Final Presentation from the CSUF Applied Math Project To GE Healthcare

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Summary of Midterm Report

- The implementation of the first step of MLEM with the composite image as the initial image is numerically similar as HYPR. In other words, the original HYPR is a heuristic derivation of MLEM for time-dependent data.
- Initial development of a simulator for HYPR showed that the algorithm did not break catastrophically when an object with time-invariant signal had vertical motion.

Results Since Midterm Report

- Better understanding of the relationship between HYPR and the first step of MLEM
- Classification of HW-HYPR as the first step of the MART algorithm applied to the normal equations
- Development of I-HW-HYPR
- Further development of the HYPR Simulator: it now implements HYPR, HW-HYPR, HYPR-LR, MLEM, I-HYPR, I-HW-HYPR for user input, several test cases and reproduces figures from papers. When all these methods were tested on the paper-clip phantom with noise, the method with the least error was HW-HYPR.
- An initial literature search of the time-dependent SPECT did not lead to a case where the composite was the initial condition.
- Initial conditions for MLEM are not considered terribly important since after a few iterations, the reconstructions look similar.

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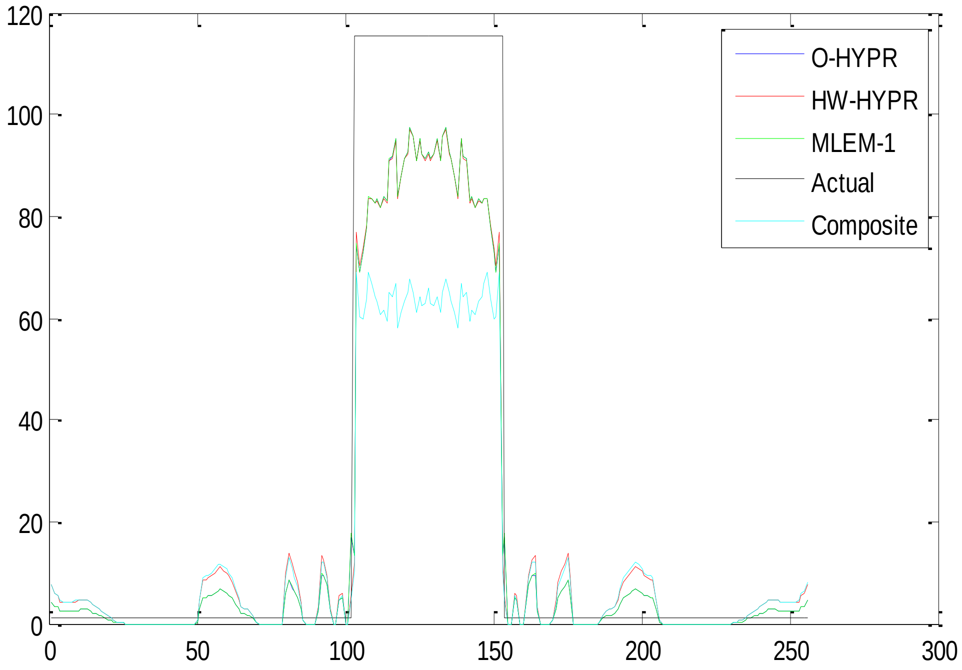
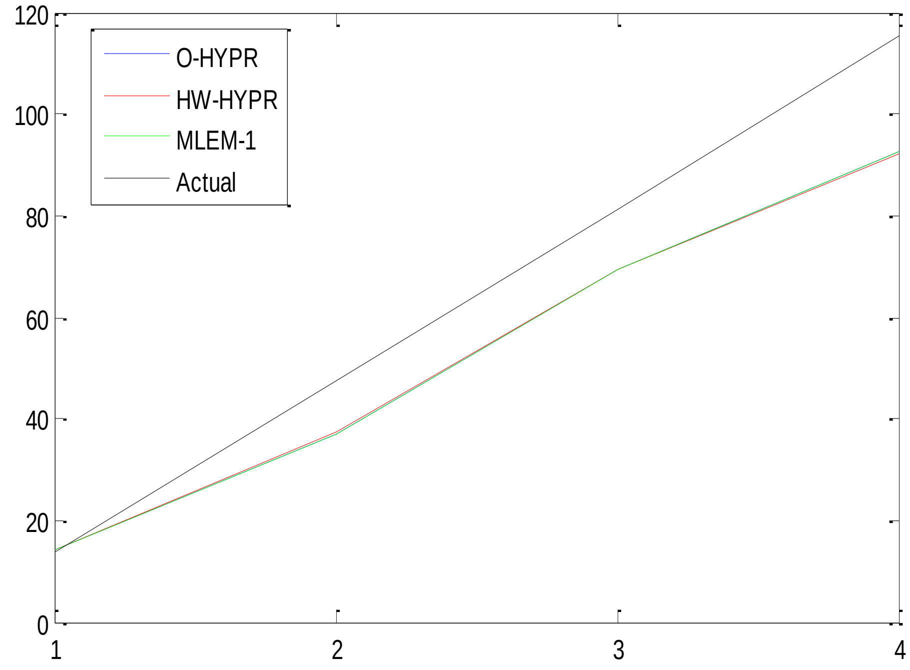
HYPR vs. MLEM

- Original HYPR (and Huang-Wright HYPR) compared to MLEM-1 in simple noise-free simulation…original HYPR same as MLEM as shown below
- Next slide shows how algorithms start out differently but ultimately produce same reconstructions via their respective normalizations (black = 0, white = 2)

Spatial Profile: Time-Varying Disk, Projections 13-16



Temporal Profile: Time Varying Disk



MLEM-1, O-HYPR indistinguishable

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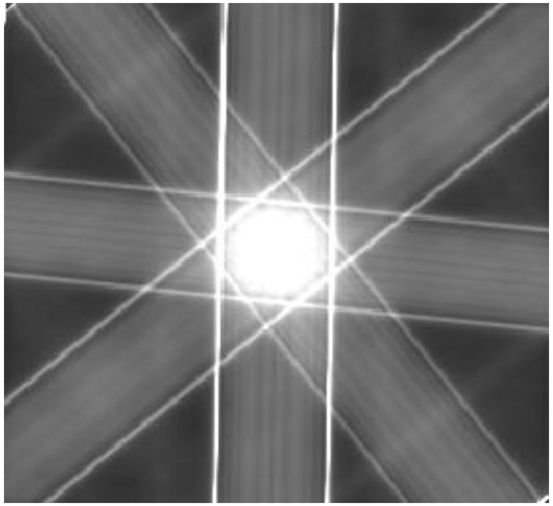
HYPR vs. MLEM

MLEM-1

$R_{\Phi}^{T}[s_{4,1}/R_{\Phi_{4,1}}C,\ldots,s_{4,4}/R_{\Phi_{4,4}}C]$

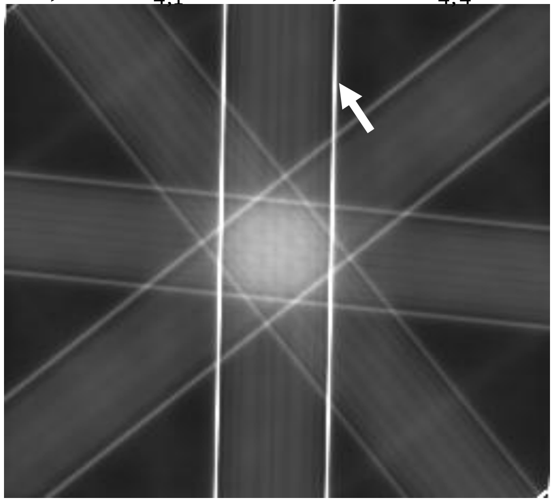


*

z: BP of Unit Projections (=2/π)

=

$R_{\Phi}^{T}[s_{4,1}/R_{\Phi_{4,1}}C,\ldots,s_{4,4}/R_{\Phi_{4,4}}C]/z$



O-HYPR

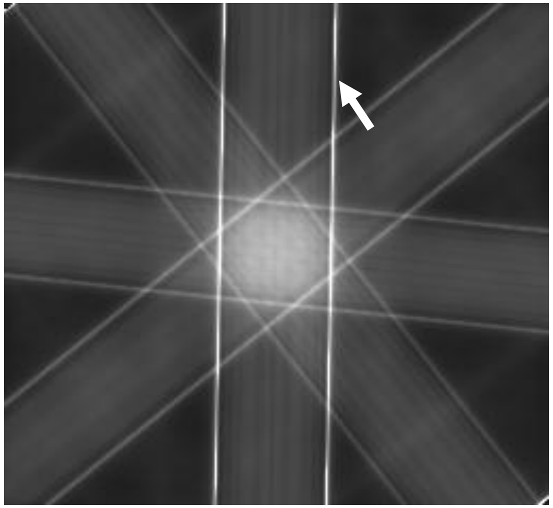
$$\sum_{j=1}^{4}\frac{R_{\Phi_{4,j}}^{T}s_{4,j}}{R_{\Phi_{4,j}}^{T}C}$$

*

$1/N_p$: One over Num Proj. (=1/4)

=

$$\frac{1}{4}\sum_{j=1}^{4}\frac{R_{\Phi_{4,j}}^{T}s_{4,j}}{R_{\Phi_{4,j}}^{T}C}$$



HW-HYPR Algorithm:

Data Acquisition:

$$s_t = R_{\phi_t}[I_t]$$

t : Time

ϕ_t : Angle of projection at time t

s_t : Projection at time t

I_t : True image at time t

R_{ϕ_t} : Radon iransform for angle ϕ_t

Image Reconstruction:

$$J_t = C\left[\frac{\sum_{i=1}^{N_p} R^u_{\phi_{t_i}}[s_{t_i}]}{\sum_{i=1}^{N_p} R^u_{\phi_{t_i}}[s_{c_{t_i}}]}\right]$$

C : Composite image

N_p : Number of projections per time frame

S_{t_i} : Projection data at time t_i

$S_{c_{t_i}}$: Projection of composite image at time t_i

R^u_ϕ : Transpose of Radon transform (unfiltered backprojection)

J_t : Reconstructed image

HW-HYPR as MART

Normal Equation:

$$R^u_{\phi_{t_i}}[s_{t_i}] = R^u_{\phi_{t_i}}[R_{\phi_{t_i}} I_{t_i}]$$

$$H^t g = H^t H f$$

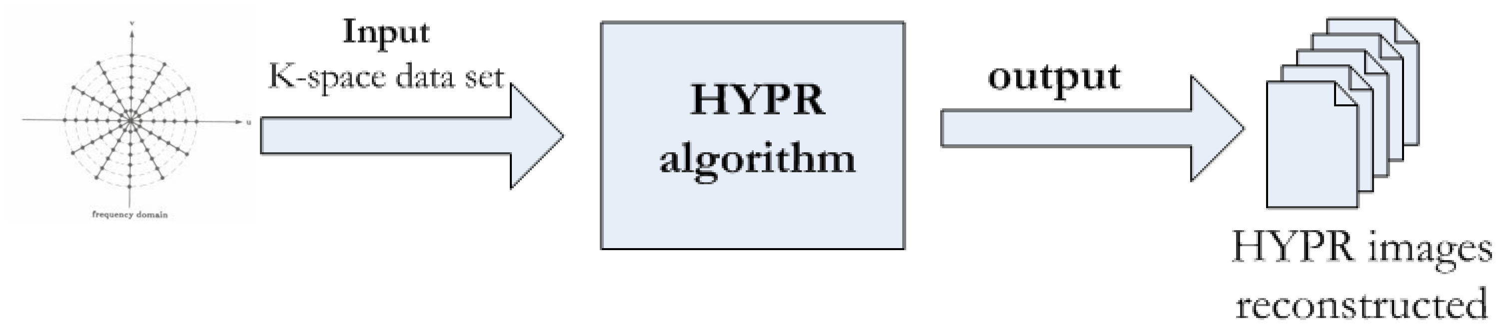
MART: Multiplicative Arithmetic Reconstruction Technique (Gordon, et. al. 1970)

$$J_t = C\left[\frac{\sum_{i=1}^{N_p} R^u_{\phi_{t_i}}[s_{t_i}]}{\sum_{i=1}^{N_p} R^u_{\phi_{t_i}}[R_{\phi_{t_i}} C]}\right]$$

$$f^1 = f^o\left[\frac{H^t g}{H^t H f^o}\right]$$

MART is a fixed point iteration that at convergence solves the normal equation using a multiplicative update. The solution to the normal equations is the least-squares solution. It is possible that this is why HW-HYPR has better noise characteristics than HYPR.

HYPR Input-Output view



The HYPR algorithm uses a composite image and a weighting image during reconstruction in order to generate images with reduced artifacts and with improved temporal characteristics

Weighting image:
- Good temporal characteristics
- low SNR
- low spatial characteristics

Composite image:
- Poor temporal characteristics
- good SNR
- good spatial characteristics
- less streak artifacts

HYPR image:
- good temporal characteristics
- good SNR
- good spatial characteristics
- reduced artifacts

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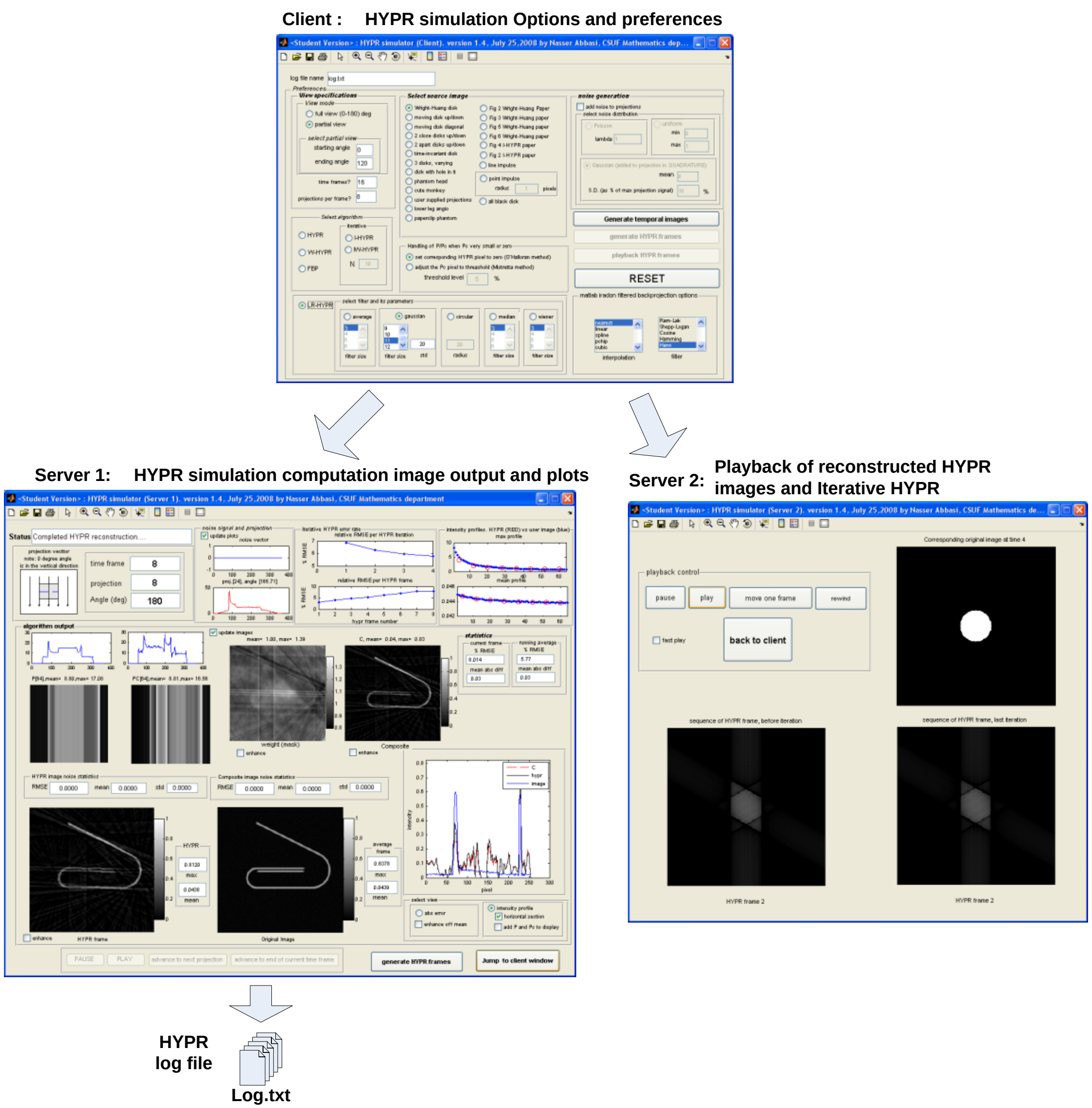
HYPR Simulator Review



- Over 8,000 lines of Matlab code.
- Implement HYPR, W-HYPR, FBP, I-HYPR, IW-HYPR, and HYPR-LR.
- Implements a noise process as a separate work flow.
- Three types of noise can be added: Normal, Poisson and Uniform
- Detailed log file contains all the results and statistics generated.
- HYPR-LR implemented with five different low pass filters.
- Allows the user to load their own set of projection data.
- More than 20 prepackaged test image cases included.
- Allows the user to play back the final reconstructed HYPR images…and many more features…

Comparing the Algorithms

Algorithm	Advantages	Disadvantages
O-HYPR	Suitable to use with images with high sparsity and limited object movements.	• Crosstalk when objects are close to each others. • Difficulty with images that exhibit significant spatial and temporal dynamics.
W-HYPR	• Better noise response than O-HYPR, higher SNR.	Similar to O-HYPR.
LR-HYPR	• Can be applied to images acquired with arbitrary k-space trajectories. • Reconstruction time is shorter than with for iterative methods or O-HYPR.	• Crosstalk still exists. Use of sliding window can reduce this problem. • Using inappropriate low pass filter type and parameters can result in worst reconstruction.
I-HYPR	Improves temporal characteristics and accuracy.	Noise amplified making reconstruction worst.
IW-HYPR	Suppresses noise amplification more than I-HYPR.	Noise is still amplified (but at lower levels).

Accuracy of Algorithms Using the GE Phantom Clip

Results of two tests cases, one with noise (zero mean, 5% S.D. of maximum projections) and one without noise. Both used 8 time frames and 8 projections per time frame. For Iterative HYPR, 10 iterations were used. HYPR-LR used the circular low pass filter with size 20 pixels.

test	O-HYPR	W-HYPR	HYPR-LR	I-HYPR 1^{st}	I-HYPR 10^{th}	IW-HYPR 1^{st}	IW-HYPR 10^{th}
No noise	6.83	6.77	6.7	6.83	5.41	6.77	5.55
With noise	10.76	9.55	13.7	10.76	14.22	9.55	11.436

Wright-Huang based HYPR algorithms have the best overall results.

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Slides from Midterm Report

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HYPR Algorithm:

Data Acquisition:

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t : Time

ϕ_t : Angle of projection at time t

s_t : Projection at time t

I_t : True image at time t

R_{ϕ_t} : Radon iransform for angle ϕ_t

Image Reconstruction:

$$J_t = C\left[\frac{1}{N_p}\sum_{i=1}^{N_p}\frac{R^u_{\phi_{t_i}}[s_{t_i}]}{R^u_{\phi_{t_i}}[s_{c_{t_i}}]}\right]$$

C : Composite image

N_p : Number of projections per time frame

S_{t_i} : Projection data at time t_i

$S_{c_{t_i}}$: Projection of composite image at time t_i

R^u_ϕ : Transpose of Radon transform (unfiltered backprojection)

J_t : Reconstructed image

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MLEM Algorithm:

Let $g = Hf + \varepsilon$

H : Matrix Projection Operator

f: Discrete Object

g: Projection

ε: Poisson Noise

MLEM maximizes the likelihood that g came from f.

MLEM Algorithm: $f_n^{(k+1)} = \left(\frac{f_n^{(k)}}{s_n}\right)\sum_m \left(\frac{g_m}{\left(Hf^{(k)}\right)_m} H_{mn}\right)$ *where* $s_n = \sum_m H_{mn}$

Using Matrix Notation Unfiltered Backprojection is H^T :

$$\Rightarrow \quad f_n^{(k+1)} = f_n^{(k)} \frac{1}{s_n}\left(H^T\left[\frac{g}{Hf^{(k)}}\right]\right)_n$$

**Notation adopted from Foundations of Image Science, by Barrett and Myers*

Comparison of MLEM & HYPR

MLEM-1 Algorithm

HYPR

$$g = Hf + \varepsilon \quad \text{-------} \quad s_t = R_{\phi_t}\left[I_t\right]$$

$$f_n^{(1)} = f_n^{(0)} \frac{1}{z_n}\left[H^{\mathrm{T}}\left[\frac{g}{(Hf^{(0)})}\right]\right]_n \quad \text{-------} \quad J_n = \frac{1}{N_p} C_n \cdot R_\phi^u \left(\frac{s}{R_\phi(C)}\right)_n$$

H ------- R_ϕ – *Radon Transform*

H^T ------- R_ϕ^u – *Unfiltered backprojection*

z_n ------- N_p – *Projections per time frame*

g ------- s – *Original projection*

$f^{(0)}$ ------- C – *Composite image*

For this method to match the original HYPR in the first iteration we need that

$$\frac{R_\phi^u(s)}{R_\phi^u(s_c)} = R_\phi^u\left(\frac{s}{s_c}\right)$$

The ratio of unfiltered backprojections is the unfiltered backprojection of the ratio.

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Relevant Properties of HYPR/MLEM

- Multiplicative update on each iteration, so if the initial estimate is zero, subsequent estimates remain zero. This property reduces streaking artifacts by using the composite image as the initial guess.

- Enforces non-negativity constraint. If initial estimate is positive and H has non-negative entries, future updates remain non-negative.

- MLEM while popular in the research community, adoption in clinical nuclear medicine was slow because of unpredictable nature of artifacts. This may be something to discuss with clinical collaborators.

- Noise properties for time resolved MRA very different than in nuclear medicine where the major source of noise is the Poisson noise in the projections.

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Computational Comparison of HYPR, HYPR-W & MLEM-1

- In the following slide we compare Original HYPR to 1-step MLEM algorithm
- Time-invariant disk used
- 128 projection angles used (bit-reversed ordering)
- Window size: 8 projections
- Also implemented HYPR-W (Huang and Wright)
- For a stationary disk, all methods give a similar result.

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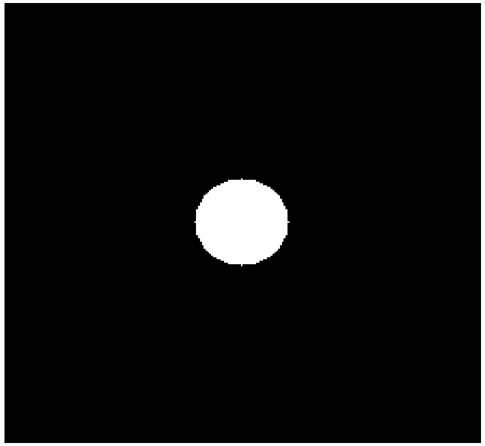
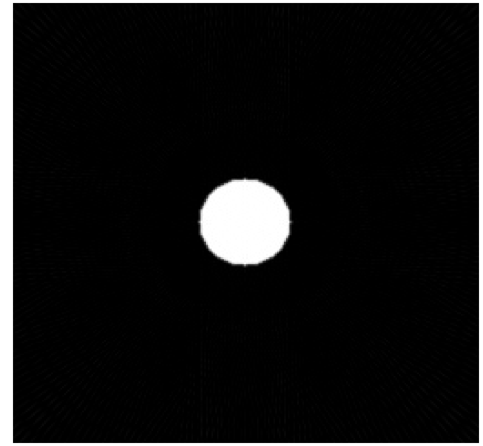
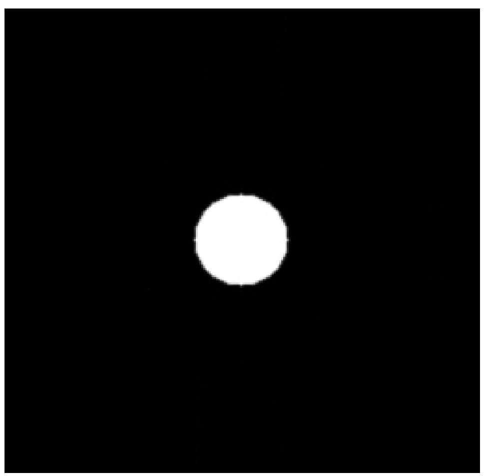
actual image



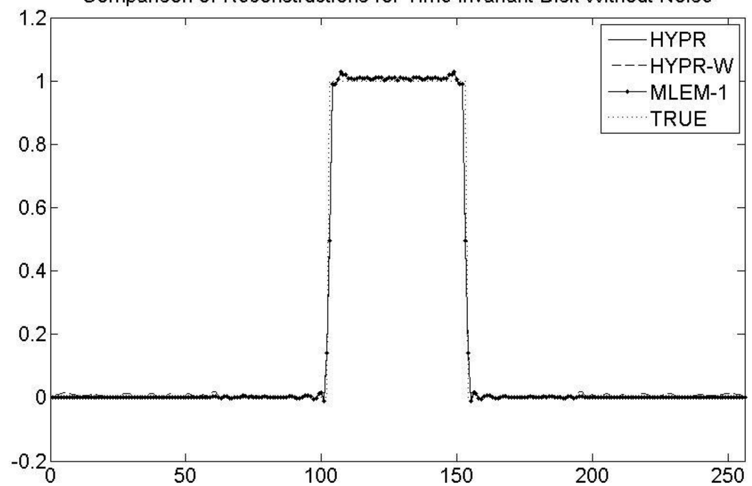
composite image



HYPR Reconstruction for Projections 1-8



Comparison of Reconstructions for Time Invariant Disk Without Noise



For a stationary disk, with no noise, all methods are similar.

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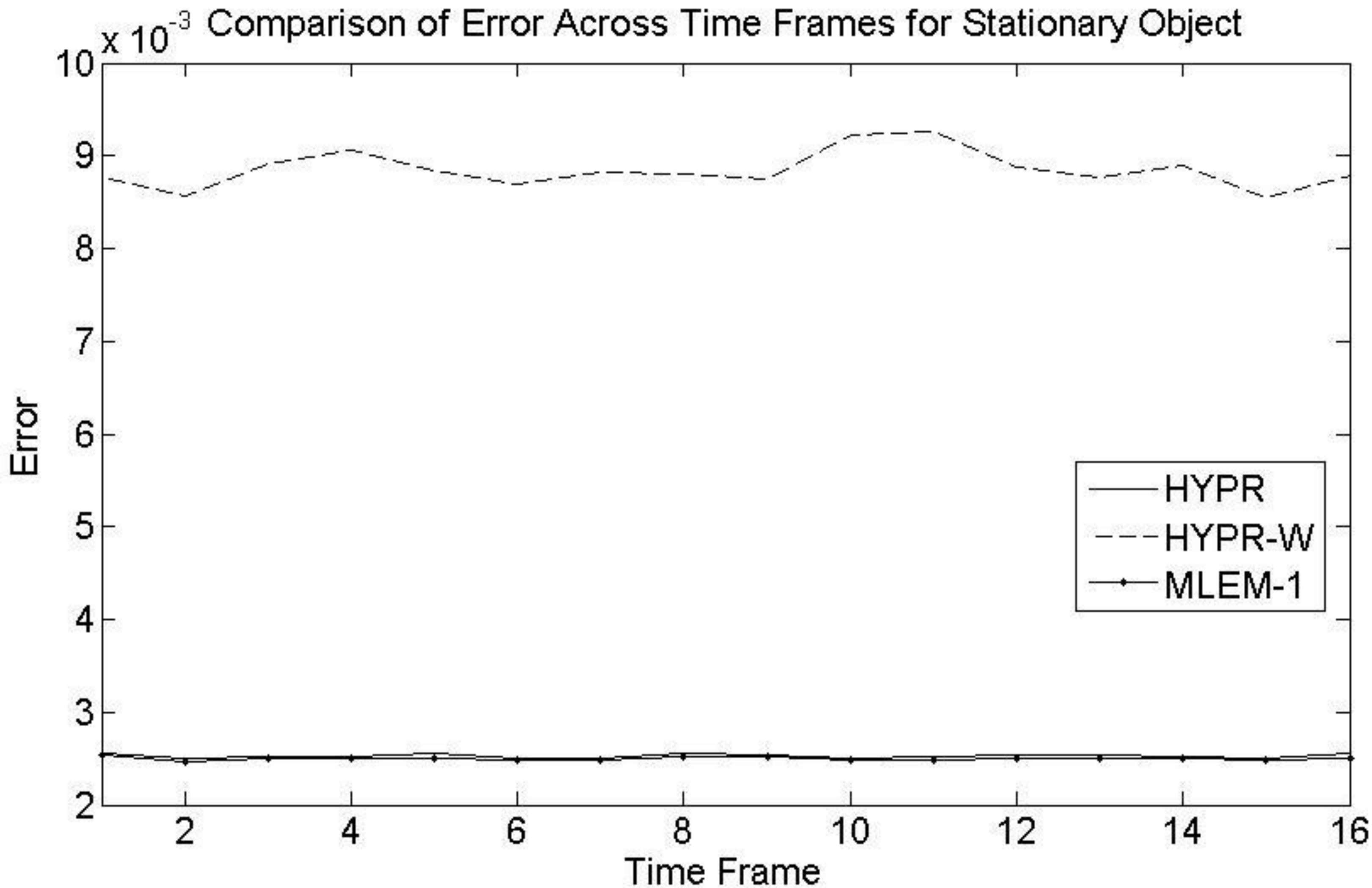


MLEM-1 and HYPR are the same method with different implementation.

MATLAB Computational Workbench

Noise distribution selection menu. Noise is added to projection from user images

The name of the log file (just the name, do not include path information)

This window is used only by the base test validation (the Wright-Huang disk)

Statistics on current frame and running average is displayed here

Use this menu to select view angles (full or limited and range)

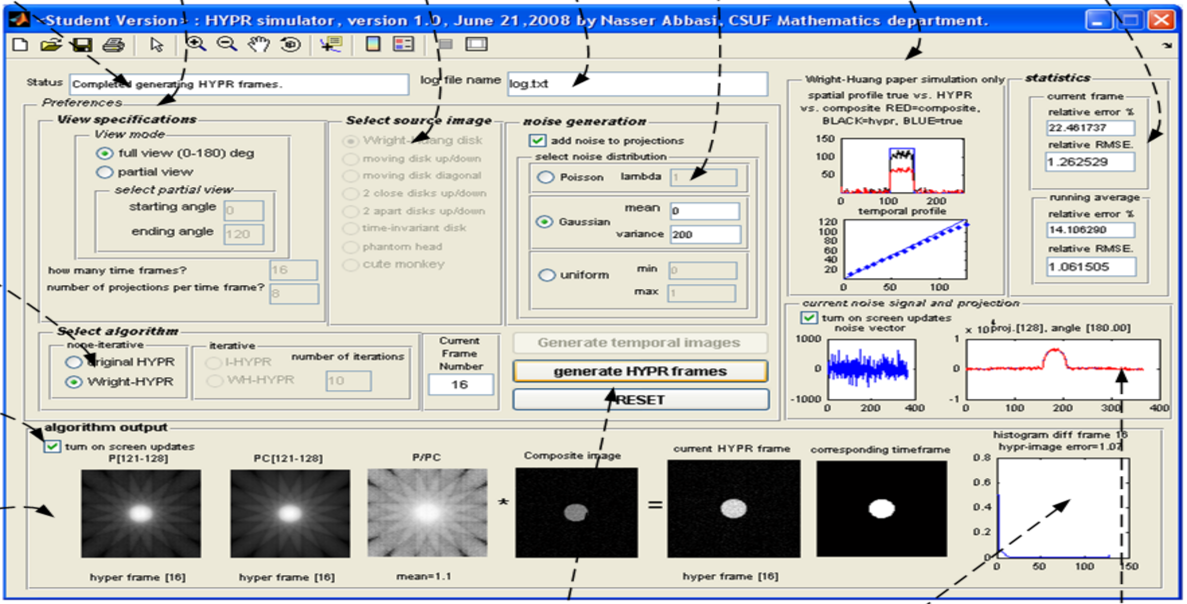
Use this menu to select the test image

Status windows. Displays current simulator state

Select the algorithm of HYPR to use. Iterative HYPR is not currently enabled as it is under development

Select this option to temporally stop displaying all outputs. This can speed up simulation.

This row of images shows the P, PC, mask, composite and HYPR frame image, and averaged time frame image. These are updated for each time frame.

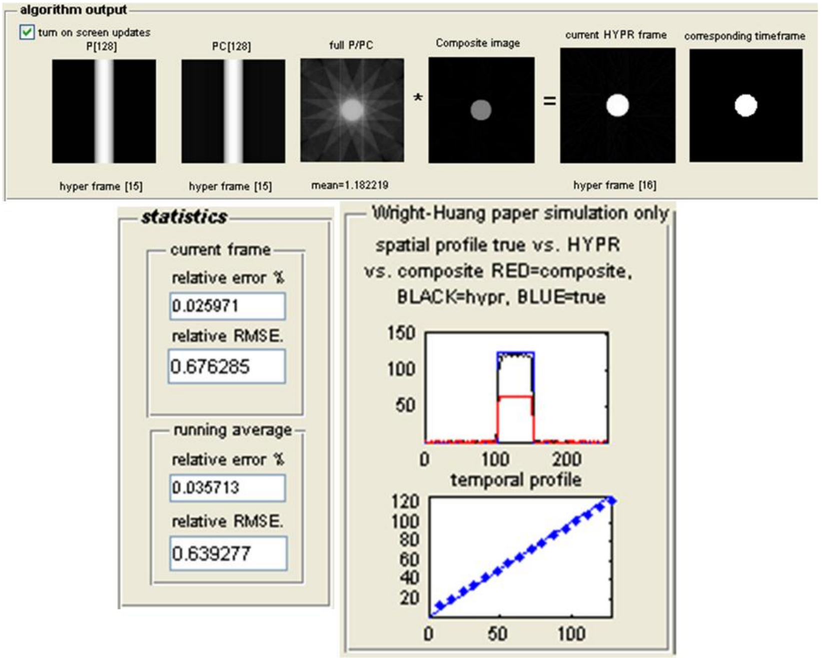


Main control of the simulator is located here. Allows one to generate user images, then HYPR images. User RESET to select a new test

Histogram difference between HYPR image and corresponding averaged time frame is displayed here (bins are gray levels). And running average of error found

Current projection vector is displayed here with noise vector if any

Time Dependent Intensity with Stationary Boundary



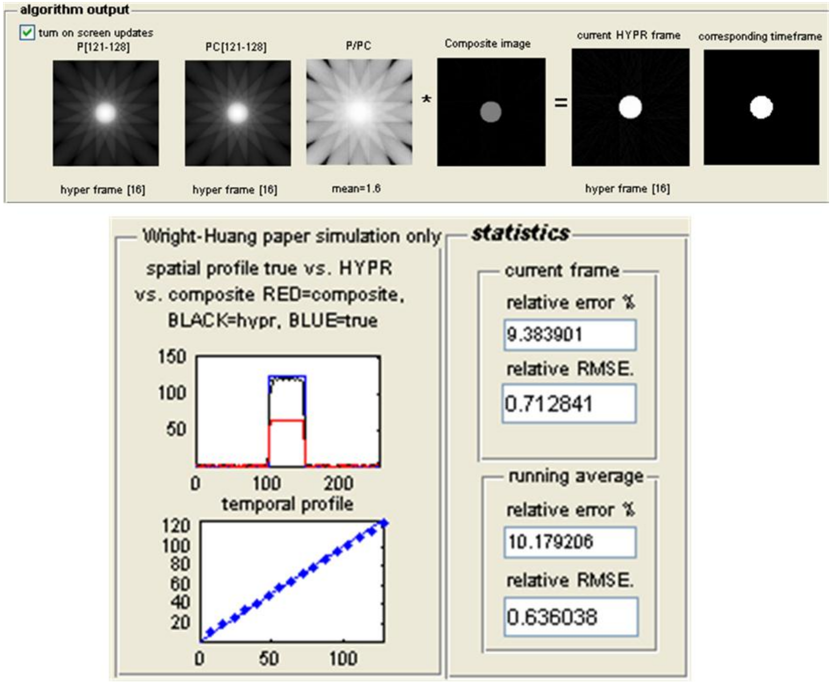

```
generated data for simulation [wrightPaperDiskTag]
Completed image generation...
number of time frames = [16]
number of projections per time frame = [8]
[22-Jun-2008 13:56:42]Enter generate HYPR ...

Enter original HYPR ...
No NOISE is being added
```

frame	rmse	relativeError	rmse_averaged	relativeError_averaged	averageMask
1	1.026815	0.124615	1.026815	0.124615	0.810999
2	0.789398	0.080273	0.908107	0.102444	0.834959
3	0.658115	0.049115	0.824776	0.084668	0.856301
4	0.579282	0.057853	0.763403	0.077964	0.888973
5	0.551421	0.015053	0.721006	0.065382	0.905486
6	0.539332	0.016687	0.690727	0.057266	0.934073
7	0.544661	0.011765	0.669861	0.050766	0.957154
8	0.555759	0.021502	0.655598	0.047108	0.988724
9	0.565009	0.019138	0.645533	0.044000	1.005649
10	0.582615	0.021568	0.639241	0.041757	1.030899
11	0.600318	0.024314	0.635702	0.040171	1.053254
12	0.618896	0.016745	0.634302	0.038219	1.085530
13	0.629853	0.033764	0.633959	0.037876	1.101912
14	0.649074	0.025293	0.635039	0.036977	1.132088
15	0.661592	0.027750	0.636809	0.036362	1.153034
16	0.676285	0.025971	0.639277	0.035713	1.182219

```
Done,  totalHistError =1.33 ...
```

Running the Wright-Huang test case using the original HYPR algorithm




```
Enter Wright-Huang HYPR ...
No NOISE is being added
```

frame	rmse	relErr	MeanRmse	MeanRelErr	MeanMask
1	0.982878	38.035191	0.982878	38.035191	0.396131
2	0.734453	26.583094	0.858665	32.309142	0.483218
3	0.604836	19.085338	0.774055	27.901208	0.557789
4	0.532656	14.054107	0.713705	24.439433	0.639344
5	0.520571	9.217883	0.675079	21.395123	0.715395
6	0.519005	5.920670	0.649066	18.816047	0.798848
7	0.535644	3.097390	0.632863	16.570525	0.877456
8	0.556542	0.955617	0.623323	14.618661	0.958114
9	0.574512	1.083784	0.617900	13.114786	1.035286
10	0.599367	2.818749	0.616046	12.085182	1.114582
11	0.621385	4.294862	0.616532	11.376971	1.193476
12	0.645831	5.560250	0.618973	10.892245	1.277558
13	0.658385	6.633323	0.622005	10.564635	1.353904
14	0.682260	7.683917	0.626309	10.358870	1.434595
15	0.695441	8.459228	0.630918	10.232227	1.508504
16	0.712841	9.383901	0.636038	10.179206	1.596524

```
Done, totalHistError =1.35 ...
```

Test 1b results

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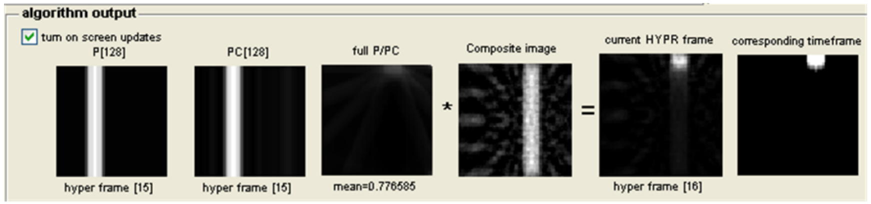
Time Dependent Boundary with Constant Intensity



generated data for simulation [movingDiskUpDownTag]
Completed image generation...
number of time frames = [16]
number of projections per time frame = [8]
[22-Jun-2008 16:03:26]Enter generate HYPR ...

Enter original HYPR ...
No NOISE is being added

frame	rmse	relErr	MeanRmse	MeanRelErr	MeanMask
1	2.930777	0.112860	2.930777	0.112860	0.776031
2	2.283642	0.032046	2.607209	0.072453	0.870046
3	2.468333	0.049740	2.560917	0.064882	0.837949
4	2.615779	0.060302	2.574633	0.063737	0.833535
5	2.693877	0.087963	2.598481	0.068582	0.829808
6	2.653286	0.004072	2.607615	0.057831	0.846449
7	2.686605	0.044205	2.618900	0.055884	0.846036
8	2.745714	0.132117	2.634751	0.065413	0.833106
9	2.756394	0.134185	2.648267	0.073055	0.833394
10	2.683672	0.051199	2.651808	0.070869	0.846576
11	2.643038	0.005699	2.651011	0.064944	0.846128
12	2.694688	0.089126	2.654650	0.066960	0.831088
13	2.621380	0.046047	2.652091	0.065351	0.833980
14	2.467277	0.046980	2.638890	0.064039	0.838185
15	2.287501	0.025182	2.615464	0.061448	0.869622
16	2.926942	0.116627	2.634932	0.064897	0.776585

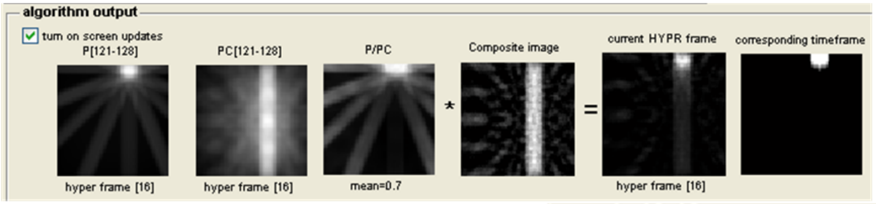
Done, totalHistError =1.16 ...

statistics

current frame
relative error %
0.116627
relative RMSE.
2.926942

running average
relative error %
0.064897
relative RMSE.
2.634932

Test 5a



Enter Wright-Huang HYPR ...
No NOISE is being added

frame	rmse	relErr	MeanRmse	MeanRelErr	MeanMask
1	3.621499	8.250962	3.621499	8.250962	0.683032
2	2.683754	2.802784	3.152626	5.526873	0.845110
3	2.913513	3.013436	3.072922	4.689061	0.876864
4	3.029160	6.509401	3.061982	5.144146	0.909015
5	3.119542	8.525960	3.073494	5.820509	0.913877
6	3.110399	7.673412	3.079645	6.129326	0.926160
7	3.120744	8.473927	3.085516	6.464269	0.938696
8	3.161076	11.227506	3.094961	7.059673	0.927934
9	3.169451	11.224866	3.103238	7.522473	0.927989
10	3.118828	8.468160	3.104797	7.617041	0.938733
11	3.101359	7.706259	3.104484	7.625152	0.926111
12	3.120562	8.506358	3.105824	7.698586	0.913934
13	3.035838	6.467908	3.100440	7.603918	0.909125
14	2.913264	3.004449	3.087071	7.275385	0.876923
15	2.684876	2.781627	3.060258	6.975801	0.845124
16	3.616383	8.230268	3.095016	7.054205	0.682970

Done, totalHistError =1.20 ...

Test 5b

statistics

current frame
relative error %
8.230268
relative RMSE.
3.616383

running average
relative error %
7.054205
relative RMSE.
3.095016