HW 4,ME 240 Dynamics, Fall 2017

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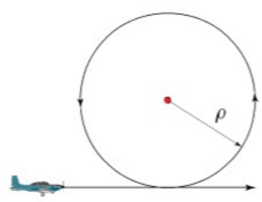
0.1 Problem 1

A 970 kg aerobatics plane initiates the basic loop maneuver at the bottom of a loop with radius ρ = 115 m and a speed of 228 km/h. At this instant, determine the magnitude of the plane's acceleration, expressed in terms of g, the acceleration due to gravity, and the magnitude of the lift provided by the wings.



a = [ ] g

F_{lift} = [ ] N

The velocity in meter per second is

$$
\begin{aligned}
v &= 228\left(\frac{1000}{km}\right)\left(\frac{h}{3600}\right) \\
&= (228)\frac{1000}{3600} \\
&= 63.333 \text{ m/s}
\end{aligned}
$$

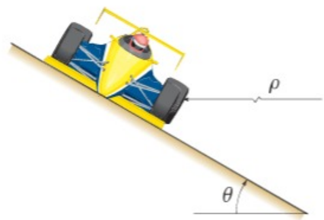
In normal direction, the acceleration is $a_n = \frac{v^2}{\rho} = \frac{(63.333)^2}{115} = 34.879 \text{ m/s}^2$ or in terms of g, it becomes $\frac{34.879}{9.81} = 3.555$ g.

Now, force balance in vertical direction gives

$$
\begin{aligned}
L - mg &= ma_n \\
L &= m\left(g + a_n\right) \\
&= 970\,(9.81 + 34.879) \\
&= 43348.33 \text{ N}
\end{aligned}
$$

0.2 Problem 2

A race car is traveling at a constant speed over a circular banked turn. Oil on the track has caused the static friction coefficient between the tires and the track to be $\mu_s = 0.2$. If the radius of the car's trajectory is $\rho = 324$ m and the bank angle is $\theta = 31°$, determine the range of speeds within which the car must travel not to slide sideways.



[ ] **km/h ≤ v ≤** [ ] **km/h**

For maximum speed, friction acts downwards as car assumed to be just about to slide upwards. Resolving forces in normal and tangential gives

$$\mu_s N\cos\theta + N\sin\theta = m\frac{v_{\text{max}}^2}{\rho} \tag{1}$$

$$-mg + N\cos\theta - \mu_s N\sin\theta = 0 \tag{2}$$

From (2)

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

From (1)

$$\begin{aligned}
v_{\text{max}}^2 &= \frac{\rho}{m}\left(\mu_s N\cos\theta + N\sin\theta\right)\\
&= \frac{\rho}{m}\left(\mu_s\left(\frac{mg}{\cos\theta - \mu_s\sin\theta}\right)\cos\theta + \left(\frac{mg}{\cos\theta - \mu_s\sin\theta}\right)\sin\theta\right)\\
&= \rho\left(\mu_s\left(\frac{g}{\cos\theta - \mu_s\sin\theta}\right)\cos\theta + \left(\frac{g}{\cos\theta - \mu_s\sin\theta}\right)\sin\theta\right)\\
&= \rho g\left(\frac{\mu_s}{1-\mu_s\tan\theta} + \frac{\tan\theta}{1-\mu_s\tan\theta}\right)\\
&= \rho g\left(\frac{\mu_s + \tan\theta}{1-\mu_s\tan\theta}\right)\\
&= 324\,(9.81)\left(\frac{0.2 + \tan\left(31\frac{\pi}{180}\right)}{1 - 0.2\tan\left(31\frac{\pi}{180}\right)}\right)\\
&= 2893.165 \text{ m/s}
\end{aligned}$$

Hence

$$\begin{aligned}
v_{\text{max}} &= 53.788 \text{ m/s}\\
&= 53.788\,(0.001)\,(3600)\\
&= 193.637 \text{ km/hr}
\end{aligned}$$

For minimum speed, friction acts upwards as car assumed to be just about to slide downwards. Resolving forces in normal and tangential gives

$$-\mu_s N\cos\theta + N\sin\theta = m\frac{v_{\text{min}}^2}{\rho} \tag{3}$$

$$-mg + N\cos\theta + \mu_s N\sin\theta = 0 \tag{4}$$

From (4)

$$N = \frac{mg}{\cos\theta + \mu_s\sin\theta}$$

From (3)

$$
\begin{aligned}
v_{\min}^2 &= \frac{\rho}{m}\left(-\mu_s N\cos\theta + N\sin\theta\right) \\
&= \frac{\rho}{m}\left(-\mu_s\left(\frac{mg}{\cos\theta+\mu_s\sin\theta}\right)\cos\theta + \left(\frac{mg}{\cos\theta+\mu_s\sin\theta}\right)\sin\theta\right) \\
&= \rho g\left(-\mu_s\left(\frac{1}{\cos\theta+\mu_s\sin\theta}\right)\cos\theta + \left(\frac{1}{\cos\theta+\mu_s\sin\theta}\right)\sin\theta\right) \\
&= \rho g\left(\frac{-\mu_s}{1+\mu_s\tan\theta} + \frac{\tan\theta}{1+\mu_s\tan\theta}\right) \\
&= \rho g\left(\frac{\tan\theta-\mu_s}{1+\mu_s\tan\theta}\right) \\
&= 324\,(9.81)\left(\frac{\tan\left(31\frac{\pi}{180}\right)-0.2}{1+0.2\tan\left(31\frac{\pi}{180}\right)}\right) \\
&= 1137.425 \text{ m/s}
\end{aligned}
$$

Hence

$$
\begin{aligned}
v_{\min} &= 33.726 \text{ m/s} \\
&= 33.726\,(0.001)\,(3600) \\
&= 121.414 \text{ km/hr}
\end{aligned}
$$

Hence

$$121.414 \le v \le 193.637$$

0.3 Problem 3

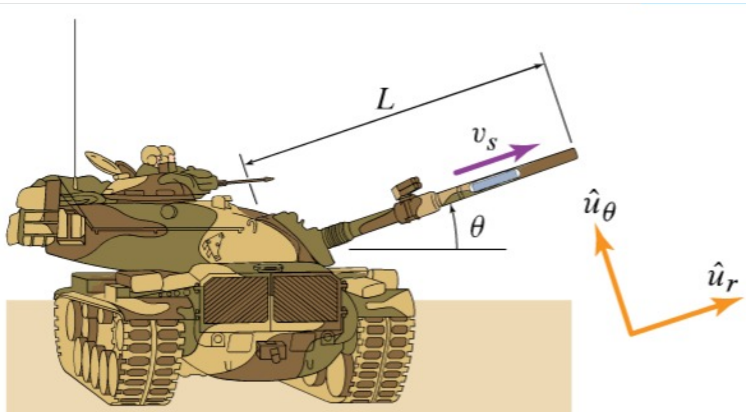
The cutaway of the gun barrel shows a projectile moving through the barrel. If the projectile's exit speed is v_s = 1,681 m/s (relative to the barrel), the projectile's mass is 16.9 kg, the length of the barrel is L = 4.2 m, the acceleration of the projectile down the gun barrel is constant, and θ is increasing at a constant rate of 0.18 rad/s, determine

(a) The acceleration of the projectile.

(b) The pressure force acting on the back of the projectile.

(c) The normal force on the gun barrel due to the projectile.

as the projectile leaves the gun, but while it is still in the barrel. Assume that the projectile exits the barrel when θ = 23°, and ignore the friction between the projectile and the barrel.



Part 1 out of 3

Part 1

(a) $\vec{a}$ = (336 $\times 10^3 \hat{u}_r$ + 605 $\hat{u}_\theta$) m/s^2

Part 2

(b) The pressure force is 5.69 $\times 10^6$ **N in the r direction.**

Part 3

(c) The normal force on the gun barrel due to the projectile is 10.4 $\times 10^3$ **N in the positive θ direction.**

0.3.1 part (1)

Since acceleration is constant along barrel, then using

$$v_f^2 = v_0^2 + 2\ddot{r}L$$

We will solve for $\ddot{r}$. Assuming $v_0 = 0$ then

$$\begin{aligned} 1681^2 &= 2\ddot{r}(4.2) \\ \ddot{r} &= \frac{1681^2}{2(4.2)} \\ &= 336400.1 \text{ m/s}^2 \end{aligned}$$

But

$$\bar{a} = \left(\ddot{r} - L\dot{\theta}^2\right)\hat{u}_r + \left(L\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{u}_\theta$$

But $\ddot{\theta} = 0$, hence the above becomes

$$\begin{aligned} \bar{a} &= \left(336400.1 - (4.2)(0.18)^2\right)\hat{u}_r + (2(1681)(0.18))\hat{u}_\theta \\ &= 336400\ \hat{u}_r + 605.16\ \hat{u}_\theta \end{aligned}$$

0.3.2 part (2)

Free body diagram for bullet gives

$$P = ma_r + mg\sin\theta$$

but $a_r = 336400$ m/s^2 and $m = 16.9$ kg and $\theta = 23^0$, hence

$$\begin{aligned} P &= (16.9)(336400) + (16.9)(9.81)\sin\left((23)\frac{\pi}{180}\right) \\ &= 5685225 \text{ N} \\ &= 5.685 \times 10^6 \text{ N} \end{aligned}$$

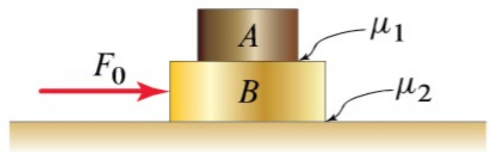
0.3.3 Part (3)

Free body diagram for bullet gives

$$\begin{aligned} N &= ma_\theta + mg\cos\theta \\ &= (16.9)(605.16) + (16.9)(9.81)\cos\left((23)\frac{\pi}{180}\right) \\ &= 10379.81 \text{ N} \\ &= 10.379 \times 10^3 \text{ N} \end{aligned}$$

0.4 Problem 4

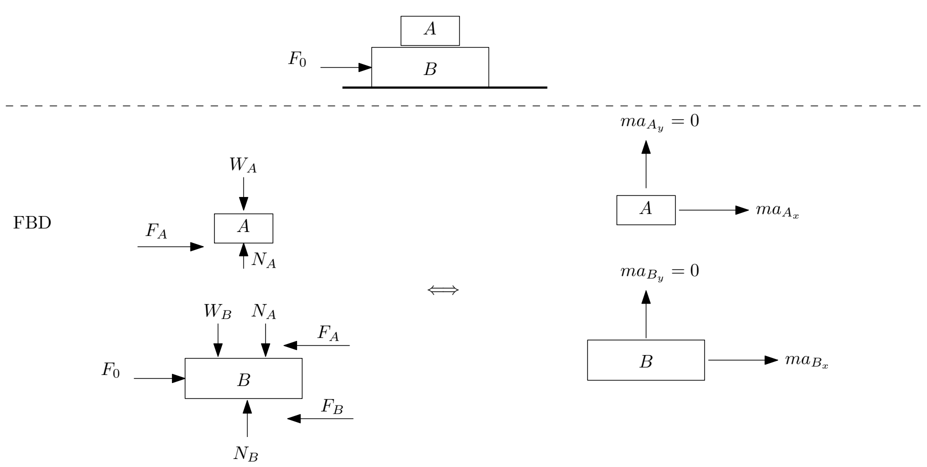
A force F_0 of 438 lb is applied to block B. Letting the weights of A and B be 50 and 71 lb, respectively, and letting the staticand kinetic friction coefficients between blocks A and B be $\mu_1 = 0.26$, and the static and kinetic friction coefficients between block B and the ground be $\mu_2 = 0.46$, determine the accelerations of both blocks.



$\vec{a}_A$ = ($\hat{\imath}$ + $\hat{\jmath}$) ft/s^2

$\vec{a}_B$ = ($\hat{\imath}$ + $\hat{\jmath}$) ft/s^2

The free body diagram is



For A,

$$\sum F_x = m_A a_{A_x}$$

$$F_A = m_A a_{A_x} \tag{2}$$

$$\sum F_y = m_A a_{A_y}$$

$$N_A - W_A = 0 \tag{3}$$

For B

$$\sum F_x = m_B a_{B_x}$$

$$F_o - F_A - F_B = m_B a_{B_x} \tag{4}$$

$$\sum F_y = m_B a_{B_y}$$

$$N_B - W_B - N_A = 0 \tag{5}$$

Hence (3) becomes

$$\begin{aligned} N_A &= W_A \\ &= m_A g \end{aligned} \tag{6A}$$

And (5) becomes

$$\begin{aligned} N_B &= W_B + N_A \\ &= m_B g + m_A g \\ &= (m_B + m_A) g \end{aligned} \tag{6B}$$

But $F_A = N_A \mu_1$ then (2) becomes

$$N_A \mu_1 = m_A a_{A_x}$$

But from (6) the above reduces to (since $N_A = m_A g$)

$$m_A g \mu_1 = m_A a_{A_x}$$

Hence

$$\begin{aligned} a_{A_x} &= g\mu_1 \\ &= (32.2)(0.26) \\ &= 8.372 \text{ ft/s}^2 \end{aligned}$$

Similarly $F_B = N_B\mu_2$ hence (4) becomes

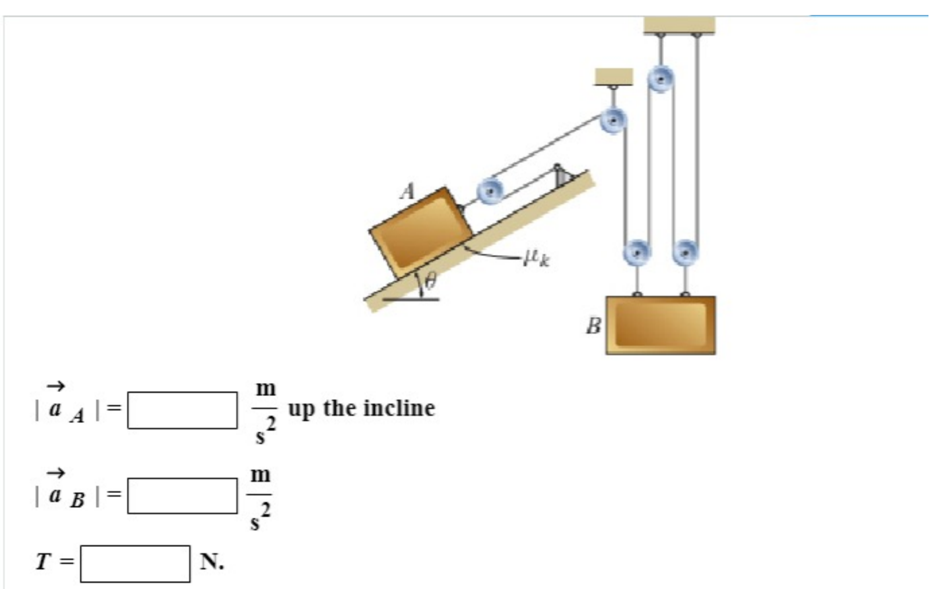
$$F_o - N_A\mu_1 - N_B\mu_2 = m_B a_{B_x}$$

But $N_B = (m_A + m_B)g$, then above becomes

$$\begin{aligned} F_o - m_A g\mu_1 - (m_A + m_B)g\mu_2 &= m_B a_{B_x} \\ a_{B_x} &= \frac{F_o - m_A g\mu_1 - (m_A + m_B)g\mu_2}{m_B} \\ a_{B_x} &= \frac{438 - (50)(0.26) - (50 + 71)(0.46)}{\frac{71}{32.2}} \\ &= 167.504 \text{ ft/sec}^2 \end{aligned}$$

0.5 Problem 5

Blocks A and B are connected by a pulley system. The coefficient of kinetic friction between the block A and the incline is $\mu_k = 0.3$ and static friction is insufficient to prevent slipping. The mass of A is $m_A = 7$ kg, the mass of B is $m_B = 20$ kg, and the angle between the incline and the horizontal is $\theta = 22°$. Determine the acceleration of A, the acceleration of B, and the tension in the rope after the system is released.



Free body diagram for block B results in

$$2g - \frac{8T}{m_B} = a_A$$

Free body diagram for block A resuts in

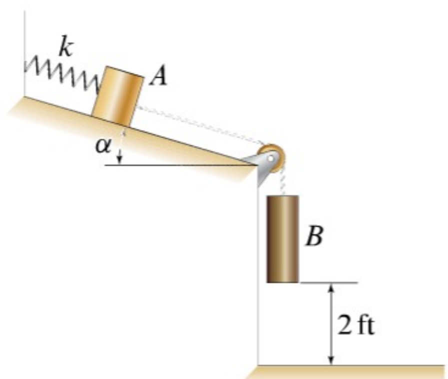
$$-m_A g \sin\theta - \mu m_A g \cos\theta + 2T = m_A a_A$$

In addision, since rope length is fixed, we find that $a_A = 2a_B$. The above are 3 equations in 3 unknowns T, a_A, a_B. Solving gives

$$\begin{aligned} a_A &= 4.439 \text{ m/s}^2 \\ a_B &= 2.2197 \text{ m/s}^2 \\ T &= 37.95 \text{ N} \end{aligned}$$

0.6 Problem 6

Two blocks A and B weighing 116 and 223 lb, respectively, are released from rest as shown. At the moment of release the spring is unstretched. In solving these problems, model A and B as particles, neglect air resistance, and assume that the cord is inextensible. *Hint:* If B hits the ground, then its maximum displacement is equal to the distance between the initial position of B and the ground. Determine the maximum displacement and the maximum speed of block B if $\alpha = 19°$, the contact between A and the incline is frictionless, and the spring constant is $k = 28$ lb/ft.



The maximum displacement of block B is [ ] ft and the maximum speed is [ ] ft/s.

maximum displacement is 2 ft. Maximum speed is 9.404 ft/sec.