Computer algebra independent integration tests

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems.

The listing of the problems used by this report are

1. MathematicaSyntaxTestFiles.zip
2. MapleSyntaxTestFiles.zip

The above zip files were downloaded from rulebasedintegration.org.

The current number of problems in this test suite is [71994].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Sympy 1.8 under Python 3.8.8 using Anaconda distribution.
7. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.52 (71651)	% 0.48 (343)
Mathematica	% 98.39 (70834)	% 1.61 (1160)
Maple	% 83.58 (60173)	% 16.42 (11821)
Fricas	% 68.7 (49460)	% 31.3 (22534)
Giac	% 52.43 (37749)	% 47.57 (34245)
Maxima	% 52.72 (37956)	% 47.28 (34038)
Sympy	% 34.47 (24814)	% 65.53 (47180)
Mupad	% 52.55 (37830)	% 47.45 (34164)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Table 1.2: Description of grading applied to integration result

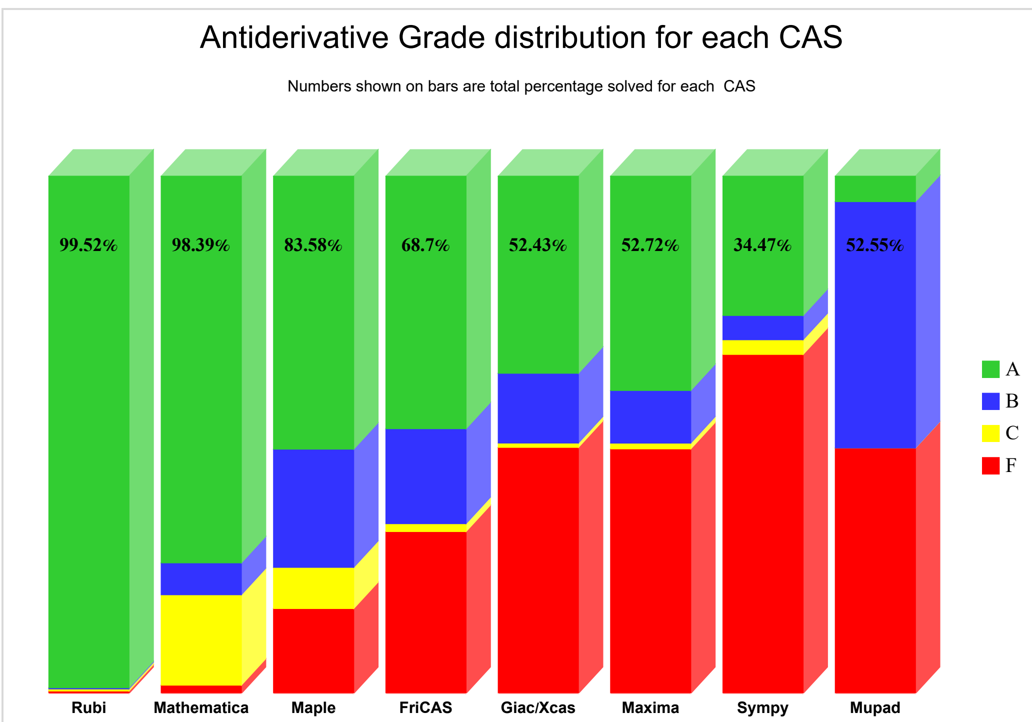
Grading is implemented for all CAS systems in this version except for CAS Mupad where a grade of B is automatically assigned as a place holder for all integrals it completes on time.

The following table summarizes the grading results.

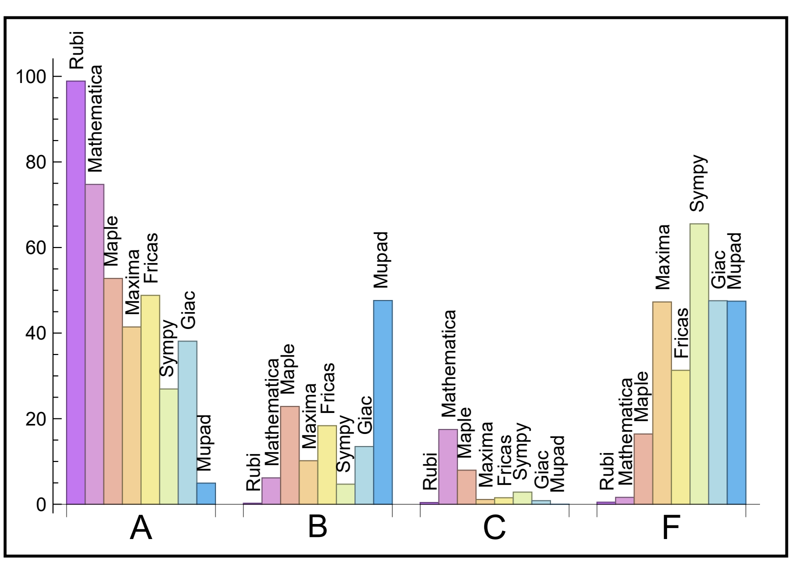
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.89	0.23	0.41	0.48
Mathematica	74.75	6.17	17.47	1.61
Maple	52.77	22.85	7.96	16.42
Maxima	41.43	10.17	1.11	47.28
Fricas	48.82	18.36	1.52	31.3
Sympy	26.94	4.69	2.84	65.53
Giac	38.1	13.49	0.84	47.57
Mupad	4.93	47.61	0.	47.45

Table 1.3: Antiderivative Grade distribution for each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.2.1 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.28	156.75	1.	107.	1.
Mathematica	1.84	799.85	2.8	92.	0.94
Maple	0.79	62768.5	746.4	132.	1.27
Maxima	0.92	209.89	1.76	81.	1.
Fricas	1.56	408.24	2.78	121.	1.37
Sympy	8.89	250.16	2.77	70.	1.13
Giac	1.05	261.	2.	92.	1.12
Mupad	2.73	743.09	3.35	76.	1.

Table 1.4: Time and leaf size performance for each CAS

1.3 Performance per integrand type

The following are the different integrand types the test suite contains.

1. Algebraic Binomial problems (products involving powers of binomials and monomials).
2. Algebraic Trinomial problems (products involving powers of trinomials, binomials and monomials).
3. Miscellaneous Algebraic functions.
4. Exponentials.
5. Logarithms.
6. Trigonometric.
7. Inverse Trigonometric.
8. Hyperbolic functions.
9. Inverse Hyperbolic functions.
10. Special functions.
11. Independent tests.

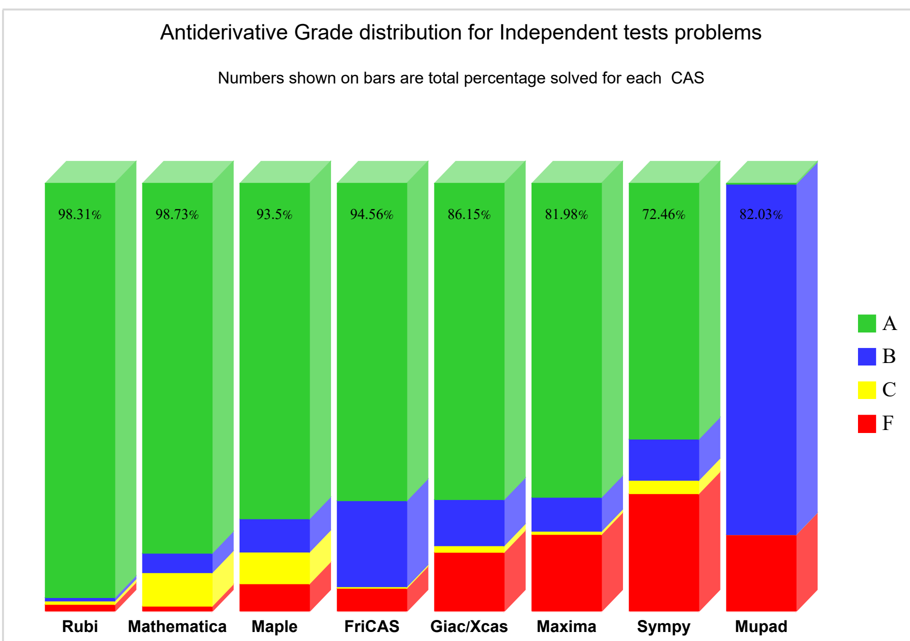
The following table gives percentage solved of each CAS per integrand type.

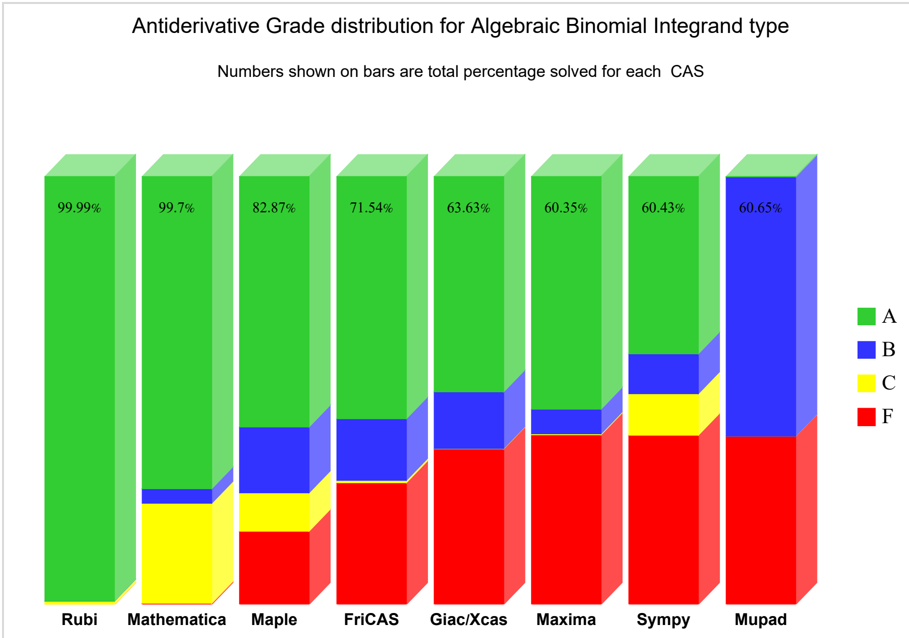
Integrand type	problems	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	M
Independent tests	1892	98.31	98.73	93.5	81.98	94.56	72.46	86.15	82
Algebraic Binomial	14276	99.99	99.7	82.87	60.35	71.54	60.43	63.63	60
Algebraic Trinomial	10187	99.99	98.94	90.73	52.97	77.	40.04	66.14	56
Algebraic Miscellaneous	1519	98.62	98.16	87.56	52.27	74.26	45.23	59.58	61
Exponentials	965	99.17	96.68	80.21	66.22	91.19	44.04	49.43	71
Logarithms	3085	98.51	97.83	54.26	56.24	58.06	32.64	45.02	43
Trigonometric	22551	99.56	97.66	85.76	47.39	64.1	15.49	42.22	49
Inverse Trigonometric	4585	99.65	98.26	83.69	36.23	48.42	35.92	40.52	38
Hyperbolic	5166	98.32	97.99	82.6	62.18	85.06	23.11	64.42	54
Inverse Hyperbolic	6626	99.52	98.46	79.94	47.75	62.56	27.3	35.69	39
Special functions	999	100.	95.6	69.97	39.54	48.85	42.84	34.73	40

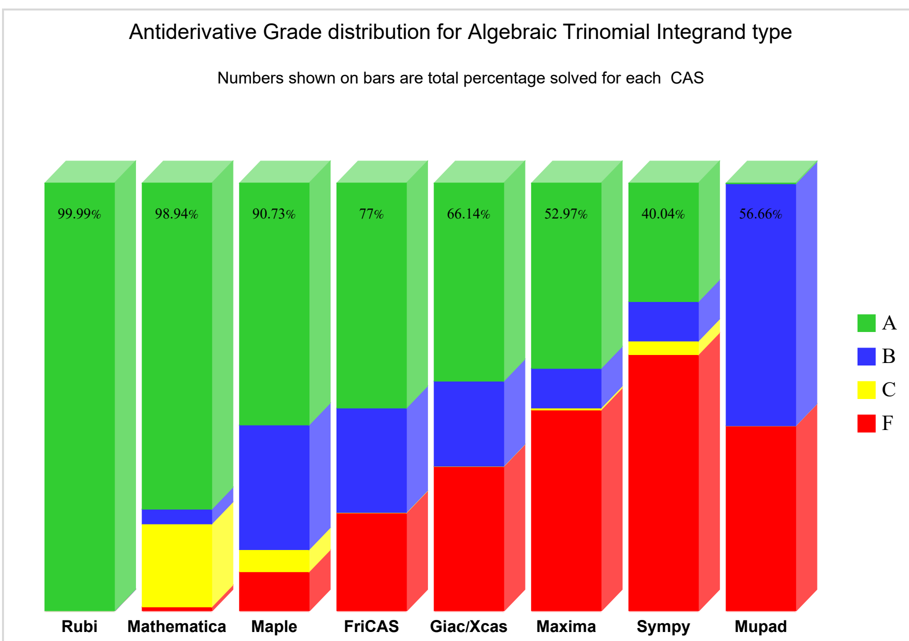
Table 1.5: Percentage solved per integrand type

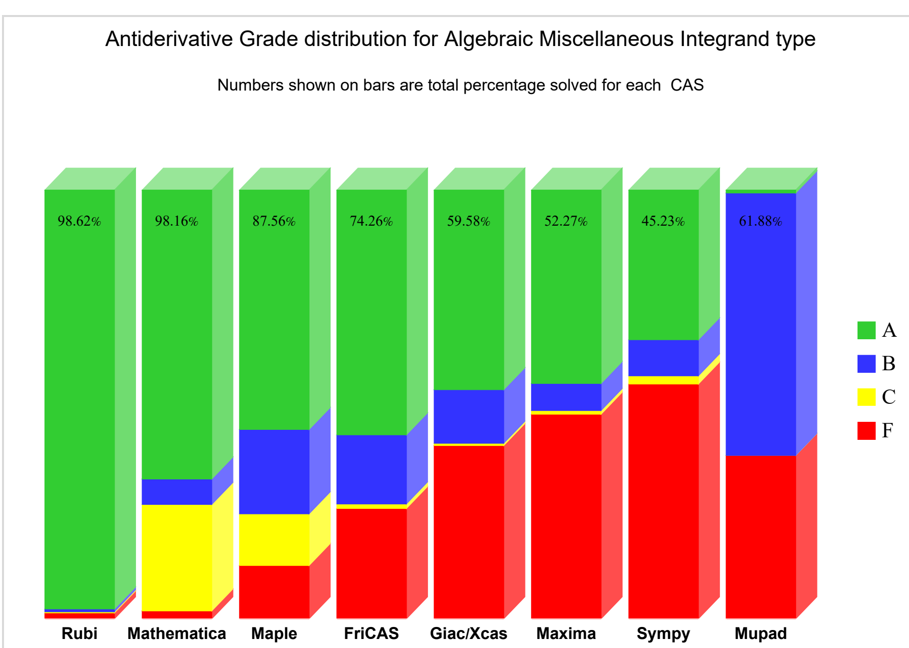
In addition to the above table, for each type of integrand listed above, 3D chart is made which shows how each CAS performed on that specific integrand type.

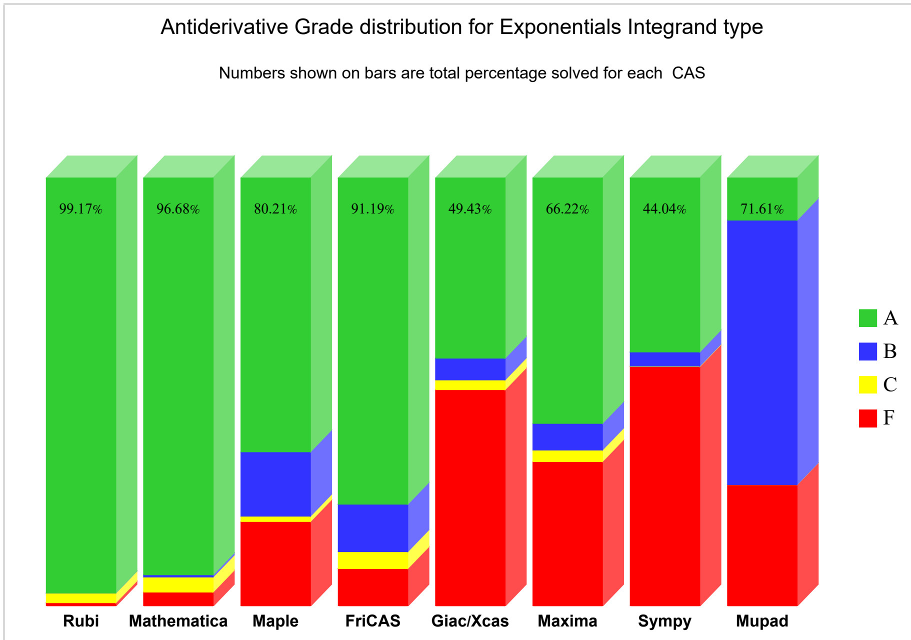
These charts and the table above can be used to show where each CAS relative strength or weakness in the area of integration.

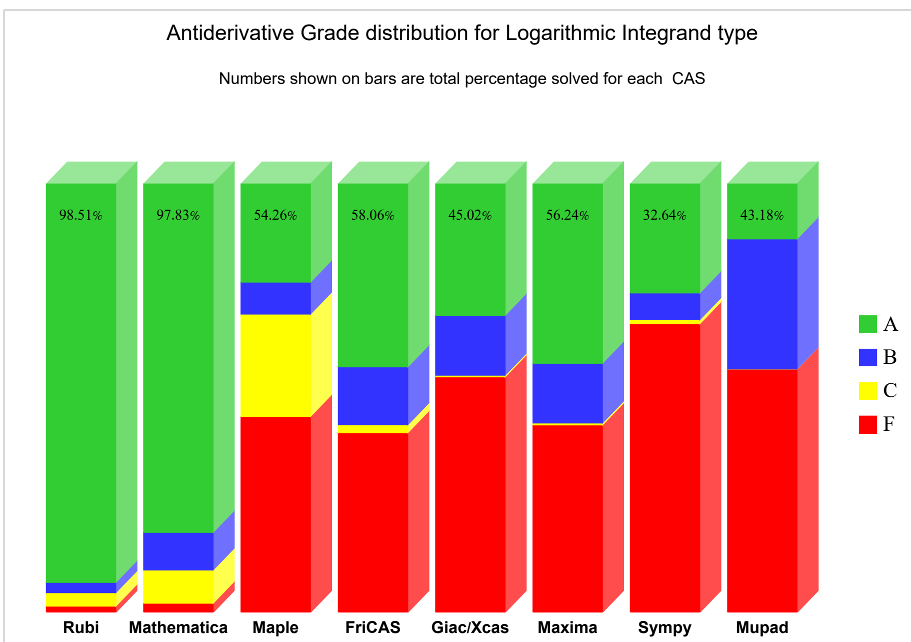


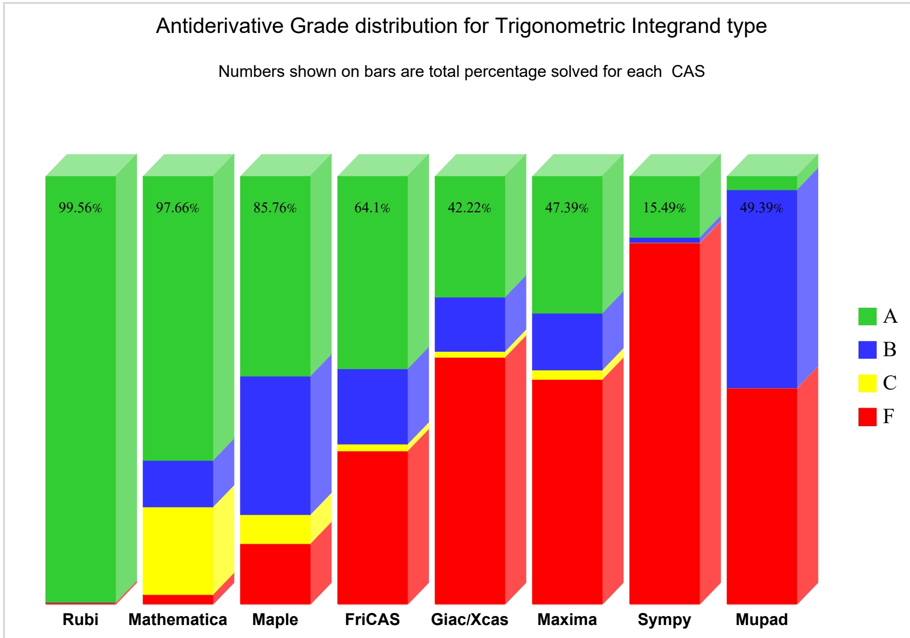

Antiderivative Grade distribution for Algebraic Binomial Integrand type
Numbers shown on bars are total percentage solved for each CAS
99.99%
99.7%
82.87%
71.54%
63.63%
60.35%
60.43%
60.65%
A
B
C
F
Rubi
Mathematica
Maple
FriCAS
Giac/Xcas
Maxima
Sympy
Mupad

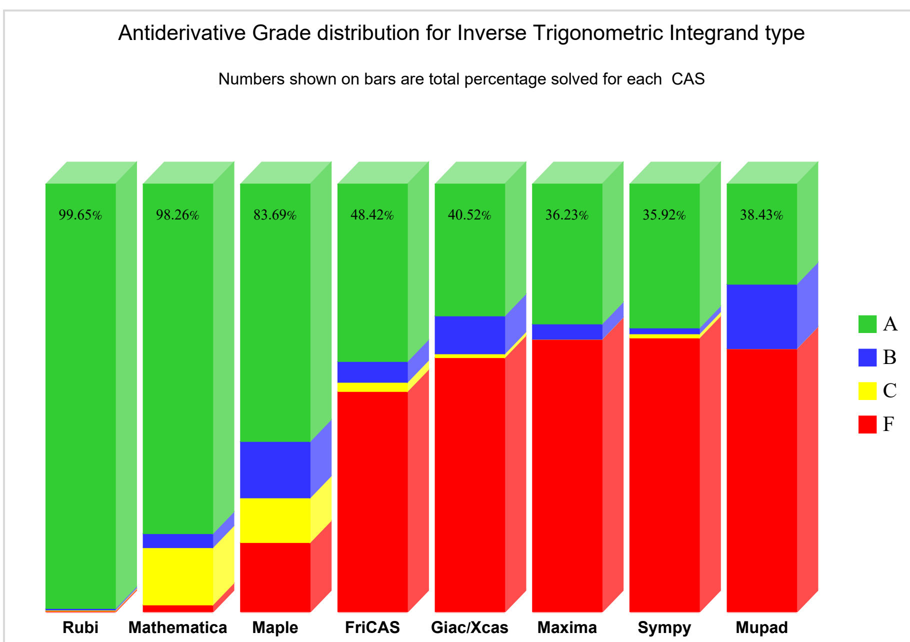

Antiderivative Grade distribution for Algebraic Trinomial Integrand type
Numbers shown on bars are total percentage solved for each CAS
99.99%
98.94%
90.73%
77%
66.14%
52.97%
40.04%
56.66%
A
B
C
F
Rubi
Mathematica
Maple
FriCAS
Giac/Xcas
Maxima
Sympy
Mupad

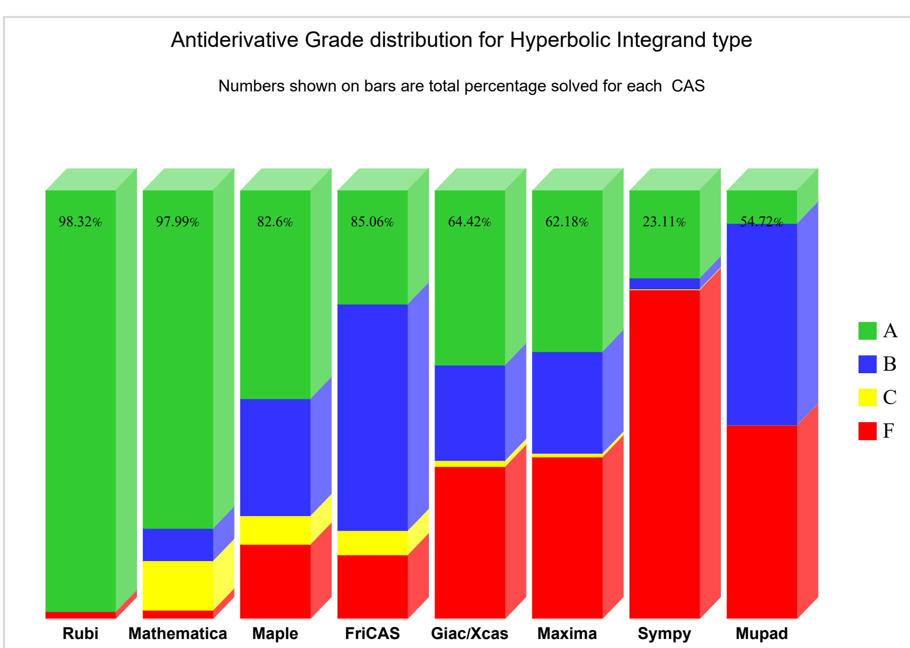

Antiderivative Grade distribution for Algebraic Miscellaneous Integrand type
Numbers shown on bars are total percentage solved for each CAS
98.62%
98.16%
87.56%
74.26%
59.58%
52.27%
45.23%
61.88%
A
B
C
F
Rubi
Mathematica
Maple
FriCAS
Giac/Xcas
Maxima
Sympy
Mupad

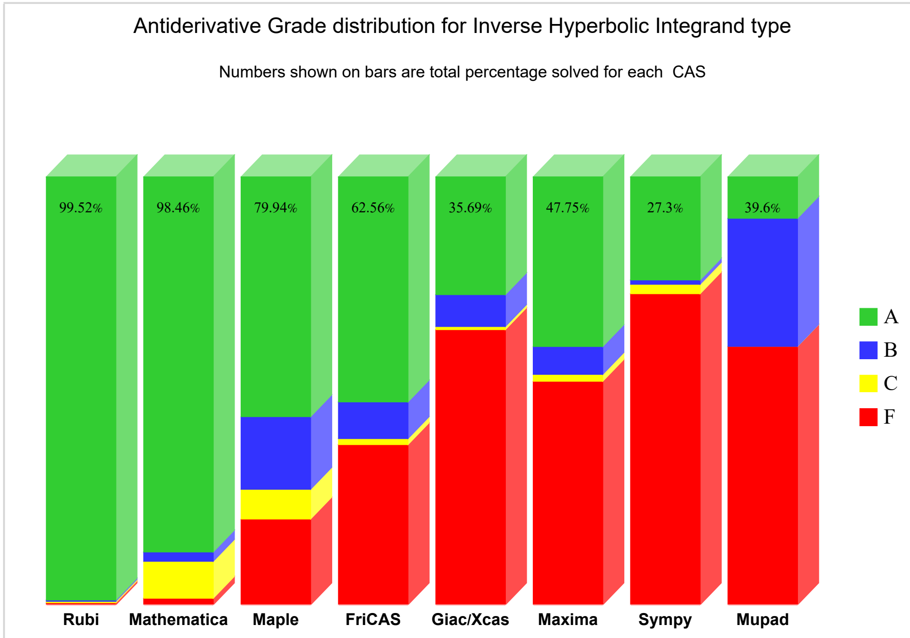

Antiderivative Grade distribution for Exponentials Integrand type
Numbers shown on bars are total percentage solved for each CAS
99.17%
96.68%
80.21%
91.19%
49.43%
66.22%
44.04%
71.61%
A
B
C
F
Rubi
Mathematica
Maple
FriCAS
Giac/Xcas
Maxima
Sympy
Mupad

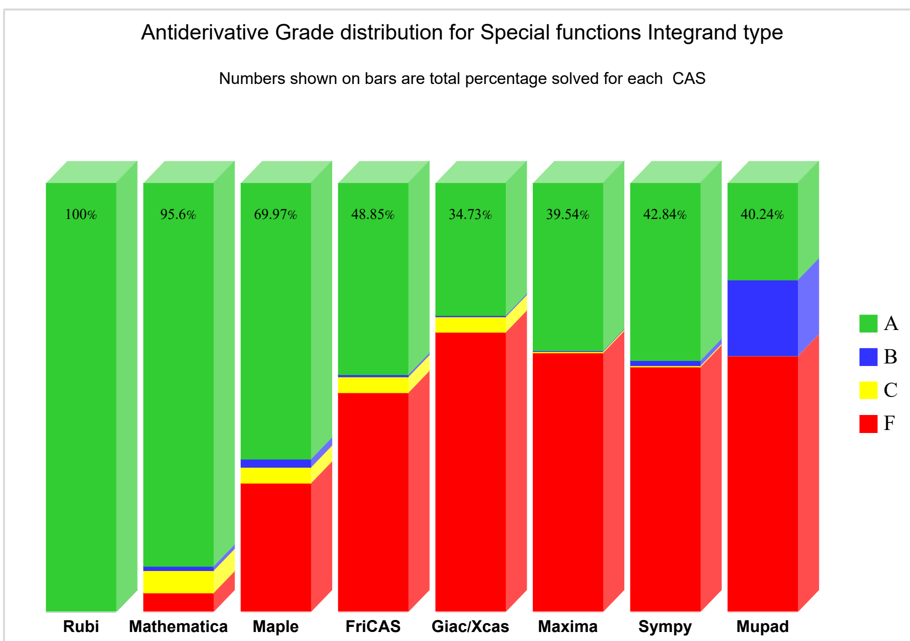

Antiderivative Grade distribution for Logarithmic Integrand type
Numbers shown on bars are total percentage solved for each CAS
98.51%
97.83%
54.26%
58.06%
45.02%
56.24%
32.64%
43.18%
A
B
C
F
Rubi
Mathematica
Maple
FriCAS
Giac/Xcas
Maxima
Sympy
Mupad


Antiderivative Grade distribution for Trigonometric Integrand type
Numbers shown on bars are total percentage solved for each CAS
99.56%
97.66%
85.76%
64.1%
42.22%
47.39%
15.49%
49.39%
A
B
C
F
Rubi
Mathematica
Maple
FriCAS
Giac/Xcas
Maxima
Sympy
Mupad


Antiderivative Grade distribution for Inverse Trigonometric Integrand type
Numbers shown on bars are total percentage solved for each CAS
99.65%
98.26%
83.69%
48.42%
40.52%
36.23%
35.92%
38.43%
A
B
C
F
Rubi
Mathematica
Maple
FriCAS
Giac/Xcas
Maxima
Sympy
Mupad


Antiderivative Grade distribution for Hyperbolic Integrand type
Numbers shown on bars are total percentage solved for each CAS
98.32%
97.99%
82.6%
85.06%
64.42%
62.18%
23.11%
54.72%
A
B
C
F
Rubi
Mathematica
Maple
FriCAS
Giac/Xcas
Maxima
Sympy
Mupad





1.4 Maximum leaf size ratio for each CAS against the optimal result

The following table gives the largest ratio found in each test file, between each CAS antiderivative and the optimal antiderivative.

For each test input file, the problem with the largest ratio $\frac{\text{CAS leaf size}}{\text{Optimal leaf size}}$ is recorded with the corresponding problem number.

In each column in the table below, the first number is the maximum leaf size ratio, and the number that follows inside the parentheses is the problem number in that specific file where this maximum ratio was found. This ratio is determined only when CAS solved the the problem and also when an optimal antiderivative is known.

If it happens that a CAS was not able to solve all the integrals in the input test file, or if it was not possible to obtain leaf size for the CAS result for all the problems in the file, then a zero is used for the ratio and -1 is used for the problem number.

This makes it easy to locate the problem. In the future, a direct link will be added as well.

Table 1.6: Maximum leaf size ratio for each CAS against the optimal result

file #	Rubi	Mathematica	Maple	Maxima	FriCAS	Sympy	Giac	Mupad
1	1. (1)	3.9 (50)	16.6 (114)	3.8 (169)	4. (45)	7.5 (169)	4.2 (164)	42.4 (169)
2	7.3 (21)	7.7 (14)	3.6 (17)	1.9 (4)	14.3 (13)	16.8 (5)	4.6 (2)	3.3 (26)
3	1. (1)	16.1 (6)	17. (6)	11.1 (7)	2. (8)	1.9 (5)	1.9 (5)	11.3 (5)
4	6.4 (5)	14.3 (13)	40.7 (46)	16.6 (43)	5.5 (43)	4.4 (40)	5.3 (1)	6.9 (4)
5	1. (65)	54.7 (278)	12737.8 (278)	8.1 (280)	7.7 (280)	16.1 (175)	19.5 (141)	14.1 (204)
6	1. (1)	1.4 (3)	2.2 (4)	1.9 (1)	1.4 (7)	0.8 (4)	2.3 (5)	1.3 (3)
7	2.2 (3)	5.6 (7)	1.8 (3)	2.8 (3)	6.7 (9)	5. (2)	1.9 (3)	1.7 (3)
8	1.6 (50)	5.3 (31)	7.9 (70)	6.5 (11)	5. (42)	26.4 (71)	5.2 (70)	22.5 (70)
9	1.2 (365)	7.2 (80)	4.3 (341)	12.1 (328)	4.2 (341)	8. (75)	15. (328)	6. (9)
10	3.2 (335)	242.6 (327)	3343.5 (327)	36.9 (399)	32.1 (595)	76.3 (215)	18.8 (537)	12.8 (253)
11	529. (82)	127. (82)	317. (82)	2.7 (2)	70. (82)	41.3 (17)	6.6 (50)	207. (82)
12	1.8 (6)	2.3 (4)	1.2 (8)	1.5 (2)	3.3 (3)	3.4 (3)	1.6 (2)	0.9 (8)
13	7.1 (369)	23.8 (1323)	30.9 (1323)	32.9 (1323)	32.9 (1323)	136.1 (671)	34. (1323)	38.1 (1323
14	2. (870)	16.5 (1101)	22.6 (1101)	22.2 (1716)	21.8 (1101)	84.5 (67)	46.7 (827)	328.9 (230
15	3.3 (97)	9.3 (99)	28.5 (100)	2.8 (119)	10.8 (21)	49.2 (119)	10. (119)	23.6 (21)
16	1. (1)	1.5 (17)	11. (25)	4. (25)	9. (25)	59.8 (27)	19.9 (25)	1.7 (3)
17	2.6 (35)	10.1 (67)	39.8 (66)	1.7 (35)	6.4 (7)	5.3 (35)	16.5 (52)	330.8 (32)
18	1. (3)	27.5 (31)	68. (35)	0. (-1)	0. (-1)	0. (-1)	0. (-1)	0. (-1)
19	8.2 (664)	6.9 (663)	7.9 (196)	10. (196)	10. (196)	55.3 (528)	8. (434)	10.1 (196)
20	1.6 (254)	6.4 (94)	147.4 (69)	4.4 (73)	19.9 (160)	10.2 (24)	5.9 (69)	37.7 (26)
21	1. (596)	12.6 (337)	46.7 (754)	3.1 (313)	17.2 (1016)	47.7 (335)	8.6 (553)	224.1 (502
22	1.3 (64)	2.6 (63)	15.2 (57)	1.3 (15)	8.1 (60)	3. (21)	3. (98)	1.7 (21)
23	1. (1)	1.1 (50)	10.4 (15)	2. (15)	7. (15)	43. (16)	13.8 (15)	2.5 (1)
24	1.2 (173)	1.9 (45)	2. (162)	3.6 (161)	5.2 (26)	47.9 (55)	4. (157)	1.8 (133)
25	8.4 (2686)	13.4 (2913)	141.8 (2913)	13.2 (2285)	23. (2913)	170.1 (2672)	28.4 (2813)	16.4 (2913
26	4.3 (116)	9.5 (335)	17.9 (265)	4. (40)	15.1 (265)	47.5 (290)	6. (292)	62.5 (172)
27	4.2 (760)	12.3 (1051)	77.4 (546)	29.1 (1063)	17.4 (317)	36.6 (124)	9.8 (1052)	78.2 (494)
28	1.2 (46)	0.9 (45)	51.1 (15)	2.5 (15)	28.4 (15)	78.7 (3)	49.5 (16)	7.2 (15)
29	1.2 (552)	3.8 (45)	10.4 (43)	10. (43)	47.2 (73)	14.9 (577)	8.1 (591)	34.3 (171)
30	1.3 (278)	10. (328)	51.5 (297)	11.2 (348)	10.2 (348)	10. (328)	10.6 (331)	12.5 (348)
31	1. (1)	6.4 (283)	4.9 (269)	3.2 (114)	4. (269)	21.6 (269)	6.3 (269)	3.4 (190)
32	2.8 (83)	3.9 (25)	5.8 (74)	2.2 (83)	7.2 (127)	16.4 (63)	3.1 (74)	3.5 (25)
33	2. (2419)	23.9 (2302)	70.8 (2351)	28.7 (557)	36.4 (2293)	67.3 (1423)	39.4 (2023)	209.3 (230
34	1.3 (1471)	15.6 (1635)	82.2 (1180)	50.9 (2170)	46.2 (1452)	64.1 (1011)	28.9 (1593)	179.2 (220
35	2.1 (833)	58.6 (507)	116.1 (801)	5.9 (579)	34. (616)	71.6 (920)	20. (925)	201.9 (818
36	1. (1)	10.3 (6)	425.1 (78)	2.7 (95)	30.2 (112)	1.2 (19)	13.3 (5)	3. (100)
37	1. (129)	9.7 (37)	14197.2 (12)	6.6 (27)	30.2 (117)	8.6 (14)	5.8 (37)	110.2 (16)
38	1.8 (76)	42.8 (204)	421. (278)	89. (278)	123.4 (278)	114.1 (278)	119.2 (278)	101.3 (278

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Table 1.6 – continued from previous page

file #	Rubi	Mathematica	Maple	Maxima	FriCAS	Sympy	Giac	Mupad
39	1.7 (636)	8.8 (109)	9.5 (885)	5.4 (515)	25.4 (1077)	28.5 (1105)	13.7 (885)	91.3 (10
40	1.7 (212)	13.9 (409)	50.7 (220)	6.5 (88)	33.5 (109)	15.8 (283)	110.2 (216)	360. (27
41	1.9 (327)	32.6 (381)	26. (136)	5.6 (70)	55.6 (305)	47.5 (220)	35.3 (309)	123.2 (3
42	1. (59)	1.5 (25)	15.8 (54)	1.4 (111)	2.6 (46)	43. (11)	21.7 (25)	107.5 (4
43	1.6 (135)	2.4 (136)	13.8 (37)	1.6 (131)	48.1 (60)	27.3 (39)	20.4 (60)	94.8 (13
44	1.9 (1)	6.3 (24)	6.4 (29)	0. (-1)	4.2 (35)	0.8 (1)	2.5 (42)	3.8 (34)
45	1. (1)	4.9 (4)	0.9 (4)	0. (-1)	0. (-1)	0. (-1)	0. (-1)	0. (-1)
46	2.1 (154)	20. (601)	54.7 (609)	6.3 (609)	46.7 (637)	21.4 (598)	46.6 (597)	99.1 (31
47	1. (1)	25.5 (83)	2.7 (37)	1.8 (68)	12.2 (37)	42.2 (68)	15.3 (37)	116. (41
48	1. (67)	25.1 (143)	2909.3 (93)	88.7 (96)	90.4 (93)	82.9 (93)	73.6 (96)	165.9 (5
49	1. (1)	11. (17)	1.7 (11)	2.1 (16)	2.2 (16)	3.2 (11)	3.3 (16)	215.3 (1
50	1. (1)	1.7 (99)	4. (72)	1.1 (72)	9.5 (102)	18.1 (72)	12.1 (79)	41.4 (99
51	6.2 (424)	11.6 (162)	1223.1 (192)	42.3 (63)	93.1 (192)	84.3 (192)	27.1 (202)	166.7 (2
52	4.1 (997)	172.1 (1010)	3059.3 (1010)	5.1 (612)	40.5 (871)	58.4 (180)	16.9 (414)	54.2 (41
53	1. (1)	1.2 (82)	9.5 (87)	2.2 (2)	2. (81)	2.5 (2)	55.8 (2)	1.8 (2)
54	1. (1)	1. (1)	16. (46)	1.9 (25)	4.6 (58)	2.2 (32)	37.5 (25)	1.8 (20)
55	1.2 (655)	5.3 (636)	38.7 (267)	125.2 (267)	28.5 (292)	11.2 (281)	54.7 (563)	14.9 (26
56	1. (1)	1.3 (133)	83.5 (150)	4.9 (149)	5. (150)	24. (150)	10.2 (149)	2.6 (61)
57	1.7 (115)	3.9 (363)	97.5 (440)	5.1 (348)	21.1 (440)	65.5 (442)	10.8 (392)	2.5 (65)
58	1.5 (176)	12.4 (64)	375.9 (87)	2.9 (166)	10.1 (237)	3.6 (165)	13.4 (237)	2.2 (166
59	7.5 (71)	39.2 (308)	376.9 (168)	7.4 (10)	7.3 (171)	6.5 (59)	49.8 (231)	6.6 (239
60	26.4 (88)	16.8 (81)	1428.6 (228)	79.5 (81)	9.2 (212)	8. (71)	26.8 (1)	13.6 (83
61	1.6 (79)	55.1 (51)	14.7 (74)	14.2 (44)	4.5 (15)	20.3 (12)	12.3 (34)	7. (34)
62	1.8 (383)	9.3 (340)	161.9 (62)	9.1 (340)	8.8 (404)	58.5 (427)	35.8 (456)	3.6 (52)
63	1.5 (390)	4.3 (45)	54.3 (175)	7.4 (390)	33.9 (197)	45.6 (183)	13.8 (45)	5.7 (197
64	1.2 (284)	13.1 (44)	2190.9 (91)	10.6 (23)	11.2 (91)	15.9 (189)	15.3 (28)	8.4 (91)
65	1. (1)	114.1 (497)	33.3 (493)	3.9 (111)	7.8 (301)	137.4 (62)	5.7 (105)	6.2 (210
66	1. (1)	8.6 (249)	7.6 (83)	18.6 (185)	13.1 (209)	29.5 (193)	55.1 (7)	12. (328
67	1. (1)	9.2 (12)	4.3 (51)	2.4 (21)	3.6 (5)	17.1 (49)	4.3 (7)	1.8 (21)
68	1. (1)	1.8 (113)	7.7 (65)	21.3 (45)	2.3 (38)	2.2 (12)	55.1 (36)	1.7 (12)
69	1. (1)	3.3 (203)	7.8 (201)	168.3 (37)	4.7 (44)	8.4 (115)	10.8 (197)	2.7 (37)
70	2. (615)	113.3 (353)	447.5 (605)	9. (151)	23.4 (476)	68.3 (344)	140.9 (122)	19.7 (46
71	1. (1)	1.1 (10)	1.4 (29)	8.1 (33)	1.1 (10)	3.9 (12)	2.3 (30)	1.1 (8)
72	1.6 (103)	56.7 (138)	3.6 (200)	4. (53)	7. (201)	2.6 (40)	128.8 (16)	14.1 (18
73	1.9 (621)	1029.2 (406)	4914.7 (790)	30.7 (256)	15. (563)	45.9 (462)	34.1 (48)	853.8 (7
74	1.6 (1108)	1478. (937)	149.3 (174)	8.6 (46)	14.7 (937)	70.5 (1236)	42.8 (257)	41.4 (11
75	1.3 (12)	3375. (37)	688.4 (48)	7.2 (16)	28.8 (35)	3.4 (1)	1.7 (39)	132.2 (3
76	1.2 (206)	85. (202)	8067.4 (353)	35.1 (48)	16.5 (327)	51.8 (79)	157.2 (15)	82. (352
77	1. (1)	6.7 (10)	3.9 (2)	12.4 (1)	2.3 (2)	412.4 (8)	5.4 (12)	2.4 (3)
78	1.4 (32)	72.5 (30)	4.4 (33)	3.3 (20)	2.2 (18)	2.3 (32)	0.9 (32)	2.9 (1)

Continued on next

Table 1.6 – continued from previous page

file #	Rubi	Mathematica	Maple	Maxima	FriCAS	Sympy	Giac	Mupad
79	1.8 (236)	228.2 (240)	51493.3 (593)	17.6 (487)	43.6 (260)	29. (236)	19.3 (510)	295.9 (392)
80	1. (1)	2.2 (2)	2.1 (4)	1.3 (2)	4.6 (1)	11.7 (4)	2.3 (2)	35.6 (4)
81	1. (1)	1.5 (16)	6. (13)	1. (19)	25.3 (1)	2.8 (11)	1.8 (14)	122.9 (1)
82	1. (1)	3.7 (284)	8.3 (12)	16.5 (170)	4.1 (42)	2.7 (64)	12.5 (64)	3.5 (41)
83	1. (1)	4. (62)	8.3 (76)	12.1 (133)	6.9 (33)	4.1 (9)	66. (8)	2.4 (1)
84	1. (1)	2.4 (61)	3.4 (50)	2. (5)	2.7 (5)	6. (41)	2. (5)	1.3 (4)
85	1. (1)	1.3 (94)	4.2 (26)	4.2 (86)	1.5 (35)	6. (61)	4.3 (35)	1.1 (87)
86	4.3 (11)	4.1 (60)	13.2 (78)	3.2 (3)	4.2 (32)	16.8 (26)	3.7 (11)	16.1 (24)
87	1. (1)	1. (10)	1.4 (29)	8.1 (32)	1.1 (10)	3.8 (12)	2.3 (30)	1.1 (8)
88	1. (1)	3.2 (1)	3.4 (3)	4.1 (3)	4.1 (20)	0. (-1)	3. (3)	14.7 (10)
89	1.4 (370)	35.3 (773)	9.3 (642)	46.6 (119)	7.2 (484)	23. (452)	57.9 (116)	29. (479)
90	1. (1)	2.8 (2)	2.9 (2)	0. (-1)	0. (-1)	0. (-1)	0. (-1)	0. (-1)
91	1. (1)	3. (1)	1.8 (1)	3.5 (1)	1.7 (1)	0. (-1)	0. (-1)	1.2 (1)
92	1.1 (40)	36.7 (454)	14.3 (436)	56.4 (95)	7.5 (278)	36.2 (252)	28.6 (78)	32.8 (267)
93	1. (1)	53.3 (393)	8. (29)	20.3 (115)	3.4 (319)	9. (35)	80.5 (35)	2.8 (122)
94	1.4 (940)	84.8 (1350)	18. (1154)	47.3 (96)	7.2 (590)	29.3 (565)	55.5 (78)	193.6 (981)
95	1.2 (81)	4.9 (91)	6.9 (70)	9.4 (53)	44.5 (85)	400.9 (20)	3.7 (91)	20.3 (14)
96	1. (1)	2.1 (9)	7.6 (21)	1. (2)	8.4 (13)	13.9 (4)	7. (4)	113.8 (15)
97	1. (1)	1.9 (5)	10.5 (13)	0.8 (11)	25.1 (13)	3.2 (12)	35.4 (15)	139.2 (13)
98	1. (1)	105. (358)	326.6 (179)	1.3 (7)	8.5 (251)	3.1 (376)	14.2 (7)	26.5 (105)
99	1. (1)	4.4 (44)	8.6 (29)	11.1 (49)	4.9 (54)	2.5 (24)	6.5 (22)	2.8 (16)
100	1. (1)	3.8 (44)	1.5 (21)	7.8 (52)	4.5 (39)	16.9 (21)	1.7 (21)	2.3 (28)
101	1.5 (562)	75.5 (641)	173.9 (617)	18.9 (393)	8.6 (80)	40. (172)	52.2 (73)	11.3 (560)
102	1. (1)	7. (46)	4.1 (61)	2.9 (67)	7.5 (75)	1.3 (2)	77.5 (13)	6.3 (58)
103	1.4 (891)	200.5 (678)	9431.6 (611)	141. (1121)	111.4 (1249)	68.2 (1213)	27.4 (1203)	547.4 (125
104	1. (1)	941.7 (463)	15282.6 (454)	144. (373)	90.9 (369)	42.2 (280)	24.3 (257)	304.5 (428)
105	1. (130)	3975.5 (145)	172.7 (123)	3. (83)	11.4 (83)	42.2 (37)	26.2 (51)	688.1 (115)
106	1. (1)	44.6 (159)	2905.5 (351)	18.1 (272)	20.7 (379)	67.8 (245)	31.7 (199)	2360. (138)
107	1. (1)	777.6 (45)	31763.8 (14)	0. (-1)	16.1 (45)	0. (-1)	0. (-1)	0. (-1)
108	1. (1)	21.6 (47)	288.2 (43)	1.3 (4)	4.9 (20)	2.6 (1)	4.2 (3)	5.9 (7)
109	1. (1)	5.5 (42)	10.1 (27)	18.6 (47)	4.9 (59)	2.4 (22)	40.3 (8)	2.2 (16)
110	1. (1)	2.5 (11)	3.4 (16)	3.3 (11)	4. (7)	1.3 (2)	2.5 (7)	6.7 (17)
111	1. (1)	2.4 (5)	4.3 (9)	4.3 (7)	3.3 (7)	1.2 (2)	2.7 (6)	8.7 (15)
112	1. (1)	3.9 (15)	69.4 (103)	1.9 (94)	4.3 (6)	35.7 (93)	2.4 (94)	51.1 (103)
113	1. (1)	23.7 (22)	35. (29)	13.6 (8)	13. (57)	64.2 (7)	19.6 (37)	32.4 (7)
114	1. (1)	1997.4 (22)	36459.7 (8)	0. (-1)	25.5 (27)	0. (-1)	0. (-1)	0. (-1)
115	1. (1)	14.7 (42)	9.6 (259)	25.9 (47)	5.7 (42)	3.3 (1)	11.7 (42)	3.9 (223)
116	1. (1)	10. (40)	4.1 (29)	14.8 (16)	5.1 (6)	0. (-1)	6.4 (18)	1.8 (18)
117	1. (1)	3.2 (18)	5.9 (73)	120.4 (20)	4.5 (68)	2.2 (53)	4.2 (20)	5.5 (15)
118	1.4 (423)	249. (874)	14.7 (578)	52.4 (255)	7. (515)	2.6 (5)	5.8 (513)	29.9 (520)

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Table 1.6 – continued from previous page

file #	Rubi	Mathematica	Maple	Maxima	FriCAS	Sympy	Giac	Mupad
119	1. (1)	45.2 (153)	12.6 (284)	2.9 (65)	5.8 (227)	0. (-1)	7. (196)	15.1 (23
120	1.7 (340)	55.8 (191)	46.5 (339)	3.7 (67)	34.3 (339)	13.1 (90)	7.1 (286)	48.1 (29
121	1.3 (115)	2602.3 (169)	1152.4 (153)	37.5 (109)	8.8 (159)	0. (-1)	5.3 (197)	42.4 (19
122	2.2 (197)	1877.2 (240)	7.1 (238)	43.2 (130)	15.3 (263)	3. (170)	4.4 (256)	112. (26
123	1.3 (265)	350.5 (634)	15.8 (385)	52.6 (259)	8.2 (336)	2.2 (47)	6.9 (335)	33.7 (33
124	1. (1)	3.6 (65)	24.1 (25)	13.5 (25)	2.6 (58)	2.9 (33)	2.7 (41)	6. (58)
125	1.2 (870)	383.4 (1373)	19.8 (970)	45.9 (1289)	7.3 (808)	3. (930)	7.6 (489)	193.5 (9
126	1.3 (231)	66.8 (138)	544.5 (433)	47.4 (379)	27.3 (461)	7.4 (459)	4.6 (15)	1013.7 (
127	1. (1)	5.6 (42)	12.4 (21)	33.4 (39)	3.8 (42)	3.1 (1)	3.1 (41)	3.7 (61)
128	1. (1)	4. (25)	5.2 (74)	39.4 (15)	4.6 (69)	2.2 (53)	2.9 (61)	23. (27)
129	1. (1)	5.3 (36)	19.6 (18)	6.4 (13)	7.8 (20)	0. (-1)	13.6 (15)	34.8 (50
130	1. (1)	2.5 (8)	4. (9)	4.9 (8)	3.7 (14)	0. (-1)	2.2 (8)	15.8 (9)
131	1.3 (20)	3.3 (10)	2.3 (22)	3.5 (1)	5. (22)	0. (-1)	2.2 (10)	28.1 (19
132	1. (1)	2.7 (3)	2.2 (8)	2.5 (8)	2.3 (9)	4.9 (18)	3.3 (12)	3.9 (4)
133	1. (1)	1.2 (1)	1.8 (1)	0. (-1)	0. (-1)	0. (-1)	0. (-1)	0. (-1)
134	1. (12)	3.1 (18)	26.8 (15)	26.6 (13)	16.6 (11)	0. (-1)	5.3 (16)	22.2 (7)
135	1. (1)	29.1 (187)	4879055.9 (170)	85. (57)	7.2 (231)	330.6 (40)	78. (71)	8. (233)
136	3.3 (23)	25.3 (272)	5.9 (146)	9.5 (209)	8.5 (143)	18.9 (124)	65.7 (238)	54.3 (22
137	1.1 (281)	9.2 (164)	14.6 (80)	58.4 (391)	13.8 (273)	10.3 (396)	81.4 (293)	3.1 (81)
138	1. (1)	2.7 (1)	6.9 (9)	0.4 (5)	12.2 (4)	1.1 (5)	0.7 (5)	2.2 (5)
139	4.3 (259)	8. (318)	12.8 (259)	90.9 (225)	3.1 (173)	7.6 (18)	73.7 (126)	4.1 (224
140	19.2 (34)	9.1 (133)	40.1 (34)	81.3 (34)	4.2 (63)	10.8 (42)	266.8 (31)	8.6 (63)
141	10.8 (759)	718.9 (434)	651.2 (860)	178.8 (64)	27.7 (503)	1334.1 (478)	84.2 (904)	47.9 (50
142	1.4 (107)	2.5 (95)	4.8 (156)	1.7 (155)	1.8 (7)	2.3 (11)	9.9 (145)	3. (150)
143	1.7 (100)	9.5 (655)	19.9 (90)	3.3 (195)	6. (642)	2.9 (413)	56.7 (620)	3. (662)
144	1.9 (147)	7. (85)	13.9 (55)	12.1 (177)	8.6 (103)	8.1 (206)	14. (233)	3. (12)
145	1.3 (168)	4.9 (41)	2.8 (156)	1.8 (155)	3. (7)	2.3 (11)	26.4 (147)	2. (150)
146	1. (1)	1.9 (10)	2.8 (13)	2.4 (11)	5.1 (33)	2. (23)	36.5 (23)	1.1 (21)
147	1. (1)	3.8 (13)	7.8 (12)	2.4 (24)	5.7 (29)	2. (58)	3.9 (31)	2.4 (27)
148	10. (146)	4.8 (83)	28.1 (148)	1.5 (165)	3.4 (112)	8.9 (105)	1.9 (134)	1.8 (21)
149	1.2 (25)	4.2 (25)	44.1 (20)	1.8 (8)	19.4 (21)	44.8 (8)	4.4 (24)	4.1 (7)
150	1.3 (152)	6.4 (429)	85.8 (146)	4.8 (218)	9.9 (1223)	4.2 (197)	2.4 (1279)	7.7 (115
151	1. (1)	3.3 (36)	80. (56)	26.3 (61)	3. (30)	9.8 (12)	1. (27)	5.2 (1)
152	2. (344)	2.7 (248)	13.6 (329)	9.6 (180)	10. (375)	11.5 (375)	8.4 (375)	4.2 (376
153	1.1 (117)	11.4 (54)	27.1 (147)	5.4 (67)	5.6 (50)	13.1 (131)	5.8 (125)	5.5 (1)
154	1.3 (109)	11.4 (164)	72.1 (110)	13.3 (107)	6.8 (64)	5.9 (106)	27.2 (135)	5.5 (149
155	1. (1)	1.2 (7)	1. (2)	1. (2)	1.1 (5)	2.7 (4)	1.1 (2)	0.9 (5)
156	1.2 (68)	2.6 (104)	11.9 (105)	3.4 (31)	8.7 (151)	2.5 (12)	75.8 (1)	2. (29)
157	1. (1)	3.3 (42)	4.2 (26)	1.7 (14)	4. (24)	2.7 (8)	2.6 (2)	1.2 (5)
158	1.4 (51)	2.8 (111)	11.9 (112)	1.9 (22)	8.7 (156)	2.6 (12)	27.3 (91)	1.9 (29)

Continued on next

Table 1.6 – continued from previous page

file #	Rubi	Mathematica	Maple	Maxima	FriCAS	Sympy	Giac	Mupad
159	1. (1)	3.3 (40)	4.9 (26)	1.6 (13)	4. (23)	2.7 (8)	3.5 (26)	1.3 (5)
160	1. (1)	16. (299)	7.5 (379)	3.7 (327)	18.6 (329)	7.4 (297)	8.7 (6)	16.6 (489)
161	1. (1)	5.4 (53)	3.4 (98)	12.9 (90)	6.6 (20)	1.9 (10)	6.9 (29)	1.7 (50)
162	1. (1)	1.5 (24)	1.9 (28)	6. (7)	5.4 (21)	0. (-1)	1.6 (29)	0. (-1)
163	1. (1)	8.7 (365)	8.4 (198)	21.3 (134)	32.5 (87)	16.3 (253)	25.9 (273)	22.4 (234)
164	1.3 (16)	9.9 (394)	15.3 (316)	21.9 (315)	64.5 (502)	103. (65)	23. (273)	45.2 (327)
165	1. (1)	13.6 (173)	6. (1)	3.6 (1)	16. (36)	4.1 (8)	8.7 (6)	2.4 (16)
166	1. (1)	1.8 (38)	3.6 (79)	3.5 (5)	4.3 (108)	2.3 (12)	18. (32)	1.7 (12)
167	1. (1)	2.1 (3)	3.4 (64)	12.9 (56)	6.6 (20)	1.9 (10)	5.2 (25)	1.9 (24)
168	1. (1)	1.5 (12)	1.9 (28)	6. (7)	5.4 (21)	0. (-1)	1.6 (29)	0. (-1)
169	1. (1)	8.7 (328)	7.4 (165)	11.4 (196)	40.1 (177)	22. (152)	25.9 (246)	21.4 (180)
170	1.3 (60)	2.5 (11)	8. (38)	7.4 (13)	56.7 (12)	414.7 (16)	5. (38)	53.8 (12)
171	1. (1)	3.7 (3)	9.8 (43)	3.2 (8)	23.5 (11)	1.7 (8)	3.4 (8)	1.6 (32)
172	1.2 (109)	3.5 (212)	6.6 (102)	12.6 (188)	65.9 (200)	42.3 (62)	4.2 (102)	11.1 (85)
173	1.3 (257)	10.5 (252)	14.4 (114)	23.3 (190)	89.5 (249)	36.6 (195)	21.1 (108)	27.4 (106)
174	1. (1)	5.1 (48)	11. (27)	3.7 (8)	20. (47)	11.7 (27)	3.2 (8)	1.6 (8)
175	1. (1)	7.6 (113)	6.2 (35)	13. (193)	66.3 (205)	11.6 (148)	7.1 (113)	11.1 (119)
176	1. (1)	6.7 (10)	9.2 (24)	6.3 (10)	89.6 (41)	6.4 (5)	8.7 (10)	19.2 (7)
177	1. (1)	3.8 (6)	2.9 (5)	2.5 (7)	16. (9)	0. (-1)	2.7 (7)	1.7 (7)
178	1. (1)	3.5 (18)	3.6 (79)	2. (15)	22.6 (82)	0. (-1)	2. (31)	4.7 (79)
179	3.5 (186)	6.4 (145)	12.7 (186)	8.6 (59)	84. (136)	2.2 (119)	5.4 (186)	7.5 (116)
180	1.4 (54)	14.4 (168)	13.9 (169)	21.9 (158)	87.8 (209)	5.4 (142)	5. (167)	18.2 (31)
181	1. (1)	9.1 (26)	5. (29)	3.1 (7)	15.7 (9)	0. (-1)	2.8 (7)	2.2 (25)
182	1. (1)	5.2 (18)	3.9 (78)	2.3 (15)	28.8 (15)	0. (-1)	1.9 (5)	9.7 (81)
183	3.3 (160)	6.7 (24)	22.3 (24)	8.9 (91)	33.8 (124)	0. (-1)	9. (24)	8.2 (120)
184	1.1 (12)	3.4 (24)	16.5 (8)	6.2 (1)	58. (14)	0. (-1)	8.8 (22)	9.5 (1)
185	1.9 (192)	515.8 (777)	140.9 (767)	26. (100)	63.7 (794)	57.3 (808)	12.2 (11)	24.9 (100)
186	1. (1)	1.9 (141)	2.7 (38)	1.4 (15)	3.3 (7)	1. (22)	2.3 (19)	0.9 (5)
187	2.1 (73)	3.5 (230)	10. (313)	3.5 (219)	6. (651)	3.5 (255)	2.6 (118)	1.9 (531)
188	1.2 (170)	4.7 (46)	6.2 (151)	12.3 (115)	5.3 (11)	8.2 (147)	8.2 (115)	2.7 (354)
189	1.1 (163)	3.2 (39)	2.3 (18)	1.2 (135)	2. (7)	1.1 (135)	2. (19)	0.9 (136)
190	1.7 (322)	5.5 (516)	17.2 (93)	2.6 (22)	7. (508)	1.7 (528)	2.2 (528)	1.6 (347)
191	1.3 (73)	6.9 (167)	26. (291)	9.2 (93)	9.7 (20)	7.6 (122)	6.1 (93)	88.8 (279)
192	8.1 (149)	1.9 (31)	49.6 (28)	5.3 (202)	9.9 (216)	29.6 (63)	7.5 (1)	3.3 (200)
193	1.6 (21)	4. (12)	56.2 (20)	2.3 (40)	27.6 (32)	62.6 (8)	19. (8)	8.9 (28)
194	1.6 (538)	4. (156)	74.3 (235)	16.1 (244)	7.1 (516)	4.5 (307)	6.7 (15)	6.9 (244)
195	1. (43)	8.2 (42)	62.1 (46)	5.2 (15)	5. (37)	20.4 (22)	28.3 (37)	16. (22)
196	2. (172)	3.7 (868)	16.4 (867)	18.3 (1152)	12.2 (1368)	30.7 (997)	9.7 (1368)	11.7 (652)
197	1.7 (81)	24. (319)	24.6 (312)	4.3 (72)	6.3 (315)	2.9 (277)	7.6 (133)	30.3 (131)
198	1.2 (78)	24. (238)	3055.9 (185)	3.9 (95)	8.4 (181)	14.4 (107)	0.8 (298)	9.4 (176)

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Table 1.6 – continued from previous page

file #	Rubi	Mathematica	Maple	Maxima	FriCAS	Sympy	Giac	Mupad
199	1.9 (172)	4.9 (430)	14.9 (117)	3.5 (37)	4.3 (130)	12.1 (767)	5.1 (235)	2.8 (584
200	1. (1)	16.3 (85)	19.3 (124)	1.4 (47)	8.9 (168)	1.5 (35)	0. (-1)	1.2 (27)
201	2.8 (38)	5.6 (18)	40.2 (80)	1. (34)	9.3 (6)	0.7 (93)	3.3 (47)	9.3 (74)
202	1.2 (75)	2.9 (111)	11.5 (112)	2.1 (10)	9.7 (156)	1.2 (9)	0. (-1)	1.2 (9)
203	1.6 (55)	8.2 (13)	5.2 (65)	2.7 (31)	7.3 (71)	2. (31)	2.8 (31)	1.5 (31)
204	1. (1)	1.7 (102)	2.5 (221)	1.1 (31)	2. (140)	2.7 (221)	1.6 (18)	4.4 (48)
205	1. (1)	2.5 (57)	1.5 (92)	0. (-1)	0. (-1)	2. (179)	0. (-1)	0. (-1)
206	1. (1)	2.6 (41)	3.3 (134)	0. (-1)	0. (-1)	7.3 (69)	62.4 (135)	0. (-1)
207	1. (1)	2.5 (131)	1.3 (35)	0. (-1)	0. (-1)	8.5 (69)	0. (-1)	0. (-1)
208	1.1 (174)	1.3 (195)	2.4 (144)	4.1 (155)	2.7 (28)	4.9 (30)	0. (-1)	1.6 (155

1.5 Pass/Fail per test file for each CAS system

The following table gives the number of passed integrals and number of failed integrals per test number. There are 208 tests. Each tests corresponds to one input file.

Table 1.7: Pass/Fail per test file for each CAS

Test #	Rubi		MMA		Maple		Maxima		FriCAS		Sympy		Giac		Mupad	
	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail
1	175	0	175	0	173	2	166	9	172	3	158	17	169	6	169	6
2	33	2	35	0	27	8	15	20	24	11	7	28	17	18	9	26
3	13	1	13	1	11	3	8	6	12	2	9	5	10	4	11	3
4	48	2	50	0	33	17	24	26	48	2	19	31	41	9	12	38
5	279	5	283	1	282	2	252	32	280	4	249	35	269	15	270	14
6	3	4	7	0	5	2	3	4	7	0	5	2	5	2	7	0
7	7	2	9	0	9	0	7	2	9	0	5	4	9	0	9	0
8	113	0	112	1	113	0	111	2	112	1	104	9	109	4	106	7
9	376	0	376	0	376	0	374	2	376	0	348	28	375	1	372	4
10	705	0	705	0	655	50	564	141	652	53	430	275	587	118	542	163
11	100	16	95	21	77	39	20	96	89	27	29	87	31	85	37	79
12	8	0	8	0	8	0	7	1	8	0	8	0	8	0	8	0
13	1917	0	1917	0	1560	357	1328	589	1603	314	1201	716	1276	641	1241	676
14	3201	0	3201	0	2870	331	2048	1153	2535	666	1547	1654	2403	798	1884	1317
15	158	1	155	4	128	31	39	120	47	112	40	119	42	117	49	110
16	34	0	34	0	28	6	16	18	28	6	13	21	28	6	4	30
17	78	0	78	0	78	0	27	51	44	34	14	64	33	45	40	38
18	35	0	35	0	35	0	0	35	0	35	0	35	0	35	0	35
19	1071	0	1071	0	755	316	632	439	674	397	1013	58	616	455	695	376
20	349	0	349	0	260	89	79	270	141	208	103	246	108	241	66	283
21	1156	0	1156	0	1041	115	682	474	856	300	594	562	822	334	730	426
22	115	0	114	1	105	10	27	88	30	85	27	88	31	84	27	88
23	51	0	51	0	14	37	14	37	14	37	25	26	14	37	14	37
24	174	0	174	0	170	4	170	4	158	16	140	34	170	4	129	45
25	3078	0	3044	34	2591	487	2196	882	2370	708	2688	390	1990	1088	2228	850

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Table 1.7 – continued from previous page

Test #	Rubi		MMA		Maple		Maxima		FriCAS		Sympy		Giac		Mupad	
	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail
26	385	0	383	2	197	188	167	218	213	172	144	241	134	251	170	215
27	1081	0	1081	0	749	332	391	690	663	418	380	701	535	546	531	550
28	46	0	46	0	12	34	12	34	12	34	14	32	11	35	12	34
29	594	0	594	0	577	17	422	172	341	253	444	150	420	174	449	145
30	454	0	454	0	385	69	153	301	256	198	114	340	238	216	193	261
31	298	0	296	2	275	23	212	86	228	70	126	172	213	85	197	101
32	143	0	143	0	113	30	108	35	113	30	47	96	106	37	132	11
33	2590	0	2584	6	2325	265	1428	1162	2096	494	1037	1553	1678	912	1589	1001
34	2646	0	2646	0	2584	62	1720	926	2299	347	1214	1432	2153	493	1685	961
35	958	0	942	16	729	229	331	627	594	364	260	698	279	679	276	682
36	123	0	123	0	121	2	67	56	111	12	43	80	90	33	53	70
37	143	0	142	1	141	2	15	128	69	74	11	132	50	93	19	124
38	400	0	394	6	388	12	290	110	330	70	141	259	335	65	195	205
39	1126	0	1126	0	1062	64	688	438	846	280	476	650	799	327	695	431
40	412	1	378	35	399	14	113	300	210	203	188	225	177	236	184	229
41	413	0	400	13	376	37	173	240	256	157	126	287	264	149	218	195
42	111	0	103	8	111	0	83	28	83	28	47	64	100	11	106	5
43	145	0	145	0	143	2	73	72	115	30	79	66	139	6	143	2
44	42	0	38	4	40	2	0	42	9	33	6	36	5	37	1	41
45	4	0	4	0	4	0	0	4	0	4	0	4	0	4	0	4
46	664	0	662	2	496	168	303	361	535	129	282	382	412	252	360	304
47	96	0	92	4	49	47	17	79	47	49	40	56	37	59	49	47
48	156	0	147	9	137	19	69	87	108	48	75	81	110	46	122	34
49	17	0	14	3	2	15	2	15	7	10	1	16	4	13	5	12
50	140	0	139	1	136	4	24	116	129	11	53	87	106	34	72	68
51	491	3	494	0	489	5	409	85	431	63	431	63	421	73	485	9
52	1007	18	997	28	841	184	385	640	697	328	256	769	484	541	455	570
53	98	0	98	0	78	20	64	34	93	5	40	58	55	43	58	40
54	93	0	84	9	75	18	72	21	93	0	50	43	52	41	53	40
55	766	8	751	23	621	153	503	271	694	80	335	439	370	404	580	194
56	193	0	193	0	98	95	106	87	123	70	76	117	102	91	60	133
57	456	0	449	7	309	147	245	211	280	176	231	225	200	256	146	310
58	249	0	243	6	78	171	68	181	90	159	42	207	58	191	46	203
59	288	26	298	16	187	127	238	76	210	104	118	196	152	162	200	114
60	249	14	249	14	98	165	179	84	156	107	50	213	88	175	127	136
61	106	2	108	0	24	84	68	40	39	69	22	86	34	74	35	73
62	543	4	543	4	309	238	223	324	221	326	164	383	214	333	209	338
63	641	0	621	20	337	304	389	252	393	248	177	464	350	291	326	315
64	314	0	314	0	234	80	219	95	279	35	127	187	191	123	183	131
65	538	0	538	0	442	96	243	295	286	252	99	439	191	347	248	290
66	348	0	348	0	264	84	194	154	322	26	113	235	162	186	143	205
67	72	0	72	0	47	25	32	40	39	33	32	40	39	33	36	36
68	113	0	113	0	113	0	53	60	113	0	26	87	65	48	20	93
69	357	0	345	12	245	112	260	97	305	52	105	252	182	175	129	228
70	653	0	638	15	562	91	288	365	358	295	96	557	278	375	258	395
71	36	0	36	0	34	2	34	2	36	0	20	16	34	2	16	20

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Table 1.7 – continued from previous page

Test #	Rubi		MMA		Maple		Maxima		FriCAS		Sympy		Giac		Mupad	
	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail
72	206	2	203	5	178	30	142	66	178	30	4	204	127	81	154	54
73	837	0	820	17	635	202	217	620	512	325	153	684	302	535	344	493
74	1560	3	1519	44	1380	183	984	579	1216	347	222	1341	1111	452	1131	432
75	51	0	51	0	50	1	16	35	30	21	4	47	6	45	13	38
76	358	0	348	10	290	68	133	225	275	83	90	268	130	228	178	180
77	19	0	15	4	12	7	13	6	13	6	8	11	12	7	13	6
78	34	0	32	2	5	29	7	27	8	26	1	33	1	33	9	25
79	590	4	583	11	521	73	332	262	397	197	64	530	321	273	334	260
80	9	0	9	0	9	0	2	7	9	0	5	4	9	0	9	0
81	19	0	19	0	19	0	5	14	15	4	6	13	9	10	19	0
82	294	0	294	0	196	98	92	202	93	201	15	279	26	268	80	214
83	189	0	187	2	135	54	137	52	133	56	54	135	111	78	74	115
84	62	0	62	0	45	17	37	25	39	23	32	30	39	23	35	27
85	99	0	99	0	87	12	81	18	91	8	33	66	51	48	30	69
86	88	0	88	0	88	0	27	61	32	56	22	66	32	56	34	54
87	34	0	34	0	32	2	32	2	34	0	18	16	32	2	15	19
88	22	0	22	0	22	0	17	5	21	1	1	21	20	2	18	4
89	932	0	923	9	854	78	291	641	443	489	95	837	269	663	310	622
90	4	0	4	0	4	0	0	4	0	4	0	4	0	4	0	4
91	1	0	1	0	1	0	1	0	1	0	0	1	0	1	1	0
92	644	0	634	10	634	10	189	455	317	327	63	581	196	448	231	413
93	393	0	389	4	236	157	119	274	121	272	9	384	17	376	75	318
94	1541	0	1534	7	1533	8	451	1090	734	807	122	1419	505	1036	629	912
95	98	0	98	0	98	0	70	28	73	25	18	80	75	23	67	31
96	21	0	21	0	21	0	2	19	16	5	6	15	15	6	19	2
97	20	0	20	0	20	0	4	16	16	4	5	15	12	8	20	0
98	387	0	386	1	264	123	137	250	153	234	13	374	67	320	122	265
99	62	1	63	0	58	5	45	18	63	0	28	35	31	32	32	31
100	66	0	66	0	36	30	49	17	48	18	36	30	36	30	38	28
101	700	0	700	0	580	120	405	295	452	248	122	578	247	453	369	331
102	91	0	90	1	83	8	79	12	83	8	8	83	75	16	83	8
103	1328	0	1210	118	1114	214	577	751	921	407	289	1039	478	850	835	493
104	855	0	798	57	780	75	428	427	583	272	205	650	225	630	529	326
105	171	0	169	2	122	49	84	87	84	87	58	113	79	92	103	68
106	499	0	497	2	406	93	269	230	406	93	91	408	209	290	283	216
107	51	0	51	0	40	11	0	51	16	35	0	51	0	51	0	51
108	52	0	52	0	37	15	37	15	21	31	8	44	16	36	26	26
109	61	0	61	0	58	3	46	15	61	0	28	33	35	26	28	33
110	23	0	23	0	23	0	19	4	23	0	6	17	22	1	23	0
111	19	0	19	0	19	0	15	4	19	0	4	15	19	0	19	0
112	106	0	105	1	103	3	79	27	31	75	2	104	3	103	103	3
113	64	0	64	0	63	1	21	43	64	0	12	52	36	28	39	25
114	32	0	32	0	25	7	0	32	16	16	0	32	0	32	0	32
115	299	0	299	0	225	74	93	206	106	193	21	278	35	264	78	221
116	46	0	45	1	42	4	29	17	46	0	20	26	21	25	24	22
117	83	0	79	4	51	32	28	55	63	20	37	46	43	40	47	36

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Table 1.7 – continued from previous page

Test #	Rubi		MMA		Maple		Maxima		FriCAS		Sympy		Giac		Mupad	
	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail
118	879	0	869	10	735	144	309	570	393	486	49	830	262	617	323	556
119	305	1	304	2	267	39	175	131	191	115	7	299	191	115	193	113
120	364	1	344	21	331	34	213	152	260	105	40	325	251	114	181	184
121	240	1	227	14	216	25	96	145	145	96	5	236	44	197	56	185
122	286	0	273	13	262	24	166	120	236	50	1	285	82	204	191	95
123	634	0	634	0	586	48	193	441	300	334	8	626	209	425	195	439
124	70	0	70	0	70	0	48	22	49	21	3	67	46	24	49	21
125	1373	0	1340	33	1263	110	459	914	732	641	11	1362	545	828	552	821
126	468	2	424	46	431	39	286	184	401	69	21	449	163	307	243	227
127	70	0	70	0	53	17	28	42	31	39	9	61	31	39	16	54
128	84	0	80	4	52	32	35	49	64	20	37	47	44	40	47	37
129	59	0	53	6	41	18	25	34	41	18	3	56	38	21	33	26
130	16	0	16	0	16	0	12	4	16	0	0	16	16	0	16	0
131	23	0	23	0	23	0	18	5	23	0	0	23	23	0	23	0
132	24	0	24	0	24	0	24	0	24	0	9	15	24	0	24	0
133	1	0	1	0	1	0	0	1	0	1	0	1	0	1	0	1
134	27	0	27	0	27	0	17	10	27	0	0	27	17	10	8	19
135	254	0	252	2	215	39	159	95	209	45	54	200	139	115	169	85
136	294	0	294	0	289	5	271	23	290	4	64	230	279	15	290	4
137	397	0	396	1	359	38	302	95	365	32	121	276	219	178	155	242
138	9	0	9	0	9	0	1	8	9	0	1	8	1	8	1	8
139	254	76	305	25	107	223	139	191	148	182	65	265	74	256	149	181
140	140	2	142	0	114	28	114	28	115	27	38	104	63	79	50	92
141	944	6	938	12	908	42	651	299	851	99	418	532	705	245	700	250
142	227	0	226	1	216	11	66	161	79	148	100	127	163	64	75	152
143	700	3	694	9	554	149	251	452	265	438	202	501	234	469	146	557
144	472	2	464	10	376	98	112	362	185	289	162	312	246	228	89	385
145	227	0	227	0	214	13	67	160	79	148	100	127	163	64	73	154
146	33	0	33	0	30	3	12	21	15	18	11	22	15	18	3	30
147	118	0	113	5	78	40	31	87	48	70	33	85	51	67	22	96
148	157	9	163	3	144	22	93	73	92	74	91	75	80	86	108	58
149	29	2	28	3	30	1	14	17	11	20	11	20	7	24	14	17
150	1301	0	1287	14	1200	101	416	885	554	747	553	748	365	936	754	547
151	70	0	67	3	69	1	37	33	28	42	23	47	6	64	30	40
152	385	0	368	17	203	182	134	251	286	99	71	314	117	268	147	238
153	153	0	153	0	133	20	86	67	131	22	50	103	61	92	55	98
154	234	0	230	4	228	6	142	92	168	66	82	152	111	123	108	126
155	12	0	12	0	6	6	1	11	6	6	3	9	1	11	6	6
156	174	0	169	5	139	35	84	90	108	66	67	107	92	82	53	121
157	50	0	49	1	37	13	18	32	28	22	13	37	27	23	10	40
158	178	0	173	5	144	34	82	96	110	68	63	115	94	84	57	121
159	49	0	49	0	36	13	15	34	27	22	12	37	25	24	12	37
160	502	0	466	36	340	162	294	208	456	46	118	384	199	303	209	293
161	102	0	101	1	80	22	84	18	78	24	31	71	54	48	29	73
162	33	0	33	0	31	2	31	2	33	0	9	24	31	2	9	24
163	369	0	369	0	318	51	266	103	304	65	115	254	276	93	221	148

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Table 1.7 – continued from previous page

Test #	Rubi		MMA		Maple		Maxima		FriCAS		Sympy		Giac		Mupad	
	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail
164	525	0	501	24	488	37	196	329	366	159	72	453	267	258	247	278
165	183	0	181	2	110	73	143	40	150	33	61	122	103	80	70	113
166	111	0	111	0	111	0	64	47	111	0	26	85	71	40	20	91
167	68	0	68	0	58	10	62	6	60	8	23	45	43	25	21	47
168	33	0	33	0	31	2	31	2	33	0	9	24	31	2	9	24
169	336	0	335	1	293	43	208	128	283	53	103	233	259	77	190	146
170	85	0	84	1	85	0	34	51	67	18	17	68	46	39	55	30
171	72	5	71	6	69	8	63	14	64	13	30	47	46	31	39	38
172	206	41	247	0	207	40	151	96	209	38	67	180	187	60	175	72
173	263	0	263	0	249	14	177	86	246	17	40	223	233	30	185	78
174	61	0	60	1	58	3	55	6	61	0	28	33	35	26	28	33
175	183	41	224	0	164	60	105	119	177	47	32	192	137	87	131	93
176	53	0	53	0	43	10	16	37	50	3	6	47	27	26	32	21
177	16	0	16	0	8	8	5	11	12	4	3	13	4	12	4	12
178	84	0	80	4	50	34	39	45	63	21	34	50	44	40	47	37
179	201	0	192	9	140	61	90	111	142	59	9	192	114	87	94	107
180	220	0	220	0	180	40	147	73	211	9	10	210	141	79	121	99
181	29	0	29	0	19	10	13	16	25	4	4	25	8	21	8	21
182	83	0	74	9	49	34	55	28	62	21	34	49	43	40	47	36
183	175	0	175	0	136	39	111	64	132	43	0	175	107	68	91	84
184	27	0	27	0	14	13	10	17	24	3	0	27	20	7	5	22
185	1059	0	1049	10	936	123	762	297	975	84	313	746	802	257	740	319
186	156	0	156	0	109	47	51	105	43	113	48	108	36	120	30	126
187	663	0	663	0	498	165	259	404	241	422	185	478	95	568	133	530
188	370	1	370	1	233	138	117	254	147	224	93	278	97	274	78	293
189	166	0	166	0	112	54	57	109	52	114	53	113	39	127	32	134
190	569	0	558	11	471	98	245	324	236	333	154	415	106	463	144	425
191	295	1	288	8	185	111	81	215	122	174	80	216	84	212	64	232
192	216	27	231	12	195	48	154	89	147	96	83	160	127	116	128	115
193	46	3	47	2	48	1	29	20	16	33	10	39	17	32	17	32
194	538	0	536	2	508	30	270	268	256	282	144	394	175	363	175	363
195	62	0	60	2	61	1	34	28	17	45	14	48	17	45	17	45
196	1378	0	1353	25	1100	278	619	759	1118	260	454	924	675	703	698	680
197	361	0	361	0	342	19	267	94	338	23	78	283	257	104	239	122
198	300	0	294	6	273	27	246	54	223	77	100	200	30	270	153	147
199	935	0	914	21	784	151	502	433	838	97	200	735	490	445	518	417
200	190	0	185	5	151	39	86	104	120	70	48	142	45	145	52	138
201	100	0	98	2	74	26	21	79	69	31	1	99	6	94	56	44
202	178	0	173	5	100	78	84	94	113	65	32	146	46	132	49	129
203	71	0	71	0	53	18	42	29	49	22	32	39	23	48	41	30
204	311	0	300	11	179	132	140	171	258	53	168	143	133	178	203	108
205	218	0	190	28	154	64	60	158	60	158	114	104	60	158	60	158
206	136	0	134	2	118	18	34	102	34	102	50	86	104	32	34	102
207	136	0	136	0	104	32	34	102	34	102	50	86	34	102	34	102
208	198	0	195	3	144	54	127	71	102	96	46	152	16	182	71	127

1.6 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.7 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.8 Important notes about some of the results

1.8.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.8.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.8.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.8.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

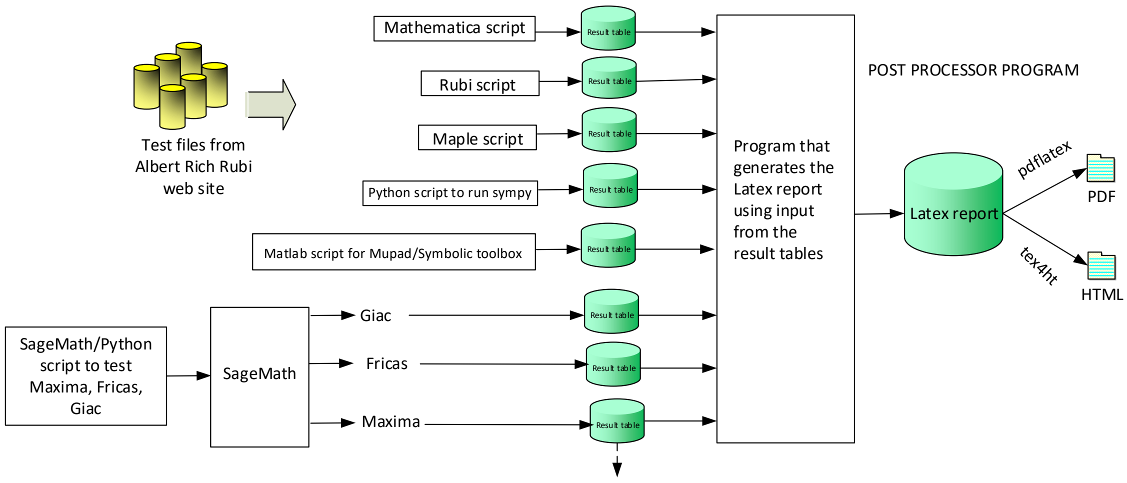
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.9 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
May 11, 2021

Chapter 2

links to individual test reports

These are links to each test report. The number in square brackets to right of the link is the number of integrals in the test. The list of numbers in the curly brackets after that (if any) is the list of the integrals in that specific test which were solved by any CAS which are known not to have antiderivative. This makes it easier to find these integrals and do more investigation into them.

2.1 Tests completed

1. 0_Independent_test_suites/Apostol_Problems [175]
2. 0_Independent_test_suites/Bondarenko_Problems [35]
3. 0_Independent_test_suites/Bronstein_Problems [14]
4. 0_Independent_test_suites/Charlwood_Problems [50]
5. 0_Independent_test_suites/Hearn_Problems [284] { **Maxima: 145.** }
6. 0_Independent_test_suites/Hebisch_Problems [7]
7. 0_Independent_test_suites/Jeffrey_Problems [9]
8. 0_Independent_test_suites/Moses_Problems [113]
9. 0_Independent_test_suites/Stewart_Problems [376]
10. 0_Independent_test_suites/Timofeev_Problems [705]
11. 0_Independent_test_suites/Welz_Problems [116]
12. 0_Independent_test_suites/Wester_Problems [8]
13. 1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/1.1.1.2-a+b_x-^m-c+d_x-^n [1917]
14. 1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/1.1.1.3-a+b_x-^m-c+d_x-^n-e+f_x-^p [3201]
15. 1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/1.1.1.4-a+b_x-^m-c+d_x-^n-e+f_x-^p-g+h_x-^q [159]
16. 1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/1.1.1.5_P-x-a+b_x-^m-c+d_x-^n [34]
17. 1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/1.1.1.6_P-x-a+b_x-^m-c+d_x-^n-e+f_x-^p [78]
18. 1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/1.1.1.7_P-x-a+b_x-^m-c+d_x-^n-e+f_x-^p-g+h_x-^q [35]
19. 1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/1.1.2.2-c_x-^m-a+b_x^2-^p [1071]

20. 1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/1.1.2.3-a+b_x^2-^p-c+d_x^2-^q [349]

21. 1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/1.1.2.4-e_x-^m-a+b_x^2-^p-c+d_x^2-^q [1156]

22. 1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/1.1.2.5-a+b_x^2-^p-c+d_x^2-^q-e+f_x^2-^r [115]

23. 1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/1.1.2.6-g_x-^m-a+b_x^2-^p-c+d_x^2-^q-e+f_x^2-^r [51]

24. 1_Algebraic_functions/1.1_Binomial_products/1.1.2_Quadratic/1.1.2.8_P-x-c_x-^m-a+b_x^2-^p [174]

25. 1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/1.1.3.2-c_x-^m-a+b_x^n-^p [3078]

26. 1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/1.1.3.3-a+b_x^n-^p-c+d_x^n-^q [385]

27. 1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/1.1.3.4-e_x-^m-a+b_x^n-^p-c+d_x^n-^q [1081]

28. 1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/1.1.3.6-g_x-^m-a+b_x^n-^p-c+d_x^n-^q-e+f_x^n-^r [46]

29. 1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/1.1.3.8_P-x-c_x-^m-a+b_x^n-^p [594]

30. 1_Algebraic_functions/1.1_Binomial_products/1.1.4_Improper/1.1.4.2-c_x-^m-a_x^j+b_x^n-^p [454]

31. 1_Algebraic_functions/1.1_Binomial_products/1.1.4_Improper/1.1.4.3-e_x-^m-a_x^j+b_x^k-^p-c+d_x^n-^q [298]

32. 1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/1.2.1.1-a+b_x+c_x^2-^p [143]

33. 1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/1.2.1.2-d+e_x-^m-a+b_x+c_x^2-^p [2590]

34. 1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/1.2.1.3-d+e_x-^m-f+g_x-a+b_x+c_x^2-^p [2646]

35. 1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/1.2.1.4-d+e_x-^m-f+g_x-^n-a+b_x+c_x^2-^p [958]

36. 1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/1.2.1.5-a+b_x+c_x^2-^p-d+e_x+f_x^2-^q [123]

37. 1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/1.2.1.6-g+h_x-^m-a+b_x+c_x^2-^p-d+e_x+f_x^2-^q [143]

38. 1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/1.2.1.9_P-x-d+e_x-^m-a+b_x+c_x^2-^p [400]

39. 1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/1.2.2.2-d_x-^m-a+b_x^2+c_x^4-^p [1126]

40. 1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/1.2.2.3-d+e_x^2-^m-a+b_x^2+c_x^4-^p [413]

41. 1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/1.2.2.4-f_x-^m-d+e_x^2-^q-a+b_x^2+c_x^4-^p [413]

42. 1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/1.2.2.5_P-x-a+b_x^2+c_x^4-^p [111]

43. 1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/1.2.2.6_P-x-d_x-^m-a+b_x^2+c_x^4-^p [145]

44. 1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/1.2.2.7_P-x-d+e_x^2-^q-a+b_x^2+c_x^4-^p [42]

45. 1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/1.2.2.8_P-x-d+e_x-^q-a+b_x^2+c_x^4-^p [4]

46. 1_Algebraic_functions/1.2_Trinomial_products/1.2.3_General/1.2.3.2-d_x-^m-a+b_x^n+c_x^-2_n-^p [664]

47. 1_Algebraic_functions/1.2_Trinomial_products/1.2.3_General/1.2.3.3-d+e_x^n-^q-a+b_x^n+c_x^-2_n-^p [96]

48. 1_Algebraic_functions/1.2_Trinomial_products/1.2.3_General/1.2.3.4-f_x-^m-d+e_x^n-^q-a+b_x^n+c_x^-2_n-^p [156]

49. 1_Algebraic_functions/1.2_Trinomial_products/1.2.3_General/1.2.3.5_P-x-d_x-^m-a+b_x^n+c_x^-2_n-^p [17]

50. 1_Algebraic_functions/1.2_Trinomial_products/1.2.4_Improper/1.2.4.2-d_x-^m-a_x^q+b_x^n+c_x^-2_n-q-^p [140]

51. 1_Algebraic_functions/1.3_Miscellaneous/1.3.1_Rational_functions [494]

52. 1_Algebraic_functions/1.3_Miscellaneous/1.3.2_Algebraic_functions [1025]

53. 2_Exponentials/2.1_u-F^-c-a+b_x-^n [98]

54. 2_Exponentials/2.2-c+d_x-^m-F^-g-e+f_x-^n-a+b-F^-g-e+f_x-^n-^p [93]

55. 2_Exponentials/2.3_Exponential_functions [774]

56. 3_Logarithms/3.1.2-d_x-^m-a+b_log-c_x^n-^p [193]

57. 3_Logarithms/3.1.4-f_x-^m-d+e_x^r-^q-a+b_log-c_x^n-^p [456] { **Mathematica: 166, 167, 168, 170, 322, 323, 406, 407, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 444, 445.** }

58. 3_Logarithms/3.1.5_u-a+b_log-c_x^n-^p [249] { **Mathematica: 138, 144, 145, 146, 148, 149, 220. Maple: 220, 221, 222.** }

59. 3_Logarithms/3.2.1-f+g_x-^m-A+B_log-e-a+b_x-over-c+d_x-^n-^p [314]

60. 3_Logarithms/3.2.2-f+g_x-^m-h+i_x-^q-A+B_log-e-a+b_x-over-c+d_x-^n-^p [263]

61. 3_Logarithms/3.2.3_u_log-e-f-a+b_x-^p-c+d_x-^q-^r-^s [108]

62. 3_Logarithms/3.3_u-a+b_log-c-d+e_x-^n-^p [547]

63. 3_Logarithms/3.4_u-a+b_log-c-d+e_x^m-^n-^p [641] { **Mathematica: 98, 99, 100, 101, 158, 159, 277, 298, 299, 485, 486, 487, 488, 528, 529, 530, 531.** }

64. 3_Logarithms/3.5_Logarithm_functions [314]

65. 4_Trig_functions/4.1_Sine/4.1.0-a_sin-^m-b_trg-^n [538]

66. 4_Trig_functions/4.1_Sine/4.1.10-c+d_x-^m-a+b_sin-^n [348]

67. 4_Trig_functions/4.1_Sine/4.1.1.1-a+b_sin-^n [72]

68. 4_Trig_functions/4.1_Sine/4.1.11-e_x-^m-a+b_x^n-^p_sin [113]

69. 4_Trig_functions/4.1_Sine/4.1.12-e_x-^m-a+b_sin-c+d_x^n-^p [357]

70. 4_Trig_functions/4.1_Sine/4.1.1.2-g_cos-^p-a+b_sin-^m [653]

71. 4_Trig_functions/4.1_Sine/4.1.13-d+e_x-^m_sin-a+b_x+c_x^2-^n [36]

72. 4_Trig_functions/4.1_Sine/4.1.1.3-g_tan-^p-a+b_sin-^m [208]

73. 4_Trig_functions/4.1_Sine/4.1.2.1-a+b_sin-^m-c+d_sin-^n [837]

74. 4_Trig_functions/4.1_Sine/4.1.2.2-g_cos-^p-a+b_sin-^m-c+d_sin-^n [1563]

75. 4_Trig_functions/4.1_Sine/4.1.2.3-g_sin-^p-a+b_sin-^m-c+d_sin-^n [51]

76. 4_Trig_functions/4.1_Sine/4.1.3.1-a+b_sin-^m-c+d_sin-^n-A+B_sin- [358]
77. 4_Trig_functions/4.1_Sine/4.1.4.1-a+b_sin-^m-A+B_sin+C_sin^2- [19]
78. 4_Trig_functions/4.1_Sine/4.1.4.2-a+b_sin-^m-c+d_sin-^n-A+B_sin+C_sin^2- [34]
79. 4_Trig_functions/4.1_Sine/4.1.7-d_trig-^m-a+b-c_sin-^n-^p [594] { **Mathematica: 399, 400, 401, 402, 403. Maple: 399, 400, 401, 402, 403.** }
80. 4_Trig_functions/4.1_Sine/4.1.8-a+b_sin-^m-c+d_trig-^n [9]
81. 4_Trig_functions/4.1_Sine/4.1.9_trig^m-a+b_sin^n+c_sin^-2_n-^p [19]
82. 4_Trig_functions/4.2_Cosine/4.2.0-a_cos-^m-b_trg-^n [294]
83. 4_Trig_functions/4.2_Cosine/4.2.10-c+d_x-^m-a+b_cos-^n [189]
84. 4_Trig_functions/4.2_Cosine/4.2.1.1-a+b_cos-^n [62]
85. 4_Trig_functions/4.2_Cosine/4.2.12-e_x-^m-a+b_cos-c+d_x^n-^p [99]
86. 4_Trig_functions/4.2_Cosine/4.2.1.2-g_sin-^p-a+b_cos-^m [88]
87. 4_Trig_functions/4.2_Cosine/4.2.13-d+e_x-^m_cos-a+b_x+c_x^2-^n [34]
88. 4_Trig_functions/4.2_Cosine/4.2.1.3-g_tan-^p-a+b_cos-^m [22]
89. 4_Trig_functions/4.2_Cosine/4.2.2.1-a+b_cos-^m-c+d_cos-^n [932]
90. 4_Trig_functions/4.2_Cosine/4.2.2.2-g_sin-^p-a+b_cos-^m-c+d_cos-^n [4]
91. 4_Trig_functions/4.2_Cosine/4.2.2.3-g_cos-^p-a+b_cos-^m-c+d_cos-^n [1]
92. 4_Trig_functions/4.2_Cosine/4.2.3.1-a+b_cos-^m-c+d_cos-^n-A+B_cos- [644]
93. 4_Trig_functions/4.2_Cosine/4.2.4.1-a+b_cos-^m-A+B_cos+C_cos^2- [393]
94. 4_Trig_functions/4.2_Cosine/4.2.4.2-a+b_cos-^m-c+d_cos-^n-A+B_cos+C_cos^2- [1541]
95. 4_Trig_functions/4.2_Cosine/4.2.7-d_trig-^m-a+b-c_cos-^n-^p [98]
96. 4_Trig_functions/4.2_Cosine/4.2.8-a+b_cos-^m-c+d_trig-^n [21]
97. 4_Trig_functions/4.2_Cosine/4.2.9_trig^m-a+b_cos^n+c_cos^-2_n-^p [20]
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128. 4_Trig_functions/4.6_Cosecant/4.6.11-e_x-^m-a+b_csc-c+d_x^n-^p [84]
129. 4_Trig_functions/4.6_Cosecant/4.6.1.2-d_csc-^n-a+b_csc-^m [59]
130. 4_Trig_functions/4.6_Cosecant/4.6.1.3-d_cos-^n-a+b_csc-^m [16]
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149. 5_Inverse_trig_functions/5.3_Inverse_tangent/5.3.3-d+e_x-^m-a+b_arctan-c_x^n-^p [31]

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151. 5_Inverse_trig_functions/5.3_Inverse_tangent/5.3.5_u-a+b_arctan-c+d_x-^p [70] { **Mathematica: 65, 66, 69, 70.** }

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156. 5_Inverse_trig_functions/5.5_Inverse_secant/5.5.1_u-a+b_arcsec-c_x-^n [174]

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162. 6_Hyperbolic_functions/6.1_Hyperbolic_sine/6.1.4-d+e_x-^m_sinh-a+b_x+c_x^2-^n [33]

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170. 6_Hyperbolic_functions/6.2_Hyperbolic_cosine/6.2.7_hyper^m-a+b_cosh^n-^p [85]

171. 6_Hyperbolic_functions/6.3_Hyperbolic_tangent/6.3.1-c+d_x-^m-a+b_tanh-^n [77]

172. 6_Hyperbolic_functions/6.3_Hyperbolic_tangent/6.3.2_Hyperbolic_tangent_functions [247]

173. 6_Hyperbolic_functions/6.3_Hyperbolic_tangent/6.3.7-d_hyper-^m-a+b-c_tanh-^n-^p [263] { **Mathematica: 74, 76, 77, 79. Maple: 74, 76, 77, 79. Giac: 74, 76, 77, 79.** }

174. 6_Hyperbolic_functions/6.4_Hyperbolic_cotangent/6.4.1-c+d_x-^m-a+b_coth-^n [61]

175. 6_Hyperbolic_functions/6.4_Hyperbolic_cotangent/6.4.2_Hyperbolic_cotangent_functions [224]

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177. 6_Hyperbolic_functions/6.5_Hyperbolic_secant/6.5.1-c+d_x-^m-a+b_sech-^n [16]

178. 6_Hyperbolic_functions/6.5_Hyperbolic_secant/6.5.2-e_x-^m-a+b_sech-c+d_x^n-^p [84]

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190. 7_Inverse_hyperbolic_functions/7.2_Inverse_hyperbolic_cosine/7.2.4-f_x-^m-d+e_x^2-^p-a+b_arccosh-c_x-^n [569]
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194. 7_Inverse_hyperbolic_functions/7.3_Inverse_hyperbolic_tangent/7.3.4_u-a+b_arctanh-c_x-^p [538]
195. 7_Inverse_hyperbolic_functions/7.3_Inverse_hyperbolic_tangent/7.3.5_u-a+b_arctanh-c+d_x-^p [62]
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202. 7_Inverse_hyperbolic_functions/7.6_Inverse_hyperbolic_cosecant/7.6.1_u-a+b_arccsch-c_x-^n [178]
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Chapter 3

Listing of integrals solved by CAS which has no known antiderivatives

3.1 Test file Number [5] 0-Independent-test-suites/Hearn-Problems

3.1.1 Maxima

Integral number [145]

$$\int x\cos(k\csc(x))\cot(x)\csc(x)\,dx$$

[B] time = 0.473911 (sec), size = 240 ,normalized size = 17.14

$$-\frac{\left(xe^{\left(\frac{4\,k\cos(2\,x)\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}+\frac{4\,k\sin(2\,x)\sin(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}\right)}+xe^{\left(\frac{4\,k\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}\right)}\right)e^{\left(-\frac{2\,k\cos(2\,x)\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}-\frac{2\,k\sin(2\,x)\sin(}{\cos(2\,x)^2+\sin(2\,x)^2-2}\right.}}{2\,k}$$

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm=""maxima"")
```

```
[Out] -1/2*(x*e^(4*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) +
 4*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)) + x*e^(4*k
*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)))*e^(-2*k*cos(2*x)*cos(x
)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*sin(2*x)*sin(x)/(cos(2*x
)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 -
2*cos(2*x) + 1))*sin(2*(k*cos(x)*sin(2*x) - k*cos(2*x)*sin(x) + k*sin(x))/(
cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))/k
```

3.2 Test file Number [57] 3-Logarithms/3.1.4-f-x-^m-d+e-x^r-^q-a+b-log-c-x^n-^p

3.2.1 Mathematica

Integral number [166]

$$\int \frac{(fx)^m\left(a+b\log\left(cx^n\right)\right)}{d+ex}\,dx$$

[B] time = 0.104714 (sec), size = 72 ,normalized size = 2.77

$$\frac{x(fx)^m\left((m+1)\,{}_2F_1\left(1,m+1;m+2;-\frac{ex}{d}\right)\left(a+b\log\left(cx^n\right)\right)-bn\,{}_3F_2\left(1,m+1,m+1;m+2,m+2;-\frac{ex}{d}\right)\right)}{d(m+1)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x),x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{1, 1 + m, 1 + m}, {2 + m, 2 + m}, -((e
*x)/d)]) + (1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((e*x)/d)]*(a + b*Lo
g[c*x^n])))/(d*(1 + m)^2)
```

Integral number [167]

$$\int \frac{(fx)^m \left(a + b\log\left(cx^n\right)\right)}{(d + ex)^2}\, dx$$

[B] time = 0.107026 (sec), size = 72 ,normalized size = 2.77

$$\frac{x(fx)^m \left((m+1)\,{}_2F_1\left(2, m+1; m+2; -\frac{ex}{d}\right)\left(a + b\log\left(cx^n\right)\right) - bn\,{}_3F_2\left(2, m+1, m+1; m+2, m+2; -\frac{ex}{d}\right)\right)}{d^2(m+1)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x)^2,x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{2, 1 + m, 1 + m}, {2 + m, 2 + m}, -((e
*x)/d)]) + (1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((e*x)/d)]*(a + b*Lo
g[c*x^n])))/(d^2*(1 + m)^2)
```

Integral number [168]

$$\int x(a + bx)^m \log\left(cx^n\right)\, dx$$

[B] time = 0.250616 (sec), size = 173 ,normalized size = 9.61

$$\frac{(a+bx)^m \left(\frac{bx}{a}+1\right)^{-m} \left(ab(m+2)nx\,{}_3F_2\left(1,1,-m-1;2,2;-\frac{bx}{a}\right) + \left(-a^2\left(\left(\frac{bx}{a}+1\right)^m - 1\right) + b^2(m+1)x^2\left(\frac{bx}{a}+1\right)^m + abmx\left(\frac{bx}{a}\right.\right.\right.}{b^2(m+1)(m+2)}$$

```
[In] Integrate[x*(a + b*x)^m*Log[c*x^n],x]
```

```
[Out] ((a + b*x)^m*(-(n*(2*a*b*x*(1 + (b*x)/a)^m + b^2*x^2*(1 + (b*x)/a)^m + a^2*
(-1 + (1 + (b*x)/a)^m))) + a*b*(2 + m)*n*x*HypergeometricPFQ[{1, 1, -1 - m}
, {2, 2}, -((b*x)/a)] + (a*b*m*x*(1 + (b*x)/a)^m + b^2*(1 + m)*x^2*(1 + (b*
x)/a)^m - a^2*(-1 + (1 + (b*x)/a)^m))*Log[c*x^n]))/(b^2*(1 + m)*(2 + m)*(1
+ (b*x)/a)^m)
```

Integral number [170]

$$\int \frac{(a + bx)^m \log\left(cx^n\right)}{x}\, dx$$

[B] time = 0.0644198 (sec), size = 89 ,normalized size = 4.45

$$\frac{\left(\frac{a}{bx}+1\right)^{-m}(a+bx)^m \left(m\log\left(cx^n\right)\,{}_2F_1\left(-m,-m;1-m;-\frac{a}{bx}\right) - n\,{}_3F_2\left(-m,-m,-m;1-m,1-m;-\frac{a}{bx}\right)\right)}{m^2}$$

```
[In] Integrate[((a + b*x)^m*Log[c*x^n])/x,x]
```

```
[Out] ((a + b*x)^m*(-(n*HypergeometricPFQ[{-m, -m, -m}, {1 - m, 1 - m}, -(a/(b*x)
)]) + m*Hypergeometric2F1[-m, -m, 1 - m, -(a/(b*x))]*Log[c*x^n]))/(m^2*(1 +
 a/(b*x))^m)
```

Integral number [322]

$$\int \frac{(fx)^m \left(a + b\log\left(cx^n\right)\right)}{d + ex^2}\, dx$$

[B] time = 0.208145 (sec), size = 108 ,normalized size = 3.86

$$\frac{x(fx)^m \left((m+1)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{ex^2}{d}\right)\left(a+b\log\left(cx^n\right)\right)-bn\,{}_3F_2\left(1,\frac{m}{2}+\frac{1}{2},\frac{m}{2}+\frac{1}{2};\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};-\frac{ex^2}{d}\right)\right)}{d(m+1)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^2),x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{1, 1/2 + m/2, 1/2 + m/2}, {3/2 + m/2,
3/2 + m/2}, -((e*x^2)/d)]) + (1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, -((e*x^2)/d)]*(a + b*Log[c*x^n])))/(d*(1 + m)^2)
```

Integral number [323]

$$\int \frac{(fx)^m \left(a+b\log\left(cx^n\right)\right)}{\left(d+ex^2\right)^2}\,dx$$

[B] time = 0.128838 (sec), size = 108 ,normalized size = 3.86

$$\frac{x(fx)^m \left((m+1)\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{ex^2}{d}\right)\left(a+b\log\left(cx^n\right)\right)-bn\,{}_3F_2\left(2,\frac{m}{2}+\frac{1}{2},\frac{m}{2}+\frac{1}{2};\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};-\frac{ex^2}{d}\right)\right)}{d^2(m+1)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^2)^2,x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{2, 1/2 + m/2, 1/2 + m/2}, {3/2 + m/2,
3/2 + m/2}, -((e*x^2)/d)]) + (1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m
)/2, -((e*x^2)/d)]*(a + b*Log[c*x^n])))/(d^2*(1 + m)^2)
```

Integral number [406]

$$\int \frac{x^3 \left(a+b\log\left(cx^n\right)\right)}{d+ex^r}\,dx$$

[B] time = 0.124222 (sec), size = 87 ,normalized size = 3.35

$$\frac{x^4 \left(4\,{}_2F_1\left(1,\frac{4}{r};\frac{r+4}{r};-\frac{ex^r}{d}\right)\left(a+b\log\left(cx^n\right)\right)-bn\,{}_3F_2\left(1,\frac{4}{r},\frac{4}{r};1+\frac{4}{r},1+\frac{4}{r};-\frac{ex^r}{d}\right)\right)}{16d}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x^r),x]
```

```
[Out] (x^4*(-(b*n*HypergeometricPFQ[{1, 4/r, 4/r}, {1 + 4/r, 1 + 4/r}, -((e*x^r)/
d)]) + 4*Hypergeometric2F1[1, 4/r, (4 + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x^
n])))/(16*d)
```

Integral number [407]

$$\int \frac{x \left(a+b\log\left(cx^n\right)\right)}{d+ex^r}\,dx$$

[B] time = 0.106702 (sec), size = 87 ,normalized size = 3.62

$$\frac{x^2 \left(2\,{}_2F_1\left(1,\frac{2}{r};\frac{r+2}{r};-\frac{ex^r}{d}\right)\left(a+b\log\left(cx^n\right)\right)-bn\,{}_3F_2\left(1,\frac{2}{r},\frac{2}{r};1+\frac{2}{r},1+\frac{2}{r};-\frac{ex^r}{d}\right)\right)}{4d}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x^r),x]
```

```
[Out] (x^2*(-(b*n*HypergeometricPFQ[{1, 2/r, 2/r}, {1 + 2/r, 1 + 2/r}, -((e*x^r)/
d)]) + 2*Hypergeometric2F1[1, 2/r, (2 + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x^
```

```
n])))/(4*d)
```

Integral number [409]

$$\int \frac{a+b\log(cx^n)}{x^3(d+ex^r)}\,dx$$

[B] time = 0.117141 (sec), size = 86 ,normalized size = 3.31

$$-\frac{bn\,{}_3F_2\left(1,-\frac{2}{r},-\frac{2}{r};1-\frac{2}{r},1-\frac{2}{r};-\frac{ex^r}{d}\right)+2\,{}_2F_1\left(1,-\frac{2}{r};\frac{r-2}{r};-\frac{ex^r}{d}\right)\left(a+b\log(cx^n)\right)}{4dx^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x^r)),x]
```

```
[Out] -1/4*(b*n*HypergeometricPFQ[{1, -2/r, -2/r}, {1 - 2/r, 1 - 2/r}, -((e*x^r)/
d)] + 2*Hypergeometric2F1[1, -2/r, (-2 + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x
^n]))/(d*x^2)
```

Integral number [410]

$$\int \frac{x^2\left(a+b\log(cx^n)\right)}{d+ex^r}\,dx$$

[B] time = 0.114966 (sec), size = 87 ,normalized size = 3.35

$$\frac{x^3\left(3\,{}_2F_1\left(1,\frac{3}{r};\frac{r+3}{r};-\frac{ex^r}{d}\right)\left(a+b\log(cx^n)\right)-bn\,{}_3F_2\left(1,\frac{3}{r},\frac{3}{r};1+\frac{3}{r},1+\frac{3}{r};-\frac{ex^r}{d}\right)\right)}{9d}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x^r),x]
```

```
[Out] (x^3*(-(b*n*HypergeometricPFQ[{1, 3/r, 3/r}, {1 + 3/r, 1 + 3/r}, -((e*x^r)/
d)]) + 3*Hypergeometric2F1[1, 3/r, (3 + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x^
n])))/(9*d)
```

Integral number [411]

$$\int \frac{a+b\log(cx^n)}{d+ex^r}\,dx$$

[B] time = 0.085968 (sec), size = 69 ,normalized size = 3.

$$\frac{x\left({}_2F_1\left(1,\frac{1}{r};1+\frac{1}{r};-\frac{ex^r}{d}\right)\left(a+b\log(cx^n)\right)-bn\,{}_3F_2\left(1,\frac{1}{r},\frac{1}{r};1+\frac{1}{r},1+\frac{1}{r};-\frac{ex^r}{d}\right)\right)}{d}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^r),x]
```

```
[Out] (x*(-(b*n*HypergeometricPFQ[{1, r^(-1), r^(-1)}, {1 + r^(-1), 1 + r^(-1)},
-((e*x^r)/d)]) + Hypergeometric2F1[1, r^(-1), 1 + r^(-1), -((e*x^r)/d)]*(a
+ b*Log[c*x^n])))/d
```

Integral number [412]

$$\int \frac{a+b\log(cx^n)}{x^2(d+ex^r)}\,dx$$

[B] time = 0.103256 (sec), size = 83 ,normalized size = 3.19

$$-\frac{bn\,{}_3F_2\left(1,-\frac{1}{r},-\frac{1}{r};1-\frac{1}{r},1-\frac{1}{r};-\frac{ex^r}{d}\right)+\,{}_2F_1\left(1,-\frac{1}{r};\frac{r-1}{r};-\frac{ex^r}{d}\right)\left(a+b\log(cx^n)\right)}{dx}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x^r)),x]
```

```
[Out] -((b*n*HypergeometricPFQ[{1, -r^(-1), -r^(-1)}, {1 - r^(-1), 1 - r^(-1)}, -
((e*x^r)/d)] + Hypergeometric2F1[1, -r^(-1), (-1 + r)/r, -((e*x^r)/d)]*(a +
 b*Log[c*x^n]))/(d*x))
```

Integral number [413]

$$\int \frac{x^3\left(a+b\log\left(cx^n\right)\right)}{\left(d+ex^r\right)^2}\,dx$$

[B] time = 0.261741 (sec), size = 140 ,normalized size = 5.38

$$\frac{x^4\left(-bn(r-4)\left(d+ex^r\right)\,{}_3F_2\left(1,\frac{4}{r},\frac{4}{r};1+\frac{4}{r},1+\frac{4}{r};-\frac{ex^r}{d}\right)+4\left(d+ex^r\right)\,{}_2F_1\left(1,\frac{4}{r};\frac{r+4}{r};-\frac{ex^r}{d}\right)\left(a(r-4)+b(r-4)\log\left(cx^n\right)-b\right.}{16d^2r\left(d+ex^r\right)}$$

```
[In] Integrate[(x^3*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] (x^4*(-(b*n*(-4 + r)*(d + e*x^r)*HypergeometricPFQ[{1, 4/r, 4/r}, {1 + 4/r,
 1 + 4/r}, -((e*x^r)/d)]) + 16*d*(a + b*Log[c*x^n]) + 4*(d + e*x^r)*Hyperge
ometric2F1[1, 4/r, (4 + r)/r, -((e*x^r)/d)]*(-(b*n) + a*(-4 + r) + b*(-4 +
r)*Log[c*x^n])))/(16*d^2*r*(d + e*x^r))
```

Integral number [414]

$$\int \frac{x\left(a+b\log\left(cx^n\right)\right)}{\left(d+ex^r\right)^2}\,dx$$

[B] time = 0.239464 (sec), size = 140 ,normalized size = 5.83

$$\frac{x^2\left(-bn(r-2)\left(d+ex^r\right)\,{}_3F_2\left(1,\frac{2}{r},\frac{2}{r};1+\frac{2}{r},1+\frac{2}{r};-\frac{ex^r}{d}\right)+2\left(d+ex^r\right)\,{}_2F_1\left(1,\frac{2}{r};\frac{r+2}{r};-\frac{ex^r}{d}\right)\left(a(r-2)+b(r-2)\log\left(cx^n\right)-b\right.}{4d^2r\left(d+ex^r\right)}$$

```
[In] Integrate[(x*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] (x^2*(-(b*n*(-2 + r)*(d + e*x^r)*HypergeometricPFQ[{1, 2/r, 2/r}, {1 + 2/r,
 1 + 2/r}, -((e*x^r)/d)]) + 4*d*(a + b*Log[c*x^n]) + 2*(d + e*x^r)*Hypergeo
metric2F1[1, 2/r, (2 + r)/r, -((e*x^r)/d)]*(-(b*n) + a*(-2 + r) + b*(-2 + r
)*Log[c*x^n])))/(4*d^2*r*(d + e*x^r))
```

Integral number [416]

$$\int \frac{a+b\log\left(cx^n\right)}{x^3\left(d+ex^r\right)^2}\,dx$$

[B] time = 3.16006 (sec), size = 205 ,normalized size = 7.88

$$\frac{4ben(r+2)x^r\left(d+ex^r\right)\,{}_3F_2\left(1,1-\frac{2}{r},1-\frac{2}{r};2-\frac{2}{r},2-\frac{2}{r};-\frac{ex^r}{d}\right)-(r-2)\left(4ex^r\left(d+ex^r\right)\,{}_2F_1\left(1,\frac{r-2}{r};2-\frac{2}{r};-\frac{ex^r}{d}\right)\left(a(r+2)+\right.\right.}{4d^3(r-2)^2rx^2\left(d+\right.}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^3*(d + e*x^r)^2),x]
```

```
[Out] (4*b*e*n*(2 + r)*x^r*(d + e*x^r)*HypergeometricPFQ[{1, 1 - 2/r, 1 - 2/r}, {
2 - 2/r, 2 - 2/r}, -((e*x^r)/d)] - (-2 + r)*(4*e*x^r*(d + e*x^r)*Hypergeome
tric2F1[1, (-2 + r)/r, 2 - 2/r, -((e*x^r)/d)]*(-(b*n) + a*(2 + r) + b*(2 +
r)*Log[c*x^n]) + d*(-2 + r)*(b*n*r*(d + e*x^r) + 2*a*(d*r + e*(2 + r)*x^r)
+ 2*b*(d*r + e*(2 + r)*x^r)*Log[c*x^n])))/(4*d^3*(-2 + r)^2*r*x^2*(d + e*x^
```

```
r))
```

Integral number [417]

$$\int \frac{x^2\left(a+b\log\left(cx^n\right)\right)}{\left(d+ex^r\right)^2}\,dx$$

[B] time = 0.249699 (sec), size = 140 ,normalized size = 5.38

$$\frac{x^3\left(-bn(r-3)\left(d+ex^r\right)\,{}_3F_2\left(1,\frac{3}{r},\frac{3}{r};1+\frac{3}{r},1+\frac{3}{r};-\frac{ex^r}{d}\right)+3\left(d+ex^r\right)\,{}_2F_1\left(1,\frac{3}{r};\frac{r+3}{r};-\frac{ex^r}{d}\right)\left(a(r-3)+b(r-3)\log\left(cx^n\right)-bn\right)}{9d^2r\left(d+ex^r\right)}$$

```
[In] Integrate[(x^2*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] (x^3*(-(b*n*(-3 + r)*(d + e*x^r)*HypergeometricPFQ[{1, 3/r, 3/r}, {1 + 3/r,
 1 + 3/r}, -((e*x^r)/d)]) + 9*d*(a + b*Log[c*x^n]) + 3*(d + e*x^r)*Hypergeo
metric2F1[1, 3/r, (3 + r)/r, -((e*x^r)/d)]*(-(b*n) + a*(-3 + r) + b*(-3 + r
)*Log[c*x^n])))/(9*d^2*r*(d + e*x^r))
```

Integral number [418]

$$\int \frac{a+b\log\left(cx^n\right)}{\left(d+ex^r\right)^2}\,dx$$

[B] time = 2.61975 (sec), size = 161 ,normalized size = 7.

$$\frac{x\left(-bn(r-1)\left(d+ex^r\right)\,{}_3F_2\left(1,\frac{1}{r},\frac{1}{r};1+\frac{1}{r},1+\frac{1}{r};-\frac{ex^r}{d}\right)+aerx^r\,{}_2F_1\left(2,\frac{1}{r};1+\frac{1}{r};-\frac{ex^r}{d}\right)+adr\,{}_2F_1\left(2,\frac{1}{r};1+\frac{1}{r};-\frac{ex^r}{d}\right)-b\left(d+ex^r\right.}{d^2r\left(d+ex^r\right)}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d + e*x^r)^2,x]
```

```
[Out] (x*(a*d*r*Hypergeometric2F1[2, r^(-1), 1 + r^(-1), -((e*x^r)/d)] + a*e*r*x^
r*Hypergeometric2F1[2, r^(-1), 1 + r^(-1), -((e*x^r)/d)] - b*n*(-1 + r)*(d
+ e*x^r)*HypergeometricPFQ[{1, r^(-1), r^(-1)}, {1 + r^(-1), 1 + r^(-1)}, -
((e*x^r)/d)] + b*d*Log[c*x^n] - b*(d + e*x^r)*Hypergeometric2F1[1, r^(-1),
1 + r^(-1), -((e*x^r)/d)]*(n - (-1 + r)*Log[c*x^n])))/(d^2*r*(d + e*x^r))
```

Integral number [419]

$$\int \frac{a+b\log\left(cx^n\right)}{x^2\left(d+ex^r\right)^2}\,dx$$

[B] time = 0.204882 (sec), size = 135 ,normalized size = 5.19

$$\frac{-bn(r+1)\left(d+ex^r\right)\,{}_3F_2\left(1,-\frac{1}{r},-\frac{1}{r};1-\frac{1}{r},1-\frac{1}{r};-\frac{ex^r}{d}\right)-\left(d+ex^r\right)\,{}_2F_1\left(1,-\frac{1}{r};\frac{r-1}{r};-\frac{ex^r}{d}\right)\left(ar+a+b(r+1)\log\left(cx^n\right)-bn\right)+a}{d^2rx\left(d+ex^r\right)}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(x^2*(d + e*x^r)^2),x]
```

```
[Out] (-(b*n*(1 + r)*(d + e*x^r)*HypergeometricPFQ[{1, -r^(-1), -r^(-1)}, {1 - r^
(-1), 1 - r^(-1)}, -((e*x^r)/d)]) + d*(a + b*Log[c*x^n]) - (d + e*x^r)*Hype
rgeometric2F1[1, -r^(-1), (-1 + r)/r, -((e*x^r)/d)]*(a - b*n + a*r + b*(1 +
 r)*Log[c*x^n]))/(d^2*r*x*(d + e*x^r))
```

Integral number [444]

$$\int \frac{(fx)^m\left(a+b\log\left(cx^n\right)\right)}{d+ex^r}\,dx$$

[B] time = 0.156347 (sec), size = 111 ,normalized size = 3.96

$$\frac{x(fx)^m\left((m+1)\left(a+b\log\left(cx^n\right)\right){}_2F_1\left(1,\frac{m+1}{r};\frac{m+r+1}{r};-\frac{ex^r}{d}\right)-bn\,{}_3F_2\left(1,\frac{m}{r}+\frac{1}{r},\frac{m}{r}+\frac{1}{r};\frac{m}{r}+\frac{1}{r}+1,\frac{m}{r}+\frac{1}{r}+1;-\frac{ex^r}{d}\right)\right)}{d(m+1)^2}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^r),x]
```

```
[Out] (x*(f*x)^m*(-(b*n*HypergeometricPFQ[{1, r^(-1) + m/r, r^(-1) + m/r}, {1 + r
^(-1) + m/r, 1 + r^(-1) + m/r}, -((e*x^r)/d)]) + (1 + m)*Hypergeometric2F1[
1, (1 + m)/r, (1 + m + r)/r, -((e*x^r)/d)]*(a + b*Log[c*x^n])))/(d*(1 + m)^
2)
```

Integral number [445]

$$\int \frac{(fx)^m\left(a+b\log\left(cx^n\right)\right)}{\left(d+ex^r\right)^2}\,dx$$

[B] time = 0.386001 (sec), size = 177 ,normalized size = 6.32

$$\frac{x(fx)^m\left(bn(m-r+1)\left(d+ex^r\right){}_3F_2\left(1,\frac{m}{r}+\frac{1}{r},\frac{m}{r}+\frac{1}{r};\frac{m}{r}+\frac{1}{r}+1,\frac{m}{r}+\frac{1}{r}+1;-\frac{ex^r}{d}\right)-(m+1)\left(\left(d+ex^r\right){}_2F_1\left(1,\frac{m+1}{r};\frac{m+r+1}{r}\right.\right.\right.}{d^2(m+1)^2r\left(d+ex^r\right)}$$

```
[In] Integrate[((f*x)^m*(a + b*Log[c*x^n]))/(d + e*x^r)^2,x]
```

```
[Out] (x*(f*x)^m*(b*n*(1 + m - r)*(d + e*x^r)*HypergeometricPFQ[{1, r^(-1) + m/r,
 r^(-1) + m/r}, {1 + r^(-1) + m/r, 1 + r^(-1) + m/r}, -((e*x^r)/d)] - (1 +
m)*(-(d*(1 + m)*(a + b*Log[c*x^n])) + (d + e*x^r)*Hypergeometric2F1[1, (1 +
 m)/r, (1 + m + r)/r, -((e*x^r)/d)]*(b*n + a*(1 + m - r) + b*(1 + m - r)*Lo
g[c*x^n]))))/(d^2*(1 + m)^2*r*(d + e*x^r))
```

3.3 Test file Number [58] 3-Logarithms/3.1.5-u-a+b-log-c-x^n-^p

3.3.1 Mathematica

Integral number [138]

$$\int (gx)^q\left(a+b\log\left(cx^n\right)\right)\log\left(d\left(e+fx^m\right)^k\right)\,dx$$

[B] time = 0.348144 (sec), size = 304 ,normalized size = 9.81

$$x(gx)^q\left(-bkmn\,{}_3F_2\left(1,\frac{q}{m}+\frac{1}{m},\frac{q}{m}+\frac{1}{m};\frac{q}{m}+\frac{1}{m}+1,\frac{q}{m}+\frac{1}{m}+1;-\frac{fx^m}{e}\right)+km\,{}_2F_1\left(1,\frac{q+1}{m};\frac{m+q+1}{m};-\frac{fx^m}{e}\right)\left(aq+a+b(q+1)\log\right.\right.$$

```
[In] Integrate[(g*x)^q*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] (x*(g*x)^q*(-(a*k*m) + 2*b*k*m*n - a*k*m*q - b*k*m*n*HypergeometricPFQ[{1,
m^(-1) + q/m, m^(-1) + q/m}, {1 + m^(-1) + q/m, 1 + m^(-1) + q/m}, -((f*x^m
)/e)] - b*k*m*Log[c*x^n] - b*k*m*q*Log[c*x^n] + k*m*Hypergeometric2F1[1, (1
 + q)/m, (1 + m + q)/m, -((f*x^m)/e)]*(a - b*n + a*q + b*(1 + q)*Log[c*x^n]
) + a*Log[d*(e + f*x^m)^k] - b*n*Log[d*(e + f*x^m)^k] + 2*a*q*Log[d*(e + f*
x^m)^k] - b*n*q*Log[d*(e + f*x^m)^k] + a*q^2*Log[d*(e + f*x^m)^k] + b*Log[c
*x^n]*Log[d*(e + f*x^m)^k] + 2*b*q*Log[c*x^n]*Log[d*(e + f*x^m)^k] + b*q^2*
Log[c*x^n]*Log[d*(e + f*x^m)^k]))/(1 + q)^3
```

Integral number [144]

$$\int x^2 \left(a + b\log\left(cx^n\right)\right)\log\left(d\left(e + fx^m\right)^k\right) dx$$

[B] time = 0.188693 (sec), size = 292 ,normalized size = 10.07

$$-\frac{x^3\left(bekm(m+3)n\,{}_3F_2\left(1,\frac{3}{m},\frac{3}{m};1+\frac{3}{m},1+\frac{3}{m};-\frac{fx^m}{e}\right) - 27ae\log\left(d\left(e+fx^m\right)^k\right) - 9aem\log\left(d\left(e+fx^m\right)^k\right) + 9afkmx^m\,{}_2F_1\left(\right.\right.}{}$$

```
[In] Integrate[x^2*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] -1/27*(x^3*(-6*b*e*k*m*n - 2*b*e*k*m^2*n + 9*a*f*k*m*x^m*Hypergeometric2F1[
1, (3 + m)/m, 2 + 3/m, -((f*x^m)/e)] + b*e*k*m*(3 + m)*n*HypergeometricPFQ[
{1, 3/m, 3/m}, {1 + 3/m, 1 + 3/m}, -((f*x^m)/e)] + b*e*k*m*(3 + m)*Hypergeo
metric2F1[1, 3/m, (3 + m)/m, -((f*x^m)/e)]*(n - 3*Log[c*x^n]) + 9*b*e*k*m*L
og[c*x^n] + 3*b*e*k*m^2*Log[c*x^n] - 27*a*e*Log[d*(e + f*x^m)^k] - 9*a*e*m*
Log[d*(e + f*x^m)^k] + 9*b*e*n*Log[d*(e + f*x^m)^k] + 3*b*e*m*n*Log[d*(e +
f*x^m)^k] - 27*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 9*b*e*m*Log[c*x^n]*Log
[d*(e + f*x^m)^k]))/(e*(3 + m))
```

Integral number [145]

$$\int x \left(a + b\log\left(cx^n\right)\right)\log\left(d\left(e + fx^m\right)^k\right) dx$$

[B] time = 0.176377 (sec), size = 292 ,normalized size = 10.81

$$-\frac{x^2\left(bekm(m+2)n\,{}_3F_2\left(1,\frac{2}{m},\frac{2}{m};1+\frac{2}{m},1+\frac{2}{m};-\frac{fx^m}{e}\right) - 8ae\log\left(d\left(e+fx^m\right)^k\right) - 4aem\log\left(d\left(e+fx^m\right)^k\right) + 4afkmx^m\,{}_2F_1\left(1\right.\right.}{}$$

```
[In] Integrate[x*(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] -1/8*(x^2*(-4*b*e*k*m*n - 2*b*e*k*m^2*n + 4*a*f*k*m*x^m*Hypergeometric2F1[1
, (2 + m)/m, 2 + 2/m, -((f*x^m)/e)] + b*e*k*m*(2 + m)*n*HypergeometricPFQ[{
1, 2/m, 2/m}, {1 + 2/m, 1 + 2/m}, -((f*x^m)/e)] + b*e*k*m*(2 + m)*Hypergeom
etric2F1[1, 2/m, (2 + m)/m, -((f*x^m)/e)]*(n - 2*Log[c*x^n]) + 4*b*e*k*m*Lo
g[c*x^n] + 2*b*e*k*m^2*Log[c*x^n] - 8*a*e*Log[d*(e + f*x^m)^k] - 4*a*e*m*Lo
g[d*(e + f*x^m)^k] + 4*b*e*n*Log[d*(e + f*x^m)^k] + 2*b*e*m*n*Log[d*(e + f*
x^m)^k] - 8*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 4*b*e*m*Log[c*x^n]*Log[d*
(e + f*x^m)^k]))/(e*(2 + m))
```

Integral number [146]

$$\int \left(a + b\log\left(cx^n\right)\right)\log\left(d\left(e + fx^m\right)^k\right) dx$$

[B] time = 0.181192 (sec), size = 165 ,normalized size = 6.35

$$x\left(-bkmn\,{}_3F_2\left(1,\frac{1}{m},\frac{1}{m};1+\frac{1}{m},1+\frac{1}{m};-\frac{fx^m}{e}\right) + km\,{}_2F_1\left(1,\frac{1}{m};1+\frac{1}{m};-\frac{fx^m}{e}\right)\left(a+b\log\left(cx^n\right)-bn\right) + a\log\left(d\left(e+fx^m\right)^k\right)\right.$$

```
[In] Integrate[(a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k],x]
```

```
[Out] b*k*m*n*x - k*m*x*(a + b*(-(n*Log[x]) + Log[c*x^n])) + x*(b*k*m*n - b*k*m*n
*HypergeometricPFQ[{1, m^(-1), m^(-1)}, {1 + m^(-1), 1 + m^(-1)}, -((f*x^m)
/e)] - b*k*m*n*Log[x] + k*m*Hypergeometric2F1[1, m^(-1), 1 + m^(-1), -((f*x
^m)/e)]*(a - b*n + b*Log[c*x^n]) + a*Log[d*(e + f*x^m)^k] - b*n*Log[d*(e +
f*x^m)^k] + b*Log[c*x^n]*Log[d*(e + f*x^m)^k])
```

Integral number [148]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^k\right)}{x^2}\,dx$$

[B] time = 0.17089 (sec), size = 282 ,normalized size = 9.72

$$\frac{bek(m-1)mn\,{}_3F_2\left(1,-\frac{1}{m},-\frac{1}{m};1-\frac{1}{m},1-\frac{1}{m};-\frac{fx^m}{e}\right)+ae\log\left(d\left(e+fx^m\right)^k\right)-aem\log\left(d\left(e+fx^m\right)^k\right)+afkmx^m\,{}_2F_1\left(1,\frac{m}{\ }\right.}{}$$

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^2,x]
```

```
[Out] (2*b*e*k*m*n - 2*b*e*k*m^2*n + a*f*k*m*x^m*Hypergeometric2F1[1, (-1 + m)/m,
 2 - m^(-1), -((f*x^m)/e)] + b*e*k*(-1 + m)*m*n*HypergeometricPFQ[{1, -m^(-
1), -m^(-1)}, {1 - m^(-1), 1 - m^(-1)}, -((f*x^m)/e)] + b*e*k*m*Log[c*x^n]
- b*e*k*m^2*Log[c*x^n] + b*e*k*(-1 + m)*m*Hypergeometric2F1[1, -m^(-1), (-1
 + m)/m, -((f*x^m)/e)]*(n + Log[c*x^n]) + a*e*Log[d*(e + f*x^m)^k] - a*e*m*
Log[d*(e + f*x^m)^k] + b*e*n*Log[d*(e + f*x^m)^k] - b*e*m*n*Log[d*(e + f*x^
m)^k] + b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - b*e*m*Log[c*x^n]*Log[d*(e + f
*x^m)^k])/(e*(-1 + m)*x)
```

Integral number [149]

$$\int \frac{(a+b\log(cx^n))\log\left(d\left(e+fx^m\right)^k\right)}{x^3}\,dx$$

[B] time = 0.162371 (sec), size = 292 ,normalized size = 10.07

$$\frac{bek(m-2)mn\,{}_3F_2\left(1,-\frac{2}{m},-\frac{2}{m};1-\frac{2}{m},1-\frac{2}{m};-\frac{fx^m}{e}\right)+8ae\log\left(d\left(e+fx^m\right)^k\right)-4aem\log\left(d\left(e+fx^m\right)^k\right)+4afkmx^m\,{}_2F_1\left(\right.}{}$$

```
[In] Integrate[((a + b*Log[c*x^n])*Log[d*(e + f*x^m)^k])/x^3,x]
```

```
[Out] (4*b*e*k*m*n - 2*b*e*k*m^2*n + 4*a*f*k*m*x^m*Hypergeometric2F1[1, (-2 + m)/
m, 2 - 2/m, -((f*x^m)/e)] + b*e*k*(-2 + m)*m*n*HypergeometricPFQ[{1, -2/m,
-2/m}, {1 - 2/m, 1 - 2/m}, -((f*x^m)/e)] + 4*b*e*k*m*Log[c*x^n] - 2*b*e*k*m
^2*Log[c*x^n] + b*e*k*(-2 + m)*m*Hypergeometric2F1[1, -2/m, (-2 + m)/m, -((
f*x^m)/e)]*(n + 2*Log[c*x^n]) + 8*a*e*Log[d*(e + f*x^m)^k] - 4*a*e*m*Log[d*
(e + f*x^m)^k] + 4*b*e*n*Log[d*(e + f*x^m)^k] - 2*b*e*m*n*Log[d*(e + f*x^m)
^k] + 8*b*e*Log[c*x^n]*Log[d*(e + f*x^m)^k] - 4*b*e*m*Log[c*x^n]*Log[d*(e +
 f*x^m)^k])/(8*e*(-2 + m)*x^2)
```

Integral number [220]

$$\int -(dx)^m\left(a+b\log(cx^n)\right)\log\left(1-ex^q\right)\,dx$$

[B] time = 0.243344 (sec), size = 266 ,normalized size = 8.87

$$-\frac{x(dx)^m\left(-bnq\,{}_3F_2\left(1,\frac{m}{q}+\frac{1}{q},\frac{m}{q}+\frac{1}{q};\frac{m}{q}+\frac{1}{q}+1,\frac{m}{q}+\frac{1}{q}+1;ex^q\right)+q\,{}_2F_1\left(1,\frac{m+1}{q};\frac{m+q+1}{q};ex^q\right)\left(am+a+b(m+1)\log(cx^n\right.\right.}{}$$

```
[In] Integrate[-((d*x)^m*(a + b*Log[c*x^n])*Log[1 - e*x^q]),x]
```

```
[Out] -((x*(d*x)^m*(-(a*q) - a*m*q + 2*b*n*q - b*n*q*HypergeometricPFQ[{1, q^(-1)
 + m/q, q^(-1) + m/q}, {1 + q^(-1) + m/q, 1 + q^(-1) + m/q}, e*x^q] - b*q*L
og[c*x^n] - b*m*q*Log[c*x^n] + q*Hypergeometric2F1[1, (1 + m)/q, (1 + m + q
)/q, e*x^q]*(a + a*m - b*n + b*(1 + m)*Log[c*x^n]) + a*Log[1 - e*x^q] + 2*a
*m*Log[1 - e*x^q] + a*m^2*Log[1 - e*x^q] - b*n*Log[1 - e*x^q] - b*m*n*Log[1
 - e*x^q] + b*Log[c*x^n]*Log[1 - e*x^q] + 2*b*m*Log[c*x^n]*Log[1 - e*x^q] +
 b*m^2*Log[c*x^n]*Log[1 - e*x^q]))/(1 + m)^3)
```

3.3.2 Maple

Integral number [220]

$$\int -(dx)^m \left(a + b\log\left(cx^n\right)\right)\log\left(1 - ex^q\right)\,dx$$

[B] time = 0.866 (sec), size = 844 ,normalized size = 28.13

$$-\frac{\left(-\frac{(-m-q-1)eq\,x^{m+q+1}(-e)^{\frac{m}{q}+\frac{1}{q}}\Phi\left(e\,x^q,1,\frac{m+q+1}{q}\right)}{(m+q+1)(m+1)}+\frac{q\,x^{m+1}(-e)^{\frac{m}{q}+\frac{1}{q}}\ln(-e\,x^q+1)}{m+1}\right)b\,x^{-m}\,(dx)^m\,(-e)^{-\frac{m}{q}-\frac{1}{q}}\ln(c)}{q}-\frac{\left(-\frac{(-m-q-1)eq\,x^{m+q+1}(-e)^{\frac{m}{q}+\frac{1}{q}}\Phi(\ldots}{(m+q+1)(m+1)}\right.}{}$$

```
[In] int(-(d*x)^m*(b*ln(c*x^n)+a)*ln(1-e*x^q),x)
```

```
[Out] -(d*x)^m*x^(-m)*(-e)^(-1/q*m-1/q)*b*ln(c)/q*(q*x^(m+1)*(-e)^(1/q*m+1/q)/(m+
1)*ln(1-e*x^q)-q/(1+m+q)*x^(1+m+q)*e*(-e)^(1/q*m+1/q)*(-q-m-1)/(m+1)*LerchP
hi(e*x^q,1,(1+m+q)/q))+((-e)^(-1/q*m-1/q)*ln(-e)/q^2*(d*x)^m*x^(-m)*b*n*(q*
x^m*(-e)^(1/q*m+1/q)/(m+1)*ln(1-e*x^q)-q/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)
*(-q-m-1)/(m+1)*LerchPhi(e*x^q,1,(1+m+q)/q))-(-e)^(-1/q*m-1/q)*(d*x)^m*x^(-
m)*b*n/q*(q*x^m*(-e)^(1/q*m+1/q)*ln(x)/(m+1)*ln(1-e*x^q)+x^m*(-e)^(1/q*m+1/
q)*ln(-e)/(m+1)*ln(1-e*x^q)-q*x^m*(-e)^(1/q*m+1/q)/(m+1)^2*ln(1-e*x^q)+q/(1
+m+q)^2*x^(q+m)*e*(-e)^(1/q*m+1/q)*(-q-m-1)/(m+1)*LerchPhi(e*x^q,1,(1+m+q)/
q)-q/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)*ln(x)*(-q-m-1)/(m+1)*LerchPhi(e*x^q
,1,(1+m+q)/q)-1/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)*ln(-e)*(-q-m-1)/(m+1)*Le
rchPhi(e*x^q,1,(1+m+q)/q)+q/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)/(m+1)*LerchP
hi(e*x^q,1,(1+m+q)/q)+q/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)*(-q-m-1)/(m+1)^2
*LerchPhi(e*x^q,1,(1+m+q)/q)+1/(1+m+q)*x^(q+m)*e*(-e)^(1/q*m+1/q)*(-q-m-1)/
(m+1)*LerchPhi(e*x^q,2,(1+m+q)/q)))*x-(d*x)^m*x^(-m)*(-e)^(-1/q*m-1/q)*a/q*
(q*x^(m+1)*(-e)^(1/q*m+1/q)/(m+1)*ln(1-e*x^q)-q/(1+m+q)*x^(1+m+q)*e*(-e)^(1
/q*m+1/q)*(-q-m-1)/(m+1)*LerchPhi(e*x^q,1,(1+m+q)/q))
```

Integral number [221]

$$\int (dx)^m \left(a + b\log\left(cx^n\right)\right)\mathrm{Li}_2\left(ex^q\right)\,dx$$

[B] time = 0.352 (sec), size = 867 ,normalized size = 4.87

$$-\frac{\left(-\frac{e\,q^2x^{m+q+1}(-e)^{\frac{m}{q}+\frac{1}{q}}\Phi\left(e\,x^q,1,\frac{m+q+1}{q}\right)}{(m+1)^2}-\frac{q^2x^{m+1}(-e)^{\frac{m}{q}+\frac{1}{q}}\ln(-e\,x^q+1)}{(m+1)^2}-\frac{q\,x^{m+1}(-e)^{\frac{m}{q}+\frac{1}{q}}\operatorname{polylog}(2,e\,x^q)}{m+1}\right)b\,x^{-m}\,(dx)^m\,(-e)^{-\frac{m}{q}-\frac{1}{q}}\ln(c)}{q}-\frac{\left(-\frac{e\,q^2x^m\ldots}{}\right.}{}$$

```
[In] int((d*x)^m*(b*ln(c*x^n)+a)*polylog(2,e*x^q),x)
```

```
[Out] -(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*b*ln(c)/q*(-q^2*x^(m+1)*(-e)^(m/q+1/q)/(m+1
)^2*ln(-e*x^q+1)-q*x^(m+1)*(-e)^(m/q+1/q)/(m+1)*polylog(2,e*x^q)-q^2*x^(m+q
+1)*e*(-e)^(m/q+1/q)/(m+1)^2*LerchPhi(e*x^q,1,(m+q+1)/q))+((-e)^(-m/q-1/q)*
ln(-e)/q^2*(d*x)^m*x^(-m)*b*n*(-q^2*x^m*(-e)^(m/q+1/q)/(m+1)^2*ln(-e*x^q+1)
-q*x^m*(-e)^(m/q+1/q)/(m+1)*polylog(2,e*x^q)-q^2*x^(m+q)*e*(-e)^(m/q+1/q)/(
m+1)^2*LerchPhi(e*x^q,1,(m+q+1)/q))-(-e)^(-m/q-1/q)*(d*x)^m*x^(-m)*b*n/q*(-
q^2*x^m*(-e)^(m/q+1/q)*ln(x)/(m+1)^2*ln(-e*x^q+1)-q*x^m*(-e)^(m/q+1/q)*ln(-
e)/(m+1)^2*ln(-e*x^q+1)+2*q^2*x^m*(-e)^(m/q+1/q)/(m+1)^3*ln(-e*x^q+1)-q*x^m
*(-e)^(m/q+1/q)*ln(x)/(m+1)*polylog(2,e*x^q)-x^m*(-e)^(m/q+1/q)*ln(-e)/(m+1
)*polylog(2,e*x^q)+q*x^m*(-e)^(m/q+1/q)/(m+1)^2*polylog(2,e*x^q)-q^2*x^(m+q
)*e*(-e)^(m/q+1/q)*ln(x)/(m+1)^2*LerchPhi(e*x^q,1,(m+q+1)/q)-q*x^(m+q)*e*(-
```

```
e)^(m/q+1/q)*ln(-e)/(m+1)^2*LerchPhi(e*x^q,1,(m+q+1)/q)+2*q^2*x^(m+q)*e*(-e
)^(m/q+1/q)/(m+1)^3*LerchPhi(e*x^q,1,(m+q+1)/q)+q*x^(m+q)*e*(-e)^(m/q+1/q)/
(m+1)^2*LerchPhi(e*x^q,2,(m+q+1)/q)))*x-(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*
(-q^2*x^(m+1)*(-e)^(m/q+1/q)/(m+1)^2*ln(-e*x^q+1)-q*x^(m+1)*(-e)^(m/q+1/q)/
(m+1)*polylog(2,e*x^q)-q^2*x^(m+q+1)*e*(-e)^(m/q+1/q)/(m+1)^2*LerchPhi(e*x^
q,1,(m+q+1)/q))
```

Integral number [222]

$$\int (dx)^m \left(a + b\log\left(cx^n\right)\right) \mathrm{Li}_3\left(ex^q\right)\, dx$$

[B] time = 2.282 (sec), size = 1065 ,normalized size = 4.35

result too large to display

```
[In] int((d*x)^m*(b*ln(c*x^n)+a)*polylog(3,e*x^q),x)
```

```
[Out] -(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*b*ln(c)/q*(q^3*x^(m+1)*(-e)^(m/q+1/q)/(m+1)
^3*ln(-e*x^q+1)+q^2*x^(m+1)*(-e)^(m/q+1/q)/(m+1)^2*polylog(2,e*x^q)-q*x^(m+
1)*(-e)^(m/q+1/q)/(m+1)*polylog(3,e*x^q)+q^3*x^(m+q+1)*e*(-e)^(m/q+1/q)/(m+
1)^3*LerchPhi(e*x^q,1,(m+q+1)/q))+((-e)^(-m/q-1/q)/q^2*ln(-e)*(d*x)^m*x^(-m
)*b*n*(q^3*x^m*(-e)^(m/q+1/q)/(m+1)^3*ln(-e*x^q+1)+q^2*x^m*(-e)^(m/q+1/q)/(
m+1)^2*polylog(2,e*x^q)-q*x^m*(-e)^(m/q+1/q)/(m+1)*polylog(3,e*x^q)+q^3*x^(
m+q)*e*(-e)^(m/q+1/q)/(m+1)^3*LerchPhi(e*x^q,1,(m+q+1)/q))-(-e)^(-m/q-1/q)*
(d*x)^m*x^(-m)*b*n/q*(q^3*x^m*(-e)^(m/q+1/q)*ln(x)/(m+1)^3*ln(-e*x^q+1)+q^2
*x^m*(-e)^(m/q+1/q)*ln(-e)/(m+1)^3*ln(-e*x^q+1)-3*q^3*x^m*(-e)^(m/q+1/q)/(m
+1)^4*ln(-e*x^q+1)+q^2*x^m*(-e)^(m/q+1/q)*ln(x)/(m+1)^2*polylog(2,e*x^q)+q*
x^m*(-e)^(m/q+1/q)*ln(-e)/(m+1)^2*polylog(2,e*x^q)-2*q^2*x^m*(-e)^(m/q+1/q)
/(m+1)^3*polylog(2,e*x^q)-q*x^m*(-e)^(m/q+1/q)*ln(x)/(m+1)*polylog(3,e*x^q)
-x^m*(-e)^(m/q+1/q)*ln(-e)/(m+1)*polylog(3,e*x^q)+q*x^m*(-e)^(m/q+1/q)/(m+1
)^2*polylog(3,e*x^q)+q^3*x^(m+q)*e*(-e)^(m/q+1/q)*ln(x)/(m+1)^3*LerchPhi(e*
x^q,1,(m+q+1)/q)+q^2*x^(m+q)*e*(-e)^(m/q+1/q)*ln(-e)/(m+1)^3*LerchPhi(e*x^q
,1,(m+q+1)/q)-3*q^3*x^(m+q)*e*(-e)^(m/q+1/q)/(m+1)^4*LerchPhi(e*x^q,1,(m+q+
1)/q)-q^2*x^(m+q)*e*(-e)^(m/q+1/q)/(m+1)^3*LerchPhi(e*x^q,2,(m+q+1)/q)))*x-
(d*x)^m*x^(-m)*(-e)^(-m/q-1/q)*a/q*(q^3*x^(m+1)*(-e)^(m/q+1/q)/(m+1)^3*ln(-
e*x^q+1)+q^2*x^(m+1)*(-e)^(m/q+1/q)/(m+1)^2*polylog(2,e*x^q)-q*x^(m+1)*(-e)
^(m/q+1/q)/(m+1)*polylog(3,e*x^q)+q^3*x^(m+q+1)*e*(-e)^(m/q+1/q)/(m+1)^3*Le
rchPhi(e*x^q,1,(m+q+1)/q))
```

3.4 Test file Number [63] 3-Logarithms/3.4-u-a+b-log-c-d+e-x^m-^n-^p

3.4.1 Mathematica

Integral number [98]

$$\int x^2 \log^3\left(c\left(a + bx^2\right)^p\right) dx$$

[B] time = 3.89493 (sec), size = 909 ,normalized size = 2.39

$$\frac{\left(-48\left(4\sqrt{bx^2}\,\tanh^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{-a}}\right)\left(\log\left(bx^2+a\right)-\log\left(\frac{bx^2}{a}+1\right)\right)-\sqrt{-a}\sqrt{-\frac{bx^2}{a}}\left(\log^2\left(\frac{bx^2}{a}+1\right)-4\log\left(\frac{1}{2}\left(\sqrt{-\frac{bx^2}{a}}+1\right)\right)\log\right.\right.\right.}{}$$

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (2*a*p*x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/b - (2*a^(3/2)*p*A
rcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/
```

```
b^(3/2) + p*x^3*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])
^2 + (x^3*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-2*p - p*Log[a +
b*x^2] + Log[c*(a + b*x^2)^p]))/3 + 3*p^2*(-(p*Log[a + b*x^2]) + Log[c*(a +
 b*x^2)^p])*((x^3*Log[a + b*x^2]^2)/3 - (4*((9*I)*a^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]]^2 + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-8 + 6*Log[(2*Sqrt[a]
)/(Sqrt[a] + I*Sqrt[b]*x)] + 3*Log[a + b*x^2]) + Sqrt[b]*x*(24*a - 2*b*x^2
+ (-9*a + 3*b*x^2)*Log[a + b*x^2]) + (9*I)*a^(3/2)*PolyLog[2, (I*Sqrt[a] +
Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/(27*b^(3/2))) + (p^3*(416*Sqrt[-a]
*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a +
b*x^2]] + (2*Sqrt[-a]*b*x^2*(624*a - 16*b*x^2 + (-288*a + 24*b*x^2)*Log[a +
 b*x^2] + 18*(3*a - b*x^2)*Log[a + b*x^2]^2 + 9*b*x^2*Log[a + b*x^2]^3))/3
+ 36*Sqrt[-a]*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*(8*Sqrt[a]*HypergeometricPF
Q[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + Log[a + b*x^2]*(4
*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sq
rt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]]*Log[a + b*x^2])) - 48*a^2*(4*
Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(Log[a + b*x^2] - Log[1 + (b*x^2)
/a]) - Sqrt[-a]*Sqrt[-((b*x^2)/a)]*(Log[1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2
)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2
- 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/2]))))/(18*Sqrt[-a]*b^2*x)
```

Integral number [99]

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)dx$$

[B] time = 3.53156 (sec), size = 789 ,normalized size = 2.72

$$\frac{p^3\left(-6\sqrt{-a^2}\sqrt{\frac{bx^2}{a+bx^2}}\left(8\sqrt{a}\ {}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right)+\log\left(a+bx^2\right)\left(4\sqrt{a}\ {}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right)+\sqrt{a+bx^2}\log\left(a+\right.\right.\right.\right.}{}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (6*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a +
b*x^2)^p])^2)/Sqrt[b] + 3*p*x*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(
a + b*x^2)^p])^2 + x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-6*p -
 p*Log[a + b*x^2] + Log[c*(a + b*x^2)^p]) - (3*p^2*(p*Log[a + b*x^2] - Log[
c*(a + b*x^2)^p])*((4*I)*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)
] + Log[a + b*x^2]) + Sqrt[b]*x*(8 - 4*Log[a + b*x^2] + Log[a + b*x^2]^2) +
 (4*I)*Sqrt[a]*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x
)]))/Sqrt[b] + (p^3*(-48*Sqrt[-a^2]*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^
2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]] + Sqrt[-a]*b*x^2*(-48 + 24*Log[a + b*x^2
] - 6*Log[a + b*x^2]^2 + Log[a + b*x^2]^3) - 6*Sqrt[-a^2]*Sqrt[(b*x^2)/(a +
 b*x^2)]*(8*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}
, a/(a + b*x^2)] + Log[a + b*x^2]*(4*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1
/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b
*x^2]]*Log[a + b*x^2])) + 24*a*Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(L
og[a + b*x^2] - Log[1 + (b*x^2)/a]) + 6*(-a)^(3/2)*Sqrt[-((b*x^2)/a)]*(Log[
1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2
*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/
2])))/(Sqrt[-a]*b*x)
```

Integral number [100]

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}dx$$

[C] time = 0.827298 (sec), size = 505 ,normalized size = 9.9

$$\frac{p^3\left(-96\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right)-48\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\log\left(a+bx^2\right){}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{bx^2+a}\right)-2\log^2\left(\right.\right.}{2\sqrt{a}\,x}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

```
[Out] (p^3*(-96*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2,
 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] - 48*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]
*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)]*Log[a + b*x^
2] - 2*Log[a + b*x^2]^2*(6*Sqrt[a + b*x^2]*Sqrt[1 - a/(a + b*x^2)]*ArcSin[S
qrt[a]/Sqrt[a + b*x^2]] + Sqrt[a]*Log[a + b*x^2])))/(2*Sqrt[a]*x) + (6*Sqrt
[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^
p])^2)/Sqrt[a] - (3*p*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^
2)^p])^2)/x - (-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3/x + 3*p^2*(-(p
*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])*(-(Log[a + b*x^2]^2/x) + (4*Sqrt[b
]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*Log[(2*I)
/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[a + b*x^2]) + I*PolyLog[2, (I*Sqrt[a] + S
qrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/Sqrt[a])
```

Integral number [101]

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$$

[B] time = 2.71026 (sec), size = 851 ,normalized size = 3.35

$$\frac{\left(-a^2\log^3\left(bx^2+a\right)-6abx^2\log^2\left(bx^2+a\right)+6\sqrt{a}\left(\frac{bx^2}{bx^2+a}\right)^{3/2}\left(bx^2+a\right)^{3/2}\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{bx^2+a}}\right)\log^2\left(bx^2+a\right)+24\sqrt{-a}\left(bx^2\right)^{3/}\right.}{}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

```
[Out] (a^2*(p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])^3 - 6*a*b*p*x^2*(-(p*Log[a +
 b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 6*Sqrt[a]*b^(3/2)*p*x^3*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 3*a^2*p*Log[
a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 3*Sqrt[a]*p^2*(
p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])*(a^(3/2)*Log[a + b*x^2]^2 + 4*b*x^
2*(I*Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + Sqrt[a]*Log[a + b*x^2] + Sqr
t[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)] + Log[a + b*x^2]) + I*Sqrt[b]*x*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)
/((-I)*Sqrt[a] + Sqrt[b]*x)])) + p^3*(48*a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*
HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + 2
4*Sqrt[-a]*(b*x^2)^(3/2)*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[a + b*x^2] + 24*
a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2,
3/2}, a/(a + b*x^2)]*Log[a + b*x^2] - 6*a*b*x^2*Log[a + b*x^2]^2 + 6*Sqrt[a
]*((b*x^2)/(a + b*x^2))^(3/2)*(a + b*x^2)^(3/2)*ArcSin[Sqrt[a]/Sqrt[a + b*x
^2]]*Log[a + b*x^2]^2 - a^2*Log[a + b*x^2]^3 - 24*Sqrt[-a]*(b*x^2)^(3/2)*Ar
cTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[1 + (b*x^2)/a] - 6*a^2*(-((b*x^2)/a))^(3/2)
*Log[1 + (b*x^2)/a]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*Log[1 + (b*x^2)/a]*Log[
(1 + Sqrt[-((b*x^2)/a)])/2] - 12*a^2*(-((b*x^2)/a))^(3/2)*Log[(1 + Sqrt[-((
b*x^2)/a)])/2]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*PolyLog[2, 1/2 - Sqrt[-((b*x
^2)/a)]/2]))/(3*a^2*x^3)
```

Integral number [158]

$$\int (fx)^m\log^3\left(c\left(d+ex^2\right)^p\right)dx$$

[B] time = 2.19947 (sec), size = 994 ,normalized size = 12.91

$$(fx)^m \left(\frac{6p^3\left(d\left(\left(-\frac{ex^2}{d}\right)^{\frac{m+1}{2}}-1\right)\log^2\left(ex^2+d\right)+(m+1)\left(ex^2+d\right){}_3F_2\left(1,1,\frac{1}{2}-\frac{m}{2};2,2;\frac{ex^2}{d}+1\right)\log\left(ex^2+d\right)-(m+1)\left(ex^2+d\right){}_4F_3\left(1,1,1,\frac{1}{2}-\frac{m}{2};2,2,2;\frac{ex^2}{d}+1\right)\right)\left(-\frac{ex^2}{d}\right)^{\frac{1}{2}-\frac{m}{2}}}{e} - \frac{3m\cdots}{\cdots}\right.$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] ((f*x)^m*((1 + m)*p^3*x^2*Log[d + e*x^2]^3 + (6*p^3*(-((e*x^2)/d))^(1/2 - m
/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2
}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2
}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/
2))*Log[d + e*x^2]^2))/e + (6*d*(1 + m)*p^3*((e*x^2)/(d + e*x^2))^(1/2 - m/
2)*(8*HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2
- m/2, 3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] + (-1 + m)*Log[d + e*x^2]*(-4*
HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}
, d/(d + e*x^2)] + (-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m
/2, d/(d + e*x^2)]*Log[d + e*x^2])))/(e*(-1 + m)^3) - (3*p^2*(-((e*x^2)/d))
^(1/2 - m/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2},
 {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1,
1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^
((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])
)/e - (3*m*p^2*(-((e*x^2)/d))^(1/2 - m/2)*(-((1 + m)*(d + e*x^2)*Hypergeome
tricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x
^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x
^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p]))/e + (3*p*x^2*(-2*e*x^2*Hypergeometric2F1[1,
(3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e*x^2])*(-(p*Log[d
+ e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + (3*m*p*x^2*(-2*e*x^2*Hyp
ergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e
*x^2])*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + x^2*(-
(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^3 + m*x^2*(-(p*Log[d + e*x^2]) +
 Log[c*(d + e*x^2)^p])^3))/((1 + m)^2*x)
```

Integral number [159]

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$

[B] time = 1.07977 (sec), size = 466 ,normalized size = 6.21

$$(fx)^m \left(\frac{4d(m+1)p^2\left(\frac{ex^2}{d+ex^2}\right)^{\frac{1}{2}-\frac{m}{2}}\left((m-1)\log\left(d+ex^2\right){}_2F_1\left(\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2};\frac{d}{ex^2+d}\right)-2\,{}_3F_2\left(\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2};\frac{d}{ex^2+d}\right)\right)}{e(m-1)^2x} + \frac{2p\left(p\log\left(d+ex^2\right)-\log\left(c\left(d+ex^2\cdots\right.\right.\right.}{\cdots}\right.$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((f*x)^m*(4*p^2*x*((2*e*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e
*x^2)/d)])/(d*(3 + m)) - Log[d + e*x^2]) + (1 + m)*p^2*x*Log[d + e*x^2]^2 +
 (4*d*(1 + m)*p^2*((e*x^2)/(d + e*x^2))^(1/2 - m/2)*(-2*HypergeometricPFQ[{
1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] +
(-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m/2, d/(d + e*x^2)]*
Log[d + e*x^2]))/(e*(-1 + m)^2*x) + (2*p*(2*e*x^3*Hypergeometric2F1[1, (3 +
 m)/2, (5 + m)/2, -((e*x^2)/d)] - d*(3 + m)*x*Log[d + e*x^2])*(p*Log[d + e*
x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) - (2*m*p*(-2*e*x^3*Hypergeometric
2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*x*Log[d + e*x^2])*(p
*Log[d + e*x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) + x*(-(p*Log[d + e*x^2
]) + Log[c*(d + e*x^2)^p])^2 + m*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)
```

```
^p])^2))/(1 + m)^2
```

Integral number [277]

$$\int \left(f+gx^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

[B] time = 4.56894 (sec), size = 1460 ,normalized size = 2.14

result too large to display

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g*p^3*x*(-18*(d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2, 2
}, (d + e*x^2)/d] + 18*(d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2
, 2}, (d + e*x^2)/d]*Log[d + e*x^2] - 9*(d + e*x^2)*HypergeometricPFQ[{-1/2
, 1, 1}, {2, 2}, (d + e*x^2)/d]*Log[d + e*x^2]^2 + 2*d*Log[d + e*x^2]^3 - 2
*d*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 + 2*(d + e*x^2)*Sqrt[1 - (d + e
*x^2)/d]*Log[d + e*x^2]^3))/(6*e*Sqrt[1 - (d + e*x^2)/d]) + (2*d*g*p*x*(-(p
*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/e + (6*Sqrt[d]*f*p*ArcTan[(Sqrt
[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] - (
2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d +
 e*x^2)^p])^2)/e^(3/2) + 3*f*p*x*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p])^2 + g*p*x^3*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(
d + e*x^2)^p])^2 + f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p
 - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]) + (g*x^3*(-(p*Log[d + e*x^2]) +
 Log[c*(d + e*x^2)^p])^2*(-2*p - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]))/
3 + 3*f*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^
2 - (4*((-I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(-2 + Log[d
+ e*x^2]) - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sq
rt[d] + I*Sqrt[e]*x)] + Log[d + e*x^2]) - I*Sqrt[d]*PolyLog[2, (I*Sqrt[d] +
 Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/Sqrt[e]) + 3*g*p^2*(-(p*Log[d + e
*x^2]) + Log[c*(d + e*x^2)^p])*((x^3*Log[d + e*x^2]^2)/3 - (4*((9*I)*d^(3/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + 3*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-8
 + 6*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 3*Log[d + e*x^2]) + Sqrt[e]
*x*(24*d - 2*e*x^2 + (-9*d + 3*e*x^2)*Log[d + e*x^2]) + (9*I)*d^(3/2)*PolyL
og[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(27*e^(3/2))) +
 (f*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^2]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt
[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*Sqrt[1 - d/(d + e*x^2)]*(8*Sqrt[d]*Hype
rgeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)] + 4*Sqr
t[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)]*Log[d +
e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^2]^2)
+ Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*
x^2]^3) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] -
Log[(d + e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)
/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(
1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)
/d]/2])))/(Sqrt[-d]*e*x)
```

Integral number [298]

$$\int \left(f+gx^3\right)^2\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

[B] time = 9.32333 (sec), size = 2727 ,normalized size = 2.42

Result too large to show

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g^2*p^3*x*(168*d^2*(d + e*x^2)*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2
}, (d + e*x^2)/d] - 280*d^2*(d + e*x^2)*HypergeometricPFQ[{-3/2, 1, 1, 1},
```

```
{2, 2, 2}, (d + e*x^2)/d] - 112*d^2*(d + e*x^2)*HypergeometricPFQ[{-5/2, 1,
 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^2)/d] + 280*d^2*(d + e*x^2)*Hypergeometri
cPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^2)/d] - 210*d^2*(d + e*x^2)
*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^2)/d] + 16*d^
3*Log[d + e*x^2] - 16*d^3*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2] + 48*d^2*(
d + e*x^2)*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2] - 48*d*(d + e*x^2)^2*Sqrt
[1 - (d + e*x^2)/d]*Log[d + e*x^2] + 16*(d + e*x^2)^3*Sqrt[1 - (d + e*x^2)/
d]*Log[d + e*x^2] + 112*d^2*(d + e*x^2)*HypergeometricPFQ[{-5/2, 1, 1, 1},
{2, 2, 2}, (d + e*x^2)/d]*Log[d + e*x^2] - 280*d^2*(d + e*x^2)*Hypergeometr
icPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^2)/d]*Log[d + e*x^2] + 210*d^2*(
d + e*x^2)*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^2)/d]*Log
[d + e*x^2] - 32*d^3*Log[d + e*x^2]^2 + 32*d^3*Sqrt[1 - (d + e*x^2)/d]*Log[
d + e*x^2]^2 - 68*d^2*(d + e*x^2)*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^2
+ 40*d*(d + e*x^2)^2*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^2 - 4*(d + e*x^
2)^3*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^2 - 105*d^2*(d + e*x^2)*Hyperge
ometricPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^2)/d]*Log[d + e*x^2]^2 + 10*d^3*L
og[d + e*x^2]^3 - 10*d^3*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 + 30*d^2*
(d + e*x^2)*Sqrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 - 30*d*(d + e*x^2)^2*S
qrt[1 - (d + e*x^2)/d]*Log[d + e*x^2]^3 + 10*(d + e*x^2)^3*Sqrt[1 - (d + e*
x^2)/d]*Log[d + e*x^2]^3 + 140*d^2*(d + e*x^2)*HypergeometricPFQ[{-3/2, 1,
1}, {2, 2}, (d + e*x^2)/d]*Log[d + e*x^2]*(2 + Log[d + e*x^2]) - 56*d^2*(d
+ e*x^2)*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (d + e*x^2)/d]*(1 + 3*Log[
d + e*x^2] + Log[d + e*x^2]^2)))/(70*e^3*Sqrt[1 - (d + e*x^2)/d]) + (f*g*p^
3*(d + e*x^2)*(-8*d*(-6 + 6*Log[d + e*x^2] - 3*Log[d + e*x^2]^2 + Log[d + e
*x^2]^3) + (d + e*x^2)*(-3 + 6*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + 4*Log[
d + e*x^2]^3)))/(8*e^2) + 6*f*g*p^2*((x^4*Log[d + e*x^2]^2)/4 - e*((3*d*x^2
)/(4*e^2) - x^4/(8*e) - (3*d^2*Log[d + e*x^2])/(4*e^3) - (d*x^2*Log[d + e*x
^2])/(2*e^2) + (x^4*Log[d + e*x^2])/(4*e) + (d^2*Log[d + e*x^2]^2)/(4*e^3))
)*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]) + (3*d*f*g*p*x^2*(-(p*Log[d
+ e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*e) - (2*d^2*g^2*p*x^3*(-(p*Log[d +
e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(7*e^2) + (6*d*g^2*p*x^5*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p])^2)/(35*e) - (3*d^2*f*g*p*Log[d + e*x^2]*(-(p*
Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*e^2) + (3*p*x*(14*f^2 + 7*f*g
*x^3 + 2*g^2*x^6)*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p
])^2)/14 + (f*g*x^4*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-3*p +
2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])))/4 + (g^2*x^7*(-(p*Log[d +
e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p + 7*(-(p*Log[d + e*x^2]) + Log[c*(d
 + e*x^2)^p])))/49 + (x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-42
*e^3*f^2*p + 6*d^3*g^2*p + 7*e^3*f^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^
2)^p])))/(7*e^3) - (6*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-7*d*e^3*f^2*p*(-(p*Log[
d + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + d^4*g^2*p*(-(p*Log[d + e*x^2]) + Lo
g[c*(d + e*x^2)^p])^2))/(7*Sqrt[d]*e^(7/2)) + 3*f^2*p^2*(-(p*Log[d + e*x^2]
) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^2 - (4*((-I)*Sqrt[d]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(-2 + Log[d + e*x^2]) - Sqrt[d]*ArcTan[(Sqr
t[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + Log[d +
 e*x^2]) - I*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqr
t[e]*x)]))/Sqrt[e]) + 3*g^2*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]
)*((x^7*Log[d + e*x^2]^2)/7 - (4*((11025*I)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]^2 + 105*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-352 + 210*Log[(2*Sqrt[d]
)/(Sqrt[d] + I*Sqrt[e]*x)] + 105*Log[d + e*x^2]) + Sqrt[e]*x*(36960*d^3 - 4
970*d^2*e*x^2 + 1512*d*e^2*x^4 - 450*e^3*x^6 - 105*(105*d^3 - 35*d^2*e*x^2
+ 21*d*e^2*x^4 - 15*e^3*x^6)*Log[d + e*x^2]) + (11025*I)*d^(7/2)*PolyLog[2,
 (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(77175*e^(7/2))) + (
f^2*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^2]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt
[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*Sqrt[1 - d/(d + e*x^2)]*(8*Sqrt[d]*Hype
rgeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)] + 4*Sqr
t[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)]*Log[d +
e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^2]^2)
+ Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*
x^2]^3) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] -
Log[(d + e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)
/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(
1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)
```

```
/d]/2])))/(Sqrt[-d]*e*x)
```

Integral number [299]

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

[B] time = 4.53962 (sec), size = 1146 ,normalized size = 2.21

result too large to display

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g*p^3*(d + e*x^2)*(45*d - 3*e*x^2 + (-42*d + 6*e*x^2)*Log[d + e*x^2] + 6*(
3*d - e*x^2)*Log[d + e*x^2]^2 - 4*(d - e*x^2)*Log[d + e*x^2]^3))/(16*e^2) -
 (3*g*p^2*(e*x^2*(-6*d + e*x^2) + (6*d^2 + 4*d*e*x^2 - 2*e^2*x^4)*Log[d + e
*x^2] - 2*(d^2 - e^2*x^4)*Log[d + e*x^2]^2)*(p*Log[d + e*x^2] - Log[c*(d +
e*x^2)^p]))/(8*e^2) + (3*d*g*p*x^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)
^p])^2)/(4*e) + (6*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x
^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] - (3*d^2*g*p*Log[d + e*x^2]*(-(p*Lo
g[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(4*e^2) + (3*p*x*(4*f + g*x^3)*Log
[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/4 - (g*x^4*(3*p
 + 2*p*Log[d + e*x^2] - 2*Log[c*(d + e*x^2)^p])*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p])^2)/8 + f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2
*(-6*p - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]) - (3*f*p^2*(p*Log[d + e*x
^2] - Log[c*(d + e*x^2)^p])*((4*I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 +
4*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*
Sqrt[e]*x)] + Log[d + e*x^2]) + Sqrt[e]*x*(8 - 4*Log[d + e*x^2] + Log[d + e
*x^2]^2) + (4*I)*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] +
 Sqrt[e]*x)]))/Sqrt[e] + (f*p^3*(-48*Sqrt[-d^2]*Sqrt[(e*x^2)/(d + e*x^2)]*S
qrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]] + Sqrt[-d]*e*x^2*(-48 + 24*L
og[d + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*x^2]^3) - 6*Sqrt[-d^2]*Sqrt[
(e*x^2)/(d + e*x^2)]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/
2, 3/2, 3/2}, d/(d + e*x^2)] + Log[d + e*x^2]*(4*Sqrt[d]*HypergeometricPFQ[
{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d
]/Sqrt[d + e*x^2]]*Log[d + e*x^2])) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/
Sqrt[-d]]*(Log[d + e*x^2] - Log[1 + (e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[-((e*x^
2)/d)]*(Log[1 + (e*x^2)/d]^2 - 4*Log[1 + (e*x^2)/d]*Log[(1 + Sqrt[-((e*x^2)
/d)])/2] + 2*Log[(1 + Sqrt[-((e*x^2)/d)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-(
(e*x^2)/d)]/2])))/(Sqrt[-d]*e*x)
```

Integral number [485]

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$$

[B] time = 9.25272 (sec), size = 3146 ,normalized size = 3.96

Result too large to show

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (b^3*n^3*x^(1/3)*(32*d^4 - 32*d^4*Sqrt[1 - (d + e*x^(2/3))/d] + 128*d^3*Sqr
t[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3)) - 192*d^2*Sqrt[1 - (d + e*x^(2/3))
/d]*(d + e*x^(2/3))^2 + 128*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3
 - 32*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4 + 1584*d^3*(d + e*x^(2/
3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] - 4536
*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x
^(2/3))/d] + 3780*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2
, 2, 2}, (d + e*x^(2/3))/d] - 864*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7
/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] + 3024*d^3*(d + e*x^(2/3)
)*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] -
```

```
3780*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2}
, (d + e*x^(2/3))/d] + 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1,
 1, 1, 1}, {2, 2, 2, 2}, (d + e*x^(2/3))/d] - 240*d^4*Log[d + e*x^(2/3)] +
240*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 672*d^3*Sqrt[1 - (
d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(2/3)] + 576*d^2*Sqrt[1 - (d
+ e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x^(2/3)] - 96*d*Sqrt[1 - (d + e
*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x^(2/3)] - 48*Sqrt[1 - (d + e*x^(2
/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2/3)] - 3780*d^3*(d + e*x^(2/3))*Hyp
ergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]
+ 864*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, (d
+ e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 3024*d^3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 37
80*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e
*x^(2/3))/d]*Log[d + e*x^(2/3)] - 1890*d^3*(d + e*x^(2/3))*HypergeometricPF
Q[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 284*d
^4*Log[d + e*x^(2/3)]^2 - 284*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(
2/3)]^2 + 668*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(
2/3)]^2 - 552*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x
^(2/3)]^2 + 236*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x
^(2/3)]^2 - 68*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2
/3)]^2 - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (
d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]^2 + 945*d^3*(d + e*x^(2/3))*Hypergeome
tricPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)]^2 - 70*
d^4*Log[d + e*x^(2/3)]^3 + 70*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(
2/3)]^3 - 280*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(
2/3)]^3 + 420*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x
^(2/3)]^3 - 280*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x
^(2/3)]^3 + 70*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2
/3)]^3 + 1512*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (
d + e*x^(2/3))/d]*(1 + 3*Log[d + e*x^(2/3)] + Log[d + e*x^(2/3)]^2) - 144*d
^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/
d]*(6 + 11*Log[d + e*x^(2/3)] + 3*Log[d + e*x^(2/3)]^2)))/(210*e^4*Sqrt[1 -
 (d + e*x^(2/3))/d]) + (b^2*n^2*x^(1/3)*(-120*d^4 + 120*d^4*Sqrt[1 - (d + e
*x^(2/3))/d] - 336*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3)) + 288*d^
2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2 - 48*d*Sqrt[1 - (d + e*x^(2
/3))/d]*(d + e*x^(2/3))^3 - 24*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^
4 - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d + e
*x^(2/3))/d] + 432*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-7/2, 1, 1, 1}, {
2, 2, 2}, (d + e*x^(2/3))/d] - 1512*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{
-5/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] + 1890*d^3*(d + e*x^(2/3))*Hy
pergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))/d] - 945*d^3*(d
 + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, (d + e*x^(2/3))
/d] + 284*d^4*Log[d + e*x^(2/3)] - 284*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[
d + e*x^(2/3)] + 668*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d
+ e*x^(2/3)] - 552*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d
+ e*x^(2/3)] + 236*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d +
e*x^(2/3)] - 68*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(
2/3)] - 1890*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, (d
 + e*x^(2/3))/d]*Log[d + e*x^(2/3)] + 945*d^3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-1/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*Log[d + e*x^(2/3)] - 105*d^4*
Log[d + e*x^(2/3)]^2 + 105*d^4*Sqrt[1 - (d + e*x^(2/3))/d]*Log[d + e*x^(2/3
)]^2 - 420*d^3*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))*Log[d + e*x^(2/3
)]^2 + 630*d^2*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^2*Log[d + e*x^(2
/3)]^2 - 420*d*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^3*Log[d + e*x^(2
/3)]^2 + 105*Sqrt[1 - (d + e*x^(2/3))/d]*(d + e*x^(2/3))^4*Log[d + e*x^(2/3
)]^2 + 756*d^3*(d + e*x^(2/3))*HypergeometricPFQ[{-5/2, 1, 1}, {2, 2}, (d +
 e*x^(2/3))/d]*(3 + 2*Log[d + e*x^(2/3)]) - 72*d^3*(d + e*x^(2/3))*Hypergeo
metricPFQ[{-7/2, 1, 1}, {2, 2}, (d + e*x^(2/3))/d]*(11 + 6*Log[d + e*x^(2/3
)]))*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n])))/(105*e^4
*Sqrt[1 - (d + e*x^(2/3))/d]) - (2*b*d^4*n*x^(1/3)*(a + b*(-(n*Log[d + e*x^
(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/e^4 + (2*b*d^3*n*x*(a + b*(-(n*Log[
d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/(3*e^3) - (2*b*d^2*n*x^(5/3
)*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/(5*e^2) +
```

```
(2*b*d*n*x^(7/3)*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n
]))^2)/(7*e) + (2*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*(-(n
*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2)/e^(9/2) + b*n*x^3*Log[
d + e*x^(2/3)]*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))
^2 + (x^3*(a + b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n]))^2*(3
*a - 2*b*n + 3*b*(-(n*Log[d + e*x^(2/3)]) + Log[c*(d + e*x^(2/3))^n])))/9
```

Integral number [486]

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{3}\,dx$$

[A] time = 1.28345 (sec), size = 598 ,normalized size = 1.23

$$\frac{3b^2n^2x\left(-a-b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)+bn\log\left(d+ex^{2/3}\right)\right)\left(3\left(d+ex^{2/3}\right)\,{}_4F_3\left(-\frac{1}{2},1,1,1;2,2,2;\frac{x^{2/3}e}{d}+1\right)+\log\left(d+ex^{2/3}\right)\cdots\right.}{d\left(-\frac{ex^{2/3}}{d}\right)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] -1/2*(b^3*n^3*x*(-18*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1, 1},
{2, 2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*(18*(d + e*x^(2/3))*H
ypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e
*x^(2/3)]*(-9*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + (
e*x^(2/3))/d] + 2*(d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))))/(
d*(-((e*x^(2/3))/d))^(3/2)) + (3*b^2*n^2*x*(3*(d + e*x^(2/3))*Hypergeometri
cPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*(-
3*(d + e*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + (e*x^(2/3))/d
] + (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))*(-a + b*n*Log[d +
 e*x^(2/3)] - b*Log[c*(d + e*x^(2/3))^n]))/(d*(-((e*x^(2/3))/d))^(3/2)) + (
6*b*d*n*x^(1/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2
)/e - (6*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d + e*x
^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/e^(3/2) + 3*b*n*x*Log[d + e*x^(2/3
)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a - b*n
*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*(a - 2*b*n - b*n*Log[d
+ e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])
```

Integral number [487]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{3}}{x^2}\,dx$$

[B] time = 2.65974 (sec), size = 646 ,normalized size = 2.03

$$-\frac{-3b^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)-bn\log\left(d+ex^{2/3}\right)\right)\left(-6d\left(d+ex^{2/3}\right)\left(-\frac{ex^{2/3}}{d}\right)^{3/2}\,{}_4F_3\left(1,1,1,\frac{5}{2};2,2,2;\frac{x^{2/3}e}{d}+1\right)-2d\cdots\right.}{\cdots}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

```
[Out] -1/2*(2*b^3*d*n^3*(-9*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*Hypergeometr
icPFQ[{1, 1, 1, 1, 5/2}, {2, 2, 2, 2}, 1 + (e*x^(2/3))/d] + 9*(d + e*x^(2/3
))*(-((e*x^(2/3))/d))^(3/2)*HypergeometricPFQ[{1, 1, 1, 5/2}, {2, 2, 2}, 1
+ (e*x^(2/3))/d]*Log[d + e*x^(2/3)] + Log[d + e*x^(2/3)]^2*(-6*e*(-1 + Sqrt
[-((e*x^(2/3))/d)])*x^(2/3) + 6*d*(-((e*x^(2/3))/d))^(3/2)*Log[(1 + Sqrt[-(
(e*x^(2/3))/d)])/2] + (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d + e*x^(2/3)]))
 - 3*b^2*n^2*(-6*d*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*HypergeometricP
FQ[{1, 1, 1, 5/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] - 2*d*Log[d + e*x^(2/3)]*(
-4*e*(-1 + Sqrt[-((e*x^(2/3))/d)])*x^(2/3) + 4*d*(-((e*x^(2/3))/d))^(3/2)*L
og[(1 + Sqrt[-((e*x^(2/3))/d)])/2] + (d - d*(-((e*x^(2/3))/d))^(3/2))*Log[d
```

```
 + e*x^(2/3)]))*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n]) +
 12*b*d*e*n*x^(2/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n
])^2 + 12*b*Sqrt[d]*e^(3/2)*n*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*
Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 6*b*d^2*n*Log[d + e*x^
(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 2*d^2*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3)/(d^2*x)
```

Integral number [488]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$$

[A] time = 2.93655 (sec), size = 803 ,normalized size = 1.27

$$\frac{-70\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 d^5-210bn\log\left(d+ex^{2/3}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

```
[Out] (35*b^3*n^3*(54*e^4*(d + e*x^(2/3))*Sqrt[-((e*x^(2/3))/d)]*x^(8/3)*Hypergeo
metricPFQ[{1, 1, 1, 1, 11/2}, {2, 2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*
x^(2/3)]*(54*d*e^3*(d + e*x^(2/3))*(-((e*x^(2/3))/d))^(3/2)*x^2*Hypergeomet
ricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] + Log[d + e*x^(2/3)]*
(27*e^4*(d + e*x^(2/3))*Sqrt[-((e*x^(2/3))/d)]*x^(8/3)*HypergeometricPFQ[{1
, 1, 11/2}, {2, 2}, 1 + (e*x^(2/3))/d] - 2*d*(d^4 + d*e^3*(-((e*x^(2/3))/d)
)^(3/2)*x^2)*Log[d + e*x^(2/3)]))) + (210*b^2*n^2*(-9*e^5*(d + e*x^(2/3))*x
^(10/3)*HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + (e*x^(2/3))/d] +
Log[d + e*x^(2/3)]*(9*e^5*(d + e*x^(2/3))*x^(10/3)*HypergeometricPFQ[{1, 1,
 11/2}, {2, 2}, 1 + (e*x^(2/3))/d] + d*(d^5*Sqrt[-((e*x^(2/3))/d)] + e^5*x^
(10/3))*Log[d + e*x^(2/3)]))*(-a + b*n*Log[d + e*x^(2/3)] - b*Log[c*(d + e*
x^(2/3))^n]))/(d*Sqrt[-((e*x^(2/3))/d)]) - 60*b*d^4*e*n*x^(2/3)*(a - b*n*Lo
g[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 84*b*d^3*e^2*n*x^(4/3)*(
a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 140*b*d^2*e^3*
n*x^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + 420*b*d
*e^4*n*x^(8/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2
+ 420*b*Sqrt[d]*e^(9/2)*n*x^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Lo
g[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 210*b*d^5*n*Log[d + e*x^
(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 70*d^5
*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3)/(210*d^5*x^3)
```

Integral number [528]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

[A] time = 4.95269 (sec), size = 764 ,normalized size = 0.6

$$-\frac{b^2n^2\left(-a-b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+bn\log\left(d+\frac{e}{x^{2/3}}\right)\right)\left(\log\left(d+\frac{e}{x^{2/3}}\right)\left(9e^5\left(dx^{2/3}+e\right)\,{}_3F_2\left(1,1,\frac{11}{2};2,2;\frac{e}{dx^{2/3}}+1\right)+dx^{2/3}\left(d^5x\right.\right.\right.}{d^6x\sqrt{-\frac{e}{dx^{2/3}}}}$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (b^3*n^3*(54*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 1, 11/2}, {2,
2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d + e/x^(2/3)]*(-54*e^5*(e + d*x^(2/3))*
HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d +
e/x^(2/3)]*(27*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 11/2}, {2, 2},
1 + e/(d*x^(2/3))] + 2*d*x^(2/3)*(e^5 + d^5*Sqrt[-(e/(d*x^(2/3)))]*x^(10/3)
```

```
)*Log[d + e/x^(2/3)]]]]])/(6*d^6*Sqrt[-(e/(d*x^(2/3)))]*x) - (b^2*n^2*(-9*e^
5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1, 1, 11/2}, {2, 2, 2}, 1 + e/(d*x^
(2/3))] + Log[d + e/x^(2/3)]*(9*e^5*(e + d*x^(2/3))*HypergeometricPFQ[{1, 1
, 11/2}, {2, 2}, 1 + e/(d*x^(2/3))] + d*x^(2/3)*(e^5 + d^5*Sqrt[-(e/(d*x^(2
/3)))]*x^(10/3))*Log[d + e/x^(2/3)]))*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[
c*(d + e/x^(2/3))^n]))/(d^6*Sqrt[-(e/(d*x^(2/3)))]*x) - (2*b*e^4*n*x^(1/3)*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^4 + (2*b*e^3
*n*x*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*d^3) -
 (2*b*e^2*n*x^(5/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n
])^2)/(5*d^2) + (2*b*e*n*x^(7/3)*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d +
 e/x^(2/3))^n])^2)/(7*d) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]
*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/d^(9/2) + b*n
*x^3*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3
))^n])^2 + (x^3*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^3
)/3
```

Integral number [529]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

[A] time = 3.21781 (sec), size = 475 ,normalized size = 0.64

$$\frac{-9b^2en^2\left(dx^{2/3}+e\right)\sqrt{-\frac{e}{dx^{2/3}}}\ {}_4F_3\left(1,1,1,\frac{5}{2};2,2,2;\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+9b^3en^3\left(dx^{2/3}+e\right)\sqrt{-\frac{e}{dx^{2/3}}}\ {}_5F_4\left(\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (9*b^3*e*n^3*(e + d*x^(2/3))*Sqrt[-(e/(d*x^(2/3)))]*HypergeometricPFQ[{1, 1
, 1, 1, 5/2}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 9*b^2*e*n^2*(e + d*x^(2/3)
)*Sqrt[-(e/(d*x^(2/3)))]*HypergeometricPFQ[{1, 1, 1, 5/2}, {2, 2, 2}, 1 + e
/(d*x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]) - 6*b*Sqrt[d]*e^(3/2)*n*x^(1
/3)*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c
*(d + e/x^(2/3))^n])^2 + d*x^(2/3)*(-2*b^3*e*n^3*Sqrt[-(e/(d*x^(2/3)))]*Log
[d + e/x^(2/3)]^3 - 12*b^2*e*n^2*Sqrt[-(e/(d*x^(2/3)))]*(1 + Log[(1 + Sqrt[
-(e/(d*x^(2/3)))])/2])*Log[d + e/x^(2/3)]*(a + b*Log[c*(d + e/x^(2/3))^n])
+ 3*b^2*e*n^2*Sqrt[-(e/(d*x^(2/3)))]*Log[d + e/x^(2/3)]^2*(a + 2*b*n + 2*b*
n*Log[(1 + Sqrt[-(e/(d*x^(2/3)))])/2] + b*Log[c*(d + e/x^(2/3))^n]) + (a +
b*Log[c*(d + e/x^(2/3))^n])^2*(6*b*e*n + a*d*x^(2/3) + b*d*x^(2/3)*Log[c*(d
 + e/x^(2/3))^n])))/(d^2*x^(1/3))
```

Integral number [530]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$$

[B] time = 2.31475 (sec), size = 1097 ,normalized size = 2.27

$$\frac{b^3\left(18\left(x^{2/3}d+e\right){}_5F_4\left(-\frac{1}{2},1,1,1,1;2,2,2,2;\frac{e}{dx^{2/3}}+1\right)-\log\left(d+\frac{e}{x^{2/3}}\right)\left(18\left(x^{2/3}d+e\right){}_4F_3\left(-\frac{1}{2},1,1,1;2,2,2;\frac{e}{dx^{2/3}}+1\right)+\right.\right.}{2e\sqrt{-\frac{e}{dx^{2/3}}}\,x}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

```
[Out] (b^3*n^3*(18*(e + d*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2
, 2}, 1 + e/(d*x^(2/3))] - Log[d + e/x^(2/3)]*(18*(e + d*x^(2/3))*Hypergeom
etricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] + Log[d + e/x^(2/3)
]*(-9*(e + d*x^(2/3))*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2
/3))] + 2*(e*Sqrt[-(e/(d*x^(2/3)))] + d*x^(2/3))*Log[d + e/x^(2/3)]))))/(2*
```

```
e*Sqrt[-(e/(d*x^(2/3)))]*x) - (6*b*d*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[
c*(d + e/x^(2/3))^n])^2)/(e*x^(1/3)) - (6*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/
3))/Sqrt[e]]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/e
^(3/2) - (3*b*n*Log[d + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d
 + e/x^(2/3))^n])^2)/x - ((a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2
/3))^n])^2*(a - 2*b*n - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n]
))/x + (b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*
(8*e^(3/2) - 96*d*Sqrt[e]*x^(2/3) + 96*d^(3/2)*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^
(1/3))] - 12*e^(3/2)*Log[d + e/x^(2/3)] + 36*d*Sqrt[e]*x^(2/3)*Log[d + e/x^
(2/3)] + 9*e^(3/2)*Log[d + e/x^(2/3)]^2 + 18*Sqrt[-d]*d*x*Log[d + e/x^(2/3)
]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 9*(-d)^(3/2)*x*Log[Sqrt[e] - Sqrt[-d]*x
^(1/3)]^2 + 18*(-d)^(3/2)*x*Log[d + e/x^(2/3)]*Log[Sqrt[e] + Sqrt[-d]*x^(1/
3)] + 9*Sqrt[-d]*d*x*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2 + 18*Sqrt[-d]*d*x*Lo
g[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 1
8*(-d)^(3/2)*x*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/
Sqrt[e])/2] + 36*(-d)^(3/2)*x*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-
d]*x^(1/3))/Sqrt[e])] + 36*Sqrt[-d]*d*x*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log
[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 36*Sqrt[-d]*d*x*PolyLog[2, 1 - (Sqrt[-d]*x^(
1/3))/Sqrt[e]] + 18*(-d)^(3/2)*x*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqr
t[e])] + 18*Sqrt[-d]*d*x*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] + 3
6*(-d)^(3/2)*x*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(3*e^(3/2)*x)
```

Integral number [531]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$$

[B] time = 8.70501 (sec), size = 2726 ,normalized size = 3.48

Result too large to show

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```

```
[Out] (b^3*n^3*(32*e^4*Sqrt[-(e/(d*x^(2/3)))] - 32*d^4*x^(8/3) - 1584*d^3*e*x^2*H
ypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] - 1584*d^4*
x^(8/3)*HypergeometricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))] +
4536*d^3*e*x^2*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/
3))] + 4536*d^4*x^(8/3)*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + e
/(d*x^(2/3))] - 3780*d^3*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}
, 1 + e/(d*x^(2/3))] - 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1, 1},
{2, 2, 2}, 1 + e/(d*x^(2/3))] + 864*d^3*e*x^2*HypergeometricPFQ[{-7/2, 1, 1
, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] + 864*d^4*x^(8/3)*HypergeometricP
FQ[{-7/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 3024*d^3*e*x^2*Hy
pergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 3024
*d^4*x^(8/3)*HypergeometricPFQ[{-5/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x
^(2/3))] + 3780*d^3*e*x^2*HypergeometricPFQ[{-3/2, 1, 1, 1, 1}, {2, 2, 2, 2
}, 1 + e/(d*x^(2/3))] + 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1, 1,
1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 1890*d^3*e*x^2*HypergeometricPFQ[{-1
/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - 1890*d^4*x^(8/3)*Hyperg
eometricPFQ[{-1/2, 1, 1, 1, 1}, {2, 2, 2, 2}, 1 + e/(d*x^(2/3))] - (288*e^4
*Log[d + e/x^(2/3)])/Sqrt[-(e/(d*x^(2/3)))] + 48*e^4*Sqrt[-(e/(d*x^(2/3)))]
*Log[d + e/x^(2/3)] + 240*d^4*x^(8/3)*Log[d + e/x^(2/3)] + 3780*d^3*e*x^2*H
ypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)
] + 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/
3))]*Log[d + e/x^(2/3)] - 864*d^3*e*x^2*HypergeometricPFQ[{-7/2, 1, 1, 1},
{2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)] - 864*d^4*x^(8/3)*Hypergeo
metricPFQ[{-7/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)]
 + 3024*d^3*e*x^2*HypergeometricPFQ[{-5/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^
(2/3))]*Log[d + e/x^(2/3)] + 3024*d^4*x^(8/3)*HypergeometricPFQ[{-5/2, 1, 1
, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)] - 3780*d^3*e*x^2*Hyp
ergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(
2/3)] - 3780*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1, 1}, {2, 2, 2}, 1 +
```

```
e/(d*x^(2/3))]*Log[d + e/x^(2/3)] + 1890*d^3*e*x^2*HypergeometricPFQ[{-1/2,
 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)] + 1890*d^4*x^(8
/3)*HypergeometricPFQ[{-1/2, 1, 1, 1}, {2, 2, 2}, 1 + e/(d*x^(2/3))]*Log[d
+ e/x^(2/3)] + (252*e^4*Log[d + e/x^(2/3)]^2)/(-(e/(d*x^(2/3))))^(3/2) - (3
6*e^4*Log[d + e/x^(2/3)]^2)/Sqrt[-(e/(d*x^(2/3)))] + 68*e^4*Sqrt[-(e/(d*x^(
2/3)))]*Log[d + e/x^(2/3)]^2 - 284*d^4*x^(8/3)*Log[d + e/x^(2/3)]^2 + 1890*
d^3*e*x^2*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d
+ e/x^(2/3)]^2 + 1890*d^4*x^(8/3)*HypergeometricPFQ[{-3/2, 1, 1}, {2, 2}, 1
 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)]^2 - 945*d^3*e*x^2*HypergeometricPFQ[{-
1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(2/3)]^2 - 945*d^4*x^(8/
3)*HypergeometricPFQ[{-1/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*Log[d + e/x^(
2/3)]^2 - 70*e^4*Sqrt[-(e/(d*x^(2/3)))]*Log[d + e/x^(2/3)]^3 + 70*d^4*x^(8/
3)*Log[d + e/x^(2/3)]^3 - 1512*d^3*(e + d*x^(2/3))*x^2*HypergeometricPFQ[{-
5/2, 1, 1}, {2, 2}, 1 + e/(d*x^(2/3))]*(1 + 3*Log[d + e/x^(2/3)] + Log[d +
e/x^(2/3)]^2) + 144*d^3*(e + d*x^(2/3))*x^2*HypergeometricPFQ[{-7/2, 1, 1},
 {2, 2}, 1 + e/(d*x^(2/3))]*(6 + 11*Log[d + e/x^(2/3)] + 3*Log[d + e/x^(2/3
)]^2)))/(210*e^4*Sqrt[-(e/(d*x^(2/3)))]*x^3) - (2*b*d*n*(a - b*n*Log[d + e/
x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(7*e*x^(7/3)) + (2*b*d^2*n*(a - b
*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(5*e^2*x^(5/3)) - (2
*b*d^3*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*e^
3*x) + (2*b*d^4*n*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])
^2)/(e^4*x^(1/3)) + (2*b*d^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a - b
*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/e^(9/2) - (b*n*Log[d
 + e/x^(2/3)]*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2)/
x^3 - ((a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*(3*a - 2
*b*n - 3*b*n*Log[d + e/x^(2/3)] + 3*b*Log[c*(d + e/x^(2/3))^n]))/(9*x^3) +
(b^2*n^2*(-a + b*n*Log[d + e/x^(2/3)] - b*Log[c*(d + e/x^(2/3))^n])*(9800*e
^(9/2) - 28800*d*e^(7/2)*x^(2/3) + 72072*d^2*e^(5/2)*x^(4/3) - 208320*d^3*e
^(3/2)*x^2 + 1418760*d^4*Sqrt[e]*x^(8/3) - 1418760*d^(9/2)*x^3*ArcTan[Sqrt[
e]/(Sqrt[d]*x^(1/3))] - 44100*e^(9/2)*Log[d + e/x^(2/3)] + 56700*d*e^(7/2)*
x^(2/3)*Log[d + e/x^(2/3)] - 79380*d^2*e^(5/2)*x^(4/3)*Log[d + e/x^(2/3)] +
 132300*d^3*e^(3/2)*x^2*Log[d + e/x^(2/3)] - 396900*d^4*Sqrt[e]*x^(8/3)*Log
[d + e/x^(2/3)] + 99225*e^(9/2)*Log[d + e/x^(2/3)]^2 - 198450*(-d)^(9/2)*x^
3*Log[d + e/x^(2/3)]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 99225*(-d)^(9/2)*x^3
*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2 + 198450*(-d)^(9/2)*x^3*Log[d + e/x^(2/3
)]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 99225*(-d)^(9/2)*x^3*Log[Sqrt[e] + Sqr
t[-d]*x^(1/3)]^2 - 198450*(-d)^(9/2)*x^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Lo
g[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 198450*(-d)^(9/2)*x^3*Log[Sqrt[e]
 - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] + 396900*(-d)^
(9/2)*x^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])
] - 396900*(-d)^(9/2)*x^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(
1/3))/Sqrt[e]] - 396900*(-d)^(9/2)*x^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sq
rt[e]] + 198450*(-d)^(9/2)*x^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[
e])] - 198450*(-d)^(9/2)*x^3*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2]
 + 396900*(-d)^(9/2)*x^3*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(9922
5*e^(9/2)*x^3)
```

3.5 Test file Number [79] 4-Trig-functions/4.1-Sine/4.1.7-d-trig-^m-a+b-c-sin-^n-^p

3.5.1 Mathematica

Integral number [399]

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

[C] time = 0.357346 (sec), size = 394 ,normalized size = 15.15

$$\frac{24\cos(c+dx)(a+b\sin(c+dx))}{4a+3b\sin(c+dx)-b\sin(3(c+dx))} - i\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&, \frac{2\#1^4b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-4i\#1^3a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+2i}{}\right.$$

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
, (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2] + (4*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 12*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^2 - (6*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 2*a*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^3 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^
5) & ] + (24*Cos[c + d*x]*(a + b*Sin[c + d*x]))/(4*a + 3*b*Sin[c + d*x] - b
*Sin[3*(c + d*x)]))/(18*a*b*d)
```

Integral number [400]

$$\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

[C] time = 0.236978 (sec), size = 273 ,normalized size = 10.5

$$\frac{\frac{12\sin(2(c+dx))}{4a+3b\sin(c+dx)-b\sin(3(c+dx))} - i\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&, \frac{2\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-6i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+\right.}{}\right.}{18ad}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
, (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2] + 12*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (6*I)*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2
*b*#1^3 + b*#1^5) & ] + (12*Sin[2*(c + d*x)])/(4*a + 3*b*Sin[c + d*x] - b*S
in[3*(c + d*x)]))/(18*a*d)
```

Integral number [401]

$$\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

[C] time = 0.476477 (sec), size = 502 ,normalized size = 29.53

$$-\frac{12b\cos(c+dx)(a\cos(2(c+dx))-3a+2b\sin(c+dx))}{(a-b)(a+b)(4a+3b\sin(c+dx)-b\sin(3(c+dx)))} + \frac{i\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&, \frac{2\#1^4b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-4i\#1^3ab\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+12i\#1^2a^2\log}{}\right.}{}$$

```
[In] Integrate[(a + b*Sin[c + d*x]^3)^(-2),x]
```

```
[Out] ((I*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & ,
(2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b^2*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2] + (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*
a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 24*a^2*ArcTan[Sin[c + d*x]/(Cos[
c + d*x] - #1)]*#1^2 + 12*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
```

```
 + (12*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^2*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^3 - 2*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2*b^2*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a^2 - b^2) - (12
*b*Cos[c + d*x]*(-3*a + a*Cos[2*(c + d*x)] + 2*b*Sin[c + d*x]))/((a - b)*(a
 + b)*(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c + d*x)])))/(18*a*d)
```

Integral number [402]

$$\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

[C] time = 1.60412 (sec), size = 845 ,normalized size = 32.5

$$-\frac{ib\,\mathrm{RootSum}\left[ib\#1^6-3ib\#1^4+8a\#1^3+3ib\#1^2-ib\&,\frac{2b^3\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+16a^2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-ib^3\log\left(\#1^2-2\cos(c+dx)\#1+1\right)\#1^4-8ia^2b\log\left(\#1^2-2\cos(c+dx)\#1\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (((-I)*b*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6
 & , (16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^3*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)] - (8*I)*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2] - I*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (20*I)*a^3*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1 + (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1 + 10*a^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 8*a*b^2*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1 - 120*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^2 + 12*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (60*I)
*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^3*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^2 - (20*I)*a^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 10*a^3
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 8*a*b^2*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^3 + 16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 2*
b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (8*I)*a^2*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^4 - I*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)
/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a*(a^2 - b^2)^2) + (18*Sin
[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (18*Sin[
(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*b*Cos
[c + d*x]*(-2*a^3 - 7*a*b^2 + 3*a*b^2*Cos[2*(c + d*x)] + 2*b*(2*a^2 + b^2)*
Sin[c + d*x]))/(a*(a - b)^2*(a + b)^2*(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c
+ d*x)])))/(18*d)
```

Integral number [403]

$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

[C] time = 1.74385 (sec), size = 1158 ,normalized size = 44.54

result too large to display

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((4*I)*b^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1
^6 & , (14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 74*a^2*b^2*ArcTan
[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x
] - #1)] - (7*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (37*I)*a^2*b^2*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2] - I*b^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] +
```

```
(144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + (36*I)*a*b^3*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 72*a^3*b*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1 + 18*a*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 180*a^4*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 372*a^2*b^2*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 + 12*b^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^2 + (90*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (186*I)*a^2*
b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^4*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^2 - (144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - (36*I)*a*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 72*a^3
*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 18*a*b^3*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^3 + 14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 7
4*a^2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 2*b^4*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (7*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1^4 - (37*I)*a^2*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - I*b^4*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b
*#1^5) & ] + (3*Sec[c + d*x]^3*(48*a^5*b + 568*a^3*b^3 + 14*a*b^5 + (78*a^5
*b + 606*a^3*b^3 + 81*a*b^5)*Cos[2*(c + d*x)] + 18*a*b^3*(4*a^2 + b^2)*Cos[
4*(c + d*x)] + 2*a^5*b*Cos[6*(c + d*x)] - 30*a^3*b^3*Cos[6*(c + d*x)] - 17*
a*b^5*Cos[6*(c + d*x)] + 48*a^6*Sin[c + d*x] - 244*a^4*b^2*Sin[c + d*x] + 2
0*a^2*b^4*Sin[c + d*x] - 4*b^6*Sin[c + d*x] + 16*a^6*Sin[3*(c + d*x)] - 194
*a^4*b^2*Sin[3*(c + d*x)] - 86*a^2*b^4*Sin[3*(c + d*x)] - 6*b^6*Sin[3*(c +
d*x)] - 14*a^4*b^2*Sin[5*(c + d*x)] - 74*a^2*b^4*Sin[5*(c + d*x)] - 2*b^6*S
in[5*(c + d*x)]))/(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c + d*x)]))/(72*a*(a^2
 - b^2)^3*d)
```

3.5.2 Maple

Integral number [399]

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

[B] time = 0.965 (sec), size = 550 ,normalized size = 21.15

$$-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\left(\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a\right)a}+\frac{}{3d\left(\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+3\right.}$$

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2
*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^5+2/3/d/(tan(1/2*d*x
+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x
+1/2*c)^2*a+a)/b*tan(1/2*d*x+1/2*c)^4+8/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1
/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*ta
n(1/2*d*x+1/2*c)^3+4/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8
*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/b*tan(1/2*d*x+1/2*c)^2+
2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*
c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)+2/3/d/(tan(1/2*d*x+1/
2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/
2*c)^2*a+a)/b+2/9/d/a/b*sum((_R^4*b+_R^3*a+_R*a+b)/(_R^5*a+2*_R^3*a+4*_R^2*
b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2
*a+a))
```

Integral number [400]

$$\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

[B] time = 0.944 (sec), size = 236 ,normalized size = 9.08

$$-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\left(\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a\right)a}+\frac{}{3d\left(\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a\right.}$$

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2
*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^5+2/3/d/(tan(1/2*d*x
+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x
+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)+2/9/d/a*sum((_R^4+1)/(_R^5*a+2*_R^3*a+4
*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+
3*_Z^2*a+a))
```

Integral number [401]

$$\int \frac{1}{\left(a+b\sin^{3}(c+dx)\right)^{2}}\,dx$$

[B] time = 0.65 (sec), size = 658 ,normalized size = 38.71

$$\frac{2b^{2}\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\left(\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a\right)a\left(a^{2}-b^{2}\right)}-\frac{}{3d\left(\left(\tan^{6}\left(\frac{dx}{2}\right.\right.\right.}$$

```
[In] int(1/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*
c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^5-2/3/d
/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+
3*tan(1/2*d*x+1/2*c)^2*a+a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)^4+8/3/d/(tan(1/2
*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2
*d*x+1/2*c)^2*a+a)*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+8/3/d/(tan(1/2*d*x+
1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+
1/2*c)^2*a+a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-2/3/d/(tan(1/2*d*x+1/2*c)^6*
a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*
a+a)*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)+2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan
(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*b/
(a^2-b^2)+1/9/d/a/(a^2-b^2)*sum(((3*a^2-2*b^2)*_R^4-2*_R^3*a*b+6*_R^2*a^2-2
*a*_R*b+3*a^2-2*b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Integral number [402]

$$\int \frac{\sec^{2}(c+dx)}{\left(a+b\sin^{3}(c+dx)\right)^{2}}\,dx$$

[B] time = 0.956 (sec), size = 1276 ,normalized size = 49.08

result too large to display

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-4/3/
d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*
tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*c)^5-2/3
```

```
/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b
*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^5-2/
3/d*b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*
tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^4*a^2+8
/3/d*b^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8
*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^4-8/
3/d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*
b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*c)^3-1
6/3/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+
8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^3
-4/3/d*b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8
*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^2*a^
2-20/3/d*b^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4
*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^
2+4/3/d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*
a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*c)
+2/3/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a
+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)-
2/3/d*b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*
b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^2-4/3/d*b^3/(a-b)^2/(a
+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*
c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)-1/9/d*b/(a-b)^2/(a+b)^2/a*sum((b*(11*a^2-2
*b^2)*_R^4+2*a*(-5*a^2-4*b^2)*_R^3+54*_R^2*a^2*b+2*a*(-5*a^2-4*b^2)*_R+11*a
^2*b-2*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=Ro
otOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Integral number [403]

$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

[B] time = 1.183 (sec), size = 1549 ,normalized size = 59.58

result too large to display

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -1/3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1
)^2-1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)*a-4/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)
*b-1/3/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)
+1)^2-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)*a+4/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+
1)*b+2/3/d*b^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)
^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^3*tan(1/2*d*x+1
/2*c)^5+14/3/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/
2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x
+1/2*c)^5+2/3/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1
/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*
x+1/2*c)^5-6/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/
2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1
/2*c)^4+16/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*
c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1
/2*c)^3+8/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c
)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/
2*c)^3+12/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c
)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*
c)^2*a^2+12/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2
*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/
2*c)^2-2/3/d*b^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*
c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^3*tan(1/2*d*x
+1/2*c)-14/3/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/
2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x
+1/2*c)-2/3/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2
```

```
*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+
1/2*c)+4/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)
^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^2+2/d*b^5/(a-b)
^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x
+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)+1/9/d*b^2/(a-b)^3/(a+b)^3/a*sum(((19*
a^4+28*a^2*b^2-2*b^4)*_R^4+18*a*b*(-4*a^2-b^2)*_R^3+6*a^2*(11*a^2+34*b^2)*_
R^2+18*a*b*(-4*a^2-b^2)*_R+19*a^4+28*a^2*b^2-2*b^4)/(_R^5*a+2*_R^3*a+4*_R^2
*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^
2*a+a))
```

3.6 Test file Number [151] 5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.5-u-a+b-arctan-c+d-x-^p

3.6.1 Mathematica

Integral number [65]

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

[B] time = 0.421783 (sec), size = 163 ,normalized size = 7.09

$$\frac{\frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{(a+bx)^2+1}\right)}{(a+bx)^2+1}+6\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{4(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{(a+bx)^2+1}\right)\tan^{-1}(a+bx)}{(a+bx)^2+1}+10(a+bx)\tan^{-1}(a+bx)+15\right)}{20b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\sqrt[3]{a^2+2abx+b^2x^2+1}}$$

```
[In] Integrate[ArcTan[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)
*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(
1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/
3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(1
+ a^2 + 2*a*b*x + b^2*x^2)^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Integral number [66]

$$\int \frac{\tan^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx$$

[B] time = 0.0997431 (sec), size = 165 ,normalized size = 6.6

$$\frac{\frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{(a+bx)^2+1}\right)}{(a+bx)^2+1}+6\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{4(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{(a+bx)^2+1}\right)\tan^{-1}(a+bx)}{(a+bx)^2+1}+10(a+bx)\tan^{-1}(a+bx)+15\right)}{20b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\sqrt[3]{c\left(a^2+2abx+b^2x^2+1\right)}}$$

```
[In] Integrate[ArcTan[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(15 + 10*(a + b*x)*ArcTan[a + b*x] + (4*(a + b*x)
*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(
1 + (a + b*x)^2)) + (5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/
3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2))/(20*b*(c*
(1 + a^2 + 2*a*b*x + b^2*x^2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

Integral number [69]

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

[B] time = 1.57668 (sec), size = 181 ,normalized size = 6.03

$$-\frac{3\left((a+bx)^2+1\right)^{2/3}\left(\frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{(a+bx)^2+1}\right)}{\left((a+bx)^2+1\right)^2}+\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{24(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{(a+bx)^2+1}\right)\tan^{-1}(a+bx)}{\left((a+bx)^2+1\right)^2}+\frac{90}{(a+bx)^2+1}+5\tan\right.\right.}{140b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)}$$

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)
,x]
```

```
[Out] (-3*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricP
FQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2
 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 + (a + b*x)^2) + (24*(a + b*x)*ArcTan
[a + b*x]*Hypergeometric2F1[1, 4/3, 11/6, (1 + (a + b*x)^2)^(-1)])/(1 + (a
+ b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a + b*x) + 6*Sin[2*ArcTan[a + b*x]])))
)/(140*b*Gamma[11/6]*Gamma[7/3])
```

Integral number [70]

$$\int \frac{(a+bx)^2\tan^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx$$

[B] time = 0.800008 (sec), size = 225 ,normalized size = 7.03

$$-\frac{3\sqrt[3]{a^2+2abx+b^2x^2+1}\left((a+bx)^2+1\right)^{2/3}\left(\frac{5\sqrt[3]{2}\sqrt{\pi}\Gamma\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{(a+bx)^2+1}\right)}{\left((a+bx)^2+1\right)^2}+\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(\frac{24(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{(a+bx)^2+1}\right)\tan^{-1}(\right.}{\left((a+bx)^2+1\right)^2}\right.\right.}{140b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\sqrt[3]{c\left(a^2+2abx+b^2x^2+1\right)}}$$

```
[In] Integrate[((a + b*x)^2*ArcTan[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^
2)^(1/3),x]
```

```
[Out] (-3*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)*(1 + (a + b*x)^2)^(2/3)*((5*2^(1/3)
*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + (a
+ b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + Gamma[11/6]*Gamma[7/3]*(15 + 90/(1 +
 (a + b*x)^2) + (24*(a + b*x)*ArcTan[a + b*x]*Hypergeometric2F1[1, 4/3, 11/
6, (1 + (a + b*x)^2)^(-1)])/(1 + (a + b*x)^2)^2 + 5*ArcTan[a + b*x]*(-4*(a
+ b*x) + 6*Sin[2*ArcTan[a + b*x]]))))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^
2))^(1/3)*Gamma[11/6]*Gamma[7/3])
```

3.7 Test file Number [154] 5-Inverse-trig-functions/5.4-Inverse-cotangent/5.4.1-Inverse-cotangent-functions

3.7.1 Mathematica

Integral number [116]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

[B] time = 0.41034 (sec), size = 177 ,normalized size = 7.7

$$\frac{6\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(4(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{a^2+2bxa+b^2x^2+1}\right)\cot^{-1}(a+bx)+5\left(a^2+2abx+b^2x^2+1\right)\left(2(a+bx)\cot^{-1}(a+bx)-3\right)\right)}{20b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(a^2+2abx+b^2x^2+1\right)^{4/3}}$$

```
[In] Integrate[ArcCot[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3),x]
```

```
[Out] (6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a + b*x
)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1, 4/3,
11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma[5/3]*
HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b^2*x^2)
^(-1)])/(20*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(4/3)*Gamma[11/6]*Gamma[7/3])
```

Integral number [117]

$$\int \frac{\cot^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx$$

[B] time = 0.112107 (sec), size = 180 ,normalized size = 7.2

$$\frac{c\left(6\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(4(a+bx)\,{}_2F_1\left(1,\frac{4}{3};\frac{11}{6};\frac{1}{a^2+2bxa+b^2x^2+1}\right)\cot^{-1}(a+bx)+5\left(a^2+2abx+b^2x^2+1\right)\left(2(a+bx)\cot^{-1}(a+bx\right.\right.\right.}{20b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(c\left(a^2+2abx+b^2x^2+1\right)\right)^{4/3}}$$

```
[In] Integrate[ArcCot[a + b*x]/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^2)^(1/3),x]
```

```
[Out] (c*(6*Gamma[11/6]*Gamma[7/3]*(5*(1 + a^2 + 2*a*b*x + b^2*x^2)*(-3 + 2*(a +
b*x)*ArcCot[a + b*x]) + 4*(a + b*x)*ArcCot[a + b*x]*Hypergeometric2F1[1, 4/
3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) - 5*2^(1/3)*Sqrt[Pi]*Gamma[5/
3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1 + a^2 + 2*a*b*x + b^2*x
^2)^(-1)]))/(20*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))^(4/3)*Gamma[11/6]*Gamma
[7/3])
```

Integral number [120]

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{1+a^2+2abx+b^2x^2}}\,dx$$

[B] time = 1.00319 (sec), size = 198 ,normalized size = 6.6

$$\frac{3\left(5\sqrt[3]{2}\sqrt{\pi}\,\Gamma\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{a^2+2bxa+b^2x^2+1}\right)+\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(5\left((a+bx)^2+1\right)\left(3\left((a+bx)^2+7\right)+4(a+bx)\left((a+\right.\right.\right.\right.}{140b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\sqrt[3]{a^2+2abx+b^2x^2+1}\left((a+b\right.}$$

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/3)
,x]
```

```
[Out] (3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(a
 + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x]*
Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5*2^
(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1
+ a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(1/
3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

Integral number [121]

$$\int \frac{(a+bx)^2\cot^{-1}(a+bx)}{\sqrt[3]{\left(1+a^2\right)c+2abcx+b^2cx^2}}\,dx$$

[B] time = 0.224771 (sec), size = 200 ,normalized size = 6.25

$$\frac{3\left(5\sqrt[3]{2}\sqrt{\pi}\,\Gamma\left(\frac{5}{3}\right){}_3F_2\left(1,\frac{4}{3},\frac{4}{3};\frac{11}{6},\frac{7}{3};\frac{1}{a^2+2bxa+b^2x^2+1}\right)+\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left(5\left((a+bx)^2+1\right)\left(3\left((a+bx)^2+7\right)+4(a+bx)\left((a+\right.\right.\right.\right.}{140b\Gamma\left(\frac{11}{6}\right)\Gamma\left(\frac{7}{3}\right)\left((a+bx)^2+1\right)\sqrt[3]{c\left(a^2+2abx+\right.}}$$

```
[In] Integrate[((a + b*x)^2*ArcCot[a + b*x])/((1 + a^2)*c + 2*a*b*c*x + b^2*c*x^
2)^(1/3),x]
```

```
[Out] (3*(Gamma[11/6]*Gamma[7/3]*(5*(1 + (a + b*x)^2)*(3*(7 + (a + b*x)^2) + 4*(a
 + b*x)*(-2 + (a + b*x)^2)*ArcCot[a + b*x]) - 24*(a + b*x)*ArcCot[a + b*x]*
Hypergeometric2F1[1, 4/3, 11/6, (1 + a^2 + 2*a*b*x + b^2*x^2)^(-1)]) + 5*2^
(1/3)*Sqrt[Pi]*Gamma[5/3]*HypergeometricPFQ[{1, 4/3, 4/3}, {11/6, 7/3}, (1
+ a^2 + 2*a*b*x + b^2*x^2)^(-1)]))/(140*b*(c*(1 + a^2 + 2*a*b*x + b^2*x^2))
^(1/3)*(1 + (a + b*x)^2)*Gamma[11/6]*Gamma[7/3])
```

3.8 Test file Number [173] 6-Hyperbolic-functions/6.3-Hyperbolic-tangent/6.3.7-d-hyper-^m-a+b-c-tanh-^n-^p

3.8.1 Mathematica

Integral number [74]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

[B] time = 0.534942 (sec), size = 826 ,normalized size = 25.03

$\cosh(3(c+dx))a^3 + 27b\sinh(c+dx)a^2 - b\sinh(3(c+dx))a^2 - 9\left(a^2+3b^2\right)\cosh(c+dx)a - b^2\cosh(3(c+dx))a - 2b\text{Root}$

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (-9*a*(a^2 + 3*b^2)*Cosh[c + d*x] + a^3*Cosh[3*(c + d*x)] - a*b^2*Cosh[3*(c
 + d*x)] - 2*a*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4
+ a*#1^6 + b*#1^6 & , (3*a^2*c + 3*a*b*c + 3*b^2*c + 3*a^2*d*x + 3*a*b*d*x
+ 3*b^2*d*x + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c
 + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*b^2*Log[-Cosh
[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2
]*#1] + 2*a^2*c*#1^2 - 2*b^2*c*#1^2 + 2*a^2*d*x*#1^2 - 2*b^2*d*x*#1^2 + 4*a
^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^2 - 4*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a^2*c*#1^4 - 3*a*b*c
*#1^4 + 3*b^2*c*#1^4 + 3*a^2*d*x*#1^4 - 3*a*b*d*x*#1^4 + 3*b^2*d*x*#1^4 + 6
*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Si
nh[(c + d*x)/2]*#1]*#1^4 - 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
 + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 6*b^2*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]
*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ] + 27*a^2*b
*Sinh[c + d*x] + 9*b^3*Sinh[c + d*x] - a^2*b*Sinh[3*(c + d*x)] + b^3*Sinh[3
*(c + d*x)])/(12*(a - b)^2*(a + b)^2*d)
```

Integral number [76]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

[B] time = 0.26943 (sec), size = 409 ,normalized size = 13.19

$b\text{RootSum}\left[\#1^6a + \#1^6b + 3\#1^4a - 3\#1^4b + 3\#1^2a + 3\#1^2b + a - b\&, \frac{4\#1^4a\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\text{cos}\right.}{}\right.$

```
[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (6*a*Cosh[c + d*x] + b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b
*#1^4 + a*#1^6 + b*#1^6 & , (2*a*c + b*c + 2*a*d*x + b*d*x + 4*a*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1] + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#
1 - Sinh[(c + d*x)/2]*#1] + 2*a*c*#1^4 - b*c*#1^4 + 2*a*d*x*#1^4 - b*d*x*#1
^4 + 4*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^4 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/
2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*
#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ] - 6*b*Sinh[c + d*x])/(6*(a - b)*(a +
 b)*d)
```

Integral number [77]

$$\int \frac{\operatorname{csch}(c + dx)}{a + b\tanh^3(c + dx)}\,dx$$

[B] time = 0.179177 (sec), size = 319 ,normalized size = 10.29

$$\frac{6\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - b\text{RootSum}\left[\#1^6a + \#1^6b + 3\#1^4a - 3\#1^4b + 3\#1^2a + 3\#1^2b + a - b\&, \frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\right.}{}\right.}{}$$

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] (6*Log[Tanh[(c + d*x)/2]] - b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^
4 - 3*b*#1^4 + a*#1^6 + b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 -
 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c +
d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ])/(6*a*d)
```

Integral number [79]

$$\int \frac{\operatorname{csch}^3(c + dx)}{a + b\tanh^3(c + dx)}\,dx$$

[B] time = 0.386272 (sec), size = 201 ,normalized size = 6.09

$$-\frac{16b\text{RootSum}\left[\#1^6a + \#1^6b + 3\#1^4a - 3\#1^4b + 3\#1^2a + 3\#1^2b + a - b\&, \frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+d\right.\right.}{\#1^4a+\#1^4b+2\#1^2a-2\#1^2b+a+}\right.}{24ad}$$

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

```
[Out] -1/24*(16*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#
1^6 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(a + b + 2*a*#1^2 -
2*b*#1^2 + a*#1^4 + b*#1^4) & ] + 3*(Csch[(c + d*x)/2]^2 + 4*Log[Tanh[(c +
d*x)/2]] + Sech[(c + d*x)/2]^2))/(a*d)
```

3.8.2 Maple

Integral number [74]

$$\int \frac{\sinh^3(c + dx)}{a + b\tanh^3(c + dx)}\,dx$$

[B] time = 0.467 (sec), size = 346 ,normalized size = 10.48

$$-\frac{ab\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\left(2a^2+b^2\right)\_R^4-6\_R^3ab+2\left(4a^2+5b^2\right)\_R^2-6ab\_R+2a^2+b^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3d\,(a+b)^2\,(a-b)^2}-\frac{1}{3d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$$

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -1/3/d*a*b/(a+b)^2/(a-b)^2*sum(((2*a^2+b^2)*_R^4-6*_R^3*a*b+2*(4*a^2+5*b^2)
*_R^2-6*a*b*_R+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1
/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-16/3/d/(tanh(1/2*
d*x+1/2*c)-1)^3/(16*a+16*b)-8/d/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d
/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*b-8/
d/(16*a-16*b)/(tanh(1/2*d*x+1/2*c)+1)^2+16/3/d/(tanh(1/2*d*x+1/2*c)+1)^3/(1
6*a-16*b)-1/2/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)*a-1/d/(a-b)^2/(tanh(1/2*d*x
+1/2*c)+1)*b
```

Integral number [76]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

[B] time = 0.528 (sec), size = 164 ,normalized size = 5.29

$$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4a-2\_R^3b+6\_R^2a-2\_Rb+a\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3d\,(a-b)\,(a+b)}+\frac{4}{d\,(4a-4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{}{d\,(4a}$$

```
[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] 1/3/d*b/(a-b)/(a+b)*sum((_R^4*a-2*_R^3*b+6*_R^2*a-2*_R*b+a)/(_R^5*a+2*_R^3*
a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^
3*b+3*_Z^2*a+a))+4/d/(4*a-4*b)/(tanh(1/2*d*x+1/2*c)+1)-4/d/(4*a+4*b)/(tanh(
1/2*d*x+1/2*c)-1)
```

Integral number [77]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

[B] time = 0.619 (sec), size = 98 ,normalized size = 3.16

$$-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

```
[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] -4/3/d/a*b*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+1/d/a*ln(tanh(1/2*d*x+1
/2*c))
```

Integral number [79]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

[B] time = 0.595 (sec), size = 144 ,normalized size = 4.36

$$\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da} - \frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)} \frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3da} - \frac{1}{8da\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2} - \frac{\ln\left(\tanh\right.}{2}$$

```
[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/3/d/a*b*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^3*
a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^
3*b+3*_Z^2*a+a))-1/8/d/a/tanh(1/2*d*x+1/2*c)^2-1/2/d/a*ln(tanh(1/2*d*x+1/2*
c))
```

3.8.3 Giac

Integral number [74]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

[B] time = 3.16393 (sec), size = 344 ,normalized size = 10.42

$$-\frac{\frac{\left(9ae^{(2dx+2c)}+9be^{(2dx+2c)}-a+b\right)e^{(-3dx)}}{a^2e^{(3c)}-2abe^{(3c)}+b^2e^{(3c)}} - \frac{a^2e^{(3dx+30c)}+2abe^{(3dx+30c)}+b^2e^{(3dx+30c)}-9a^2e^{(dx+28c)}+9b^2e^{(dx+28c)}}{a^3e^{(27c)}+3a^2be^{(27c)}+3ab^2e^{(27c)}+b^3e^{(27c)}}}{24d} - \frac{\frac{6\left(a^3be^c+a^2b^2e^c+ab^3e^c\right)dx}{a-b} - \frac{\left(a^3be^c+\right.}{}}{}$$

```
[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm=""giac"")
```

```
[Out] -1/24*((9*a*e^(2*d*x + 2*c) + 9*b*e^(2*d*x + 2*c) - a + b)*e^(-3*d*x)/(a^2*
e^(3*c) - 2*a*b*e^(3*c) + b^2*e^(3*c)) - (a^2*e^(3*d*x + 30*c) + 2*a*b*e^(3
*d*x + 30*c) + b^2*e^(3*d*x + 30*c) - 9*a^2*e^(d*x + 28*c) + 9*b^2*e^(d*x +
 28*c))/(a^3*e^(27*c) + 3*a^2*b*e^(27*c) + 3*a*b^2*e^(27*c) + b^3*e^(27*c))
)/d - (6*(a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*d*x/(a - b) - (a^3*b*e^c + a
^2*b^2*e^c + a*b^3*e^c)*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a
*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x
 + 2*c) + a - b))/(a - b))/((a^4 - 2*a^2*b^2 + b^4)*d^2)
```

Integral number [76]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

[B] time = 1.84122 (sec), size = 189 ,normalized size = 6.1

$$\frac{\frac{e^{(dx+8c)}}{ae^{(7c)}+be^{(7c)}} + \frac{e^{(-dx)}}{ae^c-be^c}}{2d} + \frac{\frac{6\left(2abe^c+b^2e^c\right)dx}{a-b} - \frac{\left(2abe^c+b^2e^c\right)\log\left(\left|ae^{(6dx+6c)}+be^{(6dx+6c)}+3ae^{(4dx+4c)}-3be^{(4dx+4c)}+3ae^{(2dx+2c)}+3be^{(2dx+2c)}+a-b\right|\right)}{a-b}}{3\left(a^2-b^2\right)d^2}$$

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm=""giac"")
```

```
[Out] 1/2*(e^(d*x + 8*c)/(a*e^(7*c) + b*e^(7*c)) + e^(-d*x)/(a*e^c - b*e^c))/d +
1/3*(6*(2*a*b*e^c + b^2*e^c)*d*x/(a - b) - (2*a*b*e^c + b^2*e^c)*log(abs(a*
e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x +
4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b))/(a - b))/((a^2 -
 b^2)*d^2)
```

Integral number [77]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

[B] time = 1.39841 (sec), size = 147 ,normalized size = 4.74

$$-\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}}{d}-\frac{\frac{6\,bdxe^{c}}{a-b}-\frac{be^{c}\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a-b}}{3\,ad^{2}}$$

```
[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm=""giac"")
```

```
[Out] -(log(e^(d*x + c) + 1)/a - log(abs(e^(d*x + c) - 1))/a)/d - 1/3*(6*b*d*x*e^
c/(a - b) - b*e^c*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*
d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c
) + a - b))/(a - b))/(a*d^2)
```

Integral number [79]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

[B] time = 0.788413 (sec), size = 68 ,normalized size = 2.06

$$\frac{\frac{\log\left(e^{(dx+c)}+1\right)}{a}-\frac{\log\left(\left|e^{(dx+c)}-1\right|\right)}{a}-\frac{2\left(e^{(3\,dx+3\,c)}+e^{(dx+c)}\right)}{a\left(e^{(2\,dx+2\,c)}-1\right)^{2}}}{2\,d}$$

```
[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm=""giac"")
```

```
[Out] 1/2*(log(e^(d*x + c) + 1)/a - log(abs(e^(d*x + c) - 1))/a - 2*(e^(3*d*x + 3
*c) + e^(d*x + c))/(a*(e^(2*d*x + 2*c) - 1)^2))/d
```

Chapter 4

Listing of grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
```

```
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,debug:=
    false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal
    );
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as
    well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
```

```
            if debug then
               print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A";
         else
            if debug then
               print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B";
         end if
     end if
  else #ExpnType(result) > ExpnType(optimal)
     if debug then
        print("ExpnType(result) > ExpnType(optimal)");
     fi;
     return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn
    ,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[
    List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
```

```
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-
    equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False
```

```
def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational
    ):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args
    [0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #
    max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(
    expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.
    args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.
    func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.
    args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.
    args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.
    args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",
    leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```