sin^2(x) - 1) \csc^2(x)$$

✓ **Mathematica** : cpu = 0.100006 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-2x} (c_2 e^{4x} + 4c_1)}{4\sqrt{\sin(x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 22

$$\left\{ y(x) = ({}_2F_1 \left( -\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \sin(x) \right) \cosh(2x) + {}_2F_1 \left( -\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \sin(x) \right) \sinh(2x)) \frac{1}{\sqrt{\sin(x)}} \right\}$$

**2.1433 ODE No. 1433**

$$y''(x) = -\frac{y(x) \sec^2(x) (2x^2 + x^2 \sin^2(x) - 24 \cos^2(x))}{4x^2} - \tan(x)y'(x) + \sqrt{\cos(x)}$$

✓ **Mathematica** : cpu = 0.242178 (sec), leaf count = 35

$$\left\{ \left\{ y(x) \rightarrow \frac{(4c_2x^5 + 20c_1 - 5x^4) \sqrt{\cos(x)}}{20x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.116 (sec), leaf count = 28

$$\left\{ y(x) = \frac{4x^5\_C1 - x^4 + 4\_C2}{4x^2} \sqrt{\cos(x)} \right\}$$

**2.1434 ODE No. 1434**

$$y''(x) = -\frac{b \cot(x)y'(x)}{a} - \frac{y(x) \csc^2(x) (c \cos^2(x) + d \cos(x) + e)}{a}$$

✓ **Mathematica** : cpu = 107.774 (sec), leaf count = 1

\$Aborted

✓ **Maple** : cpu = 0.68 (sec), leaf count = 517

$$\left\{ y(x) = (\sin(x))^{-\frac{a+b}{2a}} \left( \frac{\cos(x)}{2} - \frac{1}{2} \right)^{\frac{1}{4a} (2a + \sqrt{a^2 + (-2b - 4c - 4d - 4e)a + b^2})} \left( {}_2F_1\left(\frac{1}{4a} \left( \sqrt{a^2 + (-2b - 4c - 4d - 4e)a + b^2} \right), \cos(x) \right) \right) \right\}$$

**2.1435 ODE No. 1435**

$$y''(x) = -4y(x) \sin(3x) \csc^3(x)$$

✓ **Mathematica** : cpu = 0.149605 (sec), leaf count = 61

$$\left\{ \left\{ y(x) \rightarrow \sqrt[4]{-\sin^2(x)} \left( c_1 P_{-\frac{1}{2}+4i}^{\frac{i\sqrt{47}}{2}}(\cos(x)) + c_2 Q_{-\frac{1}{2}+4i}^{\frac{i\sqrt{47}}{2}}(\cos(x)) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.171 (sec), leaf count = 38

$$\left\{ y(x) = \sqrt{\sin(x)} \left( LegendreQ\left(-\frac{1}{2} + 4i, \frac{i}{2}\sqrt{47}, \cos(x)\right) - C2 + LegendreP\left(-\frac{1}{2} + 4i, \frac{i}{2}\sqrt{47}, \cos(x)\right) - C1 \right) \right\}$$

**2.1436 ODE No. 1436**

$$y''(x) = -\frac{1}{4}y(x) \csc^2(x) (-4n^2 + 4v(v+1) \sin^2(x) - \cos^2(x) + 2)$$

✓ **Mathematica** : cpu = 0.548315 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \sqrt[4]{-\sin^2(x)} (c_1 P_v^n(\cos(x)) + c_2 Q_v^n(\cos(x))) \right\} \right\}$$

✓ **Maple** : cpu = 0.281 (sec), leaf count = 113

$$\left\{ y(x) = 1 \sqrt{-2 \cos(2x) + 2} \sqrt[4]{2 \cos(2x) + 2} \left( \frac{\cos(2x)}{2} - \frac{1}{2} \right)^{\frac{n}{2}} \left( {}_2F_1\left(1 + \frac{v}{2} + \frac{n}{2}, \frac{1}{2} - \frac{v}{2} + \frac{n}{2}; \frac{3}{2}; \frac{\cos(2x)}{2} + \frac{1}{2}\right) \right) \right\}$$

**2.1437 ODE No. 1437**

$$y''(x) = (3 \sin^2(x) + 1) \csc(x) \sec(x) y'(x) + y(x) \tan^2(x)$$

✓ **Mathematica** : cpu = 0.285193 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow \cos^{-\frac{3}{2} - \frac{\sqrt{13}}{2}}(x) (c_1 \cos^{\sqrt{13}}(x) + c_2) \right\} \right\}$$

✓ **Maple** : cpu = 0.177 (sec), leaf count = 29

$$\left\{ y(x) = \_C1 (\cos(x))^{-\frac{3}{2} + \frac{\sqrt{13}}{2}} + \_C2 (\cos(x))^{-\frac{3}{2} - \frac{\sqrt{13}}{2}} \right\}$$

**2.1438 ODE No. 1438**

$$y''(x) = y(x) (-\csc^2(x)) \sec^2(x) (-a \sin^2(x) \cos^2(x) - (m-1)m \sin^2(x) - (n-1)n \cos^2(x))$$

✓ **Mathematica** : cpu = 0.977446 (sec), leaf count = 158

$$\left\{ \left\{ y(x) \rightarrow \frac{(-1)^{-m} \cos^2(x)^{-\frac{m}{2} - \frac{1}{4}} (-\sin^2(x))^{n/2} \left( c_1 (-1)^m \cos^2(x)^{m+\frac{1}{2}} {}_2F_1\left(\frac{1}{2}(m+n-\sqrt{-a}), \frac{1}{2}(m+n+\sqrt{-a}); \sqrt{c} \right) \right)}{\sqrt{c}} \right\} \right\}$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 102

$$\left\{ y(x) = (\sin(x))^n \left( (\cos(x))^{-m+1} {}_2F_1\left(\frac{n}{2} - \frac{m}{2} + \frac{i}{2}\sqrt{a} + \frac{1}{2}, \frac{n}{2} - \frac{m}{2} - \frac{i}{2}\sqrt{a} + \frac{1}{2}; \frac{3}{2} - m; (\cos(x))^2\right) \_C2 + (\cos(x)) \right) \right\}$$

**2.1439 ODE No. 1439**

$$y''(x) = \frac{\phi'(x)y'(x)}{\phi(x) - \phi(a)} - \frac{y(x) (\phi''(a) - n(n+1)(\phi(x) - \phi(a))^2)}{\phi(x) - \phi(a)}$$

✗ **Mathematica** : cpu = 0.820822 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == (Derivative[1][phi][x]*Derivative[1][y][x])/(-phi[a] + phi[x]) - (n*(1 + n)*(-phi[a] + phi[x])^2 + Derivative[2][phi][a])/(-phi[a] + phi[x]), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) - \frac{\left(\frac{d}{dx} \phi(x)\right) \frac{d}{dx} Y(x)}{\phi(x) - \phi(a)} + \frac{\left(-n(n+1)(\phi(x) - \phi(a))^2 + \frac{d^2}{da^2} \phi(a)\right) - Y(x)}{\phi(x) - \phi(a)} \right\}, \{ \_Y \} \right.$$

**2.1440 ODE No. 1440**

$$y''(x) = -\frac{y'(x) (-\phi''(x) - \phi(x)\phi'(x) + \phi(x^3))}{\phi'(x) + \phi(x)^2} - \frac{y(x) (-\phi(x)\phi''(x) + \phi(x)^2(-\phi'(x) + \phi'(x)^2))}{\phi'(x) + \phi(x)^2}$$

✗ **Mathematica** : cpu = 0.962321 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -((Derivative[1][y][x]*(phi[x]^3 - phi[x]*Derivative[1][phi][x]) - (phi[x]^2*Derivative[1][phi][x]) + Derivative[1][phi][x]^2 - phi[x]*Derivative[2][phi][x]))/(\phi'(x) + \phi(x)^2), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{\left(\left(\frac{d}{dx} \phi(x)\right)^2 - (\phi(x))^2 \frac{d}{dx} \phi(x) - \phi(x) \frac{d^2}{dx^2} \phi(x)\right) - Y(x)}{\frac{d}{dx} \phi(x) + (\phi(x))^2} + \frac{\left(\phi(x^3) - \phi(x) \frac{d}{dx} \phi(x) - \frac{d^2}{dx^2} \phi(x)\right)}{\frac{d}{dx} \phi(x) + (\phi(x))^2} \right\}, \{ \_Y \} \right.$$

**2.1441 ODE No. 1441**

$$y''(x) = -\frac{y'(x)(-\operatorname{cn}(x|k)\operatorname{dn}(x|k) - 2\operatorname{sn}(x|k))}{\operatorname{sn}(x|k)^2 - \operatorname{sn}(a|k)^2} - \frac{y(x) (6k^2\operatorname{sn}(a|k)^4 - 4(k^2 + 1)\operatorname{sn}(a|k)^2 + 2)}{\operatorname{sn}(x|k)^2 - \operatorname{sn}(a|k)^2} - \frac{1}{\operatorname{sn}(x|k)^2 - \operatorname{sn}(a|k)^2}$$

✗ **Mathematica** : cpu = 1.57714 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[2][y][x] == -(-JacobiSN[a, k]^2 + JacobiSN[x, k]^2)^(-1) - ((2 - 4*(1 + k^2) JacobiSN[a, k]^2 + JacobiSN[x, k]^2) - ((-JacobiCN[x, k]*JacobiDN[x, k]) - 2*JacobiSN[x, k]^2) JacobiSN[a, k]^2 + JacobiSN[x, k]^2), y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) - 2 \frac{JacobiSN(x, k) JacobiCN(x, k) JacobiDN(x, k) \frac{d}{dx} Y(x)}{(JacobiSN(x, k))^2 - JacobiSN(a, k)} - \frac{(-2 + 4(k^2 + 1) JacobiSN(a, k)^4 - 4(k^2 + 1) JacobiSN(a, k)^2 + 2)}{\operatorname{sn}(x|k)^2 - \operatorname{sn}(a|k)^2} \right\}, \{ \_Y \} \right.$$

2.1442 ODE No. 1442

$$y''(x) = \frac{y(x)}{f(x)} - \frac{xy'(x)}{f(x)}$$

✗ **Mathematica** : cpu = 0.219919 (sec), leaf count = 0 , could not solve

DSolve[Derivative[2][y][x] == y[x]/f[x] - (x\*Derivative[1][y][x])/f[x], y[x], x]

✓ **Maple** : cpu = 0.07 (sec), leaf count = 30

$$\left\{ y(x) = x \left( \int e^{\int \frac{1}{x} \left( -2 - \frac{x^2}{f(x)} \right) dx} dx \_C1 + \_C2 \right) \right\}$$

2.1443 ODE No. 1443

$$y''(x) = -\frac{f'(x)y'(x)}{2f(x)} - \frac{g(x)y(x)}{f(x)}$$

✗ **Mathematica** : cpu = 0.282281 (sec), leaf count = 0 , could not solve

DSolve[Derivative[2][y][x] == -(g[x]\*y[x])/f[x] - (Derivative[1][f][x]\*Derivative[1][y][x])

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{g(x) - Y(x)}{f(x)} + \frac{\left( \frac{d}{dx} f(x) \right) \frac{d}{dx} Y(x)}{2 f(x)} + \frac{d^2}{dx^2} Y(x) \right\}, \{ -Y(x) \} \right) \right\}$$

2.1444 ODE No. 1444

$$y''(x) = -by(x)f(x)^{2a} - \frac{af'(x)y'(x)}{f(x)}$$

✗ **Mathematica** : cpu = 1.49525 (sec), leaf count = 0 , could not solve

DSolve[Derivative[2][y][x] == -(b\*f[x]^(2\*a)\*y[x]) - (a\*Derivative[1][f][x]\*Derivative[1][y][x])

✓ **Maple** : cpu = 0.015 (sec), leaf count = 37

$$\left\{ y(x) = \_C1 e^{\int i(f(x))^a \sqrt{b} dx} + \_C2 e^{-\int i(f(x))^a \sqrt{b} dx} \right\}$$

**2.1445 ODE No. 1445**

$$y''(x) = -\frac{y'(x)(2f(x)g(x)g'(x)^2 - (g(x)^2 - 1)(2f'(x)g'(x) + f(x)g''(x)))}{f(x)(g(x)^2 - 1)g'(x)} - \frac{y(x)((g(x)^2 - 1)(f'(x)(2f'(x)g'(x) + f(x)g''(x)))}{f(x)(g(x)^2 - 1)g'(x)}$$

✗ **Mathematica** : cpu = 1.3342 (sec), leaf count = 0 , could not solve

```
DSolve[Derivative[2][y][x] == -(Derivative[1][y][x]*(2*f[x]*g[x]*Derivative[1][g][x]^2 - (1 + g[x]^2)*(2*Derivative[1][f][x]*Derivative[1][g][x] + f[x]*Derivative[2][g][x]))/(f[x]*(1 + g[x]^2)*Derivative[1][g][x]) - (y[x]*(-(f[x]*Derivative[1][g][x]^2*(2*g[x]*Derivative[1][f][x] + g[x]^2)*(-f[x]*Derivative[1][g][x]*Derivative[2][f][x]) + Derivative[1][f][x]*(2*Derivative[1][g][x]*Derivative[1][g][x])), y[x], x]
```

✓ **Maple** : cpu = 0.279 (sec), leaf count = 20

$$\{y(x) = f(x)(\text{LegendreP}(v, g(x))\_C1 + \text{LegendreQ}(v, g(x))\_C2)\}$$

**2.1446 ODE No. 1446**

$$y''(x) = -\frac{(x-1)y(x)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0255332 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow e^{-1/x} \left( c_1 - c_2 \text{Ei} \left( \frac{2}{x} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 22

$$\{y(x) = e^{-x^{-1}}(\text{Ei}(1, -2x^{-1})\_C2 +\_C1)\}$$

**2.1447 ODE No. 1447**

$$y''(x) = -\frac{(-x-1)y(x)}{x^4} - \frac{y'(x)}{x}$$

✓ **Mathematica** : cpu = 0.0229845 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{1}{x}} \left( c_1 - c_2 \text{Ei} \left( -\frac{2}{x} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 20

$$\{y(x) = e^{x^{-1}}(\text{Ei}(1, 2x^{-1})\_C2 +\_C1)\}$$



2.1448 ODE No. 1448

$$y''(x) = -\frac{b^2 y(x)}{(x^2 - a^2)^2}$$

✓ **Mathematica** : cpu = 0.339448 (sec), leaf count = 142

$$\left\{ \left\{ y(x) \rightarrow \frac{(x-a)^{\frac{1}{2}-\frac{1}{2}} \sqrt{1-\frac{b^2}{a^2}} (a+x)^{\frac{1}{2}-\frac{1}{2}} \sqrt{1-\frac{b^2}{a^2}} \left( 2ac_1 \sqrt{1-\frac{b^2}{a^2}} (x-a) \sqrt{1-\frac{b^2}{a^2}} - c_2 (a+x) \sqrt{1-\frac{b^2}{a^2}} \right)}{2a \sqrt{1-\frac{b^2}{a^2}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.103 (sec), leaf count = 77

$$\left\{ y(x) = \sqrt{(a-x)(x+a)} \left( \left( \frac{a-x}{x+a} \right)^{-\frac{1}{2a} \sqrt{a^2-b^2}} - C2 + \left( \frac{a-x}{x+a} \right)^{\frac{1}{2a} \sqrt{a^2-b^2}} - C1 \right) \right\}$$

2.1449 ODE No. 1449

$$y^{(3)}(x) - \lambda y(x) = 0$$

✓ **Mathematica** : cpu = 0.0199639 (sec), leaf count = 53

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{(-1)^{2/3} \sqrt[3]{\lambda} x} + c_2 e^{-\sqrt[3]{-1} \sqrt[3]{\lambda} x} + c_3 e^{\sqrt[3]{\lambda} x} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 47

$$\left\{ y(x) = -C1 e^{-\frac{(i\sqrt{3}+1)x}{2} \sqrt[3]{\lambda}} + -C2 e^{\frac{(i\sqrt{3}-1)x}{2} \sqrt[3]{\lambda}} + -C3 e^{\sqrt[3]{\lambda} x} \right\}$$

2.1450 ODE No. 1450

$$ax^3 y(x) - bx + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 300.01 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.233 (sec), leaf count = 1616

$$\left\{ y(x) = \int -11211200 bx^3 \left( \left( -5/8 x^6 {}_0F_2 \left( ; \frac{13}{6}, 7/3; -\frac{x^6 a}{216} \right) a + 35 {}_0F_2 \left( ; 7/6, 4/3; -\frac{x^6 a}{216} \right) \right) {}_0F_2 \left( ; 5/6, 7/6; -\frac{x^6}{216} \right) \right) dx \right\}$$

**2.1451 ODE No. 1451**

$$y^{(3)}(x) - ax^b y(x) = 0$$

✓ **Mathematica** : cpu = 0.0224092 (sec), leaf count = 164

$$\left\{ \left\{ y(x) \rightarrow (-1)^{\frac{1}{b+3}} (b+3)^{-\frac{6}{b+3}} x a^{\frac{1}{b+3}} \left( (-1)^{\frac{1}{b+3}} c_3 x a^{\frac{1}{b+3}} {}_0F_2 \left( ; 1 + \frac{1}{b+3}, 1 + \frac{2}{b+3}; \frac{ax^{b+3}}{(b+3)^3} \right) + (b+3)^{\frac{3}{b+3}} c_2 {}_0F_2 \left( ; \frac{1}{b+3}, \frac{2}{b+3}; \frac{ax^{b+3}}{(b+3)^3} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 114

$$\left\{ y(x) = -C1 {}_0F_2 \left( ; \frac{b+1}{b+3}, \frac{b+2}{b+3}; \frac{x^{b+3}a}{(b+3)^3} \right) + -C2 x {}_0F_2 \left( ; \frac{b+2}{b+3}, \frac{b+4}{b+3}; \frac{x^{b+3}a}{(b+3)^3} \right) + -C3 x^2 {}_0F_2 \left( ; \frac{b+4}{b+3}, \frac{b+5}{b+3}; \frac{x^{b+3}a}{(b+3)^3} \right) \right\}$$

**2.1452 ODE No. 1452**

$$y^{(3)}(x) + 3y'(x) - 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.00841401 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow e^{-x/2} \left( c_3 e^{3x/2} + c_1 \sin \left( \frac{\sqrt{15}x}{2} \right) + c_2 \cos \left( \frac{\sqrt{15}x}{2} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.008 (sec), leaf count = 35

$$\left\{ y(x) = -C1 e^x + -C2 e^{-\frac{x}{2}} \sin \left( \frac{\sqrt{15}x}{2} \right) + -C3 e^{-\frac{x}{2}} \cos \left( \frac{\sqrt{15}x}{2} \right) \right\}$$

**2.1453 ODE No. 1453**

$$a^2(-y'(x)) - e^{2ax} \sin^2(x) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.656161 (sec), leaf count = 128

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-ax} (-9(a^2 - 4) a^4 e^{3ax} \cos(2x) - 3(11a^2 - 4) a^3 e^{3ax} \sin(2x) + (9a^6 + 49a^4 + 56a^2 + 16) (12a^2 c_1 e^{2ax} + 12a^2 c_2 e^{2ax}))}{12a^3 (9a^6 + 49a^4 + 56a^2 + 16)} \right\} \right\}$$

✓ **Maple** : cpu = 0.138 (sec), leaf count = 122

$$\left\{ y(x) = \frac{1}{108 a^9 + 588 a^7 + 672 a^5 + 192 a^3} \left( (-9 a^6 + 36 a^4) \cos(2x) + (-33 a^5 + 12 a^3) \sin(2x) + 9 a^6 + 49 a^4 \right) \right\}$$

**2.1454 ODE No. 1454**

$$2axy'(x) + ay(x) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0110635 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow c_1 \text{Ai} \left( \sqrt[3]{-\frac{1}{2} \sqrt[3]{ax}} \right)^2 + c_3 \text{Bi} \left( \sqrt[3]{-\frac{1}{2} \sqrt[3]{ax}} \right)^2 + c_2 \text{Ai} \left( \sqrt[3]{-\frac{1}{2} \sqrt[3]{ax}} \right) \text{Bi} \left( \sqrt[3]{-\frac{1}{2} \sqrt[3]{ax}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 55

$$\left\{ y(x) = -C1 \left( \text{Ai} \left( -\frac{2^{\frac{2}{3}}x}{2} \sqrt[3]{a} \right) \right)^2 + -C2 \left( \text{Bi} \left( -\frac{2^{\frac{2}{3}}x}{2} \sqrt[3]{a} \right) \right)^2 + -C3 \text{Ai} \left( -\frac{2^{\frac{2}{3}}x}{2} \sqrt[3]{a} \right) \text{Bi} \left( -\frac{2^{\frac{2}{3}}x}{2} \sqrt[3]{a} \right) \right\}$$

**2.1455 ODE No. 1455**

$$x(a + b - 1)y'(x) - aby(x) + x^2(-y''(x)) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0290095 (sec), leaf count = 127

$$\left\{ \left\{ y(x) \rightarrow \sqrt[3]{-\frac{1}{3}} c_2 x {}_2F_2 \left( \frac{1}{3} - \frac{a}{3}, \frac{1}{3} - \frac{b}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{3} \right) + c_1 {}_2F_2 \left( -\frac{a}{3}, -\frac{b}{3}; \frac{1}{3}, \frac{2}{3}; \frac{x^3}{3} \right) + \left( -\frac{1}{3} \right)^{2/3} c_3 x^2 {}_2F_2 \left( \frac{2}{3} - \frac{a}{3}, \frac{2}{3} - \frac{b}{3}; \frac{4}{3}, \frac{2}{3}; \frac{x^3}{3} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.145 (sec), leaf count = 71

$$\left\{ y(x) = -C1 {}_2F_2 \left( -\frac{a}{3}, -\frac{b}{3}; \frac{1}{3}, \frac{2}{3}; \frac{x^3}{3} \right) + -C2 x {}_2F_2 \left( \frac{1}{3} - \frac{b}{3}, \frac{1}{3} - \frac{a}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{3} \right) + -C3 x^2 {}_2F_2 \left( -\frac{a}{3} + \frac{2}{3}, -\frac{b}{3} + \frac{2}{3}; \frac{4}{3}, \frac{2}{3}; \frac{x^3}{3} \right) \right\}$$

**2.1456 ODE No. 1456**

$$x^{2c-2}y'(x) + (c-1)x^{2c-3}y(x) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0394625 (sec), leaf count = 183

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_1F_2 \left( \frac{1}{2} - \frac{1}{2c}; 1 - \frac{1}{c}, 1 - \frac{1}{2c}; -\frac{x^{2c}}{4c^2} \right) + 4^{-1/c} c_3 c^{-2/c} (x^{2c})^{\frac{1}{c}} {}_1F_2 \left( \frac{1}{2} + \frac{1}{2c}; 1 + \frac{1}{2c}, 1 + \frac{1}{c}; -\frac{x^{2c}}{4c^2} \right) + 2^{-1/c} c_2 c^{-1/c} (x^{2c})^{\frac{1}{c}} {}_1F_2 \left( \frac{1}{2}; 1 + \frac{1}{2c}, 1 + \frac{1}{c}; -\frac{x^{2c}}{4c^2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 73

$$\left\{ y(x) = x \left( \left( J_{\frac{1}{2c}} \left( \frac{x^c}{2c} \right) \right)^2 - C1 + J_{\frac{1}{2c}} \left( \frac{x^c}{2c} \right) Y_{\frac{1}{2c}} \left( \frac{x^c}{2c} \right) - C3 + \left( Y_{\frac{1}{2c}} \left( \frac{x^c}{2c} \right) \right)^2 - C2 \right) \right\}$$

**2.1457 ODE No. 1457**

$$-3y'(x)(a + 2\wp(x; g2, g3)) + by(x) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0498492 (sec), leaf count = 0 , could not solve

DSolve[b\*y[x] - 3\*(a + 2\*WeierstrassP[x, {g2, g3}])\*Derivative[1][y][x] + Derivative[3][y][x]

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^3}{dx^3} Y(x) + (-6 \text{WeierstrassP}(x, g2, g3) - 3a) \frac{d}{dx} Y(x) + b Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

**2.1458 ODE No. 1458**

$$\frac{1}{2}y(x) ((1 - n^2) \wp'(x; g2, g3) - a) + (1 - n^2) y'(x)\wp(x; g2, g3) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.021 (sec), leaf count = 0 , could not solve

DSolve[((-a + (1 - n^2)\*WeierstrassPPrime[x, {g2, g3}])\*y[x])/2 + (1 - n^2)\*WeierstrassP[x,

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^3}{dx^3} Y(x) + (-n^2 \text{WeierstrassP}(x, g2, g3) + \text{WeierstrassP}(x, g2, g3)) \frac{d}{dx} Y(x) + \left( -\frac{a}{2} \right) Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

**2.1459 ODE No. 1459**

$$-y'(x)(a + 4n(n + 1)\wp(x; g2, g3)) - 2n(n + 1)y(x)\wp'(x; g2, g3) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0197898 (sec), leaf count = 0 , could not solve

DSolve[-2\*n\*(1 + n)\*WeierstrassPPrime[x, {g2, g3}]\*y[x] - (a + 4\*n\*(1 + n)\*WeierstrassP[x,

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \left( DESol \left( \left\{ \frac{d^2}{dx^2} Y(x) + \left( -n^2 \text{WeierstrassP}(x, g2, g3) - n \text{WeierstrassP}(x, g2, g3) - \frac{a}{4} \right) Y(x) \right\}, \{ Y(x) \} \right) \right)$$

**2.1460 ODE No. 1460**

$$y'(x)(a + A\wp(x; g2, g3)) + By(x)\wp'(x; g2, g3) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0143037 (sec), leaf count = 0 , could not solve

DSolve[B\*WeierstrassPPrime[x, {g2, g3}]\*y[x] + (a + A\*WeierstrassP[x, {g2, g3}])\*Derivative[1][y][x] + Derivative[3][y][x] == 0, y[x], x]

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^3}{dx^3} Y(x) + (A WeierstrassP(x, g2, g3) + a) \frac{d}{dx} Y(x) + B WeierstrassPPrime(x, g2, g3) Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

**2.1461 ODE No. 1461**

$$-y'(x)(a + 3k^2 \operatorname{sn}(z|x)^2) + y(x)(b + c \operatorname{sn}(z|x)^2 - 3k^2 \operatorname{cn}(z|x) \operatorname{dn}(z|x) \operatorname{sn}(z|x)) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.03114 (sec), leaf count = 0 , could not solve

DSolve[(b - 3\*k^2\*JacobiCN[z, x]\*JacobiDN[z, x]\*JacobiSN[z, x] + c\*JacobiSN[z, x]^2)\*y[x] - (a + 3\*k^2\*JacobiSN[z, x]^2)\*Derivative[1][y][x] + Derivative[3][y][x] == 0, y[x], x]

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^3}{dx^3} Y(x) + (-3k^2(\operatorname{JacobiSN}(z, x))^2 - a) \frac{d}{dx} Y(x) + (b + c(\operatorname{JacobiSN}(z, x))^2 - 3k^2 \operatorname{JacobiCN}(z, x) \operatorname{JacobiDN}(z, x) \operatorname{JacobiSN}(z, x)) Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

**2.1462 ODE No. 1462**

$$-y'(x)(a + 6k^2 \sin^2(x)) + by(x) + y^{(3)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0292223 (sec), leaf count = 0 , could not solve

DSolve[b\*y[x] - (a + 6\*k^2\*Sin[x]^2)\*Derivative[1][y][x] + Derivative[3][y][x] == 0, y[x], x]

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^3}{dx^3} Y(x) + (-6k^2(\sin(x))^2 - a) \frac{d}{dx} Y(x) + b Y(x) \right\}, \{ Y(x) \} \right) \right\}$$

**2.1463 ODE No. 1463**

$$y(x)f'(x) + 2f(x)y'(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0742663 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*Derivative[1][f][x] + 2*f[x]*Derivative[1][y][x] + Derivative[3][y][x] == 0, y[x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \left( DESol \left( \left\{ \frac{d^2}{dx^2} - Y(x) + \frac{f(x) - Y(x)}{2} \right\}, \{-Y(x)\} \right) \right)^2 \right\}$$

**2.1464 ODE No. 1464**

$$y^{(3)}(x) - 2y''(x) - 3y'(x) + 10y(x) = 0$$

✓ **Mathematica** : cpu = 0.00814074 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow e^{-2x} (c_1 e^{4x} \sin(x) + c_2 e^{4x} \cos(x) + c_3) \right\} \right\}$$

✓ **Maple** : cpu = 0.006 (sec), leaf count = 27

$$\{ y(x) = \_C1 e^{-2x} + \_C2 e^{2x} \sin(x) + \_C3 e^{2x} \cos(x) \}$$

**2.1465 ODE No. 1465**

$$-a^2 y'(x) + 2a^2 y(x) + y^{(3)}(x) - 2y''(x) - \sinh(x) = 0$$

✓ **Mathematica** : cpu = 0.0933073 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-x} - 3e^x}{6 - 6a^2} + c_1 e^{-ax} + c_3 e^{ax} + c_2 e^{2x} \right\} \right\}$$

✓ **Maple** : cpu = 0.077 (sec), leaf count = 214

$$\left\{ y(x) = \frac{1}{12a^5 - 60a^3 + 48a} (-3(a+1)((a-2)e^{-ax} + e^{ax}(a+2)) \cosh((a-1)x) + 3((a-2)e^{-ax} + e^{ax}(a+2)) \right\}$$

**2.1466 ODE No. 1466**

$$a^3(-y(x)) + 3a^2y'(x) - 3ay''(x) - e^{ax} + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0175128 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{6} e^{ax} (6c_3x^2 + 6c_2x + 6c_1 + x^3) \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 27

$$\left\{ y(x) = \frac{e^{ax}(6\_C3 x^2 + x^3 + 6\_C2 x + 6\_C1)}{6} \right\}$$

**2.1467 ODE No. 1467**

$$a_0y(x) + a_1y'(x) + a_2y''(x) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0073619 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{x \text{Root}[\#1^3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 1]} + c_2 e^{x \text{Root}[\#1^3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 2]} + c_3 e^{x \text{Root}[\#1^3 + \#1^2 a_2 + \#1 a_1 + a_0 \&, 3]} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 590

$$\left\{ y(x) = \_C1 e^{-x \left( \left( \frac{i}{12} \sqrt{3} + \frac{1}{12} \right) \left( 36 a_1 a_2 - 108 a_0 - 8 a_2^3 + 12 \sqrt{12 a_0 a_2^3 - 3 a_1^2 a_2^2 - 54 a_1 a_2 a_0 + 12 a_1^3 + 81 a_0^2} \right)^{\frac{2}{3}} + \frac{a_2}{3} \sqrt[3]{36 a_1 a_2 - 108 a_0} \right)} \right\}$$

**2.1468 ODE No. 1468**

$$2(2a + 4x^2 - 1) y'(x) - 8axy(x) + y^{(3)}(x) - 6xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.091022 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow c_2 H_{\frac{a}{2}}(x) {}_1F_1\left(-\frac{a}{4}; \frac{1}{2}; x^2\right) + c_1 H_{\frac{a}{2}}(x)^2 + c_3 {}_1F_1\left(-\frac{a}{4}; \frac{1}{2}; x^2\right)^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.082 (sec), leaf count = 59

$$\left\{ y(x) = x^2 \left( \left( U\left(\frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2\right) \right)^2 - C2 + U\left(\frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2\right) M\left(\frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2\right) - C3 + \left( M\left(\frac{1}{2} - \frac{a}{4}, \frac{3}{2}, x^2\right) \right)^2 \right) \right\}$$

**2.1469 ODE No. 1469**

$$a^3 x^3 y(x) + 3a^2 x^2 y'(x) + 3ax y''(x) + y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0323086 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow e^{-\frac{ax^2}{2} - \sqrt{3}\sqrt{ax}} \left( c_1 e^{\sqrt{3}\sqrt{ax}} + c_3 e^{2\sqrt{3}\sqrt{ax}} + c_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 37

$$\left\{ y(x) = e^{-\frac{ax^2}{2}} \left( -C1 + -C2 e^{\sqrt{3}\sqrt{ax}} + -C3 e^{-\sqrt{3}\sqrt{ax}} \right) \right\}$$

**2.1470 ODE No. 1470**

$$y^{(3)}(x) - \sin(x)y''(x) - 2\cos(x)y'(x) + y(x)\sin(x) - \log(x) = 0$$

✗ **Mathematica** : cpu = 300.03 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.087 (sec), leaf count = 36

$$\left\{ y(x) = \left( -C3 + \int \left( 2-C1 x + -C2 + \frac{x^2 \ln(x)}{2} - \frac{3x^2}{4} \right) e^{\cos(x)} dx \right) e^{-\cos(x)} \right\}$$

**2.1471 ODE No. 1471**

$$f(x)y''(x) + f(x)y(x) + y^{(3)}(x) + y'(x) = 0$$

✗ **Mathematica** : cpu = 0.0787381 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + Derivative[1][y][x] + f[x]*Derivative[2][y][x] + Derivative[3][y][x] == 0`

✓ **Maple** : cpu = 0.204 (sec), leaf count = 36

$$\left\{ y(x) = e^{ix} \left( \int e^{-2ix} \left( \int -C3 e^{\int i-f(x) dx} dx + -C2 \right) dx + -C1 \right) \right\}$$



**2.1472 ODE No. 1472**

$$f(x) (x^2 y''(x) - 2xy'(x) + 2y(x)) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0797979 (sec), leaf count = 0 , could not solve

DSolve[f[x]\*(2\*y[x] - 2\*x\*Derivative[1][y][x] + x^2\*Derivative[2][y][x]) + Derivative[3][y][x]]

✓ **Maple** : cpu = 0.285 (sec), leaf count = 33

$$\left\{ y(x) = \left( \int -C1 + -C2 \int e^{-\int x^2 f(x) + 3x^{-1} dx} dx dx + -C3 \right) x \right\}$$

**2.1473 ODE No. 1473**

$$y(x) (f(x)g(x) + g'(x)) + f(x)y''(x) + g(x)y'(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0153647 (sec), leaf count = 0 , could not solve

DSolve[y[x]\*(f[x]\*g[x] + Derivative[1][g][x]) + g[x]\*Derivative[1][y][x] + f[x]\*Derivative[2][y][x] + Derivative[3][y][x]]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^3}{dx^3} - Y(x) + f(x) \frac{d^2}{dx^2} - Y(x) + g(x) \frac{d}{dx} - Y(x) + \left( f(x)g(x) + \frac{d}{dx}g(x) \right) - Y(x) \right\}, \{-Y(x)\} \right) \right\}$$

**2.1474 ODE No. 1474**

$$y'(x) (f'(x) + 2f(x)^2 + 4g(x)) + y(x) (4f(x)g(x) + 2g'(x)) + 3f(x)y''(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0164421 (sec), leaf count = 0 , could not solve

DSolve[y[x]\*(4\*f[x]\*g[x] + 2\*Derivative[1][g][x]) + (2\*f[x]^2 + 4\*g[x] + Derivative[1][f][x])\*y'[x] + 3\*f[x]\*Derivative[2][y][x] + Derivative[3][y][x]]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \left( DESol \left( \left\{ \frac{d^2}{dx^2} - Y(x) + f(x) \frac{d}{dx} - Y(x) + g(x) - Y(x) \right\}, \{-Y(x)\} \right) \right)^2 \right\}$$

**2.1475 ODE No. 1475**

$$4y^{(3)}(x) - 8y''(x) - 11y'(x) - 3y(x) + 18e^x = 0$$

✓ **Mathematica** : cpu = 0.0284486 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow e^{-x/2} \left( c_2 x + c_3 e^{7x/2} + c_1 + e^{3x/2} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.021 (sec), leaf count = 23

$$\left\{ y(x) = (_C3 x + _C2) e^{-\frac{x}{2}} + _C1 e^{3x} + e^x \right\}$$

**2.1476 ODE No. 1476**

$$-36n^2 y'(x) \wp(x; g2, g3) - 2(n+3)(4n-3)ny(x)\phi'(x) + 27y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.128851 (sec), leaf count = 0 , could not solve

DSolve[-2\*n\*(3 + n)\*(-3 + 4\*n)\*y[x]\*Derivative[1][phi][x] - 36\*n^2\*WeierstrassP[x, {g2, g3}]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ 27 \frac{d^3}{dx^3} Y(x) - 36 n^2 WeierstrassP(x, g2, g3) \frac{d}{dx} Y(x) + (-8 WeierstrassPPrime(x, g2, g3)) \right\} \right) \right\}$$

**2.1477 ODE No. 1477**

$$xy^{(3)}(x) + 3y''(x) + xy(x) = 0$$

✓ **Mathematica** : cpu = 0.179327 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-x} + c_2 e^{\sqrt[3]{-1}x} + c_3 e^{-(-1)^{2/3}x}}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 41

$$\left\{ y(x) = \frac{1}{x} \left( -C1 e^{-x} + _C2 e^{\frac{x}{2}} \sin \left( \frac{\sqrt{3}x}{2} \right) + _C3 e^{\frac{x}{2}} \cos \left( \frac{\sqrt{3}x}{2} \right) \right) \right\}$$

**2.1478 ODE No. 1478**

$$-ax^2y(x) + xy^{(3)}(x) + 3y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0359727 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow \frac{(2-2i)c_1 {}_0F_2\left(\frac{1}{2}, \frac{3}{4}; \frac{ax^4}{64}\right)}{\sqrt[4]{ax}} + c_2 {}_0F_2\left(\frac{3}{4}, \frac{5}{4}; \frac{ax^4}{64}\right) + \left(\frac{1}{4} + \frac{i}{4}\right) \sqrt[4]{ac_3} x {}_0F_2\left(\frac{5}{4}, \frac{3}{2}; \frac{ax^4}{64}\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.127 (sec), leaf count = 48

$$\left\{ y(x) = {}_C1 {}_0F_2\left(\frac{3}{4}, \frac{5}{4}; \frac{ax^4}{64}\right) + \frac{{}_C2}{x} {}_0F_2\left(\frac{1}{2}, \frac{3}{4}; \frac{ax^4}{64}\right) + {}_C3 x {}_0F_2\left(\frac{5}{4}, \frac{3}{2}; \frac{ax^4}{64}\right) \right\}$$

**2.1479 ODE No. 1479**

$$(a+b)y''(x) - ay(x) + xy^{(3)}(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.153261 (sec), leaf count = 153

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} ic_2 x {}_1F_2\left(\frac{a}{2} + \frac{1}{2}; \frac{3}{2}, \frac{a}{2} + \frac{b}{2} + \frac{1}{2}; \frac{x^2}{4}\right) + c_1 {}_1F_2\left(\frac{a}{2}; \frac{1}{2}, \frac{a}{2} + \frac{b}{2}; \frac{x^2}{4}\right) + c_3 \left(\frac{i}{2}\right)^{-a-b+2} x^{-a-b+2} {}_1F_2\left(1 - \right. \right.$$

✓ **Maple** : cpu = 0.244 (sec), leaf count = 92

$$\left\{ y(x) = {}_C1 {}_1F_2\left(\frac{a}{2}; \frac{1}{2}, \frac{a}{2} + \frac{b}{2}; \frac{x^2}{4}\right) + {}_C2 x {}_1F_2\left(\frac{1}{2} + \frac{a}{2}; \frac{3}{2}, \frac{a}{2} + \frac{b}{2} + \frac{1}{2}; \frac{x^2}{4}\right) + {}_C3 x^{-a-b+2} {}_1F_2\left(1 - \frac{b}{2}; 2 - \frac{a}{2} - \right.$$

**2.1480 ODE No. 1480**

$$-(2v+x)y''(x) - (-2v+x-1)y'(x) + xy^{(3)}(x) + (x-1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.226519 (sec), leaf count = 91

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} e^x \left( \frac{4c_3 x^{2v+2} \Gamma(v + \frac{3}{2}) {}_1\tilde{F}_1\left(v + \frac{3}{2}; 2v + 3; -2x\right)}{\Gamma\left(\frac{1}{2} - v\right)} + c_2 4^{-v} G_{2,3}^{2,1}\left(2x \left| \begin{matrix} 1, v + \frac{3}{2} \\ 1, 2(v+1), 0 \end{matrix} \right. \right) + 4c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.267 (sec), leaf count = 35

$$\{y(x) = {}_C1 e^x + {}_C2 x^{v+1} I_{-v-1}(x) + {}_C3 x^{v+1} K_{v+1}(x)\}$$

**2.1481 ODE No. 1481**

$$-f(x) + (x^2 - 3)y''(x) + xy^{(3)}(x) + 4xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 1.03708 (sec), leaf count = 378

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{240} e^{-\frac{x^2}{2}} \left( 240x^5 \left( \int_1^x \frac{f(K[1]) \left( 2 \left( 4\sqrt{2\pi} K[1]^5 \operatorname{erfi} \left( \frac{K[1]}{\sqrt{2}} \right) + e^{\frac{K[1]^2}{2}} (-8K[1]^4 + 7K[1]^2 + 6) \right) - 15K[1]^3 \right)}{240K[1]^4} dx \right) \right) \right. \right.$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 44

$$\left\{ y(x) = \left( -C3 + \int \frac{2-C1 x + -C2 - \int \int -f(x) dx dx}{x^6} e^{\frac{x^2}{2}} dx \right) e^{-\frac{x^2}{2}} x^5 \right\}$$

**2.1482 ODE No. 1482**

$$axy(x) - b + 2xy^{(3)}(x) + 3y''(x) = 0$$

✗ **Mathematica** : cpu = 300.013 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.328 (sec), leaf count = 1616

$$\left\{ y(x) = - \int 2802800 bx \left( \left( -5/8 {}_0F_2 \left( ; \frac{13}{6}, 7/3; -\frac{ax^3}{54} \right) ax^3 + \frac{35}{4} {}_0F_2 \left( ; 7/6, 4/3; -\frac{ax^3}{54} \right) \right) {}_0F_2 \left( ; 5/6, 7/6; -\frac{ax^3}{54} \right) \right. \right.$$

**2.1483 ODE No. 1483**

$$-4(\nu + x - 1)y''(x) + (6\nu + 2x - 5)y'(x) + (1 - 2\nu)y(x) + 2xy^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.149599 (sec), leaf count = 105

$$\left\{ \left\{ y(x) \rightarrow e^x \left( \frac{2c_3 \Gamma(\frac{5}{2} - 3\nu) (\Gamma(2 - 2\nu) {}_1\tilde{F}_1(\frac{3}{2} - 3\nu; 1 - 2\nu; -x) + 2\nu - 1)}{3(2\nu - 1)\Gamma(2 - 2\nu)\Gamma(\frac{3}{2} - \nu)} + c_2 G_{2,3}^{2,1} \left( x \left| \begin{matrix} 1, 3\nu - \frac{1}{2} \\ 1, 2\nu, 0 \end{matrix} \right. \right) + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.273 (sec), leaf count = 37

$$\left\{ y(x) = -C1 e^x + -C2 e^{\frac{x}{2}} x^\nu I_\nu \left( \frac{x}{2} \right) + -C3 e^{\frac{x}{2}} x^\nu K_\nu \left( \frac{x}{2} \right) \right\}$$

**2.1484 ODE No. 1484**

$$6y'(x)(ak + bx) + 3(2ax + k)y''(x) + y(x)(3bk + 2cx) + 2xy^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 63.2145 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ (2xc + 3bk)y(x) + (6xb + 6ak)y'(x) + (6xa + 3k)y''(x) + 2xy^{(3)}(x) = 0, y(1) \right\} \right) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ (3bk + 2cx) \_Y(x) + (6ak + 6bx) \frac{d}{dx} \_Y(x) + (6ax + 3k) \frac{d^2}{dx^2} \_Y(x) + 2x \frac{d^3}{dx^3} \_Y(x) \right\}, \{ \right\} \right)$$

**2.1485 ODE No. 1485**

$$(x - 2)xy^{(3)}(x) - (x - 2)xy''(x) - 2y'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.122839 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{c_3(e^2(x^2 \log(2 - x) - x^2 \log(x) + 2x + 2) - 4e^x \text{Ei}(2 - x))}{4e^2} + c_1x^2 + c_2e^x \right\} \right\}$$

✓ **Maple** : cpu = 0.379 (sec), leaf count = 51

$$\left\{ y(x) = \_C3 \text{Ei}(1, x - 2) e^{x-2} + \frac{\_C3 x^2 \ln(x - 2)}{4} + \_C2 e^x - \frac{\_C3 \ln(x) x^2}{4} + \frac{(2x + 2) \_C3}{4} + \_C1 x^2 \right\}$$

**2.1486 ODE No. 1486**

$$(2x - 1)y^{(3)}(x) - 8xy'(x) + 8y(x) = 0$$

✓ **Mathematica** : cpu = 0.174437 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} \left( c_3 e^{2x-2} \text{Ei}(2 - 4x) - \frac{2c_3 x \text{Ei}(1 - 2x)}{e} + 4c_1 x - 4c_2 e^{2x} - c_3 e^{-2x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.25 (sec), leaf count = 51

$$\left\{ y(x) = \_C1 x + \_C2 e^{2x} - \frac{\_C3 (2xe^{-1} \text{Ei}(1, 2x - 1) - \text{Ei}(1, 4x - 2) e^{2x-2} - e^{-2x})}{4} \right\}$$

2.1487 ODE No. 1487

$$(2x - 1)y^{(3)}(x) + (x + 4)y''(x) + 2y'(x) = 0$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.086 (sec), leaf count = 38

$$\left\{ y(x) = 1 \left( -C3 + \int (2 - C1 x + -C2) e^{\frac{x}{2}} (2x - 1)^{-\frac{3}{4}} dx \right) e^{-\frac{x}{2}} \frac{1}{\sqrt[4]{2x - 1}} \right\}$$

2.1488 ODE No. 1488

$$ax^2y(x) + x^2y^{(3)}(x) - 6y'(x) = 0$$

✓ **Mathematica** : cpu = 0.562241 (sec), leaf count = 97

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\sqrt[3]{ax}} (\sqrt[3]{ax} + 2) + c_2 e^{\sqrt[3]{-1} \sqrt[3]{ax}} (\sqrt[3]{ax} + 2(-1)^{2/3}) + c_3 e^{-(-1)^{2/3} \sqrt[3]{ax}} (\sqrt[3]{ax} - 2\sqrt[3]{-1})}{x} \right\} \right\}$$

✓ **Maple** : cpu = 0.553 (sec), leaf count = 132

$$\left\{ y(x) = \frac{1}{x} \left( -C2 \left( (-i - \sqrt{3}) (-a^4)^{\frac{2}{3}} + ia^3 x \right) e^{-\frac{i(\sqrt{3}-i)x}{a} \sqrt[3]{-a^4}} + C3 \left( (\sqrt{3} - i) (-a^4)^{\frac{2}{3}} + ia^3 x \right) e^{\frac{i(\sqrt{3}+i)x}{a} \sqrt[3]{-a^4}} \right) \right\}$$

2.1489 ODE No. 1489

$$x^2y^{(3)}(x) + (x + 1)y''(x) - y(x) = 0$$

✗ **Mathematica** : cpu = 0.81506 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ y^{(3)}(x)x^2 - y(x) + (x + 1)y''(x) = 0, y(1) = c_1, y'(1) = c_2, y''(1) = c_3 \right\} \right) \right\} (x)$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ -Y(x) + (1 + x) \frac{d^2}{dx^2} Y(x) + x^2 \frac{d^3}{dx^3} Y(x) \right\}, \{ -Y(x) \} \right) \right\}$$

**2.1490 ODE No. 1490**

$$x^2 y^{(3)}(x) + (x^2 + 1) y'(x) - xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0200032 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} c_1 x^2 {}_0\tilde{F}_1\left( ; 2; -\frac{x^2}{4} \right) + c_2 x Y_1(x) + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.064 (sec), leaf count = 18

$$\{y(x) = \_C1 + \_C2 x J_1(x) + \_C3 x Y_1(x)\}$$

**2.1491 ODE No. 1491**

$$(-4a^2\nu^2 + 4a^2x^{2a} + 1) y'(x) + x^2 y^{(3)}(x) + 3xy''(x) = 4a^3 x^{2a-1} y(x)$$

✓ **Mathematica** : cpu = 0.0462804 (sec), leaf count = 102

$$\left\{ \left\{ y(x) \rightarrow c_2 (x^{2a})^{-\nu} {}_1F_2\left(-\nu - \frac{1}{2}; 1 - 2\nu, 1 - \nu; -x^{2a}\right) + c_3 (x^{2a})^\nu {}_1F_2\left(\nu - \frac{1}{2}; \nu + 1, 2\nu + 1; -x^{2a}\right) + c_1 {}_1F_2\left(\right) \right\} \right\}$$

✓ **Maple** : cpu = 0.09 (sec), leaf count = 88

$$\left\{ y(x) = \_C1 {}_1F_2\left(-\frac{1}{2}; \nu + 1, -\nu + 1; -x^{2a}\right) + \_C2 x^{-2a\nu} {}_1F_2\left(-\frac{1}{2} - \nu; 1 - 2\nu, -\nu + 1; -x^{2a}\right) + \_C3 x^{2a\nu} {}_1F_2\left(\right) \right\}$$

**2.1492 ODE No. 1492**

$$(4x(n - m) + m(2m - 1) + 2x^2) y'(x) - 2n(-2m + 2x + 1)y(x) - 3x(x - m)y''(x) + x^2 y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.414154 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow c_2 U(-n, m, x) L_n^{m-1}(x) + c_1 U(-n, m, x)^2 + c_3 L_n^{m-1}(x)^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.114 (sec), leaf count = 39

$$\left\{ y(x) = \_C1 (M(-n, m, x))^2 + \_C2 (U(-n, m, x))^2 + \_C3 M(-n, m, x) U(-n, m, x) \right\}$$

**2.1493 ODE No. 1493**

$$-f(x) + x^2 y^{(3)}(x) + (x^2 + 2) y'(x) + 4xy''(x) + 3xy(x) = 0$$

✓ **Mathematica** : cpu = 7.33606 (sec), leaf count = 872

$$\left\{ \left\{ y(x) \rightarrow \frac{x \left( J_0(x)c_1 + 2Y_0(x)c_2 + J_0(x) \int_1^x - \frac{K[1] \left( 2(8(J_1(K[1])Y_0(K[1]) - J_0(K[1])Y_1(K[1])) \right) {}_1F_2 \left( 3, \frac{5}{2}, \frac{5}{2}; -\frac{1}{4}K[1]^2 \right) K[1]^2 + 9 {}_1F_2 \right.}{\left. \right)} \right. \right. \right.$$

✓ **Maple** : cpu = 0.323 (sec), leaf count = 1033

$$\left\{ y(x) = \frac{1}{x} \left( - \int -18 \frac{(( -18x^2 J_0(x) - 72x J_1(x) + 54 J_0(x)) {}_1F_2(1; 1/2, 1/2; -1/4 x^2) + 8 (9/4 J_0(x) (x^2 + 9))}{\left. \right)} \right. \right.$$

**2.1494 ODE No. 1494**

$$x^2 y^{(3)}(x) + 5xy''(x) + 4y'(x) - \log(x) = 0$$

✓ **Mathematica** : cpu = 0.0314616 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{(x^2 - 8c_2) \log(x) - 2(-2c_3x + 2c_1 + 4c_2 + x^2)}{4x} \right\} \right\}$$

✓ **Maple** : cpu = 0.022 (sec), leaf count = 32

$$\left\{ y(x) = \frac{(x^2 + 4\_C2) \ln(x) - 2x^2 + 4\_C1x + 4\_C3}{4x} \right\}$$

**2.1495 ODE No. 1495**

$$x^2 y^{(3)}(x) + 6xy''(x) + 6y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0205094 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_2}{2x^2} - \frac{c_1}{x} + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 16

$$\left\{ y(x) = -C1 + \frac{C2}{x^2} + \frac{C3}{x} \right\}$$



**2.1496 ODE No. 1496**

$$ax^2y(x) + x^2y^{(3)}(x) + 6xy''(x) + 6y'(x) = 0$$

✓ **Mathematica** : cpu = 0.278651 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\sqrt[3]{ax}} + c_2 e^{\sqrt[3]{-1} \sqrt[3]{ax}} + c_3 e^{-(-1)^{2/3} \sqrt[3]{ax}}}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.028 (sec), leaf count = 57

$$\left\{ y(x) = \frac{1}{x^2} \left( -C1 e^{\frac{(i\sqrt{3}-1)x}{2} \sqrt[3]{-a}} + -C2 e^{-\frac{(i\sqrt{3}+1)x}{2} \sqrt[3]{-a}} + -C3 e^{\sqrt[3]{-ax}} \right) \right\}$$

**2.1497 ODE No. 1497**

$$-3x(p+q)y''(x) + 3p(3q+1)y'(x) + x^2y^{(3)}(x) + x^2(-y(x)) = 0$$

✓ **Mathematica** : cpu = 0.498455 (sec), leaf count = 127

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_0F_2\left(\frac{2}{3} - p, \frac{1}{3} - q; \frac{x^3}{27}\right) + c_2 (-1)^{p+\frac{1}{3}} 3^{-3p-1} x^{3p+1} {}_0F_2\left(p + \frac{4}{3}, p - q + \frac{2}{3}; \frac{x^3}{27}\right) + c_3 (-1)^{q+\frac{2}{3}} 3^{-3q-2} \right\} \right\}$$

✓ **Maple** : cpu = 0.277 (sec), leaf count = 77

$$\left\{ y(x) = -C1 {}_0F_2\left(-p + \frac{2}{3}, -q + \frac{1}{3}; \frac{x^3}{27}\right) + -C2 x^{1+3p} {}_0F_2\left(p + \frac{4}{3}, \frac{2}{3} - q + p; \frac{x^3}{27}\right) + -C3 x^{2+3q} {}_0F_2\left(q + \frac{5}{3}, \right)$$

**2.1498 ODE No. 1498**

$$(ax^2 + 6n)y'(x) - 2axy(x) - 2(n+1)xy''(x) + x^2y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 8.51526 (sec), leaf count = 378

$$\left\{ \left\{ y(x) \rightarrow \frac{2^{-n-\frac{3}{2}} (\sqrt{ax})^{-n} \left( \frac{\pi a^2 c_3 4^n x^4 \sec(\pi n) \Gamma(\frac{3}{2}-n) \Gamma(n+\frac{3}{2}) J_{n+\frac{1}{2}}(\sqrt{ax}) {}_1F_2\left(\frac{3}{2}-n; \frac{1}{2}-n, \frac{5}{2}-n; -\frac{ax^2}{4}\right)}{\sqrt{\sqrt{ax}}} \right) + (\sqrt{ax})^n \left( J_{n+\frac{1}{2}}(\sqrt{ax}) \right)}{\right\} \right\}$$

✓ **Maple** : cpu = 0.267 (sec), leaf count = 53

$$\left\{ y(x) = -C1 x^{n+\frac{1}{2}} J_{-n-\frac{1}{2}}(\sqrt{ax}) + -C2 x^{n+\frac{1}{2}} Y_{-n-\frac{1}{2}}(\sqrt{ax}) + -C3 (ax^2 + 4n - 2) \right\}$$

**2.1499 ODE No. 1499**

$$-\left(\nu^2 + x^2 - \frac{1}{4}\right) y'(x) + \left(\nu^2 + x^2 - 2x - \frac{1}{4}\right) y(x) + x^2 y^{(3)}(x) - (x^2 - 2x) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.244138 (sec), leaf count = 91

$$\left\{ \left\{ y(x) \rightarrow e^x \left( \frac{c_3 x^{\nu+\frac{1}{2}} \Gamma(\nu + \frac{1}{2}) {}_1\tilde{F}_1(\nu + \frac{1}{2}; 2\nu + 1; -2x)}{\Gamma(\frac{3}{2} - \nu)} + c_2 2^{-\nu-\frac{1}{2}} G_{2,3}^{2,1} \left( 2x \left| \begin{matrix} 1, 0 \\ \frac{1}{2} - \nu, \nu + \frac{1}{2}, 0 \end{matrix} \right. \right) + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.277 (sec), leaf count = 25

$$\{y(x) = \_C1 e^x + \_C2 \sqrt{x} I_\nu(x) + \_C3 \sqrt{x} K_\nu(x)\}$$

**2.1500 ODE No. 1500**

$$\nu(2x + 1)y'(x) - \nu(x + 1)y(x) - x(\nu + x)y''(x) + x^2 y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 47.1415 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ y^{(3)}(x)x^2 - (x + \nu)y''(x)x - (x + 1)\nu y(x) + (2x\nu + \nu)y'(x) = 0, y(1) = c_1 \right. \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 55

$$\{y(x) = \_C1 e^x + \_C2 x^{\frac{\nu}{2}+\frac{1}{2}} J_{-\nu-1}(2\sqrt{\nu}\sqrt{x}) + \_C3 x^{\frac{\nu}{2}+\frac{1}{2}} Y_{-\nu-1}(2\sqrt{\nu}\sqrt{x})\}$$

**2.1501 ODE No. 1501**

$$\left(-\nu^2 + x^2 - 2x + \frac{1}{4}\right) y'(x) + \left(\nu^2 - \frac{1}{4}\right) y(x) + x^2 y^{(3)}(x) - 2(x^2 - x) y''(x) = 0$$

✓ **Mathematica** : cpu = 0.191907 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow e^x \left( \frac{c_3 x^{\nu+\frac{1}{2}} \Gamma(\nu + \frac{1}{2}) {}_1\tilde{F}_1(\nu + \frac{1}{2}; 2\nu + 1; -x)}{\Gamma(\frac{3}{2} - \nu)} + c_2 G_{2,3}^{2,1} \left( x \left| \begin{matrix} 1, 0 \\ \frac{1}{2} - \nu, \nu + \frac{1}{2}, 0 \end{matrix} \right. \right) + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.246 (sec), leaf count = 37

$$\{y(x) = \_C1 e^x + \_C2 e^{\frac{x}{2}} \sqrt{x} I_\nu\left(\frac{x}{2}\right) + \_C3 e^{\frac{x}{2}} \sqrt{x} K_\nu\left(\frac{x}{2}\right)\}$$

**2.1502 ODE No. 1502**

$$-(x^4 - 6x) y''(x) - (2x^3 - 6) y'(x) + x^2 y^{(3)}(x) + 2x^2 y(x) = 0$$

✓ **Mathematica** : cpu = 0.0567634 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(-\frac{2}{3}, \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{3}\right)}{3x \Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{-\frac{1}{3}} c_3 \Gamma\left(\frac{2}{3}\right) {}_2F_2\left(-\frac{1}{3}, \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{3}\right)}{3 \Gamma\left(\frac{5}{3}\right)} + \frac{c_1}{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.49 (sec), leaf count = 103

$$\left\{ y(x) = \frac{1}{x^2} \left( -C_3 \int e^{\frac{x^3}{6}} \sqrt{x} \left( K_{\frac{5}{6}} \left( -\frac{x^3}{6} \right) x^3 - K_{\frac{1}{6}} \left( -\frac{x^3}{6} \right) x^3 + 2 K_{1/6}(-1/6 x^3) \right) dx + -C_2 \int e^{\frac{x^3}{6}} \sqrt{x} \left( I_{\frac{1}{6}} \left( -\frac{x^3}{6} \right) \right) dx \right) \right\}$$

**2.1503 ODE No. 1503**

$$(x^2 + 1) y^{(3)}(x) + \frac{1}{x^2} + 8xy''(x) + 10y'(x) - 2\log(x) - 3 = 0$$

✓ **Mathematica** : cpu = 0.116202 (sec), leaf count = 62

$$\left\{ \left\{ y(x) \rightarrow c_3 - \frac{100(3c_2 - 1)x^3 + 900c_2x + 225c_1 + 36x^5 - 60(3x^4 + 10x^2 + 15)x \log(x)}{900(x^2 + 1)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 67

$$\left\{ y(x) = \frac{(45x^5 + 150x^3 + 225x) \ln(x) - 9x^5 + 225\_C1 x^4 + (225\_C2 - 50)x^3 + 450\_C1 x^2 + (675\_C2 - 150\_C1)x - 225}{225(x^2 + 1)^2} \right\}$$

**2.1504 ODE No. 1504**

$$(x^2 + 2) y^{(3)}(x) + (x^2 + 2) y'(x) - 2xy''(x) - 2xy(x) = 0$$

✓ **Mathematica** : cpu = 0.109182 (sec), leaf count = 41

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} (2c_1 x^2 + 2ic_2 e^{-ix} - c_3 e^{ix}) \right\} \right\}$$

✓ **Maple** : cpu = 0.216 (sec), leaf count = 18

$$\{ y(x) = \_C1 x^2 + \_C2 \cos(x) + \_C3 \sin(x) \}$$

**2.1505 ODE No. 1505**

$$(2ax + b)y'(x) + ay(x) + 2(x - 1)xy^{(3)}(x) + 3(2x - 1)y''(x) = 0$$

✗ **Mathematica** : cpu = 62.1958 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}\left(\{y, x\}, \left\{ ay(x) + (2xa + b)y'(x) + (6x - 3)y''(x) + 2(x - 1)xy^{(3)}(x) = 0, y(2) = c_1, \dots \right\} \right) \right\} \right.$$

✓ **Maple** : cpu = 0.157 (sec), leaf count = 79

$$\left\{ y(x) = \_C1 \left( \text{MathieuC}\left(-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \arccos(\sqrt{x})\right) \right)^2 + \_C2 \left( \text{MathieuS}\left(-\frac{a}{2} - \frac{b}{2} + 1, \frac{a}{4}, \arccos(\sqrt{x})\right) \right) \right\}$$

**2.1506 ODE No. 1506**

$$4x^2y^{(3)}(x) + (x^2 + 14x - 1)y''(x) + 4(x + 1)y'(x) + 2y(x) = 0$$

✗ **Mathematica** : cpu = 300.13 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.06 (sec), leaf count = 43

$$\left\{ y(x) = \left( \_C3 + \int \frac{2\_C1 x + \_C2}{4} e^{\frac{x}{4}} e^{\frac{1}{4x}} x^{-\frac{5}{2}} dx \right) e^{-\frac{x}{4}} \sqrt{x} e^{-\frac{1}{4x}} \right\}$$

**2.1507 ODE No. 1507**

$$xy^{(3)}(x)(ax + b) + (\alpha x + \beta)y''(x) - f(x) + xy'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 2.47883 (sec), leaf count = 0 , could not solve

`DSolve[-f[x] + y[x] + x*Derivative[1][y][x] + (beta + alpha*x)*Derivative[2][y][x] + x*(b +`

✓ **Maple** : cpu = 0.651 (sec), leaf count = 1210

$$\left\{ y(x) = (ax + b)^{\frac{(2b+\beta)a-\alpha b}{ab}} \left( \text{HeunC}\left(0, \frac{2b - \beta}{b}, \frac{(2b + \beta)a - \alpha b}{ab}, -\frac{b}{a^2}, \frac{(4a - \alpha)b^2 - \alpha\beta b + a\beta^2}{2ab^2}, -\frac{ax}{b} \right) \left( \int - \right. \right.$$

**2.1508 ODE No. 1508**

$$y(x) (ax^3 + \nu^2 - 1) + (1 - \nu^2) xy'(x) + x^3 y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.851903 (sec), leaf count = 136

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{3} x \left( c_2 3^\nu a^{\frac{1}{3} - \frac{\nu}{3}} x^{-\nu} {}_0F_2 \left( ; 1 - \frac{2\nu}{3}, 1 - \frac{\nu}{3}; -\frac{ax^3}{27} \right) + c_3 3^{-\nu} a^{\frac{\nu+1}{3}} x^\nu {}_0F_2 \left( ; \frac{\nu}{3} + 1, \frac{2\nu}{3} + 1; -\frac{ax^3}{27} \right) + \sqrt[3]{ac_1} \right. \right.$$

✓ **Maple** : cpu = 0.153 (sec), leaf count = 81

$$\left. \left\{ y(x) = -C1 x {}_0F_2 \left( ; -\frac{\nu}{3} + 1, \frac{\nu}{3} + 1; -\frac{ax^3}{27} \right) + -C2 x^{-\nu+1} {}_0F_2 \left( ; 1 - \frac{2\nu}{3}, -\frac{\nu}{3} + 1; -\frac{ax^3}{27} \right) + -C3 x^{\nu+1} {}_0F_2 \left( ; \frac{2\nu}{3} + 1, \frac{\nu}{3} + 1; -\frac{ax^3}{27} \right) \right\}$$

**2.1509 ODE No. 1509**

$$((1 - 4\nu^2)x + 4x^3) y'(x) + (4\nu^2 - 1) y(x) + x^3 y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.0106382 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow x (c_1 J_\nu(x)^2 + c_3 Y_\nu(x)^2 + c_2 J_\nu(x) Y_\nu(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.057 (sec), leaf count = 29

$$\left\{ y(x) = x \left( (J_\nu(x))^2 - C1 + J_\nu(x) Y_\nu(x) - C3 + (Y_\nu(x))^2 - C2 \right) \right\}$$

**2.1510 ODE No. 1510**

$$y(x) (a(\nu - 1)x^{2\nu} + bx^{3\nu} + \nu^2 - 1) + x(ax^{2\nu} - \nu^2 + 1) y'(x) + x^3 y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0852642 (sec), leaf count = 0 , could not solve

`DSolve[(-1 + nu^2 + a*(-1 + nu)*x^(2*nu) + b*x^(3*nu))*y[x] + x*(1 - nu^2 + a*x^(2*nu))*Derivative[1][y[x]] + x^3*y[x]^(3) = 0, y[x]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ x^3 \frac{d^3}{dx^3} Y(x) + (x^{2\nu} ax - \nu^2 x + x) \frac{d}{dx} Y(x) + (x^{2\nu} a\nu - ax^{2\nu} + bx^{3\nu} + \nu^2 - 1) Y(x) \right\}, \{ \right.$$

**2.1511 ODE No. 1511**

$$x^3 y^{(3)}(x) + (x + 8)x^3 - 6(x - 1)x^3 \log(x) + 3x^2 y''(x) - 2xy'(x) + 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0380622 (sec), leaf count = 52

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1}{x^2} + \left( c_3 x + \frac{x^4}{9} - \frac{3x^3}{10} \right) \log(x) + c_2 x - \frac{x^4}{9} - \frac{x^3}{25} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 49

$$\left\{ y(x) = \frac{(50 x^6 - 135 x^5 + 450 \_C3 x^3) \ln(x) - 50 x^6 - 18 x^5 + 450 \_C1 x^3 + 450 \_C2}{450 x^2} \right\}$$

**2.1512 ODE No. 1512**

$$(1 - a^2) xy'(x) + x^3 y^{(3)}(x) + 3x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.038938 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow \frac{-c_1 x^{-a} + c_2 x^a + a c_3}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 18

$$\{y(x) = \_C1 + \_C2 x^{-a} + \_C3 x^a\}$$

**2.1513 ODE No. 1513**

$$x^3 y^{(3)}(x) - 4x^2 y''(x) + (x^2 + 8) xy'(x) - 2(x^2 + 4) y(x) = 0$$

✓ **Mathematica** : cpu = 0.0775481 (sec), leaf count = 23

$$\{\{y(x) \rightarrow x(c_1 x - c_2 \sin(x) + c_3 \cos(x))\}\}$$

✓ **Maple** : cpu = 0.226 (sec), leaf count = 18

$$\{y(x) = x(\cos(x) \_C3 + \sin(x) \_C2 + \_C1 x)\}$$

**2.1514 ODE No. 1514**

$$(ax^3 - 12)y(x) + x^3y^{(3)}(x) + 6x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.746265 (sec), leaf count = 97

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 e^{-\sqrt[3]{ax}} (\sqrt[3]{ax} + 2) + c_2 e^{\sqrt[3]{-1} \sqrt[3]{ax}} (\sqrt[3]{ax} + 2(-1)^{2/3}) + c_3 e^{-(-1)^{2/3} \sqrt[3]{ax}} (\sqrt[3]{ax} - 2\sqrt[3]{-1})}{x^3} \right\} \right\}$$

✓ **Maple** : cpu = 0.546 (sec), leaf count = 132

$$\left\{ y(x) = \frac{1}{x^3} \left( -_C2 \left( (-i - \sqrt{3}) (-a^4)^{\frac{2}{3}} + ia^3x \right) e^{-\frac{i(\sqrt{3}-i)x}{a} \sqrt[3]{-a^4}} + _C3 \left( (\sqrt{3} - i) (-a^4)^{\frac{2}{3}} + ia^3x \right) e^{\frac{i(\sqrt{3}+i)x}{a}} \right) \right\}$$

**2.1515 ODE No. 1515**

$$y(x) (a(4c^2\nu^2 - a^2) + 4b^2c^2(c - a)x^{2c}) + y'(x) (3(a - 1)ax + 4b^2c^2x^{2c+1} - 4c^2\nu^2 + 1) + 3(1 - a)x^2y''(x) + x^3y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.214037 (sec), leaf count = 0 , could not solve

DSolve[(a\*(-a^2 + 4\*c^2\*nu^2) + 4\*b^2\*c^2\*(-a + c)\*x^(2\*c))\*y[x] + (1 - 4\*c^2\*nu^2 + 3\*(-1 + a)\*a\*x + 4\*b^2\*c^2\*x^(1 + 2\*c))\*Derivative[1][y][x] + 3\*(1 - a)\*x^2\*Derivative[2][y][x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.1516 ODE No. 1516**

$$x^3y^{(3)}(x) + (x + 3)x^2y''(x) + 5(x - 6)xy'(x) + (4x + 30)y(x) = 0$$

✗ **Mathematica** : cpu = 300.936 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.477 (sec), leaf count = 188

$$\left\{ y(x) = \frac{C3 e^{-x} (x^8 + 28x^7 + 450x^6 + 5100x^5 + 42900x^4 + 267120x^3 + 1179360x^2 + 3326400x + 4536000)}{x^3} \right\}$$

**2.1517 ODE No. 1517**

$$x^3 y^{(3)}(x) - 2x^3 + x^2 y''(x) + 2xy'(x) - y(x) + \log(x) = 0$$

✓ **Mathematica** : cpu = 0.40514 (sec), leaf count = 1656

$$\left\{ \left\{ y(x) \rightarrow c_3 x^{\text{Root}[\#1^3 - 2\#1^2 + 3\#1 - 1\&, 3]} + c_2 x^{\text{Root}[\#1^3 - 2\#1^2 + 3\#1 - 1\&, 2]} + c_1 x^{\text{Root}[\#1^3 - 2\#1^2 + 3\#1 - 1\&, 1]} + \frac{(-2447 \dots)}{\dots} \right. \right.$$

✓ **Maple** : cpu = 0.519 (sec), leaf count = 866

$$\left\{ y(x) = - \int - \frac{\left( x^{\frac{(11-3\sqrt{69})(44+12\sqrt{69})}{1200} + \frac{\sqrt[3]{44+12\sqrt{69}}}{12} + \frac{2}{3} \right)^2 \sqrt[3]{44+12\sqrt{69}} \left( 3\sqrt{69} \sqrt[3]{44+12\sqrt{69}} - 11 \sqrt[3]{44+12\sqrt{69}} \right)}{13800 x^3} \right.$$

**2.1518 ODE No. 1518**

$$x(x^2 + 1) y^{(3)}(x) + 3(2x^2 + 1) y''(x) - 12y(x) = 0$$

✓ **Mathematica** : cpu = 0.245207 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow \frac{4c_1 x^3 + 2c_2 \sqrt{x^2 + 1} x^2 + 3c_3 x^2 + 3c_3 \sqrt{x^2 + 1} x^2 \log(x) - 3c_3 \sqrt{x^2 + 1} x^2 \log(\sqrt{x^2 + 1} + 1) + 2c_1 x + \dots}{6x} \right. \right.$$

✓ **Maple** : cpu = 0.474 (sec), leaf count = 60

$$\left\{ y(x) = \frac{1}{x} \left( 3 \operatorname{Artanh} \left( \frac{1}{\sqrt{x^2 + 1}} \right) \sqrt{x^2 + 1} - C2 x^2 + -C1 x^2 \sqrt{x^2 + 1} + 2 - C3 x^3 - 3 - C2 x^2 + -C3 x - -C2 \right) \right.$$



**2.1519 ODE No. 1519**

$$(x + 3)x^2y^{(3)}(x) - 3(x + 2)xy''(x) + 6(x + 1)y'(x) - 6y(x) = 0$$

✓ **Mathematica** : cpu = 0.0316449 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{8} (2c_1(x^3 - 3x^2 + 3x + 3) - (x - 1)(4c_2(x^2 - 2x - 1) + c_3(-3x^2 + 2x + 1))) \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 19

$$\{y(x) = \_C2 x^3 + \_C1 x^2 + \_C3 x + \_C3\}$$

**2.1520 ODE No. 1520**

$$y''(x) (-6x(a1 + a2 + a3) + 3a1a2 + 3a1a3 + 3a2a3 + 9x^2) + 2(x - a1)(x - a2)(x - a3)y^{(3)}(x) - 2(b + (n^2 + n - 3))y'(x) = 0$$

✗ **Mathematica** : cpu = 73.1532 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ -n(n + 1)y(x) - 2(xn^2 + xn - 3x + b) y'(x) + 3(3x^2 - 2a1x - 2a2x - 2a3) \right\} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.596 (sec), leaf count = 288

$$\left\{ y(x) = -\_C2 (x - a1) \left( \text{HeunG} \left( \frac{-a3 + a1}{-a2 + a1}, \frac{(-n^2 - n + 3) a1 - b}{-4 a2 + 4 a1}, \frac{n}{2} + 1, -\frac{n}{2} + \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{-x + a1}{-a2 + a1} \right) \right)^2 + \dots \right\}$$

**2.1521 ODE No. 1521**

$$(x + 1)x^3y^{(3)}(x) - (4x + 2)x^2y''(x) + (10x + 4)xy'(x) - 4(3x + 1)y(x) = 0$$

✓ **Mathematica** : cpu = 0.0793899 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow x^2 \left( c_3 \left( x + \frac{1}{x} + \log^2(x) \right) + c_2 \log(x) + c_1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.454 (sec), leaf count = 28

$$\{y(x) = x \left( (\ln(x))^2 \_C3 x + \_C2 x \ln(x) + \_C3 x^2 + \_C1 x + \_C3 \right)\}$$

**2.1522 ODE No. 1522**

$$4x^4y^{(3)}(x) - 4x^3y''(x) + 4x^2y'(x) - 1 = 0$$

✓ **Mathematica** : cpu = 0.0212998 (sec), leaf count = 42

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4}(2c_1 - c_2)x^2 + \frac{1}{2}c_2x^2 \log(x) + c_3 - \frac{1}{36x} \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 34

$$\left\{ y(x) = \frac{18x^3\_C1 \ln(x) - 1 + (-9\_C1 + 18\_C2)x^3 + 36\_C3x}{36x} \right\}$$

**2.1523 ODE No. 1523**

$$-(4x^2 + 2)x^2y''(x) + (10x^2 + 4)xy'(x) - 4(3x^2 + 1)y(x) + (x^2 + 1)x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.120876 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2}x(c_2x^2 - 2c_1(x^2 - 3x + 1) - 2c_2x + c_3x + c_3x \log(x) + c_2) \right\} \right\}$$

✓ **Maple** : cpu = 0.469 (sec), leaf count = 23

$$\{y(x) = (\ln(x) \_C2 x + \_C3 x^2 + (\_C1 + \_C2) x + \_C3) x\}$$

**2.1524 ODE No. 1524**

$$x^6y^{(3)}(x) + x^2y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.152727 (sec), leaf count = 96

$$\left\{ \left\{ y(x) \rightarrow -\frac{(-\frac{1}{3})^{2/3} c_2 x \Gamma(\frac{1}{3}) {}_2F_2(-\frac{2}{3}, \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{1}{3x^3})}{3\Gamma(\frac{4}{3})} + \frac{c_3 \Gamma(\frac{2}{3}) {}_2F_2(-\frac{1}{3}, \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{1}{3x^3})}{9\Gamma(\frac{5}{3})} + c_1 x^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.536 (sec), leaf count = 98

$$\left\{ y(x) = x^2 \left( \int 1e^{\frac{1}{6x^3}} \left( 2x^3 I_{1/6}(-1/6x^{-3}) - I_{\frac{1}{6}}\left(-\frac{1}{6x^3}\right) - I_{-\frac{5}{6}}\left(-\frac{1}{6x^3}\right) \right) x^{-\frac{11}{2}} dx \_C3 + \int 1e^{\frac{1}{6x^3}} \left( 2x^3 K_{1/6}(-\right) \right.$$

**2.1525 ODE No. 1525**

$$ay(x) + x^6 y^{(3)}(x) + 6x^5 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.448262 (sec), leaf count = 101

$$\left\{ \left\{ y(x) \rightarrow c_1 \left( -e^{\frac{\sqrt[3]{a}}{x}} \right) \left( \sqrt[3]{a} - 2x \right) + c_2 e^{\frac{(-1)^{2/3} \sqrt[3]{a}}{x}} \left( x - \frac{1}{2} (-1)^{2/3} \sqrt[3]{a} \right) + c_3 e^{-\frac{\sqrt[3]{-1} \sqrt[3]{a}}{x}} \left( \frac{1}{2} \sqrt[3]{-1} \sqrt[3]{a} + x \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.502 (sec), leaf count = 133

$$\left\{ y(x) = 4\_C3 \left( \left( -i/4 + 1/4 \sqrt{3} \right) \sqrt[3]{-a^4 + ixa} \right) e^{\frac{i/2 \sqrt[3]{-a^4} (\sqrt{3}-i)}{ax}} - 4\_C2 \left( \left( -i/4 - 1/4 \sqrt{3} \right) \sqrt[3]{-a^4 + ixa} \right) e^{-i/2}$$

**2.1526 ODE No. 1526**

$$(x^4 + 2x^2 + 2x + 1) x^2 y^{(3)}(x) - (2x^6 + 3x^4 - 6x^2 - 6x - 1) y''(x) + (x^6 - 6x^3 - 15x^2 - 12x - 2) y'(x) + (x^4 + 4x^3$$

✗ **Mathematica** : cpu = 300.1 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.238 (sec), leaf count = 19

$$\left\{ y(x) = \_C2 e^{x^{-1}} + e^x (\_C3 x + \_C1) \right\}$$

**2.1527 ODE No. 1527**

$$(x - a)^3 (x - b)^3 y^{(3)}(x) - cy(x) = 0$$

✗ **Mathematica** : cpu = 135.14 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ (a - x)^3 (b - x)^3 y^{(3)}(x) - cy(x) = 0, y(0) = c_1, y'(0) = c_2, y''(0) = c_3 \right\} \right) (x) \right\} \right\}$$

✓ **Maple** : cpu = 0.598 (sec), leaf count = 437

$$\left\{ y(x) = (x - b)^2 \frac{a}{a-b} (x - a)^{-2} \frac{b}{a-b} \left( (b - x)^{-\frac{\text{RootOf}(\_Z^3 + (-3 a - 3 b) \_Z^2 + (2 a^2 + 8 ab + 2 b^2) \_Z - 4 ba^2 - 4 ab^2 - c, \text{index}=3)}{a-b}} (a - x)^{\frac{\text{RootOf}(\_Z^3 + (-3 a - 3 b) \_Z^2 + (2 a^2 + 8 ab + 2 b^2) \_Z - 4 ba^2 - 4 ab^2 - c, \text{index}=3)}{a-b}} \right) \right\}$$

**2.1528 ODE No. 1528**

$$y^{(3)}(x) \sin(x) + (2 \cos(x) + 1)y''(x) - \sin(x)y'(x) - \cos(x) = 0$$

✓ **Mathematica** : cpu = 0.623719 (sec), leaf count = 56

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_2 x}{\sqrt{2}} - \frac{\cot\left(\frac{x}{2}\right) (c_2 \log(2(\cos(x) + 1)) + 2c_1)}{\sqrt{2}} + c_3 + \cot\left(\frac{x}{2}\right) \sin^{-1}(\cos(x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.199 (sec), leaf count = 71

$$\left\{ y(x) = \frac{1}{(\cos(x) - 1) \sin(x)} \left( \ln\left(\frac{1 - \cos(x)}{\sin(x)}\right) (\sin(x))^2 \_C1 - \ln(\sin(x)) (\sin(x))^2 \_C1 + (\sin(x))^2 \_C3 + \dots \right) \right\}$$

**2.1529 ODE No. 1529**

$$y^{(3)}(x)(x + \sin(x)) + 3(\cos(x) + 1)y''(x) - 3 \sin(x)y'(x) - y(x) \cos(x) + \sin(x) = 0$$

✗ **Mathematica** : cpu = 0.08462 (sec), leaf count = 0 , could not solve

`DSolve[Sin[x] - Cos[x]*y[x] - 3*Sin[x]*Derivative[1][y][x] + 3*(1 + Cos[x])*Derivative[2][y][x]`

✓ **Maple** : cpu = 0.087 (sec), leaf count = 25

$$\left\{ y(x) = \frac{\_C3 + \_C1 x^2 + \_C2 x - \cos(x)}{\sin(x) + x} \right\}$$

**2.1530 ODE No. 1530**

$$y'(x) (4\nu(\nu + 1) \sin^2(x) + \cos(2x)) + 2\nu(\nu + 1)y(x) \sin(2x) + y^{(3)}(x) \sin^2(x) + 3 \sin(x) \cos(x)y''(x) = 0$$

✗ **Mathematica** : cpu = 0.19218 (sec), leaf count = 0 , could not solve

`DSolve[2*nu*(1 + nu)*Sin[2*x]*y[x] + (Cos[2*x] + 4*nu*(1 + nu)*Sin[x]^2)*Derivative[1][y][x]`

✓ **Maple** : cpu = 0.252 (sec), leaf count = 113

$$\left\{ y(x) = \_C1 \left( {}_2F_1\left(-\frac{\nu}{2}, \frac{\nu}{2} + \frac{1}{2}; \frac{1}{2}; \frac{\cos(2x)}{2} + \frac{1}{2}\right) \right)^2 + \_C2 (\cos(2x) + 1) \left( {}_2F_1\left(1 + \frac{\nu}{2}, \frac{1}{2} - \frac{\nu}{2}; \frac{3}{2}; \frac{\cos(2x)}{2} + \frac{1}{2}\right) \right) \right\}$$

**2.1531 ODE No. 1531**

$$A(x) (f(x)y''(x) + g(x)y'(x) + h(x)y(x)) + f'(x)y''(x) + f(x)y^{(3)}(x) + g'(x)y'(x) + g(x)y''(x) + y(x)h'(x) + h(x)y'(x))$$

✗ **Mathematica** : cpu = 0.0319473 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*Derivative[1][h][x] + h[x]*Derivative[1][y][x] + Derivative[1][g][x]*Derivative[1][y][x], y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ f(x) \frac{d^3}{dx^3} - Y(x) + \left( \frac{d}{dx} f(x) + g(x) + A(x) f(x) \right) \frac{d^2}{dx^2} - Y(x) + \left( \frac{d}{dx} g(x) + h(x) + A(x) g(x) \right) \frac{d}{dx} - Y(x) + h(x) y(x) \right\} \right) \right.$$

**2.1532 ODE No. 1532**

$$ny(x) + y^{(3)}(x) + xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0172571 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x {}_1F_2 \left( \frac{n}{3} + \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; -\frac{x^3}{9} \right)}{3^{2/3}} + c_1 {}_1F_2 \left( \frac{n}{3}; \frac{1}{3}, \frac{2}{3}; -\frac{x^3}{9} \right) + \frac{c_3 x^2 {}_1F_2 \left( \frac{n}{3} + \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; -\frac{x^3}{9} \right)}{3\sqrt[3]{3}} \right\} \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 58

$$\left\{ y(x) = -C1 {}_1F_2 \left( \frac{n}{3}; \frac{1}{3}, \frac{2}{3}; -\frac{x^3}{9} \right) + -C2 x {}_1F_2 \left( \frac{1}{3} + \frac{n}{3}; \frac{2}{3}, \frac{4}{3}; -\frac{x^3}{9} \right) + -C3 x^2 {}_1F_2 \left( \frac{2}{3} + \frac{n}{3}; \frac{4}{3}, \frac{5}{3}; -\frac{x^3}{9} \right) \right\}$$

**2.1533 ODE No. 1533**

$$-ny(x) + y^{(3)}(x) - xy'(x) = 0$$

✓ **Mathematica** : cpu = 0.0189175 (sec), leaf count = 106

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{9} \left( 3\sqrt[3]{-3} c_2 x {}_1F_2 \left( \frac{n}{3} + \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{9} \right) + 9c_1 {}_1F_2 \left( \frac{n}{3}; \frac{1}{3}, \frac{2}{3}; \frac{x^3}{9} \right) + (-3)^{2/3} c_3 x^2 {}_1F_2 \left( \frac{n}{3} + \frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{9} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 58

$$\left\{ y(x) = -C1 {}_1F_2 \left( \frac{n}{3}; \frac{1}{3}, \frac{2}{3}; \frac{x^3}{9} \right) + -C2 x {}_1F_2 \left( \frac{1}{3} + \frac{n}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{9} \right) + -C3 x^2 {}_1F_2 \left( \frac{2}{3} + \frac{n}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{9} \right) \right\}$$

**2.1534 ODE No. 1534**

$$y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.00541959 (sec), leaf count = 22

$$\{ \{ y(x) \rightarrow x(x(c_4x + c_3) + c_2) + c_1 \} \}$$

✓ **Maple** : cpu = 0.009 (sec), leaf count = 21

$$\left\{ y(x) = \frac{C1 x^3}{6} + \frac{C2 x^2}{2} + C3 x + C4 \right\}$$

**2.1535 ODE No. 1535**

$$-f(x) + y^{(4)}(x) + 4y(x) = 0$$

✓ **Mathematica** : cpu = 1.31189 (sec), leaf count = 168

$$\left\{ \left\{ y(x) \rightarrow e^{-x} \left( \cos(x) \left( \int_1^x \frac{1}{8} e^{K[1]} f(K[1]) (\cos(K[1]) - \sin(K[1])) dK[1] \right) + e^{2x} \cos(x) \left( \int_1^x -\frac{1}{8} e^{-K[4]} f(K[4]) (s \right. \right. \right.$$

✓ **Maple** : cpu = 0.019 (sec), leaf count = 36

$$\left\{ y(x) = \frac{f}{4} + C1 e^x \cos(x) + C2 e^x \sin(x) + C3 e^{-x} \cos(x) + C4 e^{-x} \sin(x) \right\}$$

**2.1536 ODE No. 1536**

$$\lambda y(x) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.00599076 (sec), leaf count = 76

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{(-1)^{3/4} \sqrt[4]{\lambda} x} + c_2 e^{-\sqrt[4]{-1} \sqrt[4]{\lambda} x} + c_3 e^{-(-1)^{3/4} \sqrt[4]{\lambda} x} + c_4 e^{\sqrt[4]{-1} \sqrt[4]{\lambda} x} \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 50

$$\left\{ y(x) = C1 e^{-i \sqrt[4]{-\lambda} x} + C2 e^{i \sqrt[4]{-\lambda} x} + C3 e^{-\sqrt[4]{-\lambda} x} + C4 e^{\sqrt[4]{-\lambda} x} \right\}$$

**2.1537 ODE No. 1537**

$$-16e^{x^2}x^4 + y^{(4)}(x) - 12y''(x) + 12y(x) = 0$$

✓ **Mathematica** : cpu = 0.933025 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow c_1 e^{\sqrt{6-2\sqrt{6}}x} + c_2 e^{-\sqrt{6-2\sqrt{6}}x} + c_3 e^{\sqrt{2(3+\sqrt{6})}x} + c_4 e^{-\sqrt{2(3+\sqrt{6})}x} + e^{x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.147 (sec), leaf count = 67

$$\left\{ y(x) = e^{x^2} + \_C1 e^{\sqrt{6-2\sqrt{6}}x} + \_C2 e^{\sqrt{6+2\sqrt{6}}x} + \_C3 e^{-\sqrt{6-2\sqrt{6}}x} + \_C4 e^{-\sqrt{6+2\sqrt{6}}x} \right\}$$

**2.1538 ODE No. 1538**

$$a^4 y(x) + 2a^2 y''(x) - \cosh(ax) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.259547 (sec), leaf count = 47

$$\left\{ \left\{ y(x) \rightarrow \frac{4a^4((c_4 x + c_3) \sin(ax) + (c_2 x + c_1) \cos(ax)) + \cosh(ax)}{4a^4} \right\} \right\}$$

✓ **Maple** : cpu = 0.523 (sec), leaf count = 51

$$\left\{ y(x) = \frac{e^{-ax} + (8\_C3 x + 8\_C1) a^4 \cos(ax) + (8\_C4 x + 8\_C2) a^4 \sin(ax) + e^{ax}}{8a^4} \right\}$$

**2.1539 ODE No. 1539**

$$a^4 \lambda y(x) + a^2(\lambda + 1)y''(x) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.00781435 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow c_2 \sin(a\sqrt{\lambda}x) + c_1 \cos(a\sqrt{\lambda}x) + c_4 \sin(ax) + c_3 \cos(ax) \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 35

$$\left\{ y(x) = \_C1 \sin(ax) + \_C2 \cos(ax) + \_C3 \sin(a\sqrt{\lambda}x) + \_C4 \cos(a\sqrt{\lambda}x) \right\}$$

**2.1540 ODE No. 1540**

$$a(bx - 1)y''(x) + aby'(x) + \lambda y(x) + y^{(4)}(x) = 0$$

✘ **Mathematica** : cpu = 0.403119 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ \lambda y(x) + aby'(x) + a(bx - 1)y''(x) + y^{(4)}(x) = 0, y(0) = c_1, y'(0) = c_2, y''(0) = c_3 \right\} \right) \right\} \right\}$$

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ \lambda \_Y(x) + ab \frac{d}{dx} \_Y(x) + a(bx - 1) \frac{d^2}{dx^2} \_Y(x) + \frac{d^4}{dx^4} \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

**2.1541 ODE No. 1541**

$$y''(x) (ax^2 + b\lambda + c) + y(x) (ax^2 + \beta\lambda + \gamma) + y^{(4)}(x) = 0$$

✘ **Mathematica** : cpu = 80.2216 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ (ax^2 + \beta\lambda + \gamma) y(x) + (ax^2 + c + b\lambda) y''(x) + y^{(4)}(x) = 0, y(0) = c_1, y'(0) = c_2 \right\} \right) \right\} \right\}$$

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ (ax^2 + \beta\lambda + \gamma) \_Y(x) + (ax^2 + b\lambda + c) \frac{d^2}{dx^2} \_Y(x) + \frac{d^4}{dx^4} \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$

**2.1542 ODE No. 1542**

$$ay''(x)\wp(x; g2, g3) + by'(x)\wp'(x; g2, g3) + y(x) \left( c \left( 6\wp(x; g2, g3)^2 - \frac{g2}{2} \right) + d \right) + y^{(4)}(x) = 0$$

✘ **Mathematica** : cpu = 0.0291017 (sec), leaf count = 0 , could not solve

`DSolve[(d + c*(-g2/2 + 6*WeierstrassP[x, {g2, g3}]^2))*y[x] + b*WeierstrassPPrime[x, {g2, g3}]y'[x] + y[x]^4 == 0, y[x]]`

✘ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ \frac{d^4}{dx^4} \_Y(x) + a \text{WeierstrassP}(x, g2, g3) \frac{d^2}{dx^2} \_Y(x) + b \text{WeierstrassPPrime}(x, g2, g3) \frac{d}{dx} \_Y(x) + c \left( 6 \_Y(x)^2 - \frac{g2}{2} \_Y(x) \right) + d \_Y(x) \right\}, \{ \_Y(x) \} \right) \right\}$$



**2.1543 ODE No. 1543**

$$-y''(x) (a + 12k^2 \operatorname{sn}(z|x)^2) + y(x) (\alpha \operatorname{sn}(z|x)^2 + \beta) + by'(x) + y^{(4)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0951658 (sec), leaf count = 0 , could not solve

`DSolve[(beta + alpha*JacobiSN[z, x]^2)*y[x] + b*Derivative[1][y][x] - (a + 12*k^2*JacobiSN[z`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \text{DESol} \left( \left\{ \frac{d^4}{dx^4} Y(x) + \left( -12k^2 (\operatorname{JacobiSN}(z, x))^2 - a \right) \frac{d^2}{dx^2} Y(x) + b \frac{d}{dx} Y(x) + \left( \alpha (\operatorname{JacobiSN}(z, x))^2 + \beta \right) Y(x) \right\} \right) \right\}$$

**2.1544 ODE No. 1544**

$$y(x) (3f''(x) + 3f(x)^2) + 10f'(x)y'(x) + 10f(x)y''(x) + y^{(4)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0157145 (sec), leaf count = 0 , could not solve

`DSolve[10*Derivative[1][f][x]*Derivative[1][y][x] + y[x]*(3*f[x]^2 + 3*Derivative[2][f][x])`

✓ **Maple** : cpu = 0.017 (sec), leaf count = 41

$$\left\{ y(x) = \sum_{a=1}^4 e^{\operatorname{RootOf}(-Z^4 + 10fZ^2 + 10dfZ + 3f^2 + 3ddf, \operatorname{index}=_a)x} C_{-a} \right\}$$

**2.1545 ODE No. 1545**

$$y^{(4)}(x) + 2y^{(3)}(x) - 3y''(x) - 4y'(x) + 4y(x) - 32 \sin(2x) + 24 \cos(2x) = 0$$

✓ **Mathematica** : cpu = 0.178007 (sec), leaf count = 40

$$\left\{ \left\{ y(x) \rightarrow e^{-2x} (c_2 x + c_3 e^{3x} + c_4 e^{3x} x + c_1) + \sin(2x) \right\} \right\}$$

✓ **Maple** : cpu = 0.137 (sec), leaf count = 27

$$\left\{ y(x) = (_C4 x + _C2) e^{-2x} + \sin(2x) + (_C3 x + _C1) e^x \right\}$$

**2.1546 ODE No. 1546**

$$a^4 x^4 y(x) + 4a^3 x^3 y'(x) + 6a^2 x^2 y''(x) + 4axy^{(3)}(x) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.690312 (sec), leaf count = 148

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\frac{1}{2}x(ax+2\sqrt{-(\sqrt{6}-3)a})} \left( 6\sqrt{a} \left( c_2 e^{2\sqrt{-(\sqrt{6}-3)ax}} + c_1 \right) + \sqrt{18-6\sqrt{6}} e^{-\frac{(-3+\sqrt{3}+\sqrt{6})ax}{\sqrt{-(\sqrt{6}-3)a}}} \left( c_4 e^{\frac{2ax}{\sqrt{a-\sqrt{\frac{2}{3}a}}} + c_3 \right)} \right)}{6\sqrt{a}} \right. \right.$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 73

$$\left\{ y(x) = e^{-\frac{ax^2}{2}} \left( -C_2 e^{\sqrt{-a(\sqrt{6}-3)x}} + -C_4 e^{\sqrt{(3+\sqrt{6})ax}} + -C_1 e^{-\sqrt{-a(\sqrt{6}-3)x}} + -C_3 e^{-\sqrt{(3+\sqrt{6})ax}} \right) \right\}$$

**2.1547 ODE No. 1547**

$$3y(x) (2g(x)f'(x) + 5f(x)g'(x) + 6f(x)^2g(x) + g''(x) + 3g(x)^2) + y''(x) (4f'(x) + 11f(x)^2 + 10g(x)) + y'(x) (f''(x) + 2g''(x)) = 0$$

✗ **Mathematica** : cpu = 0.0329841 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x]*(6*f[x]^3 + 30*f[x]*g[x] + 7*f[x]*Derivative[1][f][x] + 10*Derivative[1][g][x]) + Derivative[2][y][x]*(4*f[x]^2 + 11*f[x]*g[x] + 10*g[x]^2) + y[x]*(f[x]*Derivative[2][f][x] + 2*g[x]*Derivative[2][g][x]) = 0, y[x], x]`

✓ **Maple** : cpu = 0.023 (sec), leaf count = 87

$$\left\{ y(x) = \sum_{a=1}^4 e^{\text{RootOf}(-Z^4+6f-Z^3+(11f^2+4df+10g)-Z^2+(6f^3+7df+30fg+ddf+10dg)-Z+18f^2g+6dfg+15dgg+9g^2+3ddg, i)} \right\}$$

**2.1548 ODE No. 1548**

$$4y^{(4)}(x) - 12y^{(3)}(x) + 11y''(x) - 3y'(x) - 4\cos(x) = 0$$

✓ **Mathematica** : cpu = 0.0924712 (sec), leaf count = 50

$$\left\{ \left\{ y(x) \rightarrow 2c_1 e^{x/2} + \frac{2}{3}c_2 e^{3x/2} + c_3 e^x + c_4 + \frac{18 \sin(x)}{65} - \frac{14 \cos(x)}{65} \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 32

$$\left\{ y(x) = -\frac{14 \cos(x)}{65} + \frac{18 \sin(x)}{65} + -C_1 e^x + 2 -C_2 e^{x/2} + \frac{2-C_3}{3} e^{\frac{3x}{2}} + -C_4 \right\}$$

**2.1549 ODE No. 1549**

$$xy^{(4)}(x) + 5y^{(3)}(x) - 24 = 0$$

✓ **Mathematica** : cpu = 0.0136173 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow c_4 x^2 - \frac{c_1}{24 x^2} + c_3 x + c_2 + \frac{4 x^3}{5} \right\} \right\}$$

✓ **Maple** : cpu = 0.02 (sec), leaf count = 26

$$\left\{ y(x) = -\frac{C1}{24 x^2} + \frac{4 x^3}{5} + \frac{C2 x^2}{2} + C3 x + C4 \right\}$$

**2.1550 ODE No. 1550**

$$12x^3 y''(x) - (6x^2 + 1) y^{(3)}(x) - (9x^2 - 7) x^2 y'(x) + 2(x^2 - 3) x^3 y(x) + xy^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 5.11206 (sec), leaf count = 214

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{x^2}{2}} \left( c_3 \int_1^x \frac{e^{\frac{K[1]^2}{2}} K[1] \left( \int \frac{e^{-\frac{1}{4}(1+\sqrt{5})K[1]^2} (K[1]^2)^{3/4} U\left(\frac{1}{20}(-5-9\sqrt{5}), -\frac{1}{2}, \frac{1}{2}\sqrt{5}K[1]^2\right) dK[1]}{K[1]^{7/2}} \right) dK[1]}{\sqrt[4]{2}} + c_4 \int_1^x \dots \right) \right. \right.$$

✓ **Maple** : cpu = 0.789 (sec), leaf count = 157

$$\left\{ y(x) = -e^{x^2} \int 1M_{\frac{9\sqrt{5}}{20}, \frac{3}{4}} \left( \frac{\sqrt{5}x^2}{2} \right) e^{-\frac{x^2}{4}} x^{-\frac{3}{2}} dx - C3 - e^{x^2} \int 1W_{\frac{9\sqrt{5}}{20}, \frac{3}{4}} \left( \frac{\sqrt{5}x^2}{2} \right) e^{-\frac{x^2}{4}} x^{-\frac{3}{2}} dx - C4 + \int 1M_{\frac{9\sqrt{5}}{20}, \frac{3}{4}} \dots \right.$$

**2.1551 ODE No. 1551**

$$-2(\nu^2 x^2 + 6) y''(x) + \nu^2(\nu^2 x^2 + 4) y(x) + x^2 y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.429926 (sec), leaf count = 84

$$\left\{ \left\{ y(x) \rightarrow \frac{e^{-\nu x} (c_3(-\nu^2 x^3 + \nu^2 - 6\nu x^2 + 6\nu - 15x + 15) + e^{2\nu x} (c_4(-\nu^2 x^3 + \nu^2 + 6\nu x^2 - 6\nu - 15x + 15) + c_2)}{x} \right\} \right.$$

✓ **Maple** : cpu = 0.265 (sec), leaf count = 62

$$\left\{ y(x) = \frac{(-C_4 \nu^2 x^3 + 6 - C_4 \nu x^2 + 15 - C_4 x + -C_2) e^{-x\nu} + e^{x\nu} (-C_3 \nu^2 x^3 - 6 - C_3 \nu x^2 + 15 - C_3 x + -C_1)}{x} \right.$$

**2.1552 ODE No. 1552**

$$ay(x) - bx^2 + x^2 y^{(4)}(x) + 2xy^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.076 (sec), leaf count = 89

$$\left\{ y(x) = \frac{bx^2}{a} + -C_1 \sqrt{x} J_1(2 \sqrt[4]{-a} \sqrt{x}) + -C_2 \sqrt{x} Y_1(2 \sqrt[4]{-a} \sqrt{x}) + -C_3 \sqrt{x} J_1\left(2 \sqrt{-\sqrt{-a}} \sqrt{x}\right) + -C_4 \sqrt{x} Y_1\left(2 \sqrt{-\sqrt{-a}} \sqrt{x}\right) \right.$$

**2.1553 ODE No. 1553**

$$x^2 y^{(4)}(x) + 4xy^{(3)}(x) + 2y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0255153 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow (c_4 - c_1) x + (c_1 x - c_2) \log(x) + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 17

$$\left\{ y(x) = (-C_2 x + -C_4) \ln(x) + -C_1 x + -C_3 \right\}$$

**2.1554 ODE No. 1554**

$$x^2 y^{(4)}(x) + 6xy^{(3)}(x) + 6y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0246878 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2}{2x} + c_4 x - c_1 \log(x) + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 18

$$\left\{ y(x) = -C_1 + -C_2 \ln(x) + -C_3 x + \frac{-C_4}{x} \right\}$$

**2.1555 ODE No. 1555**

$$\lambda^2(-y(x)) + x^2y^{(4)}(x) + 6xy^{(3)}(x) + 6y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0666385 (sec), leaf count = 156

$$\left\{ \left\{ y(x) \rightarrow c_4 G_{0,4}^{2,0} \left( \frac{\lambda^2 x^2}{16} \mid -\frac{1}{2}, \frac{1}{2}, 0, 0 \right) + c_2 G_{0,4}^{2,0} \left( \frac{\lambda^2 x^2}{16} \mid 0, 0, -\frac{1}{2}, \frac{1}{2} \right) + \frac{c_1 \left( J_1 \left( 2\sqrt{\lambda}\sqrt{x} \right) + I_1 \left( 2\sqrt{\lambda}\sqrt{x} \right) \right)}{2\sqrt{\lambda}\sqrt{x}} - \dots \right. \right.$$

✓ **Maple** : cpu = 0.181 (sec), leaf count = 61

$$\left\{ y(x) = 1 \left( -C1 J_1 \left( 2\sqrt{\lambda}\sqrt{x} \right) + -C2 Y_1 \left( 2\sqrt{\lambda}\sqrt{x} \right) + -C4 Y_1 \left( 2\sqrt{-\lambda}\sqrt{x} \right) + -C3 J_1 \left( 2\sqrt{-\lambda}\sqrt{x} \right) \right) \frac{1}{\sqrt{x}} \right\}$$

**2.1556 ODE No. 1556**

$$x^2y^{(4)}(x) + 8xy^{(3)}(x) + 12y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0255502 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{3c_2x + c_1}{6x^2} + c_4x + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.014 (sec), leaf count = 19

$$\left\{ y(x) = -C1 + \frac{C2}{x^2} + -C3 x + \frac{C4}{x} \right\}$$

**2.1557 ODE No. 1557**

$$\lambda^2(-y(x)) + x^2y^{(4)}(x) + 8xy^{(3)}(x) + 12y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0696393 (sec), leaf count = 146

$$\left\{ \left\{ y(x) \rightarrow c_4 G_{0,4}^{2,0} \left( \frac{\lambda^2 x^2}{16} \mid -1, 0, -\frac{1}{2}, \frac{1}{2} \right) + c_2 G_{0,4}^{2,0} \left( \frac{\lambda^2 x^2}{16} \mid -\frac{1}{2}, \frac{1}{2}, -1, 0 \right) - \frac{3ic_1 \left( I_2 \left( 2\sqrt{\lambda}\sqrt{x} \right) - J_2 \left( 2\sqrt{\lambda}\sqrt{x} \right) \right)}{4\lambda x} \right. \right.$$

✓ **Maple** : cpu = 0.142 (sec), leaf count = 61

$$\left\{ y(x) = \frac{1}{x} \left( -C2 Y_2 \left( 2\sqrt{\lambda}\sqrt{x} \right) + -C3 J_2 \left( 2\sqrt{-\lambda}\sqrt{x} \right) + -C4 Y_2 \left( 2\sqrt{-\lambda}\sqrt{x} \right) + -C1 J_2 \left( 2\sqrt{\lambda}\sqrt{x} \right) \right) \right\}$$

**2.1558 ODE No. 1558**

$$-\frac{1}{16}b^4y(x) + x(2n - 2\nu + 4)y^{(3)}(x) + (n - \nu + 1)(n - \nu + 2)y''(x) + x^2y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 0.164611 (sec), leaf count = 222

$$\left\{ \left\{ y(x) \rightarrow i^{-n}2^{n-3\nu-3}b^{\nu-n}x^{\frac{\nu-n}{2}} \left( i^n4^\nu(4c_1\Gamma(n - \nu + 1) - ic_2\Gamma(n - \nu + 2)) J_{n-\nu}(b\sqrt{x}) + i^n4^\nu(4c_1\Gamma(n - \nu + 1) + \right. \right. \right.$$

✓ **Maple** : cpu = 0.194 (sec), leaf count = 67

$$\left. \left. \left. y(x) = x^{-\frac{n}{2} + \frac{\nu}{2}} (K_{n-\nu}(b\sqrt{x})\_C3 + Y_{n-\nu}(b\sqrt{x})\_C4 + J_{n-\nu}(b\sqrt{x})\_C2 + I_{n-\nu}(b\sqrt{x})\_C1) \right\} \right\}$$

**2.1559 ODE No. 1559**

$$a^4(-x^3)y(x) + x^3y^{(4)}(x) + 2x^2y^{(3)}(x) - xy''(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.292362 (sec), leaf count = 100

$$\left\{ \left\{ y(x) \rightarrow c_4G_{0,4}^{2,0} \left( \frac{a^4x^4}{256} \mid 0, 0, \frac{1}{2}, \frac{1}{2} \right) + c_2G_{0,4}^{2,0} \left( \frac{a^4x^4}{256} \mid \frac{1}{2}, \frac{1}{2}, 0, 0 \right) + \frac{1}{8}ic_1(I_0(ax) - J_0(ax)) + \frac{1}{2}c_3(J_0(ax) + I_0(ax)) \right\} \right\}$$

✓ **Maple** : cpu = 0.185 (sec), leaf count = 33

$$\{y(x) = \_C1 I_0(ax) + \_C2 J_0(ax) + \_C3 K_0(ax) + \_C4 Y_0(ax)\}$$

**2.1560 ODE No. 1560**

$$x^3y^{(4)}(x) + 6x^2y^{(3)}(x) + 6xy''(x) = 0$$

✓ **Mathematica** : cpu = 0.0269658 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2}{2x} + c_4x - c_1 \log(x) + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 18

$$\left\{ y(x) = \_C1 + \_C2 \ln(x) + \_C3 x + \frac{\_C4}{x} \right\}$$

**2.1561 ODE No. 1561**

$$y(x) (ax^4 + (n-2)n(n+1)(n+3)) - 2n(n+1)x^2y''(x) + 4n(n+1)xy'(x) + x^4y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 3.90697 (sec), leaf count = 310

$$\left\{ \left\{ y(x) \rightarrow \sqrt[3]{a} 2^{-n-\frac{7}{2}} \sqrt{x} \left( 2^{2n+1} \text{ber}_{-n-\frac{1}{2}}(\sqrt[4]{ax}) \left( 4c_2 \cos\left(\frac{3}{8}\pi(2n+1)\right) \Gamma\left(\frac{1}{2}-n\right) - c_1 \cos\left(\frac{3}{8}\pi(2n-3)\right) \Gamma\left(\frac{3}{2}\right) \right) \right. \right.$$

✓ **Maple** : cpu = 0.285 (sec), leaf count = 69

$$\left. \left. y(x) = \sqrt{x} \left( Y_{n+\frac{1}{2}}(\sqrt[4]{-ax}) - C_2 + J_{n+\frac{1}{2}}(\sqrt[4]{-ax}) - C_1 + Y_{n+\frac{1}{2}}\left(\sqrt{-\sqrt{-ax}}\right) - C_4 + J_{n+\frac{1}{2}}\left(\sqrt{-\sqrt{-ax}}\right) - C_3 \right) \right\} \right\}$$

**2.1562 ODE No. 1562**

$$-(4n^2-1)x^2y''(x) + (4n^2-1)xy'(x) + x^4y^{(4)}(x) - 4x^4y(x) + 4x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 1.09892 (sec), leaf count = 140

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_0F_3\left(\frac{1}{2}, 1-\frac{n}{2}, \frac{n}{2}+1; \frac{x^4}{64}\right) + \frac{1}{8} i c_2 x^2 {}_0F_3\left(\frac{3}{2}, \frac{3}{2}-\frac{n}{2}, \frac{n}{2}+\frac{3}{2}; \frac{x^4}{64}\right) + c_3 \left(\frac{i}{2}\right)^{-n} \Gamma(1-n)^2 (\text{ber}_{-n}(x)) \right. \right.$$

✓ **Maple** : cpu = 0.408 (sec), leaf count = 77

$$\left. \left. y(x) = \left( Y_n\left(\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2x}\right) - C_3 + -C_1 J_n\left(\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{2x}\right) \right) J_n\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2x}\right) + Y_n\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{2x}\right) \right\} \right\}$$

**2.1563 ODE No. 1563**

$$(4n^2-4x^4-1)y(x) - (4n^2-1)x^2y''(x) - (4n^2-1)xy'(x) + x^4y^{(4)}(x) + 4x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 1.86166 (sec), leaf count = 193

$$\left\{ \left\{ y(x) \rightarrow \frac{\left(\frac{1}{4}+\frac{i}{4}\right) \left(x^2 \left(c_2 {}_0F_3\left(\frac{3}{2}, 1-\frac{n}{2}, \frac{n}{2}+1; \frac{x^4}{64}\right) + c_3 8^n e^{-\frac{1}{2}i\pi n} x^{-2n} {}_0F_3\left(1-n, 1-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}; \frac{x^4}{64}\right) + c_4 8^{-n} e^{\frac{1}{2}i\pi n} x^{-2n} {}_0F_3\left(\frac{3}{2}, 1-\frac{n}{2}, \frac{n}{2}+1; \frac{x^4}{64}\right)\right)}{x} \right. \right.$$

✓ **Maple** : cpu = 0.283 (sec), leaf count = 87

$$\left. \left. y(x) = \frac{1}{x} \left( -C_4 {}_0F_3\left(\frac{1}{2}, -\frac{n}{2}+\frac{1}{2}, \frac{n}{2}+\frac{1}{2}; \frac{x^4}{64}\right) + x^2 \left( -C_3 {}_0F_3\left(\frac{3}{2}, -\frac{n}{2}+1, \frac{n}{2}+1; \frac{x^4}{64}\right) + -C_2 (\text{bei}_{-n}(x))^2 + \right. \right. \right.$$

**2.1564 ODE No. 1564**

$$-(12n^2 + 4x^4 - 3)y(x) - (4n^2 + 3)x^2y''(x) + (12n^2 - 3)xy'(x) + x^4y^{(4)}(x) + 4x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 1.35275 (sec), leaf count = 196

$$\left\{ \left\{ y(x) \rightarrow \frac{\left(\frac{1}{32} + \frac{i}{32}\right) \left(8c_1 x^2 {}_0F_3\left(\frac{1}{2}, \frac{3}{2} - \frac{n}{2}, \frac{n}{2} + \frac{3}{2}; \frac{x^4}{64}\right) + i\left(c_2 x^4 {}_0F_3\left(\frac{3}{2}, 2 - \frac{n}{2}, \frac{n}{2} + 2; \frac{x^4}{64}\right) - 8^{2-n} e^{-\frac{1}{2}i\pi n} x^{-2n}\right)}{x} \right. \right.$$

✓ **Maple** : cpu = 0.263 (sec), leaf count = 88

$$\left\{ y(x) = \frac{1}{x} \left( -C_4 x^2 {}_0F_3\left(\frac{1}{2}, \frac{3}{2} - \frac{n}{2}, \frac{n}{2} + \frac{3}{2}; \frac{x^4}{64}\right) - C_3 x^4 {}_0F_3\left(\frac{3}{2}, -\frac{n}{2} + 2, \frac{n}{2} + 2; \frac{x^4}{64}\right) - C_2 (\text{bei}_{-n}(x))^2 + \dots \right)$$

**2.1565 ODE No. 1565**

$$(x(-\rho^2 - \sigma^2 + 1) + 16x^3)y'(x) + y(x)(\rho^2\sigma^2 + 8x^2) + (x^2(-\rho^2 - \sigma^2 + 7) + 4x^4)y''(x) + x^4y^{(4)}(x) + 6x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.558962 (sec), leaf count = 242

$$\left\{ \left\{ y(x) \rightarrow c_1 x^{-\rho} {}_2F_3\left(\frac{1}{2} - \frac{\rho}{2}, 1 - \frac{\rho}{2}; 1 - \rho, -\frac{\rho}{2} - \frac{\sigma}{2} + 1, -\frac{\rho}{2} + \frac{\sigma}{2} + 1; -x^2\right) + c_3 x^{-\sigma} {}_2F_3\left(\frac{1}{2} - \frac{\sigma}{2}, 1 - \frac{\sigma}{2}; 1 - \sigma, \dots \right) \right. \right.$$

✓ **Maple** : cpu = 0.437 (sec), leaf count = 71

$$\left\{ y(x) = \left( Y_{\frac{\rho}{2} + \frac{\sigma}{2}}(x) - C_3 + -C_1 J_{\frac{\rho}{2} + \frac{\sigma}{2}}(x) \right) J_{-\frac{\sigma}{2} + \frac{\rho}{2}}(x) + Y_{-\frac{\sigma}{2} + \frac{\rho}{2}}(x) \left( Y_{\frac{\rho}{2} + \frac{\sigma}{2}}(x) - C_4 + -C_2 J_{\frac{\rho}{2} + \frac{\sigma}{2}}(x) \right) \right\}$$

**2.1566 ODE No. 1566**

$$(x(-2\mu^2 - 2\nu^2 + 1) + 16x^3)y'(x) + y(x)\left((\mu^2 - \nu^2)^2 + 8x^2\right) + (x^2(-2\mu^2 - 2\nu^2 + 7) + 4x^4)y''(x) + x^4y^{(4)}(x) + 6x^3y^{(3)}(x) = 0$$

✓ **Mathematica** : cpu = 0.653549 (sec), leaf count = 237

$$\left\{ \left\{ y(x) \rightarrow x^{-\mu-\nu} \left( c_1 {}_2F_3\left(-\frac{\mu}{2} - \frac{\nu}{2} + \frac{1}{2}, -\frac{\mu}{2} - \frac{\nu}{2} + 1; 1 - \mu, 1 - \nu, -\mu - \nu + 1; -x^2\right) + c_2 x^{2\mu} {}_2F_3\left(\frac{\mu}{2} - \frac{\nu}{2} + \frac{1}{2}, \dots \right) \right. \right.$$

✓ **Maple** : cpu = 0.399 (sec), leaf count = 35

$$\{y(x) = (Y_\mu(x) - C_2 + -C_1 J_\mu(x)) J_\nu(x) + Y_\nu(x) (Y_\mu(x) - C_4 + -C_3 J_\mu(x))\}$$



**2.1567 ODE No. 1567**

$$x^4 y^{(4)}(x) + 8x^3 y^{(3)}(x) + 12x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0256433 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow \frac{3c_2 x + c_1}{6x^2} + c_4 x + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 19

$$\left\{ y(x) = \_C1 + \frac{C2}{x^2} + \_C3 x + \frac{C4}{x} \right\}$$

**2.1568 ODE No. 1568**

$$ay(x) + x^4 y^{(4)}(x) + 8x^3 y^{(3)}(x) + 12x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0124481 (sec), leaf count = 116

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1 x^{-\frac{1}{2}\sqrt{5-4\sqrt{1-a}}} + c_2 x^{\frac{1}{2}\sqrt{5-4\sqrt{1-a}}} + c_3 x^{-\frac{1}{2}\sqrt{4\sqrt{1-a}+5}} + c_4 x^{\frac{1}{2}\sqrt{4\sqrt{1-a}+5}}}{\sqrt{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 89

$$\left\{ y(x) = \_C1 x^{-\frac{1}{2}-\frac{1}{2}\sqrt{5-4\sqrt{1-a}}} + \_C2 x^{-\frac{1}{2}+\frac{1}{2}\sqrt{5-4\sqrt{1-a}}} + \_C3 x^{-\frac{1}{2}-\frac{1}{2}\sqrt{5+4\sqrt{1-a}}} + \_C4 x^{-\frac{1}{2}+\frac{1}{2}\sqrt{5+4\sqrt{1-a}}} \right\}$$

**2.1569 ODE No. 1569**

$$xy'(x) ((2a-1)C0 + 4b^2 B0 c^2 x^{2c}) + (6-4a)x^3 y^{(3)}(x) + x^2 y''(x) (A0 + 4b^2 c^2 x^{2c}) + y(x) (4b^2 c^2 D0 x^{2c} + E0) + x^4 y^{(4)}(x)$$

✗ **Mathematica** : cpu = 301.151 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.55 (sec), leaf count = 63

$$\{y(x) = ((J_\mu(x^{cb})\_C2 + Y_\mu(x^{cb})\_C3) J_\nu(x^{cb}) + Y_\nu(x^{cb})(\_C4 Y_\mu(x^{cb}) + \_C1 J_\mu(x^{cb}))) x^a\}$$

**2.1570 ODE No. 1570**

$$y(x) \left( (a^2 - c^2\nu^2) (a^2 + 4ac - c^2\nu^2 + 4c^2) - b^4 c^4 x^{4c} \right) + x^2 (2a^2 + 4(a + c - 1)^2 + 4(a - 1)(c - 1) - 2c^2\nu^2 - 1) y''(x)$$

✓ **Mathematica** : cpu = 0.148103 (sec), leaf count = 213

$$\left\{ \left\{ y(x) \rightarrow b^{a/c} (-1)^{\frac{a-c\nu}{4c}} 2^{-\frac{2a}{c} - \nu - 3} (x^{4c})^{\frac{a}{4c}} \left( 4^\nu (4c_1 \Gamma(1 - \nu) - ic_2 \Gamma(2 - \nu)) J_{-\nu} \left( b^4 \sqrt{x^{4c}} \right) + 4^\nu (4c_1 \Gamma(1 - \nu) + ic_2 \Gamma(2 - \nu)) Y_{-\nu} \left( b^4 \sqrt{x^{4c}} \right) \right) \right. \right.$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 49

$$\{y(x) = x^a (Y_\nu(x^c b) \_C2 + Y_\nu(ibx^c) \_C4 + J_\nu(ibx^c) \_C3 + J_\nu(x^c b) \_C1)\}$$

**2.1571 ODE No. 1571**

$$-\frac{1}{16} b^4 x^{2/v} y(x) + \nu^4 x^4 y^{(4)}(x) + \nu^3 (4\nu - 2) x^3 y^{(3)}(x) + (\nu - 1) \nu^2 (2\nu - 1) x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0902636 (sec), leaf count = 389

$$\left\{ \left\{ y(x) \rightarrow c_1 {}_0F_3 \left( ; 1 - \frac{v}{2}, 1 - \frac{v}{2\nu}, -\frac{v}{2\nu} - \frac{v}{2} + 1; \frac{b^4 \nu^4 x^{2/v}}{256 \nu^4} \right) + c_2 \left( \frac{i}{16} \right)^v v^{2\nu} b^{2\nu} \nu^{-2\nu} (x^{2/v})^{v/2} {}_0F_3 \left( ; \frac{v}{2} + 1, 1 - \frac{v}{2}, -\frac{v}{2} + 1; \frac{b^4 \nu^4 x^{2/v}}{256 \nu^4} \right) \right. \right.$$

✓ **Maple** : cpu = 0.204 (sec), leaf count = 143

$$\left\{ y(x) = \sqrt{x} \left( J_{(\lfloor \nu^{-1} \rfloor)^{-1}} \left( \frac{1}{\lfloor \nu^{-1} \rfloor} \sqrt{\frac{b^2}{\nu^2} x^{\frac{\lfloor \nu^{-1} \rfloor}{2}}} \right) \_C1 + Y_{(\lfloor \nu^{-1} \rfloor)^{-1}} \left( \frac{1}{\lfloor \nu^{-1} \rfloor} \sqrt{\frac{b^2}{\nu^2} x^{\frac{\lfloor \nu^{-1} \rfloor}{2}}} \right) \_C2 + Y_{(\lfloor \nu^{-1} \rfloor)^{-1}} \left( \frac{1}{\lfloor \nu^{-1} \rfloor} \sqrt{\frac{b^2}{\nu^2} x^{\frac{\lfloor \nu^{-1} \rfloor}{2}}} \right) \_C3 \right.$$

**2.1572 ODE No. 1572**

$$(-2(x^2 - 1) (\mu(\mu + 1) + \nu(\nu + 1)) + 24x^3 - 8) y''(x) - 6x(\mu(\mu + 1) + \nu(\nu + 1) - 2) y'(x) + ((\mu(\mu + 1) - \nu(\nu + 1))^2 - 4) y(x) = 0$$

✗ **Mathematica** : cpu = 93.2549 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot} \left( \{y, x\}, \left\{ (\mu - \nu - 1)(\mu - \nu + 1)(\mu + \nu)(\mu + \nu + 2)y(x) - 6x(\mu^2 + \mu + \nu^2 + \nu - 2) y'(x) + ((\mu(\mu + 1) - \nu(\nu + 1))^2 - 4)y(x) \right\} \right) \right. \right.$$

✓ **Maple** : cpu = 0.464 (sec), leaf count = 35

$$\{y(x) = (\text{LegendreQ}(\mu, x) \_C2 + \_C1 \text{LegendreP}(\mu, x)) \text{LegendreP}(\nu, x) + \text{LegendreQ}(\nu, x) (\text{LegendreQ}(\mu, x) \_C2 + \_C1 \text{LegendreP}(\mu, x))\}$$

**2.1573 ODE No. 1573**

$$-\frac{1}{x^5} + (2x + e^x)y^{(4)}(x) + 4(e^x + 2)y^{(3)}(x) + 6e^xy''(x) + 4e^xy'(x) + e^xy(x) = 0$$

✗ **Mathematica** : cpu = 0.0572722 (sec), leaf count = 0 , could not solve

DSolve[-x^(-5) + E^x\*y[x] + 4\*E^x\*Derivative[1][y][x] + 6\*E^x\*Derivative[2][y][x] + 4\*(2 + E

✓ **Maple** : cpu = 0.049 (sec), leaf count = 41

$$\left\{ y(x) = \frac{24\_C1 x^4 + 24\_C2 x^3 + 24\_C3 x^2 + 24\_C4 x + 1}{(24e^x + 48x)x} \right\}$$

**2.1574 ODE No. 1574**

$$y(x)(a^4 \sin^4(x) - 3) + y^{(4)}(x) \sin^4(x) + 2y^{(3)}(x) \sin^3(x) \cos(x) + (\sin^2(x) - 3) \sin^2(x) y''(x) + (2 \sin^2(x) + 3) \sin(x) y'(x) + y(x) \sin^6(x) = 0$$

✗ **Mathematica** : cpu = 0.20678 (sec), leaf count = 0 , could not solve

DSolve[(-3 + a^4\*Sin[x]^4)\*y[x] + Cos[x]\*Sin[x]\*(3 + 2\*Sin[x]^2)\*Derivative[1][y][x] + Sin[x]^3 + Sin[x]^2)\*Derivative[2][y][x] + 2\*Cos[x]\*Sin[x]^3\*Derivative[3][y][x] + Sin[x]^4\*Derivative[4][y][x] = 0

✓ **Maple** : cpu = 0.886 (sec), leaf count = 252

$$\left\{ y(x) = \sin(x) \left( {}_2F_1\left(\frac{3}{4}, -\frac{1}{4} \sqrt{-4 \sqrt{-(a-1)(a+1)(a^2+1)} + 5}, \frac{3}{4} + \frac{1}{4} \sqrt{-4 \sqrt{-(a-1)(a+1)(a^2+1)} + 5}; 1, x \right) \right)$$

**2.1575 ODE No. 1575**

$$-f(x) + y^{(4)}(x) \sin^6(x) + 4y^{(3)}(x) \sin^5(x) \cos(x) - 6 \sin^6(x) y''(x) - 4 \sin^5(x) \cos(x) y'(x) + y(x) \sin^6(x) = 0$$

✗ **Mathematica** : cpu = 0.0645189 (sec), leaf count = 0 , could not solve

DSolve[-f[x] + Sin[x]^6\*y[x] - 4\*Cos[x]\*Sin[x]^5\*Derivative[1][y][x] - 6\*Sin[x]^6\*Derivative[2][y][x] + 4\*Sin[x]^5\*Cos[x]\*Derivative[3][y][x] + Sin[x]^6\*Derivative[4][y][x] = 0

✓ **Maple** : cpu = 0.5 (sec), leaf count = 719

$$\left\{ y(x) = \frac{1}{48 (e^{2ix} - 1)^4 (\sin(x))^5} \left( 12 (\sin(x))^4 x \left( x^2 + \frac{20}{3} \right) f(e^{2ix} - 3/2 e^{4ix} + e^{6ix} - 1/4 e^{8ix} - 1/4) \ln(1 - e^{2ix}) \right) \right\}$$

**2.1576 ODE No. 1576**

$$2f'(x) \left( y^{(3)}(x) - a^2 y'(x) \right) + f(x) \left( a^4 y(x) - 2a^2 y''(x) + y^{(4)}(x) \right) = 0$$

✗ **Mathematica** : cpu = 0.236292 (sec), leaf count = 0 , could not solve

DSolve[2\*Derivative[1][f][x]\*(-(a^2\*Derivative[1][y][x]) + Derivative[3][y][x]) + f[x]\*(a^4\*

✓ **Maple** : cpu = 0.027 (sec), leaf count = 67

$$\left\{ y(x) = \_C1 e^{ax} + \_C2 e^{-ax} + \_C3 e^{\frac{x}{f}(-df + \sqrt{a^2 f^2 + df^2})} + \_C4 e^{-\frac{x}{f}(df + \sqrt{a^2 f^2 + df^2})} \right\}$$

**2.1577 ODE No. 1577**

$$f''(x)y''(x) + 2y^{(3)}(x)f'(x) + f(x)y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 1.13409 (sec), leaf count = 39

$$\left\{ \left\{ y(x) \rightarrow \int_1^x \left( \int_1^{K[2]} \frac{c_2 K[1] + c_1}{f(K[1])} dK[1] \right) dK[2] + c_4 x + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.007 (sec), leaf count = 21

$$\left\{ y(x) = \frac{\_C1 x^3}{6} + \frac{\_C2 x^2}{2} + \_C3 x + \_C4 \right\}$$

**2.1578 ODE No. 1578**

$$a^4 y(x) - \lambda(ax - b) (y''(x) - a^2 y(x)) - 2a^2 y''(x) + y^{(4)}(x) = 0$$

✓ **Mathematica** : cpu = 262.552 (sec), leaf count = 128

$$\left\{ \left\{ y(x) \rightarrow e^{-ax} \left( c_3 \int_1^x 2ae^{2aK[1]} \int e^{-aK[1]} \text{Ai} \left( \frac{a^2 + \lambda K[1]a - b\lambda}{(a\lambda)^{2/3}} \right) dK[1] dK[1] + c_4 \int_1^x 2ae^{2aK[2]} \int e^{-aK[2]} \text{Bi} \left( \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 1.366 (sec), leaf count = 89

$$\left\{ y(x) = e^{ax} \left( \int e^{-2ax} \left( \int e^{ax} \left( \text{Bi} \left( -\frac{\lambda(ax - b) + a^2}{a\lambda} \sqrt[3]{-a\lambda} \right) \_C4 + \text{Ai} \left( -\frac{\lambda(ax - b) + a^2}{a\lambda} \sqrt[3]{-a\lambda} \right) \_C3 \right) dx + \right) \right)$$

**2.1579 ODE No. 1579**

$$-ax - b \sin(x) - c \cos(x) + y^{(n)}(x) + 2y^{(3)}(x) + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.682601 (sec), leaf count = 80

$$\left\{ \left\{ y(x) \rightarrow \frac{ax^2}{2} + \sin(x) \left( -\frac{3bx}{8} - \frac{cx^2}{8} + c_2x + \frac{13c}{16} + c_1 + c_4 \right) + \cos(x) \left( \frac{1}{16}b(2x^2 - 9) - \frac{5cx}{8} - c_4x + c_2 - c_3 \right) \right. \right.$$

✓ **Maple** : cpu = 0.414 (sec), leaf count = 69

$$\left\{ y(x) = \frac{(bx^2 + (-4c - 8\_C4)x - 6b - 8\_C2 + 8\_C3) \cos(x)}{8} + \frac{(-cx^2 + (-4b + 8\_C3)x + 6c + 8\_C1)}{8} \right.$$

**2.1580 ODE No. 1580**

$$y^{(6)}(x) + y(x) - \sin\left(\frac{x}{2}\right) \sin\left(\frac{3x}{2}\right) = 0$$

✓ **Mathematica** : cpu = 0.980726 (sec), leaf count = 111

$$\left\{ \left\{ y(x) \rightarrow c_4 e^{-\frac{\sqrt{3}x}{2}} \sin\left(\frac{x}{2}\right) + c_6 e^{\frac{\sqrt{3}x}{2}} \sin\left(\frac{x}{2}\right) + c_5 \sin(x) + e^{-\frac{\sqrt{3}x}{2}} (c_1 e^{\sqrt{3}x} + c_3) \cos\left(\frac{x}{2}\right) + \left(c_2 + \frac{1}{4}\right) \cos(x) + \frac{1}{1} \right. \right.$$

✓ **Maple** : cpu = 0.803 (sec), leaf count = 147

$$\left\{ y(x) = \frac{1}{504} (504\_C3 \cos(x/2) + 504\_C4 \sin(x/2)) e^{-\frac{\sqrt{3}x}{2}} + \frac{1}{504} (504\_C5 \cos(x/2) + 504\_C6 \sin(x/2)) e^{\frac{\sqrt{3}x}{2}} \right.$$

**2.1581 ODE No. 1581**

$$-axy(x) - b + y^{(5)}(x) = 0$$

✗ **Mathematica** : cpu = 0.179485 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}\left(\{y, x\}, \left\{ -b - xay(x) + y^{(5)}(x) = 0, y(0) = c_1, y'(0) = c_2, y''(0) = c_3, y^{(3)}(0) = c_4, y^{(4)}(0) = c_5 \right\} \right) \right. \right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(\text{diff}(\text{diff}(\text{diff}(\text{diff}(\text{diff}(y(x), x), x), x), x), x), x) - a*x*y(x) - b = 0, y(x))$$

**2.1582 ODE No. 1582**

$$a\nu x^{\nu-1}y(x) + ax^\nu y'(x) + y^{(5)}(x) = 0$$

✓ **Mathematica** : cpu = 0.637165 (sec), leaf count = 528

$$\left\{ \left\{ y(x) \rightarrow \nu^{-\frac{16}{\nu+4}} \left( \frac{\nu+4}{\nu} \right)^{-\frac{16}{\nu+4}} a^{\frac{1}{\nu+4}} (x^\nu)^{\frac{1}{\nu}} \left( a^{\frac{1}{\nu+4}} (x^\nu)^{\frac{1}{\nu}} \left( a^{\frac{1}{\nu+4}} (x^\nu)^{\frac{1}{\nu}} \left( c_5 a^{\frac{1}{\nu+4}} (x^\nu)^{\frac{1}{\nu}} {}_1F_4 \left( 1; \frac{\nu}{\nu+4} + \frac{5}{\nu+4}, \frac{\nu}{\nu+4} \right) \right) \right) \right) \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = DESol \left( \left\{ \frac{d^5}{dx^5} Y(x) + ax^\nu \frac{d}{dx} Y(x) + a\nu x^{\nu-1} Y(x) \right\}, \{Y(x)\} \right) \right\}$$

**2.1583 ODE No. 1583**

$$ay^{(4)}(x) - f(x) + y^{(5)}(x) = 0$$

✗ **Mathematica** : cpu = 300.033 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.03 (sec), leaf count = 40

$$\left\{ y(x) = \frac{e^{-ax} C1}{a^4} + \frac{fx^4}{24a} + \frac{C2 x^3}{6} + \frac{C3 x^2}{2} + C4 x + C5 \right\}$$

**2.1584 ODE No. 1584**

$$axy(x) - 5my^{(4)}(x) + xy^{(5)}(x) = 0$$

✓ **Mathematica** : cpu = 2.83694 (sec), leaf count = 207

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{625} x \left( x \left( 5a^{3/5} c_4 x {}_0F_4 \left( ; \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{4}{5} - m; -\frac{ax^5}{3125} \right) + 25a^{2/5} c_3 {}_0F_4 \left( ; \frac{4}{5}, \frac{6}{5}, \frac{7}{5}, \frac{3}{5} - m; -\frac{ax^5}{3125} \right) + c_5 5^{-5} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.274 (sec), leaf count = 118

$$\left\{ y(x) = C1 {}_0F_4 \left( ; \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{5} - m; -\frac{x^5 a}{3125} \right) + C2 x {}_0F_4 \left( ; \frac{3}{5}, \frac{4}{5}, \frac{6}{5}, \frac{2}{5} - m; -\frac{x^5 a}{3125} \right) + C3 x^2 {}_0F_4 \left( ; \frac{4}{5}, \frac{6}{5}, \frac{7}{5}, \frac{3}{5} \right) \right\}$$

**2.1585 ODE No. 1585**

$$xy(x) \left( ay'(x) + by''(x) + cy^{(3)}(x) + ey^{(4)}(x) \right) = 0$$

✓ **Mathematica** : cpu = 0.2273 (sec), leaf count = 214

$$\left\{ \left\{ y(x) \rightarrow 0 \right\}, \left\{ y(x) \rightarrow \frac{c_1 e^{\text{Root}\left[\#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e}\right], 1}}{\text{Root}\left[\#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e}\right], 1}} + \frac{c_2 e^{\text{Root}\left[\#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e}\right], 2}}{\text{Root}\left[\#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e}\right], 2}} + \frac{c_3 e^{\text{Root}\left[\#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e}\right], 3}}{\text{Root}\left[\#1^3 + \frac{\#1^2 c}{e} + \frac{\#1 b}{e} + \frac{a}{e}\right], 3}} \right\}$$

✓ **Maple** : cpu = 0.032 (sec), leaf count = 679

$$\left\{ y(x) = 0, y(x) = \frac{x}{6e} \left( \left( 12\sqrt{3}\sqrt{27a^2e^2 + (-18abc + 4b^3)e + 4c^3a - b^2c^2e - 108ae^2 + 36bce - 8c^3} \right)^{\frac{2}{3}} - 2c\sqrt[3]{12\sqrt{3}\sqrt{27a^2e^2 + (-18abc + 4b^3)e + 4c^3a - b^2c^2e - 108ae^2 + 36bce - 8c^3}} \right) \right\}$$

**2.1586 ODE No. 1586**

$$-y^{(4)}(x)(xA(5) - A(4)) + A(5) - y^{(3)}(x)(xA(4) - A(3)) + A(4) - (xA(3) - A(2)) + A(3) - y''(x)(xA(2) - A(1)) + A(2) - (xA(1) - A(0)) + A(1) = 0$$

✗ **Mathematica** : cpu = 43.7537 (sec), leaf count = 0 , DifferentialRoot result

$$\left\{ \left\{ y(x) \rightarrow \text{DifferentialRoot}\left(\{y, x\}, \left\{ xA(0) - xA(1) - A(1) + (xA(1) - xA(2) - A(2))y'(x) + (xA(2) - xA(3) - A(3))y''(x) + (xA(3) - xA(4) - A(4))y'''(x) + (xA(4) - xA(5) - A(5))y^{(4)}(x) \right\} \right) \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \int \text{DESol} \left( \left\{ -\frac{(xA_2 - xA_1 + A_2) Y(x)}{x} - \frac{(xA_3 - xA_2 + A_3) \frac{d}{dx} Y(x)}{x} - \frac{(xA_4 - xA_3 + A_4) \frac{d^2}{dx^2} Y(x)}{x} \right\} \right) \right\}$$

**2.1587 ODE No. 1587**

$$x^5 y^{(10)}(x) - ay(x) = 0$$

✓ **Mathematica** : cpu = 0.371694 (sec), leaf count = 492

$$\left\{ \left\{ y(x) \rightarrow \frac{(-1)^{4/5} a^{9/5} c_1 x^9 {}_0F_9\left(\left(\frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, 2, \frac{11}{5}, \frac{12}{5}, \frac{13}{5}, \frac{14}{5}, \frac{ax^5}{9765625}\right)\right)}{3814697265625} + \frac{(-1)^{3/5} a^{8/5} c_3 x^8 {}_0F_9\left(\left(\frac{4}{5}, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, 2, \frac{11}{5}\right)\right)}{152587890625} \right\}$$

✓ **Maple** : cpu = 0.556 (sec), leaf count = 174

$$\left\{ y(x) = x^{\frac{5}{2}} \left( -C_3 I_5\left(2e^{i/5\pi} a^{1/10} \sqrt{x}\right) + -C_6 I_5\left(2e^{4/5i\pi} a^{1/10} \sqrt{x}\right) + -C_2 Y_5\left(2ia^{\frac{1}{10}} \sqrt{x}\right) + -C_{10} Y_5\left(2ie^{\frac{4i}{5}\pi} a^{\frac{1}{10}} \sqrt{x}\right) \right) \right\}$$

**2.1588 ODE No. 1588**

$$x^{10}y^{(5)}(x) - ay(x) = 0$$

✓ **Mathematica** : cpu = 6.15742 (sec), leaf count = 103

$$\left\{ \left\{ y(x) \rightarrow x^4 \left( c_1 e^{-\frac{\sqrt[5]{a}}{x}} + c_2 e^{\frac{\sqrt[5]{-1} \sqrt[5]{a}}{x}} + c_3 e^{-\frac{(-1)^{2/5} \sqrt[5]{a}}{x}} + c_4 e^{\frac{(-1)^{3/5} \sqrt[5]{a}}{x}} + c_5 e^{-\frac{(-1)^{4/5} \sqrt[5]{a}}{x}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 90

$$\left\{ y(x) = \_C1 {}_0F_4 \left( ; \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}; -\frac{a}{3125 x^5} \right) + \_C2 x {}_0F_4 \left( ; \frac{4}{5}, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}; -\frac{a}{3125 x^5} \right) + \_C3 x^2 {}_0F_4 \left( ; \frac{3}{5}, \frac{4}{5}, \frac{6}{5}, \frac{7}{5}; -\frac{a}{3125 x^5} \right) \right\}$$

**2.1589 ODE No. 1589**

$$x^{11/2}y^{(11)}(x) - ay(x) = 0$$

✓ **Mathematica** : cpu = 0.0489936 (sec), leaf count = 659

$$\left\{ \left\{ y(x) \rightarrow \frac{4 \sqrt[11]{-1} a^{2/11} x \left( 4 a^{2/11} x \left( 45949729863572161 (-1)^{3/11} c_3 {}_0F_{10} \left( ; -\frac{5}{11}, -\frac{3}{11}, -\frac{1}{11}, \frac{1}{11}, \frac{3}{11}, \frac{5}{11}, \frac{7}{11}, \frac{9}{11}, \frac{13}{11}, \frac{15}{11}; -\frac{a}{3125 x^5} \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 9.116 (sec), leaf count = 4339

**2.1590 ODE No. 1590**

$$(x - a)^5(x - b)^5y^{(5)}(x) - cy(x) = 0$$

✗ **Mathematica** : cpu = 300. (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 2.786 (sec), leaf count = 553

$$\left\{ y(x) = ODESolStruc \left( e^{\int -4 \frac{-g(-f) \left( (-b - f/4) e^{\int -g(-f) d_f + \_C1 (a-b) + a + f/4} \right)}{e^{\int -g(-f) d_f + \_C1 (a-b) - 1}} d_f + \_C2} \right), \left[ \frac{1}{(-g(-f))^2} \left( \left( \frac{d^3}{d_f^3} - g(-f) \right) \right) \right] \right\}$$



**2.1591 ODE No. 1591**

$$y''(x) - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0527774 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow \sqrt[3]{6} \wp \left( \frac{x + c_1}{\sqrt[3]{6}}; 0, c_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.017 (sec), leaf count = 12

$$\{y(x) = 6 \text{ WeierstrassP}(x + \_C1, 0, \_C2)\}$$

**2.1592 ODE No. 1592**

$$y''(x) - 6y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0287225 (sec), leaf count = 14

$$\{\{y(x) \rightarrow \wp(x + c_1; 0, c_2)\}\}$$

✓ **Maple** : cpu = 0.013 (sec), leaf count = 10

$$\{y(x) = \text{WeierstrassP}(x + \_C1, 0, \_C2)\}$$

**2.1593 ODE No. 1593**

$$y''(x) - 6y(x)^2 - x = 0$$

✗ **Mathematica** : cpu = 0.168391 (sec), leaf count = 0 , could not solve

`DSolve[-x - 6*y[x]^2 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x) - 6*y(x)^2 - x = 0, y(x))`

**2.1594 ODE No. 1594**

$$y''(x) - 6y(x)^2 + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.711993 (sec), leaf count = 200

$$\left\{ \left\{ y(x) \rightarrow (\text{Root}[4\#1^3 - 4\#1^2 + c_1\&, 2] - \text{Root}[4\#1^3 - 4\#1^2 + c_1\&, 3]) \operatorname{sn} \left( \sqrt{-(x + c_2)^2 (\text{Root}[4\#1^3 - 4\#1^2 + c_1\&, 2] - \text{Root}[4\#1^3 - 4\#1^2 + c_1\&, 3])} \right) \right. \right.$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 59

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{4a^3 - 4a^2 + C1}} d_{-a-x-C2} = 0, \int^{y(x)} -\frac{1}{\sqrt{4a^3 - 4a^2 + C1}} d_{-a-x-C2} = 0 \right\}$$

**2.1595 ODE No. 1595**

$$ay(x)^2 + bx + c + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.351045 (sec), leaf count = 0 , could not solve

`DSolve[c + b*x + a*y[x]^2 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x) + a*y(x)^2 + b*x + c = 0, y(x))`

**2.1596 ODE No. 1596**

$$a + y''(x) - 2y(x)^3 - xy(x) = 0$$

✗ **Mathematica** : cpu = 1.15468 (sec), leaf count = 0 , could not solve

`DSolve[a - x*y[x] - 2*y[x]^3 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x) - 2*y(x)^3 - x*y(x) + a = 0, y(x))`

**2.1597 ODE No. 1597**

$$y''(x) - ay(x)^3 = 0$$

✓ **Mathematica** : cpu = 2.24191 (sec), leaf count = 131

$$\left\{ \left\{ y(x) \rightarrow -\frac{i\sqrt[4]{2}\operatorname{sn}\left(-\frac{(1-i)\sqrt{\sqrt{a}\sqrt{c_1}(x+c_2)^2}}{2^{3/4}} \mid -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{c_1}}}} \right\}, \left\{ y(x) \rightarrow \frac{i\sqrt[4]{2}\operatorname{sn}\left(-\frac{(1-i)\sqrt{\sqrt{a}\sqrt{c_1}(x+c_2)^2}}{2^{3/4}} \mid -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{c_1}}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.026 (sec), leaf count = 21

$$\left\{ y(x) = -C2 \operatorname{JacobiSN}\left(\left(\frac{x}{2}\sqrt{-2a} + -C1\right) - C2, i\right) \right\}$$

**2.1598 ODE No. 1598**

$$-2a^2y(x)^3 + 2abxy(x) - b + y''(x) = 0$$

✗ **Mathematica** : cpu = 3.78677 (sec), leaf count = 0 , could not solve

`DSolve[-b + 2*a*b*x*y[x] - 2*a^2*y[x]^3 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)-2*a^2*y(x)^3+2*a*b*x*y(x)-b=0,y(x))`

**2.1599 ODE No. 1599**

$$ay(x)^3 + bxy(x) + cy(x) + d + y''(x) = 0$$

✗ **Mathematica** : cpu = 3.39107 (sec), leaf count = 0 , could not solve

`DSolve[d + c*y[x] + b*x*y[x] + a*y[x]^3 + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+d+b*x*y(x)+c*y(x)+a*y(x)^3=0,y(x))`



**2.1603 ODE No. 1603**

$$y''(x) - \frac{1}{(ay(x)^2 + bxy(x) + cx^2 + dy(x) + ex + k)^{3/2}} = 0$$

✗ **Mathematica** : cpu = 60.6987 (sec), leaf count = 0 , could not solve

`DSolve[-(k + e*x + c*x^2 + d*y[x] + b*x*y[x] + a*y[x]^2)^(-3/2) + Derivative[2][y][x] == 0,`

✓ **Maple** : cpu = 71.214 (sec), leaf count = 8411

**2.1604 ODE No. 1604**

$$y''(x) - e^{y(x)} = 0$$

✓ **Mathematica** : cpu = 0.0618531 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow \log \left( -\frac{1}{2} c_1 \operatorname{sech}^2 \left( \frac{1}{2} \sqrt{c_1 (c_2 + x)^2} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.353 (sec), leaf count = 23

$$\left\{ y(x) = \ln \left( \frac{1}{2 - C1^2} \left( \left( \tan \left( \frac{-C2 + x}{2 - C1} \right) \right)^2 + 1 \right) \right) \right\}$$

**2.1605 ODE No. 1605**

$$ae^x \sqrt{y(x)} + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.511531 (sec), leaf count = 0 , could not solve

`DSolve[a*E^x*Sqrt[y[x]] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.309 (sec), leaf count = 104

$$\left\{ y(x) = \text{ODESolStruc} \left( \frac{-a}{e^{-2} \int_{-b(-a)} d_{-a-2-C1}}, \left[ \left\{ \frac{d}{d_{-a}} - b(-a) = (-b(-a))^2 (-b(-a)) \sqrt{-aa} + 4_{-a} - b(-a) + 4 \right. \right. \right. \right.$$

**2.1606 ODE No. 1606**

$$y''(x) + e^x \sin(y(x)) = 0$$

✗ **Mathematica** : cpu = 1.1095 (sec), leaf count = 0 , could not solve

`DSolve[E^x*Sin[y[x]] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+exp(x)*sin(y(x))=0,y(x))`

**2.1607 ODE No. 1607**

$$a \sin(y(x)) + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.107062 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow -2am \left( \frac{1}{2} \sqrt{(2a + c_1)(x + c_2)^2} \middle| \frac{4a}{2a + c_1} \right) \right\}, \left\{ y(x) \rightarrow 2am \left( \frac{1}{2} \sqrt{(2a + c_1)(x + c_2)^2} \middle| \frac{4a}{2a + c_1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.122 (sec), leaf count = 49

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{2a \cos(\_a) + \_C1}} d\_a - x - \_C2 = 0, \int^{y(x)} -\frac{1}{\sqrt{2a \cos(\_a) + \_C1}} d\_a - x - \_C2 = 0 \right\}$$

**2.1608 ODE No. 1608**

$$a^2 \sin(y(x)) - b \sin(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0517573 (sec), leaf count = 0 , could not solve

`DSolve[-(b*Sin[x]) + a^2*Sin[y[x]] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+a^2*sin(y(x))-b*sin(x)=0,y(x))`

**2.1609 ODE No. 1609**

$$a^2 \sin(y(x)) - bf(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0348345 (sec), leaf count = 0 , could not solve

`DSolve[-(b*f[x]) + a^2*Sin[y[x]] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+a^2*sin(y(x))-b*f(x)=0,y(x))`

**2.1610 ODE No. 1610**

$$y''(x) - \frac{h\left(\frac{y(x)}{\sqrt{x}}\right)}{x^{3/2}} = 0$$

✗ **Mathematica** : cpu = 300.149 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.295 (sec), leaf count = 92

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) - 2 \int^{-z} \frac{1}{\sqrt{-C1 + 8 \int h(_g) d_g + _g^2}} d_g + 2_C2 \right) \sqrt{x}, y(x) = \text{RootOf} \left( -\ln(x) \right) \right.$$

**2.1611 ODE No. 1611**

$$y''(x) - 3y'(x) - y(x)^2 - 2y(x) = 0$$

✗ **Mathematica** : cpu = 5.226 (sec), leaf count = 0 , could not solve

`DSolve[-2*y[x] - y[x]^2 - 3*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.529 (sec), leaf count = 57

$$\left\{ y(x) = \text{ODESolStruc} \left( -a, \left[ \left\{ \left( \frac{d}{d_a} b(-a) \right) - b(-a) - 3_b(-a) - a^2 - 2_a = 0 \right\}, \left\{ -a = y(x), -b(-a) = \right. \right. \right.$$

**2.1612 ODE No. 1612**

$$y''(x) - 7y'(x) - y(x)^{3/2} + 12y(x) = 0$$

✗ **Mathematica** : cpu = 22.205 (sec), leaf count = 0 , could not solve

`DSolve[12*y[x] - y[x]^(3/2) - 7*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.318 (sec), leaf count = 57

$$\left\{ y(x) = \text{ODESolStruc} \left( -a, \left\{ \left( \frac{d}{d\_a} b(-a) \right) - b(-a) - 7\_b(-a) - \_a^{\frac{3}{2}} + 12\_a = 0 \right\}, \left\{ -a = y(x), -b(-a) \right\} \right) \right\}$$

**2.1613 ODE No. 1613**

$$6a^2y(x) + 5ay'(x) + y''(x) - 6y(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.8308 (sec), leaf count = 0 , could not solve

`DSolve[6*a^2*y[x] - 6*y[x]^2 + 5*a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.023 (sec), leaf count = 27

$$\left\{ y(x) = \text{WeierstrassP} \left( -\frac{e^{-ax}}{a} + \_C1, 0, \_C2 \right) (e^{-ax})^2 \right\}$$

**2.1614 ODE No. 1614**

$$2a^2y(x) + 3ay'(x) + y''(x) - 2y(x)^3 = 0$$

✗ **Mathematica** : cpu = 1.50119 (sec), leaf count = 0 , could not solve

`DSolve[2*a^2*y[x] - 2*y[x]^3 + 3*a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.066 (sec), leaf count = 33

$$\left\{ y(x) = \frac{\_C2}{e^{ax}} \text{JacobiSN} \left( \left( -\frac{1}{a} \sqrt{-e^{-2ax}} + \_C1 \right) \_C2, i \right) \right\}$$



**2.1615 ODE No. 1615**

$$-\frac{2(n+1)(n+2)y(x)\left(y(x)^{\frac{n}{n+1}}-1\right)}{n^2}-\frac{(3n+4)y'(x)}{n}+y''(x)=0$$

✗ **Mathematica** : cpu = 116.824 (sec), leaf count = 0 , could not solve

DSolve[(-2\*(1+n)\*(2+n)\*y[x]\*(-1+y[x]^(n/(1+n))))/n^2-((4+3\*n)\*Derivative[1][y][x])

✓ **Maple** : cpu = 4.878 (sec), leaf count = 91

$$\left\{y(x)=ODESolStruc\left(-a,\left[\frac{1}{n^2}\left(-2(n+2)(n+1)-a-a^{\frac{n}{n+1}}+\left(\frac{d}{d_a}b(-a)\right)-b(-a)n^2+(-3n^2-4n)\right.\right.\right.\right.$$

**2.1616 ODE No. 1616**

$$\frac{1}{4}(a^2-1)y(x)+ay'(x)+by(x)^n+y''(x)=0$$

✗ **Mathematica** : cpu = 25.9254 (sec), leaf count = 0 , could not solve

DSolve[((-1+a^2)\*y[x])/4+b\*y[x]^n+a\*Derivative[1][y][x]+Derivative[2][y][x]==0,y[x],x]

✓ **Maple** : cpu = 1.356 (sec), leaf count = 63

$$\left\{y(x)=ODESolStruc\left(-a,\left[\left(\frac{d}{d_a}b(-a)\right)-b(-a)+a_b(-a)+b_a^n+\frac{a a^2}{4}-\frac{a}{4}=0\right],\left\{-a=y(x),\right.\right.\right.$$

**2.1617 ODE No. 1617**

$$ay'(x)+bx^r y(x)^n+y''(x)=0$$

✗ **Mathematica** : cpu = 0.0469767 (sec), leaf count = 0 , could not solve

DSolve[b\*x^r\*y[x]^n+a\*Derivative[1][y][x]+Derivative[2][y][x]==0,y[x],x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(diff(y(x),x),x)+a\*diff(y(x),x)+b\*x^r\*y(x)^n=0,y(x))

2.1618 ODE No. 1618

$$ay'(x) - 2a + be^{y(x)} + y''(x) = 0$$

✗ **Mathematica** : cpu = 30.896 (sec), leaf count = 0 , could not solve

`DSolve[-2*a + b*E^y[x] + a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.736 (sec), leaf count = 56

$$\left\{ y(x) = \text{ODESolStruc}\left(-a, \left\{ \left( \frac{d}{d_a} b(-a) \right) b(-a) + a b(-a) + be^{-a} - 2a = 0 \right\}, \left\{ -a = y(x), b(-a) = \right. \right. \right.$$

2.1619 ODE No. 1619

$$ay'(x) + f(x) \sin(y(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0600583 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*Sin[y[x]] + a*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x) + a*diff(y(x), x) + f(x)*sin(y(x)) = 0, y(x))`

2.1620 ODE No. 1620

$$y''(x) + y(x)y'(x) - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 122.372 (sec), leaf count = 0 , could not solve

`DSolve[-y[x]^3 + y[x]*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.125 (sec), leaf count = 253

$$\left\{ \int^{y(x)} \left( \frac{a^2}{2} + \frac{1}{2} \left( \sqrt[3]{-C1 + \sqrt{-a^6 + -C1^2}} - a^2 \frac{1}{\sqrt[3]{-C1 + \sqrt{-a^6 + -C1^2}}} \right)^2 \right)^{-1} d_a - x - C2 = 0, \int^{y(x)}$$

**2.1621 ODE No. 1621**

$$ay(x) + y''(x) + y(x)y'(x) - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 100.129 (sec), leaf count = 0 , could not solve

`DSolve[a*y[x] - y[x]^3 + y[x]*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.625 (sec), leaf count = 1088

$$\left\{ \int^{y(x)} \frac{1}{-63a^2 + 63a} \left( \frac{(-\frac{1}{2} + \frac{i}{2}\sqrt{3})^3}{2} \left( 126 \frac{1}{-a^6 + 3a^4 - 3a^2a^2 + 80C1^3 + a^3} \sqrt[3]{-4 \left( -C1 \sqrt{5} \sqrt{-\dots} \right)} \right) \right.$$

**2.1622 ODE No. 1622**

$$2a^2y(x) + (3a + y(x))y'(x) + ay(x)^2 + y''(x) - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 26.3483 (sec), leaf count = 0 , could not solve

`DSolve[2*a^2*y[x] + a*y[x]^2 - y[x]^3 + (3*a + y[x])*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.527 (sec), leaf count = 817

$$\left\{ y(x) = \frac{1}{e^{ax}} \text{RootOf} \left( \int^{-z} \frac{1}{-f^6 + C1^2} \left( -(-1(-C1 f^{12} + 2C1^3 f^6 + (-f^6 + C1^2)^{\frac{5}{2}} - C1^5)) \right) \right.$$

**2.1623 ODE No. 1623**

$$y(x) (f'(x) + 2f(x)^2) + (3f(x) + y(x))y'(x) + f(x)y(x)^2 + y''(x) - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 0.456587 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x]^2 - y[x]^3 + y[x]*(2*f[x]^2 + Derivative[1][f][x]) + (3*f[x] + y[x])*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x) + (y(x) + 3*f(x))*diff(y(x), x) - y(x)^3 + f(x)*y(x)^2 + y(x)*(diff(f(x), x))) = 0, y(x))`

**2.1624 ODE No. 1624**

$$y(x) \left( af(x)^2 - \frac{f''(x)}{f(x)} + 3f'(x) + \frac{3f'(x)^2}{f(x)^2} \right) + bf(x)^3 - \left( \frac{f'(x)}{f(x)} + f(x) \right) (3y'(x) + y(x)^2) + y''(x) + y(x)y'(x) - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 1.50878 (sec), leaf count = 0 , could not solve

`DSolve[b*f[x]^3 - y[x]^3 + y[x]*Derivative[1][y][x] - (f[x] + Derivative[1][f][x]/f[x])*y[x]^2 + y''[x] + y[x]*y'[x] - y[x]^3 == 0, y[x], x]`

✓ **Maple** : cpu = 2. (sec), leaf count = 131

$$\left\{ y(x) = ODESolStruc \left( f \left( \text{RootOf} \left( \int -b(\_a) d\_a + \_C1 - \int^{-Z} f(\_f) d\_f \right) \right) \_a, \left[ \left\{ \frac{d}{d\_a} -b(\_a) = (\_b(\_a) - b(\_a)) \right. \right. \right. \right.$$

**2.1625 ODE No. 1625**

$$y'(x) \left( y(x) - \frac{3f'(x)}{2f(x)} \right) - \frac{y(x)^2 f'(x)}{2f(x)} + y(x) \left( -\frac{f''(x)}{2f(x)} + \frac{f'(x)^2}{f(x)^2} + f(x) \right) + y''(x) - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 0.984501 (sec), leaf count = 0 , could not solve

`DSolve[-y[x]^3 - (y[x]^2*Derivative[1][f][x])/(2*f[x]) + (y[x] - (3*Derivative[1][f][x])/f[x])*y'[x] + y''[x] - y[x]^3 == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)+(y(x)-3/2*diff(f(x),x)/f(x))*diff(y(x),x)-y(x)^3-1/2*diff(f(x),x)/f(x))*y(x)^2+y''(x)-y(x)^3=0,y(x))`

**2.1626 ODE No. 1626**

$$y(x)f'(x) + f(x)y'(x) + y''(x) + 2y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 39.9909 (sec), leaf count = 0 , could not solve

`DSolve[y[x]*Derivative[1][f][x] + f[x]*Derivative[1][y][x] + 2*y[x]*Derivative[1][y][x] + y''[x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.232 (sec), leaf count = 48

$$\left\{ y(x) = ODESolStruc \left( -b(\_a), \left[ \left\{ \frac{d}{d\_a} -b(\_a) = -f(\_a) -b(\_a) - (\_b(\_a))^2 - \_C1 \right. \right. \right. \right. \left. \left. \left. \left. \_a = x, -b(\_a) = -b(x) \right. \right. \right. \right.$$

**2.1627 ODE No. 1627**

$$f(x)(y'(x) + y(x)^2) - g(x) + y''(x) + 2y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 0.32094 (sec), leaf count = 0 , could not solve

`DSolve[-g[x] + 2*y[x]*Derivative[1][y][x] + f[x]*(y[x]^2 + Derivative[1][y][x]) + Derivative`

✓ **Maple** : cpu = 0.961 (sec), leaf count = 58

$$\left\{ y(x) = ODESolStruc\left(-b(-a), \left[ \left\{ -\int e^{\int f(-a) d_a} g(-a) d_a + \left( (-b(-a))^2 + \frac{d}{d_a} b(-a) \right) e^{\int f(-a) d_a} + \dots \right. \right. \right. \right.$$

**2.1628 ODE No. 1628**

$$f(x)y(x) - g(x) + y''(x) + 3y(x)y'(x) + y(x)^3 = 0$$

✗ **Mathematica** : cpu = 7.83625 (sec), leaf count = 0 , could not solve

`DSolve[-g[x] + f[x]*y[x] + y[x]^3 + 3*y[x]*Derivative[1][y][x] + Derivative[2][y][x] == 0, y`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ y(x) = \frac{\frac{d}{dx} DESol\left(\left\{-g(x) - Y(x) + f(x) \frac{d}{dx} - Y(x) + \frac{d^3}{dx^3} - Y(x)\right\}, \{-Y(x)\}\right)}{DESol\left(\left\{-g(x) - Y(x) + f(x) \frac{d}{dx} - Y(x) + \frac{d^3}{dx^3} - Y(x)\right\}, \{-Y(x)\}\right)} \right\}$$

**2.1629 ODE No. 1629**

$$(f(x) + 3y(x))y'(x) + f(x)y(x)^2 + y''(x) + y(x)^3 = 0$$

✗ **Mathematica** : cpu = 0.038699 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x]^2 + y[x]^3 + (f[x] + 3*y[x])*Derivative[1][y][x] + Derivative[2][y][x] == 0`

✓ **Maple** : cpu = 0.039 (sec), leaf count = 38

$$\left\{ y(x) = \frac{\int -C1 e^{-\int f(x) dx} dx + -C2}{\iint -C1 e^{-\int f(x) dx} dx dx + -C2 x + 1} \right\}$$

**2.1630 ODE No. 1630**

$$-4a^2y(x) - 3ay(x)^2 - b + y''(x) - 3y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 11.1919 (sec), leaf count = 1670

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{b} \sqrt{\frac{e^{-2ax}}{b}} \sqrt{c_1} \left( -i \frac{\sqrt{4a^3-3b}}{a^{3/2}} 2^{3\sqrt{4a^6-3a^3b}} + \frac{1}{2} 3^{\frac{1}{2}} \left( \frac{\sqrt{4a^3-3b}}{a^{3/2}} - 1 \right) a^{\frac{\sqrt{4a^6-3a^3b}}{a^3}} b^{\frac{1}{2}} \left( \frac{\sqrt{4a^3-3b}}{a^{3/2}} - 1 \right) \right) \right.}{2a^3 - \sqrt{4a^3 - 3ba^3}}$$

✓ **Maple** : cpu = 0.625 (sec), leaf count = 783

$$\left\{ \int^{y(x)} -6a^2 \left( -12_a a^3 - 9_a^2 a^2 + \left( \text{RootOf} \left( 2K_{1/2} \frac{4a^3-3b}{\sqrt{4a^4-3aba}} \left( -1/2 \frac{Z}{a^2} \right) - C1 a^2 + 3K_{1/2} \frac{4a^3-3b}{\sqrt{4a^4-3aba}} \left( -1/2 \right) \right) \right) \right. \right.$$

**2.1631 ODE No. 1631**

$$-(f(x) + 3y(x))y'(x) + f(x)y(x)^2 + y''(x) + y(x)^3 = 0$$

✗ **Mathematica** : cpu = 0.0342538 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x]^2 + y[x]^3 - (f[x] + 3*y[x])*Derivative[1][y][x] + Derivative[2][y][x] == 0`

✓ **Maple** : cpu = 0.049 (sec), leaf count = 38

$$\left\{ y(x) = \frac{-\int -C1 e^{\int f(x) dx} dx - C2}{\iint -C1 e^{\int f(x) dx} dx dx + -C2 x + 1} \right\}$$

**2.1632 ODE No. 1632**

$$y''(x) - 2ay(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0634166 (sec), leaf count = 34

$$\left\{ \left\{ y(x) \rightarrow \frac{\sqrt{c_1} \tan(\sqrt{a} \sqrt{c_1} (c_2 + x))}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.095 (sec), leaf count = 23

$$\left\{ y(x) = \frac{1}{a} \tan(\sqrt{a} C1 (C2 + x)) \sqrt{a} C1 \right\}$$

**2.1633 ODE No. 1633**

$$ay(x)y'(x) + by(x)^3 + y''(x) = 0$$

✗ **Mathematica** : cpu = 39.3796 (sec), leaf count = 0 , could not solve

`DSolve[b*y[x]^3 + a*y[x]*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.279 (sec), leaf count = 97

$$\left\{ \int^{y(x)} \left( \text{RootOf} \left( -2a - a^2 \text{Artanh} \left( \frac{-a^2a + 4Z}{\sqrt{-a^4(a^2 - 8b)}} \right) \right) + C1 \sqrt{-a^4(a^2 - 8b)} - \ln(-a^4b + Z - a^2a + 2Z^2) \right) \right.$$

**2.1634 ODE No. 1634**

$$y'(x)h(x, y(x)) + j(x, y(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.160399 (sec), leaf count = 0 , could not solve

`DSolve[j[x, y[x]] + h[x, y[x]]*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)+h(x,y(x))*diff(y(x), x)+j(x,y(x))=0,y(x))`

**2.1635 ODE No. 1635**

$$ay'(x)^2 + by(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 101.49 (sec), leaf count = 0 , could not solve

`DSolve[b*y[x] + a*Derivative[1][y][x]^2 + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.178 (sec), leaf count = 79

$$\left\{ \int^{y(x)} -2 \frac{a}{\sqrt{4e^{-2a} - C1 a^2 - 4a ab + 2b}} d_{-a} - x - C2 = 0, \int^{y(x)} 2 \frac{a}{\sqrt{4e^{-2a} - C1 a^2 - 4a ab + 2b}} d_{-a} \right.$$

**2.1636 ODE No. 1636**

$$ay'(x) |y'(x)| + by'(x) + cy(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 38.1695 (sec), leaf count = 0 , could not solve

DSolve[c\*y[x] + b\*Derivative[1][y][x] + a\*Abs[Derivative[1][y][x]]\*Derivative[1][y][x] + Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.89 (sec), leaf count = 59

$$\left\{ y(x) = ODESolStruc\left(-a, \left[ \left\{ \left( \frac{d}{d\_a} b(-a) \right) - b(-a) + a\_b(-a) | -b(-a) | + -b(-a) b + c\_a = 0 \right\} \right], \left\{ -a = y(x), \dots \right\} \right.$$

**2.1637 ODE No. 1637**

$$ay'(x)^2 + by'(x) + cy(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 30.4557 (sec), leaf count = 0 , could not solve

DSolve[c\*y[x] + b\*Derivative[1][y][x] + a\*Derivative[1][y][x]^2 + Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.554 (sec), leaf count = 58

$$\left\{ y(x) = ODESolStruc\left(-a, \left[ \left\{ \left( \frac{d}{d\_a} b(-a) \right) - b(-a) + a(-b(-a))^2 + -b(-a) b + c\_a = 0 \right\} \right], \left\{ -a = y(x), \dots \right\} \right.$$

**2.1638 ODE No. 1638**

$$ay'(x)^2 + b \sin(y(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 100.112 (sec), leaf count = 0 , could not solve

DSolve[b\*Sin[y[x]] + a\*Derivative[1][y][x]^2 + Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.192 (sec), leaf count = 115

$$\left\{ \int^{y(x)} (-4a^2 - 1) \frac{1}{\sqrt{16\_C1 (a^2 + 1/4)^2 e^{-2a\_a} - 16b(a \sin(\_a) - 1/2 \cos(\_a)) (a^2 + 1/4)}} d\_a - x - \_C2 = \dots \right.$$



**2.1639 ODE No. 1639**

$$ay'(x) |y'(x)| + b \sin(y(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 41.0411 (sec), leaf count = 0 , could not solve

`DSolve[b*Sin[y[x]] + a*Abs[Derivative[1][y][x]]*Derivative[1][y][x] + Derivative[2][y][x] ==`

✓ **Maple** : cpu = 3.636 (sec), leaf count = 56

$$\left\{ y(x) = \text{ODESolStruc}\left(-a, \left[ \left( \frac{d}{d\_a} b(-a) \right) -b(-a) + a\_b(-a) | -b(-a)| + b \sin(-a) = 0 \right], \left\{ -a = y(x), \dots \right\} \right\}$$

**2.1640 ODE No. 1640**

$$ay(x)y'(x)^2 + by(x) + y''(x) = 0$$

✗ **Mathematica** : cpu = 200.285 (sec), leaf count = 0 , could not solve

`DSolve[b*y[x] + a*y[x]*Derivative[1][y][x]^2 + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.191 (sec), leaf count = 70

$$\left\{ \int^{y(x)} a \frac{1}{\sqrt{a(e^{-a^2 a} C1 a - b)}} d\_a - x - C2 = 0, \int^{y(x)} -a \frac{1}{\sqrt{a(e^{-a^2 a} C1 a - b)}} d\_a - x - C2 = 0 \right\}$$

**2.1641 ODE No. 1641**

$$g(x)y'(x) + h(y(x))y'(x)^2 + y''(x) = 0$$

✓ **Mathematica** : cpu = 2.05967 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction}\left[ \int_1^{\#1} e^{-\int_1^{K[4]} -h(K[1]) dK[1]} dK[4] \& \right] \left[ \int_1^x c_1 \left( -e^{-\int_1^{K[5]} g(K[2]) dK[2]} \right) dK[5] + c_2 \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 29

$$\left\{ \int^{y(x)} e^{\int h(-b) d\_b} d\_b - C1 \int e^{-\int g(x) dx} dx - C2 = 0 \right\}$$

**2.1642 ODE No. 1642**

$$f(x)h(y(x)) + g(x)y'(x) - \frac{j(y(x))y'(x)^2}{h(y(x))} + y''(x) = 0$$

✗ **Mathematica** : cpu = 1.01217 (sec), leaf count = 0 , could not solve

DSolve[f[x]\*h[y[x]] + g[x]\*Derivative[1][y][x] - (j[y[x]]\*Derivative[1][y][x]^2)/h[y[x]] + D

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(diff(y(x),x),x)-j(y(x))/h(y(x))\*diff(y(x),x)^2+g(x)\*diff(y(x),x)+f(x)\*h(y(x)))=0,

**2.1643 ODE No. 1643**

$$f(x)y'(x) + g(x)j(y(x)) + h(y(x))y'(x)^2 + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.373476 (sec), leaf count = 0 , could not solve

DSolve[g[x]\*j[y[x]] + f[x]\*Derivative[1][y][x] + h[y[x]]\*Derivative[1][y][x]^2 + Derivative[1][y][x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve((1-D(j)(y(x)))/j(y(x))\*diff(y(x),x)^2+f(x)\*diff(y(x),x)+diff(diff(y(x),x),x)+g(x)\*j(y(x)))=0,

**2.1644 ODE No. 1644**

$$h(y(x))y'(x)^2 + j(y(x))y'(x) + k(y(x)) + y''(x) = 0$$

✗ **Mathematica** : cpu = 53.7882 (sec), leaf count = 0 , could not solve

DSolve[k[y[x]] + j[y[x]]\*Derivative[1][y][x] + h[y[x]]\*Derivative[1][y][x]^2 + Derivative[1][y][x]

✓ **Maple** : cpu = 0.556 (sec), leaf count = 56

$$\left\{ y(x) = \text{ODESolStruc} \left( -a, \left[ \left\{ \left( \frac{d}{d\_a} - b(-a) \right) - b(-a) + h(-a) (-b(-a))^2 + -b(-a) + k(-a) = 0 \right\}, \left\{ -a = y(x) \right\} \right] \right) \right\}$$

**2.1645 ODE No. 1645**

$$(y'(x)^2 + 1) (y'(x)h(x, y(x)) + j(x, y(x))) + y''(x) = 0$$

✗ **Mathematica** : cpu = 0.148054 (sec), leaf count = 0 , could not solve

DSolve[(j[x, y[x]] + h[x, y[x]]\*Derivative[1][y][x])\*(1 + Derivative[1][y][x]^2) + Derivative[1][y][x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(diff(y(x),x),x)+(diff(y(x),x)^2+1)\*(h(x,y(x))\*diff(y(x),x)+j(x,y(x))))=0,y(x))

**2.1646 ODE No. 1646**

$$ay(x) (y'(x)^2 + 1)^2 + y''(x) = 0$$

✓ **Mathematica** : cpu = 10.7758 (sec), leaf count = 262

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{\frac{\#1^2(-a)+2c_1+1}{2c_1+1}} \sqrt{2\#1^2a - 4c_1} E\left(\sin^{-1}\left(\sqrt{\frac{a}{2c_1+1}}\#1\right) \left|1 + \frac{1}{2c_1}\right.\right)}{\sqrt{\frac{a}{2c_1+1}} \sqrt{\#1^2(-a) + 2c_1 + 1} \sqrt{2 - \frac{\#1^2a}{c_1}}} \&x \right] [c_2 + x] \right\}, \left\{ \right.$$

✓ **Maple** : cpu = 0.178 (sec), leaf count = 94

$$\left\{ \int^{y(x)} a(-a^2 + 2 - C1) \frac{1}{\sqrt{-(-1 + a(-a^2 + 2 - C1)) a(-a^2 + 2 - C1)}} d_a - x - C2 = 0, \int^{y(x)} -a(-a^2 + 2 - C1) \frac{1}{\sqrt{-(-1 + a(-a^2 + 2 - C1)) a(-a^2 + 2 - C1)}} d_a - x - C2 = 0 \right\}$$

**2.1647 ODE No. 1647**

$$y''(x) - a(xy'(x) - y(x))^r = 0$$

✓ **Mathematica** : cpu = 51.5476 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow x \left( \int_1^x \left( \frac{1}{2} a K[2]^{2r} - \frac{1}{2} a r K[2]^{2r} + c_1 K[2]^{2r-2} \right)^{\frac{1}{1-r}} dK[2] + c_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.461 (sec), leaf count = 60

$$\left\{ y(x) = \left( \int -\frac{x^2(r-1)a - C1}{2x^2} 2^{\frac{r}{r-1}} \left( -(x^2(r-1)a - C1)^{-1} \right)^{\frac{r}{r-1}} dx + C2 \right) x \right\}$$

**2.1648 ODE No. 1648**

$$y''(x) - kx^a y(x)^b y'(x)^c = 0$$

✗ **Mathematica** : cpu = 0.0807754 (sec), leaf count = 0 , could not solve

`DSolve[-(k*x^a*y[x]^b*Derivative[1][y][x]^c) + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 2.059 (sec), leaf count = 205

$$\left\{ y(x) = \text{ODESolStruc} \left( -a e^{\int -b(-a) d_a + C1}, \left[ \left\{ \frac{d}{d_a} -b(-a) = \frac{(-b(-a))^2}{(a-c+2)^2} \left( -k_a^b -b(-a) (b+c-1)^2 \left( -\frac{a}{a-c+2} \right) \right) \right\} \right] \right)$$

2.1649 ODE No. 1649

$$h(x, y(x)) \left( y'(x) - \frac{y(x)}{x} \right)^a + y''(x) = 0$$

✗ **Mathematica** : cpu = 2.73722 (sec), leaf count = 0 , could not solve

`DSolve[h[x, y[x]]*(-(y[x]/x) + Derivative[1][y][x])^a + Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x) + (diff(y(x), x) - y(x)/x)^a * h(x, y(x)) = 0, y(x))`

2.1650 ODE No. 1650

$$y''(x) - a\sqrt{y'(x)^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.0221948 (sec), leaf count = 19

$$\left\{ \left\{ y(x) \rightarrow \frac{\cosh(ax + c_1)}{a} + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.238 (sec), leaf count = 16

$$\left\{ y(x) = \frac{\cosh(a(-C1 + x))}{a} + -C2 \right\}$$

2.1651 ODE No. 1651

$$a\left(-\sqrt{y'(x)^2 + 1}\right) - b + y''(x) = 0$$

✓ **Mathematica** : cpu = 0.263785 (sec), leaf count = 414

$$\left\{ \left\{ y(x) \rightarrow \frac{a \operatorname{InverseFunction} \left[ \frac{b \tan^{-1} \left( \frac{\#1 b}{\sqrt{\#1^2 + 1} \sqrt{a^2 - b^2}} \right) - \frac{b \tan^{-1} \left( \frac{\#1 a}{\sqrt{a^2 - b^2}} \right) + \sinh^{-1}(\#1)}{\sqrt{a^2 - b^2}} \right]}{a} \right] \& [c_1 + x]^2 - b \operatorname{InverseFunction} \left[ \sqrt{a^2 - b^2} \right]} \right\}$$

✓ **Maple** : cpu = 0.163 (sec), leaf count = 31

$$\left\{ y(x) = \int \operatorname{RootOf} \left( x - \int^{-Z} \left( a \sqrt{-f^2 + 1} + b \right)^{-1} d_f + -C1 \right) dx + -C2 \right\}$$

**2.1652 ODE No. 1652**

$$y''(x) - a\sqrt{by(x)^2 + y'(x)^2} = 0$$

✗ **Mathematica** : cpu = 0.871325 (sec), leaf count = 0 , could not solve

`DSolve[-(a*Sqrt[b*y[x]^2 + Derivative[1][y][x]^2]) + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.309 (sec), leaf count = 36

$$\left\{ y(x) = e^{\int \text{RootOf}\left(x - f^{-z} \left(a\sqrt{-f^2 + b - f^2}\right)^{-1} d\_f + \_C1\right) dx + \_C2} \right\}$$

**2.1653 ODE No. 1653**

$$y''(x) - a(y'(x)^2 + 1)^{3/2} = 0$$

✓ **Mathematica** : cpu = 0.0653448 (sec), leaf count = 75

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{i\sqrt{a^2x^2 + 2ac_1x + c_1^2 - 1}}{a} \right\}, \left\{ y(x) \rightarrow c_2 + \frac{i\sqrt{a^2x^2 + 2ac_1x + c_1^2 - 1}}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 41

$$\left\{ y(x) = \frac{1}{a} \left( \left( -1 + (\_C1 + x)^2 a^2 \right) \sqrt{-\left( -1 + (\_C1 + x)^2 a^2 \right)^{-1} + \_C2 a} \right) \right\}$$

**2.1654 ODE No. 1654**

$$y''(x) - 2ax(y'(x)^2 + 1)^{3/2} = 0$$

✓ **Mathematica** : cpu = 0.290395 (sec), leaf count = 308

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{\sqrt{\frac{ax^2+c_1-1}{c_1-1}} \sqrt{\frac{ax^2+c_1+1}{c_1+1}} \left( F\left(i \sinh^{-1}\left(x\sqrt{\frac{a}{c_1+1}}\right) \middle| \frac{c_1+1}{c_1-1}\right) + (c_1-1) E\left(i \sinh^{-1}\left(x\sqrt{\frac{a}{c_1+1}}\right) \middle| \frac{c_1+1}{c_1-1}\right) \right)}{\sqrt{\frac{a}{c_1+1}} \sqrt{a^2x^4 + 2ac_1x^2 + c_1^2 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.187 (sec), leaf count = 38

$$\left\{ y(x) = \int \sqrt{-\left( -1 + (x^2 + 2\_C1)^2 a^2 \right)^{-1}} a(x^2 + 2\_C1) dx + \_C2 \right\}$$

**2.1655 ODE No. 1655**

$$y''(x) - ay(x)(y'(x)^2 + 1)^{3/2} = 0$$

✓ **Mathematica** : cpu = 0.837853 (sec), leaf count = 350

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{\frac{\#1^2 a + 2c_1 - 2}{c_1 - 1}} \sqrt{\frac{\#1^2 a + 2c_1 + 2}{c_1 + 1}} \left( F \left( i \sinh^{-1} \left( \sqrt{\frac{a}{2c_1 + 2}} \#1 \right) \middle| \frac{c_1 + 1}{c_1 - 1} \right) + (c_1 - 1) E \left( i \sinh^{-1} \left( \sqrt{\frac{a}{2c_1 + 2}} \#1 \right) \right) \right)}{\sqrt{\frac{a}{2c_1 + 2}} \sqrt{\#1^4 a^2 + 4\#1^2 a c_1 + 4c_1^2 - 4}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.22 (sec), leaf count = 84

$$\left\{ \int^{y(x)} a(-a^2 + 2\_C1) \frac{1}{\sqrt{4 - a^2(-a^2 + 2\_C1)^2}} d\_a - x - \_C2 = 0, \int^{y(x)} -a(-a^2 + 2\_C1) \frac{1}{\sqrt{4 - a^2(-a^2 + 2\_C1)^2}} d\_a - x - \_C2 = 0 \right\}$$

**2.1656 ODE No. 1656**

$$y''(x) - a(y'(x)^2 + 1)^{3/2}(bx + c + y(x)) = 0$$

✗ **Mathematica** : cpu = 100.315 (sec), leaf count = 0 , could not solve

`DSolve[-(a*(c + b*x + y[x])*(1 + Derivative[1][y][x]^2)^(3/2)) + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.78 (sec), leaf count = 771

$$\left\{ y(x) = -bx + \text{RootOf} \left( -x + \int^{-Z} \frac{1}{(\_f^4 a^2 + 4\_f^3 a^2 c + 4\_f^2 a^2 c^2 - 4\_f^2 a^2 \_C1 - 8\_C1 \_f a^2 c + 4\_C1^2 a^2)} d\_f \right) \right\}$$

**2.1657 ODE No. 1657**

$$y''(x) + y(x)^3 y'(x) - y(x) y'(x) \sqrt{4y'(x) + y(x)^4} = 0$$

✓ **Mathematica** : cpu = 0.152255 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow \sqrt{2} e^{c_1} \tan \left( 2\sqrt{2} e^{3c_1} (c_2 + x) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.268 (sec), leaf count = 35

$$\left\{ y(x) = \frac{1}{\_C1} \tan \left( (\_C1^{-2})^{\frac{3}{2}} (\_C2 + x) \right), y(x) = \frac{1}{\_C1} \tanh \left( (\_C1^{-2})^{\frac{3}{2}} (\_C2 + x) \right) \right\}$$

**2.1658 ODE No. 1658**

$$y''(x) - h(y'(x), ax + by(x)) = 0$$

✗ **Mathematica** : cpu = 0.120165 (sec), leaf count = 0 , could not solve

`DSolve[-h[Derivative[1][y][x], a*x + b*y[x]] + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.143 (sec), leaf count = 115

$$\left\{ y(x) = ODESolStruc \left( -\frac{a(\int -b(-a) d_a + -C1) - b_a}{b}, \left[ \left\{ \frac{d}{d_a} - b(-a) = -h \left( \frac{-a - b(-a) + b}{-b(-a)b}, b_a \right) (-b(-a)) \right. \right. \right.$$

**2.1659 ODE No. 1659**

$$y''(x) - y(x)h \left( x, \frac{y'(x)}{y(x)} \right) = 0$$

✗ **Mathematica** : cpu = 10.5355 (sec), leaf count = 0 , could not solve

`DSolve[-(h[x, Derivative[1][y][x]/y[x]]*y[x]) + Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.093 (sec), leaf count = 60

$$\left\{ y(x) = ODESolStruc \left( e^{\int -b(-a) d_a + -C1}, \left[ \left\{ \frac{d}{d_a} - b(-a) = -(-b(-a))^2 + h(-a, -b(-a)) \right\}, \left\{ -a = x, -b(-a) \right\} \right. \right.$$

**2.1660 ODE No. 1660**

$$y''(x) - x^{n-2}h(x^{-n}y(x), x^{1-n}y'(x)) = 0$$

✗ **Mathematica** : cpu = 4.13428 (sec), leaf count = 0 , could not solve

`DSolve[-(x^(-2 + n))*h[y[x]/x^n, x^(1 - n)*Derivative[1][y][x]]) + Derivative[2][y][x] == 0,`

✓ **Maple** : cpu = 0.944 (sec), leaf count = 125

$$\left\{ y(x) = ODESolStruc \left( \frac{-a}{e^{-(\int -b(-a) d_a + -C1)n}}, \left[ \left\{ \frac{d}{d_a} - b(-a) = \left( -b(-a) h \left( -a, \frac{b(-a) - a n + 1}{-b(-a)} \right) + n_a \right. \right. \right.$$

**2.1661 ODE No. 1661**

$$8y''(x) + 9y'(x)^4 = 0$$

✓ **Mathematica** : cpu = 0.0293413 (sec), leaf count = 90

$$\left\{ \left\{ y(x) \rightarrow c_2 - \frac{1}{3} \sqrt[3]{-\frac{1}{3}(9x - 8c_1)^{2/3}} \right\}, \left\{ y(x) \rightarrow \frac{(9x - 8c_1)^{2/3}}{3\sqrt[3]{3}} + c_2 \right\}, \left\{ y(x) \rightarrow \frac{1}{9} \left( (-3)^{2/3} (9x - 8c_1)^{2/3} + 9c_2 \right) \right\} \right.$$

✓ **Maple** : cpu = 0.085 (sec), leaf count = 51

$$\left\{ y(x) = (_C1 + x)^{\frac{2}{3}} + _C2, y(x) = \frac{i\sqrt{3}-1}{2} (_C1 + x)^{\frac{2}{3}} + _C2, y(x) = -\frac{i\sqrt{3}+1}{2} (_C1 + x)^{\frac{2}{3}} + _C2 \right\}$$

**2.1662 ODE No. 1662**

$$ay''(x) + cy(x) + h(y'(x)) = 0$$

✗ **Mathematica** : cpu = 1.38411 (sec), leaf count = 0 , could not solve

`DSolve[h[Derivative[1][y][x]] + c*y[x] + a*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.4 (sec), leaf count = 56

$$\left\{ y(x) = ODESolStruc \left( _a, \left[ \left( \frac{d}{d\_a} b(_a) \right) _b(_a) + \frac{h(_b(_a)) + c\_a}{a} = 0 \right], \left\{ _a = y(x), _b(_a) = \frac{d}{dx} y \right\} \right.$$

**2.1663 ODE No. 1663**

$$-xy(x)^n + xy''(x) + 2y'(x) = 0$$

✗ **Mathematica** : cpu = 0.0332532 (sec), leaf count = 0 , could not solve

`DSolve[-(x*y[x]^n) + 2*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.959 (sec), leaf count = 125

$$\left\{ y(x) = ODESolStruc \left( _a e^{\int -b(_a) d\_a} + _C1, \left[ \frac{d}{d\_a} b(_a) = -\frac{(_b(_a))^2 (_b(_a)(n-1)^2 _a^n + 2\_a(n-1))}{4} \right] \right.$$



**2.1664 ODE No. 1664**

$$ax^m y(x)^n + xy''(x) + 2y'(x) = 0$$

✗ **Mathematica** : cpu = 0.526519 (sec), leaf count = 0 , could not solve

`DSolve[a*x^m*y[x]^n + 2*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 3.272 (sec), leaf count = 155

$$\left\{ y(x) = ODESolStruc \left( \_a e^{\int \_b(\_a) d\_a + \_C1}, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = \frac{(a\_b(\_a)(n-1)^2 \_a^n + (m+1)(\_a(m-n))}{(m+1)} \right. \right. \right. \right.$$

**2.1665 ODE No. 1665**

$$xy''(x) + 2y'(x) + xe^{y(x)} = 0$$

✗ **Mathematica** : cpu = 0.362516 (sec), leaf count = 0 , could not solve

`DSolve[E^y[x]*x + 2*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.617 (sec), leaf count = 84

$$\left\{ y(x) = ODESolStruc \left( \_a - 2 \int \_b(\_a) d\_a - 2 \_C1, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = (e^{-a} - 2) (\_b(\_a))^3 + (\_b(\_a))^2 \right. \right. \right. \right.$$

**2.1666 ODE No. 1666**

$$ay'(x) + bxe^{y(x)} + xy''(x) = 0$$

✗ **Mathematica** : cpu = 0.59694 (sec), leaf count = 0 , could not solve

`DSolve[b*E^y[x]*x + a*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.963 (sec), leaf count = 93

$$\left\{ y(x) = ODESolStruc \left( \_a - 2 \int \_b(\_a) d\_a - 2 \_C1, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = (be^{-a} - 2a + 2) (\_b(\_a))^3 + (a - 1) \right. \right. \right. \right.$$

**2.1667 ODE No. 1667**

$$bx^{5-2a}e^{y(x)} + ay'(x) + xy''(x) = 0$$

✗ **Mathematica** : cpu = 0.61455 (sec), leaf count = 0 , could not solve

`DSolve[b*E^y[x]*x^(5 - 2*a) + a*Derivative[1][y][x] + x*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.548 (sec), leaf count = 121

$$\left\{ y(x) = \text{ODESolStruc} \left( (2a - 6) \int -b(-a) d_a + 2a\_C1 + -a - 6\_C1, \left[ \left\{ \frac{d}{d\_a} - b(-a) = (be^{-a} + 2a^2 - 8a) \right. \right. \right. \right.$$

**2.1668 ODE No. 1668**

$$xy''(x) - (1 - y(x))y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0714264 (sec), leaf count = 46

$$\left\{ \left\{ y(x) \rightarrow 2 - \sqrt{2}\sqrt{c_1 + 2} \tanh \left( \frac{\sqrt{c_1 + 2}(2c_2 - \log(x))}{\sqrt{2}} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 24

$$\left\{ y(x) = \frac{1}{-C1} \left( 2\_C1 + \tanh \left( \frac{\ln(x) - \_C2}{2\_C1} \right) \right) \right\}$$

**2.1669 ODE No. 1669**

$$-x^2y'(x)^2 + xy''(x) + 2y'(x) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 125.481 (sec), leaf count = 74

$$\text{Solve} \left[ \int_1^x -\frac{c_1 e^{K[2]} + K[2] + 1}{c_1 e^{K[2]} K[2] + 2K[2]^2 + K[2]} dK[2] + c_2 = \int_1^{y(x)} -\frac{x}{c_1 e^{xK[1]} + 2xK[1] + 1} dK[1], y(x) \right]$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 32

$$\left\{ y(x) = \frac{1}{x} \text{RootOf} \left( -\ln(x) + \_C2 + \int^{-Z} -\left( e^{-f} \_C1 - 2\_f - 1 \right)^{-1} d\_f \right) \right\}$$

**2.1670 ODE No. 1670**

$$a(xy'(x) - y(x))^2 - b + xy''(x) = 0$$

✓ **Mathematica** : cpu = 83.9237 (sec), leaf count = 49

$$\left\{ \left\{ y(x) \rightarrow x \left( \int_1^x \frac{\sqrt{-\frac{b}{a}} \tan\left(\frac{bK[2]}{\sqrt{-\frac{b}{a}}} + c_1\right)}{K[2]^2} dK[2] + c_2 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.398 (sec), leaf count = 35

$$\left\{ y(x) = \left( \int \frac{i}{x^2} \tan(-i\sqrt{a}\sqrt{b}x + \_C1) \sqrt{b} \frac{1}{\sqrt{a}} dx + \_C2 \right) x \right\}$$

**2.1671 ODE No. 1671**

$$2xy''(x) + y'(x)^3 + y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0348029 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow c_2 - 2ie^{c_1} \sqrt{e^{2c_1} - x} \right\}, \left\{ y(x) \rightarrow c_2 + 2ie^{c_1} \sqrt{e^{2c_1} - x} \right\} \right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 35

$$\left\{ y(x) = -2 \frac{\sqrt{\_C1 x - 1}}{\_C1} + \_C2, y(x) = 2 \frac{\sqrt{\_C1 x - 1}}{\_C1} + \_C2 \right\}$$

**2.1672 ODE No. 1672**

$$x^2y''(x) - a(y(x)^n - y(x)) = 0$$

✗ **Mathematica** : cpu = 17.7796 (sec), leaf count = 0 , could not solve

`DSolve[-(a*(-y[x] + y[x]^n)) + x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.094 (sec), leaf count = 65

$$\left\{ y(x) = ODESolStruc \left( \_a, \left[ \left\{ \frac{d}{d\_a} \_b(\_a) = (\_b(\_a))^2 (\_a \_b(\_a) a - \_a^n \_b(\_a) a - 1) \right\} \right], \left\{ \_a = y(x), \_a \right\} \right)$$

**2.1673 ODE No. 1673**

$$a(e^{y(x)} - 1) + x^2 y''(x) = 0$$

✗ **Mathematica** : cpu = 31.306 (sec), leaf count = 0 , could not solve

`DSolve[a*(-1 + E^y[x]) + x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.813 (sec), leaf count = 60

$$\left\{ y(x) = ODESolStruc \left( -a, \left\{ \frac{d}{d_a} b(-a) = (-1 + a(e^{-a} - 1) b(-a)) (b(-a))^2 \right\}, \left\{ -a = y(x), b(-a) = \right. \right. \right.$$

**2.1674 ODE No. 1674**

$$y(x) \left( a(a + b) + b^2 c^2 x^{2b} \right) - x(2a + b - 1)y'(x) + x^2 y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0612403 (sec), leaf count = 69

$$\left\{ \left\{ y(x) \rightarrow 2^{-\frac{a+b}{b}} c^{a/b} \left( x^{2b} \right)^{\frac{a}{2b}} \left( c_2 \sin \left( c \sqrt{x^{2b}} \right) + 2c_1 \cos \left( c \sqrt{x^{2b}} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 25

$$\left\{ y(x) = x^a \left( \sin \left( x^b c \right) C1 + \cos \left( x^b c \right) C2 \right) \right\}$$

**2.1675 ODE No. 1675**

$$x^k \left( -h \left( x^k y(x), k y(x) + x y'(x) \right) \right) + (a + 1) x y'(x) + x^2 y''(x) = 0$$

✗ **Mathematica** : cpu = 3.24962 (sec), leaf count = 0 , could not solve

`DSolve[-(x^k*h[x^k*y[x], k*y[x] + x*Derivative[1][y][x]]) + (1 + a)*x*Derivative[1][y][x] +`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(x^2*diff(diff(y(x),x),x)+(a+1)*x*diff(y(x),x)-x^k*h(x^k*y(x),x*diff(y(x),x)+k*y(x)))=0`

**2.1676 ODE No. 1676**

$$a(xy'(x) - y(x))^2 - bx^2 + x^2y''(x) = 0$$

✓ **Mathematica** : cpu = 54.9174 (sec), leaf count = 117

$$\left\{ \left\{ y(x) \rightarrow x \left( c_2 + \int_1^x \frac{i\sqrt{b} \left( Y_1 \left( -i\sqrt{a}\sqrt{b}K[2] \right) - c_1 J_1 \left( i\sqrt{a}\sqrt{b}K[2] \right) \right)}{\sqrt{a}K[2] \left( c_1 J_0 \left( i\sqrt{a}\sqrt{b}K[2] \right) + Y_0 \left( -i\sqrt{a}\sqrt{b}K[2] \right) \right)} dK[2] \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.28 (sec), leaf count = 72

$$\left\{ y(x) = \left( \int -\frac{1}{ax} \sqrt{-ab} \left( -C1 Y_1(\sqrt{-abx}) + J_1(\sqrt{-abx}) \right) \left( -C1 Y_0(\sqrt{-abx}) + J_0(\sqrt{-abx}) \right)^{-1} dx + -C2 \right) a \right\}$$

**2.1677 ODE No. 1677**

$$ay(x)y'(x)^2 + bx + x^2y''(x) = 0$$

✗ **Mathematica** : cpu = 105.42 (sec), leaf count = 0 , could not solve

`DSolve[b*x + a*y[x]*Derivative[1][y][x]^2 + x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 2.267 (sec), leaf count = 101

$$\left\{ y(x) = ODESolStruc \left( -a e^{\int -b(-a) d_{-a} - C1}, \left[ \left\{ \frac{d}{d_{-a}} - b(-a) = (a_{-a}^3 + b) (-b(-a))^3 + (2_{-a}^2 a + 1) (-b(-a)) \right\} \right] \right)$$

**2.1678 ODE No. 1678**

$$x^2y''(x) - \sqrt{ax^2y'(x)^2 + by(x)^2} = 0$$

✗ **Mathematica** : cpu = 1.55151 (sec), leaf count = 0 , could not solve

`DSolve[-Sqrt[b*y[x]^2 + a*x^2*Derivative[1][y][x]^2] + x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.265 (sec), leaf count = 60

$$\left\{ y(x) - e^{\int^{\ln(x)} \text{RootOf} \left( \int^{-z-y(x)} \left( -a^2y(x) - a y(x) - \sqrt{(y(x))^2 (-a^2a+b)} \right)^{-1} d_{-a} - b + -C1 \right) d_{-b} + -C2} = 0 \right\}$$

**2.1679 ODE No. 1679**

$$(x^2 + 1)y''(x) + y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.077638 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow -x \cot(c_1) + \csc^2(c_1) \log(-x \sin(c_1) - \cos(c_1)) + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.152 (sec), leaf count = 27

$$\left\{ y(x) = \frac{x}{-C1} + \ln(-C1 x - 1) + \frac{\ln(-C1 x - 1)}{-C1^2} + -C2 \right\}$$

**2.1680 ODE No. 1680**

$$x^4(-y'(x)^2) + 4x^2y''(x) + 4y(x) = 0$$

✗ **Mathematica** : cpu = 10.8022 (sec), leaf count = 0 , could not solve

`DSolve[4*y[x] - x^4*Derivative[1][y][x]^2 + 4*x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.638 (sec), leaf count = 103

$$\left\{ y(x) = ODESolStruc \left( \frac{-a}{\left( e^{\int -b(-a) d_a + -C1} \right)^2}, \left[ \left\{ \frac{d}{d_a} - b(-a) = (-a^2 + 7_a) (-b(-a))^3 + (-a - 5) (-b(-a)) \right\} \right] \right) \right\}$$

**2.1681 ODE No. 1681**

$$ay(x)^3 + 9x^2y''(x) + 2y(x) = 0$$

✗ **Mathematica** : cpu = 3.47522 (sec), leaf count = 0 , could not solve

`DSolve[2*y[x] + a*y[x]^3 + 9*x^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.056 (sec), leaf count = 31

$$\left\{ y(x) = -C2 \operatorname{JacobiSN} \left( \left( \frac{\sqrt{2}}{2x^3} \sqrt{x^{\frac{20}{3}} a + -C1} \right) - C2, i \right) \sqrt[3]{x} \right\}$$

**2.1682 ODE No. 1682**

$$x^3(y''(x) + y(x)y'(x) - y(x)^3) + 12xy(x) + 24 = 0$$

✗ **Mathematica** : cpu = 22.5703 (sec), leaf count = 0 , could not solve

`DSolve[24 + 12*x*y[x] + x^3*(-y[x]^3 + y[x]*Derivative[1][y][x] + Derivative[2][y][x]) == 0,`

✓ **Maple** : cpu = 0.638 (sec), leaf count = 94

$$\left\{ y(x) = ODESolStruc \left( \_a e^{\int -b(\_a) d\_a + \_C1}, \left[ \left\{ \frac{d}{d\_a} b(\_a) = -(\_b(\_a))^2 ((\_a^3 + \_a^2 - 14\_a - 24) \_b(\_a) \right. \right. \right. \right.$$

**2.1683 ODE No. 1683**

$$x^3y''(x) - a(xy'(x) - y(x))^2 = 0$$

✓ **Mathematica** : cpu = 0.0782767 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow -\frac{x \log \left( -\frac{a(c_2x+c_1)}{x} \right)}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 23

$$\left\{ y(x) = -\frac{x}{a} \ln \left( \frac{a(\_C1 x - \_C2)}{x} \right) \right\}$$

**2.1684 ODE No. 1684**

$$xy(x) (a - 2x^2y(x)^2 + 3xy(x)) + b + 2x^3y''(x) + x^2(2xy(x) + 9)y'(x) = 0$$

✗ **Mathematica** : cpu = 60.9737 (sec), leaf count = 0 , could not solve

`DSolve[b + x*y[x]*(a + 3*x*y[x] - 2*x^2*y[x]^2) + x^2*(9 + 2*x*y[x])*Derivative[1][y][x] +`

✓ **Maple** : cpu = 1.717 (sec), leaf count = 100

$$\left\{ y(x) = ODESolStruc \left( \_a e^{\int -b(\_a) d\_a + \_C1}, \left[ \left\{ \frac{d}{d\_a} b(\_a) = \frac{(\_b(\_a))^2 ((-2\_a^3 + \_a^2 + (a - 5) \_a + b) \_a)}{2} \right. \right. \right. \right.$$

**2.1685 ODE No. 1685**

$$axy(x) + b - (kx^{k-1} - 12x^2) (3y'(x) + y(x)^2) + 2(4x^3 - x^k) (y''(x) + y(x)y'(x) - y(x)^3) = 0$$

✗ **Mathematica** : cpu = 5.42896 (sec), leaf count = 0 , could not solve

DSolve[b + a\*x\*y[x] - (-12\*x^2 + k\*x^(-1 + k))\*(y[x]^2 + 3\*Derivative[1][y][x]) + 2\*(4\*x^3 - y[x]^3 + y[x]\*Derivative[1][y][x] + Derivative[2][y][x]) == 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(2\*(-x^k+4\*x^3)\*(diff(diff(y(x),x),x)+y(x)\*diff(y(x),x)-y(x)^3)-(k\*x^(k-1)-12\*x^2)\*(3\*diff(y(x),x)+y(x)^2)+a\*x\*y(x)+b=0,y(x))

**2.1686 ODE No. 1686**

$$a^2y(x)^n + x^4y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0328033 (sec), leaf count = 0 , could not solve

DSolve[a^2\*y[x]^n + x^4\*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 1.18 (sec), leaf count = 128

$$\left\{ y(x) = ODESolStruc \left( \_a e^{\int -b(\_a) d\_a + \_C1}, \left[ \frac{d}{d\_a} b(\_a) = \frac{(\_b(\_a))^2 (a^2 \_b(\_a) (n-1)^2 \_a^n - 2 \_a (n-1) \_b(\_a))}{4} \right] \right) \right\}$$

**2.1687 ODE No. 1687**

$$x^4y''(x) - x(x^2 + 2y(x))y'(x) + 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0686413 (sec), leaf count = 83

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2 \left( (1 - i\sqrt{-c_1 - 1}) x^{2i\sqrt{-c_1 - 1}} + (1 + i\sqrt{-c_1 - 1}) c_2 \right)}{c_2 + x^{2i\sqrt{-c_1 - 1}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.104 (sec), leaf count = 21

$$\{y(x) = x^2(\tanh(\_C1 (\_C2 - \ln(x))) \_C1 + 1)\}$$



**2.1688 ODE No. 1688**

$$x^4 y''(x) - x^2 y'(x) (y'(x) + x) + 4y(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.012 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.113 (sec), leaf count = 32

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \_C2 - \int^{-Z} \left( e^{-f} \_C1 + 4\_f + 2 \right)^{-1} d\_f \right) x^2 \right\}$$

**2.1689 ODE No. 1689**

$$x^4 y''(x) + (xy'(x) - y(x))^3 = 0$$

✓ **Mathematica** : cpu = 0.647997 (sec), leaf count = 259

$$\left\{ \left\{ y(x) \rightarrow -ix \log \left( -\frac{\sqrt{-8ic_1 x^2 - \sinh(2c_2) - \cosh(2c_2) + i \sinh(c_2) + i \cosh(c_2)}}{4c_1 x} \right) \right\}, \left\{ y(x) \rightarrow -ix \log \left( -\frac{\sqrt{-8ic_1 x^2 - \sinh(2c_2) - \cosh(2c_2) + i \sinh(c_2) + i \cosh(c_2)}}{4c_1 x} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 37

$$\left\{ y(x) = \left( -\arctan \left( \frac{1}{\sqrt{\_C1 x^2 - 1}} \right) + \_C2 \right) x, y(x) = \left( \arctan \left( \frac{1}{\sqrt{\_C1 x^2 - 1}} \right) + \_C2 \right) x \right\}$$

**2.1690 ODE No. 1690**

$$\sqrt{x} y''(x) - y(x)^{3/2} = 0$$

✗ **Mathematica** : cpu = 22.4068 (sec), leaf count = 0 , could not solve

`DSolve[-y[x]^(3/2) + Sqrt[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.837 (sec), leaf count = 99

$$\left\{ y(x) = \text{ODESolStruc} \left( \frac{-a}{\left( e^{\int -b(\_a) d\_a + \_C1} \right)^3}, \left[ \left\{ \frac{d}{d\_a} b(\_a) = -(\_b(\_a))^3 \_a^{\frac{3}{2}} + 12(\_b(\_a))^3 \_a - 7(\_b(\_a))^3 \right\} \right] \right) \right\}$$

**2.1691 ODE No. 1691**

$$y''(x) (ax^2 + bx + c)^{3/2} - f\left(\frac{y(x)}{\sqrt{ax^2 + bx + c}}\right) = 0$$

✗ **Mathematica** : cpu = 61.6814 (sec), leaf count = 0 , could not solve

`DSolve[-f[y[x]/Sqrt[c + b*x + a*x^2]] + (c + b*x + a*x^2)^(3/2)*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 1.267 (sec), leaf count = 254

$$\left\{ y(x) = \text{RootOf}\left(-2a \arctan\left(\frac{2ax + b}{\sqrt{4ca - b^2}}\right) - 2 \int^{-Z} \frac{a}{\sqrt{4C1a^2 - 4c_g^2a + b^2_g^2 + 8 \int F(_g) d_g}} d_g \sqrt{4}\right.\right.$$

**2.1692 ODE No. 1692**

$$x^{\frac{n}{n+1}} y''(x) - y(x)^{\frac{2n+1}{n+1}} = 0$$

✗ **Mathematica** : cpu = 0.0829136 (sec), leaf count = 0 , could not solve

`DSolve[-y[x]^((1 + 2*n)/(1 + n)) + x^(n/(1 + n))*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 3.955 (sec), leaf count = 156

$$\left\{ y(x) = \text{ODESolStruc}\left(-a \left(e^{\frac{(f_b(a) d_a + C1)(n+2)}{n}}\right)^{-1}, \left[\frac{d}{d_a} b(a) = 2 \frac{(b(a))^2}{n^2} \left(-1/2 a^{\frac{2n+1}{n+1}} b(a) n^2\right.\right.\right.$$

**2.1693 ODE No. 1693**

$$-h(y(x), f(x)y'(x)) + f(x)f'(x)y'(x) + f(x)^2y''(x) = 0$$

✗ **Mathematica** : cpu = 0.986146 (sec), leaf count = 0 , could not solve

`DSolve[-h[y[x], f[x]*Derivative[1][y][x]] + f[x]*Derivative[1][f][x]*Derivative[1][y][x] + f[x]^2*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.329 (sec), leaf count = 68

$$\left\{ y(x) = \text{ODESolStruc}\left(-a, \left[\frac{d}{d_a} b(a) = -h\left(-a, (b(a))^{-1}\right) (b(a))^3\right], \left\{-a = y(x), b(a) = \frac{f(x)}{f(x)}\right.\right.\right.$$

**2.1694 ODE No. 1694**

$$y(x)y''(x) - a = 0$$

✓ **Mathematica** : cpu = 0.188895 (sec), leaf count = 111

$$\left\{ \left\{ y(x) \rightarrow \exp \left( -\frac{c_1 + 2a \operatorname{erf}^{-1} \left( -i \sqrt{\frac{2}{\pi}} \sqrt{a e^{\frac{c_1}{a}}} (c_2 + x)^2 \right)^2}{2a} \right) \right\}, \left\{ y(x) \rightarrow \exp \left( -\frac{c_1 + 2a \operatorname{erf}^{-1} \left( i \sqrt{\frac{2}{\pi}} \sqrt{a e^{\frac{c_1}{a}}} (c_2 + x)^2 \right)^2}{2a} \right) \right\} \right.$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 54

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{2 \ln(\_a) a - 2 a \_C1}} d\_a - x - \_C2 = 0, \int^{y(x)} -\frac{1}{\sqrt{-2 a (\_C1 - \ln(\_a))}} d\_a - x - \_C2 = 0 \right\}$$

**2.1695 ODE No. 1695**

$$y(x)y''(x) - ax = 0$$

✗ **Mathematica** : cpu = 25.5803 (sec), leaf count = 0 , could not solve

`DSolve[-(a*x) + y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.627 (sec), leaf count = 103

$$\left\{ y(x) = \operatorname{ODESolStruc} \left( \_a \left( e^{\int -b(\_a) d\_a + \_C1} \right)^{\frac{3}{2}}, \left[ \frac{d}{d\_a} b(\_a) = \frac{(3\_a^2 - 4a) (\_b(\_a))^3}{4\_a} + 2 (\_b(\_a))^2 \right] \right\},$$

**2.1696 ODE No. 1696**

$$y(x)y''(x) - ax^2 = 0$$

✗ **Mathematica** : cpu = 24.3657 (sec), leaf count = 0 , could not solve

`DSolve[-(a*x^2) + y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.513 (sec), leaf count = 100

$$\left\{ y(x) = \operatorname{ODESolStruc} \left( \_a \left( e^{\int -b(\_a) d\_a + \_C1} \right)^2, \left[ \frac{d}{d\_a} b(\_a) = \frac{(2\_a^2 - a) (\_b(\_a))^3}{\_a} + 3 (\_b(\_a))^2 \right] \right\}, \left\{ \right.$$

**2.1697 ODE No. 1697**

$$-a + y(x)y''(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0663563 (sec), leaf count = 68

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a^2 (c_2 + x)^2 - e^{2c_1}}}{\sqrt{a}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a^2 (c_2 + x)^2 - e^{2c_1}}}{\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.044 (sec), leaf count = 39

$$\left\{ y(x) = \sqrt{ax^2 - 2\_C1 x + 2\_C2}, y(x) = -\sqrt{ax^2 - 2\_C1 x + 2\_C2} \right\}$$

**2.1698 ODE No. 1698**

$$-ax - b + y(x)y''(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0435909 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{\frac{ax^3}{3} + bx^2 + c_2x + 2c_1} \right\}, \left\{ y(x) \rightarrow \sqrt{\frac{ax^3}{3} + bx^2 + c_2x + 2c_1} \right\} \right\}$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)*y(x)+y(x)^2-a*x-b=0, y(x))`

**2.1699 ODE No. 1699**

$$y(x)y''(x) + y'(x)^2 - y'(x) = 0$$

✓ **Mathematica** : cpu = 0.041576 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow -c_1 \left( W \left( -\frac{e^{-\frac{c_1+c_2+x}{c_1}}}{c_1} \right) + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 33

$$\left\{ y(x) = -\_C1 \left( \text{lambertW} \left( -\frac{e^{-1}}{-\_C1} \left( e^{-\frac{c_2}{\_C1}} \right)^{-1} \left( e^{-\frac{x}{\_C1}} \right)^{-1} \right) + 1 \right) \right\}$$

**2.1700 ODE No. 1700**

$$y(x)y''(x) - y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.0845452 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow -e^{-c_1} \sinh(e^{c_1}(c_2 + x)) \right\}, \left\{ y(x) \rightarrow e^{-c_1} \sinh(e^{c_1}(c_2 + x)) \right\} \right\}$$

✓ **Maple** : cpu = 0.331 (sec), leaf count = 86

$$\left\{ y(x) = \frac{1}{2} \left( -C1 \left( e^{-\frac{x}{C1}} \right)^2 \left( e^{-\frac{C2}{C1}} \right)^2 + C1 \right) \left( e^{-\frac{C2}{C1}} \right)^{-1} \left( e^{-\frac{x}{C1}} \right)^{-1}, y(x) = \frac{1}{2} \left( -C1 \left( e^{-\frac{x}{C1}} \right)^2 \left( e^{-\frac{C2}{C1}} \right)^2 - C1 \right) \right\}$$

**2.1701 ODE No. 1701**

$$y(x)y''(x) - y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.197158 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{-e^{c_1}(c_2+x)} \left( e^{2e^{c_1}(c_2+x)-2c_1} + 1 \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} e^{-e^{c_1}(c_2+x)} \left( e^{2e^{c_1}(c_2+x)} + e^{-2c_1} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.289 (sec), leaf count = 42

$$\left\{ y(x) = \frac{C1}{2} \left( \left( e^{-\frac{C2}{C1}} \right)^2 \left( e^{-\frac{x}{C1}} \right)^2 + 1 \right) \left( e^{-\frac{C2}{C1}} \right)^{-1} \left( e^{-\frac{x}{C1}} \right)^{-1} \right\}$$

**2.1702 ODE No. 1702**

$$e^{2x}(ay(x)^4 + b) + e^x y(x)(cy(x)^2 + d) + y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 7.35032 (sec), leaf count = 0 , could not solve

`DSolve[E^x*y[x]*(d + c*y[x]^2) + E^(2*x)*(b + a*y[x]^4) - Derivative[1][y][x]^2 + y[x]*Derivative[2][y][x], y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)*y(x)-diff(y(x), x)^2+exp(x)*y(x)*(c*y(x)^2+d)+exp(2*x)*(b+a*y(x)^4), y(x), x)`

**2.1703 ODE No. 1703**

$$y(x)y''(x) - y'(x)^2 + y(x)^2(-\log(y(x))) = 0$$

✓ **Mathematica** : cpu = 0.0945617 (sec), leaf count = 63

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{1}{2}(e^{c_2+x} - c_1 e^{-c_2-x})} \right\}, \left\{ y(x) \rightarrow e^{\frac{1}{2}(e^{-c_2-x} - c_1 e^{c_2+x})} \right\} \right\}$$

✓ **Maple** : cpu = 0.07 (sec), leaf count = 25

$$\left\{ y(x) = e^{-\frac{e^{2x} C_1}{2 e^x}} e^{\frac{C_2}{2 e^x}} \right\}$$

**2.1704 ODE No. 1704**

$$y(x)^2 \left( \frac{f''(x)}{f(x)} - \frac{f'(x)^2}{f(x)^2} \right) + f(x)y(x)^3 + y(x)y''(x) - y'(x)^2 - y'(x) = 0$$

✗ **Mathematica** : cpu = 43.1428 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x]^3 - Derivative[1][y][x] - Derivative[1][y][x]^2 + y[x]^2*(-Derivative[1][f[x]]/f[x])]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-diff(y(x),x)+f(x)*y(x)^3+y(x)^2*(diff(diff(f(x),x)^2/f(x)^2)=0,y(x)))`

**2.1705 ODE No. 1705**

$$-y(x)f'(x) + f(x)y'(x) + y(x)y''(x) - y'(x)^2 - y(x)^3 = 0$$

✗ **Mathematica** : cpu = 0.261782 (sec), leaf count = 0 , could not solve

`DSolve[-y[x]^3 - y[x]*Derivative[1][f][x] + f[x]*Derivative[1][y][x] - Derivative[1][y][x]^2]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+f(x)*diff(y(x),x)-diff(f(x),x)*y(x)-y(x)^3=0,y(x)))`

**2.1706 ODE No. 1706**

$$-y(x)f''(x) + f'(x)y'(x) + f(x)y(x)^3 + y(x)y''(x) - y'(x)^2 - y(x)^4 = 0$$

✗ **Mathematica** : cpu = 0.536772 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x]^3 - y[x]^4 + Derivative[1][f][x]*Derivative[1][y][x] - Derivative[1][y][x]^2 + y[x]^4, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x), x), x)*y(x)-diff(y(x), x)^2+diff(f(x), x)*diff(y(x), x)-diff(diff(f(x), x), x), y(x))^4=0, y(x))`

**2.1707 ODE No. 1707**

$$ay(x)y'(x) + by(x)^2 + y(x)y''(x) - y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0827801 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{-\frac{c_1 e^{-ax} + bx}{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.066 (sec), leaf count = 39

$$\left\{ y(x) = 1 e^{\frac{e^{-ax} C_1}{a}} e^{\frac{b}{a^2}} \left( e^{\frac{bx}{a}} \right)^{-1} \left( e^{-\frac{C_2}{a}} \right)^{-1} \right\}$$

**2.1708 ODE No. 1708**

$$ay(x)y'(x) - 2ay(x)^2 + by(x)^3 + y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 46.9325 (sec), leaf count = 0 , could not solve

`DSolve[-2*a*y[x]^2 + b*y[x]^3 + a*y[x]*Derivative[1][y][x] - Derivative[1][y][x]^2 + y[x]*Derivative[2][y][x], y[x], x]`

✓ **Maple** : cpu = 1.043 (sec), leaf count = 73

$$\left\{ y(x) = ODESolStruc \left( -a, \left[ \left\{ \left( \frac{d}{d\_a} - b(-a) \right) - b(-a) - \frac{(-b(-a))^2 - \_a - b(-a) a - b\_a^3 + 2\_a^2 a}{\_a} = 0 \right\} \right] \right) \right\}$$

**2.1709 ODE No. 1709**

$$2a^2y(x)^2 - (ay(x) - 1)y'(x) + ay(x) - 2b^2y(x)^3 + y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 62.5212 (sec), leaf count = 0 , could not solve

`DSolve[a*y[x] + 2*a^2*y[x]^2 - 2*b^2*y[x]^3 - (-1 + a*y[x])*Derivative[1][y][x] - Derivative`

✓ **Maple** : cpu = 1.977 (sec), leaf count = 84

$$\left\{ y(x) = ODESolStruc \left( -a, \left[ \left( \frac{d}{d_a} b(-a) \right) - b(-a) - \frac{2b^2_a a^3 - 2_a a^2 a^2 + _a _b(-a) a + (_b(-a))^2 - a_a}{_a} \right] \right) \right.$$

**2.1710 ODE No. 1710**

$$-y(x)(y(x) + 1)(b^2y(x)^2 - a^2) + (ay(x) - 1)y'(x) + y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 109.451 (sec), leaf count = 0 , could not solve

`DSolve[-(y[x]*(1 + y[x])*(-a^2 + b^2*y[x]^2)) + (-1 + a*y[x])*Derivative[1][y][x] - Derivati`

✓ **Maple** : cpu = 2.734 (sec), leaf count = 91

$$\left\{ y(x) = ODESolStruc \left( -a, \left[ \left( \frac{d}{d_a} b(-a) \right) - b(-a) - \frac{a^4 b^2 + b^2_a a^3 - _a a^2 a^2 - _a _b(-a) a - _a a^2 + (_b(-a))^2 - a_a}{_a} \right] \right) \right.$$

**2.1711 ODE No. 1711**

$$y(x)^2 \log(y(x)) (\cos^2(x) - n^2 \cot^2(x)) + y(x)y''(x) - y'(x)^2 + y(x)y'(x)(\tan(x) + \cot(x)) = 0$$

✗ **Mathematica** : cpu = 300.004 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.553 (sec), leaf count = 81

$$\left\{ y(x) = 1e^{\frac{J_n(\sin(x))_C1}{\sin(x)(J_{n+1}(\sin(x))Y_n(\sin(x)) - J_n(\sin(x))Y_{n+1}(\sin(x)))}} \left( e^{\frac{Y_n(\sin(x))_C2}{\sin(x)(J_{n+1}(\sin(x))Y_n(\sin(x)) - J_n(\sin(x))Y_{n+1}(\sin(x)))}} \right)^{-1} \right\}$$



**2.1712 ODE No. 1712**

$$-f(x)y(x)y'(x) - g(x)y(x)^2 + y(x)y''(x) - y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 11.0708 (sec), leaf count = 57

$$\left\{ \left\{ y(x) \rightarrow c_2 \exp \left( \int_1^x e^{\int_1^{K[3]} f(K[1]) dK[1]} \left( \int_1^{K[3]} g(K[2]) e^{-\int_1^{K[2]} f(K[1]) dK[1]} dK[2] + c_1 \right) dK[3] \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.092 (sec), leaf count = 61

$$\left\{ y(x) = \frac{-C2}{e^{-C1} \int e^{\int f(x) dx} dx} e^{\int e^{\int f(x) dx} dx \int \frac{g(x)}{e^{\int f(x) dx}} dx} \left( e^{\int \frac{e^{\int f(x) dx} dx g(x)}{e^{\int f(x) dx}} dx} \right)^{-1} \right\}$$

**2.1713 ODE No. 1713**

$$-y(x) (g'(x) - y(x)^2 f'(x)) + y'(x) (f(x)y(x)^2 + g(x)) + y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 21.011 (sec), leaf count = 0 , could not solve

`DSolve[-(y[x]*(-(y[x]^2*Derivative[1][f][x]) + Derivative[1][g][x])) + (g[x] + f[x]*y[x]^2)*`

✓ **Maple** : cpu = 0.333 (sec), leaf count = 54

$$\left\{ y(x) = ODESolStruc \left( -b(-a), \left[ \frac{f(-a) (-b(-a))^2 + -C1 -b(-a) - g(-a) + \frac{d}{d-a} -b(-a)}{-b(-a)} = 0 \right] \right), \{ -a = x, \dots \}$$

**2.1714 ODE No. 1714**

$$y(x)y''(x) + 3y(x)y'(x) - 3y'(x)^2 - y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0688579 (sec), leaf count = 25

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 e^x}{\sqrt{1 - 2e^{c_1+x}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 68

$$\left\{ y(x) = \frac{1}{2e^{-x} - C1 - 2C2} \sqrt{-2(e^{-x} - C1 - C2)e^x}, y(x) = -\frac{1}{2e^{-x} - C1 - 2C2} \sqrt{-2(e^{-x} - C1 - C2)e^x} \right\}$$

**2.1715 ODE No. 1715**

$$y(x)y''(x) - ay'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0352611 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow c_2(-ax - c_1 + x)^{\frac{1}{1-a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.045 (sec), leaf count = 25

$$\left\{ y(x) = \left( \frac{1}{(1-a)(-C1x + -C2)} \right)^{(a-1)^{-1}} \right\}$$

**2.1716 ODE No. 1716**

$$a(y'(x)^2 + 1) + y(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.678731 (sec), leaf count = 172

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\#1\sqrt{1 - e^{2c_1}\#1^{-2a}} {}_2F_1\left(\frac{1}{2}, -\frac{1}{2a}; 1 - \frac{1}{2a}; e^{2c_1}\#1^{-2a}\right)}{\sqrt{e^{2c_1}\#1^{-2a} - 1}} \& \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\#1\sqrt{1 - e^{2c_1}\#1^{-2a}} {}_2F_1\left(\frac{1}{2}, -\frac{1}{2a}; 1 - \frac{1}{2a}; e^{2c_1}\#1^{-2a}\right)}{\sqrt{e^{2c_1}\#1^{-2a} - 1}} \& \right] [c_2 + x] \right\} \right\}$$

✓ **Maple** : cpu = 0.216 (sec), leaf count = 68

$$\left\{ \int^{y(x)} \frac{1}{-a^{-a}} \frac{1}{\sqrt{-a^{2a} + -C1}} d_{-a-x} - -C2 = 0, \int^{y(x)} -\frac{1}{-a^{-a}} \frac{1}{\sqrt{-a^{2a} + -C1}} d_{-a-x} - -C2 = 0 \right\}$$

**2.1717 ODE No. 1717**

$$ay'(x)^2 + by(x)^3 + y(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 1.68565 (sec), leaf count = 290

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\sqrt{2a+3}\#1^{a+1} \sqrt{\frac{-2b\#1^{2a+3} + 2ac_1 + 3c_1}{(2a+3)c_1}} {}_2F_1\left(\frac{1}{2}, \frac{a+1}{2a+3}; \frac{a+1}{2a+3} + 1; \frac{2b\#1^{2a+3}}{2ac_1 + 3c_1}\right)}{(a+1)\sqrt{-2b\#1^{2a+3} + 2ac_1 + 3c_1}} \& \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{\sqrt{2a+3}\#1^{a+1} \sqrt{\frac{-2b\#1^{2a+3} + 2ac_1 + 3c_1}{(2a+3)c_1}} {}_2F_1\left(\frac{1}{2}, \frac{a+1}{2a+3}; \frac{a+1}{2a+3} + 1; \frac{2b\#1^{2a+3}}{2ac_1 + 3c_1}\right)}{(a+1)\sqrt{-2b\#1^{2a+3} + 2ac_1 + 3c_1}} \& \right] [c_2 + x] \right\} \right\}$$

✓ **Maple** : cpu = 0.272 (sec), leaf count = 107

$$\left\{ \int^{y(x)} (2a+3)_{-a^{2a}} \frac{1}{\sqrt{-(2a+3)_{-a^{2a}}(2_{-a^{2a+3}b} - -C1)}} d_{-a-x} - -C2 = 0, \int^{y(x)} (-2a-3)_{-a^{2a}} \frac{1}{\sqrt{-(2a+3)_{-a^{2a}}(2_{-a^{2a+3}b} - -C1)}} d_{-a-x} - -C2 = 0 \right\}$$

**2.1718 ODE No. 1718**

$$dy(x)^{1-a} + ay'(x)^2 + by(x)y'(x) + cy(x)^2 + y(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 1.56007 (sec), leaf count = 398

$$\left\{ \left\{ y(x) \rightarrow - \frac{\exp\left(-\frac{x(b\sqrt{b^2-4(a+1)c-2(a+1)c+b^2})}{\sqrt{b^2-4(a+1)c+b}}\right) \left( b^2 \left( de^{\frac{1}{2}x(\sqrt{b^2-4(a+1)c+b})} - cc_2 \exp\left(\frac{x(b\sqrt{b^2-4(a+1)c-4(a+1)c+b^2})}{\sqrt{b^2-4(a+1)c+b}}\right) \right)}{\right.} \right.$$

✓ **Maple** : cpu = 0.276 (sec), leaf count = 133

$$\left\{ y(x) = e^{-\frac{x}{2a+2}\sqrt{(-4a-4)c+b^2}} e^{-\frac{bx}{2a+2}} \left( ((-4a-4)c^3 + b^2c^2) \left( de^{\frac{x}{2}(b+\sqrt{(-4a-4)c+b^2})} \sqrt{(-4a-4)c+b^2} + (e^{x\sqrt{(-4a-4)c+b^2}}) \right) \right) \right.$$

**2.1719 ODE No. 1719**

$$ay'(x)^2 + f(x)y(x)y'(x) + g(x)y(x)^2 + y(x)y''(x) = 0$$

✗ **Mathematica** : cpu = 42.0359 (sec), leaf count = 0 , could not solve

`DSolve[g[x]*y[x]^2 + f[x]*y[x]*Derivative[1][y][x] + a*Derivative[1][y][x]^2 + y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.579 (sec), leaf count = 70

$$\left\{ y(x) = ODESolStruc \left( e^{\int -b(\_a) d\_a - C1}, \left[ \left\{ \frac{d}{d\_a} b(\_a) = (-a-1)(\_b(\_a))^2 - f(\_a)\_b(\_a) - g(\_a) \right\}, \left\{ \right. \right. \right.$$

**2.1720 ODE No. 1720**

$$ay'(x)^2 + by(x)^2y'(x) + cy(x)^4 + y(x)y''(x) = 0$$

✗ **Mathematica** : cpu = 95.7367 (sec), leaf count = 0 , could not solve

`DSolve[c*y[x]^4 + b*y[x]^2*Derivative[1][y][x] + a*Derivative[1][y][x]^2 + y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.339 (sec), leaf count = 173

$$\left\{ \int^{y(x)} (2a+4) \left( \tan \left( \text{RootOf} \left( 2\_Z b\_a^2 - 2 \ln(\_a) a \sqrt{4\_a^4 ac - \_a^4 b^2 + 8c\_a^4} - \sqrt{4\_a^4 ac - \_a^4 b^2 + 8c\_a^4} \right) \right) \right) dy \right.$$

**2.1721 ODE No. 1721**

$$-\frac{ay(x)^3 f'(x)}{a+2} + \frac{af(x)^2 y(x)^4}{(a+2)^2} - \frac{(a-1)y'(x)^2}{a} - f(x)y(x)^2 y'(x) + y(x)y''(x) = 0$$

✗ **Mathematica** : cpu = 0.970806 (sec), leaf count = 0 , could not solve

`DSolve[(a*f[x]^2*y[x]^4)/(2+a)^2 - (a*y[x]^3*Derivative[1][f][x])/(2+a) - f[x]*y[x]^2*Derivative[1][y][x]^2/a + y[x]*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(diff(diff(y(x),x),x)*y(x)-(a-1)/a*diff(y(x),x)^2-f(x)*y(x)^2*diff(y(x),x)+a/(a+2)^2*f(x)/(a+2)*diff(f(x),x)*y(x)^3=0,y(x))`

**2.1722 ODE No. 1722**

$$-2ay(x) (y'(x)^2 + 1)^{3/2} + y(x)y''(x) - y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 1.97838 (sec), leaf count = 797

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{\left( (4c_1 a^2 + \sqrt{8c_1 a^2 + 1} + 1) E\left( i \sinh^{-1} \left( \sqrt{2} \sqrt{\frac{a^2}{-4c_1 a^2 + \sqrt{8c_1 a^2 + 1} - 1}} \right) \right) \right)^{\frac{4c_1 a^2 - \sqrt{8c_1 a^2 + 1}}{4c_1 a^2 + \sqrt{8c_1 a^2 + 1}}} \right]}{2c_1} \right. \right.$$

✓ **Maple** : cpu = 0.44 (sec), leaf count = 98

$$\left\{ \int^{y(x)} (-a^2 a + _C1) \frac{1}{\sqrt{-_a^4 a^2 - 2 _C1 _a^2 a - _C1^2 + _a^2}} d_a - x - _C2 = 0, \int^{y(x)} -(_a^2 a + _C1) \frac{1}{\sqrt{-_a^4 a^2 - 2 _C1 _a^2 a - _C1^2 + _a^2}} d_a - x - _C2 = 0 \right.$$

**2.1723 ODE No. 1723**

$$(y(x) + x)y''(x) + y'(x)^2 - y'(x) = 0$$

✓ **Mathematica** : cpu = 0.840261 (sec), leaf count = 227

$$\left\{ \left\{ y(x) \rightarrow -\frac{e^{-2c_1} \sqrt{e^{2c_1} (4e^{c_1} (x - c_2) + 1)}}{\sqrt{2}} + \frac{e^{-c_1}}{2} - 2c_2 + x \right\}, \left\{ y(x) \rightarrow \frac{e^{-2c_1} \sqrt{e^{2c_1} (4e^{c_1} (x - c_2) + 1)}}{\sqrt{2}} + \frac{e^{-c_1}}{2} \right\} \right.$$

✓ **Maple** : cpu = 0.13 (sec), leaf count = 16

$$\left\{ y(x) = \sqrt{-C1 + 2x_C2 + _C1 + x} \right\}$$

**2.1724 ODE No. 1724**

$$(x - y(x))y''(x) + 2y'(x)(y'(x) + 1) = 0$$

✓ **Mathematica** : cpu = 0.219856 (sec), leaf count = 27

$$\left\{ \left\{ y(x) \rightarrow -\frac{c_2(c_2 + x) + e^{-c_1}}{c_2 + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.587 (sec), leaf count = 21

$$\left\{ y(x) = \frac{-C2^2 - C2x + C1}{-C2 - x} \right\}$$

**2.1725 ODE No. 1725**

$$(x - y(x))y''(x) - (y'(x) + 1)(y'(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.349059 (sec), leaf count = 59

$$\left\{ \left\{ y(x) \rightarrow -\sqrt{e^{2c_1} - (c_2 + x)^2} - c_2 \right\}, \left\{ y(x) \rightarrow \sqrt{e^{2c_1} - (c_2 + x)^2} - c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.577 (sec), leaf count = 105

$$\left\{ y(x) = x + \text{RootOf} \left( -x + \int^{-Z} (-C1^2 - f^2 - 1) \left( 2 - C1^2 - f^2 + C1 \sqrt{-C1^2 - f^2 + 2f} \right)^{-1} d_f + C2 \right) \right\}$$

**2.1726 ODE No. 1726**

$$(x - y(x))y''(x) - h(y'(x)) = 0$$

✓ **Mathematica** : cpu = 0.838566 (sec), leaf count = 73

$$\text{Solve} \left[ \left\{ x = \int \frac{\exp \left( -\int_1^{K[3]-1} \frac{K[3]-1}{h(K[3])} dK[3] - c_1 \right)}{h(K[3])} dK[3] + c_2, x = \exp \left( -\int_1^{K[3]-1} \frac{K[3]-1}{h(K[3])} dK[3] \right) \right\} \right]$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 39

$$\left\{ y(x) = x + \text{RootOf} \left( -x + \int^{-Z} \left( -1 + \text{RootOf} \left( \int^{-Z} \frac{a-1}{h(a)} d_a + \ln(-g) + C1 \right) \right)^{-1} d_g + C2 \right) \right\}$$

**2.1727 ODE No. 1727**

$$2y(x)y''(x) + y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.183932 (sec), leaf count = 129

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ e^{2c_1} \tan^{-1} \left( \frac{\sqrt{\#1}}{\sqrt{e^{2c_1} - \#1}} \right) - \sqrt{\#1} \sqrt{e^{2c_1} - \#1} \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ e^{2c_1} \tan^{-1} \left( \frac{\sqrt{\#1}}{\sqrt{e^{2c_1} - \#1}} \right) - \sqrt{\#1} \sqrt{e^{2c_1} - \#1} \right] [c_2 + x] \right\} \right\}$$

✓ **Maple** : cpu = 0.222 (sec), leaf count = 87

$$\left\{ -\sqrt{y(x)(-C1 - y(x))} + \frac{C1}{2} \arctan \left( 1 \left( y(x) - \frac{C1}{2} \right) \frac{1}{\sqrt{y(x)(-C1 - y(x))}} \right) - x - C2 = 0, \sqrt{y(x)(-C1 - y(x))} + \frac{C1}{2} \arctan \left( 1 \left( y(x) - \frac{C1}{2} \right) \frac{1}{\sqrt{y(x)(-C1 - y(x))}} \right) - x - C2 = 0 \right\}$$

**2.1728 ODE No. 1728**

$$a + 2y(x)y''(x) - y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.00739197 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{x^2(c_1^2 - a)}{4c_2} + c_1x + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 24

$$\left\{ y(x) = \frac{(-C1^2 - a)x^2}{4C2} + C1x + C2 \right\}$$

**2.1729 ODE No. 1729**

$$a + f(x)y(x)^2 + 2y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.0248542 (sec), leaf count = 0 , could not solve

`DSolve[a + f[x]*y[x]^2 - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+f(x)*y(x)^2+a=0,y(x))`

**2.1730 ODE No. 1730**

$$2y(x)y''(x) - y'(x)^2 - 8y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.499096 (sec), leaf count = 77

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}i\sqrt{c_1}\operatorname{ns}\left(\left(-\frac{1}{2} + \frac{i}{2}\right)\sqrt[4]{c_1}(x + c_2)\right) - 1 \right\}^2 \right\}, \left\{ y(x) \rightarrow -\frac{1}{2}i\sqrt{c_1}\operatorname{ns}\left(\left(-\frac{1}{2} + \frac{i}{2}\right)\sqrt[4]{c_1}(x + c_2)\right) - 1 \right\}$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 53

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{4a^3 + C1a}} d_{a-x-C2} = 0, \int^{y(x)} -\frac{1}{\sqrt{4a^3 + C1a}} d_{a-x-C2} = 0 \right\}$$

**2.1731 ODE No. 1731**

$$2y(x)y''(x) - y'(x)^2 - 8y(x)^3 - 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.46777 (sec), leaf count = 359

$$\left\{ \left\{ y(x) \rightarrow \operatorname{InverseFunction}\left[ \frac{2i\#1\sqrt{\frac{c_1}{\#1(2-2\sqrt{1-c_1})}} + 1\sqrt{\frac{c_1}{\#1(2\sqrt{1-c_1}+2)}} + 1F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{c_1}{2\sqrt{1-c_1}+2}}}\right)}{\sqrt{\#1}}\right)}{\sqrt{2\sqrt{1-c_1}+2}\sqrt{4\#1^2 + 4\#1 + c_1}} \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.102 (sec), leaf count = 61

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{4a^3 + aC1 + 4a^2}} d_{a-x-C2} = 0, \int^{y(x)} -\frac{1}{\sqrt{4a^2 + C1 + 4a}} d_{a-x-C2} = 0 \right\}$$

**2.1732 ODE No. 1732**

$$2y(x)y''(x) - y'(x)^2 - 4(2y(x) + x)y(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.6458 (sec), leaf count = 0 , could not solve

`DSolve[-4*y[x]^2*(x + 2*y[x]) - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0, y[x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2-4*(x+2*y(x))*y(x)^2=0,y(x))`

**2.1733 ODE No. 1733**

$$y(x)^2(ay(x) + b) + 2y(x)y''(x) - y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 2.60988 (sec), leaf count = 437

$$\left\{ \left\{ \begin{array}{l} y(x) \rightarrow \text{InverseFunction} \left[ \frac{i\sqrt{2}\#1^{3/2} \sqrt{\frac{4c_1}{\#1(\sqrt{2ac_1+b^2-b})} + 2} \sqrt{1 - \frac{2c_1}{\#1(\sqrt{2ac_1+b^2+b})}} F\left(i \sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{c_1}{\sqrt{b^2+2ac_1}}}}{\sqrt{\#1}}\right)}{\sqrt{\frac{c_1}{2ac_1+b^2-b}} \sqrt{-\#1(\#1^2a + 2\#1b - 2c_1)}}\right)} \right. \end{array} \right.$$

✓ **Maple** : cpu = 0.101 (sec), leaf count = 71

$$\left\{ \int^{y(x)} -2 \frac{1}{\sqrt{-2a_a^3 - 4b_a^2 + 4_a_C1}} d_a - x - _C2 = 0, \int^{y(x)} 2 \frac{1}{\sqrt{-2a_a^3 - 4b_a^2 + 4_a_C1}} d_a - \right.$$

**2.1734 ODE No. 1734**

$$ay(x)^3 + 2y(x)y''(x) - y'(x)^2 + 2xy(x)^2 + 1 = 0$$

✗ **Mathematica** : cpu = 2.79602 (sec), leaf count = 0 , could not solve

`DSolve[1 + 2*x*y[x]^2 + a*y[x]^3 - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0,`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+1+2*x*y(x)^2+a*y(x)^3=0,y(x))`

**2.1735 ODE No. 1735**

$$y(x)^2(ay(x) + bx) + 2y(x)y''(x) - y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.23622 (sec), leaf count = 0 , could not solve

`DSolve[y[x]^2*(b*x + a*y[x]) - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0, y[x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+(a*y(x)+b*x)*y(x)^2=0,y(x))`



**2.1736 ODE No. 1736**

$$2y(x)y''(x) - y'(x)^2 - 3y(x)^4 = 0$$

✓ **Mathematica** : cpu = 8.57608 (sec), leaf count = 285

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{2i\#1^{3/2} \sqrt{(-1)^{5/6} \left( \frac{\sqrt[3]{-c_1}}{\#1} - 1 \right) \sqrt{\frac{(-c_1)^{2/3}}{\#1^2} + \frac{\sqrt[3]{-c_1}}{\#1}} + 1} F \left( \sin^{-1} \left( \frac{\sqrt{-\frac{i\sqrt[3]{-c_1}}{\#1} - (-1)^{5/6}}}{\sqrt[4]{3}} \right)} \right)}{\sqrt[4]{3} \sqrt[3]{-c_1} \sqrt{\#1^3 + c_1}} \right. \right. \right.$$

✓ **Maple** : cpu = 0.099 (sec), leaf count = 49

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-a^4 + C1 - a}} d_{-a - x - C2} = 0, \int^{y(x)} -\frac{1}{\sqrt{-a^4 + C1 - a}} d_{-a - x - C2} = 0 \right\}$$

**2.1737 ODE No. 1737**

$$-4(a + x^2)y(x)^2 + b + 2y(x)y''(x) - y'(x)^2 - 3y(x)^4 - 8xy(x)^3 = 0$$

✗ **Mathematica** : cpu = 1.27054 (sec), leaf count = 0 , could not solve

`DSolve[b - 4*(a + x^2)*y[x]^2 - 8*x*y[x]^3 - 3*y[x]^4 - Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0, y[x]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+b-4*(x^2+a)*y(x)^2-8*x*y(x)^3-3*y(x)^4=0,y(x))`

**2.1738 ODE No. 1738**

$$2y(x)^2 (f'(x) + f(x)^2) + 3f(x)y(x)y'(x) + 2y(x)y''(x) - y'(x)^2 - 8y(x)^3 = 0$$

✗ **Mathematica** : cpu = 0.558554 (sec), leaf count = 0 , could not solve

`DSolve[-8*y[x]^3 + 2*y[x]^2*(f[x]^2 + Derivative[1][f][x]) + 3*f[x]*y[x]*Derivative[1][y][x] + 2*y[x]*Derivative[2][y][x] - (Derivative[1][y][x])^2 == 0, y[x]]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*diff(diff(y(x),x),x)*y(x)-diff(y(x),x)^2+3*f(x)*y(x)*diff(y(x),x)+2*(f(x)^2+diff(f(x),x))*y(x)^2-8*y(x)^3=0,y(x))`

**2.1739 ODE No. 1739**

$$f(x)y(x)^2 + 2y(x)y''(x) + 4y(x)^2y'(x) - y'(x)^2 + y(x)^4 + 1 = 0$$

✗ **Mathematica** : cpu = 0.0591713 (sec), leaf count = 0 , could not solve

DSolve[1 + f[x]\*y[x]^2 + y[x]^4 + 4\*y[x]^2\*Derivative[1][y][x] - Derivative[1][y][x]^2 + 2\*y

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(2\*diff(diff(y(x),x),x)\*y(x)-diff(y(x),x)^2+4\*y(x)^2\*diff(y(x),x)+1+f(x)\*y(x)^2+y(x)^4

**2.1740 ODE No. 1740**

$$2y(x)y''(x) - 3y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0292809 (sec), leaf count = 16

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2}{(2c_1 + x)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 13

$$\{y(x) = 4(-C1 x + -C2)^{-2}\}$$

**2.1741 ODE No. 1741**

$$2y(x)y''(x) - 3y'(x)^2 - 4y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.096001 (sec), leaf count = 17

$$\{ \{ y(x) \rightarrow c_2 \sec^2(2c_1 + x) \} \}$$

✓ **Maple** : cpu = 0.075 (sec), leaf count = 34

$$\left\{ y(x) = 4 \left( (-C1^2 - C2^2) (\sin(x))^2 - 2 C1 C2 \sin(x) \cos(x) + C2^2 \right)^{-1} \right\}$$

2.1742 ODE No. 1742

$$f(x)y(x)^2 + 2y(x)y''(x) - 3y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 11.2792 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x]^2 - 3*Derivative[1][y][x]^2 + 2*y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.191 (sec), leaf count = 60

$$\left\{ y(x) = ODESolStruc \left( e^{\int -b(-a) d_a + C1}, \left[ \frac{d}{d_a} - b(-a) = \frac{(-b(-a))^2}{2} - \frac{f(-a)}{2} \right] \right), \left\{ -a = x, -b(-a) = \frac{\frac{d}{dx}y(x)}{y(x)} \right. \right.$$

2.1743 ODE No. 1743

$$y(x)^2 (ay(x)^3 + 1) + 2y(x)y''(x) - 6y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 42.773 (sec), leaf count = 2761

$$\left\{ \text{Solve} \left[ \frac{4 \left( F \left( \sin^{-1} \left( \sqrt{\frac{(\text{Root}[4c_1 \#1^4 + 4a \#1^3 + 1, 2] - \text{Root}[4c_1 \#1^4 + 4a \#1^3 + 1, 4]) (\text{Root}[4c_1 \#1^4 + 4a \#1^3 + 1, 1] - y(x))}{(\text{Root}[4c_1 \#1^4 + 4a \#1^3 + 1, 1] - \text{Root}[4c_1 \#1^4 + 4a \#1^3 + 1, 4]) (\text{Root}[4c_1 \#1^4 + 4a \#1^3 + 1, 2] - y(x))} \right)} \right) \right)}{\dots} \right] \right.$$

✓ **Maple** : cpu = 0.111 (sec), leaf count = 71

$$\left\{ \int^{y(x)} -2 \frac{1}{\sqrt{4_C1_a^4 + 4a_a^3 + 1_a}} d_a - x - C2 = 0, \int^{y(x)} 2 \frac{1}{\sqrt{4_C1_a^4 + 4a_a^3 + 1_a}} d_a - x - \dots \right.$$

2.1744 ODE No. 1744

$$2y(x)y''(x) - y'(x)^2 (y'(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.962658 (sec), leaf count = 173

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -ie^{-c_1} \left( \sqrt{\#1} \sqrt{\#1 e^{2c_1} - 1} - e^{-c_1} \log \left( \sqrt{\#1} e^{2c_1} + e^{c_1} \sqrt{\#1 e^{2c_1} - 1} \right) \right) \right] [c_2 + x] \right\} \right.$$

✓ **Maple** : cpu = 0.225 (sec), leaf count = 87

$$\left\{ -\frac{C1}{2} \arctan \left( 1 \left( y(x) - \frac{C1}{2} \right) \frac{1}{\sqrt{y(x) (-C1 - y(x))}} \right) - \sqrt{y(x) (-C1 - y(x))} - x - C2 = 0, \frac{C1}{2} \arctan \dots \right.$$

**2.1745 ODE No. 1745**

$$2(y(x) - a)y''(x) + y'(x)^2 + 1 = 0$$

✓ **Mathematica** : cpu = 0.306408 (sec), leaf count = 204

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\sqrt{2} \left( \frac{1}{2} \sqrt{a - \#1} \sqrt{e^{2c_1} - 2(a - \#1)} - \frac{e^{2c_1} \tan^{-1} \left( \frac{\sqrt{2} \sqrt{a - \#1}}{\sqrt{e^{2c_1} - 2(a - \#1)}} \right)}{2\sqrt{2}} \right) \& \right] [c_2 + x] \right\} \right\}$$

✓ **Maple** : cpu = 0.386 (sec), leaf count = 117

$$\left\{ -\frac{C1}{2} \arctan \left( 1 \left( y(x) - a - \frac{C1}{2} \right) \frac{1}{\sqrt{-(-y(x) + a)(a + C1 - y(x))}} \right) - x - C2 + \sqrt{-(-y(x) + a)(a + C1 - y(x))} \right\}$$

**2.1746 ODE No. 1746**

$$-ax^2 - bx - c + 3y(x)y''(x) - 2y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.0416061 (sec), leaf count = 0 , could not solve

`DSolve[-c - b*x - a*x^2 - 2*Derivative[1][y][x]^2 + 3*y[x]*Derivative[2][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.596 (sec), leaf count = 207

$$\left\{ y(x) = \text{RootOf} \left( -2 \int^{-z} \frac{b}{\sqrt{4_f^{4/3} C1 b^2 - 36 c_f^2 a + 9 b^2_f^2 - 2}} d_f \sqrt{4 ca - b^2} + C2 \sqrt{4 ca - b^2} - 2 ba \right) \right\}$$

**2.1747 ODE No. 1747**

$$3y(x)y''(x) - 5y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0279571 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2}{(3c_1 + 2x)^{3/2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.018 (sec), leaf count = 17

$$\left\{ -\frac{3}{2}(y(x))^{-\frac{2}{3}} - C1 x - C2 = 0 \right\}$$

**2.1748 ODE No. 1748**

$$4y(x)y''(x) - 3y'(x)^2 + 4y(x) = 0$$

✓ **Mathematica** : cpu = 0.0946634 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{(c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2 - 64)^2}{256 c_1^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.125 (sec), leaf count = 67

$$\left\{ -4 \frac{\sqrt{-C1 (y(x))^{3/2} + 4y(x)}}{\sqrt{y(x)}_C1} - x - _C2 = 0, 4 \frac{\sqrt{-C1 (y(x))^{3/2} + 4y(x)}}{\sqrt{y(x)}_C1} - x - _C2 = 0, y(x) = 0 \right\}$$

**2.1749 ODE No. 1749**

$$4y(x)y''(x) - 3y'(x)^2 - 12y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.542306 (sec), leaf count = 181

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ -\frac{4\sqrt{\frac{4\#1^{3/2}}{c_1}} + 1\sqrt{\#1^{3/2}c_1 + 4\#1^3} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; -\frac{4\#1^{3/2}}{c_1}\right)}{4\#1^2 + \sqrt{\#1}c_1} \& \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \dots \right\} \right\}$$

✓ **Maple** : cpu = 0.289 (sec), leaf count = 57

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{-C1 _a^{3/2} + 4 _a^3}} d_a - x - _C2 = 0, \int^{y(x)} -\frac{1}{\sqrt{-C1 _a^{3/2} + 4 _a^3}} d_a - x - _C2 = 0 \right\}$$

**2.1750 ODE No. 1750**

$$ay(x)^3 + by(x)^2 + cy(x) + 4y(x)y''(x) - 3y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 59.2399 (sec), leaf count = 1

\$Aborted

✓ **Maple** : cpu = 0.426 (sec), leaf count = 87

$$\left\{ \int^{y(x)} -3 \frac{1}{\sqrt{9 _C1 _a^{3/2} - 3a _a^3 - 9b _a^2 + 9c _a}} d_a - x - _C2 = 0, \int^{y(x)} 3 \frac{1}{\sqrt{9 _C1 _a^{3/2} - 3a _a^3 - 9b _a^2 + 9c _a}} d_a - x - _C2 = 0 \right\}$$

**2.1751 ODE No. 1751**

$$y'(x) \left( 6y(x)^2 - \frac{2y(x)f'(x)}{f(x)} \right) + f(x)y(x) + g(x)y(x)^2 + 4y(x)y''(x) - 2y(x)^2y'(x) - 3y'(x)^2 + y(x)^4 = 0$$

✗ **Mathematica** : cpu = 0.705536 (sec), leaf count = 0 , could not solve

`DSolve[f[x]*y[x] + g[x]*y[x]^2 + y[x]^4 - 2*y[x]^2*Derivative[1][y][x] + (6*y[x]^2 - (2*y[x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(4*diff(diff(y(x),x),x)*y(x)-3*diff(y(x),x)^2+(6*y(x)^2-2*diff(f(x),x)*y(x)/f(x))*diff`  
`2*y(x)^2*diff(y(x),x)+g(x)*y(x)^2+f(x)*y(x)=0,y(x))`

**2.1752 ODE No. 1752**

$$ay(x)^2 + 4y(x)y''(x) - 5y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.129652 (sec), leaf count = 26

$$\left\{ \left\{ y(x) \rightarrow c_2 \operatorname{sech}^4 \left( \frac{1}{4} \sqrt{a} (x - 4c_1) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.096 (sec), leaf count = 33

$$\left\{ y(x) = 16 \frac{\left( e^{1/4 \sqrt{a} x} \right)^4 a^2}{\left( e^{1/2 \sqrt{a} x} - C1 - C2 \right)^4} \right\}$$

**2.1753 ODE No. 1753**

$$12y(x)y''(x) - 15y'(x)^2 + 8y(x)^3 = 0$$

✓ **Mathematica** : cpu = 0.324285 (sec), leaf count = 43

$$\left\{ \left\{ y(x) \rightarrow \frac{2304c_1^2}{\left( 3c_1^2x^2 + 6c_2c_1^2x + 3c_2^2c_1^2 + 128 \right)^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 147

$$\left\{ -12 \frac{y(x) \left( 8 \sqrt{y(x)} - C1 \right) \sqrt{8y(x) - C1} \sqrt{y(x)}}{\sqrt{-24(y(x))^3 + 3C1(y(x))^{5/2} - C1} \sqrt{\sqrt{y(x)} \left( 8 \sqrt{y(x)} - C1 \right)}} - x - C2 = 0, 12 \frac{y(x)}{\sqrt{-24(y(x))^3 + 3C1(y(x))^{5/2} - C1}} \right\}$$

**2.1754 ODE No. 1754**

$$ny(x)y''(x) - (n-1)y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0326628 (sec), leaf count = 17

$$\{ \{y(x) \rightarrow c_2(x - c_1n)^n\} \}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 15

$$\left\{ y(x) = \left( \frac{-C1 x + -C2}{n} \right)^n \right\}$$

**2.1755 ODE No. 1755**

$$ay(x)y''(x) + by'(x)^2 + c0 + c1y(x) + c2y(x)^2 + c3y(x)^3 + c4y(x)^4 = 0$$

✗ **Mathematica** : cpu = 104.055 (sec), leaf count = 0 , could not solve

DSolve[c0 + c1\*y[x] + c2\*y[x]^2 + c3\*y[x]^3 + c4\*y[x]^4 + b\*Derivative[1][y][x]^2 + a\*y[x]\*D

✓ **Maple** : cpu = 0.341 (sec), leaf count = 418

$$\left\{ \int^{y(x)} (2a+b)(3a+2b)(a+b)(a+2b)b_{-}a^{2\frac{b}{a}} \frac{dy}{\sqrt{-36(a+2/3b)b(a+b)(a+b/2)-a^{2\frac{b}{a}}(a+2b)(2/3b(a+2/3b)+b^2)}} \right\}$$

**2.1756 ODE No. 1756**

$$ay(x)y''(x) + by'(x)^2 - \frac{y(x)y'(x)}{\sqrt{c^2+x^2}} = 0$$

✓ **Mathematica** : cpu = 0.287329 (sec), leaf count = 111

$$\left\{ \left\{ y(x) \rightarrow c_2 \left( -a^2 \left( x \left( \sqrt{c^2+x^2} + x \right)^{\frac{1}{a}} + c_1 \right) + a \left( \sqrt{c^2+x^2} + x \right)^{\frac{1}{a}} \left( \sqrt{c^2+x^2} - bx \right) + b\sqrt{c^2+x^2} \left( \sqrt{c^2+x^2} - bx \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.121 (sec), leaf count = 75

$$\left\{ y(x) = \left( \left( \frac{a}{a+b} \left( \frac{-C1 \sqrt[2]{2} a x^{a-1+1}}{a+1} {}_2F_1 \left( -\frac{1}{2a}, -\frac{1}{2a} - \frac{1}{2}; 1 - a^{-1}; -\frac{c^2}{x^2} \right) + -C2 \right)^{-1} \right)^{\frac{a}{a+b}} \right)^{-1} \right\}$$

**2.1757 ODE No. 1757**

$$(a + 2)f(x)y(x)^2y'(x) + ay(x)y''(x) + ay(x)^3y'(x) - (a - 1)y'(x)^2 + f(x)^2y(x)^4 = 0$$

✗ **Mathematica** : cpu = 0.883959 (sec), leaf count = 0 , could not solve

`DSolve[f[x]^2*y[x]^4 + (2 + a)*f[x]*y[x]^2*Derivative[1][y][x] + a*y[x]^3*Derivative[1][y][x] + (1 + a)*Derivative[1][y][x]^2 + a*y[x]*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(a*y(x)*diff(diff(y(x),x),x)-(a-1)*diff(y(x),x)^2+(a+2)*f(x)*y(x)^2*diff(y(x),x)+f(x)^2*y(x)^4=0,y(x),x)`

**2.1758 ODE No. 1758**

$$y''(x)(ay(x) + b) + cy'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0598874 (sec), leaf count = 31

$$\left\{ \left\{ y(x) \rightarrow \frac{(c_1(a + c)(c_2 + x))^{\frac{a}{a+c}} - b}{a} \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 42

$$\left\{ y(x) = \frac{1}{a} \left( (-C1 x + -C2)(a + c) \left( \frac{1}{(a + c)(-C1 x + -C2)} \right)^{\frac{c}{a+c}} - b \right) \right\}$$

**2.1759 ODE No. 1759**

$$xy(x)y''(x) + xy'(x)^2 - y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0373125 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_2 \sqrt{c_1 + x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.027 (sec), leaf count = 31

$$\left\{ y(x) = \sqrt{-C1 x^2 + 2 - C2}, y(x) = -\sqrt{-C1 x^2 + 2 - C2} \right\}$$



**2.1760 ODE No. 1760**

$$ay(x)y'(x) + f(x) + xy(x)y''(x) + xy'(x)^2 = 0$$

✗ **Mathematica** : cpu = 300.044 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.083 (sec), leaf count = 114

$$\left\{ y(x) = \frac{\sqrt{2}}{a-1} \sqrt{(a-1) \left( x^{1-a} \int \frac{x^a f(x)}{x} dx + x^{1-a} C1 - \int f(x) dx - C2 \right)}, y(x) = -\frac{\sqrt{2}}{a-1} \sqrt{(a-1) \left( x^{1-a} \int \frac{x^a f(x)}{x} dx + x^{1-a} C1 - \int f(x) dx - C2 \right)} \right.$$

**2.1761 ODE No. 1761**

$$x(ay(x)^4 + d) + y(x)(by(x)^2 + c) + xy(x)y''(x) - xy'(x)^2 + y(x)y'(x) = 0$$

✗ **Mathematica** : cpu = 1.53744 (sec), leaf count = 0 , could not solve

DSolve[y[x]\*(c + b\*y[x]^2) + x\*(d + a\*y[x]^4) + y[x]\*Derivative[1][y][x] - x\*Derivative[1][y][x]^2 + y[x]\*Derivative[1][y][x] = 0, y[x], x]

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(x\*y(x)\*diff(diff(y(x),x),x)-x\*diff(y(x),x)^2+y(x)\*diff(y(x),x)+x\*(d+a\*y(x)^4)+y(x)\*(c+b\*y(x)^2),y(x),x)

**2.1762 ODE No. 1762**

$$ay(x)y'(x) + bxy(x)^3 + xy(x)y''(x) - xy'(x)^2 = 0$$

✗ **Mathematica** : cpu = 52.972 (sec), leaf count = 0 , could not solve

DSolve[b\*x\*y[x]^3 + a\*y[x]\*Derivative[1][y][x] - x\*Derivative[1][y][x]^2 + x\*y[x]\*Derivative[1][y][x] = 0, y[x], x]

✓ **Maple** : cpu = 0.911 (sec), leaf count = 108

$$\left\{ y(x) = ODESolStruc \left( \frac{-a}{\left( e^{\int -b(-a) d_{-a} + C1} \right)^2}, \left[ \frac{d}{d_{-a}} - b(-a) = -2 \frac{-b(-a) \left( 1/2 + -a^2(-1/2 b_{-a} + a - 1) \right)}{-a} \right] \right.$$

**2.1763 ODE No. 1763**

$$ay(x)y'(x) + xy(x)y''(x) + 2xy'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.142668 (sec), leaf count = 33

$$\left\{ \left\{ y(x) \rightarrow c_2 x^{-a/3} \sqrt[3]{3x - (a-1)c_1 x^a} \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 148

$$\left\{ y(x) = \frac{\sqrt[3]{3}}{(a-1)x^a} \sqrt[3]{(x^a)^2 (a-1)^2 (-C2 (a-1)x^a - C1 x)}, y(x) = \frac{\sqrt[3]{3}(i\sqrt{3}-1)}{(2a-2)x^a} \sqrt[3]{(x^a)^2 (a-1)^2 (-C2 (a-1)x^a - C1 x)} \right\}$$

**2.1764 ODE No. 1764**

$$xy(x)y''(x) - 2xy'(x)^2 + (y(x) + 1)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0635551 (sec), leaf count = 37

$$\left\{ \left\{ y(x) \rightarrow \frac{\tan\left(\frac{\sqrt{c_1}(\log(x)-c_2)}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.195 (sec), leaf count = 18

$$\left\{ y(x) = -C1 \tanh\left(\frac{\ln(x) - C2}{2C1}\right) \right\}$$

**2.1765 ODE No. 1765**

$$ay(x)y'(x) + xy(x)y''(x) - 2xy'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.127414 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x^a}{(a-1)c_1 x^a + x} \right\} \right\}$$

✓ **Maple** : cpu = 0.03 (sec), leaf count = 27

$$\left\{ y(x) = -\frac{(a-1)x^a}{-C2(a-1)x^a - C1 x} \right\}$$

**2.1766 ODE No. 1766**

$$xy(x)y''(x) - 4xy'(x)^2 + 4y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0472711 (sec), leaf count = 21

$$\left\{ \left\{ y(x) \rightarrow \frac{c_2 x}{\sqrt[3]{c_1 x^3 + 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 64

$$\left\{ y(x) = x \frac{1}{\sqrt[3]{-3\_C2 x^3 +\_C1}}, y(x) = \frac{(i\sqrt{3} - 1)x}{2} \frac{1}{\sqrt[3]{-3\_C2 x^3 +\_C1}}, y(x) = -\frac{(i\sqrt{3} + 1)x}{2} \frac{1}{\sqrt[3]{-3\_C2 x^3 +\_C1}} \right\}$$

**2.1767 ODE No. 1767**

$$\left( \frac{ax}{\sqrt{b^2 - x^2}} - x \right) y'(x)^2 + xy(x)y''(x) - y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0759833 (sec), leaf count = 54

$$\left\{ \left\{ y(x) \rightarrow c_2 e^{\frac{\sqrt{b^2 - x^2}}{a}} \left( a\sqrt{b^2 - x^2} - c_1 \right)^{\frac{c_1}{a^2}} \right\} \right\}$$

✓ **Maple** : cpu = 0.345 (sec), leaf count = 50

$$\left\{ y(x) = \_C2 e^{\int -x\sqrt{b^2 - x^2} (-C1\sqrt{b^2 - x^2} + a(b^2 - x^2))^{-1} dx} \right\}$$

**2.1768 ODE No. 1768**

$$x(y(x) + x)y''(x) + xy'(x)^2 + (x - y(x))y'(x) - y(x) = 0$$

✓ **Mathematica** : cpu = 0.115471 (sec), leaf count = 79

$$\left\{ \left\{ y(x) \rightarrow -x - \sqrt{-e^{2c_2} (x^2 - 1) + (1 - 2ic_1) x^2} \right\}, \left\{ y(x) \rightarrow -x + \sqrt{-e^{2c_2} (x^2 - 1) + (1 - 2ic_1) x^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 43

$$\left\{ y(x) = -x - \sqrt{(-\_C1 + 1)x^2 +\_C2}, y(x) = -x + \sqrt{(-\_C1 + 1)x^2 +\_C2} \right\}$$

**2.1769 ODE No. 1769**

$$2xy(x)y''(x) - xy'(x)^2 + y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.0465322 (sec), leaf count = 18

$$\left\{ \left\{ y(x) \rightarrow c_2 (c_1 + \sqrt{x})^2 \right\} \right\}$$

✓ **Maple** : cpu = 0.035 (sec), leaf count = 21

$$\left\{ y(x) = \_C1 \sqrt{x} \_C2 + \_C1^2 x + \frac{\_C2^2}{4} \right\}$$

**2.1770 ODE No. 1770**

$$x^2(y(x) - 1)y''(x) - 2x^2y'(x)^2 - 2x(y(x) - 1)y'(x) - 2(y(x) - 1)^2y(x) = 0$$

✓ **Mathematica** : cpu = 0.803533 (sec), leaf count = 22

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{c_2 x^2 - c_1 x - 1} + 1 \right\} \right\}$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 26

$$\left\{ y(x) = \frac{x(\_C1 x - \_C2)}{\_C1 x^2 - \_C2 x - 1} \right\}$$

**2.1771 ODE No. 1771**

$$x^2(y(x) + x)y''(x) - (xy'(x) - y(x))^2 = 0$$

✓ **Mathematica** : cpu = 0.0819123 (sec), leaf count = 20

$$\left\{ \left\{ y(x) \rightarrow x \left( c_2 e^{\frac{c_1}{x}} - 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.162 (sec), leaf count = 22

$$\left\{ y(x) = -\frac{x}{\_C2} \left( -e^{\frac{\_C1}{x}} e^{-1} + \_C2 \right) \right\}$$

**2.1772 ODE No. 1772**

$$a(xy'(x) - y(x))^2 + x^2(x - y(x))y''(x) = 0$$

✓ **Mathematica** : cpu = 0.934375 (sec), leaf count = 36

$$\left\{ \left\{ y(x) \rightarrow x \left( \left( -\frac{(a-1)((-1)^a c_1 + c_2 x)}{x} \right)^{\frac{1}{1-a}} + 1 \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.151 (sec), leaf count = 37

$$\{(x^a y(x) - x^{a+1})(x - y(x))^{-a} - x(a-1) \_C2 + \_C1 = 0\}$$

**2.1773 ODE No. 1773**

$$2x^2 y(x) y''(x) + x^2 (-(y'(x)^2 + 1)) + y(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.183499 (sec), leaf count = 44

$$\left\{ \left\{ y(x) \rightarrow \frac{x(c_1^2 \log^2(x) - 2c_2 c_1^2 \log(x) + c_2^2 c_1^2 + 4)}{4c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.043 (sec), leaf count = 30

$$\left\{ y(x) = \frac{x(4 \_C2^2 (\ln(x))^2 + 4 \_C1 \ln(x) \_C2 + \_C1^2 + 1)}{4 \_C2} \right\}$$

**2.1774 ODE No. 1774**

$$ax^2 y(x) y''(x) + bx^2 y'(x)^2 + cxy(x) y'(x) + dy(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.36371 (sec), leaf count = 92

$$\left\{ \left\{ y(x) \rightarrow c_2 x^{-\frac{a \left( \sqrt{\frac{a^2 - 2a(c+2d) - 4bd + c^2}{a^2}} - 1 \right) + c}{2(a+b)}} \left( x \sqrt{\frac{a^2 - 2a(c+2d) - 4bd + c^2}{a^2}} + c_1 \right)^{\frac{a}{a+b}} \right\} \right\}$$

✓ **Maple** : cpu = 0.253 (sec), leaf count = 136

$$\left\{ y(x) = x^{-\frac{1}{2a+2b} \sqrt{(-4a-4b)d+(a-c)^2}} x^{\frac{a}{2a+2b}} x^{-\frac{c}{2a+2b}} \left( \frac{a^2 + (-2c-4d)a - 4bd + c^2}{(a+b)^2} \left( x^{\frac{1}{a} \sqrt{(-4a-4b)d+(a-c)^2}} \_C1 - \right. \right. \right.$$

**2.1775 ODE No. 1775**

$$-a(x+2)y(x)^2 + x(x+1)^2y(x)y''(x) - x(x+1)^2y'(x)^2 + 2(x+1)^2y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 0.130863 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow c_2(x+1)^a e^{-\frac{a+c_1}{x}} \right\} \right\}$$

✓ **Maple** : cpu = 0.1 (sec), leaf count = 31

$$\left\{ y(x) = \frac{(1+x)^a}{-C2 e^a} e^{-\frac{C1}{x}} \left( e^{\frac{a}{x}} \right)^{-1} \right\}$$

**2.1776 ODE No. 1776**

$$8(1-x^3)y(x)y''(x) - 4(1-x^3)y'(x)^2 - 12x^2y(x)y'(x) + 3xy(x)^2 = 0$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 0.352 (sec), leaf count = 49

$$\left\{ y(x) = \frac{x}{-C1} \left( -C1 \text{LegendreQ} \left( -\frac{1}{6}, \frac{1}{3}, \sqrt{-(x-1)(x^2+x+1)} \right) + \frac{C2}{2} \text{LegendreP} \left( -\frac{1}{6}, \frac{1}{3}, \sqrt{-(x-1)(x^2+x+1)} \right) \right) \right\}$$

**2.1777 ODE No. 1777**

$$f0(x)y(x)y''(x) + f1(x)y'(x)^2 + f2(x)y(x)y'(x) + f3(x)y(x)^2 = 0$$

✗ **Mathematica** : cpu = 49.4041 (sec), leaf count = 0 , could not solve

`DSolve[f3[x]*y[x]^2 + f2[x]*y[x]*Derivative[1][y][x] + f1[x]*Derivative[1][y][x]^2 + f0[x]*y[x]^2]`

✓ **Maple** : cpu = 0.74 (sec), leaf count = 79

$$\left\{ y(x) = \text{ODESolStruc} \left( e^{\int -b(\_a) d\_a - C1}, \left[ \left\{ \frac{d}{d\_a} b(\_a) = \frac{(-f0(\_a) - f1(\_a)) (\_b(\_a))^2 - f2(\_a) \_b(\_a)}{f0(\_a)} \right\} \right] \right) \right\}$$



**2.1781 ODE No. 1781**

$$(y(x)^2 + 1) y''(x) + (1 - 2y(x)) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0825091 (sec), leaf count = 14

$$\{ \{ y(x) \rightarrow \tan(\log(c_1(c_2 + x))) \} \}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 11

$$\{ y(x) = \tan(\ln(_C1 x + _C2)) \}$$

**2.1782 ODE No. 1782**

$$(y(x)^2 + 1) y''(x) - 3y(x) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0901926 (sec), leaf count = 93

$$\left\{ \left\{ y(x) \rightarrow -\frac{ic_1(c_2 + x)}{\sqrt{c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2 - 1}} \right\}, \left\{ y(x) \rightarrow \frac{ic_1(c_2 + x)}{\sqrt{c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2 - 1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 33

$$\left\{ y(x) = \sqrt{-(_C1^2 x^2 + 2 _C1 _C2 x + _C2^2 - 1)^{-1} (_C1 x + _C2)} \right\}$$

**2.1783 ODE No. 1783**

$$(y(x)^2 + x) y''(x) - 2(x - y(x)^2) y'(x)^3 + (4y(x) y'(x) + 1) y'(x) = 0$$

✓ **Mathematica** : cpu = 1.45828 (sec), leaf count = 24

$$\text{Solve} \left[ y(x)^2 + x = c_2 e^{e^{-c_1} y(x)}, y(x) \right]$$

✓ **Maple** : cpu = 0.174 (sec), leaf count = 23

$$\left\{ \frac{-_C1 y(x) + \ln(x + (y(x))^2) + _C2 + 2}{y(x)} = 0 \right\}$$



**2.1784 ODE No. 1784**

$$(x^2 + y(x)^2) y''(x) - (xy'(x) - y(x)) (y'(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.381405 (sec), leaf count = 72

$$\text{Solve} \left[ \log(x) + \frac{1}{2} \left( i \cot(c_1) \left( \log \left( 1 - \frac{iy(x)}{x} \right) - \log \left( 1 + \frac{iy(x)}{x} \right) \right) + \log \left( 1 - \frac{iy(x)}{x} \right) + \log \left( 1 + \frac{iy(x)}{x} \right) \right) = c_2 \right]$$

✓ **Maple** : cpu = 0.569 (sec), leaf count = 82

$$\left\{ y(x) = \tan \left( \text{RootOf} \left( - \left( e^{\frac{iC_1 - Z}{-1 + -C_1}} \right)^2 \left( e^{\frac{-iZ}{-1 + -C_1}} \right)^2 \left( e^{\frac{-C_2 - C_1}{-1 + -C_1}} \right)^2 \left( x^{\frac{-C_1}{-1 + -C_1}} \right)^2 + \left( e^{\frac{-C_2}{-1 + -C_1}} \right)^2 \left( x^{(-1 + -C_1)^{-1}} \right)^2 \right) \cos(\dots) \right\}$$

**2.1785 ODE No. 1785**

$$(x^2 + y(x)^2) y''(x) - 2(xy'(x) - y(x)) (y'(x)^2 + 1) = 0$$

✓ **Mathematica** : cpu = 0.377731 (sec), leaf count = 95

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} \left( -\sqrt{4x(e^{c_2} - x) + e^{2c_2} \cot^2(c_1)} - e^{c_2} \cot(c_1) \right) \right\}, \left\{ y(x) \rightarrow \frac{1}{2} \left( \sqrt{4x(e^{c_2} - x) + e^{2c_2} \cot^2(c_1)} - e^{c_2} \cot(c_1) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.359 (sec), leaf count = 83

$$\left\{ y(x) = \frac{1}{2_{-}C_2} \left( -C_1 + 1 - \sqrt{-C_1^2 + (4i_{-}C_2 x + 2)_{-}C_1 - 4_{-}C_2^2 x^2 - 4i_{-}C_2 x + 1} \right), y(x) = \frac{1}{2_{-}C_2} \left( -C_1 + 1 + \sqrt{-C_1^2 + (4i_{-}C_2 x + 2)_{-}C_1 - 4_{-}C_2^2 x^2 - 4i_{-}C_2 x + 1} \right) \right\}$$

**2.1786 ODE No. 1786**

$$f(x)(1 - y(x))y(x)y'(x) + 2(1 - y(x))y(x)y''(x) - (1 - 2y(x))y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.02621 (sec), leaf count = 87

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} \exp \left( -i \left( \int_1^x c_1 \left( -e^{-\int_1^{K[3]} \frac{1}{2} f(K[1]) dK[1]} \right) dK[3] + c_2 \right) \right) \left( 1 + \exp \left( i \left( \int_1^x c_1 \left( -e^{-\int_1^{K[3]} \frac{1}{2} f(K[1]) dK[1]} \right) dK[3] + c_2 \right) \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.164 (sec), leaf count = 42

$$\left\{ y(x) = \frac{1}{8_{-}C_2} \left( 2e^{-C_1 \int e^{-1/2 \int f(x) dx} dx} - C_2 + 1 \right)^2 \left( e^{-C_1 \int e^{-\frac{f(x) dx}{2}} dx} \right)^{-1} \right\}$$

**2.1787 ODE No. 1787**

$$h(y(x)) + 2(1 - y(x))y(x)y''(x) - (1 - 3y(x))y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 100.264 (sec), leaf count = 0 , could not solve

`DSolve[h[y[x]] - (1 - 3*y[x])*Derivative[1][y][x]^2 + 2*(1 - y[x])*y[x]*Derivative[2][y][x]`

✓ **Maple** : cpu = 0.283 (sec), leaf count = 80

$$\left\{ \int^{y(x)} \frac{1}{-b-1} \frac{1}{\sqrt{-b \left( -C1 + \int \frac{h(\_b)}{-b^2(\_b-1)^3} d\_b \right)}} d\_b - x - \_C2 = 0, \int^{y(x)} -\frac{1}{-b-1} \frac{1}{\sqrt{-b \left( -C1 + \int \frac{h(\_b)}{-b^2(\_b-1)^3} d\_b \right)}} d\_b - x - \_C2 = 0 \right.$$

**2.1788 ODE No. 1788**

$$-4(1-y(x))y(x)^2 (-f'(x) - f(x)^2 - g'(x) + g(x)^2) + 4y(x)y'(x)(f(x)y(x)+g(x)) - 2(1-y(x))y(x)y''(x) + (1-3y(x))y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.59217 (sec), leaf count = 0 , could not solve

`DSolve[-4*(1 - y[x])*y[x]^2*(-f[x]^2 + g[x]^2 - Derivative[1][f][x] - Derivative[1][g][x]) +`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ \sqrt{y(x)} - 2 \frac{\frac{\partial}{\partial x} DESol \left( \left\{ -1/4 e^{-2 \int f(x) dx + 2 \int g(x) dx} \_C1^2 \_Y(x) - 2 g(x) \frac{d}{dx} \_Y(x) + \frac{d^2}{dx^2} \_Y(x) \right\} \right)}{DESol \left( \left\{ -1/4 e^{-2 \int f(x) dx + 2 \int g(x) dx} \_C1^2 \_Y(x) - 2 g(x) \frac{d}{dx} \_Y(x) + \frac{d^2}{dx^2} \_Y(x) \right\}, \{ \_Y(x) \} \right)} e^{\int f(x) dx + 2 \int g(x) dx} \_C1^2 \_Y(x) - 2 g(x) \frac{d}{dx} \_Y(x) + \frac{d^2}{dx^2} \_Y(x)}, \left\{ \_Y(x) \right\} e^{\int f(x) dx + 2 \int g(x) dx} \_C1^2 \_Y(x) - 2 g(x) \frac{d}{dx} \_Y(x) + \frac{d^2}{dx^2} \_Y(x)} \right.$$

**2.1789 ODE No. 1789**

$$4y(x)^2(1-y(x)) (-f'(x) + f(x)^2 - g'(x) - g(x)^2) - 4y(x)y'(x)(f(x)y(x)+g(x)) + (1-y(x))^3 (f_0(x)^2 y(x)^2 - f_1(x)^2) = 0$$

✗ **Mathematica** : cpu = 10.4643 (sec), leaf count = 0 , could not solve

`DSolve[(1 - y[x])^3*(-f1[x]^2 + f0[x]^2*y[x]^2) + 4*(1 - y[x])*y[x]^2*(f[x]^2 - g[x]^2 - Derivative[1][f][x] + f[x]^2 - Derivative[1][g][x]) - 4*y[x]*y'[x]*(f[x]*y[x] + g[x]) + (1 - y[x])^3*(f0[x]^2*y[x]^2 - f1[x]^2) = 0`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(-2*y(x)*(1-y(x))*diff(diff(y(x),x),x)+(1-3*y(x))*diff(y(x),x)^2-4*y(x)*diff(y(x),x)*(f(x)*y(x)+g(x))-4*y(x)^2*(1-y(x))*(f(x)^2-g(x)^2-diff(g(x),x)-diff(f(x),x))=0,y(x))`

**2.1790 ODE No. 1790**

$$-h(y(x)) + 3(1 - y(x))y(x)y''(x) - 2(1 - 2y(x))y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 23.3412 (sec), leaf count = 182

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} -\frac{1}{(1 - K[2])^{2/3} K[2]^{2/3} \sqrt{2 \int_1^{K[2]} -\frac{h(K[1]) \exp(-2(\frac{2}{3} \log(1 - K[1]) + \frac{2}{3} \log(K[1]))}{3(K[1] - 1)K[1]}} dK[1]}} \right. \right. \right.$$

✓ **Maple** : cpu = 0.255 (sec), leaf count = 119

$$\left\{ \int^{y(x)} -\frac{\sqrt{9}}{3} \frac{1}{\sqrt{\sqrt[3]{-b(-b-1)}(-b-1)} \left( -C1 - \frac{2}{3} \int \frac{h(-b)}{-b(-b-1)} (-b^2 - b)^{-\frac{4}{3}} d_b \right)} d_b - x - C2 = 0, \int^{y(x)}$$

**2.1791 ODE No. 1791**

$$-h(y(x)) + (1 - y(x))y''(x) - 3(1 - 2y(x))y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 23.1719 (sec), leaf count = 164

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} -\frac{e^{\frac{1}{2}(12 - 12K[2])}}{(K[2] - 1)^3 \sqrt{2 \int_1^{K[2]} -\frac{h(K[1]) \exp(-2(6(K[1] - 1) + 3 \log(K[1] - 1)))}{K[1] - 1}} dK[1] + c_1}} \right. \right. \right.$$

✓ **Maple** : cpu = 0.364 (sec), leaf count = 90

$$\left\{ \int^{y(x)} \frac{1}{(-b-1)^3 (e^{-b})^6} \frac{1}{\sqrt{-2 \int \frac{h(-b)}{(e^{-b})^{12} (-b-1)^7} d_b + C1}} d_b - x - C2 = 0, \int^{y(x)} -\frac{1}{(-b-1)^3 (e^{-b})^6} \frac{1}{\sqrt{-2 \int}}$$

**2.1792 ODE No. 1792**

$$a(y(x) - 1)y(x)y''(x) + y'(x)^2(by(x) + c) + h(y(x)) = 0$$

✓ **Mathematica** : cpu = 26.6651 (sec), leaf count = 222

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} - \frac{K[2]^{-\frac{c}{a}} (1 - K[2])^{\frac{1}{2} \left( \frac{2b}{a} + \frac{2c}{a} \right)}}{\sqrt{2 \int_1^{K[2]} - \frac{h(K[1]) \exp\left(-\frac{2(c \log(K[1]) - (b+c) \log(1-K[1]))}{a}\right)}{a(K[1]-1)K[1]} dK[1] + c_1}} dK[2] \right] \& \right\} [c_2 + \dots$$

✓ **Maple** : cpu = 0.905 (sec), leaf count = 194

$$\left\{ \int^{y(x)} a \frac{1}{\sqrt{a \left( a\_C1 - 2 \int_{-b}^{\frac{h(-b)}{-b(-b-1)}} \left( (-b-1)^{\frac{b}{a}} \right)^2 \left( (-b-1)^{\frac{c}{a}} \right)^2 \left( -b^{\frac{c}{a}} \right)^{-2} d\_b \right)}} \left( -b^{\frac{c}{a}} \right)^{-1} \left( (-b-1)^{\frac{-b-c}{a}} \right)^{-1} d\_b$$

**2.1793 ODE No. 1793**

$$a(y(x) - 1)y(x)y''(x) - (a - 1)(2y(x) - 1)y'(x)^2 + f(x)(y(x) - 1)y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 1.36385 (sec), leaf count = 113

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ - \frac{a(1 - \#1)^{-1/a} (-\#1 - 1)\#1^{\frac{1}{a}} \left( (a + 1) {}_2F_1\left(-\frac{1}{a}, \frac{1}{a}; 1 + \frac{1}{a}; \#1\right) + \#1 {}_2F_1\left(1 + \frac{1}{a}, \frac{1}{a}; 2 + \frac{1}{a}; \#1\right) \right)}{a + 1} \right] \right\}$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 40

$$\left\{ -C1 e^{-\frac{fx}{a}} - C2 + \int^{y(x)} \frac{\sqrt{-a(-a-1)}}{-a(-a-1)} d\_a = 0 \right\}$$

**2.1794 ODE No. 1794**

$$ab(y(x) - 1)y(x)y''(x) + y'(x)^2(-((2ab - a - b)y(x) + (1 - a)b)) + f(x)(y(x) - 1)y(x)y'(x) = 0$$

✓ **Mathematica** : cpu = 1.28903 (sec), leaf count = 98

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ - \frac{a\#1^{\frac{1}{a}} \left( (a + 1) {}_2F_1\left(\frac{1}{a}, -\frac{1}{b}; 1 + \frac{1}{a}; \#1\right) + \#1 {}_2F_1\left(1 + \frac{1}{a}, \frac{b-1}{b}; 2 + \frac{1}{a}; \#1\right) \right)}{a + 1} \right] \& \right\} \left[ \int_1^x c_1 \right]$$

✓ **Maple** : cpu = 0.093 (sec), leaf count = 46

$$\left\{ -C1 e^{-\frac{fx}{ab}} - C2 + \int^{y(x)} \frac{\sqrt{-a-1} \sqrt{-a}}{-a(-a-1)} d_a = 0 \right\}$$

**2.1795 ODE No. 1795**

$$xy(x)^2 y''(x) - a = 0$$

✓ **Mathematica** : cpu = 0.418292 (sec), leaf count = 103

$$\text{Solve} \left[ 2\sqrt{2} \sqrt{-\frac{y(x)(ax+c_1y(x))}{x^2}} + \frac{\sqrt{2}a \tan^{-1}\left(\frac{ax+2c_1y(x)}{2\sqrt{c_1}x\sqrt{-\frac{y(x)(ax+c_1y(x))}{x^2}}}\right)}{\sqrt{c_1}} + \frac{4c_1}{x} + 4c_1c_2 = 0, y(x) \right]$$

✓ **Maple** : cpu = 1.037 (sec), leaf count = 529

$$\left\{ y(x) = \frac{x_{C1} \left( 9a_{C1} + e^{\text{RootOf}(243 \text{csgn}(-C1^{-1})_{C1^4 a^2 x - 54_Z e^{-Z} ax_{C1^3 - 3 \text{csgn}(-C1^{-1}) (e^{-Z})^2_{C1^2 x - 6 \text{csgn}(-C1^{-1})}}}\right)}{2e^{\text{RootOf}(243 \text{csgn}(-C1^{-1})_{C1^4 a^2 x - 54_Z e^{-Z} ax_{C1^3 - 3 \text{csgn}(-C1^{-1}) (e^{-Z})^2_{C1^2 x - 6 \text{csgn}(-C1^{-1}) e^{-Z}_{C1^2 x - 6 \text{csgn}(-C1^{-1})}}}\right)}$$

**2.1796 ODE No. 1796**

$$(a^2 - x^2) (a^2 - y(x)^2) y''(x) + (a^2 - x^2) y(x) y'(x)^2 - x(a^2 - y(x)^2) y'(x) = 0$$

✓ **Mathematica** : cpu = 0.330287 (sec), leaf count = 65

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{2} e^{c_2} \left( \sqrt{x^2 - a^2} + x \right)^{-c_1} + \frac{1}{2} a^2 e^{-c_2} \left( \sqrt{x^2 - a^2} + x \right)^{c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.227 (sec), leaf count = 51

$$\left\{ y(x) = \frac{1}{2_{C2}} \left( \left( \left( x + \sqrt{-a^2 + x^2} \right)^{-C1} \right)^2 - C2^2 + a^2 \right) \left( \left( x + \sqrt{-a^2 + x^2} \right)^{-C1} \right)^{-1} \right\}$$

**2.1797 ODE No. 1797**

$$(y(x)-1)^3 (ay(x)^2 + b) + cxy(x)^2(y(x)-1) + dx^2y(x)^2(y(x)+1) + 2x^2y(x)(y(x)-1)y''(x) - x^2(3y(x)-1)y'(x)^2 + 2xy$$

✗ **Mathematica** : cpu = 14.4768 (sec), leaf count = 0 , could not solve

```
DSolve[c*x*(-1 + y[x])*y[x]^2 + d*x^2*y[x]^2*(1 + y[x]) + (-1 + y[x])^3*(b + a*y[x]^2) + 2*x*
1 + y[x])*y[x]*Derivative[1][y][x] - x^2*(-1 + 3*y[x])*Derivative[1][y][x]^2 + 2*x^2*(-
1 + y[x])*y[x]*Derivative[2][y][x] == 0, y[x], x]
```

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

```
dsolve(2*x^2*y(x)*(-1+y(x))*diff(diff(y(x),x),x)-x^2*(3*y(x)-1)*diff(y(x),x)^2+2*x*y(x)*(-
1+y(x))*diff(y(x),x)+(a*y(x)^2+b)*(-1+y(x))^3+c*x*y(x)^2*(-1+y(x))+d*x^2*y(x)^2*(1+y(x))=0,y
```

**2.1798 ODE No. 1798**

$$x^3y(x)^2y''(x) + (y(x) + x)(xy'(x) - y(x))^3 = 0$$

✗ **Mathematica** : cpu = 38.2144 (sec), leaf count = 0 , could not solve

```
DSolve[(x + y[x])*(-y[x] + x*Derivative[1][y][x])^3 + x^3*y[x]^2*Derivative[2][y][x] == 0, y
```

✓ **Maple** : cpu = 0.242 (sec), leaf count = 166

$$\left\{ y(x) = \text{RootOf} \left( -2 \ln(x) - \int^{-Z} 1 \left( i Y_{i\sqrt{3}}(2\sqrt{-f}) \sqrt{3} - C1 \sqrt{-f} + i \sqrt{3} J_{i\sqrt{3}}(2\sqrt{-f}) \sqrt{-f} + Y_{i\sqrt{3}}(2\sqrt{-f}) - C \right) \right) \right\}$$

**2.1799 ODE No. 1799**

$$y(x)^3y''(x) - a = 0$$

✓ **Mathematica** : cpu = 1.90175 (sec), leaf count = 58

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{a + c_1^2(c_2 + x)^2}}{\sqrt{c_1}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{a + c_1^2(c_2 + x)^2}}{\sqrt{c_1}} \right\} \right\}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 46

$$\left\{ y(x) = \frac{1}{-C1} \sqrt{\left( (-C2 + x)^2 - C1^2 + a \right) - C1}, y(x) = -\frac{1}{-C1} \sqrt{\left( (-C2 + x)^2 - C1^2 + a \right) - C1} \right\}$$

**2.1800 ODE No. 1800**

$$y(x) (y(x)^2 + 1) y''(x) + (1 - 3y(x)^2) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.496121 (sec), leaf count = 78

$$\left\{ \left\{ y(x) \rightarrow -\frac{\sqrt{-2c_1x - 2c_2c_1 - 1}}{\sqrt{2}\sqrt{c_1(c_2 + x)}} \right\}, \left\{ y(x) \rightarrow \frac{\sqrt{-2c_1x - 2c_2c_1 - 1}}{\sqrt{2}\sqrt{c_1(c_2 + x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 60

$$\left\{ y(x) = \frac{1}{2\_C1x + 2\_C2} \sqrt{-4(-C1x - C2)(-C1x - C2 + 1/2)}, y(x) = -\frac{1}{2\_C1x + 2\_C2} \sqrt{-4(-C1x - C2)(-C1x - C2 + 1/2)} \right\}$$

**2.1801 ODE No. 1801**

$$-a^2xy(x)^2 + 2y(x)^3y''(x) + y(x)^4 - 1 = 0$$

✗ **Mathematica** : cpu = 44.5916 (sec), leaf count = 0 , could not solve

`DSolve[-1 - a^2*x*y[x]^2 + y[x]^4 + 2*y[x]^3*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*y(x)^3*diff(diff(y(x),x),x)+y(x)^4-a^2*x*y(x)^2-1=0,y(x))`

**2.1802 ODE No. 1802**

$$-ax^2 - bx - c + 2y(x)^3y''(x) + y(x)^2y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.346346 (sec), leaf count = 0 , could not solve

`DSolve[-c - b*x - a*x^2 + y[x]^2*Derivative[1][y][x]^2 + 2*y[x]^3*Derivative[2][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve(2*y(x)^3*diff(diff(y(x),x),x)+y(x)^2*diff(y(x),x)^2-a*x^2-b*x-c=0,y(x))`

**2.1803 ODE No. 1803**

$$-a0(a-y(x))^2(b-y(x))^2(c-y(x))^2 - a2(a-y(x))^2(c-y(x))^2 - a3(a-y(x))^2(b-y(x))^2 + 2(a-y(x))(b-y(x))(c-y(x))y''(x) + y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 86.2091 (sec), leaf count = 1

\$Aborted

✓ **Maple** : cpu = 2.665 (sec), leaf count = 115620

result too large to display

**2.1804 ODE No. 1804**

$$y''(x) (-ay(x) - b + 4y(x)^3) - \left(6y(x)^2 - \frac{a}{2}\right) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 3.92158 (sec), leaf count = 415

$$\text{Solve} \left[ \frac{2 \sqrt{\frac{y(x) - \text{Root}[4\#1^3 - \#1a - b\&,1]}{\text{Root}[4\#1^3 - \#1a - b\&,3] - \text{Root}[4\#1^3 - \#1a - b\&,1]}} \sqrt{\frac{y(x) - \text{Root}[4\#1^3 - \#1a - b\&,2]}{\text{Root}[4\#1^3 - \#1a - b\&,3] - \text{Root}[4\#1^3 - \#1a - b\&,2]}} (y(x) - \text{Root}[4\#1^3 - \#1a - b\&,3])}{c_1 \sqrt{2ay(x) + 2b - 8y(x)^3}} \sqrt{\frac{y(x) - \text{Root}[4\#1^3 - \#1a - b\&,1]}{\text{Root}[4\#1^3 - \#1a - b\&,3] - \text{Root}[4\#1^3 - \#1a - b\&,1]}}$$

✓ **Maple** : cpu = 0.034 (sec), leaf count = 31

$$\left\{ \int^{y(x)} \frac{1}{\sqrt{4a^3 - a_a - b}} d_a - C1 x - C2 = 0 \right\}$$

**2.1805 ODE No. 1805**

$$(-ay(x) - b + 4y(x)^3) (f(x)y'(x) + y''(x)) - \left(6y(x)^2 - \frac{a}{2}\right) y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 3.29824 (sec), leaf count = 436

$$\text{Solve} \left[ \frac{2 \sqrt{\frac{\text{Root}[4\#1^3 - \#1a - b\&,1] - y(x)}{\text{Root}[4\#1^3 - \#1a - b\&,1] - \text{Root}[4\#1^3 - \#1a - b\&,3]}} \sqrt{\frac{\text{Root}[4\#1^3 - \#1a - b\&,2] - y(x)}{\text{Root}[4\#1^3 - \#1a - b\&,2] - \text{Root}[4\#1^3 - \#1a - b\&,3]}} (y(x) - \text{Root}[4\#1^3 - \#1a - b\&,3])}{\sqrt{ay(x) + b - 4y(x)^3}} \sqrt{\frac{\text{Root}[4\#1^3 - \#1a - b\&,1] - y(x)}{\text{Root}[4\#1^3 - \#1a - b\&,1] - \text{Root}[4\#1^3 - \#1a - b\&,3]}}$$

✓ **Maple** : cpu = 0.046 (sec), leaf count = 34

$$\left\{ -C1 e^{-fx} - C2 + \int^{y(x)} \frac{1}{\sqrt{4a^3 - a_a - b}} d_a = 0 \right\}$$



**2.1806 ODE No. 1806**

$$-f(x)((y(x)-1)y(x)(y(x)-x))^{3/2}+2(1-y(x))(x^2-2xy(x)+y(x))y(x)y'(x)-2(1-x)x(1-y(x))(x-y(x))y(x)y'(x))$$

✗ **Mathematica** : cpu = 20.1099 (sec), leaf count = 0 , could not solve

$$\text{DSolve[-((1 - y[x])^2*y[x]^2) - f[x]*((-1 + y[x])*y[x]*(-x + y[x]))^(3/2) + 2*(1 - y[x])*y[x]*y'[x], y[x], x]$$

✓ **Maple** : cpu = 5.365 (sec), leaf count = 718

$$\left\{ -\frac{C1}{2} \int \frac{1}{x-1} e^{\int \frac{1}{x(x-1)} \text{EllipticE}(\sqrt{x}) (\text{EllipticK}(\sqrt{x}))^{-1} dx} \int_1^{y(x)} \frac{1}{-a(-a-1)(-a+x)^2} \sqrt{-a(-a-1)(-a+x)} dx \right\}$$

**2.1807 ODE No. 1807**

$$a(1-y(x))^2(x-y(x))^2y(x)^2+bx(1-y(x))^2(x-y(x))^2-c(1-x)(x-y(x))^2y(x)^2-d(1-x)x(1-y(x))^2y(x)^2+2(1-x)y(x)^2$$

✗ **Mathematica** : cpu = 27.6618 (sec), leaf count = 0 , could not solve

$$\text{DSolve[b*x*(1 - y[x])^2*(x - y[x])^2 - d*(1 - x)*x*(1 - y[x])^2*y[x]^2 - c*(1 - x)*(x - y[x])^2*y[x]^2 + 2*(1 - x)*y[x]^2, y[x], x]$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

$$\text{dsolve}(2*x^2*y(x)*(1-x)^2*(1-y(x))*(x-y(x))*\text{diff}(\text{diff}(y(x), x), x)-x^2*(1-x)^2*(x-2*x*y(x)-2*y(x)+3*y(x)^2)*\text{diff}(y(x), x)^2-2*x*y(x)*(1-x)*(1-y(x))*(x^2+y(x)-2*x*y(x))*\text{diff}(y(x), x)^2*(x-y(x))^2-c*(1-x)*y(x)^2*(x-y(x))^2-d*x*y(x)^2*(1-x)*(1-y(x))^2+a*y(x)^2*(x-y(x))^2*(1-y(x))^2=0, y(x))$$

**2.1808 ODE No. 1808**

$$b\sqrt{(1-y(x)^2)(1-a^2y(x)^2)}y'(x)^2+(y(x)^2-1)(a^2y(x)^2-1)y''(x)+y(x)(-2a^2y(x)^2+a^2+1)y'(x)^2=0$$

✓ **Mathematica** : cpu = 104.936 (sec), leaf count = 155

$$\text{Solve}\left[2\left(\log\left(bc_1\sqrt{1-y(x)^2}\sqrt{1-a^2y(x)^2}+\sqrt{y(x)^2-1}\sqrt{a^2y(x)^2-1}\exp\left(\frac{b\sqrt{1-y(x)^2}\sqrt{1-a^2y(x)^2}F(\sin^{-1}\sqrt{1-a^2y(x)^2})}{\sqrt{y(x)^2-1}\sqrt{a^2y(x)^2-1}}\right)\right)\right)\right]$$

✓ **Maple** : cpu = 0.141 (sec), leaf count = 72

$$\left\{ \int^{y(x)} e^{\int \frac{1}{(-b^2-1)(-b^2a^2-1)} \left( -2_b^3a^2+_ba^2+b\sqrt{(-b^2-1)(-b^2a^2-1)}+_b \right) d_b} d_b - C1 x - C2 = 0 \right\}$$

2.1809 ODE No. 1809

$$y''(x) (ax^2 + 2bx + c + y(x)^2)^2 + dy(x) = 0$$

✗ **Mathematica** : cpu = 44.7404 (sec), leaf count = 0 , could not solve

DSolve[d\*y[x] + (c + 2\*b\*x + a\*x^2 + y[x]^2)^2\*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.705 (sec), leaf count = 336

$$\left\{ y(x) = \text{RootOf} \left( - \int^{-Z} \frac{a}{-f^4 ac + f^4 b^2 + C1 f^2 a^2 - c f^2 a + b^2 f^2 + C1 a^2 + d} \sqrt{(f^2 + 1) (-f^4 ac - \dots)} \right) \right.$$

2.1810 ODE No. 1810

$$\sqrt{y(x)} y''(x) - a = 0$$

✓ **Mathematica** : cpu = 0.102951 (sec), leaf count = 1881

$$\left\{ \left\{ y(x) \rightarrow \frac{288c_1 c_2^2 a^4 + 288x^2 c_1 a^4 + 576x c_1 c_2 a^4 + \left( \frac{10368x^4 a^8 + 10368c_2^4 a^8 + 41472x c_2^3 a^8 + 62208x^2 c_2^2 a^8 + 41472x^3 c_2 a^8 + 48 \sqrt{\frac{(x - \dots)}{a^6}}}{16a^4 \sqrt[3]{10368x^4 a^8 + \dots}} \right)}{\dots} \right. \right.$$

✓ **Maple** : cpu = 0.119 (sec), leaf count = 91

$$\left\{ \frac{1}{12a^2} \left( -3 C1 \sqrt{4a\sqrt{y(x)} - C1} - \left( 4a\sqrt{y(x)} - C1 \right)^{\frac{3}{2}} \right) - x - C2 = 0, \frac{1}{12a^2} \left( 3 C1 \sqrt{4a\sqrt{y(x)} - C1} - \dots \right) \right.$$

2.1811 ODE No. 1811

$$\sqrt{x^2 + y(x)^2} y''(x) - a(y'(x)^2 + 1)^{3/2} = 0$$

✗ **Mathematica** : cpu = 300.004 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 3.85 (sec), leaf count = 1862

$$\left\{ y(x) = \text{RootOf} \left( - \ln(x) + \int^{-Z} \frac{1}{-g^2 + 1} \left( \text{RootOf} \left( \arctan(-g) + \int^{-Z} \frac{1}{(f^2 a^2 + a^2 - 1) (f^2 + 1)} \left( 1 + \sqrt{a^2 - \dots} \right) \right) \right) \right.$$

**2.1812 ODE No. 1812**

$$y(x)y''(x)(1 - \log(y(x))) + y'(x)^2(\log(y(x)) + 1) = 0$$

✓ **Mathematica** : cpu = 0.0296293 (sec), leaf count = 29

$$\left\{ \left\{ y(x) \rightarrow e^{\frac{c_1 x + c_2 c_1 - 1}{c_1 (c_2 + x)}} \right\} \right\}$$

✓ **Maple** : cpu = 0.037 (sec), leaf count = 19

$$\left\{ y(x) = e^{\frac{C1 x + C2 - 1}{-C1 x + C2}} \right\}$$

**2.1813 ODE No. 1813**

$$Ay(x) (a \sin^2(y(x)) + c) + y''(x) (a \sin^2(y(x)) + b) + ay'(x)^2 \sin(y(x)) \cos(y(x)) = 0$$

✗ **Mathematica** : cpu = 100.845 (sec), leaf count = 0 , could not solve

DSolve[A\*(c + a\*Sin[y[x]]^2)\*y[x] + a\*Cos[y[x]]\*Sin[y[x]]\*Derivative[1][y][x]^2 + (b + a\*Sin

✓ **Maple** : cpu = 0.421 (sec), leaf count = 138

$$\left\{ \int^{y(x)} \sqrt{2} (b + a(\sin(\_a))^2) \frac{1}{\sqrt{-\left(a(\sin(\_a))^2 A - 2 a\_a \sin(\_a) A \cos(\_a) + \_a^2 (a + 2 c) A - 2 \_C1\right)} (b + a \sin(\_a))^2} d\_a \right\}$$

**2.1814 ODE No. 1814**

$$ah(y(x))y'(x)^2 + h(y(x))y''(x) + j(y(x)) = 0$$

✓ **Mathematica** : cpu = 13.4964 (sec), leaf count = 116

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} -\frac{e^{aK[2]}}{\sqrt{2 \int_1^{K[2]} -\frac{e^{2aK[1]} j(K[1])}{h(K[1])} dK[1] + c_1} dK[2]} \& \right] [c_2 + x] \right\}, \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \int_1^{\#1} -\frac{e^{aK[2]}}{\sqrt{2 \int_1^{K[2]} -\frac{e^{2aK[1]} j(K[1])}{h(K[1])} dK[1] + c_1} dK[2]} \& \right] [c_2 + x] \right\} \right\}$$

✓ **Maple** : cpu = 0.183 (sec), leaf count = 87

$$\left\{ \int^{y(x)} \frac{1}{(h(\_b))^{-a}} \frac{1}{\sqrt{-2 \int \frac{(h(\_b))^a}{h(\_b)} d\_b + \_C1}} d\_b - x - \_C2 = 0, \int^{y(x)} -\frac{1}{(h(\_b))^{-a}} \frac{1}{\sqrt{-2 \int 2 \frac{(h(\_b))^a}{h(\_b)} d\_b}} d\_b \right\}$$

**2.1815 ODE No. 1815**

$$h(y(x))^2 \left( -j \left( x, \frac{y'(x)}{h(y(x))} \right) \right) + h(y(x))y''(x) - h(y(x))y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 1.22577 (sec), leaf count = 0 , could not solve

DSolve[-(h[y[x]]^2\*j[x, Derivative[1][y][x]/h[y[x]]]) - h[y[x]]\*Derivative[1][y][x]^2 + h[y[x]]

✓ **Maple** : cpu = 0.999 (sec), leaf count = 71

$$\left\{ y(x) = ODESolStruc \left( \text{RootOf} \left( \int -b(-a) d_a + -C1 - \int^{-Z} (h(-f))^{-1} d_f \right), \left\{ \frac{d}{d_a} b(-a) = 1 \right\}, \left\{ -a = \right. \right.$$

**2.1816 ODE No. 1816**

$$x^2(-y(x))y'(x) + y'(x)y''(x) - xy(x)^2 = 0$$

✗ **Mathematica** : cpu = 64.1018 (sec), leaf count = 0 , could not solve

DSolve[-(x\*y[x]^2) - x^2\*y[x]\*Derivative[1][y][x] + Derivative[1][y][x]\*Derivative[2][y][x]

✓ **Maple** : cpu = 1.231 (sec), leaf count = 46

$$\left\{ y(x) = ODESolStruc \left( -b(-a), \left\{ -(-b(-a))^2 - a^2 + \left( \frac{d}{d_a} b(-a) \right)^2 + -C1 = 0 \right\}, \{ -a = x, -b(-a) = y(x) \right. \right.$$

**2.1817 ODE No. 1817**

$$4y'(x)^2 + (xy'(x) - y(x))y''(x) = 0$$

✗ **Mathematica** : cpu = 21.6002 (sec), leaf count = 0 , could not solve

DSolve[4\*Derivative[1][y][x]^2 + (-y[x] + x\*Derivative[1][y][x])\*Derivative[2][y][x] == 0, y

✓ **Maple** : cpu = 0.234 (sec), leaf count = 40

$$\left\{ y(x) = e^{\int \ln(x) e^{\text{RootOf}(\ln(e^{-Z}-1)e^{-Z} + -C1 e^{-Z} -_Z e^{-Z} -_b e^{-Z} +2) -1d_b + -C2}} \right\}$$

2.1818 ODE No. 1818

$$(xy'(x) - y(x))y''(x) - (y'(x)^2 + 1)^2 = 0$$

✗ **Mathematica** : cpu = 1.50373 (sec), leaf count = 0 , could not solve

DSolve[-(1 + Derivative[1][y][x]^2)^2 + (-y[x] + x\*Derivative[1][y][x])\*Derivative[2][y][x]

✓ **Maple** : cpu = 0.4 (sec), leaf count = 66

$$\left\{ y(x) = \text{RootOf} \left( -\ln(x) + \int^{-Z} \frac{-f + \text{RootOf}(-\tan(Z^{-1}) - C1 - Z + f - C1 \tan(Z^{-1}) + \tan(Z^{-1}))}{f^2 + 1} \right) \right.$$

2.1819 ODE No. 1819

$$ax^3y'(x)y''(x) + by(x)^2 = 0$$

✗ **Mathematica** : cpu = 90.7832 (sec), leaf count = 0 , could not solve

DSolve[b\*y[x]^2 + a\*x^3\*Derivative[1][y][x]\*Derivative[2][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.063 (sec), leaf count = 42

$$\left\{ y(x) = e^{\int^{\ln(x)} \text{RootOf} \left( -f - Z \frac{a-a}{a-a^3-a^2a+b} d_a - b + C1 \right) d_b + C2} \right\}$$

2.1820 ODE No. 1820

$$y''(x) (f1(x)y'(x) + f2(x)y(x)) + f3(x)y'(x)^2 + f4(x)y(x)y'(x) + f5(x)y(x)^2 = 0$$

✗ **Mathematica** : cpu = 356.668 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.209 (sec), leaf count = 88

$$\left\{ y(x) = \text{ODESolStruc} \left( e^{\int -b(a) d_a + C1}, \left[ \frac{d}{d_a} b(a) = \frac{-(-b(a))^3 f1 + (-f2 - f3)(-b(a))^2 - f4(a)}{-b(a) f1 + f2} \right] \right) \right.$$

**2.1821 ODE No. 1821**

$$(x^2 + 2y(x)^2 y'(x)) y''(x) + 2y(x) y'(x)^3 + 3x y'(x) + y(x) = 0$$

✗ **Mathematica** : cpu = 82.0945 (sec), leaf count = 0 , could not solve

`DSolve[y[x] + 3*x*Derivative[1][y][x] + 2*y[x]*Derivative[1][y][x]^3 + (x^2 + 2*y[x]^2*Deriv`

✓ **Maple** : cpu = 2.286 (sec), leaf count = 54

$$\left\{ y(x) = ODESolStruc \left( \_b(\_a), \left[ \left( \_b(\_a) \right)^2 \left( \frac{d}{d\_a} \_b(\_a) \right)^2 + \_a^2 \frac{d}{d\_a} \_b(\_a) + \_a \_b(\_a) + \_C1 = 0 \right] \right) \right\}$$

**2.1822 ODE No. 1822**

$$(y'(x)^2 + y(x)^2) y''(x) + y(x)^3 = 0$$

✓ **Mathematica** : cpu = 2.96146 (sec), leaf count = 369

$$\left. \left\{ \begin{array}{l} \left( \frac{c_2 \exp \left( \frac{\tan^{-1} \left( \frac{1 + 2 \operatorname{InverseFunction} \left[ \frac{(\sqrt{3}-i) \tan^{-1} \left( \frac{\#1}{\sqrt{\frac{1}{2}(1-i\sqrt{3})} \right)} + \frac{(\sqrt{3}+i) \tan^{-1} \left( \frac{\#1}{\sqrt{\frac{1}{2}(1+i\sqrt{3})} \right)} \right)}{\sqrt{6(1-i\sqrt{3})}} \right)}{\sqrt{3}} \right)}{2\sqrt{3}} \right)}{y(x) \rightarrow \sqrt[4]{\operatorname{InverseFunction} \left[ \frac{(\sqrt{3}-i) \tan^{-1} \left( \frac{\#1}{\sqrt{\frac{1}{2}(1-i\sqrt{3})} \right)} + \frac{(\sqrt{3}+i) \tan^{-1} \left( \frac{\#1}{\sqrt{\frac{1}{2}(1+i\sqrt{3})} \right)} \right] \& [c_1 - x]^4 + \operatorname{InverseFunction} \left[ \frac{(\sqrt{3}-i) \tan^{-1} \left( \frac{\#1}{\sqrt{\frac{1}{2}(1-i\sqrt{3})} \right)} + \frac{(\sqrt{3}+i) \tan^{-1} \left( \frac{\#1}{\sqrt{\frac{1}{2}(1+i\sqrt{3})} \right)} \right]} \right]} \right.} \right.$$

✓ Maple : cpu = 1.215 (sec), leaf count = 291

$$\left\{ y(x) = \left( -C1 + \tan(\sqrt{3}x) \right)^{(2-C1^2+2)^{-1}} - C2 \left( -C1 + \tan(\sqrt{3}x) \right)^{\frac{C1^2}{2-C1^2+2}} \left( 1 + \left( \tan(\sqrt{3}x) \right)^2 \right)^{-\frac{C1^2}{4-C1^2+4}} \left( 1 + \left( \tan(\sqrt{3}x) \right)^2 \right)^{\frac{C1^2}{4-C1^2+4}} \right\}$$

**2.1823 ODE No. 1823**

$$y''(x) (a(xy'(x) - y(x)) + y'(x)^2) - b = 0$$

✗ **Mathematica** : cpu = 0.169908 (sec), leaf count = 0 , could not solve

`DSolve[-b + (Derivative[1][y][x]^2 + a*(-y[x] + x*Derivative[1][y][x]))*Derivative[2][y][x]`

✓ **Maple** : cpu = 0.434 (sec), leaf count = 289

$$\left\{ y(x) = -\frac{ax^2}{4} + \text{RootOf}\left(-x - \int^{-Z} \frac{1}{-f^2 a^2 - 4_f b + 2_C1} \sqrt{(-f^2 a^2 - 4_f b + 2_C1) (a_f + \sqrt{4_f b - 2_C1})} dz \right) \right\}$$

**2.1824 ODE No. 1824**

$$y''(x) (a\sqrt{y'(x)^2 + 1} - xy'(x)) - y'(x)^2 - 1 = 0$$

✓ **Mathematica** : cpu = 0.457638 (sec), leaf count = 347

$$\left\{ \left\{ y(x) \rightarrow \frac{-2\sqrt{x^2 (a^2 + c_1^2 - x^2)} + c_1 x \log\left(-c_1\left(\sqrt{x^2 (a^2 + c_1^2 - x^2)} + c_1 x\right) + a^2(-x) + ax^2\right) + c_1 x \log\left(c_1\left(\sqrt{x^2 (a^2 + c_1^2 - x^2)} + c_1 x\right)\right)}{\dots} \right\} \right\}$$

✓ **Maple** : cpu = 0.988 (sec), leaf count = 96

$$\left\{ y(x) = \int \frac{1}{a^3 - ax^2} \left(-_C1 a^2 - x\sqrt{a^2 (-_C1^2 + a^2 - x^2)}\right) dx + _C2, y(x) = \int \frac{1}{a^3 - ax^2} \left(-_C1 a^2 + x\sqrt{a^2 (-_C1^2 + a^2 - x^2)}\right) dx + _C2 \right\}$$

**2.1825 ODE No. 1825**

$$f(x) + y''(x)h(y'(x)) + j(y(x))y'(x) = 0$$

✗ **Mathematica** : cpu = 0.0414342 (sec), leaf count = 0 , could not solve

`DSolve[f[x] + j[y[x]]*Derivative[1][y][x] + h[Derivative[1][y][x]]*Derivative[2][y][x] == 0,`

✓ **Maple** : cpu = 1.053 (sec), leaf count = 49

$$\left\{ y(x) = \text{ODESolStruc}\left(-f(-b), \left[\int^{-f(-b)} 1 d_a + \int^{\frac{d}{a-b} f(-b)} h(-a) d_a + _b f + _C1 = 0\right], \{-b = x, -f(-b) = y(x)\}\right) \right\}$$



**2.1826 ODE No. 1826**

$$-ay(x) - b + y''(x)^2 = 0$$

✓ **Mathematica** : cpu = 1.86028 (sec), leaf count = 119

$$\left\{ \text{Solve} \left[ \frac{(ay(x) + b)^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; -\frac{4(b+ay(x))^{3/2}}{3ac_1}\right)}{a^2c_1} = (c_2 + x)^2, y(x) \right], \text{Solve} \left[ \frac{(ay(x) + b)^2 {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \frac{4(b+ay(x))^{3/2}}{3ac_1}\right)}{a^2c_1} = (c_2 + x)^2, y(x) \right] \right\}$$

✓ **Maple** : cpu = 0.375 (sec), leaf count = 173

$$\left\{ \int^{y(x)} \sqrt{3a} \frac{1}{\sqrt{a(4 - a\sqrt{a - a + ba} + 4\sqrt{a - a + bb} - C1)}} d_{-a - x - C2} = 0, \int^{y(x)} -3 \frac{a}{\sqrt{-12((a - a + b)^{3/2} - C1)}} d_{-a - x - C2} = 0 \right\}$$

**2.1827 ODE No. 1827**

$$a^2y''(x)^2 - 2axy''(x) + y'(x) = 0$$

✗ **Mathematica** : cpu = 0.834575 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x] - 2*a*x*Derivative[2][y][x] + a^2*Derivative[2][y][x]^2 == 0, y[x], x]`

✓ **Maple** : cpu = 9.289 (sec), leaf count = 81

$$\left\{ y(x) = \int \text{RootOf}\left(-\int_{-g}^{-Z} (x\sqrt{x^2 - f} - x^2 + 2a_f)^{-1} d_f + C1\right) dx + C2, y(x) = \int \text{RootOf}\left(-\int_{-g}^{-Z} (x\sqrt{x^2 - f} - x^2 + 2a_f)^{-1} d_f + C1\right) dx + C2 \right\}$$

**2.1828 ODE No. 1828**

$$2(x^2 + 1)y''(x)^2 + 2y'(x)(y'(x) + x) - x(4y'(x) + x)y''(x) - 2y(x) = 0$$

✓ **Mathematica** : cpu = 0.0107665 (sec), leaf count = 32

$$\left\{ \left\{ y(x) \rightarrow -\frac{1}{2}\sqrt{c_2 - c_1^2x^2} + c_1x + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.597 (sec), leaf count = 59

$$\left\{ y(x) = \frac{C1x^2}{2} + C2x + C1^2 + C2^2, y(x) = \frac{x}{2} \left( -C1 + \frac{\text{Arcsinh}(x)}{4} \right) \sqrt{x^2 + 1} - \frac{3x^2}{16} + C1^2 + \frac{C1A}{2} \right\}$$

**2.1829 ODE No. 1829**

$$3x^2y''(x)^2 + 4y'(x)^2 - 2(3xy'(x) + y(x))y''(x) = 0$$

✓ **Mathematica** : cpu = 0.00687712 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1^2 x^2}{c_2} + c_1 x + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.391 (sec), leaf count = 32

$$\left\{ y(x) = x^{\frac{2\sqrt{3}}{3}} - C1 x, y(x) = \frac{-C1^2 x^2}{-C2} + -C1 x + -C2 \right\}$$

**2.1830 ODE No. 1830**

$$(2 - 9x)x^2y''(x)^2 + 6y(x)y''(x) - 36xy'(x)^2 - 6(1 - 6x)xy'(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 0.0285532 (sec), leaf count = 24

$$\left\{ \left\{ y(x) \rightarrow \frac{c_1^2 x^3}{c_2} + c_1 x + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.552 (sec), leaf count = 304

$$\left\{ y(x) = \frac{27 - C1 \sqrt{5} \sqrt{4x}}{4} \left( (9x - 1) \sqrt{9} + 9 \sqrt{9x^2 - 2x} \right)^{-\frac{2\sqrt{9}}{9}} \left( (9x - 1) \sqrt{9} + 9 \sqrt{9x^2 - 2x} \right)^{-\frac{5\sqrt{9}}{18}} \sqrt{1 \left( \frac{4}{5} + \sqrt{1} \right)} \right\}$$

**2.1831 ODE No. 1831**

$$y(x)(xF(0, 2) + xF(2, 0))y''(x) + xF(2, 2)y''(x)^2 + xF(1, 1)y''(x) + y'(x) ((xF(1, 2) + xF(2, 1))y''(x) + y(x)(xF(0, 1) + xF(1, 0))) = 0$$

✗ **Mathematica** : cpu = 300.125 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.269 (sec), leaf count = 163

$$\left\{ y(x) = ODESolStruc \left( e^{\int -b(-a) d_a - C1}, \left[ \frac{d}{d_a} - b(-a) = \frac{1}{2 (F_{2,2}) (-a)} \left( \sqrt{((F_{2,1}) (-a))^2 + 2 (F_{1,2}) (-a) (F_{1,1})} \right) \right] \right) \right\}$$

2.1832 ODE No. 1832

$$y(x)y''(x)^2 - ae^{2x} = 0$$

✗ **Mathematica** : cpu = 0.700949 (sec), leaf count = 0 , could not solve

`DSolve[-(a*E^(2*x)) + y[x]*Derivative[2][y][x]^2 == 0, y[x], x]`

✓ **Maple** : cpu = 0.976 (sec), leaf count = 117

$$\left\{ y(x) = ODESolStruc \left( -a \left( e^{-\frac{2 \int -b(-a) d_a - 2 C1}{3}} \right)^{-1}, \left[ \frac{d}{d_a} b(-a) = -\frac{(b(-a))^3}{9_a} (-4_a^2 + 9 \sqrt{a_a}) + 4(-a) \right] \right.$$

2.1833 ODE No. 1833

$$y''(x)^2 (a^2 y(x)^2 - b^2) + y'(x)^2 (a^2 y'(x)^2 - 1) - 2a^2 y(x) y'(x)^2 y''(x) = 0$$

✗ **Mathematica** : cpu = 300.121 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 3.877 (sec), leaf count = 145

$$\left\{ y(x) = -C1, y(x) = b \left( e^{-\frac{C1+x}{b} \sqrt{-C1^2 a^2 - 1}} - C1 \right) \frac{1}{\sqrt{-C1^2 a^2 - 1}}, y(x) = \frac{b}{a} \tan \left( \frac{-C1 - x}{ab} \sqrt{a^2} \right) \frac{1}{\sqrt{\left( \tan \left( \frac{-C1 - x}{ab} \sqrt{a^2} \right) \right)^2 + 1}}$$

2.1834 ODE No. 1834

$$(x^2 y(x) y''(x) + x^2 (-y'(x)^2) + y(x)^2)^2 - 4xy(x) (xy'(x) - y(x))^3 = 0$$

✗ **Mathematica** : cpu = 16.4748 (sec), leaf count = 0 , could not solve

`DSolve[-4*x*y[x]*(-y[x] + x*Derivative[1][y][x])^3 + (y[x]^2 - x^2*Derivative[1][y][x]^2 + x^2*Derivative[2][y][x]^2) == 0, y[x], x]`

✓ **Maple** : cpu = 0.497 (sec), leaf count = 82

$$\left\{ y(x) = ODESolStruc \left( e^{\int -b(-a) d_a + C1}, \left[ \frac{d}{d_a} b(-a) = \frac{1}{-a^2} \left( 2 \sqrt{-a(-a-b(-a)-1)} b(-a) - 2 \sqrt{-a(-a-b(-a)-1)} \right) \right] \right.$$

**2.1835 ODE No. 1835**

$$32y''(x) (xy''(x) - y'(x))^3 + (2y(x)y''(x) - y'(x)^2)^3 = 0$$

✓ **Mathematica** : cpu = 0.114604 (sec), leaf count = 135

$$\left\{ \left\{ y(x) \rightarrow \frac{1}{4} \left( \left( -\frac{8c_1^3}{\sqrt[3]{3\sqrt{3}\sqrt{c_1^9 c_2^9 (27c_1 c_2 - 64)} - 27c_1^5 c_2^5}} + \frac{c_1^2}{c_2} - \frac{2\sqrt[3]{\sqrt{3}\sqrt{c_1^9 c_2^9 (27c_1 c_2 - 64)} - 9c_1^5 c_2^5}}{3^{2/3}c_2^3} \right) x^2 + 4c_1 x \right. \right.$$

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception  
time expired

**2.1836 ODE No. 1836**

$$\sqrt{ay''(x)^2 + by'(x)^2} + cy(x)y''(x) + dy'(x)^2 = 0$$

✗ **Mathematica** : cpu = 14.8414 (sec), leaf count = 0 , could not solve

`DSolve[d*Derivative[1][y][x]^2 + c*y[x]*Derivative[2][y][x] + Sqrt[b*Derivative[1][y][x]^2 +`

✓ **Maple** : cpu = 0.451 (sec), leaf count = 100

$$\left\{ y(x) = ODESolStruc \left( -a, \left[ \frac{-b(-a)}{-c^2 - a^2 + a} \left( \left( \frac{d}{d-a} - b(-a) \right) (-c^2 - a^2 + a) - a c d - b(-a) + \sqrt{(-b(-a))^2 a d^2} \right. \right. \right.$$

**2.1837 ODE No. 1837**

$$y^{(3)}(x) - a^2(y'(x))^5 + 2y'(x)^3 + y'(x) = 0$$

✗ **Mathematica** : cpu = 10.9257 (sec), leaf count = 0 , could not solve

`DSolve[-(a^2*(Derivative[1][y][x] + 2*Derivative[1][y][x]^3 + Derivative[1][y][x]^5)) + Deri`

✓ **Maple** : cpu = 0.369 (sec), leaf count = 95

$$\left\{ y(x) = \int \text{RootOf} \left( -3 \int^{-z} \frac{1}{\sqrt{3a^2 - f^6 + 9 - f^4 a^2 + 9 - f^2 a^2 + 9 - C1}} d_f + x + -C2 \right) dx + -C3, y(x) = \int R$$

**2.1838 ODE No. 1838**

$$y^{(3)}(x) + y(x)y''(x) - y'(x)^2 + 1 = 0$$

✗ **Mathematica** : cpu = 0.0365836 (sec), leaf count = 0 , could not solve

`DSolve[1 - Derivative[1][y][x]^2 + y[x]*Derivative[2][y][x] + Derivative[3][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.657 (sec), leaf count = 73

$$\left\{ y(x) = \text{ODESolStruc} \left( -a, \left[ \left( \frac{d^2}{d_a^2} b(-a) \right) (-b(-a))^2 + \left( \frac{d}{d_a} b(-a) \right)^2 - b(-a) + \left( \frac{d}{d_a} b(-a) \right) - b(-a) \right] \right) \right.$$

**2.1839 ODE No. 1839**

$$y^{(3)}(x) - y(x)y''(x) + y'(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.0330413 (sec), leaf count = 0 , could not solve

`DSolve[Derivative[1][y][x]^2 - y[x]*Derivative[2][y][x] + Derivative[3][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.911 (sec), leaf count = 116

$$\left\{ y(x) = \text{ODESolStruc} \left( e^{\int -g(-f) d_f + C2}, \left[ \frac{d}{d_f} -g(-f) = 6 \frac{-g(-f) (-g(-f) -f + 1) (1/6 + (-f - 1/6) -g(-f))}{-f} \right] \right) \right.$$

**2.1840 ODE No. 1840**

$$ay(x)y''(x) + y^{(3)}(x) = 0$$

✗ **Mathematica** : cpu = 0.0292265 (sec), leaf count = 0 , could not solve

`DSolve[a*y[x]*Derivative[2][y][x] + Derivative[3][y][x] == 0, y[x], x]`

✓ **Maple** : cpu = 0.976 (sec), leaf count = 129

$$\left\{ y(x) = \text{ODESolStruc} \left( e^{\int -g(-f) d_f + C2}, \left[ \frac{d}{d_f} -g(-f) = \frac{-g(-f) (6(-g(-f))^2 -f^2 + 2(-g(-f))^2 -f a + 7)}{-f} \right] \right) \right.$$

2.1841 ODE No. 1841

$$-f(x) + x^2 y^{(3)}(x) + xy''(x) + (2xy(x) - 1)y'(x) + y(x)^2 = 0$$

✗ **Mathematica** : cpu = 0.0988267 (sec), leaf count = 0 , could not solve

DSolve[-f[x] + y[x]^2 + (-1 + 2\*x\*y[x])\*Derivative[1][y][x] + x\*Derivative[2][y][x] + x^2\*Derivative[3][y][x] == 0, y[x], x]

✓ **Maple** : cpu = 0.625 (sec), leaf count = 60

$$\left\{ y(x) = \text{ODESolStruc}\left(-b(-a), \left[ \left\{ -a^2 \frac{d^2}{d_a^2} b(-a) + -a (-b(-a))^2 - -a \frac{d}{d_a} b(-a) - \int f(-a) d_a + -C \right\} \right] \right)$$

2.1842 ODE No. 1842

$$x^2 y^{(3)}(x) + x(y(x) - 1)y''(x) + xy'(x)^2 + (1 - y(x))y'(x) = 0$$

✓ **Mathematica** : cpu = 0.170858 (sec), leaf count = 282

$$\left\{ \left\{ y(x) \rightarrow \frac{2 \left( c_3 \left( J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}} \left( -\frac{1}{2} i x \sqrt{c_1} \right) - \frac{1}{4} i \sqrt{c_1} x \left( J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}-1} \left( -\frac{1}{2} i x \sqrt{c_1} \right) - J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}+1} \left( -\frac{1}{2} i x \sqrt{c_1} \right) \right) \right) + Y_{\frac{\sqrt{c_2+2}}{\sqrt{2}}} \left( -\frac{1}{2} i x \sqrt{c_1} \right) \right)}{c_3 J_{\frac{\sqrt{c_2+2}}{\sqrt{2}}} \left( -\frac{1}{2} i x \sqrt{c_1} \right) + Y_{\frac{\sqrt{c_2+2}}{\sqrt{2}}} \left( -\frac{1}{2} i x \sqrt{c_1} \right)} \right. \right.$$

✓ **Maple** : cpu = 0.655 (sec), leaf count = 190

$$\left\{ \ln(x) + 2 \int^{y(x)} \left( 2 \left( \text{RootOf} \left( -2 Y_{1/2 \sqrt{4+CI}} \left( 1/2 \sqrt{2} Z \right) \sqrt{4+CI} - C2 + 2 Y_{1/2 \sqrt{4+CI}} \left( 1/2 \sqrt{2} Z \right) - C2 \right) \right) \right)$$

2.1843 ODE No. 1843

$$y^{(3)}(x)y(x) + y(x)^3 y'(x) - y'(x)y''(x) = 0$$

✓ **Mathematica** : cpu = 2.98056 (sec), leaf count = 409

$$\left\{ \left\{ y(x) \rightarrow \text{InverseFunction} \left[ \frac{2i \sqrt{\frac{\#1^2}{2(\sqrt{c_2^2-c_1-c_2})}} + 1 \sqrt{1 - \frac{\#1^2}{2(c_2 + \sqrt{c_2^2-c_1})}} F \left( i \sinh^{-1} \left( \frac{\sqrt{\frac{1}{\sqrt{c_2^2-c_1-c_2}} \#1}}{\sqrt{2}} \right) \right) \Big|_{\frac{c_2 - \sqrt{c_2^2}}{c_2 + \sqrt{c_2^2}}}}{\sqrt{\frac{1}{\sqrt{c_2^2-c_1-c_2}}} \sqrt{-\frac{\#1^4}{2} + 2\#1^2 c_2 - 2c_1}} \right] \right. \right.$$

✓ **Maple** : cpu = 0.316 (sec), leaf count = 77

$$\left\{ \int^{y(x)} -2 \frac{1}{\sqrt{-a^4 + 4_C2_a^2 - 4_C2^2 + 4_C1}} d_a - x - C3 = 0, \int^{y(x)} 2 \frac{1}{\sqrt{-a^4 + 4_C2_a^2 - 4_C2^2 + 4_C1}} d_a - x - C3 = 0 \right.$$

**2.1844 ODE No. 1844**

$$4y(x)^2y^{(3)}(x) + 15y'(x)^3 - 18y(x)y'(x)y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0853887 (sec), leaf count = 0 , could not solve

DSolve[15\*Derivative[1][y][x]^3 - 18\*y[x]\*Derivative[1][y][x]\*Derivative[2][y][x] + 4\*y[x]^2

✓ **Maple** : cpu = 0.292 (sec), leaf count = 17

$$\left\{ y(x) = \frac{-C3}{(-4 + (-C2 + x)^2 - C1)^2} \right\}$$

**2.1845 ODE No. 1845**

$$9y(x)^2y^{(3)}(x) + 40y'(x)^3 - 45y(x)y'(x)y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0793787 (sec), leaf count = 0 , could not solve

DSolve[40\*Derivative[1][y][x]^3 - 45\*y[x]\*Derivative[1][y][x]\*Derivative[2][y][x] + 9\*y[x]^2

✓ **Maple** : cpu = 0.27 (sec), leaf count = 17

$$\left\{ y(x) = -C3 \left( -9 + (-C2 + x)^2 - C1 \right)^{-\frac{3}{2}} \right\}$$

**2.1846 ODE No. 1846**

$$2y^{(3)}(x)y'(x) - 3y'(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0462292 (sec), leaf count = 51

$$\left\{ \left\{ y(x) \rightarrow c_1 \right\}, \left\{ y(x) \rightarrow \sqrt{\frac{2}{3}} e^{-\sqrt{\frac{3}{2}}x} \left( c_1 e^{\sqrt{6}x} - c_2 \right) + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.011 (sec), leaf count = 28

$$\left\{ y(x) = -C1, y(x) = -C1 + -C2 e^{\frac{\sqrt{6}x}{2}} + -C3 e^{-\frac{\sqrt{6}x}{2}} \right\}$$

2.1847 ODE No. 1847

$$y^{(3)}(x) (y'(x)^2 + 1) - 3y'(x)y''(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.130911 (sec), leaf count = 95

$$\left\{ \left\{ y(x) \rightarrow c_3 - \frac{i\sqrt{c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2 - 1}}{c_1} \right\}, \left\{ y(x) \rightarrow c_3 + \frac{i\sqrt{c_1^2 x^2 + 2c_2 c_1^2 x + c_2^2 c_1^2 - 1}}{c_1} \right\} \right\}$$

✓ **Maple** : cpu = 0.27 (sec), leaf count = 49

$$\left\{ y(x) = -\sqrt{-C2^2 - 2C2x - x^2 + C1} + C3, y(x) = \sqrt{-C2^2 - 2C2x - x^2 + C1} + C3 \right\}$$

2.1848 ODE No. 1848

$$y^{(3)}(x) (y'(x)^2 + 1) - y''(x)^2 (a + 3y'(x)) = 0$$

✗ **Mathematica** : cpu = 299.998 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.503 (sec), leaf count = 789

$$\left\{ y(x) = \int \frac{\sin \left( \text{RootOf} \left( e^{2a-Z} C1^2 C2^2 a^4 + 2e^{2a-Z} C1^2 C2 a^4 x + e^{2a-Z} C1^2 a^4 x^2 + 2e^{2a-Z} C1^2 C2 \right) \right)}{\cos \left( \text{RootOf} \left( e^{2a-Z} C1^2 C2^2 a^4 + 2e^{2a-Z} C1^2 C2 a^4 x + e^{2a-Z} C1^2 a^4 x^2 + 2e^{2a-Z} C1^2 C2 \right) \right)} dx \right\}$$

2.1849 ODE No. 1849

$$y^{(3)}(x)y''(x) - a\sqrt{b^2y''(x)^2 + 1} = 0$$

✓ **Mathematica** : cpu = 0.595767 (sec), leaf count = 415

$$\left\{ \left\{ y(x) \rightarrow \frac{6a^2b^5c_3x + 6a^2b^5c_2 + (a^2b^4x^2 + 2ab^4c_1x + b^4c_1^2 - 1)^{3/2} + 3\sqrt{a^2b^4x^2 + 2ab^4c_1x + b^4c_1^2 - 1} - 3b^2c_1 \log \left( \sqrt{a^2b^4x^2 + 2ab^4c_1x + b^4c_1^2 - 1} \right)}{b^2} \right\} \right\}$$

✓ **Maple** : cpu = 0.19 (sec), leaf count = 197

$$\left\{ y(x) = C2x + \int \frac{1}{2b} \left( -1 \ln \left( \sqrt{(1+b^2(C1+x)a)} (-1+b^2(C1+x)a)} + (C1+x)b^4a^2 \frac{1}{\sqrt{a^2b^4}} \right) \right) \frac{1}{\sqrt{a^2b^4}} dx \right\}$$



2.1850 ODE No. 1850

$$y^{(4)}(x)y'(x) - y^{(3)}(x)y''(x) + y^{(3)}(x)y'(x)^3 = 0$$

✗ **Mathematica** : cpu = 0.0887431 (sec), leaf count = 0 , could not solve

DSolve[Derivative[1][y][x]^3\*Derivative[3][y][x] - Derivative[2][y][x]\*Derivative[3][y][x] +

✓ **Maple** : cpu = 1.597 (sec), leaf count = 164

$$\left\{ y(x) = ODESolStruc \left( \int \frac{-j(-h)}{e^{f(-h)d_h + C2} h} d_h + C3, \left[ \frac{d}{d_h} j(-h) = \frac{-j(-h) \left( 12 (-j(-h))^2 h^2 + 3 \right)}{\dots} \right] \right. \right.$$

2.1851 ODE No. 1851

$$y'(x)^3 (f'(x)y'(x) + f(x)y''(x)) - y''(x) (f''(x)y'(x) + 2f'(x)y''(x) + f(x)y^{(3)}(x)) + y'(x) (f^{(3)}(x)y'(x) + 3f''(x)y''(x)) = 0$$

✗ **Mathematica** : cpu = 0.856455 (sec), leaf count = 0 , could not solve

DSolve[2\*q[x]\*Sin[y[x]]\*Derivative[1][y][x]^2 + Derivative[1][y][x]^3\*(Derivative[1][f][x]\*D[Derivative[1][q][x]\*Derivative[1][y][x]] + q[x]\*Derivative[2][y][x]) - Derivative[2][y][x]\*

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

dsolve(diff(y(x),x)\*(diff(diff(diff(f(x),x),x),x)\*diff(y(x),x)+3\*diff(diff(f(x),x),x)\*diff(diff(diff(y(x),x),x)\*f\*diff(diff(diff(y(x),x),x),x)+diff(y(x),x)^3\*(diff(f(x),x)\*diff(y(x),x)+diff(q(x),x)\*diff(y(x),x))\*cos(y(x))=0,y(x))

2.1852 ODE No. 1852

$$3y^{(4)}(x)y''(x) - 5y^{(3)}(x)^2 = 0$$

✓ **Mathematica** : cpu = 0.0356409 (sec), leaf count = 28

$$\left\{ \left\{ y(x) \rightarrow c_2(-\sqrt{3c_1 + 2x}) + c_4x + c_3 \right\} \right\}$$

✓ **Maple** : cpu = 0.321 (sec), leaf count = 30

$$\left\{ y(x) = 3(-C2 + x) \sqrt{6} C1^2 \sqrt{-\frac{-C1}{-C2 + x}} + C3 x + C4 \right\}$$

**2.1853 ODE No. 1853**

$$40y^{(3)}(x)^3 + 9y^{(5)}(x)y''(x)^2 - 45y^{(4)}(x)y^{(3)}(x)y''(x) = 0$$

✗ **Mathematica** : cpu = 0.0740705 (sec), leaf count = 0 , could not solve

`DSolve[40*Derivative[3][y][x]^3 - 45*Derivative[2][y][x]*Derivative[3][y][x]*Derivative[4][y][x], y[x], x]`

✓ **Maple** : cpu = 0.681 (sec), leaf count = 110

$$\left\{ y(x) = \iint \text{RootOf} \left( - \int^{-Z} \left( \text{RootOf} \left( -20 \ln(\_f) + \int^{-Z} \_k \left( e^{\text{RootOf}(81\_k^2 e^{-Z} + 20 e^{-Z} \ln(e^{-Z} + 27)) - 40 e^{-Z} \ln(2) - 20} \right) \right) \right) \right) \right.$$

**2.1854 ODE No. 1854**

$$y^{(n)}(x) - f(y^{(n-1)}(x)) = 0$$

✗ **Mathematica** : cpu = 0.00911541 (sec), leaf count = 0 , could not solve

`DSolve[-f[Derivative[-1 + n][y][x]] + Derivative[n][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

unable to handle ODEs of undefined differential order

**2.1855 ODE No. 1855**

$$y^{(n)}(x) - f(y^{(n-2)}(x)) = 0$$

✗ **Mathematica** : cpu = 0.00366543 (sec), leaf count = 0 , could not solve

`DSolve[-f[Derivative[-2 + n][y][x]] + Derivative[n][y][x] == 0, y[x], x]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

unable to handle ODEs of undefined differential order

**2.1856 ODE No. 1856**

$$\{x'(t) = ax(t), y'(t) = b\}$$

✓ **Mathematica** : cpu = 0.0056679 (sec), leaf count = 22

$$\{\{x(t) \rightarrow c_1 e^{at}, y(t) \rightarrow bt + c_2\}\}$$

✓ **Maple** : cpu = 0.033 (sec), leaf count = 19

$$\{\{x(t) = \_C1 e^{at}, y(t) = bt + \_C2\}\}$$

**2.1857 ODE No. 1857**

$$\{x'(t) = ay(t), y'(t) = -ax(t)\}$$

✓ **Mathematica** : cpu = 0.0546035 (sec), leaf count = 39

$$\{\{x(t) \rightarrow c_2 \sin(at) + c_1 \cos(at), y(t) \rightarrow c_2 \cos(at) - c_1 \sin(at)\}\}$$

✓ **Maple** : cpu = 0.048 (sec), leaf count = 35

$$\{\{x(t) = \_C1 \sin(at) + \_C2 \cos(at), y(t) = -\sin(at)\_C2 + \cos(at)\_C1\}\}$$

**2.1858 ODE No. 1858**

$$\{x'(t) = ay(t), y'(t) = bx(t)\}$$

✓ **Mathematica** : cpu = 0.0192381 (sec), leaf count = 158

$$\left\{ \left\{ x(t) \rightarrow \frac{e^{-\sqrt{a}\sqrt{bt}} \left( \sqrt{b}c_1 \left( e^{2\sqrt{a}\sqrt{bt}} + 1 \right) + \sqrt{a}c_2 \left( e^{2\sqrt{a}\sqrt{bt}} - 1 \right) \right)}{2\sqrt{b}}, y(t) \rightarrow \frac{e^{-\sqrt{a}\sqrt{bt}} \left( \sqrt{b}c_1 \left( e^{2\sqrt{a}\sqrt{bt}} - 1 \right) + \sqrt{a}c_2 \left( e^{2\sqrt{a}\sqrt{bt}} + 1 \right) \right)}{2\sqrt{a}} \right\} \right\}$$

✓ **Maple** : cpu = 0.053 (sec), leaf count = 64

$$\left\{ \left\{ x(t) = \_C1 e^{\sqrt{a}\sqrt{bt}} + \_C2 e^{-\sqrt{a}\sqrt{bt}}, y(t) = 1\sqrt{b} \left( \_C1 e^{\sqrt{a}\sqrt{bt}} - \_C2 e^{-\sqrt{a}\sqrt{bt}} \right) \frac{1}{\sqrt{a}} \right\} \right\}$$

**2.1859 ODE No. 1859**

$$\{x'(t) = ax(t) - y(t), y'(t) = ay(t) + x(t)\}$$

✓ **Mathematica** : cpu = 0.00649794 (sec), leaf count = 43

$$\{\{x(t) \rightarrow e^{at}(c_1 \cos(t) - c_2 \sin(t)), y(t) \rightarrow e^{at}(c_1 \sin(t) + c_2 \cos(t))\}\}$$

✓ **Maple** : cpu = 0.039 (sec), leaf count = 37

$$\{\{x(t) = e^{at}(\_C1 \sin(t) + \_C2 \cos(t)), y(t) = e^{at}(\sin(t)\_C2 - \cos(t)\_C1)\}\}$$

**2.1860 ODE No. 1860**

$$\{x'(t) = ax(t) + by(t), y'(t) = by(t) + cx(t)\}$$

✓ **Mathematica** : cpu = 0.0470141 (sec), leaf count = 362

$$\left\{ \left\{ x(t) \rightarrow \frac{e^{\frac{1}{2}t(-\sqrt{a^2-2ab+b^2+4bc+a+b})} \left( ac_1 \left( e^{t\sqrt{a^2-2ab+b^2+4bc}} - 1 \right) + c_1 \sqrt{a^2 - 2ab + b^2 + 4bc} \left( e^{t\sqrt{a^2-2ab+b^2+4bc}} + 1 \right) \right)}{2\sqrt{a^2 - 2ab + b(b + 4c)}} \right. \right.$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 177

$$\left\{ \left\{ x(t) = \_C1 e^{\frac{t}{2}(a+b+\sqrt{b^2+(-2a+4c)b+a^2})} + \_C2 e^{\frac{t}{2}(a+b-\sqrt{b^2+(-2a+4c)b+a^2})}, y(t) = \frac{1}{2b} \left( -\_C2 \left( a - b + \sqrt{b^2 + \dots} \right) \right) \right. \right.$$

**2.1861 ODE No. 1861**

$$\{ax'(t) + by'(t) = \alpha x(t) + \beta y(t), bx'(t) - ay'(t) = \beta x(t) - \alpha y(t)\}$$

✓ **Mathematica** : cpu = 0.013887 (sec), leaf count = 145

$$\left\{ \left\{ x(t) \rightarrow e^{\frac{t(\alpha+\beta)}{a^2+b^2}} \left( c_2 \sin \left( \frac{t(a\beta - \alpha b)}{a^2 + b^2} \right) + c_1 \cos \left( \frac{t(a\beta - \alpha b)}{a^2 + b^2} \right) \right), y(t) \rightarrow e^{\frac{t(\alpha+\beta)}{a^2+b^2}} \left( c_2 \cos \left( \frac{t(a\beta - \alpha b)}{a^2 + b^2} \right) - c_1 \sin \left( \frac{t(a\beta - \alpha b)}{a^2 + b^2} \right) \right) \right. \right.$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 152

$$\left\{ \left\{ x(t) = \_C1 e^{\frac{((i\beta+\alpha)a-b(i\alpha-\beta))t}{a^2+b^2}} + \_C2 e^{-\frac{((i\beta-\alpha)a-b(i\alpha+\beta))t}{a^2+b^2}}, y(t) = i \left( \_C1 e^{\frac{((i\beta+\alpha)a-b(i\alpha-\beta))t}{a^2+b^2}} - \_C2 e^{-\frac{((i\beta-\alpha)a-b(i\alpha+\beta))t}{a^2+b^2}} \right) \right. \right.$$

**2.1862 ODE No. 1862**

$$\{x'(t) = -y(t), y'(t) = 2x(t) + 2y(t)\}$$

✓ **Mathematica** : cpu = 0.0752874 (sec), leaf count = 46

$$\left\{ \left\{ x(t) \rightarrow e^t (c_1 \cos(t) - (c_1 + c_2) \sin(t)), y(t) \rightarrow e^t (2c_1 \sin(t) + c_2 (\sin(t) + \cos(t))) \right. \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 39

$$\left\{ \left\{ x(t) = e^t (\sin(t) \_C1 + \cos(t) \_C2), y(t) = -e^t ((\_C1 + \_C2) \cos(t) + \sin(t) (\_C1 - \_C2)) \right. \right\}$$

**2.1863 ODE No. 1863**

$$\{x'(t) + 3x(t) + 4y(t) = 0, 2x(t) + y'(t) + 5y(t) = 0\}$$

✓ **Mathematica** : cpu = 0.00799515 (sec), leaf count = 72

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{3}e^{-7t}(c_1(2e^{6t} + 1) - 2c_2(e^{6t} - 1)), y(t) \rightarrow \frac{1}{3}e^{-7t}(c_2(e^{6t} + 2) - c_1(e^{6t} - 1)) \right\} \right\}$$

✓ **Maple** : cpu = 0.038 (sec), leaf count = 35

$$\left\{ \left\{ x(t) = \_C1 e^{-7t} + \_C2 e^{-t}, y(t) = \_C1 e^{-7t} - \frac{C2 e^{-t}}{2} \right\} \right\}$$

**2.1864 ODE No. 1864**

$$\{x'(t) = -5x(t) - 2y(t), y'(t) = x(t) - 7y(t)\}$$

✓ **Mathematica** : cpu = 0.0139952 (sec), leaf count = 52

$$\left\{ \left\{ x(t) \rightarrow e^{-6t}((c_1 - 2c_2) \sin(t) + c_1 \cos(t)), y(t) \rightarrow e^{-6t}((c_1 - c_2) \sin(t) + c_2 \cos(t)) \right\} \right\}$$

✓ **Maple** : cpu = 0.041 (sec), leaf count = 44

$$\left\{ \left\{ x(t) = e^{-6t}(\sin(t) \_C1 + \cos(t) \_C2), y(t) = -\frac{e^{-6t}((\_C1 - \_C2) \cos(t) - \sin(t) (\_C1 + \_C2))}{2} \right\} \right\}$$

**2.1865 ODE No. 1865**

$$\{x'(t) = a1x(t) + b1y(t) + c1, y'(t) = a2x(t) + b2y(t) + c2\}$$

✓ **Mathematica** : cpu = 1.20413 (sec), leaf count = 930

$$\left\{ \left\{ x(t) \rightarrow \frac{2e^{-\frac{1}{2}(a1+b2+\sqrt{a1^2-2b2a1+b2^2+4a2b1})t} \left( a1e^{(a1+b2)t} (-1 + e^{\sqrt{a1^2-2b2a1+b2^2+4a2b1}t}) c1b2^2 - \left( e^{(a1+b2)t} \left( \right) \right) \right)}{\right.} \right\}$$

✓ **Maple** : cpu = 0.136 (sec), leaf count = 224

$$\left\{ \left\{ x(t) = e^{\frac{t}{2}(a1+b2+\sqrt{a1^2-2a1b2+4b1a2+b2^2})} \_C2 + e^{\frac{t}{2}(a1+b2-\sqrt{a1^2-2a1b2+4b1a2+b2^2})} \_C1 + \frac{c2b1 - c1b2}{a1b2 - b1a2}, y \right\} \right\}$$

**2.1866 ODE No. 1866**

$$\{x'(t) + 2y(t) = 3t, y'(t) - 2x(t) = 4\}$$

✓ **Mathematica** : cpu = 0.0317083 (sec), leaf count = 47

$$\left\{ \left\{ x(t) \rightarrow -c_2 \sin(2t) + c_1 \cos(2t) - \frac{5}{4}, y(t) \rightarrow c_1 \sin(2t) + c_2 \cos(2t) + \frac{3t}{2} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 39

$$\left\{ \left\{ x(t) = \sin(2t) \_C2 + \cos(2t) \_C1 - \frac{5}{4}, y(t) = -\cos(2t) \_C2 + \sin(2t) \_C1 + \frac{3t}{2} \right\} \right\}$$

**2.1867 ODE No. 1867**

$$\{-t^2 + x'(t) + y(t) + 6t + 1 = 0, y'(t) - x(t) = -3t^2 + 3t + 1\}$$

✓ **Mathematica** : cpu = 0.0875111 (sec), leaf count = 45

$$\left\{ \left\{ x(t) \rightarrow -c_2 \sin(t) + c_1 \cos(t) + 3t^2 - t - 13, y(t) \rightarrow c_1 \sin(t) + c_2 \cos(t) + (t - 12)t \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 42

$$\left\{ \left\{ x(t) = \sin(t) \_C2 + \cos(t) \_C1 + 3t^2 - t - 13, y(t) = t^2 - \cos(t) \_C2 + \sin(t) \_C1 - 12t \right\} \right\}$$

**2.1868 ODE No. 1868**

$$\{x'(t) + 3x(t) - y(t) = e^{2t}, x(t) + y'(t) + 5y(t) = e^t\}$$

✓ **Mathematica** : cpu = 0.0475309 (sec), leaf count = 83

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{900} e^{-4t} (900(c_1(t+1) + c_2 t) + 36e^{5t} + 175e^{6t}), y(t) \rightarrow \frac{1}{900} e^{-4t} (-900((c_1 + c_2)t - c_2) + 144e^{5t} - 25e^{6t}) \right\} \right\}$$

✓ **Maple** : cpu = 0.073 (sec), leaf count = 64

$$\left\{ \left\{ x(t) = e^{-4t} \_C2 + e^{-4t} t \_C1 + \frac{7e^{2t}}{36} + \frac{e^t}{25}, y(t) = -\frac{e^{2t}}{36} - e^{-4t} \_C2 - e^{-4t} t \_C1 + e^{-4t} \_C1 + \frac{4e^t}{25} \right\} \right\}$$

**2.1869 ODE No. 1869**

$$\{x'(t) + 2x(t) + y'(t) + y(t) = t + e^{2t}, x'(t) - x(t) + y'(t) + 3y(t) = e^t - 1\}$$

✓ **Mathematica** : cpu = 0.115839 (sec), leaf count = 84

$$\left\{ \left\{ x(t) \rightarrow \frac{5}{72}c_1e^{-7t/5} + \frac{3t}{7} - \frac{e^t}{6} + \frac{5e^{2t}}{17} - \frac{1}{49}, y(t) \rightarrow \frac{5}{48}c_1e^{-7t/5} + \frac{t}{7} + \frac{e^t}{4} - \frac{e^{2t}}{17} - \frac{26}{49} \right\} \right\}$$

✓ **Maple** : cpu = 0.06 (sec), leaf count = 51

$$\left\{ \left\{ x(t) = \frac{5e^{2t}}{17} - \frac{e^t}{6} + \frac{3t}{7} - \frac{1}{49} + e^{-\frac{7t}{5}}C1, y(t) = -\frac{e^{2t}}{17} + \frac{e^t}{4} + \frac{t}{7} - \frac{26}{49} + \frac{3C1}{2}e^{-\frac{7t}{5}} \right\} \right\}$$

**2.1870 ODE No. 1870**

$$\{x'(t) + y'(t) - y(t) = e^t, 2x'(t) + y'(t) + 2y(t) = \cos(t)\}$$

✓ **Mathematica** : cpu = 0.132093 (sec), leaf count = 71

$$\left\{ \left\{ x(t) \rightarrow -\frac{3}{4}c_2e^{4t} + c_1 + \frac{3c_2}{4} + e^t + \frac{5\sin(t)}{17} - \frac{3\cos(t)}{17}, y(t) \rightarrow c_2e^{4t} - \frac{2e^t}{3} - \frac{\sin(t)}{17} + \frac{4\cos(t)}{17} \right\} \right\}$$

✓ **Maple** : cpu = 0.113 (sec), leaf count = 47

$$\left\{ \left\{ x(t) = \frac{5\sin(t)}{17} - \frac{3\cos(t)}{17} + e^t + \frac{e^{4t}C1}{4} + C2, y(t) = \frac{4\cos(t)}{17} - \frac{\sin(t)}{17} - \frac{2e^t}{3} - \frac{e^{4t}C1}{3} \right\} \right\}$$

**2.1871 ODE No. 1871**

$$\{4x'(t) + 2x(t) + 9y'(t) + 31y(t) = e^t, 3x'(t) + x(t) + 7y'(t) + 24y(t) = 3\}$$

✓ **Mathematica** : cpu = 0.165582 (sec), leaf count = 83

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{442}e^{-4t}(-442(c_1 + c_2)\sin(t) + 442c_1\cos(t) + 31e^{4t}(17e^t - 78)), y(t) \rightarrow (2c_1 + c_2)e^{-4t}\sin(t) + c_2e^{-4t} \right\} \right\}$$

✓ **Maple** : cpu = 0.077 (sec), leaf count = 62

$$\left\{ \left\{ x(t) = e^{-4t}\sin(t)C2 + e^{-4t}\cos(t)C1 - \frac{93}{17} + \frac{31e^t}{26}, y(t) = \frac{((-221C1 - 221C2)\cos(t) + 221\sin(t))}{221} \right\} \right\}$$

**2.1872 ODE No. 1872**

$$\{4x'(t) + 11x(t) + 9y'(t) + 31y(t) = e^t, 3x'(t) + 8x(t) + 7y'(t) + 24y(t) = e^{2t}\}$$

✓ **Mathematica** : cpu = 0.0597601 (sec), leaf count = 81

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{900} e^{-4t} (-900(c_1(t-1) + c_2t) + 1116e^{5t} - 1225e^{6t}), y(t) \rightarrow \frac{1}{900} e^{-4t} (900((c_1 + c_2)t + c_2) - 396e^{5t} + \dots \right. \right.$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 65

$$\left\{ \left\{ x(t) = e^{-4t} \_C2 + e^{-4t} t \_C1 - \frac{49 e^{2t}}{36} + \frac{31 e^t}{25}, y(t) = \frac{19 e^{2t}}{36} - e^{-4t} \_C2 - e^{-4t} t \_C1 - e^{-4t} \_C1 - \frac{11 e^t}{25} \right\} \right\}$$

**2.1873 ODE No. 1873**

$$\{4x'(t) + 44x(t) + 9y'(t) + 49y(t) = t, 3x'(t) + 34x(t) + 7y'(t) + 38y(t) = e^t\}$$

✓ **Mathematica** : cpu = 0.0463303 (sec), leaf count = 112

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{315} e^{-6t} (63(4c_1 - c_2) e^{5t} + 63(c_1 + c_2) + 35e^{6t}(57t - 56) - 1305e^{7t}), y(t) \rightarrow \frac{1}{315} e^{-6t} (-63(4c_1 - c_2) e^{5t} + \dots \right. \right.$$

✓ **Maple** : cpu = 0.063 (sec), leaf count = 52

$$\left\{ \left\{ x(t) = \_C2 e^{-t} + e^{-6t} \_C1 - \frac{56}{9} + \frac{19t}{3} - \frac{29e^t}{7}, y(t) = -\_C2 e^{-t} + 4e^{-6t} \_C1 + \frac{55}{9} + \frac{24e^t}{7} - \frac{17t}{3} \right\} \right\}$$

**2.1874 ODE No. 1874**

$$\{x'(t) = f(t)x(t) + g(t)y(t), y'(t) = f(t)y(t) - g(t)x(t)\}$$

✓ **Mathematica** : cpu = 0.13959 (sec), leaf count = 105

$$\left\{ \left\{ x(t) \rightarrow e^{\text{Integrate}[f(K[2]),\{K[2],1,t\},\text{Assumptions}\rightarrow\text{True}]} (c_2 \sin(\text{Integrate}[g(K[1]),\{K[1],1,t\},\text{Assumptions}\rightarrow\text{True}]) - \dots \right. \right.$$

✓ **Maple** : cpu = 0.458 (sec), leaf count = 57

$$\left\{ \left\{ x(t) = e^{\int \tan(-C1 - \int g(t) dt) g(t) + f(t) dt} \_C2, y(t) = e^{\int \tan(-C1 - \int g(t) dt) g(t) + f(t) dt} \tan \left( -C1 - \int g(t) dt \right) \_C2 \right\} \right\}$$



**2.1875 ODE No. 1875**

$$\{f(t)(ax(t) + by(t)) + x'(t) = g(t), f(t)(cx(t) + dy(t)) + y'(t) = h(t)\}$$

✗ **Mathematica** : cpu = 0.00797435 (sec), leaf count = 0 , could not solve

`DSolve[{f[t]*(a*x[t] + b*y[t]) + Derivative[1][x][t] == g[t], f[t]*(c*x[t] + d*y[t]) + Deriv`

✓ **Maple** : cpu = 1.264 (sec), leaf count = 1447

$$\left\{ \left\{ x(t) = 1 \left( - \int \frac{\left(\frac{d}{dt} f(t)\right) g(t) - f(t) \left(\frac{d}{dt} g(t) - f(t) (bh(t) - g(t) d)\right)}{(f(t))^2} e^{\frac{\int f(t) dt}{2}} \left( -\sqrt{\frac{-a^2 + 2 da - 4 bc - d^2}{da - bc}} \sqrt{-da + bc + a + d} \right) dt \right. \right. \right\}$$

**2.1876 ODE No. 1876**

$$\{x'(t) = x(t) \cos(t), y'(t) = x(t) e^{-\sin(t)}\}$$

✓ **Mathematica** : cpu = 0.107135 (sec), leaf count = 41

$$\left\{ \left\{ x(t) \rightarrow c_1 e^{\sin(t)}, y(t) \rightarrow c_1 \text{Integrate}\left[e^{\sin(K[1]) - \sin(K[1])}, \{K[1], 1, t\}, \text{Assumptions} \rightarrow \text{True}\right] + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.139 (sec), leaf count = 18

$$\left\{ \left\{ x(t) = \_C2 e^{\sin(t)}, y(t) = \_C2 t + \_C1 \right\} \right\}$$

**2.1877 ODE No. 1877**

$$\{tx'(t) + y(t) = 0, x(t) + ty'(t) = 0\}$$

✓ **Mathematica** : cpu = 0.00588005 (sec), leaf count = 31

$$\left\{ \left\{ x(t) \rightarrow c_1 t + \frac{c_2}{t}, y(t) \rightarrow \frac{c_2}{t} - c_1 t \right\} \right\}$$

✓ **Maple** : cpu = 0.036 (sec), leaf count = 31

$$\left\{ \left\{ x(t) = \frac{-C1 t^2 + \_C2}{t}, y(t) = \frac{-\_C1 t^2 + \_C2}{t} \right\} \right\}$$

**2.1878 ODE No. 1878**

$$\{tx'(t) + 2x(t) = t, -(t + 2)x(t) + ty'(t) - ty(t) = -t\}$$

✓ **Mathematica** : cpu = 0.0130292 (sec), leaf count = 39

$$\left\{ \left\{ x(t) \rightarrow \frac{c_1}{t^2} + \frac{t}{3}, y(t) \rightarrow -\frac{c_1}{t^2} + c_2 e^t - \frac{t}{3} \right\} \right\}$$

✓ **Maple** : cpu = 0.058 (sec), leaf count = 39

$$\left\{ \left\{ x(t) = \frac{t}{3} + \frac{-C2}{t^2}, y(t) = \frac{3\_C1 e^{tt^2} - t^3 - 3\_C2}{3t^2} \right\} \right\}$$

**2.1879 ODE No. 1879**

$$\{tx'(t) + 2(x(t) - y(t)) = t, x(t) + ty'(t) + 5y(t) = t^2\}$$

✓ **Mathematica** : cpu = 0.0910719 (sec), leaf count = 58

$$\left\{ \left\{ x(t) \rightarrow \frac{c_1}{t^4} + \frac{c_2}{t^3} + \frac{1}{30}t(2t + 9), y(t) \rightarrow -\frac{30c_2t + 60c_1 - 8t^6 + 3t^5}{60t^4} \right\} \right\}$$

✓ **Maple** : cpu = 0.056 (sec), leaf count = 54

$$\left\{ \left\{ x(t) = \frac{2t^6 + 9t^5 + 30\_C2t + 30\_C1}{30t^4}, y(t) = \frac{8t^6 - 3t^5 - 30\_C2t - 60\_C1}{60t^4} \right\} \right\}$$

**2.1880 ODE No. 1880**

$$\{t^2(1 - \sin(t))x'(t) = t^2y(t) + tx(t)(1 - 2\sin(t)), t^2(1 - \sin(t))y'(t) = x(t)(t \cos(t) - \sin(t)) + ty(t)(1 - t \cos(t))\}$$

✗ **Mathematica** : cpu = 0.0247361 (sec), leaf count = 0 , could not solve

`DSolve[{t^2*(1 - Sin[t])*Derivative[1][x][t] == t*(1 - 2*Sin[t])*x[t] + t^2*y[t], t^2*(1 - S`

✓ **Maple** : cpu = 0.083 (sec), leaf count = 23

$$\{\{x(t) = t(-C2t + -C1), y(t) = \sin(t) - C1 + -C2t\}\}$$

**2.1881 ODE No. 1881**

$$\{x'(t) + y'(t) + y(t) = f(t), x''(t) + x(t) + y''(t) + y'(t) + y(t) = g(t)\}$$

✓ **Mathematica** : cpu = 0.0282758 (sec), leaf count = 44

$$\{\{x(t) \rightarrow -f''(t) - f'(t) - f(t) + g'(t) + g(t), y(t) \rightarrow f''(t) + f(t) - g'(t)\}\}$$

✓ **Maple** : cpu = 0.029 (sec), leaf count = 48

$$\left\{ \left\{ x(t) = -\frac{d}{dt}f(t) + g(t) - f(t) - \frac{d^2}{dt^2}f(t) + \frac{d}{dt}g(t), y(t) = f(t) + \frac{d^2}{dt^2}f(t) - \frac{d}{dt}g(t) \right\} \right\}$$

**2.1882 ODE No. 1882**

$$\{2x'(t) - 3x(t) + y'(t) = 0, x''(t) + y'(t) - 2y(t) = e^{2t}\}$$

✓ **Mathematica** : cpu = 0.454376 (sec), leaf count = 199

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{276}e^{t/2} \left( 23e^{t/2}(6c_1 + 2c_2 + 4c_3 + 3e^t) - 2\sqrt{23}(9c_1 - 11c_2 + 2c_3) \sin\left(\frac{\sqrt{23}t}{2}\right) + 46(3c_1 - c_2 - 2c_3) \cos\left(\frac{\sqrt{23}t}{2}\right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 99

$$\left\{ \left\{ x(t) = \frac{e^{2t}}{4} + \_C1 e^t + \_C2 e^{\frac{t}{2}} \cos\left(\frac{\sqrt{23}t}{2}\right) + \_C3 e^{\frac{t}{2}} \sin\left(\frac{\sqrt{23}t}{2}\right), y(t) = -\frac{7}{4}e^{\frac{t}{2}} \left( \frac{\_C3 \sqrt{23}}{7} + \_C2 \right) \cos\left(\frac{\sqrt{23}t}{2}\right) \right\} \right\}$$

**2.1883 ODE No. 1883**

$$\{x'(t) + x(t) - y'(t) = 2t, x''(t) - 9x(t) + y'(t) + 3y(t) = \sin(2t)\}$$

✓ **Mathematica** : cpu = 0.598706 (sec), leaf count = 186

$$\left\{ \left\{ x(t) \rightarrow \frac{e^{-3t}(325(e^{4t}(c_1(20t + 7) + c_2(4t + 3) + 3c_3(1 - 4t)) + 9c_1 - 3(c_2 + c_3) + 32e^{3t}(t + 2)) - 576e^{3t} \sin(2t))}{5200} \right\} \right\}$$

✓ **Maple** : cpu = 0.112 (sec), leaf count = 80

$$\left\{ \left\{ x(t) = -\frac{2 \cos(2t)}{325} + 4 - \frac{36 \sin(2t)}{325} + 2t + \_C1 e^t + \_C2 e^{-3t} + \_C3 te^t, y(t) = \frac{16 \cos(2t)}{325} - \frac{37 \sin(2t)}{325} \right\} \right\}$$

**2.1884 ODE No. 1884**

$$\{x'(t) - x(t) + 2y(t) = 0, x''(t) - 2y'(t) = 2t - \cos(2t)\}$$

✓ **Mathematica** : cpu = 0.193016 (sec), leaf count = 116

$$\left\{ \left\{ x(t) \rightarrow 8c_1 e^{t/2} + 8c_2 e^{t/2} - c_2 - t^2 - 4t + \frac{1}{34} \sin(2t) + \frac{2}{17} \cos(2t) - 8, y(t) \rightarrow 2c_1 e^{t/2} + 2c_2 e^{t/2} - \frac{c_2}{2} - \frac{t^2}{2} - t \right. \right\}$$

✓ **Maple** : cpu = 0.107 (sec), leaf count = 69

$$\left\{ \left\{ x(t) = \frac{\sin(2t)}{34} + \frac{2 \cos(2t)}{17} - t^2 + 2\_C1 e^{t/2} - 4t + \_C2, y(t) = \frac{\cos(2t)}{34} + \frac{9 \sin(2t)}{68} - t + \frac{C1}{2} e^{\frac{t}{2}} + 2 \right. \right\}$$

**2.1885 ODE No. 1885**

$$\{tx'(t) - ty'(t) - 2y(t) = 0, tx''(t) + 2x'(t) + tx(t) = 0\}$$

✗ **Mathematica** : cpu = 0.024157 (sec), leaf count = 0 , could not solve

`DSolve[{-2*y[t] + t*Derivative[1][x][t] - t*Derivative[1][y][t] == 0, t*x[t] + 2*Derivative[1][x][t]}`

✓ **Maple** : cpu = 0.098 (sec), leaf count = 47

$$\left\{ \left\{ x(t) = \frac{\sin(t)\_C2 + \_C3 \cos(t)}{t}, y(t) = \frac{(\_C3 t + 2\_C2) \cos(t) + (\_C2 t - 2\_C3) \sin(t) + \_C1}{t^2} \right. \right\}$$

**2.1886 ODE No. 1886**

$$\{ay(t) + x''(t) = 0, y''(t) - a^2 y(t) = 0\}$$

✓ **Mathematica** : cpu = 0.0219043 (sec), leaf count = 103

$$\left\{ \left\{ x(t) \rightarrow \frac{c_4(2at + e^{-at} - e^{at})}{2a^2} - \frac{c_3 e^{-at}(e^{at} - 1)^2}{2a} + c_2 t + c_1, y(t) \rightarrow \frac{e^{-at}(ac_3(e^{2at} + 1) + c_4(e^{2at} - 1))}{2a} \right. \right\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 49

$$\left\{ \left\{ x(t) = \frac{-\_C4 e^{-at} - \_C3 e^{at} + a(\_C1 t + \_C2)}{a}, y(t) = \_C3 e^{at} + \_C4 e^{-at} \right. \right\}$$

**2.1887 ODE No. 1887**

$$\{x''(t) = ax(t) + by(t), y''(t) = cx(t) + dy(t)\}$$

✓ **Mathematica** : cpu = 0.546093 (sec), leaf count = 5647

$$\left\{ \left\{ x(t) \rightarrow \frac{e^{-\frac{(\sqrt{a+d-\sqrt{a^2-2da+d^2+4bc}}+\sqrt{a+d+\sqrt{a^2-2da+d^2+4bc}})t}}{\sqrt{2}} \left( \sqrt{a^2-2da+d^2+4bc}\sqrt{a+d-\sqrt{a^2-2da+d^2+4bc}} \right)}{\right.} \right.$$

✓ **Maple** : cpu = 0.14 (sec), leaf count = 360

$$\left\{ \left\{ x(t) = \_C1 e^{-\frac{t}{2}\sqrt{-2\sqrt{a^2-2da+4bc+d^2}+2a+2d}} + \_C2 e^{\frac{t}{2}\sqrt{-2\sqrt{a^2-2da+4bc+d^2}+2a+2d}} + \_C3 e^{-\frac{t}{2}\sqrt{2\sqrt{a^2-2da+4bc}+2a+2d}} \right. \right.$$

**2.1888 ODE No. 1888**

$$\{x''(t) = a1x(t) + b1y(t) + c1, y''(t) = a2x(t) + b2y(t) + c2\}$$

✓ **Mathematica** : cpu = 26.0887 (sec), leaf count = 1

\$Aborted

✓ **Maple** : cpu = 0.269 (sec), leaf count = 457

$$\left\{ \left\{ x(t) = \_C4 e^{\frac{t}{2}\sqrt{2\sqrt{a1^2-2a1b2+4b1a2+b2^2}+2a1+2b2}} + \_C3 e^{-\frac{t}{2}\sqrt{2\sqrt{a1^2-2a1b2+4b1a2+b2^2}+2a1+2b2}} + \_C2 e^{\frac{t}{2}\sqrt{2\sqrt{a1^2-2a1b2+4b1a2+b2^2}+2a1+2b2}} \right. \right.$$

**2.1889 ODE No. 1889**

$$\{x''(t) + x(t) + y(t) = -5, -4x(t) + y''(t) - 3y(t) = -3\}$$

✓ **Mathematica** : cpu = 0.111495 (sec), leaf count = 151

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{4}e^{-t}(2c_1(t+1) - 2c_2t + c_3t - c_4t + e^{2t}(-2c_1(t-1) - 2c_2(t-2) - c_3t - c_4t + c_4) - 4c_2 - c_4 + 72e^t) \right. \right.$$

✓ **Maple** : cpu = 0.062 (sec), leaf count = 60

$$\left\{ \left\{ x(t) = (\_C4 t + \_C2) e^{-t} + 18 + (\_C3 t + \_C1) e^t, y(t) = ((-2t + 2)\_C4 - 2\_C2) e^{-t} - 23 + ((-2t - 2)\_C3 + 2\_C1) e^t \right. \right.$$

**2.1890 ODE No. 1890**

$$\left\{ x''(t) = c^2 x(t) (3 \cos^2(at + b) - 1) + \frac{3}{2} c^2 y(t) \sin(2abt), y''(t) = \frac{3}{2} c^2 x(t) \sin(2abt) + c^2 y(t) (3 \sin^2(at + b) - 1) \right\}$$

✗ **Mathematica** : cpu = 0.0101288 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[2][x][t] == c^2*(-1 + 3*Cos[b + a*t]^2)*x[t] + (3*c^2*Sin[2*a*b*t]*y[t])/1 + 3*Sin[b + a*t]^2*y[t]}, {x[t], y[t]}, t]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ \left\{ x(t) = DESol \left( \left( \frac{d^4}{dt^4} - Y(t) + \left( -2 \frac{ab \cos(atb)}{\sin(atb)} + 2 \frac{ab \sin(atb)}{\cos(atb)} \right) \frac{d^3}{dt^3} - Y(t) + \left( 2 \frac{(\sin(atb))^2 b^2 a^2}{(\cos(atb))^2} + 2 \frac{a^2 b^2}{\sin} \right) \right) \right\} \right\}$$

**2.1891 ODE No. 1891**

$$\{ x''(t) + 6x(t) + 7y(t) = 0, 3x(t) + y''(t) + 2y(t) = 2t \}$$

✓ **Mathematica** : cpu = 0.457598 (sec), leaf count = 200

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{180} e^{-t} (27c_1 e^{2t} + 27c_2 e^{2t} - 63c_3 e^{2t} - 63c_4 e^{2t} + 42(c_2 + c_4) e^t \sin(3t) + 126(c_1 + c_3) e^t \cos(3t) + 27c_1 \right\} \right\}$$

✓ **Maple** : cpu = 0.059 (sec), leaf count = 64

$$\left\{ \left\{ x(t) = \frac{14t}{9} + \_C1 e^t + \_C2 \cos(3t) + \_C3 e^{-t} + \_C4 \sin(3t), y(t) = -\_C1 e^t + \frac{3\_C2 \cos(3t)}{7} - \_C3 \right\} \right\}$$

**2.1892 ODE No. 1892**

$$\{-ay'(t) + bx(t) + x''(t) = 0, ax'(t) + by(t) + y''(t) = 0\}$$

✓ **Mathematica** : cpu = 0.399679 (sec), leaf count = 3522

$$\left\{ \left\{ x(t) \rightarrow \frac{e^{-\frac{(\sqrt{-a^2-2b-\sqrt{a^4+4ba^2}}+\sqrt{-a^2-2b+\sqrt{a^4+4ba^2}})t}}{\sqrt{2}} \left( \left( \sqrt{-a^2-2b-\sqrt{a^4+4ba^2}} e^{\frac{(\sqrt{-a^2-2b-\sqrt{a^4+4ba^2}}+2\sqrt{-a^2-2b+\sqrt{a^4+4ba^2}})t}}{\sqrt{2}} \right) \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 463

$$\left\{ \left\{ x(t) = \_C1 e^{-\frac{t}{2} \sqrt{-2a^2-2\sqrt{a^2(a^2+4b)}-4b}} + \_C2 e^{\frac{t}{2} \sqrt{-2a^2-2\sqrt{a^2(a^2+4b)}-4b}} + \_C3 e^{-\frac{t}{2} \sqrt{-2a^2+2\sqrt{a^2(a^2+4b)}-4b}} + \right\} \right\}$$

**2.1893 ODE No. 1893**

$$\{-A_0y'(t) + a_1x''(t) + b_1x'(t) + c_1x(t) = B_0e^{i\omega t}, A_0x'(t) + a_2y''(t) + b_2y'(t) + c_2y(t) = 0\}$$

✗ **Mathematica** : cpu = 300.018 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.276 (sec), leaf count = 1579

$$\left\{ \left\{ x(t) = \frac{(a_2 a_1 (\text{RootOf}(a_1 a_2 \_Z^4 + (a_1 b_2 + b_1 a_2) \_Z^3 + (A^2 + a_1 c_2 + c_1 a_2 + b_1 b_2) \_Z^2 + (c_2 b_1 + \dots))\right.}{\dots} \right. \right.$$

**2.1894 ODE No. 1894**

$$\{a(x'(t) - y'(t)) + b_1x(t) + x''(t) = c_1e^{i\omega t}, a(y'(t) - x'(t)) + b_2y(t) + y''(t) = c_2e^{i\omega t}\}$$

✗ **Mathematica** : cpu = 300.005 (sec), leaf count = 0 , timed out

\$Aborted

✓ **Maple** : cpu = 1.103 (sec), leaf count = 1056

$$\left\{ \left\{ x(t) = \frac{e^{i\omega t}(-c_1 \omega^2 + i(c_1 + c_2) a \omega + c_1 b_2)}{\omega^4 - 2 i a \omega^3 + (-b_1 - b_2) \omega^2 + i(b_1 + b_2) a \omega + b_1 b_2} + \_C1 e^{\text{RootOf}(\_Z^4 + 2 a \_Z^3 + (b_1 + b_2) \_Z^2 + (a b_1 + b_2 \dots)} \right. \right.$$

**2.1895 ODE No. 1895**

$$\{a_{11}x''(t) + a_{12}y''(t) + b_{11}x'(t) + b_{12}y'(t) + c_{11}x(t) + c_{12}y(t) = 0, a_{21}x''(t) + a_{22}y''(t) + b_{21}x'(t) + b_{22}y'(t) + \dots\}$$

✓ **Mathematica** : cpu = 0.450997 (sec), leaf count = 6816

✓ **Maple** : cpu = 0.25 (sec), leaf count = 1008

$$\left\{ \left\{ x(t) = \sum_{-a=1}^4 e^{\text{RootOf}((a_{22} a_{11} - a_{21} a_{12}) \_Z^4 + (a_{11} b_{22} - a_{12} b_{21} - a_{21} b_{12} + b_{11} a_{22}) \_Z^3 + (c_{22} a_{11} - a_{12} c_{21} - a_{21} c_{12} + a_{22} c_{11} + b \dots)} \right. \right.$$

**2.1896 ODE No. 1896**

$$\{x''(t) - 2x'(t) - y'(t) + y(t) = 0, 2x'(t) - x(t) + y^{(3)}(t) - y''(t) = t\}$$

✓ **Mathematica** : cpu = 0.196144 (sec), leaf count = 252

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{8} e^{-t} (e^{2t} (-2c_3 t^2 + 2c_5 t^2 + c_1 (2t^2 - 6t + 7) + c_2 (2t^2 + 6t + 1) - 2c_3 t + 4c_4 t - 2c_5 t + c_3 - 2c_4 + c_5) - t - 2, y(t) = -C_2 e^{-t} - 2 + (-C_5 t^3 - \dots \right. \right.$$

✓ **Maple** : cpu = 0.068 (sec), leaf count = 67

$$\left\{ \left\{ x(t) = -\frac{2-C_2 e^{-t}}{3} + \frac{(-9-C_5 t^2 - 6-C_4 t - 3-C_3 - 18-C_5) e^t}{3} - t - 2, y(t) = -C_2 e^{-t} - 2 + (-C_5 t^3 - \dots \right. \right.$$

**2.1897 ODE No. 1897**

$$\{x''(t) + y''(t) + y'(t) = \sinh(2t), 2x''(t) + y''(t) = 2t\}$$

✓ **Mathematica** : cpu = 0.109553 (sec), leaf count = 123

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{48} e^{-2t} (2e^{2t} (6(4c_2 + 2c_4 - 1)t + 24c_1 - 6c_4 + 4t^3 + 6t^2 + 3) - 6(-2c_4 + 2t + 1) - 3e^{4t}), y(t) \rightarrow \frac{1}{8} e^{-\dots} \right. \right.$$

✓ **Maple** : cpu = 0.169 (sec), leaf count = 86

$$\left\{ \left\{ x(t) = \frac{(-12t + 12-C_2 - 15) e^{-2t}}{48} + \frac{t^3}{6} + \frac{t^2}{4} + -C_3 t + -C_4 - \frac{\cosh(2t)}{16} - \frac{\sinh(2t)}{16}, y(t) = \frac{(4t - 4-C_2 - \dots}{8} \right. \right.$$

**2.1898 ODE No. 1898**

$$\{x''(t) - x'(t) + y'(t) = 0, x''(t) - x(t) + y''(t) = 0\}$$

✓ **Mathematica** : cpu = 0.0369176 (sec), leaf count = 263

$$\left\{ \left\{ x(t) \rightarrow -\frac{1}{10} e^{\frac{1}{2}(t-\sqrt{5}t)} \left( 2c_1 \left( \sqrt{5} e^{\sqrt{5}t} - 5e^{\frac{1}{2}(1+\sqrt{5})t} - \sqrt{5} \right) - 2\sqrt{5}c_2 \left( e^{\sqrt{5}t} - 1 \right) + c_4 \left( (5 + \sqrt{5}) e^{\sqrt{5}t} - 10e^{\frac{1}{2}(1+\sqrt{5})t} \right) \right. \right. \right.$$

✓ **Maple** : cpu = 0.074 (sec), leaf count = 71

$$\left\{ \left\{ x(t) = \frac{-C_4 (\sqrt{5} - 1)}{2} e^{-\frac{(\sqrt{5}-1)t}{2}} - \frac{C_3 (\sqrt{5} + 1)}{2} e^{\frac{(\sqrt{5}+1)t}{2}} + -C_1 e^t, y(t) = -C_2 + -C_3 e^{\frac{(\sqrt{5}+1)t}{2}} + -C_4 e^{-\dots} \right. \right.$$



**2.1899 ODE No. 1899**

$$\{x'(t) = 2x(t), y'(t) = 3x(t) - 2y(t), z'(t) = 2y(t) + 3z(t)\}$$

✓ **Mathematica** : cpu = 0.0131373 (sec), leaf count = 93

$$\left\{ \left\{ x(t) \rightarrow c_1 e^{2t}, y(t) \rightarrow \frac{1}{4} e^{-2t} (3c_1 (e^{4t} - 1) + 4c_2), z(t) \rightarrow \frac{1}{10} e^{-2t} (c_1 (-15e^{4t} + 12e^{5t} + 3) + 4c_2 (e^{5t} - 1) + 10c_3) \right\} \right\}$$

✓ **Maple** : cpu = 0.089 (sec), leaf count = 52

$$\left\{ \left\{ x(t) = \_C3 e^{2t}, y(t) = \frac{3\_C3 e^{2t}}{4} + e^{-2t} \_C2, z(t) = \_C1 e^{3t} - \frac{3\_C3 e^{2t}}{2} - \frac{2e^{-2t} \_C2}{5} \right\} \right\}$$

**2.1900 ODE No. 1900**

$$\{x'(t) = 4x(t), y'(t) = x(t) - 2y(t), z'(t) = x(t) - 4y(t) + z(t)\}$$

✓ **Mathematica** : cpu = 0.0102984 (sec), leaf count = 88

$$\left\{ \left\{ x(t) \rightarrow c_1 e^{4t}, y(t) \rightarrow \frac{1}{6} e^{-2t} (c_1 (e^{6t} - 1) + 6c_2), z(t) \rightarrow \frac{1}{9} e^{-2t} (c_1 (e^{3t} + e^{6t} - 2) - 12c_2 (e^{3t} - 1) + 9c_3 e^{3t}) \right\} \right\}$$

✓ **Maple** : cpu = 0.084 (sec), leaf count = 50

$$\left\{ \left\{ x(t) = \_C3 e^{4t}, y(t) = \frac{\_C3 e^{4t}}{6} + e^{-2t} \_C2, z(t) = \frac{\_C3 e^{4t}}{9} + \_C1 e^t + \frac{4e^{-2t} \_C2}{3} \right\} \right\}$$

**2.1901 ODE No. 1901**

$$\{x'(t) = y(t) - z(t), y'(t) = x(t) + y(t), z'(t) = x(t) + z(t)\}$$

✓ **Mathematica** : cpu = 0.0112027 (sec), leaf count = 93

$$\left\{ \left\{ x(t) \rightarrow (c_2 - c_3) (e^t - 1) + c_1, y(t) \rightarrow c_1 (e^t - 1) + c_2 (e^t t + 1) - c_3 (e^t (t - 1) + 1), z(t) \rightarrow c_1 (e^t - 1) + c_2 (e^t t) \right\} \right\}$$

✓ **Maple** : cpu = 0.072 (sec), leaf count = 43

$$\left\{ \left\{ x(t) = \_C2 + \_C3 e^t, y(t) = (\_C3 t + \_C1) e^t - \_C2, z(t) = ((t - 1) \_C3 + \_C1) e^t - \_C2 \right\} \right\}$$

**2.1902 ODE No. 1902**

$$\{x'(t) - y(t) + z(t) = 0, -x(t) + y'(t) - y(t) = t, -x(t) + z'(t) - z(t) = t\}$$

✓ **Mathematica** : cpu = 0.0154918 (sec), leaf count = 109

$$\{x(t) \rightarrow (c_2 - c_3)(e^t - 1) + c_1, y(t) \rightarrow c_1(e^t - 1) + t((c_2 - c_3)e^t - 1) + c_3e^t + c_2 - c_3 - 1, z(t) \rightarrow c_1(e^t - 1) - t\}$$

✓ **Maple** : cpu = 0.065 (sec), leaf count = 51

$$\{x(t) = \_C2 + \_C3 e^t, y(t) = (\_C3 t + \_C1) e^t - t - \_C2 - 1, z(t) = ((t - 1)\_C3 + \_C1) e^t - t - \_C2 - 1\}$$

**2.1903 ODE No. 1903**

$$\{ax'(t) = bc(y(t) - z(t)), by'(t) = ac(z(t) - x(t)), cz'(t) = ab(x(t) - y(t))\}$$

✓ **Mathematica** : cpu = 0.0898444 (sec), leaf count = 741

$$\left\{ \left\{ \begin{array}{l} x(t) \rightarrow \frac{e^{-it\sqrt{a^2+b^2+c^2}} \left( a \left( b^2 \left( c_1 \left( 1 + e^{2it\sqrt{a^2+b^2+c^2}} \right) - c_2 \left( -1 + e^{it\sqrt{a^2+b^2+c^2}} \right)^2 \right) + c^2 \left( c_1 \left( 1 + e^{2it\sqrt{a^2+b^2+c^2}} \right) \right) \right)}{2a(a^2 + b^2 + c^2)} \end{array} \right. \right.$$

✓ **Maple** : cpu = 0.155 (sec), leaf count = 299

$$\left\{ \left\{ \begin{array}{l} x(t) = \_C1 + \_C2 \sin(\sqrt{a^2 + b^2 + c^2}t) + \_C3 \cos(\sqrt{a^2 + b^2 + c^2}t), y(t) = \frac{1}{b(b^2 + c^2)} \left( -C1 b^3 + \left( (-a^2 b - c^2) \cos(\sqrt{a^2 + b^2 + c^2}t) - (-a^2 b - c^2) \sin(\sqrt{a^2 + b^2 + c^2}t) \right) \right) \end{array} \right. \right.$$

**2.1904 ODE No. 1904**

$$\{x'(t) = cy(t) - bz(t), y'(t) = az(t) - cx(t), z'(t) = bx(t) - ay(t)\}$$

✓ **Mathematica** : cpu = 0.0653528 (sec), leaf count = 1084

$$\left\{ \left\{ \begin{array}{l} x(t) \rightarrow \frac{e^{-\sqrt{-a^2-b^2-c^2}t} \left( 2e^{\sqrt{-a^2-b^2-c^2}t} c_1 a^2 + b^2 \left( 1 + e^{2\sqrt{-a^2-b^2-c^2}t} \right) c_1 + c^2 \left( 1 + e^{2\sqrt{-a^2-b^2-c^2}t} \right) c_1 - c \left( -1 + e^{\sqrt{-a^2-b^2-c^2}t} \right) \right)}{2a(a^2 + b^2 + c^2)} \end{array} \right. \right.$$

✓ **Maple** : cpu = 0.098 (sec), leaf count = 257

$$\left\{ \left\{ \begin{array}{l} x(t) = \_C1 + \_C2 \sin(\sqrt{a^2 + b^2 + c^2}t) + \_C3 \cos(\sqrt{a^2 + b^2 + c^2}t), y(t) = \frac{1}{a(b^2 + c^2)} \left( (-a^2 b - c^2) \cos(\sqrt{a^2 + b^2 + c^2}t) - (-a^2 b - c^2) \sin(\sqrt{a^2 + b^2 + c^2}t) \right) \end{array} \right. \right.$$

**2.1905 ODE No. 1905**

$$\{x'(t) = h(t)y(t) - g(t)z(t), y'(t) = f(t)z(t) - h(t)x(t), z'(t) = g(t)x(t) - f(t)y(t)\}$$

✗ **Mathematica** : cpu = 0.00753469 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] == h[t]\*y[t] - g[t]\*z[t], Derivative[1][y][t] == -(h[t]\*x[t]) + f[t]\*z[t], Derivative[1][z][t] == g[t]\*x[t] - f[t]\*y[t]}, {x[t], y[t], z[t]},

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , result contains DESol

$$\left\{ \left\{ x(t) = DESol \left( \left\{ \frac{d^3}{dt^3} Y(t) + \left( -2 \frac{h(t) \left( \frac{d}{dt} h(t) \right) f(t)}{(h(t))^2 f(t) + f(t) (g(t))^2 + \left( \frac{d}{dt} h(t) \right) g(t) - h(t) \frac{d}{dt} g(t)} - 2 \frac{f(t) (g(t))^2}{(h(t))^2 f(t)} \right. \right. \right. \right.$$

**2.1906 ODE No. 1906**

$$\{x'(t) = x(t) + y(t) - z(t), y'(t) = -x(t) + y(t) + z(t), z'(t) = x(t) - y(t) + z(t)\}$$

✓ **Mathematica** : cpu = 0.0525803 (sec), leaf count = 177

$$\left\{ \left\{ x(t) \rightarrow \frac{1}{3} e^t \left( \sqrt{3} (c_2 - c_3) \sin(\sqrt{3}t) + (2c_1 - c_2 - c_3) \cos(\sqrt{3}t) + c_1 + c_2 + c_3 \right), y(t) \rightarrow \frac{1}{3} e^t \left( -\sqrt{3} (c_1 - c_3) \sin(\sqrt{3}t) + (c_1 + c_2 - c_3) \cos(\sqrt{3}t) + c_1 - c_2 + c_3 \right), z(t) \rightarrow \frac{1}{3} e^t \left( \sqrt{3} (c_1 - c_2) \sin(\sqrt{3}t) + (c_1 - c_2 + c_3) \cos(\sqrt{3}t) + c_1 - c_2 + c_3 \right) \right. \right.$$

✓ **Maple** : cpu = 0.086 (sec), leaf count = 120

$$\left\{ \left\{ x(t) = e^t \left( \sin(\sqrt{3}t) \_C2 + \cos(\sqrt{3}t) \_C3 + \_C1 \right), y(t) = \frac{e^t (\_C2 \sqrt{3} - \_C3) \cos(\sqrt{3}t)}{2} + \frac{e^t (-\_C3 \sqrt{3})}{2} \right. \right.$$

**2.1907 ODE No. 1907**

$$\{x'(t) = -3x(t) + 48y(t) - 28z(t), y'(t) = -4x(t) + 40y(t) - 22z(t), z'(t) = -6x(t) + 57y(t) - 31z(t)\}$$

✓ **Mathematica** : cpu = 0.012729 (sec), leaf count = 157

$$\left\{ \left\{ x(t) \rightarrow e^t (c_1 (3 - 2e^{2t}) + 2(e^t - 1) (3c_2 (3e^t + 5) - c_3 (5e^t + 9))), y(t) \rightarrow e^t (-2c_1 (e^{2t} - 1) + c_2 (3e^t + 18e^{2t} - 12e^t - 9)) \right. \right.$$

✓ **Maple** : cpu = 0.069 (sec), leaf count = 66

$$\left\{ \left\{ x(t) = \_C1 e^t + \_C2 e^{2t} + \_C3 e^{3t}, y(t) = \frac{2 \_C1 e^t}{3} + \frac{\_C2 e^{2t}}{4} + \_C3 e^{3t}, z(t) = \_C1 e^t + \frac{\_C2 e^{2t}}{4} + \frac{3 \_C3 e^{3t}}{4} \right. \right.$$

**2.1908 ODE No. 1908**

$$\{x'(t) = 6x(t) - 72y(t) + 44z(t), y'(t) = 4x(t) - 4y(t) + 26z(t), z'(t) = 6x(t) - 63y(t) + 38z(t)\}$$

✓ **Mathematica** : cpu = 0.0215907 (sec), leaf count = 551

$$\left\{ \left\{ x(t) \rightarrow -36c_2 \text{RootSum} \left[ \#1^3 - 40\#1^2 + 1714\#1 + 1404\&, \frac{2\#1e^{\#1t} + e^{\#1t}}{3\#1^2 - 80\#1 + 1714} \& \right] + 4c_3 \text{RootSum} \left[ \#1^3 - 40\#1^2 + 1714\#1 + 1404\&, \frac{2\#1e^{\#1t} + e^{\#1t}}{3\#1^2 - 80\#1 + 1714} \& \right] \right\} \right\}$$

✓ **Maple** : cpu = 0.749 (sec), leaf count = 1213

$$\left\{ \left\{ x(t) = \_C2 e^{\frac{\left( (263474+18\sqrt{351406311})^{\frac{2}{3}} + 80\sqrt[3]{263474+18\sqrt{351406311}-3542} \right) t}{6\sqrt[3]{263474+18\sqrt{351406311}}}} \sin \left( \frac{\sqrt{3}t \left( \sqrt[3]{4} \sqrt[3]{(131737 + 9\sqrt{351406311})^2 + 131737} \right)}{6\sqrt[3]{263474 + 18\sqrt{351406311}}} \right) \right\} \right\}$$

**2.1909 ODE No. 1909**

$$\{x'(t) = ax(t) + \beta z(t) + gy(t), y'(t) = \alpha z(t) + by(t) + gx(t), z'(t) = \alpha y(t) + \beta x(t) + cz(t)\}$$

✓ **Mathematica** : cpu = 0.0590603 (sec), leaf count = 1630

$$\left\{ \left\{ x(t) \rightarrow -c_2 \text{RootSum} \left[ \#1^3 - a\#1^2 - b\#1^2 - c\#1^2 - \alpha^2\#1 - \beta^2\#1 - g^2\#1 + ab\#1 + ac\#1 + bc\#1 + a\alpha^2 + b\beta^2 + c^2\#1 \right] \right\} \right\}$$

✓ **Maple** : cpu = 27.579 (sec), leaf count = 33085

too large to display

**2.1910 ODE No. 1910**

$$\{tx'(t) = 2x(t) - t, t^3y'(t) = t^2y(t) - x(t) + t, t^4z'(t) = t^3z(t) - t^2y(t) - x(t) + t\}$$

✓ **Mathematica** : cpu = 0.0104514 (sec), leaf count = 39

$$\left\{ \left\{ x(t) \rightarrow t(c_3t + 1), y(t) \rightarrow c_2t + c_3, z(t) \rightarrow c_1t + \frac{c_3}{t} + c_2 \right\} \right\}$$

✓ **Maple** : cpu = 0.109 (sec), leaf count = 37

$$\left\{ \left\{ x(t) = \_C3 t^2 + t, y(t) = \_C2 t + \_C3, z(t) = \frac{\_C1 t^2 + \_C2 t + \_C3}{t} \right\} \right\}$$

**2.1911 ODE No. 1911**

$$\{atx'(t) = bc(y(t) - z(t)), bty'(t) = ac(z(t) - x(t)), ctz'(t) = ab(x(t) - y(t))\}$$

✗ **Mathematica** : cpu = 0.0317035 (sec), leaf count = 0 , could not solve

`DSolve[{a*t*Derivative[1][x][t] == b*c*(y[t] - z[t]), b*t*Derivative[1][y][t] == a*c*(-x[t] + z[t]), c*t*Derivative[1][z][t] == a*b*(x[t] - y[t])}, {x[t], y[t], z[t]}, t]`

✓ **Maple** : cpu = 0.159 (sec), leaf count = 308

$$\left\{ \left\{ x(t) = \_C1 + \_C2 \sin\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right) + \_C3 \cos\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right), y(t) = \frac{1}{b(b^2 + c^2)}\left(\sqrt{a^2 + b^2 + c^2} \ln(t)\right) \right\} \right\}$$

**2.1912 ODE No. 1912**

$$\{x1'(t) = ax2(t) + bx3(t) \cos(ct) + bx4(t) \sin(ct), x2'(t) = -ax1(t) + bx3(t) \sin(ct) - bx4(t) \cos(ct), x3'(t) = ax4(t) - bx3(t) \sin(ct) + bx4(t) \cos(ct)\}$$

✗ **Mathematica** : cpu = 0.00942996 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x1][t] == a*x2[t] + b*Cos[c*t]*x3[t] + b*Sin[c*t]*x4[t], Derivative[1][x2][t] == -a*x1[t] + b*Sin[c*t]*x3[t] - b*Cos[c*t]*x4[t], Derivative[1][x3][t] == -(b*Cos[c*t]*x1[t] + b*Sin[c*t]*x1[t]) + b*Cos[c*t]*x2[t] - a*x3[t]}, {x[t], y[t], z[t]}, t]`

✓ **Maple** : cpu = 2.798 (sec), leaf count = 2956

$$\left\{ \left\{ x1(t) = \_C2 + \_C3 \sin(ct) + \_C4 \cos(ct), x2(t) = -\cos(ct) \_C3 + \sin(ct) \_C4 + \_C1, x3(t) = \frac{b(\cos(ct) - \sin(ct))}{a^2 + b^2} \right\} \right\}$$

**2.1913 ODE No. 1913**

$$\{x'(t) = -x(t)(x(t) + y(t)), y'(t) = y(t)(x(t) + y(t))\}$$

✓ **Mathematica** : cpu = 0.0335962 (sec), leaf count = 52

$$\{ \{y(t) \rightarrow -\sqrt{c1} \cot(\sqrt{c1}(t - c2)), x(t) \rightarrow -\sqrt{c1} \tan(\sqrt{c1}(t - c2))\} \}$$

✓ **Maple** : cpu = 0.135 (sec), leaf count = 57

$$\left\{ \left[ \{x(t) = 0\}, \{y(t) = (\_C1 - t)^{-1}\} \right], \left[ \left\{ x(t) = \frac{1}{-\_C1} \tanh\left(\frac{-\_C2 + t}{-\_C1}\right) \right\}, \left\{ y(t) = \frac{-(x(t))^2 - \frac{d}{dt}x(t)}{x(t)} \right\} \right] \right\}$$





**2.1918 ODE No. 1918**

$$\{x'(t) = -x(t)y(t)^2 + x(t) + y(t), y'(t) = x(t)^2y(t) - x(t) - y(t)\}$$

✗ **Mathematica** : cpu = 0.0896432 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] == x[t] + y[t] - x[t]\*y[t]^2, Derivative[1][y][t] == -x[t] - y[t] + x[t]^2\*y[t]}, {x[t], y[t]}, t]

✓ **Maple** : cpu = 2.155 (sec), leaf count = 184

$$\left\{ \{x(t) = 0\}, \{y(t) = 0\}, \left[ \left\{ x(t) = ODESolStruc \left( -a, \left[ \frac{1}{2a^2} \left( \sqrt{(4a^2 - 4ab(a) + 1)(a^3 + a - b)} \right) \right] \right\} \right. \right.$$

**2.1919 ODE No. 1919**

$$\{x'(t) = x(t) (-(x(t)^2 + y(t)^2)) + x(t) + y(t), y'(t) = -y(t) (x(t)^2 + y(t)^2) - x(t) + y(t)\}$$

✗ **Mathematica** : cpu = 0.110994 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] == x[t] + y[t] - x[t]\*(x[t]^2 + y[t]^2), Derivative[1][y][t] == -x[t] + y[t] - y[t]\*(x[t]^2 + y[t]^2)}, {x[t], y[t]}, t]

✓ **Maple** : cpu = 3.442 (sec), leaf count = 203

$$\left\{ \{x(t) = 0\}, \{y(t) = 0\}, \left[ \left\{ x(t) = ODESolStruc \left( -a, \left[ \frac{1}{2a^3} \left( \sqrt{-(4a^4 + 4ab(a) - 4a^2 - 1)(2a^2 + y(t)^2)} \right) \right] \right\} \right. \right.$$

**2.1920 ODE No. 1920**

$$\{x'(t) = x(t) (x(t)^2 + y(t)^2 - 1) - y(t), y'(t) = y(t) (x(t)^2 + y(t)^2 - 1) + x(t)\}$$

✗ **Mathematica** : cpu = 0.0861429 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] == -y[t] + x[t]\*(-1 + x[t]^2 + y[t]^2), Derivative[1][y][t] == x[t] + y[t] + y[t]\*(x[t]^2 + y[t]^2)}, {x[t], y[t]}, t]

✓ **Maple** : cpu = 3.463 (sec), leaf count = 205

$$\left\{ \{x(t) = 0\}, \{y(t) = 0\}, \left[ \left\{ x(t) = ODESolStruc \left( -a, \left[ \frac{1}{2a^3} \left( \sqrt{-(4a^4 - 4a^2 - 4ab(a) - 1)(4a^2 + y(t)^2)} \right) \right] \right\} \right. \right.$$



**2.1921 ODE No. 1921**

$$\left\{ x'(t) = -y(t) (x(t)^2 + y(t)^2), y'(t) = \begin{pmatrix} x(t)^2 + y(t)^2 & x(t)^2 + y(t)^2 \geq 2x(t) \\ (x(t)^2 + y(t)^2) \left( \frac{x(t)}{2} - \frac{y(t)^2}{2x(t)} \right) & \text{True} \end{pmatrix} \right\}$$

✗ **Mathematica** : cpu = 2.45725 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x][t] == -(y[t]*(x[t]^2 + y[t]^2)), Derivative[1][y][t] == Piecewise[{`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve({diff(x(t),t) = -y(t)*(x(t)^2+y(t)^2), diff(y(t),t) = piecewise(2*x(t) <= x(t)^2+y(t)^2, 1/2*y(t)^2/x(t))*(x(t)^2+y(t)^2)})`

**2.1922 ODE No. 1922**

$$\left\{ x'(t) = \begin{pmatrix} \sin\left(\frac{1}{x(t)^2+y(t)^2}\right) x(t) (x(t)^2 + y(t)^2 - 1) & x(t)^2 + y(t)^2 \neq 1 \\ 0 & \text{True} \end{pmatrix} - y(t), y'(t) = \begin{pmatrix} \sin\left(\frac{1}{x(t)^2+y(t)^2}\right) y(t) & x(t)^2 + y(t)^2 \neq 1 \\ 0 & \text{True} \end{pmatrix} \right\}$$

✗ **Mathematica** : cpu = 11.5237 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x][t] == Piecewise[{{Sin[(x[t]^2 + y[t]^2)^(-1)]*x[t]*(-1 + x[t]^2 + y[t]^2), x[t]^2 + y[t]^2 != 1}}, 0] - y[t], Derivative[1][y][t] == Piecewise[{{Sin[(x[t]^2 + y[t]^2)^(-1)]*y[t]*(-1 + x[t]^2 + y[t]^2), x[t]^2 + y[t]^2 != 1}}, 0] + x[t]}, {x[t], y[t]}, t]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.1923 ODE No. 1923**

$$\{(t^2 + 1) x'(t) = y(t) - tx(t), (t^2 + 1) y'(t) = -x(t) - ty(t)\}$$

✓ **Mathematica** : cpu = 0.013537 (sec), leaf count = 39

$$\left\{ \left\{ x(t) \rightarrow \frac{c_2 t + c_1}{t^2 + 1}, y(t) \rightarrow \frac{c_2 - c_1 t}{t^2 + 1} \right\} \right\}$$

✓ **Maple** : cpu = 0.04 (sec), leaf count = 35

$$\left\{ \left\{ x(t) = \frac{-C1 t + -C2}{t^2 + 1}, y(t) = \frac{-C2 t + -C1}{t^2 + 1} \right\} \right\}$$

**2.1924 ODE No. 1924**

$$\{(-t^2 + x(t)^2 + y(t)^2) x'(t) = -2tx(t), (-t^2 + x(t)^2 + y(t)^2) y'(t) = -2ty(t)\}$$

✓ **Mathematica** : cpu = 0.0714162 (sec), leaf count = 179

$$\left\{ \left\{ y(t) \rightarrow -\frac{c_1 \left( \sqrt{e^{2c_2} - 4(c_1^2 + 1)t^2} - e^{c_2} \right)}{2(c_1^2 + 1)}, x(t) \rightarrow \frac{e^{c_2} - \sqrt{e^{2c_2} - 4(c_1^2 + 1)t^2}}{2(c_1^2 + 1)} \right\}, \left\{ y(t) \rightarrow \frac{c_1 \left( \sqrt{e^{2c_2} - 4(c_1^2 + 1)t^2}}{2(c_1^2 + 1)} \right) \right\} \right\}$$

✓ **Maple** : cpu = 0.331 (sec), leaf count = 180

$$\left\{ \left\{ x(t) = 0 \right\}, \left\{ y(t) = \frac{1}{2_{-}C1} \left( 1 + \sqrt{-4_{-}C1^2 t^2 + 1} \right), y(t) = \frac{1}{2_{-}C1} \left( 1 - \sqrt{-4_{-}C1^2 t^2 + 1} \right) \right\}, \left\{ x(t) = \frac{1}{2_{-}C1} \right\} \right\}$$

**2.1925 ODE No. 1925**

$$\{ay'(t) + tx'(t) - x(t) + y'(t)^2 = 0, x'(t)y'(t) + ty'(t) - y(t) = 0\}$$

✗ **Mathematica** : cpu = 8.41162 (sec), leaf count = 0 , could not solve

DSolve[{-x[t] + t\*Derivative[1][x][t] + a\*Derivative[1][y][t] + Derivative[1][y][t]^2 == 0, y[t] + t\*Derivative[1][y][t] + Derivative[1][x][t]\*Derivative[1][y][t] == 0}, {x[t], y[t]},

✓ **Maple** : cpu = 0.312 (sec), leaf count = 194

$$\left\{ \left\{ x(t) = -\frac{t^2}{3} \right\}, \left\{ y(t) = -\frac{t^3}{27a} \right\}, \left\{ x(t) = -C1 t + -C2 \right\}, \left\{ y(t) = -\frac{\left( \frac{d}{dt} x(t) + t \right) \left( \left( \frac{d}{dt} x(t) \right)^2 + t \frac{d}{dt} x(t) - x(t) \right)}{a} \right\} \right\}$$

**2.1926 ODE No. 1926**

$$\{x(t) = f(x'(t), y'(t)) + tx'(t), y(t) = g(x'(t), y'(t)) + ty'(t)\}$$

✗ **Mathematica** : cpu = 0.00689984 (sec), leaf count = 0 , could not solve

DSolve[{x[t] == f[Derivative[1][x][t], Derivative[1][y][t]] + t\*Derivative[1][x][t], y[t] ==

✓ **Maple** : cpu = 0.115 (sec), leaf count = 96

$$\left\{ \left\{ \int \text{RootOf} \left( t \frac{d}{dt} y(t) + g \left( -Z, \frac{d}{dt} y(t) \right) - y(t) \right) dt + -C1 = t \text{RootOf} \left( t \frac{d}{dt} y(t) + g \left( -Z, \frac{d}{dt} y(t) \right) - y(t) \right) + f \right\} \right\}$$

**2.1927 ODE No. 1927**

$$\left\{ x''(t) = ae^{2x(t)} + e^{-2x(t)} \cos^2(y(t)) - e^{-x(t)}, y''(t) = e^{-2x(t)} \sin(y(t)) \cos(y(t)) - \tan(y(t)) \sec^2(y(t)) \right\}$$

✗ **Mathematica** : cpu = 0.0111483 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[2][x][t] == -E^(-x[t]) + aE^(2*x[t]) + Cos[y[t]]^2/E^(2*x[t]), Derivative[2][y][t] == e^(-2*x[t]) Sin[y[t]] Cos[y[t]] - Tan[y[t]] Sec^2[y[t]]}, {x, y}, t]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , could not solve

`dsolve({diff(diff(x(t),t),t) = a*exp(2*x(t))-exp(-x(t))+exp(-2*x(t))*cos(y(t))^2, diff(diff(y(t),t),t) = exp(-2*x(t))*sin(y(t))*cos(y(t))-sin(y(t))/cos(y(t))^3})`

**2.1928 ODE No. 1928**

$$\left\{ x''(t) = \frac{kx(t)}{(x(t)^2 + y(t)^2)^{3/2}}, y''(t) = \frac{ky(t)}{(x(t)^2 + y(t)^2)^{3/2}} \right\}$$

✗ **Mathematica** : cpu = 0.00797243 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[2][x][t] == (k*x[t])/(x[t]^2 + y[t]^2)^(3/2), Derivative[2][y][t] == (k*y[t])/(x[t]^2 + y[t]^2)^(3/2)}, {x, y}, t]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

**2.1929 ODE No. 1929**

$$\left\{ x''(t) = -\frac{cy(t)x'(t)f\left(\sqrt{x'(t)^2 + y'(t)^2}\right)}{\sqrt{x'(t)^2 + y'(t)^2}}, y''(t) = -\frac{cy(t)y'(t)f\left(\sqrt{x'(t)^2 + y'(t)^2}\right)}{\sqrt{x'(t)^2 + y'(t)^2}} - g \right\}$$

✗ **Mathematica** : cpu = 0.0100008 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[2][x][t] == -((c*f[Sqrt[Derivative[1][x][t]^2 + Derivative[1][y][t]^2]]*y[t])/(x[t]^2 + y[t]^2)^(3/2)) - g, Derivative[2][y][t] == -((c*f[Sqrt[Derivative[1][x][t]^2 + Derivative[1][y][t]^2]]*y[t]*Derivative[1][y][t])/(x[t]^2 + y[t]^2)^(3/2)) - g}, {x, y}, t]`

✓ **Maple** : cpu = 4.038 (sec), leaf count = 116

$$\left\{ \left[ \left\{ y(t) = ODESolStruc \left( \_a, \left[ \left( \frac{d}{d\_a} \_b(\_a) \right) \_b(\_a) + 1 \left( C(\_a) f \left( \sqrt{(\_b(\_a))^2} \right) \_b(\_a) + g \sqrt{(\_b(\_a))^2} \right) \right] \right\} \right. \right.$$

**2.1930 ODE No. 1930**

$$\{x'(t) = y(t) - z(t), y'(t) = x(t)^2 + y(t), z'(t) = x(t)^2 + z(t)\}$$

✓ **Mathematica** : cpu = 0.0481523 (sec), leaf count = 135

$$\{ \{x(t) \rightarrow e^{t-c_3} + c_1, y(t) \rightarrow e^{-2c_3} ((c_1 + c_2) e^{c_3+t} + 2c_1 e^{c_3+t} \log(e^{t-c_3}) - e^{2c_3} c_1^2 + e^{2t}), z(t) \rightarrow e^{-2c_3} ((c_1 + c_2 - 1) e^{c_3+t} + 2c_1 e^{c_3+t} \log(e^{t-c_3}) - e^{2c_3} c_1^2 + e^{2t})\} \}$$

✓ **Maple** : cpu = 0.05 (sec), leaf count = 45

$$\left\{ \left[ \left\{ x(t) = -C_2 + -C_3 e^t \right\}, \left\{ y(t) = \left( \int (x(t))^2 e^{-t} dt + -C_1 \right) e^t \right\}, \left\{ z(t) = -\frac{d}{dt} x(t) + y(t) \right\} \right] \right\}$$

**2.1931 ODE No. 1931**

$$\{ax'(t) = (b - c)y(t)z(t), by'(t) = (c - a)x(t)z(t), cz'(t) = (a - b)x(t)y(t)\}$$

✓ **Mathematica** : cpu = 5.66106 (sec), leaf count = 1461

$$\left\{ \left\{ x(t) \rightarrow \frac{\sqrt{2}b\sqrt{a(a-c)}(c-b)c_1 \operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(c_3-t)}{\sqrt{a}\sqrt{b}} \mid -\frac{(a-b)bc_1}{(a-c)cc_2}\right)}{a(c-a)\sqrt{b(b-c)}c_1}, y(t) \rightarrow -\frac{\sqrt{2}\sqrt{-b(b-c)}c_1 \left(\operatorname{sn}\left(\frac{\sqrt{2}\sqrt{a-c}\sqrt{b-c}\sqrt{c_2}(c_3-t)}{\sqrt{a}\sqrt{b}} \mid -\frac{(a-b)bc_1}{(a-c)cc_2}\right)\right)}{\sqrt{b}} \right\} \right\}$$

✓ **Maple** : cpu = 0.828 (sec), leaf count = 1117

$$\left\{ \left[ \left\{ x(t) = 0 \right\}, \left\{ y(t) = 0 \right\}, \left\{ z(t) = -C_1 \right\} \right], \left[ \left\{ x(t) = 0 \right\}, \left\{ y(t) = -C_1 \right\}, \left\{ z(t) = 0 \right\} \right], \left[ \left\{ x(t) = -C_1 \right\}, \left\{ y(t) = 0 \right\}, \left\{ z(t) = 0 \right\} \right] \right\}$$

**2.1932 ODE No. 1932**

$$\{x'(t) = x(t)(y(t) - z(t)), y'(t) = y(t)(z(t) - x(t)), z'(t) = z(t)(x(t) - y(t))\}$$

✗ **Mathematica** : cpu = 2.32531 (sec), leaf count = 0 , could not solve

`DSolve[{Derivative[1][x][t] == x[t]*(y[t] - z[t]), Derivative[1][y][t] == y[t]*(-x[t] + z[t]), Derivative[1][z][t] == (x[t] - y[t])*z[t]}, {x[t], y[t], z[t]}, t]`

✓ **Maple** : cpu = 1.132 (sec), leaf count = 383

$$\left\{ \left[ \left\{ x(t) = 0 \right\}, \left\{ y(t) = 0 \right\}, \left\{ z(t) = -C_1 \right\} \right], \left[ \left\{ x(t) = 0 \right\}, \left\{ y(t) = \frac{-C_1 e^{-C_2} - C_1 e^{-C_1 t}}{-1 + e^{-C_2} - C_1 e^{-C_1 t}} \right\}, \left\{ z(t) = \frac{\frac{d}{dt} y(t)}{y(t)} \right\} \right], \left[ \left\{ x(t) = -C_1 \right\}, \left\{ y(t) = 0 \right\}, \left\{ z(t) = 0 \right\} \right] \right\}$$

**2.1933 ODE No. 1933**

$$\{x'(t) + y'(t) = x(t)y(t), y'(t) + z'(t) = y(t)z(t), x'(t) + z'(t) = x(t)z(t)\}$$

✗ **Mathematica** : cpu = 127.19 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] + Derivative[1][y][t] == x[t]\*y[t], Derivative[1][y][t] + Derivative[1][z][t] == y[t]\*z[t], Derivative[1][x][t] + Derivative[1][z][t] == x[t]\*z[t]}, {x[t], y[t], z[t]}, t]

✓ **Maple** : cpu = 2.514 (sec), leaf count = 17738

too large to display

**2.1934 ODE No. 1934**

$$\left\{x'(t) = \frac{x(t)^2}{2} - \frac{y(t)}{24}, y'(t) = 2x(t)y(t) - 3z(t), z'(t) = 3x(t)z(t) - \frac{y(t)^2}{6}\right\}$$

✗ **Mathematica** : cpu = 73.3333 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] == x[t]^2/2 - y[t]/24, Derivative[1][y][t] == 2\*x[t]\*y[t] - 3\*z[t], Derivative[1][z][t] == 3\*x[t]\*z[t] - y[t]^2/6}, {x[t], y[t], z[t]}, t]

✓ **Maple** : cpu = 1.358 (sec), leaf count = 377

$$\left\{\{y(t) = 0\}, \{x(t) = -2(-2\_C1 + t)^{-1}\}, \{z(t) = 0\}\right\}, \left\{y(t) = 256(-\_C1 t + \_C2)^{-4}\right\}, \left\{x(t) = \frac{1}{6y(t)}\left(-v\right)\right\}$$

**2.1935 ODE No. 1935**

$$\{x'(t) = x(t)(y(t)^2 - z(t)^2), y'(t) = y(t)(z(t)^2 - x(t)^2), z'(t) = z(t)(x(t)^2 - y(t)^2)\}$$

✗ **Mathematica** : cpu = 0.0536612 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] == x[t]\*(y[t]^2 - z[t]^2), Derivative[1][y][t] == y[t]\*(-x[t]^2 + z[t]^2), Derivative[1][z][t] == (x[t]^2 - y[t]^2)\*z[t]}, {x[t], y[t], z[t]}, t]

✓ **Maple** : cpu = 2.187 (sec), leaf count = 741

$$\left\{\{x(t) = 0\}, \{y(t) = 0\}, \{z(t) = \_C1\}\right\}, \left\{x(t) = 0\right\}, \left\{y(t) = \frac{1}{(e^{-C2\_C1})^2 (e^{-C1 t})^2 - 1} \sqrt{\left((e^{-C2\_C1})^2 (e^{-C1 t})^2 - 1\right)}\right\}$$

**2.1936 ODE No. 1936**

$$\{x'(t) = x(t)(y(t)^2 - z(t)^2), y'(t) = -y(t)(x(t)^2 + z(t)^2), z'(t) = z(t)(x(t)^2 + y(t)^2)\}$$

✗ **Mathematica** : cpu = 0.0511442 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] == x[t]\*(y[t]^2 - z[t]^2), Derivative[1][y][t] == -(y[t]\*(x[t]^2 + z[t]^2)), Derivative[1][z][t] == (x[t]^2 + y[t]^2)\*z[t]}, {x[t], y[t], z[t]}, t]

✓ **Maple** : cpu = 0.778 (sec), leaf count = 704

$$\left\{ \{x(t) = 0\}, \{y(t) = 0\}, \{z(t) = -C1\}, \{x(t) = 0\}, \left\{ y(t) = \frac{1}{(e^{-C2-C1})^2 (e^{-C1 t})^2 - 1} \sqrt{-(e^{-C1 t})^4 - C1} \right\} \right.$$

**2.1937 ODE No. 1937**

$$\{x'(t) = -x(t)y(t)^2 + x(t) + y(t), y'(t) = x(t)^2y(t) - x(t) - y(t), z'(t) = y(t)^2 - x(t)^2\}$$

✗ **Mathematica** : cpu = 0.270166 (sec), leaf count = 0 , could not solve

DSolve[{Derivative[1][x][t] == x[t] + y[t] - x[t]\*y[t]^2, Derivative[1][y][t] == -x[t] - y[t] + x[t]^2\*y[t], Derivative[1][z][t] == -x[t]^2 + y[t]^2}, {x[t], y[t], z[t]}, t]

✓ **Maple** : cpu = 0.93 (sec), leaf count = 242

$$\left\{ \{x(t) = 0\}, \{y(t) = 0\}, \{z(t) = -C1\}, \left\{ x(t) = ODESolStruc\left(-a, \left[\frac{1}{2-a^2} \left(\sqrt{(4-a^2 - 4-ab(-a) + 1)}\right)\right]\right) \right\} \right.$$

**2.1938 ODE No. 1938**

$$\left\{ x''(t) = \frac{x(t)f'(r)}{r}, y''(t) = \frac{y(t)f'(r)}{r}, z''(t) = \frac{z(t)f'(r)}{r} \right\}$$

✓ **Mathematica** : cpu = 0.0100824 (sec), leaf count = 137

$$\left\{ \left\{ x(t) \rightarrow c_1 e^{-\frac{t\sqrt{f'(r)}}{\sqrt{r}}} + c_2 e^{\frac{t\sqrt{f'(r)}}{\sqrt{r}}}, y(t) \rightarrow c_3 e^{-\frac{t\sqrt{f'(r)}}{\sqrt{r}}} + c_4 e^{\frac{t\sqrt{f'(r)}}{\sqrt{r}}}, z(t) \rightarrow c_5 e^{-\frac{t\sqrt{f'(r)}}{\sqrt{r}}} + c_6 e^{\frac{t\sqrt{f'(r)}}{\sqrt{r}}} \right\} \right\}$$

✓ **Maple** : cpu = 0.127 (sec), leaf count = 101

$$\left\{ \left\{ x(t) = -C5 e^{t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}} + -C6 e^{-t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}}, y(t) = -C3 e^{t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}} + -C4 e^{-t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}}, z(t) = -C1 e^{t\sqrt{\frac{d}{dr}F(r)}\frac{1}{\sqrt{r}}} \right\} \right\}$$

**2.1939 ODE No. 1939**

$$\{(x(t) - y(t))(x(t) - z(t))x'(t) = f(t), (y(t) - x(t))(y(t) - z(t))y'(t) = f(t), (z(t) - x(t))(z(t) - y(t))z'(t) = f(t)\}$$

✗ **Mathematica** : cpu = 0.0310926 (sec), leaf count = 0 , could not solve

`DSolve[{(x[t] - y[t])*(x[t] - z[t])*Derivative[1][x][t] == f[t], (-x[t] + y[t])*(y[t] - z[t])*Derivative[1][y][t] == f[t], (z[t] - x[t])*(z[t] - y[t])*Derivative[1][z][t] == f[t]}, {x[t], y[t], z[t]}, t]`

✓ **Maple** : cpu = 1.591 (sec), leaf count = 899

$$\left\{ \left\{ \begin{aligned} x(t) = \int 3 \frac{f(t)}{-C1^3 + 11664_C2^2 - 23328_C2 \int f(t) dt + 11664 (\int f(t) dt)^2} \left( (-i\sqrt{3} - 1) \left( \left( 1 + 108 \sqrt{\dots} \right) \right) \right) \right. \end{aligned} \right.$$

**2.1940 ODE No. 1940**

$$\{x1'(t) \sin(x2(t)) = x4(t) \sin(x3(t)) + x5(t) \cos(x3(t)), x2'(t) = x4(t) \cos(x3(t)) - x5(t) \sin(x3(t)), x1'(t) \cos(x2(t)) = x4(t) \sin(x3(t)) - x5(t) \cos(x3(t))\}$$

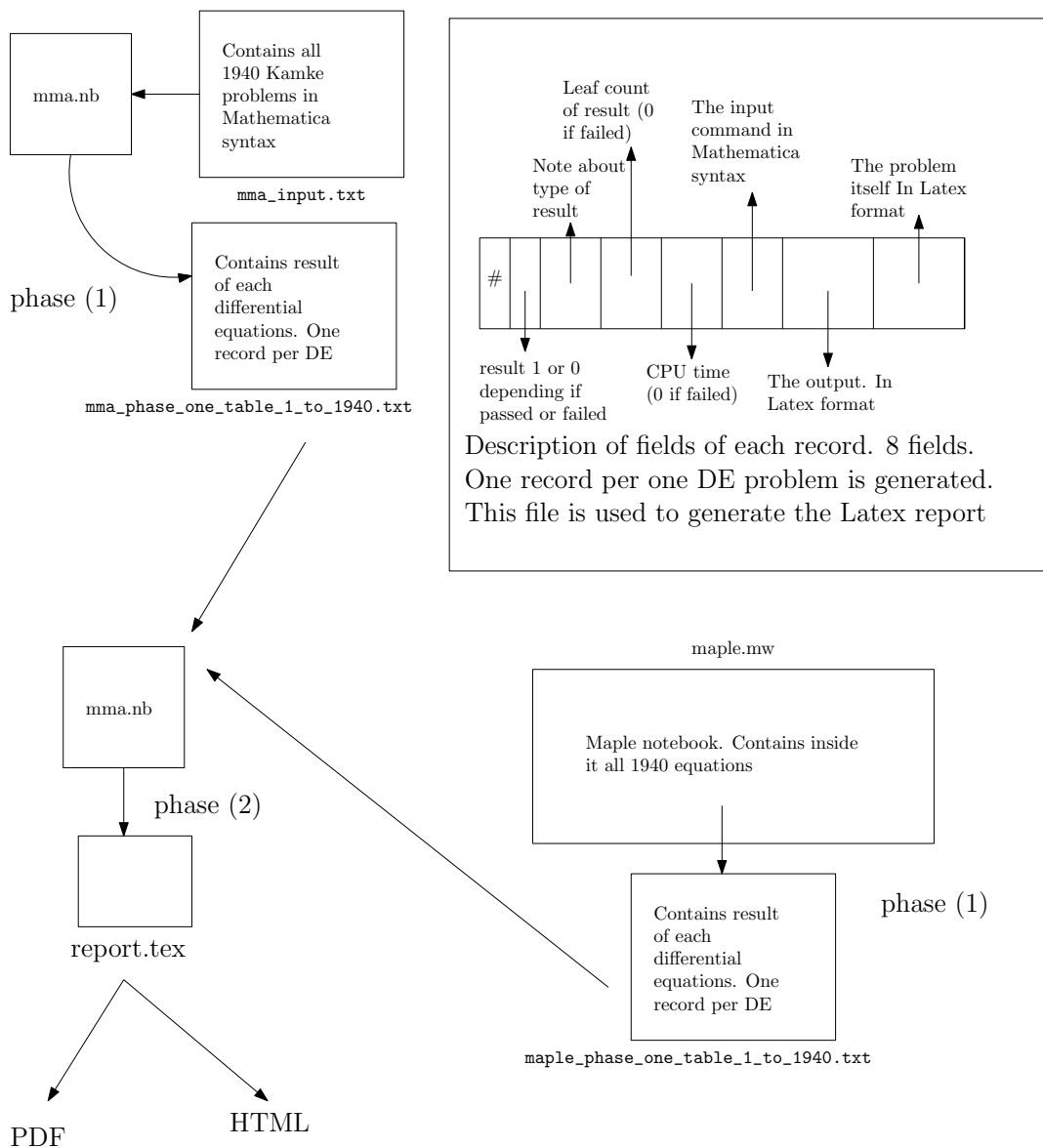
✗ **Mathematica** : cpu = 0.00923925 (sec), leaf count = 0 , could not solve

`DSolve[{Sin[x2[t]]*Derivative[1][x1][t] == Sin[x3[t]]*x4[t] + Cos[x3[t]]*x5[t], Derivative[1][x2][t] == x4[t]*Cos[x3[t]] - x5[t]*Sin[x3[t]], a*(1 - lambda)*x5[t] + Derivative[1][x4][t] == -(m*Cos[x3[t]]*Sin[x2[t]]), a*(1 - lambda)*x4[t] == m*Sin[x2[t]]}, {x1[t], x2[t], x3[t], x4[t], x5[t]}, t]`

✗ **Maple** : cpu = 0. (sec), leaf count = 0 , exception

time expired

### 3 Appendix



Kamke Differential equations build process

Nasser M. Abbasi (design.ipe)