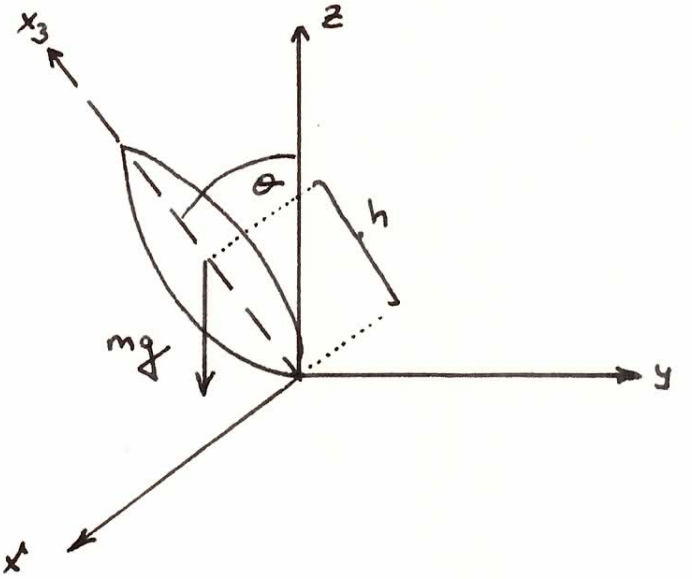
7.6 Symmetric Top in a Gravitational Field

$I_1 = I_2 \neq I_3$

Lowest point is fixed



Lagrangian: $L = T - U$ $\qquad U = mgh\cos\theta$

$$T = \frac{1}{2} I_i \omega_i^2$$

$$= \frac{1}{2} I_1 (\omega_1^2 + \omega_2^2) + \frac{1}{2} I_3 \omega_3^2$$

$$\omega_1^2 = (\dot\phi \sin\theta \sin\psi + \dot\theta \cos\psi)^2$$

$$= \dot\phi^2 \sin^2\theta \sin^2\psi + 2\dot\phi\dot\theta \sin\theta \sin\psi\cos\psi + \dot\theta^2 \cos^2\psi$$

$$\omega_2^2 = (\dot\phi \sin\theta \cos\psi - \dot\theta \sin\psi)^2$$

$$= \dot\phi^2 \sin^2\theta \cos^2\psi - 2\dot\phi\dot\theta \sin\theta\cos\psi \sin\psi + \dot\theta^2 \sin^2\psi$$

$$\Rightarrow \quad \omega_1^2 + \omega_2^2 = \dot\phi^2 \sin^2\theta + \dot\theta^2$$

$$\omega_3^2 = (\dot\phi \cos\theta + \dot\psi)^2$$