

University Course

ECE 405
Introduction to communications

Cal Poly, Pomona, California
Summer 2010

My Class Notes

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Summer 2010

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Chapter 1

Introduction

This was a hard course only because it was 5 weeks long and we meet 4 days per week and things went very quickly. We had 6 quizzes, 2 exams, and HW's and computer assignments. The instructor was Dr James Kang, EE dept, and was a very good instructor and explained things really well, but his exams were a little on the hard side.

1.1 syllabus

CALIFORNIA STATE POLYTECHNIC UNIVERSITY, POMONA

<http://www.csupomona.edu/~jskang/spring98/ece405.ht>

CALIFORNIA STATE POLYTECHNIC UNIVERSITY, POMONA
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

SYLLABUS

ECE 405 Communication Systems (4)

Prerequisite: ECE 307 and ECE 315

Instructor: Dr. James S. Kang

Office: 9 - 321

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www.csupomona.edu/~jskang

Text: Lathi and Ding, *Modern Digital and Analog Communication Systems*, 4th ed., Oxford, 2009.

Previous Text: Haykin, *Communication Systems*, 4th ed., Wiley, 2001.

Grading: Quizzes - 40%, Midterm - 25%, Final - 25%, Homework and computer assignment - 10%.

- All exams and quizzes are open book, open notes
- No make-up quizzes or tests are allowed unless approved by the instructor in advance

Topics

Review of Fourier Series and Fourier Transform

Amplitude Modulation and Demodulation

Double-sideband modulation

Amplitude modulation

Single-sideband modulation

AM receiver design

Frequency Modulation and Demodulation

Phase modulation

Frequency modulation

Superheterodyne receiver

ADC and DAC

Ideal sampling

Practical sampling

Pulse code modulation

Differential pulse code modulation

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<http://www.csupomona.edu/~jskang/spring98/ece405.htm>

Adaptive differential pulse code modulation
Delta modulation
Adaptive delta modulation



Baseband Transmission

Line coding
Pulse shaping
Equalizer design



1.2 Text Book

There was an official textbook, but it was not really needed. Taking good notes and working on the given HW's was all what is needed. The text book is below.

Text book was Lathi and Ding, Modern digital and analog communication systems, 4th edition.

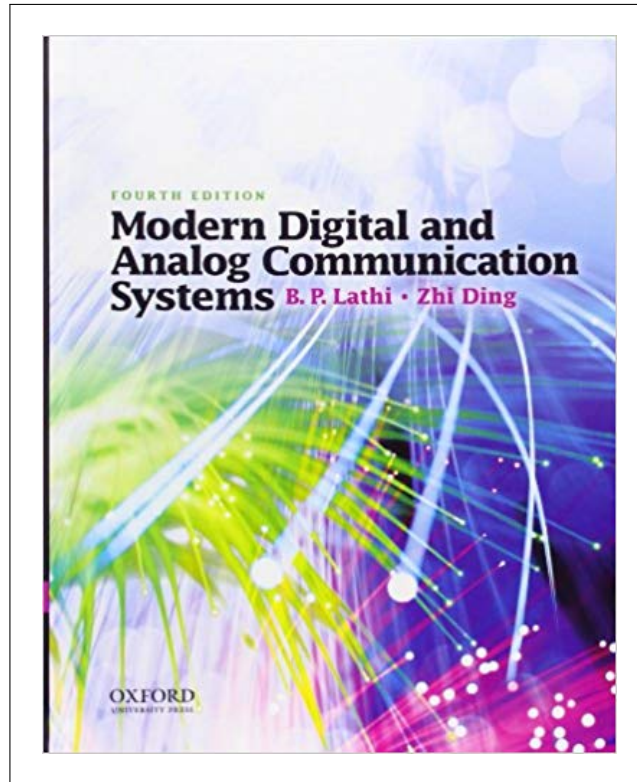


Figure 1.1: text book

Chapter 2

Quizzes

Local contents

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2.1 Quizz 1

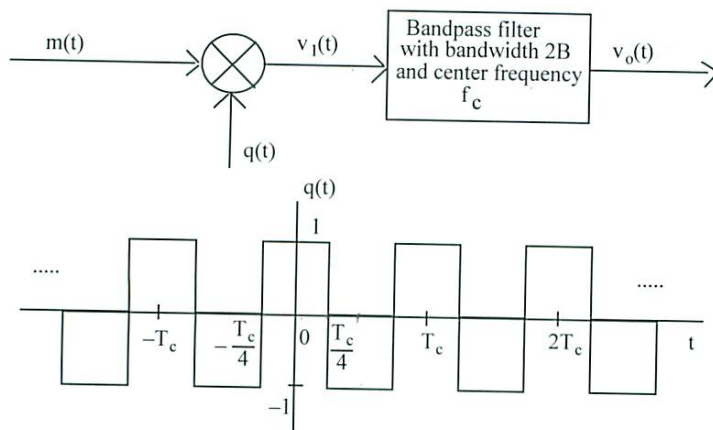
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ECE 405 QUIZ #1 20 POINTS SUMMER 2010

 $\frac{14}{20}$

A model of a balanced modulator is shown below. Let the message signal be $m(t) = 2 \cos(2\pi \times 10000t)$. The frequency of carrier is $f_c = 100000$ Hz so that $T_c = 1/100000$ s. $B = 25$ kHz.

- (a) Plot $m(t)$ in the time domain for $0 \leq t \leq 0.2$ ms.
- (b) Plot the spectrum $M(f) = F[m(t)]$ in the frequency domain.
- (c) Find the exponential Fourier coefficients Q_n of $q(t)$ and represent $q(t)$ by its exponential Fourier series.
- (d) Plot the spectrum $Q(f) = F[q(t)]$ in the frequency domain.
- (e) Plot $v_1(t)$ in the time domain for $0 \leq t \leq 0.2$ ms.
- (f) Plot the spectrum $V_1(f) = F[v_1(t)]$ in the frequency domain for -600 kHz $\leq f \leq 600$ kHz.
- (g) Plot $v_o(t)$ in the time domain for $0 \leq t \leq 0.2$ ms.
- (h) Plot the spectrum $V_o(f) = F[v_o(t)]$ in the frequency domain for -600 kHz $\leq f \leq 600$ kHz.
- (i) The center frequency of the bandpass filter is changed to $5f_c$ with bandwidth 50 kHz, find the expression for $v_o(t)$.



①

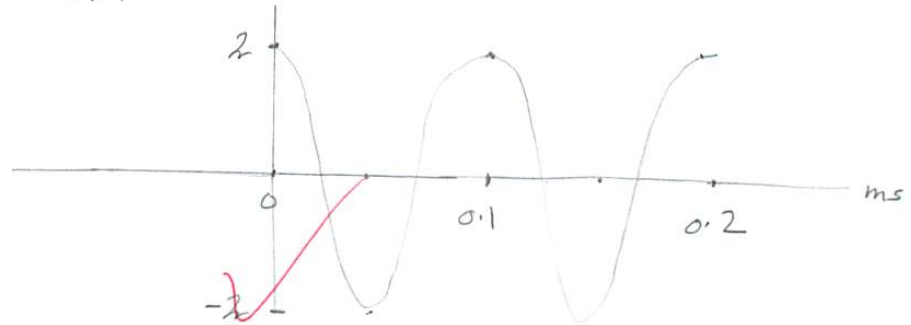
$$m(t) = 2 \cos(2\pi f_m t)$$

$$\text{where } f_m = 10,000 \text{ Hz.}$$

$$f_c = 100,000 \text{ Hz.}$$

$$\text{period of } m(t) = \frac{1}{f_m} = 0.1 \text{ ms.}$$

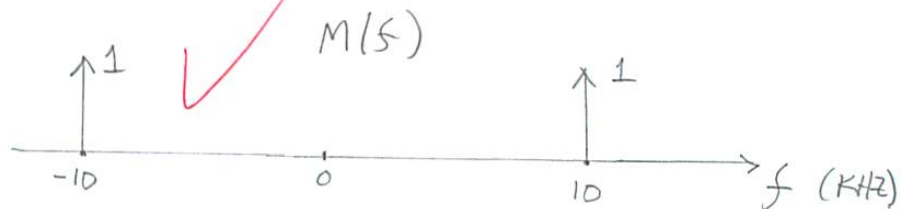
①



②

$$F(m(t)) = \frac{A_m}{2} [\delta(f - f_m) + \delta(f + f_m)] \quad \text{where } A_m = 2 \text{ here.}$$

$$= \delta(f - 10\text{k}) + \delta(f + 10\text{k})$$



$$\textcircled{3} \quad q(t): \text{ period } T_c, h=1, \quad \sigma = \frac{T_c}{2}$$

$$q(t) = \sum_{n=-\infty}^{\infty} Q_n e^{j \frac{2\pi}{T_c} n t}$$

$$\text{where } \boxed{Q_n = \frac{1}{T_c} \int_{T_c} q(t) e^{-j \frac{2\pi}{T_c} n t} dt}$$

≈

②

$$Q_n = h d \operatorname{sinc}(n d)$$

$$\text{where } h = 2, \quad d = \frac{Z}{T} = \frac{T_c}{2 \cdot T_c} = \frac{1}{2}$$

$$\therefore Q_n = \frac{2}{2} \operatorname{sinc}\left(\frac{n}{2}\right) = \operatorname{sinc}\left(\frac{n}{2}\right)$$

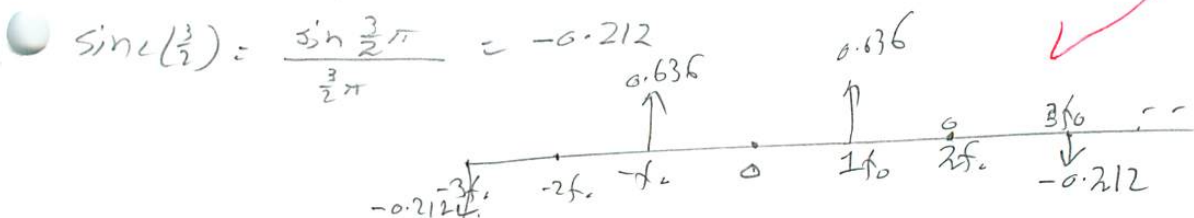
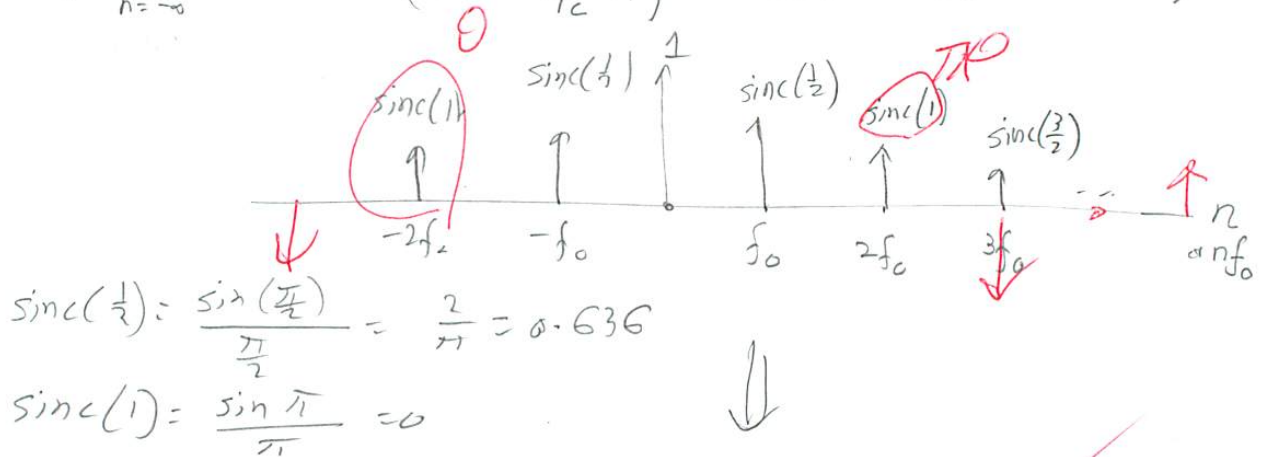
$$\text{so } q(t) = \sum Q_n e^{j \frac{2\pi}{T_c} n t}$$

$$q(t) = \sum \operatorname{sinc}\left(\frac{n}{2}\right) e^{j \frac{2\pi}{T_c} n t}$$

$$\text{where } T_c = 0.001 \text{ ms}$$

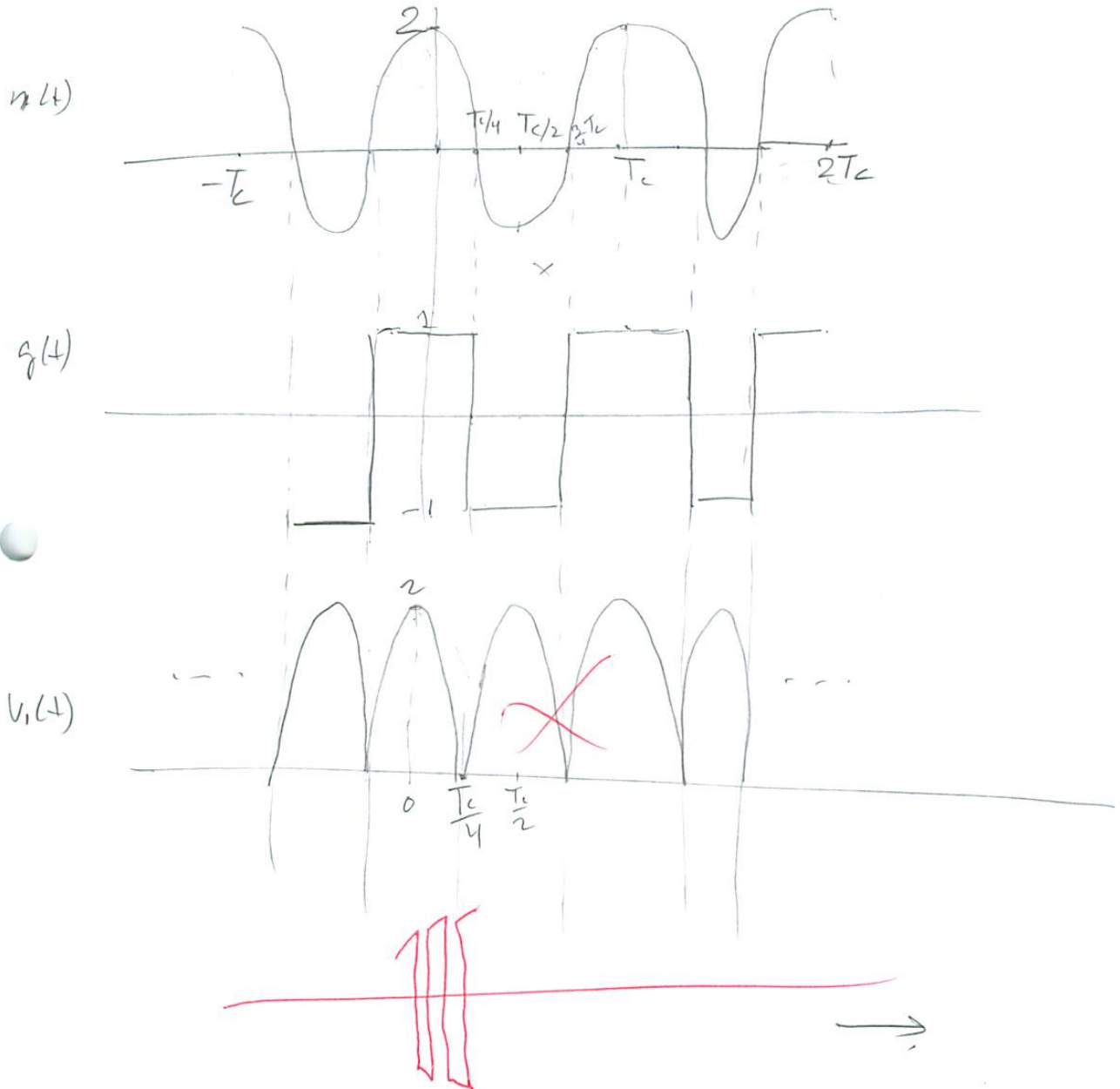
$$\text{(d)} \quad F[q(t)] = \sum \operatorname{sinc}\left(\frac{n}{2}\right) F[e^{j \frac{2\pi}{T_c} n t}] \quad f_0 = \frac{2\pi}{T_c}$$

$$F[q(t)] = \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \delta\left(f - \frac{2\pi}{T_c} n\right) = \sum \operatorname{sinc}\left(\frac{n}{2}\right) \delta(f - n f_0)$$

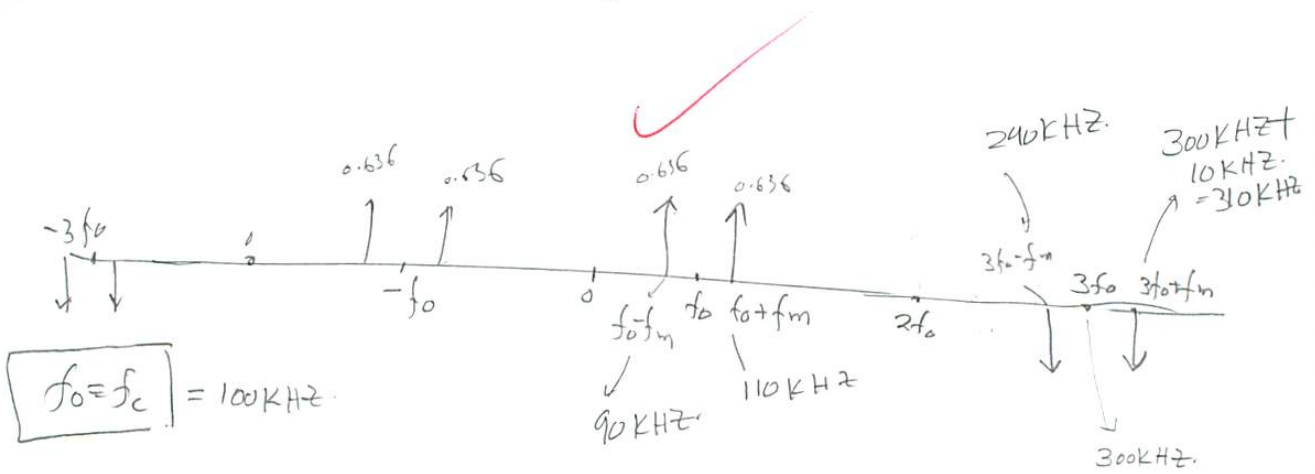
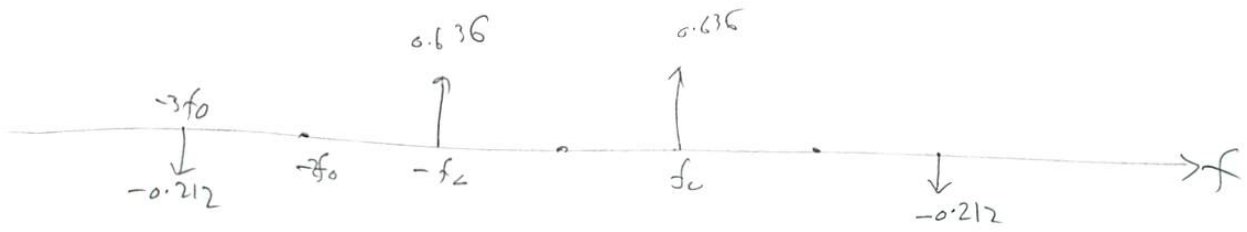
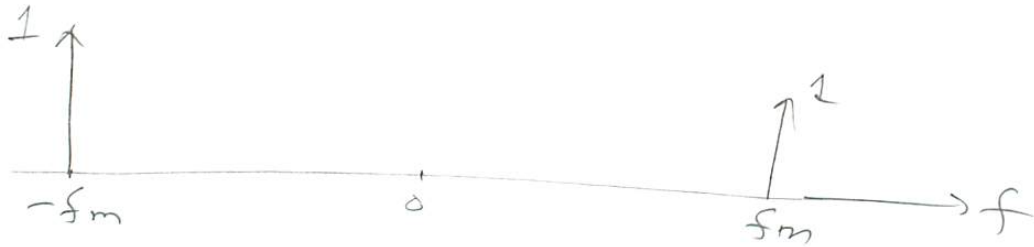


(e) $V_1 = m(t) \cdot q(t)$ (3)

$q(t) = \sum \text{sinc}\left(\frac{n}{2}\right) e^{j \frac{2\pi}{T_c} n t} = \sum \text{sinc}\left(\frac{n}{2}\right) e^{j f_0 n t}$



(5) the spectrum of $v_i(t)$ is $F(m(t)) \otimes F(q(t))$ (4)



(A) X
(B) X
(C) X

2.2 Quizz 2

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1. A message

$$m(t) = 3.0 \cos(2\pi \times 2,000t) + 6.2 \cos(2\pi \times 6,000t)$$

amplitude modulates (AM) a carrier

$$10 \cos(2\pi \times 100,000t)$$

- Plot $m(t)$ in the time domain for $0 \leq t \leq 1$ ms.
- Plot the spectrum $M(f)$ of $m(t)$ in the frequency domain.
- Find the modulation index μ of this AM modulation.
- Plot the AM waveform in the time domain for $0 \leq t \leq 1$ ms.
- Plot the spectrum of the AM waveform in the frequency domain.
- What is the bandwidth of the AM wave?

$$= \frac{1}{2} (\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$\frac{1}{2} (\cos(4) + \cos(17))$$

 $\frac{17}{20}$

$$3 \cos(2t) + 6.2 \cos(6t)$$

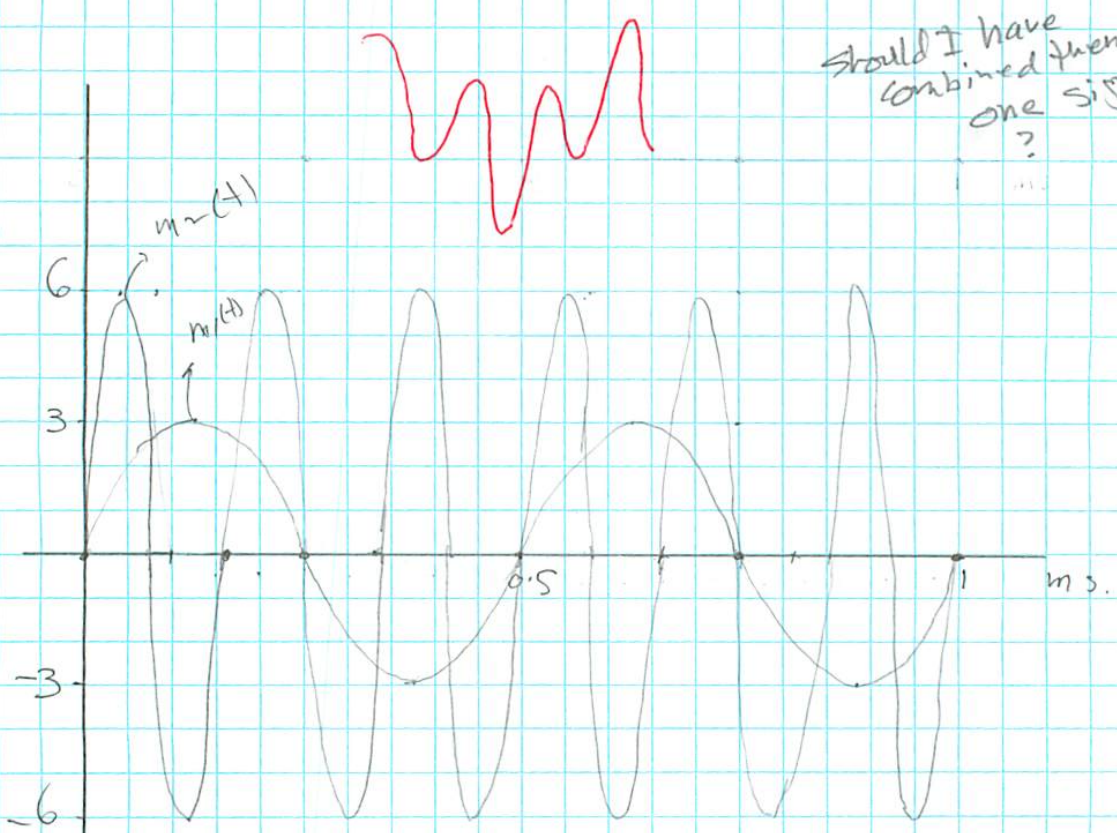
$$m(t) = \overbrace{3.0 \cos(2\pi 2000t)}^{m_1} + \overbrace{6.2 \cos(2\pi 6000t)}^{m_2}$$

$$c(t) = 10 \cos(2\pi 100000t)$$

let $m(t) = A_1 \cos(2\pi f_1 t) + A_2 \cos(2\pi f_2 t)$
 $c(t) = A_c \cos(2\pi f_c t)$

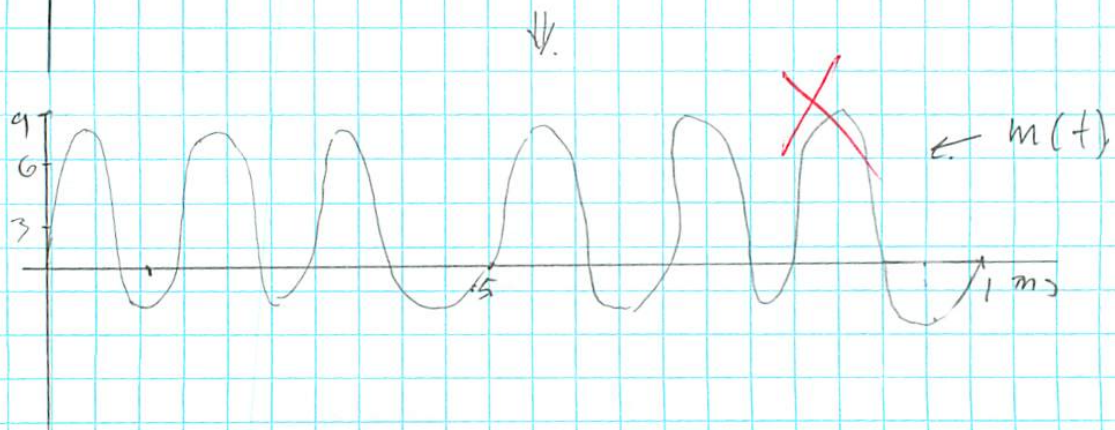
where $f_1 = 2 \text{ KHz}$, $f_2 = 6 \text{ KHz}$, $f_c = 100 \text{ KHz}$.
 $T_1 = \frac{1}{2000} = \frac{1}{2} \text{ ms} = 0.5 \text{ ms}$, $T_2 = \frac{1}{6} \text{ ms} = 0.167 \text{ ms}$

④



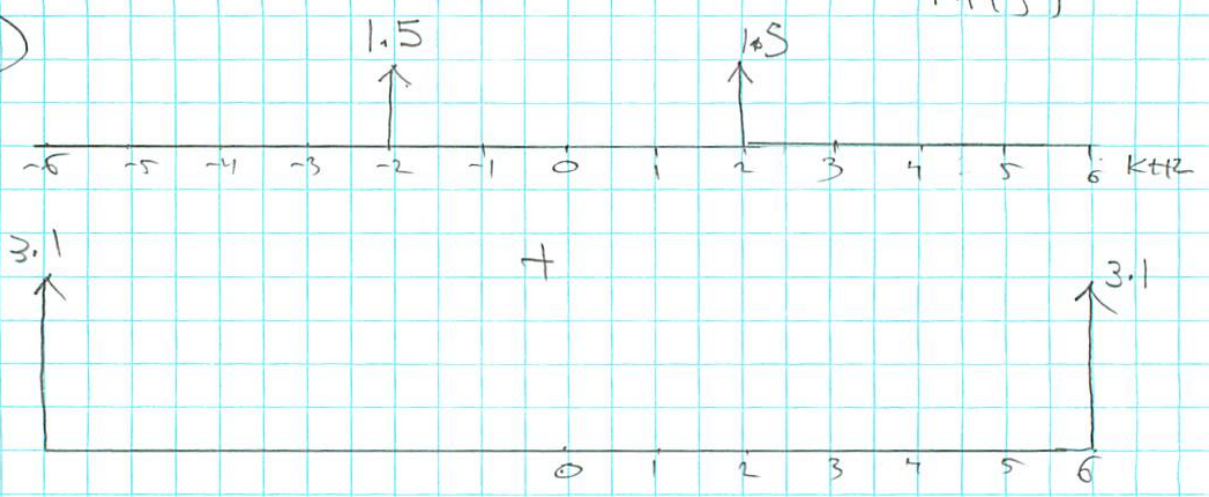
Should I have combined them into one signal?

add

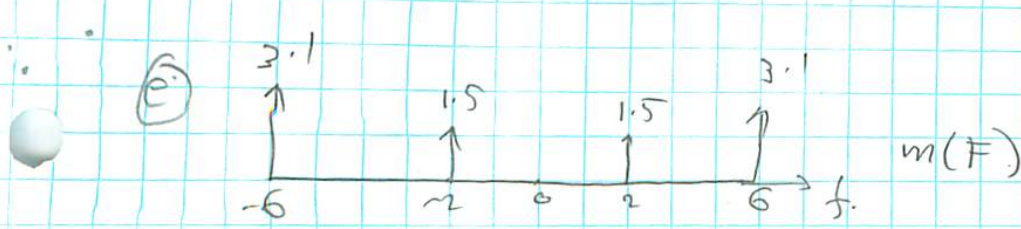


(b)

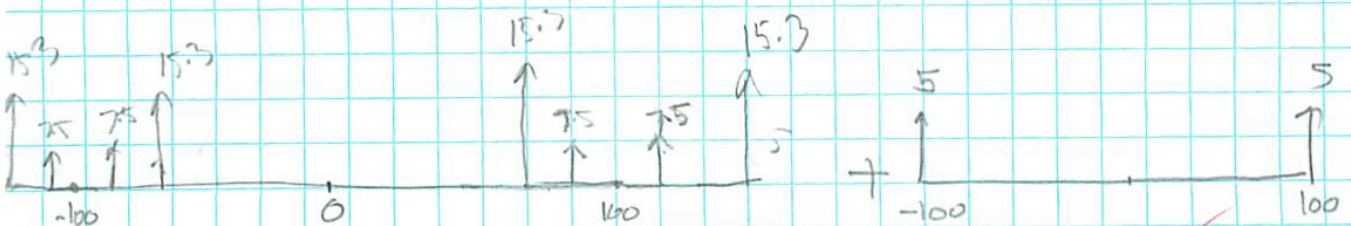
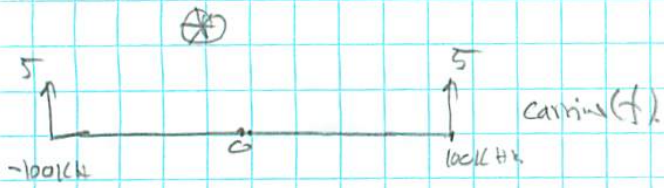
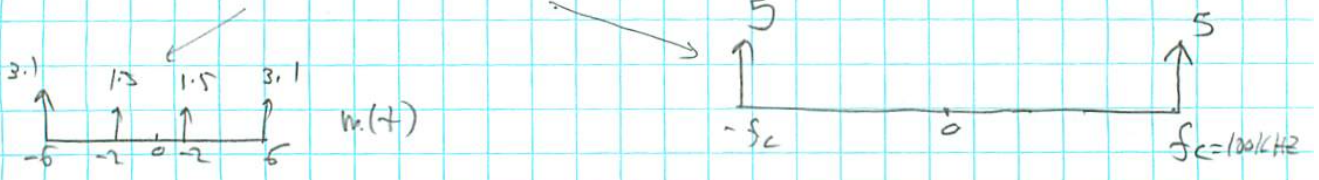
$M_1(f)$



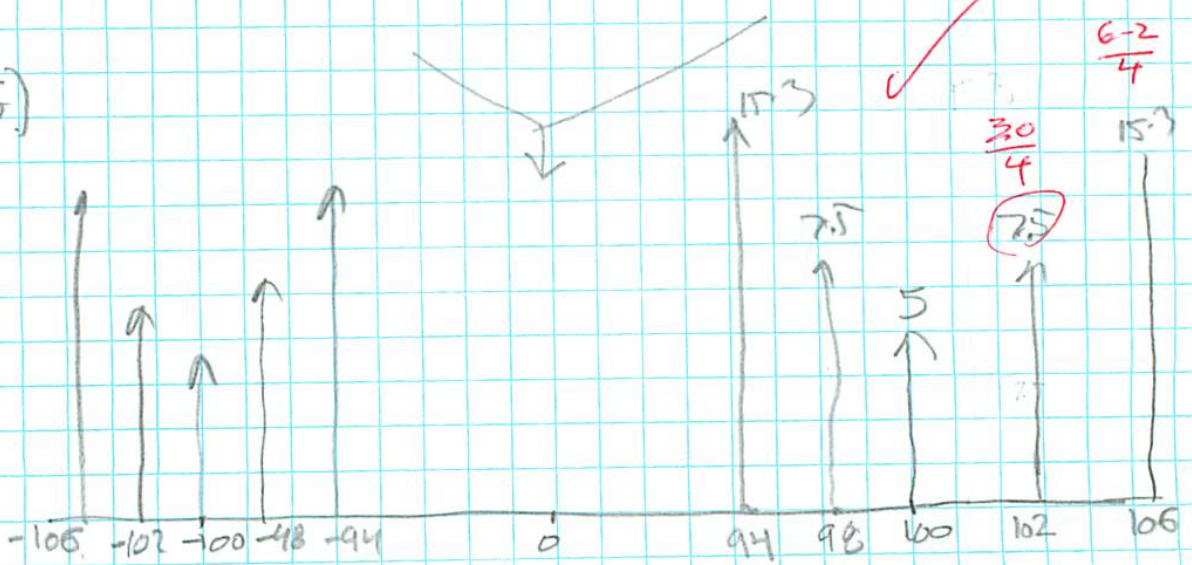
$M(f)$

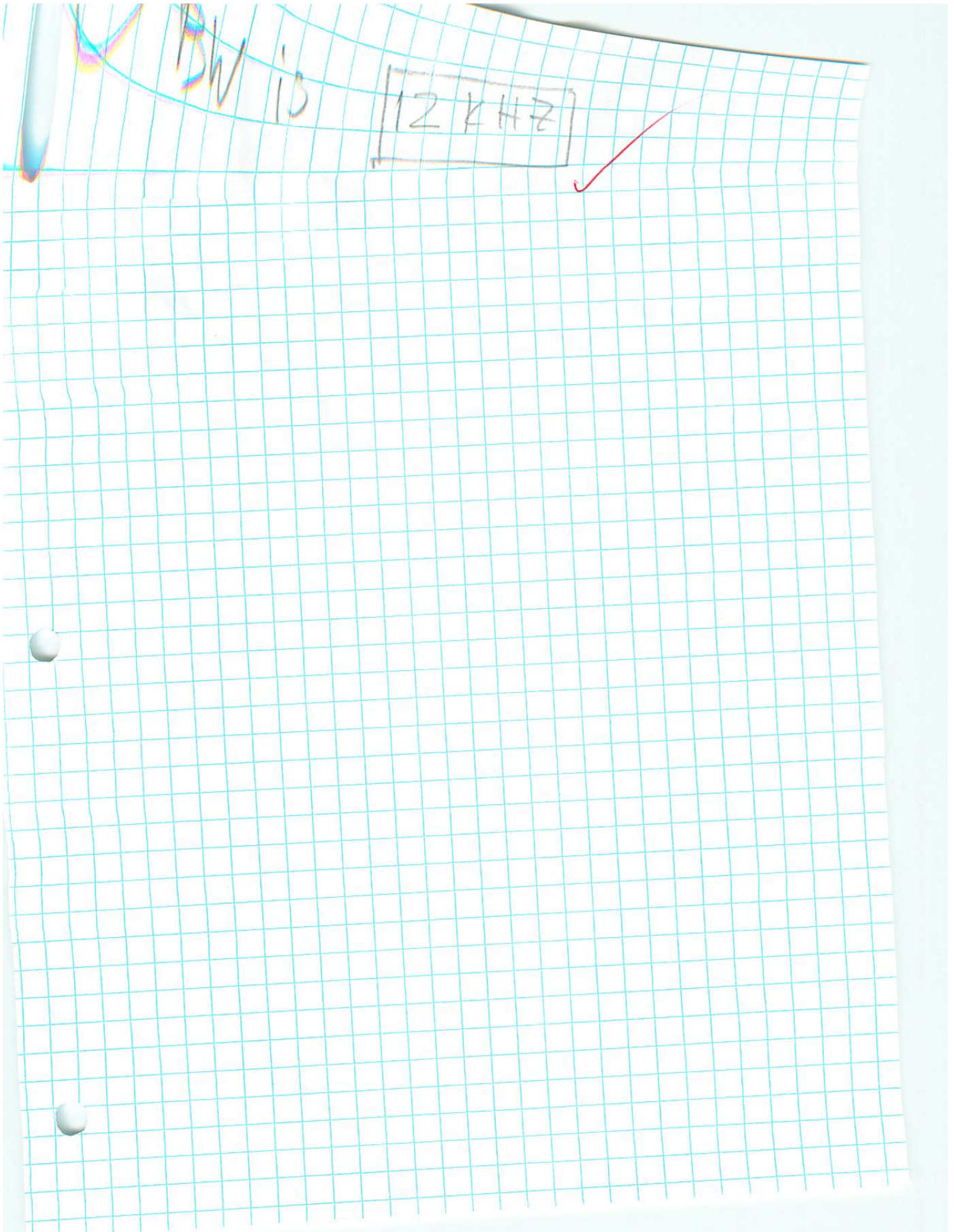


$(M(f)_{\text{convect}}) + A_{\text{convect}}$



$A_m(s)$





2.3 Quizz 3

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20

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1. The block diagram of USSB generation using phasing method is shown in Figure 1. Let $x(t) = 2 \sin(2\pi 3000t)$ and $f_c = 30000$ Hz.

- Plot $x(t)$ in the time domain for two periods starting from $t = 0$.
- Plot the spectrum $X(f)$ of $x(t)$ in the frequency domain.
- Find the Hilbert transform $\hat{x}(t)$ and plot $\hat{x}(t)$ in the time domain for two periods starting from $t = 0$.
- Plot the spectrum $\hat{X}(f)$ of $\hat{x}(t)$ in the frequency domain.
- Find the waveform at (1) and plot it in the time domain.
- Find the spectrum at (1) and plot it in the frequency domain.
- Find the waveform at (2) and plot it in the time domain.
- Find the spectrum at (2) and plot it in the frequency domain.
- Find the waveform at (3) and plot it in the time domain.
- Find the spectrum at (3) and plot it in the frequency domain.

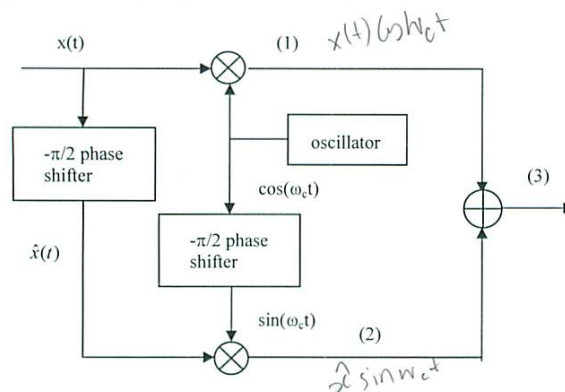


Figure 1

$$\begin{aligned} \sin(\alpha + \beta) &= \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta) \\ \sin(\alpha - \beta) &= \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta) \\ \cos(\alpha + \beta) &= \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta) \\ \cos(\alpha - \beta) &= \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta) \\ \sin(\alpha) \cos(\beta) &= (1/2) [\sin(\alpha - \beta) + \sin(\alpha + \beta)] \\ \sin(\alpha) \sin(\beta) &= (1/2) [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \cos(\alpha) \cos(\beta) &= (1/2) [\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\ \cos(\alpha) \sin(\beta) &= (1/2) [-\sin(\alpha - \beta) + \sin(\alpha + \beta)] \end{aligned}$$

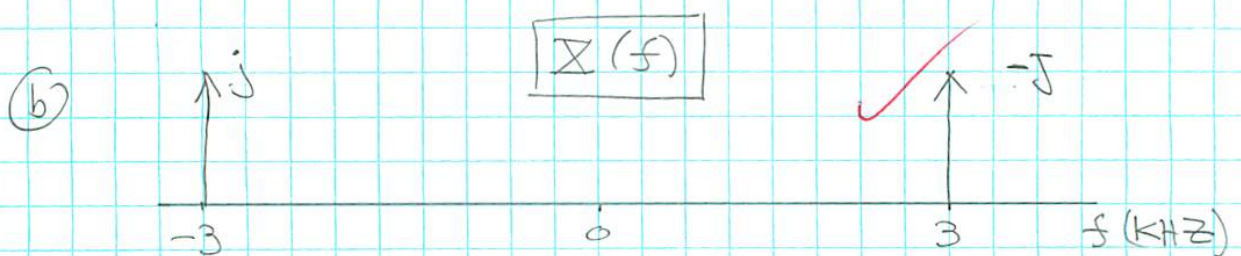
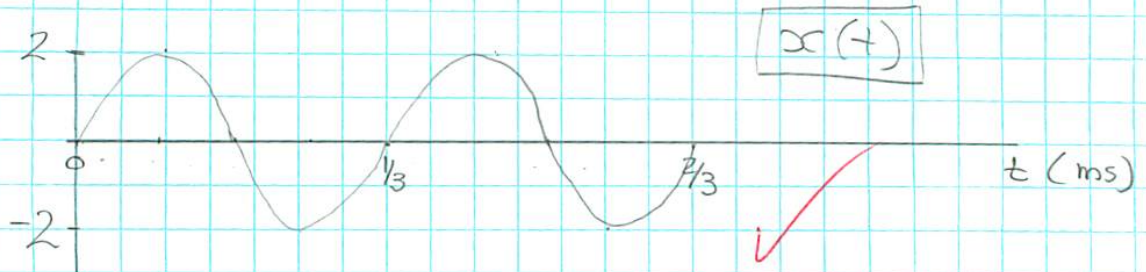
$$x(t) = 2 \sin(2\pi \cdot 3000t - \frac{\pi}{2})$$



①

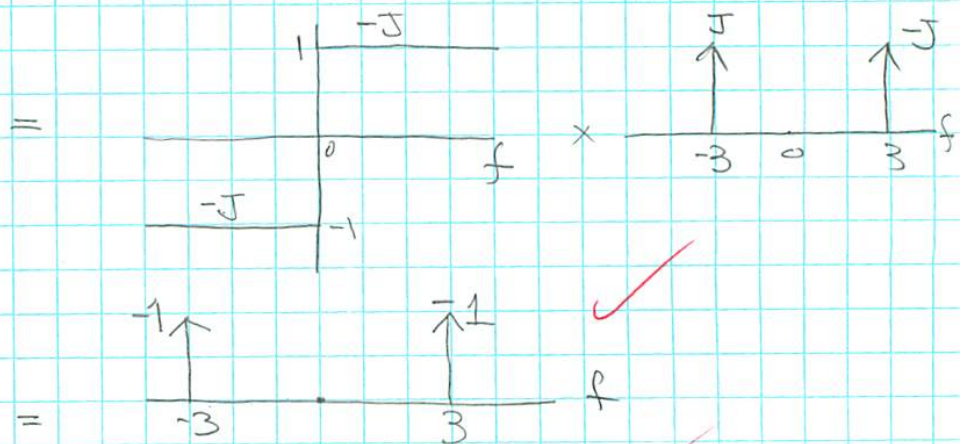
$$x(t) = 2 \sin(2\pi 3000t) \quad f_c = 30,000 \text{ Hz}$$

a) $f_m = 3000 \text{ Hz}$. so $T_0 = \frac{1}{3000} = \frac{1}{3} \text{ ms}$.



c) $\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$

or $\hat{X}(f) = -j \operatorname{sgn}(f) X(f)$

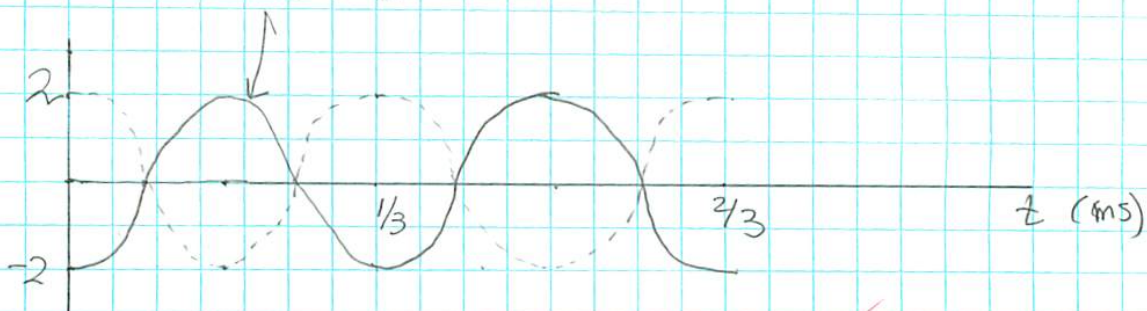


so $\hat{x}(t) = \boxed{-2 \cos(2\pi 3000t)}$

Plot $\hat{x}(t)$ for 2 periods

(2)

$$\hat{x}(t) = -2 \cos(2\pi 3000t)$$



(d) Plot $\hat{x}(f)$.

I already did this:



(e) wave-form at (1) is $\boxed{x(t) \cos \omega_c t}$

$$s_1(t) = 2 \sin(2\pi 3000t) \cdot \cos(2\pi 3000t)$$

$$= 2 \left[\frac{1}{2} \left[\sin(-27000t) + \sin(33000t) \right] \right]$$

$$\boxed{s_1(t) = -\sin(2\pi 27000t) + \sin(33000t + 2\pi)}$$

Plot
in
time
domain
next
page.

$$\text{so } S_1(f) = - \left[-j/2 \delta(f-27000) + j/2 \delta(f+27000) \right]$$

$$+ \left[-j/2 \delta(f-33000) + j/2 \delta(f+33000) \right]$$

$$= j/2 \delta(f-27^k) - j/2 \delta(f+27^k) - j/2 \delta(f-33^k) + j/2 \delta(f+33^k)$$

→

$$s_1(f) = \frac{j}{2} \left[\delta(f-27000) - \delta(f+27000) - \delta(f-33000) + \delta(f+33000) \right] \quad (3)$$

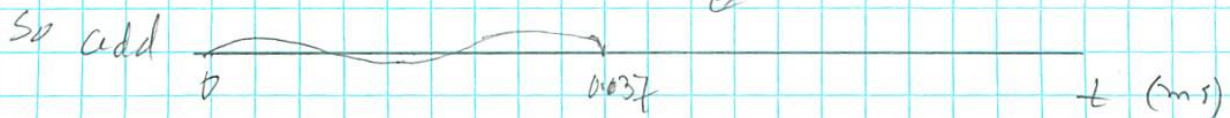
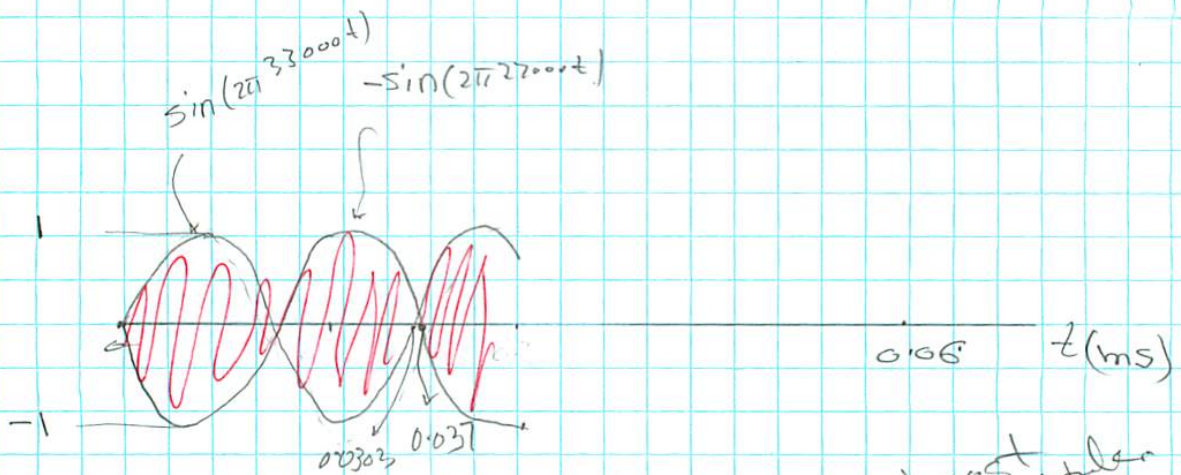


Plot $s_1(t)$ in time domain:

$$s_1(t) = -\sin(2\pi \cdot 27000t) + \sin(2\pi \cdot 33000t)$$

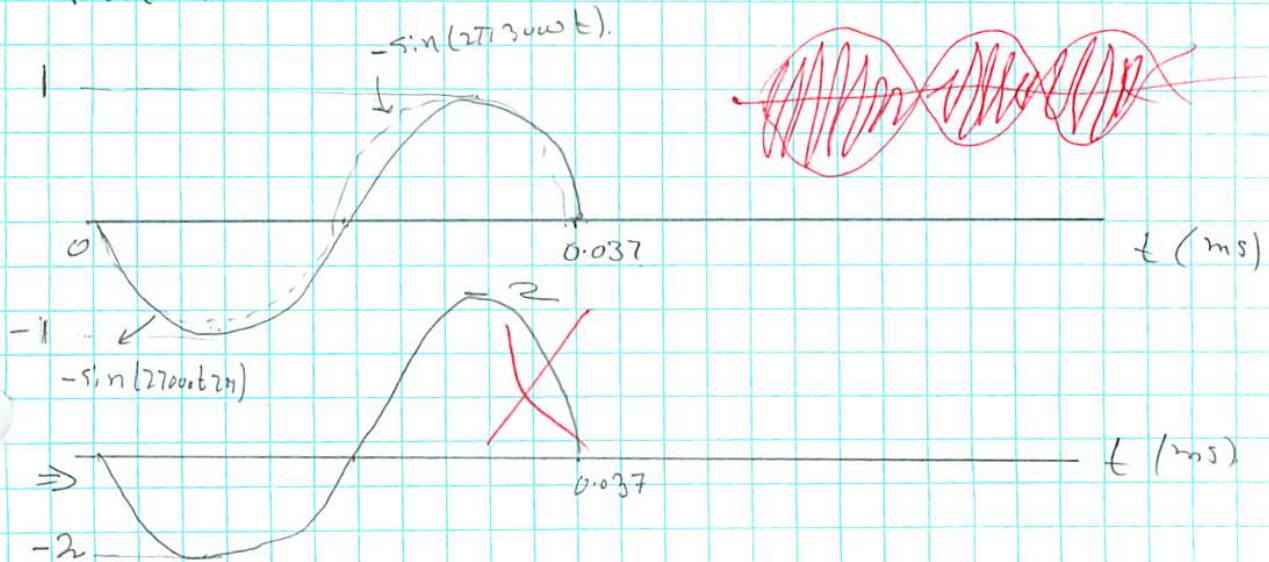
$$f = 27000 \Rightarrow T = \frac{1}{27000} = 0.037 \text{ ms} \quad \text{for period}$$

$$f = 33000 \Rightarrow T = \frac{1}{33000} = 0.0303 \text{ ms} \quad \text{for period}$$



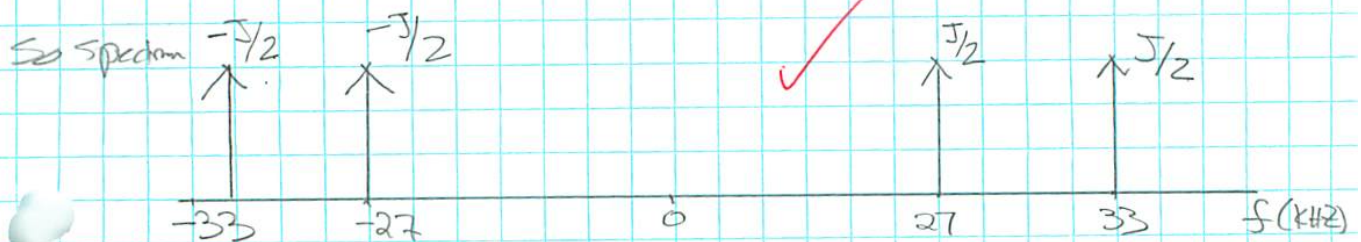
$$\begin{aligned}
 \textcircled{g} \quad S_2(t) &= \hat{x}(t) \sin(2\pi f_c t) && \textcircled{4} \\
 &= -2 \cos(2\pi 3000t) \sin(2\pi 3000t) \\
 &= -2 \left[\frac{1}{2} (-\sin(-27000t + 2\pi)) + \sin(2\pi 33000t) \right] \\
 &= -\sin(27000t + 2\pi) - \sin(2\pi 33000t)
 \end{aligned}$$

Plot in time domain.



h) Spectrum is

$$\begin{aligned}
 S_2(f) &= - \left[-\frac{j}{2} \delta(f-27000) + \frac{j}{2} \delta(f+27000) \right] \\
 &\quad - \left[-\frac{j}{2} \delta(f-33000) + \frac{j}{2} \delta(f+33000) \right] \\
 &= \frac{j}{2} \delta(f-27000) - \frac{j}{2} \delta(f+27000) + \frac{j}{2} \delta(f-33000) - \frac{j}{2} \delta(f+33000)
 \end{aligned}$$



(1)

5

Since S_{USSB} , then need to subtract

$$S_1(f) - S_2(f) \text{ to find } S_3(f).$$

First, in time domain:

$$S_3(t) = x(t) \cos \omega_c t - \hat{x}(t) \sin \omega_c t.$$

$$= S_1(t) - S_2(t)$$

$$= (-\sin(2\pi 27000t) + \sin(33000t 2\pi))$$

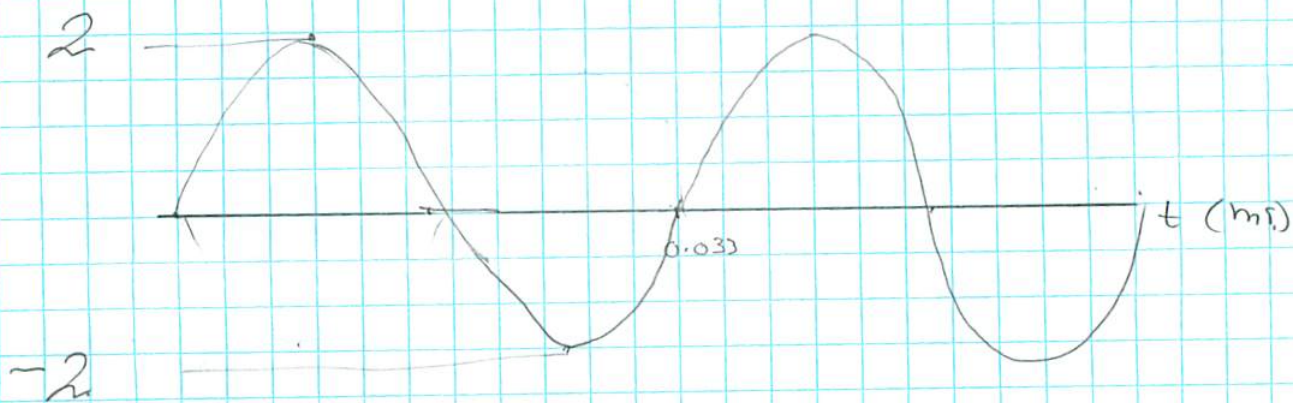
$$- (-\sin(2\pi 27000t) - \sin(2\pi 33000t))$$

$$= -\sin(2\pi 27000t) + \sin(33000t 2\pi) + \sin(2\pi 27000t) + \sin(2\pi 33000t)$$

$$S_3(t) = 2 \sin(33000t 2\pi)$$

Plot in time domain:

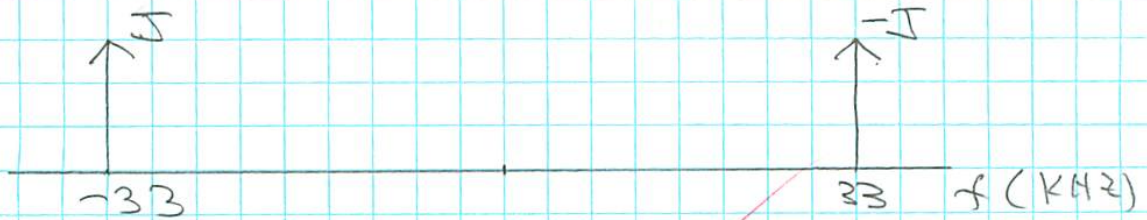
$$2 \sin(33000t)$$



(J) spectrum is

(6)

since $S_3(t) = 2 \sin(2\pi 33000t)$ then
spectrum is



$$S_3(f) = -J\delta(f-33000) + J\delta(f+33000)$$

which is also $S_1(f) - S_2(f)$ OK verified

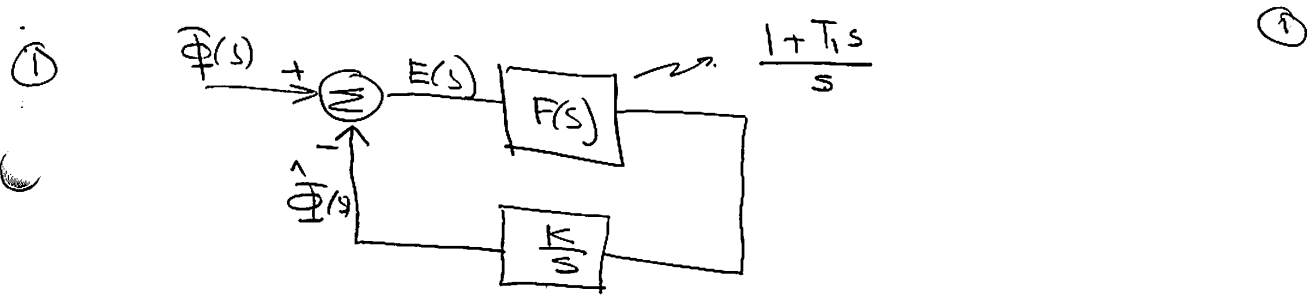
2.4 Quizz 4

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ECE 409 QUIZ #4 20 POINTS

16
20

1. The loop filter of the second-order analog PLL is given by $F(s) = \frac{1+T_1s}{s}$. Assume that $K = 0.32$, $T_1 = 15/4$.
- (a) Find the open loop transfer function $G(s) = F(s)K/s$.
 - (b) Find the closed-loop transfer function $H(s) = \frac{\hat{\Phi}(s)}{\Phi(s)}$.
 - (c) Find and plot the impulse response $h(t)$ of this APLL.
 - (d) Find and plot the step response $h_s(t)$ of this APLL.
 - (e) Find and plot the error response $h_e(t)$ to step input of this APLL.



② $G(s) = F(s) \frac{K}{s} = \left(\frac{1+T_1s}{s} \right) \left(\frac{K}{s} \right)$

③ $H(s) = \frac{\hat{\Phi}(s)}{\Phi(s)} = \frac{E(s) G(s)}{\Phi(s)} = \frac{[\Phi(s) - \hat{\Phi}(s)] G(s)}{\Phi(s)}$

$$= \frac{\Phi(s) G(s) - \hat{\Phi}(s) G(s)}{\Phi(s)} = G(s) - \frac{\hat{\Phi}(s)}{\Phi(s)} G(s)$$

$H(s) = G(s) - H(s) G(s)$

Solve for $H(s)$

$$H(s) = \frac{G(s)}{1+G(s)}$$

here $H(s) = \frac{\frac{1+T_1s}{s} \frac{K}{s}}{1 + \frac{1+T_1s}{s} \frac{K}{s}} = \frac{(1+T_1s)K}{s^2 + (1+T_1s)K}$

$$H(s) = \frac{K + K T_1 s}{s^2 + T_1 K s + K}$$

→

$$H(s) = \frac{K + K T_1 s}{s^2 + T_1 K s + K} \quad (2)$$

$$K = \frac{32}{100}, \quad T_1 = \frac{15}{4}$$

$$H(s) = \frac{\frac{32}{100} + \frac{32}{100} \frac{15}{4} s}{s^2 + \frac{15}{4} \frac{32}{100} s + \frac{32}{100}} = \frac{0.32 + 1.2s}{s^2 + 1.2s + 0.32}$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1.2 \pm \sqrt{1.2^2 - 4(0.32)}}{2}$$

$$= -0.6 \pm \frac{1}{2} \sqrt{1.44 - 1.28} = -0.6 \pm \frac{1}{2} \sqrt{0.16}$$

$$= -0.6 \pm \frac{1}{2} (0.4) = -0.6 \pm 0.2$$

$$s_1 = -0.4, \quad s_2 = -0.8$$

note poles < 0
hence stable system

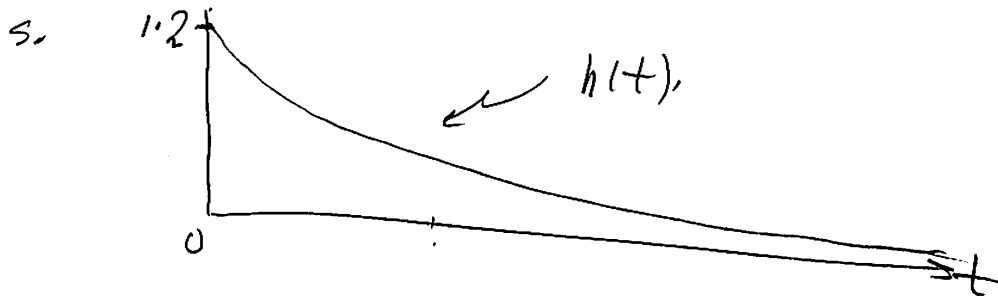
$$H(s) = \frac{0.32 + 1.2s}{(s+0.4)(s+0.8)} = \frac{A}{s+0.4} + \frac{B}{s+0.8}$$

$$A = \left. \frac{0.32 + 1.2s}{s+0.8} \right|_{s=-0.4} = \frac{(0.32) + 1.2(-0.4)}{(-0.4) + (0.8)} = \frac{-0.16}{0.4} = -0.4$$

$$B = \left. \frac{0.32 + 1.2s}{s+0.4} \right|_{s=-0.8} = \frac{(0.32) + 1.2(-0.8)}{-0.8 + 0.4} = \frac{0.32 - 0.96}{-0.4} = 1.6$$

$$H(s) = \frac{-0.45}{s+0.4} + \frac{1.6}{s+0.8}$$

$$s. \quad h(t) = \left[-0.45 e^{-0.4t} + 1.6 e^{-0.8t} \right] u(t) \quad (3)$$



(A) step response is when $\Phi(s) = u(t)$.

$$s. \quad \Phi(s) = \frac{1}{s}$$

hence $\hat{\Phi}(s) = \Phi(s) H(s)$

$$= \frac{1}{s} \frac{0.32 + 1.2s}{s^2 + 1.2s + 0.32}$$

$$= \frac{1}{s} \frac{0.32 + 1.2s}{(s+0.4)(s+0.8)}$$

$$s. \quad \hat{\Phi}(s) = \frac{A}{s} + \frac{B}{s+0.4} + \frac{C}{s+0.8}$$

$$A = \frac{0.32 + 1.2s}{(s+0.4)(s+0.8)} \Big|_{s=0} = \frac{0.32}{(0.4)(0.8)} = 1$$

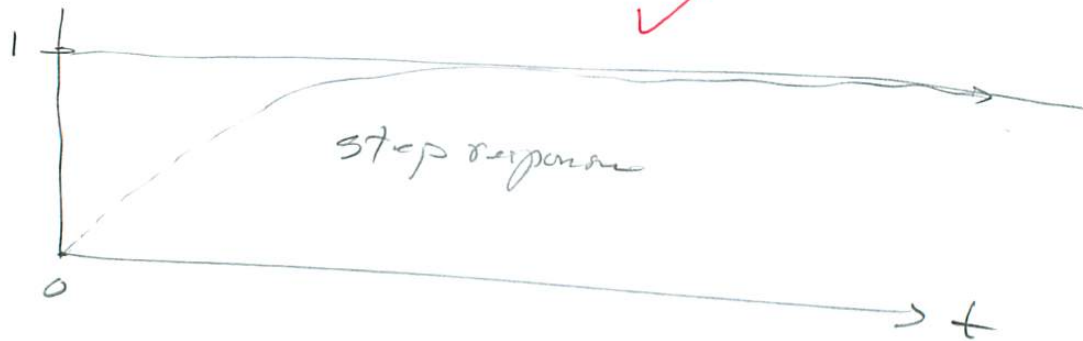
$$B = \frac{0.32 + 1.2s}{s(s+0.8)} \Big|_{s=-0.4} = \frac{(0.32)(1.2(-0.4))}{(-0.4)(-0.4+0.8)} = \frac{-0.16}{-0.16} = 1$$

$$C = \frac{0.32 + 1.2s}{s(s+0.4)} \Big|_{s=-0.8} = \frac{(0.32) - (1.2)(0.8)}{(-0.8)(-0.8+0.4)} = \frac{-0.64}{-0.32} = 2$$

$$\text{so } \hat{\Phi}(s) = \frac{1}{s} + \frac{1}{s+0.4} - \frac{2}{s+0.8}$$

⑦

$$\begin{aligned} \text{so } \hat{\Phi}(t) &= u(t) + e^{-0.4t} u(t) - 2e^{-0.8t} u(t) \\ &= (1 + e^{-0.4t} - 2e^{-0.8t}) u(t) \end{aligned}$$



$$\text{at } t=0, \quad 1 + 1 - 2 = 0$$

$$\text{at } t \rightarrow \infty \quad = 1$$

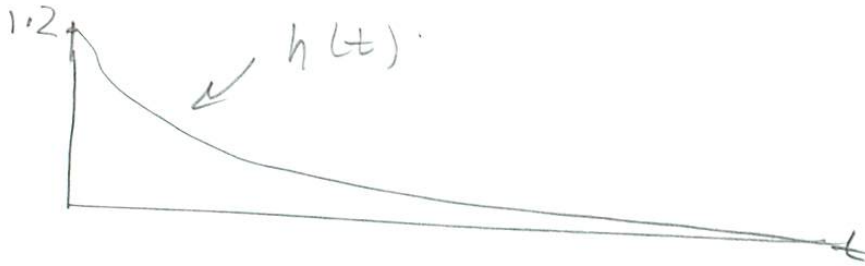
$$\text{at } t=1 \rightarrow 1 + 0.67 - 2(0.449) = 0.772$$

→

 $h_e(t)$

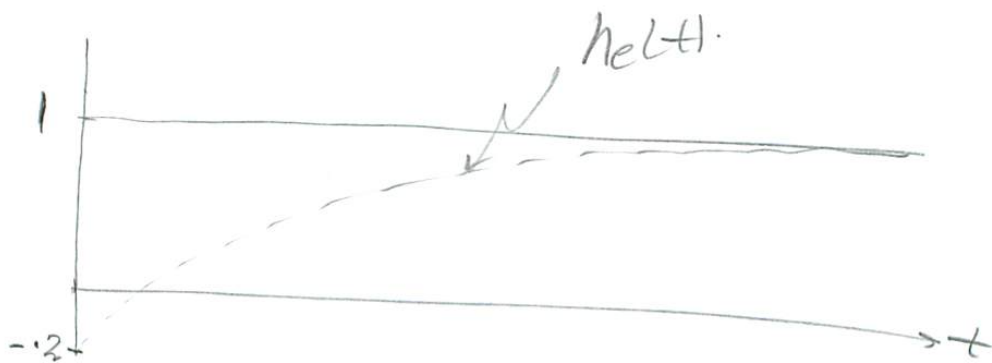
$$\text{since } H_e(s) = 1 - H(s)$$

$$\text{then } h_e(t) = 1 - h(t) \quad \longrightarrow$$



⑤

so $1-h(t)$ is



at $t=0$, $1-1.2 = -.2$

at $t=1$, $1-.8 = .2$

at $t \rightarrow \infty$, $he(t) = 1-0 = 1$

$$he(t) = \left[\cancel{1} + \cancel{0.4} e^{-0.4t} - \cancel{1.6} e^{-0.8t} \right] u(t).$$

$$(2e^{-0.8t} - e^{-0.4t}) u(t)$$

2.5 Quizz 5

Nasser M. Abbasi

20
/ 20

ECE 405 QUIZ #5 20 POINTS

1. A message $m(t) = 5 \cos(2\pi \times 3000t)$ frequency modulates a carrier of frequency 30 MHz. Let $k_f = 2\pi \times 3000$ rad/s/V and let the amplitude of the modulated wave be 0.2 V. Determine the output signal-to-noise ratio in dB if the one-sided noise power spectral density of the receiver is $N_0 = 10^{-6}$ W/Hz (no deemphasis circuit is used).
2. A message $m(t) = 5 \cos(2\pi \times 3000t)$ frequency modulates a carrier of frequency 30 MHz. Let $k_f = 2\pi \times 3000$ rad/s/V and let the amplitude of the modulated wave be 0.2 V. Determine the output signal-to-noise ratio in dB if the one-sided noise power spectral density of the receiver is $N_0 = 10^{-6}$ W/Hz and deemphasis is used at the receiver with $\omega_a = \pi f_m$ and $B = f_m$.
3. Determine the minimum value for the carrier amplitude A_c if the output signal-to-noise ratio is 25 dB, and the one-sided noise power spectral density of the receiver is $N_0 = 10^{-7}$ W/Hz, $\beta = 5$, $f_m = 15$ kHz.

①

$$m(t) = 5 \cos(2\pi 3000t)$$

$$A_m = 5, \quad f_m = 3000 \text{ Hz.}$$

$$K_f = 2\pi 3000 \text{ rad/s/V.}$$

$$A_c = 0.2 \text{ V.}$$

this is FM. \Rightarrow



$$(\text{SNR})_o = \frac{3}{4} \frac{A_c^2 \beta^2}{N_o B = f_m}$$

$$\text{But } \beta = \frac{K_f A_m}{2\pi f_m} = \frac{(2\pi 3000)(5)}{2\pi(3000)} = \boxed{5}$$

$$\Rightarrow (\text{SNR})_o = \frac{3}{4} \frac{(0.2)^2 (5)^2}{(10^{-5})(3000)} = 250$$

$$\Rightarrow (\text{SNR})_{o, \text{dB}} = 10 \log_{10}(250) = \boxed{23.979} \text{ dB}$$

(2)

#2

$$m(t) = 5 \cos(2\pi 3000t)$$

$$k_f = 2\pi 3000 \text{ rad/s/V}$$

$$A_c = 0.2 \text{ V}$$

$$(\text{SNR})_o = \frac{2\pi A_c^2 k_f^2 \overline{m^2(t)}}{2N_0 \omega_a^2 \left[2\pi B - \omega_a \tan^{-1}\left(\frac{2\pi B}{\omega_a}\right) \right]}$$

$$\overline{m^2(t)} = \frac{A_m^2}{2} = \frac{25}{2} = \boxed{12.5 \text{ W}} \cdot t$$

$$\omega_a = \pi f_m = \pi (3000)$$

$$A_c = 0.2$$

$$\begin{aligned} \text{hence } (\text{SNR})_o &= \frac{\pi (0.2)^2 (2\pi 3000)^2 (12.5)}{(10^{-6}) (\pi 3000)^2 \left[2\pi (3000) - \pi 3000 \tan^{-1}\left(\frac{2\pi 3000}{\pi 3000}\right) \right]} \\ &= \frac{\pi (0.2)^2 (12.5) (4)}{(10^{-6}) \left[2\pi (3000) - \pi 3000 \tan^{-1}(2) \right]} \end{aligned}$$

$$\boxed{\tan^{-1}(2) = 1.1071} \rightarrow \text{radians}$$

$$\begin{aligned} \therefore (\text{SNR})_o &= \frac{6.28318}{(10^{-6}) [18849.55 - 10434.17]} = \frac{6.28318}{0.008415} \\ &= 746.63 \end{aligned}$$

$$\therefore (\text{SNR})_{o, \text{dB}} = 10 \log_{10}(746.63) = \boxed{28.73}$$

#3

3

$$(SNR)_o = \frac{3}{4} \frac{A_c^2 \beta^2}{N_o B}$$

$$\Rightarrow 10 \log_{10} \left(\frac{3}{4} \frac{A_c^2 \beta^2}{N_o B} \right) = 25$$

$$\text{But } \beta = 5, B = 15 \times 10^3 \text{ Hz}, N_o = 10^{-7}$$

$$\Rightarrow \log_{10} \left(\frac{3}{4} \frac{A_c^2 (5)^2}{(10^{-7})(15 \times 10^3)} \right) = 2.5$$

$$\Rightarrow \log_{10} (12500 A_c^2) = 2.5$$

$$\Rightarrow 12500 A_c^2 = 10^{2.5}$$

$$\Rightarrow A_c^2 = \frac{10^{2.5}}{12500} \Rightarrow A_c = \sqrt{\frac{10^{2.5}}{12500}}$$

$$\therefore A_c = 0.159 \text{ Volt} \quad \checkmark$$

$$\Rightarrow \text{min } A_c = 0.159 \text{ V}$$

2.6 Quizz 6

Nasser Abbasi

ECE 405 QUIZ #6 20 POINTS

20/20

1. A sinusoidal message $x(t) = 2 \cos(2\pi 2000t)$ is sampled at a rate of 10000 samples per second ($f_s = 10000$, $T_s = 1/10000$). The sampling signal $p(t)$ is a rectangular pulse train with period $1/10000$ seconds, amplitude $h = 1V$, and duty cycle $d = 1/3$.

- ✓ (a) Plot $x(t)$ in the time domain for two periods.
- ✓ (b) Plot the spectrum $X(f)$ in the frequency domain.
- ✓ (c) Find the expression of $P(f)$ and plot the spectrum $P(f)$ of the pulse train in the frequency domain.
- ✓ (d) Find expression of the sampled waveform $x_s(t)$, and plot $x_s(t)$ in the time domain for two periods of $x(t)$.
- ✓ (e) Find the expression of the spectrum $X_s(f)$ of the sampled waveform, and plot $X_s(f)$ in the frequency domain for $-6f_s \leq f \leq 6f_s$.
- (f) The sampled signal is applied to an ideal lowpass filter with bandwidth $f_s/2$. Find the expression of the spectrum $Y(f)$ of the output signal, and plot $Y(f)$ in the frequency domain.
- (g) The sampled signal is applied to an ideal lowpass filter with bandwidth $f_s/2$. Find the expression $y(t)$ of the output signal, and plot $y(t)$ in the time domain for two periods.

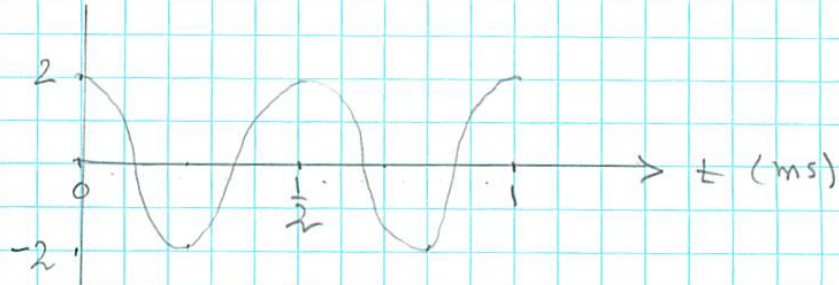
$$t = nT_s = n \frac{1}{10000}$$

$$\left(\frac{t}{T_s} - n \right) = \left(\frac{t}{1/10000} - n \right) = (30000t - 3n)$$

$$x(t) = 2 \cos(2\pi 2000t) \Rightarrow f_1 = 2000 \text{ Hz}, T_1 = \frac{1}{2} \text{ ms.} \quad (1)$$

$$f_s = 10,000 \text{ Hz.}$$

(a)



(b)



$$(c) \quad P(t) = \sum_{n=-\infty}^{\infty} h \operatorname{rect}\left(\frac{t - nT_s}{\tau}\right) \quad \therefore \begin{array}{c} \uparrow \tau \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ -T_s \quad 0 \quad T_s \quad 2T_s \\ \leftarrow T_s \rightarrow \end{array} t$$

$$= \sum \operatorname{rect}\left(\frac{t - nT_s}{\tau}\right)$$

but $\frac{\tau}{T_s} = d = \frac{1}{3}$, and $T_s = \frac{1}{10000}$ so $10,000\tau = \frac{1}{3} \Rightarrow \tau = \frac{1}{3} \frac{1}{10,000}$

$$\text{so } \hat{p}(t) = \sum_{n=-\infty}^{\infty} \frac{\tau}{T_s} \operatorname{sinc}(nd) e^{j\frac{2\pi}{T_s}nt}$$

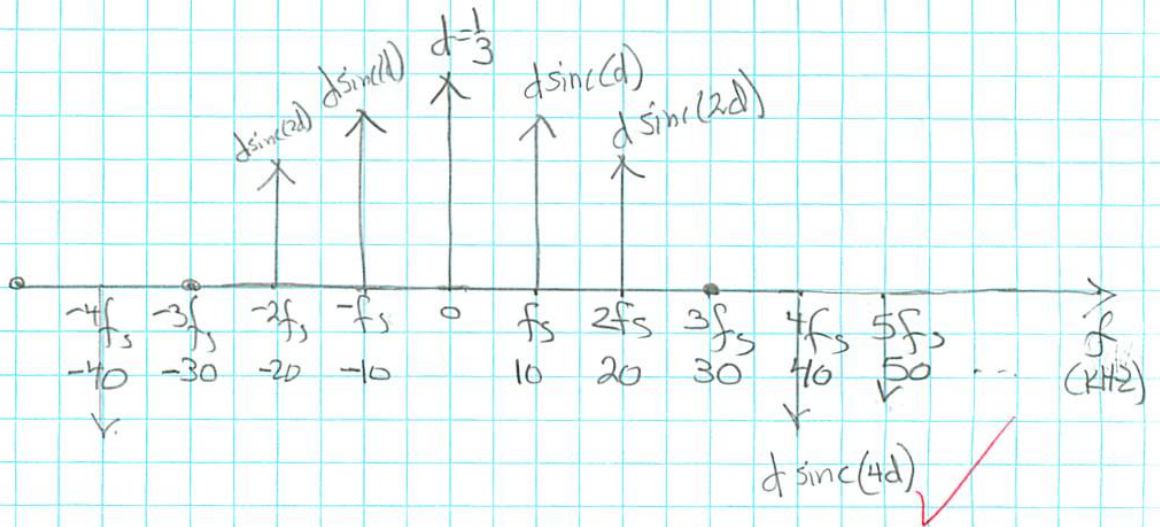
$$\text{so } P(f) = \frac{\tau}{T_s} \sum_{n=-\infty}^{\infty} \operatorname{sinc}(nd) \delta(f - n f_s)$$

$$\text{or } P(f) = \frac{1}{3} \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{n}{3}\right) \delta(f - n 10000)$$

→

Plot $P(f)$:

(2)

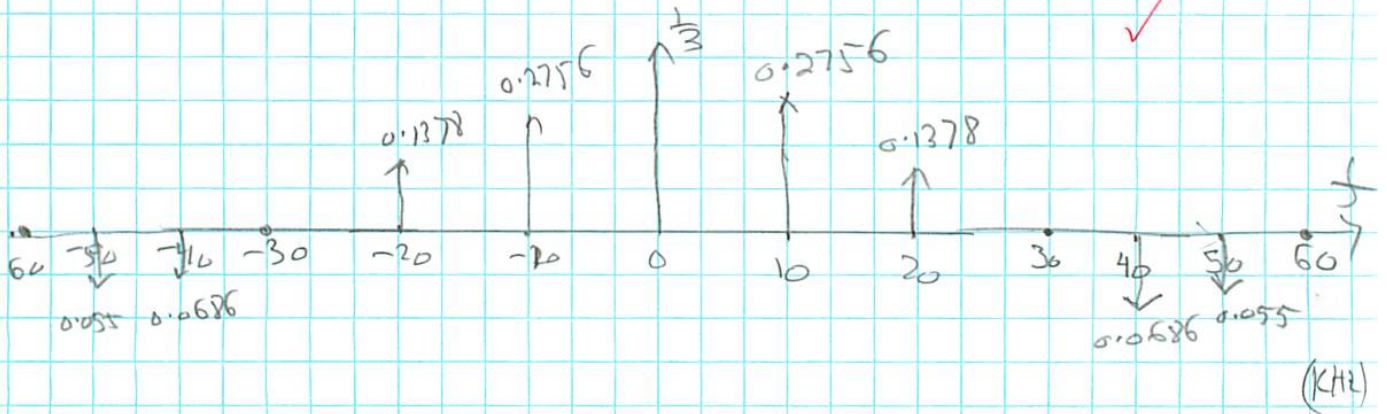


$$\text{sinc}(d) = \frac{\sin(\pi \frac{1}{3})}{\pi \frac{1}{3}} = 0.8269 (\frac{1}{3}) = 0.2756$$

$$\text{sinc}(2d) = \frac{\sin(\pi \frac{2}{3})}{\pi \frac{2}{3}} = 0.4134 (\frac{1}{3}) = 0.1378$$

$$\text{sinc}(4d) = \frac{\sin(\pi \frac{4}{3})}{\pi \frac{4}{3}} = -0.206 (\frac{1}{3}) = -0.0686$$

$$\text{sinc}(5d) = \frac{\sin(\pi \frac{5}{3})}{\pi \frac{5}{3}} = -0.165 (\frac{1}{3}) = -0.055$$



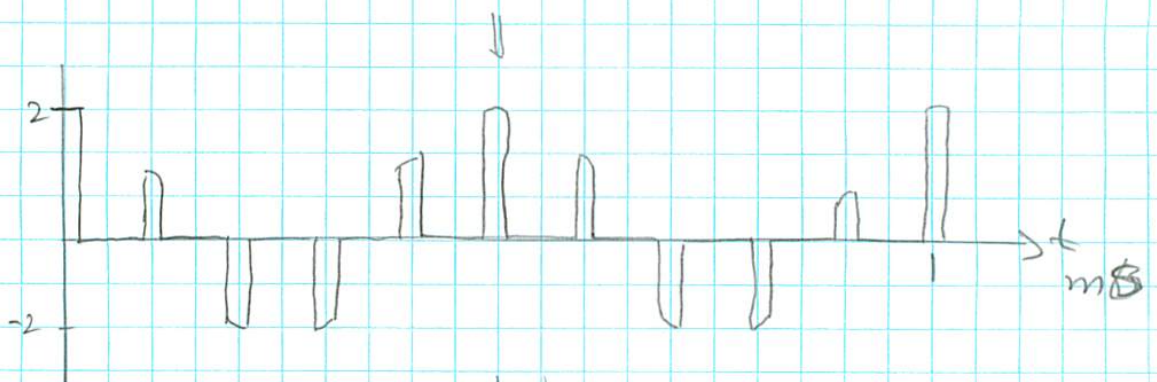
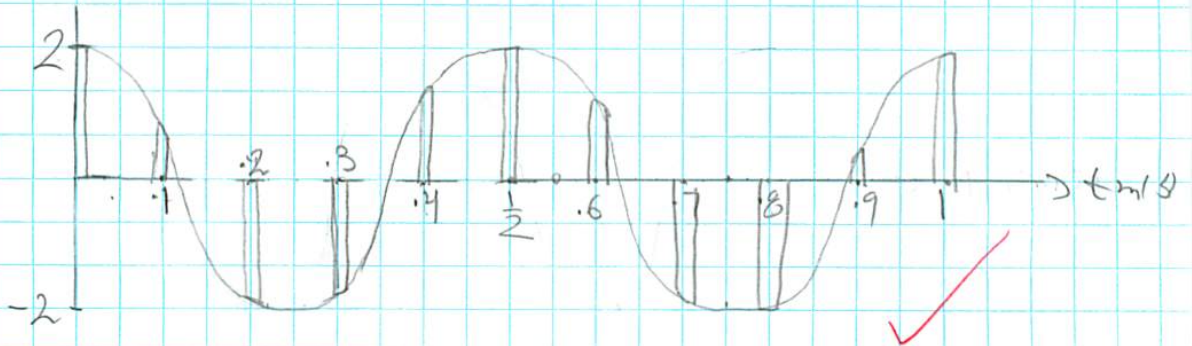
(3)

$$\begin{aligned}
 \text{(d)} \quad x_s(t) &= x(t) \left(\sum h \operatorname{rect} \left(\frac{t-nT_s}{\tau} \right) \right) \\
 &= x(t) \left(d \sum \operatorname{sinc}(nd) e^{j\frac{2\pi}{T_s}nt} \right) \\
 &= x(t) \left(\frac{1}{3} \sum \operatorname{sinc} \left(\frac{n}{3} \right) e^{j2\pi(10,000)nt} \right)
 \end{aligned}$$

$$\text{or } x_s(t) = \frac{1}{3} \sum 2 \cos(2\pi 2000t) \operatorname{sinc} \left(\frac{n}{3} \right) e^{j2\pi 10000nt}$$

$$x_s(t) = \frac{2}{3} \sum \cos(2\pi 2000 nT_s) \operatorname{sinc} \left(\frac{n}{3} \right) e^{j2\pi 10000nt}$$

$$T_s = 0.1 \text{ (ms)}, \quad T_1 = \frac{1}{2} \text{ ms}, \quad d = \frac{1}{3} = \frac{T_1}{T_s} \quad \text{so } \tau = \frac{1}{3}T_s.$$


 $x_s(t)$

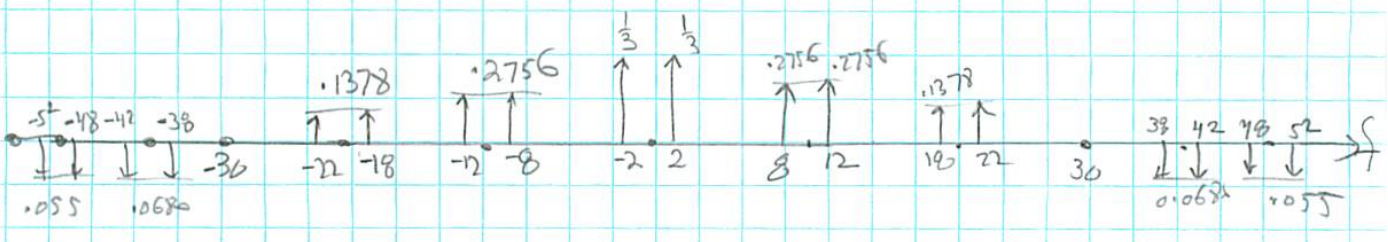
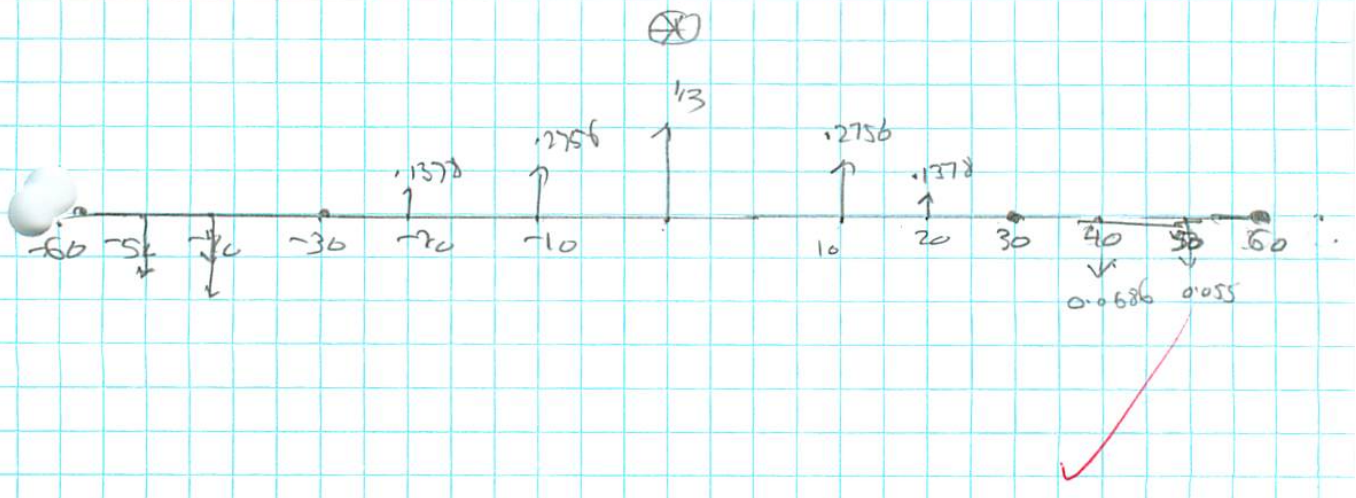
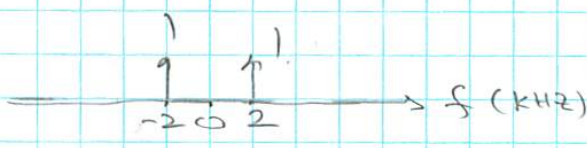
→

(2)

$$X_s(f) = X(f) \otimes G(f)$$

$$= X(f) \otimes d \sum \text{sinc}(nd) \delta(f - nfd)$$

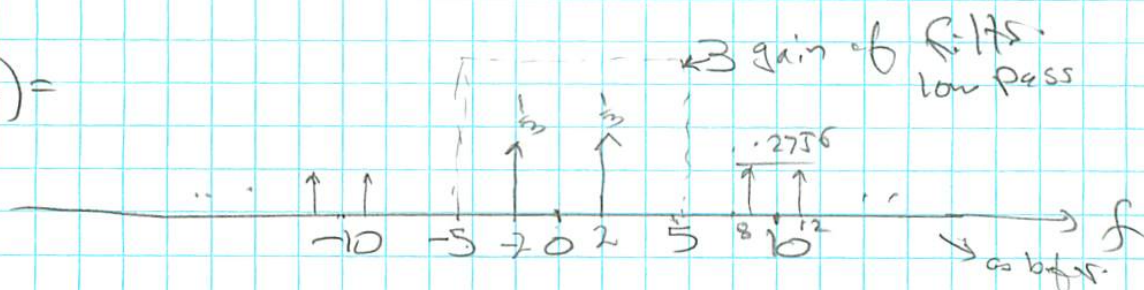
$$X_s(f) = d \sum \text{sinc}(nd) X(f - nfd)$$



$X_s(f)$

(5)

$$Y(f) =$$



$$Y(f) = X_s(f) \cdot H(f)$$

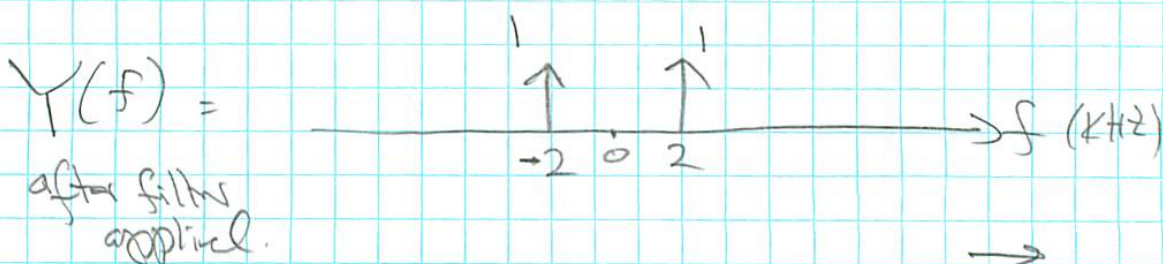
where $H(f) = \frac{1}{d} \text{Rect}\left(\frac{f}{2B}\right)$

where $d = \frac{1}{3}$, $B = 2 \text{ kHz}$.

so $H(f) = 3 \text{Rect}\left(\frac{f}{4000}\right)$

so $Y(f) = \left(d \sum \text{sinc}(nd) X(f - nfs) \right) \cdot \frac{1}{d} \text{Rect}\left(\frac{f}{2B}\right)$

$$Y(f) = \sum_{n=-\infty}^{\infty} \text{Rect}\left(\frac{f}{4000}\right) \text{sinc}\left(\frac{n}{3}\right) X(f - n1000)$$

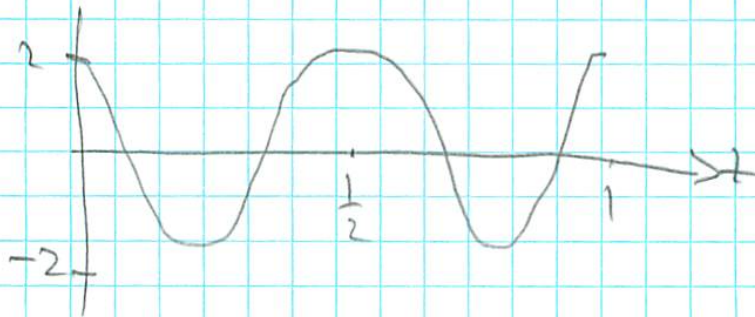


(9) so $y(t) = 2 \cos(2\pi 2000t)$

(6)

$y(t)$ is same as $x(t)$!

so same plot as (a)!



Chapter 3

Handouts

Local contents

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3.3	Properties of Fourier transform	52

3.1 Mathematical identities

Mathematical Tables

Trigonometric Identities

$$\tan(\alpha) = [\sin(\alpha)]/\cos(\alpha)$$

$$\operatorname{cosec}(\alpha) = 1/\sin(\alpha)$$

$$\sec(\alpha) = 1/\cos(\alpha)$$

$$\cot(\alpha) = 1/\tan(\alpha)$$

$$\sin(\alpha) = \cos(90^\circ - \alpha) = \sin(180^\circ - \alpha)$$

$$\cos(\alpha) = \sin(90^\circ - \alpha) = -\cos(180^\circ - \alpha)$$

$$\tan(\alpha) = \cot(90^\circ - \alpha) = -\tan(180^\circ - \alpha)$$

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$$

$$\tan(\alpha + \beta) = [\tan(\alpha) + \tan(\beta)]/[1 - \tan(\alpha) \tan(\beta)]$$

$$\tan(\alpha - \beta) = [\tan(\alpha) - \tan(\beta)]/[1 + \tan(\alpha) \tan(\beta)]$$

$$\sin(\alpha) \cos(\beta) = (1/2) [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\sin(\alpha) \sin(\beta) = (1/2) [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos(\alpha) \cos(\beta) = (1/2) [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\cos(\alpha) \sin(\beta) = (1/2) [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha) = [2 \tan(\alpha)]/[1 + \tan^2(\alpha)]$$

$$\cos(2\alpha) = 2 \cos^2(\alpha) - 1 = 1 - 2 \sin^2(\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$

$$= [1 - \tan^2(\alpha)]/[1 + \tan^2(\alpha)]$$

$$\tan(2\alpha) = [2 \tan(\alpha)]/[1 - \tan^2(\alpha)]$$

$$\sin(3\alpha) = 3 \sin(\alpha) - 4 \sin^3(\alpha)$$

$$\cos(3\alpha) = 4 \cos^3(\alpha) - 3 \cos(\alpha)$$

$$\tan(3\alpha) = [3 \tan(\alpha) - \tan^3(\alpha)]/[1 - 3 \tan^2(\alpha)]$$

$$\sin(4\alpha) = 4 \sin(\alpha) \cos(\alpha) - 8 \sin^3(\alpha) \cos(\alpha)$$

$$\cos(4\alpha) = 8 \cos^4(\alpha) - 8 \cos^2(\alpha) + 1$$

$$\tan(4\alpha) = [4 \tan(\alpha) - 4 \tan^3(\alpha)]/[1 - 6 \tan^2(\alpha) + \tan^4(\alpha)]$$

$$\sin^2(\alpha) = (1/2) [1 - \cos(2\alpha)] = 1 - \cos^2(\alpha)$$

$$\cos^2(\alpha) = (1/2) [1 + \cos(2\alpha)] = 1 - \sin^2(\alpha)$$

$$\tan^2(\alpha) = [1 - \cos(2\alpha)]/[1 + \cos(2\alpha)]$$

$$\sin^3(\alpha) = (1/4) [3 \sin(\alpha) - \sin(3\alpha)]$$

$$\cos^3(\alpha) = (1/4) [3 \cos(\alpha) + \cos(3\alpha)]$$

$$\sin^4(\alpha) = (1/8) [3 - 4 \cos(2\alpha) + \cos(4\alpha)]$$

$$\cos^4(\alpha) = (1/8) [3 + 4 \cos(2\alpha) + \cos(4\alpha)]$$

$$\cos(\alpha) = [e^{j\alpha} + e^{-j\alpha}]/2$$

$$\begin{aligned}\sin(\alpha) &= [e^{j\alpha} - e^{-j\alpha}]/2j \\ \tan(\alpha) &= (-j) [e^{j\alpha} - e^{-j\alpha}]/[e^{j\alpha} + e^{-j\alpha}] \\ e^{j\alpha} &= \cos(\alpha) + j \sin(\alpha) \\ e^{-j\alpha} &= \cos(\alpha) - j \sin(\alpha)\end{aligned}$$

$$\begin{aligned}\sin^2(\alpha) + \cos^2(\alpha) &= 1 \\ 1 + \tan^2(\alpha) &= \sec^2(\alpha) \\ 1 + \cot^2(\alpha) &= \operatorname{cosec}^2(\alpha)\end{aligned}$$

$$\begin{aligned}\sin(\alpha) + \sin(\beta) &= 2 \sin[(1/2)(\alpha + \beta)] \cos[(1/2)(\alpha - \beta)] \\ \sin(\alpha) - \sin(\beta) &= 2 \cos[(1/2)(\alpha + \beta)] \sin[(1/2)(\alpha - \beta)] \\ \cos(\alpha) + \cos(\beta) &= 2 \cos[(1/2)(\alpha + \beta)] \cos[(1/2)(\alpha - \beta)] \\ \cos(\alpha) - \cos(\beta) &= -2 \sin[(1/2)(\alpha + \beta)] \sin[(1/2)(\alpha - \beta)] \\ \tan(\alpha) + \tan(\beta) &= [\sin(\alpha + \beta)]/[\cos(\alpha) \cos(\beta)] \\ \tan(\alpha) - \tan(\beta) &= [\sin(\alpha - \beta)]/[\cos(\alpha) \cos(\beta)]\end{aligned}$$

Indefinite Integrals

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax)$$

$$\int \sin^2(ax) dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \cos^2(ax) dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int \sin(ax) \cos(ax) dx = \frac{1}{2a} \sin^2(ax)$$

$$\int x \sin(ax) dx = \frac{1}{a^2} [\sin(ax) - ax \cos(ax)]$$

$$\int x \cos(ax) dx = \frac{1}{a^2} [\cos(ax) + ax \sin(ax)]$$

$$\int x^2 \sin(ax) dx = \frac{1}{a^3} [2ax \sin(ax) + 2 \cos(ax) - a^2 x^2 \cos(ax)]$$

$$\int x^2 \cos(ax) dx = \frac{1}{a^3} [2ax \cos(ax) - 2 \sin(ax) + a^2 x^2 \sin(ax)]$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int x e^{ax} dx = \frac{1}{a^2} e^{ax} (ax - 1)$$

$$\int x^2 e^{ax} dx = \frac{1}{a^3} e^{ax} (a^2 x^2 - 2ax + 2)$$

$$\int x^n e^{ax} dx = \frac{x^n}{a} e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$\int e^{ax} \sin(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \sin(bx) - b \cos(bx)]$$

$$\int e^{ax} \cos(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \cos(bx) + b \sin(bx)]$$

$$\int \frac{dx}{a^2 + b^2 x^2} = \frac{1}{ab} \tan^{-1} \left(\frac{bx}{a} \right)$$

$$\int \frac{x^2 dx}{a^2 + b^2 x^2} = \frac{x}{b^2} - \frac{a}{b^3} \tan^{-1} \left(\frac{bx}{a} \right)$$

Sums of Powers of the First n Integers

$$\sum_{k=1}^n k = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^n k^4 = \frac{n}{30} (n+1)(2n+1)(3n^2 + 3n - 1)$$

$$\sum_{k=1}^n k^5 = \frac{n^2}{12} (n+1)^2 (2n^2 + 2n - 1)$$

If

$$\sum_{k=1}^n k^p = a_1 n^{p+1} + a_2 n^p + a_3 n^{p-1} + \dots + a_{p+1} n$$

then

$$\sum_{k=1}^n k^{p+1} = \frac{p+1}{p+2} a_1 n^{p+2} + \frac{p+1}{p+1} a_2 n^{p+1} + \frac{p+1}{p} a_3 n^p + \dots + \frac{p+1}{2} a_{p+1} n^2$$

$$+ \left[1 - (p+1) \sum_{k=1}^{p+1} \frac{a_k}{(p+3-k)} \right] n$$

Series Expansion

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots$$

$$\sin^{-1}(x) = x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \frac{x^9}{9} + \dots$$

$$\cos^{-1}(x) = \frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3x^5}{40} - \dots$$

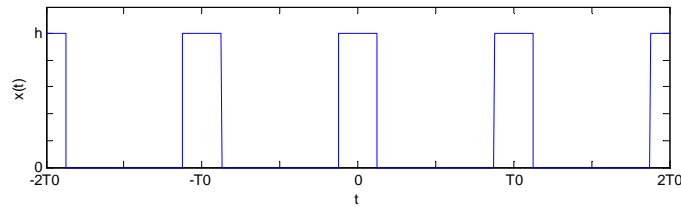
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

3.2 Fourier series representation of common signals

FOURIER SERIES REPRESENTATION OF COMMON SIGNALS

Rectangular Pulse Train



τ = pulse width ($-\tau/2$ to $\tau/2$)
 d = duty cycle = τ/T_0 .
 ω_0 = fundamental frequency = $2\pi/T_0$
 $\text{sinc}(x) = \sin(\pi x)/(\pi x)$

$$X_n = \frac{h\tau}{T_0} \text{sinc}(nd) = hd \text{sinc}(nd) = \begin{cases} hd, & n=0 \\ h \frac{\sin(n\pi d)}{n\pi}, & n \neq 0 \end{cases}$$

$$x(t) = hd + \sum_{n=1}^{\infty} 2hd \text{sinc}(nd) \cos(n\omega_0 t)$$

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$$c_0 = hd = \frac{h\tau}{T_0}, \quad c_n = |2hd \text{sinc}(nd)|, \quad \theta_n = \begin{cases} \pi, & 2hd \text{sinc}(nd) < 0 \\ 0, & \text{otherwise} \end{cases}$$

If $\tau = T_0/2$, $d = 1/2$, and the equations given above becomes

$$X_n = \frac{h}{2} \text{sinc}\left(\frac{n}{2}\right) = \begin{cases} \frac{h}{2}, & n=0 \\ \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}, & n \neq 0 \end{cases}$$

$$x(t) = \frac{h}{2} + \sum_{n=1}^{\infty} h \text{sinc}\left(\frac{n}{2}\right) \cos(n\omega_0 t)$$

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

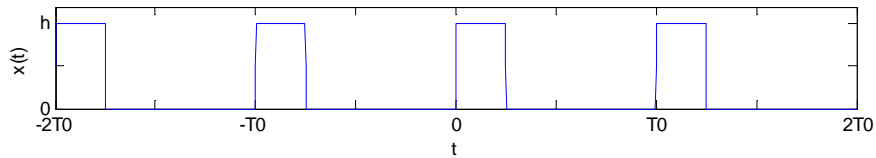
$$c_0 = \frac{h}{2}, \quad c_n = \left| h \operatorname{sinc}\left(\frac{n}{2}\right) \right|, \quad \theta_n = \begin{cases} \pi, & h \operatorname{sinc}\left(\frac{n}{2}\right) < 0 \\ 0, & \text{otherwise} \end{cases}$$

Let $y(t) = x(t - T_0/2)$. Then,

$$Y_n = X_n e^{-jn \frac{2\pi T_0}{2}} = X_n e^{-jn\pi} = X_n \cos(n\pi) = \begin{cases} \frac{h}{2}, & n = 0 \\ (-1)^n \operatorname{sinc}\left(\frac{n\pi}{2}\right), & n \neq 0 \end{cases}$$

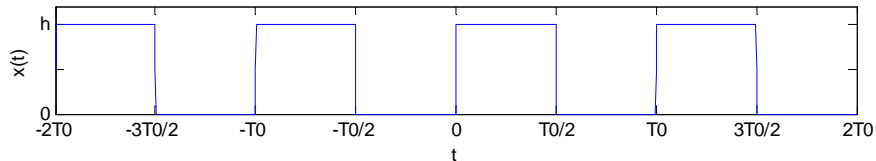
Rectangular Pulse Train with Time Shifting

$$t_0 = \tau/2.$$

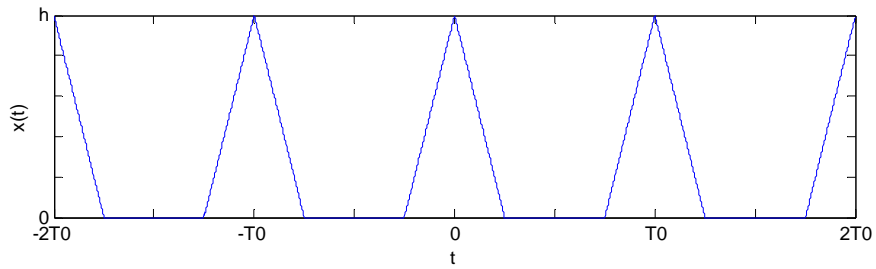


$$X_n = \frac{h\tau}{T_0} \operatorname{sinc}(nd) e^{-jn \frac{2\pi t_0}{T_0}} = h d \operatorname{sinc}(nd) e^{-jn \frac{2\pi t_0}{T_0}} = h d \operatorname{sinc}(nd) e^{-jn \frac{2\pi \tau}{T_0}} = h d \operatorname{sinc}(nd) e^{-jn\pi \frac{\tau}{T_0}}$$

If $\tau = T_0/2$, we have



$$X_n = \frac{h}{2} \operatorname{sinc}\left(\frac{n}{2}\right) e^{-jn\pi} = \frac{h}{2} \operatorname{sinc}\left(\frac{n}{2}\right) e^{-jn\pi} = \frac{h}{2} \operatorname{sinc}\left(\frac{n}{2}\right) \cos(n\pi)$$

Triangular Pulse Train

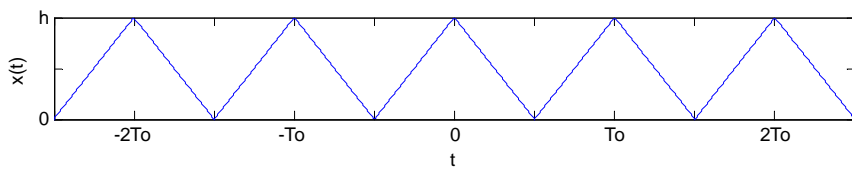
$\tau =$ half of the base of the triangle ($-\tau \leq t \leq \tau$)

$d =$ duty cycle $= \tau/T_0$.

$\omega_0 =$ fundamental frequency $= 2\pi/T_0$

$$X_n = hd \operatorname{sinc}^2(nd) = \frac{h\tau}{T_0} \operatorname{sinc}^2\left(\frac{n\omega_0\tau}{2\pi}\right)$$

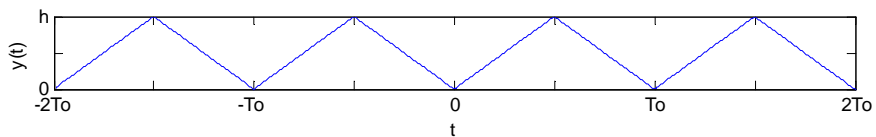
If $\tau = T_0/2$, then the pulse train looks like



and

$$X_n = \frac{h}{2} \operatorname{sinc}^2\left(\frac{n}{2}\right) = \begin{cases} \frac{h}{2}, & n = 0 \\ 0, & n = \text{even} \\ \frac{2h}{n^2\pi^2}, & n = \text{odd} \end{cases}$$

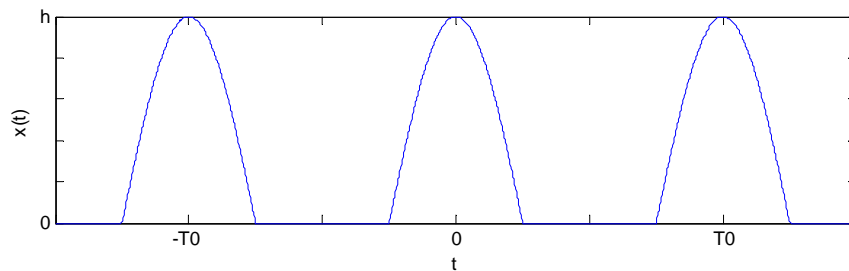
Let $y(t) = x(t - T_0/2)$.



Then,

$$Y_n = X_n e^{-jn\frac{2\pi T_0}{2}} = X_n e^{-jn\pi} = X_n \cos(n\pi) = \begin{cases} \frac{h}{2}, & n=0 \\ 0, & n = \text{even} \\ \frac{-2h}{n^2\pi^2}, & n = \text{odd} \end{cases}$$

Half-Wave Rectified Cosine

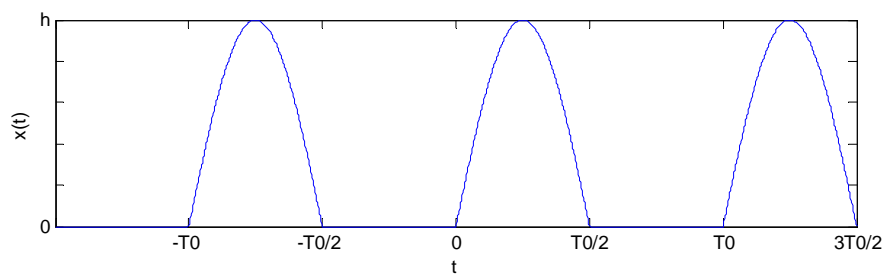


$$X_1 = \frac{h}{4}$$

$$X_{-1} = \frac{h}{4}$$

$$X_n = \frac{h}{\pi} \frac{\cos\left(\frac{n\pi}{2}\right)}{1-n^2}, \quad n \neq \pm 1$$

Half-Wave Rectified Sine



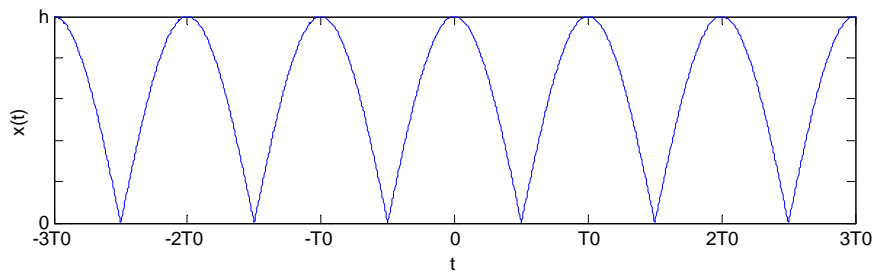
$$h = \text{amplitude}, \quad j = \sqrt{-1}$$

$$X_1 = \frac{-jh}{4}$$

$$X_{-1} = \frac{jh}{4}$$

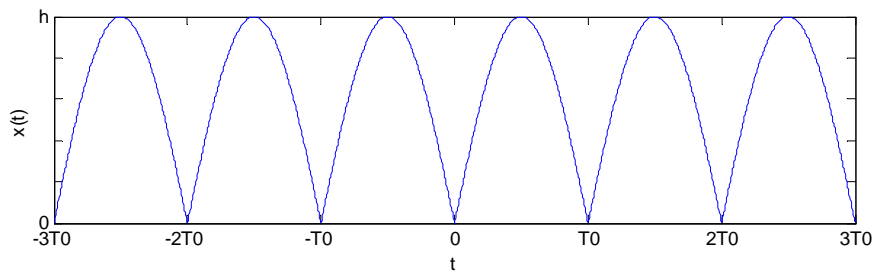
$$X_n = \frac{h}{\pi} \frac{\cos^2\left(\frac{n\pi}{2}\right)}{1-n^2} = \begin{cases} \frac{h}{\pi}, & n=0 \\ 0, & n = \pm 3, \pm 5, \pm 7, \dots \\ \frac{h}{\pi} \frac{1}{1-n^2}, & n = \pm 2, \pm 4, \pm 6, \dots \end{cases}$$

Full-Wave Rectified Cosine



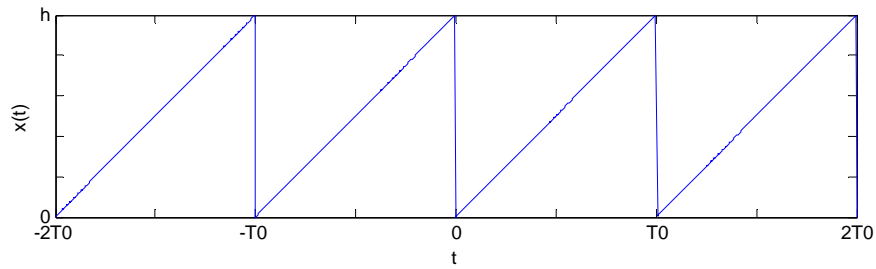
$$X_n = \frac{2h}{\pi} \frac{\cos(n\pi)}{1-4n^2} = \frac{2h}{\pi} \frac{(-1)^n}{1-4n^2}$$

Full-Wave Rectified Sine



$$X_n = \frac{2h}{\pi} \frac{\cos^2(n\pi)}{1-4n^2} = \frac{2h}{\pi} \frac{1}{1-4n^2}$$

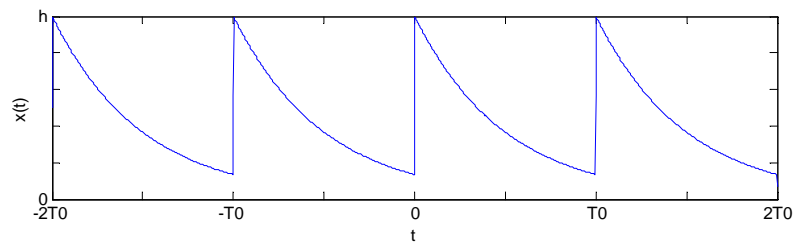
Sawtooth



$$X_0 = \frac{h}{2}$$

$$X_n = \frac{jh}{2\pi n}, \quad n \neq 0, \quad j = \sqrt{-1}$$

Exponential Decay



$$X_n = \frac{h}{T_0} \frac{1 - e^{-aT_0}}{a + jn \frac{2\pi}{T_0}}$$

3.3 Properties of Fourier transform

Properties of the Fourier Transform

Property	$f(t)$	$F(\omega)$
Linearity (Superposition)	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Time Shifting	$f(t - t_d)$	$e^{-j\omega t_d} F(\omega)$
Time Scaling	$f(ct)$	$\frac{1}{ c } F\left(\frac{\omega}{c}\right)$
Symmetry (Duality)	$F(t)$	$2\pi f(-\omega)$
Time Reversal	$f(-t)$	$F(-\omega)$
Frequency Scaling	$f(t)e^{j\omega_c t}$	$F(\omega - \omega_c)$
Modulation	$f(t)\cos(\omega_c t)$	$\frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c)$
Time Differentiation	$\frac{df(t)}{dt}$	$j\omega F(\omega)$
Frequency Differentiation	$tf(t)$	$j\frac{dF(\omega)}{d\omega}$
Conjugate	$f^*(t)$	$F^*(-\omega)$
Integration	$\int_{-\infty}^t f(\lambda)d\lambda$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
Convolution	$\int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$	$H(\omega)X(\omega)$
Multiplication	$f_1(t)f_2(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(v)F_2(\omega-v)dv$
Parseval's Theorem	$\int_{-\infty}^{\infty} f(t) ^2 dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) ^2 d\omega$

Table 16.2 Fourier Transform Pairs ($a > 0$)

$f(t)$	$F(\omega)$	$F(f)$
$\Pi\left(\frac{t}{a}\right) = \text{rect}\left(\frac{t}{a}\right)$	$a \text{sinc}\left(\frac{\omega a}{2\pi}\right)$	$a \text{sinc}(fa)$
$\Lambda\left(\frac{t}{a}\right) = \text{tri}\left(\frac{t}{a}\right)$	$a \text{sinc}^2\left(\frac{\omega a}{2\pi}\right)$	$a \text{sinc}^2(fa)$
$e^{-at}u(t)$	$\frac{1}{j\omega + a}$	$\frac{1}{j2\pi f + a}$
$e^{at}u(-t)$	$\frac{1}{-j\omega + a}$	$\frac{1}{-j2\pi f + a}$
$e^{-a t }$	$\frac{2a}{\omega^2 + a^2}$	$\frac{2a}{4\pi^2 f^2 + a^2}$
$e^{-at}u(t) - e^{at}u(-t)$	$\frac{-2j\omega}{\omega^2 + a^2}$	$\frac{-j4\pi f}{4\pi^2 f^2 + a^2}$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$tu(t)$	$\frac{\pi}{j\omega}\delta(\omega) + \frac{1}{(j\omega)^2}$	$\frac{1}{j4\pi f}\delta(f) + \frac{1}{(j2\pi f)^2}$
$te^{-at}u(t)$	$\frac{1}{(j\omega + a)^2}$	$\frac{1}{(j2\pi f + a)^2}$
$\cos(\omega_c t) = \cos(2\pi f_c t)$	$\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$

$$\begin{array}{lll}
\sin(\omega_c t) = \sin(2\pi f_c t) & -j\pi[\delta(\omega - \omega_c) - \delta(\omega + \omega_c)] & \frac{-j}{2}[\delta(f - f_c) - \delta(f + f_c)] \\
e^{-at}u(t)\cos(\omega_c t) & \frac{j\omega + a}{(j\omega + a)^2 + \omega_c^2} & \frac{j2\pi f + a}{(j2\pi f + a)^2 + (2\pi f_c)^2} \\
e^{-at}u(t)\sin(\omega_c t) & \frac{\omega_c}{(j\omega + a)^2 + \omega_c^2} & \frac{2\pi f_c}{(j2\pi f + a)^2 + (2\pi f_c)^2} \\
\text{sgn}(t) & \frac{2}{j\omega} & \frac{1}{j\pi f} \\
\text{sinc}(ct) & \frac{1}{c} \text{rect}\left(\frac{\omega}{2\pi c}\right) & \frac{1}{c} \text{rect}\left(\frac{f}{c}\right) \\
\text{sinc}^2(ct) & \frac{1}{c} \text{tri}\left(\frac{\omega}{2\pi c}\right) & \frac{1}{c} \text{tri}\left(\frac{f}{c}\right) \\
\cos\left(\frac{\pi t}{a}\right) \text{rect}\left(\frac{t}{a}\right) & \frac{2a}{\pi} \frac{\cos\left(\frac{\omega a}{2}\right)}{1 - \left(\frac{\omega a}{\pi}\right)^2} & \frac{2a}{\pi} \frac{\cos(\pi a f)}{1 - (2af)^2} \\
\frac{1}{2} \left[1 + \cos\left(\frac{\pi t}{a}\right) \right] \text{rect}\left(\frac{t}{2a}\right) & a \frac{\sin(\omega a)}{\omega a \left[1 - \left(\frac{\omega a}{\pi}\right)^2 \right]} & a \frac{\sin(2\pi f a)}{2\pi f a [1 - (2af)^2]}
\end{array}$$

Chapter 4

HW's, and computer assignments

Local contents

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4.1 HW's

Nasser M. Abbasi

HW1, ECE 405

By Nasser M. Abbasi
Cal Poly Pomona, ECE 405, first session, summer 2010.

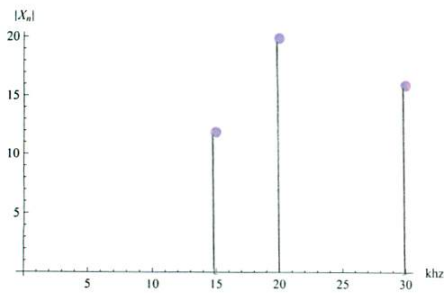
Problem 1

part(a)

$$x[t_] := 12 \cos\left[2\pi 15000 t - \frac{60}{180}\pi\right] - 20 \cos\left[2\pi 20000 t + \frac{30}{180}\pi\right] - 16 \cos\left[2\pi 30000 t - \frac{70}{180}\pi\right];$$

one sided magnitude spectrum

```
data = {{15, 12}, {20, -20}, {30, 16}};
ListPlot[Abs[data], Filling -> Axis, AxesOrigin -> {0, 0},
PlotMarkers -> {Automatic, 12}, AxesLabel -> {"khz", "|X_n|"}]
```



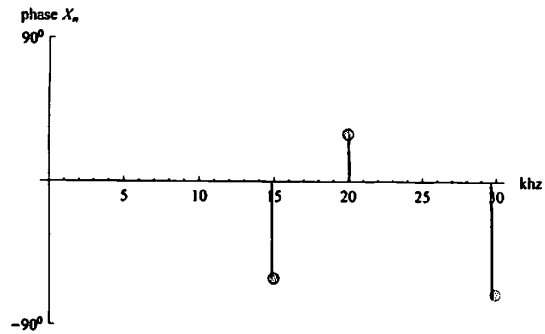
2 | hw1.nb

■ part(b)

```

data = {{15, -60}, {20, 30}, {30, -70}};
ListPlot[data, Filling -> Axis, AxesOrigin -> {0, 0},
  Ticks -> {Automatic, {{-90, "-90°"}, {90, "90°"}}}, PlotMarkers -> {Automatic, 12},
  AxesLabel -> {"khz", "phase Xn"}, PlotRange -> {Automatic, {-90, 90}}]

```



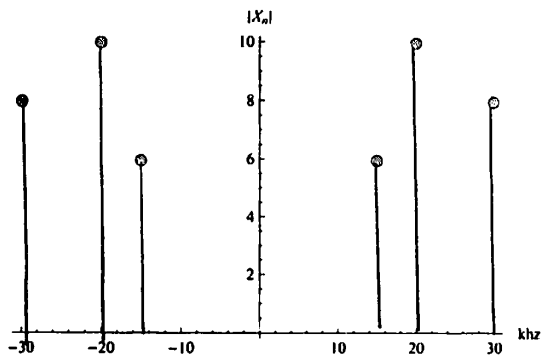
question: As Dr Kang why key solution has angles summed in different way

■ part(c)

```

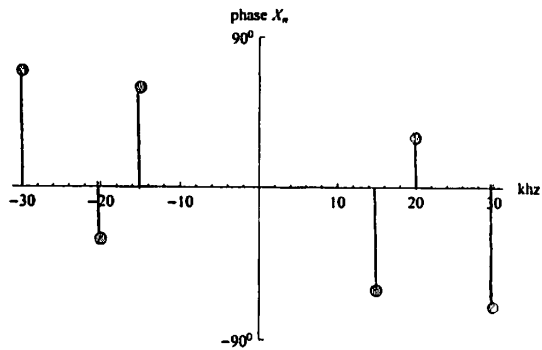
data = {{15, 6}, {20, 10}, {30, 8}, {-15, 6}, {-20, 10}, {-30, 8}};
ListPlot[data, Filling -> Axis, AxesOrigin -> {0, 0},
  PlotMarkers -> {Automatic, 12}, AxesLabel -> {"khz", "|Xn|"}]

```



■ part(d)

```
data = {{15, -60}, {20, 30}, {30, -70}, {-15, 60}, {-20, -30}, {-30, 70}};
ListPlot[data, Filling -> Axis, AxesOrigin -> {0, 0},
  Ticks -> {Automatic, {{-90, "-90°"}, {90, "90°"}}}, PlotMarkers -> {Automatic, 12},
  AxesLabel -> {"kHz", "phase Xn"}, PlotRange -> {Automatic, {-90, 90}}
```



Problem 2

■ part(a)

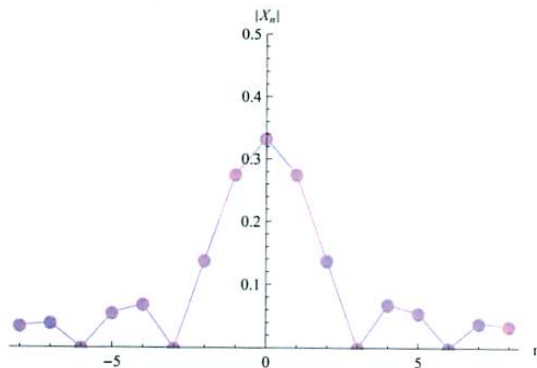
```
ClearAll["Global`*"];
xn[n_] := h d Sinc[Pi n d] Exp[-I 2 Pi f0 n t0];
parameters = {h -> 1, d -> 1/3, f0 -> 1/(3 * 10^-3), t0 -> 10^-3};
xn[n] /. parameters
```

$$\frac{1}{3} e^{i n \pi} \text{Sinc}\left[\frac{n \pi}{3}\right]$$

4 | hw1.nb

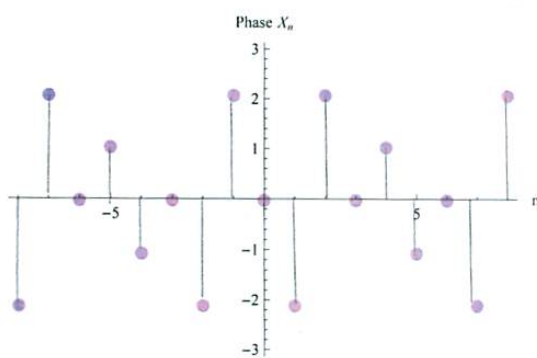
■ part(b)

```
data = Table[{n, Abs[xn[n] /. parameters]}, {n, -8, 8}];
Show[ListPlot[data, Filling -> Axis, PlotRange -> {Automatic, {0, .5}},
      PlotMarkers -> {Automatic, 12}, AxesLabel -> {"n", "|Xn|"}], ListPlot[data, Joined -> True]]
```



■ part(c)

```
data = Table[{n, Arg[xn[n] /. parameters]}, {n, -8, 8}];
ListPlot[data, Filling -> Axis, PlotRange -> {Automatic, {-Pi, Pi}},
          PlotMarkers -> {Automatic, 12}, AxesLabel -> {"n", "Phase Xn"}]
```



■ part(d)

Power in the n^{th} harmonic is $2 |X_n|^2$ where we multiply by 2 to take care of both sides of the spectrum. Hence for $n = 2$

```
Abs[xn[2] /. parameters];
Row[{N[2 * %^2], " watt"}]
0.0379954 watt
```

■ part(e)

Fourier series of $x(t)$ is $\sum_{n=-\infty}^{\infty} x_n \text{Exp}[I 2 \pi f_0 n t]$
at $n=0$

`xn[0] /. parameters`

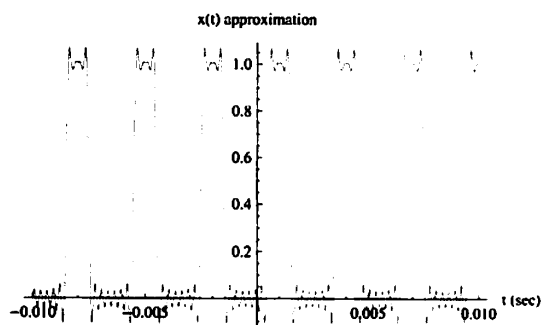
$$\frac{1}{3}$$

substituting values, we obtain $x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{3} e^{-\frac{2}{3} i n \pi + \frac{200}{3} i n \pi t} \text{Sinc}\left[\frac{n \pi}{3}\right]$

To verify, here is a plot of $x(t)$ for $n=10$ terms for $t=-10$ ms to $t=10$ ms. Notice the delay which is 1 ms

`fourier = \sum_{n=-10}^{10} xn[n] Exp[I 2 \pi f_0 n t];`

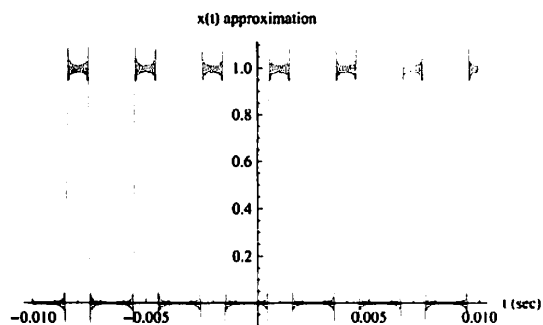
`Plot[fourier /. parameters, {t, -0.01, .01}, AxesLabel -> {"t (sec)", "x(t) approximation"}]`



Change for numbers for terms to say $n=30$, we obtain

`fourier = \sum_{n=-30}^{30} xn[n] Exp[I 2 \pi f_0 n t];`

`Plot[fourier /. parameters, {t, -0.01, .01}, AxesLabel -> {"t (sec)", "x(t) approximation"}]`



The approximation is better. But notice Gibbs phenomena at the corners.

6 | hw1.nb

part(f)

$y(t)=x(t)*\cos(2\pi 30000t)$. To find $y(t)$, convolve the fourier transform of $x(t)$ with the fourier transform of $\cos(2\pi 30000t)$. The fourier transform of $\cos(2\pi 30000t)$ is $\frac{1}{2}$ times impulse at frequency -30kHz and at frequency 30kHz . The effect of convolving the fourier transform of $x(t)$ with these 2 impulses is to shift the fourier transform of $x(t)$ and center it over the impulses. Hence $Y_n = \frac{1}{2} X_{n-m} + \frac{1}{2} X_{n+m}$ where m is amount of shift needed to center X_n over 30kHz and -30kHz

The amount of shift is given by $m = \frac{30 \text{ kHz}}{\frac{1}{3} \text{ kHz}} = 90$, hence 90 spectral lines are needed to shift X_n , hence

$$Y_n = \frac{1}{2} X_{n-90} + \frac{1}{2} X_{n+90}$$

$$Y_n = \frac{1}{2} h d \text{Sinc}[(n-90)d] \text{Exp}[-l 2 \text{Pi} f_0 (n-90)t] + \frac{1}{2} h d \text{Sinc}[(n+90)d] \text{Exp}[-l 2 \text{Pi} f_0 (n+90)t]$$

simplify, the above becomes

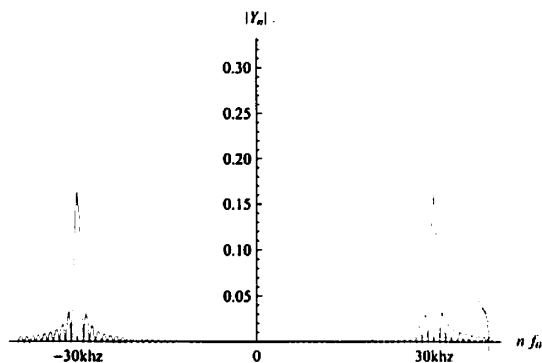
$$\text{yn}[n_] := \frac{1}{6} e^{-\frac{1}{3} i (n-90) \pi} \text{Sinc}\left[\text{Pi} \frac{1}{3} (-90+n)\right] + \frac{1}{6} e^{-\frac{1}{3} i (90+n) \pi} \text{Sinc}\left[\text{Pi} \frac{90+n}{3}\right]$$

Simplify[yn[n]]

$$\frac{1}{6} e^{-\frac{1}{3} i n \pi} \left(\text{Sinc}\left[\frac{1}{3} (-90+n) \pi\right] + \text{Sinc}\left[\frac{1}{3} (90+n) \pi\right] \right)$$

Here is a plot of the magnitude and phase of Y_n

```
data = Table[{n, Abs[yn[n]]}, {n, -120, 120, 1}];
ListPlot[data, Joined -> True, AxesLabel -> {Style["n f_0", Italic], "|Y_n|"},
PlotRange -> {Automatic, {0, 2/6}}, Ticks -> {{(-90, "-30kHz"), 0, {90, "30kHz"}}, Automatic}]
```



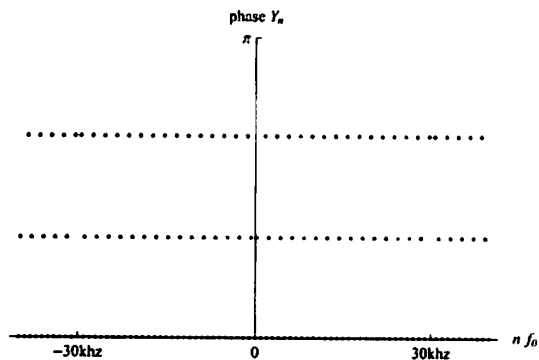
Now plot the phase

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```

data = Table[{n, Arg[yn[n]]}, {n, -120, 120, 1}];
ListPlot[data, Joined -> False, Filling -> Axis,
  AxesLabel -> {Style["n f_0", Italic], "phase Y_n"}, PlotRange -> {Automatic, {0, Pi}},
  Ticks -> {{(-90, "-30khz"), 0, (90, "30khz")}, {0, {Pi, Pi}}}

```



■ part(g)

Applying the filter to $y(t)$ in the frequency domain: The filter has width of 5kHz, hence 2.5kHz on each side of the center of the filter. The filter is centered at 30kHz, hence frequencies of 30+2.5 kHz and 30-2.5 kHz will be allowed through. Since each f_0 is $\frac{1}{3}$ kHz, then the number of spectral lines that will be allowed through is

$$5 / (1 / 3)$$

$$15$$

Hence there will be 7 spectral lines on each side of the center of the filter. Hence n will run from 90 to 97, and also run from 83 to 89. Let $w(t)$ be the signal whose spectrum is those spectral lines obtained from the filter. Hence we write

$$w(t) = \sum_{n=83}^{97} Y_n e^{-i2\pi f_0 t} \text{ where } f_0 = \frac{1}{3} \text{ kHz and } Y_n = \frac{1}{6} e^{-\frac{2}{3}i(-90+n)\pi} \text{Sinc}\left[\frac{\pi}{3}(-90+n)\right] + \frac{1}{6} e^{-\frac{2}{3}i(90+n)\pi} \text{Sinc}\left[\frac{\pi}{3}(90+n)\right]$$

Using the second form of the fourier series, compute to obtain

$$w1[n_] := \frac{1}{3} \text{Sinc}\left[\frac{\pi n}{3}\right] \text{Cos}\left[2\pi(30000 + n333)t - n\frac{2}{3}\pi\right]$$

$$w2[n_] := \frac{1}{3} \text{Sinc}\left[\frac{\pi n}{3}\right] \text{Cos}\left[2\pi(30000 - n333)t + n\frac{2}{3}\pi\right]$$

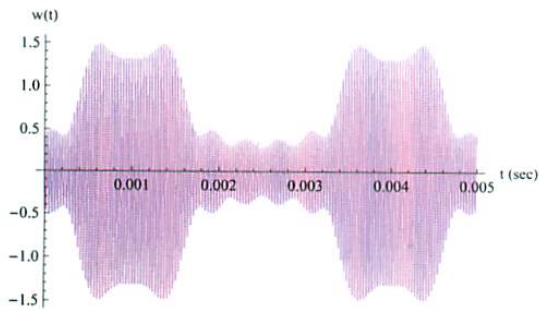
$$\text{filterOutput} = \left(\sum_{n=1}^7 w1[n] + \sum_{n=-7}^{-1} w2[n] + w1[0] + w2[0] \right) // N$$

$$0.666667 \text{Cos}[188496.t] - 0.0787613 \text{Sin}[0.523599 - 203142.t] + \\ 0.137832 \text{Sin}[0.523599 - 196865.t] - 0.551329 \text{Sin}[0.523599 - 190588.t] - \\ 0.275664 \text{Sin}[0.523599 + 192680.t] + 0.110266 \text{Sin}[0.523599 + 198957.t]$$

plot w(t)

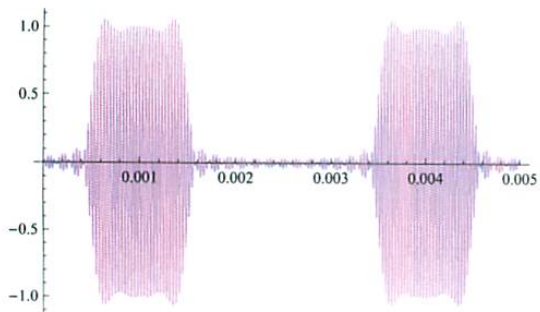
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```
Plot[filterOutput, {t, 0, .005}, PlotRange -> All, AxesLabel -> {"t (sec)", "w(t)"}]
```



compare $w(t)$, the signal from the bandpass filter, with the original signal $y(t)$ to see the effect of the filtering

```
Plot[(fourier /. parameters) * Cos[2 π * 30 000 t], {t, 0, .005}, PlotRange -> All]
```



part(h)

using the shifting property, $Z_n = Y_n e^{-12\pi f_0 t_0 n}$ where f_0 is the fundamental frequency of $y(t)$ and t_0 is the delay amount

```
parameters = {h -> 1, d -> 1/3, f0 -> 1/(3 * 10^-3), t0 -> 10^-3/4};
zn[n_] := yn[n] Exp[-I 2 π f0 t0 n] /. parameters
zn[n] // Simplify
```

$$\frac{1}{6} e^{-\frac{1}{2} i n \pi} \left(\text{Sinc}\left[\frac{1}{3} (-90 + n) \pi\right] + \text{Sinc}\left[\frac{1}{3} (90 + n) \pi\right] \right)$$

Problem 3

part(a)

```
width = 0.25 * 10^-3; period = 10^-3; h = 1; f0 = 1000;
```

X_n for triangular pulse the width term used to find the duty cycle is taken as 1/2 of the width of the base of the triangle.

d = width / period
0.25

Hence $X_n = d \left(\text{Sinc}[\text{Pi } n d] \right)^2 = \frac{1}{2} \left(\text{Sinc}\left[\text{Pi } \frac{n}{2}\right] \right)^2$

$$x[n_] := \frac{1}{2} \left(\text{Sinc}\left[\text{Pi } \frac{n}{2}\right] \right)^2$$

■ **part(b)**

The fourier series approximation is given by $\sum_{n=-\infty}^{\infty} X_n e^{-i 2 \pi f_0 n t}$ where $f_0 = 1 \text{ kHz}$ in this example.
Hence

$$x(t) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(\text{Sinc}\left[\text{Pi } \frac{n}{2}\right] \right)^2 e^{-i 2 \pi 1000 n t}$$

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■ part(c)

$$H(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} = \frac{1}{RCs+1} \text{ where } R=1000 \text{ ohm, } C = 10^{-6}, \text{ hence } RC = 10^{-3}, \text{ then } H(s) = \frac{1}{10^{-3}s+1} = \frac{1000}{s+1000},$$

$$\text{hence } H(j\omega) = \frac{1000}{j\omega+1000}$$

$$H(j\omega) = |H(j\omega)| \text{ Arg}(H(j\omega)).$$

Now, $y(t)=H(j\omega) x(t)$, hence in terms of the fourier coefficients, we write

$$Y_n = H(j\omega_0 n) X_n = (|H(j\omega)| \text{ Arg}(H(j\omega)))^* (|X_n| \text{ Arg}(X_n))$$

hence

$$Y_n = |H(j\omega_0 n)|^* |X_n| (\text{Arg}(H(j\omega_0 n)) + (\text{Arg}(X_n)))$$

$$\text{Hence } |Y_n| = |H(j\omega_0 n)|^* |X_n|$$

and

$$\text{Arg}(Y_n) = \text{Arg}(H(j\omega_0 n)) + \text{Arg}(X_n)$$

$$\text{But } |H(j\omega_0 n)| = \frac{1000}{\sqrt{\omega_0^2 n^2 + 1000^2}}$$

and

$$\text{Arg}(H(j\omega_0 n)) = -\arctan\left(\frac{n\omega_0}{1000}\right)$$

Now, $|X_n| = \frac{1}{2} \left(\text{Sinc}\left[\text{Pi} \frac{n}{2}\right]\right)^2$ and $\text{Arg}(X_n) = 0$ since there is no delay term.

$$\text{Hence } Y_n = \frac{1000}{\sqrt{\omega_0^2 n^2 + 1000^2}} \frac{1}{2} \left(\text{Sinc}\left[\text{Pi} \frac{n}{2}\right]\right)^2 \text{Exp}\left[-j \arctan\left(\frac{n\omega_0}{1000}\right)\right]$$

Now, $\omega_0 = 2\pi f_0 = 2\pi 1000$, hence

$$Y_n = \frac{1000}{\sqrt{(2\pi 1000 n)^2 + 1000^2}} \frac{1}{2} \left(\text{Sinc}\left[\text{Pi} \frac{n}{2}\right]\right)^2 \text{Exp}\left[-j \arctan(n 2\pi)\right]$$

$$y_n[n_] := \frac{1}{2} \left(\text{Sinc}\left[\text{Pi} \frac{n}{2}\right]\right)^2 \text{Exp}\left[-I \text{ArcTan}[n 2\pi]\right] \frac{1000}{\sqrt{(2\pi 1000 n)^2 + 1000^2}}$$

Now that Y_n is found, we can find

$$y(t) = \sum_{n=-\infty}^{\infty} Y_n \text{Exp}[I 2\pi f_0 n t]$$

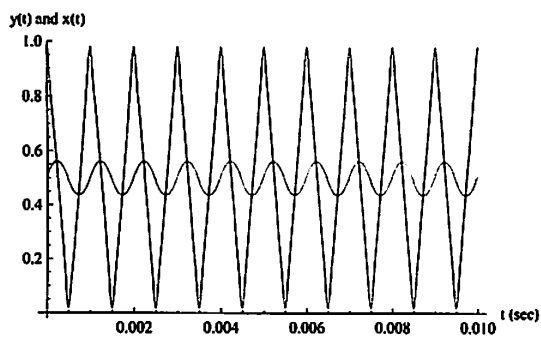
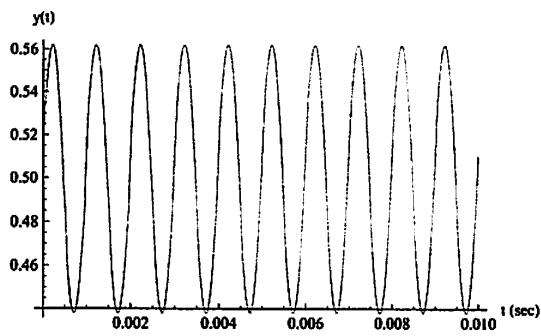
where $f_0 = 1$ khz, hence to plot $y(t)$ using say 10 terms in fourier series and compare to $x(t)$

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```

y[t_] := Sum[y[n] Exp[I 2 Pi 1000 n t], {n, -10, 10}]
x[t_] := 1/2 Sum[Sinc[Pi n/2]^2 Exp[-I 2 Pi 1000 n t], {n, -10, 10}]
Plot[y[t], {t, 0, .01}, AxesLabel -> {"t (sec)", "y(t)"}]
Plot[{y[t], x[t]}, {t, 0, .01}, AxesLabel -> {"t (sec)", "y(t) and x(t)"}]

```

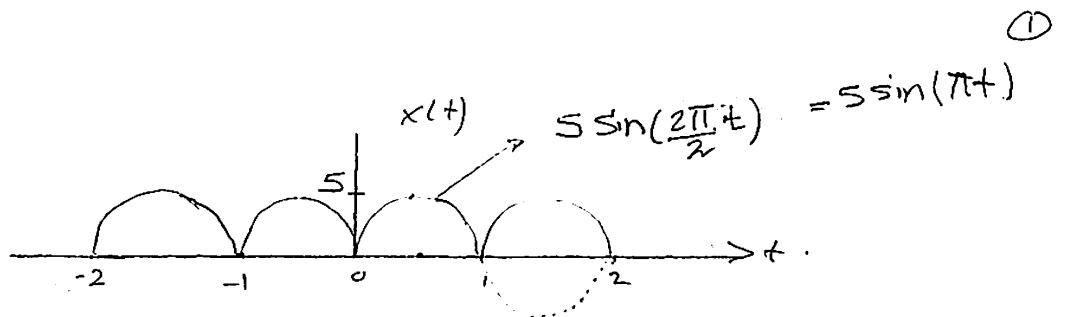
**problem 4**

$$5 \int_0^1 \sin[\pi t] \exp[I 2 \pi n t] dt$$

$$\frac{5 (1 + e^{2 i n \pi})}{\pi - 4 n^2 \pi}$$

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$$T_0 = 1 \text{ sec}, f_0 = 1 \text{ Hz}, h = 5, \omega_0 = 2\pi$$

Since $x(t)$ is even, it will only have a_n terms. so using.

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\omega_0 n t)$$

$$\begin{aligned} a_0 &= \frac{1}{T} \int_T x(t) dt = \frac{1}{1} \int_0^1 5 \sin(\pi t) dt = 5 \left[\frac{-\cos(\pi t)}{\pi} \right]_0^1 \\ &= -\frac{5}{\pi} [\cos \pi - \cos 0] = -\frac{5}{\pi} [-1 - 1] = \boxed{\frac{10}{\pi}} \end{aligned}$$

$$\begin{aligned} a_n &= \frac{2}{T} \int_T x(t) e^{j \frac{2\pi}{T} n t} dt = 2 \int_0^1 5 \sin(\pi t) e^{j 2\pi n t} dt \\ &= 10 \int_0^1 \sin(\pi t) e^{j 2\pi n t} dt \quad \text{integration by parts. gives} \\ &= \frac{10 \cdot (1 + e^{j 2n\pi})}{\pi - 4n^2 \pi} \end{aligned}$$

for n integer, $e^{j 2n\pi} = \overset{=0}{\cos 2n\pi + j \sin 2n\pi} = 1$.

$$\text{so } a_n = \frac{10(2)}{\pi(1-4n^2)} = \frac{20}{\pi(1-4n^2)}$$

$$\text{so } x(t) = \frac{10}{\pi} + \sum_{n=1}^{\infty} \frac{20}{\pi(1-4n^2)} \cos(2\pi n t)$$

 $n = -\infty$
 $n = -\infty \quad \pi(1-n^2)$

$$\textcircled{5} \quad F[x(t)] = X(\omega)$$

$$\textcircled{a} \quad 2x(t+5) \rightarrow 2F[x(t+5)] = 2X(\omega)e^{j\omega 5}$$

$$\textcircled{b} \quad 10x[(t-7)/3] \rightarrow 10F[x(\frac{t-7}{3})] = 30X(3\omega)e^{-j\omega 7}$$

using property that $F[x(at)] = aX(a\omega)$

$$\textcircled{c} \quad t x\left(\frac{t+2}{5}\right) \otimes \frac{d}{dt}x(t) \rightarrow \text{conv.}$$

using property that $F\left[\frac{d}{dt}x(t)\right] = j\omega X(\omega)$

$$\text{and } F\left[x\left(\frac{t+2}{5}\right)\right] = 5X(5\omega)e^{j2\omega}$$

$$\text{and } F[t x(t)] = j \frac{d}{d\omega} X(\omega)$$

$$\text{Then } F\left[t x\left(\frac{t+2}{5}\right)\right] = j \frac{d}{d\omega} (5X(5\omega)e^{j2\omega})$$

$$\text{so } F\left[\left(\frac{d}{dt}x(t)\right) \otimes t x\left(\frac{t+2}{5}\right)\right] = F\left[\left(\frac{d}{dt}x(t)\right)\right] F\left[t x\left(\frac{t+2}{5}\right)\right]$$

i.e. $F[\text{convolution}] \Rightarrow \text{multiplication.}$

$$\textcircled{e} \quad F\left[t x\left(\frac{t+2}{5}\right) \otimes \frac{d}{dt}x(t)\right] = j \frac{d}{d\omega} (5X(5\omega)e^{j2\omega}) \cdot j\omega X(\omega)$$

$$= \boxed{-\frac{d}{d\omega} (5X(5\omega)e^{j2\omega}) \omega X(\omega)}$$

④ $t x^*(8t)$ → Complex Conjugate

$$F[t x^*(8t)]$$

Using property $F[x^*(t)] = X^*(-\omega)$.

Then $F[x^*(8t)] = \frac{1}{8} X^*\left(-\frac{\omega}{8}\right)$.

Using property $F[t X(t)] = j \frac{d}{d\omega} X(\omega)$, then

$$F[t x^*(8t)] = j \frac{d}{d\omega} \left(\frac{1}{8} X^*\left(-\frac{\omega}{8}\right) \right) = \boxed{j \frac{d}{d\omega} \left(X^*\left(-\frac{\omega}{8}\right) \right)}$$

⑤ Find Fourier transform of

$$-x\left(-\frac{t+20}{12}\right) e^{j1000t}$$

Using property that $F[x(t) e^{j\alpha t}] = X(\omega - \alpha)$

Then $F\left[-x\left(-\frac{t+20}{12}\right) e^{j1000t}\right]$

$$= +12 X(12\omega) e^{j\omega 20} \Big|_{\omega = \omega - 1000}$$

$$= \boxed{12 X(-12(\omega - 1000)) e^{j(\omega - 1000)20}}$$

Problem 5

Part (f). $\frac{d}{dt} (x^*(-2t)) \cos 500t.$

Use Property $F(x(at)) = \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$ ——— (1)

and Property $F\left(\frac{d}{dt} x(t)\right) = j\omega X(\omega)$ ——— (2)

and Property $F(x(t) \cos(bt)) = \frac{1}{2} [X(\omega-a) + X(\omega+a)]$ — (3)

Use Property $F(x'(t)) = -X^*(-\omega)$ — (4)

Then we obtain:

$$F[x(-2t)] = \frac{1}{2} X\left(\frac{\omega}{-2}\right) \quad \text{rule (1)}$$

$$F[x^*(-2t)] = \frac{1}{2} X^*\left(\frac{\omega}{2}\right) \quad \text{rule (4)}$$

$$F\left[\frac{d}{dt} x^*(-2t)\right] = \frac{j\omega}{2} X^*\left(\frac{\omega}{2}\right) \quad \text{rule (2)}$$

↓ rule (3)

$$\frac{j(\omega-500)}{(2)(2)} \left[X^*\left(\frac{\omega-500}{2}\right) + X^*\left(\frac{\omega+500}{2}\right) \right]$$

$$= \frac{j(\omega-500)}{4} \left[X^*\left(\frac{\omega-500}{2}\right) + X^*\left(\frac{\omega+500}{2}\right) \right]$$

Problem 5.

Part (a)

$$\text{using rule } F[x_1(t)x_2(t)] = [X_1(\omega) \otimes X_2(\omega)] \frac{1}{2\pi}$$

Then

$$F[x^2(t)] = F[x(t)x(t)] = \frac{1}{2\pi} X(\omega) \otimes X(\omega)$$

$$F[x^3(t)] = F[x(t)x(t)x(t)] = \frac{1}{(2\pi)^2} X(\omega) \otimes X(\omega) \otimes X(\omega)$$

$$F[\text{constant}] = C \cdot \delta(\omega)$$

So the result is

$$\frac{1}{(2\pi)^3} X(\omega) \otimes X(\omega) \otimes X(\omega) + \frac{3}{2\pi} X(\omega) \otimes X(\omega) - 9 X(\omega) + 15 \delta(\omega)$$

Part b

$$(t-5)x(3-t)$$

$$\text{using rule } F[x(-t)] = X(-\omega)$$

$$\text{using rule } F[tx(t)] = j \frac{d}{d\omega} X(\omega)$$

$$\text{using rule } F[x(t-a)] = X(\omega) e^{-j\omega a}$$

Then we obtain

$$\begin{aligned} F[(t-5)x(3-t)] &= F[tx(-(t-3)) - 5x(-(t-3))] \\ &= F[tx(-(t-3))] - 5 F[x(-(t-3))] \\ &= j \frac{d}{d\omega} [X(-\omega) e^{-j\omega 3}] - 5 X(-\omega) e^{-j3\omega} \end{aligned}$$

C

Problem 5, Part (1)

$x(\frac{t}{5}) \otimes x(-5t)$

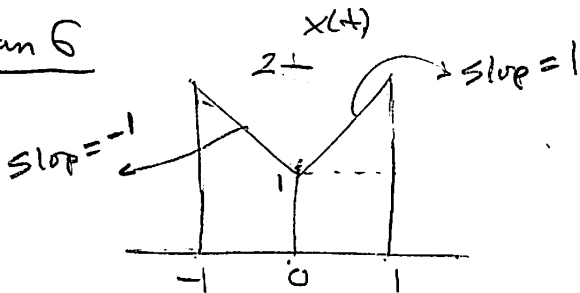
using property $F[x_1(t) \otimes x_2(t)] = X_1(\omega) X_2(\omega)$

and using property $F[x(\frac{t}{a})] = F[x(\frac{1}{a}t)] = |a| X(a\omega)$

and property $F[X(t)] = 2\pi x(-\omega)$

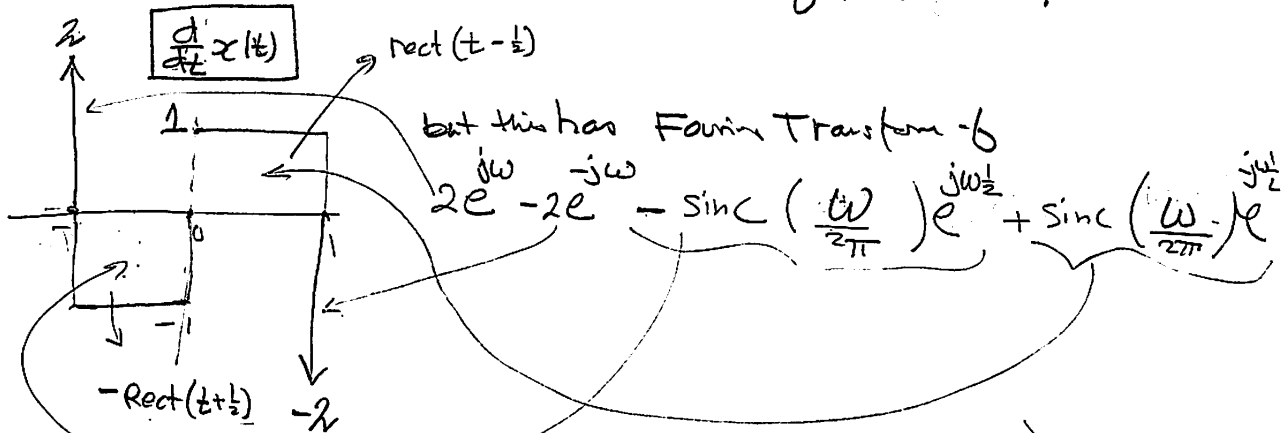
$$\left(2X(2\omega) \right) \cdot \left(\frac{2\pi}{5} x\left(\frac{\omega}{5}\right) \right) = \frac{4\pi}{5} X(2\omega) x\left(\frac{\omega}{5}\right)$$

Problem 6



The time differentiation property is $F\left[\frac{d}{dt}x(t)\right] = j\omega X(\omega)$

make a function whose is the derivative of the above:

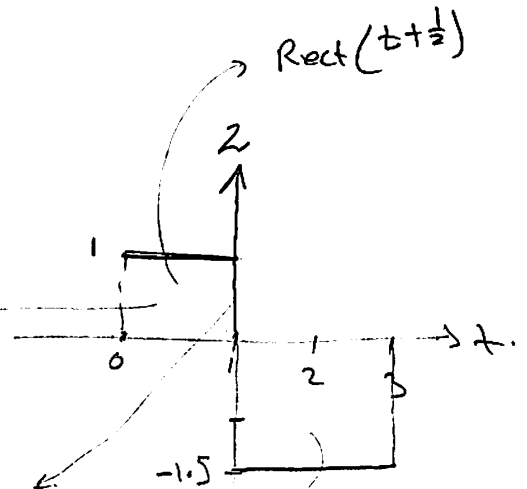
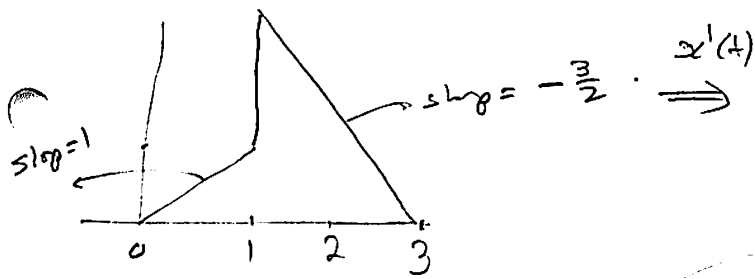


but this has Fourier Transform -6

$$2e^{j\omega} - 2e^{-j\omega} = \text{sinc}\left(\frac{\omega}{2\pi}\right) e^{j\omega/2} + \text{sinc}\left(\frac{\omega}{2\pi}\right) e^{j\omega/2}$$

$$j\omega X(\omega) = [\text{the above}] \Rightarrow X(\omega) = \frac{1}{j\omega} [\dots]$$

Problem 6 Part (b)



$$\begin{aligned} \Rightarrow F(x'(t)) &= \text{sinc}\left(\frac{\omega}{2\pi}\right) e^{-j\frac{\omega}{2}} + 2e^{-j\omega} \\ &\quad - 1.5(2) \text{sinc}\left(\frac{\omega}{\pi}\right) e^{-2j\omega} \end{aligned}$$

$$= \text{sinc}\left(\frac{\omega}{\pi}\right) e^{-j\frac{\omega}{2}} - 3 \text{sinc}\left(\frac{\omega}{\pi}\right) e^{-2j\omega} + 2e^{-j\omega}$$

Problem 7

$$(a) x(t) = t e^{-at} u(t)$$

$$\text{use rule } F\{t x(t)\} = j \frac{d}{d\omega} X(\omega).$$

$$\text{but } F\{e^{-at} u(t)\} = \frac{1}{a+j\omega}$$

$$\text{so } j \frac{d}{d\omega} \left(\frac{1}{a+j\omega} \right) = j \frac{d}{d\omega} \left((a+j\omega)^{-1} \right) = j \left[-(a+j\omega)^{-2} \cdot j \right]$$

$$= j \left(\frac{-j}{(a+j\omega)^2} \right) = \boxed{\frac{1}{(a+j\omega)^2}}$$

$$(b) x(t) = t e^{-at} u(t) \cos(\omega_c t)$$

$$= \frac{1}{(a+j\omega)^2} \otimes \frac{1}{2} \left[\delta(\omega - \omega_c) + \delta(\omega + \omega_c) \right]$$

$$= \frac{1}{2} \frac{1}{(a+j(\omega - \omega_c))^2} + \frac{1}{2} \frac{1}{(a+j(\omega + \omega_c))^2}$$

$$(c) 10 e^{-10|t|} \cos(200t)$$

$$\text{use rule } F\{e^{-a|t|}\} = \frac{2a}{a^2 + \omega^2} \Rightarrow 10 F\{e^{-10|t|}\} = \frac{200}{100 + \omega^2}$$

$$\text{Then } 10 \frac{200}{(-10)^2 + \omega^2} \otimes \frac{1}{2} \left[\delta(\omega - 200) + \delta(\omega + 200) \right]$$

$$= \frac{1}{2} \frac{200}{100 + (\omega - 200)^2} + \frac{1}{2} \frac{200}{100 + (\omega + 200)^2} = \boxed{\frac{100}{100 + (\omega - 200)^2} + \frac{100}{100 + (\omega + 200)^2}}$$

Problem 7. Part (a)

$$x(t) = \text{rect}\left(\frac{t}{12}\right) e^{j200t}$$

use property $F[x(t) e^{jat}] = X(\omega - a)$

use property $F[\text{rect}(\frac{t}{a})] = a \text{sinc}(\frac{a\omega}{\pi})$

so $12 \text{sinc}\left(\frac{12\omega}{2\pi}\right) \Big|_{\omega = \omega - 200}$

$$= \boxed{12 \text{sinc}\left(\frac{6(\omega - 200)}{\pi}\right)}$$

Part e $x(t) = \text{sgn}(t) \cos(100t)$

use property $F[\text{sgn}(t)] = \frac{2}{j\omega}$

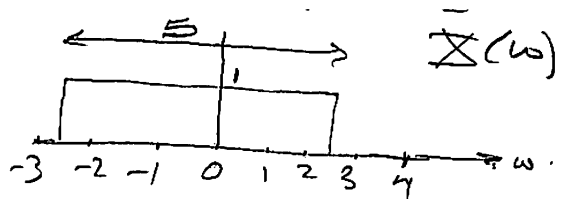
Then $\frac{2}{j\omega} \oplus \frac{1}{2} [\delta(\omega - 100) + \delta(\omega + 100)]$

$$= \boxed{\frac{1}{j(\omega - 100)} + \frac{1}{j(\omega + 100)}}$$

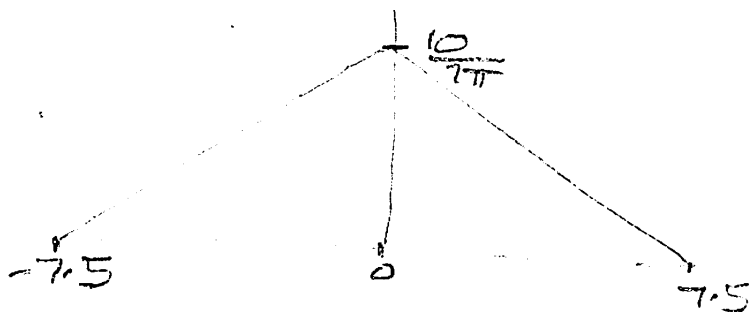
Problem 8

$$F[x(t)] = \text{rect}\left(\frac{\omega}{5}\right)$$

$$F[x^2(t)] = \frac{1}{2\pi} X(\omega) \otimes X(\omega)$$



So need to convolve 2 rect. This gives a triangle.
 extent is from -7.5 to $+7.5$
 max is at $\omega=0$, height is $\frac{10}{2\pi}$ (total area when
 both rects are on top of each other.)



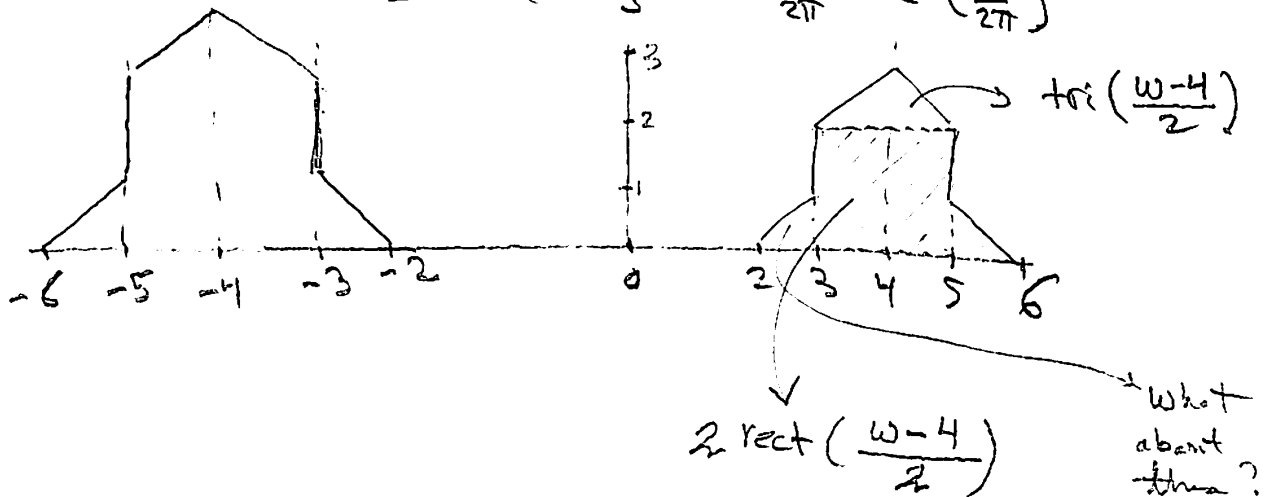
Problem 9

Use property $\mathcal{F}^{-1}[\text{tri}(\frac{\omega}{2\pi})] = \text{sinc}^2(t)$.

and $\mathcal{F}^{-1}[\text{rect}(\frac{\omega}{2\pi})] = \text{sinc}(t)$

then $\mathcal{F}^{-1}[\text{rect}(\omega)] = \frac{1}{2\pi} \text{sinc}(\frac{t}{2\pi})$

and $\mathcal{F}^{-1}[\text{tri}(\omega)] = \frac{1}{2\pi} \text{sinc}^2(\frac{t}{2\pi})$



Need to check this more.

Problem 10

$$(a) \int_{-\infty}^{\infty} \frac{\sin(\frac{\pi t}{2})}{5t} e^{-2t} \delta(t-1) dt. \quad \leftarrow \text{at } \underline{t=1} \quad \delta(t-1) = 1.$$

$$= \int_{-\infty}^{\infty} \frac{\sin(\frac{\pi t}{2})}{5} e^{-2t} \delta(t) dt = \frac{\sin(\frac{\pi}{2})}{5} e^{-2} \int_{-\infty}^{\infty} \delta(t) dt.$$

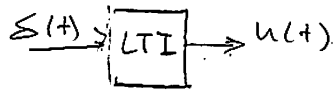
$$= \frac{\sin(\frac{\pi}{2})}{5} e^{-2} = \frac{1}{5} e^{-2} = 0.027067.$$

$$(b) \int_{-\infty}^{\infty} e^{j2\pi t} \delta(t - \frac{1}{8}) dt.$$

note at $\boxed{t = \frac{1}{8}}$ $\delta(t - \frac{1}{8})$ is not zero. hence

$$\int_{-\infty}^{\infty} e^{j2\pi t} \delta(t) dt = e^{j\frac{\pi}{4}}$$

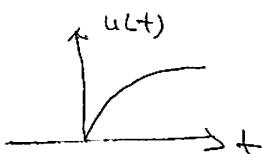
$$= \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = 0.707 + j 0.707$$

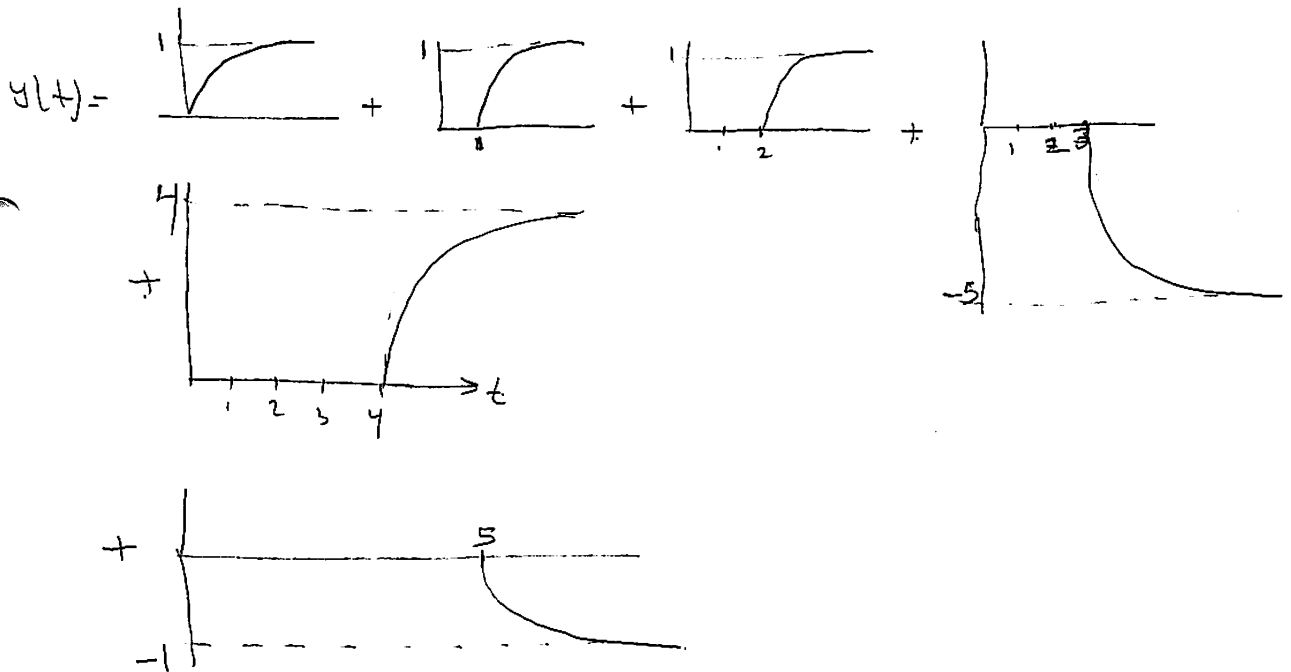
Problem 11

scaling the input causes scaling in the output.

delay in the input causes delay in the output.

$$\text{so } y(t) = u(t) + u(t-1) + u(t-2) - 5u(t-3) + 3u(t-4) - u(t-5)$$

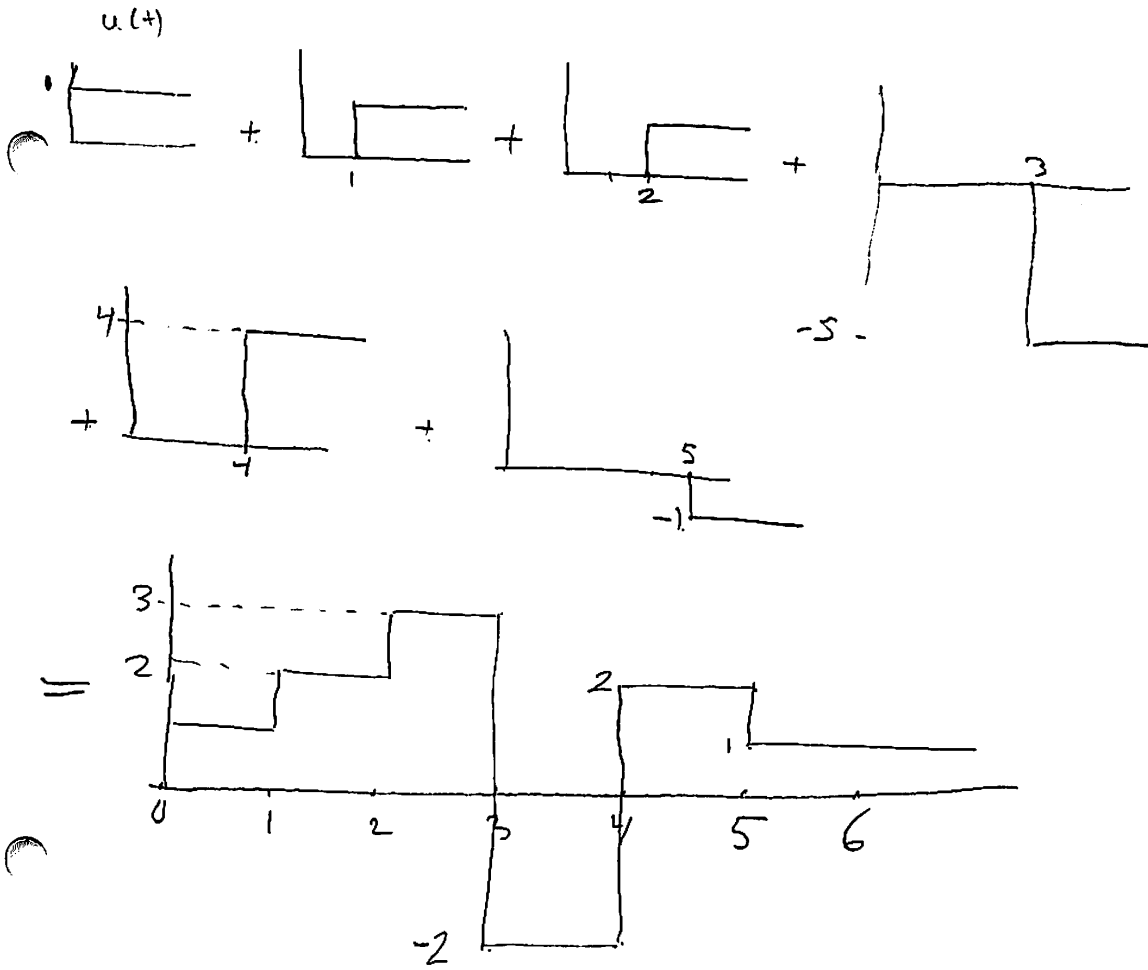
so, assume $u(t) =$  then.



so need to add all the above together to see the

find result.

if $u(t)$ is the unit step, then see next page \rightarrow



(13) $x(t)$ is band limited to B Hz.

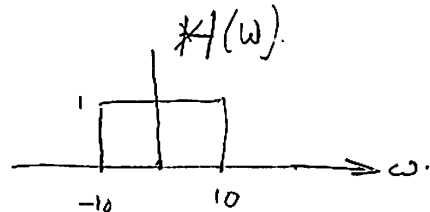
What is bandwidth of $x^3(t)$?

from convolution prop^y: $F(x^3(t)) = \frac{1}{2\pi} X(\omega) \otimes X(\omega) \otimes X(\omega)$.

When convolving, the bandwidth become to sum of the bandwidth of the $X(\omega)$. hence final bandwidth is

3B

(14) $H(\omega) = \text{rect}\left(\frac{\omega}{40\pi}\right)$
 $= \text{rect}\left(\frac{\omega}{20(2\pi)}\right)$



when $\delta(t)$ is applied, the the output is the $F^{-1}[H(\omega)]$

which is

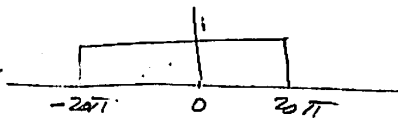
$20 \text{sinc}(20t)$

(b) if input is $10 \text{sinc}(10t)$, then output is

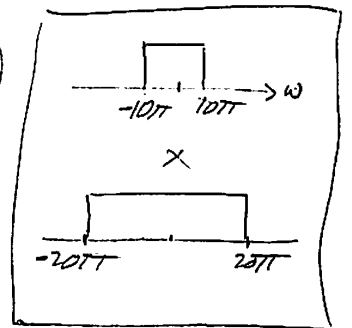
$$F[10 \text{sinc}(10t)] \quad H(\omega)$$

$$= \text{rect}\left(\frac{\omega}{10(2\pi)}\right) \quad \text{rect}\left(\frac{\omega}{20(2\pi)}\right)$$

so result is (in ω domain)



so the $F^{-1}[\text{---}]$ is a sinc.



so **$y(t) = 20 \text{sinc}(20t)$**

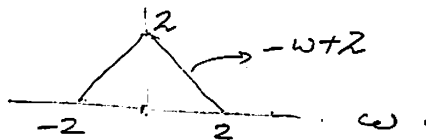
problem 15

$$|X(\omega)| = 2 \operatorname{tri}\left(\frac{\omega}{2}\right). \quad \text{Find energy of } x(t).$$

from Parseval, energy in ω domain = energy in time domain.

$$\text{so energy of } x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

the triangle is



$$\text{so } \int_{-2}^2 \frac{1}{2\pi} \int_0^2 (-\omega+2)^2 d\omega.$$

this 2 since adding both sides.

$$\text{so } \frac{1}{\pi} \int_0^2 \omega^2 + 4 - 4\omega d\omega = \frac{1}{\pi} \left(\left[\frac{\omega^3}{3} \right]_0^2 + 4[\omega]_0^2 - 4 \left[\frac{\omega^2}{2} \right]_0^2 \right)$$

$$= \frac{1}{\pi} \left(\frac{1}{3}(8) + 4(2) - 2(4) \right)$$

$$= \frac{1}{\pi} \left(\frac{8}{3} + 8 - 8 \right) = \boxed{\frac{1}{\pi} \left(\frac{8}{3} \right)}$$

HW # 2
Nasser M. Abbasi

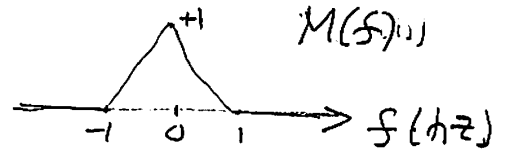
Problem 1

1

$$m(t) = \text{sinc}^2(t)$$

$$c(t) = 2 \cos(2\pi 10 t)$$

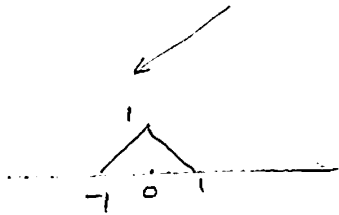
$$\begin{aligned} \text{a) } F(m(t)) &= F(\text{sinc}(t)) \otimes F(\text{sinc}(t)) \\ &= \text{rect}(f) \otimes \text{rect}(f) \\ &= \text{tri}(f) \end{aligned}$$



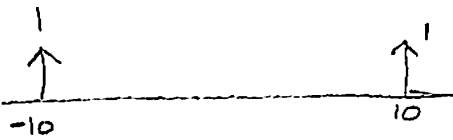
b) AM:

$$s(t) = (m(t) \cdot 2 \cos(2\pi 10 t)) + 2 \cos(2\pi 10 t)$$

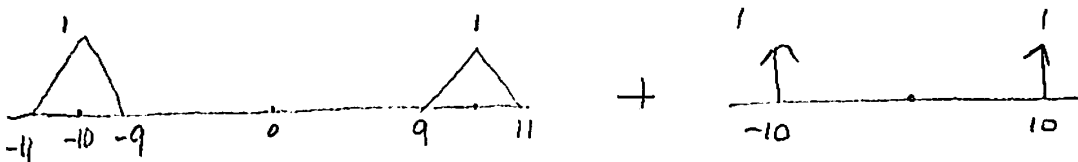
$F(2 \cos(2\pi 10 t))$



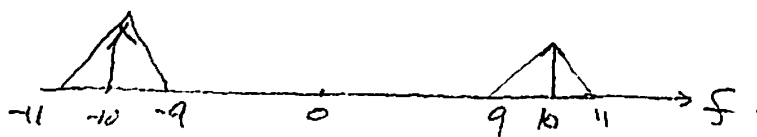
\otimes



\Downarrow



\Downarrow



$S(f)$

②

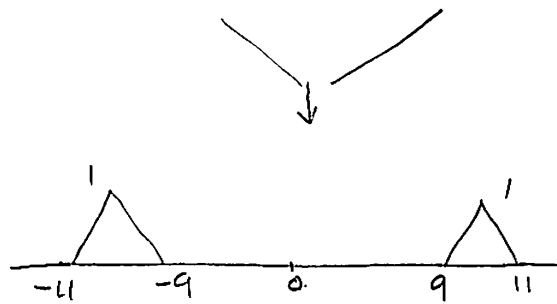
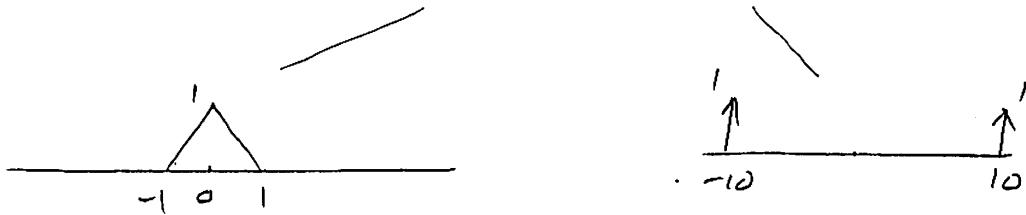
Problem 6.

⑤ ② DSB-SC.

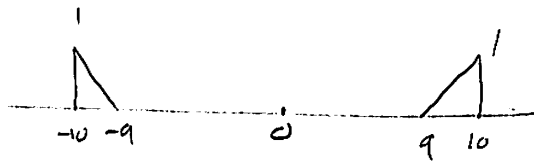
For DSB-SC, the modulated carrier is given by:

$$s(t) = m(t) \cdot c(t) = \sin^2(t) \cdot 2 \cos(2\pi 10t)$$

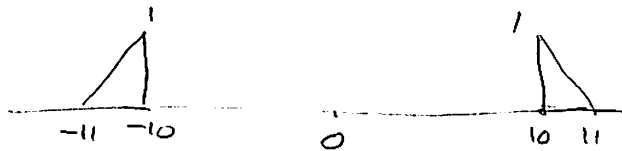
$$\text{So } F(s(t)) = F(\sin^2(t)) \otimes F(2 \cos(2\pi 10t))$$



③ LSB. here, we remove USB from DSB-SC to obtain:



③ USB. remove LSB to obtain:

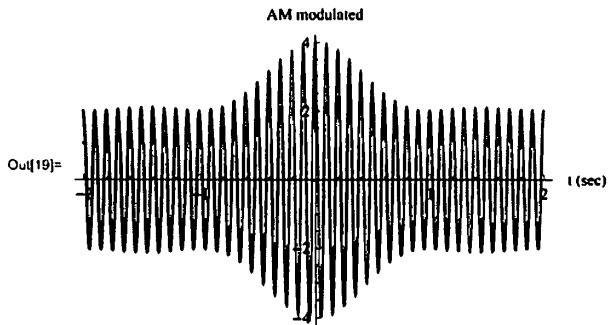


③

Problem ①. Part ①

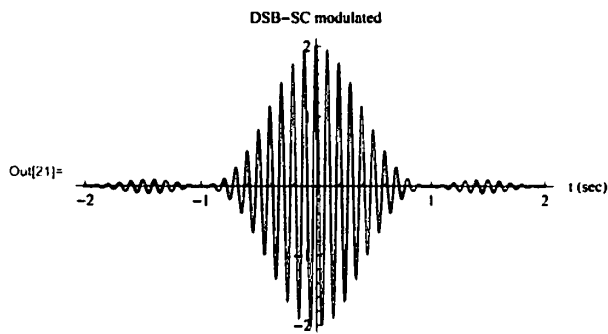
```
In[18]= s = Sinc[Pi t]^2 2 Cos[2 Pi 10 t] + 2 Cos[2 Pi 10 t]
Plot[s, {t, -2, 2}, PlotRange -> All,
AxesLabel -> {"t (sec)", "AM modulated"}, PlotStyle -> Thick]
```

```
Out[18]= 2 Cos[20 Pi t] + 2 Cos[20 Pi t] Sinc[Pi t]^2
```



```
In[20]= s = Sinc[Pi t]^2 2 Cos[2 Pi 10 t]
Plot[s, {t, -2, 2}, PlotRange -> All,
AxesLabel -> {"t (sec)", "DSB-SC modulated"}, PlotStyle -> Thick]
```

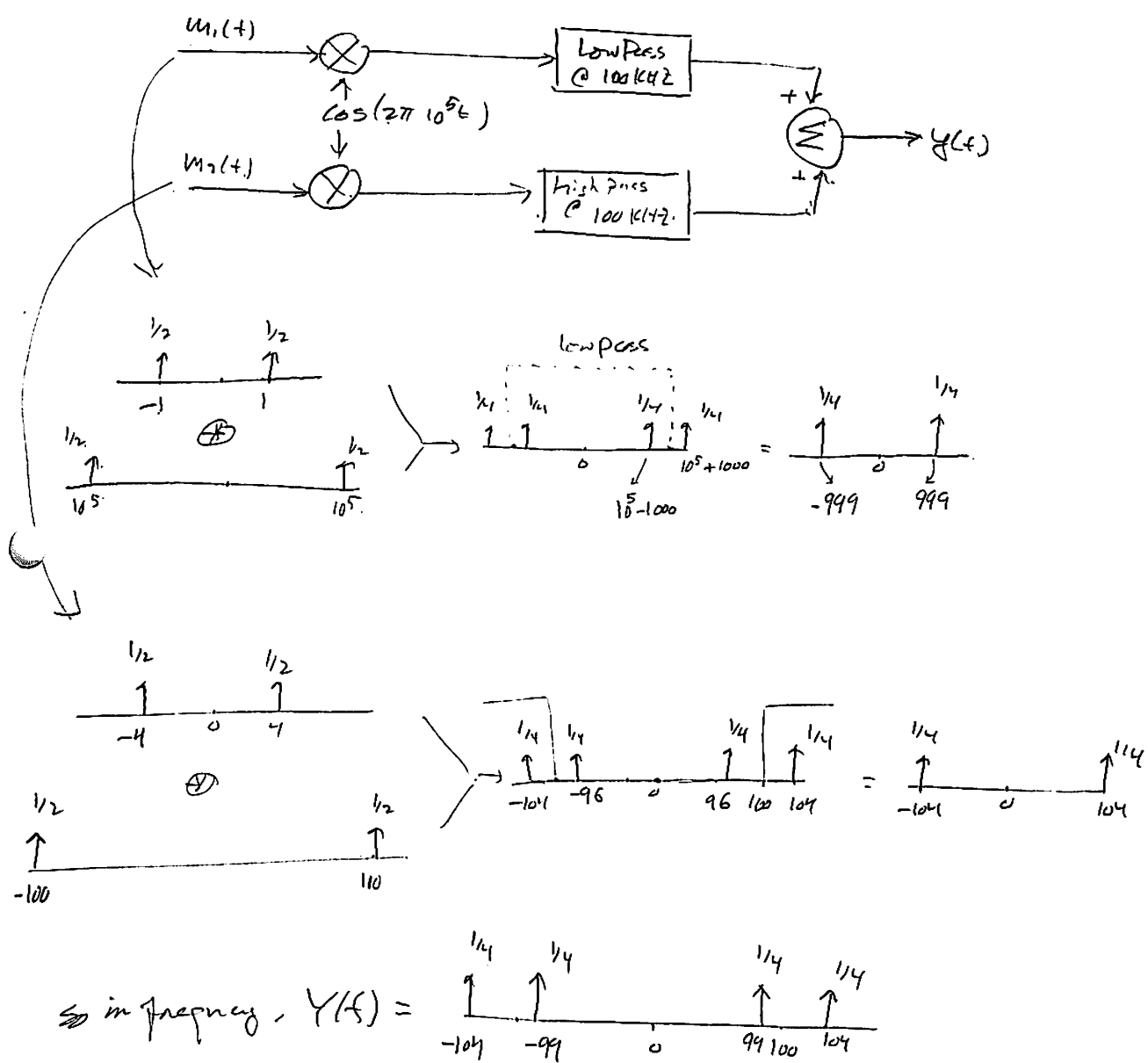
```
Out[20]= 2 Cos[20 Pi t] Sinc[Pi t]^2
```



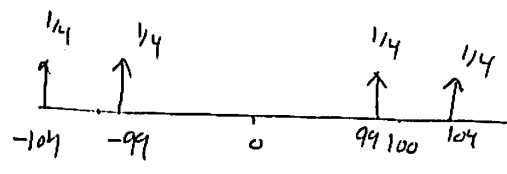
4

Problem 3

ⓐ $m_1(t) = \cos(2\pi 100t)$, $m_2(t) = \cos(2\pi 400t)$.
Find $y(t)$.



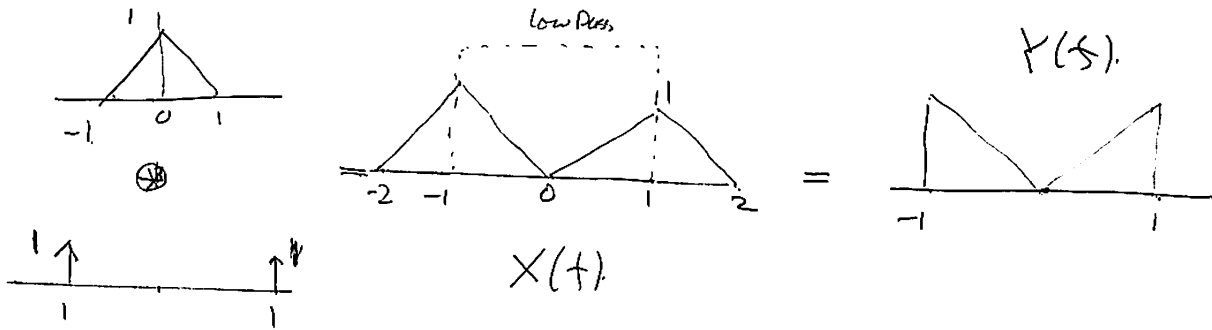
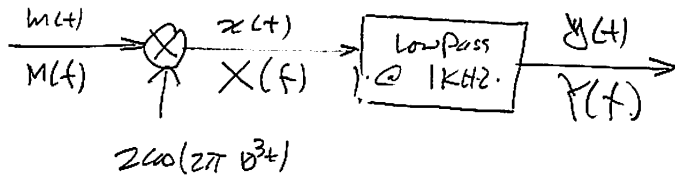
∴ in frequency, $Y(f) =$



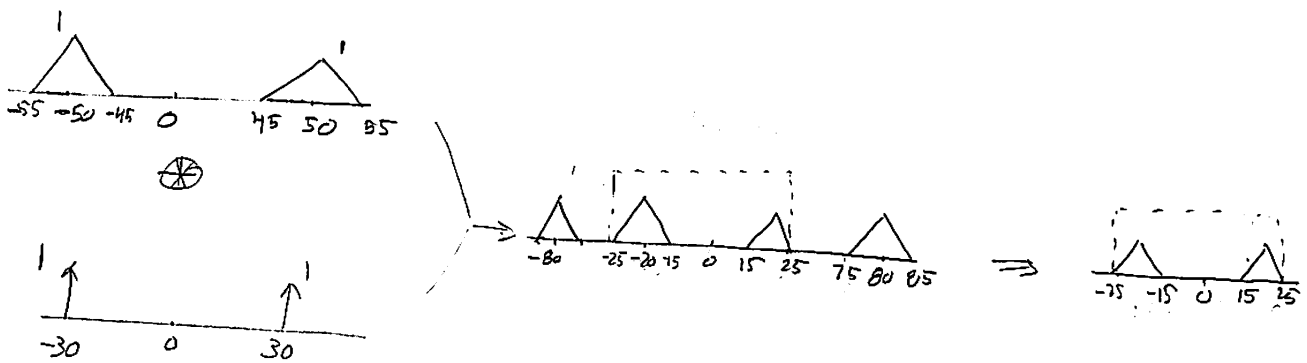
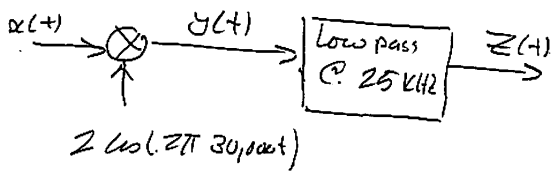
∴ in time domain

$$y(t) = \frac{1}{2} \cos(2\pi 99000t) + \frac{1}{2} \cos(2\pi 10400t)$$

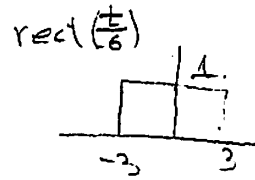
③ sketch the spectrum $X(f)$ and $Y(f)$ in the system below



④ sketch the spectrum



HW# 3
Problem ①



$$m(t) = 10 \operatorname{rect}\left(\frac{t}{6}\right).$$

① Find $\hat{m}(t)$

$$\hat{m}(t) = \frac{10}{\pi} \int_{-\infty}^{\infty} m(\tau) \frac{1}{t-\tau} d\tau = \frac{10}{\pi} \int_{-\infty}^{\infty} \frac{\operatorname{rect}\left(\frac{\tau}{6}\right)}{t-\tau} d\tau.$$

$$= \frac{10}{\pi} \left[\int_{-3}^3 \frac{1}{t-\tau} d\tau \right] = \frac{10}{\pi} \ln(t-\tau) \Big|_{-3}^3 = \frac{10}{\pi} \left[\ln(t-3) - \ln(t+3) \right]$$

$$= \frac{10}{\pi} \ln\left(\frac{t-3}{t+3}\right).$$

② $m_{\text{SSB}}(t) = m(t) \cos \omega_c t - \hat{m}(t) \sin \omega_c t.$

$$= 10 \operatorname{rect}\left(\frac{t}{6}\right) \cos(2\pi 1000t) - \frac{10}{\pi} \ln\left(\frac{t-3}{t+3}\right) \sin(2\pi 1000t)$$

② $m(t) = 4 \sin(12t) \cos(7t) \cos(3t)$

$$= 4 \left[\frac{1}{2} (\sin 5t + \sin 19t) \right] \cos(3t)$$

$$= 2 \left[\sin 5t \cos 3t + \sin 19t \cos 3t \right]$$

$$= \sin 2t + \sin 8t + \sin 16t + \sin 22t.$$

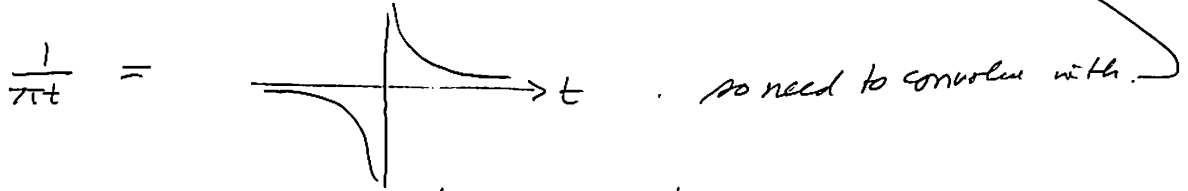
$$\hat{m}(t) = \sin\left(2t - \frac{\pi}{2}\right) + \sin\left(8t - \frac{\pi}{2}\right) + \sin\left(16t - \frac{\pi}{2}\right) + \sin\left(22t - \frac{\pi}{2}\right)$$

$$= -\cos 2t - \cos 8t - \cos 16t - \cos 22t.$$

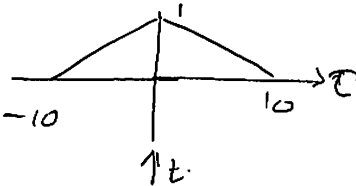
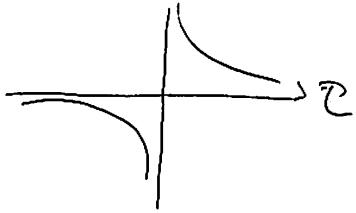
(3) $m(t) = \text{tri}\left(\frac{t}{10}\right)$.



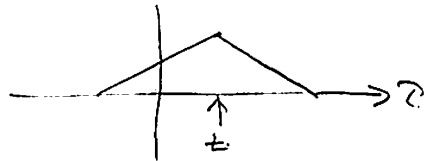
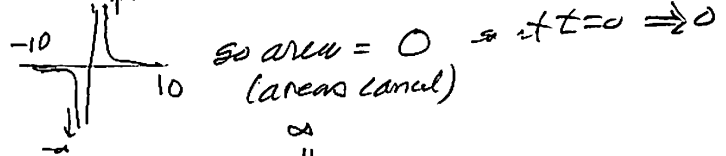
$w_1(t) = m(t) \otimes \frac{1}{\pi t}$.



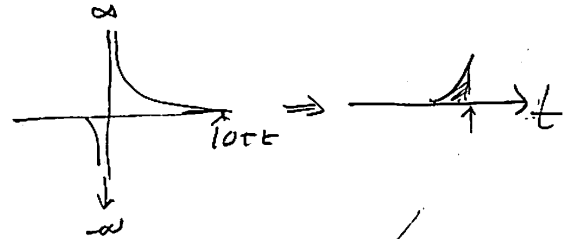
flip the tri function (even). stay the same;



at $t=0$, multiply and integrate



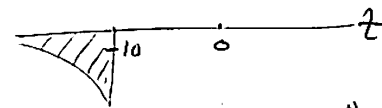
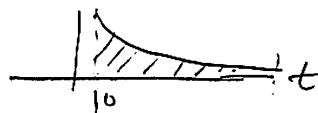
at $t > 0$,



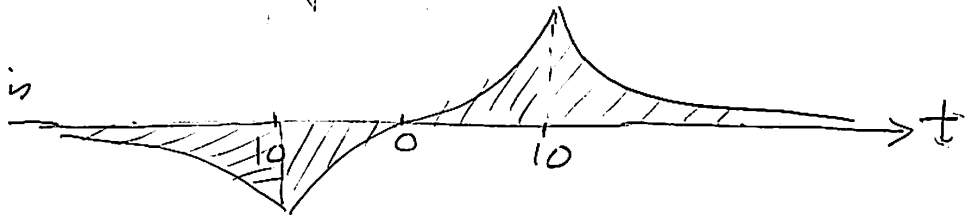
so for $-10 \leq t \leq 10$ we have

for $t > 10$, we set

for $t < -10$, we set



so final answer is



5) $m(t) = 2 \sin(5000t) \sin(7000t)$
 $c(t) = \cos(2\pi \cdot 10^7 t)$

(a) Find $m_{LSSB}(t)$

$$m_{LSSB}(t) = m(t) \cos \omega_c t + \hat{m}(t) \sin \omega_c t$$

$$m(t) = 2 \left[\frac{1}{2} \cos(-2000t) - \frac{1}{2} \cos(12000t) \right]$$

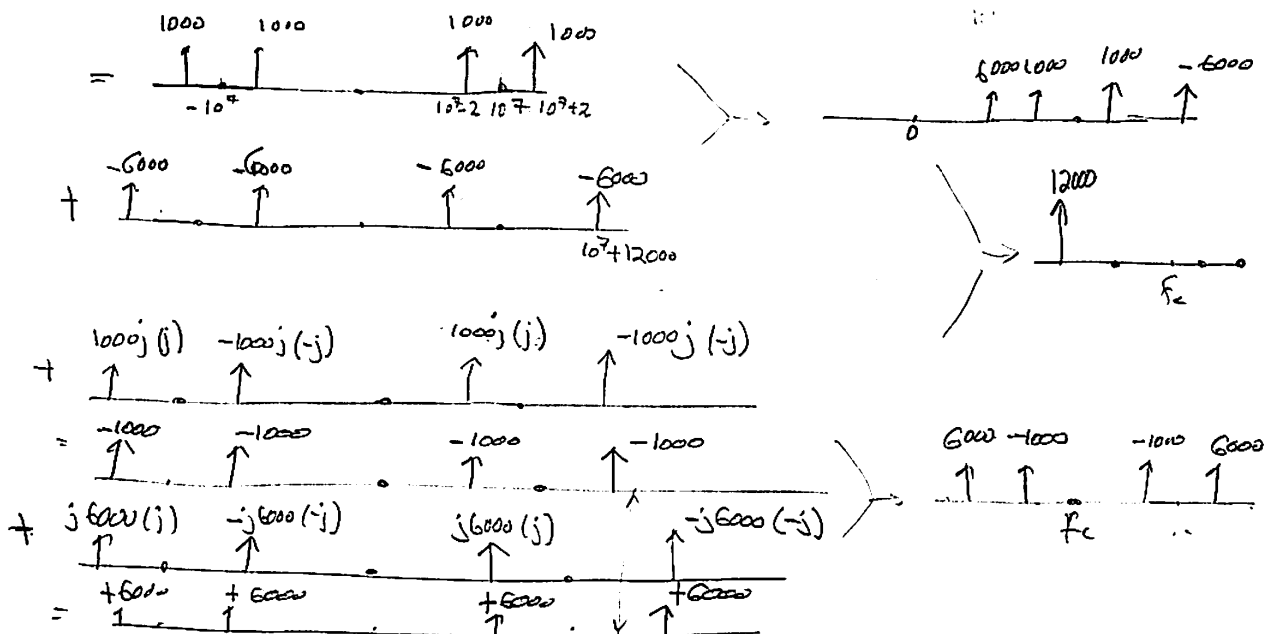
$$= \boxed{\cos(2000t) - \cos(12000t)}$$

$$\hat{m}(t) = \cos\left(2000t - \frac{\pi}{2}\right) - \cos\left(12000t - \frac{\pi}{2}\right)$$

$$= \boxed{\sin(2000t) - \sin(12000t)}$$

$$m_{LSSB}(t) = [\cos(2000t) - \cos(12000t)] \cos \omega_c t + [\sin(2000t) - \sin(12000t)] \sin \omega_c t$$

$$= \cos 2000t \cos \omega_c t - \cos 12000t \cos \omega_c t + \sin 2000t \sin \omega_c t - \sin(12000t) \sin \omega_c t$$

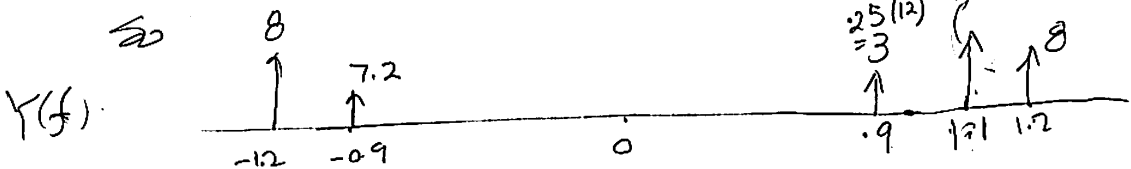
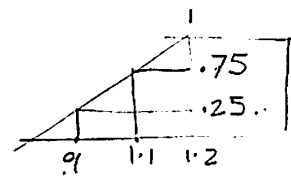
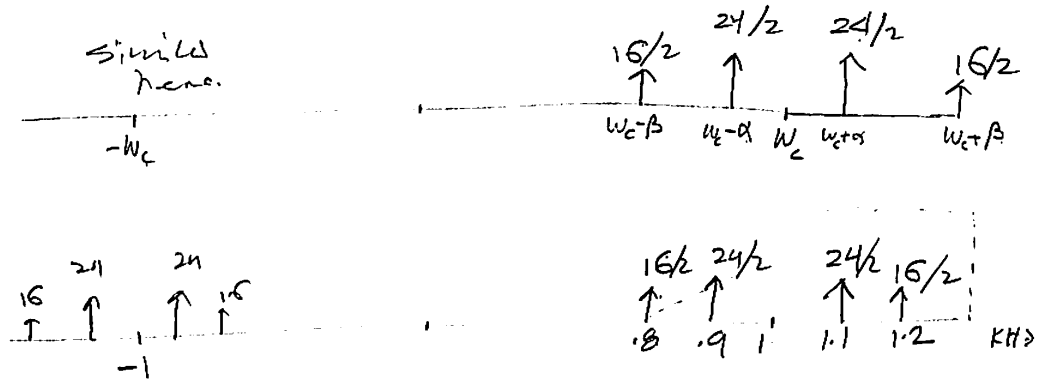


(6) $m(t) = 24 \cos(2\pi 100t) + 16 \cos(2\pi 200t)$
 $c(t) = 2 \cos(2\pi 1000t)$

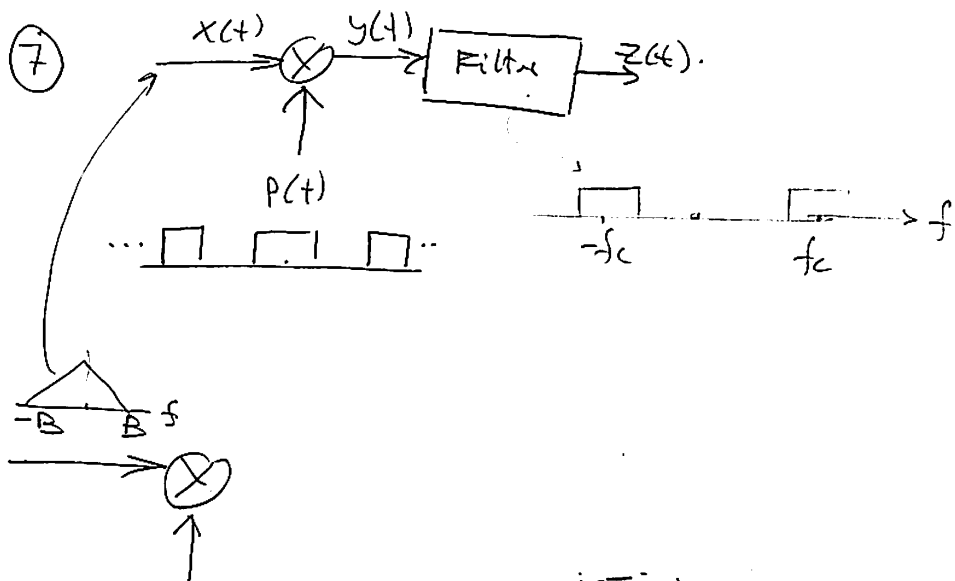
let $m(t) = 24 \cos(\alpha t) + 16 \cos(\beta t)$
 $c(t) = 2 \cos(\omega_c t)$

so $x(t) = 48 \cos(\alpha t) \cos \omega_c t + 32 \cos(\beta t) \cos \omega_c t$
 $= 48 \left[\frac{1}{2} \cos(\omega_c - \alpha)t + \frac{1}{2} \cos(\omega_c + \alpha)t \right]$
 $+ 32 \left[\frac{1}{2} \cos(\omega_c - \beta)t + \frac{1}{2} \cos(\omega_c + \beta)t \right]$
 $= 24 \cos(\omega_c - \alpha)t + 24 \cos(\omega_c + \alpha)t$
 $+ 16 \cos(\omega_c - \beta)t + 16 \cos(\omega_c + \beta)t$

so $X(f) =$



so $y(t) = 16 \cos(1200t) + 18 \cos(1100t) + 6 \cos(900t)$



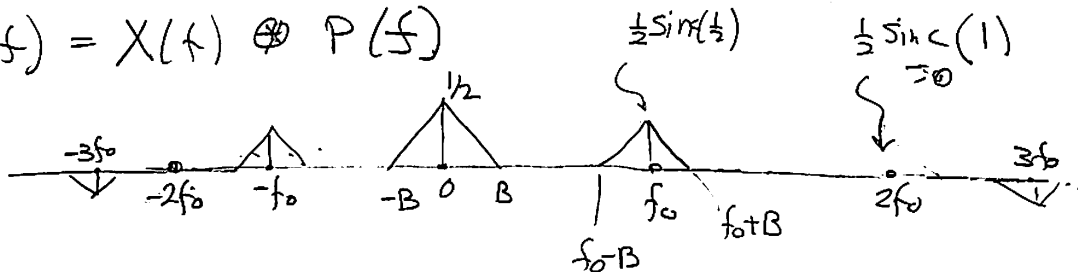
$$p(t) = \sum \frac{1}{2} \text{sinc}\left(\frac{n}{2}\right) e^{j\frac{2\pi}{T_c} n t}$$

$$\frac{n=1}{d=\frac{1}{2}} \Rightarrow \boxed{p(f) = \frac{1}{2} \text{sinc}\left(\frac{n}{2}\right) \sum \delta(f - n f_0)}$$

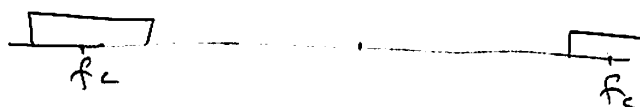
$$\Rightarrow p(f) = \dots \delta(f - 2f_0) \delta(f - f_0) \delta(f) \delta(f + f_0) \delta(f + 2f_0) \dots$$

$$\Rightarrow y(t) = x(t) \cdot p(t)$$

$$\text{or } Y(f) = X(f) \otimes P(f)$$



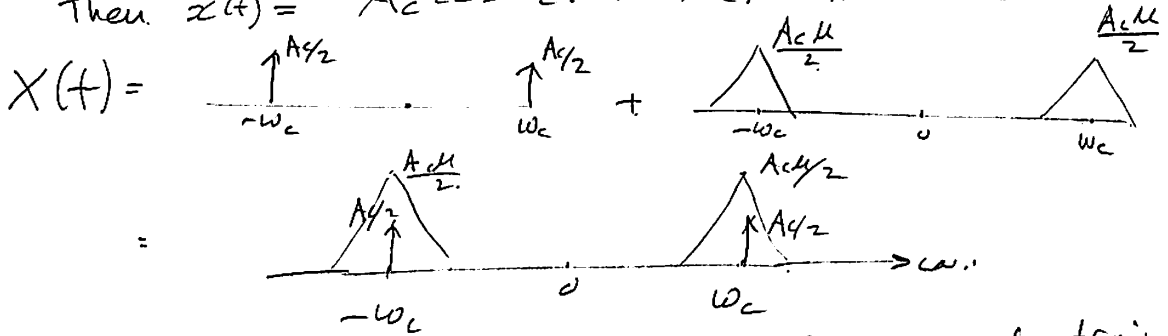
So when we multiply the above with $H(f)$ which is



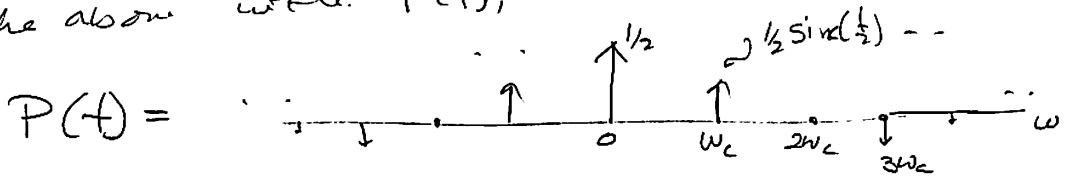
we will get, in the frequency domain $X(f_c - f) \times \underbrace{\frac{1}{2} \text{sinc}(f - f_c)}_{\text{Scale factor}}$

(b) if $x(t) = A_c [1 + \mu m_n(t)] \cos(\omega_c t)$.

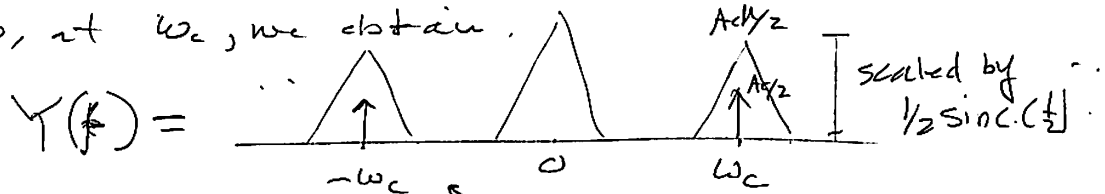
Then $x(t) = A_c \cos \omega_c t + A_c \mu m_n(t) \cos \omega_c t$.



convolve the above with $P(f)$, which is a pulse train.



so, at ω_c , we obtain



so $Z(f)$ will only contain these bandpass regions for bandpass.

$$\text{hence } Z(f) = \frac{1}{2} \text{sinc}\left(\frac{1}{2}\right) \left[\frac{A_c}{2} (f - f_c) + \frac{A_c \mu}{2} X(f - f_c) \right] + \frac{1}{2} \text{sinc}\left(\frac{1}{2}\right) \left[\frac{A_c}{2} (f + f_c) + \frac{A_c \mu}{2} X(f + f_c) \right]$$

$$\text{so } z(t) = \frac{1}{2} \text{sinc}\left(\frac{1}{2}\right) [A_c \cos(\omega_c t) + A_c \mu m_n(t)]$$

If low Pass, then only this will be used. here $-B$ 0 B

$$Z(f) = \frac{1}{2} \text{sinc}\left(\frac{1}{2}\right) [X(f)] = \frac{1}{2} \text{sinc}\left(\frac{1}{2}\right) [A_c \mu m_n(f)]$$

$$\text{so } z(t) = \frac{1}{2} \text{sinc}\left(\frac{1}{2}\right) A_c \mu m_n(t) = \frac{A_c}{\pi} \mu m_n(t)$$

⑧

$$v_1(t) = \frac{5}{1000} \cos(2\pi 600t) + \frac{1}{1000} \cos(2\pi 1000t)$$

$$v_2(t) = a_1 v_1(t) + a_2 v_2^2(t)$$

$$\begin{aligned} v_2(t) &= a_1 \left[\frac{5}{1000} \cos(2\pi 600t) + \frac{1}{1000} \cos(2\pi 1000t) \right] \\ &\quad + a_2 \left[\frac{5}{1000} \cos(2\pi 600t) + \frac{1}{1000} \cos(2\pi 1000t) \right]^2 \\ &= a_1 \frac{5}{1000} \cos(2\pi 600t) + a_1 \frac{1}{1000} \cos(2\pi 1000t) \\ &\quad + a_2 \left[\left(\frac{5}{1000}\right)^2 \cos^2(2\pi 600t) + \left(\frac{1}{1000}\right)^2 \cos^2(2\pi 1000t) \right. \\ &\quad \left. + 2 \left(\frac{5}{1000}\right) \left(\frac{1}{1000}\right) \cos(2\pi 600t) \cos(2\pi 1000t) \right] \end{aligned}$$

expand, use trig identity, look for coeffs. of $\cos(2\pi 1000t)$ and $\cos(2\pi 600t)$. equate to 1 and 0.001 to solve for a_1, a_2 .

$$\textcircled{9} m_{Am}(t) = 10 \left[1 + 0.8 \cos(2\pi 2000t) \right] \cos(2\pi 10^6 t)$$

① Final modulation index μ .

$$\text{rewrite as } m_{Am}(t) = A_c \left[1 + \mu m_p(t) \right] \cos(2\pi \omega_c t)$$

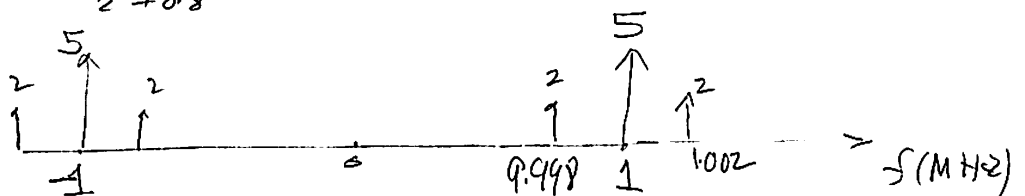
$$\text{so } \boxed{\mu = 0.8}$$

② transmission bandwidth. 2B. where $B = 2 \text{ kHz}$.

$$\text{hence } B_T = \boxed{4 \text{ kHz}}$$

$$\begin{aligned} \textcircled{3} \text{ efficiency} &= \frac{\text{Power in signal}}{\text{Total power}} = \frac{\frac{A_c^2 \mu^2}{2}}{A_c^2 + \frac{A_c^2 \mu^2}{2}} = \frac{\mu^2}{2 + \mu^2} \\ &= \frac{0.8^2}{2 + 0.8^2} = 24.24\% \end{aligned}$$

④



$$\textcircled{a} \quad m(t) = -6 \cos(2\pi 10t) - 4 \cos(2\pi 30t)$$

$$c(t) = A_c \cos(2\pi 100t)$$

$$\mu = 0.8$$

$$\textcircled{b} \quad AM(t) = m(t) \cos \omega_c t + A_c \cos \omega_c t$$

$$= [A_c + m(t)] \cos \omega_c t$$

$$= A_c \left[1 + \frac{1}{A_c} m(t) \right] \cos \omega_c t$$

$$= A_c \left[1 + \frac{1}{A_c} (-6 \cos(2\pi 10t) - 4 \cos(2\pi 30t)) \right] \cos \omega_c t$$

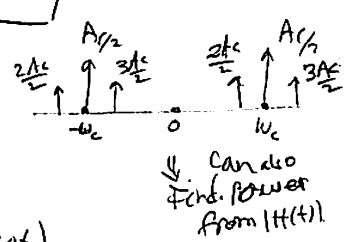
$$= A_c \left[1 + \frac{\max(m(t))}{A_c} \cdot \frac{m(t)}{\max(m(t))} \right] \cos \omega_c t$$

$$AM(t) = A_c \left[1 + \mu m_N(t) \right] \cos \omega_c t$$

$$\text{So } m_N(t) = \frac{-6}{10} \cos(2\pi 10t) - \frac{4}{10} \cos(2\pi 30t)$$

$$\textcircled{b} \quad \overline{m_N^2(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} m_N^2(t) dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[\frac{36}{100} \cos^2(2\pi 10t) + \frac{16}{100} \cos^2(2\pi 30t) + \frac{80}{100} \cos(2\pi 10t) \cos(2\pi 30t) \right] dt$$



$$= \frac{36}{100} \left(\frac{1}{2} \right) + \frac{16}{100} \left(\frac{1}{2} \right) + \frac{80}{100} \int \cancel{dt} = 0$$

$$= \boxed{0.26}$$

$$\textcircled{c} \quad E_{\text{eff}} = \frac{\text{Av. Power in signal}}{\text{Total av. power}} = \frac{\frac{1}{2} A_c^2 \mu^2 \overline{m_N^2(t)}}{\frac{A_c^2}{2} + \frac{1}{2} A_c^2 \mu^2 \overline{m_N^2(t)}}$$

$$= \frac{0.8^2 (0.26)}{1 + 0.8^2 (0.26)} = 14.27\%$$

HW#4

$$\textcircled{1} M_{EM}(t) = 50 \cos [2\pi 10^7 t + 6 \sin(2\pi 5000 t)]$$

$$\textcircled{a} \text{ For PM, } M_{EM}(t) = 50 \cos [2\pi f_c t + K_p m(t)]$$

$$K_p = 3, \text{ hence } M_{EM}(t) = 50 \cos [2\pi f_c t + 3 m(t)]$$

$$\text{hence } 3 m(t) = 6 \sin(2\pi 5000 t)$$

$$\text{hence } \boxed{m(t) = 2 \sin(2\pi 5000 t)}$$

$$\textcircled{b} \text{ for FM, } M_{EM}(t) = 50 \cos [2\pi f_c t + K_f \int m(\lambda) d\lambda]$$

$$\text{For } K_f = 2\pi 3000 \text{ rad/s/V, then}$$

$$M_{EM}(t) = 50 \cos [2\pi f_c t + 2\pi 3000 \int m(\lambda) d\lambda]$$

$$\text{hence. } 2\pi 3000 \int m(\lambda) d\lambda = 6 \sin(2\pi 5000 t)$$

$$\text{as } \int m(\lambda) d\lambda = \frac{6}{2\pi 3000} \sin(2\pi 5000 t)$$

$$\text{as } m(t) = \frac{6}{2\pi 3000} \frac{d}{dt} \sin(2\pi 5000 t)$$

$$= \frac{6}{2\pi 3000} \cdot 2\pi 5000 \cdot \cos(2\pi 5000 t)$$

$$\boxed{m(t) = 10 \cos(2\pi 5000 t)}$$

HW 4

② BW for FM = 220 kHz.
using Carson Rule.

① $\Delta f = 80 \text{ kHz}$. (max freq. deviation)

$$(B_T)_{\text{Carson}} = 2(B + \Delta f) = 2(B + 80 \text{ kHz}).$$

so $220 \text{ kHz} = 2B + 160 \text{ kHz}$.

so $B = \frac{60}{2} = \boxed{30 \text{ kHz}}$

⑥ $(B_T)_{\text{Carson}} = 2(B + \Delta f)$.

Given $B = 20 \text{ kHz}$, then

$$220 \text{ kHz} = 2(20 + \Delta f)$$

$$\frac{220 - 40}{2} = \Delta f$$

so $\boxed{\Delta f = 90 \text{ kHz}}$

③ $F_m(t) = \cos(2\pi \cdot 30 \cdot 10^6 t + k_f \int m(t) dx)$

$$\omega_i(t) = 2\pi \cdot 30 \cdot 10^6 + k_f m(t)$$

$$\Delta\omega = k_f \max |m(t)|$$

$$\boxed{\Delta\omega = k_f (6)} \Rightarrow \Delta f = 6(2000) = 12 \text{ kHz}$$

so Bandwidth is $(B_T)_{\text{Carson}} = 2(f_m + \Delta f) = 2(6 + 12) = 36 \text{ kHz}$

HW 4

$$(4) m(t) = 6 \cos(2\pi 1000 t)$$

$$f_c = 50 \text{ kHz}$$

$$\beta = 9$$

$$\text{unmodulated carrier power} = 32 \text{ Watts}$$

(a) Find frequency deviation constant k_f .

$$A_m = 6$$

$$f_m = 1000 \text{ Hz}$$

$$\beta = \frac{k_f A_m}{\omega_m}$$

$$\text{so } k_f = \frac{\beta \omega_m}{A_m} = \frac{(9)(2\pi 1000)}{6} = \pi 3000 \text{ rad/s/V}$$

(b) 60 kHz is 10 kHz away from carrier

$$\text{so } \begin{array}{ccccccc} & n=0 & n=1 & & & & n=10 \\ & f_c & f_c + f_m & \dots & & & f_c + f_m \\ & & & & & & \end{array}$$

$$\text{so for one sideband, power is } \frac{A_c^2}{2} J_{10}^2(\beta=9)$$

$$= \frac{A_c^2}{2} (0.1247)^2 \text{ Watts}$$

$$\text{so 2 sided power is } \boxed{0.1247^2} A_c^2$$

$$\text{but } \frac{A_c^2}{2} = 32 \Rightarrow A_c^2 = 64$$

$$\text{so 2 sided power} = (0.1247)^2 64$$

$$\begin{array}{ccccccccc} 50 & & 51 & & 52 & & 53 & & 59 & & 60 \\ \hline n=0 & & n=1 & & n=2 & & n=3 & & n=9 & & n=10 \end{array}$$

HW 4

⑤. $m_c(t) = 50 \cos[2\pi 10^6 t + 20t + 5 \sin(2\pi 10t)]$

① $\omega_i(t) = \frac{d}{dt} \phi_i(t) = 2\pi 10^6 + 20 + 5(2\pi \times 10) \cos(2\pi 10t)$

⑥ $m_{EM}(t) = 50 \cos[2\pi 10^7 t + 8 \cos(2\pi 2000t)]$

What is max freq. deviation?

frequency deviation = $\frac{d}{dt} \phi(t) = \frac{d}{dt} [8 \cos(2\pi 2000t)]$

= $-8 \sin(2\pi 2000t) (2\pi 2000)$

so max is $8(2\pi 2000) = 2\pi 16000$

= 16 KHz

⑦ 30 MHz carrier.

single tone $f_m = 11$ KHz.

max freq. deviation = 99 KHz.

① 1% side bandwidth.

$\cos(\omega_c t + \beta \sin \omega_m t)$

$B = \frac{\Delta f}{f_m} = \frac{99}{11} = \boxed{9}$

From table, we find $n = 13$

so $(BT)_{1\%} = 2 f_m n_{max} = 2(11)(13) = 286$ KHz.

② $(BT)_{carson} = 2 f_m (1 + \beta) = 2(11)(1 + 9) = 220$ KHz.

③ $(BT)_{carson} = 2(f_m + \Delta f)$

so $(BT)_{carson} \rightarrow 2(3f_m + 3\Delta f) = 2(3 \times 11 + 3 \times 99) = 660$

HW 4

$$\textcircled{a} \quad m(t) = 60 \cos(2\pi 1000t) + 20 \cos(2\pi 3000t) \\ c(t) = \cos(2\pi 10^5 t) \\ K_f = 2\pi 100 \text{ rad/s/V.}$$

$$\textcircled{b} \quad M_{FM}(t) = \cos\left(2\pi 10^5 t + K_f \int \left[60 \cos(2\pi 1000t) + 20 \cos(2\pi 3000t) \right] dt\right) \\ = \cos\left(2\pi 10^5 t + K_f \left[\frac{60 \sin(2\pi 1000t)}{2\pi 1000} + \frac{20 \sin(2\pi 3000t)}{2\pi 3000} \right]\right) \\ = \cos\left(2\pi 10^5 t + 6 \sin(2\pi 1000t) + \frac{2}{3} \sin(2\pi 3000t)\right)$$

$$\text{so } A_{m_1} = 6, \quad f_{m_1} = 1000 \text{ Hz} \\ A_{m_2} = \frac{2}{3}, \quad f_{m_2} = 3000 \text{ Hz.}$$

$$(B_T)_{\text{Carson}} = 2(f_m + \Delta f)$$

$$\Delta f = K_f \max|m(t)|$$

$$\text{but } \max|m(t)| = 60 + 20 = 80.$$

$$\text{so } \Delta f = 2\pi 100 (80) = 2\pi 8000 \text{ rad/sec.} = \boxed{8 \text{ kHz}}$$

$$\text{so } (B_T)_{\text{Carson}} = 2(3 + 8) = 22 \text{ kHz}$$

we take the larger of the frequencies.

HW# 4

(12) FM transmitter A_m, f_m

$$m(t) = 8 \cos(2\pi 200t), \beta = 6.$$

Unmodulated power is 12 Watt. across 50Ω.

a) find frequency deviation K_f .

$$\begin{aligned} FM(t) &= A_c \cos(\omega_c t + K_f \int m(x) dx) \\ &= A_c \cos(\omega_c t + 8 K_f \frac{\sin(2\pi 200t)}{2\pi 200}) \end{aligned}$$

compare to canonical form

$$FM(t) = A_c \cos(\omega_c t + \beta \sin(\omega_m t))$$

$$\text{so } \beta = \frac{8 K_f}{2\pi 200}$$

$$\text{so } K_f = \frac{(6)(2\pi 200)}{8} = \boxed{942.477} \text{ rad/s/Volt}$$

b) to Find A_c , use Power specifications

$$\text{since power} = \frac{1}{2} \frac{A_c^2}{Z}$$

$$\text{Then } 12 = \frac{1}{50} \frac{A_c^2}{2} \text{ solve for } A_c = \sqrt{(12)(50)(2)} = 34.64 \text{ Volt}$$

Peak amplitude. at $f_c - 200$. hence. $n = -1$ since $f_m = 200$.

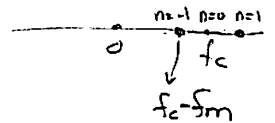
$$s_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

$$\text{From table } \boxed{J_{-1}(6) = -0.2767}$$

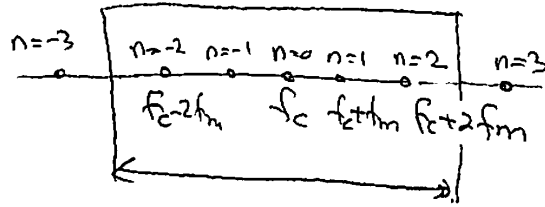
so for $n = -1$, we have

$$= A_c (J_{-1}(6)) \cos(\omega_c + n\omega_m)t$$

$$\text{Amplitude} = (34.64)(-0.2767) = \boxed{9.584 \text{ Volt}}$$



⑩ $\beta = 1,$



$J_0(1) = 0.7652$

$J_1(1) = 0.4401$

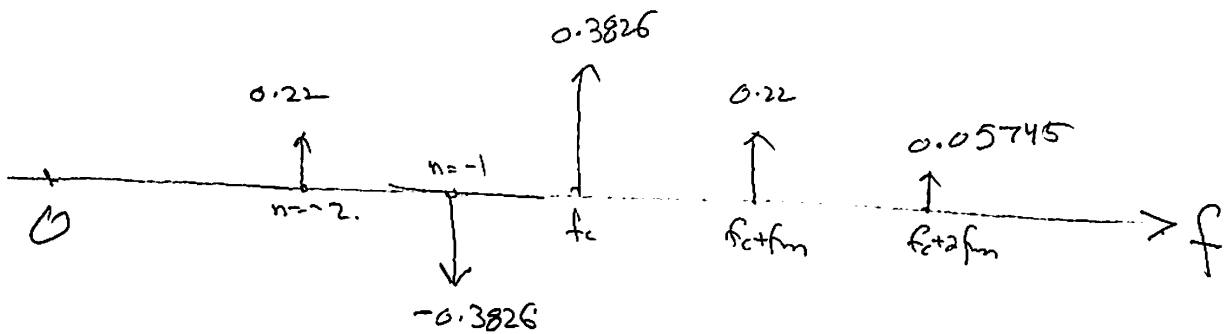
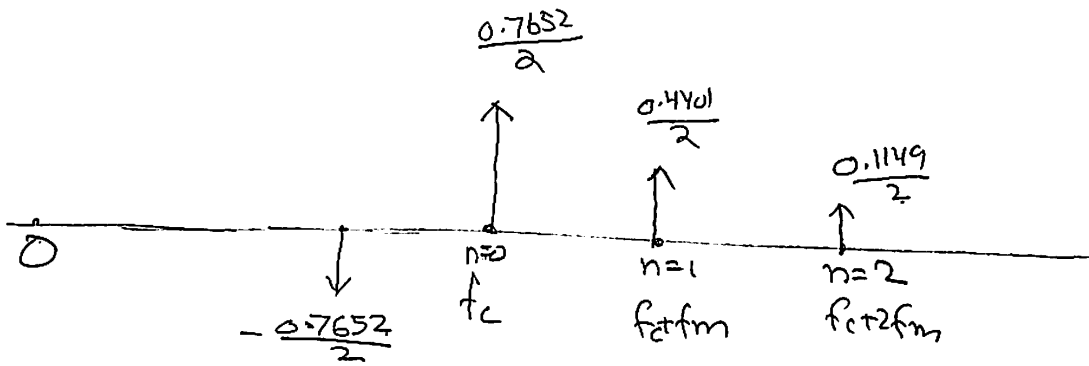
$J_2(1) = 0.1149$

$F_m(t) = \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

so output $F_m(f)$ is

$$1 = \frac{-A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - (f_c + n f_m)) + \delta(f + (f_c + n f_m)) \right]$$

need to go to $n=2$ on right side.



HW#4

#11

$$m_{FM}(t) = 25 \cos(2\pi 10^6 t + 8 \sin(2\pi 3000 t))$$

(a) Total average power = $\frac{(25^2)}{2} / 50 = 6.25 \text{ Watt}$.

(b) The Fourier Series of the above is

$$\hat{m}_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m t)$$

where $\omega_c = 2\pi 10^6 \text{ rad/sec}$

$\omega_m = 2\pi 3000 \text{ rad/sec}$

$$\boxed{\beta = 8}$$

so at $n=0$, we have $A_c J_0(\beta) \cos(2\pi 10^6 t)$

$$\text{so the power of this is } \frac{[A_c J_0(\beta)]^2}{2} = \frac{[(25)(0.1717)]^2}{2}$$

$$= 9.212$$

so over $50 \Omega \rightarrow \frac{9.212}{50} = 0.18425 \text{ Watt}$.

so % is $\frac{0.18425}{6.25} \times 100 = 2.9\%$.

(c) to find Peak freq. deviation:

$$\text{Frequency deviation} = \frac{d}{dt} \text{ Phase deviation}$$

$$= \frac{d}{dt} (8 \sin(2\pi 3000 t))$$

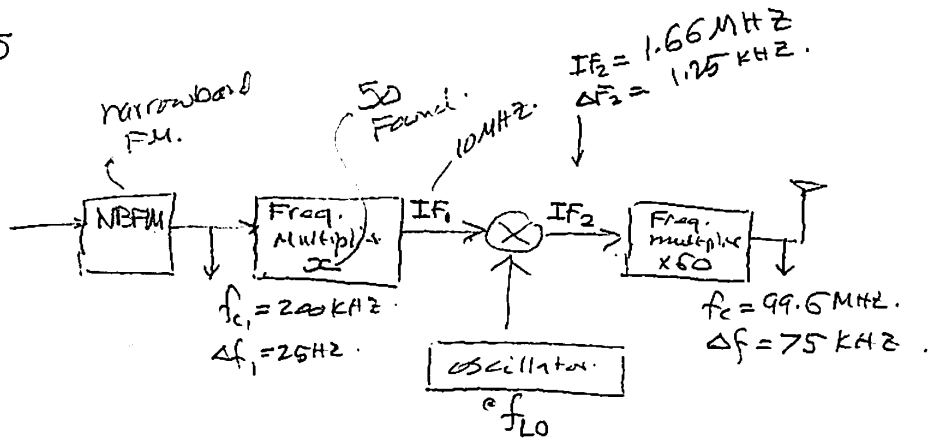
$$= 8 \cos(2\pi 3000 t) (2\pi 3000)$$

so max of the above is $(8)(2\pi 3000) = 150,796 \text{ rad/sec}$
 $= \boxed{24 \text{ kHz}}$

(d) $n_{max} = 11$

so $(BT)_{\%} = 2 f_m n_{max} = 2(3 \text{ kHz})(11) = 66 \text{ kHz}$.

HW 5
①



∴ we need $IF_1 = \frac{99.6 \text{ MHz}}{60} = 1.66 \text{ MHz}$.

Oscillator do not affect Δf .

∴ hence $(\Delta f_1) \times (60) = \Delta f$

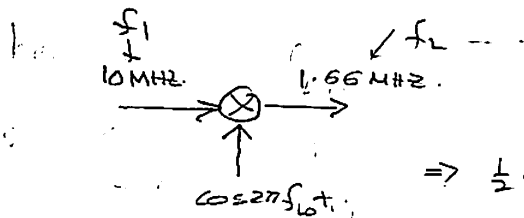
so $(25) \times (60) = 75,000$.

then $x = \frac{75,000}{(25)(60)} = \boxed{50}$ so multiplier factor = 50

now we use this to find the rest.

$IF_1 = (200 \text{ kHz})(50) = 10 \text{ MHz}$.

$IF_2 = \frac{99.6 \text{ MHz}}{60} = 1.66 \text{ MHz}$.



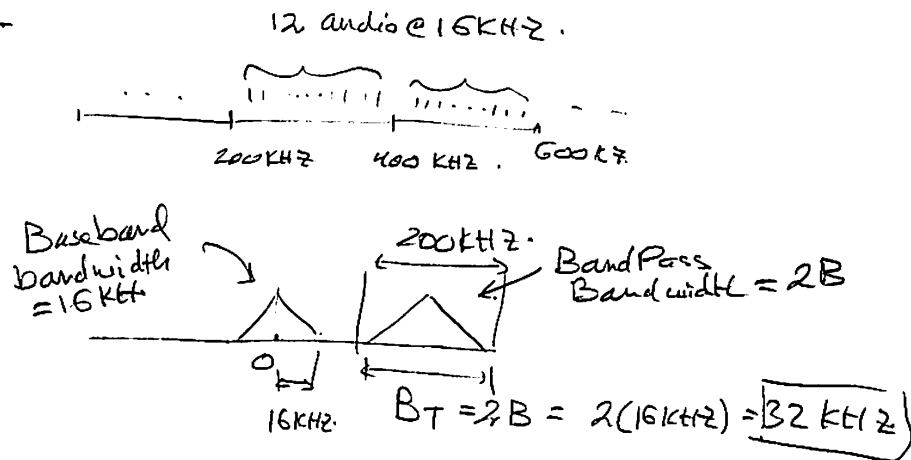
$\Rightarrow \frac{1}{2} \cos(2\pi(f_{LO} + f_1)t) + \frac{1}{2} \cos(2\pi(f_{LO} - f_1)t)$

need $f_{LO} + f_1 = 1.66 \text{ MHz}$.

or $f_{LO} - f_1 = 1.66 \text{ MHz}$.

∴ $f_{LO} = 10 \text{ MHz} - 1.66 \text{ MHz} = \boxed{8.34 \text{ MHz}}$

HW 5
3



so $200 \text{ kHz} \geq 32 \text{ kHz} (1 + \beta)$ ← ask about this.

$$= B \leq 5.25$$

(b) $B_T = 2(B + \Delta f) = 2(16 \text{ kHz} + 2 \text{ MHz})$

Carson. \downarrow 16 kHz.

$$= 4.032 \text{ MHz.}$$

(Why Carson 9 MHz?)

HW#5

⑨

FM

$$\rightarrow \boxed{} \rightarrow (SNR)_0 = \frac{3}{4} \frac{A_c^2 \beta^2}{N_0 f_m} = \frac{3}{4} \frac{A_{FM}^2 \beta^2}{N_0 f_m}$$

For AM - The modulation index μ is defined as $\mu = \frac{A_m}{A_c}$

$$A_m(t) = A_c \cos \omega_c t + m(t) \cos \omega_c t$$

$$= A_c (1 + \mu m(t)) \cos \omega_c t$$

let $\mu=1$

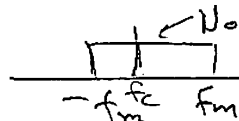
$$= A_c (1 + m(t)) \cos \omega_c t$$

$$= A_c (1 + A_m \cos(2\pi f_m t)) \cos \omega_c t$$

$$(S_0)_{AM} = \frac{A_m^2}{2}$$

$$(P_0)_{AM} = 2 N_0 f_m$$

$$\approx (SNR)_{0,AM} = \boxed{\frac{A_m^2}{4 N_0 f_m}}$$



so need.

$$\frac{A_m^2}{4 N_0 f_m} = \frac{3}{4} \frac{A_{FM}^2 \beta^2}{N_0 f_m}$$

$$\frac{A_m}{A_{FM}} = \sqrt{3 \beta^2} \quad \text{but } \beta = 9.$$

$$= \sqrt{3 \cdot 9^2} = \sqrt{3} \cdot 9 = \boxed{15.58}$$

HW#5

(10)

$$m(t) = 8 \cos(2\pi 5000t)$$

$$K_f = 2\pi 4000 \text{ rad/s/V}$$

$$A_c = 0.1 \text{ V}$$

$$(SNR)_o = \frac{3}{4} \frac{A_c^2 \beta^2}{N_o f_m} = \frac{3}{4} \frac{(0.1)^2 \beta^2}{(10^{-6})(5000)} \quad \text{--- (1)}$$

$$\text{but } \beta = \frac{K_f A_m}{2\pi f_m} = \frac{(2\pi 4000)(8)}{2\pi (5000)}$$

Plug all this into (1) and calculate

$$(SNR)_o = 614.4$$

$$S_r \text{ in db} = 10 \log_{10}(614.4) = 27.88 \text{ db}$$

HW5
11

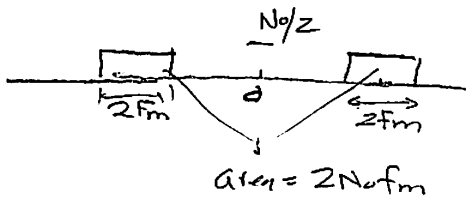
FM, $(SNR)_i = 30 \text{ db.}$
 $(SNR)_o = 48 \text{ db.}$

$$(SNR)_i \frac{S_i}{P_i} \rightarrow \boxed{\text{FM receiver}} \rightarrow (SNR)_o$$

$$S_i = \frac{A_c^2}{2}$$

$$P_i = 2 N_0 f_m$$

$$\downarrow \frac{3}{4} \frac{A_c^2 \beta^2}{N_0 f_m}$$



here. $10 \log \left(\frac{A_c^2}{2 N_0 f_m} \right) = 30$

$$10 \log \left(\frac{3}{4} \frac{A_c^2 \beta^2}{N_0 f_m} \right) = 48$$

so $\frac{A_c^2}{2 N_0 f_m} = 10^3$

$\therefore 10 \log \left(\frac{3}{2} \beta^2 (10^3) \right) = 48$

$\therefore \frac{3}{2} \beta^2 (10^3) = 10^{4.8}$

$\therefore \beta^2 = \frac{\left(\frac{2}{3} 10^{4.8} \right)}{10^3} \Rightarrow \boxed{\beta = 6.485}$

HW#5

⑦

Power in signal after filter =

$$\int_{-2,000}^{2,000} S_m(f) df = (2 \text{ kHz})(10^{-2}) + (1 \text{ kHz})(10^{-2})$$

$$= (3,000)(10^{-2}) = 30 \text{ watt}$$

$$\text{Power in noise of the filter} = \int_{-2,000}^{2,000} \frac{1}{2} \times 10^{-6} df$$

$$= \left(\frac{1}{2} \times 10^{-6}\right)(4,000) = (10^{-6})(2,000) = 2 \times 10^{-3} \text{ watt}$$

$$\text{so } \text{SNR} = \frac{30}{2 \times 10^{-3}} = 1.5 \times 10^4$$

so in db :

$$(\text{SNR})_{\text{db}} = 10 \log_{10} (1.5 \times 10^4)$$

$$= 10 [\log_{10} 1.5 + \log_{10} 10^4]$$

$$= 10 (\log_{10} 1.5 + 4)$$

$$= 40 + 10 \log_{10} 1.5$$

$$= \boxed{41.76}$$

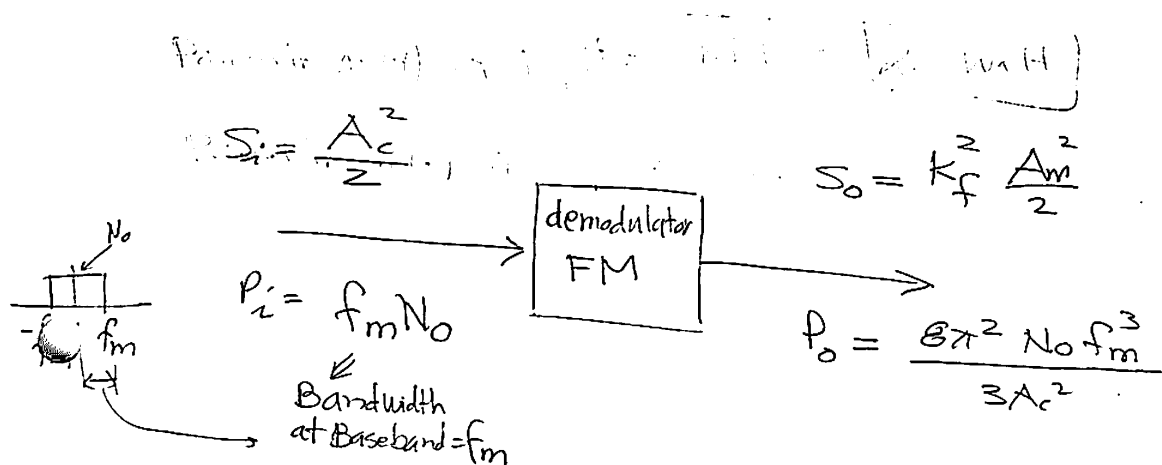
HW#5

$$\textcircled{8} \quad m(t) = 2 \cos(2\pi 5000t)$$

$$c(t) \Rightarrow \omega_c = 2\pi 30 \times 10^3$$

$$k_f = 2\pi 15000 \text{ rad/sec per Volt.}$$

$$\text{SNR}_i \rightarrow \boxed{\phantom{\text{demodulator FM}}} \rightarrow \text{SNR}_o = 40 \text{ db.}$$



$$\text{so } (\text{SNR})_i = \frac{A_c^2}{2f_m N_o}$$

$$(\text{SNR})_o = \frac{3}{4} \frac{A_c^2 \beta^2}{N_o f_m}$$

hence we see that

$$\boxed{(\text{SNR})_o = \frac{3}{2} \beta^2 (\text{SNR})_i}$$

$$\text{so } (\text{SNR})_o \text{ (in db)} = 10 \log_{10} \left(\frac{3}{2} \beta^2 \right) + (\text{SNR})_i \text{ (in db)}$$

$$\text{so } (\text{SNR})_i \text{ (in db)} = 40 - 10 \log_{10} \left(\frac{3}{2} \beta^2 \right)$$

$$\text{But } \beta = \frac{k_f A_m}{2\pi f_m} = \frac{2\pi (15000) (2)}{2\pi (5000)} = \frac{30,000}{5,000} = \boxed{6}$$

$$\text{so } (\text{SNR})_i \text{ (in db)} = 40 - 10 \log_{10} \left(\frac{3}{2} 6^2 \right) = \boxed{22.6761}$$

HW 5

$$\underline{12}. \text{ AM: } A_c \cos \omega_c t + m(t) \cos \omega_c t.$$

$$\text{FM: } A_c \cos(\omega_c t + K_f \int m(\lambda) d\lambda)$$

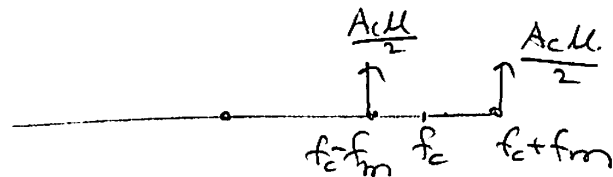
$$\textcircled{a} \text{ BT for AM} = 2f_m.$$

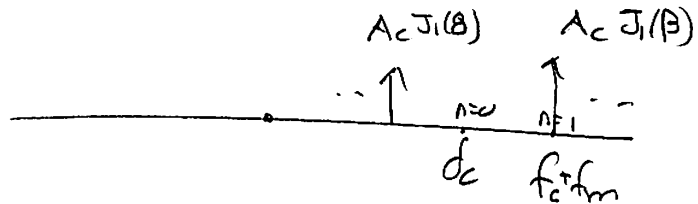
Frequency deviation for FM is $\frac{d}{dt} (K_f \int m(\lambda) d\lambda) = K_f m(t)$.

$$\text{So max of this} = K_f (m(t))_{\max} = K_f A_m = \Delta f$$

$$\text{So } \frac{\Delta f}{f_m} = 4(2f_m)$$

$$\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m} = \frac{8f_m}{f_m} = \boxed{8}$$

$$\textcircled{b} \text{ For AM}$$


$$\text{For FM}$$


$$\text{So}$$

$$A_c J_1(\beta) = \frac{A_c \mu}{2}$$

$$\text{So}$$

$$\mu = 2 J_1(\beta) = \boxed{0.469}$$

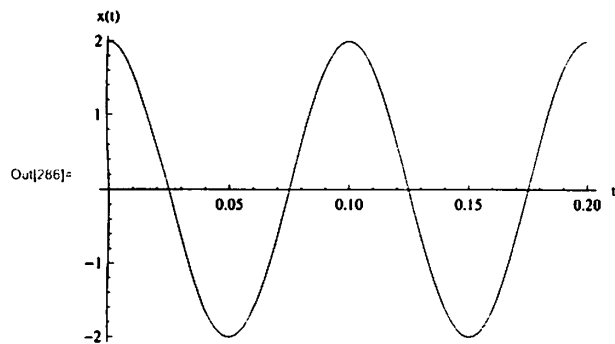
HW 6, Problem 1

by Nasser M. Abbasi

In[281]= << dsp`

part (a)

```
In[282]= Clear[w, t];
          fm = 10;
          period = 1 / fm;
          x[t_] := 2 Cos[2 Pi fm t]
          Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}]
```



part (b)

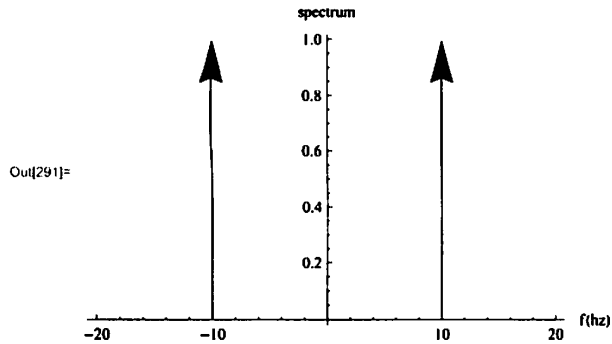
```
In[287]= ft = FourierTransform[x[t], t, f, FourierParameters -> {0, -2 Pi}];
          Print["Fourier Transform of x(t) is"];
          ft
```

Fourier Transform of x(t) is

```
Out[289]= DiracDelta[-10 + f] + DiracDelta[10 + f]
```

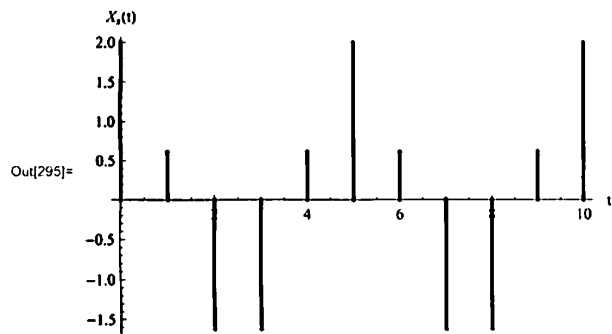
2 | *probl.nb*

```
In[290] = dsp`plotFourierTransform[ft, f, -2 fm, 2 fm, 0, .5, Large];
Show[%, PlotRange -> All, AxesLabel -> {"f (hz)", "spectrum"}]
```



■ part(c)

```
In[292] = Ts = 0.02;
nSamples = 2 * period / Ts;
data = Table[{n, x[n Ts]}, {n, 0, nSamples}];
ListPlot[data, Filling -> Axis, FillingStyle -> Thick, AxesLabel -> {"t", "Xn(t)"}]
```



■ part(d)

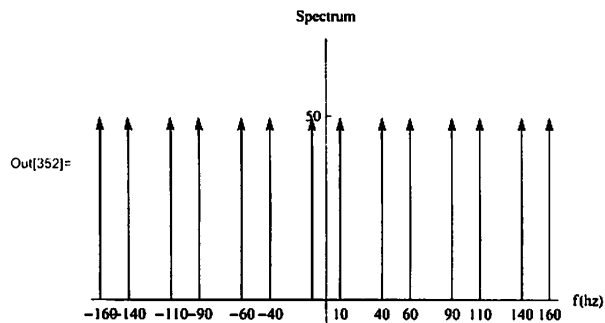
```
In[296] = Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
Sampling frequency = 50. hz
```

```
In[390] = spectrum = Expand[fs Sum[ft /. f -> (f - n * fs), {n, -3, 3}]];
Print["spectrum=", spectrum];
```

```
spectrum=50. DiracDelta[-160. + f] + 50. DiracDelta[-140. + f] + 50. DiracDelta[-110. + f] +
50. DiracDelta[-90. + f] + 50. DiracDelta[-60. + f] + 50. DiracDelta[-40. + f] +
50. DiracDelta[-10 + f] + 50. DiracDelta[10 + f] + 50. DiracDelta[40. + f] + 50. DiracDelta[60. + f] +
50. DiracDelta[90. + f] + 50. DiracDelta[110. + f] + 50. DiracDelta[140. + f] + 50. DiracDelta[160. + f]
```

probl.nb | 3

```
In[352]= Show[First@dsp`plotFourierTransform[spectrum, f, -3 * fs, 3 * fs, -.1 fs, 1.4 fs, Small],
  AxesLabel -> {"f(hz)", "Spectrum"},
  Ticks -> {{-160, -140, -110, -90, -60, -40, -10, 10, 40, 60, 90, 110, 140, 160}, {50}}]
```



■ part(e)

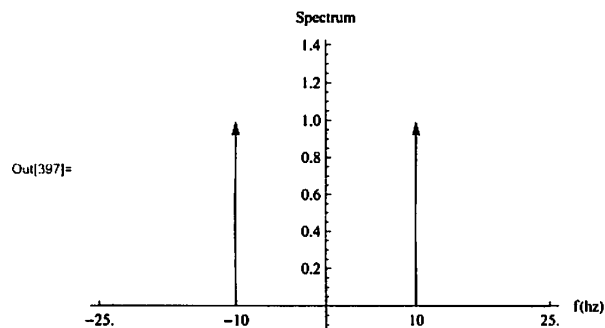
```
In[374]= bandwidth = 0.5 fs;
  Print["bandwidth = ", bandwidth, " hz"];
bandwidth = 25. hz
```

bandwidth = 25. hz

```
In[345]= gain = Ts;
  Print["Gain=", gain];
```

Gain=0.02

```
In[396]= spectrum = fs DiracDelta[-10 + f] + fs DiracDelta[10 + f];
  Show[First@dsp`plotFourierTransform[
  Expand[gain * spectrum], f, -bandwidth, bandwidth, -.1, 1.4, Small],
  AxesLabel -> {"f(hz)", "Spectrum"}, Ticks -> {{-bandwidth, -10, 0, 10, bandwidth}}]
```



■ part(f)

From the output above, we conclude that $y(t) = 2 \text{Cos}[2 \pi 10 t]$

HW 6, Problem 2

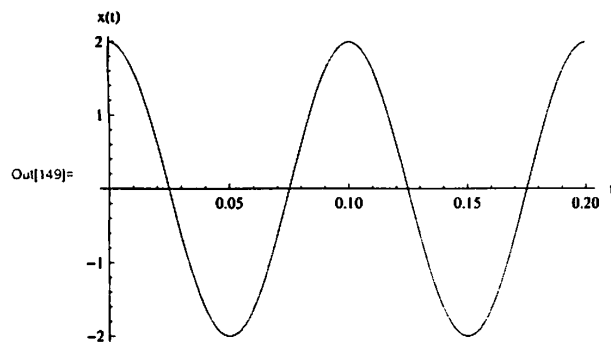
by Nasser M. Abbasi

In[96] = << dsp`

■ part (a)

```
In[144] = Clear[w, t, f];
          fm = 10;
          period = 1 / fm;
          Print["Period of message = ", N@period, " seconds"];
          Period of message = 0.1 seconds
```

```
In[148] = x[t_] := 2 Cos[2 Pi fm t]
          Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}]
```



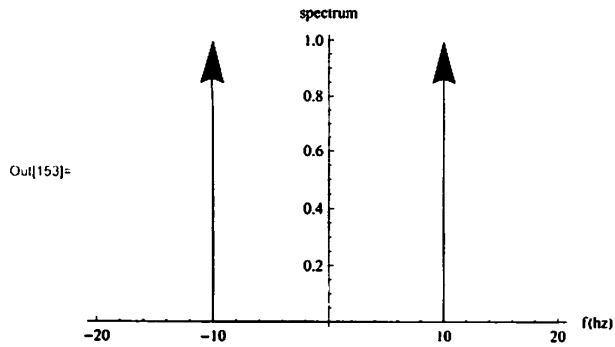
■ part(b)

```
In[150] = ft = FourierTransform[x[t], t, f, FourierParameters -> {0, -2 Pi}];
          Print["Fourier Transform of x(t) is", ft];
```

```
Fourier Transform of x(t) is DiracDelta[-10 + f] + DiracDelta[10 + f]
```

2 | prob2.nb

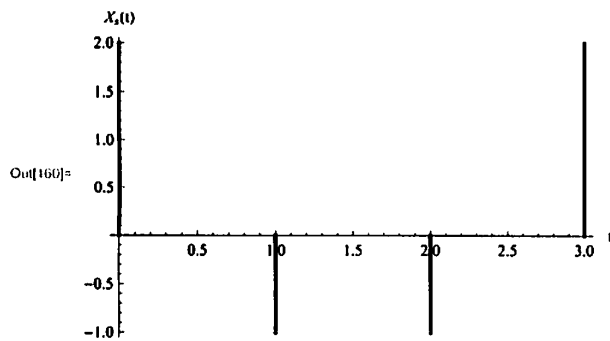
```
In[152] = Dep`PlotFourierTransform[ft, f, -2 fm, 2 fm, 0, .5, Large];
Show[%, PlotRange -> All, AxesLabel -> {"f(hz)", "spectrum"}]
```



■ part(c)

```
In[156] = Ts = 1 / 15;
Print["sampling period = ", N@Ts, " seconds"];
sampling period = 0.0666667 seconds
```

```
In[158] = nSamples = 2 * period / Ts;
data = Table[{n, x[n Ts]}, {n, 0, nSamples}];
ListPlot[data, Filling -> Axis, FillingStyle -> Thick, AxesLabel -> {"t", "X_s(t)"}]
```



■ part(d)

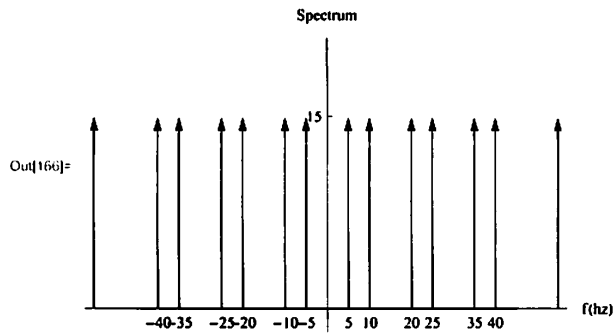
```
In[161] = Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
Sampling frequency = 15 hz
```

```
In[164] = spectrum = Expand[fs Sum[ft /. f -> (f - n * fs), {n, -3, 3}]];
Print["spectrum=", spectrum];
```

```
spectrum=15 DiracDelta[-55 + f] + 15 DiracDelta[-40 + f] + 15 DiracDelta[-35 + f] +
15 DiracDelta[-25 + f] + 15 DiracDelta[-20 + f] + 15 DiracDelta[-10 + f] +
15 DiracDelta[-5 + f] + 15 DiracDelta[5 + f] + 15 DiracDelta[10 + f] + 15 DiracDelta[20 + f] +
15 DiracDelta[25 + f] + 15 DiracDelta[35 + f] + 15 DiracDelta[40 + f] + 15 DiracDelta[55 + f]
```

prob2.nb | 3

```
In[166] = Show[First@dsp`plotFourierTransform[spectrum, f, -3 * fs, 3 * fs, -.1 fs, 1.4 fs, Small],
  AxesLabel -> {"f (hz)", "Spectrum"},
  Ticks -> {{-40, -35, -25, -20, -10, -5, 5, 10, 20, 25, 35, 40}, {fs}}]
```

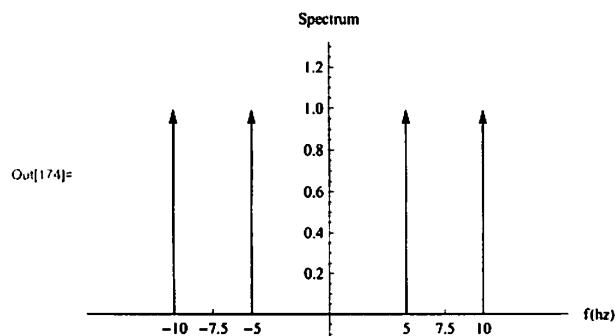


■ part(e)

```
In[167] = bandwidth = 0.5 fs;
  Print["bandwidth = ", bandwidth, " hz"];
bandwidth = 7.5 hz
```

```
In[169] = gain = Ts;
  Print["Gain=", N@gain];
Gain=0.0666667
```

```
In[173] = spectrum =
  (15 DiracDelta[-10 + f] + 15 DiracDelta[-5 + f] + 15 DiracDelta[5 + f] + 15 DiracDelta[10 + f]);
  Show[First@dsp`plotFourierTransform[Expand[gain * spectrum],
  f, -2 * bandwidth, 2 * bandwidth, -.1, 1.3, Small],
  AxesLabel -> {"f (hz)", "Spectrum"}, Ticks -> {{-bandwidth, -10, -5, 0, 5, 10, bandwidth}}]
```

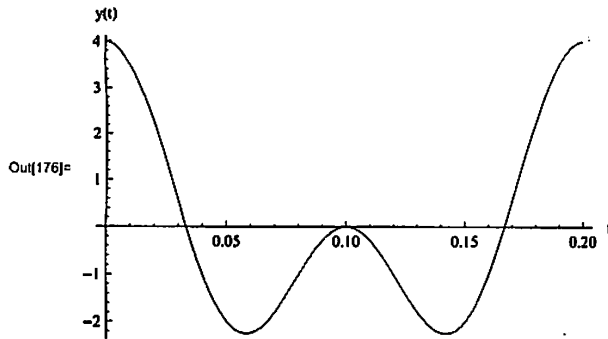


■ part(f)

From the output above, we conclude that $y(t) = 2 \cos[2\pi 10 t] + 2 \cos[2\pi 5 t]$

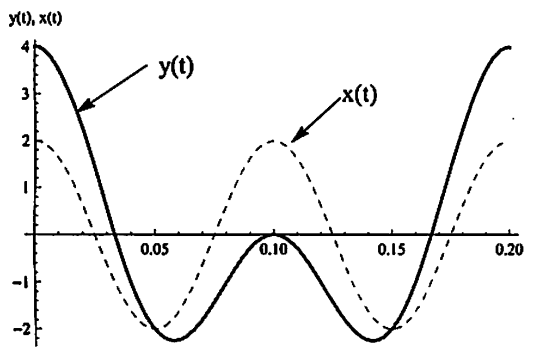
4 | prob2.nb

```
In[175] = y[t_] := 2 Cos[2 Pi 10 t] + 2 Cos[2 Pi 5 t];
Plot[y[t], {t, 0, 2 period}, AxesLabel -> {"t", "y(t)"}]
```



Compare this with the original signal $x(t)$ to see aliasing

```
In[178] = Plot[{x[t], y[t]}, {t, 0, 2 period},
  AxesLabel -> {"t", "y(t), x(t)"}, PlotStyle -> {Dashed, Thick}]
```



HW 6, Problem 3

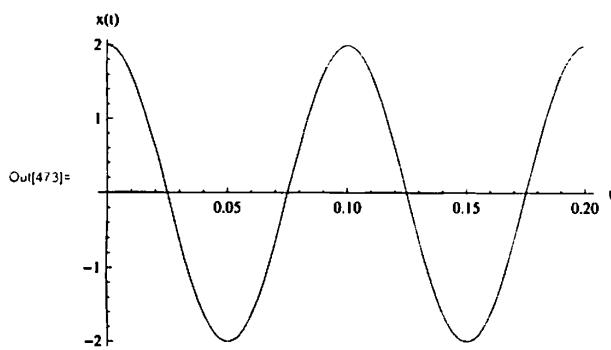
by Nasser M. Abbasi

<< dep`

part (a)

```
In[468]= Clear[w, t, t, f];
fm = 10;
period = 1 / fm;
Print["Period of message = ", N@period, " seconds"];
Period of message = 0.1 seconds
```

```
In[472]= x[t_] := 2 Cos[2 Pi fm t]
Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}]
```



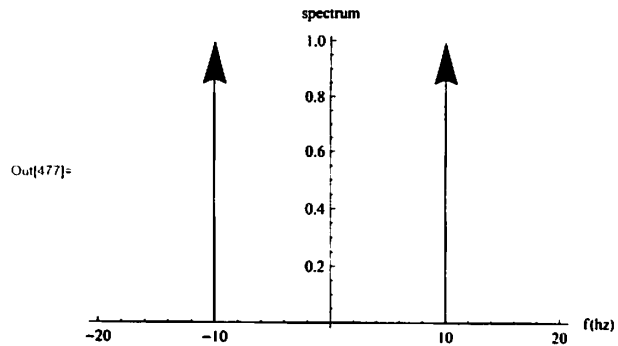
part (b)

```
In[485]= ft = FourierTransform[x[t], t, f, FourierParameters -> {0, -2 Pi}];
Print["Fourier Transform of x(t) = ", ft];
```

Fourier Transform of $x(t) = \delta(f - 10) + \delta(f + 10)$

2 | *prob3.nb*

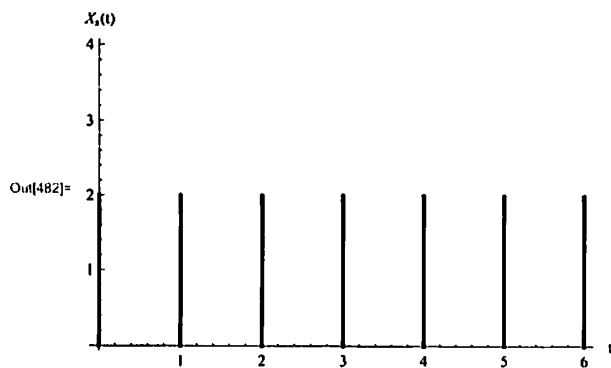
```
In[476]= dsp`plotFourierTransform[ft, f, -2 fm, 2 fm, 0, .5, Large];
Show[%, PlotRange -> All, AxesLabel -> {"f(hz)", "spectrum"}]
```



■ part(c)

```
In[478]= Ts = 1 / 10;
Print["sampling period = ", N@Ts, " seconds"];
sampling period = 0.1 seconds
```

```
In[480]= nSamples = 6 * period / Ts;
data = Table[{n, x[n Ts]}, {n, 0, nSamples}];
ListPlot[data, Filling -> Axis, FillingStyle -> Thick, AxesLabel -> {"t", "Xs(t)"}]
```

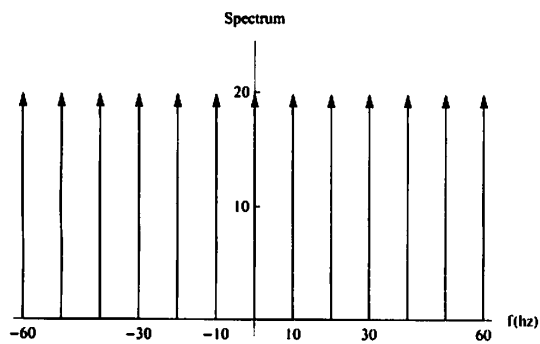


■ part(d)

```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
Sampling frequency = 10 hz
```

prob3.nb | 3

```
spectrum = fs Sum[ft /. f -> (f - n * fs), {n, -7, 7}];
spectrum = Expand[spectrum];
spectrum = 20 DiracDelta[-60 + f] + 20 DiracDelta[-50 + f] + 20 DiracDelta[-40 + f] +
  20 DiracDelta[-30 + f] + 20 DiracDelta[-20 + f] + 20 DiracDelta[-10 + f] +
  20 DiracDelta[f] + 20 DiracDelta[10 + f] + 20 DiracDelta[20 + f] + 20 DiracDelta[30 + f] +
  20 DiracDelta[40 + f] + 20 DiracDelta[50 + f] + 20 DiracDelta[60 + f];
Show[First@dsp`plotFourierTransform[spectrum, f, -6 * fs, 6 * fs, -.1 fs, 2.4 fs, Small],
  AxesLabel -> {"f (hz)", "Spectrum"}, Ticks -> {{-60, -30, -10, 10, 30, 60}, {fs, 2 * fs}}]
```

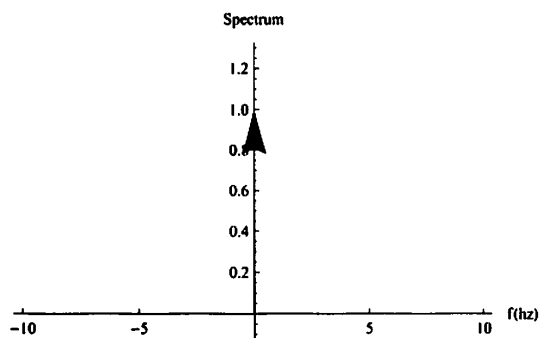


• part(e)

```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
bandwidth = 5. hz

gain = Ts;
Print["Gain=", N@gain];
Gain=0.1

spectrum = (20 DiracDelta[f]);
Show[First@dsp`plotFourierTransform[
  Expand[gain * spectrum], f, -2 * bandwidth, 2 * bandwidth, -.1, 1.3, Large],
  AxesLabel -> {"f (hz)", "Spectrum"}, Ticks -> {{-10, -5, 0, 5, 10}}]
```

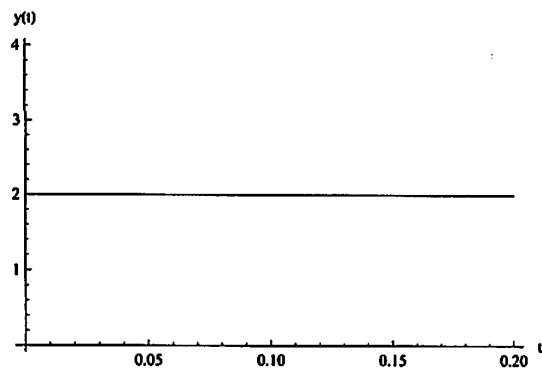


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4 | *prob3.nb*■ `part(f)`

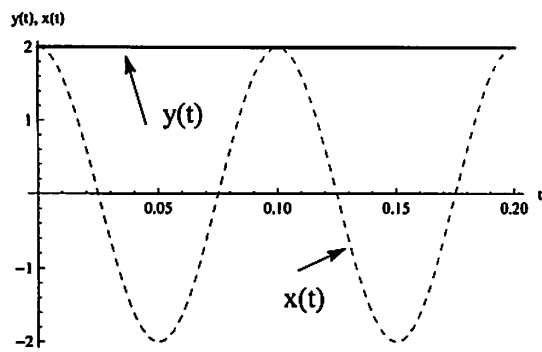
From the output above, we conclude that $y(t) = 2 \cos[2\pi 0 t] = 2$

```
y[t_] := 2 ;
Plot[y[t], {t, 0, 2 period}, AxesLabel -> {"t", "y(t)"}]
```



Compare this with the original signal $x(t)$

```
Plot[{x[t], y[t]}, {t, 0, 2 period},
  AxesLabel -> {"t", "y(t), x(t)"}, PlotStyle -> {Dashed, Thick}]
```



HW 6, Problem 4

by Nasser M. Abbasi

In[266] = << dsp`

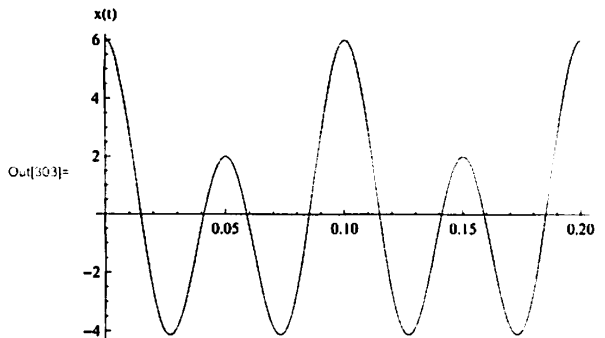
• part (a)

```
In[269] = Clear[w, t, t, f];
          f1 = 20;
          f2 = 10;
          period1 = 1 / f1;
          period2 = 1 / f2;
```

```
Print["Period of message 1 = ", N@period1, " seconds"];
Period of message 1 = 0.05 seconds
```

```
In[275] = Print["Period of message 2 = ", N@period2, " seconds"];
Period of message 2 = 0.1 seconds
```

```
In[302] = x[t_] := 4 Cos[2 Pi f1 t] + 2 Cos[2 Pi f2 t];
          p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}]
```



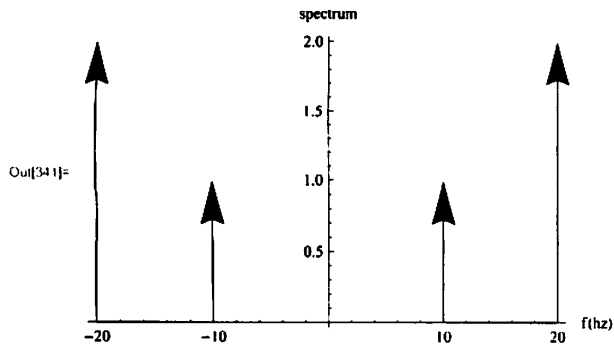
• part (b)

```
In[338] = ft = FourierTransform[x[t], t, f, FourierParameters -> {0, -2 Pi}];
          Print["Fourier Transform of x(t) = ", ft];
```

```
Fourier Transform of x(t) =
2 DiracDelta[-20 + f] + DiracDelta[-10 + f] + DiracDelta[10 + f] + 2 DiracDelta[20 + f]
```

2 | *prob4.nb*

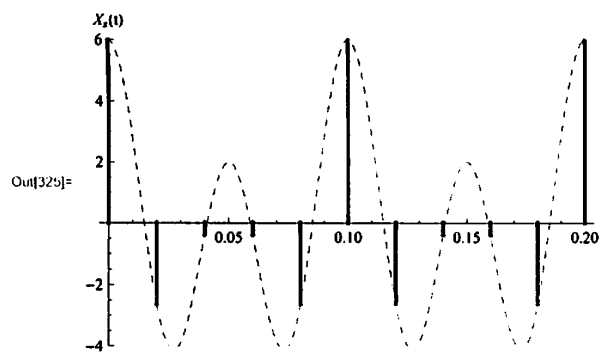
```
In[340] = dsp`plotFourierTransform[ft, f, -2 f2, 2 f2, 0, .5, Large];
Show[%, PlotRange -> All, AxesLabel -> {"f(hz)", "spectrum"}]
```



■ part(c)

```
In[282] = Ts = 1 / 50;
Print["sampling period = ", N@Ts, " seconds"];
sampling period = 0.02 seconds
```

```
In[322] = nSamples = 2 * period2 / Ts;
p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}, PlotStyle -> Dashed];
data = Table[{n * Ts, x[n Ts]}, {n, 0, nSamples}];
Show[{ListPlot[data, Filling -> Axis, FillingStyle -> Thick,
  AxesLabel -> {"t", "X_s(t)"}, PlotRange -> {Automatic, {-4, 6}}], p1}]
```



■ part(d)

```
In[326] = Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
Sampling frequency = 50 hz
```

prob4.nb | 3

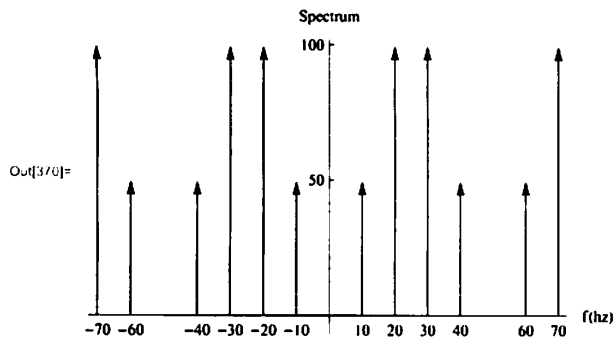
```

In[365]= spectrum = fs Sum[ft /. f -> (f - n * fs), {n, -1, 1}];
spectrum = Expand[spectrum]

Out[366]= 100 DiracDelta[-70 + f] + 50 DiracDelta[-60 + f] + 50 DiracDelta[-40 + f] +
100 DiracDelta[-30 + f] + 100 DiracDelta[-20 + f] + 50 DiracDelta[-10 + f] +
50 DiracDelta[10 + f] + 100 DiracDelta[20 + f] + 100 DiracDelta[30 + f] +
50 DiracDelta[40 + f] + 50 DiracDelta[60 + f] + 100 DiracDelta[70 + f]

In[370]= Show[First@dsp`plotFourierTransform[spectrum, f, -fs, fs, -.1 fs, 2 fs, Small],
AxesLabel -> {"f (hz)", "Spectrum"},
Ticks -> {{-70, -60, -40, -30, -20, -10, 10, 20, 30, 40, 60, 70}, {fs, 2 fs}}]

```



■ part(e)

```

In[371]= bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];

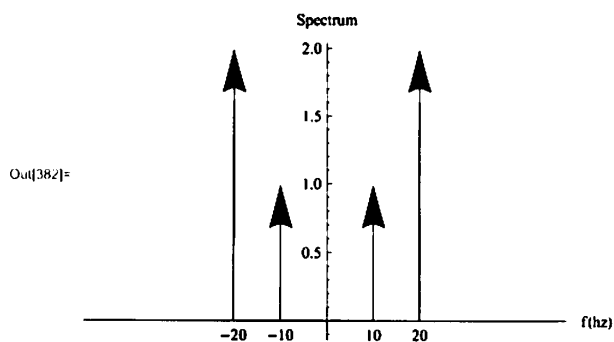
bandwidth = 25. hz

In[373]= gain = Ts;
Print["Gain=", N@gain];

Gain=0.02

In[381]= spectrum = (100 DiracDelta[-20 + f] +
50 DiracDelta[-10 + f] + 50 DiracDelta[10 + f] + 100 DiracDelta[20 + f]);
Show[First@dsp`plotFourierTransform[Expand[gain * spectrum], f,
-2 * bandwidth, 2 * bandwidth, -.1, 2, Large],
AxesLabel -> {"f (hz)", "Spectrum"}, Ticks -> {{-20, -10, 0, 10, 20}}]

```



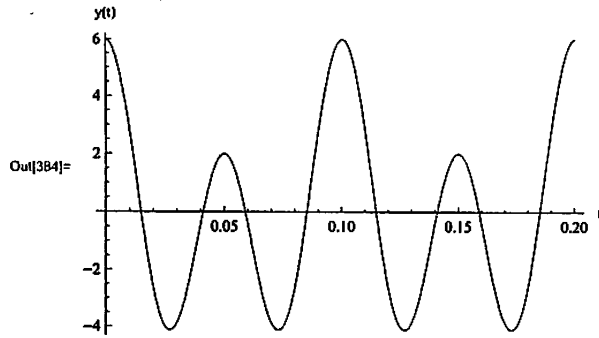
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4 | prob4.nb

part(f)

From the output above, we conclude that $y(t) = 2 \cos[2\pi 10 t] + 4\cos[2\pi 20t]$

```
In[383]:= y[t_] := 2 Cos[2 Pi 10 t] + 4 Cos[2 Pi 20 t];  
Plot[y[t], {t, 0, 2 period}, AxesLabel -> {"t", "y(t)"}]
```



which is the same as original x(t)

HW 6, Problem 5

by Nasser M. Abbasi

In[385] = << dep`

part (a)

```
In[498] = Clear[w, t, f];
          f1 = 20;
          f2 = 10;

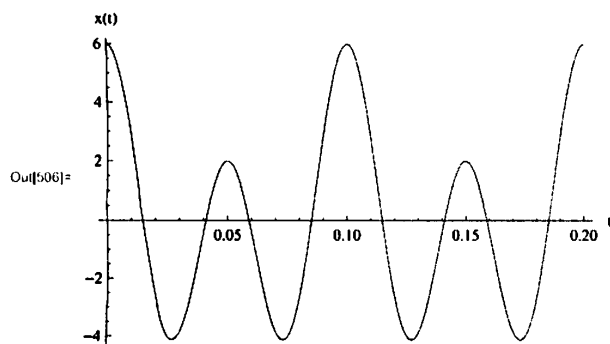
          period1 = 1/f1;
          period2 = 1/f2;
          Print["Period of message 1 = ", N[period1], " seconds"];
```

Period of message 1 = 0.05 seconds

```
In[504] = Print["Period of message 2 = ", N[period2], " seconds"];
```

Period of message 2 = 0.1 seconds

```
In[505] = x[t_] := 4 Cos[2 Pi f1 t] + 2 Cos[2 Pi f2 t];
          p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}]
```



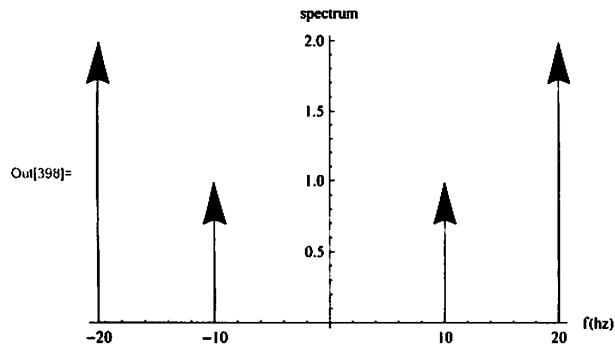
part (b)

```
In[507] = ft = FourierTransform[x[t], t, f, FourierParameters -> {0, -2 Pi}];
          Print["Fourier Transform of x(t) = ", ft];
```

Fourier Transform of $x(t) = 2 \delta(f - 20) + \delta(f - 10) + \delta(f + 10) + 2 \delta(f + 20)$

2 | prob5.nb

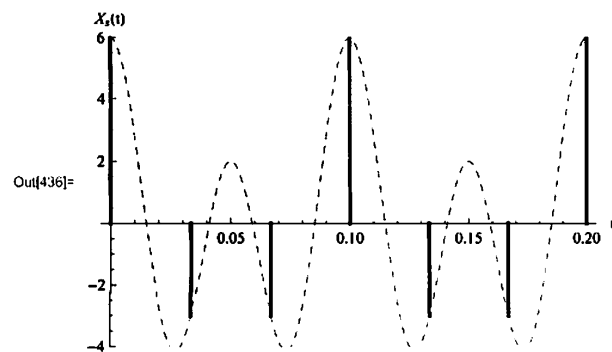
```
In[397] = Dep`PlotFourierTransform[ft, f, -2 f2, 2 f2, 0, .5, Large];
Show[%, PlotRange -> All, AxesLabel -> {"f(hz)", "spectrum"}]
```



■ part(c)

```
In[431] = Ts = 1 / 30;
Print["sampling period = ", N@Ts, " seconds"];
sampling period = 0.0333333 seconds
```

```
In[433] = nSamples = 2 * period2 / Ts;
p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}, PlotStyle -> Dashed];
data = Table[{n * Ts, x[n Ts]}, {n, 0, nSamples}];
Show[{ListPlot[data, Filling -> Axis, FillingStyle -> Thick,
  AxesLabel -> {"t", "X_s(t)"}, PlotRange -> {Automatic, {-4, 6}}], p1}]
```



■ part(d)

```
In[509] = Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
Sampling frequency = 10 hz
```

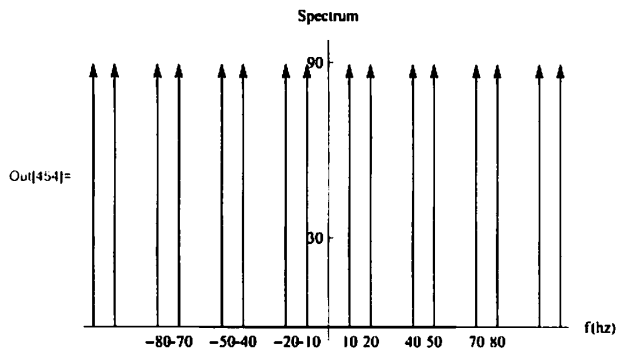
```

In[515]= spectrum = fs Sum[ft /. f -> (f - n * fs), {n, -4, 4}];
spectrum = Expand[spectrum];
spectrum = 90 DiracDelta[-110 + f] + 90 DiracDelta[-100 + f] + 90 DiracDelta[-80 + f] +
  90 DiracDelta[-70 + f] + 90 DiracDelta[-50 + f] + 90 DiracDelta[-40 + f] +
  90 DiracDelta[-20 + f] + 90 DiracDelta[-10 + f] + 90 DiracDelta[10 + f] +
  90 DiracDelta[20 + f] + 90 DiracDelta[40 + f] + 90 DiracDelta[50 + f] + 90 DiracDelta[70 + f] +
  90 DiracDelta[80 + f] + 90 DiracDelta[100 + f] + 90 DiracDelta[110 + f]

Out[517]= 90 δ(f - 110) + 90 δ(f - 100) + 90 δ(f - 80) + 90 δ(f - 70) + 90 δ(f - 50) + 90 δ(f - 40) + 90 δ(f - 20) + 90 δ(f - 10) +
  90 δ(f + 10) + 90 δ(f + 20) + 90 δ(f + 40) + 90 δ(f + 50) + 90 δ(f + 70) + 90 δ(f + 80) + 90 δ(f + 100) + 90 δ(f + 110)

In[454]= Show[First@dsp`plotFourierTransform[spectrum, f, -2 fs, 2 fs, -.1 fs, 3.2 fs, Small],
  AxesLabel -> {"f (hz)", "Spectrum"},
  Ticks -> {{-80, -70, -50, -40, -20, -10, 10, 20, 40, 50, 70, 80}, {fs, 3 fs}}]

```



■ part(e)

```

In[455]= bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];

bandwidth = 15. hz

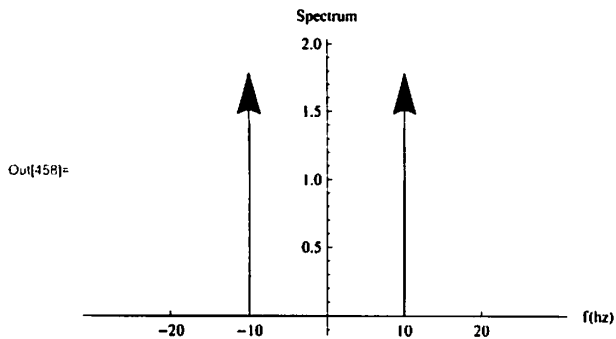
In[373]= gain = Ts;
Print["Gain=", N@gain];

Gain=0.02

```

4 | prob5.nb

```
In[457] = spectrum = (90 DiracDelta[-10 + f] + 90 DiracDelta[10 + f]);
Show[First@dep`plotFourierTransform[
  Expand[gain * spectrum], f, -2 * bandwidth, 2 * bandwidth, -.1, 2, Large],
  AxesLabel -> {"f(hz)", "Spectrum"}, Ticks -> {{-20, -10, 0, 10, 20}}]
```



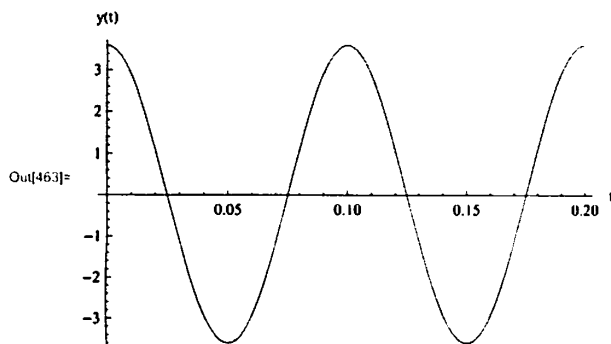
```
In[460] = h = 0.02 * 90;
v = 2 * 1.8
```

```
Out[461] = 3.6
```

▪ part(f)

From the output above, we conclude that $y(t) = 3.6 \cos[2\pi 10 t]$

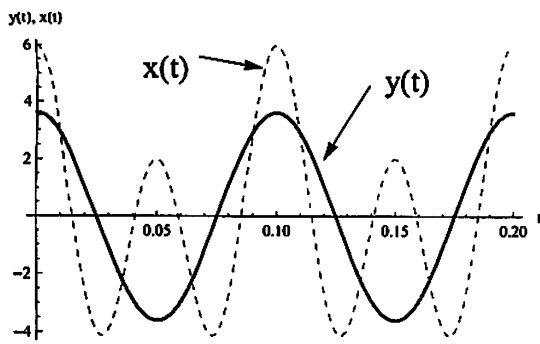
```
In[462] = y[t_] := 3.6 Cos[2 Pi 10 t];
Plot[y[t], {t, 0, 2 period}, AxesLabel -> {"t", "y(t)"}]
```



compare to original x(t)

```
In[464] = Plot[{x[t], y[t]}, {t, 0, 2 period},
  AxesLabel -> {"t", "y(t), x(t)"}, PlotStyle -> {Dashed, Thick}]
```

prob5.nb | 5



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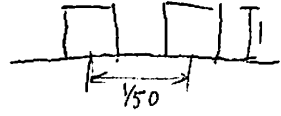
HW#6
6

$T = 0.1 \text{ sec.}$

$$\frac{\tau}{T} = \frac{1}{4} \text{ so } \tau = \frac{T}{4}$$

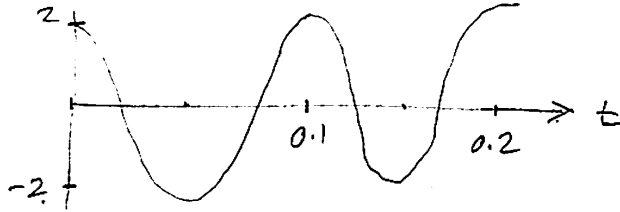
$$x(t) = 2 \cos(2\pi 10t)$$

sampled uniformly by

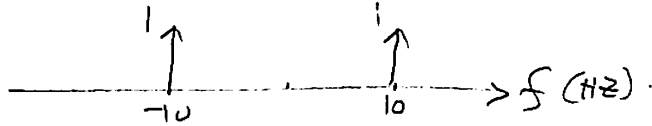


duty cycle = $\frac{1}{4}$, $T_s = \frac{1}{50}$
 $\frac{1}{50} \text{ sec}$

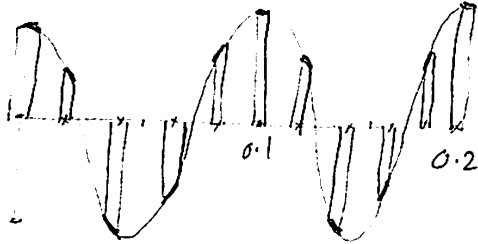
(a)



(b)

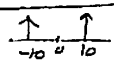


(c)



(d)

$$x_s(t) = x(t)g(t) = x(t) \sum h \text{rect}\left(\frac{t-nT_s}{\tau}\right)$$

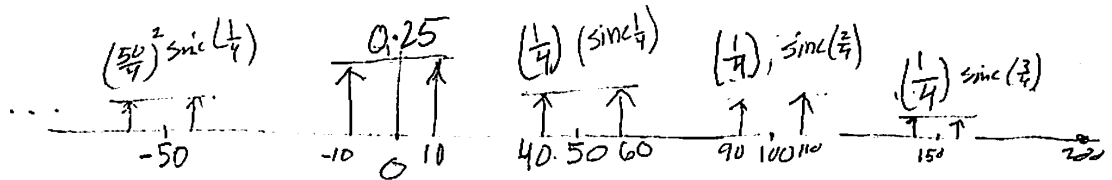


$$X_s(f) = X(f) \otimes f_s h \sum \text{sinc}\left(\frac{nT}{T_s}\right) \delta(f - n f_s)$$

$$g(t) = \sum h \text{rect}\left(\frac{t-nT_s}{\tau}\right)$$

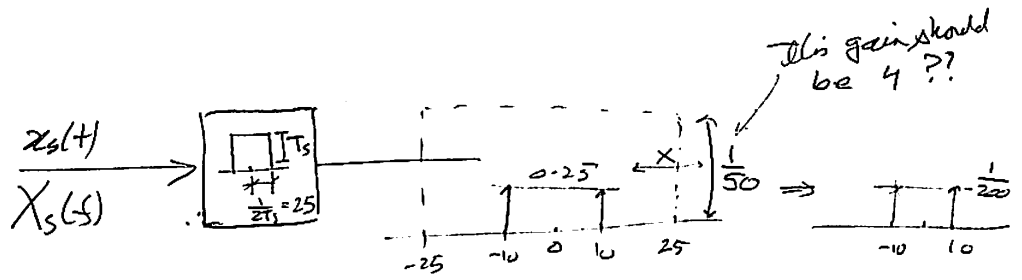
$$= f_s h \tau \sum \text{sinc}\left(\frac{nT}{T_s}\right) X(f - n f_s)$$

$$G(f) = f_s \sum h \tau \text{sinc}\left(\frac{nT}{T_s}\right) \delta(f - n f_s) = (50) \left(\frac{1}{50}\right) \sum \text{sinc}\left(\frac{n}{4}\right) X(f - n f_s)$$



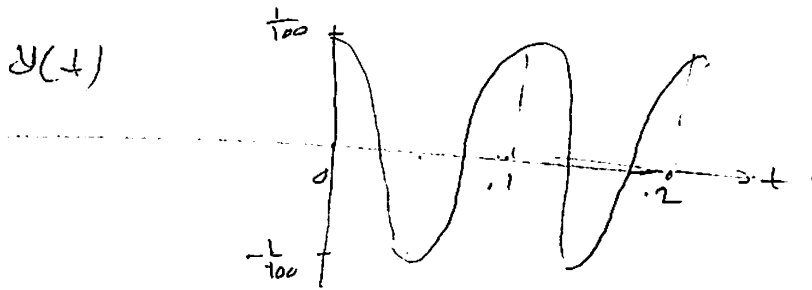
HW6
6.

⊕



⊕ we see that

$$y(t) = \frac{1}{100} \cos(2\pi 10 t)$$

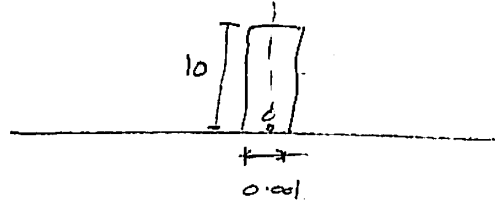


note amplitude is different, because gain we are asked to use is T_s which is not what normally used for pulse rect train which should have been d .

HW's

⑦ determine Nyquist rate for

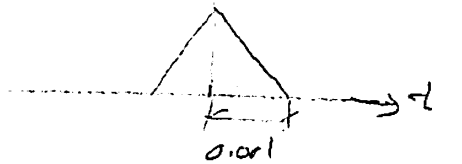
Ⓐ $x(t) = 10 \operatorname{rect}\left(\frac{t}{0.001}\right)$.



$$\approx \tau = 0.001$$

this signal is not periodic. hence its bandwidth is ∞ . hence require ∞ sampling frequency.

Ⓑ $x(t) = \operatorname{tri}\left(\frac{t}{0.001}\right)$.



for similar reasoning as Ⓐ. this is not periodic, hence ∞ bandwidth $\Rightarrow \infty$ sampling freq.

Ⓒ $\operatorname{sinc}(1000t) = \frac{\sin(\pi 1000t)}{\pi 1000}$

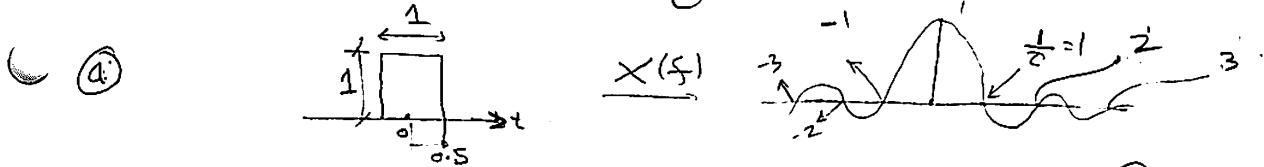
so $f_m = 500 \text{ Hz}$.

hence $N_{\text{Nyquist}} = 1000 \text{ Hz}$.

Ⓓ $\operatorname{sinc}(2000t) + \operatorname{sinc}^2(1100t)$
 $= \frac{\sin(2\pi 1000t)}{\pi 2000} + \left[\frac{\sin(2\pi 550t)}{2\pi 550} \right]^2$
 $= \frac{\sin 2\pi 1000t}{\pi 2000} + \left(\frac{1}{2\pi 550} \right)^2 \left[\frac{1}{2} - \frac{1}{2} \cos(2\pi 1100t) \right] \Rightarrow N_{\text{Nyquist}} = 2200 \text{ Hz}$

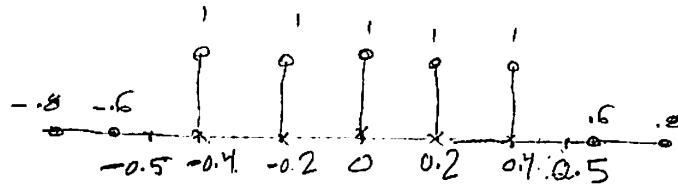
HW 6

3. $x(t) = \text{rect}(t)$ is ideally sampled at rate 5 samples/s.



Nyquist rate = ∞ . see. Problem 7, part (a).

(b) $T_s = \frac{1}{5} = 0.2$



(c) need to print \rightarrow spectrum of \uparrow .

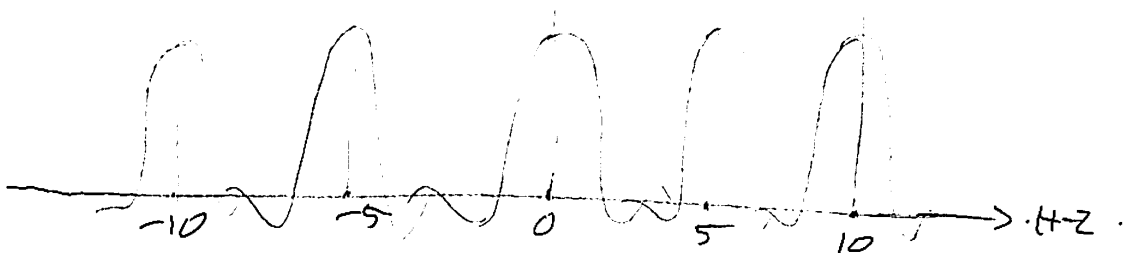
$$x_s(t) = \delta(t) + \delta(t-0.2) + \delta(t-0.4) + \delta(t+0.2) + \delta(t+0.4)$$

$$\text{so } F[x_s(t)] = X_s(f) = \frac{1}{T_s} \left[X(f) + X(f-f_s) + X(f-2f_s) + X(f+f_s) + X(f+2f_s) \right]$$

but $X(f) = 1 \cdot \text{sinc}(f)$

$$\text{so } X_s(f) = 5 \left[\text{sinc}(f) + \text{sinc}(f-5) + \text{sinc}(f-10) + \text{sinc}(f+5) + \text{sinc}(f+10) \right]$$

so, place a sinc function of height = 5 at $f = 0, 5, \pm 10$.

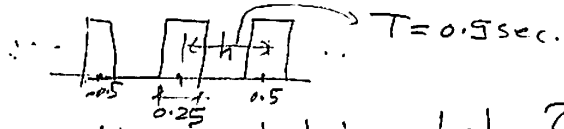


(d) it is not possible to recover, since sinc function extent for $\pm\infty$. so there will always be a loss.

HW6

9. $x(t) = \text{sinc}(t)$. sampled by rect train, $h=1$, $T=0.25$ s,

$T=0.5$ sec.

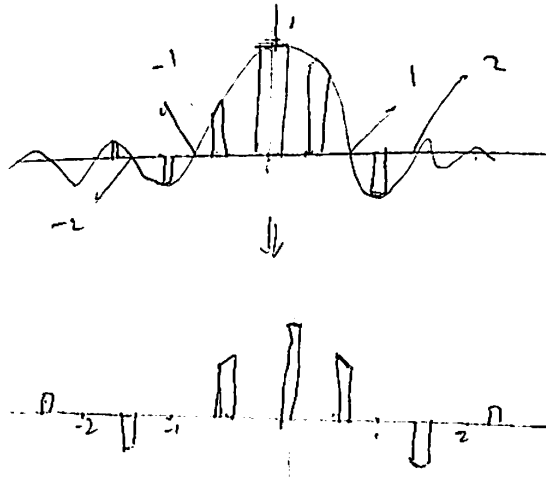


(a) Nyquist rate for $x(t)$ is ∞ since it is a sinc function with ∞ bandwidth? but suppose say 1 Hz? I do not understand.

If we apply this to the rect pulse, then

$$I \text{ set } (2)(f_m) = 2\left(\frac{1}{T}\right) = 2\left(\frac{1}{0.5}\right) = 2(2) = 4 \text{ Hz.}$$

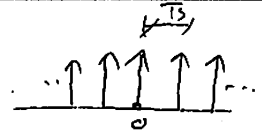
(b)



HW 6

(10) $x(t) = 2 \cos(2\pi 10t) + \cos(2\pi 20t)$

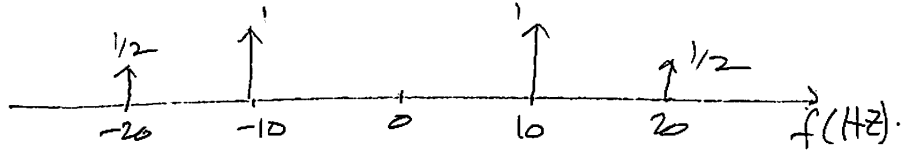
$f_s = 40 \text{ Hz}$



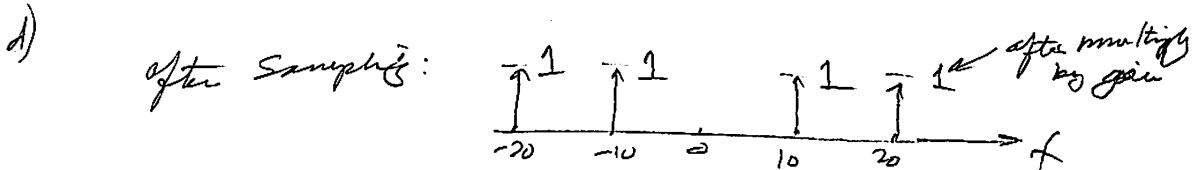
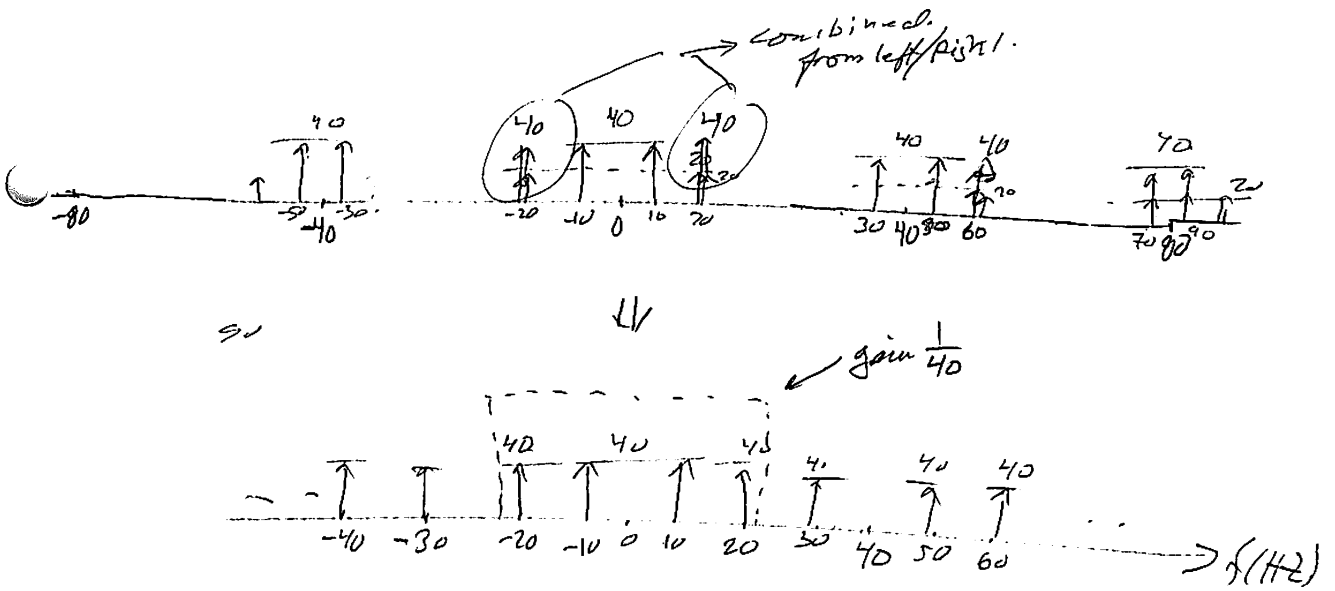
$T_s = 0.025 \text{ sec}$

(a) see plot next page.

$X(f)$



(b) $X_s(f) = f_s \sum X(f - n f_s)$

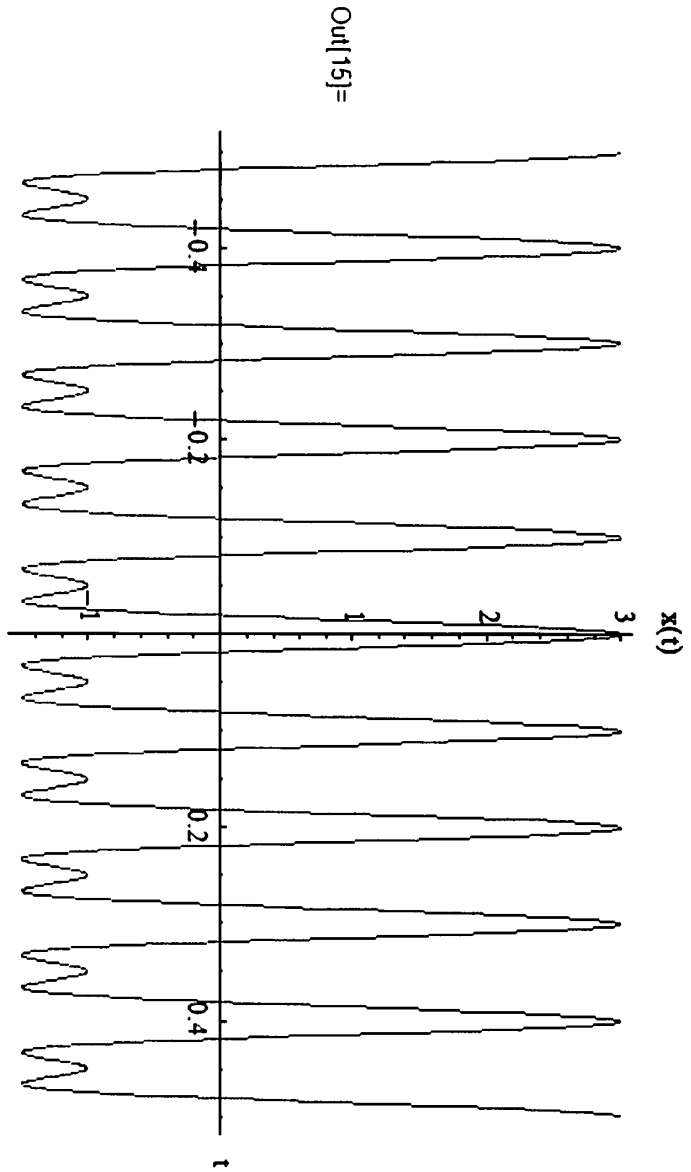


so time domain is

$$2 \cos(2\pi 10t) + 2 \cos(2\pi 20t)$$

(c) $2 \cos(2\pi 10t)$

```
In[15]:= Plot[2 Cos[2 Pi 10 t] + Cos[2 Pi 20 t], {t, -.5, .5}, AxesLabel -> {"t", "x(t)"}]
```



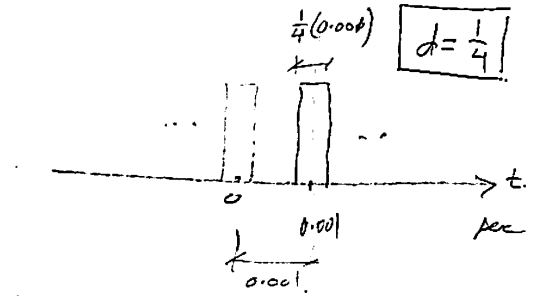
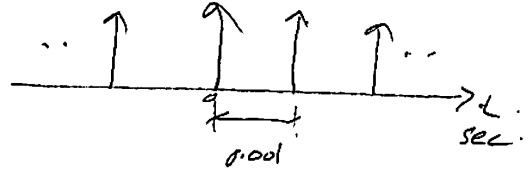
HW6

12

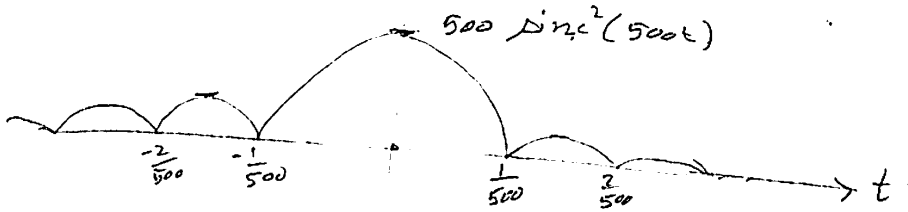
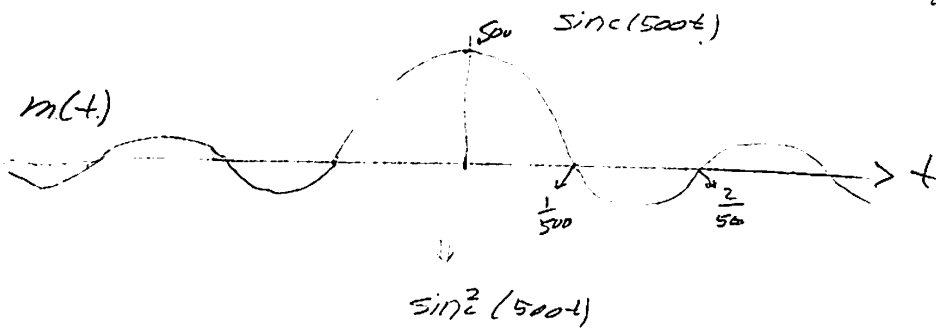
$m_1(t) = 500 \text{ sinc}^2(500t)$

$P_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - 0.001k)$

$P_2(t) = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t - 0.001k}{0.00025}\right)$
 (Labels: delay, width)



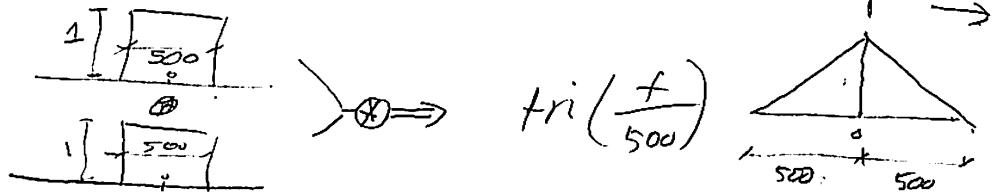
$x_1(t) = m_1(t) P_1(t)$



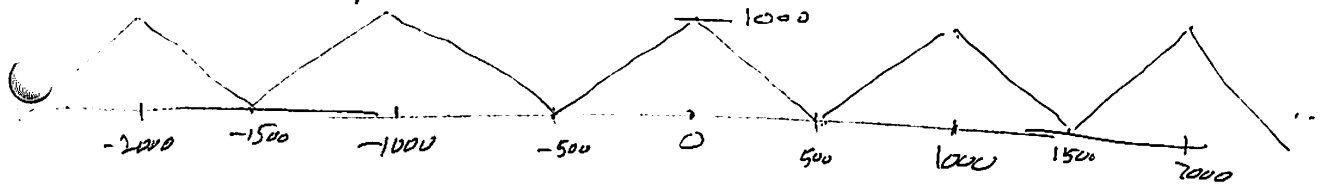
to find spectrum of \uparrow we can use relation:

$F[\text{sinc}^2] = F[\text{sinc}(\text{sinc})]$
 $= F(\text{sinc}) \otimes F(\text{sinc})$

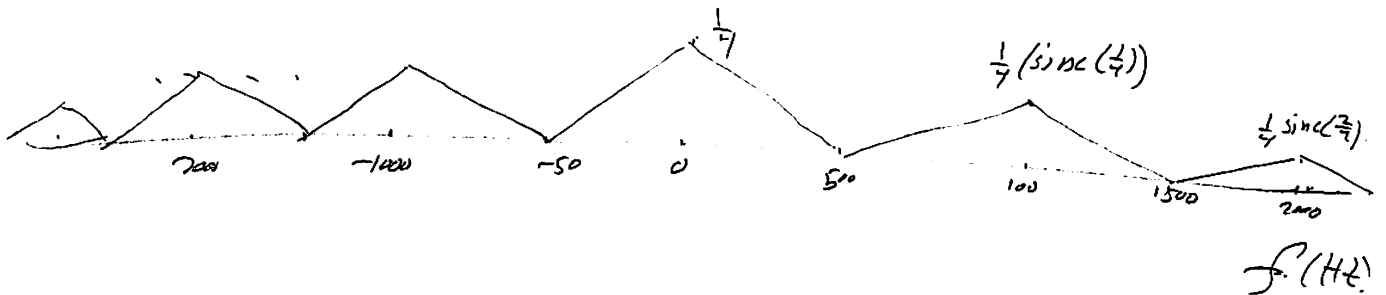
so need to convolve 2 rect.



so ④ has spectrum (note $f_s = \frac{1}{0.001} = 1000$.)



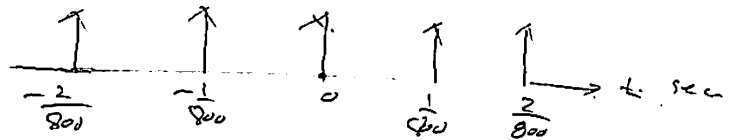
⑤ note that $d = \frac{1}{7}$ so,



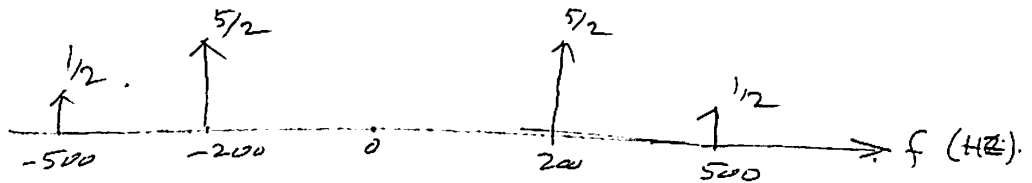
HW 6

$\frac{13}{13}$ $m(t) = 5 \cos(2\pi 200t) + \cos(2\pi 500t)$

$\Rightarrow \frac{c}{s} = \frac{1}{T_s} = 800 \text{ Hz}$

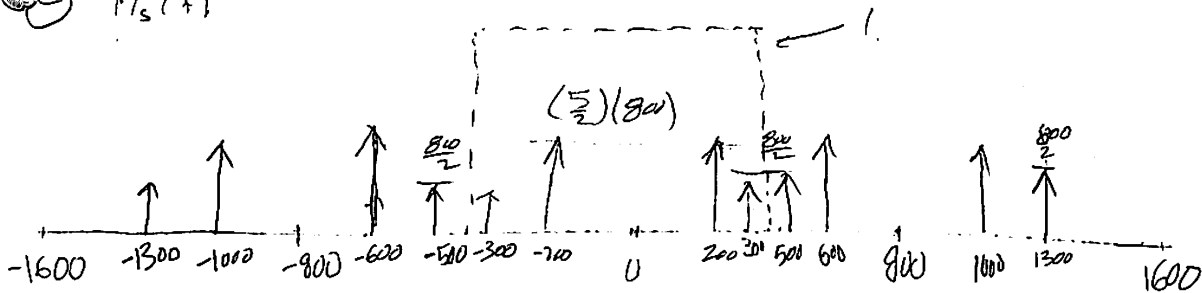


(a) $F(m(t)) = \frac{5}{2} (\delta(f-200) + \delta(f+200)) + \frac{1}{2} (\delta(f-500) + \delta(f+500))$

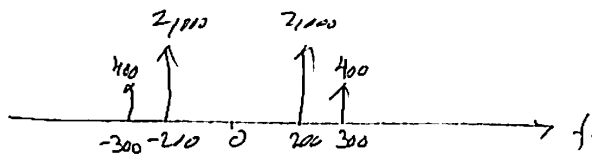


(b) $B = 500$, hence $2B = 1000 \text{ Hz} \Rightarrow \boxed{1 \text{ kHz}}$

(c) $M_s(f)$



(d) $s_D Y(t)$



$s_D y(t) = 4000 \cos(2\pi 200t) + 800 \cos(2\pi 300t)$

HW7

(1) $m_p = 16V$.
 $x = -8.7V$.

(2) $N = 8$ bits. Find offset binary code.

$$\Delta = \frac{16}{2^7} = \boxed{0.125}$$

$$\text{quantization level} = \left(\frac{\text{Abs}(x)}{\Delta} \right) = 69.6 \rightarrow \boxed{70}$$

Since x is negative, then $\text{Code} = 2^7 - 70 = 128 - 70 = \boxed{58}$

in binary, this is $\boxed{00111010}$

(3) Sign/magnitude..

since $x < 0$ then $\text{Code} = 2^7 + 70 = 128 + 70 = 198$

which in binary is $\boxed{11000110}$

(4) 2's complement.

since $x < -\frac{\Delta}{2}$ then $\text{Code} = 2^8 - 70 = 256 - 70 = 186$

which in binary is $\boxed{10111010}$

(5) 1's complement.

since $x < 0$ then

$$\text{Code} = (2^8 - 1) - 70 = 255 - 70 = 185$$

which in binary is $\boxed{10111001}$

extra: to illustrate this more, this is the calculations assuming $x = +8.7V$.

offset binary

$$\Delta = \frac{15}{2^7} = 0.125$$

$$\text{Level} = \text{round} \left(\frac{8.7}{\Delta} \right) = 69.6 \rightarrow \boxed{70}$$

Since $x > 0$ then code $(70)_2 = 0100\ 0110$

sign magnitude

Since $x > 0$ then code $(70)_2 = 0100\ 0110$

2's Complement

Since $x > 0$ then code = $(70)_2 = 0100\ 0110$

1's complement

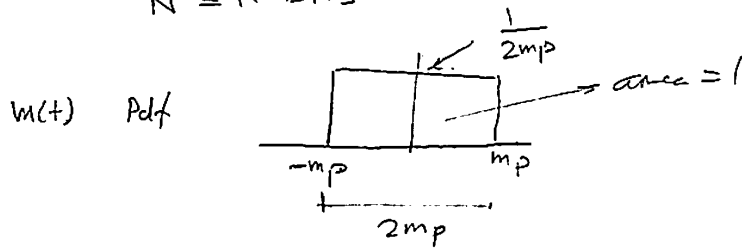
Since $x > 0$, then code = $(70)_2 = 0100\ 0110$.

HW 7

③

$$m_p = 16.$$

$$N = 11 \text{ bits.}$$



$$\textcircled{a} \quad \text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\overline{m^2(t)}}{\frac{1}{12} S^2}$$

When $S = \frac{m_p}{2^{N-1}}$ hence Noise $P_{av} = \frac{1}{12} \left(\frac{m_p}{2^{10}} \right)^2$

and since this is a Random message, then

$$\begin{aligned} \overline{m^2(t)} &= E(m^2(t)) = \int x^2 f_x dx \\ &= \int_{-m_p}^{m_p} \left(\frac{1}{2m_p} \right) x^2 dx \\ &= \frac{1}{2m_p} \left[\frac{x^3}{3} \right]_{-m_p}^{m_p} = \frac{1}{6m_p} [m_p^3 - (-m_p)^3] \\ &= \frac{1}{6m_p} [m_p^3 + m_p^3] = \left[\frac{1}{3} m_p^2 \right] \end{aligned}$$

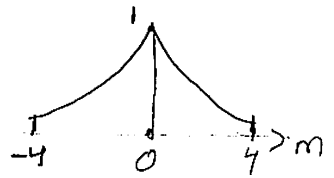
$$\text{So } \text{SNR} = \frac{\frac{1}{3} m_p^2}{\frac{1}{12} \left(\frac{m_p}{2^{10}} \right)^2} = \frac{(12)(2^{20})}{3} = (4)(2^{20})$$

$$= \boxed{66.226 \text{ db}}$$

HW 7

⑥

$$f(m) = \begin{cases} k e^{-|m|} & -4 < m < 4 \\ 0 & \text{o.w.} \end{cases}$$

① Find k .

$$\int_{-4}^4 k e^{-|m|} dm = 1 \Rightarrow \int_{-4}^0 k e^m dm + \int_0^4 k e^{-m} dm$$

$$= k \left[\left[e^m \right]_{-4}^0 + \frac{\left[e^{-m} \right]_0^4}{-1} \right]$$

$$= k \left[e^0 - e^{-4} + \frac{e^{-4} - e^0}{-1} \right] = k \left[1 - e^{-4} + \frac{e^{-4} - 1}{-1} \right]$$

$$= k \left[(1 - e^{-4}) + (1 - e^{-4}) \right] = k (2 - 2e^{-4})$$

$$= \boxed{2k(1 - e^{-4}) = 1}$$

$$\text{so } k = \frac{1}{2(1 - e^{-4})} = \boxed{0.50932}$$

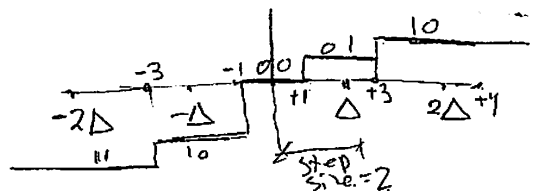
②

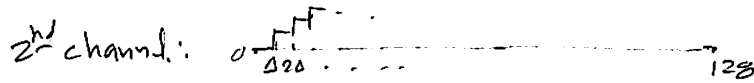
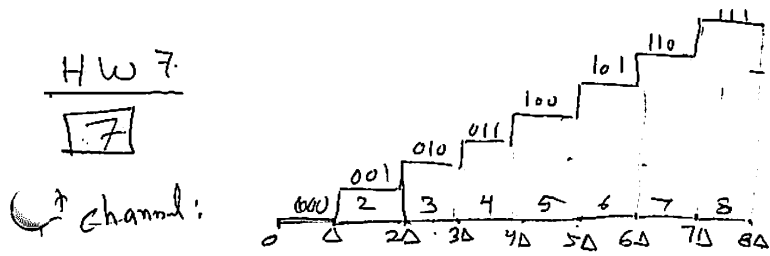
$m_p = 4$. so $\Delta = \frac{m_p = 4}{2^{N-1}}$ where N is number of bits. or $\Delta = \frac{8}{2^N}$

but $2^N = \text{number of levels}$.

but we are told to use 4 levels. hence $\boxed{N=2}$

$$\text{so } \Delta = \frac{8}{2^2} = \frac{8}{4} = \boxed{2}$$





$$m_p = 2 \text{ Volt.}$$

$$\text{noise power} = \frac{1}{12} S^2 \quad \text{where } S \text{ is step size.}$$

$$\text{for channel 1, } S_1 = \frac{m_p}{8} = \frac{2}{8}.$$

$$\text{for channel 2, } S_2 = \frac{m_p}{128} = \frac{2}{128}.$$

$$\text{so for channel 1, } \overline{e_1^2} = \frac{1}{12} \left(\frac{2}{8}\right)^2 = \frac{4}{(12)(64)}.$$

$$\text{for channel 2, } \overline{e_2^2} = \frac{1}{12} \left(\frac{2}{128}\right)^2$$

$$\text{so } \left(\frac{\overline{e_1^2}}{\overline{e_2^2}}\right)_{\text{db}} = 10 \log_{10} \frac{\overline{e_1^2}}{\overline{e_2^2}} = 10 \left[\log_{10} \overline{e_1^2} - \log_{10} \overline{e_2^2} \right]$$

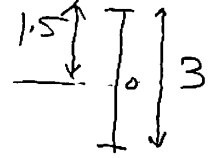
$$= 10 \left[\log_{10} \frac{4}{(12)(64)} - \log_{10} \frac{1}{12} \frac{4}{128^2} \right] =$$

$$= 10 \left[-22.833 + 4.6915 \right] = \boxed{24.082 \text{ db}}$$

HW 7.

$$\boxed{8} \quad m_p = 3V.$$

$$\text{levels} = 64.$$



$$\text{RMS of noise} = \sqrt{e^2} = \sqrt{\frac{1}{12} S^2}$$

$$\text{but } S = \frac{3}{64}, \text{ so } \text{RMS} = \sqrt{\frac{1}{12} \left(\frac{3}{64}\right)^2} = 0.01353$$

so peak signal to RMS ratio is

$$\frac{1.5}{0.01353} = 110.85$$

$$\boxed{9}$$

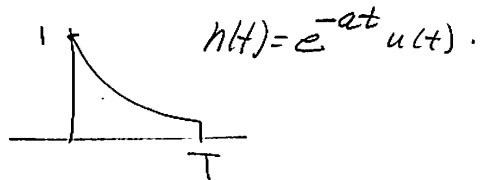
$$\text{number of levels} = 512$$

$$B = 4.2 \text{ MHz} \cdot \text{ so } 2B = 8.4 \times 10^6 \text{ Hz}.$$

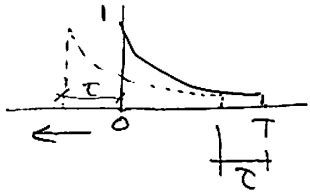
$$\text{here } N = 9. \quad (2^9 = 512).$$

$$\text{so binary pulses} = \underbrace{(9)}_{\substack{\text{bits per} \\ \text{sample}}} \underbrace{(8.4 \times 10^6)}_{\text{samples/sec.}} = \boxed{75600} \text{ binary pulses/sec.}$$

HW 8
1.



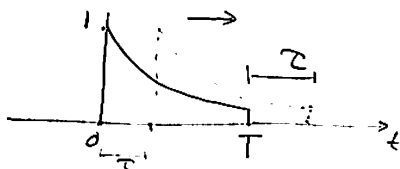
Case 1



$$0 \leq \tau \leq T.$$

$$\begin{aligned} R(\tau) &= \int_0^{T-\tau} h(t) h(t+\tau) dt = \int_0^{T-\tau} e^{-at} e^{-a(t+\tau)} dt \\ &= e^{-a\tau} \int_0^{T-\tau} e^{-2at} dt = \frac{e^{-a\tau}}{-2a} \left[e^{-2at} \right]_0^{T-\tau} \\ &= \frac{e^{-a\tau}}{-2a} \left[e^{-2a(T-\tau)} - 1 \right] = \boxed{\frac{e^{-a\tau}}{2a} \left(1 - e^{-2a(T-\tau)} \right)} \end{aligned}$$

Case 2



$$T \leq \tau \leq 2T$$

$$\begin{aligned} R(\tau) &= \int_{\tau}^T h(t) h(t-\tau) dt = \int_{\tau}^T e^{-at} e^{-a(t-\tau)} dt \\ &= e^{a\tau} \int_{\tau}^T e^{-2at} dt = \frac{e^{a\tau}}{-2a} \left[e^{-2at} \right]_{\tau}^T \\ &= \frac{e^{a\tau}}{-2a} \left[e^{-2aT} - e^{-2a\tau} \right] = \frac{e^{a\tau}}{2a} \left[e^{-2aT} - e^{-2a\tau} \right] \\ &= \frac{1}{2a} \left[e^{-aT} - e^{-2aT+a\tau} \right] = \frac{e^{-a\tau}}{2a} \left[1 - e^{-2a(T-\tau)} \right] \\ &= \boxed{\frac{e^{-a\tau}}{2a} \left[1 - e^{-2a(T-\tau)} \right]} \end{aligned}$$

We see that $R_-(\tau) = R_+(\tau)$ as expected.

$$\text{so } R(\tau) = \begin{cases} \frac{e^{-a\tau}}{2a} [1 - e^{-2a(T-\tau)}] & |\tau| \leq T \\ 0 & \text{o.w.} \end{cases}$$

$$\Rightarrow R_x(\tau) = \frac{1}{T} R(\tau)$$

$$\text{now } S_x(f) = \text{F.T.}[R_x(\tau)] \cdot \text{F.T.}[R_x(\tau)]^*$$

$$\text{or } S_x(f) = |\text{F.T.}[R_x(\tau)]|^2$$

so need to find F.T. of $R_x(\tau)$ first.

$$\text{F.T.}(R_x(\tau)) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

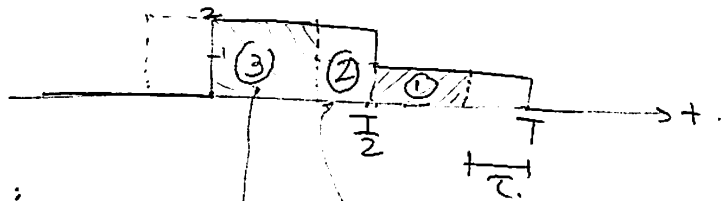
$$= \int_{-T}^T \frac{e^{-a|\tau|}}{2a} (1 - e^{-2a(T-|\tau|)}) e^{j2\pi f\tau} d\tau$$

$$H(\omega) \rightarrow \frac{1 - e^{-aT} [\cos(2\pi fT) - j \sin(2\pi fT)]}{a + j\omega}$$

$$\text{so } S_x(f) = H(\omega) H^*(\omega)$$

$$\Rightarrow \frac{1}{T} \frac{1}{a^2 + (2\pi f)^2} (1 - 2e^{-aT} \cos(\omega T) + e^{-2aT})$$

HW 8
②



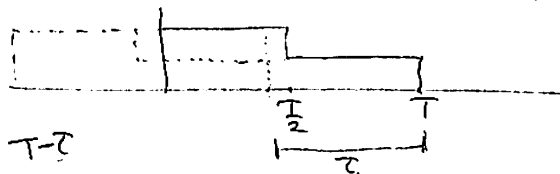
① $0 < \tau < \frac{T}{2}$:
3 regions as above.

$$\begin{aligned}
 R_p(\tau) &= \int_0^{\frac{T}{2}-\tau} (2)(2) d\lambda + \int_{\frac{T}{2}-\tau}^{\frac{T}{2}} (1)(2) d\lambda + \int_{\frac{T}{2}}^{T-\tau} (1)(1) d\lambda \\
 &= 4 \left[\lambda \right]_0^{\frac{T}{2}-\tau} + 2 \left[\lambda \right]_{\frac{T}{2}-\tau}^{\frac{T}{2}} + 1 \left[\lambda \right]_{\frac{T}{2}}^{T-\tau} \\
 &= 4\left(\frac{T}{2}-\tau\right) + 2\left(\frac{T}{2}-\frac{T}{2}+\tau\right) + \left(T-\tau-\frac{T}{2}\right)
 \end{aligned}$$

$$= 4\frac{T}{2} - 4\tau + 2\tau + \frac{T}{2} - \tau = \frac{5}{2}T - 3\tau$$

$$= \boxed{\frac{5}{2}T \left(1 - \frac{6}{5}\frac{\tau}{T}\right)}$$

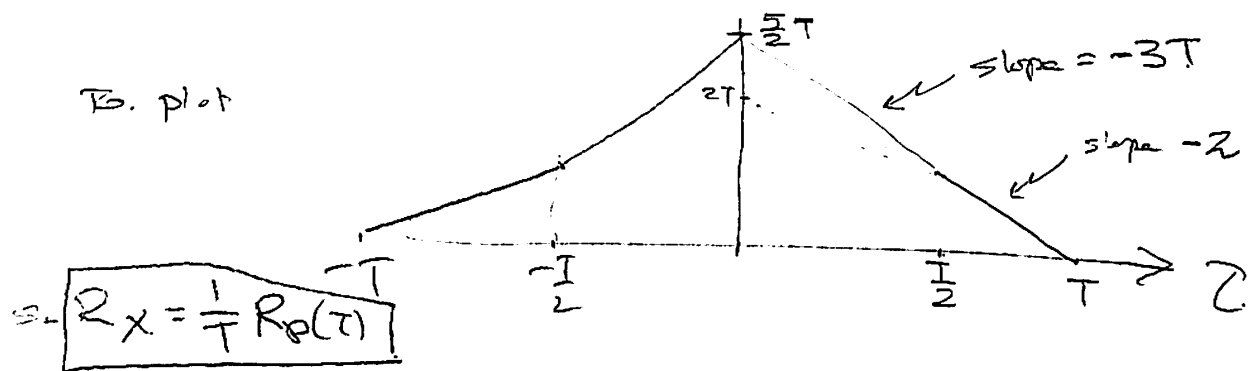
$\frac{T}{2} < \tau \leq T$



$$R_p(\tau) = \int_0^{T-\tau} (1)(2) d\lambda = 2 \left[\lambda \right]_0^{T-\tau} = \boxed{2(T-\tau)}$$

$$\text{hence } R_p(\tau) = \begin{cases} \frac{5}{2}T \left(1 - \frac{6}{5}\frac{\tau}{T}\right) & 0 < |\tau| < \frac{T}{2} \\ 2(T - |\tau|) & \frac{T}{2} \leq |\tau| \leq T \\ 0 & \text{o.w.} \end{cases}$$

→



to find F.T. of this, use tri function.

$\Rightarrow H(f) \rightarrow \text{sinc}$.

$$\text{and } S_x(f) = |H(f)|^2$$

HW7

(1) $m_p = 16V$.
 $x = -8.7V$.

(a) $N = 8$ bits. Find offset binary code.

$$\Delta = \frac{16}{2^7} = \boxed{0.125}$$

$$\text{quantization level} = \left(\frac{\text{Abs}(x)}{\Delta} \right) = 69.6 \rightarrow \boxed{70}$$

Since x is negative, then $\text{code} = 2^7 - 70 = 128 - 70 = \boxed{58}$

in binary, this is $\boxed{00111010}$

(b) Sign/magnitude..

Since $x < 0$ then $\text{code} = 2^7 + 70 = 128 + 70 = 198$

which in binary is $\boxed{11000110}$

(c) 2's complement.

Since $x < -\frac{\Delta}{2}$ then $\text{code} = 2^8 - 70 = 256 - 70 = 186$

which in binary is $\boxed{10111010}$

(d) 1's complement.

Since $x < 0$ then

$$\text{code} = (2^8 - 1) - 70 = 255 - 70 = 185$$

which in binary is $\boxed{10111001}$

extra: to illustrate this more, this is the
Calculations assuming $x = +8.7V$.

offset binary

$$\Delta = \frac{16}{2^7} = 0.125$$

$$\text{Level} = \text{round} \left(\frac{8.7}{\Delta} \right) = 69.6 \rightarrow \boxed{70}$$

Since $x > 0$ then code $(70)_2 = 0100\ 0110$

sign magnitude

since $x > 0$ then code $(70)_2 = 0100\ 0110$

2's complement

since $x > 0$ then code = $(70)_2 = 0100\ 0110$

1's complement

since $x > 0$, then code = $(70)_2 = 0100\ 0110$.

4.2 Computer assignments



Computer Assignment #1

ECE 405, Summer session 1, Cal Poly Pomona, CA
By Nasser M. Abbasi

PART(1) LOW PASS

- Load my DSP functions that I wrote for this course

```
in[2]= << dsp`
```



2 | ca1_v2.nb

■ Plot the pulse train

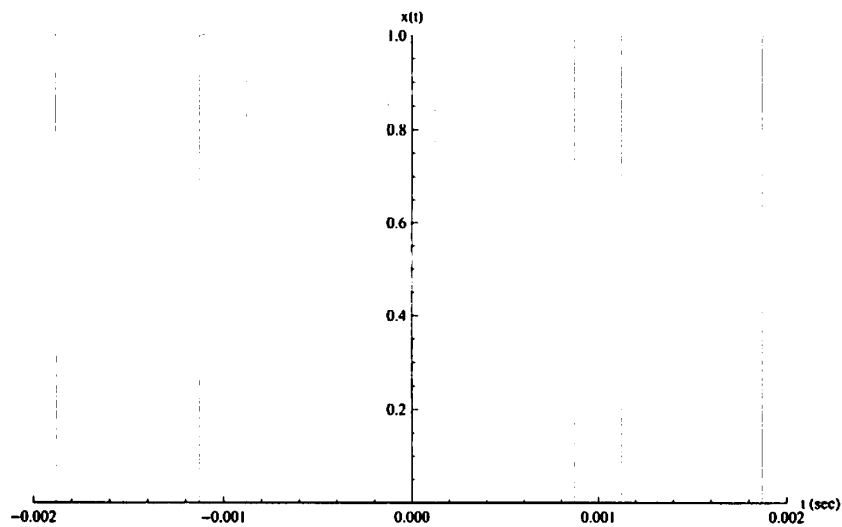
```

In[95] = delay = 0;
        period = 1 * 10^-3;
        range = 2 * 10^-3;
        tao = .25 * 10^-3;
        h = 1;
        w0 =  $\frac{2 \text{ Pi}}{\text{period}}$ ; (*rad/sec*)
        f0 =  $\frac{1}{\text{period}}$ ; (*hz*)
        dutyCycle = tao / period;
        numberOfCoeff = 20;
        currentPulses = dsp`makePulseTrain[delay, period, range, tao, h];

        Plot[0, {x, -range, range},
             PlotRange -> {{-range, range}, {0, h}}, AxesLabel -> {"t (sec)", "x(t)"},
             Epilog -> {Thin, Red, currentPulses}
        ]

```

Out[105]=



■ Find the fourier series coefficients of the above pulse train

```

In[106] = xn = getFourierCoeffPulseTrain[h, tao, period, numberOfCoeff]

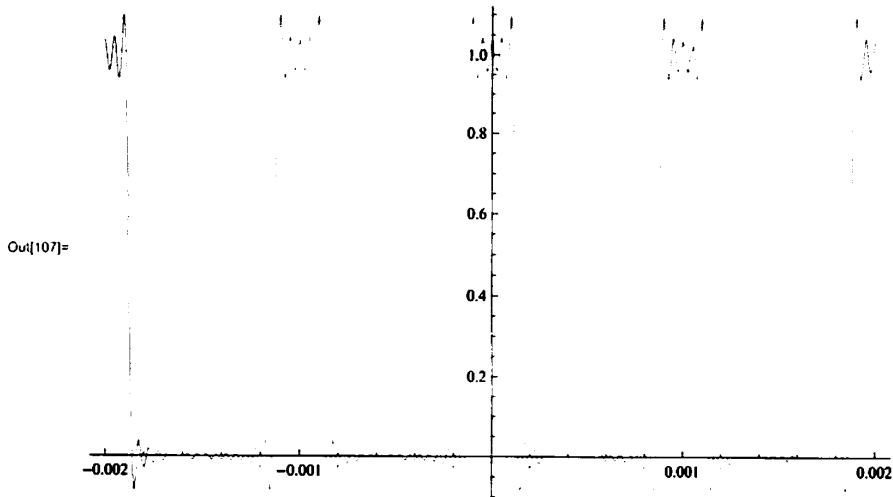
Out[106] = {0.25, 0.225079, 0.159155, 0.0750264, 9.74543 * 10^-18, -0.0450158, -0.0530516,
            -0.0321542, -9.74543 * 10^-18, 0.0250088, 0.031831, 0.0204617, 9.74543 * 10^-18,
            -0.0173138, -0.0227364, -0.0150053, -9.74543 * 10^-18, 0.0132399, 0.0176839, 0.0118463}

```

ca1_v2.nb | 3

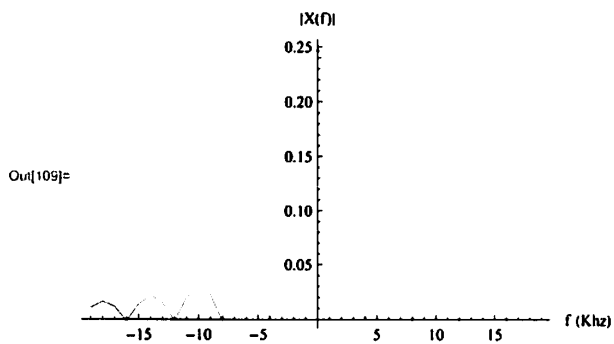
Plot fourier series approximation to the above pulse on top of it to compare

```
In[107]= Plot[getFourierApproximation[t, xn, period], {t, -range, range}]
```



Plot the spectrum of the pulse

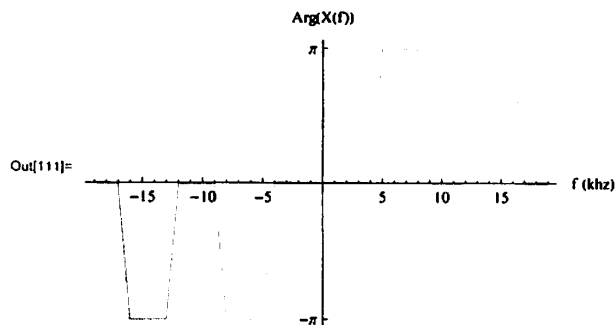
```
In[108]= data = getMagnitudeOfPulseTrainFourierCoeff[delay, period, range, dutyCycle, numberOfCoeff];
ListPlot[data, Joined -> True, AxesLabel -> {"f (Khz)", "|X(f)|"}]
```



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4 | ca1_v2.nb

```
In[110] = data = getPhaseOfPulseTrainFourierCoeff [delay, period, range, dutyCycle, numberOfCoeff];
ListPlot[data, Joined -> True,
  AxesLabel -> {"f (khz)", "Arg(X(f))"}, Ticks -> {Automatic, {-Pi, Pi}}]
```



■ Generate normalized low pass butterworth of order 4

```
In[112] = Clear[s, form];
order = 4;
cutoff = 1;
{poles, hs} = dsp`getButterworthPolynomial [order, cutoff, s];
TraditionalForm@hs
```

Out[116]/TraditionalForm=

$$\frac{1}{s^4 + 2.61313 s^3 + 3.41421 s^2 + 2.61313 s + 1.}$$

■ convert the above to low pass butterworth with specified cutoff

```
In[117] = newHs = dsp`butterToLowPass [hs, \frac{2 Pi}{tao}, s];
TraditionalForm@newHs
```

Out[118]/TraditionalForm=

$$\frac{1}{2.50634 \times 10^{-18} s^4 + 1.64604 \times 10^{-13} s^3 + 5.40519 \times 10^{-9} s^2 + 0.000103973 s + 1.}$$

■ Multiply $H(j\omega)$ with Pulse fourier series $y(n)$, and plot $Y(f)$

```

In[119]= Clear[w];
xnFourier[n_] := h dutyCycle Sinc[Pi n dutyCycle]

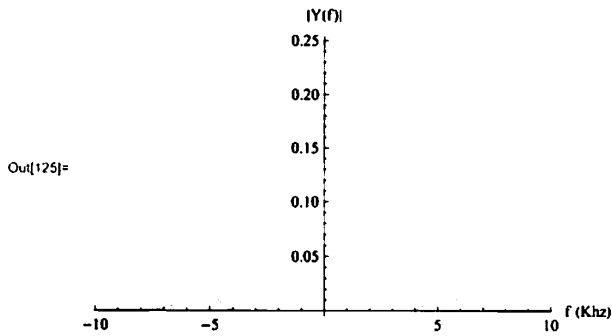
tf[n_, w0_] := newHs /. s -> (I w0 n)

yn[n_, w0_] := xnFourier[n] * tf[n, w0]

y[t_, w0_, numberOfCoeff_] := Sum[If[n == 0, yn[n, w0] * Exp[I w0 n t],
  (yn[n, w0] * Exp[I w0 n t] + yn[-n, w0] * Exp[-I w0 n t])], {n, 0, numberOfCoeff}]

data = Table[{m, Abs[yn[m, w0]]}, {m, -20, 20}];
ListPlot[data, Joined -> True,
  PlotRange -> {{-10, 10}, All}, AxesLabel -> {"f (Khz)", "|Y(f)|"}]

```

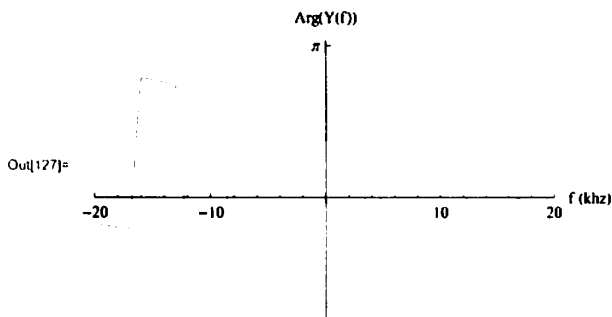


■ Plot the phase spectrum

```

In[126]= data = Table[{m, Arg[yn[m, w0]]}, {m, -20, 20}];
ListPlot[data, Joined -> True, PlotRange -> {{-20, 20}, All},
  AxesLabel -> {"f (khz)", "Arg(Y(f))"}, Ticks -> {Automatic, {-Pi, Pi}}]

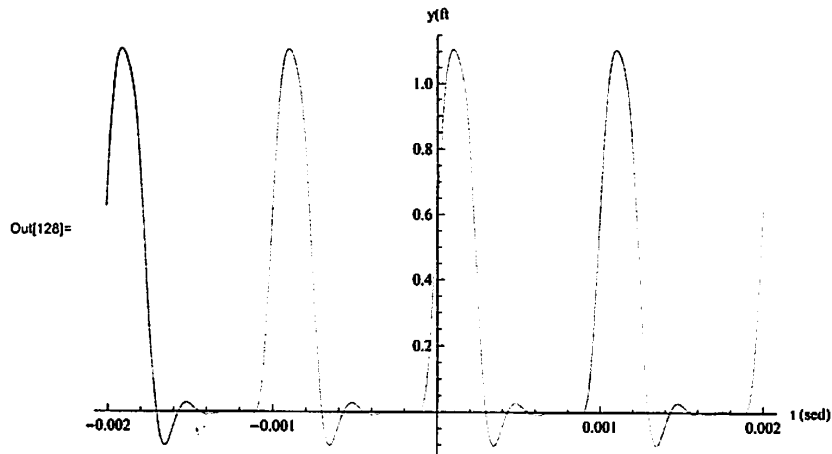
```



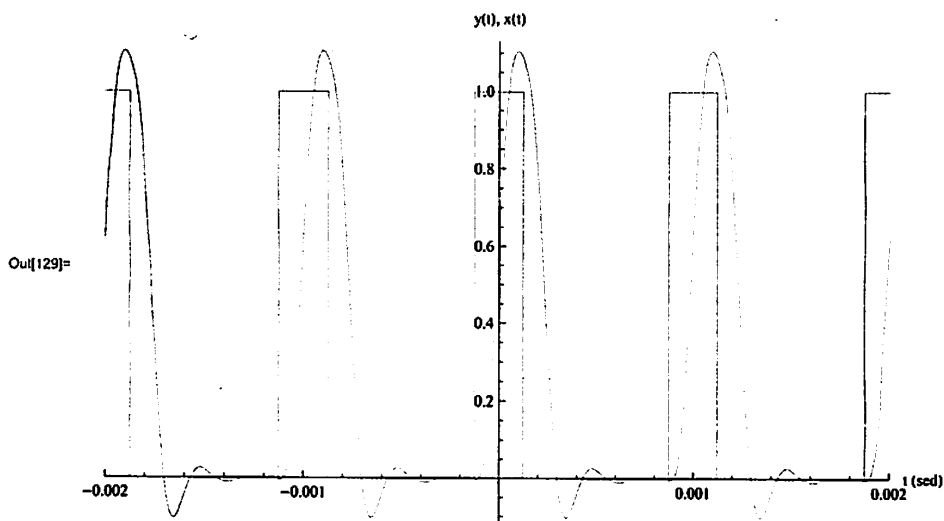
6 | ca1_v2.nb

■ Plot $y(t)$

```
In[128]:= Plot[y[t, w0, 10], {t, -range, range}, PlotRange -> All, AxesLabel -> {"t (sed)", "y(ft)"}]
```

■ Plot $y(t)$ and $x(t)$ on same plot to compare

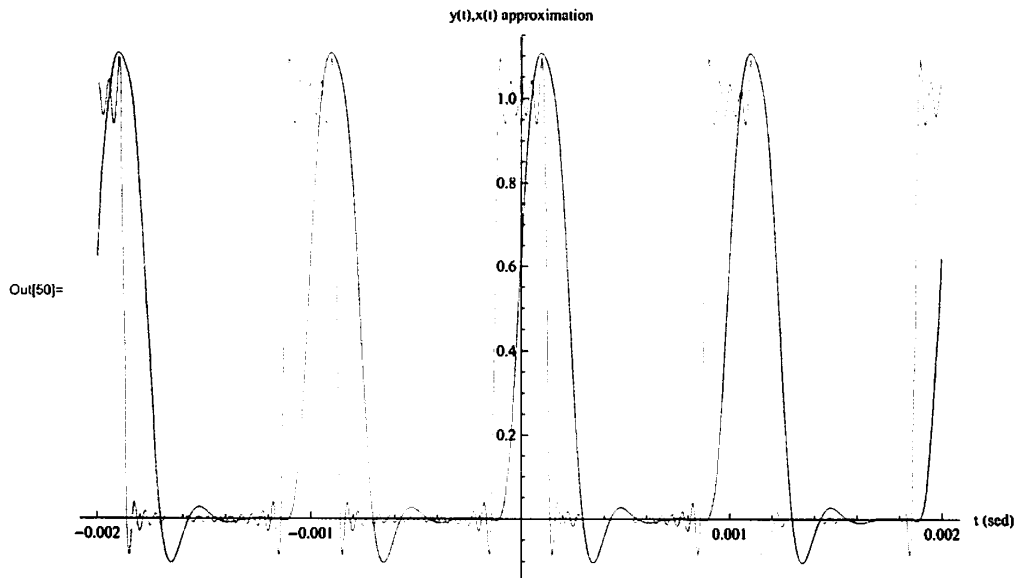
```
In[129]:= Plot[y[t, w0, 10], {t, -range, range},
  PlotRange -> {{-range, range}, All}, AxesLabel -> {"t (sed)", "y(t), x(t)"},
  Epilog -> {Thin, Red, currentPulses}
]
```



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- Plot $y(t)$ on top of approximation of $x(n)$ used

```
In[50]= Plot[{getFourierApproximation[t, xn, period], y[t, w0, 10]}, {t, -range, range},
  PlotRange -> All, AxesLabel -> {"t (sec)", "y(t), x(t) approximation"}]
```



Part (2) High Pass

- convert normalized butterworth to high pass butterworth

```
In[282]= newHs = dsp`butterToHighPass[hs,  $\frac{2 \text{ Pi}}{\text{tao}}$ , s];
  newHs = Numerator[newHs] / Together[Denominator[newHs]];
  TraditionalForm@newHs
```

Out[284]/TraditionalForm=

$$\frac{1.s^4}{1.s^4 + 65.675.s^3 + 2.1566 \times 10^9.s^2 + 4.14839 \times 10^{13}.s + 3.98988 \times 10^{17}}$$

8 | *ca1_v2.nb*

■ Multiply $H(j\omega)$ with Pulse fourier series $y(n)$, and plot $Y(f)$

```

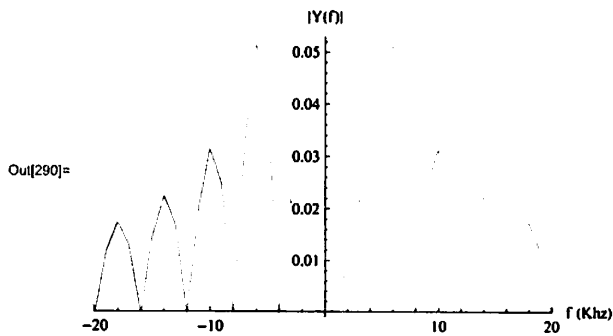
In[285]= Clear[w];
         tf[n_, w0_] := newHs /. s -> (I w0 n)

         yn[n_, w0_] := dsp`fcPulseTrain[n, h, tao, period] * tf[n, w0]

         y[t_, w0_, numberOfCoeff_] := Sum[If[n == 0, yn[n, w0] * Exp[I w0 n t],
         (yn[n, w0] * Exp[I w0 n t] + yn[-n, w0] * Exp[-I w0 n t])], {n, 0, numberOfCoeff}]

         data = Table[{m, Abs[yn[m, w0]]}, {m, -40, 40}];
         ListPlot[data, Joined -> True,
         PlotRange -> {{-20, 20}, All}, AxesLabel -> {"f (Khz)", "|Y(f)|"}]

```

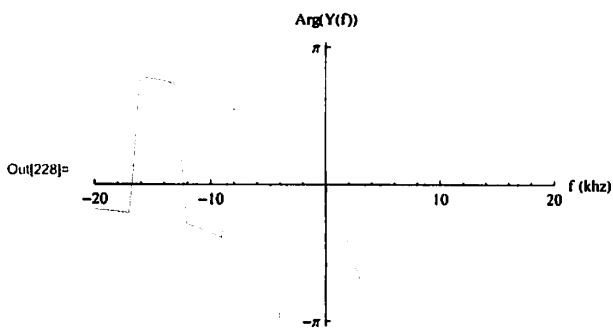


■ Plot the phase spectrum

```

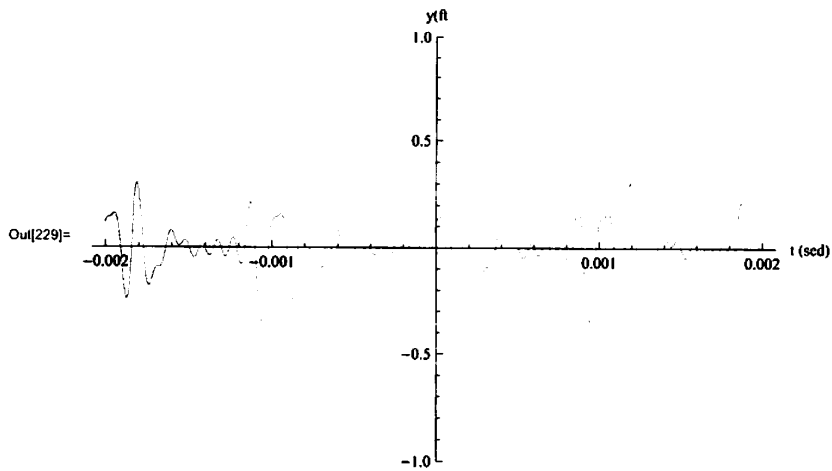
In[227]= data = Table[{m, Arg[yn[m, w0]]}, {m, -20, 20}];
         ListPlot[data, Joined -> True, PlotRange -> {{-20, 20}, All},
         AxesLabel -> {"f (kHz)", "Arg(Y(f))"}, Ticks -> {Automatic, {-Pi, Pi}}]

```



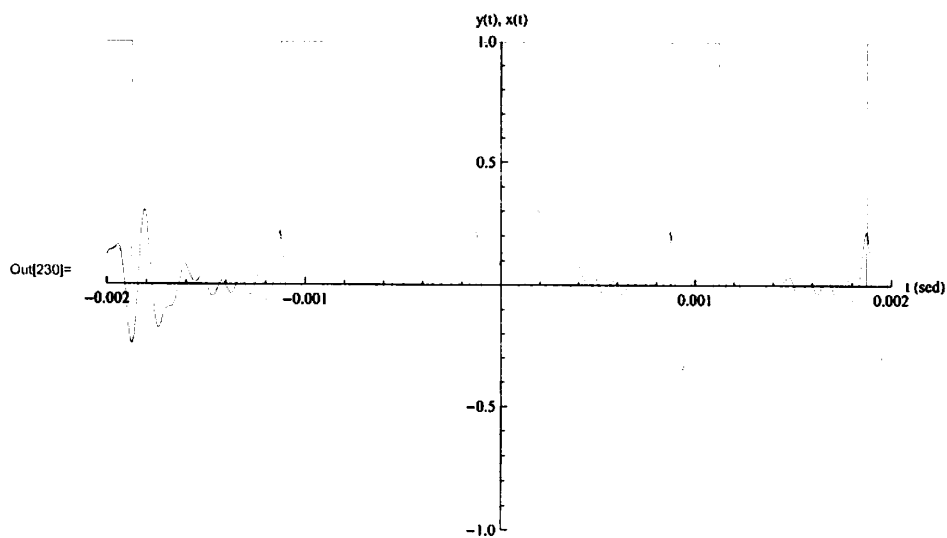
Plot $y(t)$

```
In[229]= Plot[y[t, w0, 10], {t, -range, range},
  PlotRange -> {Automatic, {-h, h}}, AxesLabel -> {"t (sed)", "y(ft)"}]
```



Plot $y(t)$ and $x(t)$ on same plot to compare

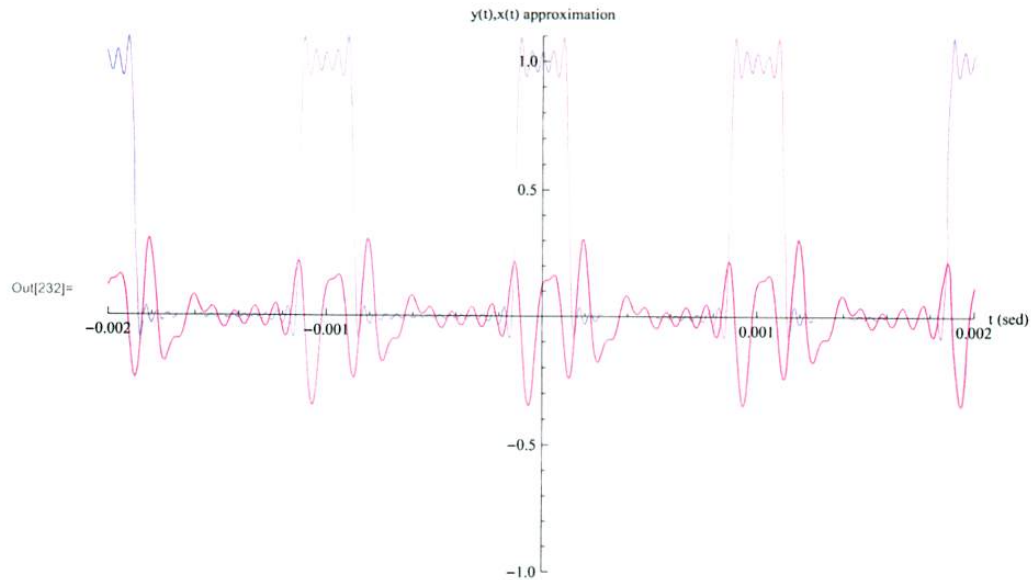
```
In[230]= Plot[y[t, w0, 10], {t, -range, range},
  PlotRange -> {{-range, range}, {-h, h}}, AxesLabel -> {"t (sed)", "y(t), x(t)"},
  Epilog -> {Thin, Red, currentPulses}
]
```



10 | ca1_v2.nb

Plot $y(t)$ on top of approximation of $x(n)$ used

```
In[232]= Plot[{getFourierApproximation[t, xn, period], y[t, w0, 10]},
  {t, -range, range}, PlotRange -> {{-range, range}, {-h, 1.1 h}},
  AxesLabel -> {"t (sed)", "y(t),x(t) approximation"}]
```



Part (3) BandPass filter

- convert normalized butterworth to band pass butterworth

```
In[273]= newHs = dsp^butterToBandPass [hs,  $\frac{2 \text{ Pi}}{\text{tao}}$ ,  $\frac{4 \text{ Pi}}{\text{tao}}$ , s];
  newHs = Numerator[newHs] / Together[Denominator[newHs]];
  TraditionalForm@newHs
```

```
Out[275]/TraditionalForm=

$$(1. s^4) / (2.50634 \times 10^{-18} s^8 + 1.64604 \times 10^{-13} s^7 + 1.80703 \times 10^{-8} s^6 + 0.000727811 s^5 + 38.6569 s^4 + 919.450 s^3 + 2.88394 \times 10^{10} s^2 + 3.31871 \times 10^{14} s + 6.3838 \times 10^{18})$$

```

ca1_v2.nb | 11

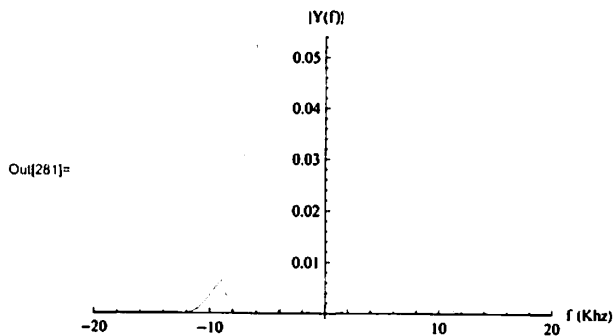
• Multiply $H(j\omega)$ with Pulse fourier series $y(n)$, and plot $Y(f)$

```
In[276]= Clear[w];
         tf[n_, w0_] := newHs /. s -> (I w0 n)

         yn[n_, w0_] := dsp`fcPulseTrain[n, h, tao, period] * tf[n, w0]

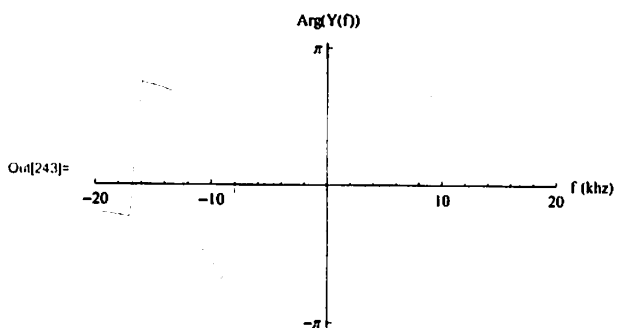
         y[t_, w0_, numberOfCoeff_] := Sum[If[n = 0, yn[n, w0] * Exp[I w0 n t],
         (yn[n, w0] * Exp[I w0 n t] + yn[-n, w0] * Exp[-I w0 n t]), {n, 0, numberOfCoeff}]

         data = Table[{m, Abs[yn[m, w0]]}, {m, -40, 40}];
         ListPlot[data, Joined -> True,
         PlotRange -> {{-20, 20}, All}, AxesLabel -> {"f (Khz)", "|Y(f)|"}]
```



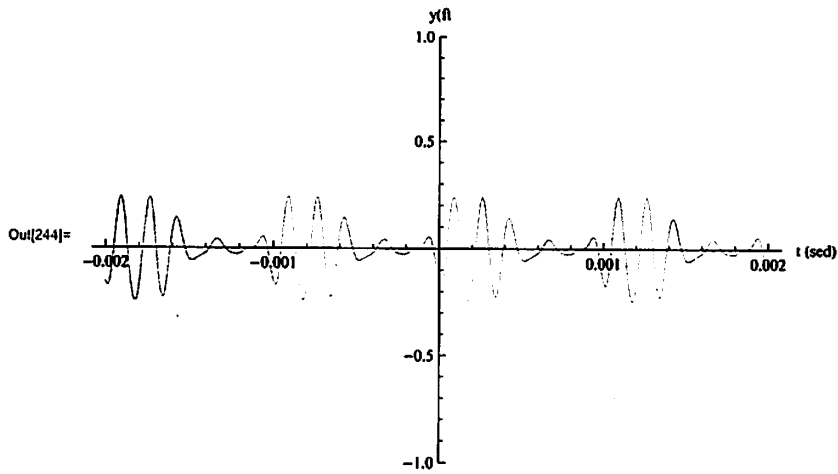
• Plot the phase spectrum

```
In[242]= data = Table[{m, Arg[yn[m, w0]]}, {m, -20, 20}];
         ListPlot[data, Joined -> True, PlotRange -> {{-20, 20}, All},
         AxesLabel -> {"f (khz)", "Arg(Y(f))"}, Ticks -> {Automatic, {-Pi, Pi}}]
```

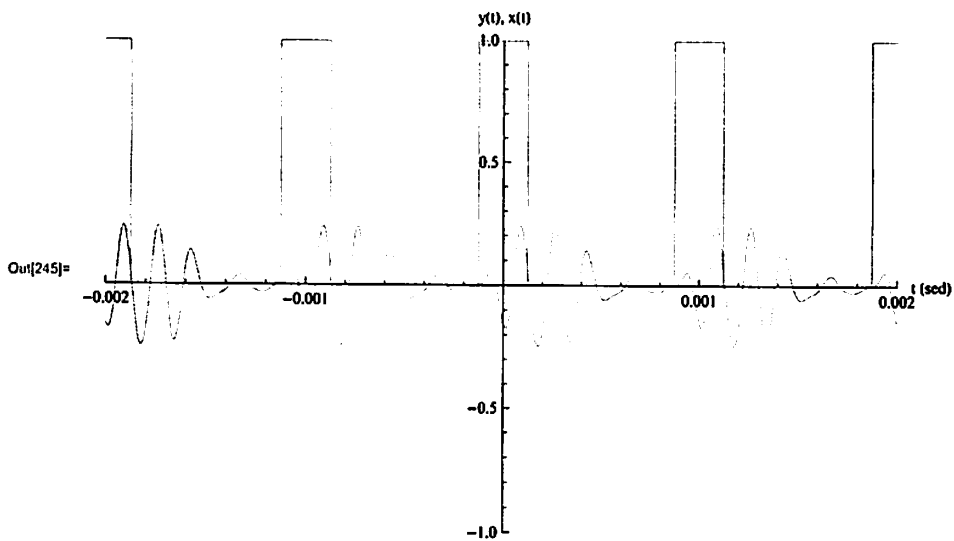


12 | *ca1_v2.nb*■ Plot $y(t)$

```
In[244]:= Plot[y[t, w0, 10], {t, -range, range},
  PlotRange -> {Automatic, {-h, h}}, AxesLabel -> {"t (sed)", "y(ft)"}]
```

■ Plot $y(t)$ and $x(t)$ on same plot to compare

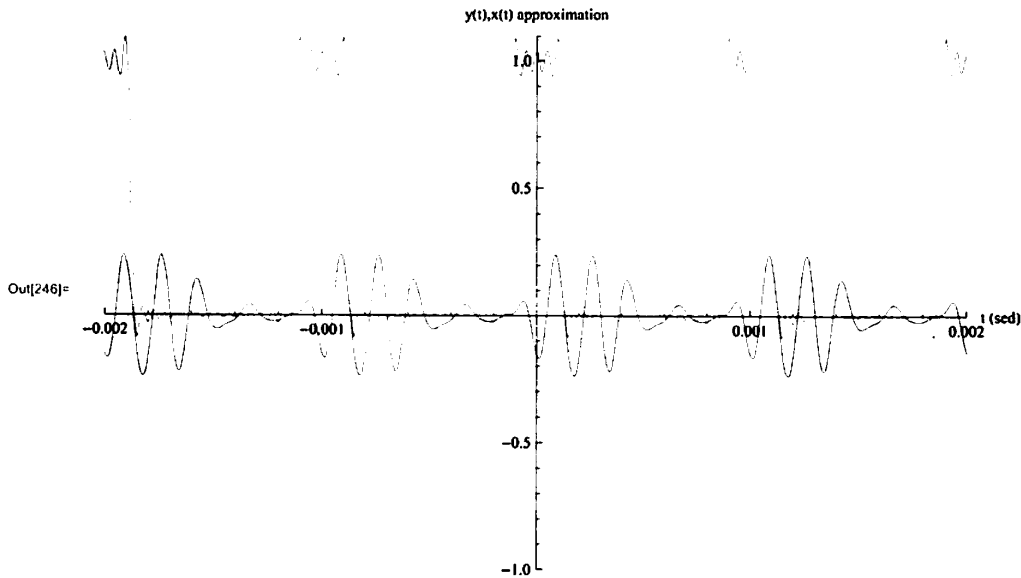
```
In[245]:= Plot[y[t, w0, 10], {t, -range, range},
  PlotRange -> {{-range, range}, {-h, h}}, AxesLabel -> {"t (sed)", "y(t), x(t)"},
  Epilog -> {Thin, Red, currentPulses}
]
```



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Plot $y(t)$ on top of approximation of $x(n)$ used

```
In[246]= Plot[{getFourierApproximation[t, xn, period], y[t, w0, 10]},
  {t, -range, range}, PlotRange -> {{-range, range}, {-h, 1.1 h}},
  AxesLabel -> {"t (sec)", "y(t),x(t) approximation"}]
```



Part (4) BandStop filter

- convert normalized butterworth to band stop butterworth

```
In[293]= newHs = dsp`butterToBandStop[hs,  $\frac{2 \text{ Pi}}{\text{tao}}$ ,  $\frac{4 \text{ Pi}}{\text{tao}}$ , s];
newHs = Numerator[newHs] / Together[Denominator[newHs]];
TraditionalForm@newHs
```

```
Out[295]/TraditionalForm=

$$\frac{(1.(s^2 + 1.26331 \times 10^9))}{(1.s^8 + 65.675.s^7 + 7.20984 \times 10^9.s^6 + 2.90388 \times 10^{14}.s^5 + 1.54236 \times 10^{19}.s^4 + 3.66849 \times 10^{23}.s^3 + 1.15066 \times 10^{28}.s^2 + 1.32413 \times 10^{32}.s + 2.54706 \times 10^{36})}$$

```


14 | ca1_v2.nb

■ Multiply $H(j\omega n)$ with Pulse fourier series $y(n)$, and plot $Y(f)$

```

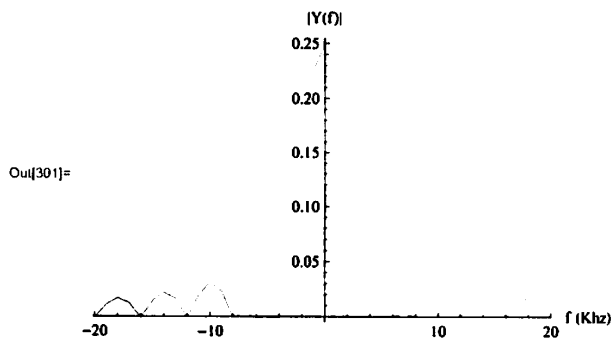
In[296]= Clear[w];
         tf[n_, w0_] := newEs /. s -> (I w0 n)

         yn[n_, w0_] := dsp`fcPulseTrain[n, h, tao, period] * tf[n, w0]

         y[t_, w0_, numberOfCoeff_] := Sum[If[n == 0, yn[n, w0] * Exp[I w0 n t],
         (yn[n, w0] * Exp[I w0 n t] + yn[-n, w0] * Exp[-I w0 n t]), {n, 0, numberOfCoeff}]

         data = Table[{m, Abs[yn[m, w0]]}, {m, -40, 40}];
         ListPlot[data, Joined -> True,
         PlotRange -> {{-20, 20}, All}, AxesLabel -> {"f (Khz)", "|Y(f)|"}]

```

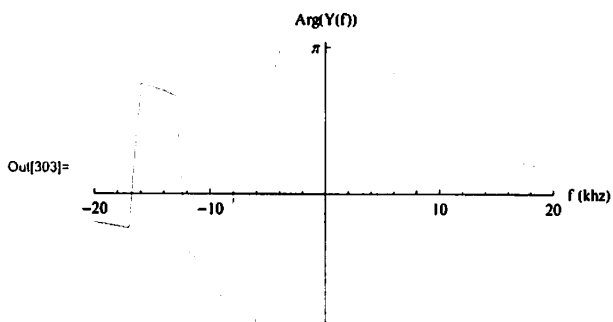


■ Plot the phase spectrum

```

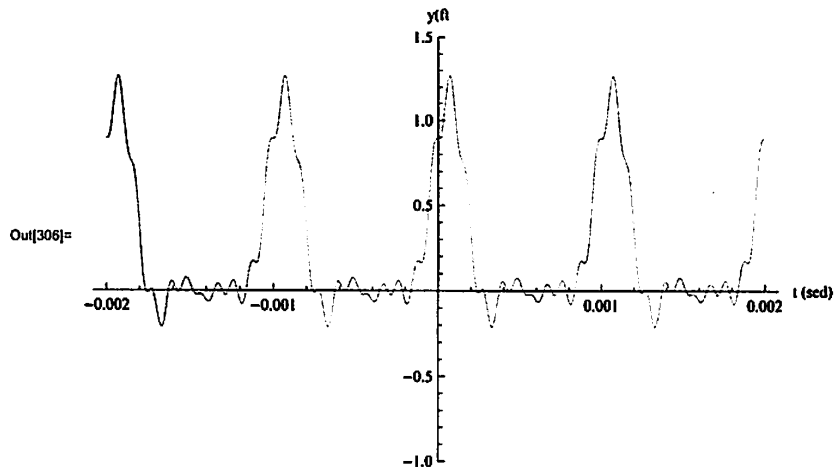
In[302]= data = Table[{m, Arg[yn[m, w0]]}, {m, -20, 20}];
         ListPlot[data, Joined -> True, PlotRange -> {{-20, 20}, All},
         AxesLabel -> {"f (khz)", "Arg(Y(f))"}, Ticks -> {Automatic, {-Pi, Pi}}]

```



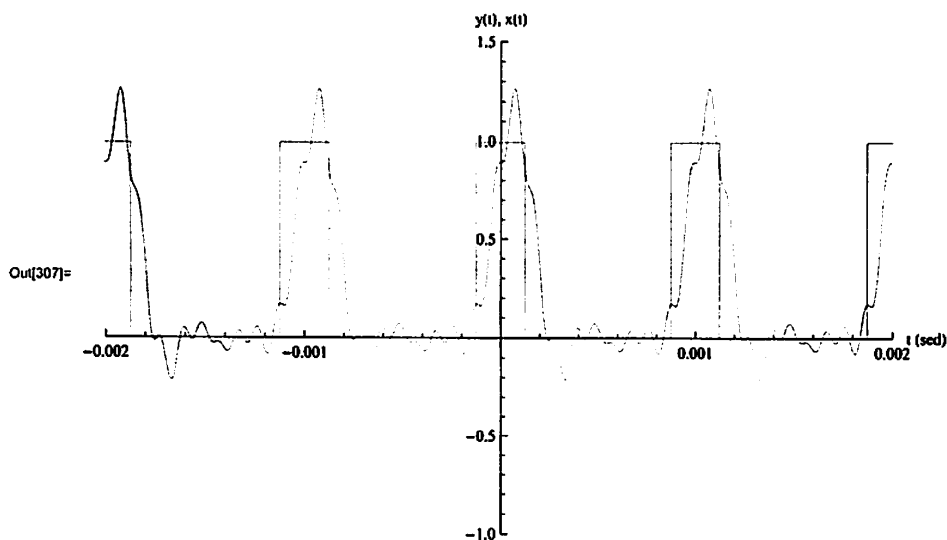
Plot y(t)

```
In[306]= Plot[y[t, w0, 10], {t, -range, range},
  PlotRange -> {Automatic, {-h, 1.5 h}}, AxesLabel -> {"t (sec)", "y(ft)"}]
```



Plot y(t) and x(t) on same plot to compare

```
In[307]= Plot[y[t, w0, 10], {t, -range, range},
  PlotRange -> {{-range, range}, {-h, 1.5 h}}, AxesLabel -> {"t (sec)", "y(t), x(t)"},
  Epilog -> {Thin, Red, currentPulses}
]
```

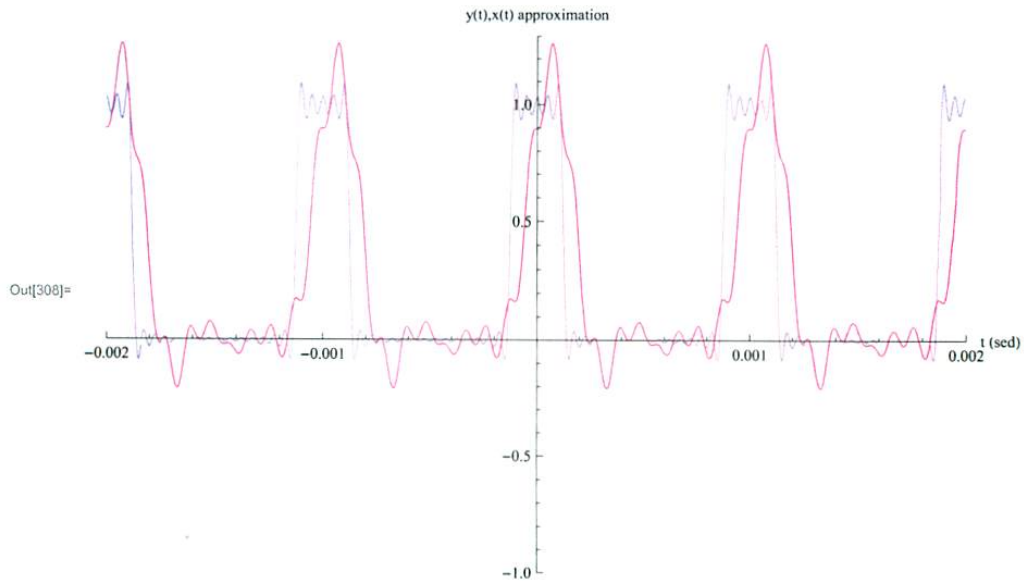


16 | ca1_v2.nb

Plot $y(t)$ on top of approximation of $x(n)$ used

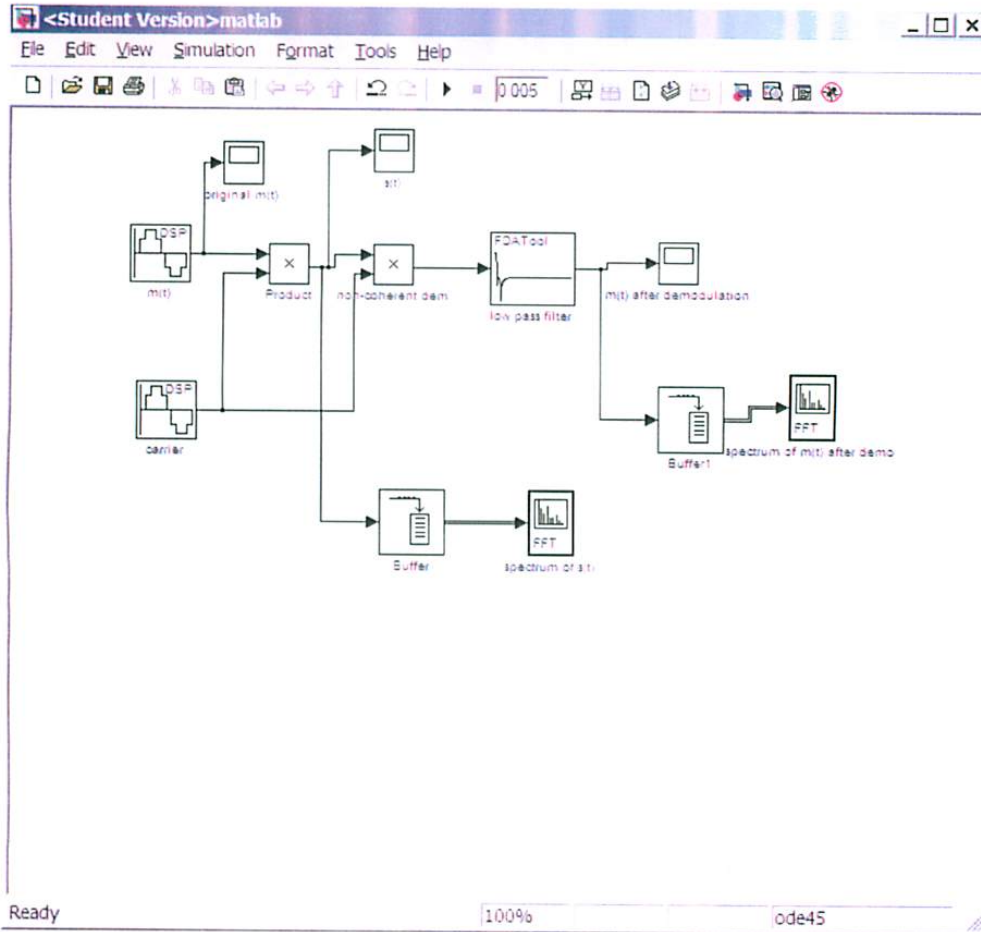
In[308]=

```
Plot[{getFourierApproximation[t, xn, period], y[t, w0, 10]},  
  {t, -range, range}, PlotRange -> {{-range, range}, {-h, 1.3 h}},  
  AxesLabel -> {"t (sed)", "y(t),x(t) approximation"}]
```



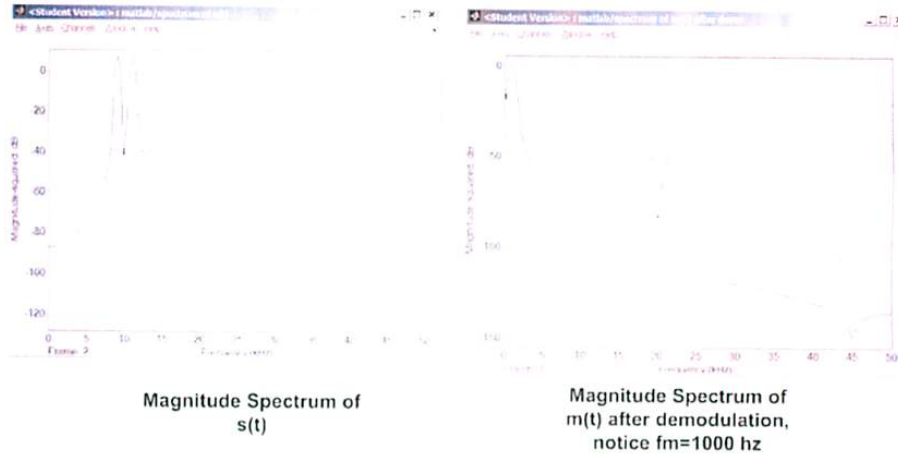
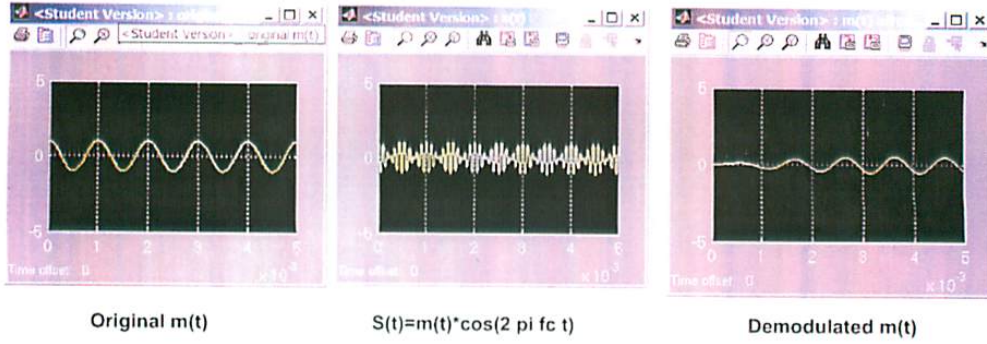
Computer Assignment #2
by Nasser M. Abbasi
ECE 405, summer session 1, Cal Poly Pomona

Simulink setup



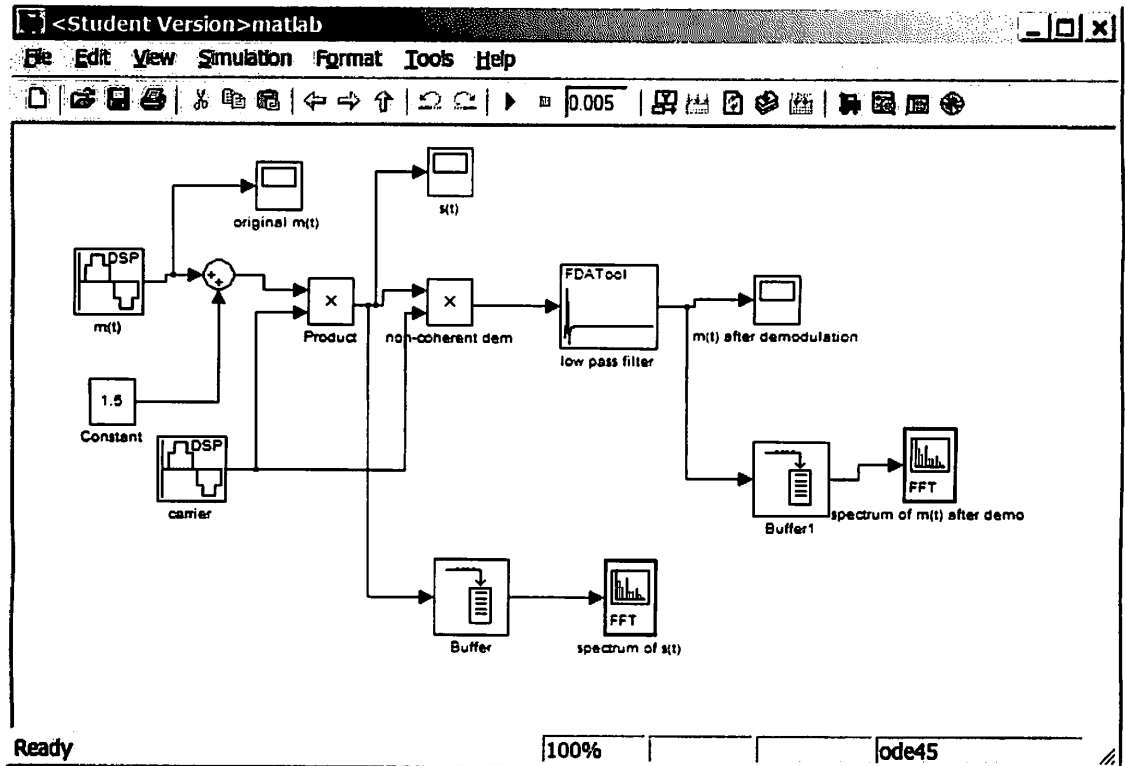
Part (1) non-coherent demodulation

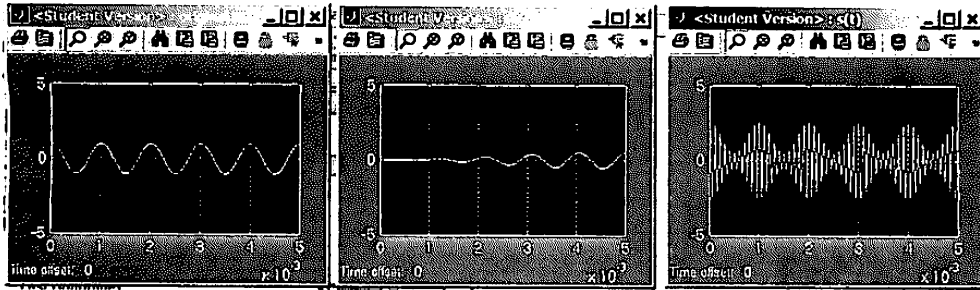
After run of the simulation, the following are the outputs time scope:



Computer Assignment #3
 by Nasser M. Abbasi
 ECE 405, summer session 1, Cal Poly Pomona

Simulink setup

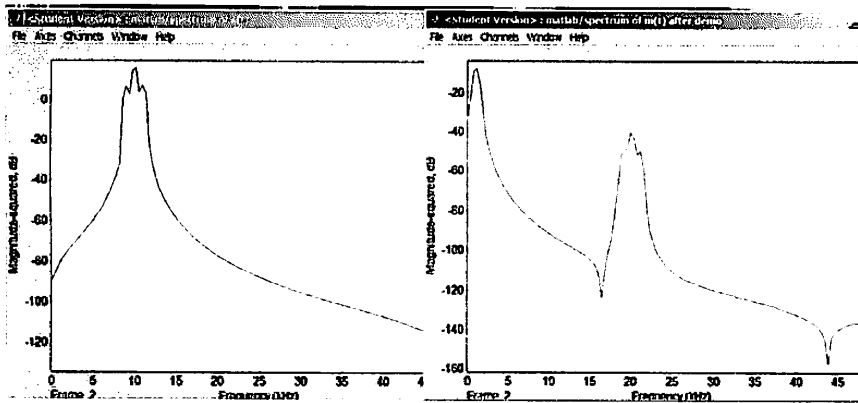




Original m(t)

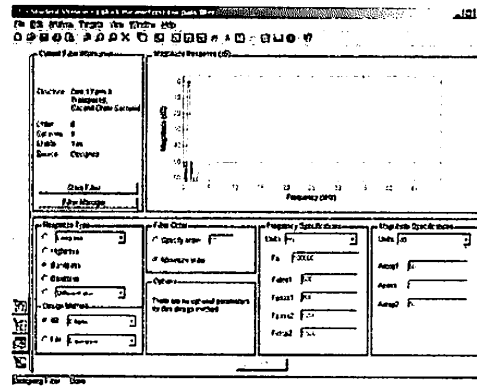
Demodulated m(t)

$S(t) = (A_c + \cos(2\pi f_m t)) \cos(2\pi f_c t)$



Magnitude Spectrum of s(t)

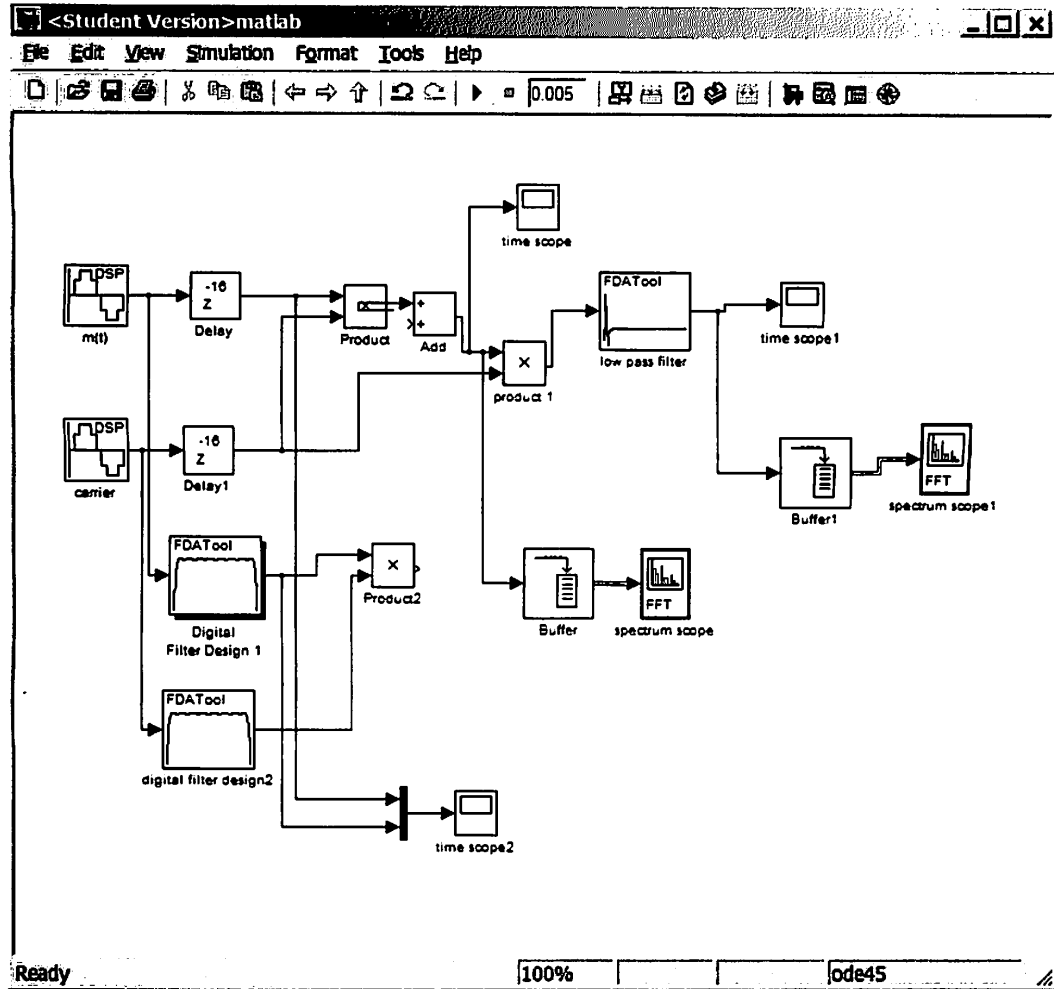
Magnitude Spectrum of m(t) after demodulation, notice $f_m=1000$ hz

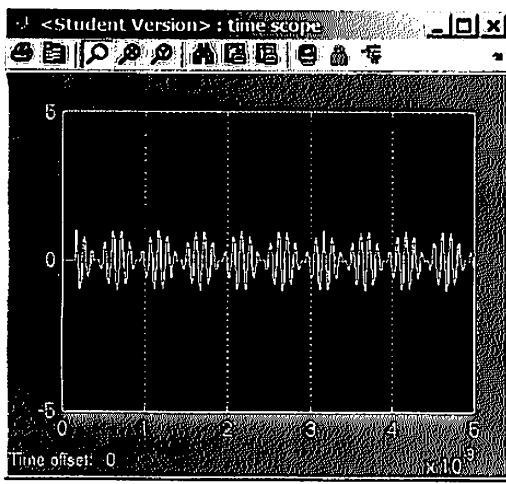


Computer #3 Part (1)

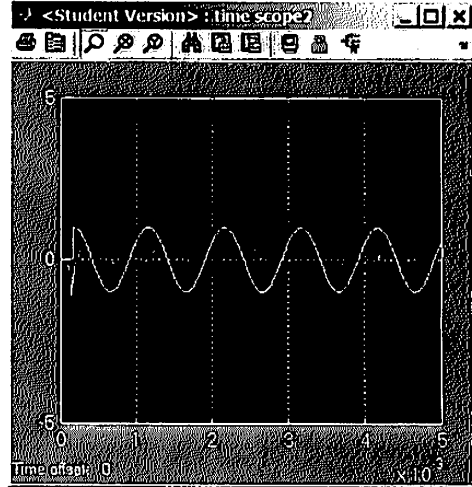
Computer Assignment #4
by Nasser M. Abbasi
ECE 405, summer session 1, Cal Poly Pomona

Simulink setup

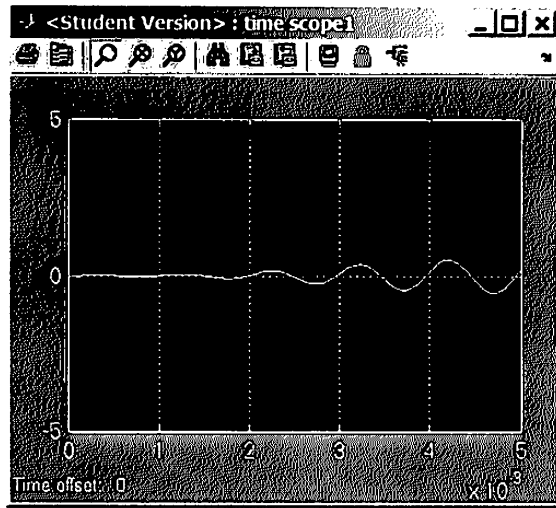




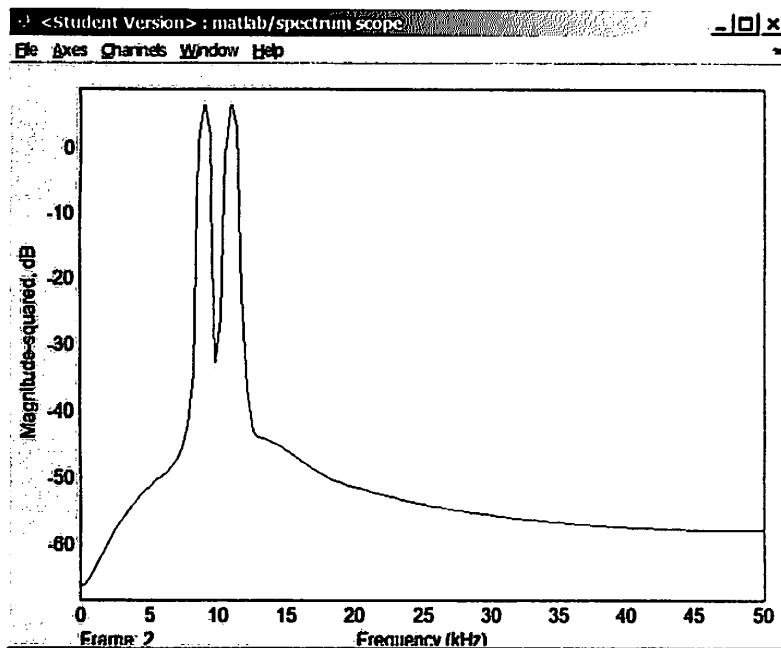
Time scope output



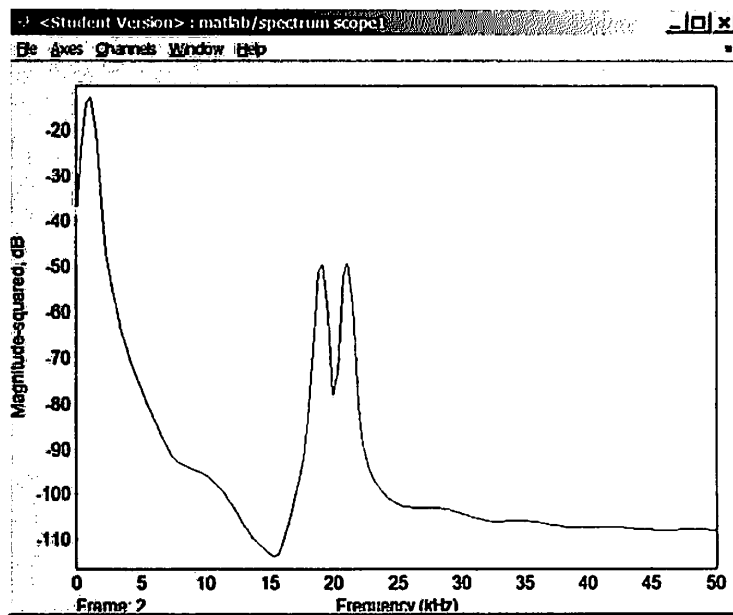
Time scope 2 output



Time scope 1 output



Spectrum scope output



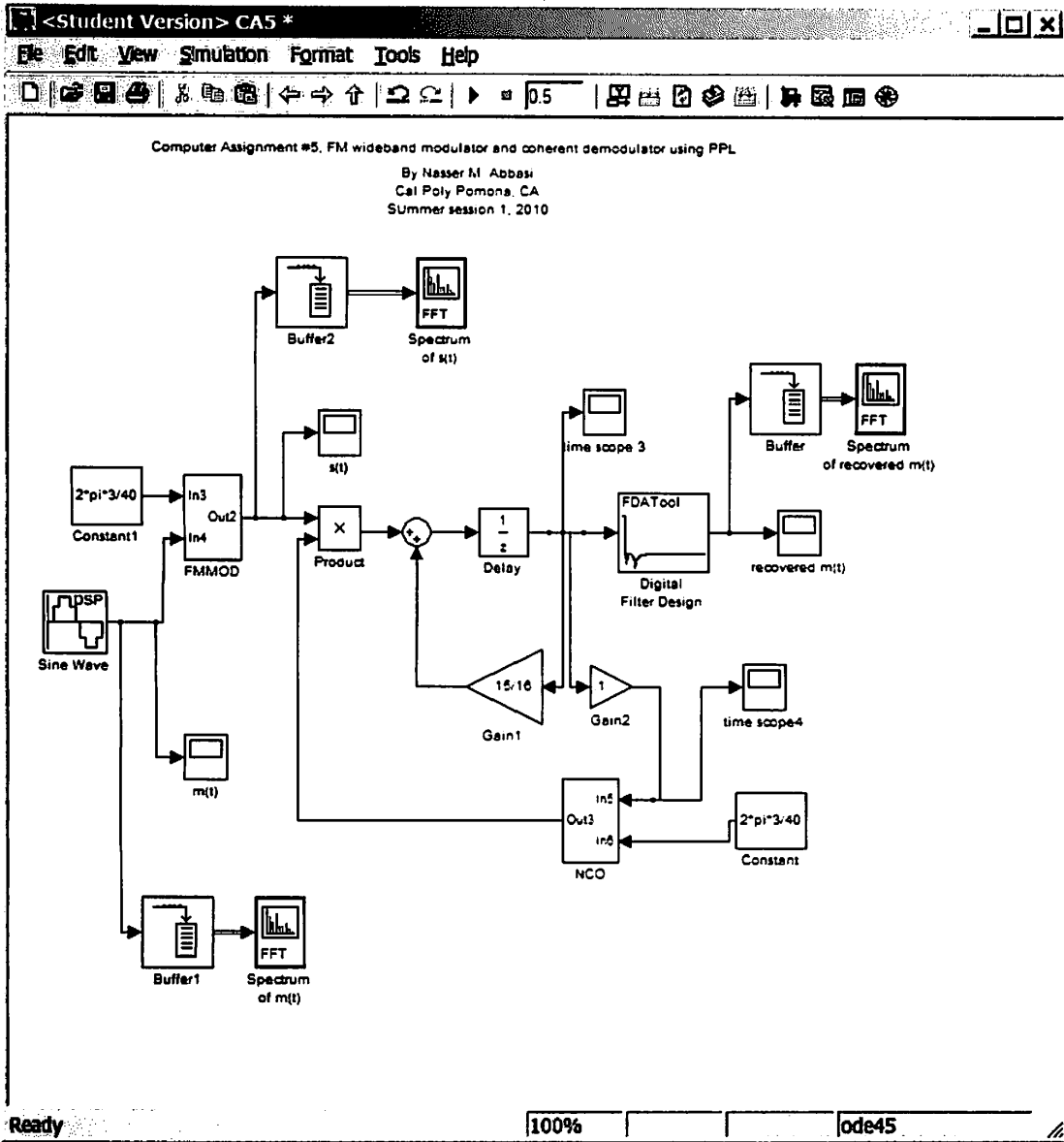
Spectrum scope 1 output

Computer Assignment #5, FM wideband modulator and coherent demodulator using PPL

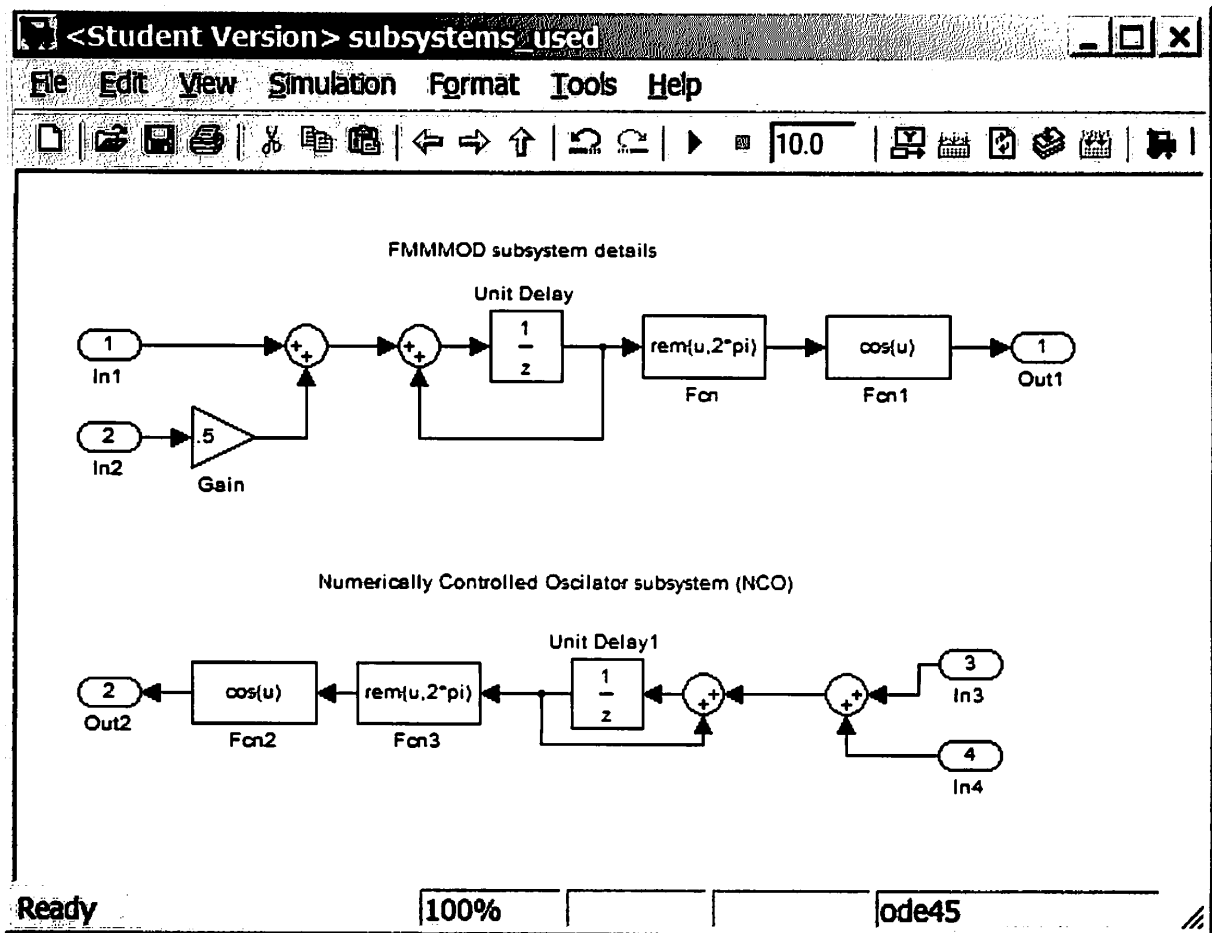
Nasser M. Abbasi
 Cal Poly Pomona, CA
 Summer session 1, 2010

Simulink model

model is here

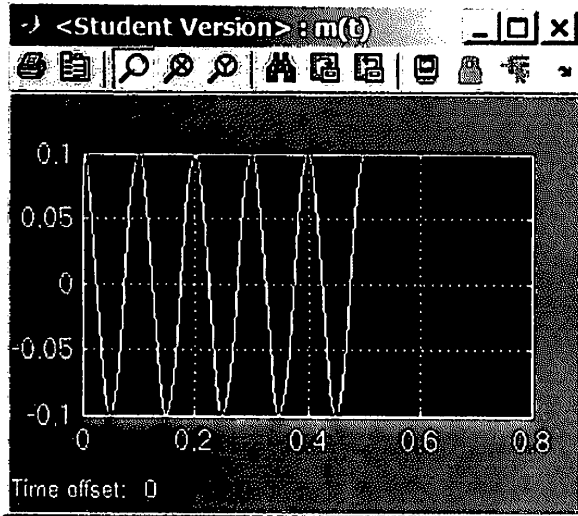


Models of subsystems

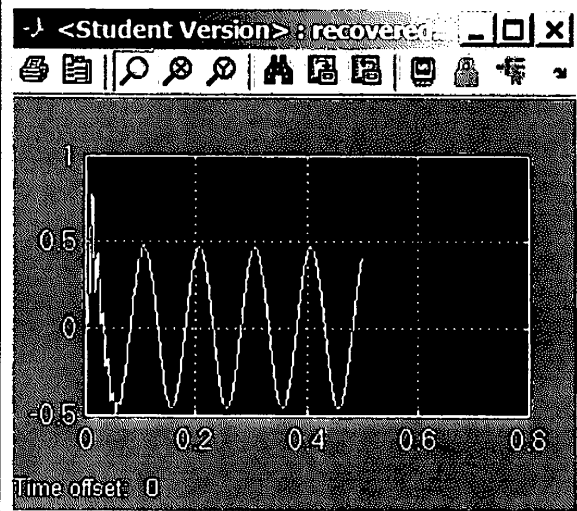


model file of the above is [here](#)

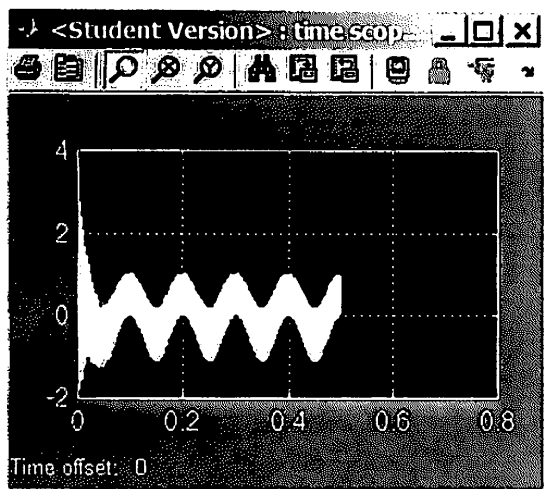
output



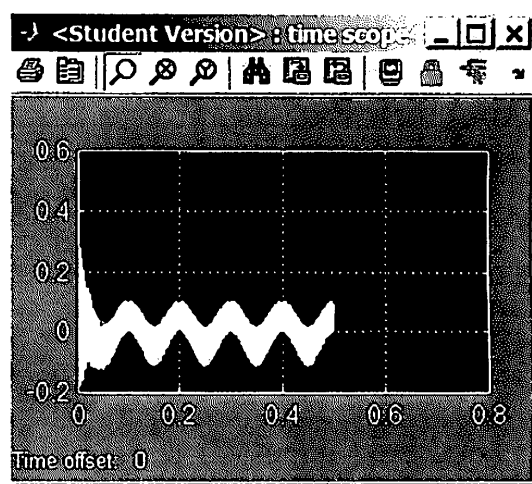
Original message $m(t)$



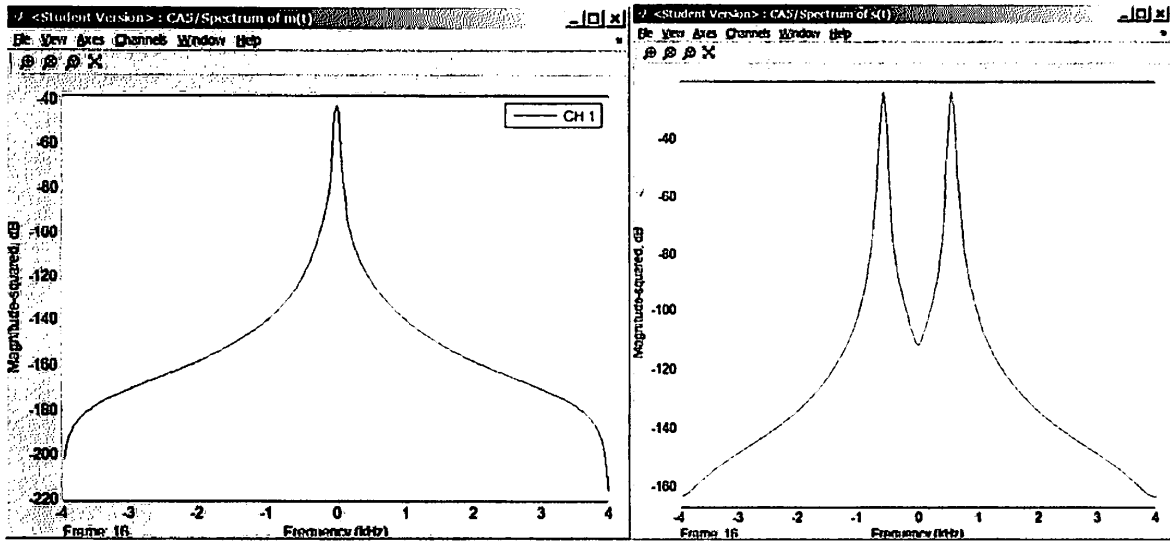
Recovered (demodulated) message $m(t)$



Time scope (3) in the model

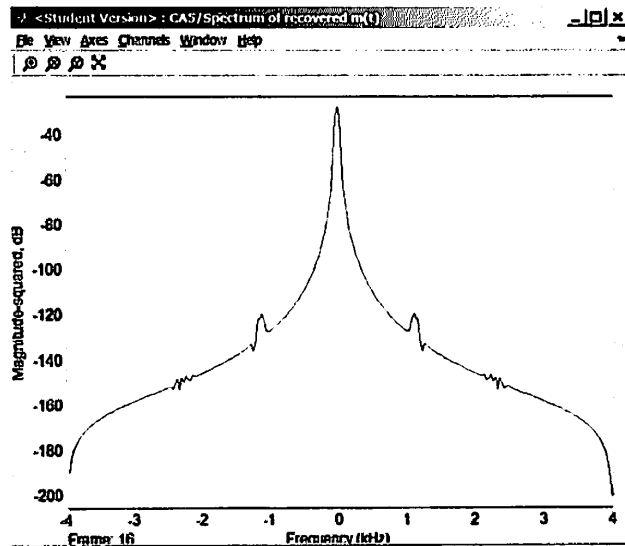


Time scope (4) in the model



Spectrum of original message $m(t)$

Spectrum of $s(t)$, the modulated carrier (FM)



Spectrum of recovered message $m(t)$ (compare to spectrum of original $m(t)$)

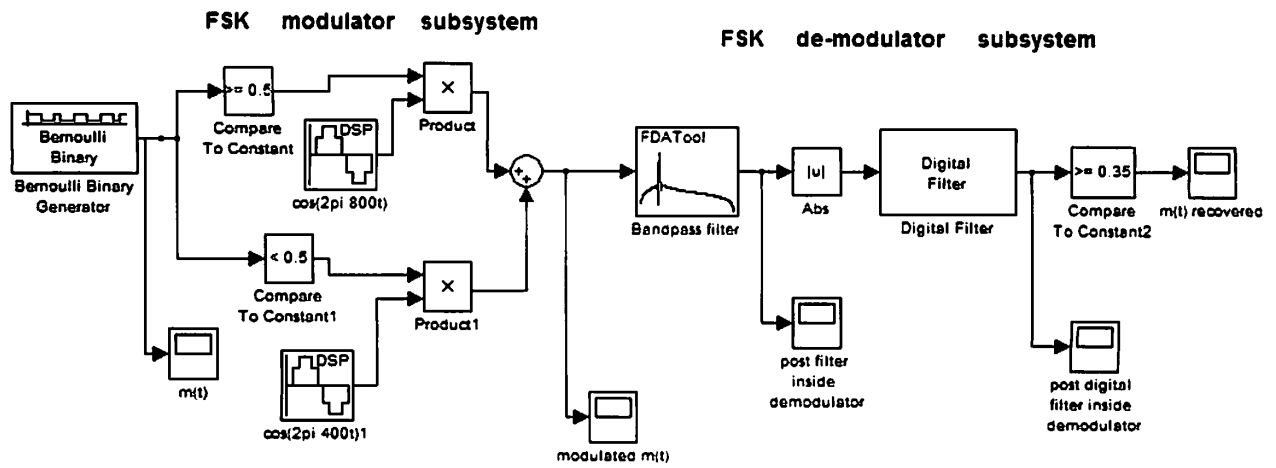
Computer Assignment #6
Nasser M. Abbasi
 Cal Poly Pomona, CA
 SUMmer session 1, 2010

problem description is [here](#)

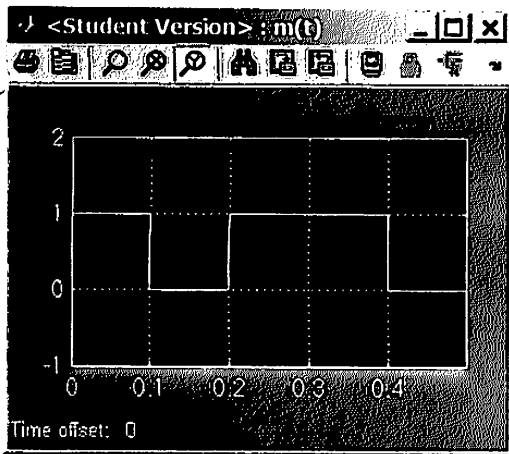
Simulink model

[model is here](#)

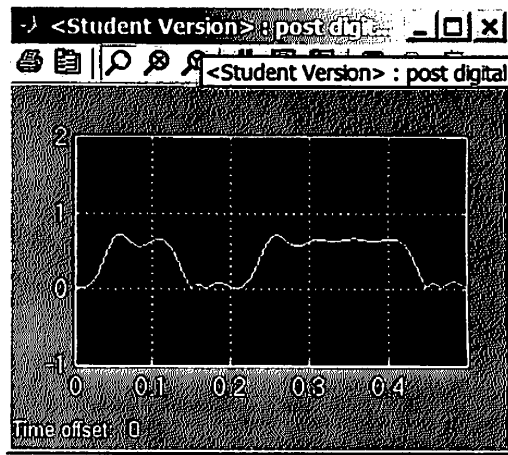
Computer assignment #6
 modulation and demodulation of Binary FSK
 by Nasser M. Abbasi, cal poly, summer session 1, 2010



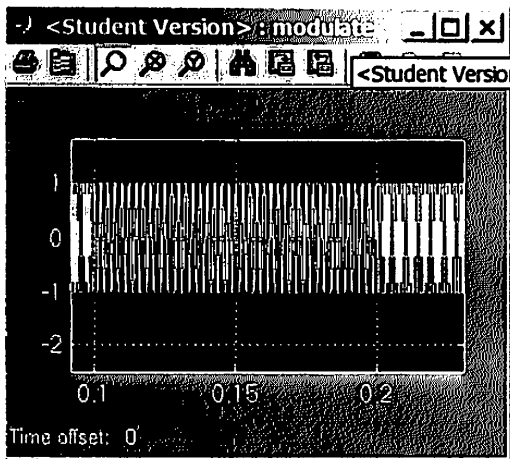
Output and result



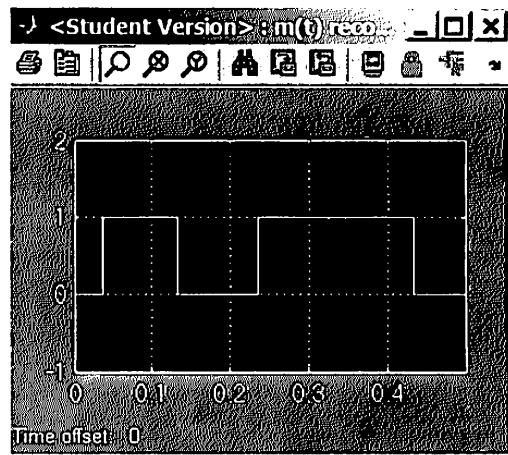
Original message $m(t)$



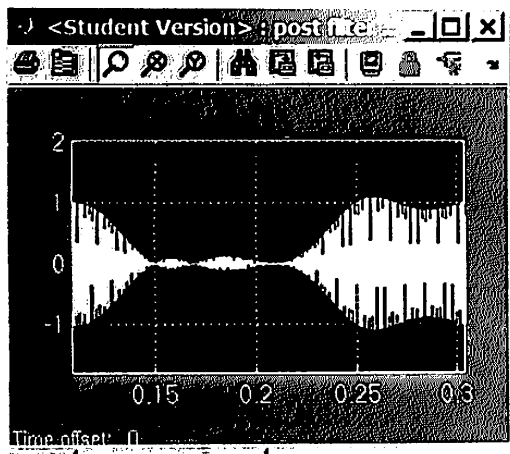
Output of bandpass filter in the demodulator



$S(t)$, the modulated carrier message



demodulated message $m(t)$. Notice some delay at the start compared to original $m(t)$



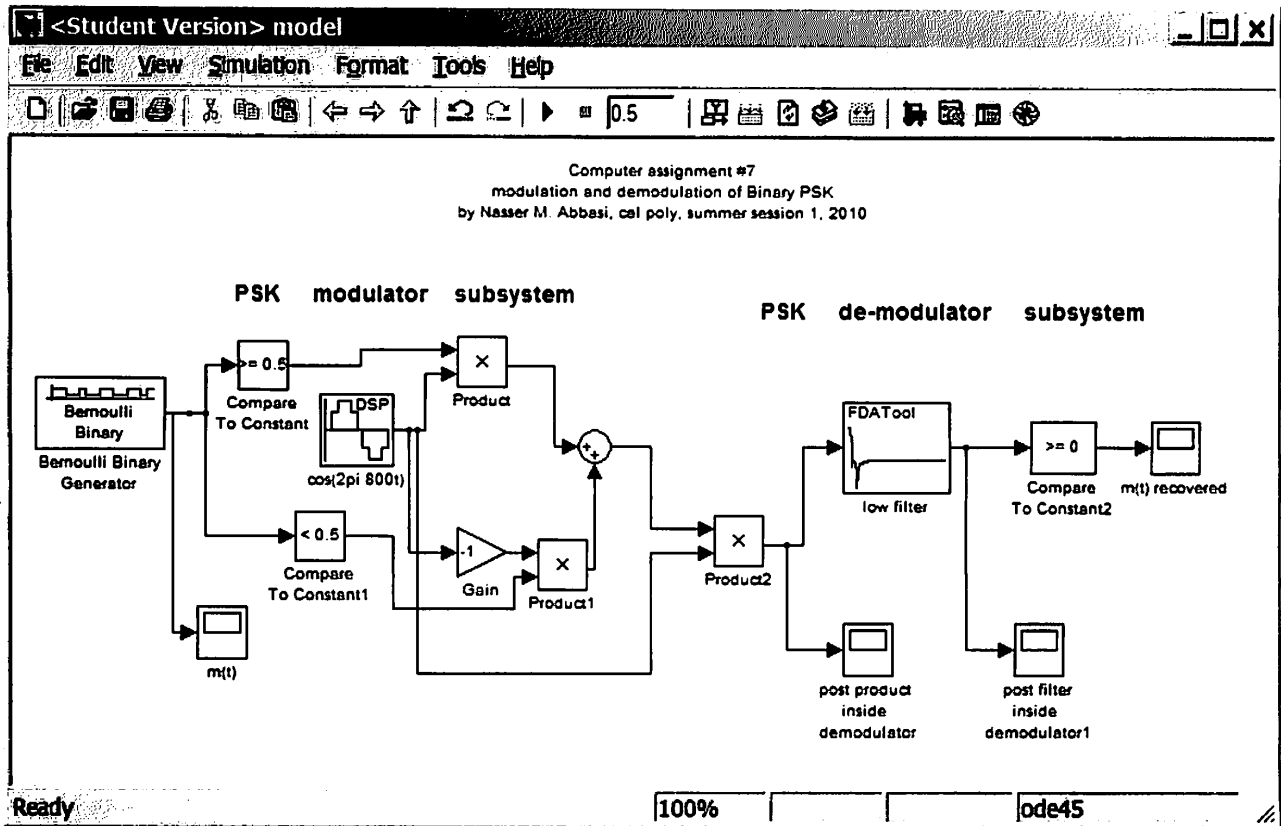
Output of digital filter inside the demodulator

Computer Assignment #7
 Nasser M. Abbasi
 Cal Poly Pomona, CA
 Summer session 1, 2010

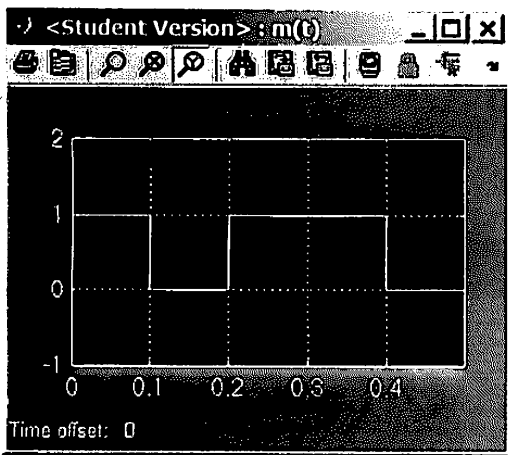
problem description is [here](#)

Simulink model

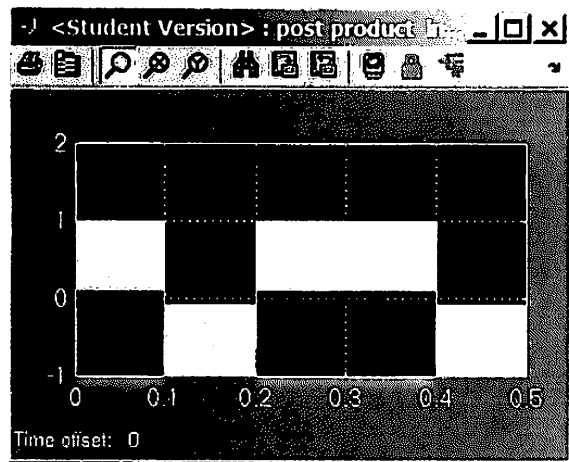
model is [here](#)



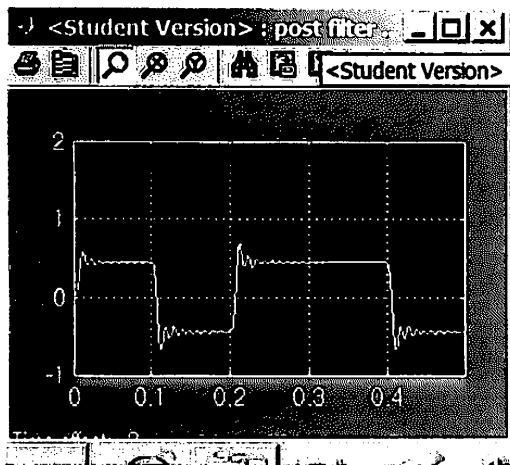
Output and result



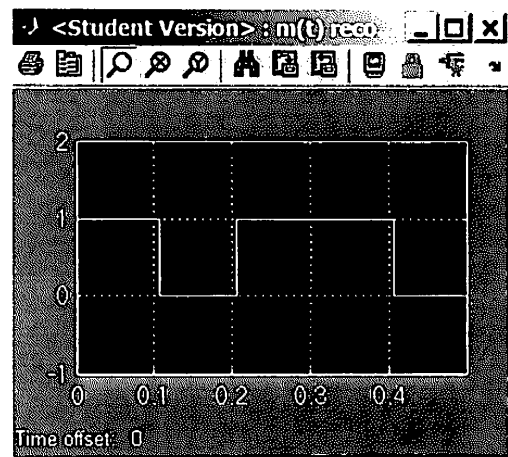
Original binary message $m(t)$



Modulated carrier $s(t)$



Message after passing the low pass filter, before compare



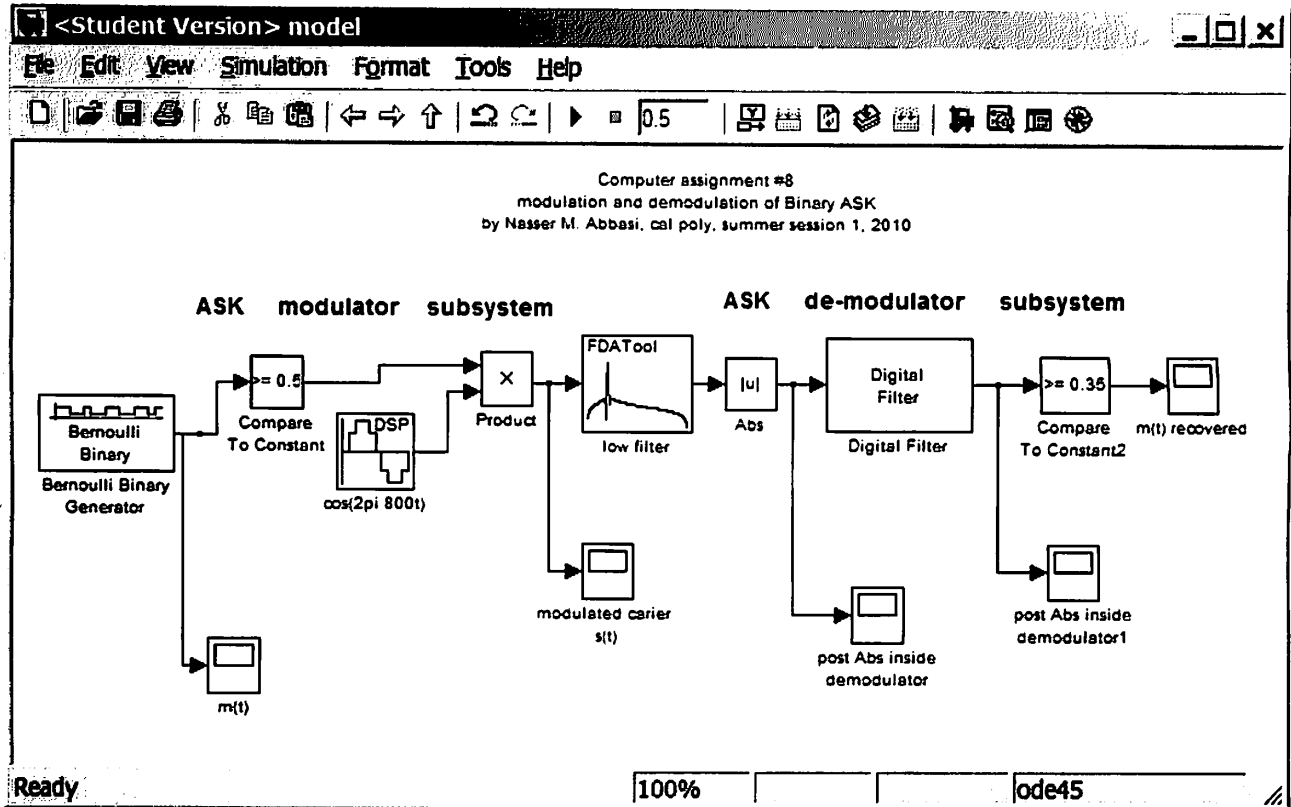
Recovered (demodulated) binary message $m(t)$

Computer Assignment #8
 Nasser M. Abbasi
 Cal Poly Pomona, CA
 Summer session 1, 2010

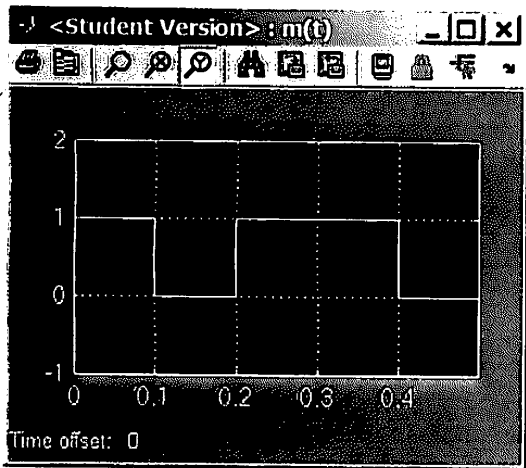
problem description is [here](#)

Simulink model

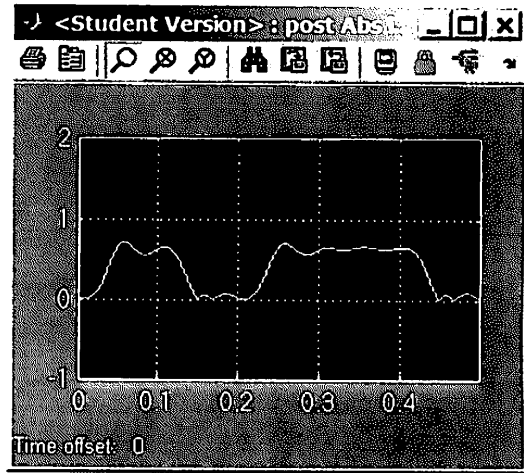
model is [here](#)



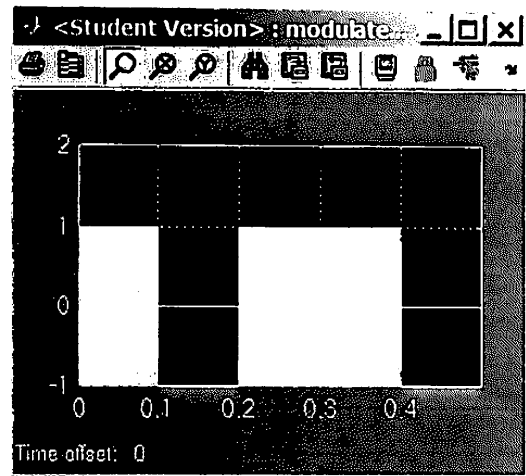
Output and result



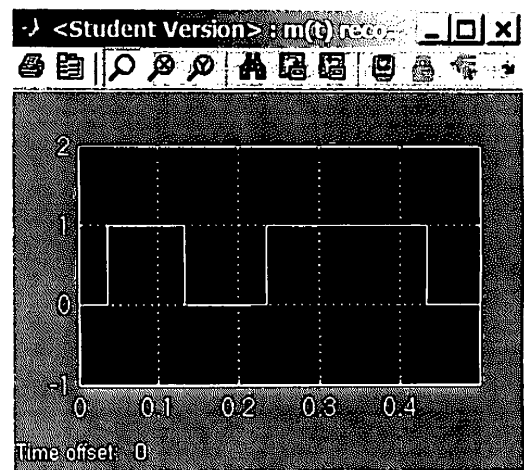
Original message $m(t)$



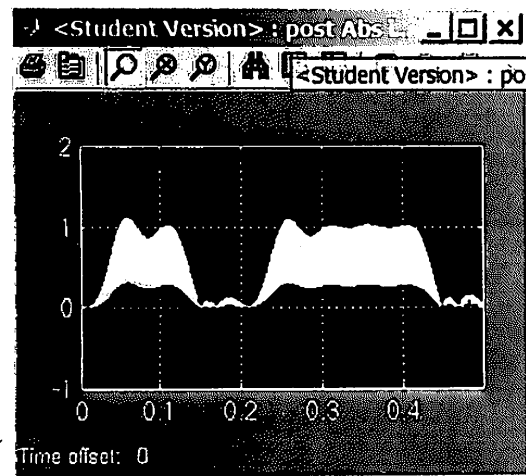
After the digital filter



Modulated carrier $s(t)$



Demodulated message $m(t)$ recovered. Notice delay at the start compared to original message $m(t)$



After check on Abs value

Chapter 5

Study notes, cheat sheets

Local contents

5.1	sheet 1	190
5.2	sheet 2	204
5.3	sheet 3	220

5.1 sheet 1

single tone FM modulation

$$x_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - (f_c + n f_m)) + \delta(f + (f_c + n f_m)) \right]$$

for one sided

$$= A_c \sum_{n=0}^{\infty} J_n(\beta) \left[\delta(f - (f_c + n f_m)) + \dots \right]$$

$$(BT)_{\text{Carson}} = 2(f_m + \Delta f)$$

$$= 2 f_m (\beta + 1)$$

$$(BT)_{\%} = 2 n_{\text{max}} f_m$$

$$P_{\text{Power in carrier}} = \frac{A_c^2}{2} J_0^2(\beta)$$

$$P_{\text{Total}} = \frac{A_c^2}{2} \left[\underbrace{J_0^2(\beta) + 2J_1^2(\beta) + \dots}_{=1} \right]$$

$$J_{-n}(\beta) = J_n(\beta) \quad \text{even}$$

$$J_{-n}(\beta) = -J_n(\beta) \quad \text{n odd.}$$

single tone

$$FM(t) = A_c \cos(\omega_c t + \underbrace{\frac{k_f A_m}{\omega_m} \sin(\omega_m t)}_{\phi(t) \text{ (phase deviation)}}$$

$\theta_c(t)$

freq deviation $\frac{d\phi(t)}{dt} = k_f m(t)$

So $\Delta\omega = \text{max freq. deviation} = k_f A_m$

$$\beta = \frac{k_f A_m}{\omega_m} = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$$

$k_f = 2\pi f_d$ called deviation constant

$$\beta = \frac{f_d A_m}{f_m}$$

HW7

(a) $m_p = 16V$.
 $x = -8.7V$.

(b) $N = 8$ bits. Find offset binary code.

$$\Delta = \frac{16}{2^7} = \boxed{0.125}$$

$$\text{quantization level} = \left(\frac{\text{Abs}(x)}{\Delta} \right) = 69.6 \rightarrow \boxed{70}$$

Since x is negative, then $\text{code} = 2^7 - 70 = 128 - 70 = \boxed{58}$

in binary, this is $\boxed{00111010}$

(c) Sign/magnitude..

since $x < 0$ then $\text{code} = 2^7 + 70 = 128 + 70 = 198$

which in binary is $\boxed{11000110}$

(d) 2's complement.

since $x < -\frac{\Delta}{2}$ then $\text{code} = 2^8 - 70 = 256 - 70 = 186$

which in binary is $\boxed{10111010}$

(e) 1's complement.

since $x < 0$ then

$$\text{code} = (2^8 - 1) - 70 = 255 - 70 = 185$$

which in binary is $\boxed{10111001}$

extra: to illustrate this more, this is the calculations assuming $x = +8.7V$.

offset binary

$$\Delta = \frac{16}{2^7} = 0.125$$

$$\text{Level} = \text{round}\left(\frac{8.7}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$$

$$\text{Since } x > 0 \text{ then code } (70)_2 = 0100\ 0110$$

sign magnitude

$$\text{Since } x > 0 \text{ then code } (70)_2 = 0100\ 0110$$

2's complement

$$\text{since } x > 0 \text{ then code} = (70)_2 = 0100\ 0110$$

1's complement

$$\text{since } x > 0, \text{ then code} = (70)_2 = 0100\ 0110.$$

$$\text{SNR} = \frac{m^2 \epsilon + 1}{\text{noise pow}} = \frac{E(m^2)}{\frac{m_p^2}{(3)(2^{2N})}}$$

USE range f. noise

$$\int_{-m_p}^{m_p} m^2 f(x) dx$$

Handout 7/6/2010. ECE 405

n	11.0000	12.0000	13.0000	14.0000	15.0000
0	-0.1712	0.0477	0.2069	0.1711	-0.0142
1.0000	-0.1768	-0.2234	-0.0703	0.1334	0.2051
2.0000	0.1390	-0.0849	-0.2177	-0.1520	0.0416
3.0000	0.2273	0.1951	0.0033	-0.1768	-0.1940
4.0000	-0.0150	0.1825	0.2193	0.0762	-0.1192
5.0000	-0.2383	-0.0735	0.1316	0.2204	0.1305
6.0000	-0.2016	-0.2437	-0.1180	0.0812	0.2061
7.0000	0.0184	-0.1703	-0.2406	-0.1508	0.0345
8.0000	0.2250	0.0451	-0.1410	-0.2320	-0.1740
9.0000	0.3089	0.2304	0.0670	-0.1143	-0.2200
10.0000	0.2804	0.3005	0.2338	0.0850	-0.0901
11.0000	0.2010	0.2704	0.2927	0.2357	0.1000
12.0000	0.1216	0.1953	0.2615	0.2855	0.2367
13.0000	0.0643	0.1201	0.1901	0.2536	0.2787
14.0000	0.0304	0.0650	0.1188	0.1855	0.2464
15.0000	0.0130	0.0316	0.0656	0.1174	0.1813
16.0000	0.0051	0.0140	0.0327	0.0661	0.1162
17.0000	0.0019	0.0057	0.0149	0.0337	0.0665
18.0000	0.0006	0.0022	0.0063	0.0158	0.0346
19.0000	0.0002	0.0008	0.0025	0.0068	0.0166
20.0000	0.0001	0.0003	0.0009	0.0028	0.0074
21.0000	0.0000	0.0001	0.0003	0.0010	0.0031
22.0000	0.0000	0.0000	0.0001	0.0004	0.0012
23.0000	0.0000	0.0000	0.0000	0.0001	0.0004
24.0000	0.0000	0.0000	0.0000	0.0000	0.0002
25.0000	0.0000	0.0000	0.0000	0.0000	0.0001

n	16.0000	17.0000	18.0000	19.0000	20.0000
0	-0.1749	-0.1699	-0.0134	0.1466	0.1670
1.0000	0.0904	-0.0977	-0.1880	-0.1057	0.0668
2.0000	0.1862	0.1584	-0.0075	-0.1578	-0.1603
3.0000	-0.0438	0.1349	0.1863	0.0725	-0.0989
4.0000	-0.2026	-0.1107	0.0696	0.1806	0.1307
5.0000	-0.0575	-0.1870	-0.1554	0.0036	0.1512
6.0000	0.1667	0.0007	-0.1560	-0.1788	-0.0551
7.0000	0.1825	0.1875	0.0514	-0.1165	-0.1842
8.0000	-0.0070	0.1537	0.1959	0.0929	-0.0739
9.0000	-0.1895	-0.0429	0.1228	0.1947	0.1251
10.0000	-0.2062	-0.1991	-0.0732	0.0916	0.1865
11.0000	-0.0682	-0.1914	-0.2041	-0.0984	0.0614
12.0000	0.1124	-0.0486	-0.1762	-0.2055	-0.1190
13.0000	0.2368	0.1228	-0.0309	-0.1612	-0.2041
14.0000	0.2724	0.2364	0.1316	-0.0151	-0.1464
15.0000	0.2399	0.2666	0.2356	0.1389	-0.0008
16.0000	0.1775	0.2340	0.2611	0.2345	0.1452
17.0000	0.1150	0.1739	0.2286	0.2559	0.2331
18.0000	0.0668	0.1138	0.1706	0.2235	0.2511
19.0000	0.0354	0.0671	0.1127	0.1676	0.2189
20.0000	0.0173	0.0362	0.0673	0.1116	0.1647
21.0000	0.0079	0.0180	0.0369	0.0675	0.1106
22.0000	0.0034	0.0084	0.0187	0.0375	0.0676
23.0000	0.0013	0.0037	0.0089	0.0193	0.0380
24.0000	0.0005	0.0015	0.0039	0.0093	0.0199
25.0000	0.0002	0.0006	0.0017	0.0042	0.0098
26.0000	0.0001	0.0002	0.0007	0.0018	0.0045
27.0000	0.0000	0.0001	0.0003	0.0007	0.0020
28.0000	0.0000	0.0000	0.0001	0.0003	0.0008
29.0000	0.0000	0.0000	0.0000	0.0001	0.0003
30.0000	0.0000	0.0000	0.0000	0.0000	0.0001

Bessel Function Table

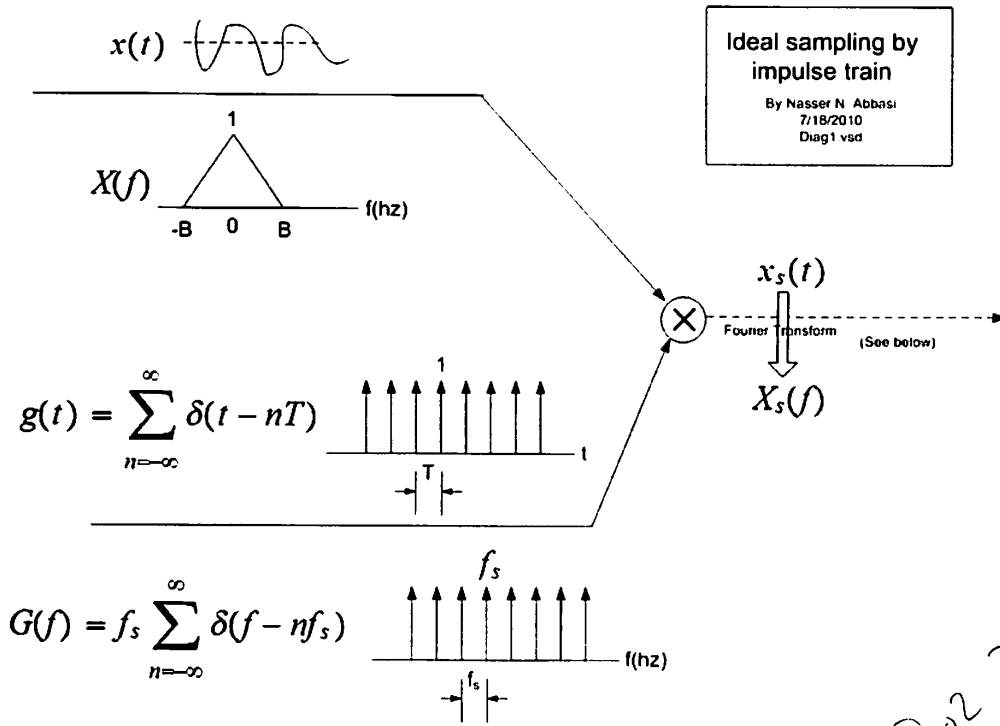
Dr. James S. Kang, Professor, Cal Poly Pomona

n	β					
	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000
0	0.9975	0.9900	0.9776	0.9604	0.9385	0.9120
1.0000	0.0499	0.0995	0.1483	0.1960	0.2423	0.2867
2.0000	0.0012	0.0050	0.0112	0.0197	0.0306	0.0437
3.0000	0.0000	0.0002	0.0006	0.0013	0.0026	0.0044
4.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003
5.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

n	β					
	0.7000	0.8000	0.9000	1.0000	1.1000	1.2000
0	0.8812	0.8463	0.8075	0.7652	0.7196	0.6711
1.0000	0.3290	0.3688	0.4059	0.4401	0.4709	0.4983
2.0000	0.0588	0.0758	0.0946	0.1149	0.1366	0.1593
3.0000	0.0069	0.0102	0.0144	0.0196	0.0257	0.0329
4.0000	0.0006	0.0010	0.0016	0.0025	0.0036	0.0050
5.0000	0.0000	0.0001	0.0001	0.0002	0.0004	0.0006
6.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

n	β					
	1.0000	2.0000	3.0000	4.0000	5.0000	
0	0.7652	0.2239	-0.2601	-0.3971	-0.1776	
1.0000	0.4401	0.5767	0.3391	-0.0660	-0.3276	
2.0000	0.1149	0.3528	0.4861	0.3641	0.0466	
3.0000	0.0196	0.1289	0.3091	0.4302	0.3648	
4.0000	0.0025	0.0340	0.1320	0.2811	0.3912	
5.0000	0.0002	0.0070	0.0430	0.1321	0.2611	
6.0000	0.0000	0.0012	0.0114	0.0491	0.1310	
7.0000	0.0000	0.0002	0.0025	0.0152	0.0534	
8.0000	0.0000	0.0000	0.0005	0.0040	0.0184	
9.0000	0.0000	0.0000	0.0001	0.0009	0.0055	
10.0000	0.0000	0.0000	0.0000	0.0002	0.0015	
11.0000	0.0000	0.0000	0.0000	0.0000	0.0004	
12.0000	0.0000	0.0000	0.0000	0.0000	0.0001	

n	β					
	6.0000	7.0000	8.0000	9.0000	10.0000	
0	0.1506	0.3001	0.1717	-0.0903	-0.2459	
1.0000	-0.2767	-0.0047	0.2346	0.2453	0.0435	
2.0000	-0.2429	-0.3014	-0.1130	0.1448	0.2546	
3.0000	0.1148	-0.1676	-0.2911	-0.1809	0.0584	
4.0000	0.3576	0.1578	-0.1054	-0.2655	-0.2196	
5.0000	0.3621	0.3479	0.1858	-0.0550	-0.2341	
6.0000	0.2458	0.3392	0.3376	0.2043	-0.0145	
7.0000	0.1296	0.2336	0.3206	0.3275	0.2167	
8.0000	0.0565	0.1280	0.2235	0.3051	0.3179	
9.0000	0.0212	0.0589	0.1263	0.2149	0.2919	
10.0000	0.0070	0.0235	0.0608	0.1247	0.2075	
11.0000	0.0020	0.0083	0.0256	0.0622	0.1231	
12.0000	0.0005	0.0027	0.0096	0.0274	0.0634	
13.0000	0.0001	0.0008	0.0033	0.0108	0.0290	
14.0000	0.0000	0.0002	0.0010	0.0039	0.0120	
15.0000	0.0000	0.0001	0.0003	0.0013	0.0045	
16.0000	0.0000	0.0000	0.0001	0.0004	0.0016	
17.0000	0.0000	0.0000	0.0000	0.0001	0.0005	
18.0000	0.0000	0.0000	0.0000	0.0000	0.0002	



Q023311

Alternative way to write the sampled signal $x_s(t)$

$$x_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_{n=-\infty}^{\infty} x(t) \delta(t - nT) = \sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT)$$

Fourier series approx

$$x_s(t) \approx x(t) \left(f_s \sum_{n=-\infty}^{\infty} e^{j \frac{2\pi}{T} n t} \right)$$

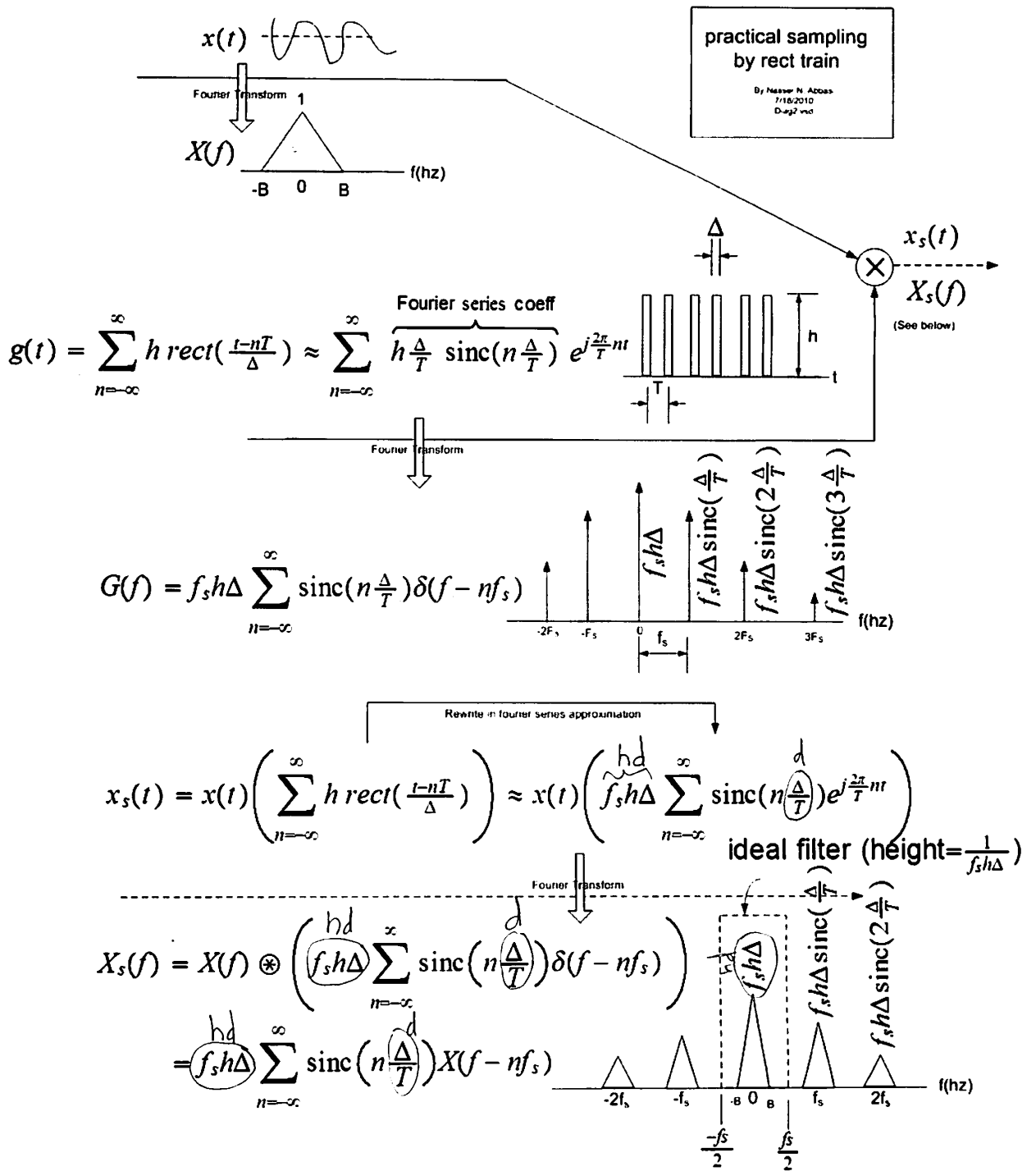
Fourier series approximation of the pulse train

Fourier Transform

$$X_s(f) = X(f) \otimes \left(f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \right) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \text{ ideal filter (height} = \frac{1}{f_s} \text{)}$$

$G(f)$

2 Practical sampling



Welcome to Vibration Data Laplace Transform Table

Laplace transforms are used to solve differential equations.
As an example, Laplace transforms are used to determine the response of a harmonic oscillator to an input signal.

By Tom Irvine Email: tomirvine@aol.com

Operation Transforms

N	F(s)	f(t), t > 0
1.1	$Y(s) = \int_0^{\infty} \exp(-st)y(t) dt$	definition of a Laplace transform $y(t)$
1.2	$Y(s)$	inversion formula $y(t) = \frac{1}{j2\pi} \int_{c-j\infty}^{c+j\infty} \exp(st)Y(s) ds$
1.3	$sY(s) - y(0)$	first derivative $y'(t)$
1.4	$s^2 Y(s) - sy(0) - y'(0)$	second derivative $y''(t)$
1.5	$s^n Y(s) - s^{n-1} [y(0)]$ $-s^{n-2} [y'(0)] - \dots - s [y^{(n-2)}(0)]$ $- [y^{(n-1)}(0)]$	nth derivative $y^{(n)}(t)$
1.6	$(1/s) F(s)$	integration $\int_0^t Y(\tau) d\tau$
1.7	$F(s)G(s)$	convolution integral $\int_0^t f(t-\tau)g(\tau) d\tau$
1.8	$\frac{1}{\alpha} F\left(\frac{s}{\alpha}\right)$	$f(\alpha t)$
1.9	$F(s - a)$	shifting in the s-plane

Table of Laplace Transforms

<http://www.vibrationdata.com/Laplace.htm>

		$\exp(-at) f(t)$
1.10	$\frac{1}{1 - \exp(-sT)} \int_0^T \exp(-st) f(t) dt$	$f(t)$ has period T , such that $f(t + T) = f(t)$
1.11	$\frac{1}{1 + \exp(-sT)} \int_0^T \exp(-st) g(t) dt$	$g(t)$ has period T , such that $g(t + T) = -g(t)$

Function Transforms

N	F(s)	f(t), t > 0
2.1	1	$\delta(t)$ unit impulse at t = 0
2.2	s	$\frac{d}{dt} \delta(t)$ double impulse at t = 0
2.3	$\exp(-\alpha s), \alpha \geq 0$	$\delta(t-a)$
2.4a	1/s	unit step u(t)
2.4b	$\frac{1}{s} [\exp(-as) - \exp(-bs)]$	0 t < a 1 a < t < b 0 t > b
2.5	$\frac{1}{s} \exp(-\alpha s)$	u(t-a)
2.6	$\frac{1}{s^2}$	t
2.7a	$\frac{1}{s^n}, n = 1, 2, 3, \dots$	$\frac{t^{n-1}}{(n-1)!}$
2.7b	$\frac{n!}{s^{n+1}}, n=1, 2, 3, \dots$	t^n
2.8	$\frac{1}{s^k}, k$ is any real number > 0	

Table of Laplace Transforms

<http://www.vibrationdata.com/Laplace.htm>

		$\frac{t^{k-1}}{\Gamma(k)}$ the Gamma function is given in Appendix A
2.9	$\frac{1}{s + \alpha}$	$\exp(-at)$
2.10	$\frac{1}{(s + \alpha)^2}$	$t \exp(-at)$

2.11	$\frac{1}{(s + \alpha)^n}, n = 1, 2, 3, \dots$	$\left[\frac{t^{n-1}}{(n-1)!} \right] \exp(-\alpha t)$
2.12	$\frac{\alpha}{s(s + \alpha)}$	$1 - \exp(-at)$
2.13	$\frac{1}{(s + \alpha)(s + \beta)}, \beta \neq \alpha$	$\frac{1}{(\beta - \alpha)} [\exp(-\alpha t) - \exp(-\beta t)]$
2.14	$\frac{1}{s(s + \alpha)(s + \beta)}, \beta \neq \alpha$	$\frac{1}{\alpha\beta} + \frac{\exp(-\alpha t)}{\alpha(\alpha - \beta)} + \frac{\exp(-\beta t)}{\beta(\beta - \alpha)}$
2.15	$\frac{s}{(s + \alpha)(s + \beta)}, \beta \neq \alpha$	$\frac{1}{(\alpha - \beta)} [\alpha \exp(-\alpha t) - \beta \exp(-\beta t)]$
2.16a	$\frac{\alpha}{s^2 + \alpha^2}$	$\sin(at)$
2.16b	$\frac{[\sin(\phi)]s + \alpha[\cos(\phi)]}{s^2 + \alpha^2}$	$\sin(at + f)$
2.17	$\frac{s}{s^2 + \alpha^2}$	$\cos(at)$
2.18	$\frac{s^2 - \alpha^2}{[s^2 + \alpha^2]^2}$	$t \cos(at)$

Table of Laplace Transforms

<http://www.vibrationdata.com/Laplace.htm>

2.19	$\frac{1}{s(s^2 + \alpha^2)}$	$\frac{1}{\alpha^2} [1 - \cos(\alpha t)]$
2.20	$\frac{1}{(s^2 + \alpha^2)^2}$	$\frac{1}{2\alpha^3} [\sin(\alpha t) - \alpha t \cos(\alpha t)]$
2.21	$\frac{s}{(s^2 + \alpha^2)^2}$	$\frac{1}{2\alpha} [t \sin(\alpha t)]$
2.22	$\frac{s^2}{(s^2 + \alpha^2)^2}$	$\frac{1}{2\alpha} [\sin(\alpha t) + \alpha t \cos(\alpha t)]$
2.23	$\frac{1}{(s^2 + \omega^2)(s^2 + \alpha^2)}, \alpha \neq \omega$	$\left\{ \frac{1}{\omega^2 - \alpha^2} \right\} \left\{ \frac{1}{\alpha} \sin(\alpha t) - \frac{1}{\omega} \sin(\omega t) \right\}$
2.24	$\frac{\alpha}{s^2(s + \alpha)}$	$t - \frac{1}{\alpha} [1 - \exp(-\alpha t)]$
2.25	$\frac{\beta}{(s + \alpha)^2 + \beta^2}$	$\exp(-\alpha t) \sin(\beta t)$
2.26	$\frac{s + \alpha}{(s + \alpha)^2 + \beta^2}$	$\exp(-\alpha t) \cos(\beta t)$
2.27	$\frac{s + \lambda}{(s + \alpha)^2 + \beta^2}$	$\exp(-\alpha t) \left\{ \cos(\beta t) + \left[\frac{\lambda - \alpha}{\beta} \right] \sin(\beta t) \right\}$
2.28	$\frac{s + \alpha}{s^2 + \beta^2}$	$\frac{\sqrt{\alpha^2 + \beta^2}}{\beta} \sin(\beta t + \phi), \phi = \arctan\left(\frac{\beta}{\alpha}\right)$
2.29	$\frac{1}{s^2 - \alpha^2}$	$\frac{1}{\alpha} \sinh(\alpha t)$
2.30	$\frac{s}{s^2 - \alpha^2}$	$\cosh(\alpha t)$
2.31	$\arctan\left(\frac{\alpha}{s}\right)$	$\frac{1}{t} \sin(\alpha t)$
2.32	$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$

Table of Laplace Transforms

<http://www.vibrationdata.com/Laplace.htm>

2.33	$\frac{1}{\sqrt{s + \alpha}}$	$\frac{1}{\sqrt{\pi t}} \exp[-\alpha t]$
2.34	$\frac{1}{\sqrt{s^3}}$	$2 \sqrt{\frac{t}{\pi}}$
2.35	$\frac{1}{\sqrt{s^2 + \alpha^2}}$	$J_0(\alpha t)$ Bessel function given in Appendix A
2.36	$\frac{1}{(s^2 + \alpha^2)^{3/2}}$	$\left(\frac{t}{\alpha}\right) J_1(\alpha t)$
2.37	$\frac{1}{\sqrt{s^2 - \alpha^2}}$	$I_0(\alpha t)$ Modified Bessel function given in Appendix A
2.38	$\frac{1}{(s^2 - \alpha^2)^{3/2}}$	$\left(\frac{t}{\alpha}\right) I_1(\alpha t)$
2.39	$\sqrt{s - \alpha} - \sqrt{s - \beta}$	$\frac{1}{2t\sqrt{\pi t}} [\exp(\beta t) - \exp(\alpha t)]$

Examples of the Laplace Transform as a Solution for Mechanical Shock and Vibration Problems:

Free Vibration of a Single-Degree-of-Freedom System: [free.pdf](#)

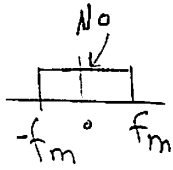
Response of a Single-degree-of-freedom System Subjected to a Unit Step Displacement: [unit_step.pdf](#)

Response of a Single-degree-of-freedom System Subjected to a Classical Pulse Base Excitation: [sbase.pdf](#)

Partial Fractions in Shock and Vibration Analysis: [partial.pdf](#)

References

1. Jan Tuma, [Engineering Mathematics Handbook](#), McGraw-Hill, New York, 1979.
2. F. Oberhettinger and L. Badii, [Table of Laplace Transforms](#), Springer-Verlag, N.Y., 1972.

FM SNR

$$\Sigma_i = \frac{A_c^2}{2} \text{ watt}$$

$$P_i = N_0 f_m$$

FM demod

$$S_o = K_f^2 \frac{A_m^2}{2}$$

$$P_o = \frac{8\pi^2 N_0 f_m^3}{3 A_c^2}$$

$$(SNR)_i = \frac{A_c^2}{2 N_0 f_m}$$

$$SNR_o = \frac{3}{4} \frac{A_c^2 \beta^2}{N_0 f_m}$$

$$\beta = \frac{K_f A_m}{2\pi f_m}$$

AM, $\mu=1$

$$\Sigma_i = \frac{A_c^2}{2}$$

$$P_i = N_0 f_m$$

AM demod

$$S_o = \frac{A_c^2}{2}$$

$$P_o = 2 N_0 f_m$$

$$(SNR)_{o,AM} = \frac{A_c^2}{4 N_0 f_m}$$

5.2 sheet 2

6 What is the relation between variance and power for a random signal $x(t)$?

Variance is the sum of the total average normalized power and the DC power.

$$\sigma_x^2 = \overbrace{E[x^2(t)]}^{\text{total Power}} + \overbrace{E[x(t)]^2}^{\text{DC power}}$$

For the a signal whose mean is zero,

$$\sigma_x^2 = \overbrace{E[x^2(t)]}^{\text{total Power}}$$

How to find average, power, PEP, effective value (or the RMS) of a periodic function?

Let $x(t)$ be a periodic function, of period T , then

$$\text{average of } x(t) = \langle x(t) \rangle = \frac{1}{T} \int_0^T x(t) dt$$

The average power is

$$p_{av} = \langle x^2(t) \rangle = \frac{1}{T} \int_0^T |x(t)|^2 dt$$

Effective value, or the RMS value is

$$x_{rms}(t) = \sqrt{\langle x^2(t) \rangle} = \sqrt{p_{av}} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt}$$

For example, for $x(t) = \cos(x)$, $\langle x(t) \rangle = 0$, $p_{av} = \frac{1}{2}$, $x_{rms}(t) = 0.707$

To find PEP (which is the peak envelope power), find the complex envelope $\tilde{x}(t)$, then find the average power of it. i.e.

$$PEP = \frac{1}{2} \tilde{x}_{\max}^2(t)$$

7 How to derive the Phase and Frequency modulation signals?

For any bandpass signal, we can write it as

$$x(t) = \text{Re}(\tilde{x}(t) e^{j\omega_c t})$$

Where $\tilde{x}(t)$ is the complex envelope of $x(t)$. For PM and FM, the baseband modulated signal, $\tilde{x}(t)$ has the form $A_c e^{j\theta(t)}$ Hence the above becomes

$$\begin{aligned} x(t) &= \text{Re}(A_c e^{j\theta(t)} e^{j\omega_c t}) \\ &= A_c (\cos \omega_c t \cos \theta(t) - \sin \omega_c t \sin \theta(t)) \end{aligned}$$

But $\cos(A + B) = \cos A \cos B - \sin A \sin B$, hence the above becomes

$$x(t) = \cos(\omega_c t + \theta(t)) \quad (1)$$

The above is the general form for PM and FM. Now, for PM, $\theta(t) = k_p m(t)$ and for FM, $\theta(t) = k_f \int_0^t m(t_1) dt_1$. Hence, substituting in (1) we obtain

$$x_{FM}(t) = \cos\left(\omega_c t + k_f \int_0^t m(t_1) dt_1\right)$$

and

$$x_{PM}(t) = \cos(\omega_c t + k_p m(t))$$

I) Amplitude Modulation short sheet 443 CSUF

a) AM wave $s_{AM}(t) = A_c [1 + K_a m(t)] \cos \omega_c t$

• modulation index $\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$, where A_{max} is the max. of envelope

b) DSB-SC $s(t) = A_c m(t) \cos \omega_c t$

c) SSB $s(t) = \frac{A_c}{2} m(t) \cos \omega_c t \mp \frac{A_c}{2} \hat{m}(t) \sin \omega_c t$

where (-) for USB and (+) for LSB

$$\hat{m}(t) = H.T[m(t)] = m(t) \otimes \frac{1}{j\omega} \quad \text{or}$$

$$M(\hat{m}(f)) = -j \text{sgn}(f) M(f)$$

II) PM wave ;

$$s(t) = A_c \cos(\omega_c t + K_p m(t))$$

III) FM wave :

$$s(t) = A_c \cos \left[2\pi f_c t + 2\pi K_f \int_0^t m(x) dx \right] \quad (1)$$

• If $m(t)$ is a sine or cosine wave for example
if $m(t) = A_m \cos \omega_m t$ then eq (1) becomes single tone modulated signal;

$$s(t) = A_c \cos [2\pi f_c t + \beta \sin 2\pi \omega_m t], \quad \text{where}$$

$$\beta = \frac{\omega_f}{f_m} = \frac{K_f A_m}{f_m}, \quad \beta \text{ is modulation index}$$

• $\omega_f = K_f A_m$ is the freq. deviation

$$f_i(t) = f_c + K_f m(t) \quad \text{inst. freq.}$$

$$\phi_i(t) = 2\pi \int_0^t f_i(t) dt, \quad \text{or} \quad \dot{\phi}_i(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt}$$

$\phi_i(t)$ in the inst. phase.

IV) Narrow Band Noise $n(t)$: Note if $E\{m(t)\} = 0 \Rightarrow E\{n_1\} = E\{n_2\} = 0$

$$n(t) = n_1(t) \cos \omega_c t - n_2(t) \sin \omega_c t$$

$$S_{N_1}(f) = S_{N_2}(f) = [S_N(f-f_c) + S_N(f+f_c)] \text{rect}\left(\frac{f}{2B}\right)$$

where these are p.s.d of the narrowband noise and its in-phase and quadrature components.

$$\text{The envelope of } n(t) \text{ is } a(t) = \sqrt{n_1^2 + n_2^2}$$

Table A11.1 Summary of Properties of the Fourier Transform

Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) \Rightarrow aG_1(f) + bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \Rightarrow \frac{1}{ a } G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \Rightarrow G(f)$, then $G(t) \Rightarrow g(-f)$
4. Time shifting	$g(t - t_0) \Rightarrow G(f) \exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t) g(t) \Rightarrow G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t) dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f) df$
8. Differentiation in the time domain	$\frac{d}{dt} g(t) \Rightarrow j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^t g(\tau) d\tau \Rightarrow \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$
10. Conjugate functions	If $g(t) \Rightarrow G(f)$, then $g^*(t) \Rightarrow G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \Rightarrow \int_{-\infty}^{\infty} G_1(\lambda)G_2(f - \lambda) d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t - \tau) d\tau \Rightarrow G_1(f)G_2(f)$

instant. phase

$A_c \cos(\Theta_i(t)) \rightarrow$ phase deviation
 $\Theta_i(t) = \omega_c t + \Phi(t)$

$\frac{d\Theta_i}{dt} = \omega_i(t) = \omega_c + \frac{d\Phi}{dt}$
 instant. Frequency \rightarrow Frequency deviation

so inst. Phase = $\omega_c t +$ Phase deviation
 inst. Freq = $\omega_c +$ freq. deviation

Table A11.4 Trigonometric Identities

$\exp(\pm j\theta) = \cos\theta \pm j \sin\theta$
$\cos\theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$
$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$
$\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta - \sin^2\theta = \cos(2\theta)$
$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)]$
$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$
$2 \sin\theta \cos\theta = \sin(2\theta)$
$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$
$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$
$\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin\alpha \cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

$\cos\alpha \sin\beta = -\frac{1}{2} \sin(\alpha - \beta) + \frac{1}{2} \sin(\alpha + \beta)$

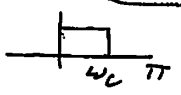
$\sin\left(t - \frac{\pi}{2}\right) = -\cos t$
 $\cos\left(t - \frac{\pi}{2}\right) = \sin t$

$W_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & 1 & -j \end{bmatrix}; W_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -\frac{1}{2} - \frac{j\sqrt{3}}{2} & -\frac{1}{2} + \frac{j\sqrt{3}}{2} & 1 \\ 1 & -\frac{1}{2} + \frac{j\sqrt{3}}{2} & -\frac{1}{2} - \frac{j\sqrt{3}}{2} & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}; W_3 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

properties of DFT $X(k)$ periodic, period = N , Linear: $a_1 x_1 + a_2 x_2 \leftrightarrow a_1 X_1 + a_2 X_2$

$x_1(n) = \sum_{l=-\infty}^{\infty} x_2[n-lN]$
 $X_1 X_2 = \text{circular convolution of } x_1, x_2$

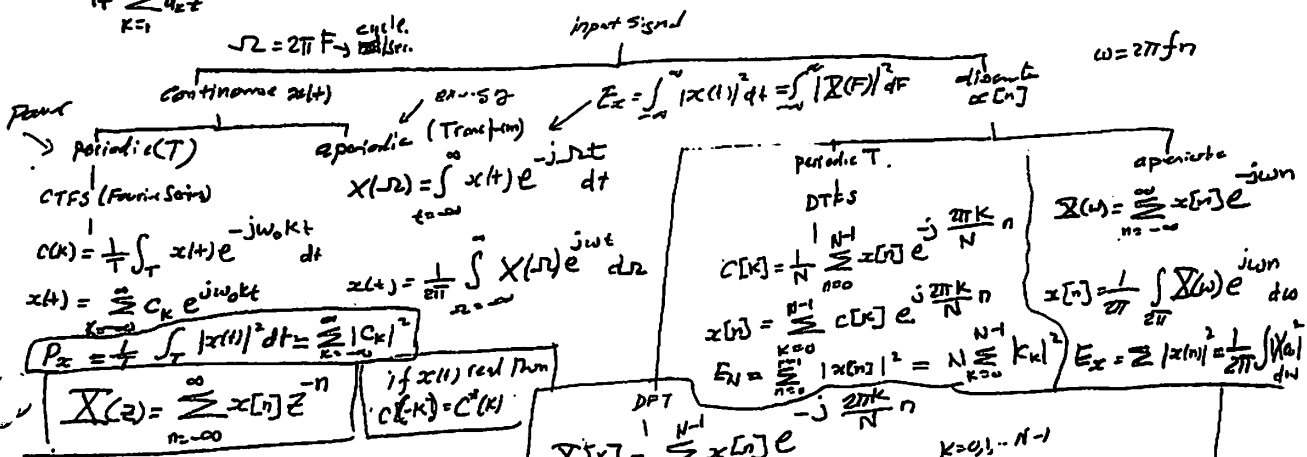
$W_N^k = e^{-j \frac{2\pi k n}{N}}$

ideal low pass  $\Rightarrow h(n) = \begin{cases} \frac{\omega_c}{\pi} & n=0 \\ \frac{\omega_c}{\pi} \frac{\sin \omega_c n}{\omega_c n} & n \neq 0 \end{cases}$

LTI is causal if specified by difference equation $y(n) = -\sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k)$

$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}}$

To do circular convolution, do same as linear convolution, just shift right!



properties of Z transform

$x(n) \leftrightarrow \frac{1}{1-z^{-1}}$ ROC $|z| > 1$
 $x(-n) \leftrightarrow \frac{1}{1-z}$ ROC $|z| < 1$
 $ax(n) \leftrightarrow -z \frac{dX(z)}{dz}$
 $a^n x(n) \leftrightarrow \frac{1}{1-az^{-1}}$
 $a^n x(n) \leftrightarrow X(az)$

$x^*(n) \leftrightarrow X^*(z^*)$ if $y(n) = -\sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k)$
 Then $H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}}$

Correlation $r_{x_1 x_2}(l) = \sum_{n=0}^{N-l} x_1(n) x_2(n-l) \leftrightarrow X_1(z) X_2(z^{-1})$

$r_{x_1 x_2}(l) = x_1(l) * x_2(-l)$ (no correlation is convolution but without flipping)

$S(n) \leftrightarrow \frac{1}{1-z^{-1}}$ All z
 $u(n) \leftrightarrow \frac{1}{1-z^{-1}}$ $|z| > 1$
 $a^n u(n) \leftrightarrow \frac{1}{1-az^{-1}}$ $|z| > |a|$

$\sum_{n=0}^{\infty} (a)^n = \frac{1}{1-a}$ $|a| < 1$
 $\sum_{n=0}^N (a)^n = \frac{1-a^{N+1}}{1-a}$ or $\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$

Symmetry relationships for $X(\omega)$

$x^*(n) \leftrightarrow X^*(-\omega)$
 $x^*(-n) \leftrightarrow X^*(\omega)$
 IF $x[n]$ is real then:
 $X(\omega) = X^*(-\omega)$
 $|X(\omega)| = |X^*(\omega)|$

convolution $y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$

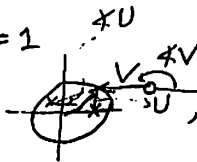
IIR $\Rightarrow y(n) = -\sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k)$
 $H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}}$

FIR = all zero $\Rightarrow y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$
 $H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}} = b_0 \frac{\prod_{k=1}^M (1 - z_k z^{-1})}{\prod_{k=1}^N (1 - p_k z^{-1})} = \frac{(z - z_1)(z - z_2) \dots}{(z - p_1)(z - p_2) \dots}$$

b_0 selected such that $|H(\omega)| = 1$

$$|H(\omega)| = |b_0| \frac{|V_1(\omega)V_2(\omega) \dots V_M(\omega)|}{|U_1(\omega)U_2(\omega) \dots U_N(\omega)|}$$



$$\angle H(\omega) = \angle b_0 + \omega(N-M) + \angle V_1 + \angle V_2 + \dots - [\angle U_1 + \angle U_2 + \dots]$$

$$|H(\omega)|^2 = H(\omega) H^*(\omega) = H(\omega) H(-\omega) \xrightarrow{\text{Real}} = H(z) H(z^{-1}) \Big|_{z=e^{j\omega}}$$

Property of DFT $X(N-k) = X^*(k) = X(-k)$ for real $x[n]$, $|X(N-k)| = |X(k)|$, $\angle X(N-k) = \angle X(k)$

if $x[n]$ real & even, then $X(k) = \sum_{n=0}^{N-1} x[n] \cos \frac{2\pi k n}{N}$, $x[n] = \frac{1}{N} \sum X(k) \cos \frac{2\pi k n}{N}$

if $x[n]$ is real & odd, then $X(k) = \sum_{n=0}^{N-1} (-j \sin \frac{2\pi k n}{N})$, $x[n] = \frac{j}{N} \sum X(k) \sin \frac{2\pi k n}{N}$

$$x[N-n] = X^*(k), \quad x[n] \leftrightarrow X(k) e^{-j \frac{2\pi k n}{N}} \Leftrightarrow X((k-0))$$

$$\tilde{x}_y(l) = x(l) \otimes y^*(-l) \leftarrow \text{circular correlation} \quad x_1, x_2 \leftrightarrow \frac{1}{N} X_1 \otimes X_2$$

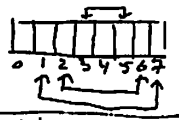
Given $x[n]$ of length L , $h[n]$ of length M , how to know length of DFT? $N = L + M - 1$

if x has $L=4$, length $h[n]=3$, then we need $N=6$

note we can X_1, X_2 to find X_3 , then IDFT to find response of system. like Linear convolution, as long as we pad sequences.

$$\int_0^{\infty} e^{-t(1+j2\pi F)} dt = \frac{1}{1+j2\pi F}$$

if we are given some points of DFT, we can find rest using properties $X(N-k) = X^*(k) = X(k)$



$$N=8, \quad X(6) = X(8-2) = X^*(2)$$

$$\sum_{k=0}^{N-1} e^{-j \frac{2\pi k n}{N}} = N \delta(k-n)$$

$$\sum_{k=0}^{N-1} e^{-j \frac{2\pi k n}{N}} = N \delta(k+n)$$

$$\cos \alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$\sin \alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

$$\text{if } x(n) = \cos \frac{2\pi n}{N} \rightarrow \text{DFT} = \frac{N}{2} [\delta(k-1) + \delta(k+1)]$$

$$\text{if } x(n) = \sin \frac{2\pi n}{N} \rightarrow \text{DFT} = \frac{N}{2j} [\delta(k-1) - \delta(k+1)]$$

$$\text{if } X(k) = \frac{N^2}{4j} [\delta(k-1) + \delta(k+1)] \rightarrow x(n) = \frac{N}{2} \sin \left(\frac{2\pi n}{N} \right)$$

DFT if given sequence $x(n)$ and we want to find its Energy do $E = \sum_{n=0}^{N-1} x(n) x^*(n)$. Example: $x(n) = \cos \frac{2\pi k n}{N} \Rightarrow x(n) x^*(n) = \frac{1}{4} (2 + e^{j \frac{4\pi k n}{N}} + e^{-j \frac{4\pi k n}{N}})$

$$E = \sum_{n=0}^{N-1} x(n) x^*(n) = \frac{1}{4} 2N = \frac{N}{2}$$

$x_1 = [1, 1, 1, 1, 1, 1, 1, 1]$ $x_2 = x_1((n-5))$

$x_2 = [1, 1, 1, 1, 1, 1, 1, 1]$ $\Rightarrow X_2 = X_1 e^{-j \frac{2\pi 5k}{N}}$

Least squares method for discrete IIR & FIR

Properties of the Fourier Transform

Property	$f(t)$	$F(\omega)$
Linearity (Superposition)	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Time Shifting	$f(t - t_d)$	$e^{-j\omega t_d} F(\omega)$
Time Scaling	$f(ct)$	$\frac{1}{ c } F\left(\frac{\omega}{c}\right)$
Symmetry (Duality)	$F(t)$	$2\pi f(-\omega)$
Time Reversal	$f(-t)$	$F(-\omega)$
Frequency Scaling	$f(t)e^{j\omega_c t}$	$F(\omega - \omega_c)$
Modulation	$f(t)\cos(\omega_c t)$	$\frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c)$
Time Differentiation	$\frac{df(t)}{dt}$	$j\omega F(\omega)$
Frequency Differentiation	$tf(t)$	$j\frac{dF(\omega)}{d\omega}$
Conjugate	$f^*(t)$	$F^*(-\omega)$
Integration	$\int_{-\infty}^{\infty} f(\lambda)d\lambda$	$\frac{1}{j\omega} F(\omega) + \pi F(0)\delta(\omega)$
Convolution	$\int_{-\infty}^{\infty} h(\lambda)x(t - \lambda)d\lambda$	$H(\omega)X(\omega)$
Multiplication	$f_1(t)f_2(t)$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(\nu)F_2(\omega - \nu)d\nu$
Parseval's Theorem	$\int_{-\infty}^{\infty} f(t) ^2 dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) ^2 d\omega$

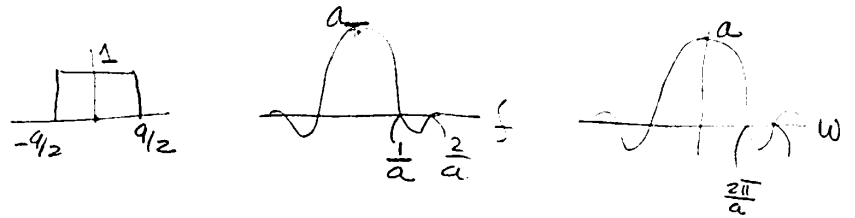
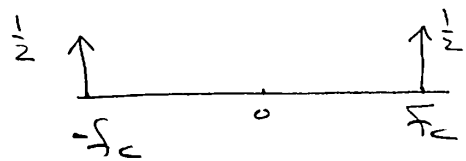
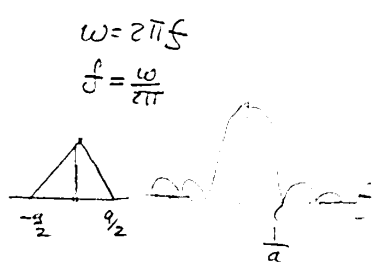
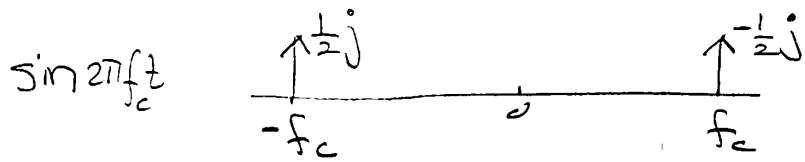


Table 16.2 Fourier Transform Pairs ($a > 0$)

$f(t)$	$F(\omega)$	$F(f)$
$\Pi\left(\frac{t}{a}\right) = \text{rect}\left(\frac{t}{a}\right)$	$a \text{sinc}\left(\frac{\omega a}{2\pi}\right)$	$a \text{sinc}(fa)$
$\Lambda\left(\frac{t}{a}\right) = \text{tri}\left(\frac{t}{a}\right)$	$a \text{sinc}^2\left(\frac{\omega a}{2\pi}\right)$	$a \text{sinc}^2(fa)$
$e^{-at}u(t)$	$\frac{1}{j\omega + a}$	$\frac{1}{j2\pi f + a}$
$e^{at}u(-t)$	$\frac{1}{-j\omega + a}$	$\frac{1}{-j2\pi f + a}$
$e^{-a t }$	$\frac{2a}{\omega^2 + a^2}$	$\frac{2a}{4\pi^2 f^2 + a^2}$
$e^{-at}u(t) - e^{at}u(-t)$	$\frac{-2j\omega}{\omega^2 + a^2}$	$\frac{-j4\pi f}{4\pi^2 f^2 + a^2}$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$tu(t)$	$\frac{\pi}{j\omega}\delta(\omega) + \frac{1}{(j\omega)^2}$	$\frac{1}{j4\pi f}\delta(f) + \frac{1}{(j2\pi f)^2}$
$te^{-at}u(t)$	$\frac{1}{(j\omega + a)^2}$	$\frac{1}{(j2\pi f + a)^2}$
$\cos(\omega_c t) = \cos(2\pi f_c t)$	$\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$





$$\sin(\omega_c t) = \sin(2\pi f_c t) \quad -j\pi[\delta(\omega - \omega_c) - \delta(\omega + \omega_c)] \quad \frac{-j}{2}[\delta(f - f_c) - \delta(f + f_c)]$$

$$e^{-a}u(t)\cos(\omega_c t) \quad \frac{j\omega + a}{(j\omega + a)^2 + \omega_c^2} \quad \frac{j2\pi f + a}{(j2\pi f + a)^2 + (2\pi f_c)^2}$$

$$e^{-a}u(t)\sin(\omega_c t) \quad \frac{\omega_c}{(j\omega + a)^2 + \omega_c^2} \quad \frac{2\pi f_c}{(j2\pi f + a)^2 + (2\pi f_c)^2}$$

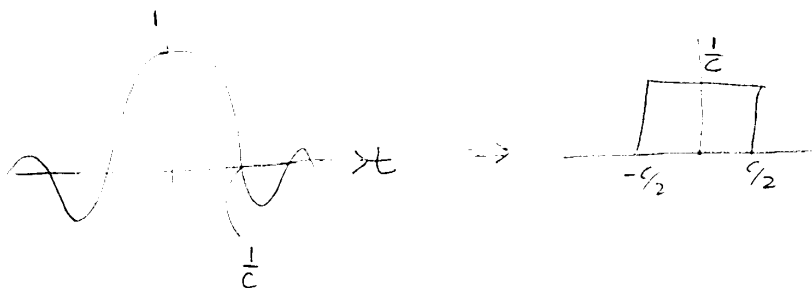
$$\text{sgn}(t) \quad \frac{2}{j\omega} \quad \frac{1}{j\pi f}$$

$$\text{sinc}(ct) \quad \frac{1}{c} \text{rect}\left(\frac{\omega}{2\pi c}\right) \quad \frac{1}{c} \text{rect}\left(\frac{f}{c}\right)$$

$$\text{sinc}^2(ct) \quad \frac{1}{c} \text{tri}\left(\frac{\omega}{2\pi c}\right) \quad \frac{1}{c} \text{tri}\left(\frac{f}{c}\right)$$

$$\cos\left(\frac{\pi}{a}\right)\text{rect}\left(\frac{t}{a}\right) \quad \frac{2a}{\pi} \frac{\cos\left(\frac{\omega a}{2}\right)}{1 - \left(\frac{\omega a}{\pi}\right)^2} \quad \frac{2a}{\pi} \frac{\cos(\pi f a)}{1 - (2af)^2}$$

$$\frac{1}{2}\left[1 + \cos\left(\frac{\pi}{a}\right)\right]\text{rect}\left(\frac{t}{2a}\right) \quad a \frac{\sin(\omega a)}{\omega a \left[1 - \left(\frac{\omega a}{\pi}\right)^2\right]} \quad a \frac{\sin(2\pi f a)}{2\pi f a [1 - (2af)^2]}$$

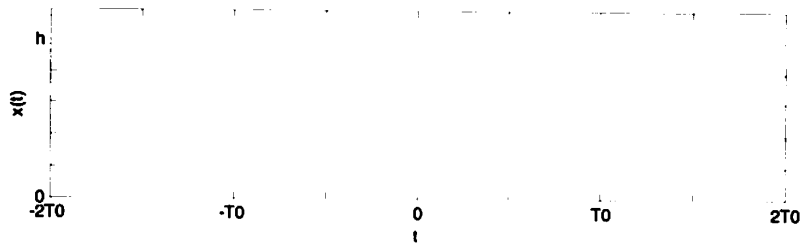


$$\text{Sinc}(\pi ct) = \text{Rect}\left(\frac{t}{c}\right)$$

$$t = \frac{c}{2}$$

FOURIER SERIES REPRESENTATION OF COMMON SIGNALS

Rectangular Pulse Train



τ = pulse width ($-\tau/2$ to $\tau/2$)

d = duty cycle = τ/T_0 .

ω_0 = fundamental frequency = $2\pi/T_0$

$\text{sinc}(x) = \sin(\pi x)/(\pi x)$

$$X_n = \frac{ha}{T_0} \text{sinc}(nd) = hd \text{sinc}(nd) = \begin{cases} hd, & n=0 \\ h \frac{\sin(n\pi d)}{n\pi}, & n \neq 0 \end{cases}$$

$$x(t) = hd + \sum_{n=1}^{\infty} 2hd \text{sinc}(nd) \cos(n\omega_0 t)$$

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$$c_0 = hd = \frac{h\tau}{T_0}, \quad c_n = |2hd \text{sinc}(nd)|, \quad \theta_n = \begin{cases} \pi, & 2hd \text{sinc}(nd) < 0 \\ 0, & \text{otherwise} \end{cases}$$

If $\tau = T_0/2$, $d = 1/2$, and the equations given above becomes

$$X_n = \frac{h}{2} \text{sinc}\left(\frac{n}{2}\right) = \begin{cases} \frac{h}{2}, & n=0 \\ h \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}, & n \neq 0 \end{cases}$$

$$x(t) = \frac{h}{2} + \sum_{n=1}^{\infty} h \text{sinc}\left(\frac{n}{2}\right) \cos(n\omega_0 t)$$

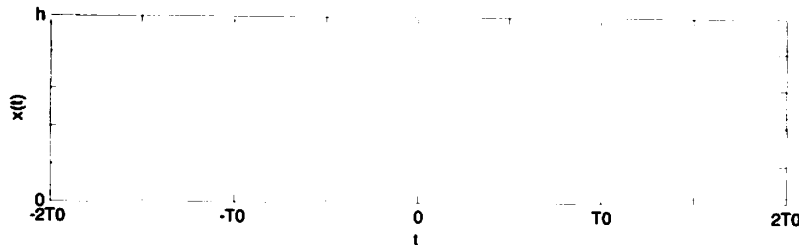
$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$$c_0 = \frac{h}{2}, \quad c_n = \left| h \operatorname{sinc}\left(\frac{n}{2}\right) \right|, \quad \theta_n = \begin{cases} \pi, & h \operatorname{sinc}\left(\frac{n}{2}\right) < 0 \\ 0, & \text{otherwise} \end{cases}$$

Let $y(t) = x(t - T_0/2)$. Then,

$$Y_n = X_n e^{-jn \frac{2\pi T_0}{2}} = X_n e^{-jn\pi} = X_n \cos(n\pi) = \begin{cases} \frac{h}{2}, & n = 0 \\ h \frac{(-1)^n \sin\left(\frac{n\pi}{2}\right)}{n\pi}, & n \neq 0 \end{cases}$$

Triangular Pulse Train



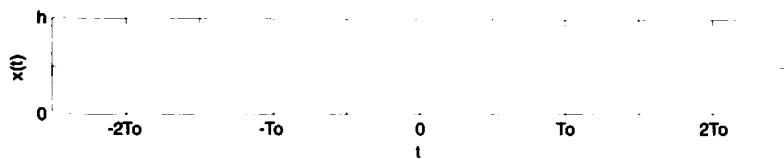
τ = half of the base of the triangle ($-\tau \leq t \leq \tau$)

d = duty cycle = τ/T_0 .

ω_0 = fundamental frequency = $2\pi/T_0$

$$X_n = hd \operatorname{sinc}^2(nd) = \frac{h\tau}{T_0} \operatorname{sinc}^2\left(\frac{n\omega_0\tau}{2\pi}\right)$$

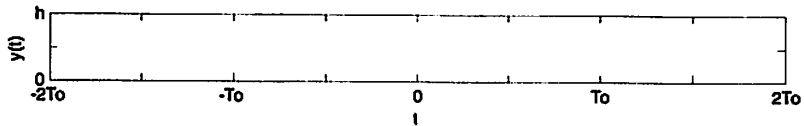
If $\tau = T_0/2$, then the pulse train looks like



and

$$X_n = \frac{h}{2} \operatorname{sinc}^2\left(\frac{n}{2}\right) = \begin{cases} \frac{h}{2}, & n=0 \\ 0, & n=\text{even} \\ \frac{2h}{n^2\pi^2}, & n=\text{odd} \end{cases}$$

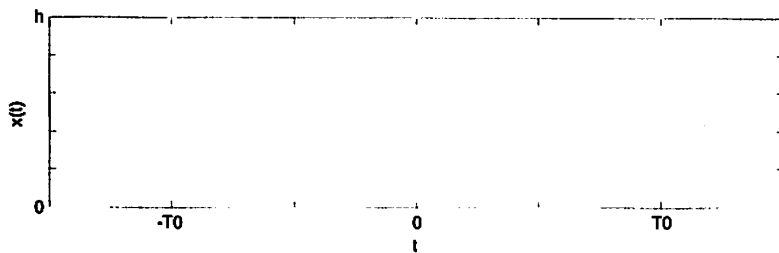
Let $y(t) = x(t - T_0/2)$.



Then,

$$Y_n = X_n e^{-jn\frac{T_0}{2}} = X_n e^{-jn\pi} = X_n \cos(n\pi) = \begin{cases} \frac{h}{2}, & n=0 \\ 0, & n=\text{even} \\ \frac{-2h}{n^2\pi^2}, & n=\text{odd} \end{cases}$$

Half-Wave Rectified Cosine

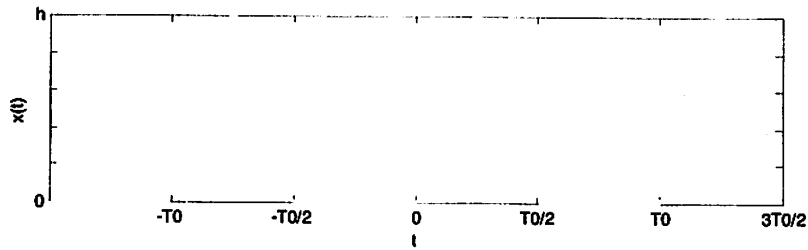


$$X_1 = \frac{h}{4}$$

$$X_{-1} = \frac{h}{4}$$

$$X_n = \frac{h}{\pi} \frac{\cos\left(\frac{n\pi}{2}\right)}{1-n^2}, \quad n \neq \pm 1$$

Half-Wave Rectified Sine



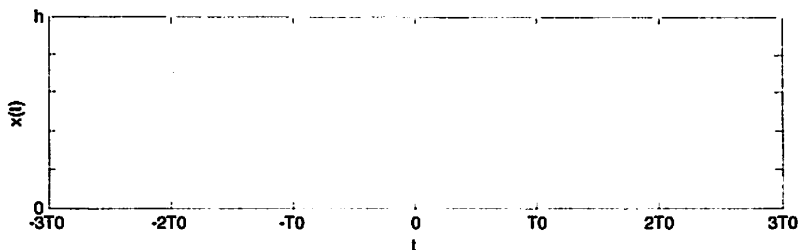
$h = \text{amplitude}, j = \sqrt{-1}$

$$X_1 = \frac{-jh}{4}$$

$$X_{-1} = \frac{jh}{4}$$

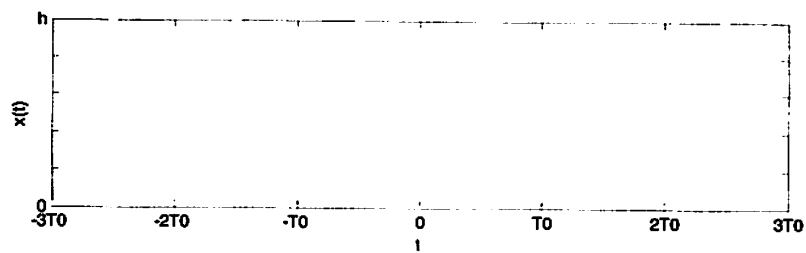
$$X_n = \frac{h}{\pi} \frac{\cos^2\left(\frac{n\pi}{2}\right)}{1-n^2} = \begin{cases} \frac{h}{\pi}, & n=0 \\ 0, & n = \pm 3, \pm 5, \pm 7, \dots \\ \frac{h}{\pi} \frac{1}{1-n^2}, & n = \pm 2, \pm 4, \pm 6, \dots \end{cases}$$

Full-Wave Rectified Cosine



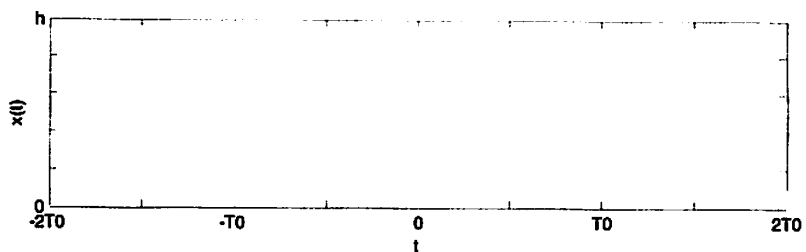
$$X_n = \frac{2h}{\pi} \frac{\cos(n\pi)}{1-4n^2} = \frac{2h}{\pi} \frac{(-1)^n}{1-4n^2}$$

Full-Wave Rectified Sine



$$X_n = \frac{2h}{\pi} \frac{\cos^2(n\pi)}{1-4n^2} = \frac{2h}{\pi} \frac{1}{1-4n^2}$$

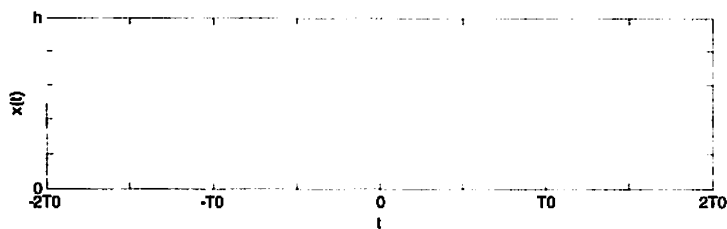
Sawtooth



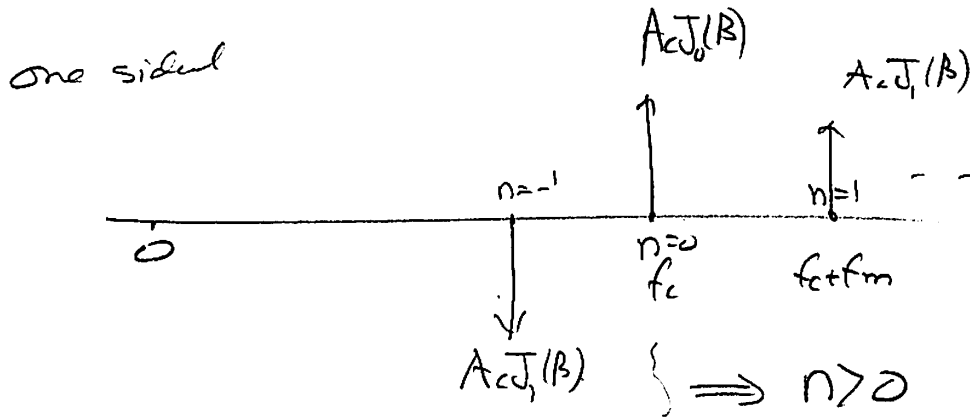
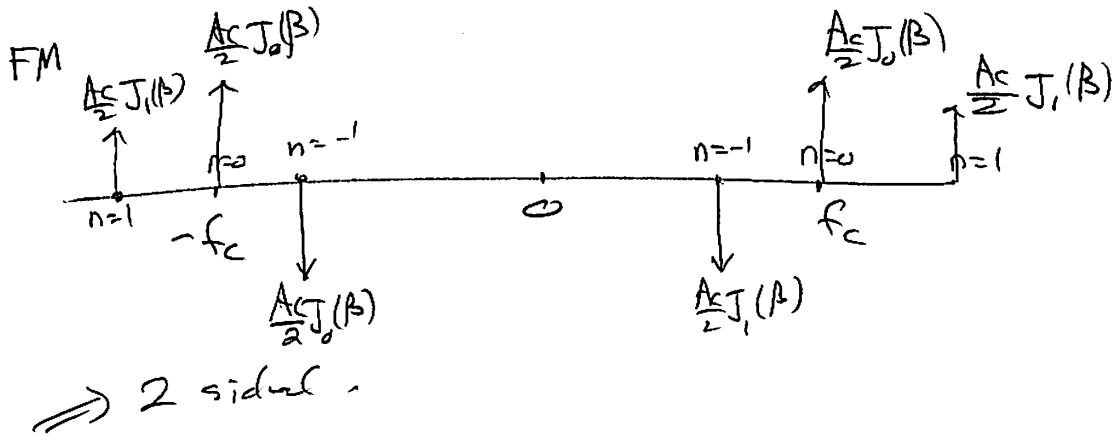
$$X_0 = \frac{h}{2}$$

$$X_n = \frac{jh}{2\pi n}, \quad n \neq 0, \quad j = \sqrt{-1}$$

Exponential Decay

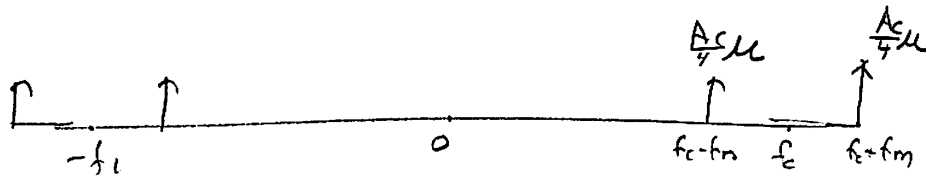


$$X_n = \frac{h}{T_0} \frac{1 - e^{-aT_0}}{a + jn \frac{2\pi}{T_0}}$$

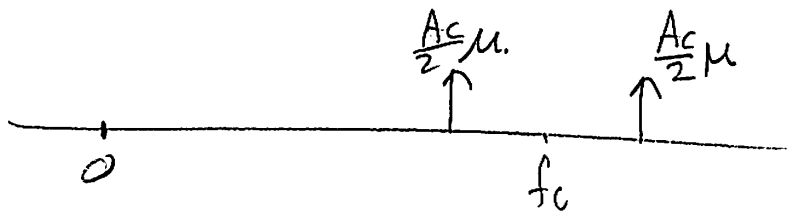


AM

2 sideband.



one sideband



FM

$$FM(t) = A_c \cos(\omega_c t + \underbrace{K_f \int m(\lambda) d\lambda}_{\Phi(t) \text{ Phase deviation}})$$

Frequency deviation = $\frac{d}{dt} \Phi(t) = K_f m(t)$

max frequency deviation = $\max |K_f m(t)|$
 $\Delta\omega = K_f \max |m(t)|$

$\omega_i(t)$ = instant. frequency $\Phi'(t)$
 $= \frac{d}{dt} \Theta_i(t) = \omega_c + K_f m(t)$

PM

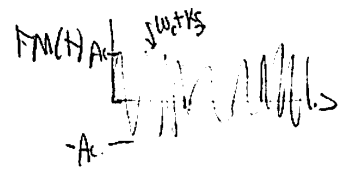
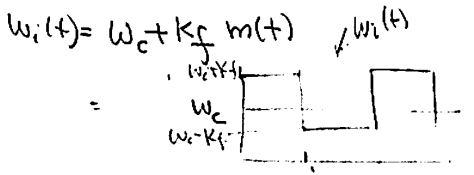
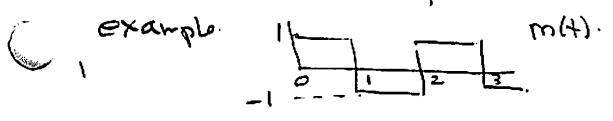
$$PM(t) = A_c \cos(\omega_c t + \underbrace{K_p m(t)}_{\Phi(t) = \text{phase deviation}})$$

Frequency deviation: $\frac{d}{dt} \Phi(t) = K_p \frac{d}{dt} m(t)$

max frequency deviation = $K_p \max \left| \frac{dm}{dt} \right|$

$\omega_i(t) = \frac{d}{dt} \Theta_i(t) = \omega_c + K_p \frac{dm}{dt}$

to plot FM(t), do
 Find $\omega_i(t) = \omega_c + K_f m(t)$.



5.3 sheet 3

PM modulation

perfectly angle $\Rightarrow \omega_c t \Rightarrow \omega_c t + k_p m(t)$
 phase deviation
 instant. freqs $\frac{d}{dt}(\omega_c t + k_p m(t))$
 the general signal is $\cos(\omega_c t + k_p m(t))$
 frequency deviation \propto PM

FM

time = t time = t

perfectly ω_c instantaneous freq $\omega_c \rightarrow \omega_c + k_f m(t)$
 called frequency deviation \propto FM.

here the radial freq. is by general signal is $\omega_c + k_f m(t)$

and the phase is $\omega_c t + k_f \int m(t) dt$

instantaneous

