my digital communications cheat sheet

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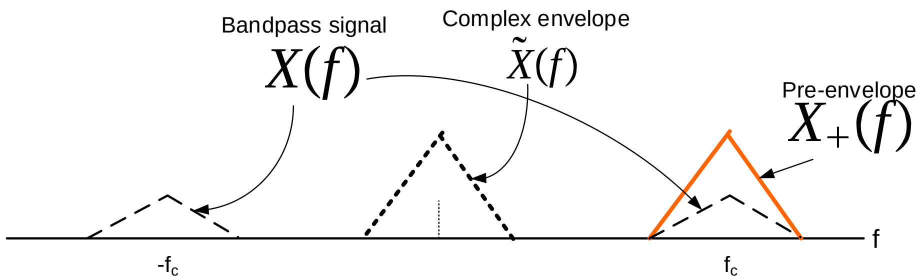
Contents

1 What is the relation between bandpass, baseband ,complex envelop and pre envelop?



$$\tilde{X}(f) = X_+(f + f_c)$$
$$\tilde{x}(t) = x_+(t)e^{-j\omega_c t}$$

$$X_+(f) = 2X(f) \times U(f)$$
$$x_+(t) = 2\left(x(t) \circledast \left(\frac{\delta(t)}{2} + \frac{j}{2\pi t}\right)\right)$$
$$x_+(t) = x(t) + j\hat{x}(t)$$

$$\tilde{x}(t) = [x(t) + j\hat{x}(t)]e^{-j\omega_c t}$$
$$\tilde{x}(t)e^{j\omega_c t} = x(t) + j\hat{x}(t)$$
$$\mathrm{Re}[\tilde{x}(t)e^{j\omega_c t}] = x(t) \longrightarrow \text{BandPass signal}$$

BasePass signal **Shifts frequency To band pass**

Complex envelope

$$\tilde{x}(t) = x_I(t) + jx_Q(t)$$

2 ways to find Hilbert transform :

$$\hat{x}(t) = x(t) \circledast \frac{1}{\pi t}$$

Hilbert transform of x(t)

$$\hat{x}(t) = F^{-1}\left[-j\ sgn(f)\ X(f)\right]$$

In phase component **quadrature component**

$$s(t) = s_I(t)\cos\omega_c t - s_Q(t)\sin\omega_c t$$

$$s_I(t) = s(t)\cos\omega_c t + \hat{s}(t)\sin\omega_c t$$
$$s_Q(t) = \hat{s}(t)\cos\omega_c t - s(t)\sin\omega_c t$$

$$s(t) = a(t)\cos(\omega_c t + \Theta(t))$$

Envelope of s(t) Phase of s(t)

$$a(t) = \sqrt{s_I^2(t) + s_Q^2(t)} \qquad \Theta(t) = \tan^{-1}\left(\frac{s_Q(t)}{s_I(t)}\right)$$

Figure 1: bandpass, baseband ,complex envelop

2 Some useful Fourier Transforms

$x(t)$	$X(f)$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$
$\cos(2\pi f_c t+\theta)$	$\frac{1}{2}\left[e^{j\theta}\delta(f-f_c)+e^{-j\theta}\delta(f+f_c)\right]$
$\sin(2\pi f_c t+\theta)$	$\frac{1}{2}\left[e^{j\theta}\delta(f-f_c)-e^{-j\theta}\delta(f+f_c)\right]$

3 Random process definitions

$X(t)$ is Stationary: If all its statistics do not change with shift of origin

$X(t)$ is Wide Sense Stationary: If the mean is constant, and $R_x(t,t+\tau)=R_x(\tau)$

where autocorrelation $R_x(\tau)$ is defined as $E[x(t)\,x^*(t+\tau)]$. Note, if $X(t)$ is real, then $R_x(\tau)$ is real and even

Note: $R(x)$ must be WSS if it is ergodic.So ergodic process has constant mean.

4 How to determine Hilbert transform of a signal?

input $x(t)$. Find $\hat{x}(t)$ which is Hilbert transform of $x(t)$ defined as $\hat{x}(t)=x(t)\otimes\frac{1}{\pi t}$

An easy way is to first find $\hat{G}(f)$ which is the Fourier transform of $\hat{x}(t)$ and then inverse it to find $\hat{x}(t)$

$$\hat{G}(f)=-j\ sgn(f)\ \ G(f)$$

Where $G(f)$ is Fourier transform of $x(t)$

5 How to find Power Spectrum (PSD) of a random signal $x(t)$

input: random signal $x(t)$

output: PSD of $x(t)$

Algorithm:

1. Find autocorrelation $R_x(\tau)$ of $x(t)$
2. Find the Fourier Transform of $R_x(\tau)$. The result is the PSD of $x(t)$ called $S_x(f)$

Another method (this below works if not random $x(t)$) , why? can't find FT for random process?

1. Find Fourier Transform $X(f)$ of $x(t)$
2. Find the $|X(f)|^2 = X(f)\,X^*(f)$

6 What is the relation between variance and power for a random signal $x(t)$?

Variance is the sum of the total average normalized power and the DC power.

$$\sigma_x^2 = \overbrace{E\left[x^2(t)\right]}^{\text{total Power}} + \overbrace{E[x(t)]^2}^{\text{DC power}}$$

For the a signal whose mean is zero,

$$\sigma_x^2 = \overbrace{E\left[x^2(t)\right]}^{\text{total Power}}$$

How to find average, power, PEP, effective value (or the RMS) of a periodic function?

Let $x(t)$ be a periodic function, of period T, then

$$\text{average of } x(t) = \langle x(t) \rangle = \frac{1}{T}\int_0^T x(t)\,dt$$

The average power is

$$p_{av} = \left\langle x^2(t) \right\rangle = \frac{1}{T}\int_0^T |x(t)|^2\,dt$$

Effective value, or the RMS value is

$$x_{rms}(t) = \sqrt{\langle x^2\,(t) \rangle} = \sqrt{p_{av}} = \sqrt{\frac{1}{T}\int_0^T x^2\,(t)\,dt}$$

For example, for $x(t) = A\cos(x)\,,\langle x(t) \rangle = 0, P_{av} = \frac{A^2}{2}, x_{rms}(t) = 0.707A$

To find PEP (which is the peak envelope power), find the complex envelope $\tilde{x}(t)$, then find the average power of it. i.e.

$$PEP = \frac{1}{2}\tilde{x}_{\max}^2(t)$$

7 How to find the SNR for sampling quantization?

Suppose we have a message $m(t)$ that is sampled. Assume we have n bits to use for encoding the sample levels. Hence there are 2^n levels of quantizations. We want to find the ration of the signal to the noise power. Noise here is generated due to quantization (i.e. due to the rounding off values of $m(t)$ during sampling).

This is the algorithm:

Input: n, the number of bits for encoding, m_p absolute maximum value of the message $m(t)$, the pdf $f_X(t)$ of the message $m(t)$ is $m(t)$ is random message or $m(t)$ function if it is deterministic (such as $\cos(t)$)

1. Find the quantization step size $S = \frac{2m_p}{2^2}$
2. Find P_{av} of the error is $\frac{1}{12}S^2$ where S is the step size found in (1), hence $P_{av} = \frac{1}{12}S^2 = \frac{1}{12}\left(\frac{2m_p}{2^2}\right)^2 = \frac{m_p^2}{3\times 2^{2n}}$
3. If $m(t)$ is deterministic find $p_{av} = \langle m^2(t)\rangle = \frac{1}{T}\int_0^T |m(t)|^2\, dt$
4. If $m(t)$ is random, find $p_{av} = E(m(t)) = \int m^2(t)\, f_X(t)\, dt$, this is called the second moment of the pdf
5. $SNR = \frac{E(m(t))}{\frac{m_p^2}{3\times 2^{2n}}}$

Hence find SNR for noise quantisation comes down to finding the power in the message $m(t)$.

Examples: For sinusoidal message $m(t)$, $SNR_{db} = 6n + 1.761$. For random $m(t)$ with PDF which is uniform distributed $SNR_{db} = 6n$, for random $m(t)$ which is AWGN. Do this later

8 How to determine coding of a number from quantization?

Given an analog value say x and given a maximum absolute possible value to be m_p, and given the number of bits available for coding to be N, the following are the algorithm to generate the quantized version of x, called $\hat{x}$

8.1 sign magnitude

Input: x, m_p, N

output: $\hat{x}$

Let $\Delta = \frac{m_p}{2^{N-1}}$ called the step size

Let $q = round\left[\frac{abs(x)}{\Delta}\right]$ which is the quantization level

If $q \geq 2^{N-1} - 1$ then $q = 2^{N-1}$ end if

if $x < 0$ then $code = q + 2^{N-1}$ else $code = q$ endif

return $code$ in base 2

8.2 ones complement

Input: x, m_p, N

output: $\hat{x}$

Let $\Delta = \frac{m_p}{2^{N-1}}$ called the step size

Let $q = round\left[\frac{abs(x)}{\Delta}\right]$ which is the quantization level

If $q \geq 2^{N-1} - 1$ then $q = 2^{N-1} - 1$ end if

If $x > 0$ then $code = q$ else $code = \left(2^N - 1\right) - q$ endif

return $code$ in base 2

8.3 offset binary

Input: x, m_p, N

output: $\hat{x}$

Let $\Delta = \frac{m_p}{2^{N-1}}$ called the step size

Let $q = round\left[\frac{abs(x)}{\Delta}\right]$ which is the quantization level

If $x \geq -\frac{\Delta}{2}$ then

  if $q \geq 2^{N-1} - 1$ then

    $q = 2^{N-1} - 1$

  end if

  $code = 2^{N-1} + q$

else

if $q \geq 2^{N-1} - 1$ then

$q = 2^{N-1}$

end if

$code = 2^{N-1} - q$

end if

return $code$ in base 2

8.4 2's complement

Input: x, m_p, N

output: $\hat{x}$

Let $\Delta = \frac{m_p}{2^{N-1}}$ called the step size

Let $q = round\left[\frac{abs(x)}{\Delta}\right]$ which is the quantization level

If $x \geq -\frac{\Delta}{2}$ then

if $q \geq 2^{N-1} - 1$ then

$q = 2^{N-1} - 1$

end if

$code = q$

else

if $q \geq 2^{N-1} - 1$ then

$q = 2^{N-1}$

end if

$code = 2^N - q$

end if

return $code$ in base 2

9 How to derive the Phase and Frequency modulation signals?

For any bandpass signal, we can write it as

$$x(t) = \text{Re}\left(\tilde{x}(t)\, e^{j\omega_c t}\right)$$

Where $\tilde{x}(t)$ is the complex envelope of $x(t)$. For PM and FM, the baseband modulated signal, $\tilde{x}(t)$ has the form $A_c e^{j\theta(t)}$ Hence the above becomes

$$\begin{aligned} x(t) &= \text{Re}\left(A_c e^{j\theta(t)} e^{j\omega_c t}\right) \\ &= A_c(\cos\omega_c t\cos\theta(t) - \sin\omega_c t\sin\theta(t)) \end{aligned}$$

But $\cos(A+B) = \cos A\cos B - \sin A\sin B$, hence the above becomes

$$x(t) = \cos(\omega_c t + \theta(t)) \tag{1}$$

The above is the general form for PM and FM. Now, for PM, $\theta(t) = k_p m(t)$ and for FM, $\theta(t) = k_f \int_0^t m(t_1)\, dt_1$. Hence, substituting in (1) we obtain

$$x_{FM}(t) = \cos\left(\omega_c t + k_f \int_0^t m(t_1)\, dt_1\right)$$

and

$$x_{PM}(t) = \cos(\omega_c t + k_p m(t))$$

10 How to obtain the phase deviation and the frequency deviation for angle modulated signal?

From the general form for angle modulated signal (see above note)

$$x(t) = \cos(\omega_c t + \theta(t))$$

The phase deviation is $\theta(t)$. And the maximum phase deviation is simply the maximum of $\theta(t)$

Now, to find the frequency deviation, we need a little bit more work. Start with

$$f_i = f_c + \Delta f$$

Where f_i is the instantaneous frequency in Hz. But

$$\begin{aligned} f_i &= \frac{1}{2\pi}\frac{d}{dt}(\omega_c t + \theta(t)) \\ &= f_c + \overbrace{\frac{1}{2\pi}\frac{d}{dt}\theta(t)}^{\Delta f} \end{aligned}$$

11 How to quickly determine SNR$_i$ from SNR_c?

First find SNR_c, for to find SNR_i use the following

$SNR_i = SNR_c \frac{B}{B_T}$, where B_T is the transmission bandwidth, and B is the baseband bandwidth. For AM, $B_T = 2B$. For $DSB - SC$, $B_T = 2B$. For $DSB - SS$, $B_T = B$.

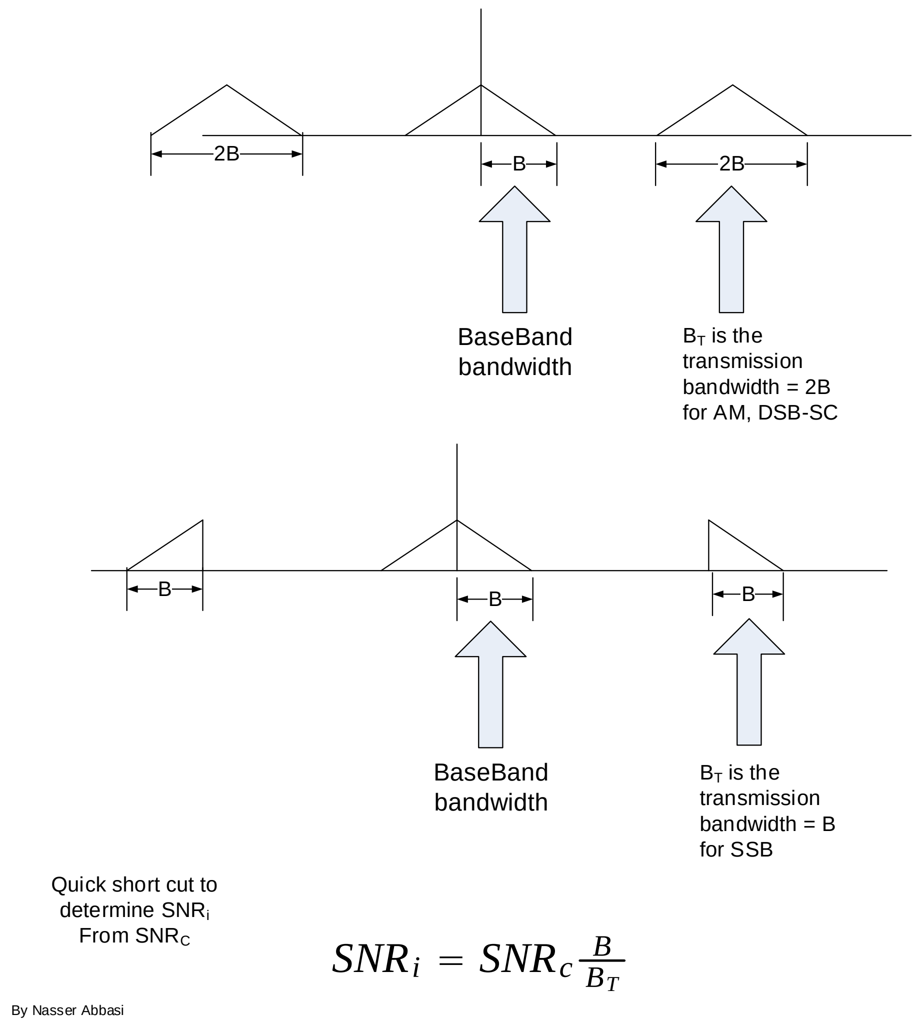
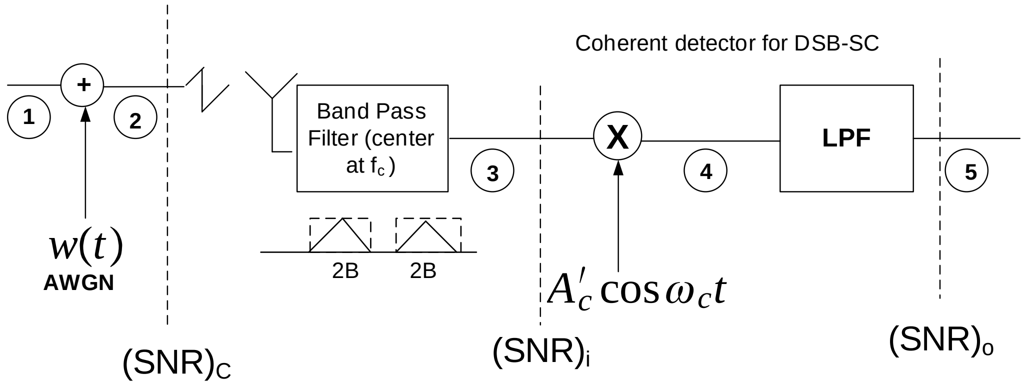


Figure 2: bandwidth

12 How to determine figure of merit for DSB-SC using coherent detector?

Figure of merit, γ is defined as $\frac{SNR_o}{SNR_c}$ where SNR_o is the signal-to-noise ratio on output from modulator, and SNR_c is signal-to-noise ratio for the channel, assuming channel has AWGN added. The following diagram shows the calculations. I used a coherent demodulator.



$$s_1(t) = A_c m(t)\cos\omega_c t$$
$$s_2(t) = A_c m(t)\cos\omega_c t + w(t)$$

$$SNR_c = \frac{\langle A_c^2 m^2(t)\cos^2\omega_c t\rangle}{BN_0} = \frac{\frac{A_c^2}{2}P_m}{BN_0}$$

$$\begin{aligned}
s_3(t) &= A_c m(t)\cos\omega_c t + \overbrace{n(t)}^{\text{narrow band noise}} \\
&= A_c m(t)\cos\omega_c t + [n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t] \\
&= \overbrace{(A_c m(t) + n_I(t))}^{\text{in phase}} \cos\omega_c t - \overbrace{n_Q(t)}^{\text{quadrature}} \sin\omega_c t
\end{aligned}$$

$$SNR_i = \frac{\langle A_c^2 m^2(t)\cos^2\omega_c t\rangle}{2BN_0} = \frac{\frac{A_c^2}{2}P_m}{2BN_0}$$

$$\begin{aligned}
s_4(t) &= s_3(t) A_c' \cos\omega_c t \\
&= [(A_c m(t) + n_I(t))\cos\omega_c t - n_Q(t)\sin\omega_c t] A_c' \cos\omega_c t \\
&= A_c'(A_c m(t) + n_I(t))\cos^2\omega_c t - A_c' n_Q(t)\sin\omega_c t\cos\omega_c t \\
&= \left(\frac{1}{2} + \frac{1}{2}\cos 2\omega_c t\right) A_c'(A_c m(t) + n_I(t)) - A_c' n_Q(t)(\sin(0) + \sin(2\omega_c t)) \\
&= \frac{A_c'[A_c m(t) + n_I(t)]}{2} + \frac{A_c'}{2}[A_c m(t) + n_I(t)]\cos 2\omega_c t - A_c' n_Q(t)\sin(2\omega_c t)
\end{aligned}$$

$$s_5(t) = \frac{A_c'[A_c m(t) + n_I(t)]}{2}$$

$$SNR_o = \frac{\left\langle \frac{A_c'^2 A_c^2}{4} m^2(t) \right\rangle}{E\left[\left(\frac{n_I(t)}{2}\right)^2\right]} = \frac{\frac{A_c'^2 A_c^2}{4}P_m}{\frac{1}{4}E[(n_I^2(t))]} = \frac{\frac{A_c'^2 A_c^2}{4}P_m}{\frac{1}{4}2BN_0} = \frac{A_c'^2 A_c^2 P_m}{2BN_0}$$

$$\frac{SNR_o}{SNR_i} = \frac{\frac{A_c'^2 A_c^2 P_m}{2BN_0}}{\frac{\frac{A_c^2}{2}P_m}{2BN_0}} = 2A_c'^2$$

$$\gamma = \frac{SNR_o}{SNR_c} = \frac{\frac{A_c'^2 A_c^2 P_m}{2BN_0}}{\frac{\frac{A_c^2}{2}P_m}{BN_0}} = A_c'^2$$

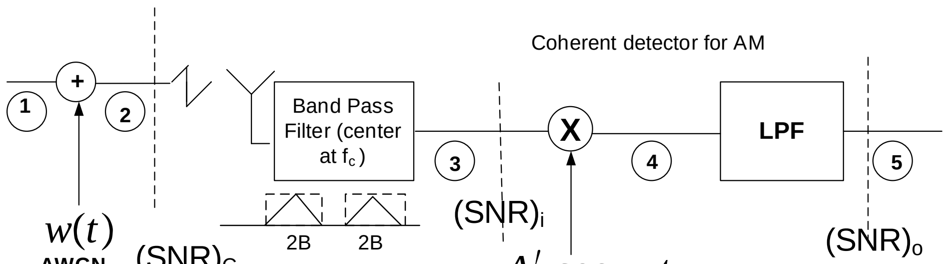
Figure of Merit for DSB-SC signal

By Nasser Abbasi
11/30/2008
Study_notes_2.vsd

Figure 3: figure of merit for DSB-SC

Question: Verify the above.

13 How to determine figure of merit for AM transmission using coherent detector?



$$A_c' \cos \omega_c t$$

Figure of merit for AM

$$s_1(t) = A_c(1 + k_a m(t)) \cos \omega_c t$$
$$s_2(t) = A_c(1 + k_a m(t)) \cos \omega_c t + w(t)$$

$$\boxed{\begin{aligned} SNR_c &= \frac{\langle (A_c(1 + k_a m(t)) \cos \omega_c t)^2 \rangle}{BN_0} = \frac{\frac{A_c^2}{2}\langle (1 + k_a m(t))^2 \rangle}{BN_0} \\ &= \frac{\frac{A_c^2}{2}\langle 1 + k_a^2 m^2(t) + 2k_a m(t) \rangle}{BN_0} = \frac{\frac{A_c^2}{2}(1 + k_a^2 P_m)}{BN_0} \end{aligned}}$$

Assuming <m(t)>=0

$$\boxed{SNR_i = SNR_c \frac{B}{B_t} = SNR_c \frac{B}{2B} = \frac{A_c^2 \langle 1 + k_a^2 P_m \rangle}{4BN_0}}$$

$$\boxed{\begin{aligned} s_3(t) &= A_c(1 + k_a m(t)) \cos \omega_c t + [n_I(t) \cos \omega_c t - n_Q(t) \sin \omega_c t] \\ &= [A_c(1 + k_a m(t)) + n_I(t)] \cos \omega_c t - n_Q(t) \sin \omega_c t \end{aligned}}$$

$$\boxed{\begin{aligned} s_4(t) &= s_3(t)\left(A_c' \cos \omega_c t\right) = A_c'[A_c(1 + k_a m(t)) + n_I(t)] \cos^2 \omega_c t - n_Q(t) \sin \omega_c t \cos \omega_c t \\ &= \frac{A_c'}{2}[A_c + A_c k_a m(t) + n_I(t)] + \frac{A_c'}{2} \cos 2\omega_c t[A_c + A_c k_a m(t) + n_I(t)] - \frac{1}{2} n_Q(t)(\sin 2\omega_c t) \end{aligned}}$$

$$s_5(t) = \overbrace{\frac{A_c' A_c}{2}}^{ignore} + \overbrace{\frac{A_c' A_c k_a}{2} m(t)}^{message} + \overbrace{\frac{A_c'}{2} n_I(t)}^{noise}$$

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AM_coherent.vsdx

Figure 4: figure of merit for AM coherent

$$
\boxed{SNR_o = \frac{\left\langle \left(\frac{A_c' A_c k_a}{2} m(t) \right)^2 \right\rangle}{E\left[\left(\frac{A_c'}{2} n_I(t) \right)^2 \right]} = \frac{\frac{A_c'^2 A_c^2 k_a^2}{4} P_m}{\frac{A_c'^2}{4} E[n_I^2(t)]} = \frac{\frac{A_c'^2 A_c^2 k_a^2}{4} P_m}{\frac{A_c'^2}{4} 2BN_0} = \frac{A_c^2 k_a^2 P_m}{2BN_0}}
$$

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AM_coherent_2.vsdx

$$
\boxed{\gamma = \frac{SNR_o}{SNR_c} = \frac{\frac{A_c^2 k_a^2 P_m}{2BN_0}}{\frac{\frac{A_c^2}{2}\left(1+k_a^2 P_m\right)}{BN_0}} = \frac{k_a^2 P_m}{1+k_a^2 P_m}}
$$

Figure 5: figure of merit for AM coherent (2)

14 How to determine figure of merit for AM using envelope detector?

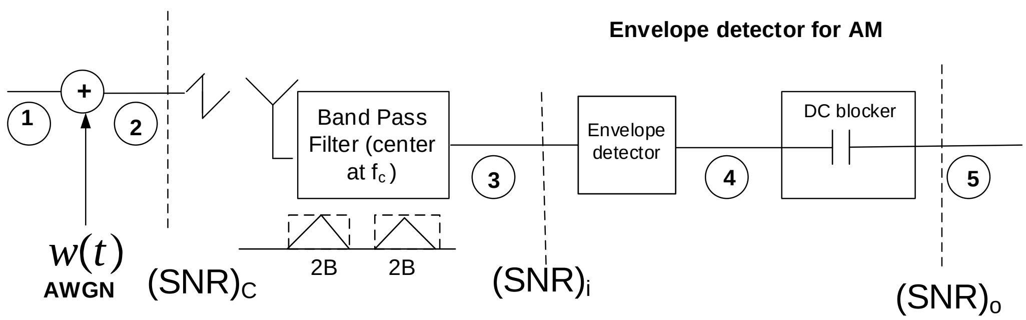

Figure 6: figure of merit for AM using envelope detector

$$
\begin{aligned}
s_1(t) &= A_c(1 + k_a m(t)) \cos \omega_c t \\
s_2(t) &= A_c(1 + k_a m(t)) \cos \omega_c t + w(t)
\end{aligned}
$$

And

$$
\begin{aligned}
SNR_c &= \frac{\left\langle \left(A_c(1 + k_a m(t)) \cos \omega_c t\right)^2 \right\rangle}{BN_0} \\
&= \frac{\frac{A_c^2}{2} \left\langle (1 + k_a m(t))^2 \right\rangle}{BN_0} \\
&= \frac{\frac{A_c^2}{2} \langle 1 + k_a^2 m^2(t) + 2k_a m(t) \rangle}{BN_0}
\end{aligned}
$$

Now assuming $\langle m(t)\rangle = 0$, the above simplifies to

$$SNR_c = \frac{\frac{A_c^2}{2}(1+k_a^2P_m)}{BN_0}$$

Hence

$$\begin{aligned} SNR_i &= SNR_c\frac{B}{B_T} \\ &= \frac{\frac{A_c^2}{2}(1+k_a^2P_m)}{BN_0}\frac{B}{2B} \\ &= \frac{\frac{A_c^2}{2}(1+k_a^2P_m)}{2BN_0} \end{aligned}$$

Now find $s_3(t)$

$$\begin{aligned} s_3(t) &= A_c(1+k_am(t))\cos\omega_ct + \overbrace{n(t)}^{\text{narrow band noise}} \\ &= A_c(1+k_am(t))\cos\omega_ct + [n_I(t)\cos\omega_ct - n_Q(t)\sin\omega_ct] \\ &= \overbrace{[A_c(1+k_am(t)) + n_I(t)]}^{\text{in phase}}\cos\omega_ct - \overbrace{n_Q(t)}^{\text{quadrature}}\sin\omega_ct \end{aligned}$$

Now, to find $s_4(t)$, which is the envelope of $s_3(t)$.

$$\begin{aligned} s_4(t) &= \text{envelope}(s_3(t)) \\ &= \sqrt{(s_3)_I^2 + (s_3)_Q^2} \\ &= \sqrt{\left(A_c\left(1+k_am\left(t\right)\right) + n_I\left(t\right)\right)^2 + n_Q^2\left(t\right)} \end{aligned}$$

Now, assuming $A_c \gg |n_I(t)|$ and $A_c \gg |n_Q(t)|$, then the above simplifies to

$$s_4(t) = A_c(1+k_am(t)) + n_I(t)$$

now apply the DC blocker, we obtain

$$s_5(t) = A_ck_am(t) + n_I(t)$$

$$SNR_o = \frac{\left\langle (A_ck_am(t))^2\right\rangle}{E\left[n_I^2\left(t\right)\right]} = \frac{A_c^2k_a^2P_m}{2BN_0}$$

$$\gamma = \frac{SNR_o}{SNR_c} = \frac{\frac{A_c^2k_a^2P_m}{2BN_0}}{\frac{\frac{A_c^2}{2}(1+k_a^2P_m)}{BN_0}} = \frac{k_a^2P_m}{1+k_a^2P_m}$$

We notice, that for Large SNR_i, this detector gives the same result as coherent detector. For small SNR_i, it is better to use the coherent detector than the envelope detector.

15 How to determine figure of merit for SSB using coherent detector?

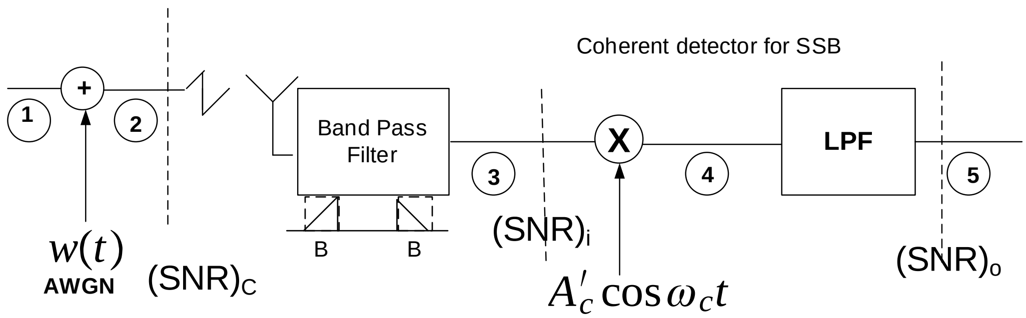

Figure 7: figure of merit for SSB

The difference here is that SSB signal has transmission bandwidth $B_T = B$ and not $2B$ as in all the previous signals. Assume we are working with upper sideband. Analysis is the same for lower sideband.

$$s_1(t) = k[m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t]$$

Where k is a constant. Usually $\frac{A_c}{2}$ but we will leave it as k for now. $\hat{m}(t)$ is the Hilbert transform of $m(t)$

$$s_2(t) = k[m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t] + w(t)$$

$$\begin{aligned}
SNR_c &= \frac{\left\langle (k[m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t])^2 \right\rangle}{BN_0} \\
&= \frac{k^2\langle m^2(t)\cos^2\omega_c t\rangle + k^2\langle \hat{m}^2(t)\sin^2\omega_c t\rangle - 2k^2\langle m(t)\,\hat{m}(t)\cos(\omega_c t)\sin(\omega_c t)\rangle}{BN_0} \\
&= \frac{k^2\langle m^2(t)\rangle\,\langle\cos^2\omega_c t\rangle + k^2\langle m^2(t)\rangle\,\langle\sin^2\omega_c t\rangle - 2k^2\langle m(t)\,\hat{m}(t)\cos(\omega_c t)\sin(\omega_c t)\rangle}{BN_0}
\end{aligned}$$

Assume $\langle m(t)\rangle = 0$, we obtain

$$\begin{aligned}
SNR_c &= \frac{k^2\frac{P_m}{2} + k^2\frac{P_m}{2}}{BN_0} \\
&= \frac{k^2 P_m}{BN_0}
\end{aligned}$$

$$s_3(t) = k[m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t] + n(t)$$

Hence

$$\begin{aligned} SNR_i &= SNR_c \frac{B}{B_T} \\ &= \frac{k^2 P_m}{B N_0} \frac{B}{B} \\ &= \frac{k^2 P_m}{B N_0} \end{aligned}$$

$$\begin{aligned} s_4(t) &= [k[m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t] + n(t)]\, A_c^{'} \cos\omega_c t \\ &= A_c^{'} k m(t)\cos^2\omega_c t - A_c^{'} k\hat{m}(t)\sin\omega_c t\cos\omega_c t + A_c^{'}[n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t]\cos\omega_c t \\ &= A_c^{'} k m(t)\left(\frac{1}{2} + \frac{1}{2}\cos 2\omega_c t\right) - A_c^{'} k\hat{m}(t)\,\frac{1}{2}(\sin(0) + \sin(2\omega_c t)) \\ &+ A_c^{'}\left[n_I(t)\cos^2\omega_c t - n_Q(t)\sin\omega_c t\cos\omega_c t\right] \\ &= \frac{1}{2}A_c^{'} k m(t) + \frac{1}{2}A_c^{'} k m(t)\cos 2\omega_c t - A_c^{'} k\hat{m}(t)\,\frac{1}{2}\sin(2\omega_c t) \\ &+ A_c^{'}\left[n_I(t)\left(\frac{1}{2} + \frac{1}{2}\cos 2\omega_c t\right) - n_Q(t)\,\frac{1}{2}(\sin(0) + \sin(2\omega_c t))\right] \\ &= \frac{1}{2}A_c^{'} k m(t) + \frac{1}{2}A_c^{'} k m(t)\cos 2\omega_c t - \frac{1}{2}A_c^{'} k\hat{m}(t)\sin(2\omega_c t) \\ &+ \frac{A_c^{'}}{2} n_I(t) + \frac{A_c^{'}}{2} n_I(t)\cos 2\omega_c t - \frac{A_c^{'}}{2} n_Q(t)\sin 2\omega_c t \end{aligned}$$

After low pass filter, we obtain

$$s_5(t) = \frac{1}{2}A_c^{'} k m(t) + \frac{A_c^{'}}{2} n_I(t)$$

Hence,

$$\begin{aligned} SNR_o &= \frac{\left\langle \left(\frac{1}{2}A_c^{'} k m(t)\right)^2 \right\rangle}{E\left(\left[\frac{A_c^{'}}{2} n_I(t)\right]^2\right)} \\ &= \frac{\frac{1}{4}\left(A_c^{'}\right)^2 k^2 P_m}{\frac{1}{4}\left(A_c^{'}\right)^2 N_0 B} \\ &= \frac{k^2 P_m}{N_0 B} \end{aligned}$$

Hence

$$\begin{aligned} \frac{SNR_o}{SNR_i} &= \frac{\frac{k^2 P_m}{N_0 B}}{\frac{k^2 P_m}{B N_0}} \\ &= 1 \end{aligned}$$

Hence

$$\begin{aligned}\gamma &= \frac{SNR_o}{SNR_c}\\ &= \frac{\frac{k^2 P_m}{N_0 B}}{\frac{k^2 P_m}{B N_0}}\\ &= 1\end{aligned}$$

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$$s(t) = \frac{A_c}{2}[m(t)\cos\omega_c t \mp m_Q(t)\sin\omega_c t]$$

$m_Q(t)$ is the output of VSB filter when input is $m(t)$