Small note on finding Euler circuits in connected graphs using Fleury algorithm

Nasser M. Abbasi

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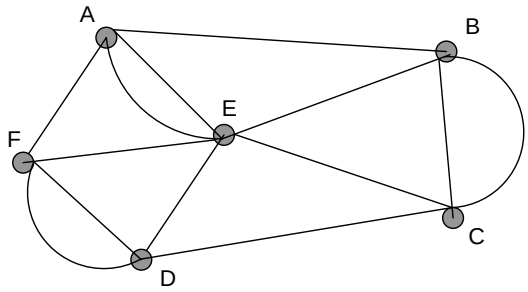
1 Example 1



Only connected graphs with has no vertices of odd degree can have an Euler circuit.
So the above graphs must have at least one Euler circuit. To find, use Fleury algorithm:
1. Start at any vertex.
2. Pick an edge to travel. If one of the edges is a bridge (Going along it, there is no way to come back to the vertex other than on it, then do not select it, unless it is the one edge left from that vertex).
2. Remove the edge traveled. If a vertex left with no edges leaving it, remove the vertex.
3. repeat step 2 until no more edges left.
4. The order of the edges traveled is the Euler circuit.

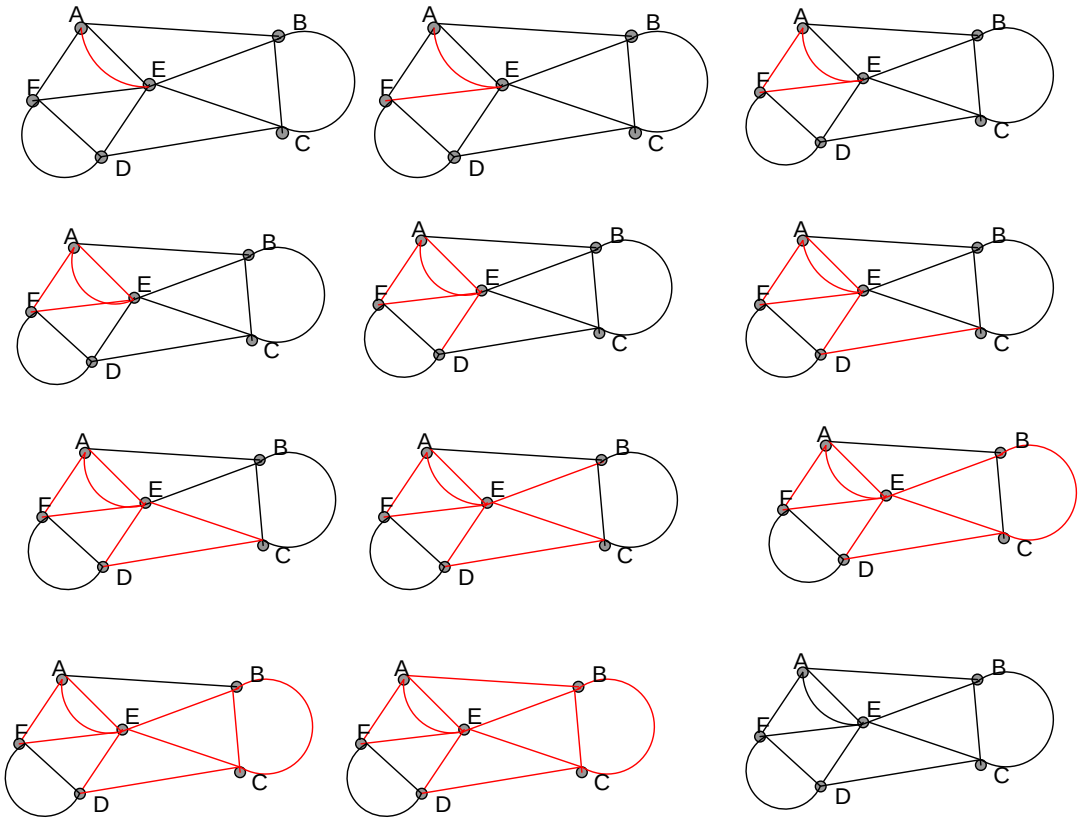


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Figure 1: Example 1

2 Example 2

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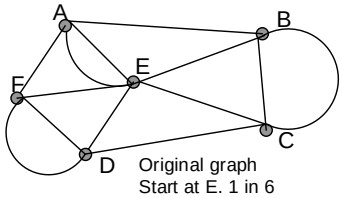
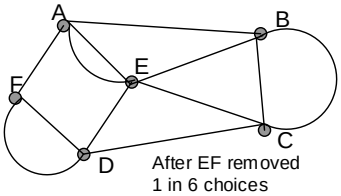
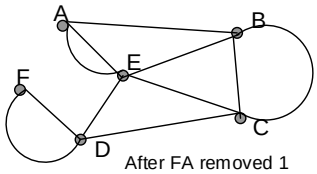


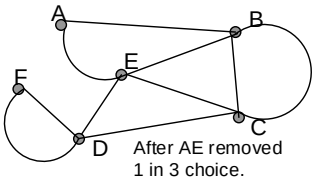
Original graph
Start at E. 1 in 6 choice.



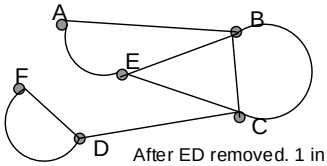
After EF removed
1 in 6 choices



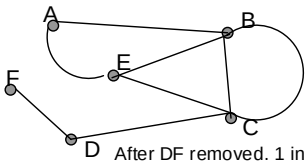
After FA removed 1 in 4 choices.



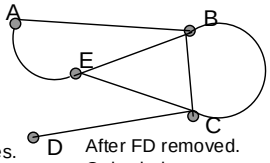
After AE removed
1 in 3 choice.



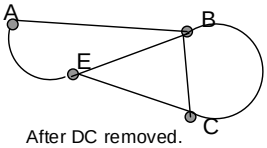
After ED removed. 1 in 3 choice. Notice now have option of going DC or DF.



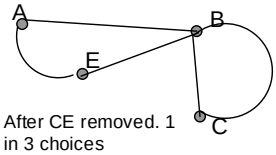
After DF removed. 1 in 2 choices. Notice, can not travel DC, since DC is a bridge.



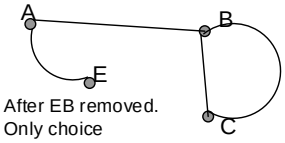
After FD removed.
Only choice.



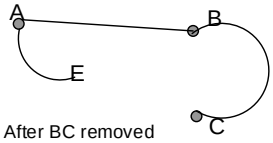
After DC removed.
Only choice



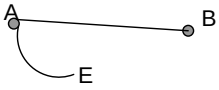
After CE removed. 1 in 3 choices



After EB removed.
Only choice



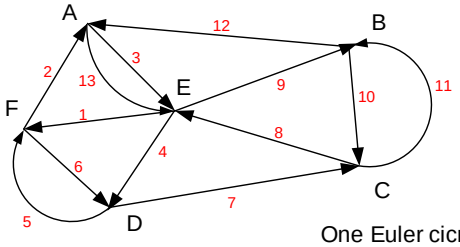
After BC removed
1 in 2 choices



After CB removed

After BA removed

After AE removed. Back to starting vertex, and no edges left. Euler circuit.



One Euler cicruit.

Showing Euler circuit. Visit each edge once, and visit all edges. Starting from E and back to E.

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Figure 2: Example 2

3 Example 3

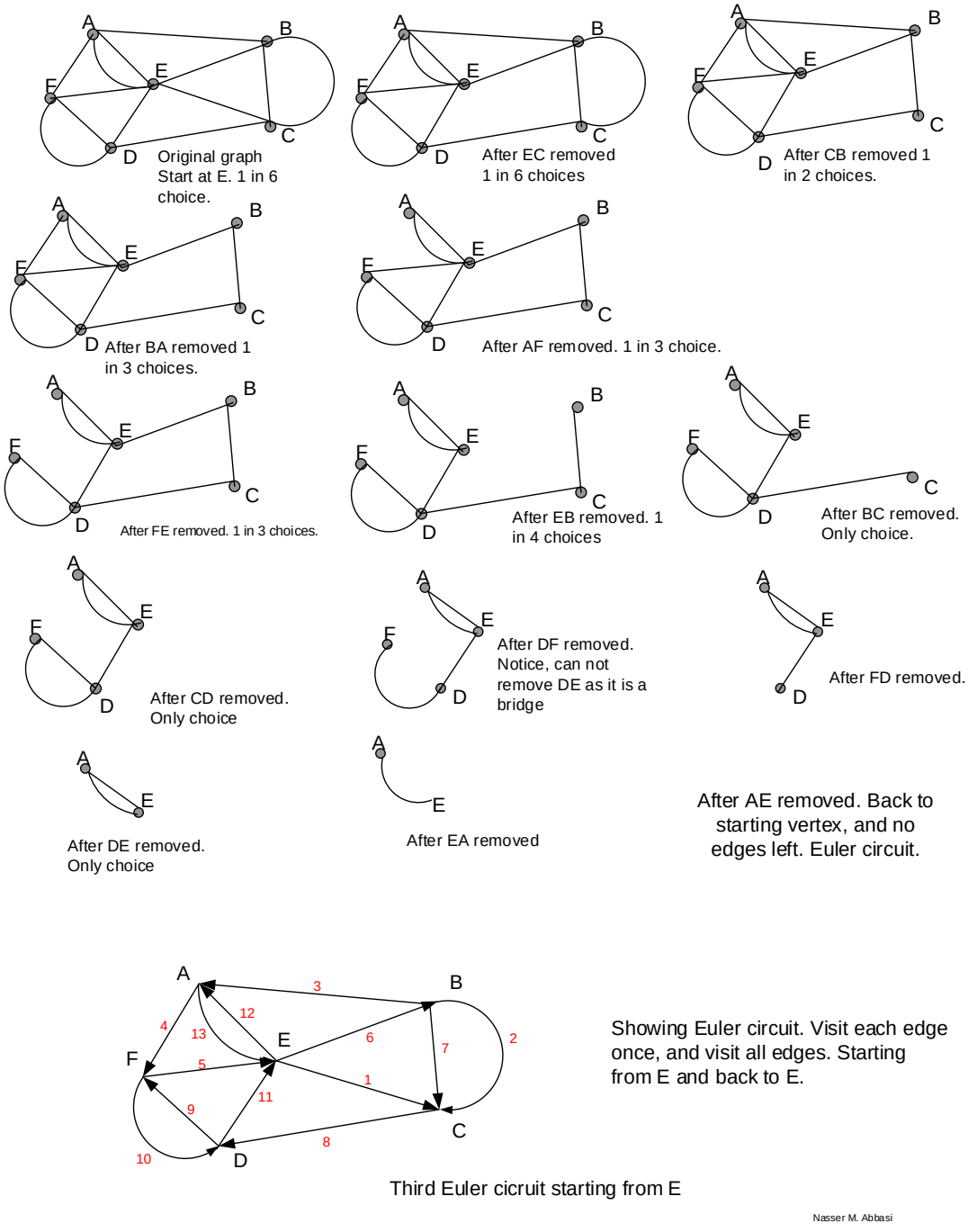
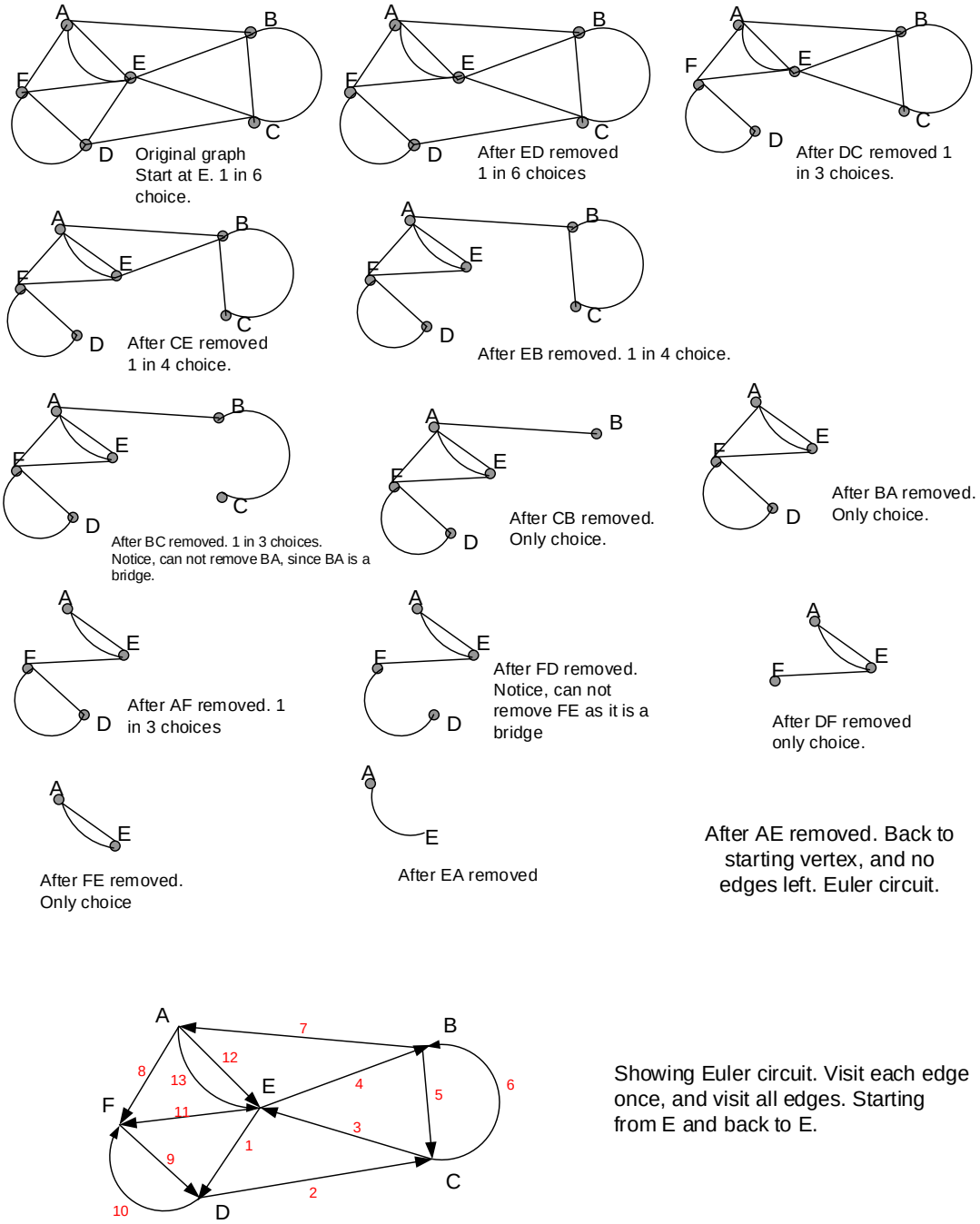


Figure 3: Example 3

4 Example 4



Showing Euler circuit. Visit each edge once, and visit all edges. Starting from E and back to E.

second Euler cicruit starting from E

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Figure 4: Example 4

Reference: `https://en.wikipedia.org/wiki/Eulerian_path` is reference on Euler circuits.