HW 7, ME 440 Intermediate Vibration, Fall 2017

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0.1 Problem 1

Problem 1

Download the ANSYS input file "*MODAL_pipe_flywheel.txt*" from Canvas, run this input file in ANSYS and go through the file line by line to figure out what the system parameters are for this modal analysis. (Hint: When viewing the mode shapes within ANSYS, try plotting all 3 displacements and all 3 rotations (1 at a time) available under the "Nodal Solu" / "DOF Solution" option; this should be helpful in determining the type of displacement associated with each specific frequency.

A) List the 4 frequencies from ANSYS and label each as longitudinal, transverse, or torsional.

B) Using the parameters defined in the text file, analytically determine 3 of the 4 natural frequencies of this system. Show ALL your work for these calculations and then compare the analytical and finite element frequencies in a table with % errors.

The following is diagram of the model of the problem to solve
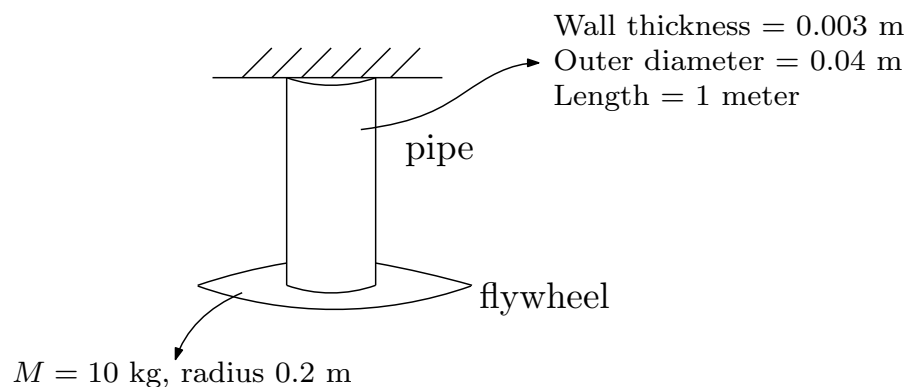
The ANSYS APDL (input file) listing was provided to us to use and is given in the text file below for reference

```
/filnam, pipe_flywheel_modal
/title, Flywheel on torsional spring example
/prep7
!element type
et,1,mass21,,0,0  !element type no.1 is mass21 (",,,0" signifies
!that this is a 3-D mass with rotary inertia)
et,2,pipe288        !element type no.2 is pipe288 (this is a pipe element)
mp,ex,1,200e9         ! elastic modulus for steel is 200 GPa
mp,gxy,1,77.2e9        ! shear modulus for steel is 77.2 GPa
mp,prxy,1,0.295        ! poisson's ratio for steel is 0.295
! model parameters
mass = 10                     ! mass of flywheel (kg)
rad_f = 0.2                     ! outer radius of flywheel
izz = 0.5*mass*rad_f*rad_f      ! mass moment of inertia
```

1

```
outer_d = 0.04                          ! outer diameter of pipe (m)
wall_t = 0.003                          ! wall thickness of pipe (m)
pipe_l = 1                    ! pipe length (m)
n_modes = 10                         ! number of modes wanted
!real constants
r,1,mass,mass,mass,0.5*izz,izz,0.5*izz        ! real constant set 1 is for the mass21 element
sectype,1,pipe                ! section type 1 is "pipe"
secdata,outer_d,wall_t        ! section data for pipe is outer diameter and wall thickness
!create nodes
k,1,0,0,0          ! keypoint 1 is at x=0, y=0, z=0, this will be the fixed end of the pipe
k,2,0,-pipe_l,0       ! keypoint 2 is at x=0, y=-pipe_l, z=0, this will be the free
! end of the pipe with the flywheel
!create elements
type,2               ! specify element type of subsequently defined elements
secnum,1        ! specify section type number of subsequently defined elements
l,1,2               ! creates a line from keypoint 1 to keypoint 2
lesize,1,,,10        ! specifies that line 1 will consist of 10 elements when meshed
lmesh,1               ! take line 1 and mesh it, resulting in elements representing the pipe
type,1               ! specify element type of subsequently defined elements
real,1               ! specify real constant set of subsequently defined elements
e,2               ! create element to be created at node 2
nsel,all        ! selects all nodes
d,all,uz,0        ! sets the z displacements on selected nodes to be 0, thereby
! limiting our modal analysis to modes in the xy plane
d,all,rotx        ! sets the rotx displacments on selected nodes to be 0
!displacement boundary conditions
nsel,s,loc,y,0               ! select node at x = 0
d,all,ux,0                ! displacement of selected node in x-dir is 0
d,all,uy,0                ! displacement of selected node in y-dir is 0
d,all,uz,0                ! displacement of selected node in z-dir is 0
d,all,rotx,0               ! rotations of selected node about x axis is 0
d,all,roty,0               ! rotations of selected node about y axis is 0
d,all,rotz,0               ! rotations of selected node about z axis is 0
allsel
finish
/solu                    !select static load solution
antype,modal
modopt,lanb,n_modes
solve
finish
/post1
```

0.1.1 Part 1

The following 4 modal frequencies were generated by ANSYS after running the above APDL file.

mode	Mode number	Frequency (Hz)
transverse	1	9.4438
torsional	2	34.272
transverse	3	111.41
longitudinal	4	420.31

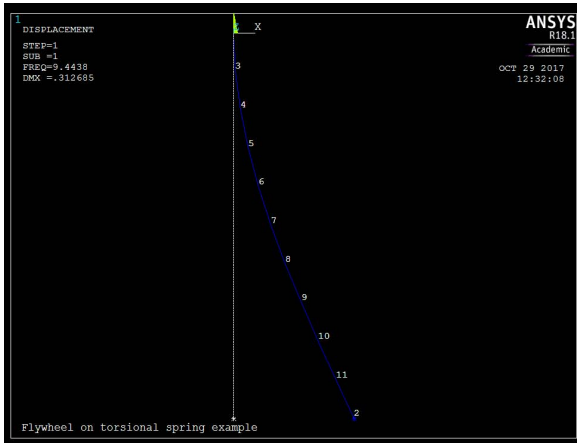The modal shapes were then plotted using ANSYS. They are given below for each mode

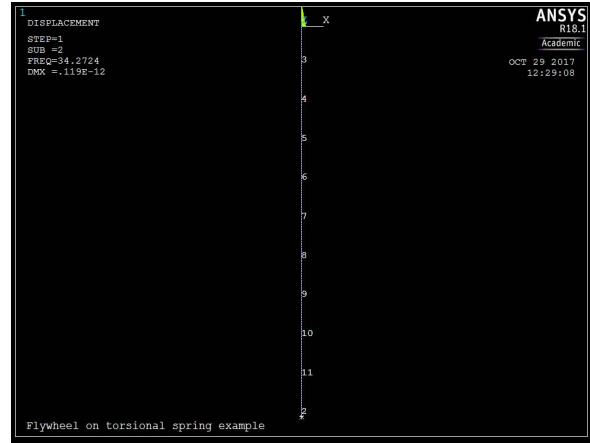Figure 1: First mode: Transverseat 9.443 Hz
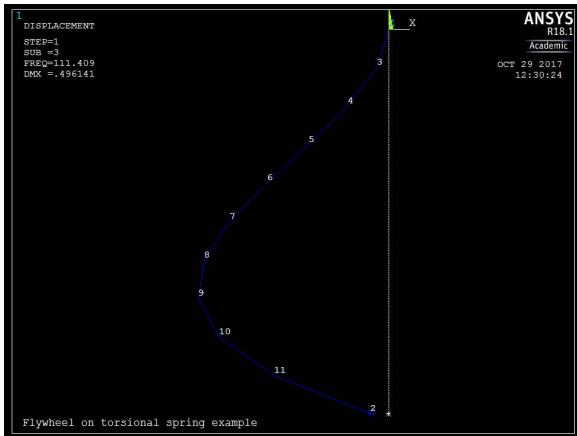

Figure 2: Second mode: Torsional at 34.2724 Hz

Figure 3: Third mode: Transverse at 111.408 Hz

Figure 4: Fourth mode: longtitudinal at 420.312 Hz

The system parameters are

```
PARAMETER STATUS-          (      13 PARAMETERS DEFINED)
               (INCLUDING      6 INTERNAL PARAMETERS)

NAME                          VALUE                    TYPE   DIMENSIONS
IZZ                           0.200000000              SCALAR
MASS                          10.0000000               SCALAR
N_MODES                       10.0000000               SCALAR
OUTER_D                       4.000000000E-002         SCALAR
PIPE_L                        1.00000000               SCALAR
RAD_F                         0.200000000              SCALAR
WALL_T
```
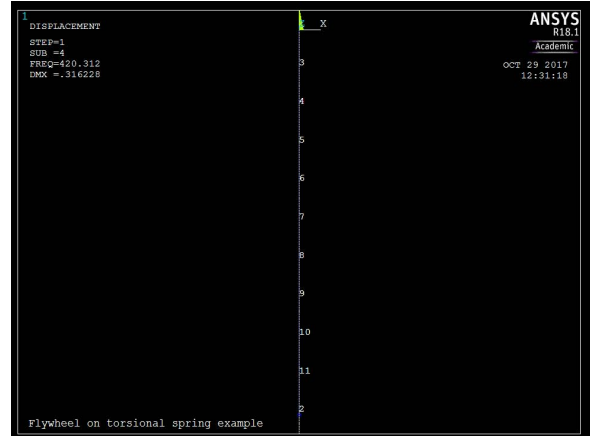
Total U displacement by ANSYS for mode 1 is

```
PRINT U    NODAL SOLUTION PER NODE

  ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

  LOAD STEP=     1  SUBSTEP=     1
   FREQ=     9.4438      LOAD CASE=   0

  THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

    NODE      UX             UY             UZ             USUM
       1   0.0000         0.0000         0.0000          0.0000
       2   0.31268       -0.15719E-019   0.0000          0.31268
       3   0.45277E-002-0.15719E-020    0.0000          0.45277E-002
       4   0.17442E-001-0.31438E-020    0.0000          0.17442E-001
       5   0.37825E-001-0.47156E-020    0.0000          0.37825E-001
       6   0.64762E-001-0.62875E-020    0.0000          0.64762E-001
       7   0.97336E-001-0.78594E-020    0.0000          0.97336E-001
```

```
       8  0.13463     -0.94313E-020  0.0000       0.13463
       9  0.17573     -0.11003E-019  0.0000       0.17573
      10  0.21972     -0.12575E-019  0.0000       0.21972
      11  0.26567     -0.14147E-019  0.0000       0.26567

 MAXIMUM ABSOLUTE VALUES
 NODE          2            2             0            2
 VALUE    0.31268     -0.15719E-019  0.0000       0.31268
```

Total ROT displacement by ANSYS for mode 1 is

```
 PRINT ROT   NODAL SOLUTION PER NODE

   ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

  LOAD STEP=     1  SUBSTEP=     1
   FREQ=    9.4438      LOAD CASE=   0

  THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

     NODE      ROTX          ROTY          ROTZ          RSUM
        1  0.0000       0.0000        0.0000        0.0000
        2  0.0000       0.54021E-017 0.47205       0.47205
        3  0.0000       0.54021E-018 0.88444E-001 0.88444E-001
        4  0.0000       0.10804E-017 0.16772       0.16772
        5  0.0000       0.16206E-017 0.23784       0.23784
        6  0.0000       0.21609E-017 0.29879       0.29879
        7  0.0000       0.27011E-017 0.35058       0.35058
        8  0.0000       0.32413E-017 0.39320       0.39320
        9  0.0000       0.37815E-017 0.42666       0.42666
       10  0.0000       0.43217E-017 0.45095       0.45095
       11  0.0000       0.48619E-017 0.46608       0.46608

 MAXIMUM ABSOLUTE VALUES
 NODE          0            2             2            2
 VALUE    0.0000       0.54021E-017 0.47205       0.47205
```

Total U displacement by ANSYS for mode 2 is

```
  PRINT U     NODAL SOLUTION PER NODE

   ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

  LOAD STEP=     1  SUBSTEP=     2
   FREQ=    34.272      LOAD CASE=   0

  THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

     NODE      UX           UY           UZ            USUM
        1  0.0000       0.0000        0.0000        0.0000
        2  0.11870E-012 0.57718E-018  0.0000        0.11870E-012
        3  0.17191E-014 0.57718E-019  0.0000        0.17191E-014
        4  0.66222E-014 0.11544E-018  0.0000        0.66222E-014
        5  0.14361E-013 0.17316E-018  0.0000        0.14361E-013
        6  0.24588E-013 0.23087E-018  0.0000        0.24588E-013
        7  0.36955E-013 0.28859E-018  0.0000        0.36955E-013
        8  0.51113E-013 0.34631E-018  0.0000        0.51113E-013
        9  0.66715E-013 0.40403E-018  0.0000        0.66715E-013
       10  0.83412E-013 0.46175E-018  0.0000        0.83412E-013
       11  0.10086E-012 0.51947E-018  0.0000        0.10086E-012

 MAXIMUM ABSOLUTE VALUES
 NODE          2            2             0            2
 VALUE    0.11870E-012 0.57718E-018  0.0000        0.11870E-012
```

Total ROT displacement by ANSYS for mode 2 is

```
 PRINT ROT   NODAL SOLUTION PER NODE

   ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****
```

```
   LOAD STEP=     1  SUBSTEP=     2
    FREQ=    34.272      LOAD CASE=   0

   THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

      NODE       ROTX          ROTY          ROTZ          RSUM
        1   0.0000        0.0000        0.0000        0.0000
        2   0.0000        2.2361        0.17917E-012  2.2361
        3   0.0000        0.22361       0.33581E-013  0.22361
        4   0.0000        0.44721       0.63680E-013  0.44721
        5   0.0000        0.67082       0.90299E-013  0.67082
        6   0.0000        0.89443       0.11344E-012  0.89443
        7   0.0000        1.1180        0.13309E-012  1.1180
        8   0.0000        1.3416        0.14927E-012  1.3416
        9   0.0000        1.5652        0.16197E-012  1.5652
       10   0.0000        1.7889        0.17118E-012  1.7889
       11   0.0000        2.0125        0.17691E-012  2.0125

   MAXIMUM ABSOLUTE VALUES
   NODE         0           2           2           2
   VALUE     0.0000        2.2361        0.17917E-012  2.2361
```

Total U displacement by ANSYS for mode 3 is

```
PRINT U    NODAL SOLUTION PER NODE

  ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

  LOAD STEP=     1  SUBSTEP=     3
   FREQ=    111.41      LOAD CASE=   0

  THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

     NODE       UX            UY            UZ            USUM
       1    0.0000        0.0000        0.0000        0.0000
       2  -0.47205E-001-0.10687E-014  0.0000        0.47205E-001
       3  -0.29904E-001-0.10687E-015  0.0000        0.29904E-001
       4  -0.10556      -0.21374E-015  0.0000        0.10556
       5  -0.20770      -0.32061E-015  0.0000        0.20770
       6  -0.31709      -0.42748E-015  0.0000        0.31709
       7  -0.41446      -0.53435E-015  0.0000        0.41446
       8  -0.48056      -0.64122E-015  0.0000        0.48056
       9  -0.49614      -0.74809E-015  0.0000        0.49614
      10  -0.44194      -0.85497E-015  0.0000        0.44194
      11  -0.29872      -0.96184E-015  0.0000        0.29872

  MAXIMUM ABSOLUTE VALUES
  NODE          9           2           0           9
  VALUE   -0.49614      -0.10687E-014  0.0000        0.49614
```

Total ROT displacement by ANSYS for mode 3 is

```
PRINT ROT  NODAL SOLUTION PER NODE

  ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

  LOAD STEP=     1  SUBSTEP=     3
   FREQ=    111.41      LOAD CASE=   0

  THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

     NODE       ROTX          ROTY          ROTZ          RSUM
       1   0.0000        0.0000        0.0000        0.0000
       2   0.0000       -0.49528E-013  3.1268        3.1268
       3   0.0000       -0.49528E-014-0.55375        0.55375
       4   0.0000       -0.99056E-014-0.91496        0.91496
       5   0.0000       -0.14858E-013 -1.0836        1.0836
       6   0.0000       -0.19811E-013 -1.0598        1.0598
       7   0.0000       -0.24764E-013-0.84334        0.84334
```

```
    8   0.0000      -0.29717E-013-0.43438      0.43438
    9   0.0000      -0.34670E-013 0.16711      0.16711
   10   0.0000      -0.39622E-013 0.96115      0.96115
   11   0.0000      -0.44575E-013  1.9477       1.9477

MAXIMUM ABSOLUTE VALUES
NODE            0          2          2          2
VALUE      0.0000      -0.49528E-013 3.1268     3.1268
```

Total U displacement by ANSYS for mode 4 is

```
PRINT U    NODAL SOLUTION PER NODE

  ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=    1  SUBSTEP=    4
  FREQ=    420.31      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

    NODE      UX          UY          UZ          USUM
     1   0.0000      0.0000      0.0000      0.0000
     2  -0.14600E-009 0.31623     0.0000      0.31623
     3  -0.21370E-011 0.31623E-001 0.0000      0.31623E-001
     4  -0.82246E-011 0.63246E-001 0.0000      0.63246E-001
     5  -0.17820E-010 0.94868E-001 0.0000      0.94868E-001
     6  -0.30480E-010 0.12649     0.0000      0.12649
     7  -0.45762E-010 0.15811     0.0000      0.15811
     8  -0.63223E-010 0.18974     0.0000      0.18974
     9  -0.82420E-010 0.22136     0.0000      0.22136
    10  -0.10291E-009 0.25298     0.0000      0.25298
    11  -0.12425E-009 0.28460     0.0000      0.28460

MAXIMUM ABSOLUTE VALUES
NODE            2          2          0          2
```

And total ROT displacement by ANSYS for mode 4 is

```
PRINT ROT   NODAL SOLUTION PER NODE

  ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=    1  SUBSTEP=    4
  FREQ=    420.31      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

    NODE      ROTX         ROTY         ROTZ         RSUM
     1   0.0000      0.0000       0.0000       0.0000
     2   0.0000      0.15287E-011-0.21790E-009 0.21790E-009
     3   0.0000      0.15287E-012-0.41720E-010 0.41721E-010
     4   0.0000      0.30573E-012-0.79011E-010 0.79012E-010
     5   0.0000      0.45860E-012-0.11187E-009 0.11187E-009
     6   0.0000      0.61146E-012-0.14031E-009 0.14031E-009
     7   0.0000      0.76433E-012-0.16431E-009 0.16431E-009
     8   0.0000      0.91720E-012-0.18389E-009 0.18389E-009
     9   0.0000      0.10701E-011-0.19903E-009 0.19904E-009
    10   0.0000      0.12229E-011-0.20975E-009 0.20975E-009
    11   0.0000      0.13758E-011-0.21604E-009 0.21604E-009

MAXIMUM ABSOLUTE VALUES
NODE            0          2          2          2
VALUE      0.0000      0.15287E-011-0.21790E-009 0.21790E-009
```

0.1.2 Part 2

To verify ANSYS solution, this was solved in two ways. By taking into account the mass m of the pipe and then by ignoring the mass m. Both hand solutions are given below. ANSYS do not take the mass of the pipe into account, since it was not told the density of the pipe

material in the APDL input file. The first solution below is the recommend one to use to compare the ANSYS result against and it the method which gave more agreement with ANSYS result.

First solution. Not accounting for mass of pipe

Finding the longitudinal (axial) natural frequency.

Using $k_{eq} = \frac{AE}{L}$ where A is the cross sectional area of the pipe and L is the pipe length and using $m_{eq} = M$, then the longitudinal natural frequency is

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{AE}{LM}} \tag{1}$$

The cross sectional area of the pipe is

$$\begin{aligned} A &= \frac{\pi}{4}\left(D_o^2 - D_i^2\right) \\ &= \frac{\pi}{4}\left((0.04)^2 - (0.034)^2\right) \\ &= 3.487\,2 \times 10^{-4}\ \text{m}^2 \end{aligned}$$

The length of pipe is 1 meter and $M = 10$ kg. Equation (1) becomes

$$\begin{aligned} \omega_n &= \sqrt{\frac{\left(3.487\,2 \times 10^{-4}\right)\left(200 \times 10^9\right)}{(1)\,(10)}} \\ &= 2640.9\ \text{rad/sec} \end{aligned}$$

The cycle frequency is

$$\begin{aligned} f_n &= \frac{\omega_n}{2\pi} \\ &= \frac{2640.9}{2\pi} \\ &= 420.31\ \text{Hz} \end{aligned}$$

ANSYS gives 420.31 Hz. So the error is 0.

Finding the torsional natural frequency.

Torsional stiffness k_t is

$$k_t = \frac{GJ}{L}$$

Where G is the shear modulus (given in handout), and J is the polar area moment of inertia of the cross section given by

$$\begin{aligned} J &= \frac{\pi}{32}\left(D_o^4 - D_i^4\right) \\ &= \frac{\pi}{32}\left(0.04^4 - 0.034^4\right) \\ &= 1.201\,3 \times 10^{-7}\ \text{m}^4 \end{aligned}$$

Therefore

$$k_t = \frac{\left(77.2 \times 10^9\right)\left(1.201\,3 \times 10^{-7}\right)}{1} = 9274\ \text{N-m per radian}$$

The equivalent mass is just the mass moment of inertia of the flywheel $\frac{1}{2}Mr_f^2$ (since the pipe assumed to have no mass). Hence the torsional frequency is

$$\begin{aligned} \omega &= \sqrt{\frac{k_t}{\frac{1}{2}Mr_f^2}} \\ &= \sqrt{\frac{9274}{\frac{1}{2}\,(10)\,(0.2)^2}} \\ &= 215.34\ \text{rad/sec} \end{aligned}$$

Therefore the torsional frequency in Hz is

$$f = \frac{215.34}{2\pi}$$
$$= 34.272 \text{ hz}$$

ANSYS gives this as 34.272 Hz. The the <u>error is 0</u>.

<u>Finding the transverse natural frequency:</u>

Using $k = \frac{3EI}{L^3}$ and $M = 10$. Where I is the area moment of inertia given by

$$I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$$
$$= \frac{\pi}{64}\left(0.04^4 - 0.034^4\right)$$
$$= 6.0066 \times 10^{-8} \text{ m}^4$$

The transverse natural frequency is therefore

$$\omega = \sqrt{\frac{3EI}{ML^3}}$$
$$= \sqrt{\frac{3\left(200 \times 10^9\right)\left(6.0066 \times 10^{-8}\right)}{(10)(1)^3}}$$
$$= 60.033 \text{ rad/sec}$$

Hence

$$f_n = \frac{60.033}{2\pi} = 9.5545 \text{ Hz}$$

ANSYS gives 9.4438 for the first transverse natural frequency. Hence error is $\left(\frac{|9.4438 - 9.5545|}{9.4438}\right)100 = 1.1722\%$

<u>Summary of results</u>

mode	ANSYS result	Hand calculation	%error
First transverse	9.4438	9.5545	1.1722%
First torsional	34.272	34.272	0%
First longitudinal (axial)	420.31	420.31	0%

All the analytical solutions gave exact agreement with ANSYS except for the transverse case. The transverse case uses stiffness $\frac{3EI}{L^3}$ due to load at end of fixed-free beam. This does not account for bending rotation in the beam. That is why ANSYS result is more accurate, as its finite elements account for the small bending associated with the transverse vibration. In the other two cases (Torsional and axial), there is no associated bending, hence the solutions agree.

Second solution. Accounting for mass of pipe

<u>Finding the longitudinal natural frequency.</u>

Following the example given in the textbook, at page 715, the (first) longitudinal natural frequency is found to be

$$\omega_1 = \frac{\alpha_1\sqrt{\frac{E}{\rho}}}{L} \tag{E.4}$$

Where α_1 is the (first) root of

$$\alpha \tan \alpha = \beta$$

Where β is the mass ratio $\beta = \frac{m}{M}$ where m is mass of pipe and M is end mass (flywheel). To find mass of pipe m, using steel density <u>$\rho = 7800 \text{ kg/m}^3$</u>, we first find the volume of the pipe.

Let D_i be the inner diameter and D_o the outer diameter. $D_o = 0.04$ meter and $D_i = 0.04 - 2\,(0.003) = 0.034$ meter , therefore the cross sectional area of the pipe is

$$A = \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$$
$$= \frac{\pi}{4}\left((0.04)^2 - (0.034)^2\right)$$
$$= 3.487\,2 \times 10^{-4}\ \text{m}^2$$

And since length of pipe is 1 meter, the mass of pipe is

$$m = \rho A L$$
$$= (7800)\left(3.487\,2 \times 10^{-4}\right)(1)$$
$$= 2.72\ \text{kg.}$$

The mass at the end is given as $M = 10$ kg. Therefore the mass ratio

$$\beta = \frac{m}{M} = \frac{2.72}{10} = 0.272$$

To find α_1 we now need to solve $\alpha_1 \tan \alpha_1 = 0.272$. This was solved numerical using root finder. The first root was found to be

$$\alpha_1 = 0.499$$

Therefore from equation E.4 in textbook (page 715)

$$\omega_1 = \frac{\alpha_1 c}{L}$$
$$= \frac{\alpha_1 \sqrt{\frac{E}{\rho}}}{L}$$
$$= \frac{(0.499)\sqrt{\frac{200 \times 10^9}{7800}}}{1}$$
$$= 2526.8\ \text{rad/sec}$$

Therefore

$$f_1 = \frac{\omega_1}{2\pi}$$
$$= \frac{2526.8}{2\pi}$$
$$= 402.15\ \text{Hz}$$

ANSYS gives 420.31 Hz. So the error is $\left(\frac{420.31 - 402.\,15}{420.31}\right)100 = 4.321\%$

Finding the torsional natural frequency.

k_t is

$$k_t = \frac{GJ}{L}$$

Where G is the shear modulus (given in handout), and J is the polar area moment of inertia of the cross section given by

$$J = \frac{\pi}{32}\left(D_o^4 - D_i^4\right)$$
$$= \frac{\pi}{32}\left(0.04^4 - 0.034^4\right)$$
$$= 1.201\,3 \times 10^{-7}\ \text{m}^4$$

Hence

$$k_t = \frac{\left(77.2 \times 10^9\right)\left(1.201\,3 \times 10^{-7}\right)}{1} = 9274$$

To find equivalent mass, using kinetic energy method

$$\frac{1}{2}I_{flywheel}\dot{\theta}^2 + \frac{1}{2}I_{pipe}\dot{\theta}^2 = \frac{1}{2}I_{eq}\dot{\theta}^2 \tag{1}$$

For a hollow pipe, where now m is replaced by $\frac{1}{3}m$ from continuous system derivation.

$$I_{pipe} = \frac{1}{2}\left(\frac{1}{3}m\right)\left(\left(\frac{D_o}{2}\right)^2 + \left(\frac{D_i}{2}\right)^2\right)$$
$$= \frac{1}{24}m\left(D_o^2 + D_i^2\right)$$

And for the flywheel, $I_{fly} = \frac{1}{2}Mr_f^2$ where $r_f = 0.2$ meter. Hence from (1)

$$I_{eq} = \frac{1}{2}Mr_f^2 + \frac{1}{24}m\left(D_o^2 + D_i^2\right)$$
$$= \frac{1}{2}(10)(0.2)^2 + \frac{1}{24}(2.72)\left(0.04^2 + 0.034^2\right)$$
$$= 0.20031 \text{ kg-m}^2$$

Hence the torsional frequency is

$$\omega = \sqrt{\frac{k_t}{I_{eff}}}$$
$$= \sqrt{\frac{9274}{0.20031}}$$
$$= 215.17 \text{ rad/sec}$$

Therefore the torsional frequency in Hz is

$$f = \frac{215.17}{2\pi}$$
$$= 34.245 \text{ hz}$$

ANSYS gives this as 34.272 Hz. The the error is $\left(\frac{34.272 - 34.245}{34.272}\right)100 = 0.079\%$

Finding the transverse natural frequency:

From textbook, table 8.15 page 726, it gives for fixed-end beam the value $\beta_1 L = 1.875104$. But since there is a mass attach to the end in our problem, I did not know how to add this using the table.

So I used the other method we used before, which is the Rayleigh energy method, where we assume motion is simple harmonic motion. Taking the displacement as the transverse motion of the free end of the pipe (where the large mass is attached), measured from equilibrium then the kinetic energy is

$$T = \frac{1}{2}\overbrace{(M + 0.23m)}^{m_{eq}}\dot{x}^2$$

Where we added $0.23m$, where m is the mass of the pipe, since this is continuous mass. For the potential energy, we use the stiffness formula for the fixed-free beam which is $k = \frac{3EI}{L^3}$, hence

$$U = \frac{1}{2}kx^2$$

Now, assuming $x = X \sin \omega_n t$, then $\dot{x} = X\omega_n \cos \omega_n t$. Therefore when

$$U_{\max} = T_{\max}$$

We obtain

$$\frac{1}{2}\frac{3EI}{L^3}X^2 = \frac{1}{2}(M + 0.23m)(X\omega_n)^2$$
$$\frac{3EI}{L^3} = (M + 0.23m)\omega_n^2$$
$$\omega_n^2 = \frac{3EI}{L^3(M + 0.23m)}$$
$$\omega_n = \sqrt{\frac{3EI}{L^3(M + 0.23m)}} \tag{1}$$

Where I now is the area moment of inertia[1] is given by

$$I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$$
$$= \frac{\pi}{64}\left(0.04^4 - 0.034^4\right)$$
$$= 6.0066 \times 10^{-8} \text{ m}^4$$

And

$$M + 0.23m = 10 + 0.23\,(2.72)$$
$$= 10.626 \text{ kg}$$

Substituting the numerical values in (1) gives

$$\omega_n = \sqrt{\frac{3\left(200 \times 10^9\right)\left(6.0066 \times 10^{-8}\right)}{10.626}}$$
$$= 58.239 \text{ rad/sec}$$

Hence

$$f_n = \frac{58.239}{2\pi} = 9.269 \text{ Hz}$$

ANSYS gives 9.4438 for the first transverse natural frequency. Hence error is $\left(\frac{9.4438 - 9.269}{9.4438}\right)100 = 1.851\,\%$

Summary of results

| mode	ANSYS result	Hand calculation	%error
First transverse	9.4438	9.269	1.851 %
First torsional	34.272	34.245	0.079%
First longitudinal	420.31	402.15	4.321%

Comparing the above table to the first solution, it shows that ignoring the mass of the pipe gave result which agree with ANSYS result much better. This is because ANSYS did not take into the account the mass of the pipe. It will be interesting exercise to find how to change the APDL input file to make ANSYS account for the mass of the pipe and then compare the above results with ANSYS.

0.2 Problem 2

Problem 2

The signpost of a fast food restaurant consists of a hollow steel cylinder of height h, inside diameter d, and outside diameter D, fixed to the ground and carries a concentrated mass M at the top. It can be modeled as a single degree of freedom spring-mass-damper system with an equivalent viscous damping ratio of 0.1 for analyzing its transverse vibration characteristics under wind excitation. Assume the signpost mass (m) and concentrated mass (M) have an equivalent mass (m_{eq}) as defined below. (this equivalent mass equation was from a lecture example earlier in the semester). The specific weight (ρg) and the elastic modulus (E) of the steel are 76,500 N/m³ and 207 GPa, respectively. For the density and viscosity of air, use 1.20 kg/m³ and 1.80×10^{-5} N-s/m², respectively. For the remaining parameters, assume $h = 10$ m, $D = 25$ cm, $d = 20$ cm and $M = 200$ kg.
D
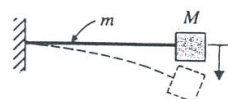
Cantilever beam of mass m
carrying an end mass M

$$m_{eq} = M + 0.23\,m$$

Determine the following:
A) the natural frequency of transverse vibration of the signpost,
B) the wind velocity at which the signpost undergoes maximum steady-state displacement, and
C) the maximum wind induced steady-state displacement of the signpost.

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[1]Notice that the polar area moment of ineria has $\frac{1}{32}$ factor, while the area moment of interia, the factor is $\frac{1}{64}$

0.2.1 Part A

The first step is to determine the natural frequency ω_n for the transverse vibration. Rayleigh energy method was used to find the transverse frequency. Taking the displacement as the transverse motion of the free end of the sigpost (where the large mass M is attached), measured from equilibrium, then the kinetic energy is

$$T = \frac{1}{2} \overbrace{(M + 0.23m)}^{m_{eq}} \dot{x}^2$$

m is the mass of the sigpost. For the potential energy, the bending stiffness formula for the fixed-free beam with load at the end was used, which is

$$k = \frac{3EI}{L^3}$$

The potential energy is therefore

$$U = \frac{1}{2}kx^2$$

Assuming $x = X \sin \omega_n t$, then $\dot{x} = X\omega_n \cos \omega_n t$. Using

$$U_{\max} = T_{\max}$$

Then the above reduces to

$$\frac{1}{2}\left(\frac{3EI}{L^3}\right)X^2 = \frac{1}{2}(M + 0.23m)(X\omega_n)^2$$

$$\frac{3EI}{L^3} = (M + 0.23m)\,\omega_n^2$$

$$\omega_n^2 = \frac{3EI}{L^3\,(M + 0.23m)}$$

$$\omega_n = \sqrt{\frac{3EI}{L^3\,(M + 0.23m)}} \tag{1}$$

I is the area moment of inertia of the pipe cross section. Since $D_o = 0.25$ m and $D_i = 0.2$ m, then

$$I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$$

$$= \frac{\pi}{64}\left(0.25^4 - 0.2^4\right)$$

$$= 1.1321 \times 10^{-4} \text{ m}^4$$

$M = 200$ kg, and $L = 10$ meter. Using $\rho_{steel}g = 76500$ N/m^3 and $E = 207 \times 10^9$ Pa. To find the mass m of the post, the cross sectional area is first found

$$A = \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$$

$$= \frac{\pi}{4}\left(0.25^2 - 0.2^2\right)$$

$$= 0.017671 \text{ m}^2$$

Hence the mass m is

$$m = \frac{\left(\rho_{steel}g\right)}{g}AL$$

$$= \frac{76500}{9.81}(0.017671)(10)$$

$$= 1378 \text{ kg}$$

Substituting the numerical values in (1) gives

$$\omega_n = \sqrt{\frac{3EI}{L^3(M+0.23m)}}$$

$$= \sqrt{\frac{3(207\times10^9)(1.1321\times10^{-4})}{(10)^3(200+0.23(1378))}}$$

$$= 11.662 \text{ rad/sec}$$

Or

$$f_n = \frac{11.662}{2\pi}$$

Therefore

$$\boxed{f_n = 1.8561 \text{ Hz}}$$

0.2.2 Part B

Maximum steady state displacement occurs at resonance. This is when the frequency of vortex shedding is the same as the natural frequency f_n of the post found above. Using Strouhal formula

$$v = \frac{f_n D_o}{S}$$

Where in the above v is the wind velocity and the vortex shedding frequency is set to be the natural frequency in order to obtain the maximum displacement. Assume $S = 0.21$ gives

$$v = \frac{(1.8561)(0.25)}{0.21}$$

Hence

$$\boxed{v = 2.2096 \text{ m/s}}$$

<u>Checking Reynold number</u>

$$\text{Re} = \frac{vD_o\rho_{air}}{\mu}$$

ρ_{air} is density of air and μ is viscosity of air. Using the numerical values given the above becomes

$$\text{Re} = \frac{(2.2096)(0.25)(1.2)}{(1.8\times10^{-5})}$$

$$= 36827$$

Since $400 \leq \text{Re} \leq 300000$ then the assumption of Strouhal $S = 0.21$ was <u>valid</u>.

0.2.3 Part C

The lateral force exerted by the wind on the sigpost is given by

$$F(t) = \frac{1}{2}c\rho_{air}v^2A\sin\omega t$$

$$= F_0\sin\omega t$$

Where $c \approx 1$ for cylinder and v is the wind speed found in last part and A is the projected area $A = D_oL$. Hence

$$F_0 = \frac{1}{2}c\rho_{air}v^2A$$

$$= \frac{1}{2}(1.2)(2.2096)^2(0.25)(10)$$

$$= 7.3235 \text{ N}$$

Using the steady state displacement formula for damped single degree of freedom system, which is

$$y_{ss} = \frac{F_0}{k} \frac{1}{\sqrt{\left(1 - r^2\right)^2 + (2\xi r)^2}}$$

Where F_0 is total force from the wind over the whole span. Assuming this force acts at the end of a fixed-free beam (This is an over estimation. The wind force actually acts over the whole length of the sigpost, but it is now taken as acting on the end). Therefore $k = \frac{3EI}{L^3}$ can be used based on this. Since $r = 1$ (resonance) and $\xi = 0.1$, then y_{ss} is now evaluated

$$y_{ss} = \frac{F_0}{k} \frac{1}{\sqrt{4\xi^2}}$$

$$= \frac{F_0 L^3}{3EI} \frac{1}{2\xi}$$

$$= \frac{(7.3235)(10)^3}{3\left(207 \times 10^9\right)\left(1.1321 \times 10^{-4}\right)} \frac{1}{2(0.1)}$$

$$= 5.2085 \times 10^{-4} \text{ meter}$$

Or

$$y_{ss} \approx 0.5 \text{ mm}$$