
HW 7 EMA 471 Intermediate Problem Solving for Engineers

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ENGINEERING MECHANICS DEPARTMENT
UNIVERSITY OF WISCONSIN, MADISON
INSTRUCTOR: PROFESSOR ROBERT J. WITT

BY
NASSER M. ABBASI

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0.1 Problem 1

EP 471 – Homework #7 Solns: Parabolic PDEs Due: Thursday, May 5th, 2016

Use Matlab's pdepe utility for both problems.

- (1) (20 pts) When a body experiences very high temperatures, radiative heating/cooling becomes an important mechanism for heat transfer. Consider the case of a slab of steel, six inches thick, that has been heated in a furnace to a uniform temperature of 2350 °F. The steel is removed from the furnace, and we'd like to know how fast it cools. Specifically, we'd like to know how long it takes for the centerline temperature of the steel to fall below 1000 °F.

Take the material properties of the steel as constant ($k = 25$ Btu/hr-ft-°F, $\rho = 500$ lbm/ft³, and $c = 0.12$ Btu/lbm-°F). Perform two simulations. In the first, ignore radiative heat transfer and use a convective heat transfer coefficient to ambient air ($T_\infty = 60$ °F) of 50 Btu/hr-ft²-°F. In the second, include radiative heat transfer. In addition to the convective cooling, radiative cooling includes a term of the form:

$$q'' = \epsilon\sigma(T_s^4 - T_\infty^4)$$

Here ϵ is the emissivity (dimensionless, take equal to 0.8). The parameter σ is the Stefan-Boltzmann constant, equal to 0.171×10^{-8} Btu/hr-ft²-°R⁴. Here T_s is the surface temperature of the steel and T_∞ is again the ambient air temperature. Note that in this radiative heat transfer expression, temperatures must be expressed in absolute values (°R, not °F). Calculate and plot the evolution of the temperature profile in the steel and report the times required for the centerline temperature to fall below 1000 °F without and with the effect of radiative cooling.

Figure 1: problem 1 description

The PDE to solve is the parabolic PDE

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$$

There is no source term. We consider only convective heat loss in the first part of the problem, then for the second part, add the radiative heat loss.

Before we solve this problem, it is a good idea to write down all the units. This table summarizes this

| term | name | dimensional | SI units | Imperial units | SI convert |
|----------------------------------|------------------|--------------------------|---|--|------------|
| $\alpha = \frac{k}{(\rho)(c_p)}$ | diffusivity | $\frac{L^2}{T}$ | $\frac{\text{meter}^2}{\text{sec}}$ | $\frac{\text{ft}^2}{\text{sec}}$ | 0.3048 |
| k | conductivity | $\frac{ML}{T^3\Theta}$ | $\frac{\text{Watt}}{\text{meter-Kelvin}}$ | $\frac{\text{Btu}}{\text{hr-ft-F}^0}$ | 1.73 |
| ρ | mass density | $\frac{M}{L^3}$ | $\frac{\text{kg}}{\text{meter}^3}$ | $\frac{\text{lbm}}{\text{ft}^3}$ | 16.019 |
| c_p | specific heat | $\frac{L^2M}{T^2\Theta}$ | $\frac{\text{joule}}{\text{kg-Kelvin}}$ | $\frac{\text{Btu}}{\text{lbm-F}^0}$ | 4188 |
| σ | Stefan-Boltzmann | | $\frac{\text{Watt}}{\text{meter}^2 \text{ Kelvin}^4} (5.670367 \times 10^{-8})$ | $\frac{\text{Btu}}{\text{hr-ft}^2 \text{ R}^4} (0.171 \times 10^{-8})$ | |

And to convert F^0 to C^0 us $C^0 = (F^0 - 32) \frac{5}{9}$. To convert from F^0 to absolute R^0 (Rankine) add 459.67. Hence $R^0 = F^0 + 459.67$.

The Matlab program nma_HW_7_problem_1.m solves both parts. Summary of result

| | time to cool to 1000 F (minutes) |
|-------------------------------|----------------------------------|
| convective only loss | 20.18 |
| convective and radiative loss | 17.26 (85.5% of above time) |

The following shows the plot for the first part (convective heat loss only).

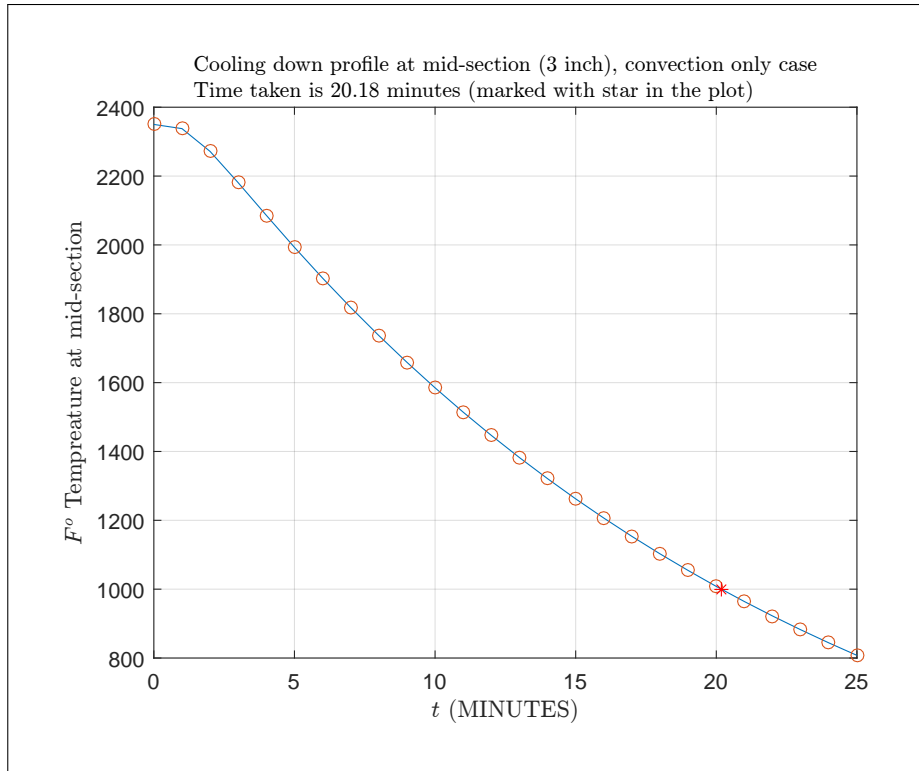


Figure 2: First part of first problem. Convective only heat loss

The plot below shows the profile of the heat loss across the whole section. Time is increasing going down in this figure. From initial time $t = 0$ to $t = 25$ minutes.

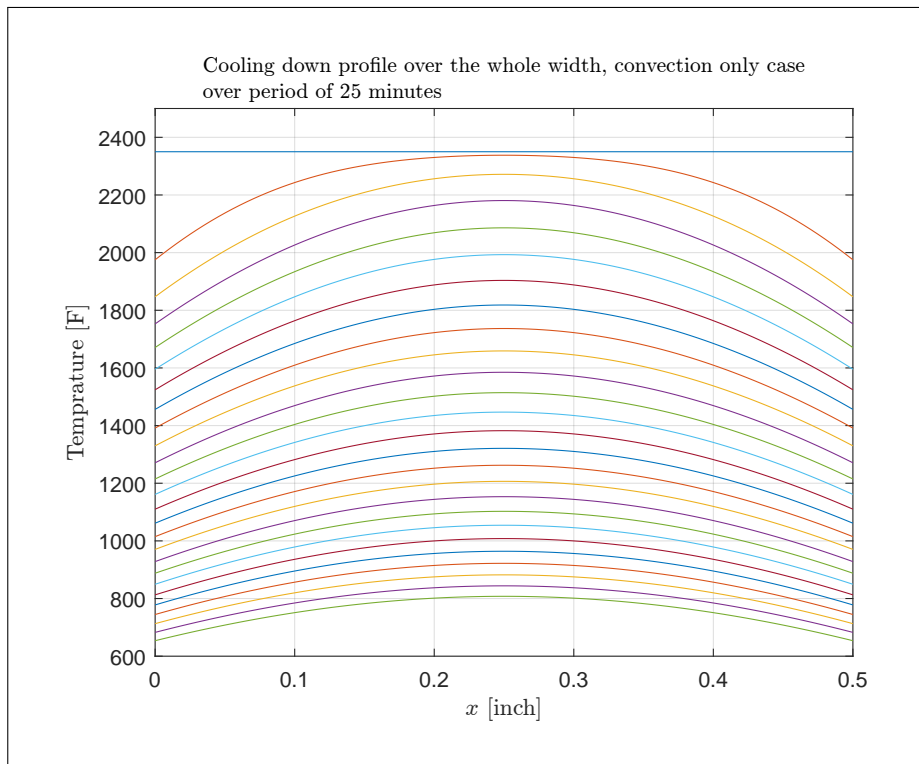


Figure 3: problem 1, first part. Profile across the whole section

Now radiative heat loss was added. It was found that steel will cool faster, by about 85.5 percent of the time it took without radiative heat.

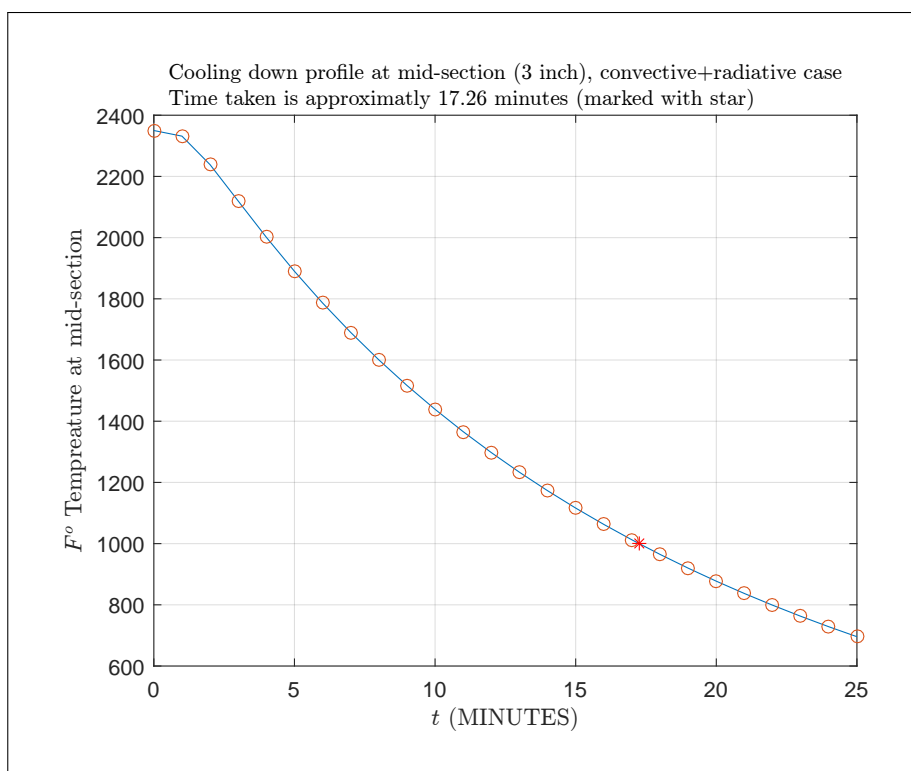


Figure 4: Second part of first problem. Convective and radiative heat loss

A plot is given below which compare both cases on same plot to make it more clear to see the difference.

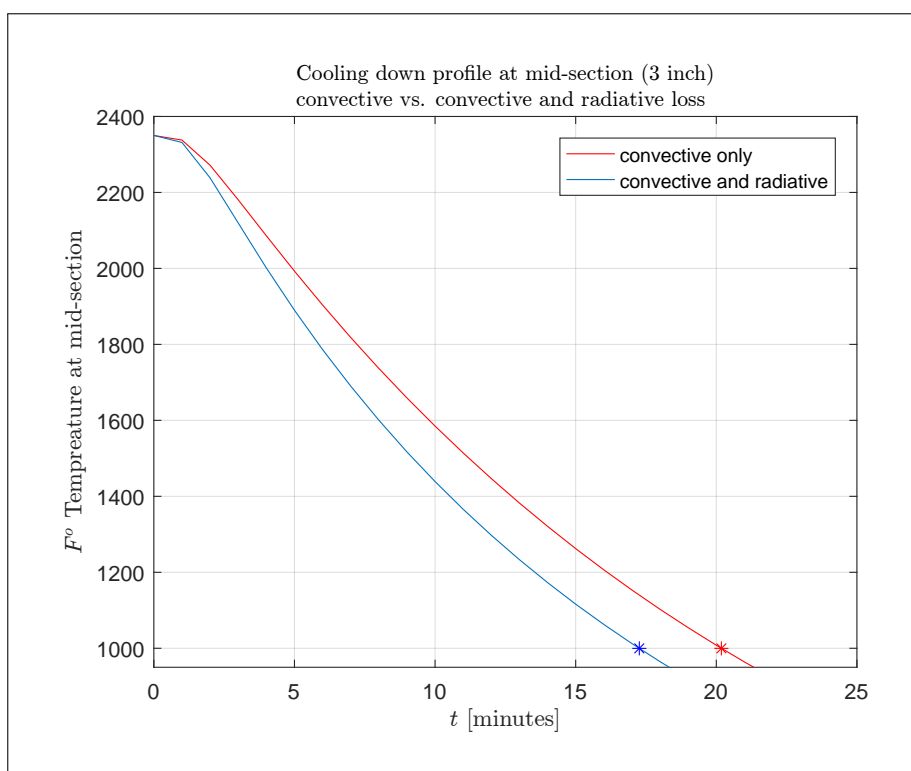


Figure 5: Both parts one and two on same plot, comparing heat loss at mid-section

```

1 function nma_HW_7_problem_1()
2 %solves problem 1, HW 7. EMA 471
3 %adopted from trans_heat_cond_pdepe.m
4 %
5 clear; clc; close all;
6
7 % parameters of interest:
8 % thermal diffusivity is declared as a global variable
9 global g_alpha;
10
11 % thermal conductivity is declared as a global variable
12 global g_k;
13 global g_h; % heat transfer coefficients to ambient air
14 global g_T_inf; % ambient (asymptotic) temperature
15
16 g_k = 25; % thermal conductivity of steel, [Btu/(hr-ft-F)]
17 c_p = 0.12; % specific heat capacity of steel, [Btu/(lbm-F)]
18 rho = 500; % density of steel, [lbm/ft^3]

```

```

19 g_h      = 50;    % convective heat transfer coefficient to air
20                % [Btu/(hr-ft^2-F)]
21 g_T_inf = 60;    % ambient temperature, [F]
22 g_alpha = g_k/(rho*c_p); % thermal diffusivity [ft^2/hr]
23
24 [soln_pde_1,x,t] = process_part_one();
25 [soln_pde_2,~,~] = process_part_two();
26
27 % plots of numerical solution as points:
28 figure;
29 x_idx = round(length(x)/2);
30 plot(t*60,soln_pde_1(:,x_idx),'r');
31 hold on;
32 plot(t*60,soln_pde_2(:,x_idx));
33 title({'Cooling down profile at mid-section (3 inch)', ...
34       'convective vs. convective and radiative loss'},...
35       'Interpreter', 'latex','fontsize',10);
36 xlabel('$t$ [minutes]', 'Interpreter', 'latex');
37 ylabel('$F^o$ Temperature at mid-section','Interpreter', 'latex');
38 %set(gca,'TickLabelInterpreter', 'Latex','fontsize',10);
39 legend('convective only','convective and radiative');
40 grid;
41 ylim([1000-50,2400]);
42 hold on;
43 plot(20.18,1000,'r*');
44 plot(17.26,1000,'b*');
45
46 end
47 %=====
48 function [soln_pde,x,t] = process_part_one()
49 %This part for the convective heat transfer loss only
50
51 x = linspace(0,0.5,100); %units in feet, so this is 6 inch
52 t = 0:1/60:25/60; %units in hrs. So this is up to 25 minutes
53                %this time scale was found to be enough
54                %to see the cooling to 1000 F.
55 m = 0; %for Matlab pdepe since this
56                %is catersian coordinates
57
58 soln_pde = pdepe(m,@pde_trans_heat,@pde_trans_ic,...
59                @pde_trans_bc_part_1,x,t);
60
61 figure;
62 x_idx = round(length(x)/2); %to find mid-point
63
64 %change time to minutes
65 plot(t*60,soln_pde(:,x_idx),t*60,soln_pde(:,x_idx),'o');
66 grid;
67 title({'Cooling down profile at mid-section (3 inch), convection only case',...
68       'Time taken is $20.18$ minutes (marked with star in the plot)'},...
69       'Interpreter', 'latex','fontsize',10);
70
71 xlabel('$t$ (MINUTES)', 'Interpreter', 'latex');
72 ylabel('$F^o$ Temperature at mid-section','Interpreter', 'latex');
73 hold on;
74 plot(20.18,1000,'r*');
75 %set(gca,'TickLabelInterpreter', 'Latex','fontsize',10);
76
77 figure;
78 for i=1:length(t)
79     plot(x,soln_pde(i,:));
80     hold on;
81 end
82 title({'Cooling down profile over the whole width, convection only case',...
83       'over period of 25 minutes'},...
84       'Interpreter', 'latex','fontsize',10);
85 xlabel('$x$ [inch]', 'Interpreter', 'latex');
86 ylabel('Temperature [F]', 'Interpreter', 'latex');
87 %set(gca,'TickLabelInterpreter', 'Latex','fontsize',10);
88 grid;
89 ylim([600,2500]);
90 end
91 %=====
92 function [soln_pde,x,t] = process_part_two()
93 %This part for the convective heat
94 %transfer loss + radiative heat loss
95
96 x = linspace(0,0.5,100); % units feet

```

```

97 t = 0:1/60:25/60;
98 m = 0;
99
100 soln_pde = pdepe(m,@pde_trans_heat,@pde_trans_ic,...
101                 @pde_trans_bc_part_2,x,t);
102
103 figure;
104 x_idx=round(length(x)/2);
105 plot(t*60,soln_pde(:,x_idx),t*60,soln_pde(:,x_idx),'o');
106 grid;
107 title({'Cooling down profile at mid-section (3 inch), convective+radiative case',...
108       'Time taken is approximatly $17.26$ minutes (marked with star)'},...
109       'Interpreter', 'latex','fontsize',10);
110 xlabel('$t$ (MINUTES)', 'Interpreter', 'latex');
111 ylabel('$F^o$ Tempreature at mid-section','Interpreter', 'latex');
112 hold on;
113 plot(17.26,1000,'r*');
114 %set(gca,'TickLabelInterpreter', 'Latex','fontsize',10);
115
116 figure;
117 for i=1:length(t)
118     plot(x,soln_pde(i,:));
119     hold on;
120 end
121 title({'Cooling down profile over the whole width, convective+radiative case',...
122       'over period of 25 minutes'},...
123       'Interpreter', 'latex','fontsize',10);
124 xlabel('$x$ [inch]', 'Interpreter', 'latex');
125 ylabel('Temprature [F]', 'Interpreter', 'latex');
126 %set(gca,'TickLabelInterpreter', 'Latex','fontsize',10);
127 grid;
128 end
129 %=====
130 function [c,f,s] = pde_trans_heat(~,~,~,DTDx)
131 global g_alpha; % thermal diffusivity. global variable
132
133 %THESE COMMENT FROM CLASS CODE
134 % because k is a constant in this problem we pull it out of
135 % the divergence operator and divide both sides by k.
136 % Then our "capacity" for this problem is
137 % rho*c_p/k = 1/alpha. Note that if we had
138 % to leave k inside the operator, the capacity would
139 % just be rho*c_p
140 c = 1/g_alpha;
141
142 % again, because k has been pulled out of the operator,
143 % the flux in this problem is just dT/dx. If it were
144 % left inside, the flux would be k*dT/dx
145 f = DTDx;
146
147 s = 0; % the source term is the power density/thermal conductivity
148 end
149 %=====
150 function [pL,qL,pr,qr] = pde_trans_bc_part_1(~,TL,~,Tr,~)
151 %case one, convective heat loss only
152 global g_h g_k g_T_inf;
153
154 pL = g_h*(TL - g_T_inf);
155 qL = -g_k;
156
157 pr = g_h*(Tr - g_T_inf);
158 qr = g_k;
159 end
160 %=====
161 function [pL,qL,pr,qr] = pde_trans_bc_part_2(~,TL,~,Tr,~)
162 %case where have convective coefficient and radiative
163 global g_h g_k g_T_inf;
164 Stefan_Blottzmann = 0.171*10^(-8);
165 emissivity = 0.8;
166 c0 = 459.67; %convert from F to R
167
168 pL = g_h*(TL - g_T_inf)+emissivity*Stefan_Blottzmann*...
169         ((TL+c0)^4-(g_T_inf+c0)^4);
170 qL = -g_k;
171 pr = g_h*(Tr - g_T_inf)+emissivity*Stefan_Blottzmann*...
172         ((Tr+c0)^4-(g_T_inf+c0)^4);
173 qr = g_k;
174 end

```

```

175 %=====
176 function T_0 = pde_trans_ic(~)
177 T_0 = 2350; %initial temp, [F]
178 end
179 %-----

```

0.2 Problem 2

- (2) (20 pts) A hollow tube 20 cm in length is filled with air containing 2% of ethyl alcohol vapors. At the bottom of the tube ($x = 20$ cm) is a pool of alcohol which evaporates into the stagnant gas above. The concentration at this end should be assumed constant and equal to 10%. At the top of the tube ($x = 0$ cm), the alcohol vapors dissipate to the outside air, so the concentration is essentially zero. Considering only the effects of molecular diffusion, determine the spatial evolution of the alcohol concentration as a function of time. In particular, plot the time-dependent evolution of the concentration at $x = 4, 8, 12$ and 16 cm. How long does it take for the concentration at $x = 16$ cm to reach 7.5%? The governing equation for the concentration takes the form:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

Here D is the diffusion coefficient and has a value of $0.119 \text{ cm}^2/\text{s}$.

Figure 6: problem 2 description

In this problem, the independent variable is the concentration of alcohol, called $c(x, t)$. The initial conditions is that $c(x, 0) = 0.02$ and the boundary conditions is the left side (where $x = 0$) is $c(L, t) = 0$ and on the right side (where $x = 20$ cm), then $c(R, t) = 0.1$.

This is a parabolic pde. It is solved using pdepe. The program `nma_HW7_problem_2.m` solves this and the following plot shows the result. It took

486 seconds

For the concentration at $x = 16$ cm to reach 7.5%. The first plot below show all profiles on same plot, then each one on a separate plot

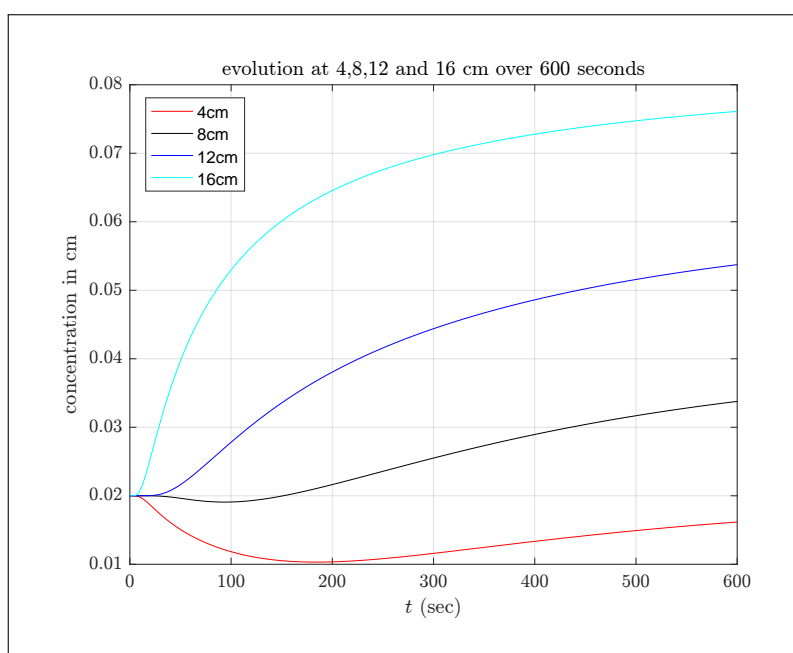


Figure 7: profile of concentrationn at 4,8,12,16 cm combined in one plot

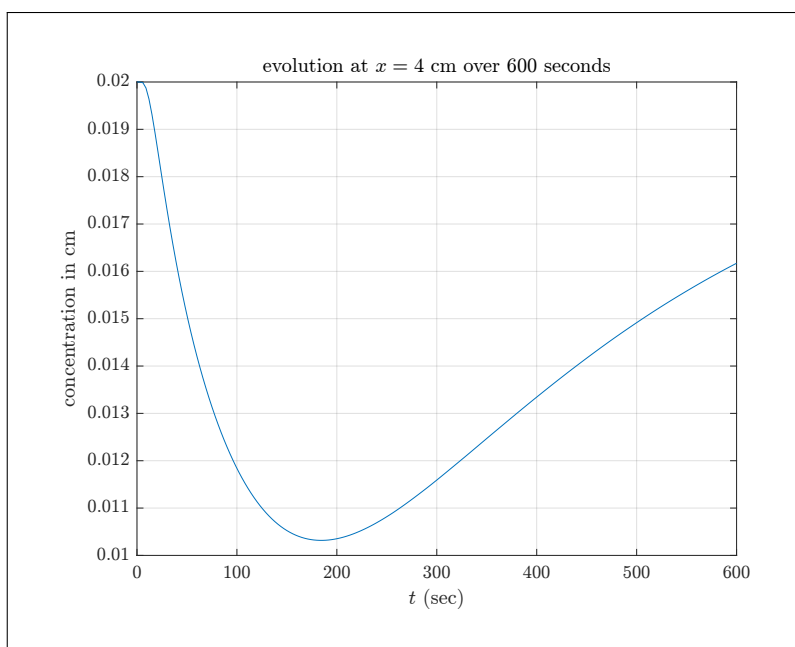


Figure 8: profile of concentrationn at 4 cm

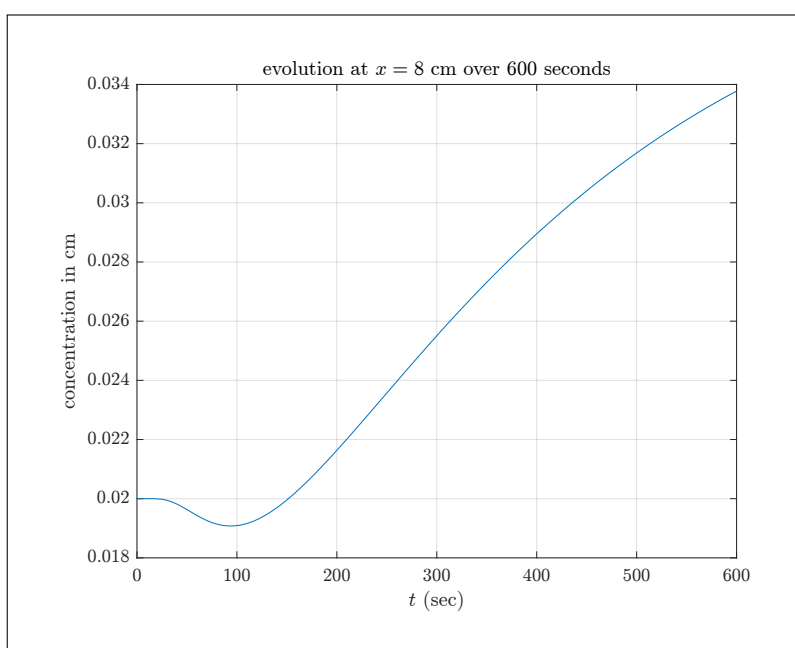


Figure 9: profile of concentrationn at 8 cm

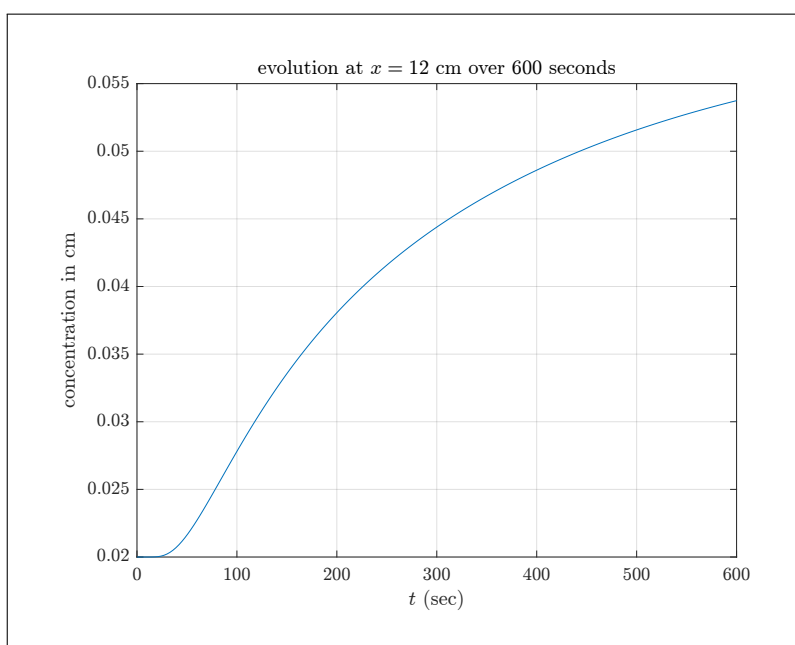


Figure 10: profile of concentrationn at 12 cm

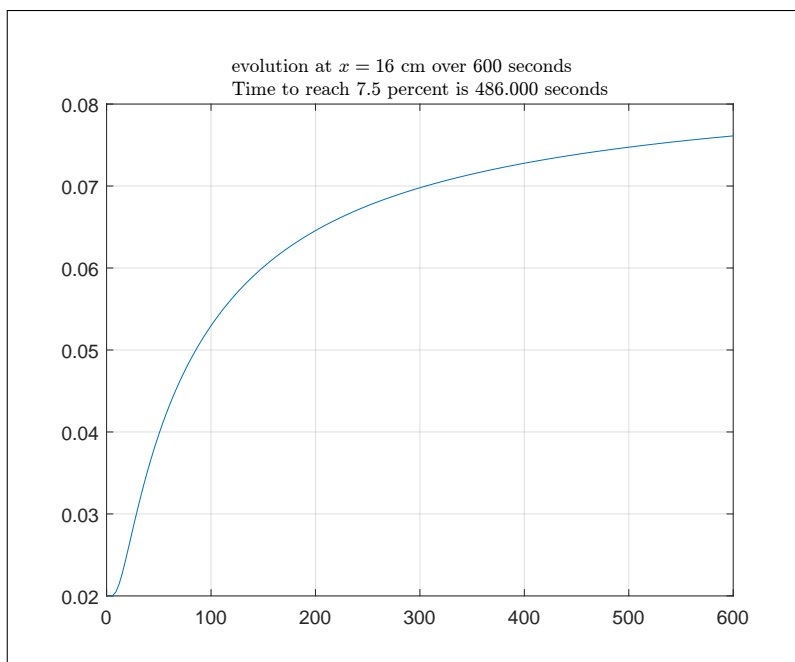


Figure 11: profile of concentrationn at 16 cm

```

1 function nma_HW_7_problem_2()
2 %solves problem 2, HW 7. EMA 471
3 %
4 clear; clc; close all;
5
6 % parameters of interest:
7 global g_D; % diffusion coefficient declared as a global
8 g_D = 0.119; % cm^2/sec
9
10 x = 0:0.1:20;
11 t = linspace(0,600,length(x));
12 m = 0;
13
14 soln_pde = pdepe(m,@pde_trans_heat,@pde_trans_ic, ...
15 @pde_trans_bc,x,t);
16
17 plot_at(4,t,x,soln_pde);
18 plot_at(8,t,x,soln_pde);
19 plot_at(12,t,x,soln_pde);
20 plot_at(16,t,x,soln_pde);
21
22 %plot all on same figure now
23
24 figure;
25 idx = x==4;
26 plot(t,soln_pde(:,idx),'r');
27 hold on;
28 idx = x==8;
29 plot(t,soln_pde(:,idx),'k');
30 idx = x==12;
31 plot(t,soln_pde(:,idx),'b');
32 idx = x==16;
33 plot(t,soln_pde(:,idx),'c');
34 grid;
35 legend('4cm','8cm','12cm','16cm');
36 title(sprintf('evolution at 4,8,12 and 16 cm over %d$ seconds',...
37 t(end)),...
38 'Interpreter','latex','fontsize',10);
39 xlabel('$t$ (sec)', 'Interpreter', 'latex');
40 ylabel('concentration in cm','Interpreter', 'latex');
41 set(gca,'TickLabelInterpreter', 'Latex','fontsize',10);
42 end
43 %=====
44 function plot_at(x0,t,x,soln_pde)
45
46 idx = x==x0;
47 figure;
48 plot(t,soln_pde(:,idx));
49 if x0==16
50 I=find(round(soln_pde(:,idx),3)==0.075);
51 title(sprintf('evolution at $x=%d$ cm over %d$ seconds',...

```

```

52     x0,t(end)),...
53     sprintf('Time to reach $7.5$ percent is $%5.3f$ seconds',...
54             t(I(1))),...
55     },'Interpreter', 'latex','fontsize',10);
56 end
57 title(sprintf('evolution at $x=%d$ cm over $%d$ seconds',...
58             x0,t(end)),...
59             'Interpreter', 'latex','fontsize',10);
60 xlabel('$t$ (sec)', 'Interpreter', 'latex');
61 ylabel('concentration in cm','Interpreter', 'latex');
62 set(gca,'TickLabelInterpreter', 'Latex','fontsize',10);
63 end
64 grid;
65
66
67 end
68 %=====
69 function [c,f,s] = pde_trans_heat(~,~,~,DTDx)
70 global g_D; % diffusion coefficient declared as a global variable
71
72 c = 1/g_D;
73 f = DTDx;
74 s = 0;
75 end
76 %=====
77 function [pL,qL,pr,qr] = pde_trans_bc(~,TL,~,Tr,~)
78 pL = TL;
79 qL = 0;
80
81 pr = Tr - 0.1;
82 qr = 0;
83 end
84 %=====
85 function T_0 = pde_trans_ic(~)
86 T_0 = 0.02; %initial concentration
87 end
88 %-----

```