My control systems cheat sheet

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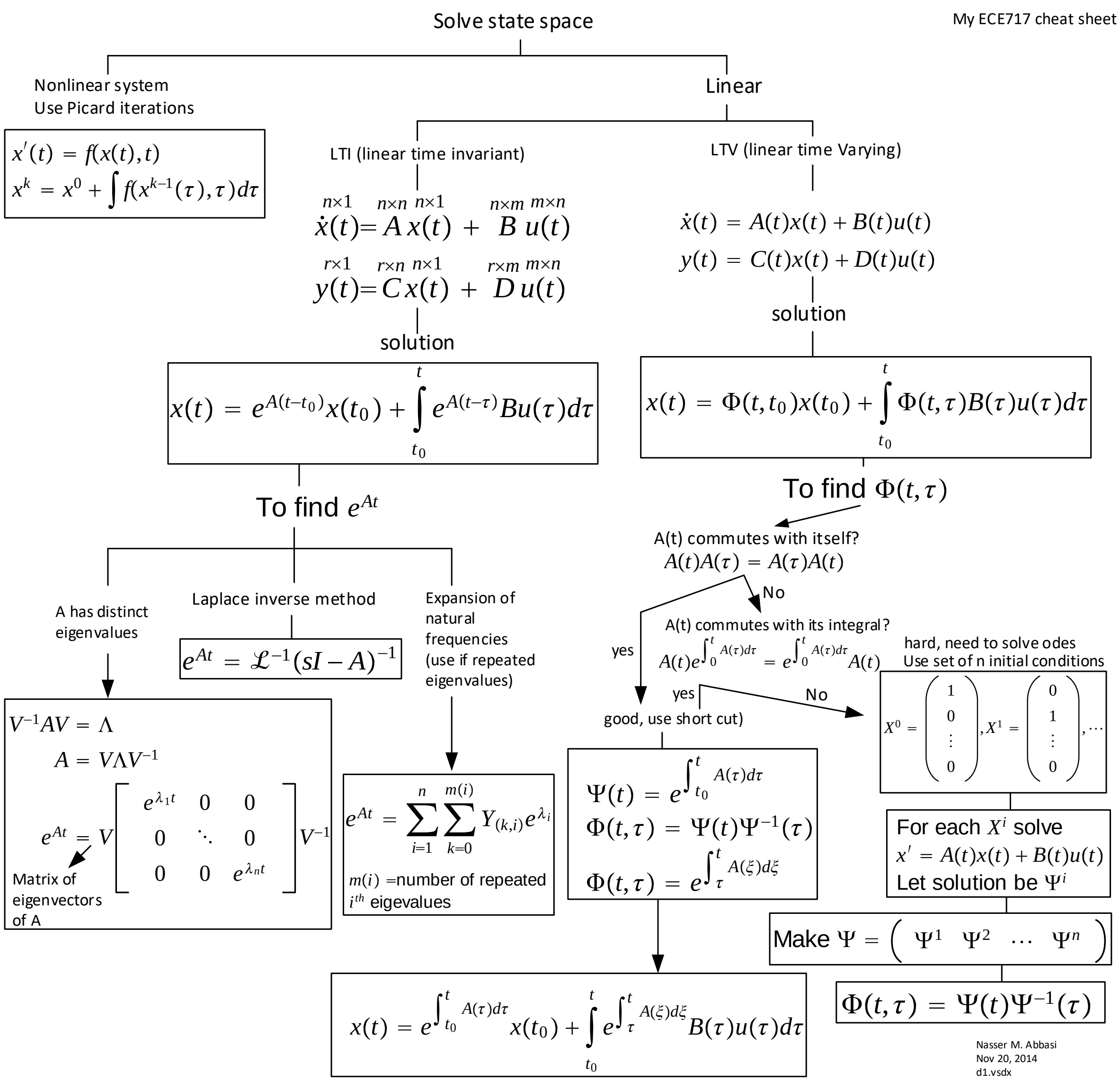

Figure 1: Flow chart to solve state space $x' = Ax$

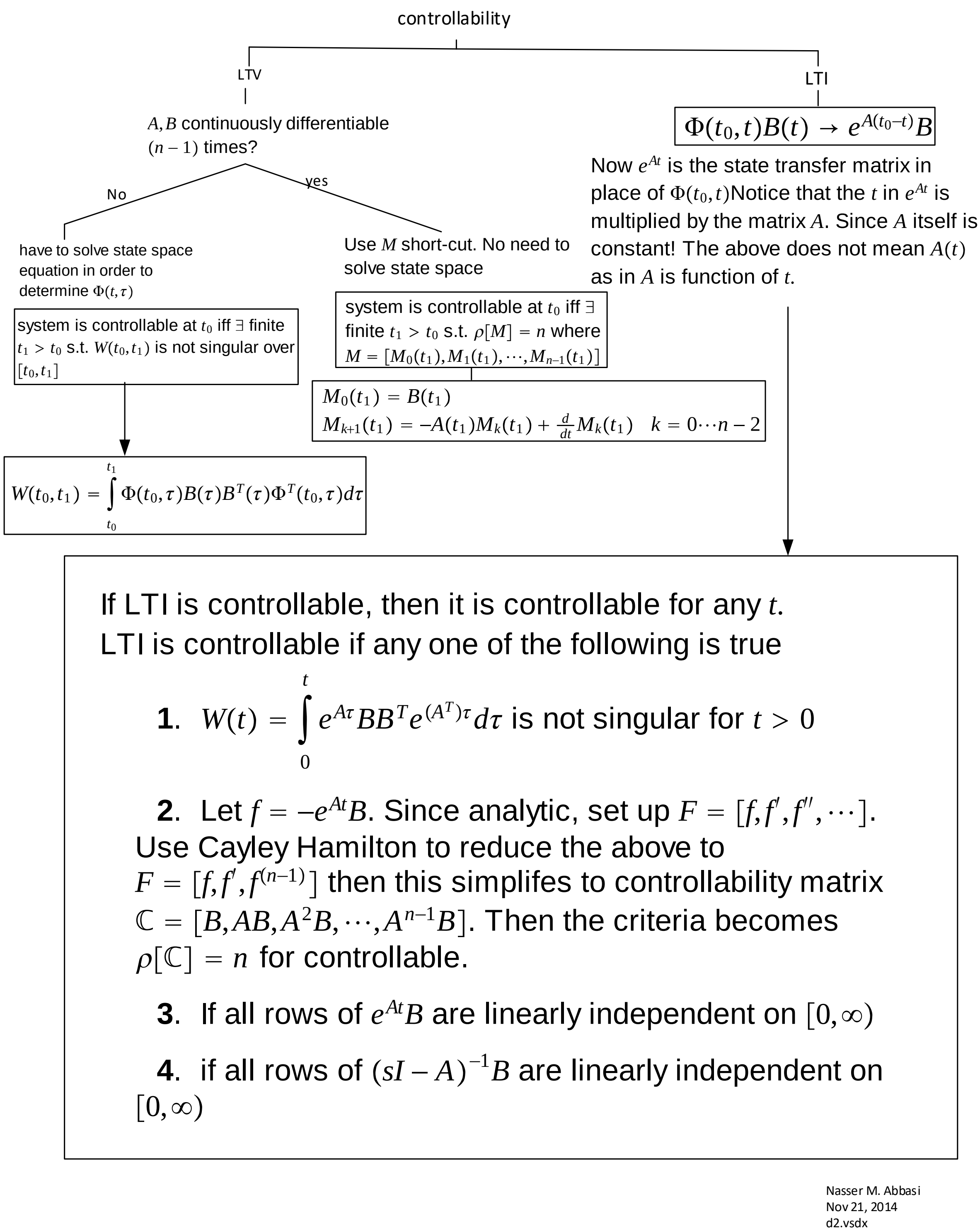

Figure 2: Controllability flow chart

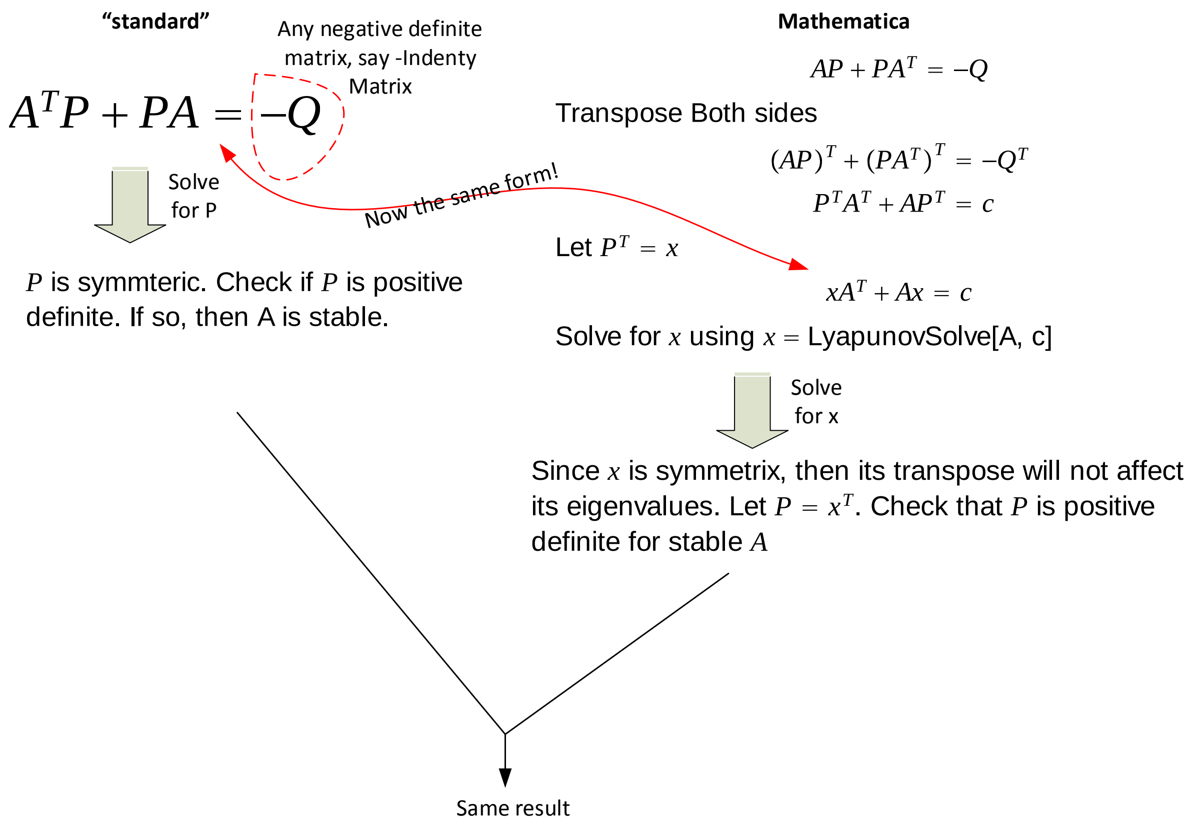

Figure 3: Lyupnov flow chart

Lyapunov stability. All what this says, is that if we start with any initial conditions $x_0(t_0)$ at time t_0 near the origin, then if solution $x(t)$ is always bounded from above for any future time t, then we say that the origin is stable equilibrium point.

To make this more mathematically precise, we say that for any $||x_0|| \leq \delta(t_0,)$ we can find $\epsilon(\delta)$ such that $||x(t)|| \leq \epsilon$ for any $t \geq t_0$.

In this both δ and ϵ are some positive quantities. And ϵ depends on choice of δ

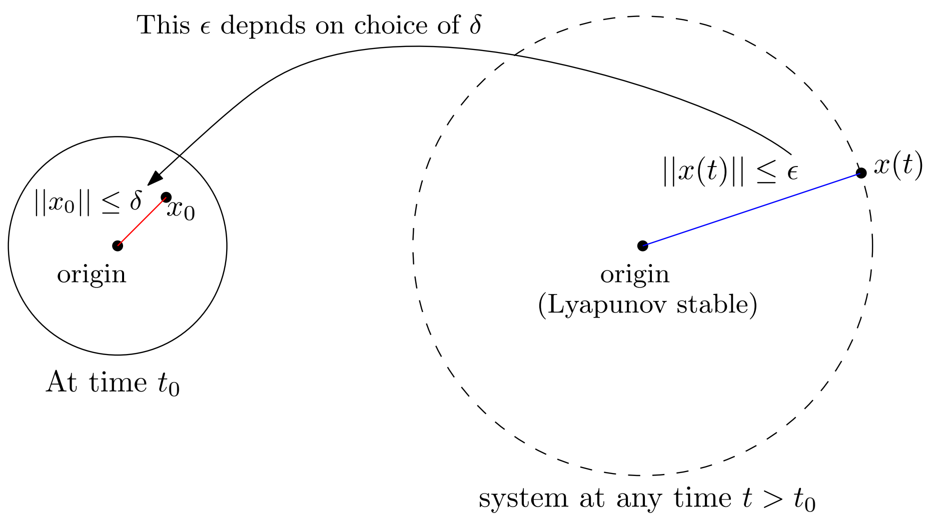

Figure 4: Lyupnov stability definition

Duality in linear time varying systems

Primal

$$x'(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$
$$\Phi(t_0,\tau) = \Psi(t_0)\Psi^{-1}(\tau)$$
$$\frac{d}{dt}\Psi^{-1}(t) = -\Psi^{-1}(t)A(t)$$

Transpose both sides

$$\frac{d}{dt}\left[\Psi^{-1}(t)\right]^T = \left[-\Psi^{-1}(t)A(t)\right]^T$$
$$= -A^T(t)\left[\Psi^{-1}(t)\right]^T$$

Duel

$$\tilde{x}'(t) = \tilde{A}(t)\tilde{x}(t) + \tilde{B}(t)u(t)$$
$$\tilde{y}(t) = \tilde{C}(t)\tilde{x}(t) + \tilde{D}(t)u(t)$$
$$\tilde{\Phi}(t_0,\tau) = \tilde{\Psi}(t_0)\left[\tilde{\Psi}(\tau)\right]^{-1}$$

$$\frac{d}{dt}\tilde{\Psi}(t) = \tilde{A}(t)\tilde{\Psi}(t)$$

if $\boxed{\tilde{A}(t) = -A(t)}$ then

$$\frac{d}{dt}\tilde{\Psi}(t) = -A(t)\tilde{\Psi}(t)$$

compare

$$\tilde{\Psi}(t) = \left[\Psi^{-1}(t)\right]^T = \left[\Psi^T(t)\right]^{-1}$$

Hence $\left[\tilde{\Psi}(\tau)\right]^{-1} = \Psi^T(\tau)$

combine

$$\tilde{\Phi}(t_0,\tau) = \left[\Psi^{-1}(t_0)\right]^T\Psi^T(\tau)$$
$$= \left[\Psi(\tau)\Psi^{-1}(t_0)\right]^T$$

Hence

$$\tilde{\Phi}(t_0,\tau) = \Phi^T(\tau,t_0)$$

Summary

$$\begin{aligned}
\tilde{\Phi}(t_0,\tau) &= \Phi^T(\tau,t_0)\\
&= \left[\Psi(\tau)\Psi^{-1}(t_0)\right]^T\\
&= \left[\Psi^{-1}(t_0)\right]^T\Psi^T(\tau)\\
\frac{d}{dt}\tilde{\Psi}(t) &= -A(t)\tilde{\Psi}(t)\\
\tilde{A}(t) &= -A(t)\\
\tilde{\Psi}(t) &= \left[\Psi^{-1}(t)\right]^T\\
&= \left[\Psi^T(t)\right]^{-1}\\
\left[\tilde{\Psi}(t)\right]^{-1} &= \Psi^T(t)
\end{aligned}$$

References:
1. Principles of linear systems, Sarachick, pages 160-161
2. Linear system theory and design by Chen, first edition, pages 195-196

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Figure 5: Duality in Linear time varying

Graphical methods for linear control systems analysis (SISO)

root locus (time domain)

Bode plot (frequency domain)

Nyquist plot (frequency domain)

Nichols plot (frequency domain)

Performace specifications

frequency based specifications

time domain specification

specification	note
Gain margin	measure absolute stability.
Phase margin	measure relative stability, tendency to oscillate during damped response
delay time T_d	speed of response as function of frequency ω, defined as $T_d(\omega) = -\frac{d\gamma}{d\omega}$ where γ is phase of open loop transfer function

Figure 6: Graphical methods for linear control system analysis