University Course

EMA 550
Astrodynamics

University of Wisconsin, Madison
Spring 2014

My Class Notes

Nasser M. Abbasi

Spring 2014

Contents

Chapter 1

Introduction

Local contents

Took this course in spring 2014. Part of MSc. in Engineering Mechanics.

Instructor: professor Suzannah Sandrik

Class link moodle internal course site

1.1 Syllabus

EMA 550 Astrodynamics
Spring 2014

Instructor: Dr. Suzannah Sandrik, Department of Engineering Physics
811 Engineering Research Building
sandrik@engr.wisc.edu, (608) 262-0764

Class sessions: TuTh 2:30-3:45 pm. The lecture room may change and we will have at least one class in a computer lab, so watch your email. Exams will be in-class or take-home; no evening exams are expected.

Office hours: After class or by appointment.

Course web site: Moodle, https://courses.moodle.wisc.edu/prod/my/. The course site will have lecture notes, homework assignments, and other material related to the course.

Catalog course description: Coordinate system transformations, central force motion, two body problem, three and *n*-body problem, theory of orbital perturbations, artificial satellites, elementary transfer orbits, and elementary rocket dynamics. Prerequisite: EMA 202 or 221; or Physics 311 or con reg; or cons inst.

What do astrodynamicists/orbital mechanicists do?
Astrodynamicists design and optimize trajectories (paths through space defined by a sequence of rocket burns) to move a spacecraft from an initial orbit to a desired final orbit. They work in teams with other engineers who are responsible for different parts of the mission (propulsion, payload, etc.). Astrodynamicists use equations with simplifying assumptions to estimate the required orbit and software like Systems Tool Kit (STK, formerly Satellite Tool Kit) to refine the trajectory, account for perturbing effects, and create visualizations of the planned mission.

Expectations for the course:
Preparation: Course notes and slides from lectures will be posted to the course web site. You'll get the most from lecture if you read the notes for the day's topic before class .There are some spaces in the notes for you to complete examples based on what we discuss in class.

Homework: You can expect homework assignments on an approximately weekly basis. Working together on weekly homework assignments for EMA 550 is acceptable and encouraged, but each student is expected to work through the problems individually and will be responsible for being able to complete similar problems on exams. Larger projects will be completed in pairs or teams.

Projects: You will be asked to design two trajectories in EMA 550. The first is a trajectory from the Earth to the Moon, and the second is an interplanetary trajectory involving a gravity assist fly-by. The lunar project is best done in pairs and the interplanetary project will have teams of four. You will also be asked to research a current space mission or program and present information about it to the class.

Exams: There will be three exams. All exams are open-notes (open-book) and must be completed individually. Two exams will be held in-class and will not require the use of specialized software. One exam will be take-home and may involve problems needing software like Matlab or EES. Laptops may be used on in-class midterms *for open-note purposes ONLY*. For fairness, the use of Mathcad, Maple, Matlab, EES, etc. on in-class exams is prohibited. Students observed using their laptops for anything other than notes on in-class exams will receive a zero for the exam. Students observed collaborating on exams will receive a zero for the exam.

Course notes: EMA 550 uses course notes prepared by Engineering Physics department professors in place of a published textbook.

Math software: Familiarity with math software (Matlab, Mathcad, EES, etc.) is helpful and will be assumed. Matlab and EES tutors are generally available in Wendt library during walk-in tutoring on Sunday, Monday, Tuesday, and Wednesday nights. See http://studentservices.engr.wisc.edu/classes/tutoring/ for more details.

Dynamics/modeling software: I will use STK for in-class demonstrations. It is available on the CAE server and can be downloaded from agi.com. STK is used by NASA, Boeing, Lockheed, Northrup Grumman, other companies, and private citizens engaged in the pursuit of space applications. As a UW-Madison student, you can take a certification exam in STK for free, if you choose.

Grading policy: The final course grade will be based on weekly homework assignments (10%), a lunar project (10%), an interplanetary project (15%), three exams (20% each), and a space mission/program presentation (5%). You can access your grades during the semester on the course website. The grading scale will be approximately 100-92 A, 92-87 AB, 87-82 B, 82-77 BC, 77-72 C, 72-62 D, and < 62 F.

McBurney accommodations: Please contact the instructor during the first two weeks of class regarding McBurney passport accommodations.

Textbook and references: No required textbook. Lecture notes will be posted on the course web site. Additional useful references on astrodynamics include:

1. John Prussing and Bruce Conway, *Orbital Mechanics*, Oxford Univ. Press, 1993. The orbital mechanics textbook at Purdue University and the University of Illinois Urbana-Champaign.

2. Vladimir A. Chobotov, *Orbital Mechanics*, AIAA Education Series, 3rd ed., 2002. The textbook for EMA 550 several years ago.

3. Howard D. Curtis, *Orbital Mechanics for Engineering Students*, 2nd ed., Elsevier, 2010. Written by a professor at Embry-Riddle Aeronautical University, used there.

4. Jerry Jon Sellers et al, *Understanding Space*, McGraw-Hill Primis Custom Publishing, 2005. A less technical introduction to many space topics, including orbital mechanics, launch and entry, and spacecraft subsystems. Has been used to teach orbital mechanics to practicing engineers at NASA's Johnson Space Center.

5. Richard Battin, *An Introduction to the Mathematics and Methods of Astrodynamics*, Revised ed., AIAA Education Series, 1999. An advanced orbital mechanics reference book for graduate students and professionals.

6. Roger Bate, Donald Mueller, and Jerry White, *Fundamentals of Astrodynamics*, Dover Publications, 1971. A classic.

7. Charles D. Brown, *Spacecraft Mission Design*, AIAA Education Series, 2nd ed., 1998. Brown teaches short courses on orbital mechanics for professionals in the aerospace industry. His book is sort of a cookbook of techniques for approximate analyses, especially patched conics, but weak on the underlying theory.

8. A.E. Roy, *Orbital Motion*, Inst. Of Physics Publishing, 4th ed., 2005. Earlier editions of this text were used for this course by previous instructors. It went out of print for a while until the new edition came out. It emphasizes celestial mechanics, as opposed to astrodynamics, more than most texts.

EMA 550 Astrodynamics
Spring 2014

	Date	Mtg	Topics	Homework
Tu	1/21	1	Introduction and Two-Body Gravitation	
Th	1/23	2	Two-Body Gravitation (Equations of Motion)	
Tu	1/28	3	Two-Body Gravitation (Elliptical Orbits)	
Th	1/30	4	Two-Body Gravitation (Elliptical Orbits, continued)	HW 1 Due
Tu	2/4	5	Two-Body Gravitation (Parabolic and Hyperbolic Orbits)	
Th	2/6	6	Orbit Elements, Classical-to-Cartesian Conversion	HW 2 Due
Tu	2/11	7	Cartesian-to-Classical Conversion, Orbit Usage	
Th	2/13	8	Orbit Maneuvers (In-Plane Hohmann and Bi-Elliptic)	HW 3 Due
Tu	2/18	9	Orbit Maneuvers (In-Plane Semi-Tan.) and Interplanetary Trajectories (Sph. of Grav. and Influence)	
Th	2/20	10	Interplanetary Trajectories (Patched Conics)	HW 4 Due
Tu	2/25	11	Review	
Th	2/27	12	**IN-CLASS MIDTERM** (through Orbit Maneuvers)	
Tu	3/4	13	Interplanetary Trajectories (Gravity Assist)	
Th	3/6	14	Orbital Position (Walking Orbits, 2D Rendezvous)	HW 5 Due, Lunar project assigned
Tu	3/11	15	STK Tutorial - Computer lab TBA	Lunar project assigned
Th	3/13	16	Orbital Position (Semi-Tangential Rendezvous)	
Tu	3/18		**SPRING BREAK**	
Th	3/20			
Tu	3/25	17	Orbital Position (Lambert's Theorem)	HW 6 Due
Th	3/27	18	Orbit Maneuvers (Out-Of-Plane Maneuvers)	
Tu	4/1	19	Rocket Equation, Fixed Impulses, Launch Windows	HW 7 Due
Th	4/3	20	Orbital Position (3D Rendezvous)	Lunar project due, Interplanetary project assigned
Tu	4/8	21	Relative Motion (Terminal Rendezvous, Fly-Around)	HW 8 Due
Th	4/10	22	Relative Motion (Ejected Particles)	
Tu	4/15	23	Orbit Perturbations	HW 9 Due
Th	4/17	24	Orbit Perturbations, continued	**Take-home exam assigned**
Tu	4/22	25	Three-Body Gravitation (Lagrange Points)	**Take-home exam due**
Th	4/24	26	Low/Continuous Thrust	
Tu	4/29	27	TBA	HW 10 Due
Th	5/1	28	Presentations	
Tu	5/6	29	Presentations	
Th	5/8	30	Presentations	Interplanetary project due
Su	5/11		**FINAL EXAM (10:05 AM - 12:05 PM)**	

Planetary Constants

Earth
Mass = 5.974 x 10^{24} kg
Equatorial radius = 6378 km
$\mu_{Earth} = Gm_{Earth}$ = 3.986 x 10^5 km^3/s^2
Mean distance from the Sun = 1 AU = 1.495978 x 10^8 km

Sun
Mass = 1.989 x 10^{30} kg
Mean radius = 695,990 km
$\mu_{Sun} = Gm_{Sun}$ = 1.327 x 10^{11} km^3/s^2

	Mean distance from the Sun (AU)	**Orbit eccentricity**	**Orbit inclination to the ecliptic plane (deg)**	**Mass (units of M_{Earth})**	**Equatorial radius (km)**	**Sphere of influence radius (km)**
Mercury	0.3871	0.2056	7.005	0.0553	2440	1.13 x 10^5
Venus	0.7233	0.006777	3.395	0.8149	6052	6.17 x 10^5
Earth	1.000	0.01671	0.000	1.000	6378	9.24 x 10^5
Mars	1.524	0.09339	1.850	0.1074	3396	5.74 x 10^5
Jupiter	5.203	0.04839	1.304	317.9	71,492	4.83 x 10^7
Saturn	9.537	0.05386	2.486	95.18	60,268	3.47 x 10^7
Uranus	19.19	0.04726	0.7726	14.53	25,559	5.19 x 10^7
Neptune	30.07	0.008590	1.770	17.14	24,764	8.67 x 10^7
Pluto	39.48	0.2488	17.14	0.0022	1195	3.17 x 10^7

Moon
Mass = 7.3483 x 10^{22} kg
Mean planetary radius = 1738 km
$\mu_{Moon} = Gm_{Moon}$ = 4902.8 km^3/s^2
Mean distance from the Earth = 384,400 km
Orbit eccentricity = 0.05490
Orbit inclination to ecliptic = 5.15°
Orbit inclination to the Earth's equatorial plane ranges from 18° to 29°
Sphere of influence radius: 6.61 x 10^4 km

Universal Constant of Gravitation
G = 6.674 x 10^{-11} $m^3/(kg\ s^2)$

1.2 STK tutorial emailed to class

STK software can be downloaded for free.

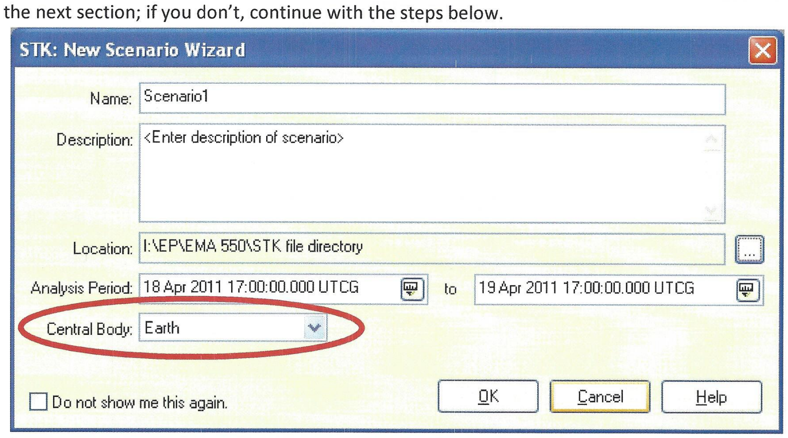
EMA 550 Astrodynamics STK Tutorial
University of Wisconsin-Madison
Creating Interplanetary Ellipse and Gravity Assist Flyby Figures

Opening STK

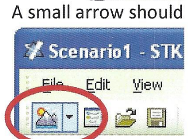
1) Log in with your CAE login.
2) Open STK from Start Menu → All Programs → Course Software → M-S → STK 9.2

Creating a Sun-Centered Scenario

1) STK will ask you where you would like your files to be saved. Choose a convenient location and click OK.
2) When STK opens, click the "Create a New Scenario" button.
3) By default, STK is configured to open with an Earth-centered scenario, but we would like to make a heliocentric ellipse. Look for a pull-down at the bottom-left of the New Scenario Wizard window labeled "Central Body:". You might not have this pull-down. If you do have the pull-down, skip to the next section; if you don't, continue with the steps below.



4) If you do not see the Central Body pull-down, click OK to create an Earth-centered scenario. You will not be keeping this scenario, so you do not need to worry about a name or other information.
5) When the scenario opens, close the Insert STK Objects dialog box that opens automatically.
6) Go to the View menu and click on Planetary Options.
7) A small arrow should appear next to the New Scenario icon in the toolbar.



8) Click on the arrow and choose "Sun." The New Scenario Wizard will open and allow you to create a Sun-centered scenario. Continue to the next section.

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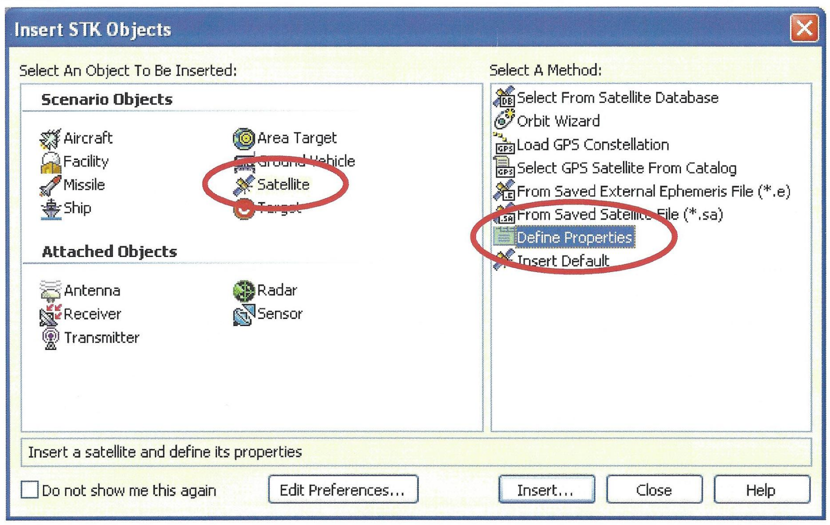
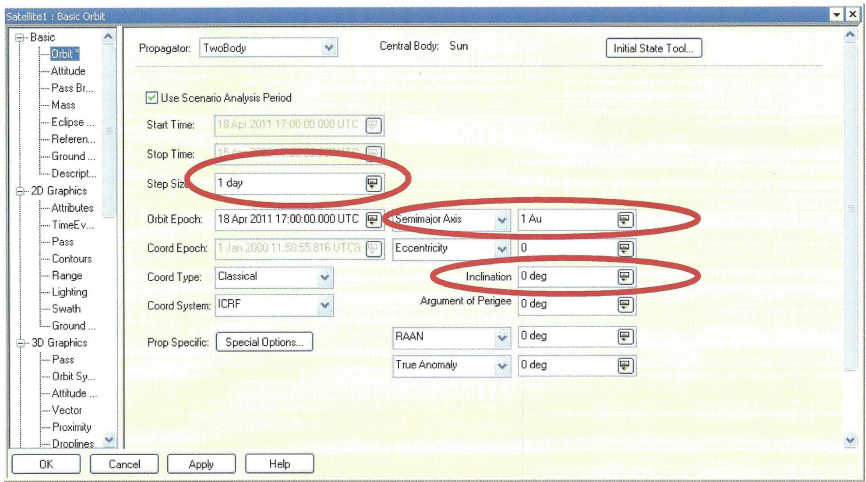
Sun-Centered Scenario

1) In the New Scenario Wizard, make sure that the Central Body pull-down is set to Sun.
2) We are going to create a window that shows us the orbits of the Earth and Jupiter. Since STK only shows the portions of orbits in the scenario time-frame, we will need a much longer time period than the one-day default analysis period. In the second box for the Analysis Period, the end time, type "+12 years" (without the quotation marks).
3) Give your Scenario a name and click OK. If you had to open an Earth-centered scenario to get to this point, STK will ask if you want to save that scenario. You do not need to save it.
4) STK will show graphics windows for the Sun and launch the Insert STK Objects dialog box for your sun-centered scenario. We are going to insert planets as if they are satellites, so make sure Satellite is selected (it is selected by default) and click "Define Properties" on the right, then the Insert… button.



5) Change the step size from the default value of 60 sec to 1 day. Use the measuring tool button on the right edge of the Step Size box to change your time unit. (Figure below)
6) Change the Semimajor Axis from its default value to 1 Au. Use the measuring tool button on the right edge of the Semimajor Axis box to change your distance unit. (Figure below)
7) Change the Inclination from its default value of 45 degrees to 0 degrees. (Note: this is not actually the ecliptic plane, which is oriented ~7° from the Sun's equator, but it works for our model.) (Figure below)



8) Click the OK button.
9) You will be returned to the Insert STK Objects window. Click the Insert… button to insert a satellite to represent Jupiter.
10) Change the step size to 1 day again. Change the Semimajor Axis to 5.203 Au and the Inclination to 0 degrees. Click OK.
11) On the Insert STK Objects dialog box, click the Insert… button one more time to insert a satellite that will have an orbit that connects Earth's orbit and Jupiter's orbit.
12) Let's say that the orbit you found for your interplanetary mission has a semimajor axis of 4 Au and an eccentricity of 0.8. In the satellite properties, adjust the step size (1 day), the Semimajor Axis (4 Au), the Eccentricity (0.8), and the Inclination (0 deg).
13) Click OK to return to the STK Objects window, then Close.
14) Close the 2D Graphics window if you have one open; you will not need it.
15) Maximize the 3D Graphics window.
16) The left-most window pane should be called the Object Browser and should list three satellites. Click on Satellite1 twice, with a bit of time between clicks (not a double-click) so that the name is editable. Rename Satellite 1 "Earth."
17) In the same manner, rename Satellite2 "Jupiter" and Satellite3 "Satellite" or whatever name you would like to give your project satellite.
18) Save your scenario somewhere that you can find it again. I recommend creating a folder for your scenario, because it will have multiple files.

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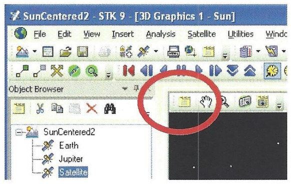
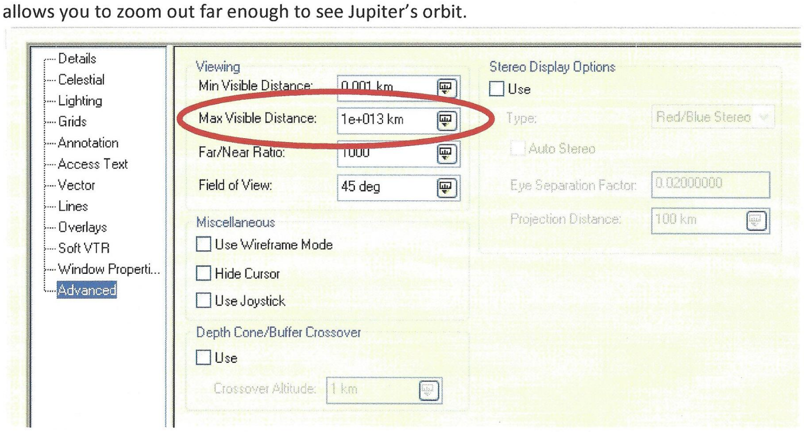
Adjusting the View

1) Once you are in the 3D Graphics window, click the Properties button just above the Graphics window to open the properties for the 3D graphics.



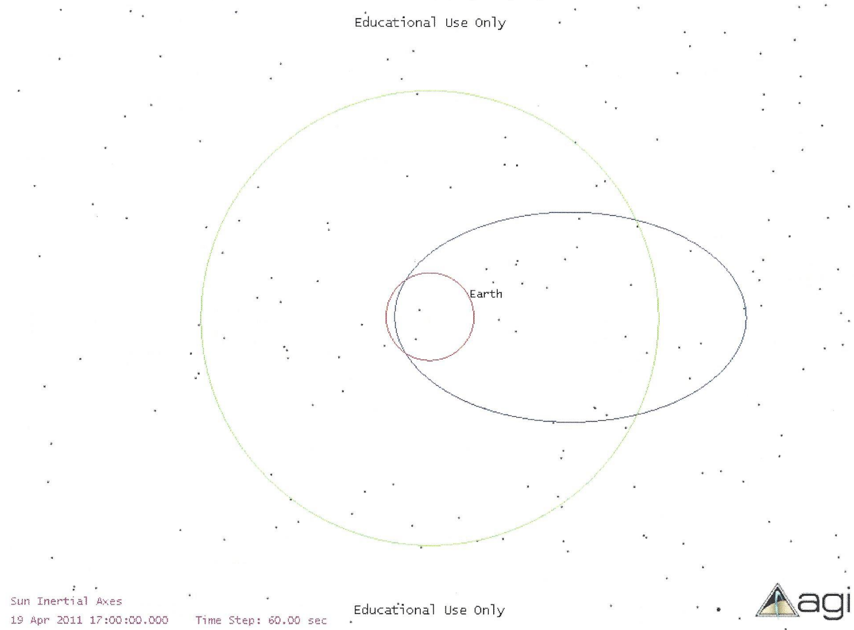
2) In the left frame of the Properties window, choose Advanced.
3) In the Viewing area, change the Max Visible Distance to the maximum STK allows, 1e+027 km. This allows you to zoom out far enough to see Jupiter's orbit.



4) Click OK.
5) In the 3D Graphics window, use the left button of your mouse to click and drag to change the orbit view from looking at the edge of the orbits to looking down on the orbit plane.
6) Click the right button of your mouse and drag to zoom in and out.
7) Zoom out until you can see the Earth's orbit, Jupiter's orbit, and your satellite's orbit. You might notice at this point that you have an Earth labeled. This is the built-in Earth in STK. Leaving the label is less problematic than trying to remove it, so just let it mark which orbit is the Earth's orbit.
8) Voila! At this point, you should have two circles and an ellipse connecting them. Clicking the left mouse button and dragging in the window allows you to spin the view around to whatever orientation you like. When you are satisfied, you can copy the 3D graphics window with Ctrl-C or Edit → Copy. You can paste this figure into a document or presentation.
9) Note: for a printed report, you might want to paste the figure into a graphics editor that allows you to make a negative of the image (even Paint will do this, but newer versions of Microsoft Office do not have the option to make a negative of an image). The negative of the image will save your

printer from using all of its toner printing a black background. You can also use the Recolor function in Microsoft Office to set black as transparent for a similar effect.

10) Your finished (negative) figure should look something like this (if you chose to spin the view around to make the semimajor axis horizontal, which is entirely up to you):



11) Note: if you want to change the orbit colors, double-click on the satellite in the Object Browser to bring up its Properties window. Find the 2D Graphics heading (3D graphics are inherited from the 2D graphics properties) and the Attributes subheading. Change the line color, line style, and line width to whatever you prefer.

Creating a Flyby Figure (2)

HW5

1) Let's say that you to show a flyby of a satellite in Jupiter's frame of reference. In HW6, we calculated flybys of Jupiter with an incoming Jupiter-centric speed of 3 km/s. Let's model a flyby with v_∞ = 3 km/s that just grazes the surface of Jupiter.
2) After saving your Sun-centered scenario, click the arrow next to the New Scenario icon in the toolbar and choose Jupiter as the central body.

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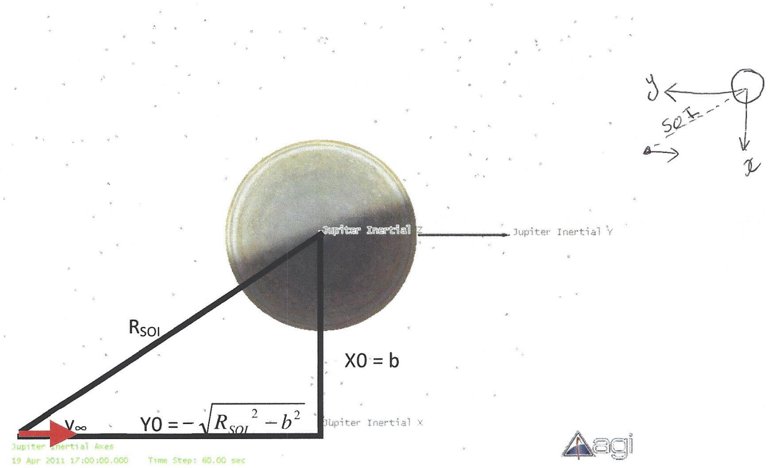
3) In the New Scenario Wizard, name the scenario and choose an Analysis Period that will show the flyby. For the close flyby described above we will need a time period of 233 days (enter "+233 days" without quotation marks in the Analysis Period end time). For other trajectories, you can calculate the time that you need from these steps:
   a. Find *a* for the hyperbolic flyby from $r_{bo} = a_{hyp}(e_{hyp} - 1)$.
   b. Find *F* at the sphere of influence from $r_{SOI} = a_{hyp}(e_{hyp}\cosh F_{SOI} - 1)$.
   c. Find the time from entering the sphere of influence to periapse from

$$\Delta t = \sqrt{\frac{a_{hyp}^{3}}{\mu_{planet}}}(e_{hyp}\sinh F_{SOI} - F_{SOI}).$$

   d. Double the time to reach periapse to get the time from entering the SOI to leaving it again.
4) In the Insert STK Objects window, select Satellite, Define Properties, and Insert… as you did before.
5) This time, change the Coord Type from Classical to Cartesian. (Figure on following page)
6) The default Cartesian coordinates are inertial coordinates centered at Jupiter. The Z axis is Jupiter's spin axis, and the X and Y axes are in Jupiter's equatorial plane. Even though Jupiter's spin axis is inclined to the ecliptic plane, we will model the hyperbolic trajectory as in Jupiter's equatorial plane, as we are just looking to show the shape of the flyby. The position and velocity coordinates to enter on this screen (for our purposes) are those of the satellite as it enters Jupiter's SOI.



   a. Allow the X coordinate to equal the impact parameter, b. Let's model a flyby with b = 1.42e6 km.
   b. Set the Y coordinate equal to $-\sqrt{R_{SOI}^{2} - b^2}$ (-4.83e7 km in our example)
   c. Zero the Z position and velocity.

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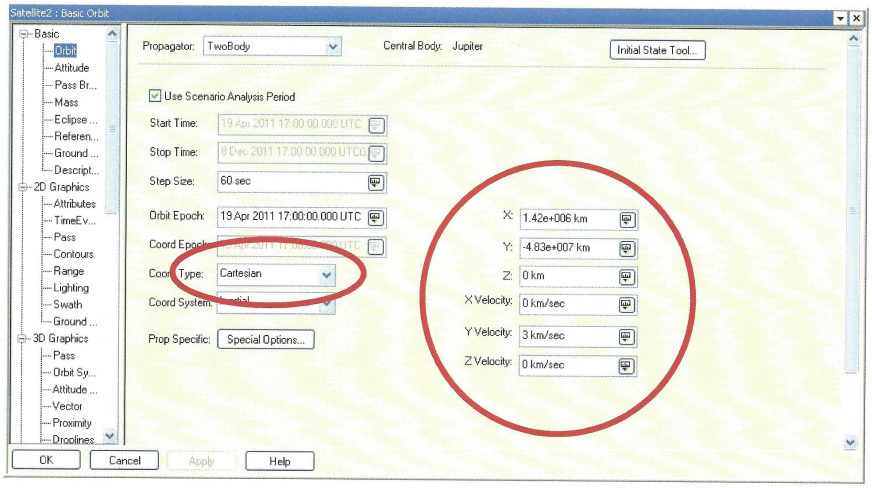
d. Zero the X velocity.

e. Set the Y velocity equal to v_∞ (3 km/s in our example). The settings should look like those in the figure below.



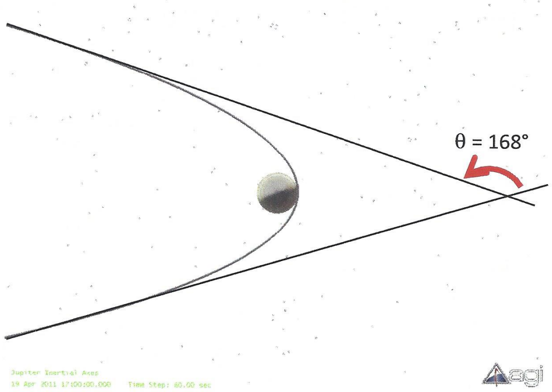
7) Click Apply and OK.

8) Close the Insert STK Objects window.

9) Close the 2D Graphics window and maximize the 3D Graphics window.

10) In the 3D Graphics window, click and drag with the left mouse button to change the view, and the right mouse button to zoom in and out.

11) Voila! You should now have a hyperbolic flyby image that you can copy (Ctrl-C or Edit → Copy), paste, and annotate as desired. (Asymptote lines and turning angle added below as an example.)



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Chapter 2

presentation project and Final project

Local contents

2.1 presentation project. Indian PSLV-C25 Mars orbiter mission

2.1.1 Description of task

Space Mission Research Project
EMA/ASTRO 550: Astrodynamics, Spring 2014

Research a current space mission or space-related topic and deliver a one-page handout, a description of the project's orbital mechanics, a list of resources, and an in-class presentation.

1. One-page handout

Create a one-page handout with the key pieces of information for your topic, presented in an easy-to-read, visually appealing style. Bullet-points are likely. Headings are recommended. It may include a photo (with proper attribution unless you take the photo yourself). Samples are on the course website. The types of questions your handout should answer include (but are not limited to) the following:

> Programs/directives (e.g. Augustine Commission, COTS): Who was involved in the decision-making process? Why was the group convened? What were the main findings or decisions? When were the decisions made? When will they take effect? When are spacecraft that arise as a result of them anticipated to be complete? How has the aerospace industry been affected by the decisions or findings?
>
> Past, current, and future spacecraft: Who (people, companies) was involved in the development? What is the goal of the project? What is the timeline of the project (start dates, completion dates, launch dates, arrival dates, etc.)? Where were the spacecraft built and launched? Where are they going?

2. Orbital mechanics

Describe the orbital mechanics of the project. This will likely be a single paragraph, about half a page. Include information like the following:

> Programs/directives: Which areas of space are affected by these decisions? What vehicles or programs have arisen as a result? What orbits do these vehicles use?
>
> Spacecraft: Which orbits do the satellites use? Are they launched directly to the target orbit or are they launched to a parking orbit? How do they transfer to their destination orbit? If relevant, find a picture of the trajectory and describe the transfer.

3. Annotated references

List the five best non-Wikipedia sources that discuss your program or mission. For each, give the citation information so that an interested reader could find that source for him or herself. Also provide a few sentences of description regarding the information that each source provides. The goal here is to really provide your space-loving classmates with genuinely helpful information. Wikipedia doesn't count because your classmates can find that easily enough without your help. What else is out there for them? (Note: you may list a Wikipedia page as a 6^{th} source if it is particularly good or has helpful graphics that

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2.1.2 Sample of projects to select from

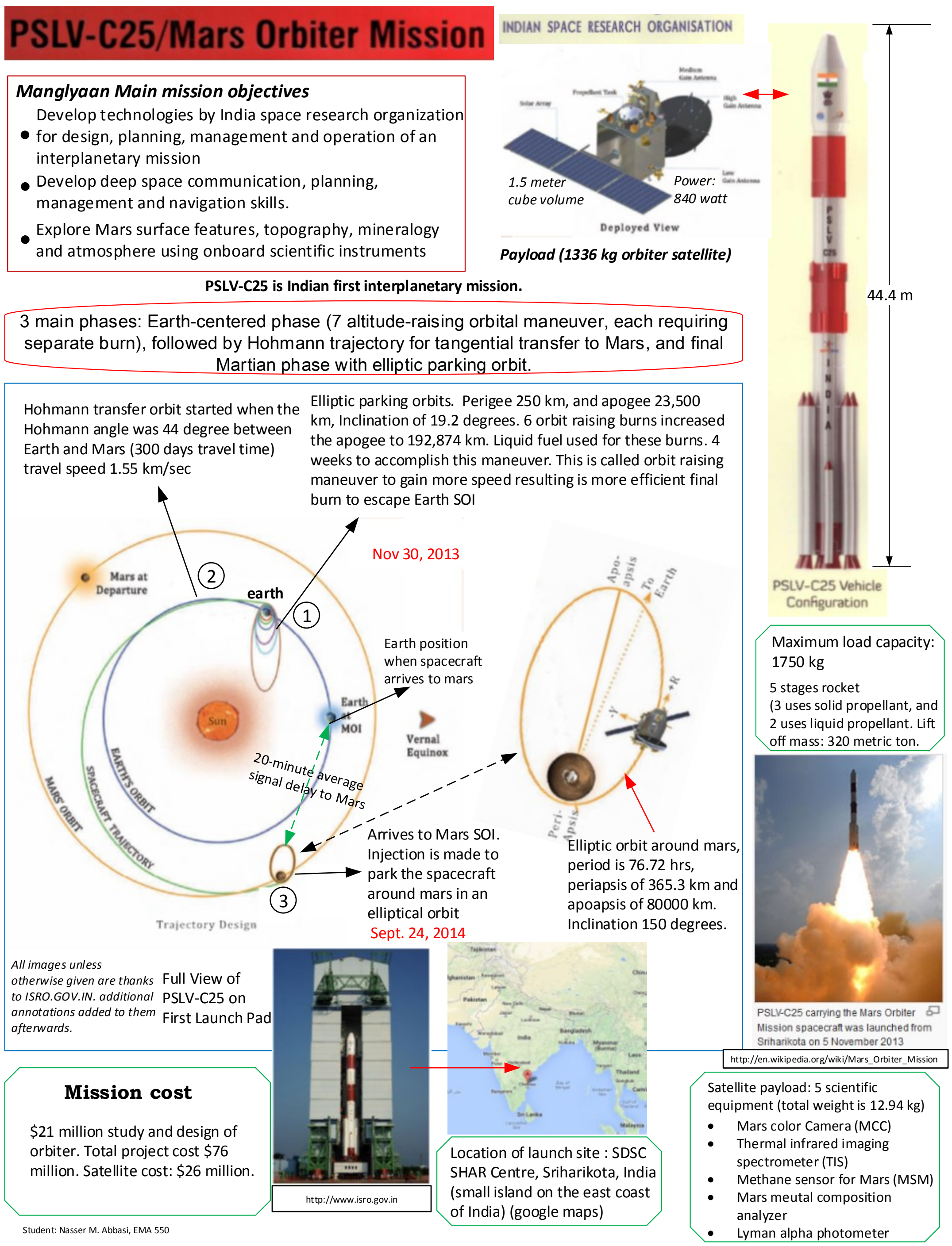
EMA 550 Project Sign Up Sheet 2014

EMA/Astronomy 550 - Astrodynamics

Space Mission Research Project Sign Up Sheet -- FEEL FREE TO ADD A TOPIC!!!

Projects/programs/vehicles	Name	Name	Pres. Date (May 1, 6, or 8)
Dawn spacecraft			
SpaceX Dragon Capsule			6/5/2014
Commercial Resupply Services (CRS)			
Orbital Sciences Cygnus spacecraft			5/6/2014
New Horizons			5/1/2014
Mercury Messenger			
Globalstar Constellation			
SpaceX Red Dragon mission			5/6/2014
Chinese Tiangong 1 spacecraft			
Van Allen Probes			5/1/2014
X-37B Space Plane			
Lunar Atmosphere and Dust Environment Explorer			
MAVEN Mars mission			
Indian Mars Orbiter Mission			
Rosetta comet mission			5/6/2014
Japanese Akatsuki mission to Venus			
Juno mission to Jupiter			
Reaction Engines Skylon Spaceplane/SABRE Engine			5/1/2014

2.1.3 my presentation



References

[1] http://www.isro.org/mars/home.aspx This is the official web site for the Indian Mars mission. It is part of the ISRO web site (below) and contains all technical material about the mission.

[2] http://www.isro.gov.in/

The official website of the Indian Space Research Organization where most of the material were obtained including the images in the first page. ISRO is equivalent to NASA Organization in the US.

[3] http://www.isro.gov.in/pslv-c25/pdf/pslv-c25-brochure.pdf and http://www.isro.gov.in/pslv-c25/pdf/pslv-c25.pdf

These two PDF documents contain technical information about the Earth to Mars orbit and about the launch rocket used (PSLV-C25) and description of the satellite and its instrumentation Both are published by the Indian Space Research Organization

[4] http://www.spaceflight101.com/mars-orbiter-mission.html

This article contains more information about the actual scientific experiments to be performed by PSLV-C25 about about the instrumentation carried aboard the satellite and information about the orbital mechanics part.

[5] http://www.space.com/23802-india-mars-probe-red-planet-journey.html This article on space.com gives a general overview description of the mission, giving reasons for using PSLV as launch instead of using GLSV (Geosynchronous Satellite Launch Vehicle) which encountered few problems in earlier missions.

Orbital mechanics highlights

The transfer trajectory from Earth to Mars was a classical Hohmann transfer. The spacecraft left Earth tangentially from the perigee of the final parking orbit it had and will arrive tangentially at the apogee of the Hohmann ellipse. Rendezvous was accomplished by waiting the the required Hohmann angle to occur between the Earth and Mars before initiating the Hohmann transfer. The Hohmann angle can be found as follows. Let $r_a = 1\text{AU}$ be the distance of Earth from Sun, and $r_b = 1.524\text{AU}$ the distance from Mars to Sun, then the Hohmann angle is

$$\theta_H = \pi \left(1 - \left(\frac{r_a + r_b}{2 r_b} \right)^{\frac{3}{3}} \right)$$

Substituting numerical values results in $\theta_H = 44.36^\circ$. On November 30,2013 when the initial rocket was launched, the angular longitudes on the ecliptic plane of Earth and Mars were (from JPL) $\theta_{earth} = 66.7^\circ$ and $\theta_{mars} = 140.8^\circ$

Small simulation showing the Hohmann transfer to Mars will now be given. What was more interesting is the initial maneuver around Earth before starting the Hohmann transfer.

The spacecraft started in an elliptical parking orbit with perigee of 250 km and apogee of 23500 km. Next, and over a period of 4 weeks, 6 separate burns, all using its liquid fuel engine, were made at the perigee to increases the semi-major of the parking ellipse all the way to 192000 km. This method is called orbit raising maneuver When the spacecraft was in the final and largest elliptical orbit, it initiated the final burn to escape the Earth SOI from the perigee in order to enter the heliocentric Hohmann transfer ellipse. All elliptical orbits shared the same perigee.

Orbit raising maneuvers allows the spacecraft to gradually gain speed resulting in smaller final burn to escape the earth using its solid rocket engine. All burns done to raise the orbit are done when the probe is at the perigee. From an article http://www.spacenews.com/article/launch-report/38111indian-mars-probes-orbit-raising-maneuver-falls-short it says that by the

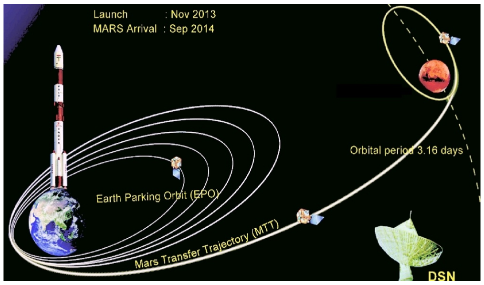


Figure 1: Showing the gradual enlargement of the elliptical parking orbits over period of 4 weeks. Image due to `http://www.spaceflight101.com/mars-orbiter-mission.html`

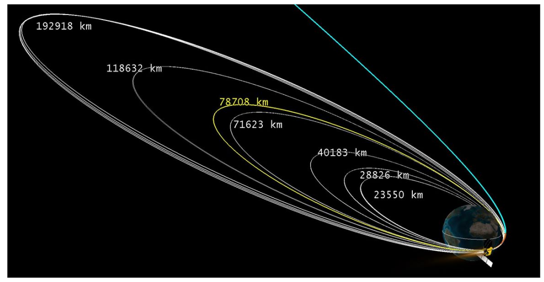


Figure 2: Showing the size of each ellipse during the initial parking maneuver used to gain speed. Image due to `https://www.facebook.com/isromom`

end of the sixth and final orbit raising maneuver, the probe would have the required escape velocity when it arrived back at the perigee of the final ellipse, and that no additional ΔV was needed to escape Earth.

The reason given in the literature about this initial maneuvers, is that it reduced the final burn needed to escape Earth, since the spacecraft will have much higher speed at the perigee in the final parking orbit due to its much larger semi-major axes.

The launch rocket (PSLV-C25) is a five stages rocket. This diagram shows a break down of the sequence of the rocket launch stages.

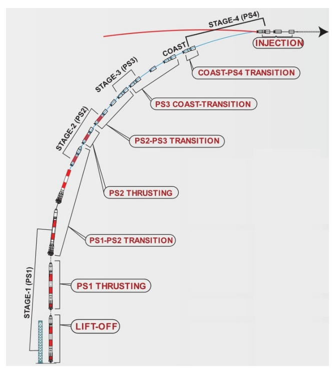


Figure 3: Ascent Profile of PSLV-25 showin all rocket stages. Image due to `http://www.spaceflight101.com/uploads/6/4/0/6/6406961/4674618_orig.jpg`

Some facts about the PSLV-C25 fuel From `http://www.spaceflightnow.com/pslv/c25/131104preview/`

1. "Two-thirds of the orbiter's mass at the time of launch is propellant."
2. "The launcher's liquid-fueled fourth stage will coast for 25 minutes before igniting for the mission's final burn."
3. "Almost all of the mission's 390 liters, or 103 gallons, of liquid fuel will be consumed to accelerate the spacecraft out of Earth orbit and to slow its velocity for capture into orbit around Mars."

2

2.1.4 Power points

Indian Mars Orbiter Mission

India's first interplanetary mission

http://www.isro.gov.in

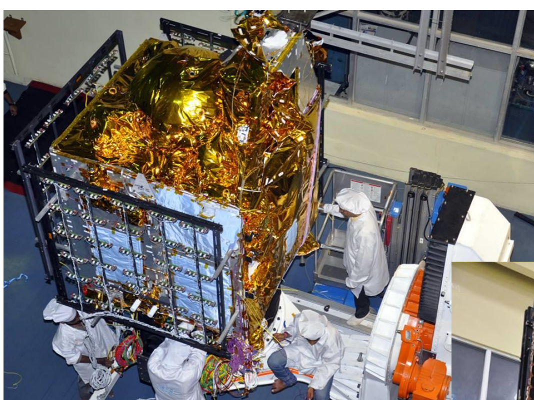
Location of launch site : SDSC SHAR Centre, Sriharikota, India (small island on the east coast of India) (google maps)

Mission Objectives

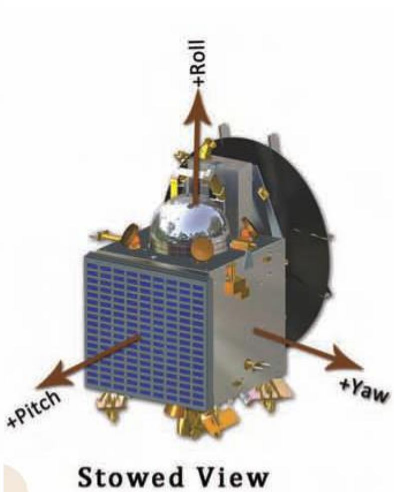
- Develop technologies by India space research organization for design, planning, management and operation of an interplanetary mission
- Develop deep space communication, planning, management and navigation skills.
- Explore Mars surface features, topography, mineralogy and atmosphere using onboard scientific instruments

Deployed View of Orbiter Satellite

Satellite being packaged to move to rocket payload

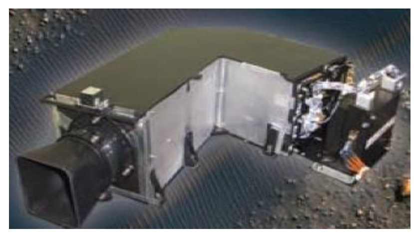


Orbiter stowed vs. deployed



Satellite 5 main scientific equipment

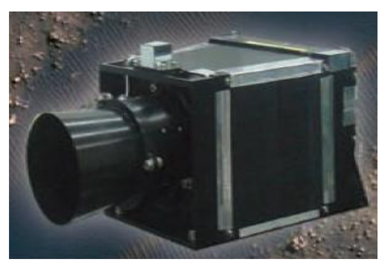
Layman alpha photometer



Exospheric neutral composition analyser

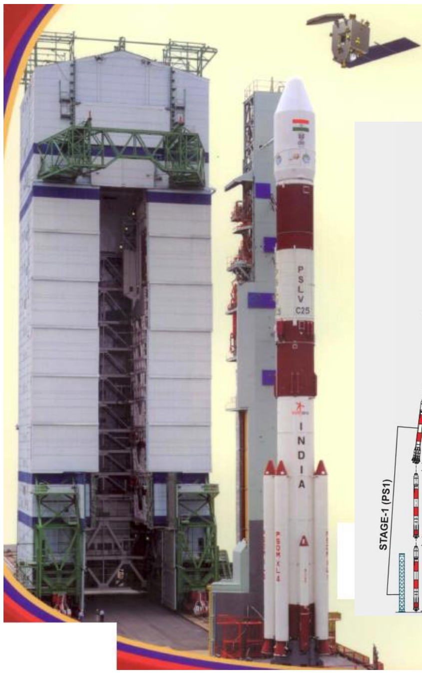
Methane sensor

Exospheric neutral composition analyser

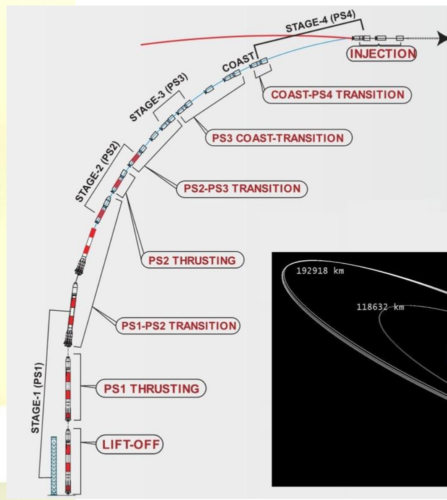


MCC color camera

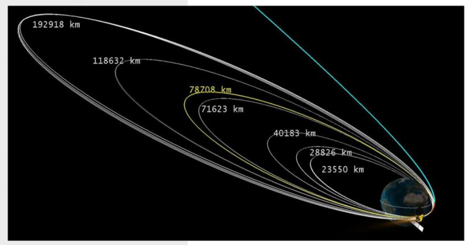
PSLV-C25 Launch rocket



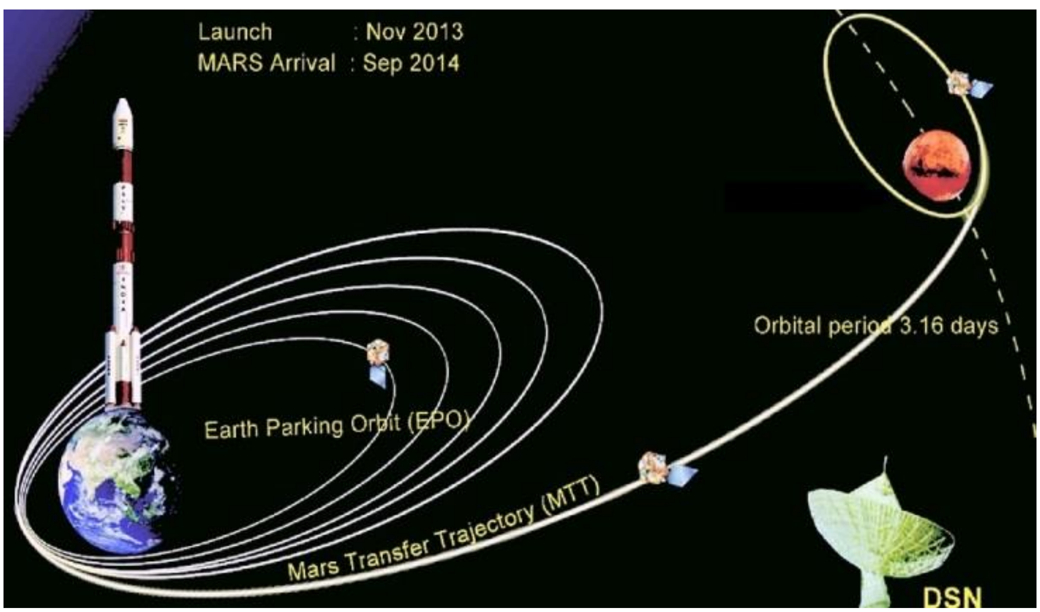
HEIGHT 44.4 METER
LIFT OFF MASS: 320 TON
5 STAGES (PAYLOAD AT TOP)



Once launched, orbiter spends 2 months in orbit raising maneuver around the earth to gain speed

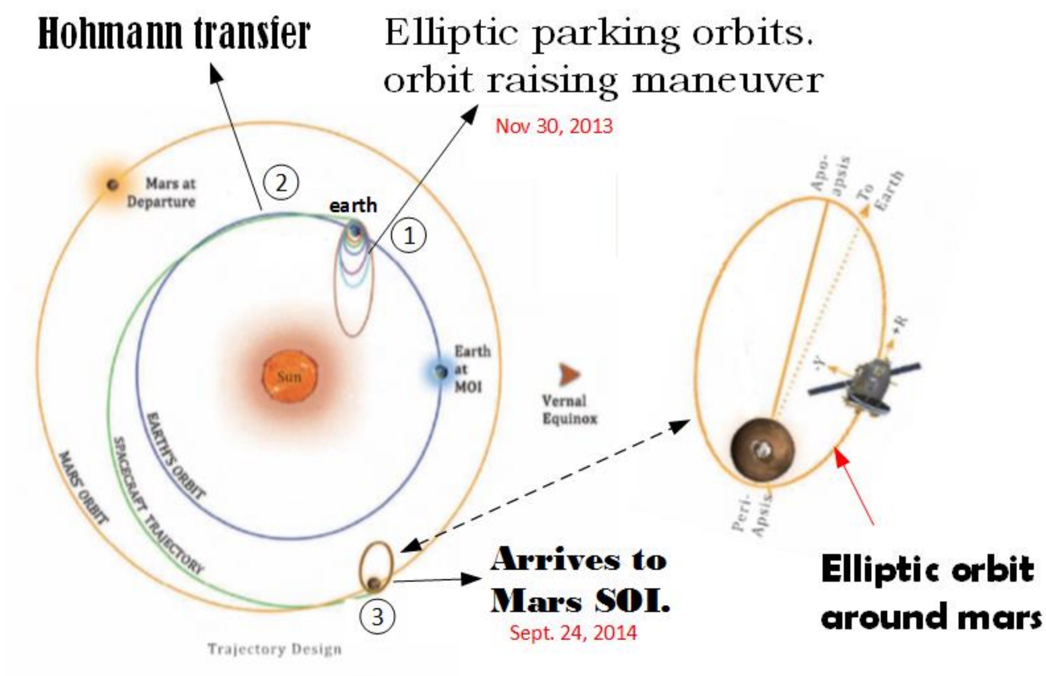


ANOTHER VIEW OF ORBIT RAISING MANEUVER



7 elliptical orbits, lasting 2 months, each requiring burn at perigee, designed to gain speed and reduce cost of fuel needed for final escape from Earth to Mars

PSLV-C25 Mars Trajectory Outline



2.2 Final project. Earth to Neptune via Gravity assist flyby Jupiter

2.2.1 project description

EMA 550 Interplanetary Project
Spring 2014

Calculate, model, and present an interplanetary trajectory from the Earth to Neptune via a gravity assist flyby of Jupiter.

General Guidelines

1. The work is to be done in teams of four, with the various tasks delegated to the team members. Submit one report for the team. If you have difficulty with a team member, attempt to resolve the issue. If a problem persists, refer the issue to me, along with what you have done to resolve it. If you would like to be assigned to a team or you need additional members for your team, let me know.

2. Focus on the orbital mechanics aspects of the project once the spacecraft is in its initial orbit. You may start your analysis with the spacecraft in a circular, 300 km altitude parking orbit around the Earth in the ecliptic plane.

3. Determine launch dates, arrival dates, and ΔV using the actual positions of planets. The JPL Horizons web site has data on the heliocentric coordinates of the planets. **Assume the planetary orbits are circular orbits in the ecliptic plane.** This means you may choose one date at which you find the planets' positions from JPL Horizons, then write code to propagate their positions forward or backward in time assuming circular orbits. Choose launch and arrival dates that will allow the project to be completed within your professional lifetimes.

planet's moons, etc., at your discretion, but make sure to include what happens when your spacecraft reaches Neptune.

8. Your goal is not necessarily to optimize the trajectory. Focus your early efforts on finding one trajectory that works given where the planets are in the Solar System. Once you have one solid option, you may consider variations on the launch, flyby, and/or arrival dates to identify the effect they have on the ΔV for the mission.

Grading
The reports will be graded for thoroughness of the analysis, accuracy, completeness, readability, and visuals. All team members will receive the same grade unless there is a problem with a team member.

Project Schedule and Deadlines
Monday, April 7: By this date, email Dr. Sandrik with the names of your group members or to indicate that you are looking for group members.

Thursday, May 8: Reports are due by the end of the day. Turn in paper reports in class or to Dr. Sandrik's mailbox (in the walkway between ME and ERB, near the loading dock) or electronic documents online through the course website's Interplanetary Project dropbox.

2.2.2 finding planet with JPL handout

JPL site `http://ssd.jpl.nasa.gov/horizons.cgi`

Finding Planet Positions with JPL Horizons

EMA 550: Astrodynamics, University of Wisconsin-Madison

Website: JPL Horizons, http://ssd.jpl.nasa.gov/horizons.cgi

Introduction

The JPL Horizons website offers a rich set of data about planets, satellites, and other celestial bodies. For the purposes of EMA 550, and for the Interplanetary Project in particular, the orbits of the planets are generally assumed to be circular and in the ecliptic plane. All that is needed, then, is a singular angular measurement to identify their positions relative to each other. The **heliocentric longitude** is just such a measurement. A planet's heliocentric longitude at a given time is the angle between the "first point in Aries," or "x," direction, and the planet's location at the specified time. Since all of the planets' heliocentric longitudes are measured from a common direction and in the same direction of motion, the heliocentric longitude of each can be used to find their relative positions.

Finding the Heliocentric Longitude on JPL Horizons

The JPL Horizons web interface at the address above allows the user to modify six categories of settings to find the information of interest. Each category is discussed below with regard to finding heliocentric longitude. Each setting can be changed in JPL Horizons by clicking the "change" link next to the category title.

a) **Ephemeris Type:** for heliocentric longitude, choose **Observer.**
b) **Target Body:** choose the planet you would like to locate.
c) **Center:** for heliocentric, this must be the Sun. Choose the Sun by entering **@sun** in the box that appears when you click the change link.
d) **Time Span:** to get a common time for locating all of your planets (which you can then propagate by assuming a constant mean motion for each) find the link for the discrete times form and choose one common time.
e) **Table Settings:** if you are in the Observer mode, the table settings page should provide you with a list of check boxes (40 of them in three columns). The only one that you need for heliocentric longitude is **#18, Helio eclip. lon & lat**. You can uncheck all of the others.
f) **Display Output**: To get a single longitude for each planet, there is nothing here that you need to change.

Finding Orbital Elements on JPL Horizons

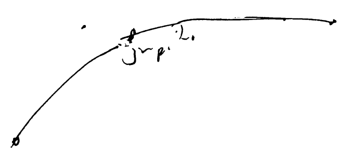
You can use values from the class Planetary Constants sheet or JPL Horizons to set the radius for your planets' orbits. Use the semimajor axis of the orbit as the average radius for the assumed circular orbit. To find the semimajor axis on JPL Horizons, set **Ephemeris Type** to **Orbital Elements**.

Interplanetary Project Intro Work Day

EMA 550: Astrodynamics, University of Wisconsin-Madison

Tasks to Complete Today

1) Find your group members.
2) Follow the instructions on the reverse for the heliocentric position of the Earth on today's date via JPL Horizons (see reverse).
3) Find the heliocentric longitudes for Jupiter and Neptune on today's date and sketch (by hand) their relative positions. To sketch them well, determine the radii of their heliocentric orbits from JPL Horizons or from the course Planetary Constants sheet and draw them roughly to scale.
4) Confirm your sketch against the JPL Solar System Simulator (SSS) (http://space.jpl.nasa.gov). To see the solar system, choose "Show me **the Solar System** as seen from **above**" with a field of view taking up 100% of the image width. If you need to zoom in to see the inner planets, note the field of view in the upper left corner of the current image and choose a smaller field of view angle on the previous page. (Note: JPL SSS seems to put the first point in Aries direction as vertically upward, which will help you align your sketch with theirs.)
5) Write a code that will
   a. Take positions of Earth, Jupiter, and Neptune today and return their positions at a future date (such as a launch date, flyby date, or arrival date) assuming circular orbits with radii equal to the planets' semimajor axis distances
   b. Return the differences between the planets' angular positions on a common date (i.e., "On the launch date, Jupiter leads Earth by ___ degrees.")
   c. Return the angle between two planets on different dates (the θ angle in the Lambert method, also equal to the change in true anomaly on the transfer orbit, i.e. "The angle between Earth at launch and Jupiter at arrival is ___ degrees.")
6) Verify the positions returned by your code against the JPL Solar System Simulator.



2.2.3 my final report

Final Interplanetary Project EMA 550

by Nasser M. Abbasi

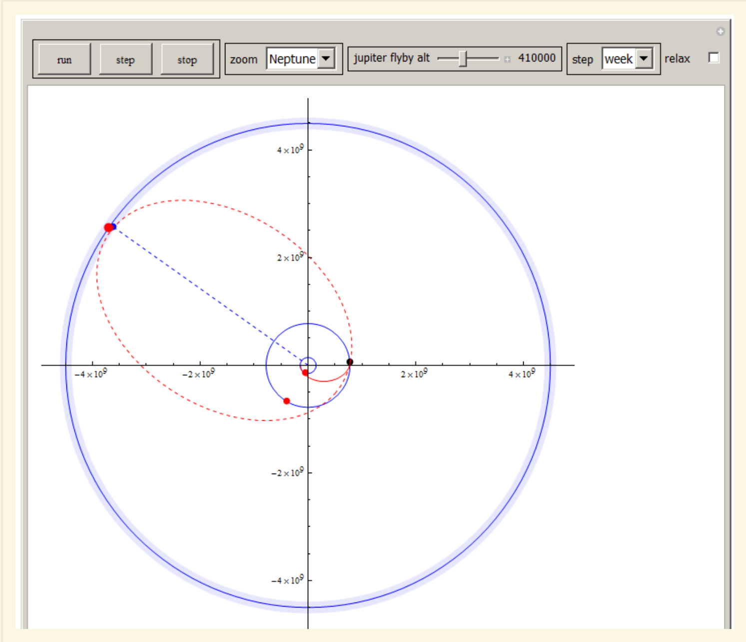
Introduction

The project was broken into 6 phases. This the high level summary of each phase.
1) The first phase was the waiting period to synchronize earth with Jupiter with the correct Hohmann angle. Once this was achieved, the probe is launched from LEO orbit.
2) The second phase is the escape from earth SOI using hyperbolic escape trajectory
3) The third phase is the travel over a Hohmann ellipse to reach Jupiter at the apogee location of the Hohmann transfer ellipse.
4) This stage the probe enters Jupiter SOI and performs a hyperbolic fly-by trajectory. The burnout distance used was based on trial and error experiments using the simulation written for this project in order to obtain a post fly-by ellipse that allowed the probe to reach Neptune orbit at the same time when Neptune was there.
5) This is the post-flyby stage, leaving Jupiter SOI and traveling on an ellipse to Neptune.
6) This is the final phase, the probe is now inside Neptune SOI. It enters a circular orbit around Neptune and remains there.

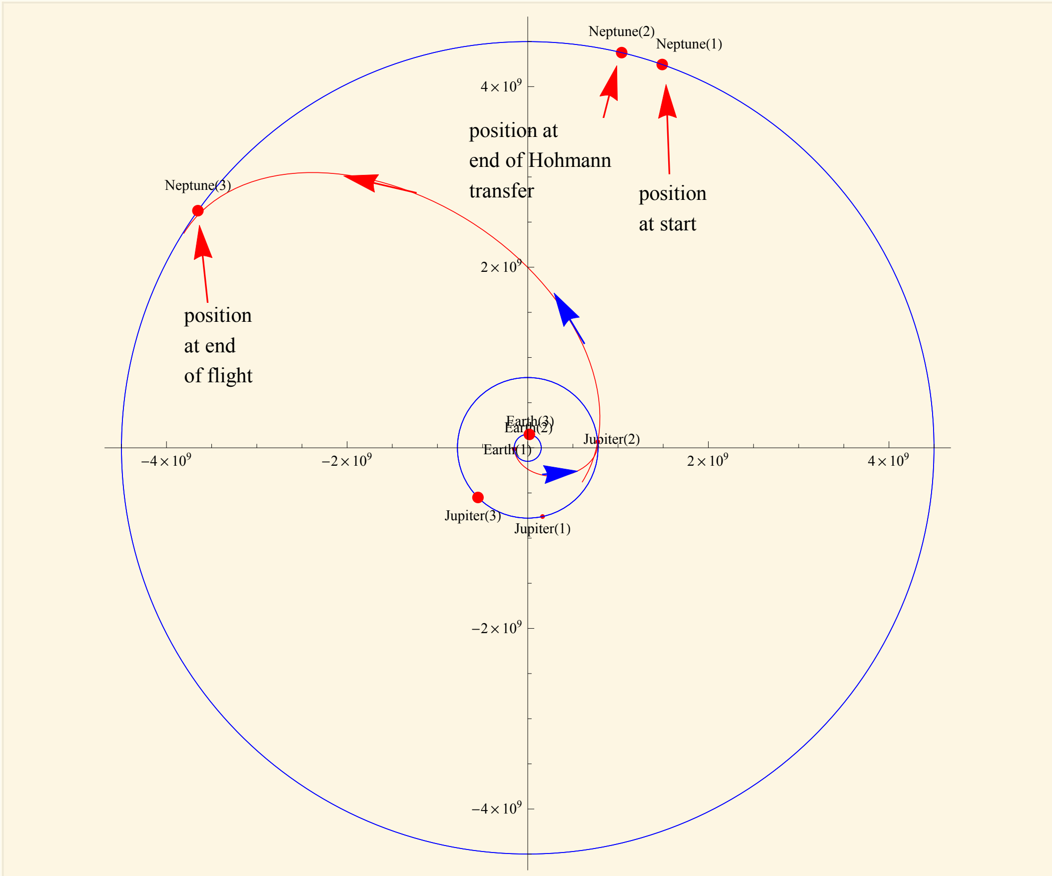
The final results will be shown here, followed by the step by step calculations done in each phase, then the simulation program will be described.

How was the final trajectory found?

One week of full time work was spend on writing the simulator, as this was the only method to find if a chosen input will lead to the probe meeting Neptune when it arrives to its orbit. The simulator takes as input the initial angular positions of Earth, Jupiter and Neptune in the ecliptic plane and using time step, advances the positions of the planets and the probe on its orbit. This is screen shot of the GUI of the simulator. It allows one to stop, run, and make one step at a time. The step size can be changed from one day to one week to one month.



Once the simulator was completed, different starting positions for Earth, Jupiter and Neptune were tried. Each position used was obtained from the JPL Horizon web site. Different dates were selected. In addition, for each selected initial position, the altitude that the probe will be closest to Jupiter in its fly-by was modified using a slider in simulator. This resulted in different ellipse since the burn out distance r_{bo} is different. The closest altitude to Jupiter (r_{bo}) was modified from 200 000 km to 500 000 km above the surface of Jupiter. When none of the resulting trajectories found to be acceptable, if they did not lead to acceptable rendezvous with Neptune, another starting date was selected and the process was repeated. Acceptable rendezvous with Neptune is one which reaches Neptune within distance less that Neptune's SOI. This is the final trajectory selected *project.nb* 3



To speed the process of finding the final trajectory, the simulator used a varying time step. The simulation time step can be one day, one week, one month or even one year. However the accuracy of the resulting trajectories will become worst if the time step was made large. When a candidate trajectory was found using large time step (month for example) it was repeated again using one week time step, and then again using one day time step. Using the one day time step, the simulation will take about 15 minutes to complete. So this was a very time consuming part of the project to find the correct trajectory.

This table shows some of the dates and corresponding ecliptic longitude angles showing which initial position was selected

Printed by Wolfram Mathematica Student Edition

selected?	date	Earth	Jupiter	Neptune	Altitude above Jupiter (KM)
NO	3/21/2014	270	111.3	334.996	many
No	9/21/2014	356.76	126	336	many
No	10/01/2014	7.5	127	336	many
No	03/21/2016	180	166	339	many
No	03/21/2017	180	196	304	many
YES	3/21/2016	180	169	339	410 000

Initial positions tried in simulation

- **The following table shows the time history for all the phases on the project**

phase	date started	date completed
waiting for correct Hohmann angle between Earth/Jupiter	3/21/2016	12/26/2016
Start on Hohmann transfer, travel to Jupiter SOI	12/26/2016	9/20/2019
Enter and exist Jupiter SOI	9/20/2019	2/29/2020
travel on Ellipse from Jupiter to Neptune	2/29/2020	12/25/2054

Time schedule of complete trajectory

- **Show ΔV for fly-by and compare to Hohmann transfer**

Trajectory	ΔV1 (km/s)	ΔV2 (km/s)	Total (km/s)
Fly–by	6.267	13.44	19.71
Direct Hohmann	8.22	14.91	23.133

Compare total ΔV using Fly–by and Direct Hohmann. Saving is over 3 km/sec

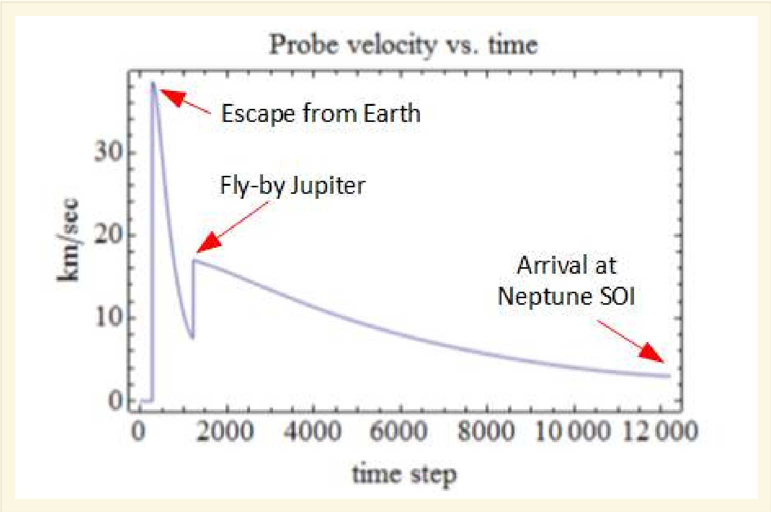
- **Show trajectory information for each phase (relevant data is shown)**

Item	Earth escape Hyperbola	Hohmann transfer Earth/Jupiter	Fly–by Jupiter Hyperbola	Elliptical orbit Jupiter/Neptune
eccentricity e	2.291	0.6775	1.1199	0.726
semi–major a (km)	–	4.639×10^{8}	4.01×10^{6}	2.6×10^{9}
V_∞ (km/sec)	8.79	–	5.64	–
Departure speed V_D (km/s)	–	–	17.024	–
η (deg)	115.88	–	153.24	–
Turn angle θ (deg)	64.12	–	126.48	–
Flight path angle γ_d (deg)	–	–	15.45	–
True anomaly f (deg)	–	–	36.98	–

Orbits data found

- **Velocity profile of probe showing speed gain due to flyby**

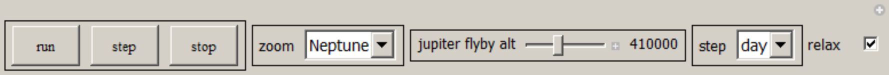
The simulator keeps track of current velocity of probe as it travels starting from Earth all the way to Neptune. It then plots the velocity vs. Time of the probe. This plot below was generated by the simulator and shows the speed gained during the fly-by phase. ΔV gained due to flyby was found to be 10.077 km/sec. This is free ΔV due to gravity assist.



The above shows that the fly-by Jupiter gave the probe almost 8 km/sec boost in speed relative to Sun.

- **Trajectory data gathering**

The simulator contains an option to display all the information about the trajectory during its running. This display can be turned off if needed. This allows one to monitor each aspect of the orbit as it runs. Here is a screen show showing typical display during one simulation run



Timings and angles as simulation runs						
θ_E	θ_J	θ_N	$\theta_{Hohmann}$	State	Phase	θ Hohmann
087.38	221.28	118.11	097.16	RUN	2	097.16

mean speeds (km/sec)				
Earth	Jupiter	Neptune	probe on Hohmann	probe to Neptune
29.78	13.06	05.43	16.91	07.18

Dimenstions data and current probe speed			
rES	rJS	rNS	current ProbeSpeed (km/sec)
$150. \times 10^6$	$778. \times 10^6$	4.5×10^9	05.45

current positions in space				
xN	yN	x probe	y probe	dist. probe to Neptuen
-2.12×10^9	3.97×10^9	-1.25×10^9	3.02×10^9	1.28×10^9

Hohmann transfer from Earth to Jupiter data					
a	rp	ra	e	current f	current E
$464. \times 10^6$	$150. \times 10^6$	$778. \times 10^6$	0.67758	149.67	111.79

hyperbolic Jupiter flyby					
V_∞ (km/s)	e Hyper	η Hyper (deg)	γ_d (degree)	θ_{turn}	V_d (km/s)
005.643	1.1210238	153.130	+015.5	+126.3	+17.014

Post fly–by ellipse, Jupiter to Neptune					
aJN	rpJN	raJN	eJN (eccentricity)	f_{new} (true anamoly)	mean probe speed deg/day
2.58×10^9	$712. \times 10^6$	4.44×10^9	0.72381	{−037.2, +037.2}	+00.0138

current E (spacecraft)	current f	nHuhmannToJupiter (deg/day)	nJN(deg/day)
+111.8	+149.7	+00.1804	

Step by step calculations

Constants used

In[7]:=

```
<< Calendar`
AU = 1.495978 * 10^8;
rE  = 6378; (*Earth radius*)
rJ = 71 492;(*Jupiter radius*)
rN = 24 764;(*Neptune radius*)
rES = 1 AU;(*Earth distance from sun*)
rJS = 5.203 AU;(*Jupiter distance from sun*)
rNS = 30.07 AU; (*Neptune distance from sun*)
(* SOI for each planet *)
eSOI = 9.24 * 10^5;
jSOI = 4.82 * 10^7;
nSOI = 8.67 * 10^7;
(*mu for each planet*)
μSun = 1.327 * 10^11;
μE = 3.986 * 10^5;
μJ = 126 686 534;
μN = 6 836 529;

(*speed of each planet, all relative to sun*)
```

$sE = \sqrt{\frac{\mu Sun}{rES}}$;(*km/sec*)

$sJ = \sqrt{\frac{\mu Sun}{rJS}}$;(*km/sec*)

$sN = \sqrt{\frac{\mu Sun}{rNS}}$;(*km/sec*)

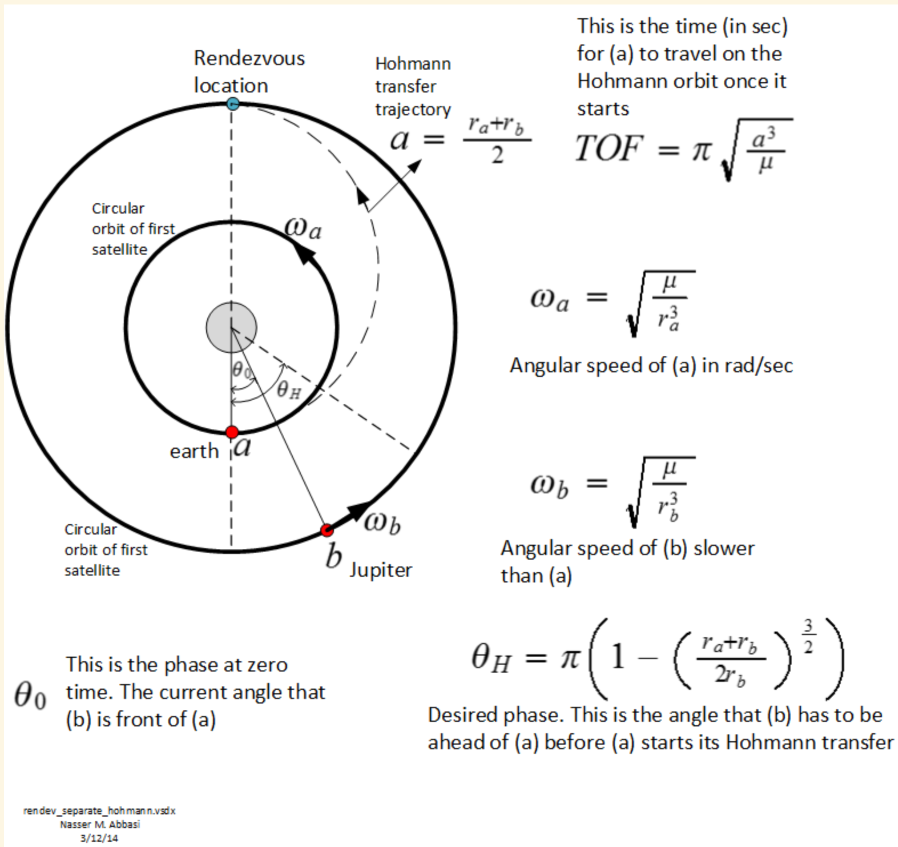
(*angular velocity of each planet*)

$\omega E = \sqrt{\frac{\mu Sun}{rES^3}}$;(*angular vecloity of earth*)

$\omega J = \sqrt{\frac{\mu Sun}{rJS^3}}$;(*angular vecloity of earth*)

$\omega N = \sqrt{\frac{\mu Sun}{rNS^3}}$;(*angular vecloity of earth*)

Find the Hohmann angle needed rendezvous between Earth and Jupiter



In[28]:=

```
θEarthJupiter = Pi (1 - ((rES + rJS)/(2 rJS))^(3/2));
N@θEarthJupiter*180/Pi
```

Out[29]=

```
97.15821569
```

- Enter the initial positions. These have been found by simulation first. The simulation includes all these steps build into it. There are shown here in order to be able to show each step done outside of the simulation code.
  Note that 90 degrees were added to each position to make it compatible with standard coordinate system with positive x points to the right

```
θE0 = Mod[180 + 90, 360] Degree; (*Earth*)
θJ0 = (169 + 90) Degree; (*Jupiter*)
θN0 = Mod[(339 + 90), 360] Degree; (*Neptune*)
```

- **find wait time between Earth and Jupiter in order to find date when start Hohmann transfer.**

- Find θ_0 the initial angle between earth and Jupiter at initial configuration

```
θ0 = θJ0 − θE0;
θ0 * 180. / Pi
```

```
-11.
```

- Adjust θ_0 if θ_H is larger than θ_0 by adding 2π so not to get negative time

```
If [θ0 ≤ θEarthJupitor, θ0 = θ0 + 2 Pi];
θ0 * 180. / Pi
```

```
349.
```

- calculate wait time before starting Hohmann transfer. This is the time needed to sync with Jupiter

$$\text{waitTimeEarthJupiter0} = \frac{\theta 0 - \theta\text{EarthJupitor}}{\omega E - \omega J};$$

```
waitTimeEarthJupiter0 / (60 * 60 * 24) (*days*)
```

```
279.0431558
```

- Display the date the Hohmann transfer starts

```
currentDate = {2016, 3, 21};
currentDate = DaysPlus[currentDate, Ceiling[waitTimeEarthJupiter0 / (60 * 60 * 24)]]
```

```
{2016, 12, 26}
```

- Find *a* for the Hohmann transfer

$$\text{aEJ} = \frac{\text{rES} + \text{rJS}}{2}$$

$4.639775767 \times 10^{8}$

- find time of flight on the Hohmann transfer

```
tof = π Sqrt[aEJ^3/μSun];
tof/(60*60*24*365) (*years*)
```

2.73308597

- Find total wait time which includes sync time and time of flight over Hohmann transfer

```
waitTimeEarthJupiter = waitTimeEarthJupiter0 + tof;
waitTimeEarthJupiter/((60*60*24*365)) (*years*)
```

3.497587767

- display the date probe arrives to Jupiter SOI

```
currentDate = DaysPlus[currentDate, Round[tof/(60*60*24)]]
```

{2019, 9, 20}

- **Make function to convert Gregorian date to Julian day (Not used at this time)**

```
toJD[d_, m_, y_] := 367 y - IntegerPart[(7 (y + IntegerPart[(m+9)/12]))/4] + IntegerPart[(275 m)/9] + d + 1 721 013.5;
toJD[20, 10, 2014]
```

2.4569505×10^6

Hyperbolic escape from Earth

- Find eccentricity of Hohmann transfer ellipse

```
eEJ = (rJS - rES)/(rES + rJS)
```

0.6775753668

Find semi-minor axes for Hohmann ellipse (km)

$bEJ = aEJ \sqrt{1 - eEJ^2}$

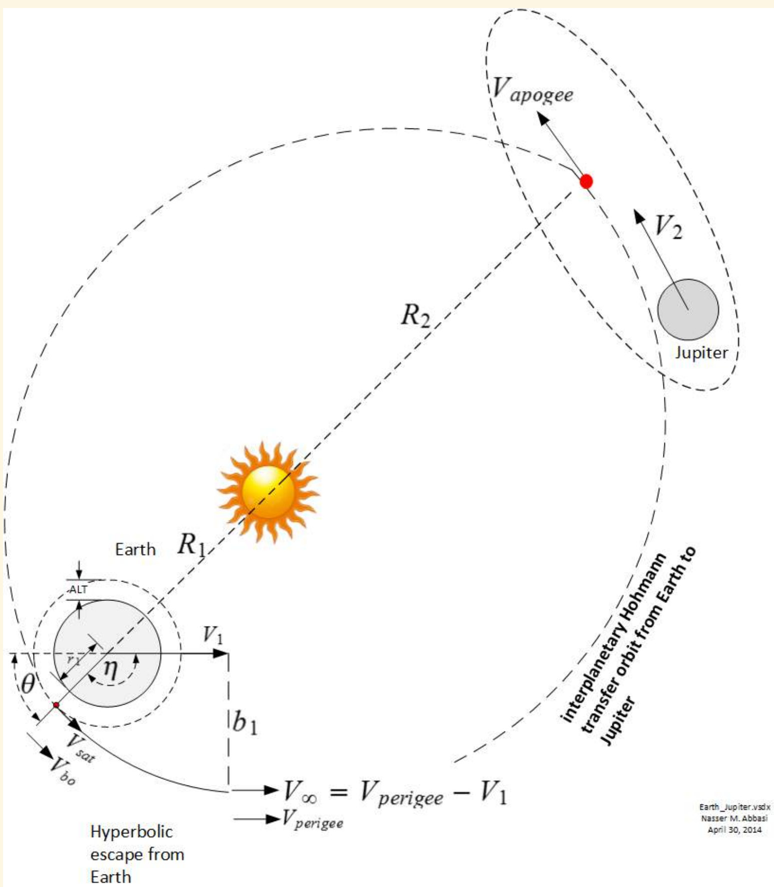
3.412338607×10^8

Find velocity at perigee V_p (KM/sec)

$vp = \sqrt{\mu Sun \left(\frac{2}{rES} - \frac{1}{aEJ}\right)}$

38.57570557

Show drawing (not to scale) of Earth escape hyperbolic trajectory



- Find V_∞ to escape earth (km/sec)

```
vInf = vp - Sqrt[μSun/rES]
```

8.792402687

- Find r_{bo} the burn out radius (km)

```
rbo = rE + 300 (*300 KM is altitude*)
```

6678

- Find V_{bo} the burn out speed using the energy equation (km/sec)

```
Clear[vbo];
eq = vbo^2/2 - μE/rbo == vInf^2/2 - μE/eSOI;
vbo = First@Select[vbo /. NSolve[eq, vbo], # > 0 &]
```

13.99359259

- Find ΔV_1 needed to escape earth

```
delV1 = Abs[vbo - Sqrt[μE/rbo]]
```

6.267757388

- Calculate the eccentricity of the hyperbolic escape from earth

```
e = Sqrt[1 + vInf^2 vbo^2 rbo^2/μE^2]
```

2.291080512

- Calculate angle η where ΔV should be applied

```
η = ArcCos[-1/e];
η*180/Pi
```

115.8792052

- Find the turn angle θ

```
θ = Pi − η;
θ * 180 / Pi
```

```
64.12079477
```

Hohmann transfer between Earth and Jupiter

- Find mean angular velocity on the Hohmann ellipses (rad/sec)

$$\text{nHuhmannToJupiter} = \sqrt{\frac{\mu\text{Sun}}{\text{aEJ}^3}}$$

$3.644936553 \times 10^{-8}$

- Find the angular positions that earth and Jupiter will have at the end of the Hohmann transfer. We calculated the time of flight from above. So using this time, and knowing the angular velocity of Earth the Jupiter, we can find the new angular positions in ecliptic plane.

  First display time of flight to Jupiter in days (this is half the period of the Hohmann transfer ellipse)

```
(tof) / (60 * 60 * 24)
```

```
997.5763791
```

- Find the angle the earth will be at when probe starts Hohmann orbit

```
θE1 = θE0 + ωE * waitTimeEarthJupiter0;
Mod[θE1, 2 Pi] * 180 / Pi
```

```
185.0143788
```

- Find the angle Jupiter will be at when probe starts Hohmann orbit

```
θJ1 = θJ0 + ωJ * waitTimeEarthJupiter0;
Mod[θJ1, 2 Pi] * 180 / Pi
```

```
282.1725945
```

- **Find the angle the earth will be at when probe reach end of Hohmann to Jupiter**

```
θE2 = θE0 + ωE * (waitTimeEarthJupiter0 + tof);
Mod[θE2, 2 Pi] * 180 / Pi
```

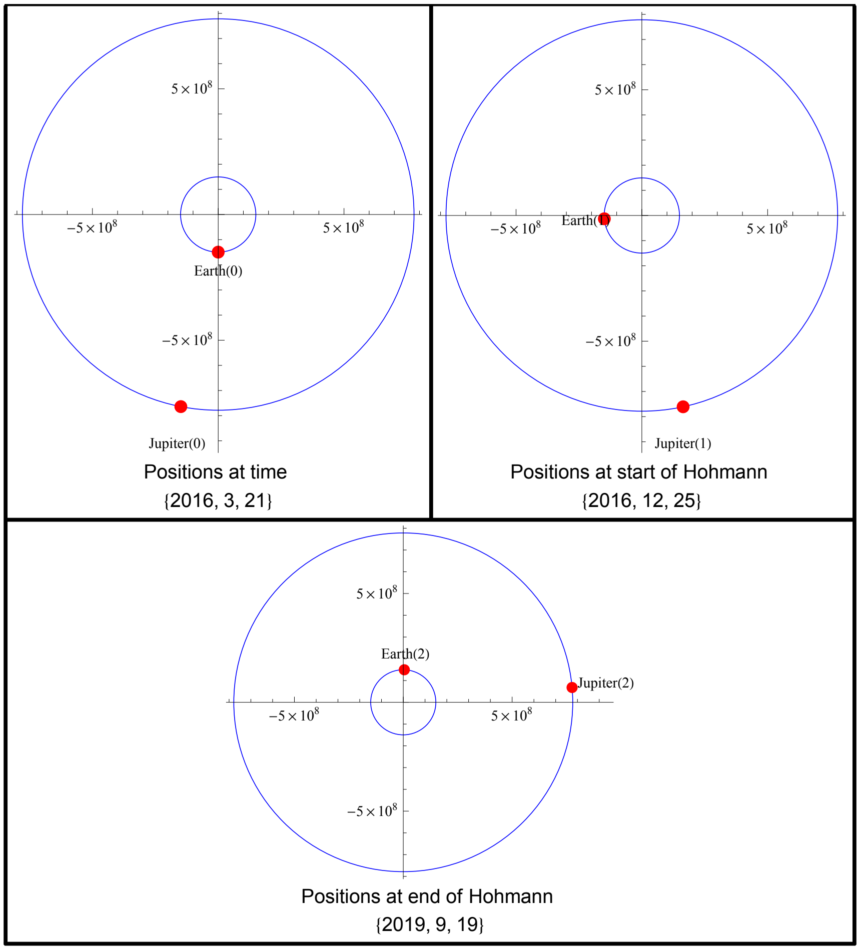
```
88.18792204
```

- **Find the angle Jupiter will be at with probe reach Jupiter**

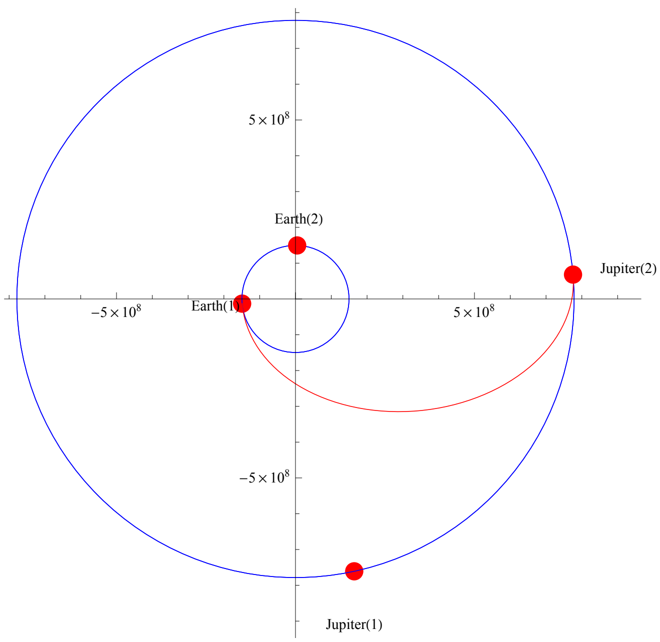
```
θJ2 = θJ0 + ωJ * (waitTimeEarthJupiter0 + tof);
Mod[θJ2, 2 Pi] * 180 / Pi
```

```
5.014378828
```

- **Draw diagram showing the initial Earth/Jupiter positions at t=0 and at start of Hohmann transfer and at end of Hohmann transfer** p


5 × 10^8
−5 × 10^8
5 × 10^8
Earth(0)
−5 × 10^8
Jupiter(0)
Positions at time
{2016, 3, 21}
5 × 10^8
−5 × 10^8
Earth(1)
5 × 10^8
−5 × 10^8
Jupiter(1)
Positions at start of Hohmann
{2016, 12, 25}
5 × 10^8
Earth(2)
Jupiter(2)
−5 × 10^8
5 × 10^8
−5 × 10^8
Positions at end of Hohmann
{2019, 9, 19}

- **Draw diagram showing the Hohmann elliptic transfer orbit showing initial positions of planets and final positions all on one diagram**



Configuration at end of Hohmann transfer
{2019, 9, 19}
with positions indicated by (1),(2) for start/end of transfer

- **Before making the fly-by Jupiter calculations, lets show the above diagram along with the position of Neptune as well. All to scale.**
  Find the angle Neptune will be at when probe starts on Hohmann transfer from Earth to Jupiter

```
θN1 = θN0 + ωN * waitTimeEarthJupiter0;
Mod[θN1, 2 Pi] * 180 / Pi
```

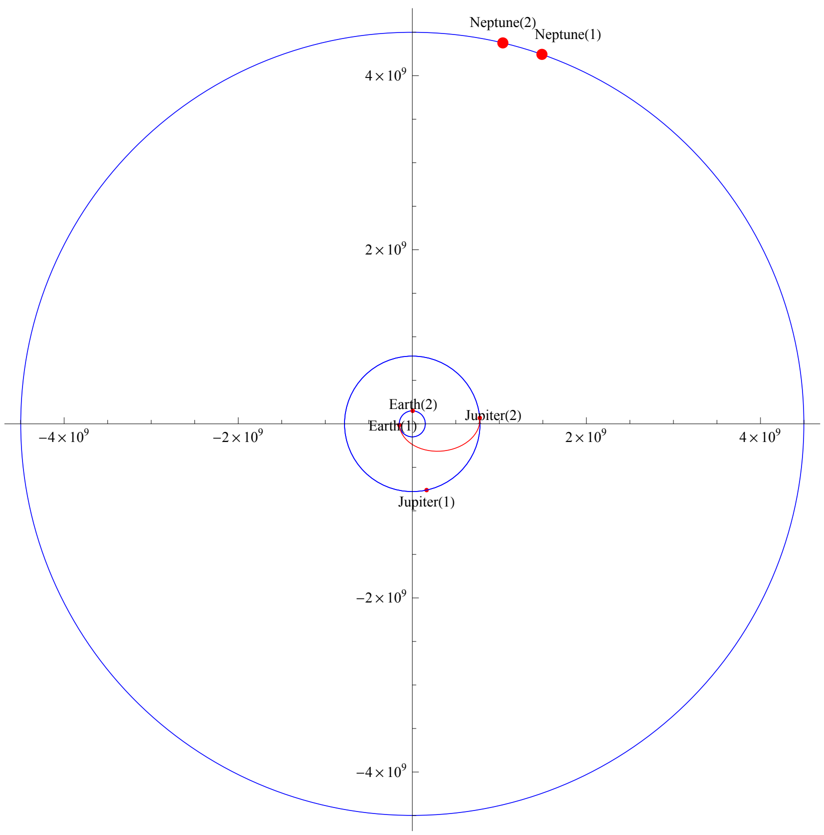
```
70.66784335
```

- **Find the angle Neptune will be at when probe ends the Hohmann transfer from Earth to Jupiter**

```
θN2 = θN0 + ωN * (waitTimeEarthJupiter0 + tof);
Mod[θN2, 2 Pi] * 180 / Pi (*degree*)
```

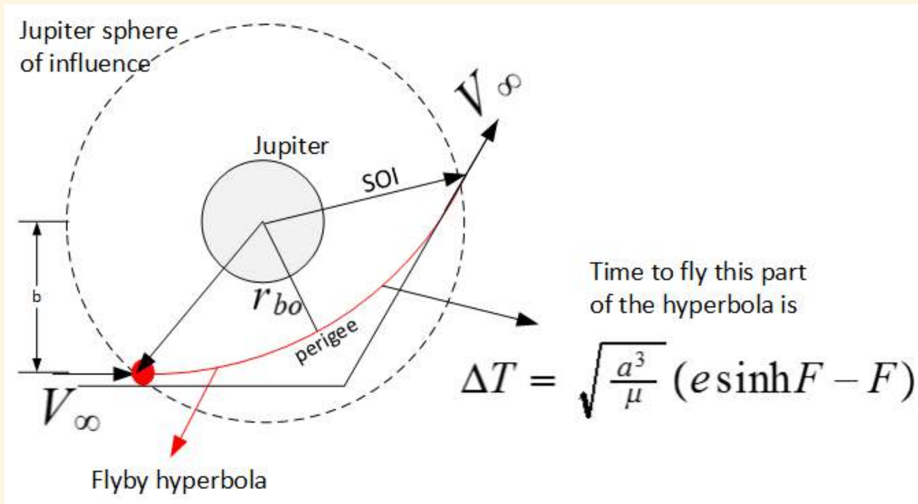
```
76.630366
```

▫ Draw diagram of the three planets at end of Hohmann transfer



Configuration at end of Hohmann transfer, all planets
{2019, 9, 19}
with positions indicated by (1),(2) for start/end of transfer

Fly-by Jupiter, Hyperbolic flyby



- **Set the burn out radius r_{bo} . This was found by simulation below in order to obtain the rendezvous with Neptune (km)**

```
alt = 410000;
rbo = alt + rJ
```

```
481492
```

- **Find probe speed at entrance to Jupiter SOI**

$$va = \sqrt{\mu Sun\left(\frac{2}{rJS} - \frac{1}{aEJ}\right)} \ (*velocity\ of\ craft\ atJupiter\ entrance*)$$

```
7.414127535
```

- **Find V_∞ for Jupiter flyby (km/sec)**

```
vInf = sJ - va
```

```
5.642948859
```

Find V_{bo} the burn out speed using the energy equation (km/sec)

```
Clear[vbo];
eq = vbo^2/2 - μJ/rbo == vInf^2/2 - μJ/jSOI;
vbo = First@Select[vbo /. NSolve[eq, vbo], # > 0 &]
```

23.5119341

Calculate the eccentricity of the hyperbolic escape from Jupiter

$$e = \sqrt{1 + \frac{vInf^2\, vbo^2\, rbo^2}{\mu J^2}}$$

1.119944854

Find η angle

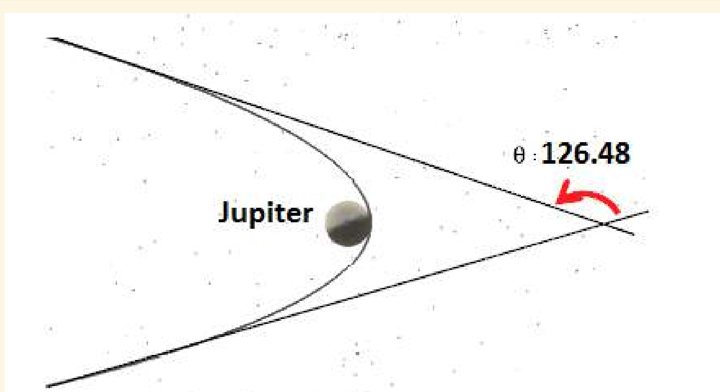
```
η = ArcCos[-1/e];
η * 180/Pi
```

153.2400935

Find the turn angle

```
θ = 2 η - Pi;
θ * 180/Pi
```

126.4801869



- **Find impact parameter *b* (km)**

```
Clear[b]
eq = b vInf == rbo vbo;
b /. First@Solve[eq, b];
b = %
```

2.006186563×10^6

- **Find the departure velocity (km/sec)**

```
vdJN = √(sJ^2 + vInf^2 - 2 sJ vInf Cos[θ])
```

17.02770468

- **Find semi-major axes (km) of the Hyperbolic fly-by trajectory. Since r_{bo} is r_p for the Hyperbolic, we can use $r_{bo} = a(e-1)$ to solve for *a***

```
Clear[aHyper]
eq = rbo == aHyper (e - 1);
aHyper = aHyper /. First@Solve[eq, aHyper]
```

4.014278105×10^6

- **Find the time probe is inside Jupiter SOI during fly-by. First, find the eccentric anomaly *F* of the hyperbolic trajectory when probe at SOI**

```
Clear[F0];
eq = jSOI == aHyper (e Cosh[F0] - 1);
F0 = First@Select[(F0 /. NSolve[eq, F0, Reals]), # > 0 &]
```

3.143507611

- **Find the time inside Jupiter SOI. More than 4 months are spent inside Jupiter SOI. Yet, in the patched conic approximation, we assume the fly-by happens instantly and this time in simulation is not accounted for. But this is approximation.**

$$tJ = 2\sqrt{\frac{aHyper^3}{\mu J}}\,(e\,Sinh[F0] - F0);$$

```
tJ / (60 * 60 * 24) (*days*)
```

162.3555079

Find the date when probe leaves Jupiter SOI

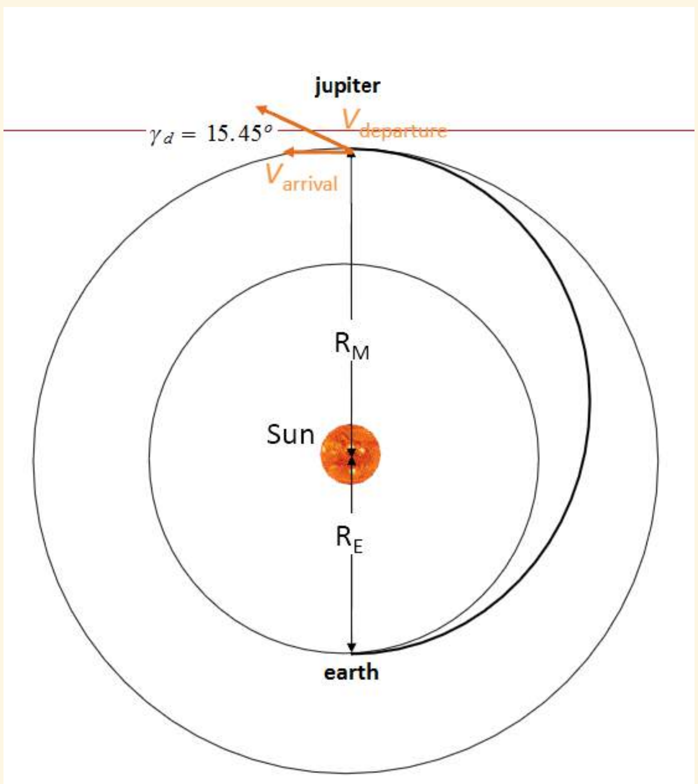
```
currentDate = DaysPlus[currentDate, Round[tJ / (60 * 60 * 24)]]
```

```
{2020, 2, 29}
```

Find the flight path angle at Jupiter for the new ellipse (post-fly ellipse) relative to sun

```
Clear[z];
sol = Quiet[Solve[Sin[z] == vInf Sin[θ] / vdJN, z]];
γ = z /. First@sol;
γ * 180 / Pi
```

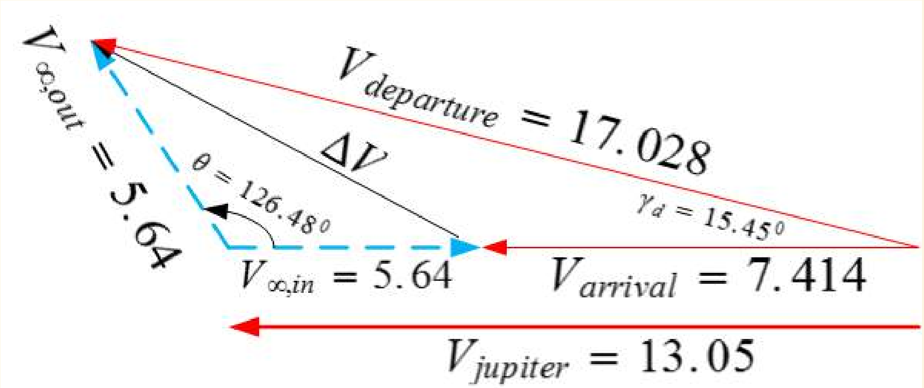
```
15.45400848
```



Departure from Jupiter. (Image edited from class handouts)

Velocity diagram of the fly-by Jupiter

A summary of the above calculations is now given in terms of velocity diagram



Velocity diagram for fly-by Jupiter. Red color means speed is relative to Sun. Blue color means relative to Jupiter. All values are in KM/SEC

- Find ΔV due to fly-by (km/sec)

```
delV = 2 vInf Sin[θ/2]
```

10.07719057

Post-fly by calculations of new Ellipse

- Find the semi-major axes *a* of the post-fly ellipses (KM)

```
Clear[z];
eq = vdJN == Sqrt[μSun (2/rJS - 1/z)];
aJN = z /. First@NSolve[eq, z]
```

2.600341362×10^{9}

- **Find the eccentricity of the post-fly ellipse, to transfer to Neptune**

```
Clear[z];
eq = Cos[γ] == Sqrt[(aJN^2 (1 - z^2))/(rJS (2 aJN - rJS))];
sol = NSolve[eq, z];
sol = z /. sol;
e = First@Select[sol, # > 0 &]
```

$$\text{eq} = \text{Cos}[\gamma] == \sqrt{\frac{\text{aJN}^2\,(1-\text{z}^2)}{\text{rJS}\,(2\,\text{aJN}-\text{rJS})}};$$

0.7260062019

- **Find true anomaly at Jupiter for the new ellipse**

```
Clear[z];
eq = rJS == (aJN (1 - e^2))/(1 + e Cos[z]);
(*sol=z/.First@FindRoot[eq,{z,θJ2}];*)
sol = z /. FindRoot[eq, {z, Pi/8}];
fJN = sol;
fJN*180/Pi
```

$$\text{eq} = \text{rJS} == \frac{\text{aJN}\,(1-\text{e}^2)}{1+\text{e}\,\text{Cos}[\text{z}]};$$

36.98646614

- **Since γ was positive (from above) then the true anomaly will be between zero and 180**
- **Find r_p of the new ellipse (km)**

```
rpJN = aJN (1 - e)
```

7.12477406×10^{8}

- **Find r_a of the new ellipse (km)**

```
raJN = aJN (1 + e)
```

4.488205318×10^{9}

- **Find semi-minor axes of new ellipse (km)**

```
bJN = aJN Sqrt[1 - e^2]
```

$$\text{bJN} = \text{aJN}\sqrt{1-\text{e}^2}$$

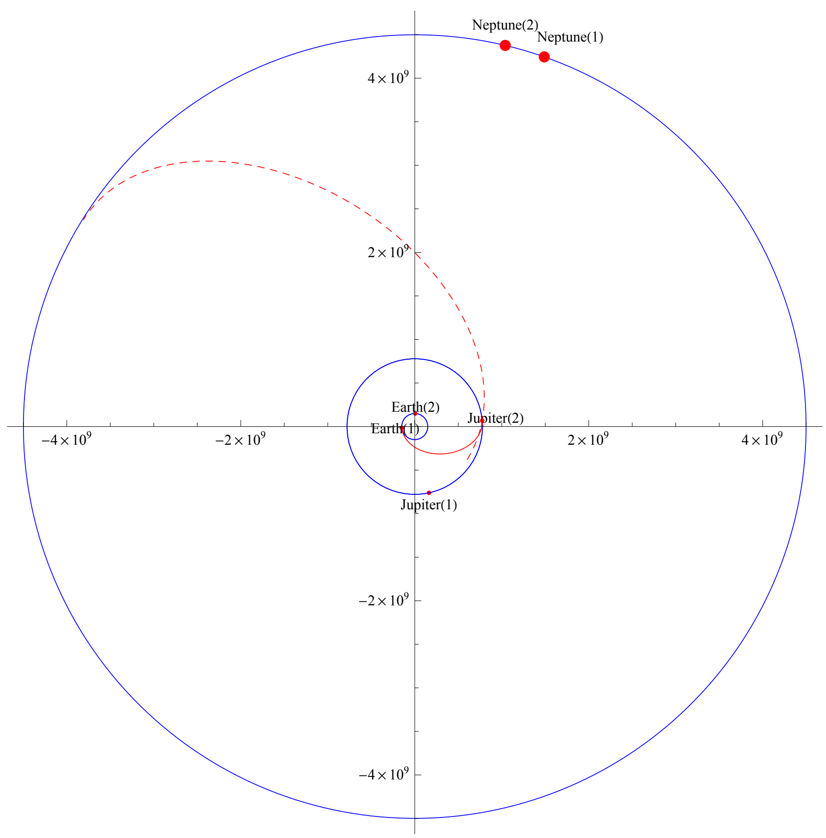
1.788223946×10^{9}

- Find center of new ellipse

```
xc2 = -aJN * e;
yc2 = 0;
```

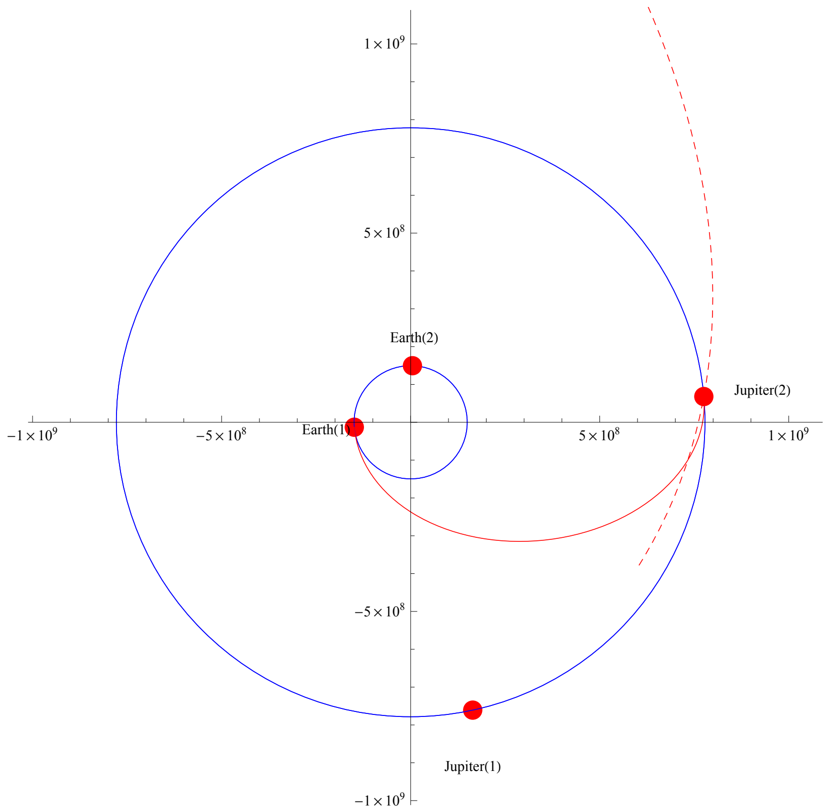
Transfer on new ellipse from Jupiter to Neptune, post-flyby

- Now that the new ellipse is found, it can be drawn to scale to show all trajectories found so far. This shows at the time when the Hohmann transfer was just completed with the new Ellipse draw showing the trajectory from Jupiter to Neptune, but the actual transfer has not started yet



Configuration at end of Hohmann transfer, all planets and final ellipse
{2019, 9, 19}
with positions indicated by (1),(2) for start/end of Hohmann transfer

▫ **zoom into the above diagram showing the flyby Jupiter area**



Showing the fly–by effect and the flight path angle
{2020, 2, 29}
with positions indicated by (1),(2) for start/end of Hohmann transfer

- **Show the positions of planets at end of trajectory when probe enters Neptune SOI. First find the time it takes to travel from Jupiter to Neptune on the new ellipse**
- **Find E1, and E2 for new ellipse**

```
Clear[E1];
eq = rJS == aJN (1 - e Cos[E1]);
E1 = First@Select[E1 /. Quiet[NSolve[eq, E1]], # > 0 &];
E1 * 180 / Pi
```

```
15.18098712
```

```
Clear[E2];
eq = rNS == aJN (1 - e Cos[E2]);
E2 = (E2 /. Quiet[FindRoot[eq, {E2, Pi / 5}]]);
E2 * 180 / Pi
```

```
179.9999997
```

$$\text{timeOfFlyOnNewEllipse} = \sqrt{\frac{aJN^3}{\mu Sun}}\,((E2 - E1) - e\,\text{Sin}[E2 - E1])$$

9.779114032×10^8

- **In days**

```
tof2 = timeOfFlyOnNewEllipse / (60 * 60 * 24) + 1400
```

```
12718.41902
```

- **Find date it arrives to Neptune SOI**

```
currentDate = DaysPlus[currentDate, Round[tof2]]
```

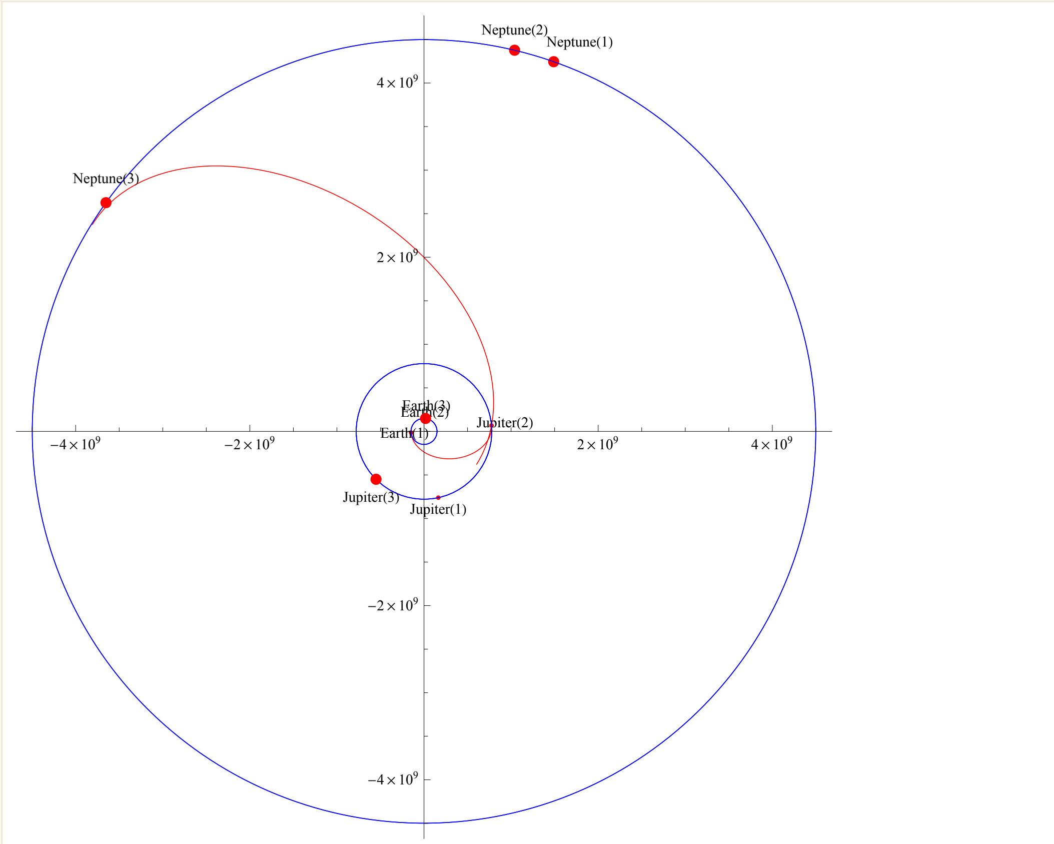
```
{2054, 12, 25}
```

- **Time on new ellipse in years**

```
timeOfFlyOnNewEllipse / (60 * 60 * 24 * 365)
```

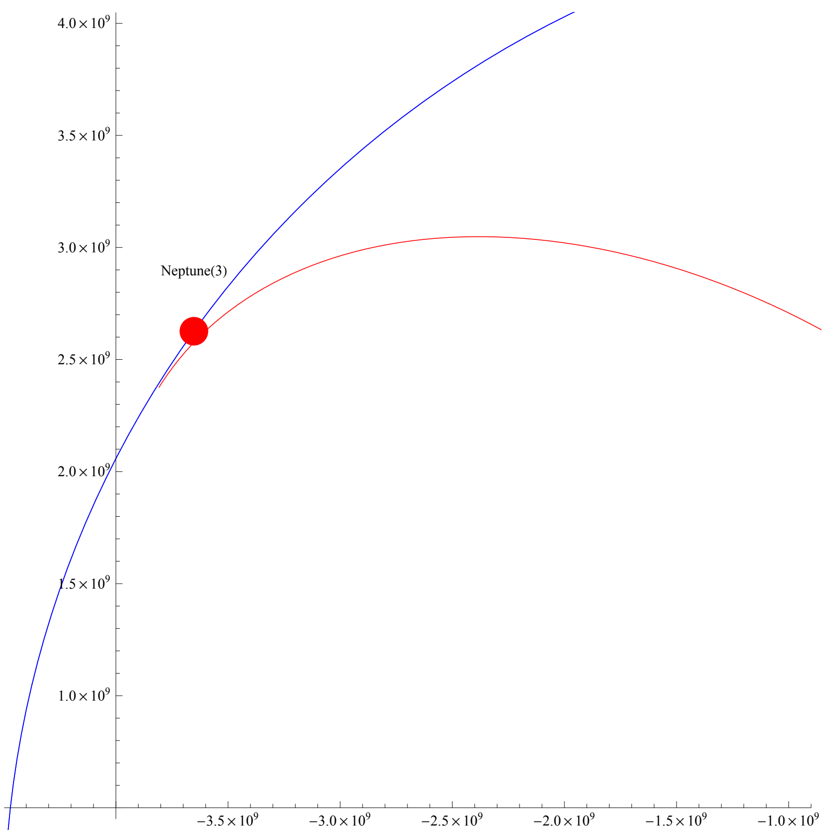
```
31.00936717
```

▫ Find positions of all planets at the final day



- **Zoom in at the area where the probe enters Neptune SOI**

```
Show[p1, p2, p3, p41, p7, p6, ImageSize → 500, PlotRange → {{−rNS, −0.2 rNS}, {0.1 rNS, 0.9 rNS}}]
```



Move probe into final circular orbit around Neptune, final ΔV applied

- **Now that probe is inside Neptune SOI, we use make a burn out to slow it down into a circular orbit around Neptune. First find the speed the probe is at when it enters Neptune SOI using the ellipse equation (km/sec). Simulation stops when probe is just inside Neptune SOI. Let the altitude above Neptune be 1000 KM as the final parking orbit. The probe arrive on tangential approach to Neptune, hence the speed at apogee is**

$$v0 = \sqrt{\mu Sun \left(\frac{2}{raJN} - \frac{1}{aJN}\right)}$$

2.846226578

- **Find the required speed the probe in a circular orbit around Neptune (KM/sec) assuming 1000 km altitude above the surface**

$$v1 = \sqrt{\mu N \left(\frac{1.}{rN + 1000}\right)}$$

16.28962869

- **Find ΔV needed (km)**

```
delV2 = v1 - v0
```

13.44340211

Find total ΔV needed for the whole interplanetary trip and compare to if Hohmann transfer was used all the way from Earth to Neptune

```
delV = Abs[delV1] + Abs[delV2]
```

19.7111595

- **The above is ΔV using fly-by Jupiter. Now lets find ΔV assuming Hohmann transfer from Earth to Neptune. First find *a* for this new ellipse (km)**

$$aDirect = \frac{rES + rNS}{2}$$

2.324001823×10^9

- **Find V_p needed (km/sec)**

$$vp = \sqrt{\mu Sun \left(\frac{2}{rES} - \frac{1}{aDirect}\right)}$$

41.43658381

- **Find needed V_∞ to escape Earth**

```
vInf = vp - Sqrt[μSun/rES]
```

11.65328093

- **Find r_{bo} the burn out radius**

```
rbo = rE + 300 (*300 KM is altitude*)
```

6678

- **Find V_{bo} the burn out speed using the energy equation (km/sec)**

```
Clear[vbo];
eq = vbo^2/2 - μE/rbo == vInf^2/2 - μE/eSOI;
vbo = First@Select[vbo /. NSolve[eq, vbo], # > 0 &]
```

15.94720179

- **Find ΔV_1 needed to escape earth**

```
delV1Direct = Abs[vbo - Sqrt[μE/rbo]]
```

8.22136659

- **Now find V_a at the apogee at Neptune end of the ellipse (km/sec)**

```
va = Sqrt[μSun (2/rNS - 1/aDirect)]
```

1.378004117

- **Find the needed circular speed around Neptune (using SOI since that is what was used above)**

```
v3 = N@Sqrt[μN/(rN + 1000)]
```

16.28962869

- **Find ΔV2 needed at Neptune**

```
delV2Direct = v3 - va
```

```
14.91162457
```

- **Find total delV**

```
Abs[delV1Direct] + Abs[delV2Direct]
```

```
23.13299116
```

- **Therefore, when using flyby, total ΔV was 19.71 km/sec, and using direct Hohmann transfer, total ΔV is 23.13 The saving is about 3.4 km/sec.**

Find the time to travel from Earth to Neptune if direct Hohmann transfer was made

The time in this case is half the period of the Hohmann transfer ellipse, which can be found as follows

$$\text{time} = \text{Pi}\sqrt{\frac{\text{aDirect}^3}{\mu\text{Sun}}};$$

```
time/(60*60*24*365) (*years*)
```

```
30.63814738
```

Appendix

Simulation program source code

In[93]:=

```
(*NotebookDelete[Cells[EvaluationNotebook[],GeneratedCell→True]];*)
Manipulate[
  tick;
  Module[
   {xE, yE, xJ, yJ, xN, yN, eq, sol, xcc, ycc, slope, eq1, eq2, debug = False, va, ra, z, delt, rbo, r, g0, now, x0, y0},

   If[(state == "RUN" || state == "STEP" || state == "INITIAL"),

    delt = Which[timeStep == "day", 60*60*24,
       timeStep == "week", 60*60*24*7,
       timeStep == "month", 60*60*24*30,
       timeStep == "year", 60*60*24*365
      ];

    xE = rES Cos[θE]; yE = rES Sin[θE];
    (*xJ=rJS Cos[θJ];yJ=rJS Sin[θJ];*)
    xJ = rJS Cos[θJx]; yJ = rJS Sin[θJx];
    xN = rNS Cos[θN]; yN = rNS Sin[θN];

    date = DaysPlus[date,
       Which[timeStep == "day", 1, timeStep == "week", 7, timeStep == "month", 30, timeStep == "year", 365]];
    now = Grid[{
        {"year", "month", "day"},
        {padIt2[date[[1]], 4], padIt2[date[[2]], 2], padIt2[date[[3]], 3]}
       }, Frame → All];

    If[showStats, g0 =
       Grid[{
         {Grid[{
             {Style["Timings and angles as simulation runs", Bold], SpanFromLeft},
             {"θE", "θJ", "θN", "θHohmann", "State", "Phase", "θ Hohmann"},
             {padIt2[θE*180./Pi, {5, 2}],
              (*padIt2[θJ*180./Pi,{5,2}],*)
              padIt2[θJx*180./Pi, {5, 2}],
              padIt2[θN*180./Pi, {5, 2}],
              padIt2[θEarthJupitor*180./Pi, {5, 2}],
              state,
              padIt2[phase, 1],
              padIt2[θEarthJupitor*180/Pi, {5, 2}]
             }
            }, Frame → All]
         },
         {Grid[{
             {Style["mean speeds (km/sec)", Bold], SpanFromLeft},
```

```
          {"Earth", "Jupiter", "Neptune", "probe to Neptune"},
          {padIt2[sE, {4, 2}],
            padIt2[sJ, {4, 2}],
            padIt2[sN, {4, 2}],
            padIt2[nJN * aJN, {4, 2}]
          }
        }, Frame → All]
    },
    {Grid[{
          {Style["Dimenstions data and current probe speed", Bold], SpanFromLeft},
          {"rES", "rJS", "rNS", "current ProbeSpeed (km/sec)"},
          {EngineeringForm[rES, 3],
            EngineeringForm[rJS, 3],
            EngineeringForm[rNS, 3],
            padIt2[currentProbeSpeed, {4, 2}]
          }
        }, Frame → All]
    },
    {Grid[{
          {Style["current positions in space", Bold], SpanFromLeft},
          {"xN", "yN", "x probe", "y probe", "dist. probe to Neptuen"},
          {EngineeringForm[xN, 3],
            EngineeringForm[yN, 3],
            EngineeringForm[x, 3],
            EngineeringForm[y, 3],
            EngineeringForm[EuclideanDistance[{xN, yN, 0}, {x, y, 0}], 3]
          }
        }, Frame → All]
    },
    {Grid[{
          {Style["Hohmann transfer from Earth to Jupiter data", Bold], SpanFromLeft},
          {"a", "rp", "ra", "e", "current f", "current E"},
          {EngineeringForm[aEJ, 3],
            EngineeringForm[rES, 3],
            EngineeringForm[rJS, 3],
            padIt2[eEJ, {6, 5}],
            padIt2[currentf * 180 / Pi, {5, 2}],
            padIt2[currentE * 180 / Pi, {5, 2}]
          }
        }, Frame → All]
    },
    {Grid[{
          {Style["hyperbolic Jupiter flyby", Bold], SpanFromLeft},
          {"V∞ (km/s)", "e Hyper", "η Hyper (deg)", "γd (degree)", "θturn", "Vd (km/s)"},
          {padIt2[vInfinityHyperJ, {6, 3}],
            padIt2[eHyperJ, {8, 7}],
            padIt2[ηHyperJ * 180 / Pi, {5, 3}],
            padIt1[γJN * 180 / Pi, {4, 1}],
            padIt1[θJNoriginal * 180 / Pi, {4, 1}],
            padIt1[vdJN, {5, 3}]}
        }, Frame → All]
    },
```

```
            {Grid[{
                {Style["Post fly-by ellipse, Jupiter to Neptune", Bold], SpanFromLeft},
                {"aJN", "rpJN", "raJN", "eJN (eccentricity)",
                  "f_new (true anamoly)", Style["mean probe speed deg/day", 9]},
                {EngineeringForm[aJN, 3],
                  EngineeringForm[rpJN, 3],
                  EngineeringForm[raJN, 3],
                  padIt2[eJN, {6, 5}],
                  padIt1[fJNoriginal * 180 / Pi, {4, 1}],
                  padIt1[nJN * 180 / Pi * 60 * 60 * 24, {6, 4}]
                }
              }, Frame → All]
            },
            {Grid[{{"current E (spacecraft)",
                  "current f",
                  "nHuhmannToJupiter (deg/day)",
                  "nJN(deg/day)"},
                {padIt1[currentE * 180 / Pi, {4, 1}],
                  padIt1[currentf * 180 / Pi, {4, 1}],
                  padIt1[nHuhmannToJupiter * 180 / Pi * 60 * 60 * 24, {6, 4}]}
              }, Frame → All]
            }
          }]
    ];

    g = Grid[{{Graphics[{
              (*{White,EdgeForm[Directive[Blue]],Disk[{0,0},rNS]},*)
              {White, Opacity[0], EdgeForm[Directive[Blue]], Disk[{0, 0}, rNS]},
              {White, Opacity[0], EdgeForm[Directive[Blue]], Disk[{0, 0}, rJS]},
              {White, Opacity[0], EdgeForm[Directive[Blue]], Disk[{0, 0}, rES]},
              {Blue, Opacity[.1], Thickness[0.022], EdgeForm[Gray], Circle[{0, 0}, rNS]},

              Which[phase == 0,
                {LightBlue, Opacity[.5], EdgeForm[Gray], Disk[{0, 0}, rJS, {θE, θE + θEarthJupiter }]}
                ,
                phase == 1,
                {
                  Clear[currentE];
                  currentE = currentE /. First@
                      Quiet[NSolve[tPhase1 == Sqrt[aEJ^3/μSun] (currentE - eEJ Sin[currentE]), currentE, Reals]];
                  currentE = Mod[currentE, 2 Pi];
                  currentR = aEJ * (1 - eEJ * Cos[currentE]);
                  x0 = aEJ Cos[currentE];
                  x0 = x0 - (aEJ - rES);
                  y0 = aEJ Sqrt[1 - eEJ^2] Sin[currentE];
                  r = RotationMatrix[-initialHohmann];
                  {x, y} = {x0, y0}.r;
```

```
      currentProbeSpeed = Sqrt[μSun (2/currentR - 1/aEJ)];

      tPhase1 = tPhase1 + delt;
      {
        (*Rotate[{Blue,Disk[{x,y},size]},initialHohmann,{0,0}],*)
        {Blue, Disk[{x, y}, size/4]},
        Rotate[{Red, Circle[{xc, yc}, {aEJ, bEJ}, {0, Pi}]}, initialHohmann, {0, 0}],
        Rotate[{Red, Dashed, Line[{{-aEJ (1 + eEJ), 0}, {aEJ (1 - eEJ), 0}}]}, initialHohmann, {0, 0}]
      }
    },
    phase == 2,
    {
      Clear[currentE];
      currentE = currentE /. First@
          Quiet[NSolve[ tPhase2 == Sqrt[aJN^3/μSun] (currentE - eJN Sin[currentE]), currentE, Reals]];
      currentE = Mod[currentE, 2 Pi];
      currentR = aJN*(1 - eJN*Cos[currentE]);
      x0 = aJN Cos[currentE];
      x0 = x0 - (aJN - rJS);
      y0 = aJN Sqrt[1 - eJN^2] Sin[currentE];
      r = RotationMatrix[-(θJForPhase2)];
      {x, y} = {x0, y0}.r;
      currentProbeSpeed = Sqrt[μSun (2/currentR - 1/aJN)];

      If[EuclideanDistance[{xN, yN, 0}, {x, y, 0}] < nSOI,
        state = "STOP"
      ];

      tPhase2 = tPhase2 + delt;

      {Blue, Disk[{x, y}, size/4]}, (*moving spacecraft*)
      {Blue, Dashed, Line[{{0, 0}, {x, y}}]}, (*moving spacecraft*)

      (*rendevouze location Jupiter and earth*)
      (*{Black,Disk[{rJS Cos[θJForPhase2+fJN],rJS Sin[θJForPhase2+fJN]},size]},*)
      {
        (*original Hohmann Jupiter earth*)
        Rotate[{Red, Circle[{xc, yc}, {aEJ, bEJ}, {0, Pi}]}, initialHohmann, {0, 0}],
        (*new ellipse post flyby*)
        Rotate[{Red, Dashed, Circle[{xc2, yc2}, {aJN, bJN}]}, θJForPhase2, {0, 0}]
      }
    }
  ],
  {Opacity[.4], Red, Disk[{xE, yE}, size]},
```

```
                {Opacity[.4], Red, Disk[{xJ, yJ}, size]},
                {Red, Disk[{xN, yN}, nSOI]}
              }
              , PlotRange → {{-maxX, maxX}, {-maxX, maxX}},
              If[showStats, ImageSize → 400, ImageSize → 600], Axes → True
            ]
          }}]
  ];

  If[state == "RUN" || state == "STEP",
    t = t + delt;

    θE = Mod[θE + ωE * delt, 2 Pi];
    θJx = Mod[θJx + ωJ * delt, 2 Pi];
    θN = Mod[θN + ωN * delt, 2 Pi];

    Which[phase == 0,
      If[Abs[(Mod[θE + θEarthJupitor, 2 Pi] - Mod[θJx, 2 Pi])] ≤ 5 Degree ,
        If[debug, Print["detected Hohmann lock in,Mod[θE+θEarthJupitor,2 Pi]=",
            Mod[θE + θEarthJupitor, 2 Pi], " Mod[θJx,2 Pi]=", Mod[θJx, 2 Pi]]];
        If[debug, Print["setting phase=1"]];
        phase = 1;
        aEJ = (rES + rJS)/2;
        eEJ = (rJS - rES)/(rES + rJS);
        bEJ = aEJ Sqrt[1 - eEJ^2] ;
        lockAngleWithJupiter = Mod[θE + Pi, 2 Pi];
        xf = rES Cos[θE];
        yf = rES Sin[θE];
        If[debug, Print["e Hohmann=", eEJ]];
        nHuhmannToJupiter = Sqrt[μSun/aEJ^3] ;
        initialHohmann = θE;
        xc = -aEJ eEJ;
        yc = 0;
        tPhase1 = 0
      ],
      phase == 1,
      If[Abs[lockAngleWithJupiter - θJx] ≤ Pi/100,
        phase = 2;
        va = Sqrt[μSun (2/rJS - 1/aEJ)] ; (*velocity of craft atJupiter entrance*)
```

```
vInfinityHyperJ = sJ − va;
rbo = rJ + SOIrb0; (*use this KM*)
Clear[vbo];
eq = vbo^2/2 − μJ/rbo == vInfinityHyperJ^2/2 − μJ/jSOI;
vbo = First@Select[vbo /. NSolve[eq, vbo], # > 0 &];
eHyperJ = 1 + (rbo vInfinityHyperJ^2)/μJ;
ηHyperJ = ArcCos[−1/eHyperJ];
θJN = 2 ηHyperJ − Pi;
θJNoriginal = θJN;
vdJN = Sqrt[(sJ^2 + vInfinityHyperJ^2 − 2 sJ vInfinityHyperJ Cos[θJN])];
Clear[z];
sol = Solve[Sin[z] == (vInfinityHyperJ Sin[θJN])/vdJN, z];
γJNoriginal = z /. sol;
γJN = z /. First@sol;
Clear[z];
eq = vdJN == Sqrt[μSun (2/rJS − 1/z)];
aJN = z /. First@NSolve[eq, z];
Clear[z];
eq = Cos[γJN] == Sqrt[(aJN^2 (1 − z^2))/(rJS (2 aJN − rJS))];
sol = NSolve[eq, z];
sol = z /. sol;
eJN = First@Select[sol, # > 0 &];
Clear[z];
eq = rJS == (aJN (1 − eJN^2))/(1 + eJN Cos[z]);
sol = z /. NSolve[eq, z];
fJNoriginal = sol;
fJN = If[γJN ≥ 0, First@Select[sol, # > 0 &], First@Select[sol, # < 0 &]];
rpJN = aJN (1 − eJN);
raJN = aJN (1 + eJN);
bJN = aJN Sqrt[1 − eJN^2];
θJForPhase2 = θJx − fJN;
xc2 = −aJN*eJN;
yc2 = 0;
```

```
          nJN = Sqrt[μSun / aJN^3];
          currentE = ArcCos[(1 - rJS/aJN) / eJN];
          currentf = 0;
          Clear[z];
          eq = Tan[fJN/2] == Sqrt[(1 + eJN)/(1 - eJN)] Tan[z/2];
          z = z /. First@NSolve[eq, z];
          Clear[tPhase2];
          tPhase2 = tPhase2 /. First@Quiet[NSolve[tPhase2 == Sqrt[aJN^3/μSun] (z - eJN Sin[z]), tPhase2, Reals]];
          If[debug, Print["currentE for JN is =", currentE * 180/Pi]];
        ](*Entered SOI jupiter*)
      ]
    ];

    If[state == "RUN",
      vp[[vpIdx, 1]] = t;
      vp[[vpIdx, 2]] = currentProbeSpeed;
      vpIdx++;
      tick = Not[tick]
    ];

    If[showStats,
      Grid[{{g0}, {g}, {now}}],
      Grid[{{g}, {now}}]
    ]

  ],
  Grid[{
    {Grid[{
        {Button[Text[Style["run", 12]], state = "RUN"; tick = Not[tick], ImageSize → {60, 35}],
         Button[Text[Style["step", 12]], state = "STEP"; tick = Not[tick], ImageSize → {60, 35}],
         Button[Text[Style["stop", 12]], state = "STOP"; tick = Not[tick], ImageSize → {60, 35}]
        }}, Frame → True],
     Grid[{
        {"zoom",
         PopupMenu[Dynamic[zoom, {zoom = #; Which[zoom == "Earth", maxX = 1.2 rES, zoom == "Jupiter",
                maxX = 1.2 rJS, True, maxX = 1.2 rNS]; tick = Not[tick]} &], {"Earth", "Jupiter", "Neptune"},
           ImageSize → Tiny, ContinuousAction → False]
        }
       }, Frame → True],
     Grid[{{
         "jupiter flyby alt",
         Manipulator[Dynamic[SOIrb0, {SOIrb0 = #; tick = Not[tick]} &],
```

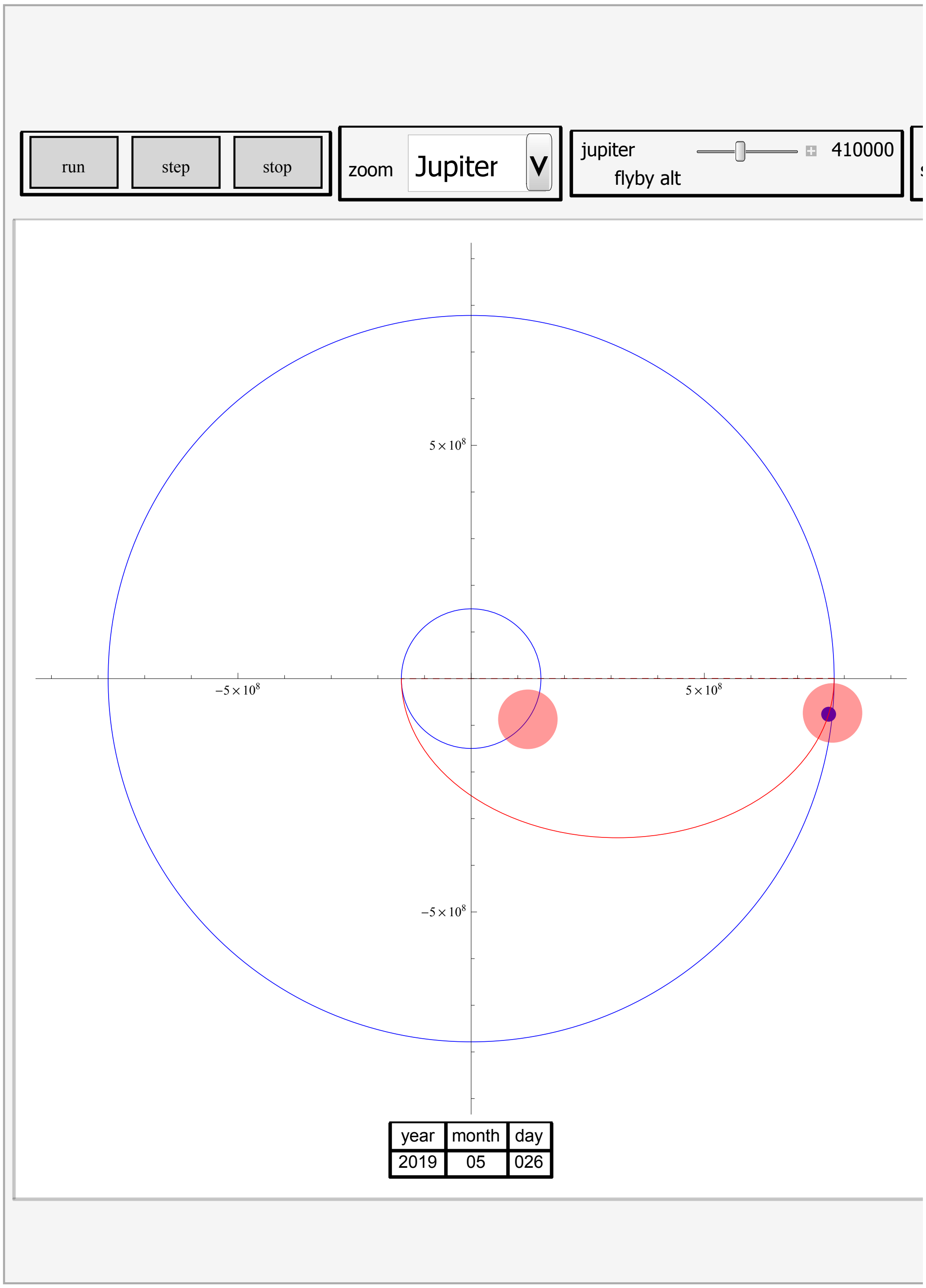
```
                {1000, 10^6, 1000}, ImageSize → Tiny, ContinuousAction → True],
              Dynamic[padIt2[SOIrb0, 6]]
            }}, Frame → True],
      Grid[{
          {"step",
              PopupMenu[Dynamic[timeStep, {timeStep = #; tick = Not[tick]} &], {"day", "week", "month", "year"},
                ImageSize → Tiny, ContinuousAction → False]
          }
        }, Frame → True],
      Grid[{
          {"relax", Spacer[2], Checkbox[Dynamic[showStats, {showStats = #; tick = Not[tick]} &]]}
        }]
    }
  }, Spacings → {0.4, .2}, Alignment → Center
],

{{showStats, False}, None},
(*hyper flyby Jupiter parameters*)
{{eHyperJ, 0}, None},
{{ηHyperJ, 0}, None},
{{vInfinityHyperJ, 0}, None},

{{x, 0}, None},
{{y, 0}, None},

{{maxX, 1.1 rNS}, None},
{{zoom, "Neptune"}, None},
{{SOIrb0, 410 000}, None},
(*{{SOIrb0,395000},None},*)
(*{{SOIrb0,390000},None},*)
{{timeStep, "week"}, None},
{{size, 10 000 rE}, None},
{{tick, False}, None},
{{state, "INITIAL"}, None},
{{phase, 0}, None},
(*set  03/21/2014 θE=180+90, θJ=111.30+90, θN=334.9963+90 *)
(*{{θJ,(111.30 +90)Degree},None},
{{θN,Mod[(334.9963 +90),360]Degree},None},
{{θE,270 Degree},None},*)

(*set 09/21/2014 very close *)
(*{{θJx,(126.2818+90)Degree},None},
{{θN,Mod[(336.1014+90),360]Degree},None},
{{θE,Mod[356.7575+90,360] Degree},None},*)


(*set 10/21/2014 very very close=========> *)
(*{{θJx,(128.6882+90)Degree},None},
{{θN,Mod[(336.2816+90),360]Degree},None},
{{θE,Mod[26.3119+90,360] Degree},None},*)

(*set 10/01/2014 ok, with 340,000 *)
```

```
(*{{θJx,(127.1652+90)Degree},None},
{{θN,Mod[(336.167+90),360]Degree},None},
{{θE,Mod[7.5386+90,360] Degree},None},*)

(*set 03/21/2016 *)
{{θJx, (169 + 90) Degree}, None},
{{θN, Mod[(339 + 90), 360] Degree}, None},
{{θE, 270 Degree}, None},

(*set 03/15/2016 *)
(*{{θJx,(168.5740+90)Degree},None},
{{θN,Mod[(339.3579+90),360]Degree},None},
{{θE,(174.6131+90) Degree},None},*)


(*set 03/30/2016 *)
(*{{θJx,(169.7170+90)Degree},None},
{{θN,Mod[(339.4480+90),360]Degree},None},
{{θE,(189.4931+90) Degree},None},*)

(*set 04/15/2016 *)
(*{{θJx,(170.9351+90)Degree},None},
{{θN,Mod[(339.5442+90),360]Degree},None},
{{θE,(205.2323+90) Degree},None},*)

(*set 05/15/2016 *)
(*{{θJx,(173.2164+90)Degree},None},
{{θN,Mod[(339.7246+90),360]Degree},None},
{{θE,(234.3716+90) Degree},None},*)

(*set 06/15/2016 *)
(*{{θJx,(175.5701+90)Degree},None},
{{θN,Mod[(339.9109+90),360]Degree},None},
{{θE,(264.1023+90) Degree},None},*)

(*set 01/01/2016 *)
(*{{θJx,(162.9198+90)Degree},None},
{{θN,Mod[(338.9131+90),360]Degree},None},
{{θE,(99.7590+90) Degree},None},*)

(*set 03/21/2000 θE=179.5877, θJ=43.4305+90, θN=304.3955+90*)
(*{{θJ,(43.4305+90)Degree},None},
{{θN,Mod[(304.3955+90),360]Degree},None},
{{θE,270 Degree},None},*)


(*set 03/21/2017 *)
(*{{θJ,(196.5839 +90)Degree},None},
{{θN,Mod[(341.5831 +90),360]Degree},None},
{{θE,270 Degree},None},*)

(*set 03/21/2020  OK *)
```

```
(*{{θJ,Mod[(282.3034+90),360]Degree},None},
{{θN,Mod[(348.1929 +90),360]Degree},None},
{{θE,270 Degree},None},*)

(*{{θJx,Pi/4},None},
{{θJxx,45},None},*)

{{t, 0}, None},
{{tPhase1, 0}, None},
{{tPhase2, 0}, None},
{{date, {2016, 03, 21}}, None},
{{a, 0}, None},
{{e, 0}, None},
{{nHuhmannToJupiter, 0}, None},
{{nJN, 0}, None},
{{currentE, 0}, None},
{{currentf, 0}, None},
{{initialHohmann, 0}, None},
{{xf, 0}, None},
{{yf, 0}, None},
{{xc, 0}, None},
{{yc, 0}, None},
{{xc2, 0}, None},
{{yc2, 0}, None},
{{currentR, 0}, None},
{{lockAngleWithJupiter, 0}, None},
{{lockAngleWithNeputon, 0}, None},
{{aJN, 0}, None},
{{raJN, 0}, None},
{{rpJN, 0}, None},
{{bJN, 0}, None},
{{eJN, 0}, None},
{{fJN, 0}, None},
{{fJNoriginal, 0}, None},

{{aEJ, 0}, None},
{{bEJ, 0}, None},
{{eEJ, 0}, None},
{{fEJ, 0}, None},
{{θJForPhase2, 0}, None},

{{γJN, 0}, None},
{{γJNoriginal, 0}, None},
{{θJN, 0}, None},
{{θJNoriginal, 0}, None},
{{vdJN, 0}, None},
{{currentProbeSpeed, 0}, None},
{{g, 0}, None},
{{g0, 0}, None},
{{vp, Table[{0, 0}, {50*365}]}, None},
{{vpIdx, 1}, None},
TrackedSymbols :→ {tick},
```

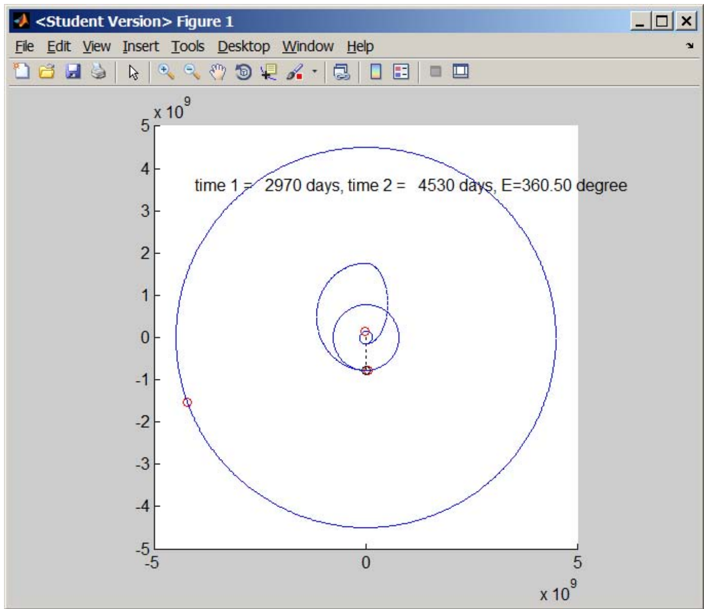
```
ControlPlacement → Top,
Initialization :→
  (
    integerStrictPositive = (IntegerQ[#] && # > 0 &);
    integerPositive = (IntegerQ[#] && # ≥ 0 &);
    numericStrictPositive = (Element[#, Reals] && # > 0 &);
    numericPositive = (Element[#, Reals] && # ≥ 0 &);
    numericStrictNegative = (Element[#, Reals] && # < 0 &);
    numericNegative = (Element[#, Reals] && # ≤ 0 &);
    bool = (Element[#, Booleans] &);
    numeric = (Element[#, Reals] &);
    integer = (Element[#, Integers] &);
    (*------------------------------------------*)
    padIt1[v_?numeric, f_List] := AccountingForm[v,
        f, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True];
    (*------------------------------------------*)
    padIt1[v_?numeric, f_Integer] := AccountingForm[Chop[v],
        f, NumberSigns → {"-", "+"}, NumberPadding → {"0", "0"}, SignPadding → True];
    (*------------------------------------------*)
    padIt2[v_?numeric, f_List] := AccountingForm[v,
        f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True];
    (*------------------------------------------*)
    padIt2[v_?numeric, f_Integer] := AccountingForm[Chop[v],
        f, NumberSigns → {"", ""}, NumberPadding → {"0", "0"}, SignPadding → True]
  )
]
```

Out[93]=


run
step
stop
zoom
Jupiter
jupiter
flyby alt
410000
5 × 10^8
−5 × 10^8
5 × 10^8
−5 × 10^8
year
month
day
2019
05
026

2.2.4 Matlab code

I do not now remember why I wrote this for. I think it was an initial attempt in Matlab, because in the final report I used Mathematica. But here is the listing. It seems to be do something. I should make an animation of this.



nma_project2_EMA550_v3.m

```
function nma_project2_EMA550_v3

close all;
MODE=1;

earthToSun = 1.495978*10^8;
jupiterToSun = 1.495978*10^8*5.203;
neptuneToSun = 30.07*1.495978*10^8;
jupiterR=71492;
muSun = 1.327*10^11;
muJupiter = 126686534;
jupiterSOI=4.83*10^7;
rE = 6378;
earthCurrentP=10*pi/180;
earthInitialP=10*pi/180;
neptuneCurrentP=mod((335.0023 + 180), 360)*pi/180;
neptuneInitialP=mod((335.0023 + 180), 360)*pi/180;
jupiterCurrentP=(1)*pi/180;
jupiterInitialP=(1)*pi/180;

if jupiterCurrentP<earthInitialP
    initialPhasEarthJupitor=2*pi-(earthInitialP-jupiterCurrentP);
```

```
else
    initialPhasEarthJupitor=jupiterCurrentP-earthInitialP;
end

if MODE==0
    p=hohmannPeriod(earthToSun, jupiterToSun, muSun);
    a=semiMajor(earthToSun, jupiterToSun);
    e=hohmannEnergy(earthToSun, jupiterToSun, muSun);
    a=hohmannAngle(earthToSun, jupiterToSun);
    v=vperigee(earthToSun, jupiterToSun, muSun);
    w=angularVelocity(earthToSun,  muSun);
    p=updatePosition(earthToSun,0,1, muSun);
    [rt,transferTime]=biEllipticalTransfer(rE,earthToSun,  ...
                      jupiterToSun,initialPhasEarthJupitor,muSun)
end

[rTransferToJupiter,transferTime]=biEllipticalTransfer(...
  2*rE,earthToSun, jupiterToSun,initialPhasEarthJupitor,muSun);

huhmannToJupiterSemiMajor1 = (earthToSun + rTransferToJupiter)/2;
eHuhmannToJupiter1 = (rTransferToJupiter - earthToSun)/...
                        (rTransferToJupiter + earthToSun);

nHuhmannToJupiter1 = 1/ sqrt(huhmannToJupiterSemiMajor1^3/muSun);%rad/sec
periodhuhmannToJupiter1 = 2*pi/nHuhmannToJupiter1;

huhmannToJupiterSemiMajor2 = (rTransferToJupiter + jupiterToSun)/2;
if rTransferToJupiter<jupiterToSun
    eHuhmannToJupiter2 = (jupiterToSun-rTransferToJupiter)/...
                             (rTransferToJupiter + jupiterToSun);
else
    eHuhmannToJupiter2 = (rTransferToJupiter-jupiterToSun)/...
                             (rTransferToJupiter + jupiterToSun);
end
nHuhmannToJupiter2 = 1/ sqrt(huhmannToJupiterSemiMajor2^3/muSun);%rad/sec*)
periodhuhmannToJupiter2 = 2*pi/nHuhmannToJupiter2;

wEarth = angularVelocity(earthToSun, muSun);
wJupiter = angularVelocity(jupiterToSun, muSun);
wNeptune = angularVelocity(neptuneToSun, muSun);
hohmannAngleJupiter = hohmannAngle(earthToSun, jupiterToSun);
hohmannAngleNeptune = hohmannAngle(earthToSun, neptuneToSun);
ndays1=0;
ndays2=0;
ndays3=0;

currentTimeInSec=0;
```

```
time1=0;
time2=0;
time3=0;

currentF=0;
currentE=0;

figure;
axis square
hold on;
syms EE currentTheta;
firstTime=true;
firstTimeFlyBy=true;
stepSize=60*60*24*60; %month
doneLoop=false;

for i=0:10000


    if  currentTimeInSec < periodhuhmannToJupiter1/2
        ndays1=currentTimeInSec/(60*60*24);
        currentE = nHuhmannToJupiter1*currentTimeInSec;
        currentR = huhmannToJupiterSemiMajor1*...
                        (1 - eHuhmannToJupiter1*cos(currentE));

        eq = cos(currentTheta) == (eHuhmannToJupiter1 - ...
           cos(currentE))/(eHuhmannToJupiter1*cos(currentE) - 1);

        solCurrentTheta =  double(vpa(solve(eq, currentTheta)));
        solCurrentTheta = solCurrentTheta(solCurrentTheta==...
                                        real(solCurrentTheta));

        solCurrentTheta = solCurrentTheta(solCurrentTheta>0);
        solCurrentTheta = min(solCurrentTheta)+earthInitialP;
    else %on second ellipse, long one
        if currentTimeInSec >= periodhuhmannToJupiter1/2 && ...
                                  time2<periodhuhmannToJupiter2

            time2 = time2 +stepSize;
            if firstTime
                time2=time2+periodhuhmannToJupiter2/2;
                firstTime=false;
            end
            ndays2=(time2-periodhuhmannToJupiter2/2)/(60*60*24);
            currentE = nHuhmannToJupiter2*time2;
            currentR = huhmannToJupiterSemiMajor2*...
                             (1 - eHuhmannToJupiter2*cos(currentE));
```

```
            eq = cos(currentTheta) == (eHuhmannToJupiter2 - ...
           cos(currentE))/(eHuhmannToJupiter2*cos(currentE) - 1);

            solCurrentTheta = double(vpa(solve(eq, currentTheta)));

            solCurrentTheta = solCurrentTheta(solCurrentTheta==...
                                  real(solCurrentTheta));

            z=solCurrentTheta(solCurrentTheta<0);
            if length(z)>=1
                if abs(z)<pi
                    z=2*pi+z;
                else
                    z=pi-z;
                end
            else
                z=max(solCurrentTheta);
            end
            solCurrentTheta = z+earthInitialP;
        else %third legg calculate flyby
            if firstTimeFlyBy
                firstTimeFlyBy=false;

                %this assume jupiter is at perigee!!
                vp=sqrt(muSun*(2/jupiterToSun  - 1/...
                     huhmannToJupiterSemiMajor2));

                vJupiter = sqrt(muSun/jupiterToSun);
                vinf= abs(vp-vJupiter);
                rbo=500+jupiterR;
                vbo= sqrt(2*( muJupiter/rbo + vinf^2/2 - ...
                                  muJupiter/jupiterSOI));

                eHyper=1+ (rbo*vinf^2)/muJupiter;

                eta = acos(-1/eHyper);
                theta=2*eta-pi;

                %vD = sqrt(vJupiter^2+vinf^2-2*vJupiter*vinf*cos(theta));
                vD = sqrt(vp^2+vinf^2-2*vp*vinf*cos(theta));

                gamma=asin(vinf*sin(theta)/vD);

                syms anew;
                eq=vD==sqrt(muSun*(2/jupiterToSun - 1/anew));
                anew=double(solve(eq,anew));

```

```
                syms e;
                eq=cos(gamma)==sqrt((anew^2*(1-e^2))/...
                            (jupiterToSun*(2*anew-jupiterToSun)));

                sol=double(solve(eq,e));
                e=sol(sol>0);

                syms fNew;
                if e>1
                   %find true anamoly in the new hyperbola
                   eq= jupiterToSun == abs(anew)*(e^2-1)/...
                                                (1+e*cos(fNew));

                   sol=double(solve(eq,fNew));
                   fNew=sol(sol>0);
                   rpNew=abs(anew)*(e-1);
                   bNew = sqrt(anew^2*(e^1-1));

                   %http://mathforum.org/kb/message.jspa?messageID=6230348
                   plot_hyper(fNew,bNew,anew,e,neptuneToSun);
                else
                   eq = tan(gamma) == e*sin(fNew)/(1+e*cos(fNew));
                   sol=double(solve(eq,fNew));
                   z2=max(real(sol));
                   rpNew=anew*(1-e);
                   bNew=anew*sqrt(1-e^2);
                   raNew=2*anew-rpNew;
                   plot_ellipse(fNew,bNew,anew,e,neptuneToSun,x2,y2);
                end

            end
            if time2>periodhuhmannToJupiter2
                doneLoop=true;
            end
        end
    end


    earthCurrentP = mod((wEarth*...
                           currentTimeInSec+earthInitialP),2*pi);

    jupiterCurrentP = mod( wJupiter*currentTimeInSec+...
                                          jupiterInitialP, 2*pi);

    neptuneCurrentP = mod( wNeptune* currentTimeInSec+...
                                          neptuneInitialP, 2*pi);

```

```
    x1 = earthToSun*cos(earthCurrentP );
    y1 = earthToSun*sin(earthCurrentP);
    x2 = jupiterToSun*cos(jupiterCurrentP);
    y2 = jupiterToSun*sin(jupiterCurrentP);
    x3 = neptuneToSun*cos(neptuneCurrentP);
    y3 = neptuneToSun*sin(neptuneCurrentP);
    ej = abs(earthCurrentP - jupiterCurrentP);
    en = abs(earthCurrentP - neptuneCurrentP);

    plot( earthToSun*exp((0:.01:2*pi)*1i));
    h1=plot(x1,y1,'or');
    h2=plot(x2,y2,'or');
    if i==0
        plot([0 5.5*x1],[0 5.5*y1],':k');
    end
    plot( jupiterToSun*exp((0:.01:2*pi)*1i));
    h3=plot(x3,y3,'or');

    plot( neptuneToSun*exp((0:.01:2*pi)*1i));
    plot(currentR*cos(solCurrentTheta),currentR*...
         sin(solCurrentTheta),'ob','LineWidth',1,'MarkerSize',1);

    h4=plot(currentR*cos(solCurrentTheta),currentR*...
                                      sin(solCurrentTheta),'ok');

    h5=text(-.9*neptuneToSun,.8*neptuneToSun,...
        sprintf('time 1 = %6.0f days, time 2 = %6.0f days, E=%5.2f degree',...
                                ndays1, ndays2,currentE*180/pi));

    pause(.01);
    if ~doneLoop
        delete(h1);delete(h2);delete(h3);delete(h4); delete(h5);
    else
        break;
    end
    currentTimeInSec = currentTimeInSec+stepSize;
    %hold off;


end
end

function p=hohmannPeriod(r1, r2, mu)
a = semiMajor(r1, r2);
p=2*pi*sqrt(a^3/mu);
end
```

```
function a=semiMajor(r1, r2)
a=(r1 + r2)/2;
end

function e=hohmannEnergy(r1, r2, mu)
e= -mu/(r1 + r2);
end

function a=hohmannAngle(sourceR, targetR)
a=pi*(1 - ((sourceR + targetR)/(2*targetR))^(3/2));
end


function v=vperigee(sourceR, targetR, mu)
a = semiMajor(sourceR, targetR);
v=sqrt(mu*(2/sourceR - 1/a));
end

function w=angularVelocity(r, mu)
w = sqrt(mu/r^3);
end


function v=linearVelocity(r, mu)
v = sqrt(mu/r);
end

function p=updatePosition(r, currentPos, nDays, mu)
w = angularVelocity(r, mu);
w = w*60*60*24; %convert to radians per day
p=currentPos + (w*nDays*180/pi);
end

function [sol, t1]=biEllipticalTransfer(rMin,sourceR,...
                                        targetR,initialTheta,mu)

hTheta = hohmannAngle(sourceR, targetR);
syms rt;
a1 = (sourceR + rt)/2;
a2 = (targetR + rt)/2;
t1 =  ((rt+sourceR)/2)^(3/2) + ((rt+targetR)/2)^(3/2) ;
wUpper = angularVelocity(targetR, mu);

n = 0;
if initialTheta < hTheta
    t2  = ( (2*pi*(n+1) - initialTheta )*targetR^(3/2) /pi);
    sol = double(vpa(solve(t1==t2, rt)));
```

```
    sol = sol(sol==real(sol));
    sol = sol(sol>rMin&sol>targetR);
    sol = min(sol);
else
    sol=0;
    n=1;
    foundSolution=false;
    while n<10 && ~foundSolution
        t2  = ( (2*pi*(n+1) - initialTheta )*targetR^(3/2) /pi);
        sol = solve(t1 == t2, rt);
        sol = double(vpa(solve(t1==t2, rt)));
        sol = sol(sol==real(sol));
        sol = sol(sol>rMin);
        if length(sol)>=1
            sol = min(sol);
            foundSolution=true;
        else
            n=n+1;
        end
    end
end

t1=double(vpa(subs(t1,rt,sol)));

end

function  plot_hyper(f,b,a,e,neptuneToSun)
%Q=[cos( fNew) -sin( fNew);sin( fNew) cos( fNew)];
syms y x;
c=a*e;

ezplot( (x-c)^2/a^2 - y^2/b^2 - 1,[-neptuneToSun neptuneToSun-...
                                      neptuneToSun neptuneToSun]);
end

function plot_ellipse(f,b,a,e,neptuneToSun,x2,y2)
syms y x;
c=a*e;

ezplot( (x-x2)^2/a^2 + (y-y2)^2/b^2 - 1 ,...
       [-neptuneToSun neptuneToSun -neptuneToSun neptuneToSun]);
end
```

nma_project2_EMA550_driver

```

clear all;
syms rt;
```

```
t1=pi*(((rt/2 + 74798900)^3/132700000000)^(1/2) + ...
       ((rt/2 + 1632333680397517/4194304)^3/132700000000)^(1/2));
t2=3.7455e+08;
sol1=solve(t1 == t2, rt)
sol2=solve(t1 == t2, rt)
```

Chapter 3

Lunar project

Local contents

3.1 project description

From the Earth to the Moon

EMA 550 Astrodynamics - Spring 2014

Due Date: Thursday, April 3, 2014 (PDF of report to online Lunar Project dropbox by 11:55 pm)

Your job is to design a variety of trajectories from the Earth to the Moon. Submit a detailed and well-written technical report with the parts specified below. For each part, your report should describe the maneuver and answer any questions asked in full paragraphs. Include all requested illustrations. Show clearly how you arrived at your answers so that you could easily reference this document again in the future and follow your steps again. **Please complete this project in pairs and submit one report for the team.**

For all parts of the project, assume the following regarding the Moon and the Moon's orbit:

Radius of the Moon: r_{Moon} = 1738 km
Gravitational parameter of the Moon: μ_{Moon} = 4902.8 km^3/s^2
Moon's sphere of influence radius: 6.6 x 10^4 km
Moon's orbit about the Earth:
a = 384,400 km
e = 0 (actual mean eccentricity = 0.05490)
i = 23.5° relative to the Earth's equatorial plane (average of its range from 18° to 29°)
ω undefined because of circular orbit assumption
Ω = 0° (oscillates ±14° about Ω = 0° with a period of 18.6 years)

For all parts of the project, assume that the Moon orbits the Earth's gravitational and geometrical center and that the Earth is gravitationally and geometrically spherically symmetric with r_{Earth} = 6378 km and μ_{Earth} = 3.986 x 10^5 km^3/s^2.

Also assume that the spacecraft starts in a 300 km altitude circular orbit about the Earth (LEO) in the same plane and in the same direction as the Moon's orbit about the Earth.

Part I: Hohmann Transfer

Mission: Find a Hohmann transfer from a 300 km altitude initial circular orbit about the Earth to a circular orbit about the Earth at the same distance as the Moon's orbit.

Details to include:

1) Report the semi-major axis and eccentricity of the Hohmann transfer orbit.
2) Report the ΔV for each burn and the total ΔV required.
3) Report the transfer time required for the transfer (in days)

Part II: Tangential Flyby

Mission: With a single ΔV in LEO, perform a Hohmann transfer from LEO to the vicinity of the Moon, performing a close flyby of the surface of the Moon. According to the JPL Lunar Constants and Models Document, the highest peak on the Moon's surface is 8 km above an average spherical radius of 1737.4 km. For safety considerations, set the burnout radius for the close flyby to 1760 km.

Details to include:

1) Calculate the impact parameter required to achieve a lunar burnout radius of 1760 km.
2) Include a Moon-centered figure that shows the hyperbolic flyby of the spacecraft in the Moon's frame of reference. The Moon's radius, the impact parameter, and the burnout radius should all be shown to scale relative to each other. Include the turning angle of the asymptotes. The curved part of the hyperbola may be approximated, but should connect the asymptotes and the burnout radius.
3) Assuming that the spacecraft approaches the Moon on the side between the Moon and the Earth, calculate the *a* and *e* of the spacecraft's orbit relative to the Earth after it leaves the Moon's sphere of influence.
4) Calculate the true anomaly *f* of the Moon's position (that is, the position it shares with the spacecraft from the Earth's perspective during the flyby) on the spacecraft's post-flyby orbit about the Earth. Use this true anomaly to locate perigee of the post-flyby orbit.
5) Include a figure that shows the velocity triangles for the flyby ($V_{\text{Moon wrt Earth}}$, $V_{\text{Arrival wrt Earth}}$, $V_{\text{Departure wrt Earth}}$, $v_{\infty \text{ in wrt Moon}}$, $v_{\infty \text{ out wrt Moon}}$, turning angle θ).
6) Include an Earth-centered figure that shows the LEO orbit, the Moon's orbit, the Hohmann trajectory from LEO to the Moon, and the post-tangential-flyby orbit to scale with accurate sizes and shapes.

Part III: Non-Tangential Flyby

Mission: With a single ΔV in LEO, send the spacecraft on a transfer ellipse that is tangent to LEO and has a semi-major axis equal to 300,000 km. Perform the same close flyby of the lunar surface.

Details to include:

1) Assuming that the spacecraft flies behind the Moon at the intersection of the two orbits where $0 \leq f \leq 180°$ on the pre-flyby ellipse, calculate the *a* and *e* of the spacecraft's orbit relative to the Earth after the flyby.
2) Calculate the true anomaly *f* of the Moon's position on the post-flyby orbit about the Earth.
3) Include a figure that shows the velocity triangles for the flyby.
4) Repeat steps 1-3 assuming that the spacecraft flies in front of the Moon.

Part IV: Free-Return Trajectory

Mission: Create a trajectory that uses a single burn in Earth LEO to reach the Moon, performs the same close flyby of the Moon (same burnout radius), and achieves a post-flyby elliptical orbit about the Earth with a perigee radius between 6678 km and 6878 km (300 to 500 km altitude). This is called a *free-return trajectory* because the spacecraft reaches the Moon and returns to the Earth without needing to burn fuel for the return trip. For an animated illustration of a free-return lunar trajectory, see http://www.braeunig.us/apollo/free-return.htm.

To accomplish this automatic return, you get to choose the size and shape of the pre-flyby trajectory and the arrival position with respect to the Moon (between the Moon and the Earth, outside the Moon's orbit, fly behind the Moon, fly in front of the Moon).

Details to include:

1) Describe any assumptions or design decisions used to limit the available variables.
2) Determine the *a* and *e* of the initial orbit and the ΔV needed in LEO to start the maneuver.
3) Describe the arrival position with respect to the Moon that you chose and illustrate it using velocity triangles.
4) Show that the spacecraft will return to the required perigee without any burns beyond the one required to start the transfer.

Part V: Rendezvous and Timing Considerations

Mission: Calculate the timing and positions required for your free-return trajectory.

Details to include:

1) Treating the SOI of the Moon as a single point at the location of the Moon, how long does your spacecraft take to reach the Moon (i.e., what is the transfer time on the pre-flyby piece of your free-return trajectory)?
2) What angle must your spacecraft and the Moon have relative to each other at the time of the LEO ΔV in order for the Moon to be at the required location at the time of the flyby?
3) How often do the spacecraft in LEO and the Moon have the correct alignment?
4) The patched conic approach treats the flyby as an instantaneous ΔV from the Earth's frame of reference. Evaluate and discuss this assumption. How long does the flyby really take (i.e., how long is the spacecraft within the Moon's SOI)? How does the time in the SOI compare to the total time required for the trip (time to get to the Moon plus the flyby time and the return time)? What percentage of an orbit does the Moon complete during the time that the spacecraft is within the Moon's SOI?

3.2 fact check

This is a check on some selected values for the first parts of the Lunar Project. There are multiple solutions to the fourth part, the free-return trajectory, but you can use this worksheet to verify your code for the first three parts. This is entirely optional and not part of your project grade.

NOTE: The write-up requests more values than those shown here.

3.2.1 Part I: Hohmann

Semi-major axis: Answer km

Total ΔV Answer km/s

3.2.2 Part II: Tangential Flyby

Turning angle of the asymptotes: Answer degrees

Speed after the flyby relative to the Earth: Answer km/s

Eccentricity on post-flyby trajectory: Answer

3.2.3 Part III: Non-Tangential Flyby Behind the Moon

Turning angle of the asymptotes: Answer degrees

Speed after the flyby relative to the Earth: Answer km/s

Eccentricity on post-flyby trajectory: Answer

3.2.4 Part III: Non-Tangential Flyby In Front of the Moon

Turning angle of the asymptotes: Answer degrees

Speed after the flyby relative to the Earth: Answer km/s

Eccentricity on post-flyby trajectory: Answer

3.3 report

3.3.1 Part 1, Hohmann Transfer

3.3.2 problem description

Mission: Find a Hohmann transfer from a 300 km altitude initial circular orbit about the Earth to a circular orbit about the Earth at the same distance as the Moon's orbit.

Details to include:

1. Report the semi-major axis and eccentricity of the Hohmann transfer orbit.
2. Report the DV for each burn and the total DV required.
3. Report the transfer time required for the transfer (in days)

3.3.2.1 part 1

Figure 3.1 shows the steps used. The satellite perigee r_p is found from

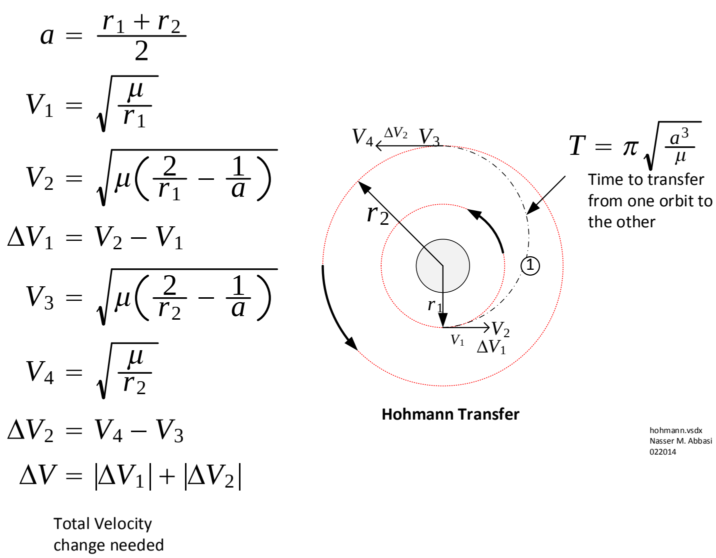


Figure 3.1: Steps to preform Hohmann orbit transfer

$$
\begin{aligned}
r_p &= r_{earth} + \text{alt} \\
&= 6378 + 300 \\
&= 6678\,\text{km}
\end{aligned}
$$

The apogee distance r_a is the moon's distance from center of earth given by $r_a = 384400$ km. Therefore the semi-major axis a is

$$
\begin{aligned}
a &= \frac{r_a + r_p}{2} \\
&= \frac{384400 + 6678}{2} \\
&= \boxed{195539\,\text{km}}
\end{aligned}
$$

The eccentricity e is

$$
\begin{aligned}
e &= \frac{r_a - r_p}{r_a - r_p} \\
&= \frac{384400 + 6678}{384400 - 6678} \\
&= \boxed{0.96585}
\end{aligned}
$$

3.3.2.2 part 2

V_1 is the spacecraft velocity in LEO and is given by

$$
\begin{aligned}
V_1 &= \sqrt{\frac{\mu_{earth}}{r_p}} \\
&= \sqrt{\frac{3.986 \times 10^5}{6678}} \\
&= \boxed{7.7258\,\text{km per second}}
\end{aligned}
$$

The spacecraft required speed at perigee of the Hohmann transfer orbit V_p is

$$
\begin{aligned}
V_p &= \sqrt{\mu_{earth}\left(\frac{2}{r_p} - \frac{1}{a}\right)} \\
&= \sqrt{3.986 \times 10^5 \left(\frac{2}{6678} - \frac{1}{195539}\right)} \\
&= \boxed{10.8323\,\text{km per second}}
\end{aligned}
$$

Since the moon is inside the sphere of influence of the earth, the difference of the above two speeds is all that is needed to send the spacecraft to the moon using a Hohmann orbit. Therefore

$$
\begin{aligned}
\Delta V_1 &= V_p - V_1 \\
&= 10.8323 - 7.7251 \\
&= \boxed{3.1065\,\text{km per second}}
\end{aligned}
$$

When the spacecraft reaches the apogee of the Hohmann orbit, its speed V_a will be

$$
\begin{aligned}
V_a &= \sqrt{\mu_{earth}\left(\frac{2}{r_a} - \frac{1}{a}\right)} \\
&= \sqrt{3.986 \times 10^5 \left(\frac{2}{384400} - \frac{1}{195539}\right)} \\
&= \boxed{0.1882\,\text{km per second}}
\end{aligned}
$$

The required speed V_2 to put the satellite in the moon's circular orbit is

$$
\begin{aligned}
V_2 &= \sqrt{\frac{\mu_{earth}}{r_a}} \\
&= \sqrt{\frac{3.986 \times 10^5}{3844008}} \\
&= \boxed{1.0183\ \ \text{km per second}}
\end{aligned}
$$

Therefore the impulse needed is

$$\begin{aligned}\Delta V_2 &= 1.0183 - 0.1882\\ &= \boxed{0.83\,\text{km per second}}\end{aligned}$$

The total ΔV is found from

$$\begin{aligned}\Delta V &= |\Delta V_1| + |\Delta V_2|\\ &= 3.1065 + 0.83\\ &= \boxed{3.937\,\text{km per second}}\end{aligned}$$

3.3.2.3 part 3

The transfer time ΔT in seconds from the earth's LEO orbit to the moon's circular orbit is half the period of the Hohmann ellipse. Therefore

$$\begin{aligned}\Delta T &= \pi\sqrt{\frac{a^3}{\mu_{earth}}}\\ &= \pi\sqrt{\frac{195539^3}{3.986\times 10^5}}\\ &= 4.3026e5\,\text{second}\\ &= \boxed{4.9798\,\text{day}}\end{aligned}$$

Figure 3.2 shows the final orbit which is to scale and was generated from STK.

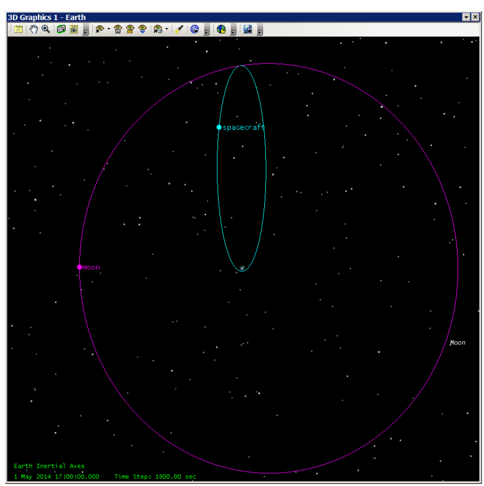


Figure 3.2: Hohmann orbit to scale from STK

3.3.3 Part II, Tangential flyby

The following parameters are used in the calculations that follows

$$
\begin{aligned}
\mu_{earth} &= 3.986e5\,\text{km}^3 \text{ per second squared}\\
\mu_{moon} &= 4902.8\,\text{km}^3 \text{ per second squared}\\
r_a &= 384400\,\text{km}\\
r_{earth} &= 6378\,\text{km}\\
r_{moon} &= 1737.4\,\text{km}\\
r_{bo} &= 1760\,\text{km}\\
SOI_{moon} &= 6.61e4\,\text{km}
\end{aligned}
$$

Figure 3.3 shows a more detailed Hohmann transfer orbit used as a guide in the calculations that follows. This diagram is not drawn to scale. A diagram drawn to scale is given at the end of this section.

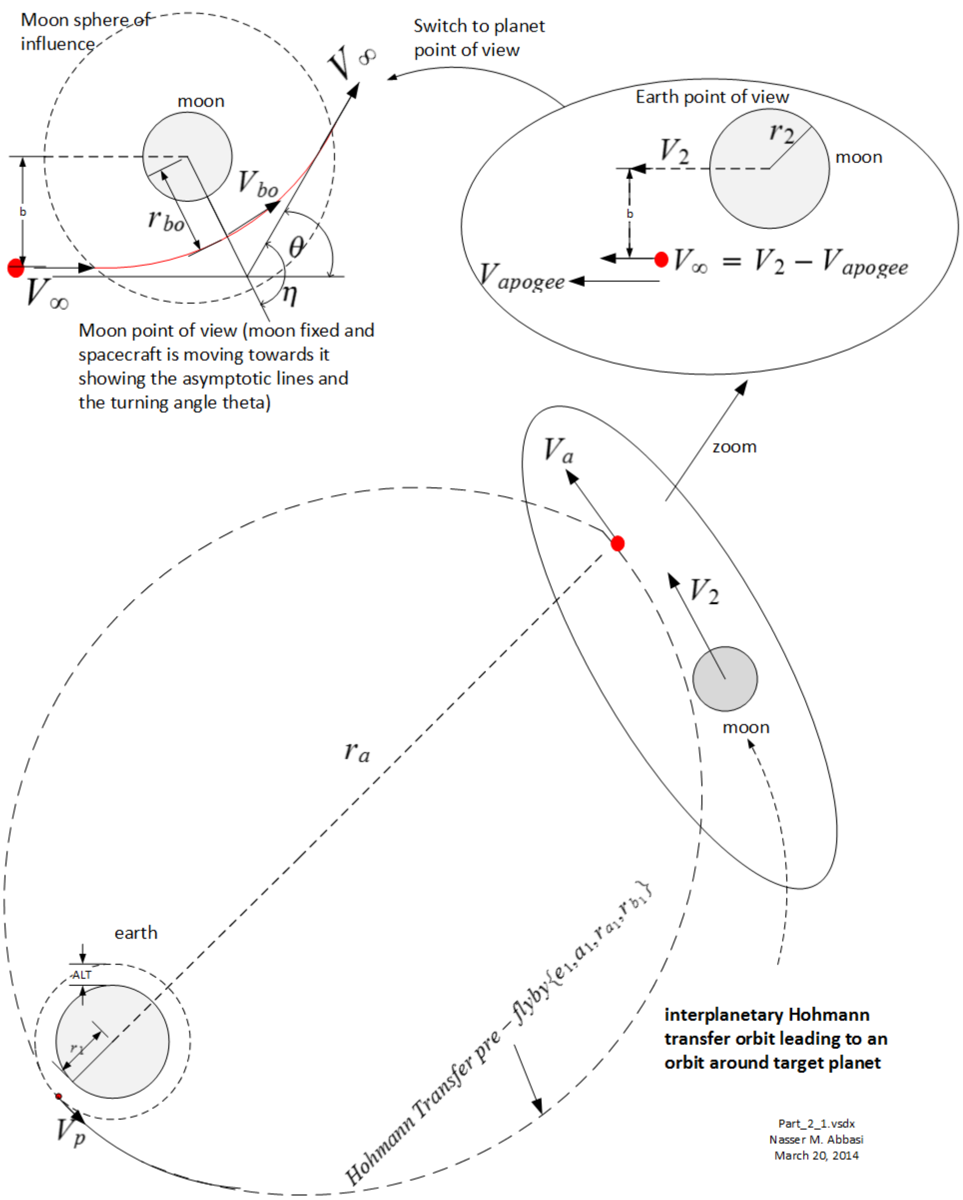


Figure 3.3: Showing Hohmann transfer from earth to the moon

3.3.3.1 part 1

The velocity of the spacecraft at the apogee of the Hohmann transfer was found in part (I) as $V_a = 0.188184$ km per second. The speed of the moon relative to earth is $V_{moon} = \sqrt{\frac{\mu_{earth}}{r_a}} = 1.0183$ km per second, therefore the speed of the spacecraft relative to the moon at the entry of the moon's sphere of influence is

$$\boxed{V_\infty = V_{moon} - V_a = 0.830119 \text{ km per second}}$$

Using the energy equation we can solve for the burn out speed V_{bo}, which is the speed of the spacecraft at r_{bo}, the closest distance from the moon surface

$$\frac{V_{bo}^2}{2} - \frac{\mu_{moon}}{r_{bo}} = \frac{V_\infty^2}{2} - \frac{\mu_{moon}}{SOI_{moon}}$$
$$\frac{V_{bo}^2}{2} - \frac{4902.8}{1760} = \frac{0.830119^2}{2} - \frac{4902.8}{6.6\times 10^4}$$

Solving gives

$$\boxed{V_{bo} = 2.47222 \text{ km per second}}$$

The impact distance b is found by solving

$$b\,V_\infty = r_{bo}\,V_{bo}$$
$$b(0.830119) = (1760)(2.47222)$$

Giving

$$\boxed{b = 5241.56 \text{ km}}$$

3.3.3.2 part 2

Figure 3.4 drawn to scale shows a moon centered fly-by of the spacecraft. We now determine the angle η and θ and the final speed V_D which is the speed relative to earth when the spacecraft exits the moon's SOI.

The eccentricity of the flyby hyperbolic orbit is found as follows

$$\begin{aligned} e &= 1 + \frac{r_{bo} V_\infty^2}{\mu_{moon}} \\ &= 1 + \frac{(1760)(0.830119)}{4902.8} \\ &= \boxed{1.24737} \end{aligned}$$

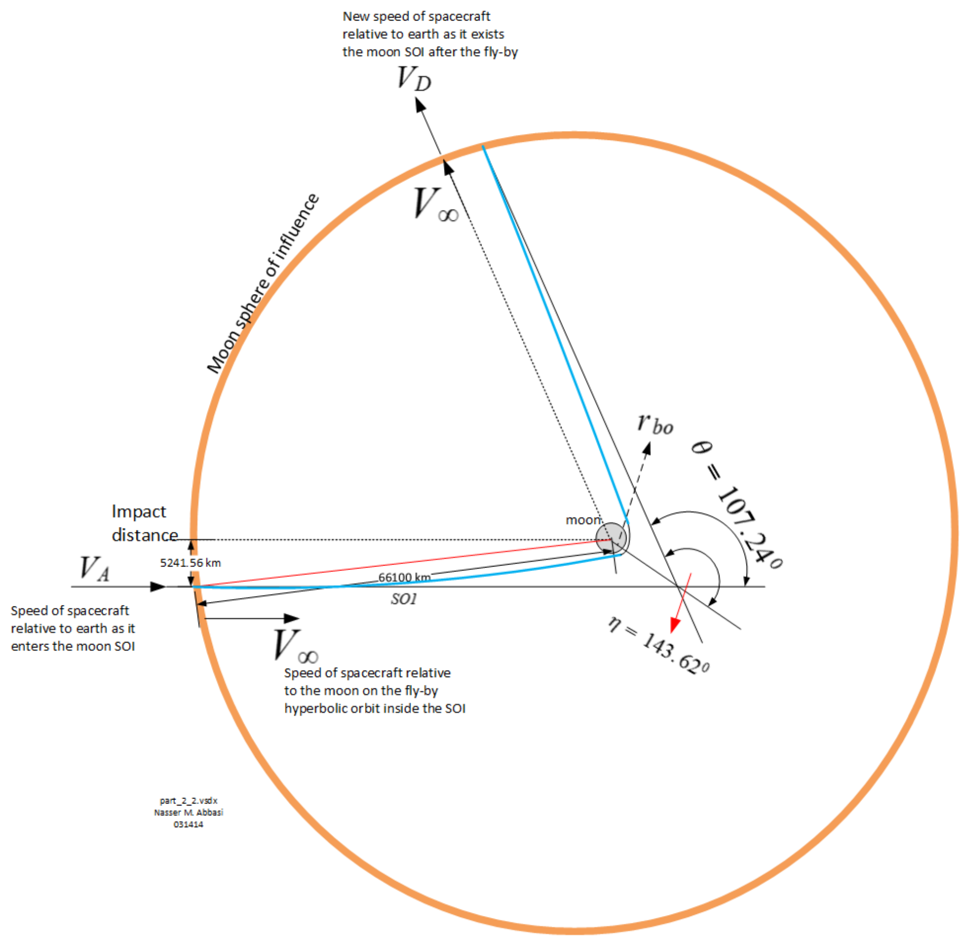


Figure 3.4: Moon-centered fly-by hyperbolic trajectory of the spacecraft

Hence

$$\begin{aligned}\eta &= \arccos\left(\frac{-1}{e}\right)\\ &= \arccos\left(\frac{-1}{1.24737}\right)\\ &= 2.50665\,\text{radian}\\ &= \boxed{143.621\text{ degree}}\end{aligned}$$

Therefore

$$\begin{aligned}\theta &= 2\eta - 180\,\text{degree}\\ &= 2(143.621) - 180\,\text{degree}\\ &= \boxed{107.241\text{ degree}}\end{aligned}$$

We now calculate V_D, the departure speed relative to earth, using figure 3.5 that shows the change in speed and direction of the spacecraft as it enters and exists the moon's sphere of influence.

$$\begin{aligned}V_D^2 &= V_{moon}^2 + V_\infty^2 - 2V_{moon}V_\infty\cos\theta\\ &= 1.0183^2 + 0.830119^2 - 2(1.0183)(0.830119)\cos(107.241\text{ degree})\end{aligned}$$

Hence

$$\boxed{V_D = 1.49236\,\text{km per second}}$$

The angle γ_d is found from the law of sines

$$\begin{aligned}\frac{V_D}{\sin\theta} &= \frac{V_\infty}{\sin\gamma_d}\\ \sin\gamma_d &= \frac{V_\infty\sin\theta}{V_D}\\ &= \frac{(0.830119)\sin(107.241\text{ degree})}{1.49236}\end{aligned}$$

Hence $\sin\gamma_d = 0.531252$ and

$$\boxed{\gamma_d = 32.0901\text{ degree}}$$

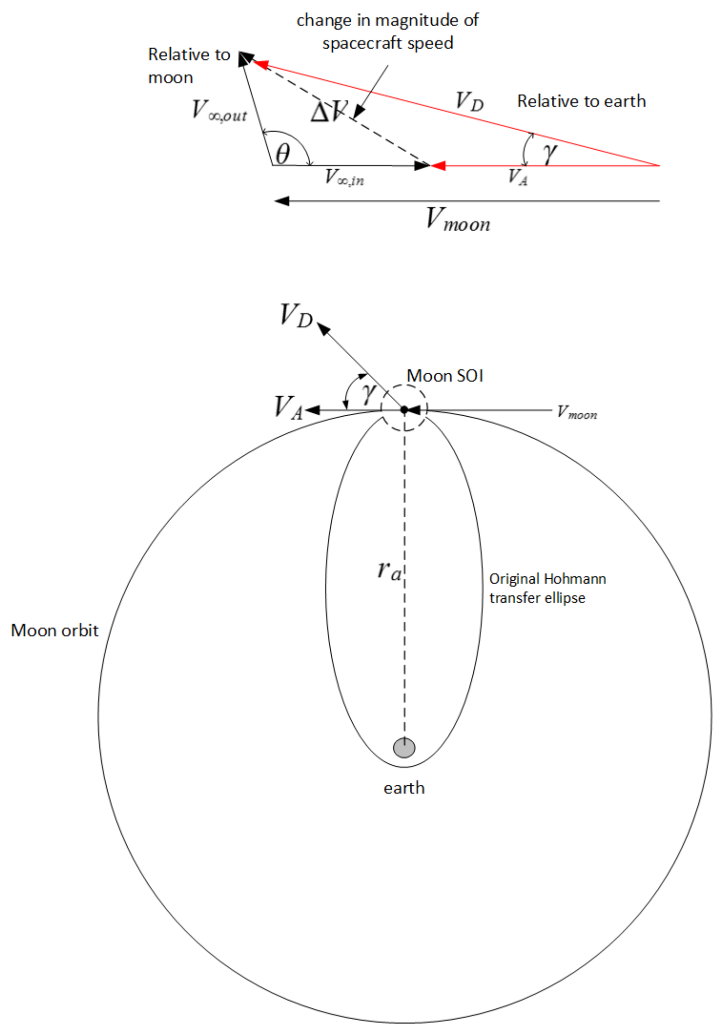


Figure 3.5: Spacecraft after fly-by and finding the new ellipse parameters

3.3.3.3 part 3

The semi-major axis of the new orbit a_{new} is found from

$$
\begin{aligned}
V_D &= \sqrt{\mu_{earth}\left(\frac{2}{r_a} - \frac{1}{a_{new}}\right)} \\
1.49236 &= \sqrt{3.986\times 10^5\left(\frac{2}{384400} - \frac{1}{a_{new}}\right)}
\end{aligned}
$$

Solving numerically for a_{new} gives

$$
\boxed{a_{new} = -2.60104e6\,\text{km}}
$$

The new eccentricity is found from

$$
\begin{aligned}
\cos\gamma_d &= \sqrt{\frac{a_{new}^2(1-e^2)}{r_a(2a_{new}-r_a)}} \\
\cos(32.0901\text{ degree}) &= \sqrt{\frac{(-2.60104\times 10^6)^2(1-e^2)}{384400(2(-2.60104\times 10^6)-384400)}}
\end{aligned}
$$

Solving numerically for the new e and taking the positive root gives

$$
\boxed{e = 1.10808}
$$

Therefore, the new trajectory is hyperbolic when the spacecraft exits the moon's sphere of influence.

3.3.3.4 part 4

Since the new trajectory is hyperbolic, the true anomaly f can be found using the hyperbolic equation

$$
\begin{aligned}
r_1 &= \frac{a_{new}(e^2-1)}{1+e\cos f} \\
384400 &= \frac{2.60104\times 10^6(1.10808^2-1)}{1+(1.10808)\cos f}
\end{aligned}
$$

Solving for f and taking the positive value since the spacecraft is in the positive half plane gives

$$
\begin{aligned}
f &= 1.06009\,\text{radian} \\
&= 60.7387\,\text{degree}
\end{aligned}
$$

This value of the true anomaly is used to locate the new perigee of the post flyby orbit. The r_p of the hyperbola is first found from

$$
\begin{aligned}
r_p &= a_{new}(e-1) \\
&= 2.60104\times 10^6(1.10808-1) \\
&= 281109\,\text{km}
\end{aligned}
$$

Figure 3.6 shows the new post flyby hyperbolic trajectory

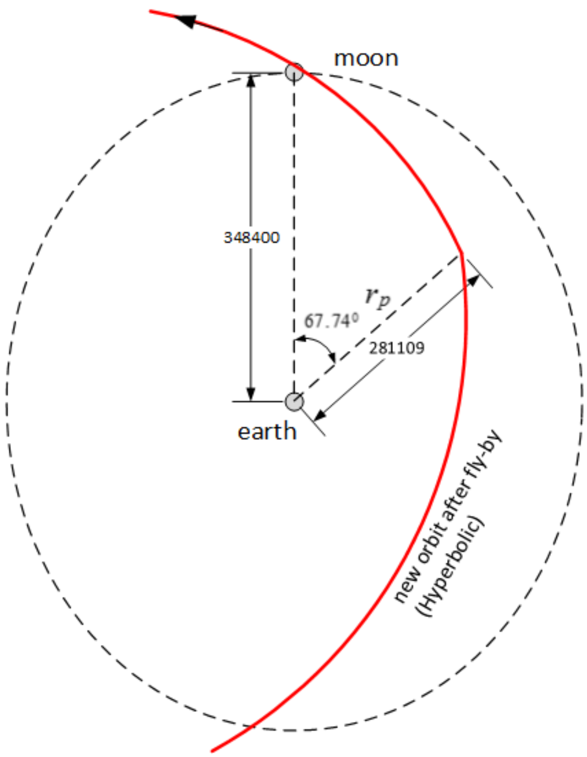


Figure 3.6: Showing the perigee on the post fly-by hyperbolic orbit (not to scale)

3.3.3.5 part 5

Figure 3.7 shows the velocity vector diagram

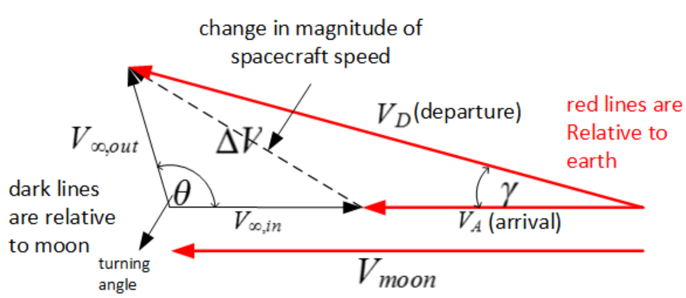


Figure 3.7: Velocity diagram

3.3.3.6 part 6

Figure 3.8 was generated from STK showing the LEO and small part of the Hohmann transfer orbit with the moon orbit at a distance. This is to scale. Figure 3.9 was drawn using

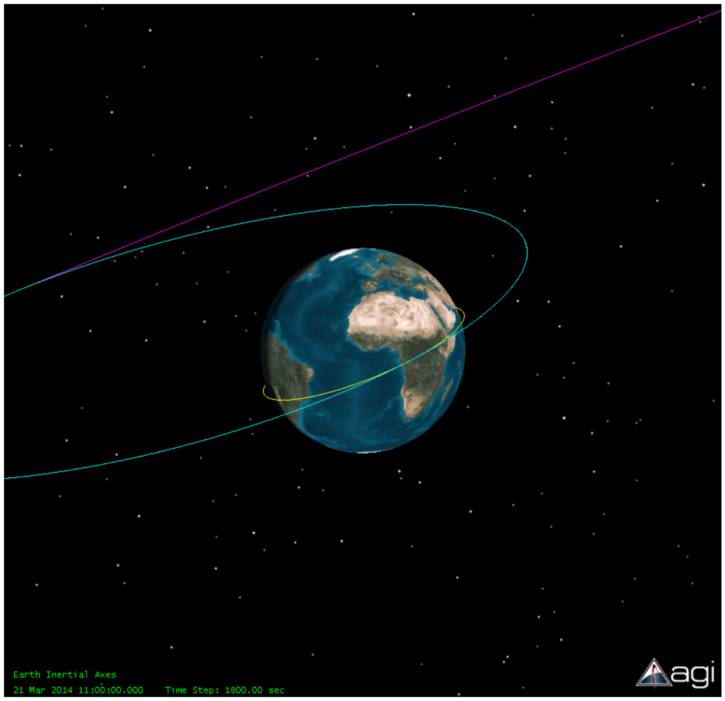


Figure 3.8: zoomed version of the final orbit for part II

VISIO showing the LEO, Hohmann, and post flyby orbit. Drawn to scale.

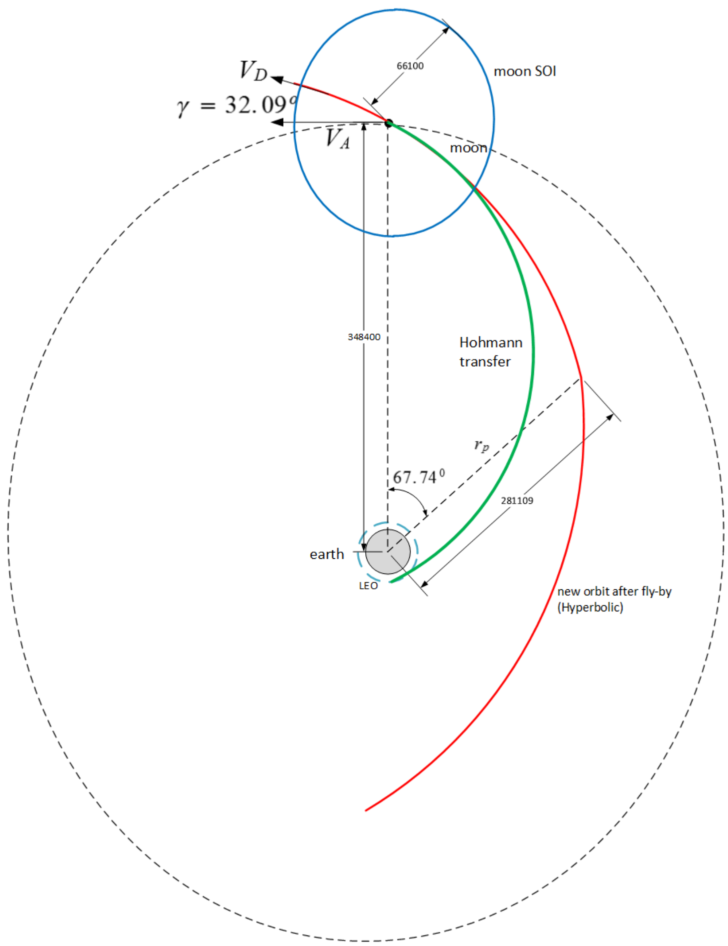


Figure 3.9: Final Part II earth centered figure. Drawn to scale

3.3.4 summary of tangential flyby

The above results are summarized in table 3.1

variable	pre flyby	post flyby
orbit type	elliptical	hyperbolic
e	0.96585	1.10808
semi-major axis a	195539 km	-2.60104e6 km
true anomaly f	180 degree	60.7387 degree
r_p	6678 km	281109 km

Table 3.1: Summary table for tangential pre and post flyby

The above results for the flyby hyperbolic trajectory are summarized in table 3.2

parameter	value
e	1.24737
V_A	0.188184 km per second
V_D	1.49236 km per second
γ_A	77.37 degree
γ_D	32.09 degree
b	5241.56 km
V_∞	0.83 km per second
η	143.621 degree
θ	107.241 degree

Table 3.2: Summary table for tangential flyby hyperbolic

3.3.5 Part III Non-Tangential flyby

The following parameters are used in the calculations that follows

$$
\begin{aligned}
\mu_{earth} &= 3.986e5\,\text{km}^3 \text{ per second squared}\\
\mu_{moon} &= 4902.8\,\text{km}^3 \text{ per second squared}\\
r_{earth} &= 6378\,\text{km}\\
r_{moon} &= 1737.4\,\text{km}\\
SOI_{moon} &= 6.61e4\,\text{km}\\
r_p &= r_{earth} + 300\\
&= 6378 + 300\\
&= 6678\,\text{km}\\
v_{moon} &= \sqrt{\frac{mu_{earth}}{r_{moon}}}\\
&= \sqrt{\frac{3.986\times 10^5}{1737.4}}\\
&= 1.0183\,\text{km per second (velocity of moon relative to earth)}\\
r_1 &= 384400\,\text{km (distance from earth to the moon)}\\
a &= 300000\,\text{km (semi-major axis of the Hohmann transfer ellipse)}
\end{aligned}
$$

Figure 3.10 gives a general view of the initial phase of the orbit showing the non-Tangential approach to the moon's circular orbit using the initial Hohmann transfer ellipse. This is not scale.

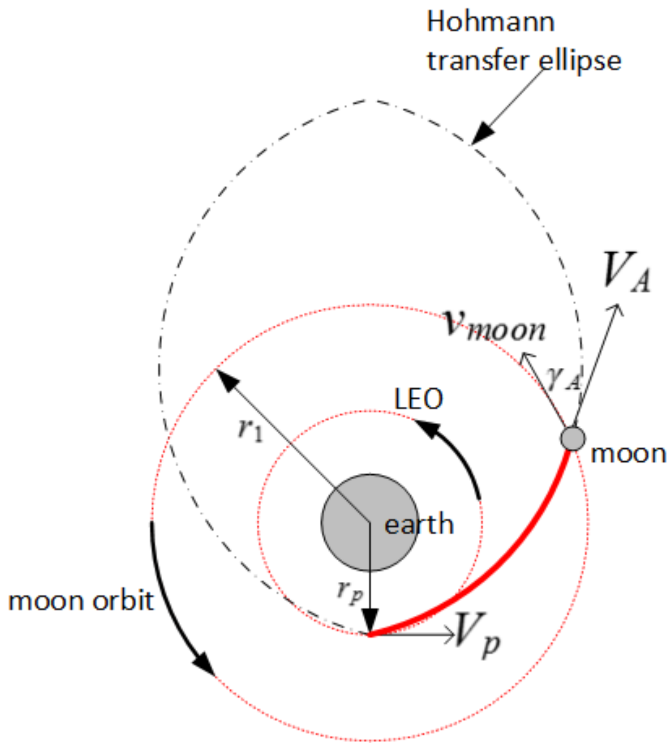


Figure 3.10: General view of the non-Tangential flyby orbit (not to scale)

3.3.5.1 Flying behind the moon

3.3.5.1.1 part 1 The given a is used to solve for r_a. Since $a = \frac{r_p+r_a}{2}$ hence

$$
\begin{aligned}
r_a &= 2a - r_p \\
&= (2)(300000) - 6678 \\
&= \boxed{593322\,\text{km}}
\end{aligned}
$$

The eccentricity of the Hohmann ellipse is now found as follows

$$
\begin{aligned}
e &= \frac{r_a - r_p}{r_a + r_p} \\
&= \frac{593322 - 6678}{593322 + 6678} \\
&= \boxed{0.97774}
\end{aligned}
$$

The speed of the spacecraft at the location where Hohmann orbit intersects the the moon's circular orbit is called V_A and found as follows

$$
\begin{aligned}
V_A &= \sqrt{\mu_{earth}\left(\frac{2}{r_1}-\frac{1}{a}\right)}\\
&= \sqrt{3.986\times10^5\left(\frac{2}{384400}-\frac{1}{300000}\right)}\\
&= 0.863258\,\text{km}
\end{aligned}
$$

γ_A is the angle between the path of the spacecraft and the moon's velocity vector direction

$$
\begin{aligned}
\cos\gamma_A &= \sqrt{\frac{a^2(1-e^2)}{r_1(2a-r_1)}}\\
&= \sqrt{\frac{300000^2(1-0.97774^2)}{384400(2(300000)-384400)}}\\
&= 0.218651
\end{aligned}
$$

Hence

$$
\begin{aligned}
\gamma_A &= 1.35036\,\text{radian}\\
&= \boxed{77.3702\text{ degree}}
\end{aligned}
$$

The true anomaly f of the pre flyby Hohmann transfer at the above location can now be found

$$
\begin{aligned}
\tan\gamma_A &= \frac{e\sin f}{1+e\cos f}\\
\tan(77.3702\text{ degree}) &= \frac{(0.97774)\sin f}{1+(0.97774)\cos f}
\end{aligned}
$$

Solving for f gives

$$
\begin{aligned}
f &= 2.8582\,\text{radian}\\
&= \boxed{163.763\text{ degree}}
\end{aligned}
$$

Relative to the moon, and at the entry to the moon's sphere of influence, the velocity of the spacecraft is given by V_{∞_a} as shown in figure 3.11 V_{∞_a} is found as follows

$$
\begin{aligned}
V_{\infty_a} &= \sqrt{V_A^2+v_{moon}^2-2V_A v_{moon}\cos\gamma_A}\\
&= \sqrt{0.863258^2+1.0183^2-2(0.863258)(1.0183)\cos 1.35036}\\
&= 1.18226\,\text{km per second}
\end{aligned}
$$

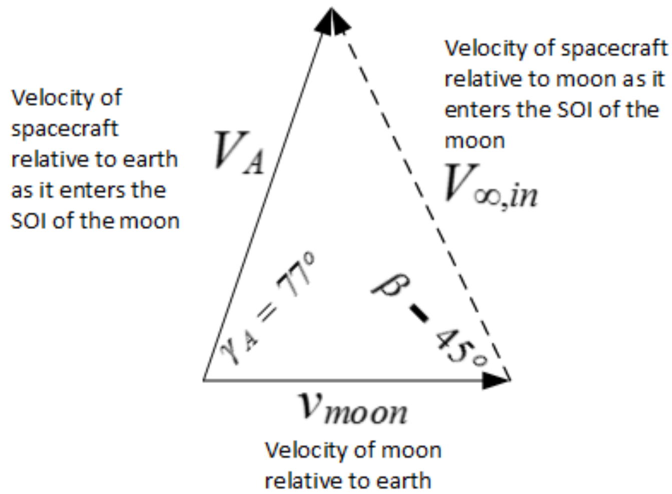


Figure 3.11: Velocity vector diagram at entry of SOI of the moon

And the angle β is

$$\frac{V_A}{\sin\beta} = \frac{V_{\infty_a}}{\sin\gamma_A}$$
$$\frac{0.863258}{\sin\beta} = \frac{1.18226}{\sin 1.35036}$$

Solving for β gives

$$\beta = 1.21158\,\text{radian}$$
$$= \boxed{45.439\text{ degree}}$$

The eccentricity of the flyby hyperbolic trajectory e_{flyby} inside the moon's sphere of influence can be found from the energy equation, using the burn out distance $r_{bo} = 1760\,\text{km}$[1].

$$\frac{V_{\infty_a}^2}{2} - \frac{\mu_{moon}}{SOI_{moon}} = \frac{V_{bo}^2}{2} - \frac{\mu_{moon}}{r_{bo}}$$
$$\frac{1.18226^2}{2} - \frac{4902.8}{6.61\times 10^4} = \frac{V_{bo}^2}{2} - \frac{4902.8}{1760}$$

Solving for V_{bo} gives

$$\boxed{V_{bo} = 2.6116\,\text{km per second}}$$

[1]the same value used in part II

Therefore

$$
\begin{aligned}
e_{flyby} &= \sqrt{1+\frac{V_{bo}^2 V_{\infty_a}^2 r_{bo}^2}{\mu_{moon}^2}}\\
&= \sqrt{1+\frac{(2.6116^2)(1.18226)^2(1760)^2}{4902.8^2}}\\
&= \boxed{1.4928}
\end{aligned}
$$

The angle η is

$$
\begin{aligned}
\eta &= \arccos\left(\frac{-1}{e_{flyby}}\right)\\
&= \arccos\left(\frac{-1}{1.4928}\right)\\
&= 2.3048\,\text{radian}\\
&= \boxed{132.05\,\text{degree}}
\end{aligned}
$$

The turning angle of the asymptotic is θ as shown in figure 3.12. The angle θ is found from

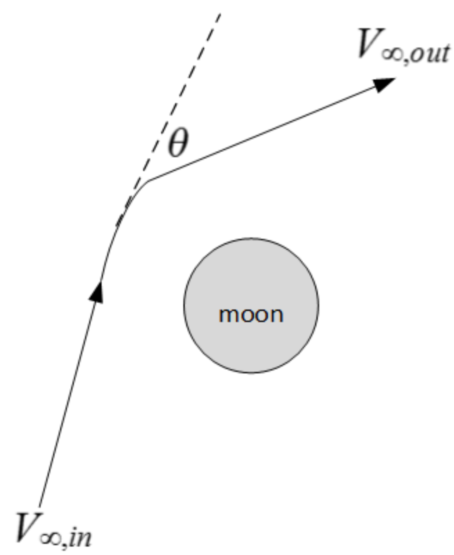


Figure 3.12: Turning angle θ when flying behind the moon

$$
\begin{aligned}
\theta &= 2\eta - 180\,\text{degree}\\
&= (2)132.05\,\text{degree} - 180\,\text{degree}\\
&= 1.468\,\text{radian}\\
&= \boxed{84.11\,\text{degree}}
\end{aligned}
$$

The departure speed of the spacecraft V_D relative to earth is found from the velocity vector in figure 3.15 as follows Hence

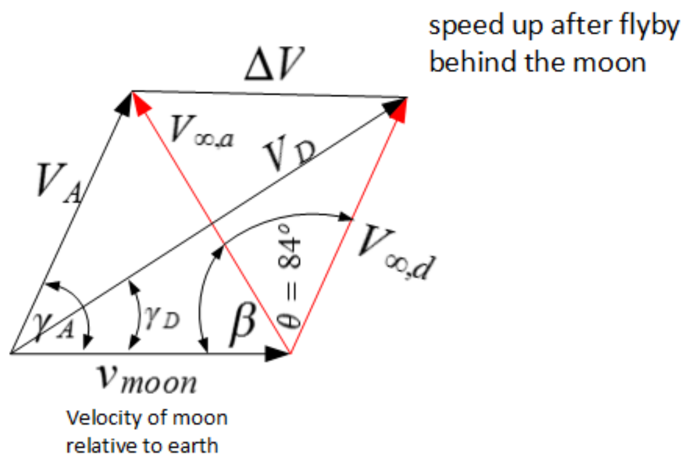


Figure 3.13: Finding departure velocity of spacecraft when flying behind the moon

$$
\begin{aligned}
V_D &= \sqrt{v_{moon}^2 + V_{\infty_d}^2 - 2 v_{moon} V_{\infty_d} \cos(\beta + \theta)} \\
&= \sqrt{1.0183^2 + 1.18226^2 - (2)(1.0183)(1.18226)\cos(45.439 \text{ degree} + 84.11 \text{ degree})} \\
&= \boxed{1.99197 \text{ km per second}}
\end{aligned}
$$

The semi-major axis a_{new} of the post flyby orbit is found from

$$
\begin{aligned}
V_D &= \sqrt{\mu_{earth}\left(\frac{2}{r_1} - \frac{1}{a_{new}}\right)} \\
1.99197 &= \sqrt{398600\left(\frac{2}{384400} - \frac{1}{a_{new}}\right)}
\end{aligned}
$$

Solving for a_{new} gives

$$
\boxed{a_{new} = -2.10444e5 \text{ km}}
$$

Therefore the departure angle γ_D is

$$
\frac{V_D}{\sin(\beta + \theta)} = \frac{V_{\infty_d}}{\sin \gamma_D}
$$

Solving for γ_D

$$
\begin{aligned}
\sin \gamma_D &= \frac{v_{\infty_d} \sin(\beta + \theta)}{V_D} \\
&= \frac{(1.18226)\sin(45.439 \text{ degree} + 84.114 \text{ degree})}{1.99197}
\end{aligned}
$$

Hence

$$\gamma_D = 0.4753\,\text{radian}$$
$$= \boxed{27.233\,\text{degree}}$$

The eccentricity of the post flyby orbit is

$$\cos\gamma_D = \sqrt{\frac{a_{new}^2(1-e^2)}{r_1(2a_{new}-r_1)}}$$
$$0.8891 = \sqrt{\frac{(-2.10444\times10^5)^2(1-e^2)}{384400(2(-2.10444\times10^5)-384400)}}$$

Solving for e gives

$$\boxed{e = 2.5546}$$

3.3.5.1.2 part 2 Since the new trajectory after the flyby is found to be a hyperbola, then the hyperbolic equation is used to obtain the true anomaly f

$$r_1 = \frac{a_{new}(e^2-1)}{1+e\cos f}$$
$$384400 = \frac{(2.10444\times10^5)(2.5546^2-1)}{1+(2.5546)\cos f}$$

Solving for f and taking the positive value since the spacecraft is in the positive half plane gives

$$f = 0.665415\,\text{radian}$$
$$= \boxed{37.552\,\text{degree}}$$

This value of the true anomaly is used to locate the new value of perigee of the post flyby orbit. The r_p of the hyperbola is found from

$$r_p = a_{new}(e-1)$$
$$= (2.10444\times10^5)(2.55546-1)$$
$$= 3.27157e5\,\text{km}$$

Figure 3.14 shows the pre flyby and the new post flyby changes to the orbit. The effect of the flyby is to produce an instantaneous ΔV that comes from the change of energy of the spacecraft due to its going and leaving the moon's sphere of influence.

3.3.5.1.3 part 3 Figure 3.15 shows the velocity triangles of the flyby trajectory.

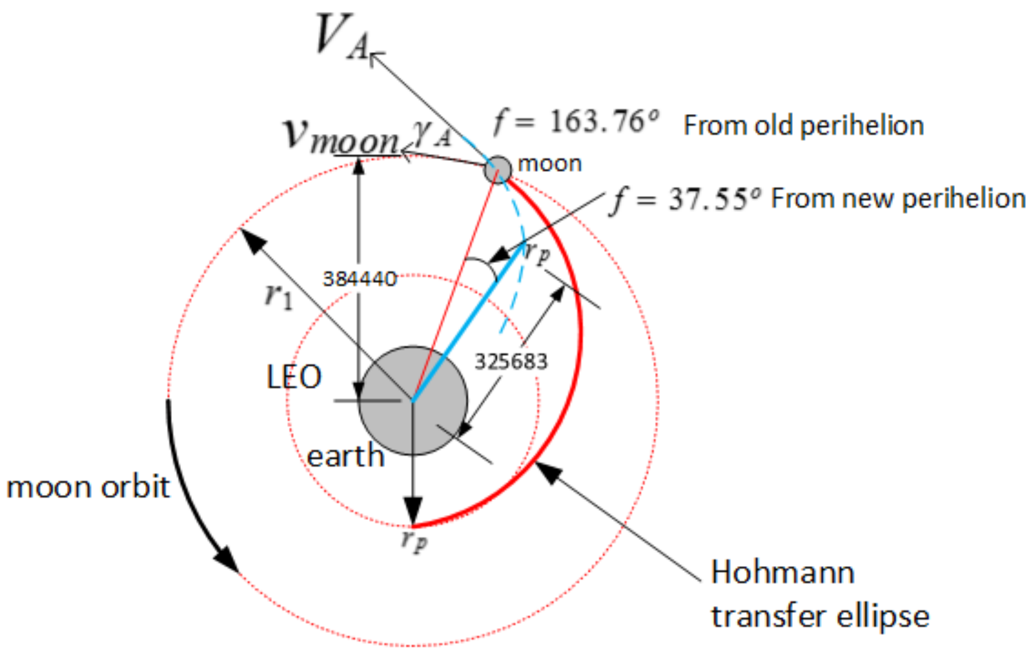


Figure 3.14: Finding departure velocity of spacecraft when flying behind the moon (not to scale)

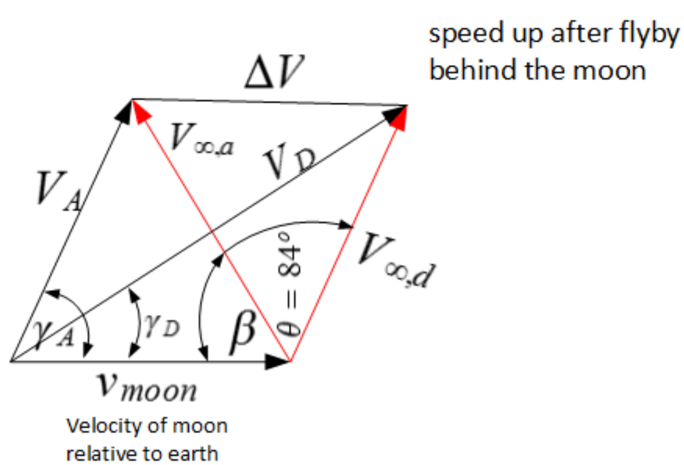


Figure 3.15: Finding departure velocity of spacecraft when flying behind the moon

3.3.5.1.4 summary for non-tangential flyby. Behind the moon case The above results for the pre and post flyby trajectories are summarized in table 3.5 The results for the

variable	pre flyby	post flyby
orbit type	elliptical	hyperbolic
e	0.97774	2.5546
semi-major axis a	300000 km	-2.10444e5 km
true anomaly f	163.76 degree	37.552 degree
r_p	6678 km	3.27157e5 km

Table 3.3: Summary table for non-tangential pre and post flyby the moon. Behind the moon case

flyby hyperbolic trajectory are summarized in table 3.6 When the spacecraft flies by the

parameter	value
e	1.4928
V_A	0.863 km per second
V_D	1.99197 km per second
γ_A	77.37 degree
γ_D	27.233 degree
V_∞	1.18226 km per second
β	45.439 degree
η	132.05 degree
θ	84.11 degree

Table 3.4: Summary table for non-tangential flyby hyperbolic. Behind the moon case

moon from behind it gains energy and the new speed relative to earth V_D is larger than the arrival speed V_A relative to earth. The reverse happens when the spacecraft flies in front of the moon. Its new velocity V_D will be smaller than V_A.

3.3.5.2 flying in front of the moon

The computation for this part follows closely what was done for the case of flying behind the moon. The difference is in how the velocity vector diagram is constructed to make sure the correct angles are used. This results in a velocity of the spacecraft V_D after leaving the moon sphere of influence slower than the above case.

The computation that follows starts from the new velocity vector diagram as follows.

The turning angle of the asymptotic θ is shown in figure 3.16. The angle θ is found from

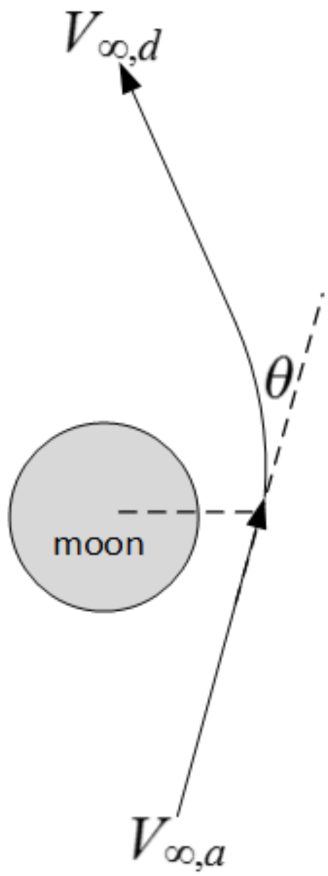


Figure 3.16: Turning angle θ when flying front of the moon

$$
\begin{aligned}
\theta &= 2\eta - 180 \text{ degree} \\
&= (2)132.05 \text{ degree} - 180 \text{ degree} \\
&= 1.468 \text{ radian} \\
&= \boxed{84.11 \text{ degree}}
\end{aligned}
$$

The departure speed of the spacecraft V_D relative to earth is found from the velocity vector in figure 3.19 Hence

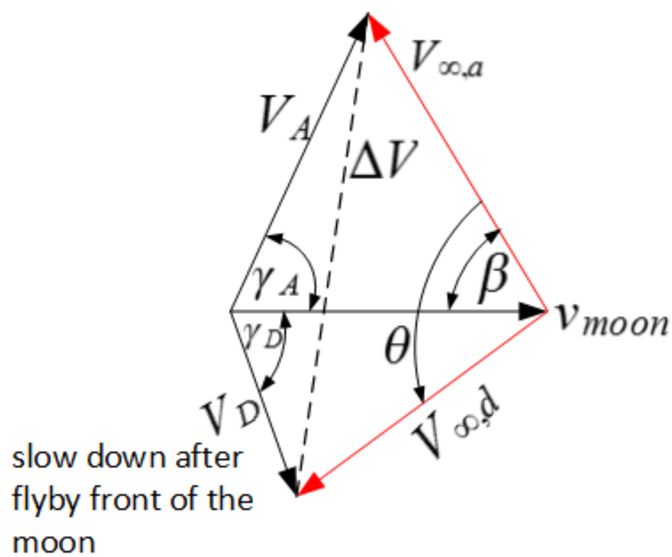


Figure 3.17: Finding departure velocity of spacecraft when flying front the moon

$$
\begin{aligned}
V_D &= \sqrt{v_{moon}^2 + V_{\infty_d}^2 - 2v_{moon}V_{\infty_d}\cos(\beta - \theta)} \\
&= \sqrt{1.0183^2 + 1.18226^2 - (2)(1.0183)(1.18226)\cos(45.439 \text{ degree} - (84.11 \text{ degree})} \\
&= \boxed{0.74492 \text{ km per second}}
\end{aligned}
$$

The semi-major axis a_{new} of the post flyby orbit is

$$
\begin{aligned}
V_D &= \sqrt{\mu_{earth}\left(\frac{2}{r_1} - \frac{1}{a_{new}}\right)} \\
0.74492 &= \sqrt{3.986 \times 10^5 \left(\frac{2}{384400} - \frac{1}{a_{new}}\right)}
\end{aligned}
$$

Solving for a_{new} gives

$$
\boxed{a_{new} = 2.62413e5 \text{ km}}
$$

The departure angle γ_D is found from

$$\begin{aligned}\sin\gamma_D &= \frac{v_\infty \sin(\beta-\theta)}{V_D}\\ &= \frac{(1.18226)\sin(45.439\,\text{degree} - (84.11\,\text{degree})}{1.99197}\end{aligned}$$

Solving for γ_D gives

$$\begin{aligned}\gamma_D &= -1.4425\,\text{radian}\\ &= \boxed{-82.649\,\text{degree}}\end{aligned}$$

The eccentricity of the post flyby orbit is found from

$$\begin{aligned}\cos\gamma_D &= \sqrt{\frac{a_{new}^2(1-e^2)}{r_1(2a_{new}-r_1)}}\\ 0.1279 &= \sqrt{\frac{(2.62413\times10^5)^2(1-e^2)}{384400(2(2.62413\times10^5)-384400)}}\end{aligned}$$

Solving for e gives

$$\boxed{e = 0.9935}$$

3.3.5.2.1 part 2 Since the new trajectory after the flyby is elliptic in this case, the elliptic equation is used to obtain the new true anomaly f

$$\begin{aligned}r_1 &= \frac{a_{new}(1-e^2)}{1+e\cos f}\\ 384400 &= \frac{(2.62413\times10^5)(1-0.9935^2)}{1+(0.9935)\cos f}\end{aligned}$$

Solving for f gives

$$\begin{aligned}f &= -3.0728\,\text{radian}\\ &= -176.06\,\text{degree}\end{aligned}$$

Since $\gamma_D < 0$ then the post flyby true anomaly is between 180 and 360 degrees. Therefore,

$$\begin{aligned}f &= 3.21\,\text{radian}\\ &= \boxed{183.94\,\text{degree}}\end{aligned}$$

This value of the true anomaly is now used to locate the new value of perigee of the post flyby orbit. The r_p of the new ellipse is found from

$$\begin{aligned}r_p &= a_{new}(1-e)\\ &= (2.62413\times10^5)(1-0.9935)\\ &= 1689.13\,\text{km}\end{aligned}$$

Figure 3.18 shows the pre flyby and the post flyby changes to the orbit.

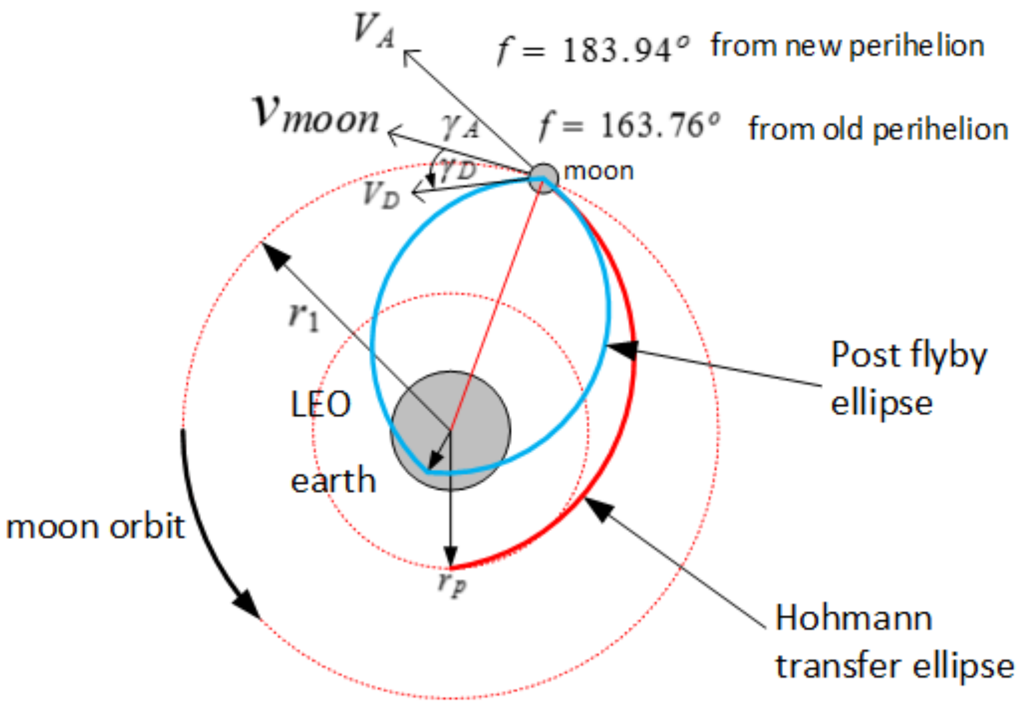


Figure 3.18: Finding departure velocity of spacecraft when flying front of the moon (not to scale)

3.3.5.2.2 part 3 Figure 3.19 shows the velocity triangle of the flyby.

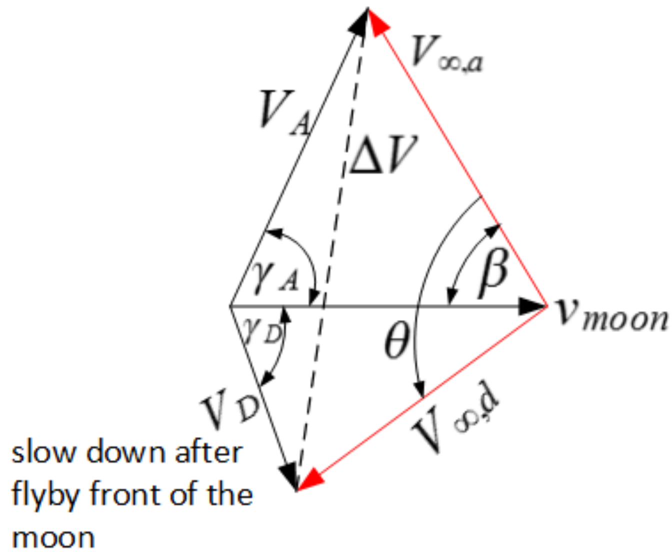


Figure 3.19: Finding departure velocity of spacecraft when flying front of the moon

3.3.5.2.3 summary of non-tangential flyby. Front of the moon case The above results for the pre and post flyby trajectories are summarized in table 3.5 The above results for the

variable	pre flyby	post flyby
orbit type	elliptical	elliptical
e	0.97774	0.9935
semi-major axis a	300000 km	262413 km
true anomaly f	163.76 degree	183.94 degree
r_p	6678 km	1689 km

Table 3.5: Summary table for non-tangential pre and post flyby the moon. Front of the moon case

flyby hyperbolic trajectory are summarized in table 3.6 Since new r_p is smaller than r_{earth}, the spacecraft will hit earth on way back on way back on the new post flyby trajectory.

When the spacecraft flies by the moon from front, it losses energy and the new speed relative to earth V_D is smaller than the arrival speed V_A relative to earth.

parameter	value
e	1.4928
V_A	0.863 km per second
V_D	0.7449 km per second
γ_A	77.37 degree
γ_D	-82.649 degree
V_∞	1.18226 km per second
β	45.439 degree
η	132.05 degree
θ	84.11 degree

Table 3.6: Summary table for non-tangential flyby hyperbolic. Front of the moon case

3.3.6 Part IV Free return trajectory

3.3.6.1 part 1

The trajectory that was selected for the pre flyby part is to send the spacecraft to front of the moon. The reason is because the post flyby velocity of the spacecraft V_D in this case will be smaller that the approach velocity V_D and the new flight path angle γ_D will be negative and the post flyby trajectory being an ellipse. This insures the the spacecraft will return back to earth.

It is assumed that the spacecraft will rendezvous with the moon when it reaches the moon's orbit. Timing considerations are discussed in part (IV).

Since the original altitude above earth of the spacecraft was fixed by the project requirement to be in LEO at 300 km, the other free variable that can be used to adjust the trajectory is the semi-major axis a of the pre flyby orbit.

Changing a is the same as changing the initial ΔV_1. The lunar burn out radius r_{bo} was also fixed by project requirement to be 1760 km.

A program was written to make it easier to change the semi-major axis a using a flyby in front of the moon approach. The program calculates all the parameters of the new post flyby trajectory.

The resulting post flyby ellipse was checked after each simulation run to see if it meets the requirement of having a return altitude on earth of between 300 km and 500 km. In addition, The selected trajectory was required to have its velocity at perigee (closest point to earth) to be below 12 km per second to ensure safety of the spacecraft as it enters earth.

The selected trajectory had a new V_p of 10.8079 km per second. This is faster than the initial elliptical orbit V_p which was 7.725 km per second but it is still a safe entry velocity back to earth.

Figure 3.20 shows the user interface of the program with the final selected trajectory. The pre flyby semi−major axis (a) 415000

altitude at moon 22

zoom 1

Out[2]=

parameter	value	units
f pre flyby	168.91	degree
f post flyby	185.979	degree
r_p pre flyby	6678	km
r_p post flyby	6725.03	km
r_a pre flyby	823322	km
r_a post flyby	453929.	km
a pre flyby	415000	km
a hyper	2823.51	km
a post flyby	230327.	km
delV1	3.1561	km/sec
e pre flyby	0.983908	
e Hypereblic at moon	1.62334	
e post flyby	0.970802	
V_A	1.05518	km/sec
V_D	0.585916	km/sec
γ_A	79.6791	degree
γ_D	-71.1728	degree
V_∞	1.32866	km/sec
θ	76.0514	degree
η	128.026	degree
β	51.3815	degree
V_{bo}	2.68107	km/sec
LEO pre flyby	300	km
new Altitude on earth	347.028	km
LEO v outbound	7.72584	km/sec
LEO v return	10.8079	km/sec
E (pre flyby)	1.49579	radian
E (pre flyby)	85.7022	degree
E (post flyby)	2.33098	radian
E (post flyby)	133.555	degree
ΔT (time to reach moon)	217926.	sec
ΔT (time to reach moon)	60.5351	hrs
ΔT (time to reach moon)	2.5223	days
ΔT (time to return from moon)	79.1491	hrs
moon angle traveled	0.577303	radians
moon angle traveled	33.077	degree
moon phase angle	135.833	degree
moon period	658.846	hrs
LEO period	1.50861	hrs
moon travel in one LEO period	0.824322	degree
ω_{moon}	2.64907×10^{-6}	rad/sec
ω_{LEO}	0.00115691	rad/sec
synodic period	1.51208	hrs
time in flyby	24.9029	hrs
totaltime	164.587	hrs
FF for flyby hyper	3.40112	radian
FF for flyby hyper	194.87	degree
fHyper	124.954	degree
moon traveled in SOI	13.6072	degree

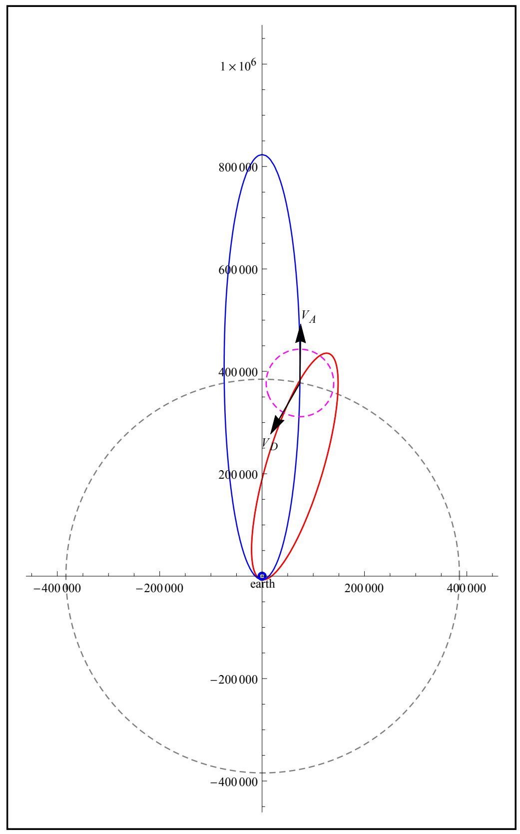


Figure 3.20: screen shot of the user interface of the program used to find the free return trajectory

program source code in the appendix. The program plots the pre and post flyby orbits and displays all the detailed parameters of each trial.

3.3.6.2 part 2

Figure 3.21 shows the final result of the trajectory selected. This table was generated by the simulation program written for this project. The semi-major axis of the initial orbit is $\boxed{a = 415000\text{km}}$ and the eccentricity is $\boxed{e = 0.983908}$ and $\boxed{\Delta V_1 = 3.1561\text{km per second}}$

The altitude at the perigee of the post flyby ellipse is $\boxed{\text{ALT} = 347.028\text{km}}$ which meets the requirements

parameter	value	units
f pre flyby	168.91	degree
f post flyby	185.979	degree
r_p pre flyby	6678	km
r_p post flyby	6725.03	km
r_a pre flyby	823 322	km
r_a post flyby	453 929.	km
a pre flyby	415 000	km
a post flyby	230 327.	km
delV1	3.1561	km/sec
e pre flyby	0.983908	
e Hypereblic at moon	1.62334	
e post flyby	0.970802	
V_A	1.05518	km/sec
V_D	0.585916	km/sec
γ_A	79.6791	degree
γ_D	-71.1728	degree
V_∞	1.32866	km/sec
θ	76.0514	degree
η	128.026	degree
β	51.3815	degree
V_{bo}	2.68107	km/sec
LEO pre flyby	300	km
new Altitude on earth	347.028	km
LEO v outbound	7.72584	km/sec
LEO v return	10.8079	km/sec

Figure 3.21: Table of results of selected free return trajectory

3.3.6.3 part 3

The program developed for this project plots the final selected trajectory to scale. It shows both the pre flyby ellipse and the post flyby ellipse.

Figure 3.22 shows the selected trajectory generated by the simulation program (to scale). Figure 3.23 shows the velocity triangle for the selected trajectory. Figure 3.24 shows the

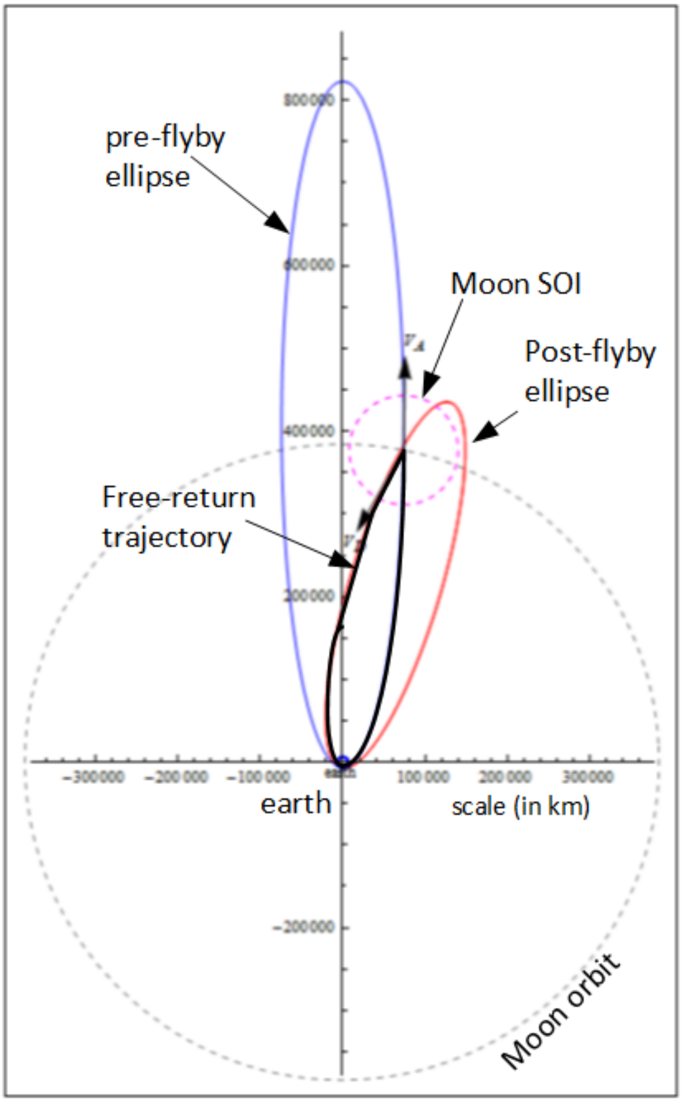


Figure 3.22: free return selected trajectory (to scale)

true anomaly angle f at the intersection with the moon's orbit for the pre and post flyby trajectories. Since $\gamma_D < 0$ then true anomaly for the post flyby trajectory is between π and 2π

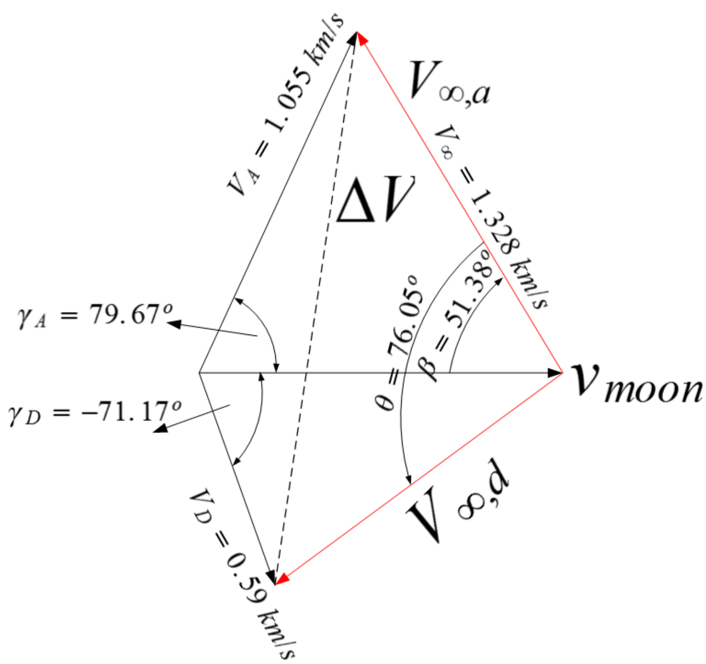


Figure 3.23: velocity triangle for the selected free return trajectory

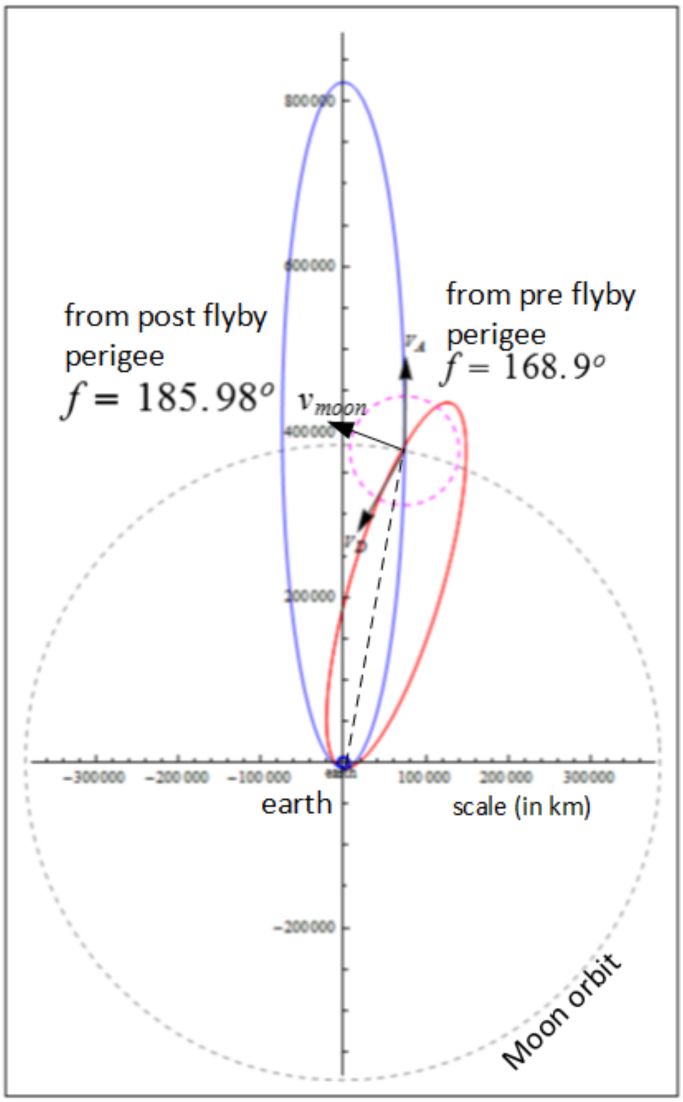


Figure 3.24: Showing the effect on true anomaly angle for return free trajectory

3.3.6.4 part 4

Figure 3.25 shows a zoomed version of the selected trajectory near earth. The new perigee is 6725.03 km which represents an altitude of 347.028 km. The above shows that the spacecraft

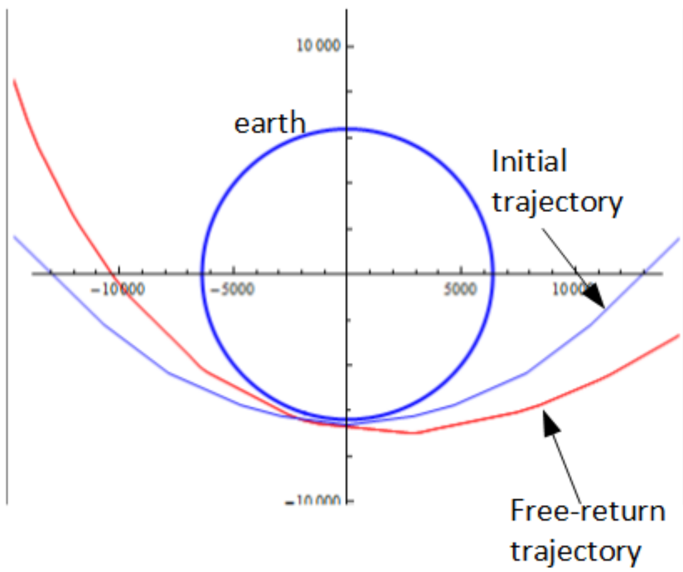


Figure 3.25: zoomed view of the free return selected trajectory near earth

returns to the required perigee with an altitude of 347.028 km and with safe entry velocity back to earth of 10.08079 km per second.

It was found during simulation that finding the return ellipse with the required final altitude was very sensitive to small changes in value of the semi-major axis a for the initial orbit. There was a small range of values of a which generated an acceptable free return trajectories. Using a simulation program helped in finding this small range of values of a easier.

3.3.7 Part V Rendezvous and timing consideration

3.3.7.1 part 1

The time to reach the moon is given by $\Delta T = \sqrt{\frac{a^3}{\mu_{earth}}}(E - e\sin E)$ where E is the eccentric anomaly of the pre flyby trajectory.

E is found by solving $r = a(1 - e\cos E)$ where r here is the distance between earth and the moon and e is the eccentricity of the pre flyby orbit. Using the result of the selected trajectory of part (4)

$$r = a(1 - e\cos E)$$
$$384400 = 415000(1 - (0.9839)\cos E)$$

Solving gives $E = 1.4957$ radian or $\boxed{E = 85.7\text{degree}}$. Therefore the time to reach the moon is

$$\begin{aligned}\Delta T &= \sqrt{\frac{a^3}{\mu_{earth}}}\left(E - e\sin E\right)\\ &= \sqrt{\frac{(415000)^3}{3.986\times 10^5}}\left(1.4957 - (0.9839)\sin(1.4957)\right)\\ &= 217926\,\text{second}\\ &= 60.535\,\text{hour}\\ &= \boxed{2.523\,\text{day}}\end{aligned}$$

3.3.7.2 part 2

The angular velocity of the moon in its orbit around earth is given by $\omega = \sqrt{\frac{\mu_{earth}}{r^3}}$ where r is the distance from earth to the moon. During the ΔT found in part (1), the moon will travel

$$\begin{aligned}\theta &= \omega(\Delta T)\\ &= \sqrt{\frac{\mu_{earth}}{r^3}}(\Delta T)\\ &= \sqrt{\frac{3.986\times 10^5}{384400}}(217926)\\ &= 0.5773\,\text{radian}\\ &= \boxed{33.077\text{ degree}}\end{aligned}$$

Since the true anomaly was found in part (1) for the pre flyby to be 168.9 degree, therefore the moon has to be at angle $\theta_0 = 168.9 - 33.077$ or

$$\theta_0 = 135.833\,\text{degree}$$

In front of the spacecraft initial position as shown in figure 3.26

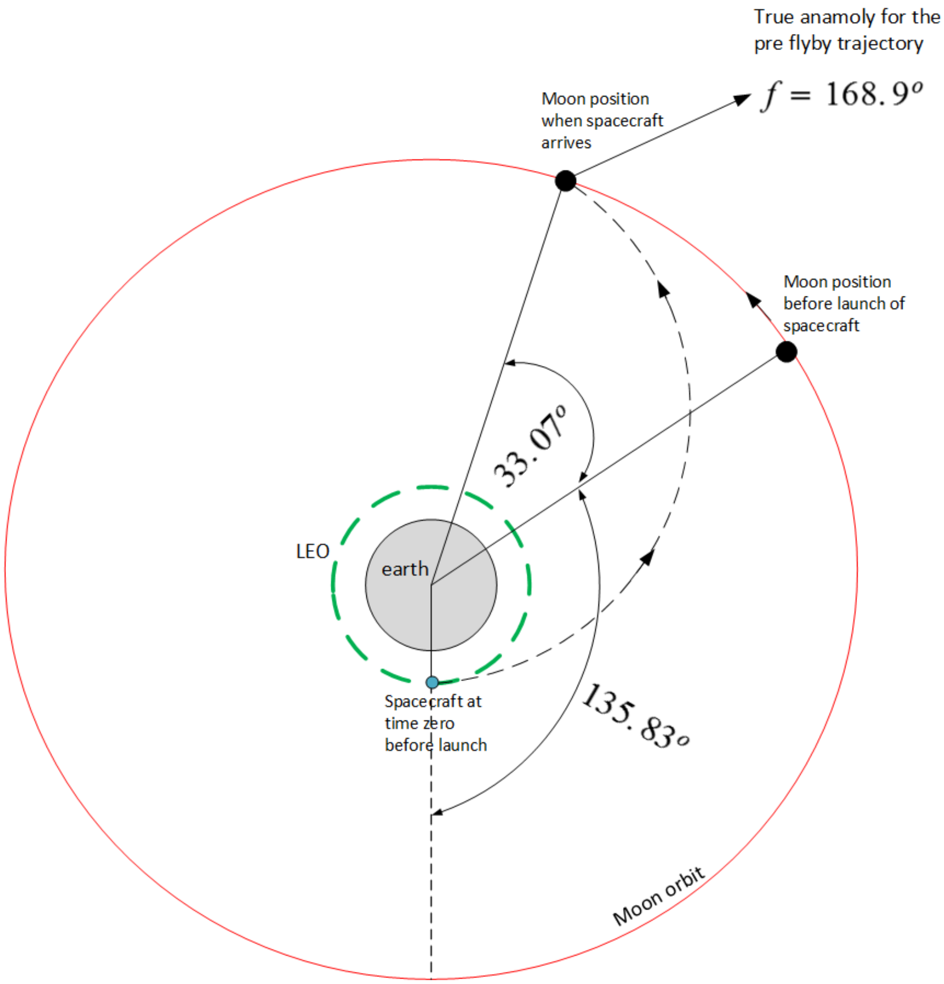


Figure 3.26: Phase angle between moon and spacecraft for rendezvous

3.3.7.3 part 3

The angular velocity of the spacecraft around earth

$$\begin{aligned}\omega_1 &= \sqrt{\frac{\mu_{earth}}{r_p}}\\ &= \sqrt{\frac{3.986\times 10^5}{6678}}\\ &= 0.00115691 \text{ radian per second}\end{aligned}$$

The angular velocity of the moon around earth is

$$\begin{aligned}\omega_{moon} &= \sqrt{\frac{\mu_{earth}}{r_1}}\\ &= \sqrt{\frac{3.986\times 10^5}{384400}}\\ &= 2.64907e(-6) \text{ radian per second}\end{aligned}$$

The synodic period of the moon relative to the spacecraft is how often the space craft and the moon have the correct alignment, which is given by

$$\begin{aligned}\tau_s &= \frac{2\pi}{|\omega_1 - \omega_{moon}|}\\ &= \frac{2\pi}{|0.00115691 - 2.64907\times 10^{-6}|}\\ &= \boxed{1.51208 \text{ hour}}\end{aligned}$$

3.3.7.4 part 4

Figure 3.27 shows the flyby hyperbola. The time during the flyby can be determined from the hyperbolic equation as follows. The semi-major axis a for the flyby hyperbolic trajectory is found from

$$\begin{aligned}r_{bo} &= a(e-1)\\ a &= \frac{r_{bo}}{e-1}\\ &= \frac{1760}{1.62334-1}\\ &= 2823.51 \text{ km}\end{aligned}$$

The eccentric anomaly F for the hyperbolic trajectory is found from

$$\begin{aligned}r_{bo} &= a(e\cosh F - 1)\\ 1760 &= 2823.51(1.62334\cosh F - 1)\end{aligned}$$

Solving gives

$$\begin{aligned}F &= 3.40112 \text{ radian}\\ &= 194.87 \text{ degree}\end{aligned}$$

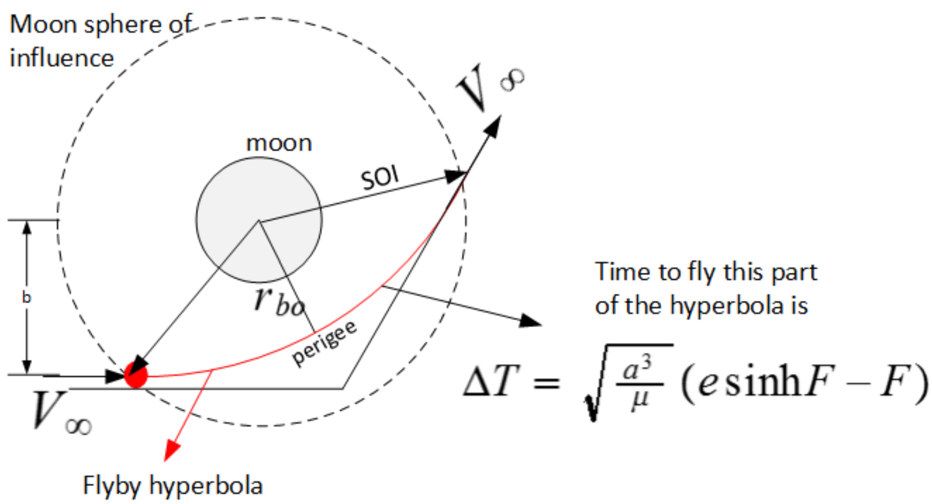


Figure 3.27: Flyby hyperbola used for calculation of flyby duration

Therefore the time for the overall flyby, which is the time that the spacecraft is inside the moon's sphere of influence is

$$\begin{aligned}\Delta T &= 2\sqrt{\frac{a^3}{\mu_{moon}}}(e\sinh F - F)\\ &= 2\sqrt{\frac{2823.51^3}{4902.8}}((1.62334)\sinh(3.40112) - 3.40112)\\ &= 24.9029 \text{ hour}\end{aligned}$$

The time to fly back to earth from the moon after the flyby phase is complete is found from the elliptical equation for the return flight solution found above.

$$\begin{aligned}r &= a(1 - e\cos E)\\ 384400 &= 230327(1 - (0.970802)\cos E)\end{aligned}$$

Solving gives $E = 2.33098$ radian or $\boxed{E = 133.55 \text{ degree}}$. Therefore the time is

$$\begin{aligned}\Delta T_2 &= \sqrt{\frac{a^3}{\mu_{earth}}}(E - e\sin E)\\ &= \sqrt{\frac{(230327)^3}{3.986 \times 10^5}}(2.33098 - (0.97080)\sin(2.33098))\\ &= 217926 \text{ second}\\ &= 79.149 \text{ hour}\\ &= \boxed{3.298 \text{ day}}\end{aligned}$$

Using the time to flyby the moon found earlier in part (1) above, the total flight time for the whole journey is therefore

$$\begin{aligned} T &= 60.535 + 24.9029 + 79.149 \\ &= \boxed{164.587\,\text{hour}} \end{aligned}$$

Hence, the percentage of time in flyby around the moon is $\frac{24.9029}{164.587}$ or $\boxed{15.13\%}$.

During the time the spacecraft is inside the moon's sphere of influence, the moon will have traveled

$$\begin{aligned} \Delta\theta &= \omega_{moon}\,(\text{flyby time}) \\ &= \sqrt{\frac{\mu_{earth}}{r_1^3}} \times (24.9029\,\text{hour}) \\ &= \sqrt{\frac{3.986\times 10^5}{384400^3}} \times (24.9029\,\text{hour}) \\ &= \boxed{13.607\,\text{degree}} \end{aligned}$$

This is $\frac{13.607}{360} = \boxed{3.78\%}$ of the full orbit of the moon around the earth. This shows that the change of speed ΔV that occurs due to the flyby is not instantaneous and takes about 3.8% of the period of the moon around the earth.

Therefore, the conic method can be considered only to be a first order approximation, and therefore, for practical spacecraft trajectory design, numerical methods can be used to obtain a more accurate solutions.

Chapter 4

Exams

Local contents

4.1 first exam

4.1.1 Key solution

EMA 550/Astronomy 550

Exam #1, Spring 2014

75 Minutes, Open Notes Feb. 27, 2014

Name ____________KEY________________

For the purposes of this exam, assume the Earth is spherical with a radius of 6378 km and $\mu = 3.986 \times 10^5$ km^3/s^2. These values are reprinted in the footer of each page for your convenience.

Show all of your work to get credit for your answer. To maximize your opportunities for partial credit, write down all of the equations you are using.

Include units with all numerical answers.

If you can't do one section of a multi-part problem and the following parts depend on your answer, make a reasonable assumption, write on your paper that you are assuming a value and what it is, and continue on with the problem using your assumed value.

	Points	Score
Question 1	20	______
Question 2	20	______
Question 3	20	______
Total Score	60	______

Question 1

A satellite is to be placed in an elliptical orbit about the Earth with a period of 8 hours.

(a) For what range of eccentricities will the orbit NOT impact the Earth?

In order for the satellite not to impact the Earth, the perigee radius rp = a (1-e) must be greater than or equal to the Earth's radius.

The semi-major axis can be determined from the orbit period.

$$T = 2\cdot\pi\cdot\sqrt{\frac{a^3}{\mu}} \qquad a := \left[\left(\frac{T}{2\cdot\pi}\right)^2\cdot\mu\right]^{\frac{1}{3}} = 20307\ \text{km}$$

Then the allowable eccentricities can be determined from

$$rp = a\cdot(1-e) \geq 6378\cdot\text{km}$$

$$e \leq 1 + \frac{6378\cdot\text{km}}{a}$$

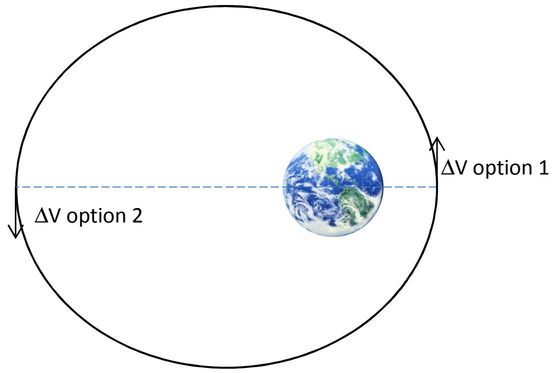
$$emax := 1 - \frac{6378\cdot\text{km}}{a} = 0.686$$

The range of eccentricities for which the elliptical orbit will not impact the Earth are eccentricities between 0 (circular orbit) and 0.686.

Earth radius = 6378 km Page 2 of 8 $\mu_{Earth} = 3.986 \times 10^5\ km^3/s^2$

Earth-Sun distance = 1 AU = 1.495978×10^8 km $\mu_{Sun} = 1.327 \times 10^{11}\ km^3/s^2$

(b) For a satellite in an elliptical orbit about the Earth with a period of 8 hours and an eccentricity of 0.5, determine whether it would cost less to escape the Earth on a parabolic trajectory by doing a tangential burn at perigee or by doing a tangential burn at apogee by calculating the ΔV required for each option.



The period is only dependent on the semi-major axis, so the semi-major axis is the same as in part (a).

Perigee location: $rp := a \cdot (1 - e) = 10154\ \text{km}$

Speed at perigee: $vp := \sqrt{\mu \cdot \left(\frac{2}{rp} - \frac{1}{a}\right)} = 7.674\ \frac{\text{km}}{\text{s}}$

Escape speed at perigee: $vesc\_p := \sqrt{\frac{2 \cdot \mu}{rp}} = 8.861\ \frac{\text{km}}{\text{s}}$

Cost of escaping from perigee: $\Delta V1 := vesc\_p - vp = 1.187\ \frac{\text{km}}{\text{s}}$

Apogee location: $ra := a \cdot (1 + e) = 30461\ \text{km}$

Speed at apogee: $va := \sqrt{\mu \cdot \left(\frac{2}{ra} - \frac{1}{a}\right)} = 2.558\ \frac{\text{km}}{\text{s}}$

Escape speed at apogee: $vesc\_a := \sqrt{\frac{2 \cdot \mu}{ra}} = 5.116\ \frac{\text{km}}{\text{s}}$

Cost of escaping from apogee: $\Delta V2 := vesc\_a - va = 2.558\ \frac{\text{km}}{\text{s}}$

It would cost less to do the parabolic escape burn at perigee than at apogee.

Question 2

A spacecraft is in a circular orbit about the Earth at a distance *r* = 16,000 km. An instantaneous tangential burn of ΔV = 3 km/s is performed.

(a) What type of trajectory (circular, elliptical, parabolic, hyperbolic) is the spacecraft on after the burn? Show your reasoning.

$$vcirc := \sqrt{\frac{\mu}{r1}} = 4.991\,\frac{km}{s}$$

$$vnew := vcirc + \Delta V = 7.991\,\frac{km}{s}$$

$$vesc := \sqrt{\frac{2\mu}{r1}} = 7.059\,\frac{km}{s}$$

The speed is greater than the escape speed, so the spacecraft is on a hyperbolic trajectory.

Alternatively, looking at the specific orbit energy,

$$\varepsilon new := \frac{vnew^2}{2} - \frac{\mu}{r1} = 7.017\times 10^{6}\cdot\frac{N\cdot m}{kg}$$

The energy is positive, so the spacecraft is on a hyperbolic trajectory.

(b) How long does it take the spacecraft to reach a distance of *r* = 32,000 km? (Additional workspace is available on the following page if needed.)

To determine time on a hyperbolic trajectory, use the hyperbolic corollary of Kepler's equation:

$$\Delta t = \sqrt{\frac{a^3}{\mu}}\cdot(e\cdot\sinh(F) - F)$$

To calculate the time, we need a and e for the new trajectory, and F when r = 32,000 km.

a can be found from the velocity equation for a hyperbola:

$$vnew = \sqrt{\mu\cdot\left(\frac{2}{r1} + \frac{1}{a}\right)} \qquad a := \frac{1}{\frac{vnew^2}{\mu} - \frac{2}{r1}} = 28401\text{ km}$$

The eccentricity can be found from applying the knowledge that the burn occurs at perigee of the new trajectory:

$$r1 = rp = a\cdot(e - 1) \qquad e := 1 + \frac{r1}{a} = 1.563$$

The true anomaly can be found from the polar equation for a hyperbola and then converted to F, or F can be found directly from the hyperbolic polar form of the position equation:

$$r = \frac{a\cdot(e^2-1)}{1+e\cdot\cos(f)}$$

$$r2 := 32000\cdot km$$

$$f2 := acos\left[\frac{1}{e}\cdot\left[\frac{a\cdot(e^2-1)}{r2}-1\right]\right] = 1.39\cdot rad$$

$$f2 = 79.62\cdot deg$$

$$\tan\left(\frac{f}{2}\right) = \sqrt{\frac{e+1}{e-1}}\cdot\tanh\left(\frac{F}{2}\right)$$

$$F2 := 2\cdot atanh\left[\left(\tan\left(\frac{f2}{2}\right)\right)\cdot\sqrt{\frac{e-1}{e+1}}\right] = 0.825$$

or

$$r = a\cdot(e\cdot\cosh(F)-1)$$

$$F2\_alt := acosh\left[\frac{1}{e}\cdot\left(\frac{r2}{a}+1\right)\right] = 0.825$$

Once these parameters are found, the time to reach r = 32,000 km can be calculated:

$$\Delta t := \sqrt{\frac{a^3}{\mu}}\cdot(e\cdot\sinh(F2)-F2) = 4674\,s$$

$$\Delta t = 77.9\cdot min$$

$$\Delta t = 1.3\cdot hr$$

(c) What is the flight path angle of the spacecraft's trajectory when *r* = 16,000 km, and what is the flight path angle of the spacecraft's trajectory when *r* = 32,000 km?

At the burn location, the velocity is perpendicular to the position vector, so the flight path angle at r = 16,000 km is simply zero.

At r = 32,000 km, the flight path angle can be calculated using the equation for flight path angle on a hyperbola:

Hyperbolic flight path angle: $$\cos\gamma = \sqrt{\frac{a^2\cdot(e^2-1)}{r\cdot(2\cdot a+r)}}$$

$$\gamma 2 := acos\left[\sqrt{\frac{a^2\cdot(e^2-1)}{r2\cdot(2\cdot a+r2)}}\right] = 0.876\cdot rad$$

$$\gamma 2 = 50.19\cdot deg$$

Question 3

A spacecraft is in a circular orbit about the Sun at a distance of 1.25 AU (between the Earth's orbit about the Sun and Mars' orbit about the Sun; see below for conversions between AU and km and for μ_{Sun}).

The spacecraft is to be placed eventually into a circular orbit about the Sun at a distance of 8 AU (between Jupiter and Saturn).

(a) How long (in years) will each of the following transfers take?
   i. A Hohmann transfer
   ii. A bi-elliptic transfer with an aphelion of 10 AU
   iii. A semi-tangential elliptical transfer, tangent at perihelion to the 1.25 AU circular orbit, with an aphelion distance of 10 AU

The semimajor axis on the Hohmannn transfer is the average of the initial and final circular radii. The time required for the Hohmann transfer is one half the period of the Hohmann ellipse.

$$aH := \frac{r1 + r2}{2} = 6.919 \times 10^{8}\,\text{km} \qquad aH\_au := \frac{aH}{au} = 4.625$$

$$\Delta tH := \frac{1}{2}\cdot\left(2\cdot\pi\cdot\sqrt{\frac{aH^3}{\mu}}\right) = 1.57 \times 10^{8}\,\text{s} \qquad \Delta tH = 1817\cdot\text{day}$$

$$\boxed{\Delta tH = 4.974\cdot\text{yr}}$$

The bi-elliptic transfer consists of two half ellipses: the first has a semimajor axis that is the average of r1 and rb, and the second has a semimajor axis that is the average of rb and r2. The time required for the entire transfer is half the period of each ellipse.

$$a1 := \frac{r1 + rb}{2} = 8.415 \times 10^{8}\,\text{km} \qquad a1\_au := \frac{a1}{au} = 5.625$$

$$a2 := \frac{rb + r2}{2} = 1.346 \times 10^{9}\,\text{km} \qquad a2\_au := \frac{a2}{au} = 9$$

$$\Delta tB := \frac{1}{2}\cdot\left(2\cdot\pi\cdot\sqrt{\frac{a1^3}{\mu}}\right) + \frac{1}{2}\cdot\left(2\cdot\pi\cdot\sqrt{\frac{a2^3}{\mu}}\right) = 6.366 \times 10^{8}\,\text{s} \qquad \Delta tB = 7367.7\cdot\text{day}$$

$$\boxed{\Delta tB = 20.172\cdot\text{yr}}$$

The time on the semi-tangential transfer is calculated from Kepler's Equation:

$$\Delta t = \sqrt{\frac{a^3}{\mu}}\cdot(E - e\cdot\sin(E))$$

The semimajor axis is the same as a1 of the bi-elliptic transfer.

The eccentricity can be determined from perihelion:

$$r1 = rp = a\cdot(1 - e) \qquad eS := 1 - \frac{r1}{a1} = 0.778$$

The eccentric anomaly can be determined by finding the true anomaly from the polar equation from an ellipse, then converting that to eccentric anomaly, or the eccentric anomaly may be found directly from the form of the polar equation for an ellipse that has the eccentric anomaly.

$$r = \frac{a\left(1 - e^2\right)}{1 + e\cdot\cos(f)}$$

$$f2 := \operatorname{acos}\left[\left[\frac{a1\cdot\left(1 - eS^2\right)}{r2} - 1\right]\cdot\frac{1}{eS}\right] = 2.761\,\text{rad}$$

$$f2 = 158.213\,\text{deg}$$

$$\tan\left(\frac{f}{2}\right) = \sqrt{\frac{1 + e}{1 - e}}\cdot\tan\left(\frac{E}{2}\right)$$

$$E2 := 2\cdot\operatorname{atan}\left(\tan\left(\frac{f2}{2}\right)\cdot\sqrt{\frac{1 - eS}{1 + eS}}\right) = 2.145\,\text{rad}$$

$$E2 = 122.9\,\text{deg}$$

$$r = a\cdot(1 - e\cdot\cos(E))$$

$$E2\_alt := \operatorname{acos}\left[\frac{1}{eS}\cdot\left(1 - \frac{r2}{a1}\right)\right] = 2.145\cdot\text{rad}$$

$$E2\_alt = 122.9\cdot\text{deg}$$

$$\Delta tS := \sqrt{\frac{a1^3}{\mu}}\cdot(E2 - eS\cdot\sin(E2)) = 9.994\times 10^7\,\text{s} \qquad \Delta tS = 1157\cdot\text{day}$$

$$\Delta tS = 3.167\cdot\text{yr}$$

Earth radius = 6378 km | Page 7 of 8 | μ_{Earth} = 3.986 x 10^5 km^3/s^2

Earth-Sun distance = 1 AU = 1.495978 x 10^8 km | μ_{Sun} = 1.327 x 10^{11} km^3/s^2

(b) Compute the ΔV for the final burn of the semi-tangential elliptical transfer option.

The second burn on the semi-tangential transfer is the non-tangential burn. The ΔV is computed using the law of cosines and the flight path angle:

$$\Delta V2 = \sqrt{v\_arrival^2 + vcirc2^2 - 2 \cdot v\_arrival \cdot vcirc2 \cdot \cos(\gamma)}$$

The arrival speed is the speed of the spacecraft with respect to the Sun on the elliptical transfer at a distance of r2 = 8 AU.

$$v\_arrival := \sqrt{\mu \cdot \left(\frac{2}{r2} - \frac{1}{a1}\right)} = 8.004 \frac{km}{s}$$

The circular orbit speed is the required speed after the burn to stay in a circular orbit at r2.

$$vcirc2 := \sqrt{\frac{\mu}{r2}} = 10.53 \frac{km}{s}$$

The flight path angle is determined from

$$\cos(\gamma) = \sqrt{\frac{a^2 \cdot \left(1 - e^2\right)}{r \cdot (2 \cdot a - r)}}$$

$$\gamma := \operatorname{acos}\left[\sqrt{\frac{a1^2 \cdot \left(1 - eS^2\right)}{r2 \cdot (2 \cdot a1 - r2)}}\right] = 0.805 \cdot rad \qquad \gamma = 46.102 \cdot deg$$

$$\cos(\gamma) = 0.693$$

$$\Delta V2 := \sqrt{v\_arrival^2 + vcirc2^2 - 2 \cdot v\_arrival \cdot vcirc2 \cdot \cos(\gamma)} = 7.62 \frac{km}{s}$$

Earth radius = 6378 km

Page 8 of 8

μ_{Earth} = 3.986 x 10^5 km^3/s^2

Earth-Sun distance = 1 AU = 1.495978 x 10^8 km

μ_{Sun} = 1.327 x 10^{11} km^3/s^2

4.1.2 Some Maple versification on first exam

problem 1

```
> restart;
  parms:={mu=3.986*10^5, T=8*60*60,r__p=6378};
```

$$parms := \{T = 28800, \mu = 3.98600000\ 10^5, r_p = 6378\}$$

```
> #define the equation to solve
  eq:=T = 2*Pi*sqrt(a^3/mu):
  `eq`,subs(parms,eq);
```

$$T = 2\pi\sqrt{\frac{a^3}{\mu}},\ 28800 = 0.003167826216\,\pi\sqrt{a^3}$$

```
> sol := solve(subs(parms,eq),a);
```

$$sol := 20307.39319, -10153.69659 + 17586.71839\,I, -10153.69659 - 17586.71839\,I$$

```
> sol:=select(x->type(x,'realcons'),[sol]);
```

$$sol := [20307.39319]$$

```
> parms:={op(parms),a=op(sol)};
```

$$parms := \{T = 28800, a = 20307.39319, e = 0.5, \mu = 3.98600000\ 10^5\}$$

```
> eq:=a*(1-e)>= r__p;
```

$$eq := r_p \le a\,(1 - e)$$

```
> solve(subs(parms,eq),e);
```

$$RealRange(-\infty, 0.6859271921)$$

Problem 2

```
> parms:={T=8*60*60,e=0.5,mu=3.986*10^5,a=op(sol)};
```

$$parms := \{T = 28800, a = 20307.39319, e = 0.5, \mu = 3.98600000\ 10^5\}$$

```
> rp:=a*(1-e);
```

$$rp := a\,(1 - e)$$

```
> vp := sqrt(mu*(2/rp - 1/a));
```

$$vp := \sqrt{\mu\left(\frac{2}{a\,(1 - e)} - \frac{1}{a}\right)}$$

```
> vesc__rp:=sqrt(2*mu/rp);
```

$$vesc_{rp} := \sqrt{2}\sqrt{\frac{\mu}{a\,(1 - e)}}$$

```
> ra:=a*(1+e);
```

$$ra := a\,(e + 1)$$

```
> va := sqrt(mu*(2/ra - 1/a));
```

$$va := \sqrt{\mu\left(\frac{2}{a\,(e + 1)} - \frac{1}{a}\right)}$$

```
> vesc__ra:=sqrt(2*mu/ra);
```

$$vesc_{ra} := \sqrt{2}\sqrt{\frac{\mu}{a\,(e+1)}}$$

```
> delta__p:=vesc__rp-vp;
```

$$\delta_p := \sqrt{2}\sqrt{\frac{\mu}{a\,(1-e)}} - \sqrt{\mu\left(\frac{2}{a\,(1-e)} - \frac{1}{a}\right)}$$

```
> delta__a:=vesc__ra-va;
```

$$\delta_a := \sqrt{2}\sqrt{\frac{\mu}{a\,(e+1)}} - \sqrt{\mu\left(\frac{2}{a\,(e+1)} - \frac{1}{a}\right)}$$

```
> evalf(subs(parms,delta__a));
```

2.557884502

```
> evalf(subs(parms,delta__p));
```

1.187118329

4.2 midterm

4.2.1 questions

EMA 550

Exam#2, Spring 2014

Take home exam Due 2:30 p.m., Tuesday April 22, 2014

Name ___________________________________

Show all of your work to get credit for your answer. Include units with all answers.

You may use mathematical software such as Matlab, MathCad, EES, etc., but include a printout of your worksheets with your solution.

Since time is not an issue, please present your solution in a neat form that is easily readable.

Use this page as a cover sheet for the work you turn in.

You are allowed to consult your notes and homework (i.e., all the class materials you would have during an in-class exam) but are not allowed to consult or collaborate with classmates or anyone other than the instructor. It is permissible to ask the instructor for clarification of the exam questions.

Points Score

Problem 1 (20 points)

(a) Find the semi-major axis and eccentricity of the heliocentric orbit that connects Mars on April 17, 2014 ($r_{Sun\text{-}Mars}$ = 1.524 AU = 2.280 x 10^8 km) with Uranus 20 years later on April 17, 2034 ($r_{Sun\text{-}Uranus}$ = 19.19 AU = 2.871 x 10^9 km). Use data from JPL Horizons at 00:00 UT on the given days to determine their angular positions. Assume the planets are in circular orbits in the ecliptic plane.

(b) For solutions using Lambert's method (whether you are using Lambert's method or not), should the variable α be calculated as $\alpha = 2\,\text{asin}\sqrt{\frac{s}{2a}}$ or $\alpha = 2\pi - 2\,\text{asin}\sqrt{\frac{s}{2a}}$? Why?

(c) For solutions using Lambert's method (whether you are using Lambert's method or not), should the variable β be calculated as $\beta = 2\,\text{asin}\sqrt{\frac{s-c}{2a}}$ or $\beta = -2\,\text{asin}\sqrt{\frac{s-c}{2a}}$? Why?

(d) What is the true anomaly of Mars on the transfer orbit at the time the transfer begins?

(e) What is the true anomaly of Uranus on the transfer orbit at the time the transfer ends?

(f) Draw to scale, on a single figure, the circular heliocentric orbits of Mars and Uranus and the transfer orbit. Clearly label Mars's position at the start of the transfer, Uranus's position at the end of the transfer, the transfer angle, and the direction of motion.

Problem 2 (20 points)

An astronaut is working on the Hubble Space Telescope (HST), which orbits the Earth in a circular orbit at an altitude of 570 km. The astronaut kicks a tool backward, giving it a speed of 0.5 m/s in the positive x-direction of an HST-centered rotating coordinate system.

(a) Plot the trajectory of the tool on *xy*-axes relative to the Hubble Space Telescope over the next two orbit periods of the HST circling the Earth. Clearly label your axes, including units. The origin of the *xy*-system should be the Hubble Space Telescope.

(b) At what time is the tool directly between the HST and the Earth (i.e., directly below the Hubble)?

(c) What is the lowest altitude that the tool reaches while drifting?

(d) How far ahead or behind the HST does the tool drift during each orbit period of the HST about the Earth? Is the tool getting ahead of the HST or drifting behind it?

Problem 3 (20 points)

Following a burn, a spacecraft has the following Earth-centered Cartesian position and velocity vectors:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7000 \\ 14000 \\ 7000 \end{pmatrix} \text{km} \qquad \begin{pmatrix} vx \\ vy \\ vz \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \frac{\text{km}}{\text{s}}$$

(a) Compute the orbital elements of the spacecraft's trajectory: a, e, i, Ω, ω, and f.

(b) What are the Earth-centered Cartesian position and velocity vectors of the spacecraft 6 hours later?

Chapter 5

practice exams

5.1 First exam practice

5.1.1 questions

EMA 550

Exam #1, Spring 2011

75 Minutes, Open Notes February 24, 2011

Name ________________________________

For the purposes of this exam, assume the Earth is spherical with a radius of 6378 km and $\mu = 3.986 \times 10^5$ km^3/s^2. Show all your work to get credit for your answer. Include units with all answers.

	Points	Score
Problem 1	40	______
Problem 2	20	______
Problem 3	20	______
Total Score	80	______

If you can't do one section of a multi-part problem and the following parts depend on your answer, make a reasonable assumption, write on your paper that you are assuming an answer for that part, and then continue on with the problem using that assumption.

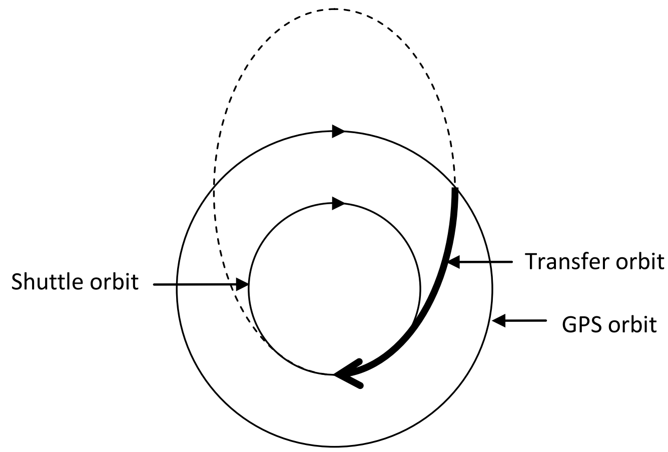
Problem 1 (40 points)

A GPS satellite orbiting the Earth has malfunctioned and is to be brought back to the Space Shuttle for servicing. The GPS satellite is initially in a circular orbit with a radius of 26,610 km. The Space Shuttle is in a circular orbit in the same plane at an altitude of 200 km.

(a) (15 points) Calculate the ΔV (magnitude and sign) for each burn that will bring the GPS satellite to the Space Shuttle on a Hohmann transfer.

(b) (4 points) Explain why the signs (positive or negative) for the burns you calculated in part (a) are correct.

(c) (15 points) Instead of returning on a Hohmann trajectory, the mission managers decide to send the GPS satellite back to the Space Shuttle's orbit along an ellipse that is tangent to the Space Shuttle's orbit and has an apogee radius of 40,000 km. Calculate the ΔVs needed for this semi-tangential return.



(d) (6 points) Calculate the transfer times in minutes for both the Hohmann return and the semi-tangential return.

Problem 2 (20 points)

A satellite is on an elliptical orbit about the Earth with a 6 hour orbital period. At perigee, the satellite is 5000 km from the center of the Earth.

(a) (15 points) At apogee, a burn is performed that allows the satellite to escape the Earth's gravitational pull. What is the smallest Δv that will accomplish Earth escape from the original orbit's apogee?

(b) (5 points) If a burn is made at perigee on the original orbit instead of at apogee and has a Δv of 2 km/s, what type of trajectory (ellipse, parabola, hyperbola) is the spacecraft on after the burn? Show your reasoning and supporting calculations.

Problem 3 (20 points)

Halley's Comet is on an elliptical orbit about the Sun. If its perihelion distance is 0.586 AU (astronomical units), its aphelion distance is 35.1 AU, and it was last at perihelion in the February of 1986, in what future year will Halley's Comet next cross the Earth's orbit about the Sun? (Assume for this problem that the orbit of Halley's Comet is in the same plane as the Earth's orbit about the Sun and that the Earth's orbit about the Sun is circular.)

Useful constants: μSun = 1.327×10^{11} km^3/s^2

1 AU = Earth's distance from the Sun = 1.495×10^8 km

5.1.2 key solution

Problem 1 (40 points)

A GPS satellite orbiting the Earth has malfunctioned and is to be brought back to the Space Shuttle for servicing. The GPS satellite is initially in a circular orbit with a radius of 26,610 km. The Space Shuttle is in a circular orbit in the same plane at an altitude of 200 km.

(a) (15 points) Calculate the ΔV (magnitude and sign) for each burn that will bring the GPS satellite to the Space Shuttle on a Hohmann transfer.

$$aH := \frac{r1 + r2}{2} \qquad aH = 16594\text{km}$$

$$v1 := \sqrt{\frac{\mu}{r1}} \qquad v1 = 3.87\frac{\text{km}}{\text{s}}$$

$$v2 := \sqrt{\mu\cdot\left(\frac{2}{r1} - \frac{1}{aH}\right)} \qquad v2 = 2.437\frac{\text{km}}{\text{s}}$$

$$v3 := \sqrt{\mu\cdot\left(\frac{2}{r2} - \frac{1}{aH}\right)} \qquad v3 = 9.858\frac{\text{km}}{\text{s}}$$

$$v4 := \sqrt{\frac{\mu}{r2}} \qquad v4 = 7.784\frac{\text{km}}{\text{s}}$$

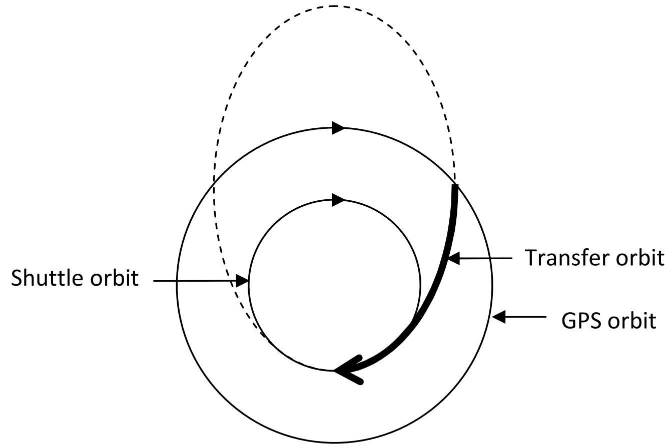
$$\Delta v1 := v2 - v1 \qquad \boxed{\Delta v1 = -1.434\frac{\text{km}}{\text{s}}}$$

$$\Delta v2 := v4 - v3 \qquad \boxed{\Delta v2 = -2.073\frac{\text{km}}{\text{s}}}$$

(b) (4 points) Explain why the signs (positive or negative) for the burns you calculated in part (a) are correct.

The satellite is starting on the largest orbit. The first Δv is negative, indicating that the spacecraft has to slow down to drop down to a smaller orbit, one with a perigee equal to the Shuttle's orbit. The second Δv is also negative, because the final circular orbit is smaller than the transfer orbit, so staying on the Shuttle's orbit requires reducing the orbit energy further.

(c) (15 points) Instead of returning on a Hohmann trajectory, the mission managers decide to send the GPS satellite back to the Space Shuttle's orbit along an ellipse that is tangent to the Space Shuttle's orbit and has an apogee radius of 40,000 km. Calculate the ΔVs needed for this semi-tangential return.



$rS := 40000\cdot km$

$aS := \frac{rS + r2}{2} = 23289\,km$

$rp := r2$

$eS := 1 - \frac{rp}{aS} = 0.718$

$v2S := \sqrt{\mu\cdot\left(\frac{2}{r1} - \frac{1}{aS}\right)} = 3.584\,\frac{km}{s}$

$\cos\gamma := \sqrt{\frac{aS^2\left(1 - eS^2\right)}{r1\cdot(2\cdot aS - r1)}} = 0.704$ $\gamma := \text{acos}(\cos\gamma) = 45.275\cdot deg$ $\gamma = 0.79\,rad$

Note: since the spacecraft is coming into perigee, the flight path angle is the negative of the value shown above. Since calculating the ΔV uses the cosine of the flight path angle, the cosine and thus the ΔV are the same for either a postivie or negative flight path angle.

$\Delta V1S := \sqrt{v1^2 + v2S^2 - 2\cdot v1\cdot v2S\cdot\cos\gamma} = 2.881\,\frac{km}{s}$ $\Delta V1S = 2.881\,\frac{km}{s}$

$v3S := \sqrt{\mu\cdot\left(\frac{2}{r2} - \frac{1}{aS}\right)} = 10.202\,\frac{km}{s}$

$\Delta V2S := v4 - v3S = -2.417\,\frac{km}{s}$ $\Delta V2S = -2.417\,\frac{km}{s}$

Page 2 of 7

(d) (6 points) Calculate the transfer times in minutes for both the Hohmann return and the semi-tangential return.

$$\Delta tH := \frac{1}{2}\cdot\left(2\cdot\pi\cdot\sqrt{\frac{aH^3}{\mu}}\right) = 1.064\times 10^4\,s \qquad \Delta tH = 177.279\,min$$

$$f1 := acos\left[\frac{1}{eS}\cdot\left[\frac{aS\cdot\left(1-eS^2\right)}{r1} - 1\right]\right] = 2.501\cdot rad \qquad f1 = 143.314\,deg$$

Note: we can use this value of f to correctly solve the problem, but since f is measure in the direction of motion, the most accurate description of the satellite's position at the start of the transfer is 2*pi-f1.

$$E1 := 2\cdot atan\left(tan\left(\frac{f1}{2}\right)\cdot\sqrt{\frac{1-eS}{1+eS}}\right) = 1.771\cdot rad \qquad E1 = 101.463\,deg$$

Could also calculate E directly from r = a*(1-e*cos(E))

$$E2 := acos\left[\frac{1}{eS}\cdot\left(1-\frac{r1}{aS}\right)\right] = 1.771\cdot rad \qquad E2 = 101.463\,deg$$

$$\Delta tS := \sqrt{\frac{aS^3}{\mu}}\cdot(E1 - eS\cdot sin(E1)) = 6.01\times 10^3\,s \qquad \Delta tS = 100.167\,min$$

Problem 2 (20 points)

A satellite is on an elliptical orbit about the Earth with a 6 hour orbital period. At perigee, the satellite is 5000 km from the center of the Earth.

Note: this is a poorly written problem, as r = 5000 km is within the 6378 km radius of the Earth. Students were instructed to treat the Earth as a point mass and ignore that the perigee radius is inside the Earth.

(a) (15 points) At apogee, a burn is performed that allows the satellite to escape the Earth's gravitational pull. What is the smallest Δv that will accomplish Earth escape from the original orbit's apogee?

$$\mu := 3.986 \cdot 10^5 \frac{km^3}{s^2}$$

$$T := 6 \cdot hr$$

$$a := \left[\left(\frac{T}{2 \cdot \pi}\right)^2 \cdot \mu\right]^{\frac{1}{3}} = 16763\ km$$

$$rp := 5000 \cdot km$$

$$e := 1 - \frac{rp}{a} = 0.702$$

$$ra := a \cdot (1 + e) = 28527\ km$$

The smallest Earth-escape ΔV comes from placing the spacecraft on a parabolic trajectory.

$$\Delta Va := \sqrt{\frac{2 \cdot \mu}{ra}} - \sqrt{\mu \cdot \left(\frac{2}{ra} - \frac{1}{a}\right)} = 3.245 \frac{km}{s}$$

$$\boxed{\Delta Va = 3.245 \frac{km}{s}}$$

(b) (5 points) If a burn is made at perigee on the original orbit instead of at apogee and has a Δv of 2 km/s, what type of trajectory (ellipse, parabola, hyperbola) is the spacecraft on after the burn? Show your reasoning and supporting calculations.

Method 1: compute the ΔV needed for a parabolic trajectory from the perigee distance. A smaller ΔV indicates an elliptical orbit, and a larger ΔV indicates a hyperbolic orbit.

$$\Delta Vp := \sqrt{\frac{2\cdot\mu}{rp}} - \sqrt{\mu\cdot\left(\frac{2}{rp} - \frac{1}{a}\right)} = 0.98\,\frac{km}{s}$$

The given ΔV is larger than the ΔV needed for a parabolic trajectory, so the spacecraft is on a hyperbolic trajectory.

Method 2: calculate the circular orbit speed at perigee and see if the given ΔV results in a post-burn speed of more than sqrt(2) times the circular orbit speed at the perigee distance.

$$\Delta Vgiven := 2\cdot\frac{km}{s}$$

$$vp := \sqrt{\mu\cdot\left(\frac{2}{rp} - \frac{1}{a}\right)} = 11.647\,\frac{km}{s} \qquad vcircp := \sqrt{\frac{\mu}{rp}} = 8.929\,\frac{km}{s}$$

$$vp + \Delta Vgiven = 13.647\,\frac{km}{s} \qquad \sqrt{2}\cdot vcircp = 12.627\,\frac{km}{s}$$

The speed is greater than sqrt(2)*vcirc, so the spacecraft is on a hyperbola.

Note: it is not accurate to apply this method using sqrt(2)*vp. The speed on a parabola is sqrt(2) times the circular orbit speed at the same distance, not sqrt(2) times any other orbit speed at that distance.

Making that mistake on this problem would indicate the the spacecraft is on an ellipse, since vp + ΔV is less than vp* sqrt(2).

$$vp\cdot\sqrt{2} = 16.472\,\frac{km}{s}$$

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Problem 3 (20 points)

Halley's Comet is on an elliptical orbit about the Sun. If its perihelion distance is 0.586 AU (astronomical units), its aphelion distance is 35.1 AU, and it was last at perihelion in the February of 1986, in what future year will Halley's Comet next cross the Earth's orbit about the Sun? (Assume for this problem that the orbit of Halley's Comet is in the same plane as the Earth's orbit about the Sun and that the Earth's orbit about the Sun is circular.)

Useful constants: μSun = 1.327×10^{11} km^3/s^2
1 AU = Earth's distance from the Sun = 1.495×10^8 km

$$\mu := 1.327 \cdot 10^{11} \frac{km^3}{s^2}$$

$$au := 1.495 \cdot 10^8 km$$

$$rp := 0.586 \cdot au = 8.761 \times 10^7 \, km$$

$$ra := 35.1 \cdot au = 5.247 \times 10^9 \, km$$

Orbit Characteristics

$$e := \frac{ra - rp}{rp + ra} = 0.967$$

$$a := \frac{ra}{1 + e} = 2.668 \times 10^9 \, km \qquad \frac{a}{au} = 17.843$$

(see next page)

(Additional workspace for Problem 3)

Crossing Earth's Orbit

$r := 1 \cdot au = 1.495 \times 10^{8}\ km$

$E1 := acos\left[\frac{1}{e}\cdot\left(1 - \frac{r}{a}\right)\right] = 0.219\ rad$ or $E2 := 2\cdot\pi - E1 = 6.064\ rad$

$E1 = 12.576\ deg$ $E2 = 347.424\ deg$

$T := 2\cdot\pi\cdot\sqrt{\frac{a^3}{\mu}} = 2.376\times 10^{9}\ s$ $T = 75.303\ yr$

$n := \frac{2\cdot\pi}{T}$ $n = 2.644\times 10^{-9}\ \frac{1}{s}$

$\Delta t1 := \frac{1}{n}\cdot(E1 - e\cdot \sin(E1)) = 3.369\times 10^{6}\ s$ $\Delta t1 = 0.107\ yr$

$\Delta t2 := \frac{1}{n}\cdot(E2 - e\cdot \sin(E2)) = 2.373\times 10^{9}\ s$ $\Delta t2 = 75.196\ yr$

On current cycle of orbit, first crossed Earth's path in the year 1986 + Δt1 = 1986. Next crossing of Earth's orbit will be Δt2 since latest perigee.

$(1986\cdot yr + \Delta t2) = 2061\ yr$

Halley's Comet will next cross the Earth's path in 75.2 years (75 years and 2.4 months) from February, 1986, placing the crossing in April or May of 2061, depending on when in February it was at perihelion.

5.1.3 my solution

my Solution to practice exam 1, EMA 550

021914, Nasser M. Abbasi (EMA 550)
Up
notebook
PDF

problem 1

question

For the purposes of this exam, assume the Earth is spherical with a radius of 6378 km and $\mu = 3.986 \times 10^5$ km^3/s^2. Show all your work to get credit for your answer. Include units with all answers.

A GPS satellite orbiting the Earth has malfunctioned and is to be brought back to the Space Shuttle for servicing. The GPS satellite is initially in a circular orbit with a radius of 26,610 km. The Space Shuttle is in a circular orbit in the same plane at an altitude of 200 km.

(a) (15 points) Calculate the ΔV (magnitude and sign) for each burn that will bring the GPS satellite to the Space Shuttle on a Hohmann transfer.

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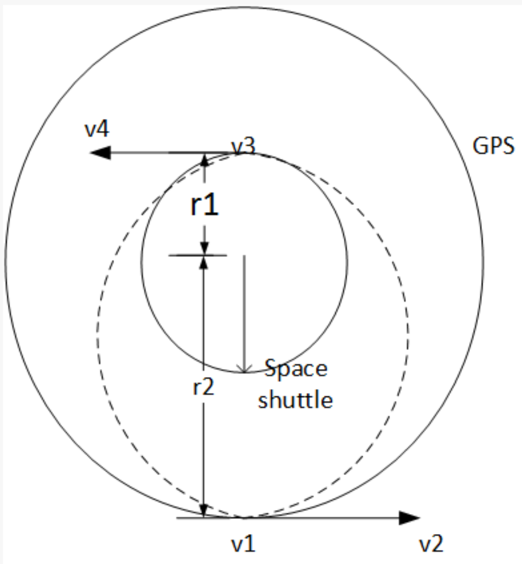
2 | *sol.nb*

answer part (a)



In[32]:=
```
r1 = 200 + 6378;(*space shuttle orbit*)
r2 = 26 610;(*satellitem GPS*)
mu = 3.986 * 10^5;
```
$$a = \frac{r1 + r2}{2}$$

Out[35]=
```
16 594
```

$$v1 = \sqrt{\frac{mu}{r2}}$$

```
3.87031
```

$$v2 = \sqrt{mu\left(\frac{2}{r2} - \frac{1}{a}\right)}$$

```
2.43679
```

$$v3 = \sqrt{mu\left(\frac{2}{r1} - \frac{1}{a}\right)}$$

```
9.85754
```

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```
v4 = Sqrt[mu/r1]
```

$$v4 = \sqrt{\frac{mu}{r1}}$$

```
7.78434
```

```
v2 - v1
```

```
-1.43353
```

```
v4 - v3
```

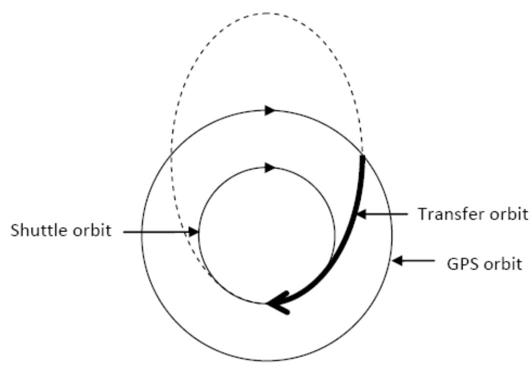
```
-2.0732
```

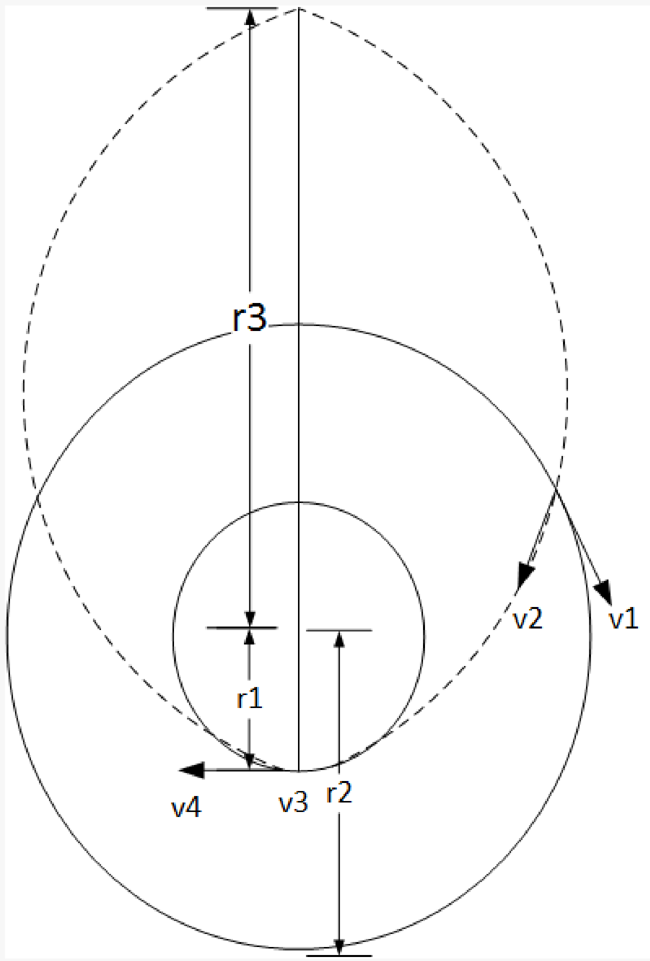
```
delV = Abs[v2 - v1] + Abs[v4 - v3]
```

```
3.50673
```

part (c)

(c) (15 points) Instead of returning on a Hohmann trajectory, the mission managers decide to send the GPS satellite back to the Space Shuttle's orbit along an ellipse that is tangent to the Space Shuttle's orbit and has an apogee radius of 40,000 km. Calculate the ΔVs needed for this semi-tangential return.





In[40]:=

```
r3 = 40 000; (*new transfer orbit*)
a = (r3 + r1)/2
```

Out[41]=

```
23 289
```

In[10]:=

```
(e = (r3 - r1)/(r3 + r1)) // N
```

Out[10]=

```
0.717549
```

In[12]:=
$$v1 = \sqrt{\frac{mu}{r2}}$$
Out[12]= 3.87031

In[13]:=
$$v2 = \sqrt{mu\left(\frac{2}{r2} - \frac{1}{a}\right)}$$
Out[13]= 3.58375

In[15]:=
$$\left(CosGamma = \sqrt{\frac{a^2\left(1 - e^2\right)}{r2\ (2\ a - r2)}}\right) \text{ // N}$$
Out[15]= 0.703699

In[16]:=
$$delV1 = \sqrt{v1^2 + v2^2 - 2\ v1\ v2\ CosGamma}$$
Out[16]= 2.88126

In[17]:=
$$\left(v3 = \sqrt{mu\left(\frac{2}{r1} - \frac{1}{a}\right)}\right) \text{ // N}$$
Out[17]= 10.2018

In[18]:=
$$v4 = \sqrt{\frac{mu}{r1}}$$
Out[18]= 7.78434

In[19]:=
```
delV2 = v4 - v3
```
Out[19]= -2.41745

In[20]:=
```
totalDelV = Abs@delV1 + Abs@delV2
```
Out[20]= 5.29871

part(d)

(d) (6 points) Calculate the transfer times in minutes for both the Hohmann return and the semi-tangential return.

For hohmann, the time is half the period

In[87]:=
```
delT1 = Pi Sqrt[a^3/mu]
```
Out[87]=
```
17 685.1
```

in minutes

In[88]:=
```
delT1 = delT1 / (60)
```
Out[88]=
```
294.752
```

For the semi tangential

In[129]:=
```
Clear[a, e, r, EE]
a = (r1 + r3)/2;
e = (r3 - r1)/(r1 + r3);
r2 = 26 610;
r2 == a (1 - e Cos[EE]);
Cos[EE] /. First@NSolve[%, Cos[EE]]
```
Out[134]=
```
-0.198731
```

In[135]:=
```
EE = ArcCos[%]
```
Out[135]=
```
1.77086
```

In[136]:=
```
delT2 = Sqrt[a^3/mu] (EE - e Sin[EE])
```
Out[136]=
```
6010.01
```

In[137]:=
```
delT2 = delT2 / 60
```
Out[137]=
```
100.167
```

Total time in minutes

In[138]:=
```
totalDelT = delT1 + delT2
```
Out[138]=
```
394.919
```

problem 2

Problem 2 (20 points)

A satellite is on an elliptical orbit about the Earth with a 6 hour orbital period. At perigee, the satellite is 5000 km from the center of the Earth.

(a) (15 points) At apogee, a burn is performed that allows the satellite to escape the Earth's gravitational pull. What is the smallest Δv that will accomplish Earth escape from the original orbit's apogee?

part(a)

In[168]:=

```
Clear[a]
rp = 5000;
mu = 3.986 * 10^5;
6 * 60 * 60 == 2 Pi Sqrt[a^3/mu]
```

Out[171]= $21\,600 == 0.00995202 \sqrt{a^3}$

In[172]:=

```
aCube = (21 600 / 0.009952019565792981`)^2
```

Out[172]= 4.7107×10^{12}

In[173]:=

```
(aCube)^(1/3)
```

Out[173]= 16 763.4

In[174]:=

```
a = %
```

Out[174]= 16 763.4

In[175]:=

```
ra = 2 a - rp
```

Out[175]= 28 526.8

In[179]:=

```
va = Sqrt[mu (2/ra - 1/a)]
```

Out[179]= 2.04149

In[180]:=
```
escape = Sqrt[2 mu / ra]
```
$$escape = \sqrt{\frac{2\,mu}{ra}}$$

Out[180]= 5.28637

In[181]:=
```
smallestV = escape - va
```

Out[181]= 3.24488

(b) (5 points) If a burn is made at perigee on the original orbit instead of at apogee and has a Δv of 2 km/s, what type of trajectory (ellipse, parabola, hyperbola) is the spacecraft on after the burn? Show your reasoning and supporting calculations.

In[182]:=
$$vp = \sqrt{mu \left(\frac{2}{rp} - \frac{1}{a}\right)}$$

Out[182]= 11.6474

In[183]:=
```
v2 = 2 + vp
```

Out[183]= 13.6474

In[184]:=
$$vecsp = \sqrt{\frac{2\,mu}{rp}}$$

Out[184]= 12.627

since v2>vescape, hence hyperbolic

problem 3

Halley's Comet is on an elliptical orbit about the Sun. If its perihelion distance is 0.586 AU (astronomical units), its aphelion distance is 35.1 AU, and it was last at perihelion in the February of 1986, in what future year will Halley's Comet next cross the Earth's orbit about the Sun? (Assume for this problem that the orbit of Halley's Comet is in the same plane as the Earth's orbit about the Sun and that the Earth's orbit about the Sun is circular.)

Useful constants: μSun = 1.327×10^{11} km^3/s^2
1 AU = Earth's distance from the Sun = 1.495×10^{8} km

In[32]:=
```
mu = 1.327 * 10^11;
r1 = 1.495 * 10^8;
au = r1;
rp = 0.586 * au;
ra = 35.1 * au;
a = (rp + ra)/2
```
Out[37]= 2.66753×10^9

In[38]:=
```
Clear[e, EE];
e = (ra - rp)/(ra + rp)
```
Out[39]= 0.967158

In[40]:=
```
r1 == a (1 - e Cos[EE])
```
Out[40]= $1.495 \times 10^8 == 2.66753 \times 10^9 (1 - 0.967158 \text{Cos}[EE])$

In[41]:=
```
Cos[EE] /. First@Solve[%, Cos[EE]]
```
Out[41]= 0.97601

In[42]:=
```
EE = ArcCos[%]
```
Out[42]= 0.219485

In[44]:=
```
delT = Sqrt[a^3/mu] (EE - e Sin[EE])
```
Out[44]= 3.36928×10^6

In[45]:=
```
period = 2 Pi Sqrt[a^3/mu]
```
Out[45]= 2.37634×10^9

In[48]:=
```
(period) / (60 * 60 * 24 * 365)
```
Out[48]= 75.3531

In[46]:=
```
delT / (60 * 60 * 24 * 365)
(period - delT) / (60 * 60 * 24 * 365)
```
Out[46]=
```
0.106839
```
Out[47]=
```
75.2463
```


5.2 Mid term practice exam

5.2.1 questions

Problem 1 (20 points)

(a) Find the semi-major axis and eccentricity of the heliocentric orbit that connects the Earth on April 20, 2013 ($r_{Sun\text{-}Earth}$ = 1.496 x 10^8 km) with Saturn on May 20, 2016 ($r_{Sun\text{-}Saturn}$ = 9.537* $r_{Sun\text{-}Earth}$). Use data from JPL Horizons at 00:00 UT on the given days to determine their positions. Assume the planets are in circular orbits in the ecliptic plane.

(b) What is the true anomaly of the Earth on the transfer orbit at the time the transfer begins?

(c) What is the true anomaly of Saturn on the transfer orbit at the time the transfer ends?

(d) Draw to scale, on a single figure, the circular heliocentric orbits of Earth and Saturn and the transfer orbit. Clearly label the Earth's position at the start of the transfer, Saturn's position at the end of the transfer, the transfer angle, and the direction of motion.

Problem 2 (20 points)

You are running the maneuvers desk at mission control for a Clean Sweep satellite that is collecting orbital debris when the satellite's sensors spot a piece of debris 1000 m ahead of the satellite's current position and 500 m above the satellite's current position. The piece of debris is moving away from the satellite with a velocity relative to the satellite at the instant observed of 1 m/s in both the in-track and vertical directions.

(a) Assuming that Clean Sweep starts in a circular orbit with a 100 minute orbit period, what instantaneous ΔV vector (i.e. components along the rotating relative coordinate x and *y* directions) will allow the Clean Sweep satellite to reach the debris in exactly 15 minutes? (Note: The debris is also moving during that time.)

(b) Plot the trajectory of both the debris and the Clean Sweep satellite during the 15 minute maneuver on a single plot in rotating relative *xy*-coordinates and show that your ΔV will allow Clean Sweep to reach the piece of debris. Clearly label your axes, units, and which line corresponds to which object. The origin of the *xy*-system should be Clean Sweep's position at the start of the maneuver.

Problem 3 (20 points)

Following a satellite collision, a piece of debris is spotted by NORAD with the Earth-centered Cartesian position and velocity below.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9000 \\ 7000 \\ -8000 \end{pmatrix} \text{km} \qquad \begin{pmatrix} vx \\ vy \\ vz \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \frac{\text{km}}{\text{s}}$$

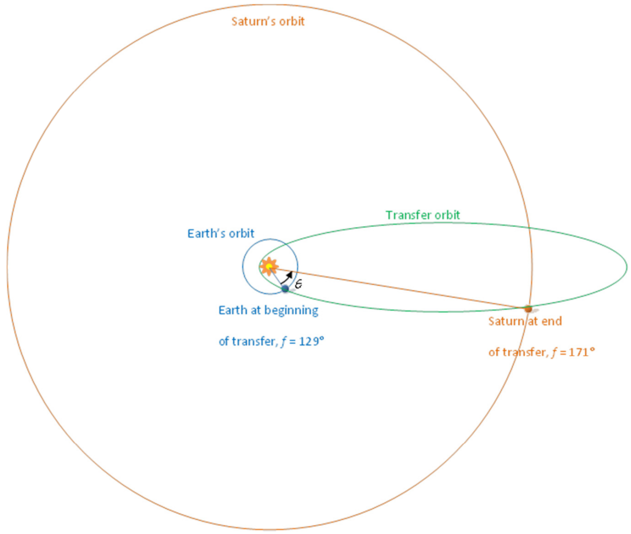
(a) Compute the orbital elements of the debris' orbit: *a*, *e*, *i*, Ω, ω, and *f*.

(b) What are the Earth-centered Cartesian position and velocity vectors of the piece of debris when it collides with the Earth? What is the speed of the debris when it collides with the Earth?

5.2.2 key

Problem 1 (20 points)
Earth data (4/20/2013):
Helio long: 209.89°
[x,y,z]=[-0.872,-0.503,-0.0000377] au
[x,y,z]=[-1.304,-0.753,-0.0000565] 10^8 km
[Ω,ω,f]=[171.7,289.9,108.4]° (sum = 210°)

Saturn data (5/20/2016)
Helio long: 252.45° (Δθ = 42.56°)
[x,y,z]=[-3.018,-9.555,0.286] au
[x,y,z]=[-4.515,-14.29,0.428] 10^8 km
[Ω,ω,f]=[113.5,340.4,158.5]° (sum = 252°)



(a) a = 9.988 x 10^8 km, e = 0.97

(b) f1Earth = 129° (2.246 rad)

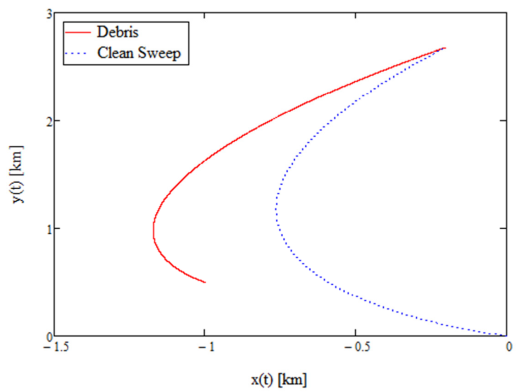
(c) f2Saturn = 171° (2.988 rad)

(d) Figure: Earth, Saturn, transfer orbit, Earth's position at start, Saturn's position at end, transfer angle, direction of motion

Problem 2 (20 points)

(a) ΔV1x = -2.459 m/s, ΔV1y = 0.961 m/s

(b) Figure:



Problem 3 (20 points)

(a) Compute the orbital elements of the satellite's orbit: *a*, *e*, *i*, Ω, ω, and *f*.

a = 11000 km
e = 0.844
i = 35.4° (0.617 rad)

Ω = 119.1° (2.078 rad)
ω = 120.9° (2.111 rad)
f = 156.3° (2.727 rad)

(b) What are the satellite's Cartesian position and velocity vectors when it collides with the Earth?

E2 = 5.234 rad (299.9°)
f2 = 4.074 rad (233.4°)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2638 \\ 5796 \\ -361 \end{pmatrix} \text{km} \qquad \begin{pmatrix} Vx \\ Vy \\ Vz \end{pmatrix} = \begin{pmatrix} -1.079 \\ -8.621 \\ 3.643 \end{pmatrix} \frac{\text{km}}{\text{s}}$$

Note: If using wrong E,
E2 = 1.049 rad = 60.1°, then
f = 2.209 rad = 126.6° and

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5385 \\ 200 \\ -3412 \end{pmatrix} \text{km} \qquad \begin{pmatrix} Vx \\ Vy \\ Vz \end{pmatrix} = \begin{pmatrix} 5.430 \\ 5.587 \\ -5.297 \end{pmatrix} \frac{\text{km}}{\text{s}}$$

5.3 practice exams for finals

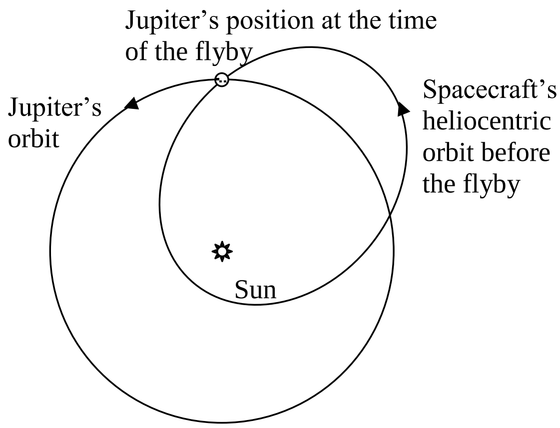
5.3.1 2011

5.3.1.1 questions

Problem 1

A spacecraft is orbiting the Sun on the elliptical heliocentric orbit shown. The spacecraft's orbit crosses Jupiter's orbit twice each revolution, and at one of the crossings (as shown on the figure), Jupiter is positioned close enough to the crossing that the spacecraft enters Jupiter's sphere of influence. Relative to the Sun, the spacecraft arrives at Jupiter's sphere of influence with a speed of 11.5 km/s and a flight path angle of -35°. Jupiter is moving with a speed of 13 km/s relative to the Sun in a circular orbit.

Heliocentric view:



(a) Calculate the speed of the spacecraft with respect to Jupiter when it enters Jupiter's sphere of influence (v_∞).

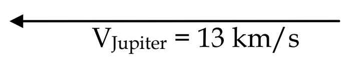
radius of the Earth = 6378 km $\quad$ μEarth = $3.986 * 10^5$ km^3/s^2 $\quad$ g = $9.81 * m/s^2$

(b) The spacecraft enters Jupiter's sphere of influence with an impact parameter equal to ten times the radius of Jupiter, resulting in a turning angle (θ) relative to Jupiter of 145°. On the figures below, draw and label

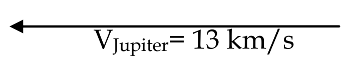
a. velocity vector $\mathbf{v}_a$, the spacecraft's arrival at Jupiter's sphere of influence with respect to the Sun
b. velocity vector $\mathbf{v}_{\infty,in}$, the incoming asymptote relative to Jupiter
c. velocity vector $\mathbf{v}_{\infty,out}$, the outgoing asymptote relative to Jupiter
d. velocity vector $\mathbf{v}_d$, the spacecraft's departure from Jupiter's sphere of influence with respect to the Sun
e. the turning angle θ with respect to Jupiter
f. the heliocentric flight path angle at arrival (γ_a)
g. the heliocentric flight path angle at departure (γ_d).

The lengths of the vectors should be drawn approximately to scale and the required angles should be drawn approximately accurately. You do not need to calculate all of the unknown velocity values and angle values. Note: it may help you to sketch the flybys of Jupiter in Jupiter's frame of reference.

Velocity diagram for a flyby ***in front of Jupiter:***



Velocity diagram for a flyby ***behind Jupiter:***



radius of the Earth = 6378 km μEarth = $3.986 * 10^5$ km^3/s^2 g = 9.81*m/s^2

(c) Calculate the speed of the spacecraft relative to the Sun after the flyby *behind* Jupiter.

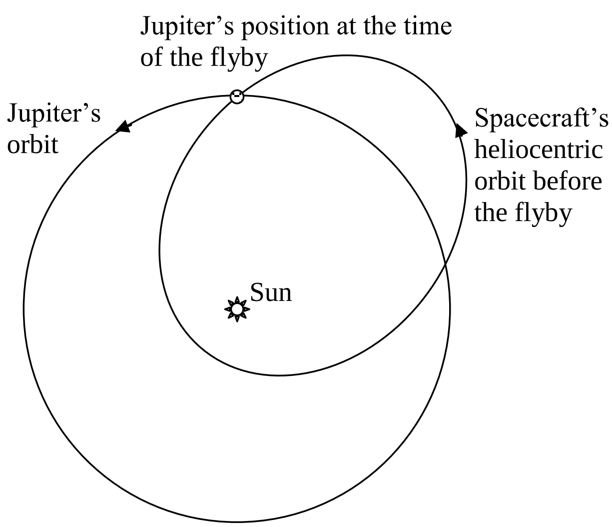
radius of the Earth = 6378 km μEarth = $3.986 * 10^5$ km^3/s^2 g = $9.81 * m/s^2$

(d) Calculate the flight path angle of the spacecraft relative to the Sun after the flyby *behind* Jupiter.

radius of the Earth = 6378 km μEarth = $3.986 * 10^5$ km^3/s^2 g = 9.81*m/s^2

(f) On the heliocentric view below, draw and label the following:
   a. velocity vector v_a, the spacecraft's arrival at Jupiter's sphere of influence with respect to the Sun
   b. velocity vector $v_{d,in\ front}$, the spacecraft's departure from Jupiter's sphere of influence with respect to the Sun following a flyby in front of Jupiter
   c. velocity vector $v_{d,behind}$, the spacecraft's departure from Jupiter's sphere of influence with respect to the Sun following a flyby behind Jupiter.

Heliocentric view:



Problem 2

A rocket engine that generates 3000 N of thrust by burning 60 kg of fuel at a constant rate over 1 minute is attached to a satellite orbiting the earth with the following orbital parameters:

perigee distance r_p = 7000 km
apogee distance r_a = 14,000 km
inclination i = 28.5°
right ascension of ascending node Ω = 90°
argument of perigee ω = 0°

(a) At what true anomaly in the satellite's orbit must the engine be fired in order to achieve the maximum inclination change? Why?

radius of the Earth = 6378 km   μEarth = $3.986 * 10^5$ km^3/s^2 g = 9.81*m/s^2

(b) If the combination of the satellite and rocket prior to burning the engine has a mass of 120 kg, what is the maximum degree of inclination change that the satellite can achieve?

radius of the Earth = 6378 km μEarth = $3.986 * 10^5$ km^3/s^2 g = $9.81*m/s^2$

Questions

For this section, answer the questions in complete sentences. Use equations and minor calculations where appropriate, but the emphasis is on explaining course concepts rather than solving for numerical values.

(a) The equation that describes the drift in right ascension of an Earth-orbiting satellite due to the oblateness of the Earth is

$$\frac{d\Omega}{dt} = -\frac{9.969}{\left(1-e^2\right)^2}\left(\frac{R_E}{R_E+\bar{h}}\right)^{3.5}\cos i \quad \text{deg/day}$$

What inclination orbits experience the maximum drift in right ascension of ascending node? What is the physical reason that the effect is greatest for those inclinations?

What inclination orbits experience the least drift in right ascension of the ascending node? What is the physical reason that the effect is least for those inclinations?

radius of the Earth = 6378 km μEarth = 3.986 * 10^5 km^3/s^2 g = 9.81*m/s^2

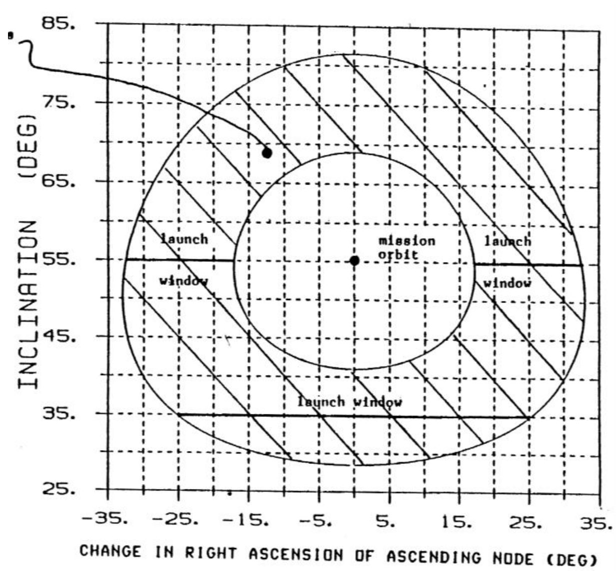
(b) A satellite is orbiting the Earth on a circular orbit with a radius of 8059 km.

At time t = 0, a small explosion aboard a satellite sends three pieces flying away from the main body of the satellite.

Piece A speeds up by 2 m/s in the original satellite's direction of motion.
Piece B attains a velocity of 1 m/s in the direction perpendicular to the orbit plane.
Piece C receives a 3 m/s ΔV toward the Earth.

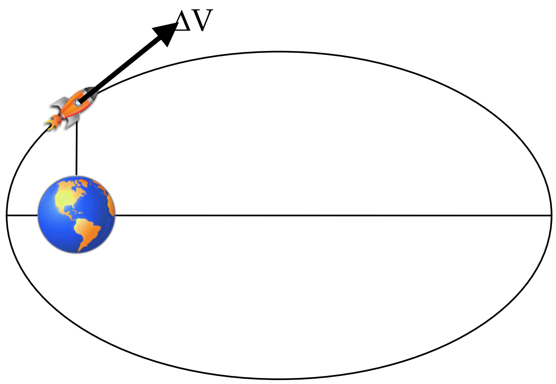
Which piece will be farthest away from the main satellite 6 hours later? Justify your answer.

(c) On page 15-9 of the course notes, you have the following figure, which illustrates right ascensions and inclinations of r_1 = 6656 km circular parking orbits that will allow an Earth-orbiting satellite with fixed-impulse rocket engines providing ΔV1 = 2.107 km/s and ΔV2 = 1.888 km/s to reach a desired mission orbit with r_2 = 26,565 km, Ω = 0°, and i = 55°.



The shaded region of the figure indicates parking orbits that will allow the rocket to reach the mission orbit. The mission orbit is in the middle of the unshaded region. Why is this okay (and expected)?

(d) A spacecraft is in an elliptical orbit about the Earth with a semimajor axis of a = 40,000 km and eccentricity e = 0.8, as shown to the right. An instantaneous tangential ΔV is applied to the spacecraft, but rather than being applied at perigee or apogee, the tangential ΔV is applied when the spacecraft has a true anomaly of f = 90°.



Provide (but do not solve) the complete set of equations needed to find the semimajor axis (a_{new}) and the eccentricity (e_{new}) of the resulting orbit and the spacecraft's true anomaly (f_{new}) on that orbit. Next to each equation, indicate why it is important (i.e., what variable(s) is (are) found from each equation or system of equations). You may assume that the orbit remains elliptical after the impulse.

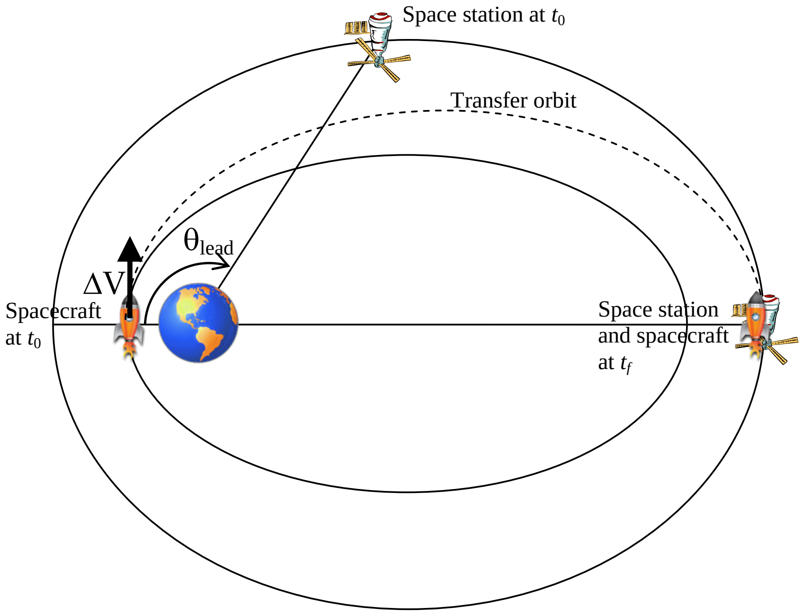
radius of the Earth = 6378 km μEarth = $3.986 * 10^5$ km^3/s^2 g = 9.81*m/s^2

How would you be able to tell if the post-ΔV orbit was hyperbolic instead of elliptical?

How would your equations for calculating a_{new}, e_{new}, and f_{new} change if the post-ΔV orbit was hyperbolic?

radius of the Earth = 6378 km μEarth = $3.986 * 10^5$ km^3/s^2 g = $9.81*m/s^2$

(e) At time t_0, a spacecraft is at perigee on an elliptical orbit with semimajor axis a_1 and eccentricity e_1. It completes an impulsive, tangential burn that will allow it to rendezvous with a space station on a larger orbit with semimajor axis a_2 and eccentricity e_2. The rendezvous occurs at apogee on the station's orbit, so the transfer orbit is tangential to the final orbit as well as the spacecraft's initial orbit. The transfer ellipse is shown on the figure below as the dashed line.



Provide (but do not solve) all of the equations necessary to find θ_{lead}, the angle by which the space station must lead the spacecraft at t_0, the time of the initial rocket firing. Indicate why you included each equation and simplify where possible using the properties of perigee and apogee. Additional space is provided on the following page.

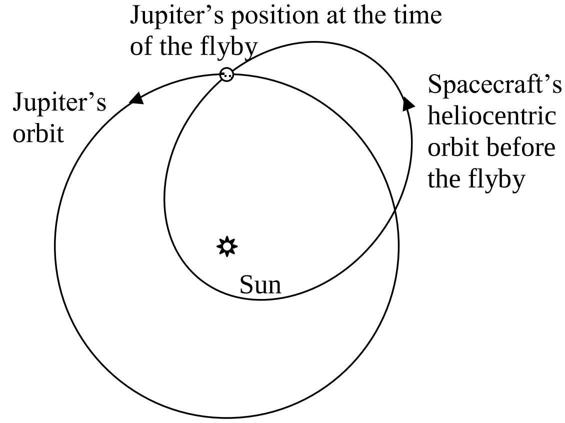
radius of the Earth = 6378 km μEarth = 3.986 * 10^5 km^3/s^2 g = 9.81*m/s^2

5.3.1.2 key

Problem 1

A spacecraft is orbiting the Sun on the elliptical heliocentric orbit shown. The spacecraft's orbit crosses Jupiter's orbit twice each revolution, and at one of the crossings (as shown on the figure), Jupiter is positioned close enough to the crossing that the spacecraft enters Jupiter's sphere of influence. Relative to the Sun, the spacecraft arrives at Jupiter's sphere of influence with a speed of 11.5 km/s and a flight path angle of -35°. Jupiter is moving with a speed of 13 km/s relative to the Sun in a circular orbit.

Heliocentric view:



(a) Calculate the speed of the spacecraft with respect to Jupiter when it enters Jupiter's sphere of influence (v_∞).

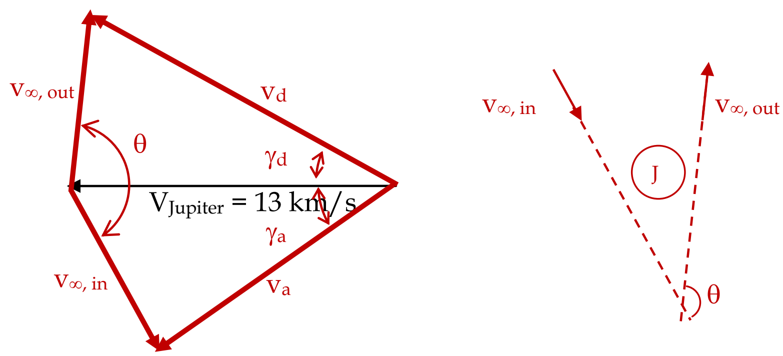
$$vinf := \sqrt{VJ^2 + va^2 - 2\cdot VJ\cdot va\cdot\cos(\gamma)} = 7.505\,\frac{km}{s}$$

(b) The spacecraft enters Jupiter's sphere of influence with an impact parameter equal to ten times the radius of Jupiter, resulting in a turning angle (θ) relative to Jupiter of 145°. On the figures below, draw and label

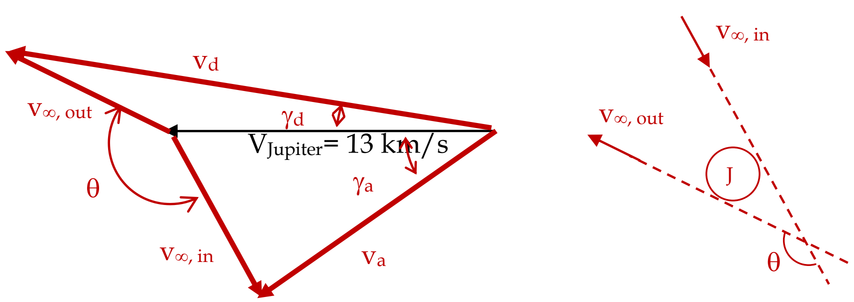
a. velocity vector $\mathbf{v}_a$, the spacecraft's arrival at Jupiter's sphere of influence with respect to the Sun
b. velocity vector $\mathbf{v}_{\infty,in}$, the incoming asymptote relative to Jupiter
c. velocity vector $\mathbf{v}_{\infty,out}$, the outgoing asymptote relative to Jupiter
d. velocity vector $\mathbf{v}_d$, the spacecraft's departure from Jupiter's sphere of influence with respect to the Sun
e. the turning angle θ with respect to Jupiter
f. the heliocentric flight path angle at arrival (γ_a)
g. the heliocentric flight path angle at departure (γ_d).

The lengths of the vectors should be drawn approximately to scale and the required angles should be drawn approximately accurately. You do not need to calculate all of the unknown velocity values and angle values. Note: it may help you to sketch the flybys of Jupiter in Jupiter's frame of reference.

Velocity diagram for a flyby ***in front of Jupiter:***

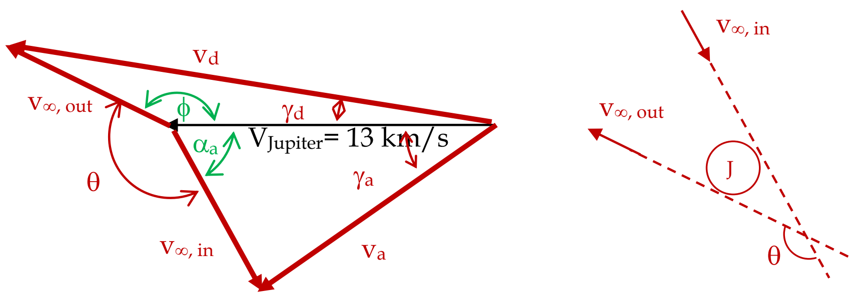


Velocity diagram for a flyby ***behind Jupiter:***



radius of the Earth = 6378 km μEarth = $3.986 * 10^5$ km^3/s^2 g = $9.81*m/s^2$

(c) Calculate the speed of the spacecraft relative to the Sun after the flyby *behind* Jupiter.



$$\gamma a := |\gamma| = 35 \cdot deg$$

$$\alpha a := asin\left(\frac{va}{vinf} \cdot sin(\gamma a)\right) = 61.511 deg$$

$$\phi := 2 \cdot \pi - \alpha a - \theta = 153.733 deg$$

$$vd := \sqrt{VJ^2 + vinf^2 - 2 \cdot VJ \cdot vinf \cdot cos(\phi)} = 20.008 \frac{km}{s}$$

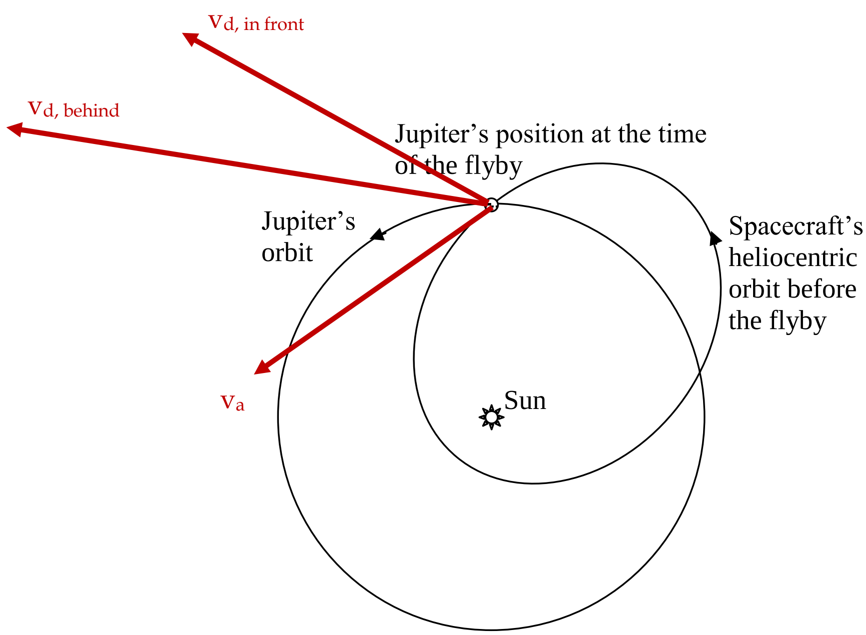
radius of the Earth = 6378 km μEarth = 3.986 * 10^5 km^3/s^2 g = 9.81*m/s^2

(d) Calculate the flight path angle of the spacecraft relative to the Sun after the flyby *behind* Jupiter.

$$\gamma d := \operatorname{asin}\left(\frac{vinf}{vd}\cdot\sin(\phi)\right) = 9.556\text{deg}$$

(f) On the heliocentric view below, draw and label the following:
   a. velocity vector v_a, the spacecraft's arrival at Jupiter's sphere of influence with respect to the Sun
   b. velocity vector $v_{d,in\ front}$, the spacecraft's departure from Jupiter's sphere of influence with respect to the Sun following a flyby in front of Jupiter
   c. velocity vector $v_{d,behind}$, the spacecraft's departure from Jupiter's sphere of influence with respect to the Sun following a flyby behind Jupiter.

Heliocentric view:



radius of the Earth = 6378 km   μEarth = $3.986 * 10^5\ km^3/s^2$ g = $9.81*m/s^2$

Problem 2

A rocket engine that generates 3000 N of thrust by burning 60 kg of fuel at a constant rate over 1 minute is attached to a satellite orbiting the earth with the following orbital parameters:

perigee distance r_p = 7000 km
apogee distance r_a = 14,000 km
inclination i = 28.5°
right ascension of ascending node Ω = 90°
argument of perigee ω = 0°

(a) At what true anomaly in the satellite's orbit must the engine be fired in order to achieve the maximum inclination change? Why?

In class, we discussed that the inclination change is maximized by firing the impulses at the equatorial crossings, because then the entire impulse goes into changing inclination and not changing the right ascension.

Because this orbit has an argument of perigee = 0, the equatorial crossings are the perigee and apogee. A greater amount of plane change is achievable if the rocket is going more slowly, so the greatest plane change is achieved at apogee, where the true anomaly f = 180 degrees.

radius of the Earth = 6378 km μEarth = $3.986 * 10^5$ km^3/s^2 g = 9.81*m/s^2

(b) If the combination of the satellite and rocket prior to burning the engine has a mass of 120 kg, what is the maximum degree of inclination change that the satellite can achieve?

$T := 3000\,N$ $\quad g = 9.807\,\frac{m}{s^2}$

$mfuel := 60 \cdot kg$

$t := 1 \cdot min$

$$Isp := \frac{T}{g \cdot \left(\frac{mfuel}{t}\right)} = 305.915\,s$$

$$mi := \frac{mfuel}{.5} = 120\,kg$$

$mf := mi - mfuel = 60\,kg$

$$\Delta V := g \cdot Isp \cdot \ln\left(\frac{mi}{mf}\right) = 2.079 \cdot \frac{km}{s}$$

Plane change: ΔV = 2*v*sin(θ/2)

$rp := 7000 \cdot km$

$ra := 2 \cdot rp = 14000 \cdot km$

$rp = a \cdot (1 - e)$

$ra = a \cdot (1 + e)$

$a = 10500 \cdot km$

$e = 0.333$

$$\mu := 3.986 \cdot 10^5 \cdot \frac{km^3}{s^2}$$

$$v := \sqrt{\mu \cdot \left(\frac{2}{ra} - \frac{1}{a}\right)} = 4.357\,\frac{km}{s}$$

$$\theta := 2 \cdot asin\left(\frac{\Delta V}{2 \cdot v}\right) = 0.482 \cdot rad \qquad \theta = 27.614 \cdot deg$$

radius of the Earth = 6378 km $\quad$ μEarth = $3.986 * 10^5\ km^3/s^2$ $\quad$ g = $9.81*m/s^2$

Questions

For this section, answer the questions in complete sentences. Use equations and minor calculations where appropriate, but the emphasis is on explaining course concepts rather than solving for numerical values.

(a) The equation that describes the drift in right ascension of an Earth-orbiting satellite due to the oblateness of the Earth is

$$\frac{d\Omega}{dt} = -\frac{9.969}{\left(1-e^2\right)^2}\left(\frac{R_E}{R_E+\bar{h}}\right)^{3.5}\cos i \quad \text{deg/day}$$

What inclination orbits experience the maximum drift in right ascension of ascending node? What is the physical reason that the effect is greatest for those inclinations?

The maximum drift of right ascension due to J2 effects is experienced by orbits in low inclinations. Not zero inclination, as the right ascension is undefined for orbits that are equatorial (no ascending or descending node), but low inclination. The effect is greatest at those inclinations because they spend the majority of their orbital period in the vicinity of the greater mass near the equator that torques the orbit's angular momentum vector.

What inclination orbits experience the least drift in right ascension of the ascending node? What is the physical reason that the effect is least for those inclinations?

The least drift of right ascension due to J2 effects is experienced by orbits in high inclinations, polar or nearly polar. The effect is least at those inclinations because the orbit is perpendicular (or nearly perpendicular) to the oblate gravitational effect that would otherwise cause a torque on the angular momentum vector of the orbit.

radius of the Earth = 6378 km   μEarth = $3.986 * 10^5$ km^3/s^2 g = 9.81*m/s^2

(b) A satellite is orbiting the Earth on a circular orbit with a radius of 8059 km.

At time t = 0, a small explosion aboard a satellite sends three pieces flying away from the main body of the satellite.

Piece A speeds up by 2 m/s in the original satellite's direction of motion.
Piece B attains a velocity of 1 m/s in the direction perpendicular to the orbit plane.
Piece C receives a 3 m/s ΔV toward the Earth.

Which piece will be farthest away from the main satellite 6 hours later? Justify your answer.

$r := 8059 \cdot km$

$T := 2 \cdot \pi \sqrt{\frac{r^3}{\mu}} = 7200.008\ s \qquad T = 120 \cdot min \qquad T = 2.000 \cdot hr$

Since the period of the orbit is 2 hrs, the time of 6 hrs is equal to three orbit periods.

Piece A receives a negative x-direction relative ΔV. The x-direction is the only direction that has a secular drift, so each period, the piece moves farther away from its original position.

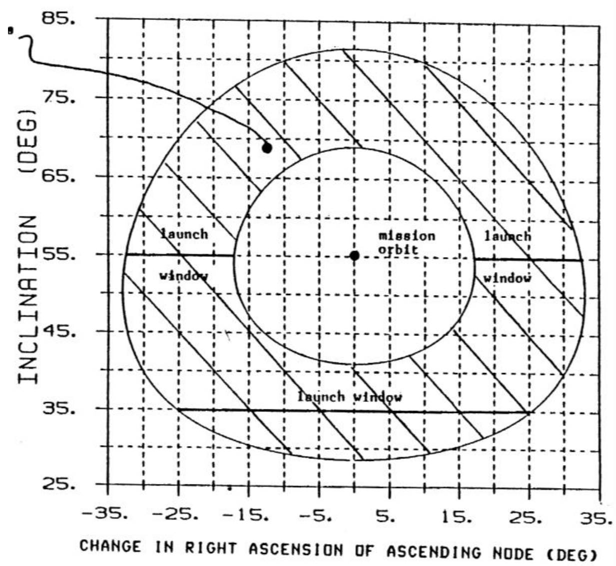
Piece B receives a z-direction relative ΔV. The piece will oscillate back and forth, perpendicular to the orbit plane, but it will return to its original position each orbit period, so at 6 hours, its distance from its original location will be zero.

Piece C receives a negative y-direction relative ΔV. The piece will move forward, backward, above, and below its original position, but it will return to its original position each orbit period, so at 6 hours, its distance from its original location will be zero.

➔ Piece A will be the farthest away 6 hours later.

radius of the Earth = 6378 km $\qquad$ μEarth = $3.986 * 10^5\ km^3/s^2$ $\qquad$ g = $9.81 * m/s^2$

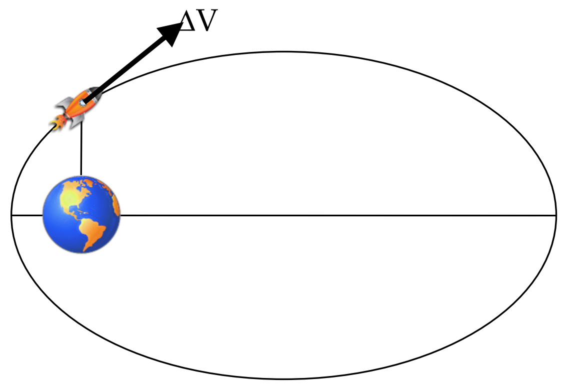
(c) On page 15-9 of the course notes, you have the following figure, which illustrates right ascensions and inclinations of r_1 = 6656 km circular parking orbits that will allow an Earth-orbiting satellite with fixed-impulse rocket engines providing ΔV1 = 2.107 km/s and ΔV2 = 1.888 km/s to reach a desired mission orbit with r_2 = 26,565 km, Ω = 0°, and i = 55°.



The shaded region of the figure indicates parking orbits that will allow the rocket to reach the mission orbit. The mission orbit is in the middle of the unshaded region. Why is this okay (and expected)?

The mission orbit is in the unshaded region because the rockets being used have too much fuel to complete an in-plane transfer and burn all of their fuel. The rocket must change inclination and/or right ascension in order to burn all of its fuel and reach the mission orbit.

(d) A spacecraft is in an elliptical orbit about the Earth with a semimajor axis of a = 40,000 km and eccentricity e = 0.8, as shown to the right. An instantaneous tangential ΔV is applied to the spacecraft, but rather than being applied at perigee or apogee, the tangential ΔV is applied when the spacecraft has a true anomaly of f = 90°.



Provide (but do not solve) the complete set of equations needed to find the semimajor axis (a_{new}) and the eccentricity (e_{new}) of the resulting orbit and the spacecraft's true anomaly (f_{new}) on that orbit. Next to each equation, indicate why it is important (i.e., what variable(s) is (are) found from each equation or system of equations). You may assume that the orbit remains elliptical after the impulse.

The equations needed are as follows:

***a*, *e*, and *f* are known, so the radius can be calculated from $r = \frac{a(1-e^2)}{1+e\cos f}$. In the particular case where *f* = 90°, this simplifies to $r = p = a(1-e^2)$.**

The speed before the ΔV is applied is $v_1 = \sqrt{\mu\left(\frac{2}{r}-\frac{1}{a}\right)}$.

The new speed is $v_2 = v_1 + \Delta V$.

The new semimajor axis can be found from $v_2 = \sqrt{\mu\left(\frac{2}{r}-\frac{1}{a_{new}}\right)}$, where the radius *r* has not changed.

Since the impulse is tangential, the flight path angle also has not changed, so the eccentricity can be found from setting $\gamma_1 = \gamma_2$, where γ is calculated in both cases from $\cos\gamma = \sqrt{\frac{a^2(1-e^2)}{r\,(2a-r)}}$.

The true anomaly can be found from either $r = \frac{a(1-e^2)}{1+e\cos f}$ or $an\ \gamma = \frac{e\sin f}{1+e\cos f}$.

radius of the Earth = 6378 km μEarth = 3.986 * 10^5 km^3/s^2 g = 9.81*m/s^2

How would you be able to tell if the post-ΔV orbit was hyperbolic instead of elliptical?

If the vis-viva equation returned a negative *a* or if the energy was calculated and found to be greater than zero, the orbit would be hyperbolic.

How would your equations for calculating a_{new}, e_{new}, and f_{new} change if the post-ΔV orbit was hyperbolic?

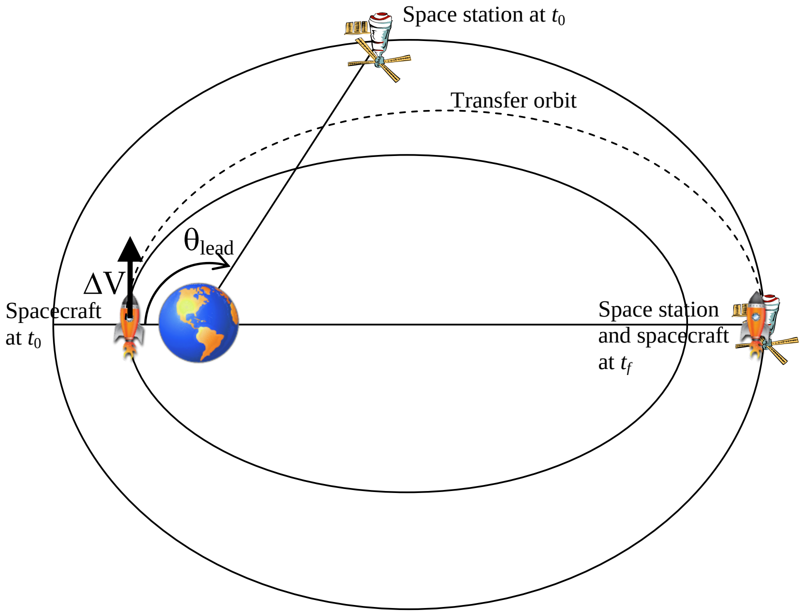
The radius would be calculated the same way, since the original orbit is elliptical. The new speed would be found the same way as well. From that point on, hyperbolic equations would need to be used for the post-burn orbit.

The hyperbolic velocity equation, $v_2 = \sqrt{\mu\left(\frac{2}{r} + \frac{1}{a_{new}}\right)}$, would give the new *a*.

The flight path angle is still equal before and after the impulse, and γ before the impulse would be calculated from the elliptical equation, as before, but finding the eccentricity after the burn would require using the hyperbolic equation for the post-burn flight path angle, $\cos\gamma = \sqrt{\frac{a^2(e^2-1)}{r\,(2a+r)}}$.

The true anomaly could then be found from the polar equation for a hyperbola, $= \frac{a(e^2-1)}{1+e\cos f}$.

(e) At time t_0, a spacecraft is at perigee on an elliptical orbit with semimajor axis a_1 and eccentricity e_1. It completes an impulsive, tangential burn that will allow it to rendezvous with a space station on a larger orbit with semimajor axis a_2 and eccentricity e_2. The rendezvous occurs at apogee on the station's orbit, so the transfer orbit is tangential to the final orbit as well as the spacecraft's initial orbit. The transfer ellipse is shown on the figure below as the dashed line.



Provide (but do not solve) all of the equations necessary to find θ_{lead}, the angle by which the space station must lead the spacecraft at t_0, the time of the initial rocket firing. Indicate why you included each equation and simplify where possible using the properties of perigee and apogee. Additional space is provided on the following page.

Since *a* and *e* are given for each orbit, use them to calculate the perigee and apogee radii of the transfer orbit.

$$r_p = a_1(1 - e_1)$$
$$r_a = a_2(1 - e_2)$$

The semimajor axis of the transfer orbit is $= \frac{r_p + r_a}{2}$.

The time it takes for the spacecraft to travel on the transfer orbit is one-half the transfer orbit period.

$$\Delta t = \frac{1}{2} 2\pi \sqrt{\frac{a^3}{\mu}} = \pi \sqrt{\frac{a^3}{\mu}}$$

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(additional workspace)

During the same time period, the space station moves through a true anomaly change of Δf, which is found by using Kepler's equation to find the initial and final eccentric anomalies (E_1, E_2) and converting the eccentric anomalies to true anomalies.

Make sure to use the a and e for the space station's orbit, which are given in the problem statement as a_2 and e_2.

Then Kepler's Equation for the time difference is

$$\sqrt{\frac{\mu}{{a_2}^3}}\Delta t = (E_2 - e_2 \sin(E_2)) - (E_1 - e_2 \sin(E_1))$$

The simplification that can be noted is that the eccentric anomaly at arrival (E_2) is equal to π radians because that point is apogee. Then the above equation can be solved for E_1 directly.

Once E_1 is known, the true anomaly f_1 can be found from

$$\tan\left(\frac{f_1}{2}\right) = \sqrt{\frac{1+e_2}{1-e_2}}\tan\left(\frac{E_1}{2}\right)$$

The lead angle, θ_{lead}, is just equal to f_1, the true anomaly that the space station must have at the time of the launch.

End of the exam. Congratulations, Rocket Scientist!

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5.3.2 2013

5.3.2.1 questions

EMA 550

Final Exam, Spring 2013

May 13, 2013, 7:45-9:45 am Open notes

Name ________________________________

Show all of your work to get credit for your answers. Include units with all answers.

Useful astronomical constants are found at the bottom of each page.

	Points	Score
Question 1	10	_______
Question 2	10	_______
Question 3	20	_______
Question 4	20	_______
Total Score	60	_______

If you are unable to find a value that is needed in subsequent sections of a problem, use a reasonable guess value (and clearly state what it is).

radius of the Earth = 6378 km μEarth = $3.986 * 10^5$ km^3/s^2 g = 9.81 m/s^2
Sun-Earth distance = 1 AU μSun = $1.327 * 10^{11}$ km^3/s^2
1 AU = $1.495978 * 10^8$ km

Question 1

We saw in class that the altitude difference between active Iridium satellites and spare Iridium satellites was intended to cause a relative drift in right ascension due to the oblateness of the Earth that would allow the orbit planes of the spares to align periodically with the orbit planes of the active satellites. While good in theory, the effect was minimal.

A different satellite constellation has been proposed that will have active satellites in 40° inclination circular orbits at an altitude of 700 km. You are asked to implement the same idea regarding spare satellites, but in a more effective way than with the Iridium constellation.

Determine the altitude required for spare satellites in circular orbits at 40° inclination that would close a 60° difference in right ascension between the actives and the spares in 45 days through the mechanism of right ascension drift due to the Earth's oblateness.

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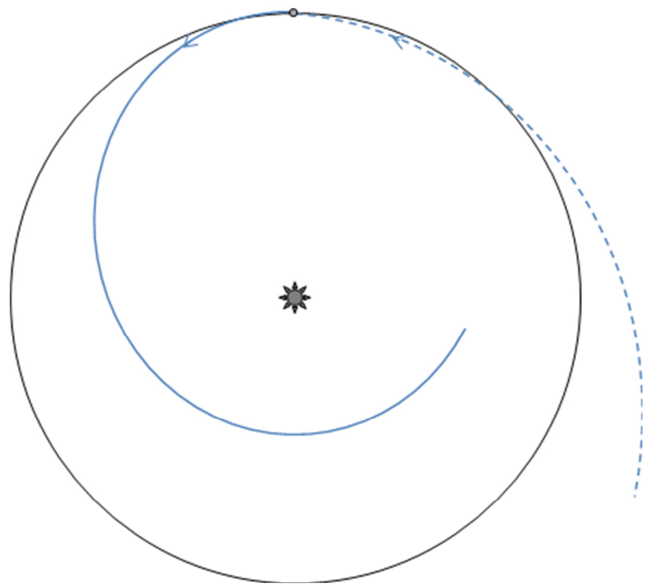
Question 2



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You study the mission, fire up your interplanetary project code, and determine the following:

Speed of Mars wrt Sun	24 km/s
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v_∞ wrt to Mars starting and ending the flyby	4.8 km/s
Turning angle during flyby	30°
Flight path angle of spacecraft wrt Sun after flyby	2.8°
Speed of spacecraft wrt Sun after flyby	19.5 km/s

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radius of the Earth = 6378 km   µEarth = $3.986 * 10^5$ km^3/s^2 g = 9.81 m/s^2

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Question 3

A satellite in a circular 8000 km radius orbit about the Earth needs to transfer quickly to a circular orbit in the same plane with a radius of 16000 km to reach an orbiting refueling station.

(a) Determine the angle by which the fueling station must lead the satellite if the satellite is to complete the transfer on a parabolic trajectory that is tangent to the initial orbit. Draw a sketch of the transfer showing the lead angle and the true anomaly of the fuel station on the transfer orbit at rendezvous.

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Question 3, continued

(b) Determine the angle by which the fueling station (r_f = 16,000 km) must lead the satellite (r_i = 8,000 km) if the satellite begins the transfer with twice the speed as at the start of the parabolic transfer (still tangential to the initial orbit). Draw a sketch of the transfer showing the lead angle and the true anomaly of the fuel station on the transfer orbit at rendezvous.

radius of the Earth = 6378 km | μEarth = $3.986 * 10^5$ km^3/s^2 | g = 9.81 m/s^2
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Question 4

A high-thrust rocket engine on an orbiting spacecraft will be fired for three minutes as it flies above Madison, WI (43°N latitude, 89°W longitude). Information about the rocket, the initial orbit, and the final orbit is as follows:

Rocket	**Initial Orbit**	**Final Orbit**
Thrust = 10 kN	a_1 = 7000 km	a_2 = ?
Specific impulse = 300 s	e_1 = 0	e_2 = ?
Initial mass = 1000 kg	i_1 = 60°	i_2 = 50°
	Ω_1 = 150°	Ω_2 = 131.086°
	ω_1 undefined	ω_2 = ?

Treating the burn as impulsive (all ΔV occurring at a single location) and firing the rocket in such a way that the burn location becomes the perigee of the new orbit, determine the *a*, *e*, and *ω* of the spacecraft's orbit after the burn.

radius of the Earth = 6378 km | μEarth = $3.986 * 10^5$ km^3/s^2 | g = 9.81 m/s^2
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5.3.2.2 key

Question 1

We saw in class that the altitude difference between active Iridium satellites and spare Iridium satellites was intended to cause a relative drift in right ascension due to the oblateness of the Earth that would allow the orbit planes of the spares to align periodically with the orbit planes of the active satellites. While good in theory, the effect was minimal.

A different satellite constellation has been proposed that will have active satellites in 40° inclination circular orbits at an altitude of 700 km. You are asked to implement the same idea regarding spare satellites, but in a more effective way than with the Iridium constellation.

Determine the altitude required for spare satellites in circular orbits at 40° inclination that would close a 60° difference in right ascension between the actives and the spares in 45 days through the mechanism of right ascension drift due to the Earth's oblateness.

The active satellites drift in right ascension is as follows:

$h1 := 700 \cdot km \qquad i := 40 \cdot deg$

$$\Omega dot1 := \left(-9.969 \cdot \frac{deg}{day}\right) \cdot \left(\frac{RE}{RE + h1}\right)^{3.5} \cdot \cos(i) = -5.304 \cdot \frac{deg}{day}$$

We want the spare satellites to close a given gap in a given amount of time, which requires a relative drift in the right ascensions of the orbital planes of the active satellites and the spare satellites.

$time\_all := 45 \cdot day$

$\Delta\Omega := 60 \cdot deg$

$$\frac{\Delta\Omega}{time\_all} = 1.333 \cdot \frac{deg}{day}$$

If the spare satellites are in a lower orbit at the same inclination, the right ascensions of their orbit planes will drift at a faster rate than that of the active satellites.

$$\Omega dot2 := \Omega dot1 - \frac{\Delta\Omega}{time\_all} = -6.637 \cdot \frac{deg}{day}$$

$$h2 := \left[\frac{1}{\left[\frac{\Omega dot2}{\left(-9.969 \cdot \frac{deg}{day}\right) \cdot \cos(i)}\right]^{\frac{1}{3.5}}} - 1 \right] \cdot 6378 \cdot km = 260.7 \cdot km$$

$h2 = 261 \cdot km$

$h1 - h2 = 439 \cdot km$

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(Additional workspace for Question 1)

If the spare satellites are in a higher orbit at the same inclination, the right ascensions of their orbit planes will drift more slowly than that of the active satellites. Both options have their advantages; the lower orbit requires less energy to reach, and the outer orbit is less crowded than close-in LEO.

$$\Omega dot3 := \Omega dot1 + \frac{\Delta\Omega}{time\_all} = -3.971 \cdot \frac{deg}{day}$$

$$h3 := \left[\frac{1}{\left[\frac{\Omega dot3}{\left(-9.969 \cdot \frac{deg}{day}\right)\cdot \cos(i)}\right]^{\frac{1}{3.5}}} - 1\right] \cdot 6378 \cdot km = 1310.4 \cdot km$$

$h3 = 1310 \cdot km$

$h3 - h1 = 610 \cdot km$

Note that the key here is RELATIVE right ascension drift. Neither of the satellites (active nor spare) drifts at 45 degrees in 60 days, but their relative drift closes that gap. If a student uses 45 degrees in 60 days as the right ascension drift, they will find the following altitude:

$$\Omega dot4 := -\frac{\Delta\Omega}{time\_all} = -1.333 \cdot \frac{deg}{day}$$

$$h4 := \left[\frac{1}{\left[\frac{\Omega dot4}{\left(-9.969 \cdot \frac{deg}{day}\right)\cdot \cos(i)}\right]^{\frac{1}{3.5}}} - 1\right] \cdot 6378 \cdot km = 4123 \cdot km$$

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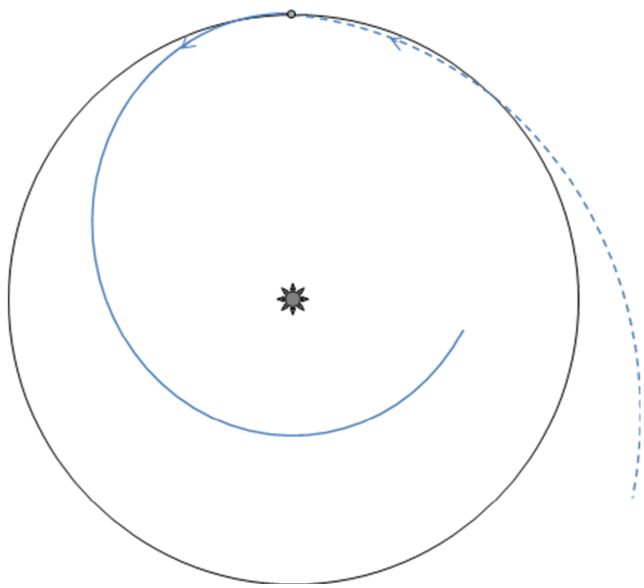

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v_∞ wrt to Mars starting and ending the flyby	4.8 km/s
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Speed of spacecraft wrt Sun after flyby	19.5 km/s

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On the next page, write a one-page response supporting or refuting the site member's comment. Your response can be scanned and uploaded, so draw velocity triangles and figures from Mars' frame of reference to illustrate your argument.

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Page 5 of 9

Response for Question 2

Note: Unfortunately, there was an error in this problem, which was unintentional. The given values should have been

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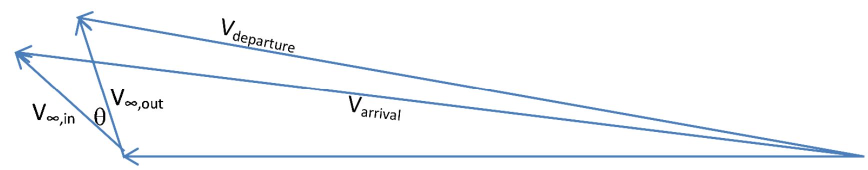
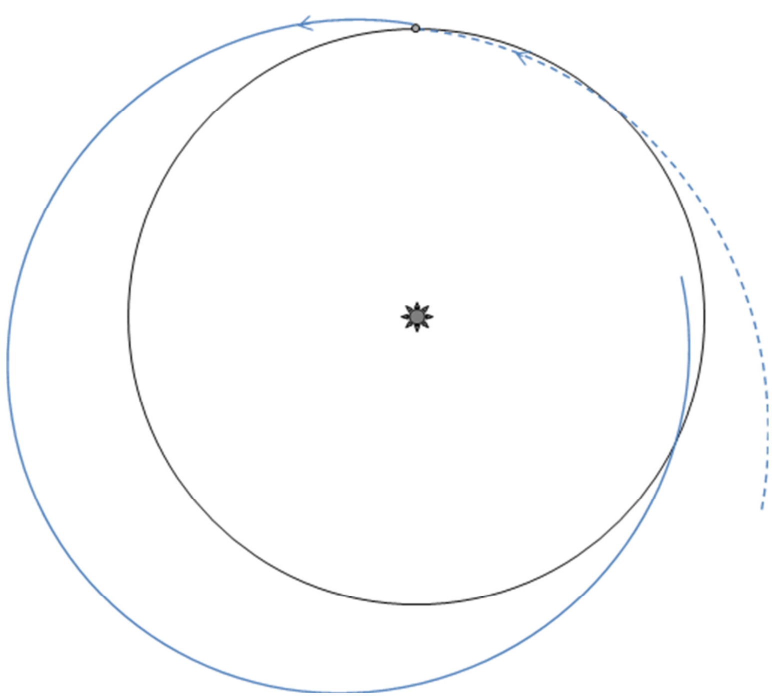
Speed of spacecraft wrt Sun before flyby	28 km/s
Flight path angle of spacecraft wrt Sun before flyby	6.5°
v_∞ wrt to Mars starting and ending the flyby	4.8 km/s
Turning angle during flyby	30°
Flight path angle of spacecraft wrt Sun after flyby	~~2.8°~~ **10°**
Speed of spacecraft wrt Sun after flyby	~~19.5 km/s~~ **26 km/s**

The correct values would have led to this velocity triangle:



The correct values also would have changed the orbit picture to look like this:



radius of the Earth = 6378 km
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1 AU = 1.495978 * 10^8 km

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Parabola

$p := 2 \cdot ri$ $\quad p = 16000\,\text{km}$

$fcp := \operatorname{acos}\left(\frac{p}{rf} - 1\right)$ $\quad fcp = 1.571 \cdot \text{rad}$

$fcp = 90 \cdot \text{deg}$

$$\Delta tp := \frac{\tan\left(\frac{fcp}{2}\right) + \frac{1}{3} \cdot \left(\tan\left(\frac{fcp}{2}\right)\right)^3}{2 \cdot \sqrt{\frac{\mu}{p^3}}}$$

$\Delta tp = 2137.076\,\text{s}$

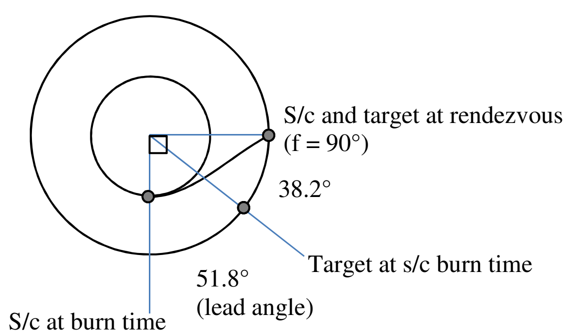
$\Delta tp = 35.618 \cdot \text{min}$

Target moves through $\Delta tp \cdot nf = 0.667 \cdot \text{rad}$

$\Delta tp \cdot nf = 38.2 \cdot \text{deg}$

Lead angle is $fcp - \Delta tp \cdot nf = 0.904 \cdot \text{rad}$

$fcp - \Delta tp \cdot nf = 51.8 \cdot \text{deg}$



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Question 3, continued

(b) Determine the angle by which the fueling station (r_f = 16,000 km) must lead the satellite (r_i = 8,000 km) if the satellite begins the transfer with twice the speed as at the start of the parabolic transfer (still tangential to the initial orbit). Draw a sketch of the transfer showing the lead angle and the true anomaly of the fuel station on the transfer orbit at rendezvous.

$$vpi := \sqrt{\frac{2\cdot\mu}{ri}} = 9.982\,\frac{km}{s}$$

$$vhi := 2\cdot vpi = 19.965\,\frac{km}{s}$$

By definition, the new ellipse must be a hyperbola because the speed is faster than the parabolic speed at that same distance.

Hyperbola

$$ah := \left(\frac{vhi^2}{\mu} - \frac{2}{ri}\right)^{-1} = 1333\,km$$

$$eh := 1 + \frac{ri}{ah} = 7.000$$

$$Ff := acosh\left[\frac{1}{eh}\cdot\left(\frac{rf}{ah} + 1\right)\right] = 1.23$$

$$\Delta th := \sqrt{\frac{ah^3}{\mu}}\cdot(eh\cdot\sinh(Ff) - Ff) = 749.883\,s$$

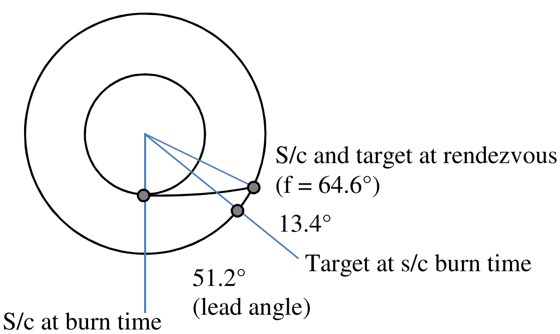
$$fch := 2\cdot atan\left(\sqrt{\frac{eh+1}{eh-1}}\cdot\tanh\left(\frac{Ff}{2}\right)\right) = 1.128\cdot rad \qquad fch = 64.623\cdot deg$$

Target moves through $\Delta th\cdot nf = 0.234\cdot rad$

$\Delta th\cdot nf = 13.4\cdot deg$

Lead angle is $fch - \Delta th\cdot nf = 0.894\cdot rad$

$fch - \Delta th\cdot nf = 51.2\cdot deg$



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	Ω_1 = 150°	Ω_2 = 131.086°
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Treating the burn as impulsive (all ΔV occurring at a single location) and firing the rocket in such a way that the burn location becomes the perigee of the new orbit, determine the *a*, *e*, and *ω* of the spacecraft's orbit after the burn.

From spherical trigonometry, we can determine the change in angle (θ) between the velocity vectors before the burn and after the burn. Since the burn location is perhielion on the new orbit, the velocity vector is simply rotated through the angle θ, not moved out of the plane of motion.

$$u1 := \operatorname{asin}\left(\frac{\sin(\phi)}{\sin(i1)}\right) = 0.907 \cdot \text{rad} \qquad u1 = 51.953 \cdot \text{deg}$$

$$\theta := \operatorname{asin}\left(\sin(\Omega 1 - \Omega 2) \cdot \frac{\sin(i2)}{\sin(u1)}\right) = 0.321 \cdot \text{rad} \qquad \theta = 18.38 \cdot \text{deg}$$

The rocket equation can be used to determine the magnitude of the applied ΔV.

$$mdot := \frac{T}{g \cdot Isp} = 3.398 \frac{kg}{s}$$

$$mf := mi - mdot \cdot \Delta t = 388.379 kg$$

$$\Delta V := Isp \cdot g \cdot \ln\left(\frac{mi}{mf}\right) = 2.783 \cdot \frac{km}{s}$$

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g = 9.81 m/s^2

(Additional workspace for Question 4)

The combination of the initial speed, the ΔV magnitude, and the turning angle θ provide the speed on the new orbit at the same location (impulsive burn assumption).

$$v1 := \sqrt{\frac{\mu}{a1}} = 7.546 \cdot \frac{km}{s}$$

From the quadratic formula,

$$v2 := \frac{2 \cdot v1 \cdot \cos(\theta) + \sqrt{(2 \cdot v1 \cdot \cos(\theta))^2 - 4 \cdot \left(v1^2 - \Delta V^2\right)}}{2} = 8.605 \cdot \frac{km}{s}$$

Or, from the planar laws of sines and cosines,

$$\psi := \operatorname{asin}\left(\sin(\theta) \cdot \frac{v1}{\Delta V}\right) = 58.743 \text{ deg}$$

$$v2\_alt := \sqrt{v1^2 + \Delta V^2 - 2 \cdot v1 \cdot \Delta V \cdot \cos(180 \cdot deg - \theta - \psi)} = 8.605 \frac{km}{s}$$

The semimajor axis can be determined from the new speed at the given distance.

$$a2 := \left(\frac{2}{a1} - \frac{v2^2}{\mu}\right)^{-1} = 10007 \cdot km \qquad \boxed{a2 = 10007 \text{ km}}$$

The eccentricity can be determined from the position and the new semimajor axis, taking advantage of the information that the burn location is perigee on the new orbit.

$$e2 := 1 - \frac{a1}{a2} = 0.3 \qquad \boxed{e2 = 0.3}$$

The argument of perigee can be calculated from spherical trigonometry. Since the burn location is the new perigee and u2 measures the distance from the equator (line of nodes) to the burn location, u2 and the argument of perigee are the same value.

$$u2 := \operatorname{asin}\left(\sin(180 \cdot deg - i1) \cdot \frac{\sin(u1)}{\sin(i2)}\right) = 1.098 \cdot rad$$

$$\omega 2 := u2 \qquad \boxed{\omega 2 = 62.9 \cdot deg}$$

radius of the Earth = 6378 km | μEarth = $3.986 * 10^5$ km^3/s^2 | g = 9.81 m/s^2

Sun-Earth distance = 1 AU | μSun = $1.327 * 10^{11}$ km^3/s^2

1 AU = $1.495978 * 10^8$ km

Chapter 6

HWs

6.1 HW1

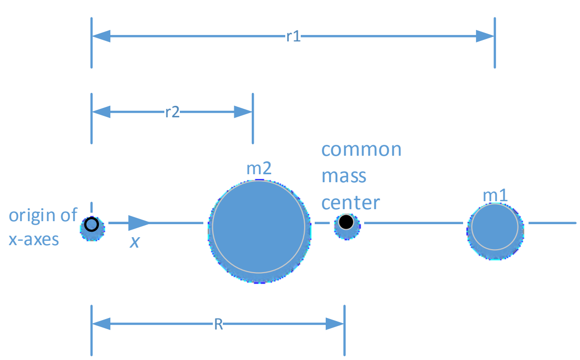
6.1.1 Problem 1

Let us examine the accuracy of the assumption that planets orbit the Sun rather than the Sun and planet orbiting the mass center of the Sun-planet system. We'll start with Earth:

What is the distance between the center of a spherical Sun with the radius given on your Planetary Constants sheet and the center of mass of the Sun-Earth system? Assume that the Earth is in a circular orbit about the Sun and that the "Mean distance from the Sun" given on your Planetary Constants sheet is the distance between the mass centers of the two bodies.

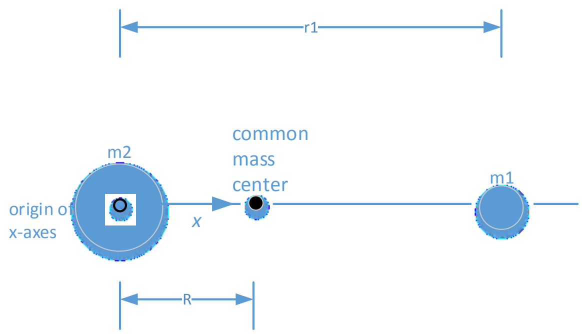
Answer:

Common mass center,measured from the origin of the coordinates system is given by solving for R in



$$(m_1 + m_2) R = m_2 r_2 + m_1 r_1$$
$$R = \frac{m_2 r_2 + m_1 r_1}{(m_1 + m_2)}$$

If we now put m_2 at the center of origin, then $r_2 = 0$. Hence the above simplifies to



$$(m_1 + m_2) R = m_1 r_1$$
$$R = \frac{m_1 r_1}{(m_1 + m_2)}$$

In our case, m_2 is the sun and m_1 is the earth, and r_1 is AU. Hence

$$R = \frac{5.974 \times 10^{24} \left[kg\right] \left(1.495978 \times 10^{8} \left[km\right]\right)}{\left(5.974 \times 10^{24} \left[kg\right] + 1.989 \times 10^{30} \left[kg\right]\right)}$$
$$= 449.32 \left[km\right]$$

The above is the distance of the common center of mass of the sun-earth, measured from the center of the sun. As a percentage of the sun radius, it is $\frac{449.32}{695990} \times 100 = 6.4558 \times 10^{-2}$ and as a percentage of the distance between the mass centers of the Sun and the Earth it is $\frac{449.32}{AU} \times 100 = \frac{449.32}{1.495978 \times 10^{8}} \times 100 = 3.0035 \times 10^{-4}\%$

Summary of answers

1. kilometers: Answer 449.319 km
2. percent of the Sun's radius: Answer 0.0645%
3. percent of the distance between the mass centers of the Sun and the Earth: Answer 0.000300351%

6.1.2 question 2

Repeat the analysis above for the most massive planet in our solar system, Jupiter.

What is the distance between the center of a spherical Sun with the radius given on your Planetary Constants sheet and the center of mass of the Sun-Jupiter system? Assume that

Jupiter is in a circular orbit about the Sun and that the "Mean distance from the Sun" given on your Planetary Constants sheet is the distance between the mass centers of the two bodies.

Answer

Now m_1 is mass of sun, but m_2 is mass of Jupiter which is 317.9 that of the earth mass, and r_1 now is the distance from center of Jupitor to center of sun (which is the origin of the coordinates systems), which is $5.203 \times AU$, hence from

$$\begin{aligned}
(m_1 + m_2) R &= m_1 r_1 \\
R &= \frac{m_1 r_1}{(m_1 + m_2)} \\
&= \frac{317.9 \times \left(5.974 \times 10^{24}\right)\left(5.203 \times \left(1.495978 \times 10^{8}\right)\right)}{\left(317.9 \times \left(5.974 \times 10^{24}\right) + 1.989 \times 10^{30}\right)} \\
&= 7.4248 \times 10^{5} [km]
\end{aligned}$$

The above is the distance of the common center of mass of the sun-Jupiter, measured from the center of the sun. As a percentage of the sun radius, it is $\frac{7.4248\times10^5}{695990} \times 100 = 106.68$ % and as a percentage of the distance between the mass centers of the Sun and the Jupitor it is $\frac{7.4248\times10^5}{5.203\times1.495978\times10^8} \times 100 = 9.5391 \times 10^{-4}\%$

Summary

1. kilometers: Answer 742481 km
2. percent of the Sun's radius: Answer 106.68%
3. percent of the distance between the Sun and Jupiter: Answer 0.095%

6.1.3 question 3

A satellite is in an elliptical orbit around the Earth; at perigee its altitude is 400 km. The eccentricity of the orbit is 0.10.

6.1.3.1 part 1

What is the speed of the satellite at perigee in km/s?

answer:

$$\begin{aligned}
r_p &= r_E + ALT \\
&= 6378 + 400 \\
&= 6778.0
\end{aligned}$$

But $r_p = \frac{a(1-e^2)}{1+e}$ hence $a = \frac{r_p(1+e)}{1-e^2} = \frac{6778(1.1)}{1-0.1^2} = 7531.1[km]$, hence

$$v_p = \sqrt{\frac{\mu}{a}\left(\frac{1+e}{1-e}\right)} = \sqrt{\frac{3.986\times 10^5}{7531.1}\left(\frac{1.1}{0.9}\right)} = 8.0429\,[km/s]$$

6.1.3.2 Part 2

What is the altitude of the satellite at apogee in km?

Answer

$$r_a = \frac{a\left(1-e^2\right)}{1-e} = \frac{7531.1\left(1-0.1^2\right)}{0.9} = 8284.2\,[km]$$

Hence altitude $8284.2 - r_E = 8284.2 - 6378 = 1906.2\,[km]$

6.1.3.3 Part 3

What is the speed of the satellite at apogee in km/s?

Answer

$$v_a = \sqrt{\frac{\mu}{a}\left(\frac{1-e}{1+e}\right)} = \sqrt{\frac{3.986\times 10^5}{7531.1}\left(\frac{0.9}{1.1}\right)} = 6.5806\,[km/s]$$

6.1.3.4 Part 4

What is the period of the orbit in hrs?

Answer

$$\begin{aligned} T &= 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{7531.1^3}{3.986\times 10^5}} = 6504.3\,[\text{sec}] \\ &= \frac{6504.3}{60\times 60} = 1.8068[hr] \end{aligned}$$

6.2 HW2

6.2.1 Problem 1

A satellite is in an orbit with a period $T = 205$ minutes and eccentricity $e = 0.40$ about the Earth. When the true anomaly of the satellite is $f = 70$ degrees, find the time $t - \tau$ since perigee passage, in minutes.

Answer

$$n(t-\tau) = E - e\sin E$$

But $n = \frac{2\pi}{T}$ hence

$$t - \tau = \frac{E - e\sin E}{n} = \frac{T(E - e\sin E)}{2\pi} \tag{1}$$

But $\tan\left(\frac{f}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$, hence E can be found. Substituting it in the above, solves for $t-\tau$

$$\begin{aligned}
\tan\left(\frac{70\pi}{2(180)}\right) &= \sqrt{\frac{1+0.4}{1-0.4}}\tan\left(\frac{E}{2}\right)\\
0.70021 &= 1.5275\tan\left(\frac{E}{2}\right)\\
\tan\left(\frac{E}{2}\right) &= \frac{0.70021}{1.5275} = 0.4584\\
\frac{E}{2} &= \arctan(0.4584) = 0.42982\\
E &= 0.85964
\end{aligned}$$

Hence from Eq (1)

$$\begin{aligned}
t - \tau &= \frac{205(0.85964 - 0.40\sin(0.85964))}{2\pi}\\
&= 18.16 \text{ min}
\end{aligned}$$

6.2.2 Problem 2

A satellite is in an orbit with a period $T = 205$ minutes and eccentricity $e = 0.40$ about the Earth. Find the true anomaly of the satellite, in degrees, when it is 50 minutes past perigee passage.

Answer

$$\begin{aligned}
n(t-\tau) &= E - e\sin E\\
\frac{2\pi}{T}(t-\tau) &= E - e\sin E\\
\frac{2\pi}{205}(50) &= E - 0.4\sin E\\
1.5325 &= E - 0.4\sin(E)
\end{aligned}$$

Solving for E

$$E = 1.9097 \text{ rad}$$

Hence

$$\begin{aligned}
\tan\left(\frac{f}{2}\right) &= \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)\\
\tan\left(\frac{f}{2}\right) &= \sqrt{\frac{1+0.4}{1-0.4}}\tan\left(\frac{1.9097}{2}\right)\\
&= 2.1581
\end{aligned}$$

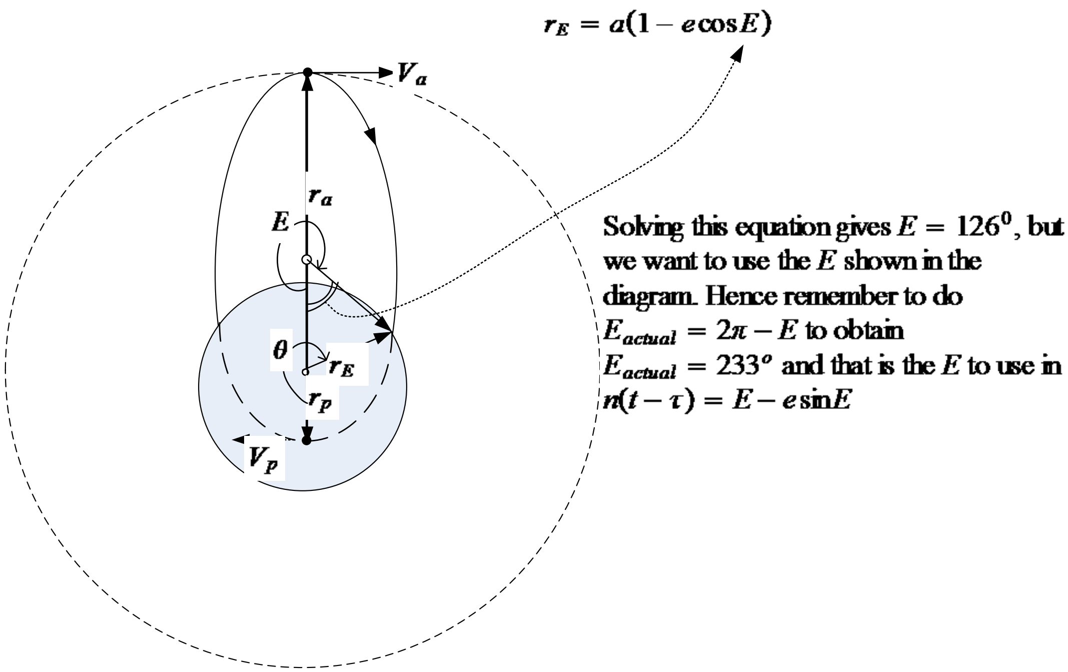
Hence

$$\begin{aligned}
\frac{f}{2} &= \arctan(2.1581) = 1.1369\\
f &= (1.1369)\,2 = 2.2738\\
&= 2.2738\left(\frac{180}{\pi}\right)\\
&= 130.28 \text{ deg}
\end{aligned}$$

6.2.3 Problem 3

A spaceship in a circular orbit above the Earth at an altitude of 300 km. At time $t = 0$, it retrofires its engine, reducing its speed by 500 m/s. How long (in minutes) does it take to impact the Earth? Neglect atmospheric drag.

Answer



$$\mu = 3.986 \times 10^5 \ \ \text{km}^3/\text{s}^2$$

But

$$\Delta V = V_2 - V_1$$

Where $V_1 = \sqrt{\frac{\mu}{r_a}} = \sqrt{\frac{\mu}{r_E + alt}}$ where r_E is earth radius and alt is the altitude at $t = 0$ when the spaceship was in circular orbit. Hence $V_1 = \sqrt{\frac{3.986 \times 10^5}{6378+300}} = 7.7258$ km/s hence $V_2 = V_1 - 500 \times 10^{-3} = 7.7258 - 0.5 = 7.2258$ km/sec. This is the speed at apogee for the new orbit.

$$V_a = 7.2258 \text{ km/sec}$$

But

$$V_a = \sqrt{\frac{\mu}{a}\left(\frac{1-e}{1+e}\right)}$$

$$7.2258 = \sqrt{\frac{3.986 \times 10^5}{a}\left(\frac{1-e}{1+e}\right)} \tag{1}$$

But also we know that $r_a = a(1+e)$, hence

$$
\begin{aligned}
6378 + 300 &= a\left(1+e\right)\\
a &= \frac{6678}{1+e} \qquad (2)
\end{aligned}
$$

Substitute (2) in (1)

$$
\begin{aligned}
7.2258 &= \sqrt{\frac{3.986\times 10^5}{6678}\left(1-e\right)}\\
52.212 &= \frac{3.986\times 10^5}{6678}\left(1-e\right)\\
\frac{(52.212)(6678)}{3.986\times 10^5} &= 1-e\\
0.87474 &= 1-e\\
e &= 1-0.87474\\
&= 0.12526
\end{aligned}
$$

Hence from (2) we find a

$$
a = \frac{6678}{1+0.12526} = 5934.6
$$

Hence n the mean speed is

$$
\begin{aligned}
n &= \sqrt{\frac{\mu}{a^3}}\\
&= \sqrt{\frac{3.986\times 10^5}{5934.6^3}}\\
&= 1.381\times 10^{-3} rad/s
\end{aligned}
$$

At impact $r = r_E$, hence

$$
\begin{aligned}
r_E &= a\left(1-e\cos E\right)\\
6378 &= 5934.6\left(1-0.12526\cos E\right)\\
\frac{6378}{5934.6} &= 1-0.12526\cos E\\
\cos E &= \frac{1-\frac{6378}{5934.6}}{0.12526} = -0.59647\\
E &= \arccos\left(-0.59647\right)\\
E &= 2.2099
\end{aligned}
$$

Solving this equation gives $E = 126^0$, but we want to use the E shown in the diagram. Hence remember to do $E_{actual} = 2\pi - E$ to obtain $E_{actual} = 233^o$ and that is the E to use in

$n(t-\tau) = E - e\sin E$.Hence, measured from perigee,

$$E = 2\pi - 2.2099$$

Using Kepler equation

$$\begin{aligned}
n(t-\tau) &= E - e\sin E \\
1.381\times10^{-3}(t-\tau) &= (2\pi - 2.2099) - 0.12526\sin(2\pi - 2.2099) \\
(t-\tau) &= \frac{(2\pi - 2.2099) - 0.12526\sin(2\pi - 2.2099)}{1.381\times10^{-3}} \\
&= 3022.\text{ sec} \\
&= 50.37\text{ min}
\end{aligned}$$

But the period is $T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\frac{1}{n} = 2\pi\frac{1}{1.381\times10^{-3}} = 4549.7\text{ sec} = 75.828\text{ min}$

Hence the time to impact is

$$50.37 - \frac{75.828}{2} = 12.456\text{ min}$$

6.2.4 Problem 4

Russians use Molniya orbits for their communications satellites. A typical Molniya orbit has a perigee altitude of 500 km and a period of 12 hr.

6.2.4.1 part a

What is the eccentricity of a Molniya orbit?

Answer

$$\begin{aligned}
T &= 2\pi\sqrt{\frac{a^3}{\mu}} \\
12\times60\times60 &= 2\pi\sqrt{\frac{a^3}{3.986\times10^5}} \\
(12\times60\times60)^2 &= (2\pi)^2\frac{a^3}{3.986\times10^5} \\
a^3 &= \frac{(12\times60\times60)^2\left(3.986\times10^5\right)}{(2\pi)^2} = 1.8843\times10^{13} \\
a &= \left(1.8843\times10^{13}\right)^{1/3} = 26610\text{ km}
\end{aligned}$$

We are given that $r_p = 6378 + 500 = 6878$, but $r_p = \frac{a(1-e^2)}{1+e} = a(1-e)$, hence

$$
\begin{aligned}
e &= 1 - \frac{r_p}{a} \\
&= 1 - \frac{6878}{26610} \\
&= 0.741\,53
\end{aligned}
$$

6.2.4.2 part b

What is the apogee radius of a Molniya orbit, in km?

Answer

$$
\begin{aligned}
r_p &= a(1+e) \\
&= 26610\,(1 + 0.741\,53) \\
&= 46342 \text{ km}
\end{aligned}
$$

6.2.4.3 part c

Determine the time, in hours, that a satellite on a Molniya orbit has a true anomaly greater than 135^o and less than 225^o

Answer

Let θ_1, θ_2 be the true anomaly angles at position 1 and 2, and let E_1, E_2 be the corresponding circular angles. We first find E_1, E_2

$$
\begin{aligned}
\tan\left(\frac{E_1}{2}\right) &= \tan\left(\frac{\theta_1}{2}\right)\sqrt{\frac{1-e}{1+e}} \\
&= \tan\left(\frac{135\pi}{2\times 180}\right)\sqrt{\frac{1-0.741\,53}{1+0.741\,53}} = 0.930\,07 \\
\frac{E_1}{2} &= \arctan(0.93007) = 0.749\,18 \\
E_1 &= 0.749\,18 \times 2 \\
&= 1.498\,4 \text{ rad} \\
&= 1.498\,4 \times \left(\frac{180}{\pi}\right) = 85.85^0
\end{aligned}
$$

Similarly

$$\begin{aligned}\tan\left(\frac{E_2}{2}\right) &= \tan\left(\frac{\theta_2}{2}\right)\sqrt{\frac{1-e}{1+e}}\\ &= \tan\left(\frac{225\pi}{2\times 180}\right)\sqrt{\frac{1-0.741\,53}{1+0.741\,53}} = -0.930\,07\\ \frac{E_1}{2} &= \arctan\left(-0.930\,07\right) = -0.749\,18\\ E_1 &= -0.749\,18\times 2\\ &= -1.498\,4 \text{ rad}\end{aligned}$$

Hence $E_2 = -1.49836$ rad or -85.75^o, Measured anticlockwise from perigee, it becomes $E_2 = 360 - 85.75 = 274.15^o$

Now the time to reach point 1, is

$$\begin{aligned}n\left(t_1\right) &= E_1 - e\sin E_1\\ t_1 &= \frac{E_1 - e\sin E_1}{\sqrt{\frac{\mu}{a^3}}} = \frac{1.498\,4 - 0.741\,53\sin\left(1.498\,4\right)}{\sqrt{\frac{3.986\times 10^5}{26610^3}}} = 5217.1 \text{ sec}\end{aligned}$$

and

$$\begin{aligned}n\left(t_2\right) &= E_2 - e\sin E_2\\ t_1 &= \frac{\left(2\pi - 1.49836\right) - 0.741\,53\sin\left(2\pi - 1.49836\right)}{\sqrt{\frac{3.986\times 10^5}{26610^3}}} = 37983 \text{ sec}\end{aligned}$$

Hence the difference is 37983 −5217.1 = 32766 sec or $\frac{32766}{60\times 60} = 9.1017$ hr

6.3 HW3

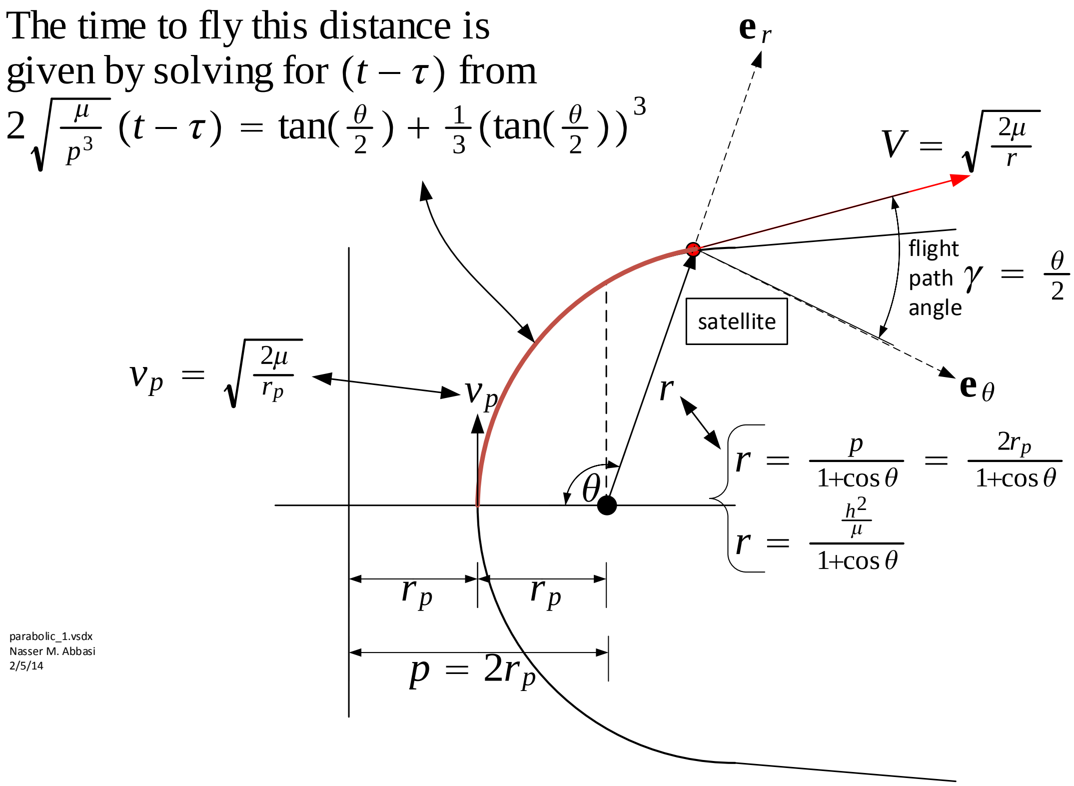
6.3.1 Problem 1

A comet is on a parabolic orbit about the Sun. At its point of closest approach, the distance between the comet and the center of the Sun is 5 million km.

6.3.1.1 part a

What is the speed of the comet, in km/s, relative to the Sun at its point of closest approach?

Answer



$$v_p = \sqrt{\frac{2\mu}{r_p}} = \sqrt{\frac{2\left(1.327\times10^{11}\right)}{5\times10^6}}$$
$$= 230.39\ [km/\sec]$$

6.3.1.2 part b

How long is the comet within 150 million km of the Sun?

Answer

$$r = \frac{p}{1+\cos\theta} = \frac{2r_p}{1+\cos\theta}$$

$p = 2r_p = 2\times5\times10^6 = 10\times10^6$. Hence

$$\cos\theta = \frac{p}{r} - 1$$
$$= \frac{10\times10^6}{150\times10^6} - 1 = -0.933\,33$$

The above can also be found using

$$r = \frac{\frac{h^2}{\mu}}{1 + \cos\theta}$$

Where $h = r_p v_p = 5 \times 10^6 \times 230.39 = 1.152\,0 \times 10^9 \left[km^2/s\right]$

Hence

$$\begin{aligned}\cos\theta &= \frac{h^2}{r\mu} - 1 \\ &= \frac{\left(1.152\,0 \times 10^9\right)^2}{150 \times 10^6 \times 1.327 \times 10^{11}} - 1 = -0.933\,33\end{aligned}$$

Therefore, $\theta = \arccos(-0.933\,33) = 2.774\,4\,[rad] = 158.96^o$. Now, from

$$\begin{aligned}2\sqrt{\frac{\mu}{p^3}}(t-\tau) &= \tan\left(\frac{\theta}{2}\right) + \frac{1}{3}\left(\tan\left(\frac{\theta}{2}\right)\right)^3 \\ (t-\tau) &= \frac{\tan\left(\frac{-2.7744}{2}\right) + \frac{1}{3}\left(\tan\left(\frac{-2.7744}{2}\right)\right)^3}{2\sqrt{\frac{1.327\times 10^{11}}{\left(10\times 10^6\right)^3}}} \\ &= 2.493\,5 \times 10^6\,[\text{sec}] \\ &= \frac{2.493\,5 \times 10^6}{60 \times 60 \times 24} \\ &= 28.8558\left[day\right]\end{aligned}$$

To account for both sides of the trajectory, then number of days is doubled, hence $28.8558 \times 2 = 57.712\left[days\right]$

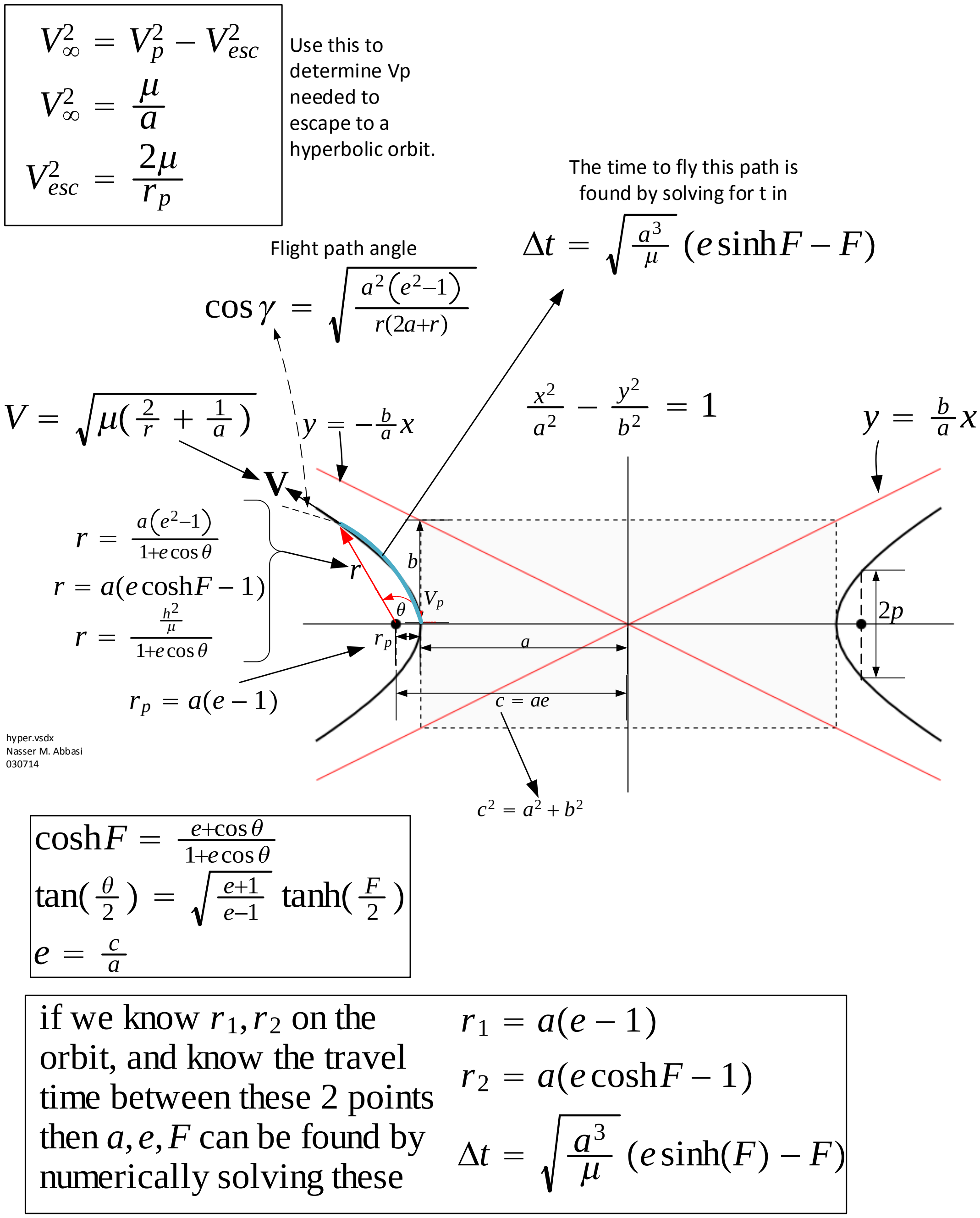
6.3.2 Problem 2

A spaceship is in a circular orbit about the Earth at an altitude of 700 km. It fires its rocket engine for a short time to instantaneously increase its speed by 75% and boost the spaceship to a hyperbolic orbit.

6.3.2.1 part a

What is the speed increase (del V) of the spaceship in km/s as a result of the rocket burn?

Answer:



$$V_{cir} = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986\times 10^5}{6378+700}} = 7.5044\ [km/s]$$

Hence

$$
\begin{aligned}
V_2 &= V_1 + \Delta V \\
1.75V_1 &= V_1 + \Delta V \\
\Delta V &= 0.75V_1 \\
&= 0.75\,(7.5044) \\
&= 5.6283\,[km/s]
\end{aligned}
$$

6.3.2.2 part b

What is the semimajor axis a of the resulting hyperbolic orbit in km?

Answer:

The new speed at the point of the firing is $V = V_1 + \Delta V = 7.5044 + 5.6283 = 13.133\,[km/s]$

But

$$
\begin{aligned}
V &= \sqrt{\mu\left(\frac{2}{r}+\frac{1}{a}\right)} \\
V^2 &= \mu\left(\frac{2}{r}+\frac{1}{a}\right) \\
\frac{1}{a} &= \frac{V^2}{\mu} - \frac{2}{r} \\
a &= \frac{1}{\frac{V^2}{\mu}-\frac{2}{r}} = \frac{1}{\frac{13.133^2}{3.988\times 10^5} - \frac{2}{6378+700}} \\
&= 6670.2\,[km]
\end{aligned}
$$

6.3.2.3 part c

What is the eccentricity e of the resulting hyperbolic orbit?

Answer

These are 3 ways to find e, the first is using $r = \frac{a\left(e^2-1\right)}{1+e\cos\theta}$, where we can use that $\theta = 0$ at the time of firing since that is when $r = r_p$ for the hyperbolic orbit. This is always the case, when an orbit changes to new orbit, we use the point of firing as perigee of the new orbit, and the true anamoly is hence zero at that point. This means $r_p = \frac{a\left(e^2-1\right)}{1+e}$ and since we know a and

r_p we can solve for e

$$\begin{aligned}6378+700 &= \frac{6670.2\left(e^2-1\right)}{1+e}\\ 7078.0 &= \frac{6670.2\left(e^2-1\right)}{1+e}\\ 7078.0+7078.0e &= 6670.2e^2-6670.2\\ 6670.2e^2-6670.2-7078.0-7078.0e &= 0\\ 6670.2e^2-7078.0e-13748 &= 0\end{aligned}$$

Hence $e=2.0614$ or $e=-1$, and since e is positive, we use $e=2.0614$ as the solution.

Another way, is to note that since $e=\frac{c}{a}$ and $c=r_p+a$, hence

$$e=\frac{(6378+700)+6670.2}{6670.2}=2.0611$$

Another way to find e is using $e=\sqrt{1+\frac{2\mathscr{E}h^2}{\mu^2}}$ where Energy $\mathscr{E}=\frac{v^2}{2}-\frac{\mu}{r}$ and $h=rv$, hence

$$\begin{aligned}e &= \sqrt{1+\frac{2\left(\frac{v^2}{2}-\frac{\mu}{r}\right)(rv)^2}{\mu^2}}\\ &= \sqrt{1+\frac{2\left(\frac{13.133^2}{2}-\frac{3.988\times10^5}{6378+700}\right)((6378+700)\,13.133)^2}{\left(3.988\times10^5\right)^2}}\\ &= 2.0614\end{aligned}$$

6.3.2.4 part d

How long (in hours) does it take the spacecraft to reach the Moon's orbit, a distance of 384,000 km from the center of the Earth?

Answer

$$r_2=384000\,[km]$$

Using

$$\sqrt{\frac{\mu}{a^3}}\left(t-\mu\right)=e\sinh(F)-F$$

Where F is found from

$$
\begin{aligned}
r &= a\left(e\cosh\left(F\right)-1\right)\\
384000 &= 6670.2\left(2.0611\cosh\left(F\right)-1\right)\\
\cosh\left(F\right) &= \frac{\frac{384000}{6670.}+1}{2.0611}=28.417\\
F &= 4.03983
\end{aligned}
$$

Hence

$$
\begin{aligned}
\sqrt{\frac{\mu}{a^3}}\left(t-\tau\right) &= e\sinh\left(F\right)-F\\
\sqrt{\frac{3.988\times10^5}{6670.2^3}}\left(t-\tau\right) &= 2.0611\sinh\left(4.03983\right)-4.03983\\
\left(t-\tau\right) &= \frac{2.0611\sinh\left(4.03983\right)-4.03983}{\sqrt{\frac{3.988\times10^5}{6670.2^3}}}\\
&= 47009\,[\text{sec}]\\
&= \frac{47009}{60\times60}=13.058\,[hr]
\end{aligned}
$$

6.3.3 Problem 3

Using Matlab, EES, Mathcad, Maple or similar software, create a program to calculate the position and velocity components of a satellite in an x,y,z coordinate system given its classical orbital elements (a, e, i, GAMMA, OMEGA, f). Use the examples in the course notes to test your program, then apply it to the set of elements below. (Save your program somewhere you can find it again; you will need it later in the semester.)

a: 9000 km

e: 0.02

i: 28.5 degrees

GAMMA: 50 degrees

OMEGA: 20 degrees

f: 40 degrees

x = Answer km

y = Answer km

z = Answer km

vx = Answer km/s

vy = Answer km/s

vz = Answer km/s

```
toXYZ[a_, e_, i_, gamma_, omega_, theta_] := Module[{r, p, x, y, z, vx, vy, vz, mu = 3.
  r = (a (1 - e^2))/(1 + e Cos[theta]);
  p = a (1 - e^2);
  t1 = {{Cos[omega], -Sin[omega], 0}, {Sin[omega], Cos[omega], 0}, {0, 0, 1}};
  t2 = {{1, 0, 0}, {0, Cos[i], -Sin[i]}, {0, Sin[i], Cos[i]}};
  t3 = {{Cos[gamma], -Sin[gamma], 0}, {Sin[gamma], Cos[gamma], 0}, {0, 0, 1}};
  {x, y, z} = t3.t2.t1.{r Cos[theta], r Sin[theta], 0};
  {vx, vy, vz} = t3.t2.t1.{-Sqrt[mu/p] Sin[theta], Sqrt[mu/p] (e + Cos[theta]), 0};
  {{x, y, z}, {vx, vy, vz}}
  ]
a = 9000;
e = 0.02;
theta = 40 Degree;
i = 28.5 Degree;
gamma = 50 Degree;
omega = 20 Degree;
toXYZ[a, e, i, gamma, omega, theta]
{{-2318.17, 7728.55, 3661.5}, {-6.05942, -2.50006, 1.64775}}
```

6.4 HW4

6.4.1 Problem 1

Create a program to calculate the classical orbital elements $(a, e, i, \Omega, \omega, f)$ of a satellite given its Cartesian position and velocity components (x, y, z, v_x, v_y, v_z). Use the examples in the course notes to test your program, then apply it to the state vector below. (Save your program somewhere you can find it again; you will need it later in the semester.)

$$
\begin{aligned}
x &= -3000 \text{ km}\\
y &= -6000 \text{ km}\\
z &= 4000 \text{ km}\\
v_x &= 6 \text{ km/s}\\
vy &= -1 \text{ km/s}\\
vz &= -3 \text{ km/s}
\end{aligned}
$$

Answer is

$$
\begin{aligned}
a &= 7108.84 \text{ km}\\
e &= 0.4615 \text{ km}\\
i &= 34.32^o\\
\Omega &= 124.287^o\\
\omega &= 242.65^o\\
f &= 232.07^o
\end{aligned}
$$

6.4.2 Problem 2

What combination of launch latitude and azimuth angle will allow a spacecraft to be launched directly into an equatorial geostationary orbit about the Earth?

Since

$$\cos i = \sin A_z \cos \phi$$

Where i is inclination and A_z is the azimath and ϕ is the latitude. Then for $i = 0^0$

Latitude: 0^0

Azimuth: 90^0

Can a spacecraft be launched directly into an equatorial geostationary orbit about the Earth from the ETR (Eastern Test Range, Cape Canaveral)? No Since i is not zero.

Can a spacecraft be launched directly into an equatorial geostationary orbit about the Earth from the WTR (Western Test Range, Vandenburg AFB)? No, same reason.

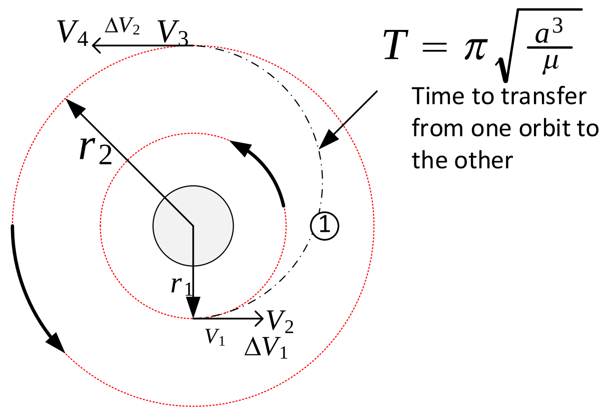
6.4.3 Problem 3

A satellite is initially in a circular orbit about the Earth at an altitude of 200 km. Its target orbit is a circular orbit in the same plane with a radius of 130,000 km. Calculate the total ΔV and transfer time (in hours) required to complete each of the orbit transfers below.

6.4.3.1 Part(a) Hohmann transfer

$$a = \frac{r_1 + r_2}{2}$$
$$V_1 = \sqrt{\frac{\mu}{r_1}}$$
$$V_2 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)}$$
$$\Delta V_1 = V_2 - V_1$$
$$V_3 = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)}$$
$$V_4 = \sqrt{\frac{\mu}{r_2}}$$
$$\Delta V_2 = V_4 - V_3$$
$$\Delta V = |\Delta V_1| + |\Delta V_2|$$

Total Velocity change needed



Hohmann Transfer

hohmann.vsdx
Nasser M. Abbasi
022014

$$a = \frac{r_1 + r_2}{2} = \frac{200 + 6378 + 130000}{2} = 68289 \text{ km}$$
$$V_1 = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{3.986\left(10^5\right)}{200 + 6378}} = 7.7843 \text{ km/s}$$
$$V_2 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{200 + 6378} - \frac{1}{68289}\right)} = 10.74 \text{ km/s}$$
$$\Delta V_1 = V_2 - V_1 = 10.74 - 7.7843 = 2.9557 \text{ km/s}$$
$$V_3 = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{130000} - \frac{1}{68289}\right)} = 0.54346 \text{ km/s}$$
$$V_4 = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{3.986\left(10^5\right)}{130000}} = 1.751 \text{ km/s}$$
$$\Delta V_2 = V_4 - V_3 = 1.751 - 0.54346 = 1.2075$$
$$\Delta V = |\Delta V_1| + |\Delta V_2| = 2.9557 + 1.2075 = 4.1632 \text{ km/s}$$

Time of transfer

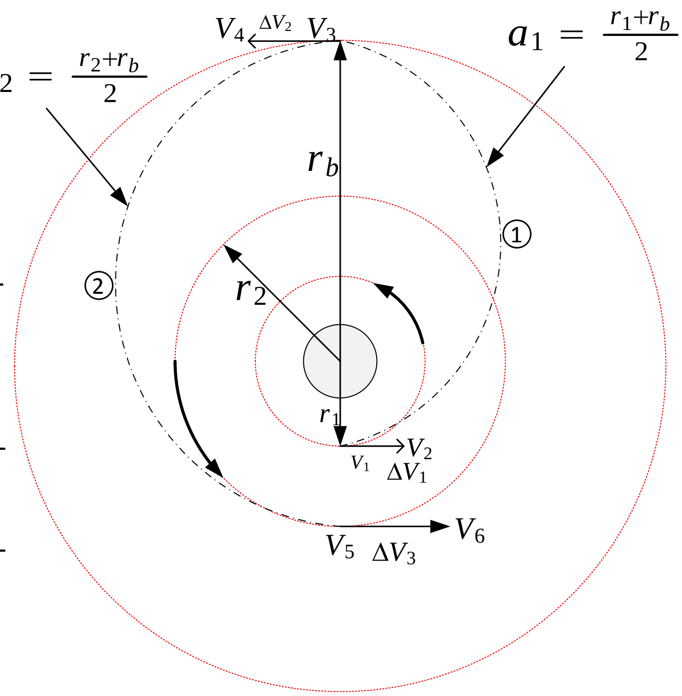
$$
\begin{aligned}
T &= \pi\sqrt{\frac{a^3}{\mu}}\\
&= \pi\sqrt{\frac{68289^3}{3.986\left(10^5\right)}} = 88799 \text{ sec}\\
&= \frac{88799}{60\times 60} = 24.666 \text{ hr}
\end{aligned}
$$

6.4.3.2 Part (b) bi-elliptic transfer

with an intermediate transfer radius of 200,000 km

$$
\begin{aligned}
a_1 &= \frac{r_1+r_b}{2}\\
a_2 &= \frac{r_2+r_b}{2}\\
V_1 &= \sqrt{\frac{\mu}{r_1}}\\
V_2 &= \sqrt{\mu\left(\frac{2}{r_1}-\frac{1}{a_1}\right)}\\
\Delta V_1 &= V_2 - V_1\\
V_3 &= \sqrt{\mu\left(\frac{2}{r_b}-\frac{1}{a_1}\right)}\\
V_4 &= \sqrt{\mu\left(\frac{2}{r_b}-\frac{1}{a_2}\right)}\\
\Delta V_2 &= V_4 - V_3\\
V_5 &= \sqrt{\mu\left(\frac{2}{r_2}-\frac{1}{a_2}\right)}\\
V_6 &= \sqrt{\frac{\mu}{r_2}}\\
\Delta V_3 &= V_6 - V_5\\
\Delta V &= |\Delta V_1| + |\Delta V_2| + |\Delta V_3|
\end{aligned}
$$

Total Velocity
change needed



Bi-Elliptic Transfer

$$T = \pi\sqrt{\frac{a_1^3}{\mu}} + \pi\sqrt{\frac{a_2^3}{\mu}}$$

Time to transfer
from one orbit to
the other

bi_ellptic.vsdx
Nasser M. Abbasi
022314

$r_b = 200000$ km, $r_1 = 200 + 6378$ km, $r_2 = 130000$ km

$$a_1 = \frac{r_1 + r_b}{2} = \frac{200 + 6378 + 200000}{2} = 1.0329 \times 10^5 \text{ km}$$
$$a_2 = \frac{r_2 + r_b}{2} = \frac{130000 + 200000}{2} = 1.65 \times 10^5 \text{ km}$$
$$V_1 = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{3.986\left(10^5\right)}{200 + 6378}} = 7.7843 \text{ km/s}$$
$$V_2 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_1}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{200 + 6378} - \frac{1}{1.0329 \times 10^5}\right)} = 10.832 \text{ km/s}$$
$$\Delta V_1 = V_2 - V_1 = 10.832 - 7.7843 = 3.0477 \text{ km/s}$$
$$V_3 = \sqrt{\mu\left(\frac{2}{r_b} - \frac{1}{a_1}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{200000} - \frac{1}{1.0329 \times 10^5}\right)} = 0.35632 \text{ km/s}$$
$$V_4 = \sqrt{\mu\left(\frac{2}{r_b} - \frac{1}{a_2}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{200000} - \frac{1}{1.65 \times 10^5}\right)} = 1.2531 \text{ km/s}$$
$$\Delta V_2 = V_4 - V_3 = 1.2531 - 0.35632 = 0.89678 \text{ km/s}$$
$$V_5 = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_2}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{130000} - \frac{1}{1.65 \times 10^5}\right)} = 1.9278 \text{ km/s}$$
$$V_6 = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{3.986\left(10^5\right)}{130000}} = 1.751 \text{ km/s}$$
$$\Delta V_3 = V_6 - V_5 = 1.751 - 1.9278 = -0.1768 \text{ km/s}$$
$$\Delta V = |\Delta V_1| + |\Delta V_2| + \left|\Delta V_3\right| = 3.0477 + 0.89678 + 0.1768 = 4.1213 \text{ km/s}$$

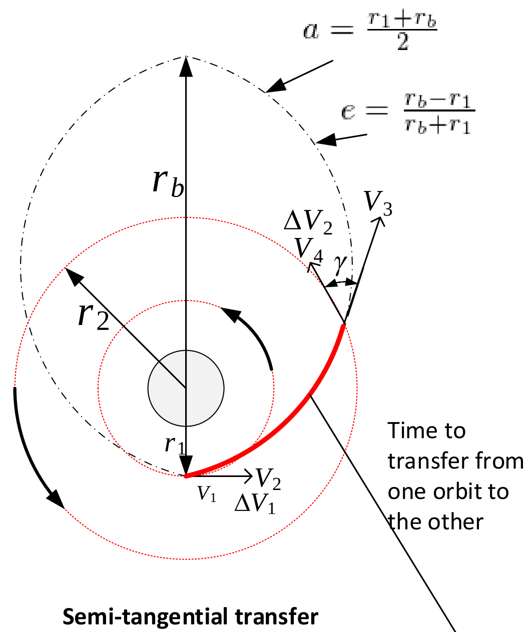
Transfer time

$$\begin{aligned} T &= \pi\sqrt{\frac{a_1^3}{\mu}} + \pi\sqrt{\frac{a_2^3}{\mu}} \\ &= \pi\sqrt{\frac{\left(1.0329 \times 10^5\right)^3}{3.986\left(10^5\right)}} + \pi\sqrt{\frac{\left(1.65 \times 10^5\right)^3}{3.986\left(10^5\right)}} \\ &= 4.9869 \times 10^5 \text{ sec} \\ &= \frac{4.9869 \times 10^5}{60 \times 60} = 138.53 \text{ hr} \end{aligned}$$

6.4.3.3 Part (c) semi-tangential elliptical transfer

$$
\begin{aligned}
a &= \frac{r_1 + r_b}{2} \\
V_1 &= \sqrt{\frac{\mu}{r_1}} \\
V_2 &= \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} \\
\Delta V_1 &= V_2 - V_1 \\
V_3 &= \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)} \\
V_4 &= \sqrt{\frac{\mu}{r_2}} \\
e &= \frac{r_b - r_1}{r_b + r_1} \\
\cos\gamma &= \sqrt{\frac{a^2(1-e^2)}{r_2(2a - r_2)}} \\
\Delta V_2 &= \sqrt{V_4^2 + V_3^2 - 2V_4V_3\cos\gamma} \\
\Delta V &= |\Delta V_1| + |\Delta V_2|
\end{aligned}
$$

Total Velocity change needed



$$
\begin{aligned}
r_2 &= a(1 - e\cos E) \\
n &= \sqrt{\frac{\mu}{a^3}} \\
\Delta t &= \frac{1}{n}(E - e\sin E)
\end{aligned}
$$

semi_tangential.vsdx
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022314

$r_b = 200000$ km, $r_1 = 200 + 6378$ km, $r_2 = 130000$ km,

$$
\begin{aligned}
a &= \frac{r_1 + r_b}{2} = \frac{200 + 6378 + 200000}{2} = 1.0329 \times 10^5 \text{ km} \\
V_1 &= \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{3.986\left(10^5\right)}{200 + 6378}} = 7.7843 \text{ km/s} \\
V_2 &= \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{200 + 6378} - \frac{1}{1.0329 \times 10^5}\right)} = 10.832 \text{ km/s} \\
\Delta V_1 &= V_2 - V_1 = 10.832 - 7.7843 = 3.0477 \text{ km/s} \\
V_3 &= \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{130000} - \frac{1}{1.0329 \times 10^5}\right)} = 1.5077 \\
V_4 &= \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{3.986\left(10^5\right)}{130000}} = 1.751 \text{ km/s} \\
e &= \frac{r_b - r_1}{r_b + r_1} = \frac{200000 - (200 + 6378)}{200000 + (200 + 6378)} = 0.93631 \\
\cos\gamma &= \sqrt{\frac{a^2\left(1 - e^2\right)}{r_2\left(2a - r_2\right)}} = \sqrt{\frac{\left(1.0329 \times 10^5\right)^2\left(1 - 0.93632^2\right)}{130000\left(2\left(1.0329 \times 10^5\right) - 130000\right)}} = 0.36351 \\
\Delta V_2 &= \sqrt{V_4^2 + V_3^2 - 2V_4V_3\cos\gamma} = \sqrt{1.751^2 + 1.5077^2 - 2\left(1.751\right)\left(1.5077\right)\left(0.36351\right)} = 1.8493 \\
\Delta V &= \left|\Delta V_1\right| + \left|\Delta V_2\right| = 3.0477 + 1.8493 = 4.897
\end{aligned}
$$

To find transfer time, we first must find E, which is found by solving $r = a\left(1 - e\cos E\right)$ where r is the radius we want to find E at which is r_2 in this case. Hence

$$
\begin{aligned}
r_2 &= a\left(1 - e\cos E\right) \\
130000 &= 1.0329 \times 10^5\left(1 - 0.93631\cos E\right) \\
0.93631\cos E &= 1 - \frac{130000}{1.0329 \times 10^5} \\
0.93631\cos E &= -0.25859 \\
\cos E &= -0.27618 \\
E &= \arccos\left(-0.27618\right) = 1.8506 \text{ rad} \\
n &= \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{3.986\left(10^5\right)}{\left(1.0329 \times 10^5\right)^3}} = 1.9019 \times 10^{-5} \\
\Delta t &= \frac{1}{n}\left(E - e\sin E\right) \\
\Delta t &= \frac{1}{1.9019 \times 10^{-5}}\left(1.8506 - \left(0.93632\right)\sin\left(1.8506\right)\right) = 49987 \text{ sec} \\
&= \frac{49987}{60 \times 60} = 13.885 \text{ hr}
\end{aligned}
$$

6.4.3.4 Part (d) a semi-tangential hyperbolic transfer

with a transfer time half that required for a semi-tangential parabolic transfer

Semi-tangential parabolic transfer time: Answer hours

Semi-tangential hyperbolic transfer time: Answer hours

Semi-tangential hyperbolic total ΔV: Answer km/s

Answer:

$r_1 = 200 + 6378$ km, $r_2 = 130000$ km. For a parabolic orbit, the true anamoly θ is found when $r = r_2$. From

$$
\begin{aligned}
r_2 &= \frac{2r_p}{1+\cos\theta}\\
\theta &= \arccos\left(\frac{2r_p}{r_2} - 1\right)\\
&= \arccos\left(\frac{2r_p}{r_2} - 1\right)
\end{aligned}
$$

But $r_p = r1$ hence

$$
\begin{aligned}
\theta &= \arccos\left(\frac{2r_1}{r_2} - 1\right)\\
&= \arccos\left(\frac{2\,(200+6378)}{130000} - 1\right)\\
&= 2.6878 \text{ rad}\\
&= 154^0
\end{aligned}
$$

So the time for transfer if we are using a parabolic orbit is

$$
\begin{aligned}
\Delta t &= \frac{\tan\left(\frac{\theta}{2}\right) + \frac{1}{3}\left(\tan\left(\frac{\theta}{2}\right)\right)^3}{2\sqrt{\frac{\mu}{2r_1}}}\\
&= \frac{\tan\left(\frac{2.6878}{2}\right) + \frac{1}{3}\left(\tan\left(\frac{2.6878}{2}\right)\right)^3}{2\sqrt{\frac{3.986\left(10^5\right)}{(2(200+6378))^3}}}\\
&= 37547 \text{ sec}\\
&= \frac{37547}{60\times 60} = 10.430 \text{ hr}
\end{aligned}
$$

Hence required time for hyperbolic is

$$
\Delta t_{hyper} = \frac{1}{2}(10.430) = 5.215 \text{ hr}
$$

Now to obtain ΔV for hyperbolic orbit.

If we know r_1, r_2 on the orbit, and know the travel time between these 2 points then a, e, F

can be found by numerically solving these equations

$$r_1 = a\left(e-1\right)$$
$$r_2 = a\left(e\cosh F-1\right)$$
$$\Delta t = \sqrt{\frac{a^3}{\mu}}\left(e\sinh\left(F\right)-F\right)$$

The above are 3 equations with 3 unknowns

$$200+6378 = a\left(e-1\right)$$
$$\sqrt{\frac{3.986\left(10^5\right)}{a^3}}\left(5.215\times 60\times 60\right) = e\sinh\left(F\right)-F$$
$$130000 = a\left(e\cosh F-1\right)$$

Solving gives

$$e = 1.5468$$
$$a = 12029.4\text{ km}$$
$$F = 2.7213\ rad$$

Hence

$$a = 12029.4\text{ km}$$
$$V_1 = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{3.986\left(10^5\right)}{200+6378}} = 7.7843\text{ km/s}$$
$$V_2 = \sqrt{\mu\left(\frac{2}{r_1}+\frac{1}{a}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{200+6378}+\frac{1}{12029.4}\right)} = 12.423\text{ km/s}$$
$$\Delta V_1 = V_2 - V_1 = 12.423 - 7.7843 = 4.6387\text{ km/s}$$
$$V_3 = \sqrt{\mu\left(\frac{2}{r_2}+\frac{1}{a}\right)} = \sqrt{3.986\left(10^5\right)\left(\frac{2}{130000}+\frac{1}{12029.4}\right)} = 6.2664$$
$$V_4 = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{3.986\left(10^5\right)}{130000}} = 1.751\text{ km/s}$$
$$\cos\gamma = \sqrt{\frac{a^2\left(e^2-1\right)}{r_2\left(2a+r_2\right)}} = \sqrt{\frac{\left(12029.4\right)^2\left(1.5468^2-1\right)}{130000\left(2\left(12029.4\ \right)+130000\right)}} = 0.10031$$
$$\Delta V_2 = \sqrt{V_4^2+V_3^2-2V_4V_3\cos\gamma} = \sqrt{1.751^2+6.2664^2-2\left(1.751\right)\left(6.2664\right)\left(0.10031\right)} = 6.335$$
$$\Delta V = \left|\Delta V_1\right| + \left|\Delta V_2\right| = 4.6387 + 6.335 = 10.974$$

6.5 HW5

6.5.1 Problem 1

A spacecraft is initially in a 300 km altitude circular orbit about the Earth in the ecliptic plane. It is to be sent on a Hohmann transfer to Saturn, also in the ecliptic plane. Assume that Saturn is in the correct position in its orbit for a flyby to occur when the spacecraft gets there.

6.5.1.1 part(a)

Calculate the initial ΔV_1 required to start the trip to Saturn.

$$r_{b0} = r_E + alt$$

Where r_E is radius of earth and alt is spacecraft altitude. Hence

$$r_{b0} = 6378 + 300 = 6678 \text{ km}$$

The distance from earth to sun is $R_E = 1.496 \times 10^8$ km and the distance from saturn to sun is $R_s = 9.536 \times 1.496 \times 10^8 = 1.4266 \times 10^9$ km therefore $a = \frac{R_E + R_s}{2} = \frac{1.496\times10^8 + 1.4266\times10^9}{2} = 7.8815 \times 10^8$ km.

The earth speed around the sun is $V_e = \sqrt{\frac{\mu_s}{r_e}} = \sqrt{\frac{1.327\times10^{11}}{1.496\times10^8}} = 29.783$ km/sec. When the spacecraft escape the earth it has to be at speed

$$V_{perigee} = \sqrt{\mu_s \left(\frac{2}{R_E} - \frac{1}{a}\right)} = \sqrt{1.327 \times 10^{11} \left(\frac{2}{1.496 \times 10^8} - \frac{1}{7.8815 \times 10^8}\right)} = 40.07 \text{ km/sec}$$

Therefore, V_∞ is the escape speed found from

$$\begin{aligned} V_\infty &= V_{perigee} - V_e \\ &= 40.07 - 29.783 \\ &= 10.287 \text{ km/sec} \end{aligned}$$

Now the burn out speed is found

$$\frac{V_{bo}^2}{2} - \frac{\mu_E}{r_{b0}} = \frac{V_\infty^2}{2} - \frac{\mu_E}{r_{SOI}}$$

Where r_{SOI} is the earth sphere of influense given by 9.24×10^5 km. Solving for V_{bo}

$$\frac{V_{bo}^2}{2} - \frac{3.986 \times 10^5}{6678} = \frac{10.287^2}{2} - \frac{3.986 \times 10^5}{9.24 \times 10^5}$$
$$V_{bo} = 14.978 \text{ km/sec}$$

Hence

$$\begin{aligned}\Delta V_1 &= V_{bo} - \sqrt{\frac{\mu_E}{r_{bo}}} \\ &= 14.97 - \sqrt{\frac{3.986 \times 10^5}{6678}} \\ &= 7.2442\end{aligned}$$

6.5.1.2 part (b)

Calculate the angle past the Earth's dawn-dusk line where the ΔV should be applied.

$$\begin{aligned}e &= \sqrt{1 + \frac{V_\infty^2 V_{bo}^2 r_{bo}^2}{\mu_E^2}} \\ &= \sqrt{1 + \frac{\left(10.287^2\right)\left(14.978^2\right)\left(6678^2\right)}{\left(3.986 \times 10^5\right)^2}} \\ &= 2.7683\end{aligned}$$

Hence

$$\begin{aligned}\eta &= \arccos\left(\frac{-1}{e}\right) = \arccos\left(\frac{-1}{2.7683}\right) = 1.9404 \text{ radian} \\ &= 111.18^0\end{aligned}$$

Hence $\theta = 180 - 111.18 = 68.82^0$

6.5.1.3 part (c)

For how long is the spacecraft on the heliocentric Hohmann transfer between Earth and Saturn? (Note: you do not need to calculate the time within either planet's sphere of influence, as that will be small relative to the Hohmann transfer time, but you are welcome to do so and compare those values for yourself.)

The time is half the period of the elliptical orbit. Hence

$$
\begin{aligned}
T &= \pi\sqrt{\frac{a^3}{u_s}} = \pi\sqrt{\frac{\left(7.8815\times10^8\right)^3}{1.327\times10^{11}}} = 1.9082\times10^8 \text{ sec} \\
&= \frac{1.908\,2\times10^8}{60\times60\times24\times365} = 6.051 \text{ year}
\end{aligned}
$$

6.5.1.4 part(d)

After crossing into the sphere of influence of Saturn, the spacecraft is to be placed in a circular orbit about Saturn with an orbital radius of 150,000 km. Calculate the ΔV_2 required to place the spacecraft on this orbit.

Solution completed in the Mathematica solution. See above for links.

6.5.2 Problem 2

A spacecraft on an interplanetary mission in the same plane as Jupiter's orbit about the Sun enters Jupiter's sphere of influence. The spacecraft has a speed of 10 km/s relative to the Sun at this point, which you can estimate as the Jupiter's average orbital radius about the Sun. (See the Planetary Constants sheet in your notes for values.) Assume that Jupiter is in a circular orbit about the Sun.

6.5.2.1 part(a)

The largest possible value for the impact parameter, b, that will still result in a hyperbolic orbit about Jupiter in the patched conic method is Jupiter's SOI radius. Find that value on the Planetary Constants sheet in the course notes and enter it here for reference.

$b_{\max} = R_{SOI,Jupitor}$ = Answer km

For parts (b) through (g), assume that, relative to the Sun, the spacecraft is moving in the same direction as Jupiter when it enters Jupiter's SOI.

6.5.2.2 part(b)

What is the speed of the satellite relative to Jupiter when it enters Jupiter's SOI?

V_∞ = Answer km/s

6.5.2.3 part(c)

What is the smallest possible value for the impact parameter b? This value of impact parameter will result in a burnout radius that just grazes the surface of Jupiter, $r_{bo} = r_{Jupiter}$

$b_{min} = km$

6.5.2.4 part(d)

Select as your impact parameter the value halfway between b_{min} and b_{max}. Note that value here for reference and use it as your impact parameter for the rest of the problem.

b = Answer km

6.5.2.5 part(e)

Given the impact parameter from part (d), calculate the turning angle of the spacecraft relative to Jupiter during the flyby.

θ = Answer degrees

6.5.2.6 part(f)

What is the spacecraft's heliocentric speed following the flyby?

$V_D = km/s$

6.5.2.7 part(g)

What is the spacecraft's heliocentric flight path angle following the flyby?

$\gamma_D = deg$

For the remaining parts, assume that, relative to the Sun, the spacecraft DOES NOT arrive at Jupiter's SOI moving in the same direction at Jupiter. The spacecraft still has a heliocentric speed of 10 km/s at the distance of Jupiter's orbit from the Sun. But now it has a heliocentric eccentricity of 0.5. (What was the heliocentric eccentricity when the spacecraft arrived in the same direction as Jupiter, assuming that point was aphelion?)

6.5.2.8 part(h)

What is the spacecraft's heliocentric flight path angle when it arrives at Jupiter's SOI?

$\gamma_A = deg$

6.5.2.9 part(i)

What is the spacecraft's speed relative to Jupiter?

V_∞ = km/s

part(j)

Using the same impact parameter as in part (d), calculate the turning angle of the spacecraft relative to Jupiter.

$\theta = deg$

part(k)

Assuming that the spacecraft flies behind Jupiter, what is the spacecraft's heliocentric speed following the flyby?

V_D =km/s

6.5.2.10 part(L)

Assuming that the spacecraft flies behind Jupiter, what is the spacecraft's heliocentric flight path angle following the flyby?

$\gamma_D = deg$

6.5.3 Appendix

6.5.3.1 solution in Maple

HW5 by Nasser M. Abbasi, EMA 550

Problem 1

A spacecraft is initially in a 300 km altitude circular orbit about the Earth in the ecliptic plane. It is to be sent on a Hohmann transfer to Saturn, also in the ecliptic plane. Assume that Saturn is in the correct position in its orbit for a flyby to occur when the spacecraft gets there.

```
> local `~`:= proc(f::uneval, `$`::identical(` $`), expr::uneval)
> local x, opr:= op(procname);
>      if opr <> `<` then  return :-`~`[opr](args)  end if;
>      x:= eval(expr);
>      print(op(1,
>           subs(
>                _F_= nprintf("%a", f), _X_= x,
>                proc(_F_:= expr=_X_) end proc
>           )
>      ));
>      assign(f,x)
> end proc:
```

part(a)

These below are from tables

```
> AU := 1.496*10^8;
  saturn_sun_distance := 9.537*1.496*10^8;
  sun_mu          := 1.327*10^11;
  earth_mu        := 3.986*10^5;
  earth_soi       := 9.24*10^5;
  satellite_earth_altitude := 300;
  earth_radius    := 6378;
```

$$AU := 1.496000000\ 10^{8}$$

$$saturn\_sun\_distance := 1.426735200\ 10^{9}$$

$$sun\_mu := 1.327000000\ 10^{11}$$

$$earth\_mu := 3.98600000\ 10^{5}$$

$$earth\_soi := 9.2400000\ 10^{5}$$

$$satellite\_earth\_altitude := 300$$

$$earth\_radius := 6378$$

Find burn out radius

```
> rb0_earth <~ satellite_earth_altitude+earth_radius;
```

$$rb0\_earth := satellite\_earth\_altitude + earth\_radius = 6678$$

find "a" for the Hohmann ellipse in sun centric space

```
> a <~ (AU+saturn_sun_distance)/2;
```

$$a := \frac{1}{2}\ AU + \frac{1}{2}\ saturn\_sun\_distance = 7.881676000\ 10^{8}$$

Find velocity of earth relative to the sun

```
> earth_speed <~ sqrt(sun_mu/AU);
```

$$earth\_speed := \sqrt{\frac{sun\_mu}{AU}} = 29.78308388$$

Find velocity of spacecraft relative to earth

```
> satellite_speed_relative_to_earth <~ sqrt(earth_mu/rb0_earth);
```

$$satellite\_speed\_relative\_to\_earth := \sqrt{\frac{earth\_mu}{rb0\_earth}} = 7.725835198$$

find what the velocity of spacecraft should be at the perigee of the Hohmann orbit in sun centeric space

```
> velocity_perigee <~ sqrt(sun_mu*(2/AU - 1/a));
```

$$velocity\_perigee := \sqrt{sun\_mu\left(\frac{2}{AU} - \frac{1}{a}\right)} = 40.07117375$$

Find excess speed V infinity out, to escape earth

```
> velocity_infinity_entering_saturn <~  velocity_perigee-
  earth_speed;
```

$$velocity\_infinity\_entering\_saturn := velocity\_perigee - earth\_speed = 10.28808987$$

set up the energy equation and solve for V_b0

```
> saturn_vb0 := 'saturn_vb0';
  saturn_vb0 <~ sqrt(2 * ((velocity_infinity_entering_saturn^2/2
  -earth_mu/earth_soi)+  earth_mu/rb0_earth ));
```

$$saturn\_vb0 := saturn\_vb0$$

$$saturn\_vb0 := \sqrt{velocity\_infinity\_entering\_saturn^2 - \frac{2\ earth\_mu}{earth\_soi} + \frac{2\ earth\_mu}{rb0\_earth}} = 14.97862082$$

```
> delta_v1 <~  saturn_vb0 - satellite_speed_relative_to_earth ;
```

$$delta\_v1 := saturn\_vb0 - satellite\_speed\_relative\_to\_earth = 7.252785622$$

part(b)

Calculate the angle past the Earth's dawn-dusk line where the ΔV should be applied.
Find escape hyperbolic trajectory eccentricity

```
> e <~ sqrt(1+ (velocity_infinity_entering_saturn^2*saturn_vb0^2*
  rb0_earth^2)/earth_mu^2 );
```

$$e := \sqrt{1 + \frac{velocity\_infinity\_entering\_saturn^2\ saturn\_vb0^2\ rb0\_earth^2}{earth\_mu^2}} = 2.768660225$$

find angle eta

```
> eta <~ arccos(- 1/e);
```

$$\eta := \arccos\left(-\frac{1}{e}\right) = 1.940335258$$

```
> theta <~ evalf(180 - eta*180/Pi);
```

$$\theta := evalf\left(180 - \frac{180\,\eta}{\pi}\right) = 68.8269789$$

Part (c)

For how long is the spacecraft on the heliocentric Hohmann transfer between Earth and Saturn? (Note: you do not need to calculate the time within either planet's sphere of influence, as that will be

small relative to the Hohmann transfer time, but you are welcome to do so and compare those values for yourself.)
The time is half the period of the elliptical orbit. Hence

```
> T <~ evalf(Pi*sqrt(a^3/sun_mu));
```

$$T := evalf\left(\pi \sqrt{\frac{a^3}{sun\_mu}}\right) = 1.908280789\ 10^8$$

```
> T <~ T/(60*60*24*365);
```

$$T := \frac{1}{31536000}\ T = 6.051118687$$

Part (d)

After crossing into the sphere of influence of Saturn, the spacecraft is to be placed in a circular orbit about Saturn with an orbital radius of 150,000 km. Calculate the ΔV2 required to place the spacecraft on this orbit. When spacecraft reaches saturn is has speed relative to sun of

```
> saturn_vb0 := 'saturn_vb0';
  rb0_saturn := 150000;
  v_apogee   <~ sqrt(sun_mu*(2/saturn_sun_distance-1/a));
  satellite_speed_relative_to_earthurn <~ sqrt(sun_mu*
  (1/saturn_sun_distance));
  velocity_infinity_entering_jupitor <~
  satellite_speed_relative_to_earthurn - v_apogee;
  saturn_mu  := 37931187;
  saturn_SOI := 3.47*10^7;
  eq := saturn_vb0^2/2 - saturn_mu/rb0_saturn =
  velocity_infinity_entering_jupitor^2/2 - saturn_mu/saturn_SOI;
  saturn_vb0 := op(select( is, [solve(eq,saturn_vb0)], positive))
  ;
  satellite_speed_relative_to_earth <~ sqrt(saturn_mu/rb0_saturn)
  ;
  del_v2     <~ evalf(satellite_speed_relative_to_earth -
  saturn_vb0);
  total_delV <~ abs(delta_v1) + abs(del_v2);
```

$$saturn\_vb0 := saturn\_vb0$$

$$rb0\_saturn := 150000$$

$$v\_apogee := \sqrt{sun\_mu \left(\frac{2}{saturn\_sun\_distance} - \frac{1}{a}\right)} = 4.201653949$$

$$satellite\_speed\_relative\_to\_earthurn := \sqrt{\frac{sun\_mu}{saturn\_sun\_distance}} = 9.644145932$$

$$velocity\_infinity\_entering\_jupitor := satellite\_speed\_relative\_to\_earthurn - v\_apogee = 5.442491983$$

$$saturn\_mu := 37931187$$

$$saturn\_SOI := 3.470000000\ 10^7$$

$$eq := \frac{1}{2}\ saturn\_vb0^2 - \frac{12643729}{50000} = 13.71724171$$

$$saturn\_vb0 := 23.09076966$$

$$satellite\_speed\_relative\_to\_earth := \sqrt{\frac{saturn\_mu}{rb0\_saturn}} = \frac{1}{500}\sqrt{63218645}$$

$$del\_v2 := evalf(satellite\_speed\_relative\_to\_earth - saturn\_vb0) = -7.18873897$$

$$total\_delV := |delta\_v1| + |del\_v2| = 14.44152459$$

Problem 2

A spacecraft on an interplanetary mission in the same plane as Jupiter's orbit about the Sun enters Jupiter's sphere of influence. The spacecraft has a speed of 10 km/s relative to the Sun at this point, which you can estimate as the Jupiter's average orbital radius about the Sun. (See the Planetary Constants sheet in your notes for values.) Assume that Jupiter is in a circular orbit about the Sun.

part(a)

The largest possible value for the impact parameter, b, that will still result in a hyperbolic orbit about Jupiter in the patched conic method is Jupiter's SOI radius. Find that value on the Planetary Constants sheet in the course notes and enter it here for reference.

```
> jupitor_SOI  := 4.83*10^7;
  sun_mu       := 1.327*10^11;
  jupitor_mu   := 126686534;
  b_max       <~ jupitor_SOI;
```

$$jupitor\_SOI := 4.830000000\ 10^{7}$$

$$sun\_mu := 1.327000000\ 10^{11}$$

$$jupitor\_mu := 126686534$$

$$b\_max := jupitor\_SOI = 4.830000000\ 10^{7}$$

part(b)

For parts (b) through (g), assume that, relative to the Sun, the spacecraft is moving in the same direction as Jupiter when it enters Jupiter's SOI

What is the speed of the satellite relative to Jupiter when it enters Jupiter's SOI?

```
> satellite_speed_relative_to_sun :=10;
  jupitor_sun_distance := 5.203*1.495978*10^8;
  jupitor_speed <~ sqrt((sun_mu)/(jupitor_sun_distance));
  velocity_infinity_entering_jupitor <~ jupitor_speed -
  satellite_speed_relative_to_sun;
```

$$satellite\_speed\_relative\_to\_sun := 10$$

$$jupitor\_sun\_distance := 7.783573534\ 10^{8}$$

$$jupitor\_speed := \sqrt{\frac{sun\_mu}{jupitor\_sun\_distance}} = 13.05707640$$

$$velocity\_infinity\_entering\_jupitor := jupitor\_speed - satellite\_speed\_relative\_to\_sun = 3.05707640$$

part(c)

What is the smallest possible value for the impact parameter b? This value of impact parameter will result in a burnout radius that just grazes the surface of Jupiter

```
> jupitor_radius :=71492;
  jupitor_vb0_min <~ sqrt(jupitor_mu/jupitor_radius);
```

```
b_min <~ evalf(jupitor_radius*
jupitor_vb0_min/velocity_infinity_entering_jupitor);
```

$$jupitor\_radius := 71492$$

$$jupitor\_vb0\_min := \sqrt{\frac{jupitor\_mu}{jupitor\_radius}} = \frac{1}{35746}\sqrt{2264268422182}$$

$$b\_min := evalf\left(\frac{jupitor\_radius\ jupitor\_vb0\_min}{velocity\_infinity\_entering\_jupitor}\right) = 9.844363876\ 10^5$$

part(d)

Select as your impact parameter the value halfway between b_{min} and b_{max}. Note that value here for reference and use it as your impact parameter for the rest of the problem

```
> b <~ (b_max+b_min)/2;
```

$$b := \frac{1}{2}\ b\_max + \frac{1}{2}\ b\_min = 2.464221819\ 10^7$$

part(e)

Given the impact parameter from part (d), calculate the turning angle of the spacecraft relative to Jupiter during the flyby.

```
> saturn_vb0 := 'saturn_vb0': rb0_earth := 'rb0_earth':
  rb0_jupitor <~ b*
  velocity_infinity_entering_jupitor/jupitor_vb0;
  eq <~ (jupitor_vb0^2/2 - jupitor_mu/rb0_jupitor =
  velocity_infinity_entering_jupitor^2/2 -
  jupitor_mu/jupitor_SOI);
  sol <~ solve(eq,jupitor_vb0);
  jupitor_vb0 <~ op(select( is, [sol], positive));
```

$$rb0\_jupitor := \frac{b\ velocity\_infinity\_entering\_jupitor}{jupitor\_vb0} = \frac{7.533314367\ 10^7}{jupitor\_vb0}$$

$$eq := \left(\frac{1}{2}\ jupitor\_vb0^2 - \frac{jupitor\_mu}{rb0\_jupitor} = \frac{1}{2}\ velocity\_infinity\_entering\_jupitor^2 - \frac{jupitor\_mu}{jupitor\_SOI}\right) = \left(\frac{1}{2}\ jupitor\_vb0^2 - 1.681683889\ jupitor\_vb0 = 2.049948451\right)$$

$$sol := solve(eq, jupitor\_vb0) = (4.313785256, -0.9504174777)$$

$$jupitor\_vb0 := op(select(is, [sol], positive)) = 4.313785256$$

```
> rb0_jupitor;
```

$$1.746335045\ 10^7$$

```
> e <~ sqrt(1+(velocity_infinity_entering_jupitor^2*
  jupitor_vb0^2*rb0_jupitor^2)/jupitor_mu^2 );
  eta &= arccos(-1/e);
  evalf(eta*180/Pi);
  theta &= (2*eta-Pi);
  evalf(theta*180/Pi);
```

$$e := \sqrt{1 + \frac{velocity\_infinity\_entering\_jupitor^2\ jupitor\_vb0^2\ rb0\_jupitor^2}{jupitor\_mu^2}} = 2.074762092$$

$$(1.940335258)\ \&= (2.073712835)$$

$$111.1730211$$
$$(68.8269789)\ \&= (3.880670516 - \pi)$$
$$3943.495406$$

part(f)

What is the spacecraft's heliocentric speed following the flyby? (11.73 is correct)

```
> vd <~ sqrt(jupitor_speed^2+
  velocity_infinity_entering_jupitor^2-2*jupitor_speed*abs
  (velocity_infinity_entering_jupitor)*cos(theta));
```

$$vd := \left(jupitor\_speed^2 + velocity\_infinity\_entering\_jupitor^2 - 2\,jupitor\_speed\,|velocity\_infinity\_entering\_jupitor|\cos(\theta)\right)^{1/2} = 10.16313731$$

part(g)

What is the spacecraft's heliocentric flight path angle following the flyby

```
> gamma_d <~ arcsin(velocity_infinity_entering_jupitor*sin
  (theta)/vd);
  evalf(gamma_d*180/Pi);
```

$$gamma\_d := \arcsin\left(\frac{velocity\_infinity\_entering\_jupitor\sin(\theta)}{vd}\right) = -0.08555941389$$
$$-4.902193312$$

Hohmann from earth to moon (for project)

```
> satellite_earth_altitude := 300;
  earth_radius   := 6378;
  r__p           <~ satellite_earth_altitude+earth_radius;
  r__a           <~ 384400;
  a              <~ ((r__p+r__a)/2);
  earth_mu      := 3.986*10^5;
  satellite_speed_relative_to_earth  <~ sqrt(earth_mu/r__p);
  velocity_perigee  <~ sqrt(earth_mu*(2/r__p - 1/a));
  del__V1 <~ velocity_perigee -
  satellite_speed_relative_to_earth;
  e <~ evalf((r__a-r__p)/(r__a+r__p));
```

$$satellite\_earth\_altitude := 300$$
$$earth\_radius := 6378$$
$$r_p := satellite\_earth\_altitude + earth\_radius = 6678$$
$$r_a := 384400 = 384400$$
$$a := \frac{1}{2}\,r_a + \frac{1}{2}\,r_p = 195539$$
$$earth\_mu := 3.98600000\ 10^5$$
$$satellite\_speed\_relative\_to\_earth := \sqrt{\frac{earth\_mu}{r_p}} = 7.725835198$$

$$velocity\_perigee := \sqrt{earth\_mu\left(\frac{2}{r_p} - \frac{1}{a}\right)} = 10.83229389$$

$$del_{V1} := velocity\_perigee - satellite\_speed\_relative\_to\_earth = 3.106458692$$

$$e := evalf\left(\frac{r_a - r_p}{r_a + r_p}\right) = 0.9658482451$$

```
> velocity_apogee <~  sqrt(earth_mu*(2/r__a - 1/a));
```

$$velocity\_apogee := \sqrt{earth\_mu\left(\frac{2}{r_a} - \frac{1}{a}\right)} = 0.1881843356$$

```
> v2 <~ sqrt(earth_mu/r__a);
```

$$v2 := \sqrt{\frac{earth\_mu}{r_a}} = 1.018302846$$

```
> delV2 <~ v2-velocity_apogee;
```

$$delV2 := v2 - velocity\_apogee = 0.8301185104$$

```
> totalDelV <~ abs(del__V1)+abs(delV2);
```

$$totalDelV := |del_{V1}| + |delV2| = 3.936577202$$

```
> delT:=Pi* sqrt(a^3/earth_mu);
```

$$delT := 1.369561180\ 10^5\ \pi$$

```
> evalf(delT);
```

$$4.302603342\ 10^5$$

```
> evalf(delT/(60*60*24));
```

$$4.979864981$$

6.5.3.2 solution in Mathematica

HW5 EMA 550, University of Wisconsin, Madison

Nasser M. Abbasi
March 11,2014

problem 1

A spacecraft is initially in a 300 km altitude circular orbit about the Earth in the ecliptic plane. It is to be sent on a Hohmann transfer to Saturn, also in the ecliptic plane. Assume that Saturn is in the correct position in its orbit for a flyby to occur when the spacecraft gets there.

Part (a)

Find ΔV_1 for Hohmann transfer

define constants to use

```
Clear["Global`*"];
AU = 1.495978 * 10^8;
r_earth = 6378;
μ_sun = 1.327 * 10^11;
μ_earth = 3.986 * 10^5;
R_earth = 1.496 * 10^8;
R_earth_SOI = 9.24 * 10^5;
R_saturn = 9.537 AU;
```

Velocity of earth relative to the sun

$$V_{earth} = \sqrt{\frac{\mu_{sun}}{R_{earth}}}$$

```
29.7831
```

spacecraft altitude over earth

```
alt = 300;
```

$$r_{b0} = r_{earth} + alt$$

6678

Find Hohmann paramters for trip to Saturn

$$a = \frac{R_{earth} + R_{saturn}}{2}$$

7.88157×10^{8}

Find V_p the velocity are perigee

$$V_{perigee} = \sqrt{\mu_{sun}\left(\frac{2}{R_{earth}} - \frac{1}{a}\right)}$$

40.0711

Find V_∞ the excess velocity to escape from Earth

$$V_{out} = V_{perigee} - V_{earth}$$

10.2881

Find V_{b0} at earth

$$V_{b0} = \sqrt{2\left(\left(\frac{V_{out}^2}{2} - \frac{\mu_{earth}}{R_{earth_{SOI}}}\right) + \frac{\mu_{earth}}{r_{b0}}\right)}$$

14.9786

Find V_{sat} the spacecraft speed around eath

$$V_{sat} = \sqrt{\frac{\mu_{earth}}{r_{b0}}}$$

7.72584

find ΔV_1

```
delV1 = Vb0 - Vsat
```

```
7.25277
```

Part (b) Angle calculation at departure

Calculate the angle past the Earth's dawn-dusk line where the ΔV should be applied.

find *e* the eccentricty for the escape hyperbola

$$e = \sqrt{1 + \frac{V_{out}^2 V_{b0}^2 r_{b0}^2}{\mu_{earth}^2}}$$

```
2.76865
```

```
η = ArcCos[-1/e];
Row[{"η Degree = ", η * 180/π}]
```

```
η Degree = 111.173
```

```
θ = Pi - η;
Row[{"θ Degree = ", θ * 180/π}]
```

```
θ Degree = 68.8269
```

Part (c)

For how long is the spacecraft on the heliocentric Hohmann transfer between Earth and Saturn? (Note: you do not need to calculate the time within either planet's sphere of influence, as that will be small relative to the Hohmann transfer time, but you are welcome to do so and compare those values for yourself.)

find time to fly, which is half the period

```
T = 2 π Sqrt[a^3/μsun];
Row[{"time to fly in years = ", (1/2) T/(60*60*24*365)}]
```

$$T = 2\pi\sqrt{\frac{a^3}{\mu_{sun}}};$$

```
time to fly in years = 6.051
```

Part (d)

After crossing into the sphere of influence of Saturn, the spacecraft is to be placed in a circular orbit about Saturn with an orbital radius of 150,000 km. Calculate the ΔV2 required to place the spacecraft on this orbit. When spacecraft reaches saturn is has speed relative to sun of

Paramters to use

$$r_{b0} = 150000;$$
$$\mu_{saturn} = 37931187;$$
$$R_{saturn_{SOI}} = 3.47 * 10\text{^}7;$$

Find $V_{apegree}$ of the Hohmann transfer

$$V_{apegee} = \sqrt{\mu_{sun}\left(\frac{2}{R_{saturn}} - \frac{1}{a}\right)}$$

```
4.20171
```

find saturn speed relative to sun

$$V_{saturn} = \sqrt{\frac{\mu_{sun}}{R_{saturn}}}$$

```
9.64422
```

Find V_{in} the speed by which spacecraft enters saturn SOI

$$V_{in} = V_{saturn} - V_{apegee}$$

```
5.4425
```

Use energy equation to solve for V_{b0} at Saturn

$$V_{b0} = \sqrt{2\left(\left(\frac{V_{in}^2}{2} - \frac{\mu_{saturn}}{R_{saturn_{SOI}}}\right) + \frac{\mu_{saturn}}{r_{b0}}\right)}$$

```
23.0908
```

Since spacecrasft will end up in an orbit around saturn, find its parking speed

$$\left(V_{sat} = \sqrt{\frac{\mu_{saturn}}{r_{b0}}}\right) \text{ // N}$$

```
15.902
```

find ΔV_2

$$delV_2 = V_{sat} - V_{b0}$$

```
-7.18874
```

Find total speed change needed

$$totalV = Abs[delV_1] + Abs[delV_2]$$

```
14.4415
```

Problem 2

A spacecraft on an interplanetary mission in the same plane as Jupiter’s orbit about the Sun enters Jupiter’s sphere of influence. The spacecraft has a speed of 10 km/s relative to the Sun at this point, which you can estimate as the Jupiter’s average orbital radius about the Sun. (See the Planetary Constants sheet in your notes for values.) Assume that Jupiter is in a circular orbit about the Sun.

Part (a)

The largest possible value for the impact parameter, b, that will still result in a hyperbolic orbit about Jupiter in the patched conic method is Jupiter’s SOI radius. Find that value on the Planetary Constants sheet in the course notes and enter it here for reference.

Paramters

```
ClearAll["Global`*"];
AU = 1.495978 * 10^8;
r_earth = 6378;
μ_sun = 1.327 * 10^11;
μ_earth = 3.986 * 10^5;
μ_jupiter = 126 686 534;
R_earth = 1.496 * 10^8;
R_earthSOI = 9.24 * 10^5;
R_jupiter = 5.203 AU;
r_jupiter = 71 492;
jupiter_SOI = 4.83 * 10^7;
bmax = jupiter_SOI;
```

Part(b)

For parts (b) through (g), assume that, relative to the Sun, the spacecraft is moving in the same direction as Jupiter when it enters Jupiter's SOI

What is the speed of the satellite relative to Jupiter when it enters Jupiter's SOI?

```
Vin = 10;
```

find Jupitor speed relative to sun

$$V_{jupitor} = \sqrt{\frac{\mu_{sun}}{R_{jupiter}}}$$

```
13.0571
```

Find speed of spacecraft relative to Jupitor

```
VinRelative = V_jupitor - Vin
```

```
3.05708
```

Part(c)

What is the smallest possible value for the impact parameter b? This value of impact parameter will result in a burnout radius that just grazes the surface of Jupiter

```
eq = bmin VinRelative == r_jupitor Sqrt[μ_jupitor / r_jupitor];
bmin /. First@Solve[eq, bmin];
(bmin = %) // N
```

```
984436.
```

Part(d)

Select as your impact parameter the value halfway between b_{min} and b_{max}. Note that value here for reference and use it as your impact parameter for the rest of the problem

```
b = Mean[{bmin, bmax}]
```

```
2.46422 × 10^7
```

Part(e)

Given the impact parameter from part (d), calculate the turning angle of the spacecraft relative to Jupiter during the flyby.

```
eq1 = (rb0) (vb0) == (b) (VinRelative);
rb0 = (b) (VinRelative) / vb0
```

```
7.53331 × 10^7 / vb0
```

setup the energy equation at Jupitor

```
eq2 = vb0^2/2 - μ_jupitor/rb0 == VinRelative^2/2 - μ_jupitor/jupitor_SOI
```

```
-1.68168 vb0 + vb0^2/2 == 2.04995
```

Solve for V_{b0}

```
sol = vb0 /. NSolve[eq2, vb0]
```

```
{-0.950417, 4.31379}
```

```
vb0 = First@Select[%, Positive]
```

```
4.31379
```

check the correspoding r_{b0}

```
rb0
```

1.74634×10^7

Find e at jupitor and find η and θ

$$e = \sqrt{1 + \frac{(\text{VinRelative})^2 (\text{vb0})^2 (\text{rb0})^2}{\mu_{\text{jupitor}}^2}}$$

```
2.07476
```

$$\eta = \text{ArcCos}\left[-\frac{1}{e}\right];$$

$$\text{Row}\left[\left\{\text{"}\eta\text{ Degree = "}, \eta * \frac{180}{\pi}\right\}\right]$$

```
η Degree = 118.815
```

$$\theta = 2\eta - \text{Pi};$$

$$\text{Row}\left[\left\{\text{"}\theta\text{ Degree = "}, \theta * \frac{180}{\pi}\right\}\right]$$

```
θ Degree = 57.63
```

Part(f)

What is the spacecraft's heliocentric speed following the flyby?

$$\text{vd} = \sqrt{V_{\text{jupitor}}^2 + \text{VinRelative}^2 - 2\, V_{\text{jupitor}}\, \text{VinRelative}\, \text{Cos}[\theta]}$$

```
11.7086
```

Part (g)

What is the spacecraft's heliocentric flight path angle following the flyby

```
γd = ArcSin[VinRelative Sin[θ] / vd];
Row["γd in degree ", γd 180 / Pi]
```

```
Row[γd in degree , 12.7398]
```

For the remaining parts, assume that, relative to the Sun, the spacecraft DOES NOT arrive at Jupiter's SOI moving in the same direction at Jupiter. The spacecraft still has a heliocentric speed of 10 km/s at the distance of Jupiter's orbit from the Sun. But now it has a heliocentric eccentricity of 0.5. (What was the heliocentric eccentricity when the spacecraft arrived in the same direction as Jupiter, assuming that point was aphelion?)

Part(h)

What is the spacecraft's heliocentric flight path angle when it arrives at Jupiter's SOI?

```
Clear[a];
e = 0.5;
eq = Vin == Sqrt[μsun (2/Rjupiter - 1/a)]
```

$$10 == 364280.\sqrt{2.56951\times 10^{-9}-\frac{1}{a}}$$

```
a = a /. First@NSolve[eq, a]
```

5.50681×10^{8}

```
γ = ArcCos[Sqrt[a^2 (1 - e^2) / (Rjupiter (2 a - Rjupiter))]];
Row[{"angle is ", γ 180 / Pi, " degree"}]
```

```
angle is 17.9875 degree
```

Part(i)

What is the spacecraft's speed relative to Jupiter

```
VinRelative = Sqrt[Vjupiter^2 + Vin^2 - 2 Vjupiter Vin Cos[γ]]
```

$$\text{VinRelative} = \sqrt{V_{jupiter}^2 + Vin^2 - 2\, V_{jupiter}\, Vin\, \text{Cos}[\gamma]}$$

```
4.70206
```

part(j)

Using the same impact parameter as in part (d), calculate the turning angle of the spacecraft relative to Jupiter.

```
Clear[vb0];
eq1 = rb0 vb0 == b VinRelative;
rb0 = (b VinRelative)/vb0
```

$$\frac{1.15869\times10^{8}}{vb0}$$

setup the energy equation at Jupiter

```
Clear[vb0];
```

$$eq2 = \frac{vb0^2}{2} - \frac{\mu_{jupiter}}{rb0} == \frac{\text{VinRelative}^2}{2} - \frac{\mu_{jupiter}}{jupiter_{SOI}}$$

$$-1.09336\, vb0 + \frac{vb0^2}{2} == 8.43177$$

Solve for V_{b0}

```
sol = vb0 /. NSolve[eq2, vb0]
```

```
{-3.15623, 5.34294}
```

```
vb0 = First@Select[%, Positive]
```

```
5.34294
```

check the correspoding r_{b0}

```
rb0
```

$$2.16864\times10^{7}$$

Find *e* at jupitor and find η and θ

```
e = Sqrt[1 + ((VinRelative)^2 (vb0)^2 (rb0)^2)/(μjupitor^2)]
```

$$e = \sqrt{1 + \frac{(\text{VinRelative})^2 (\text{vb0})^2 (\text{rb0})^2}{\mu_{\text{jupitor}}^2}}$$

```
4.4153
```

```
η = ArcCos[-1/e];
Row[{"η Degree = ", η * 180/π}]
```

```
η Degree = 103.09
```

```
θ = 2 η - Pi;
Row[{"θ Degree = ", θ * 180/π}]
```

```
θ Degree = 26.1805
```

Part(k)

Assuming that the spacecraft flies behind Jupiter, what is the spacecraft's heliocentric speed following the flyby?

V_{jupitor}

```
13.0571
```

```
VinRelative
```

```
4.70206
```

```
Vin
```

```
10
```

$$\beta = \text{ArcSin}\left[\frac{\text{Vin}\,\text{Sin}[\gamma]}{\text{VinRelative}}\right]$$

```
0.716508
```

```
vd = Sqrt[Vjupitor^2 + VinRelative^2 - 2 Vjupitor VinRelative Cos[β + θ]]
```

$$vd = \sqrt{V_{jupitor}{}^2 + VinRelative^2 - 2\, V_{jupitor}\, VinRelative\, Cos[\beta + \theta]}$$

```
12.0449
```

Part(L)

Assuming that the spacecraft flies behind Jupiter, what is the spacecraft's heliocentric flight path angle following the flyby?

$$\gamma_d = ArcSin\left[\frac{VinRelative\ Sin[\beta + \theta]}{vd}\right];$$

```
Row["γd in degree ", γd 180/Pi]
```

```
Row[γd in degree , 21.0979]
```


6.6 HW6

```
period = 120 * 60
ω = 2 π / period
```

```
7200
```

$\frac{\pi}{3600}$

```
Clear[h]
mu = 324 859;
eq = h ω == Sqrt[mu / h]
```

$\frac{h\,\pi}{3600} == \sqrt{324\,859}\ \sqrt{\frac{1}{h}}$

```
h = h /. First@NSolve[eq, h]
```

```
7527.776558
```

```
radius = 6052;
alt = h - radius
```

```
1475.776558
```

```
ra = h
```

```
7527.776558
```

part(b)

Calculate ΔV Mal required to start the maneuver (magnitude and sign).

```
leadAngle = 500 / h
```

```
0.06642067497
```

```
% * 180 / Pi
```

```
3.805624348
```

```
orbitTotalCircumference = 2 Pi h
```

47 298.41507

```
n = 1
```

1

```
timeRequired1 = (n - leadAngle/(2 Pi)) period
```

7123.887513

```
timeRequiredInMinutes = timeRequired1 / 60
```

118.7314586

```
eq = timeRequired1 == 2 Pi Sqrt[a^3/mu]
```

$$7123.887513 == \frac{2\sqrt{a^3}\,\pi}{\sqrt{324\,859}}$$

```
a = a /. First@NSolve[eq, a]
```

7474.630999

```
eq = a == (ra + rp)/2
```

$$7474.630999 == \frac{1}{2}(7527.776558 + rp)$$

```
Clear[rp]
rp = rp /. First@NSolve[eq, rp]
```

7421.48544

part(c)

```
lowestAlt = rp - radius
```

1369.48544

```
speedOnEllipse = Sqrt[mu (2/ra - 1/a)]
```

$$\text{speedOnEllipse} = \sqrt{\text{mu}\left(\frac{2}{\text{ra}} - \frac{1}{\text{a}}\right)}$$

6.545828652

$$\left(\text{speedOnCircle} = \sqrt{\frac{\text{mu}}{\text{ra}}}\right) \text{ // N}$$

6.569224315

```
delV = speedOnEllipse - speedOnCircle
```

-0.0233956631

```
delVTotal = 2 * delV
```

-0.0467913262

part(d)

```
n = 1;
```

$$\left(\text{n} + \frac{\text{leadAngle}}{\text{2 Pi}}\right)$$

$$\text{timeRequired2} = \left(\text{n} + \frac{\text{leadAngle}}{\text{2 Pi}}\right) \text{period}$$

1.010571179

7276.112487

```
Clear[a]
```

$$\text{eq} = \text{timeRequired2} == 2\ \text{Pi} \sqrt{\frac{\text{a\^{}3}}{\text{mu}}}$$

$$7276.112487 == \frac{2\sqrt{a^3}\,\pi}{\sqrt{324\,859}}$$

```
NSolve[eq, a]
```

{{a → -3790.367586 + 6565.109239 ⅈ}, {a → -3790.367586 - 6565.109239 ⅈ}, {a → 7580.735173}}

```
a = 7580.73517267325`
```

```
7580.735173
```

```
Clear[ra, rp];
rp = h;
eq = a == (ra + rp)/2
```

$$7580.735173 == \frac{1}{2}(7527.776558 + ra)$$

```
ra = ra /. First@NSolve[%, ra]
```

```
7633.693787
```

part(e)

```
largestAlt = ra - radius
```

```
1581.693787
```

$$speedOnEllipse = \sqrt{mu\left(\frac{2}{rp} - \frac{1}{a}\right)}$$

```
6.592130506
```

$$\left(speedOnCircle = \sqrt{\frac{mu}{h}}\right) \text{ // N}$$

```
6.569224315
```

```
delV = speedOnEllipse - speedOnCircle
```

```
0.02290619083
```

part(f)

```
(timeRequired1 - timeRequired2) / 60
```

```
-2.537082899
```

problem 2, new method

A spacecraft is in a circular orbit about the Earth with a radius of 6678 km. Its mission is to rendezvous with the International Space Station (ISS), which is in a circular orbit with a radius of 6878 km. Both orbits are in the same plane with the same direction of motion. At t = 0, the ISS leads the spacecraft by an angle θ. Calculate the required transfer time, including the wait time, for the spacecraft to accomplish the rendezvous with a Hohmann transfer if

(a) θ = 0°

35.2348 ✓ hours

(b) θ = 280°

27.492 ✓ hours

part(a)

```
r1 = 6678;
r2 = 6878;
mu = 3.986 * 10^5;
```

```
a = (r1 + r2)/2
```

```
6778
```

```
angularVelocityInLowerOrbit = Sqrt[mu/r1^3]
```

```
0.001156908535
```

```
angularVelocityInUpperOrbit = Sqrt[mu/r2^3]
```

```
0.001106815901
```

```
angularVelocityInUpperOrbit - angularVelocityInLowerOrbit
```

```
-0.0003147412999
```

```
initialAngle = 0;
hohmannAngle = Pi (1 - ((r1 + r2)/(2 r2))^(3/2));
If[initialAngle <= hohmannAngle, initialAngle = initialAngle + 2 Pi];
TOF = Pi Sqrt[a^3/mu]
```

```
2776.729487
```

```
waitTimeToSync = (initialAngle - hohmannAngle) / (angularVelocityInLowerOrbit - angularVelocityInUpperOrbit)

124 068.5615
```

```
waitTime = waitTimeToSync + TOF

126 845.291
```

```
waitTime / (60 * 60)

35.23480306
```

27.492 correct for second

part(b)

if initialAngle=280 degree

```
r1 = 6678;
r2 = 6878;
mu = 3.986 * 10^5;
a = (r1 + r2) / 2

6778
```

```
angularVelocityInLowerOrbit = Sqrt[mu / r1^3]

0.001156908535
```

```
angularVelocityInUpperOrbit = Sqrt[mu / r2^3]

0.001106815901
```

```
angularVelocityInUpperOrbit - angularVelocityInLowerOrbit

-0.0003147412999
```

```
initialAngle = 280 * Pi / 180;
hohmannAngle = Pi (1 - ((r1 + r2)/(2 r2))^(3/2));
If[initialAngle <= hohmannAngle, initialAngle = initialAngle + 2 Pi];
TOF = Pi Sqrt[a^3/mu]
```

```
2776.729487
```

```
waitTimeToSync = (initialAngle - hohmannAngle)/(angularVelocityInLowerOrbit - angularVelocityInUpperOrbit)
```

```
96194.93424
```

```
waitTime = waitTimeToSync + TOF
```

```
98971.66373
```

```
waitTime / (60 * 60)
```

```
27.49212881
```

Problem 3

For the same circular near-Earth orbits as the previous question (r_1 = 6678 km, r_2 = 6878 km), calculate the bielliptic transfer time for the two lead angles below (θ = angle by which the target (ISS) leads the spacecraft at time t = 0). Add additonal orbits to the target satellite as necessary to ensure that the intermediate transfer radius of the active satellite does not fall below 6578 km.

(a) θ = 0°

1.577 ✓ hours

(b) θ = 160°

2.453 ✓ hours

part(a) theta=0

```
r1 = 6678;
r2 = 6878;
rmin = 6578;
mu = 3.986 * 10^5;
a = (r1 + r2)/2
```

```
6778
```

```
angularVelocityInLowerOrbit = Sqrt[mu/r1^3]
```

```
0.001156908535
```

```
angularVelocityInUpperOrbit = Sqrt[mu/r2^3]
```

```
0.001106815901
```

```
initialAngle = 0
hohmannAngle = Pi (1 - ((r1 + r2)/(2 r2))^(3/2));
Clear[rt];
a1 = (r1 + rt)/2;
a2 = (r2 + rt)/2;
t1 = Pi (Sqrt[a1^3/mu] + Sqrt[a2^3/mu]);
t2 = ((2 Pi - initialAngle) + 2 Pi n)/angularVelocityInUpperOrbit;
eq = t1 == t2;
n = 0;
NSolve[eq, rt]
```

```
0
```

```
{{rt → -13 655.90913 + 11 913.20284 i},
 {rt → -13 655.90913 - 11 913.20284 i}, {rt → 6977.818259}}
```

```
t1 /. rt -> 6977.818258721281`
```

```
5676.811563
```

```
waitTime = % / (60 * 60)
```

```
1.576892101
```

part(b) theta=160

```
r1 = 6678;
r2 = 6878;
rmin = 6578;
mu = 3.986 * 10^5;
a = (r1 + r2)/2
```

```
6778
```

```
angularVelocityInLowerOrbit = Sqrt[mu/r1^3]
```

```
0.001156908535
```

```
angularVelocityInUpperOrbit = Sqrt[mu/r2^3]
```

```
0.001106815901
```

```
(initialAngle = 160 * Pi / 180) // N
(hohmannAngle = Pi (1 - ((r1 + r2)/(2 r2))^(3/2))) // N
```

```
2.792526803
```

```
0.06826430301
```

```
Clear[rt, n];
a1 = (r1 + rt)/2;
a2 = (r2 + rt)/2;
t1 = Pi (Sqrt[a1^3/mu] + Sqrt[a2^3/mu]);
t2 = ((2 Pi - initialAngle) + 2 Pi n)/angularVelocityInUpperOrbit;
eq = t1 == t2;
n = 1;
NSolve[eq, rt]
```

```
{{rt → -16 011.84696 + 15 993.72655 i},
 {rt → -16 011.84696 - 15 993.72655 i}, {rt → 11 689.69391}}
```

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```
t1 /. rt -> 11689.693913121537`
```

```
8830.595764
```

```
waitTime = % / (60 * 60)
```

```
2.452943268
```

function to solve hohman transfer on same orbit

```
hohmannRendezvousSameOrbit[θ00_, r_, alt_, mu_] :=
 Module[{θ0 = θ00 * Pi / 180, n = 1, delT, v1,
   v2, period, a, rp, ra, done = False, vBefore, vAfter},
  ra = r + alt;
  period = 2 Pi Sqrt[ra^3 / mu];
  While[Not[done],
   delT = (n - θ0/(2 Pi)) period ;
   a = First@Select[a /. NSolve[delT == 2 Pi Sqrt[a^3 / mu], a], Element[#, Reals] &];
   rp = 2 a - ra;
   If[rp < r, (*we hit the earth, try again*)
    n = n + 1,
    done = True
   ]
  ];
  vBefore = Sqrt[mu / h];
  vAfter = Sqrt[mu (2 / h - 1 / a)];

  {delT, 2 (vAfter - vBefore)}

 ]
```

```
mu = 324 859;
alt = 1475.7765582577413`;
r = 6052;
θ0 = 3.80562; (*degree*)

hohmannRendezvousSameOrbit[θ0, r, alt, mu]
```

```
{7123.8876, -0.04679127217}
```

```
? Select
```

Select[*list*, *crit*] picks out all elements e_i of *list* for which *crit*[e_i] is True.
Select[*list*, *crit*, *n*] picks out the first *n* elements for which *crit*[e_i] is True. >>

6.6.2 HW6 in Maple

```
> hohmann_rendezvous_2:= proc({
      theta::numeric:=0,
      r1::numeric:=0,
      r2::numeric:=0,
      N::nonnegint:=0,
      mu::numeric:=3.986*10^5})

      local theta0,thetaH,TOF;
      theta0   := theta*Pi/180;
      thetaH   := Pi*(1-((r1+r2)/(2*r2))^(3/2));

      if is(theta0 = thetaH) and N = 0 then
         proc()
           local a:=(r1+r2)/2;
           TOF    := Pi*(sqrt(a^3/mu));
         end proc()
      else
         proc()
           local t2,a1,a2,rt,omega2;
              omega2 := sqrt(mu/r2^3);
              t2     := ((2*Pi-theta0)+2*Pi*N)/omega2;
              a1     := (rt+r1)/2;
              a2     := (rt+r2)/2;
              TOF    := Pi*(sqrt(a1^3/mu)+sqrt(a2^3/mu));
              rt     := op(select(is, [solve(t2=TOF,rt)], real));
         end proc()
      fi;

      eval(TOF);
  end proc:
> %stopat(hohmann_rendezvous_2);
  TOF:=hohmann_rendezvous_2(theta=0,r1=6678,r2=6878,N=0):
  evalf(TOF/(60*60)); #in hrs
```

%stopat(hohmann_rendezvous_2)

1.576892101

```
> TOF:=hohmann_rendezvous_2(theta=160,r1=6678,r2=6878,N=1):
  evalf(TOF/(60*60));  #in hrs
```

2.452943266

```
> hohmann_rendezvous_1:= proc({
      theta::numeric:=0,
      r1::numeric:=0,
      r2::numeric:=0,
      mu::numeric:=3.986*10^5})

      local theta0,thetaH,TOF,a,omega1,omega2,wait_time;
      theta0   := theta*Pi/180;
      a        := (r1+r2)/2;
      TOF      := Pi*(sqrt(a^3/mu));
      omega1   := sqrt(mu/r1^3);
      omega2   := sqrt(mu/r2^3);
      thetaH   := Pi*(1-((r1+r2)/(2*r2))^(3/2));
```

```
        if is(theta0 <= thetaH) then
           theta0 := theta0+2*Pi;
        fi;
        wait_time := TOF+(theta0-thetaH)/(omega1-omega2);
        eval(wait_time);
     end proc:
> TOF:=hohmann_rendezvous_1(r1=6678,r2=6878,theta=0):
  evalf(TOF/(60*60));
                                        35.23480353
> TOF:=hohmann_rendezvous_1(r1=6678,r2=6878,theta=280):
  evalf(TOF/(60*60));
                                        27.49212918
> walking_rendezvous_1:= proc({
      theta::numeric:=0,
      alt::numeric:=0,
      r   ::numeric:=6378,
      N   ::posint:=1,
      mu::numeric:=3.986*10^5})

      local   TOF,a,T,theta0,time_on_ellipse,Va,Vcir;
      T        := 2*Pi*sqrt((r+alt)^3/mu);
      theta0   := theta*Pi/180;

      TOF      :=(N- theta0/(2*Pi))*T;
      time_on_ellipse := 2*Pi*sqrt(a^3/mu);
      a        := op(select(is, [solve(time_on_ellipse=TOF,a)],real))
  ;
      Va    := sqrt(mu*(2/(r+alt) - 1/a));
      Vcir  := sqrt(mu/(r+alt));
      {TOF,2*(Va-Vcir)};
  end proc:
> res:=walking_rendezvous_1(theta=evalf(500/(7527.78)*180/Pi),alt=
  1475.78,r=6052,mu=324859):
> res[1];
                                        -0.04679130
```

6.7 HW7

6.7.1 HW7 in Mathematica

HW7 EMA 550, Spring 2014

by Nasser M. Abbasi

problem 1

Use Lambert's method to find the elliptical orbit that connects a starting point in a circular, equatorial LEO (r_1 = 6678 km) and a target point in a geostationary orbit (circular, equatorial, r_2 = 42,164 km). The allowed transfer time is 6 hours and the target point r_2 at the end of the transfer is 210 degrees ahead of point r_1's location at the beginning of the transfer.

(a) Calculate the semimajor axis for the transfer orbit.

Semimajor axis a = 24493.4 ✓ km

(b) Calculate the eccentricity of the transfer orbit.

Eccentricity e = 0.738 ✓

```
s = (r1 + r2 + c) / 2
```

```
48452.71961
```

```
tp = Sqrt[2] / 3 * (s^(3 / 2) - Sign[Sin[theta]] * (s - c)^(3 / 2)) / Sqrt[mu];
tp / (60 * 60 * 24)
tp / (60 * 60)
```

```
0.09223616371
```

```
2.213667929
```

```
alpha0 = 2 * ArcSin[Sqrt[s / (2 * a)]]
```

$2\,\mathrm{ArcSin}\left[155.6481924\sqrt{\frac{1}{a}}\right]$

```
beta0 = 2 * ArcSin[Sqrt[(s - c) / (2 * a)]]
```

$2\,\mathrm{ArcSin}\left[13.95135093\sqrt{\frac{1}{a}}\right]$

```
eq = Sqrt[mu] * 6 * 60 * 60 ==
    a^(3 / 2) * ((2 Pi - alpha0) - beta0 - (Sin[(2 Pi - alpha0)] - Sin[beta0]));
```

```
(amin = s / 2) // N
```

```
timeTravel[a_, flag_] := a^(3 / 2) * (If[flag, alpha0, 2 Pi - alpha0] -
      beta0 - (Sin[If[flag, alpha0, 2 Pi - alpha0]] - Sin[beta0])) / Sqrt[mu];
data1 = Table[{a, timeTravel[a, True] / (60 * 60)}, {a, amin, 1.2 * amin, 0.1 amin}];
data2 = Table[{a, timeTravel[a, False] / (60 * 60)}, {a, amin, 1.2 * amin, 0.1 amin}];
```

```
48 063.43923
```

```
48 452.71961
```

```
7969.204545
```

```
2.213667929
```

$$2\,\text{ArcSin}\left[155.6481924\,\sqrt{\frac{1}{a}}\right]$$

$$2\,\text{ArcSin}\left[13.95135093\,\sqrt{\frac{1}{a}}\right]$$

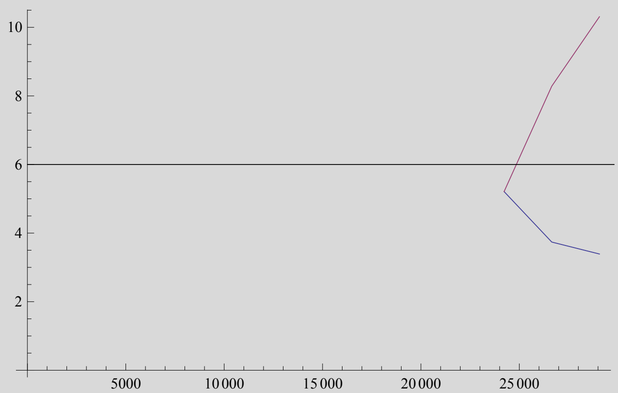
```
24 226.35981
```

```
amin // N
```

```
24 226.35981
```

$$\left\{a \to 24\,493.38502 + 8.158718813\times 10^{-13}\,i\right\}$$

```
p1 = ListLinePlot[{data1, data2}, AxesOrigin → {0, 0}];
p2 = Graphics@Line[{{0, 6}, {60 000, 6}}];
Show[p1, p2]
```




```
aFound = Re[a /. FindRoot[eq, {a, 1.1 amin}, MaxIterations → Infinity, PrecisionGoal → 5]]
```

```
24 493.38502
```

find p

```
p = (4 aFound (s - r1) (s - r2))/(c^2) Sin[((2 Pi - alpha0 /. a → aFound) + beta0 /. a → aFound)/2]
NSolve[p == aFound (1 - e^2), e]
```

```
10 933.06754
```

```
{{e → -0.7440643966}, {e → 0.7440643966}}
```

find true anaomolies

```
Clear[f2, f1];
e = 0.73838
aFound =24 491
r1
r2
(f1 /. NSolve[r1 == aFound (1 - e^2) / (1 + e Cos[f1]), f1]) * 180 / Pi
(f2 /. NSolve[r2 == aFound (1 - e^2) / (1 + e Cos[f2]), f2]) * 180 / Pi
```

```
0.73838
```

```
24 491
```

```
6678.
```

```
42 164.
```

```
{-25.23353146, 25.23353146}
```

```
{-175.2386421, 175.2386421}
```

```
Clear[f1];
(f1 /. NSolve[r1 == aFound (1 - e^2) / (1 + Cos[f1]), f1]) * 180 / Pi
```

```
{-48.09310707, 48.09310707}
```

```
Clear[f1, f2];
eq = Cos[f1] == aFound (1 - e^2) / (r1 * e) - 1 / e
(f1 /. Solve[eq, f1]) * 180 / Pi
```

```
Cos[f1] == 0.8563464842
```

Solve::ifun : Inverse functions are being used by Solve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
{-31.09119158, 31.09119158}
```

```
eq = Cos[f2] == aFound (1 - e^2) / (r2 * e) - 1 / e
(f2 /. Solve[eq, f2]) * 180 / Pi
```

```
Cos[f2] == -0.9954801908
```

Solve::ifun : Inverse functions are being used by Solve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
{-174.5504406, 174.5504406}
```

```
360 - 210
```

```
150
```

```
210 + 175
```

```
35
```

```
r
```

```
r
```

```
aFound
EE1 = First@Select[EE1 /. NSolve[r1 == aFound (1 - e Cos[EE1]), EE1], Positive]
EE2 = First@Select[EE2 /. NSolve[r2 == aFound (1 - e Cos[EE2]), EE2], Positive]
```

```
24 493.38502
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
0.212327763
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
2.894384602
```

```
(f1 = ArcCos[(e - Cos[EE1])/(e Cos[EE1] - 1)]) * 180 / Pi
(f2 = ArcCos[(e - Cos[EE2])/(e Cos[EE2] - 1)]) * 180 / Pi
```

```
31.09119158
```

```
174.5504406
```

```
(f1 = 2 * ArcTan[Sqrt[(1 + e)/(1 - e)] Tan[EE1/2]]) * 180 / Pi
```

```
31.09119158
```

```
0.21232776299399742` * 180 / Pi
```

```
12.16548469
```

```
NSolve[r2 == aFound (1 - e Cos[true2]), true2]
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
{{true2 → -2.894384602}, {true2 → 2.894384602}}
```

```
2.894384601664499` * 180 / Pi
```

```
165.836022
```

Semimajor axis a = Answer km

(b) Calculate the eccentricity of the transfer orbit.

Eccentricity e = Answer

(c) Knowing the true anomalies of the burn points allows you to draw the transfer orbit between the two points in the correct orientation. Calculate the true anomaly of the initial burn point on the transfer orbit.

True anomaly f1 = Answer degrees

(d) Calculate the true anomaly of the final burn point on the transfer orbit.

True anomaly f2 = Answer degrees

probem 2

The space shuttle is initially in a 28.5° inclination orbit. It changes to a 40° inclination orbit using a simple single-impulse plane change. If the transfer occurs at a latitude of 25°,

(a) By what angle should the velocity vector be rotated at the impulse point?

θ = 18.1547 ✓ degrees

(b) What is the resulting change in the right ascension of the ascending node?

ΔΩ = 25.425 ✓ degrees

Check

Correct
Marks for this submission: 10.00/10.00.

find Az

```
i1 = 28.5 Degree;
latitude = 25 Degree;
Clear[az]
eq = Cos[i1] == Sin[az] Cos[latitude]
az = az /. First@NSolve[eq, az]
```

```
0.8788171127 == Cos[25 °] Sin[az]
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
1.323866377
```

```
az * 180 / Pi
```

```
75.85195602
```

find u

```
Clear[u];
i2 = 40 Degree;
eq = 0 == -Cos[az] Cos[i1] + Sin[az] Sin[i1] Cos[u]
```

```
0 == -0.2148076749 + 0.462685293 Cos[u]
```

```
u /. NSolve[eq, u]
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
{-1.087993979, 1.087993979}
```

```
u = 1.0879939787936352`
```

```
1.087993979
```

```
u * 180 / Pi
```

```
62.33746312
```

```
Clear[theta]
eq = Cos[Pi - i2] == -Cos[i1] Cos[theta] + Sin[i1] Sin[theta] Cos[u]
```

```
-Cos[40 °] == -0.8788171127 Cos[theta] + 0.2215271706 Sin[theta]
```

```
NSolve[eq, theta]
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
{{theta → -0.810718662}, {theta → 0.3168587618}}
```

```
theta = 0.3168587617678044`;
theta * 180 / Pi
```

```
18.15466975
```

(b) What is the resulting change in the right ascension of the ascending node?

$\Delta\Omega$ = Answer degrees

```
i2 = 40 Degree;
Clear[delta]
eq = Cos[theta] == Cos[i1] Cos[i2] + Sin[i1] Sin[i2] Cos[delta]
```

```
0.9502188617 == 0.6732129657 + 0.3067117389 Cos[delta]
```

```
NSolve[eq, delta]
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
{{delta → -0.4437515757}, {delta → 0.4437515757}}
```

```
0.44375157574265334` * 180 / Pi
```

```
25.42509244
```

problem 3

A spacecraft starts in an intial circular orbit about the Earth that has a radius of 7000 km and an inclination of 30°. The desired orbit for the spacecraft is a circular orbit with a radius of 130,000 km and an inclination of 0° (equatorial). Calculate and compare the total ΔV for the five orbit transfer options below, all of which involve a Hohmann transfer. Assume that all of the burns take place when the spacecraft is crossing the equator.

(a) A simple plane change followed by a Hohmann transfer (3 impulses)

ΔV_{tot} = 7.94687 ✓ km/s

(b) A Hohmann transfer followed by a simple plane change (3 impulses)

ΔV_{tot} = 4.94716 ✓ km/s

(c) A Hohmann transfer that includes the plane change with the first impulse (2 impulses)

ΔV_{tot} = 6.589 ✓ km/s

(d) A Hohmann transfer that includes the plane change with the last impulse (2 impulses)

ΔV_{tot} = 4.146 ✓ km/s

(e) A Hohmann transfer with optimally split plane change (2 impulses)

ΔV_{tot} = 4.143 ✓ km/s

Check

(a) A simple plane change followed by a Hohmann transfer (3 impulses)

```
mu = 3.986 * 10^5
r1 = 7000;
r2 = 130 000;
i1 = 30 Degree;
v1 = Sqrt[mu / r1]
delV1 = 2 v1 Sin[i1 / 2]
```

```
398 600.
```

```
7.546049108
```

```
3.906122449
```

```
a = (r1 + r2) / 2;
vp = Sqrt[mu (2 / r1 - 1 / a)];
delV2 = vp - v1
```

```
2.849466088
```

```
va = Sqrt[mu (2 / r2 - 1 / a)];
v4 = Sqrt[mu / r2];
delV3 = v4 - va
```

```
1.191285134
```

```
total = delV1 + delV2 + delV3
```

```
7.946873671
```

tot=

(b) A Hohmann transfer followed by a simple plane change (3 impulses)

```
mu = 3.986 * 10^5;
r1 = 7000;
r2 = 130 000;
i1 = 30 Degree;
v1 = Sqrt[mu / r1];
a = (r1 + r2) / 2;
vp = Sqrt[mu (2 / r1 - 1 / a)];
delV1 = vp - v1;
va = Sqrt[mu (2 / r2 - 1 / a)];
v4 = Sqrt[mu / r2];
delV2 = v4 - va;
delV3 = 2 v4 Sin[i1 / 2];
total = delV1 + delV2 + delV3
```

```
4.94715811
```

(c) A Hohmann transfer that includes the plane change with the first impulse (2 impulses)

```
vp = Sqrt[mu (2 / r1 - 1 / a)];
delV1 = Sqrt[vp^2 + v1^2 - 2 vp v1 Cos[i1]]
va = Sqrt[mu (2 / r2 - 1 / a)];
v4 = Sqrt[mu / r2];
delV2 = v4 - va;
total = delV1 + delV2
```

```
5.398032016
```

```
6.589317151
```

(d) A Hohmann transfer that includes the plane change with the last impulse (2 impulses)

```
vp = Sqrt[mu (2 / r1 - 1 / a)];
delV1 = vp - v1
delV2 = Sqrt[va^2 + v4^2 - 2 va v4 Cos[i1]]
total = delV1 + delV2
```

```
2.849466088
```

```
1.296839919
```

```
4.146306007
```

ΔVtot = Answer km/s

(e) A Hohmann transfer with optimally split plane change (2 impulses)

```
mu = 3.986 * 10^5;
r1 = 7000;
r2 = 130 000;
θ = 30 Degree;
vc1 = Sqrt[mu / r1];
a = (r1 + r2) / 2;
vp = Sqrt[mu (2 / r1 - 1 / a)];
va = Sqrt[mu (2 / r2 - 1 / a)];
vc2 = Sqrt[mu / r2];
eq = (vp vc1 Sin[alpha1]) / (Sqrt[vp^2 + vc1^2 - 2 vc1 vp Cos[alpha1]]) ==
  (va vc2 Sin[θ - alpha1]) / (Sqrt[va^2 + vc2^2 - 2 vc2 va Cos[θ - alpha1]])
NSolve[
 eq,
 alpha1]
```

$$\frac{78.44506817\,\mathrm{Sin}[\mathrm{alpha1}]}{\sqrt{165.0095933 - 156.8901363\,\mathrm{Cos}[\mathrm{alpha1}]}} == -\frac{0.9801615825\,\mathrm{Sin}[\mathrm{alpha1} - 30\,°]}{\sqrt{3.379483436 - 1.960323165\,\mathrm{Cos}[\mathrm{alpha1} - 30\,°]}}$$

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
{{alpha1 → -3.096012837}, {alpha1 → 0.01347011678},
 {alpha1 → 0.5221111538 - 1.146893256 i}, {alpha1 → 0.5221111538 + 1.146893256 i}}
```

```
alpha1 = 0.01347011678252641
```

```
0.01347011678
```

```
delV1 = Sqrt[vp^2 + vc1^2 - 2 vc1 vp Cos[alpha1]]
```

```
2.85196251
```

```
delV2 = Sqrt[va^2 + vc2^2 - 2 vc2 va Cos[θ - alpha1]]
```

```
1.291799249
```

```
total = delV1 + delV2
```

```
4.143761759
```

ΔVtot = Answer km/s

6.7.2 HW7 in Maple

```
> restart;
  a:= 24493.4:
  e:= 0.738:
  r1:= 6678:
  r2:= 42164:
  eq:=r1=a*(1-e^2)/(1+e*cos(f1));
  sol:=solve(eq,f1);
  f1:=evalf(180/Pi*sol):
  sol:=solve(r2=a*(1-e^2)/(1+e*cos(f2)),f2):
  f2:=evalf(180/Pi*sol);
```

$$eq := 6678 = \frac{11153.21665}{1 + 0.738\cos(f1)}$$

$$sol := 0.4321838737$$

$$f2 := 175.2638667$$

```
> f2:=185:
  f1:='f1':
  solve(f1+210=f2,f1);
```

$$-25$$

6.7.2.1 Matlab code for problem 1

```
%script to solve HW7, problem 1
%EMA 550, by Nasser M. Abbasi
%Using Matlab 2013a

mu   = 3.986*10^5;
delT = 6*60*60;
r1   = 6678;
r2   = 42164;
theta= 210*pi/180;

c    = sqrt(r1^2+r2^2-2*r1*r2*cos((2*pi-theta)));
s    = (r1+r2+c)/2;

tp   = sqrt(2)/3*(s^(3/2)-sign(sin(theta)*(s-c)^(3/2)))/sqrt(mu);
fprintf('Tp = %f hrs\n',tp/(60*60));


alpha0 = @(a) 2*asin(sqrt(s/(2*a)));
beta0  = @(a) 2*asin(sqrt((s - c)/(2*a)));
eq     = @(a) sqrt(mu)*6*60*60 - a^(3/2)*((2*pi - alpha0(a)) - beta0(a) - ...
              (sin((2*pi - alpha0(a))) - sin(beta0(a))));
```

```
amin   = 24226.4;
a      = fzero(eq,1.2*amin);

fprintf('a = %f km',a);

p = (4*a*(s - r1)*(s - r2))/c^2 * sin(( (2*pi - alpha0(a)) + beta0(a))/2);

eq = @(e)  p - a*(1 - e^2);
e  = fsolve(eq,.5);

fprintf('e = %f km',e);

r=[r1,r2];
for i=1:2
    eq = @(f) r(i)-a*(1-e^2)/(1+e*cos(f))
    f  = fsolve(eq,pi/2);
    fprintf('f=%f\n',f*180/pi);
end

%problem 2

i1=28.5*pi/180;
theta1=61.5*pi/180;
eq=@(u) -cos(i1)*cos(theta1)+sin(i1)*sin(theta1)*cos(u)
u  = fsolve(eq,pi/2);
```

6.8 HW8

HW8 EMA 550 spring 2014

by Nasser M. Abbasi

A spacecraft starts in an intial circular orbit about the Earth that has a radius of 7000 km. The desired orbit for the spacecraft is a circular orbit with a radius of 130,000 km and an inclination of 0° (equatorial). The rocket engines that the spacecraft must use have fixed ΔVs of $\Delta V_1 = 4$ km/s and $\Delta V_2 = 1.5$ km/s.

(a) What plane changes must occur at the impulses so that the spacecraft can complete a two-impulse Hohmann transfer between the initial and desired orbits while using the fixed ΔV at each impulse?

$\alpha_1 =$ 18.237 ✓ degrees

$\alpha_2 =$ 54.818 ✓ degrees

(b) What initial inclinations would allow the Hohmann transfer to reach the desired orbit with these fixed impulse rocket engines? (Note that the burns must happen while crossing the equator for the spacecraft to end up on an equatorial orbit afterward.)

larger magnitude inclinations: i1 = ± 73.055 ✓ degrees

smaller magnitude inclinations: i1 = ± 36.581 ✓ degrees

Check

Correct
Marks for this submission: 10.00/10.00.

Answer

part(a)

```
r1 = 7000
r2 = 130 000
mu = 3.986 * 10^5;
v1 = Sqrt[mu / r1]
```

```
7000
```

```
130 000
```

```
7.546049108
```

```
a = (r1 + r2) / 2
vp = Sqrt[mu (2 / r1 - 1 / a)]
```

```
68 500
```

```
10.3955152
```

```
delV = vp - v1
```

```
2.849466088
```

```
haveV = 4;
excess = haveV - delV
```

```
1.150533912
```

```
eq = haveV^2 == v1^2 + vp^2 - 2 * v1 * vp * Cos[alpha];
NSolve[eq, alpha]
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. ≫

```
{{alpha → -0.3182952069}, {alpha → 0.3182952069}}
```

```
0.3182952068827841` * 180 / Pi
```

```
18.23697199
```

```
vh = vp = Sqrt[mu (2 / r2 - 1 / a)]
```

```
0.5597585105
```

```
v2 = Sqrt[mu / r2]
```

```
1.751043645
```

```
delV2 = v2 - vh
```

```
1.191285134
```

```
haveV2 = 1.5
eq = haveV2^2 == v2^2 + vh^2 - 2 * v2 * vh * Cos[alpha];
NSolve[eq, alpha]
```

```
1.5
```

NSolve::ifun : Inverse functions are being used by NSolve,
so some solutions may not be found; use Reduce for complete solution information. >>

```
{{alpha -> -0.9567588888}, {alpha -> 0.9567588888}}
```

```
0.9567588888438405` * 180 / Pi
```

```
54.81824634
```

part (b)

In[1]:=

```
a2 = 54.8182;
a1 = 18.237;
a1 + a2
```

Out[3]=

```
73.0552
```

```
a1 - a2
```

```
-36.5812
```

```
-a1 + a2
```

```
36.5812
```

```
-a1 - a2
```

```
-73.0552
```


6.9 HW9

6.9.1 Hint emailed to class from instructor

```
From:Suzannah Sandrik <sandrik@engr.wisc.edu>
Date:4/11/2014 11:45 AM
To:ema550-1-s14@lists.wisc.edu

I have a couple of suggestions on how to approach HW 9.

Debris avoidance is a little bit different from the examples we did in
lecture yesterday. There is a debris avoidance type of example in the
notes, so give that a read.

A good way to approach debris avoidance problems is to use the
satellite's original position as the target. Then the satellite moves
away from the target to avoid the debris, then back to the target so
that it has the position after the maneuver that it would have had if it
had never done the maneuver in the first place. Since the satellite
started on a circular orbit, keeping that same orbit as the target
reference also means that omega, the angular velocity of the target,
stays constant throughout the problem.

If you use that strategy, then the problem you are trying to solve is this:

(x0,y0) are (0,0). The satellite starts at the origin.
At time t1, being 10 km away means [x(t1)]^2 + [y(t1)]^2 = [10 km]^2.
What x0_dot and y0_dot are required for this to happen?
(And, since the problem specifies only an x-component delta-v, y0_dot is
zero.)

After performing delta-V #1, the satellite drifts away from its original
orbital position at the origin.

At time t1, the debris has passed and it's time to do a maneuver to return.

If the goal is to return to the origin at time t2, set x(t2) and y(t2)
equal to zero. Then what velocities x_dot and y_dot at time t1 are
required to accomplish reaching the origin at time t2? Compare those to
the velocities that the spacecraft already has at time t1 to find the
required second delta-V.

In the figure shown on the homework, the satellite starts and ends at
the origin, (0,0).

Hope that helps!

-- Dr. Suzannah Sandrik Department of Engineering Physics University of
Wisconsin-Madison 811 Engineering Research Building (608) 262-0764
```

6.9.2 my solution

HW9 EMA 550, Spring 2014

by Nasser M. Abbasi

HW9 EMA 550

by Nasser M. Abbasi

It is discovered that a piece of space debris will approach dangerously close to a GPS satellite (12 hour period, 55° inclination) in 4 hours. To avoid the debris, give the satellite an in-track (negative *x*-direction) ΔV such that 4 hours from now it is 10 km from the position it would have if it didn't perform the ΔV.

Note: This is a *relative motion* problem, so any distances or velocities that are asked for are positions and velocities measured in a rotating coordinate system attached to an orbiting point. In this rotating coordinate system, positive *x* is behind the reference position; positive *y* is above the reference position; and positive *z* is defined by the right-hand rule from *x* and *y*.

(a) Calculate the tangential ΔV that will allow the satellite to miss the debris by 10 km 4 hours from now. Include the correct sign for the ΔV as defined by the rotating coordinate system. Since maneuvering ΔV's are small, report your answer in m/s instead of km/s.

ΔV = -0.3533+0+0

i + 0 **j** + 0 **k** m/s

```
w = 2 Pi / (12 * 60 * 60);
ClearAll[xDot0];
yDot0 = 0; y0 = 0; x0 = 0;
x[t_, yDot0_, xDot0_, y0_, x0_] :=
  x0 + 2 yDot0 / w (1 - Cos[w t]) + (4 xDot0 / w - 6 y0) Sin[w t] + (6 w y0 - 3 xDot0) t;
y[t_, yDot0_, xDot0_, y0_, x0_] :=
  4 y0 - 2 xDot0 / w + (2 xDot0 / w - 3 y0) Cos[w t] + yDot0 / w Sin[w t];
xDot[t_, yDot0_, xDot0_, y0_, x0_] :=
  2 yDot0 Sin[w t] + (4 xDot0 - 6 w y0) * Cos[w t] + 6 w y0 - 3 xDot0;
yDot[t_, yDot0_, xDot0_, y0_, x0_] := (3 w y0 - 2 xDot0) * Sin[w t] + yDot0 Cos[w t];
t1 = 4 * 60 * 60;
eq = Sqrt[x[t1, yDot0, xDot0, y0, x0]^2 + y[t1, yDot0, xDot0, y0, x0]^2];
```

part(a)

```
xDot0 = xDot0 /. First@NSolve[eq == 10 * 1000, xDot0]
```

```
-0.3533025102
```

Printed by Wolfram Mathematica Student Edition

(b) What are the x and y coordinates of the satellite after 4 hours, measured in km from the position the satellite would have had without the ΔV?

x at 4 hours = 6.84792

km

y at 4 hours = 7.28739

km

part(b)

```
newx0 = x[t1, yDot0, xDot0, y0, x0]
```

```
6847.918308
```

```
newy0 = y[t1, yDot0, xDot0, y0, x0]
```

```
7287.387381
```

(c) What is the velocity of the satellite 4 hours after the ΔV, measured in m/s in the rotating coordinate system?

$\mathbf{V}_{\mathbf{t=4hrs}}$ = 1.76651

i + 0.611938 **j** + 0 **k** m/s

part(c)

```
newxDot0 = xDot[t1, yDot0, xDot0, y0, x0]
```

```
1.766512551
```

```
newyDot0 = yDot[t1, yDot0, xDot0, y0, x0]
```

```
0.6119378981
```

part(d)

(d) Now that the debris has safely passed, the satellite is to be returned to its original position in the GPS orbit using a two-impulse maneuver. What velocity does it need at t = 4 hours in order to return to its original orbital position 4 hours later (t = 8 hours after the initial burn)?

$V_{required}$ = 1.50348

i + -1.5231 **j** + 0 **k** m/s

```
eq1 = x[t1, requiredyDot0, requiredxDot0, newy0, newx0];
eq2 = y[t1, requiredyDot0, requiredxDot0, newy0, newx0];
sol = First@NSolve[{eq1 == 0, eq2 == 0}, {requiredyDot0, requiredxDot0}];
{requiredyDot0, requiredxDot0} = {requiredyDot0 /. sol, requiredxDot0 /. sol}
```

```
{-1.523097952, 1.5034833}
```

part(e)

(e) What is the magnitude of the ΔV that will change the velocity of the satellite from what it has (part c) to what is required (part d)?

ΔV = 2.15118

m/s

```
(*why this did not work?
   Vbefore=Sqrt[newxDot0^2+newyDot0^2]
   Vafter=Sqrt[requiredxDot0^2+requiredyDot0^2]
*)
delV = Sqrt[(newxDot0 - requiredxDot0)^2 + (newyDot0 - requiredyDot0)^2]
```

```
2.151176996
```

part(f)

(f) At t = 8 hours from the initial burn, the satellite is back at its original position. What velocity does it have when it gets there?

$V_{t=8hrs}$ = -0.616332

i + 0.91116 **j** + 0 **k** m/s

```
returnxDot = xDot[t1, requiredyDot0, requiredxDot0, newy0, newx0]
```

```
-0.6163317615
```

```
returnyDot = yDot[t1, requiredyDot0, requiredxDot0, newy0, newx0]
```

0.9111600542

part(g)

(g) What is the magnitude of the ΔV the satellite needs at t = 8 hours to zero out its relative velocity so that it stays in the correct orbital position?

ΔV = 1.10004

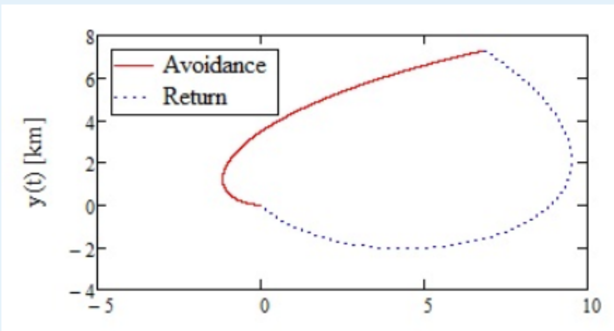
m/s

```
delV = Sqrt[returnxDot^2 + returnyDot^2]
```

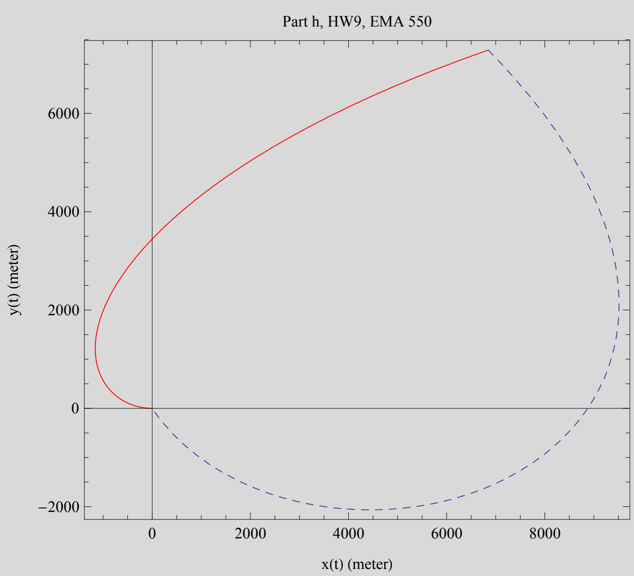
1.10003522

part(h) (plot)

(h) Plot the trajectory taken by the satellite in the *x*-*y* plane for both the avoidance phase and the return phase. Really, do it. It should look like the plot below, starting and ending at (*x*,*y*) = (0,0). If you cannot make a plot that looks like the one below, ask me for help.



```
p1 = ParametricPlot[{x[t, yDot0, xDot0, y0, x0], y[t, yDot0, xDot0, y0, x0]},
   {t, 0, 4 * 60 * 60}, PlotStyle → Red];
p2 = ParametricPlot[{x[t, requiredyDot0, requiredxDot0, newy0, newx0],
    y[t, requiredyDot0, requiredxDot0, newy0, newx0]},
   {t, 0, 4 * 60 * 60}, PlotStyle → Dashed];
Show[p1, p2, PlotRange → All, Frame → True, FrameLabel →
  {{"y(t)  (meter)", None}, {"x(t)  (meter)", "Part h, HW9, EMA 550"}}]
```



6.10 HW10

```
In[12]:= i * 180 / Pi

Out[12]= 116.5650512

In[6]:= h = 4793.490054264077`;

In[7]:= eq3 = e == ((hmax + rE) - (hmin + rE)) / ((hmax + rE) + (hmin + rE));
        eq4 = h == (hmax + hmin) / 2
        {hmin, hmax} = {hmin, hmax} /. First@NSolve[{eq3, eq4}, {hmin, hmax}]

Out[8]= 4793.490054 == (hmax + hmin)/2

Out[9]= {-233.6804702, 9820.660579}

In[10]:= rp = hmin + rE;
         Solve[rp == a (1 - e), a]

Out[11]= {{a → 11171.49005}}
```

6.10.2 second part

HW10 part 2, EMA 550, Spring 2014

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question 1

Question 1
Not complete
Marked out of 10.00
Flag question

A spacecraft wants to leave the Earth to visit one of the Trojan asteroids at the Sun-Jupiter L4 Lagrange point. The asteroid is too small for the satellite to see, so the satellite has to time its departure from the Earth based o Jupiter's location. Calculate the angle by which Jupiter must lead the Earth when the spacecraft departs the Earth's sphere of influence so that the spacecraft can rendezvous with the L4 asteroid via a heliocentric Hohmann transfer.

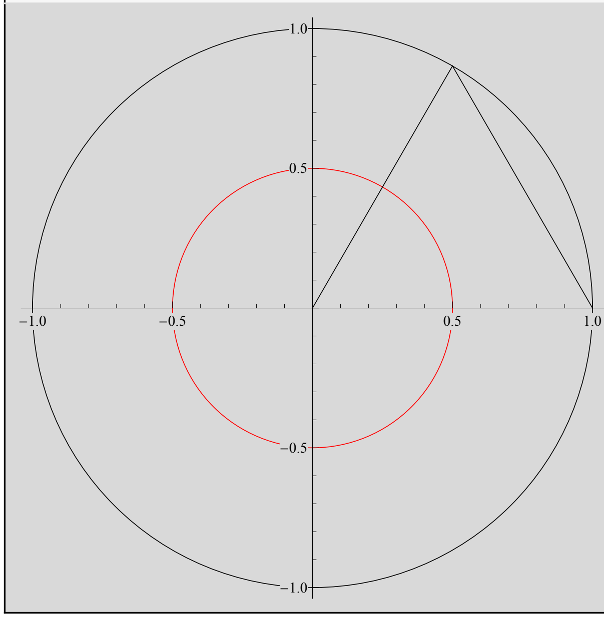
L4: Jupiter must lead the Earth by [ ] degrees when the spacecraft departs Earth's SOI.

How would your answer change if the goal was to rendezvous with a Trojan asteroid at the Sun-Jupiter L5 Lagrange point instead of the L4 point?

L5: Jupiter must lead the Earth by [ ] degrees when the spacecraft departs Earth's SOI.

Check

```
rJS = 1;
rES = 1 / 2;
Graphics[
 {
  {Circle[{0, 0}, rJS]},
  {Red, Circle[{0, 0}, rES]},
  {Line[{{0, 0},
     {rJS Cos[60 Degree], rJS Sin[60 Degree]},
     {rJS, 0}
    }]}
 }, Axes → True
]
```



L4 is 60 degree ahead of Jupiter all the time, and on the same circle Jupiter is on. Therefore, we find Hohmann angle with L4 by adding 60 degrees to Jupiter all the time.

```
AU = 1.495978 * 10^8;
rES = 1 AU;
rJS = 5.203 AU;
θH = Pi (1 - ((rES + rJS) / (2 rJS))^(3/2));
θH * 180 / Pi
```

```
97.15821569
```

```
θH - 60 Degree
```

```
0.6485332079
```

Part 1

```
% * 180 / Pi
```

```
37.15821569
```

Part 2

```
θH + 60 Degree
```

```
2.74292831
```

```
% * 180 / Pi
```

```
157.1582157
```

question 2

Question 2
Not complete
Marked out of 10.00
Flag question

A 2000 kg spacecraft in geostationary orbit (*r* = 42,241 km) turns on a low-thrust, continuous thruster. The thruster produces a thrust force of 0.25 N tangent to the spacecraft's velocity vector and has a specific impulse of 2000 s.

Assuming circular orbits throughout the transfer, as done in the Edelbaum article, determine the following:

(a) By how much does the semi-major axis change during the first revolution about the Earth after the thruster turns on?

Δa0 = ____ km

(b) By how much does the semi-major axis change during the next revolution about the Earth?

Δa1 = ____ km

(c) The spacecraft had a total of 100 kg of thruster fuel at the start of the maneuver. How many revolutions does it take (assuming the thruster is always on) to use up all of the fuel?

____ revolutions

(d) What is the final semi-major axis of the spacecraft?

a = ____ km

Check

part(a)

Find a

```
a = 42 241
```

```
42 241
```

Find period T0

```
uE = 3.986 * 10^5;
T0 = 2 Pi Sqrt[a^3/uE];
T0 / (60 * 60) (*hrs*)
```

$$T0 = 2\,Pi\,\sqrt{\frac{a^3}{uE}};$$

```
23.99993176
```

find mass burn rate

```
f = 0.25;(*N*)
Isp = 2000; (*sec*)
g = 9.81;
massBurnRate = (f / Isp)/g
```

$$massBurnRate = \frac{f / Isp}{g}$$

```
0.0000127420999
```

find change in mass (kg)

```
Δm = massBurnRate * T0
```

```
1.100914301
```

find initial speed

$$v0 = \sqrt{\frac{uE}{a}}$$

```
3.07186094
```

find change in V

```
mi = 2000 ;(*kg*)
Δv = g Isp Log[mi/(mi - Δm)]
```

$$\Delta v = g\,Isp\,Log\left[\frac{mi}{(mi - \Delta m)}\right]$$

```
10.80294284
```

Divide by 1000 since the above is in meters and not KM

```
Δv = Δv / 1000
0.01080294284
```

find Δa

$$\Delta a = \frac{\Delta v}{v0}\, 2\, a$$

```
297.1014103
```

Part (b). One more revolution

Initial conditions for next revolution

```
a = a + Δa
42 538.10141
```

```
mi = mi - Δm
1998.899086
```

```
v0 = v0 - Δv
3.061057997
```

find new period

$$T0 = 2\, Pi \sqrt{\frac{a^3}{uE}};$$

```
T0 / (60 * 60) (*hrs*)
24.25358118
```

find change in mass (kg)

```
Δm = massBurnRate * T0
1.112549596
```

find change in V

```
Δv = g Isp Log[mi/(mi - Δm)]
```

```
10.92316269
```

Divide by 1000 since the above is in meters and not KM

```
Δv = Δv / 1000
```

```
0.01092316269
```

find Δa

```
Δa = Δv/v0 2 a
```

```
303.5882382
```

Part (c)

To do this, I wrote a function which makes one revolution and update the new initial configuration from last state of last revoluton. It runs untill mass is exchaused.

```
makeOneRev[mi_, ai_, Isp_, f_] :=
 Module[{uE = 3.986 * 10^5, g = 9.81, T0, massBurnRate, Δm, v0, Δv, Δa},
  T0 = 2 Pi Sqrt[ai^3/uE];
  massBurnRate = (f / Isp)/g;
  Δm = massBurnRate * T0;
  v0 = Sqrt[uE/ai];
  Δv = g Isp Log[mi/(mi - Δm)];
  Δv = Δv / 1000;
  Δa = Δv/v0 2 ai;

  {Δm, Δa}

 ]
```

```
{Δm, Δa} = makeOneRev[2000, 42 241, 2000, 0.25]
```

```
{1.100914301, 297.1014103}
```

```
mi = 2000;
ai = 42 241;
keepRunning = True;
n = 0;
While[keepRunning,
 {Δm, Δa} = makeOneRev[mi, ai, 2000, 0.25];
 If[mi - Δm ≤ 1900,
  keepRunning = False,
  n++;
  ai = ai + Δa;
  mi = mi - Δm
 ]
]
```

```
n
```
```
56
```

```
ai
```
```
91 619.46394
```

```
ai = 42 241 + 297.1014102908317`
```
```
42 538.10141
```

Test the function on the notes example below

```
{Δm, Δa} = makeOneRev[1000, 6678, 2500, 1]
```
```
{0.2214480331, 9.389877037}
```

Try it on the notes problem

Find a

```
a = 6678
```
```
6678
```

Find period T0

```
uE = 3.986 * 10^5;
```

$$T0 = 2\,Pi\sqrt{\frac{a^3}{uE}};$$

```
T0 / (60 * 60) (*hrs*)
```
```
1.508614725
```

find mass burn rate

```
f = 1;(*N*)
Isp = 2500; (*sec*)
g = 9.81;
massBurnRate = (f / Isp) / g
```

```
0.00004077471967
```

find change in mass (kg)

```
Δm = massBurnRate * T0
```

```
0.2214480331
```

find initial speed

```
v0 = Sqrt[uE / a]
```

```
7.725835198
```

find change in V

```
mi = 1000 ;(*kg*)
Δv = g Isp Log[mi / (mi - Δm)]
```

```
5.431614444
```

Divide by 1000 since the above is in meters and not KM

```
Δv = Δv / 1000
```

```
0.005431614444
```

find Δa

```
Δa = (Δv / v0) 2 a
```

```
9.389877037
```