Mathematical formulation of the HYPR algorithms

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1 Introduction

This report is a summary of the HIghly constrained Back PRojection (HYPR) team work performed so far relating to the HYPR research project. We will describe the work done and results found.

The goals set for the HYPR project included formulating the HYPR algorithm and some of its variations (such as Wright-Huang HYPR (WH-HYPR), I-HYPR and HYPR-LR) in a mathematical framework which would allow the study and analyze of these algorithms in relation to other well known non-linear methods such as maximum a-posteriori (MAP) estimation and Maximum Likelihood Expectation Maximization (MLEM). These algorithms, like HYPR, use prior information on the object being reconstructed and they are extensively used in nuclear medicine where the data is intrinsically under sampled.

The initial period of this project, which this report reflects on, was spent becoming familiar with the HYPR algorithm and its connection to MLEM. Towards this goal, the HYPR algorithm was formulated mathematically and schematic diagrams created which helped in its implementation. MATLAB simulation software was developed to enable more understanding of the algorithm and its behavior by running it on a number of test cases. Initial comparison between the original HYPR and the WH-HYPR made on a number of different test configuration which are described in detail in the simulation section below. The MLEM algorithm was implemented and compared the HYPR algorithm.

In addition, A mathematical connection between HYPR and Expectation Maximization (EM) is described and formulated.

2 Mathematical formulation of the HYPR algorithms

2.1 Original HYPR

2.1.1 Mathematical formulation

Please see the appendix for a complete description of the notation used in this section and throught the rest of the report.

The mathematics of this algorithm will be presented by using the radon transform R notation and not the matrix projection matrix H notation.

The projection s_t is obtained by applying radon transform R on the image I_t at some angle ϕ_t

$$s_t = R_{\phi_t}[I_t]$$

When the original object image does not change with time one can drop the subscript t from I_t and just write $s_t = R_{\phi_t}[I]$

Next, the composite image C is found from the filtered back projection applied to all the s_t as follows

$$C = \sum_{i=1}^{N} R_{\phi_{t_i}}^f[s_{t_i}]$$

Notice that the sum above is taken over N and not over N_p . Next, a projection s_c is taken from C at angle ϕ as follows

$$s_{c_t} = R_{\phi_t}[C]$$

Then the unfiltered back projection 2-D image P_t is generated

$$P_t = R_{\phi_t}^u[s_t]$$

And the unfiltered back projection 2-D image P_{c_t} is generated

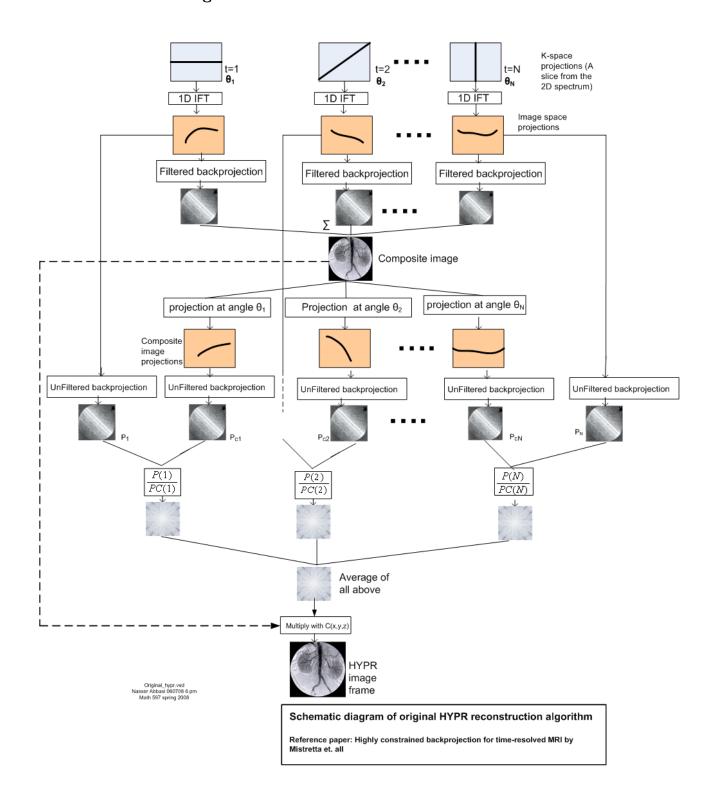
$$P_{c_t} = R_{\phi_t}^u[s_{c_t}]$$

Then the ratio of $\frac{P_t}{P_{c_t}}$ is summed and averaged over the time frame and the result multiplied by C to generate a HYPR frame J for the time frame. Hence for the k^{th} time

frame we obtain

$$\begin{split} J_k &= C \left(\frac{1}{N_p} \sum_{i=1}^{N_p} \frac{P_{t_i}}{P_{c_{t_i}}} \right) \\ &= \frac{1}{N_p} C \sum_{i=1}^{N_p} \frac{R_{\phi_{t_i}}^u \left[s_{t_i} \right]}{R_{\phi_{t_i}}^u \left[s_{c_{t_i}} \right]} \\ &= \frac{1}{N_p} \left(\sum_{i=1}^{N} R_{\phi_{t_i}}^f \left[s_{t_i} \right] \right) \sum_{j=1}^{N_p} \frac{R_{\phi_{t_j}}^u \left[s_{t_j} \right]}{R_{\phi_{t_j}}^u \left[s_{c_{t_j}} \right]} \end{split}$$

2.1.2 Schematic diagram



2.2 Wright-Huang variation of HYPR

2.2.1 Mathematical formulation

This mathematics of this algorithm will be presented by using the radon transform R notation and not the matrix projection matrix H notation.

The projection s_t is obtained by applying radon transform R on the image I_t at some angle ϕ_t

$$s_t = R_{\phi_t}[I_t]$$

The composite image C is found from the filtered back projection applied to all the s_t

$$C = \sum_{i=1}^{N} R_{\phi_{t_i}}^f[s_{t_i}]$$

Notice that the sum above is taken over N and not over N_p . Next a projection s_c is taken from C at angle ϕ as follows

$$s_{c_t} = R_{\phi_t}[C]$$

Then the unfiltered back projection 2-D image P_t is generated

$$P_t = R_{\phi_t}^u[s_t]$$

And the unfiltered back projection 2-D image P_{c_t} is generated

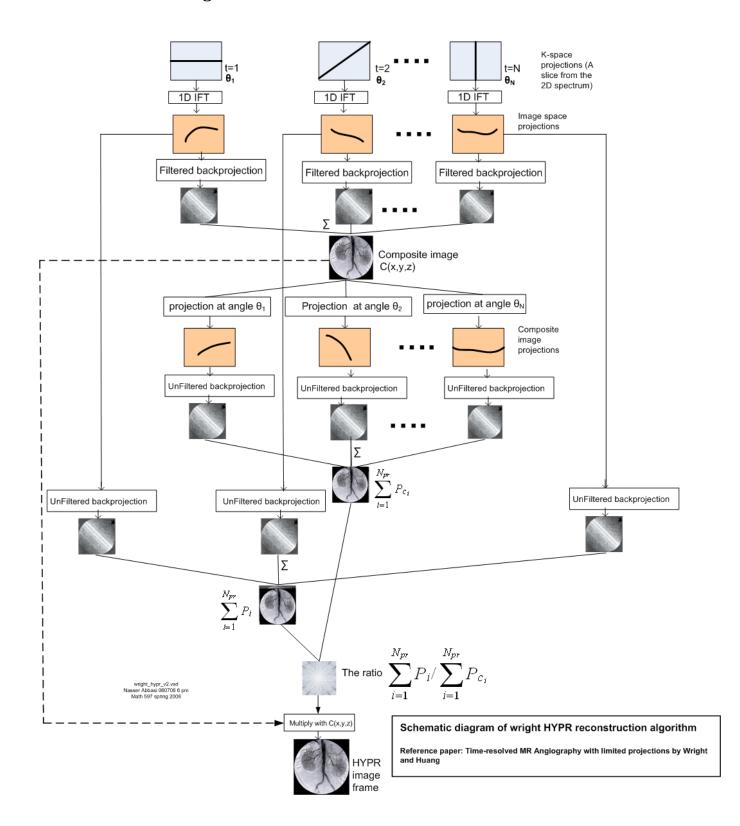
$$P_{c_t} = R_{\phi_t}^u[s_{c_t}]$$

Now the set of P_t and P_{c_t} over one time frame are summed the their ratio multiplied by C to obtain the k^{th} HYPR frame

$$J_{k} = C \frac{\sum_{i=1}^{N_{p}} P_{t_{i}}}{\sum_{i=1}^{N_{p}} P_{c_{t_{i}}}}$$

$$= C \frac{\sum_{i=1}^{N_{pr}} R_{\phi_{t}}^{u}[s_{t}]}{\sum_{i=1}^{N_{pr}} R_{\phi_{t}}^{u}[s_{c_{t}}]}$$

2.2.2 Schematic diagram



3 HYPR connection to Expectation Maximization

The following is a discussion of the Mathematics that connects the MLEM algorithm to HYPR.

According to O'Halloran's paper[1] the for Maximum-Likelihood Expectation-Maximization (MLEM) algorithm is mathematically equivalent to HYPR. The MLEM algorithm can be used in image reconstruction for medical purposes. Positron Emission Tomography (PET) and Single-Photon Emission Computed Tomography (SPECT) are two types of image reconstruction processes where the MLEM algorithm is used. The purpose here is to show that the MLEM algorithm will work for HYPR reconstructions.

The MLEM algorithm is a process that approximates the solution to

$$g = H\theta \tag{1}$$

In connection to HYPR, we can view H as a forward projection matrix, θ as the original image being projected and g the projection produced. The goal is to relate the above matrix based formulation to the radon transform based formulation seen above in the HYPR mathematical section, which is

$$s_t = R_{\phi_t}[I_t] \tag{2}$$

We formulate the first iteration of the MLEM algorithm based on equation (1) and see how it can be translated into the HYPR process of image reconstruction. The first step of MLEM is

$$\hat{\theta}_n^{(1)} = \hat{\theta}_n^{(0)} * \frac{1}{z_n} \sum_{m=0}^M H_{mn} \frac{g_m}{\left(H\hat{\theta}^{(0)}\right)_m}$$
(3)

This can be rewritten in matrix form

$$\hat{ heta}_n^{(1)} = \hat{ heta}_n^{(0)} rac{1}{z_n} \left[H^T \left[rac{g}{\left(H \hat{ heta}^{(0)}
ight)}
ight]
ight]_n$$

If we replace $\frac{1}{z_n}$ by $\frac{1}{[H^T[1]]_n}$ in (4), we obtain

$$\hat{\theta}_{n}^{(1)} = \hat{\theta}_{n}^{(0)} \frac{1}{[H^{T}[1]]_{n}} \left[\underbrace{H^{T} \left[\frac{g}{\left(H \hat{\theta}^{(0)} \right)} \right]}_{n} \right]$$

$$(4)$$

The marked portion of the above equation can be viewed as the vector that is produced from unfiltered back projection on the image produced by the ratio

$$rac{g}{H\hat{ heta}^{(0)}}$$

Here the division is done element by element to produce the vector whose elements are the ratios of the respective elements of g and $H\hat{\theta}^{(0)}$.

In HYPR the equation we want to tie to equation (3) above is as follows

$$J_t = \frac{1}{N_p} C \cdot R_{\phi_t}^u \left[\frac{s_t}{R_{\phi_t}(C)} \right]$$
 (5)

Where the \cdot represents an element by element multiplication, and the terms in (5) as defined in the section of the HYPR mathematical derivation shown earlier. Hence for (5) and (4) to be equivalent, We must have

$$\hat{\theta}_n^{(0)} = \frac{\left[H^T[1]\right]_n}{N_n} C$$

Which represents the initial guess for the user image. Therefore, by using for $\hat{\theta}^{(0)}$ as an initial guess for the MLEM algorithm the above weighted term of the composite image C, the MLEM algorithm will produce $\hat{\theta}^{(1)}$ which is a better approximation to the original image from that of the composite C. And this is what the HYPR algorithm does. It uses the composite image C to produce the HYPR image J to approximate the original user image I. Hence a one step of MLEM is equivalent to running HYPR for one time. Therefore, iterative HYPR algorithms can be seen as a multi step application of MLEM.

4 Software simulation and results

4.1 HYPR simulation

A software simulation written in MATLAB was designed and developed to enable more extensive HYPR testing of different test configurations. The software is GUI based and all test results are saved in a tab-delimited plain text file to allow one further statistical analysis of the data generated by other software. The appendix contains a screen shot of the current version of the simulator (version 1.0).

4.1.1 Description of HYPR simulation and test results

This is a description of the different tests performed. Both the original HYPR and the Wright-Huang HYPR (WH-HYPR) were run and results compared. In the following discussion, we use N_p to mean number of projections in one time frame, and N_w to mean the number of time frames. Hence the total number of projections is N_pN_w

The table below describes each test. In this table, a test with the letter 'a' represents the test being run using the original HYPR and a test with the letter 'b' represents the test being run using WH-HYPR. Each test was run under both the original HYPR and WH-HYPR.

The first set of tests are designed to detect the effect of Poisson noise on the accuracy of the HYPR algorithm as compared to the WH-HYPR. This was done for different geometry of objects while keeping the number of projections per time frame and the Poisson noise parameter λ fixed.

The second set of tests are designed to detect the effect of Gaussian noise on the accuracy of the HYPR algorithm as compared to the WH-HYPR. This was done for different geometry of objects while keeping the number of projections per time frame and the Gaussian distribution parameters (mean and variance) fixed. The set of tests used here is smaller than the first set due time limitation.

The third set of tests are designed to detect the effect of increasing the number of projections on the accuracy of the HYPR and WH-HYPR. This was done under one fixed configuration and with the absence of noise.

The main measure of accuracy used was relative RMSE. This was calculated as follows: For each HYPR frame image generated, the set of user images which make up the time frame from which the HYPR frame was generated are averaged to obtain an average time frame image. Then the RMSE was obtained between these 2 images as follows: Assuming these are N total pixels in each image, the error between each corresponding pixels is found as $H_i - I_i$ where H_i is a pixel in the HYPR frame image and I_i is the corresponding pixel in the averaged time frame image. This error is then squared. This was done for each pixel. The average of these square values is found, and the square root of the result is found. Hence

$$RMSE = \sqrt{rac{1}{N} {\sum_{i}^{N}} \sqrt{\left(H_{i} - I_{i}
ight)^{2}}}$$

This quantity is normalized by dividing it by the mean intensity of the averaged time frame image found earlier. This gives a normalized RMSE value for each time frame

generated. When there are more than one time frame generated then the average all these RMSE values are used to obtain one representative value of the RMSE for the test, and that is the value showed in the tables below for the purpose of comparing different tests.

Other statistics are calculated to determine the algorithms accuracy. The relative error between the HYPR image and the averaged time frame is found using the standard formula for relative error. This measure however did not appear to be a good indicator for determining the accuracy of the HYPR image. Another statistical measure used is the histogram difference, which is found as follows. The histogram for each HYPR image frame and the corresponding histogram for the averaged time frame image are calculated and the difference between these histograms found. This measure appear to give a good indication of the performance of each test and correlated well with the RMSE measure used. These results are all written to the log file for further analysis, but are not currently taken into account in the following tests due to time limitation. Only the RMSE measure is currently used to determine the accuracy of the algorithm. The following sections describe each set of tests in more details.

4.1.2 The first set of tests

Test	Test description
1a	HYPR[4] algorithm validation. Using the same parameters as the Wright-Huang paper[2] and validate output as it was described and shown in the paper. This is a fixed disk in the center of the image whose density changes linearly with time. $N_p=8, N_w=16$
1b	As above, but use the WH-HYPR algorithm.
2(a,b)	Repeat test 1 but with the addition of Poisson noise with $\lambda=500$ to the projection s generated from the user images
3(a,b)	A non-time varying two small white disks close to each others in black background. $N_p=8, N_w=16$
4(a,b)	As above, but with Poisson noise with $\lambda = 500$ added to projection s.
5(a,b)	Small disk that moves in vertical motion off the center of image. $N_p = 8, N_w = 16$
6(a,b)	As above, but with Poisson noise with $\lambda = 500$ added to projection s.
7(a,b)	2 small disks close to each others that move in vertical motion. $N_p = 8, N_w = 16$
8(a,b)	As above, but with Poisson noise with $\lambda = 500$ added to projection s.
9(a,b)	2 small disks further apart from each others that move in vertical motion.
10(a,b)	As above, but with Poisson noise with $\lambda = 500$ added to projection s.
11(a,b)	one small disk that move across the image in the diagonal direction. $N_p = 8, N_w = 16$
12(a,b)	As above, but with Poisson noise with $\lambda = 500$ added to projection s.

The appendix shows the output obtained from the above set of tests. We now present a summary of the results

Test	(a) Original HYPR RMSE	(b) Wright HYPR RMSE	Selected algorithm
1	0.639	0.636	WH-HYPR
2 (noise)	1.7298	1.2079	WH-HYPR
3	1.0329	1.0411	Original HYPR
4 (noise)	1.9879	1.4917	WH-HYPR
5	2.6349	3.095	Original HYPR
6 (noise)	4.9216	4.3288	WH-HYPR
7	2.1157	2.3496	Original HYPR
8 (noise)	2.99	2.7793	WH-HYPR
9	2.151	2.3524	Original HYPR
10 (noise)	2.9983	2.818	WH-HYPR
11	2.558	3.083	Original HYPR
12 (noise)	4.881	4.3884	WH-HYPR

Observation from running the first set of tests The original HYPR algorithm performed better in each test when noise is absent from projection. This occurs in either time varying or non-time varying configuration. On the other hand, WH-HYPR performed better in each case when noise was present. This occurs in either time varying or non-time varying configuration.

4.1.3 The second set of tests

These tests are a repeat of the first set of tests, but with noise generated from normal distribution. Due to time limitation only test 2,6 and 10 described above are repeated since these 3 tests are good representative of the overall tests. The letter N is added to the test name to indicate the use of Normal distribution.

Test	Test description
2N (a,b)	Repeat test 1 but with the addition of Normal noise with $\mu=0$ and $\sigma^2=500$ to the projection s generated from the user images
6N(a,b)	Repeat test 5, but with the addition of Normal noise with $\mu = 0$ and $\sigma^2 = 500$
10N(a,b)	Repeat test 9, but with the addition of Normal noise with $\mu = 0$ and $\sigma^2 = 500$

The appendix shows the output obtained from the above set of tests. Summary of the results is shown below. To clarify the nature of tests below a short description is given again below.

- 1. Test 2N is a small fixed disk, changes intensity linearly with time. $N_p = 8, N_w = 16$
- 2. Test 6N is one disk which moves vertically, off center. $N_p = 8, N_w = 16$
- 3. Test 10N is two disks separated from each others that move vertically across the image. $N_p = 8, N_w = 16$

Test	(a) Original HYPR RMSE	(b) Wright HYPR RMSE	Abs difference	Selected algorithm
2N	1.7583	1.7179	0.0404	WH-HYPR
6N	4.0069	3.9797	0.0272	WH-HYPR
10N	2.7754	2.7737	0.0017	WH-HYPR

Observations from running the second set of tests WH-HYPR performed better in all 3 cases. This correlated well with results found from the first set of tests where it was observed that WH-HYPR performed better each time Poisson noise was added. In the above 3 tests, normal noise was added and it is observed that WH-HYPR performed better.

4.1.4 Third set of tests

As was mentioned earlier, the goal of these tests is to measure the relative accuracy of the algorithms on the same configuration but with increasing number of projections per time frame. It is expected that the accuracy of each algorithm will improve, and we wish to obtain the measure of this improvement as a function of the number of projections per time frame.

For this purpose, the following test configuration was used: small white disk moving vertically and off center, no noise added. One time frame was used and the following number of projections $\{8, 16, 32, 64, 128, 256, 512, 1024\}$. These tests as named 8r, 16r, 32r, 128r, 256r, 512r and 1024r respectively. The table below show the result of the tests.

Test	(a) Original HYPR RMSE	(b) WH-HYPR RMSE	Abs difference	Selected algorithm
8r	1.6879	2.0836	0.3957	Original HYPR
16r	1.3772	1.59	0.2128	Original HYPR
32r	1.0994	1.18845	0.0891	Original HYPR
64r	0.774	0.8315	0.0575	Original HYPR
128r	0.5095	0.5355	0.0260	Original HYPR
256r	0.3722	0.3765	0.0043	Original HYPR
512r	0.2847	0.2825	0.0022	WH HYPR
1024r	0.2469	0.2459	0.0010	WH HYPR

Observations from running the third set of tests As the number of projections per time frame increased, the accuracy of WH-HYPR improved. At high number of projections (over 512 per time frame) WH-HYPR bypassed original HYPR and became more accurate. It is not clear at this time if such high number of projections per time frame will conflict with other MRI requirements (sampling rate limitation or other issues), but the above shows that, even with the absence of noise, the WH-HYPR can become more accurate than the Original HYPR but at a cost of having large number of projections per time frame.

4.1.5 Conclusions drawn from HYPR test results

Original HYPR performed better than WH-HYPR when the number of projections is relatively low (below 256 per time frame) and when there was no noise present (noise added to projections taken from the original images). This occurred in all configurations (both objects moving in time or fixed).

WH-HYPR performed better when noise is present (both Poisson and Normal noise) and for all number of projections and for all configurations.

In addition, WH-HYPR performed better when there was no noise added, but when the number of projections per time frame was increased.

These results seem to be a direct consequences of the fact that WH-HYPR sums the backprojection images over a time frame period before taking the ratio of these sums in order to obtain the mask image, while in the original HYPR the ratio for each backprojection images is first found and the ratios added and averaged. More analysis will be needed to better understand this difference and to explain mathematically this observed difference between HYPR and WH-HYPR.

Since real MRI data is characterized by low SNR, this leads one to conclude that

WH-HYPR should be the preferred choice between these 2 algorithms.

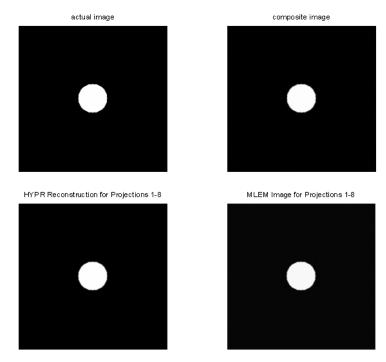
4.2 Expectation Maximization simulation

4.2.1 Description of simulation

The original HYPR algorithm was compared to 1 step of the MLEM algorithm. A time-invariant white disk with radius 25 pixels centered in a 256 by 256 black image. 128 different projection angles were used (ordered using bit-reversed ordering), and the size of the window was set to 8 projections.

4.2.2 Results of simulation

The figures below are the actual images produced. The composite image, the HYPR reconstruction for the first HYPR frame, and the corresponding MLEM image. The HYPR and the MLEM images are indistinguishable, although the mean absolute error is slightly higher for HYPR than for MLEM. More detailed comparisons of MLEM and HYPR are planned.



5 Future work

- 1. Continue research into iterative HYPR and its connection to Expectation Maximization. Both analytically and through simulation.
- 2. Work more on understanding the artifacts (modeling errors) and in the extreme, the pathological cases in which the HYPR algorithms will fail (worst-case scenario).
- 3. Characterizing the noise amplification and resolution of the HYPR algorithm through. Simulate HYPR algorithm with projection subjected to different noise distributions and determine which variations of the algorithm are most accurate and under which conditions.
- 4. Investigate and implement a new Iterative HYPR variation proposed during work on this project which uses the Wright-Huang as its iterative step and compare to the current standard I-HYPR which uses the original HYPR and compare performance.
- 5. Investigate possibility of a better measure to compare the accuracy of a HYPR image to a time frame image than was used in this report (RMSE), and if one is found, use the new measure for future testing.
- 6. Obtained a mathematical description of the HYPR method based on the matrix formulation and not based on the radon transform. Apply for a simple geometrical shape which is time varying.

6 Appendix

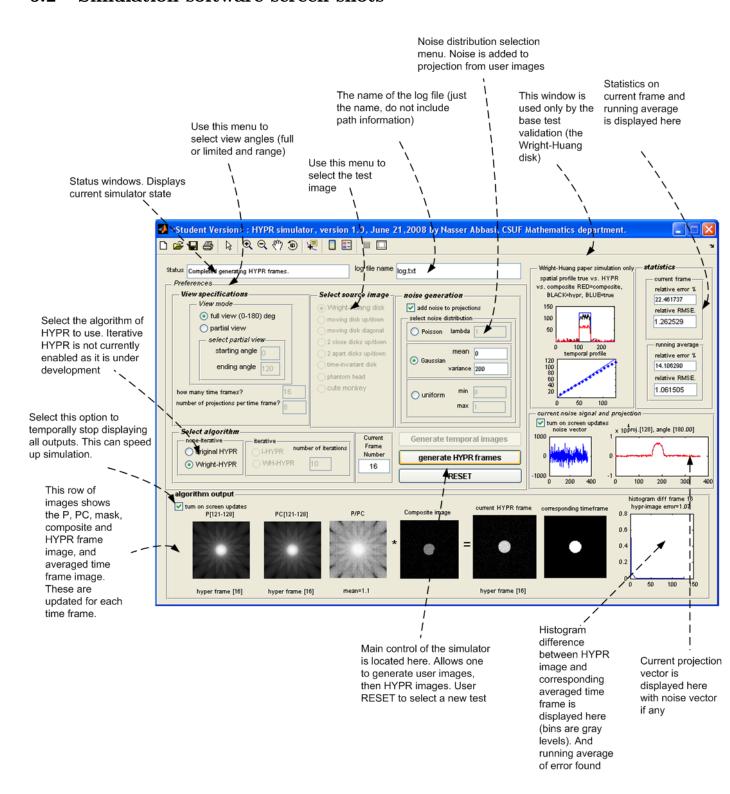
6.1 nomenclature

- 1. MLEM Maximum-Likelihood Expectation-Maximization
- 2. PET Positron Emission Tomography
- 3. SPECT Single-Photon Emission Computed Tomography
- 4. I A 2-D image. This represent the original user image at which the HYPR algorithm is applied to.
- 5. I_t When the original image content changes during the process, we add a subscript to indicate the image I at time instance t.
- 6. R radon transform.

- 7. R_{ϕ} radon transform invoked at a projection angle ϕ .
- 8. ϕ_t When the projection angle ϕ is not constant but changes with time during the MRI acquisition process, we add a subscript to indicate the angle at time instance t.
- 9. R_{ϕ_t} radon transform invoked at a projection angle ϕ_t .
- 10. $s = R_{\phi}[I]$. radon transform applied to an image I at angle ϕ . This results in a projection vector s.
- 11. H Forward projection matrix. The Matrix equivalent to the radon transform R.
- 12. θ Estimate of an image I.
- 13. $H\theta$ Multiply the forward projection matrix H with an image estimate θ .
- 14. $g = H\theta$ Multiply the forward projection matrix H with an image estimate θ to obtain a projection vector g. Notice that for the inner dimensions of the matrix multiplication operation $H\theta$ to be equal, this requires that the 2D image θ be linearized. In other words, the 2D image θ be written as a column vector.
- 15. $R_{\phi}^{u}[s]$ The inverse radon transform applied in unfiltered mode to a projection s which was taken at angle ϕ . This results in a 2D image.
- 16. $R_{\phi}^{f}[s]$ The inverse radon transform applied in filtered mode to a projection s which was taken at angle ϕ . This results in a 2D image.
- 17. H^Tg The transpose of the forward projection matrix H multiplied by the projection vector g. This is the matrix equivalent of applying the inverse radon transform in an unfiltered mode to a projection s (see item 12 above).
- 18. H^+g The pseudo inverse of the forward projection matrix H being multiplied by the projection vector g. This is the matrix equivalent of applying the inverse radon transform in filtered mode to a projection s (see item 13 above).
- 19. C Composite image generated by summing all the filtered back projections from projections s_t of the original images I_t . Hence $C = \sum_{i=1}^{N} R_{\phi_{t_i}}^f[s_{t_i}]$
- 20. P_t The unfiltered backprojection 2D image as a result of applying $R_{\phi_t}^u[s_t]$ where s_t is projection from user image I_t taken at angle ϕ_t .
- 21. P_{c_t} The unfiltered backprojection 2D image as a result of applying $R_{\phi_t}^u[s_t]$ where s_t is projection from the composite image C taken at angle ϕ_t .
- 22. N_p Number of projections used to generate one HYPR frame image. This is the same as the number of projections per one time frame.

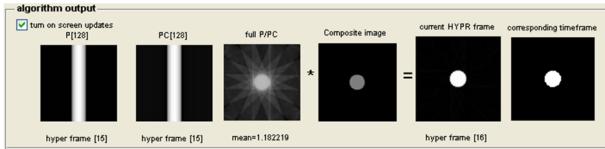
- 23. N The total number of projections used. This is the number of time frames multiplied by N_p
- 24. J_k The k^{th} HYPR frame image. A 2-D image generate at the end of the HYPR algorithm. There will be as many HYPR frame images J_k as there are time frames.

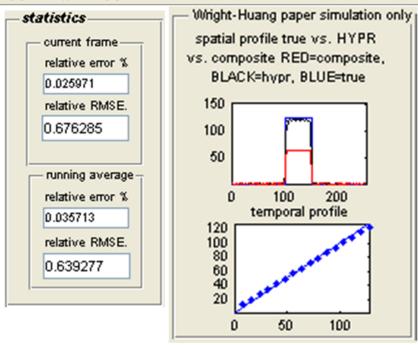
6.2 Simulation software screen shots



6.3 HYPR simulation results

6.3.1 Test 1a

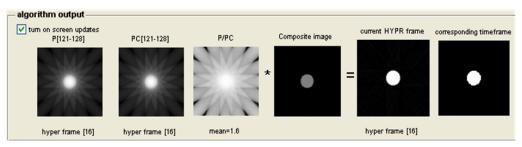


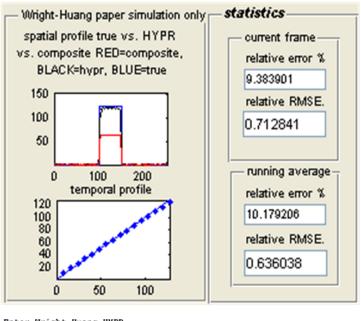


```
generated data for simulation [wrightPaperDiskTag]
Completed image generation...
number of time frames = [16]
number of projections per time frame = [8]
[22-Jun-2008 13:56:42]Enter generate HYPR ...
```

```
Enter original HYPR ...
No NOISE is being added
frame rmse relativeError
                              rmse_averaged
                                                 relativeError_averaged averageMask
      1.026815
                  0.124615
                              1.026815
                                           0.124615
                                                       0.810999
      0.789398
                  0.080273
                              0.908107
                                           0.102444
                                                       0.834959
      0.658115
                  0.049115
                              0.824776
                                           0.084668
                                                       0.856301
      0.579282
                  0.057853
                              0.763403
                                           0.077964
                                                       0.888973
                                           0.065382
                                                       0.905486
      0.551421
                  0.015053
                              0.721006
                                           0.057266
      0.539332
                  0.016687
                              0.690727
                                                       0.934073
                                                       0.957154
      0.544661
                  0.011765
                              0.669861
                                           0.050766
      0.555759
                  0.021502
                              0.655598
                                           0.047108
                                                       0.988724
      0.565009
                  0.019138
                                           0.044000
                                                       1.005649
                              0.645533
10
      0.582615
                  0.021568
                              0.639241
                                           0.041757
                                                       1.030899
11
      0.600318
                  0.024314
                              0.635702
                                           0.040171
                                                       1.053254
      0.618896
                  0.016745
                              0.634302
                                           0.038219
                                                       1.085530
      0.629853
                  0.033764
                              0.633959
                                           0.037876
                                                       1.101912
14
      0.649074
                  0.025293
                              0.635039
                                           0.036977
                                                       1.132088
15
      0.661592
                  0.027750
                              0.636809
                                           0.036362
                                                       1.153034
16
      0.676285
                  0.025971
                              0.639277
                                           0.035713
                                                       1.182219
      totalHistError =1.33 ...
```

6.3.2 Test 1b

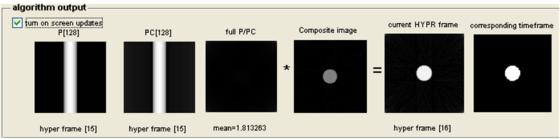


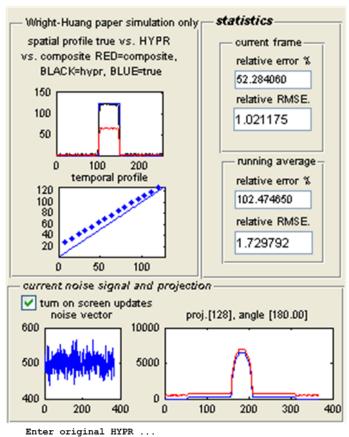


Enter Wright-Huang HYPR					
No NO	ISE is being	added			
frame	rmse	relErr	MeanRmse	MeanRelErr	MeanMask
1	0.982878	38.035191	0.982878	38.035191	0.396131
2	0.734453	26.583094	0.858665	32.309142	0.483218
3	0.604836	19.085338	0.774055	27.901208	0.557789
4	0.532656	14.054107	0.713705	24.439433	0.639344
5	0.520571	9.217883	0.675079	21.395123	0.715395
6	0.519005	5.920670	0.649066	18.816047	0.798848
7	0.535644	3.097390	0.632863	16.570525	0.877456
8	0.556542	0.955617	0.623323	14.618661	0.958114
9	0.574512	1.083784	0.617900	13.114786	1.035286
10	0.599367	2.818749	0.616046	12.085182	1.114582
11	0.621385	4.294862	0.616532	11.376971	1.193476
12	0.645831	5.560250	0.618973	10.892245	1.277558
13	0.658385	6.633323	0.622005	10.564635	1.353904
14	0.682260	7.683917	0.626309	10.358870	1.434595
15	0.695441	8.459228	0.630918	10.232227	1.508504
16	0.712841	9.383901	0.636038	10.179206	1.596524
Done,	totalHistEr	ror =1.35			

Test 1b results

6.3.3 Test 2a



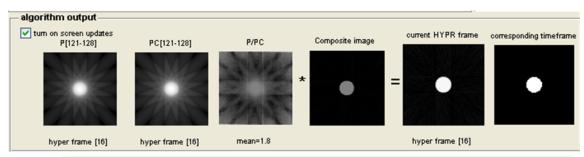


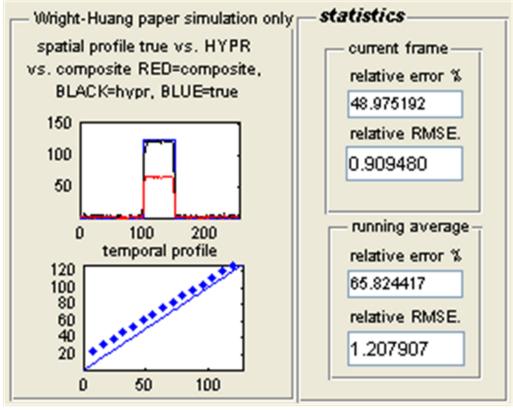
NOISE is being added
Using Poisson noise, lambda=500.000000

frame rmse relErr MeanRmse MeanRelErr MeanMask 3.794379 221.763762 3.794379 221.763762 1.474255 185.358053 3.480673 203.560907 1.508603 2 3.166966 156.888440 3.189968 188.003418 1.527144 2.608558 3 136.067387 2.947765 175.019410 1.554850 2.221155 5 1.930149 119.771893 2.744241 163.969907 1.567517 1.747583 108.147509 2.578132 154.666174 1.599652 1.590956 98.060255 2.437106 146.579614 1.621458 8 1.416717 88.445097 2.309558 139.312799 1.640420 9 1.320464 81.505980 2.199658 132.889820 1.654478 10 1.276373 76.376848 2.107330 127.238522 1.687263 1.208729 11 71.138666 2.025639 122.138535 1.708348 66.167579 117.474289 1.733409 12 1.145390 1.952285 62.349480 13 1.108661 1.887391 113.233919 1.749685 14 1.083650 58.885952 1.829981 109.351922 1.777842 15 1.062933 55.908463 1.778844 105.789024 1.799599 16 1.019940 52.258520 1.731413 102.443368 1.813701 Done, totalHistError =1.71 ...

Test 2a

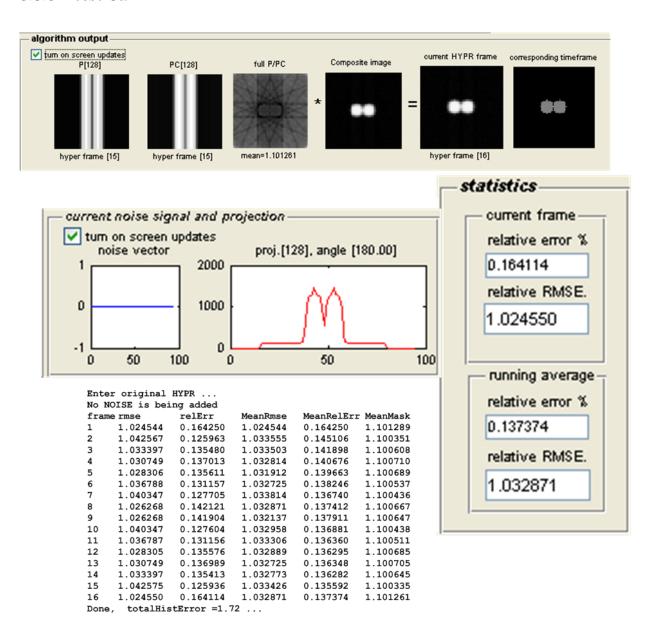
6.3.4 Test 2b





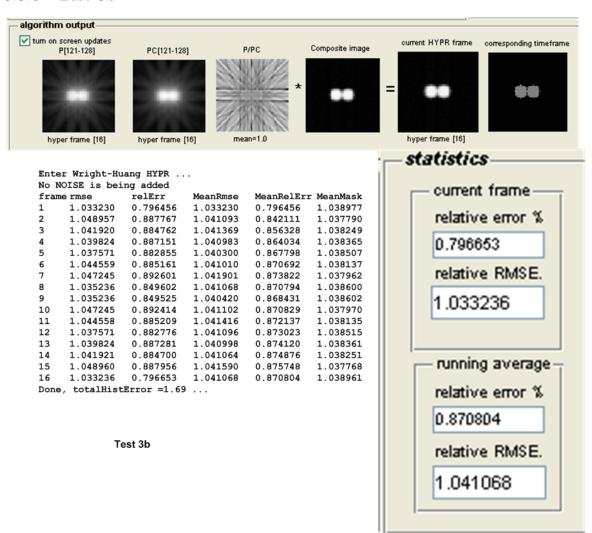
Enter Wright-Huang HYPR ... NOISE is being added Using Poisson noise, lambda=500.000000 frame rmse relErr MeanRmse MeanRelErr MeanMask 2.537686 103.394281 2.537686 103.394281 0.852526 2.063792 95.447458 2.300739 99.420870 0.934455 1.673412 84.548348 2.091630 94.463363 0.986322 1.384630 75.808737 1.914880 89.799706 1.047637 1.241278 71.417355 1.780160 86.123236 1.101549 1.126402 68.027509 1.671200 83.107281 1.174521 80.520359 1.236801 1.051424 64.998822 1.582660 0.956893 60.108881 1.504440 77.968924 1.286315 Test 2b 0.936989 58.614646 1.441389 75.818449 1.345422 10 0.928753 57.735384 1.390126 74.010142 1.419706 0.912443 55.866211 1.346700 72.360694 1.478030 11 70.780884 1.535107 12 0.894692 53.402971 1.309033 13 0.897935 52.682674 1.277410 69.388714 1.595649 68.107038 1.660646 0.902629 51.445255 1.250640 0.908081 50.716949 1.227803 66.947699 1.723549 15 48.975192 1.207907 65.824417 1.773860 0.909480 16 Done, totalHistError =1.80 ...

6.3.5 test 3a

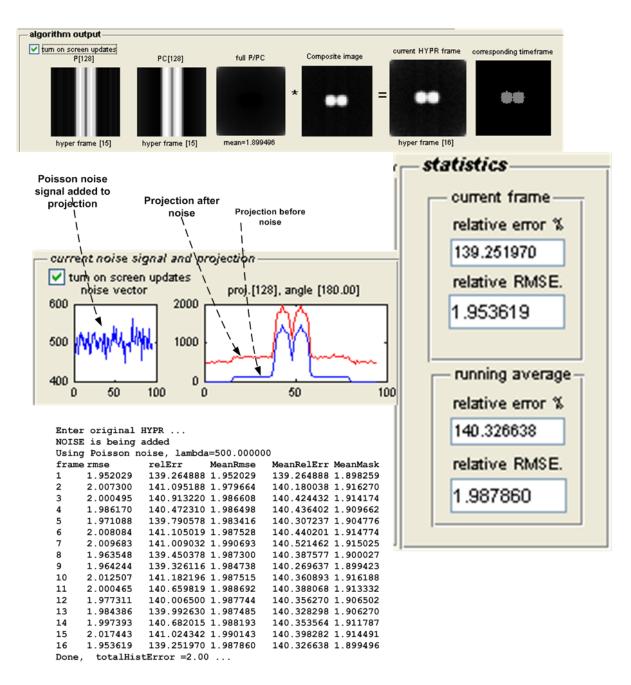


Test 3a

6.3.6 Test 3b

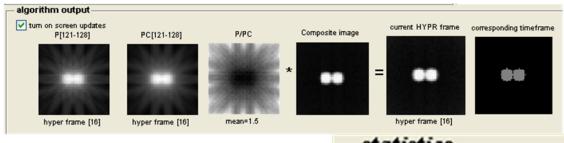


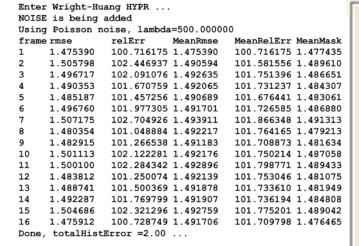
6.3.7 Test 4a



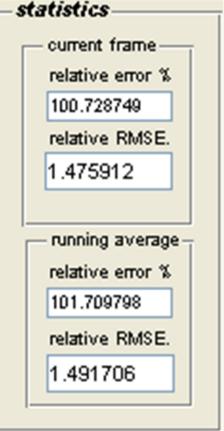
Test 4a

6.3.8 Test 4b

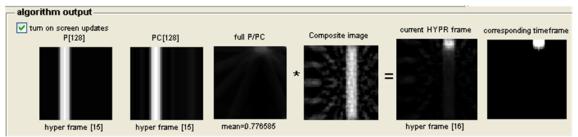




Test 4b



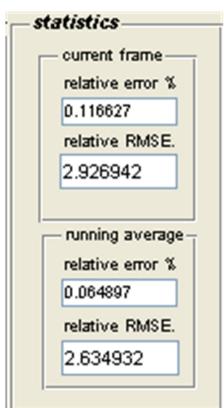
6.3.9 Test 5a



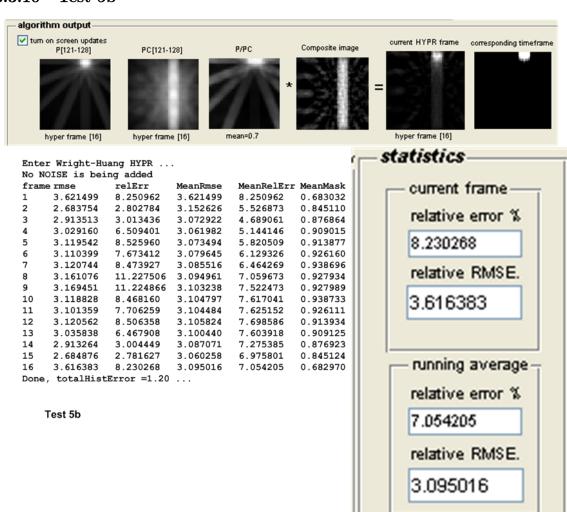
generated data for simulation [movingDiskUpDownTag]
Completed image generation...
number of time frames = [16]
number of projections per time frame = [8]
[22-Jun-2008 16:03:26]Enter generate HYPR ...

Enter original HYPR						
No NOISE is being added						
frame rmse	relErr	MeanRmse	MeanRelErr	MeanMask		
1 2.930777	0.112860	2.930777	0.112860	0.776031		
2 2.283642	0.032046	2.607209	0.072453	0.870046		
3 2.468333	0.049740	2.560917	0.064882	0.837949		
4 2.615779	0.060302	2.574633	0.063737	0.833535		
5 2.693877	0.087963	2.598481	0.068582	0.829808		
6 2.653286	0.004072	2.607615	0.057831	0.846449		
7 2.686605	0.044205	2.618900	0.055884	0.846036		
8 2.745714	0.132117	2.634751	0.065413	0.833106		
9 2.756394	0.134185	2.648267	0.073055	0.833394		
10 2.683672	0.051199	2.651808	0.070869	0.846576		
11 2.643038	0.005699	2.651011	0.064944	0.846128		
12 2.694688	0.089126	2.654650	0.066960	0.831088		
13 2.621380	0.046047	2.652091	0.065351	0.833980		
14 2.467277	0.046980	2.638890	0.064039	0.838185		
15 2.287501	0.025182	2.615464	0.061448	0.869622		
16 2.926942	0.116627	2.634932	0.064897	0.776585		
Done, totalHistError =1.16						

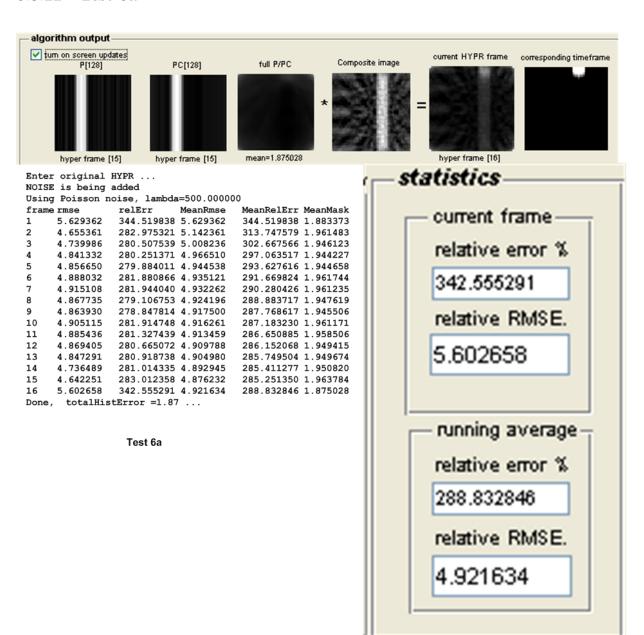
Test 5a



6.3.10 Test 5b

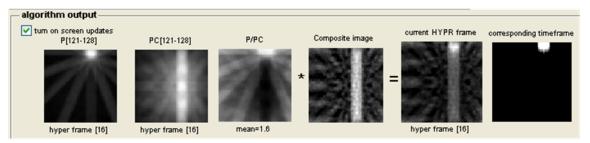


6.3.11 Test 6a



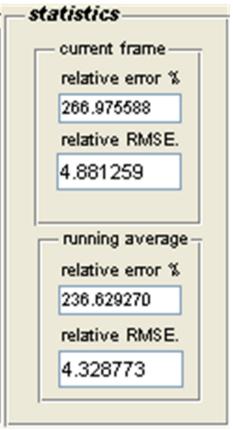
6.3.12 Test 6b

Enter Wright-Huang HYPR ...

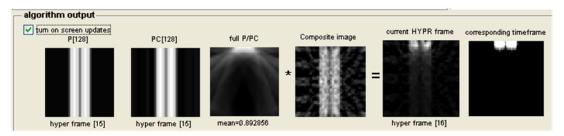


NOISE is being added Using Poisson noise, lambda=500.000000 frame rmse relErr MeanRmse MeanRelErr MeanMask 4.912966 268.785319 4.912966 268.785319 1.560698 225.748902 4.422590 3.932215 247.267111 1.664708 4.125427 229.843134 4.323536 241.459118 1.679669 4.276462 232.844267 4.311767 239.305406 1.694026 4.325068 232.786972 4.314427 238.001719 1.695276 4.339908 234.444884 4.318674 237.408913 1.709300 235.816020 4.326462 237.181357 1.716338 4.373188 233.454795 4.331138 236.715537 1.699426 4.363874 4.368241 234.046549 4.335261 236.418983 1.702199 4.376689 236.131141 4.339404 236.390198 1.718209 11 4.349609 235.220193 4.340331 236.283834 1.713632 232.516203 4.338514 235.969865 1.696569 12 4.318524 13 4.274776 233.071864 4.333611 235.746942 1.697233 14 4.117936 228.700516 4.318206 235.243626 1.677402 15 3.924222 225.681976 4.291940 234.606182 1.665489 4.881259 266.975588 4.328773 236.629270 1.555684 16 Done, totalHistError =1.88 ...

Test 6b

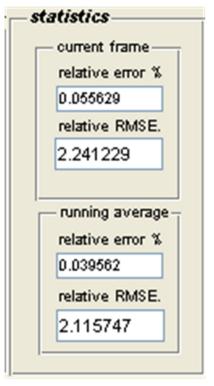


6.3.13 Test 7a

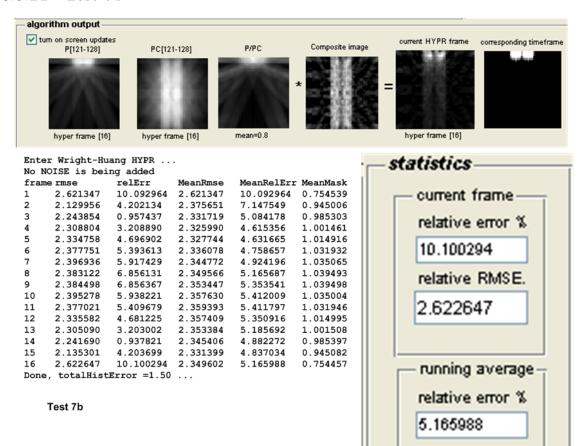


Enter original HYPR ... No NOISE is being added frame rmse relErr MeanRmse MeanRelErr MeanMask 2.239817 0.032813 2.239817 0.032813 0.893149 0.037924 1.870331 2.055074 0.035368 1.007503 2.050843 0.037437 0.964871 3 2.042381 0.041574 2.115184 0.049842 2.066928 0.040538 0.949393 2.130986 0.048243 2.079740 0.042079 0.944004 0.036704 0.950613 6 2.169312 0.009831 2.094668 0.948241 2.195184 0.017919 2.109028 0.034021 7 0.037925 0.065252 2.115624 0.945542 8 2.161797 9 2.163640 0.065947 2.120959 0.041038 0.945540 10 2.194526 0.015931 2.128316 0.038528 0.947925 11 2.168012 0.014524 2.131924 0.036345 0.950493 12 2.131513 0.049517 2.131890 0.037443 0.944035 13 2.111168 0.048591 2.130296 0.038301 0.949333 14 2.039746 0.039543 2.123828 0.038389 0.964977 1.877134 0.039910 0.038491 1.007446 15 2.107382 16 2.241229 0.055629 2.115747 0.039562 0.892856 Done, totalHistError =1.49 ...

Test 7a



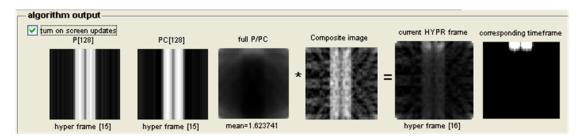
6.3.14 Test 7b



relative RMSE.

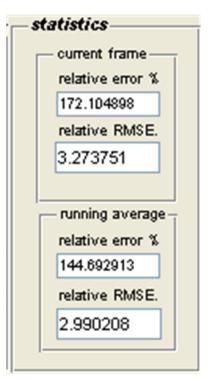
2.349602

6.3.15 Test 8a

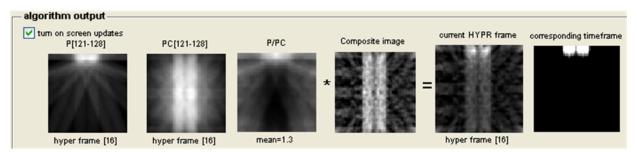


Enter original HYPR ... NOISE is being added Using Poisson noise, lambda=500.000000 frame rmse relErr MeanRmse MeanRelErr MeanMask 3.267275 171.041839 3.267275 171.041839 1.616434 2.782265 141.708531 3.024770 156.375185 1.727043 141.034861 2.983981 3 2.902405 151.261744 1.710669 2.958066 148.543885 1.703421 140.390308 2.977503 4 5 2.961299 140.228664 2.974262 146.880841 1.701117 3.022684 141.334702 2.982332 145.956484 1.711392 3.050837 141.541755 2.992119 145.325808 1.710639 2.980434 139.871123 2.990658 144.643973 1.703263 8 9 2.968799 139.257653 2.988229 144.045493 1.698596 10 3.041277 141.477476 2.993534 143.788691 1.711121 11 3.026870 141.259407 2.996565 143.558756 1.711672 140.178735 2.993621 143.277088 1.703472 2.961240 12 140.759986 2.990874 143.083465 1.706737 13 2.957911 14 2.902314 141.076983 2.984548 142.940144 1.713614 15 2.785898 141.819689 2.971305 142.865447 1.729559 172.104898 2.990208 144.692913 1.623741 16 3.273751 Done, totalHistError =1.91 ...

Test 8a

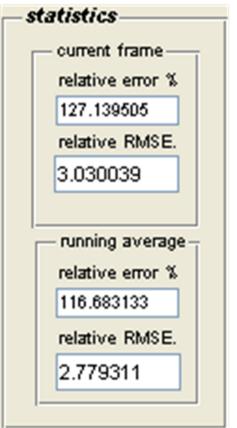


6.3.16 Test 8b

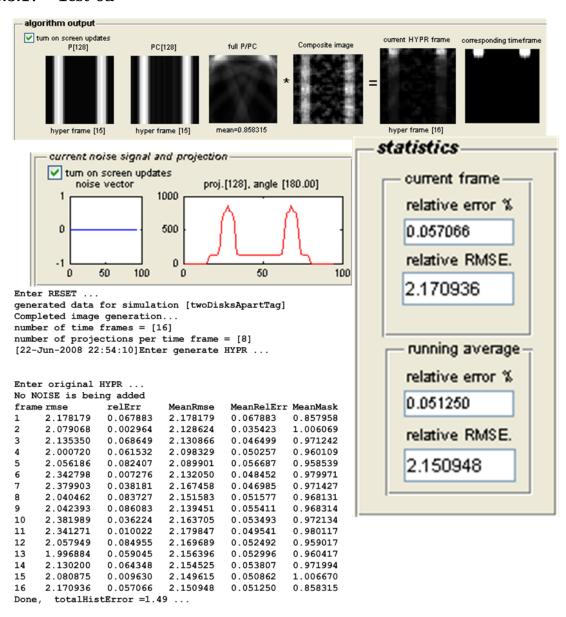


Enter Wright-Huang HYPR ... NOISE is being added Using Poisson noise, lambda=500.000000 frame rmse relErr MeanRmse MeanRelErr MeanMask 3.030838 126.908348 3.030838 126.908348 1.310844 2.520443 108.549650 2.775641 117.728999 1.450055 2.665623 113.431873 2.738968 116.296624 1.479040 3 2.746139 114.895869 2.740761 115.946435 1.488146 2.774767 115.966281 2.747562 115.950404 1.497615 2.826919 117.630822 2.760788 116.230474 1.512274 2.846058 118.046073 2.772970 116.489845 1.515011 2.824241 117.515312 2.779379 116.618029 1.509978 8 9 2.829996 117.494837 2.785003 116.715452 1.509945 10 2.851925 118.388246 2.791695 116.882731 1.516825 11 2.830234 118.019289 2.795199 116.986055 1.515664 12 2.770225 116.011994 2.793117 116.904883 1.498614 13 2.729285 114.772491 2.788207 116.740853 1.487686 14 2.658650 113.043908 2.778953 116.476785 1.477716 15 2.533590 109.115636 2.762596 115.986042 1.455856 127.139505 2.779311 116.683133 1.313645 3.030039 16 Done, totalHistError =1.91 ...

Test 8b

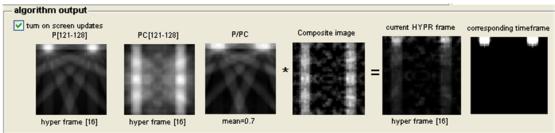


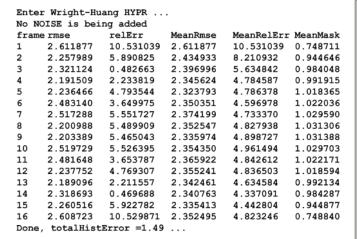
6.3.17 Test 9a



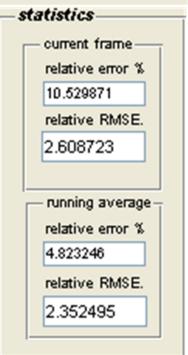
Test 9a

6.3.18 Test 9b

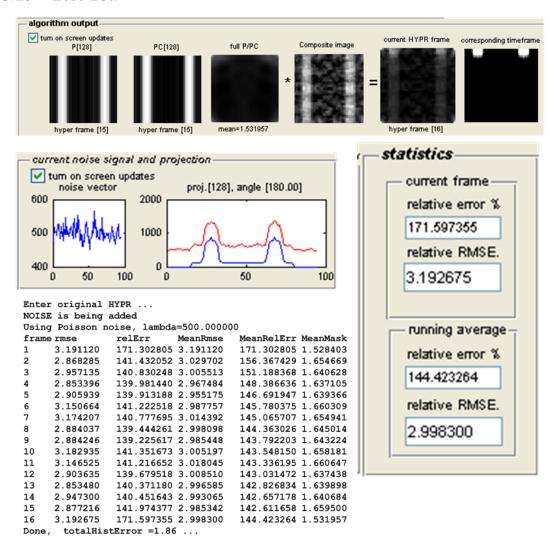




Test 9b

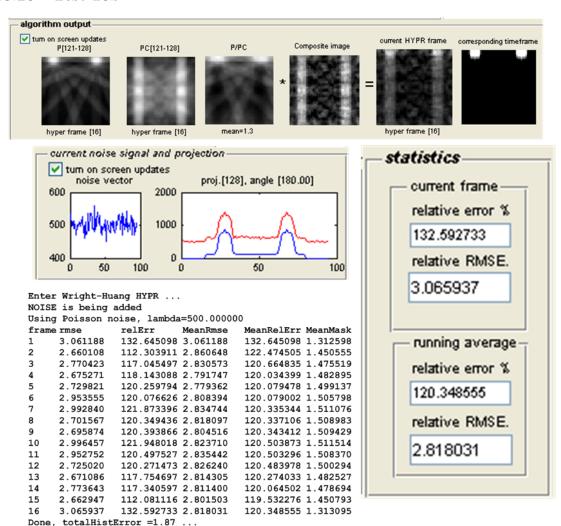


6.3.19 Test 10a



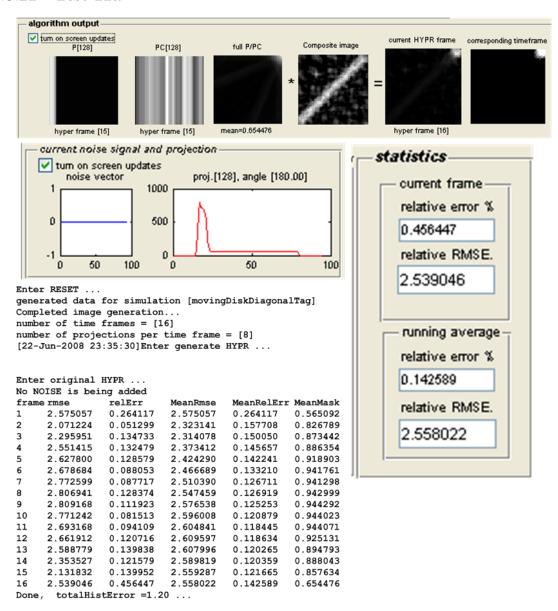
Test 10a

6.3.20 Test 10b



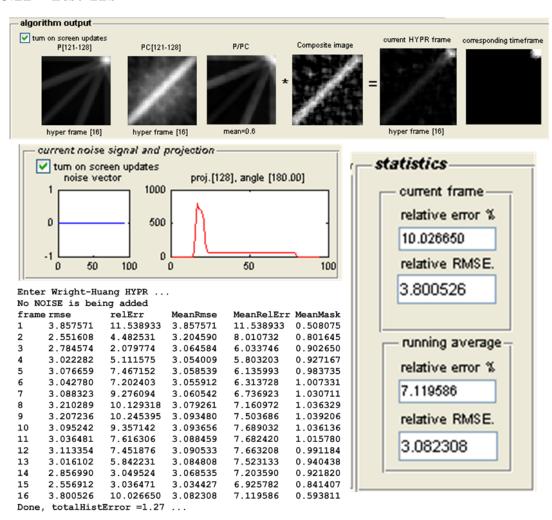
Test 10b

6.3.21 Test 11a



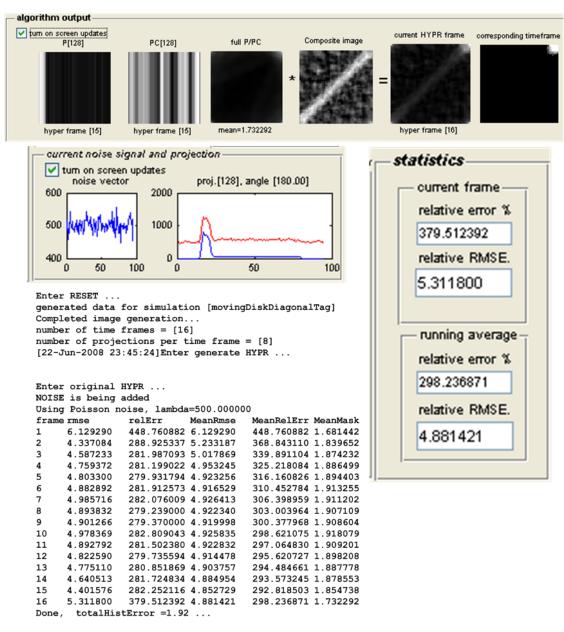
Test 11a

6.3.22 Test 11b



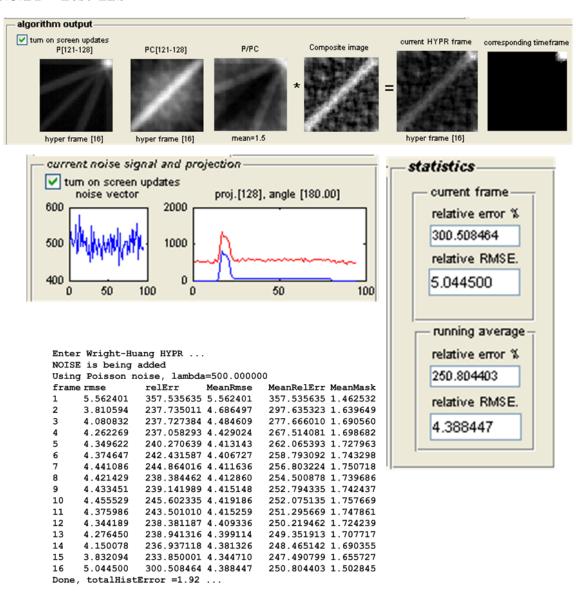
Test 11b

6.3.23 Test 12a



Test 12a

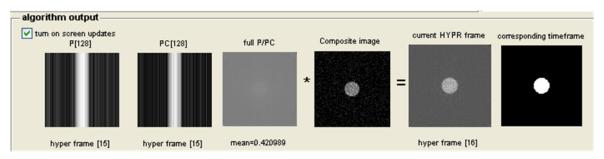
6.3.24 Test 12b

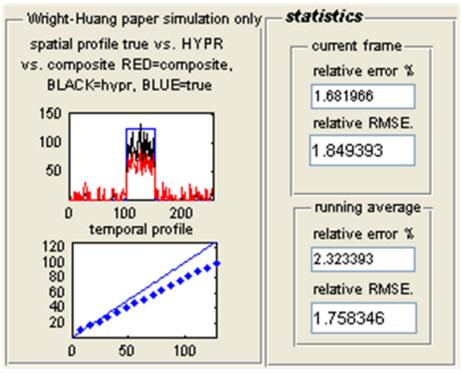


Test 12b

6.3.25 Test 2N a

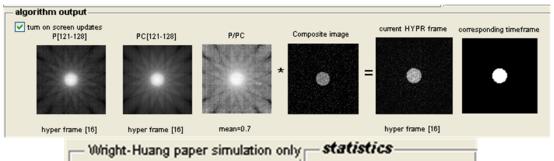
Test 2N a

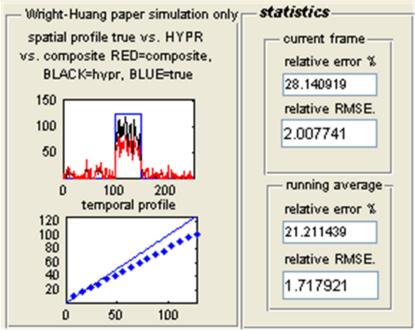




[23-Jun-2008 04:10:12]Enter generate HYPR ... Enter original HYPR ... NOISE is being added Using Gaussian noise, mean=0.000000, variance=500.000000 frame rmse relErr MeanRmse MeanRelErr MeanMask 2.188933 3.875660 2.188933 3.875660 0.158916 1.874251 1.206180 2.031592 2.540920 0.166914 1.854996 4.388939 1.972727 3.156926 0.191856 1.613788 2.585697 1.882992 3.014119 0.188856 1.687870 4.927170 1.843968 3.396729 0.238000 1.609411 2.800479 1.804875 3.297354 0.240811 1.590070 4.321904 1.774188 3.443718 0.264438 1.642190 1.669267 1.757689 3.221912 0.261340 8 2.877002 0.288657 9 1.657622 0.117722 1.746570 10 1.680792 3.418464 1.739992 2.931148 0.320764 1.762216 1.713408 1.742013 2.820444 0.335904 1.025052 1.741313 2.670828 0.350833 12 1.733620 13 1.757616 1.101132 1.742567 2.550083 0.362874 1.806749 2.018542 1.747152 2.512115 0.383195 14 15 1.824024 0.322700 1.752277 2.366154 0.392007 1.849393 1.681966 1.758346 2.323393 0.420989 Done, totalHistError =2.00 ...

6.3.26 Test 2N b



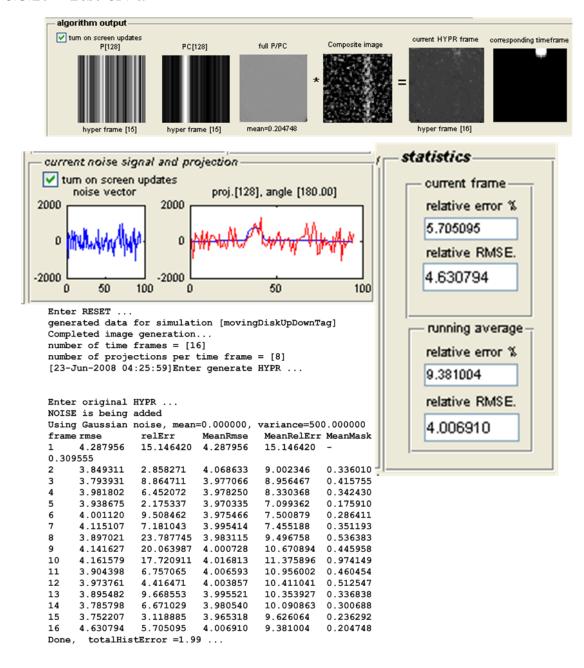


[23-Jun-2008 04:18:52]Enter generate HYPR ... Enter Wright-Huang HYPR ... NOISE is being added

Using Gaussian	noise, mean=0.000000,		variance=500.000000	
frame rmse	relErr	MeanRmse	MeanRelErr	MeanMask
1 1.482931	3.721894	1.482931	3.721894	0.144877
2 1.397441	7.324329	1.440186	5.523112	0.189556
3 1.379407	9.686641	1.419927	6.910955	0.215936
4 1.493886	16.842449	1.438417	9.393828	0.263701
5 1.521554	20.616767	1.455044	11.638416	0.299181
6 1.587763	18.948936	1.477164	12.856836	0.324298
7 1.662723	23.517241	1.503672	14.379751	0.369251
8 1.738649	25.757471	1.533044	15.801966	0.410752
9 1.753492	21.540356	1.557539	16.439565	0.420345
10 1.833851	25.927331	1.585170	17.388342	0.470976
11 1.843377	24.999919	1.608643	18.080303	0.494303
12 1.909228	26.233916	1.633692	18.759771	0.532417
13 1.933356	28.306206	1.656743	19.494112	0.577308
14 1.954653	28.587085	1.678022	20.143610	0.604636
15 1.986688	29.231571	1.698600	20.749474	0.642963
16 2.007741	28.140919	1.717921	21.211439	0.664368
Done, totalHistError =2.00				

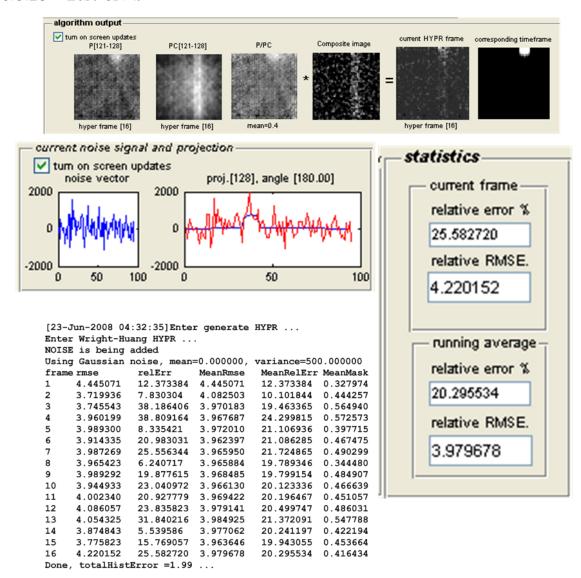
Test 2N b

6.3.27 Test 6N a



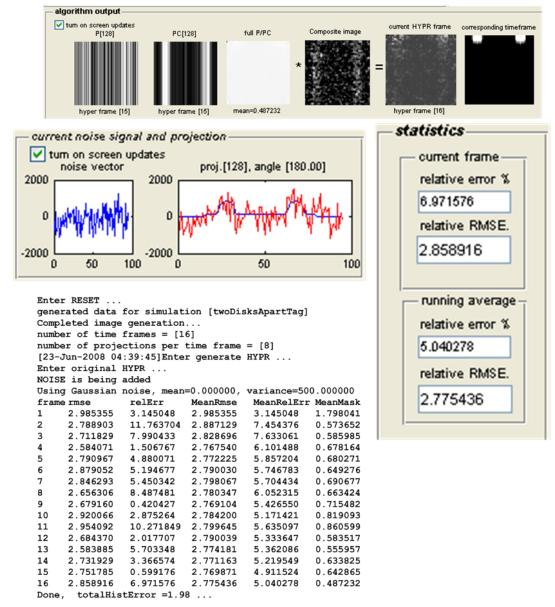
Test 6N a

6.3.28 Test 6N b



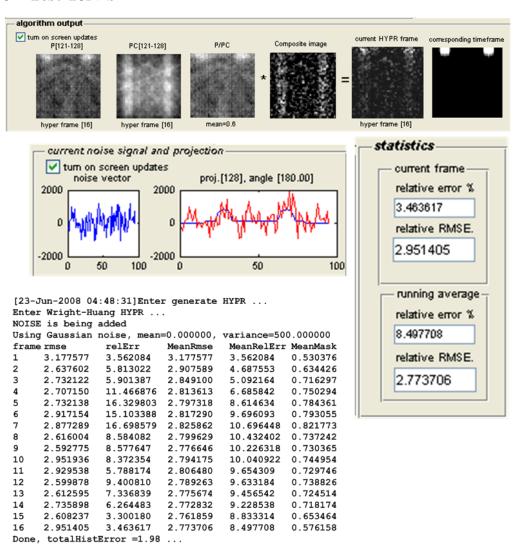
Test 6N b

6.3.29 Test 10N a



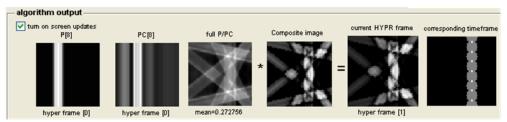
Test 10N a

6.3.30 Test 10N b



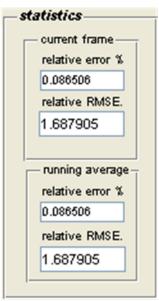
Test 10N b

6.3.31 Test 8r a

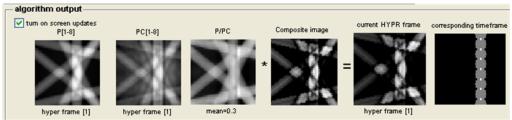


Enter RESET ...
generated data for simulation [movingDiskUpDownTag]
Completed image generation...
number of time frames = [1]
number of projections per time frame = [8]
[23-Jun-2008 06:05:26]Enter generate HYPR ...
Enter original HYPR ...

Test 8r a



6.3.32 Test 8r b



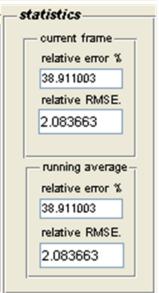
Enter Wright-Huang HYPR ...

No NOISE is being added frame rmse relErr MeanRmse MeanRelErr MeanMask

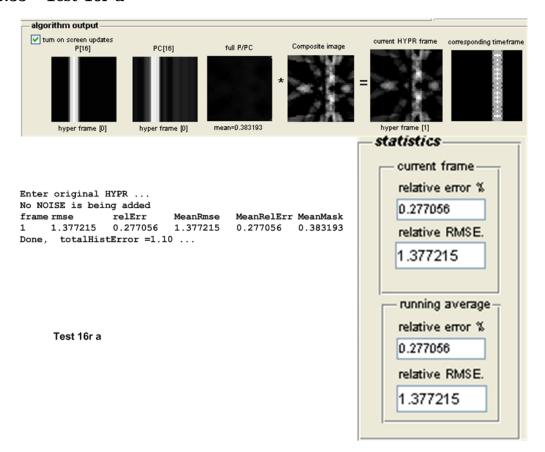
1 2.083663 38.911003 2.083663 38.911003 0.344109

Done, totalHistError =1.08 ...

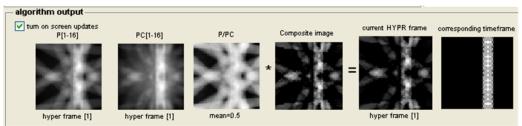
Test 8r b



6.3.33 Test 16r a



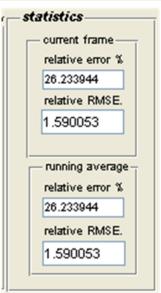
6.3.34 Test 16r b



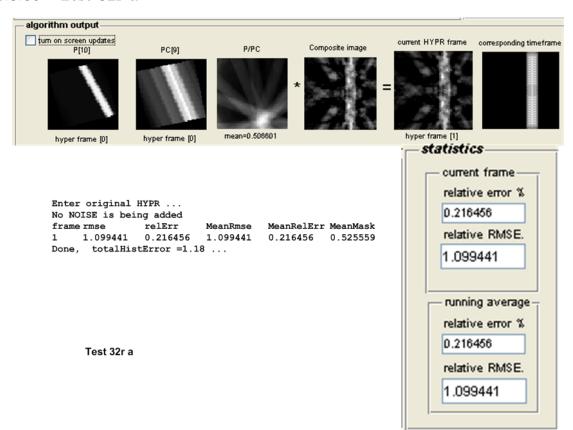
Enter Wright-Huang HYPR ...

No NOISE is being added
frame rmse relErr MeanRmse MeanRelErr MeanMask
1 1.590053 26.233944 1.590053 26.233944 0.456270
Done, totalHistError =1.10 ...

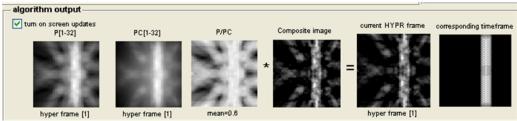
Test 16r b



6.3.35 Test 32r a



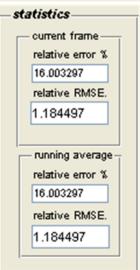
6.3.36 Test 32r b



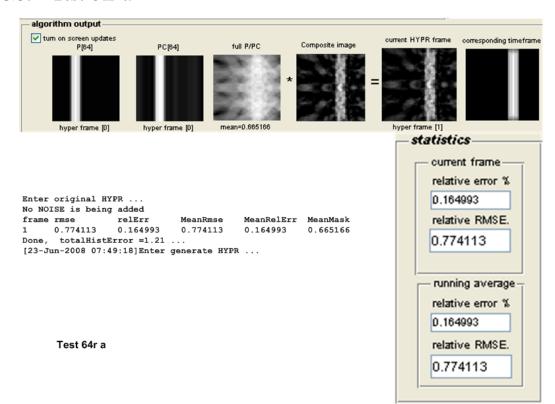
Enter Wright-Huang HYPR ...

No NOISE is being added
frame rmse relErr MeanRmse MeanRelErr MeanMask
1 1.184497 16.003297 1.184497 16.003297 0.596317
Done, totalHistError =1.18 ...

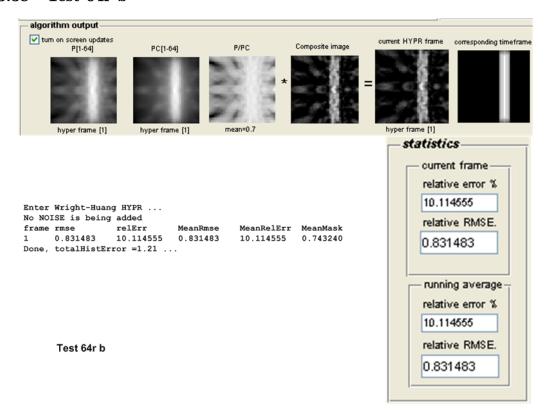
Test 32r b



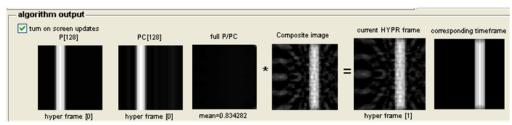
6.3.37 Test 64r a



6.3.38 Test 64r b



6.3.39 Test 128r a

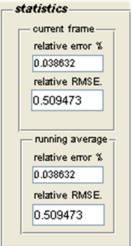


Enter original HYPR ...

No NOISE is being added frame rmse relErr MeanRmse MeanRelErr MeanMask

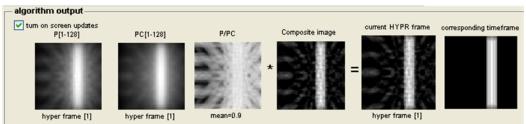
1 0.509473 0.038632 0.509473 0.038632 0.834282

1 0.509473 0.038632 0.509473 0.038632 0.834282
Done, totalHistError =1.28 ...



Test 128r a

6.3.40 Test 128r b



Enter Wright-Huang HYPR ...

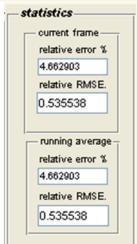
No NOISE is being added

frame rmse relErr MeanRmse MeanRelErr MeanMask

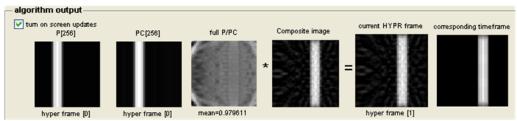
1 0.535538 4.662903 0.535538 4.662903 0.878450

Done, totalHistError =1.28 ...

Test 128r b



6.3.41 Test 256r a



Enter original HYPR ...

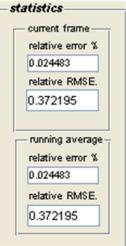
No NOISE is being added

frame rmse relErr MeanRmse MeanRelErr MeanMask

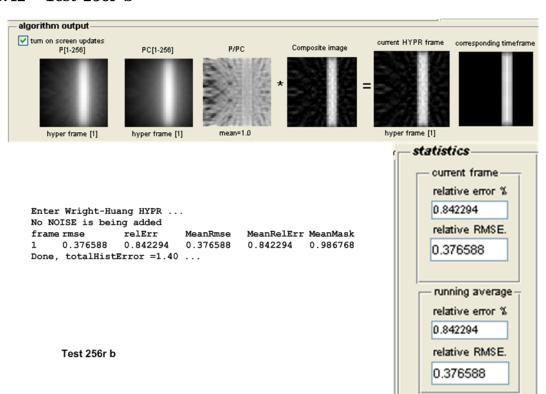
1 0.372195 0.024483 0.372195 0.024483 0.979611

Done, totalHistError =1.40 ...

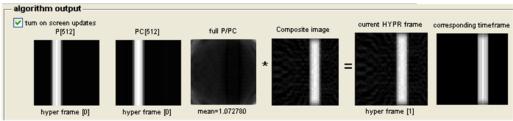
Test 256r a



6.3.42 Test 256r b



6.3.43 Test 512r a



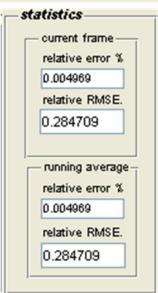
Enter original HYPR ...

No NOISE is being added
frame rmse relErr MeanRmse MeanRelErr MeanMask

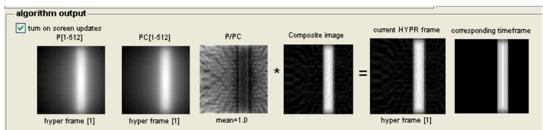
1 0.284709 0.004969 0.284709 0.004969 1.072780

Done, totalHistError =1.65 ...

Test 512r a



6.3.44 Test 512r b

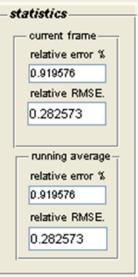


Enter Wright-Huang HYPR ...

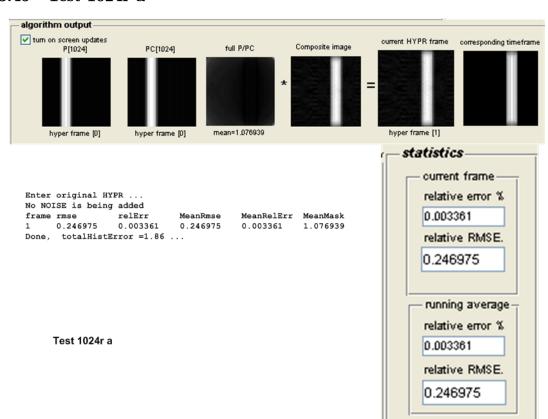
No NOISE is being added frame rmse relErr MeanRmse MeanRelErr MeanMask

1 0.282573 0.919576 0.282573 0.919576 1.039760
Done, totalHistError =1.66 ...

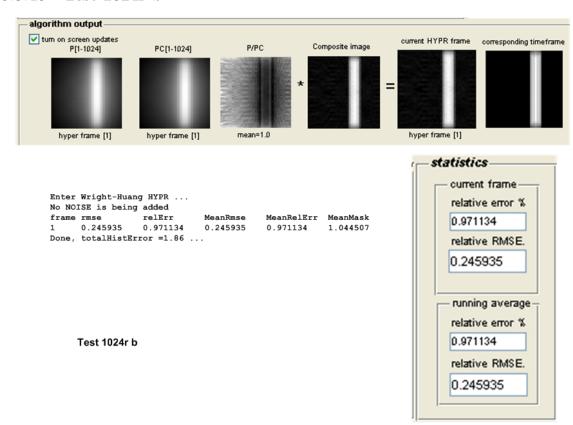
Test 512r b



6.3.45 Test 1024r a



6.3.46 Test 1024r b



6.4 References

References

- [1] Iterative projection reconstruction of time-resolved images using highly-constrained back-projection (HYPR) by Rafael L. O'Halloran, Zhifei Wen, James H. Holmes, Sean B. Fain
- [2] Highly Constrained Back projection for Time-Resolved MRI by C. A. Mistretta, O. Wieben, J. Velikina, W. Block, J. Perry, Y. Wu, K. Johnson, and Y. Wu
- [3] Principles of computerized Tomographic imaging by Kak and Staney
- [4] Highly Constrained Backprojection for Time-Resolved MRI by C. A. Mistretta, Wieben, z J, Velikina, W. Block, J. Perry, Y. Wu. K. ohnson and Y. Wu.