University Course

ECE 405
Introduction to communications

Cal Poly, Pomona, California
Summer 2010

My Class Notes

Nasser M. Abbasi

Summer 2010

Contents

Chapter 1

Introduction

This was a hard course only because it was 5 weeks longs and we meet 4 days per week and things went very quickly. We had 6 quizes, 2 exams, and HW's and computer assignments. The instructor was Dr James Kang, EE dept, and was a very good instructor and explained things really well, but his exams were a little on the hard side.

1.1 syllabus

CALIFORNIA STATE POLYTECHNIC UNIVERSITY, POMONA
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

SYLLABUS

ECE 405 Communication Systems (4)

Prerequisite: ECE 307 and ECE 315
Instructor: Dr. James S. Kang
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Text: Lathi and Ding, *Modern Digital and Analog Communication Systems,* 4th ed., Oxford, 2009.

Previous Text: Haykin, Communication Systems, 4th ed., Wiley, 2001.

Grading: Quizzes - 40%, Midterm - 25%, Final - 25%, Homework and computer assignment - 10%.

- All exams and quizzes are open book, open notes
- No make-up quizzes or tests are allowed unless approved by the instructor in advance

Topics

Review of Fourier Series and Fourier Transform

Amplitude Modulation and Demodulation

- Double-sideband modulation
- Amplitude modulation
- Single-sideband modulation
- AM receiver design

Frequency Modulation and Demodulation

- Phase modulation
- Frequency modulation
- Superheterodyne receiver

ADC and DAC

- Ideal sampling
- Practical sampling
- Pulse code modulation
- Differential pulse code modulation

Adaptive differential pulse code modulation
Delta modulation
Adaptive delta modulation

Baseband Transmission

Line coding
Pulse shaping
Equalizer design

1.2 Text Book

There was an official textbook, but it was not really needed. Taking good notes and working on the given HW's was all what is needed. The text book is below.

Text book was Lathi and Ding, Modern digital and analog communication systems, 4ht edition.

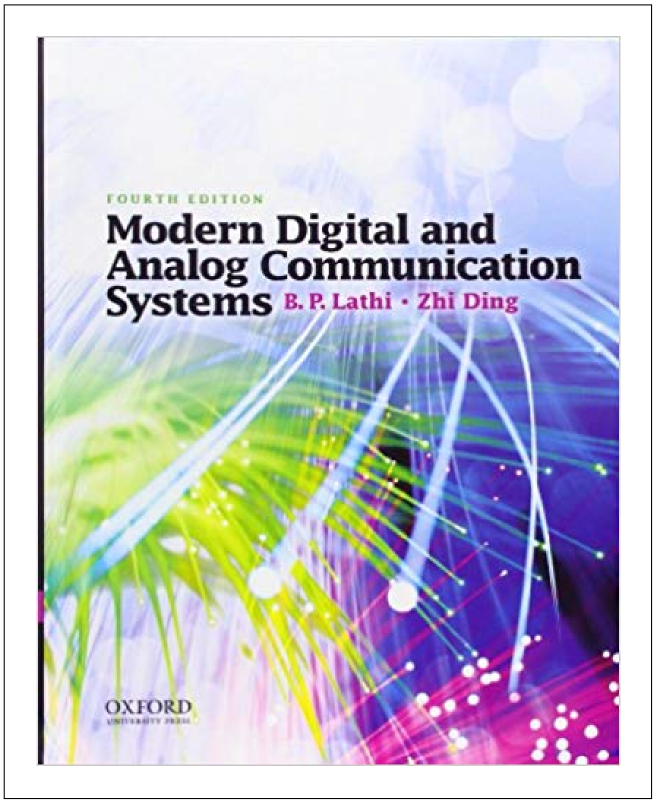

Figure 1.1: text book

Chapter 2

Quizes

Local contents

2.1 Quizz 1

Nasser Abbasi

ECE 405 QUIZ #1 20 POINTS SUMMER 2010

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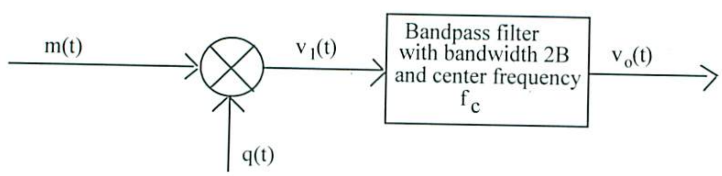
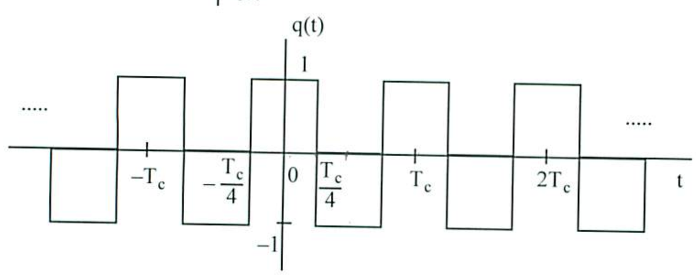
A model of a balanced modulator is shown below. Let the message signal be $m(t) = 2\cos(2\pi \times 10000t)$. The frequency of carrier is $f_c = 100000$ Hz so that $T_c = 1/100000$ s. B = 25 kHz.

(a) Plot m(t) in the time domain for $0 \le t \le 0.2$ms.
(b) Plot the spectrum $M(f) = F[m(t)]$ in the frequency domain.
(c) Find the exponential Fourier coefficients Q_n of q(t) and represent q(t) by its exponential Fourier series.
(d) Plot the spectrum $Q(f) = F[q(t)]$ in the frequency domain.
(e) Plot $v_1(t)$ in the time domain for $0 \le t \le 0.2$ms.
(f) Plot the spectrum $V_1(f) = F[v_1(t)]$ in the frequency domain for -600 kHz $\le f \le$ 600 kHz.
(g) Plot $v_o(t)$ in the time domain for $0 \le t \le 0.2$ms.
(h) Plot the spectrum $V_o(f) = F[v_o(t)]$ in the frequency domain for -600 kHz $\le f \le$ 600 kHz.
(i) The center frequency of the bandpass filter is changed to $5f_c$ with bandwidth 50 kHz, find the expression for $v_o(t)$.



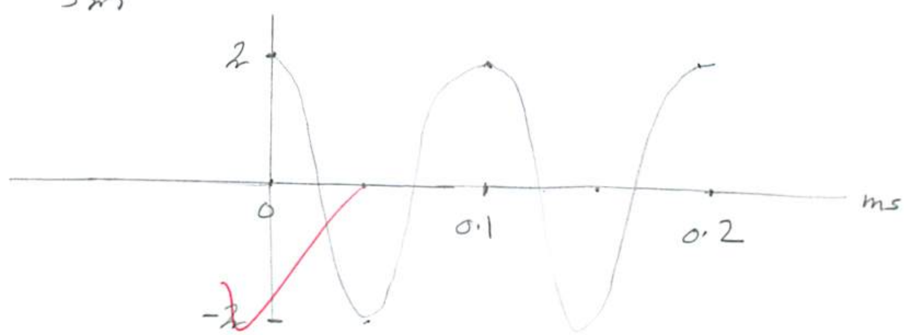


1

$m(t) = 2\cos(2\pi f_m t)$ where $f_m = 10{,}000$ Hz.
$f_c = 100{,}000$ HZ

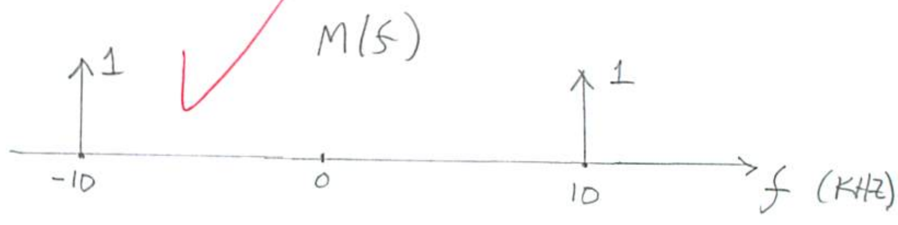
period of $m(t) = \frac{1}{f_m} = 0.1$ ms.

(a)



(b) $F(m(t)) = \frac{A_m}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right]$ where $A_m = 2$ here

$= \delta(f-10k) + \delta(f+10k)$



(c) $q(t)$: period T_c, $h = 1$, $\tau = \frac{T_c}{2}$.

$$q(t) = \sum_{n=-\infty}^{\infty} Q_n e^{j\frac{2\pi}{T_c}nt}$$

where $\boxed{Q_n = \frac{1}{T_c}\int_{T_c} q(t)\, e^{-j\frac{2\pi}{T_c}nt}\, dt}$

so

③

$Q_n = h\,d\,\mathrm{sinc}(nd)$

where $h = 2$, $d = \frac{\tau}{T} = \frac{T_c}{2\,T_c} = \frac{1}{2}$

so $Q_n = \frac{2}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right) = \mathrm{sinc}\left(\frac{n}{2}\right)$.

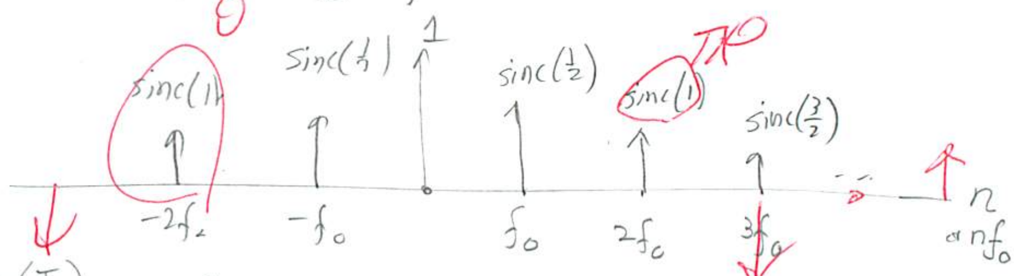
so $q(t) = \sum Q_n\, e^{j\frac{2\pi}{T_c}nt}$.

$$\boxed{q(t) = \sum \mathrm{sinc}\left(\frac{n}{2}\right) e^{j\frac{2\pi}{T_c}nt}}$$

where $T_c = 0.001$ ms

(d) $F[q(t)] = \sum \mathrm{sinc}\left(\frac{n}{2}\right) F\left[1 \cdot e^{j\frac{2\pi}{T_c}nt}\right]$ $\qquad f_0 = \frac{2\pi}{T_c}$

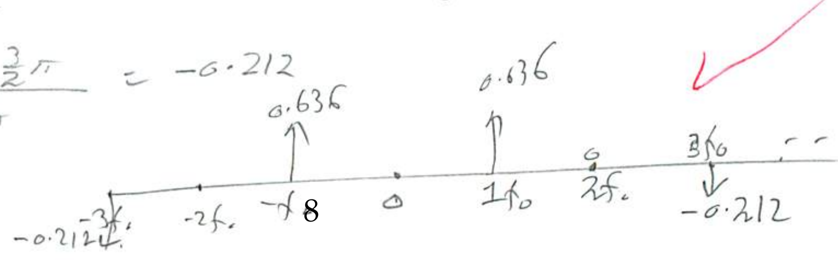
$$F[q(t)] = \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(\frac{n}{2}\right) \delta\left(f - \frac{2\pi}{T_c}n\right) = \sum \mathrm{sinc}\left(\frac{n}{2}\right) \delta(f - nf_0)$$



$\mathrm{sinc}\left(\frac{1}{2}\right) = \frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}} = \frac{2}{\pi} = 0.636$

$\mathrm{sinc}(1) = \frac{\sin \pi}{\pi} = 0$

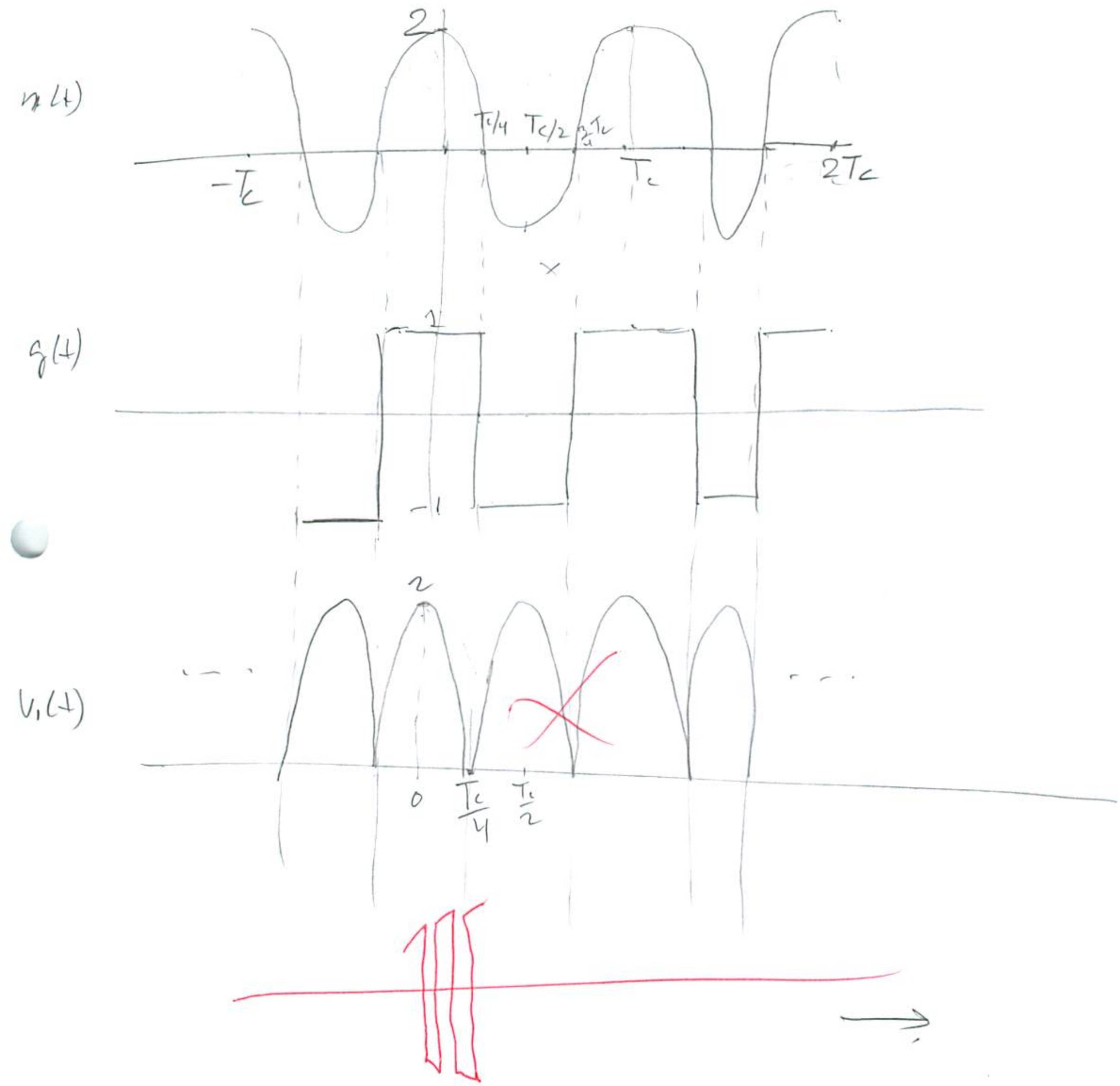
$\mathrm{sinc}\left(\frac{3}{2}\right) = \frac{\sin \frac{3}{2}\pi}{\frac{3}{2}\pi} = -0.212$



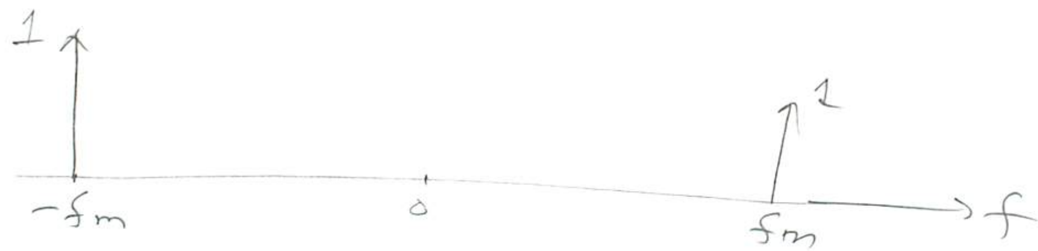
3

(e) $v_1 = m(t) \cdot g(t)$.

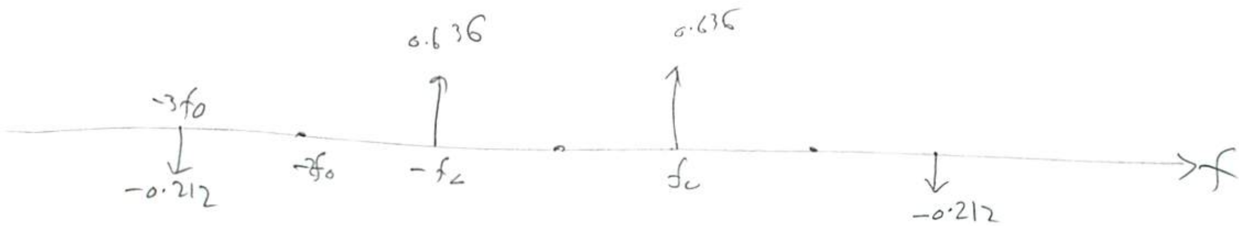
$$g(t) = \sum \text{sinc}\left(\frac{n}{2}\right) e^{j\frac{2\pi}{T_c}nt} = \sum \text{sinc}\left(\frac{n}{2}\right) e^{jf_0 nt}.$$



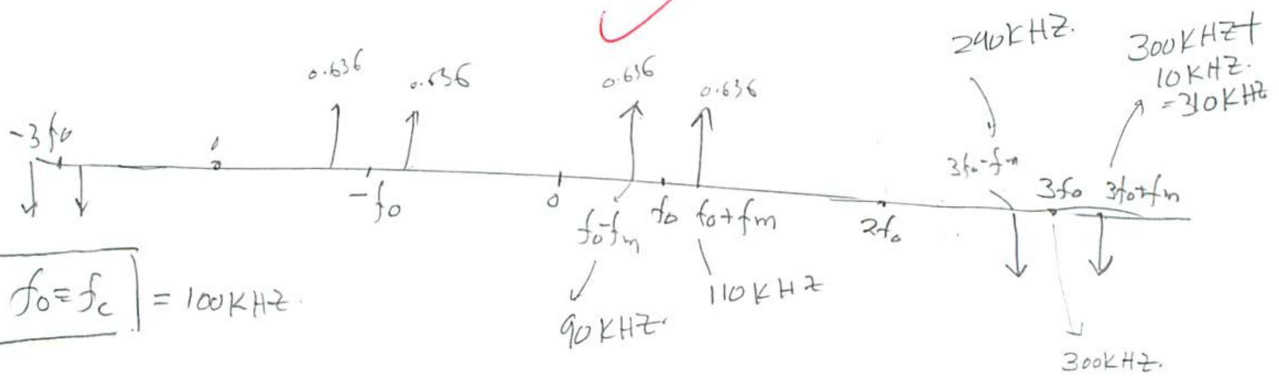
(f) the spectrum of $v_1(t)$ is $F(m(t)) \circledast F(g(t))$ 4



$\circledast$



$\Downarrow$



$\boxed{f_0 = f_c} = 100KHZ$

(a) ✓
(b) ✓
(c) ✓

2.2 Quizz 2

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ECE 405 QUIZ #2 20 POINTS SUMMER 2010

1. A message

m(t) = 3.0 cos(2π×2,000t)+6.2 cos(2π×6,000t)

amplitude modulates (AM) a carrier

10 cos(2π×100,000t)

$= \frac{1}{2}(\cos(\alpha-\beta) + \cos(\alpha+\beta))$

$\frac{1}{2}(\cos(4) + \cos($

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(a) Plot m(t) in the time domain for 0 ≤ t ≤ 1ms.
(b) Plot the spectrum M(f) of m(t) in the frequency domain.
(c) Find the modulation index μ of this AM modulation.
(d) Plot the AM waveform in the time domain for 0 ≤ t ≤ 1ms.
(e) Plot the spectrum of the AM waveform in the frequency domain.
(f) What is the bandwidth of the AM wave?

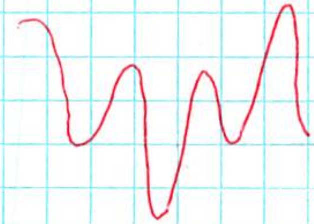
$3\cos(\overset{2}{\alpha} t) + 6.2\cos(\overset{6}{\beta} t)$.

$$m(t) = \underbrace{3.0\cos(2\pi\, 2000t)}_{m_1} + \underbrace{6.2\cos(2\pi\, 6000t)}_{m_2}$$
$$C(t) = 10\cos(2\pi\ 100{,}000t)$$

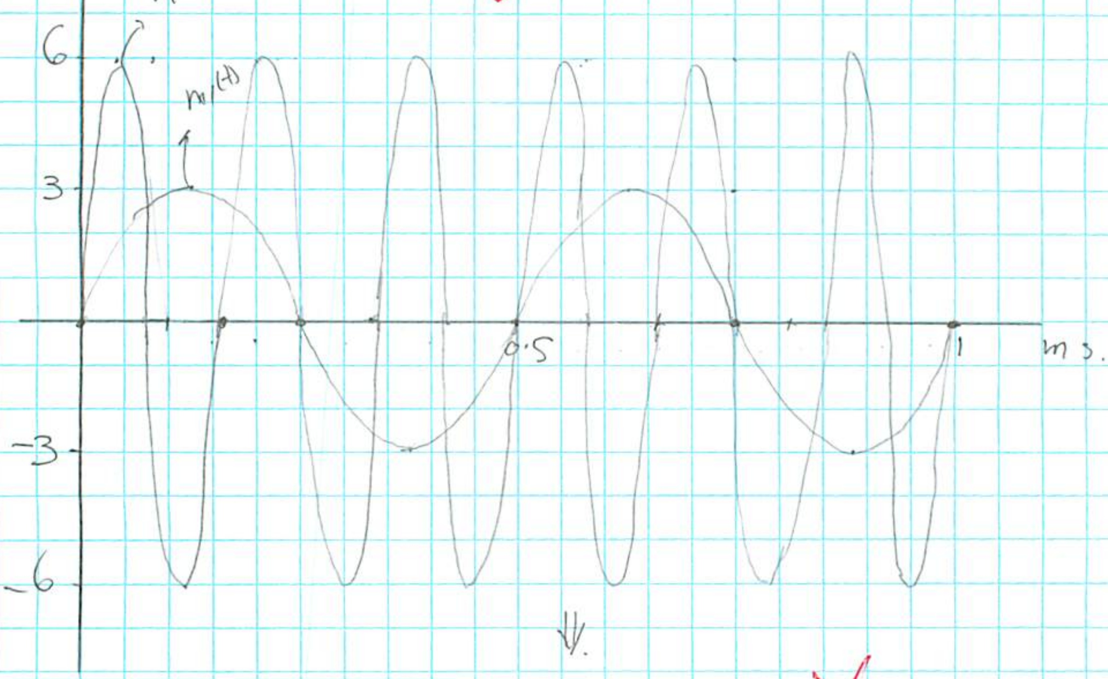
let $m(t) = A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t)$
$C(t) = A_c\cos(2\pi f_c t)$

where $f_1 = 2\text{ KHz}$, $f_2 = 6\text{ KHz}$, $f_c = 100\text{KHZ}$.
$T_1 = \frac{1}{2000} = \frac{1}{2}\text{ ms} = 0.5\text{ ms}$, $T_2 = \frac{1}{6}\text{ ms} = 0.167\text{ ms}$

(a)



Should I have combined them into one signal?



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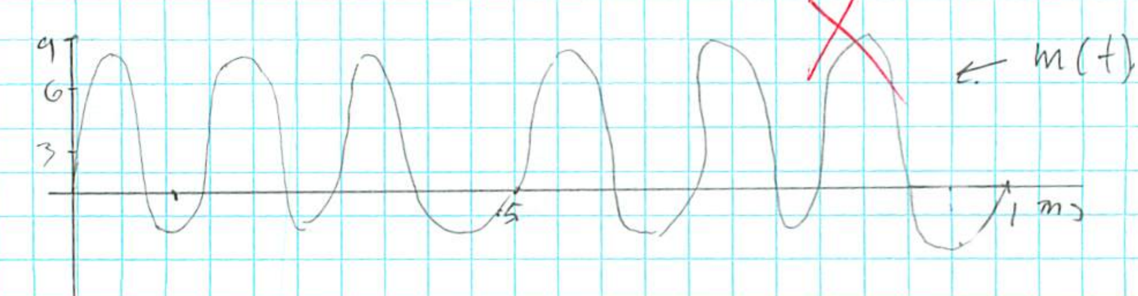
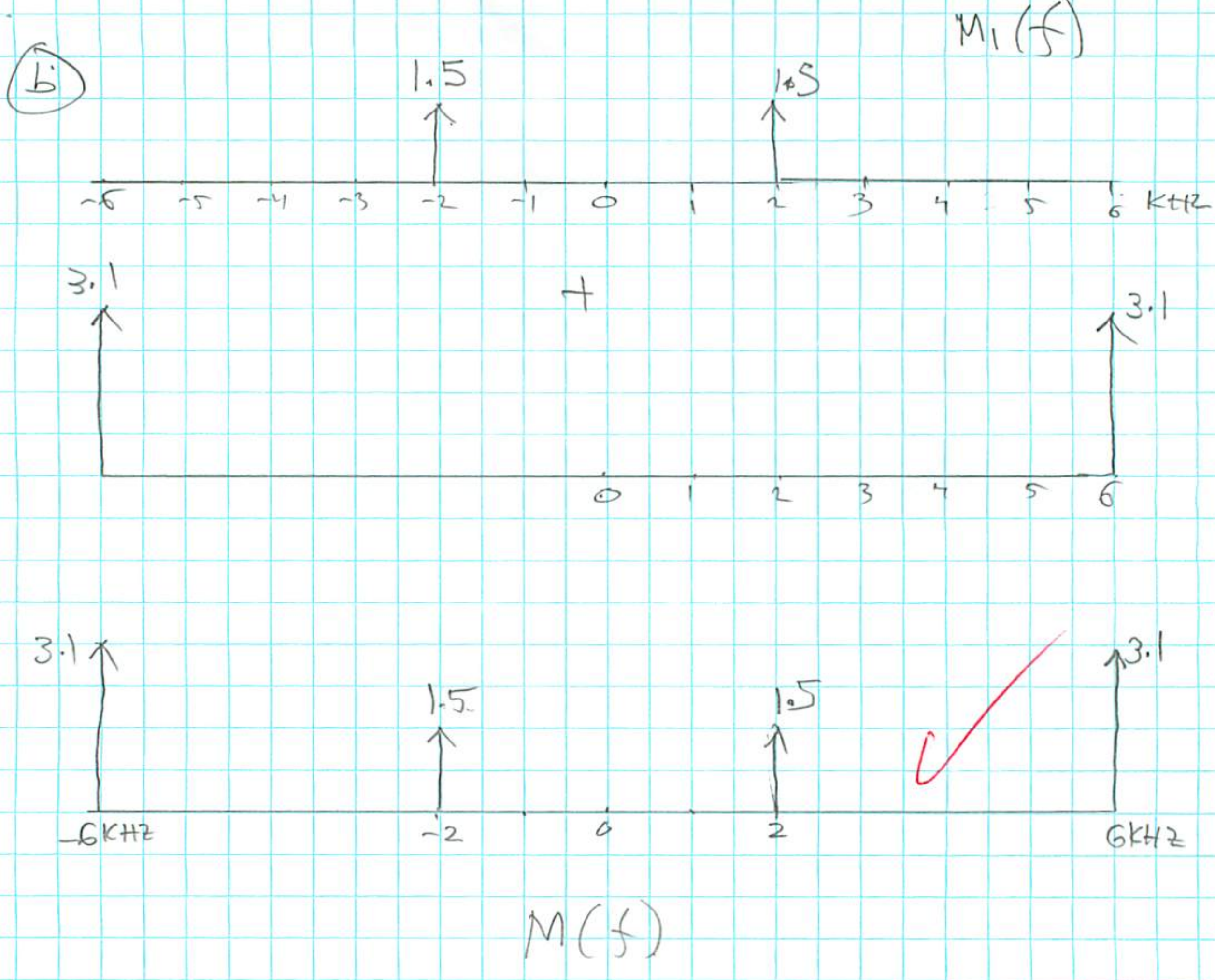
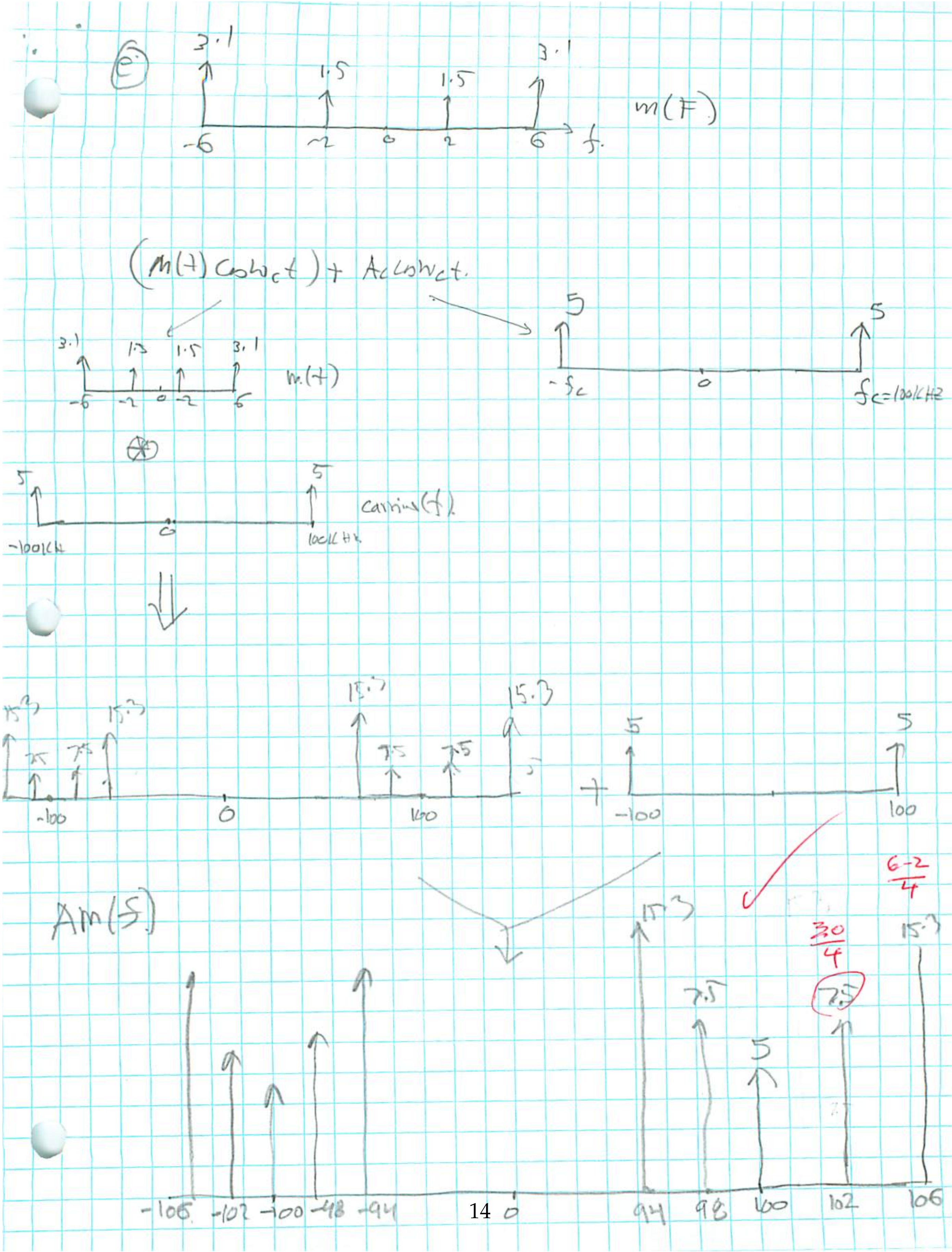
add




b
$M_1(f)$
1.5
1.5
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 KHZ
+
3.1
3.1
0 1 2 3 4 5 6
3.1
1.5
1.5
3.1
-6KHZ
-2
0
2
6KHZ
$M(f)$


e
3.1
1.5
1.5
3.1
m(F)
-6
-2
0
2
6
f.
(m(t) cos w_c t) + A_c cos w_c t.
3.1
1.5
1.5
3.1
m(t)
-6
-2
0
2
6
5
5
-f_c
0
f_c = 100KHZ
5
5
carrier(f)
-100KH
0
100KHz
15.3
7.5
7.5
15.3
-100
0
15.3
7.5
7.5
15.3
100
+
5
5
-100
100
Am(S)
15.3
7.5
5
6-2
4
30
4
7.5
15.3
-106
-102
-100
-98
-94
0
94
98
100
102
106

BW is $\boxed{12\ KHZ}$

2.3 Quizz 3

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ECE 405 QUIZ #3 20 POINTS SUMMER 2010

1. The block diagram of USSB generation using phasing method is shown in Figure 1. Let $x(t) = 2\sin(2\pi 3000t)$ and $f_c = 30000$ Hz.

(a) Plot x(t) in the time domain for two periods starting from t = 0.
(b) Plot the spectrum X(f) of x(t) in the frequency domain.
(c) Find the Hilbert transform $\hat{x}(t)$ and plot $\hat{x}(t)$ in the time domain for two periods starting from t = 0.
(d) Plot the spectrum $\hat{X}(f)$ of $\hat{x}(t)$ in the frequency domain.
(e) Find the waveform at (1) and plot it in the time domain.
(f) Find the spectrum at (1) and plot it in the frequency domain.
(g) Find the waveform at (2) and plot it in the time domain.
(h) Find the spectrum at (2) and plot it in the frequency domain.
(i) Find the waveform at (3) and plot it in the time domain.
(j) Find the spectrum at (3) and plot it in the frequency domain.

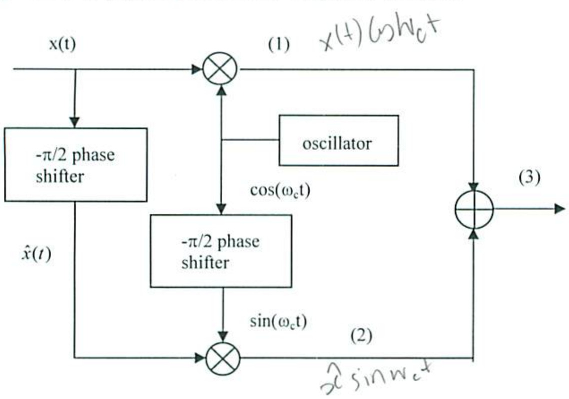


Figure 1

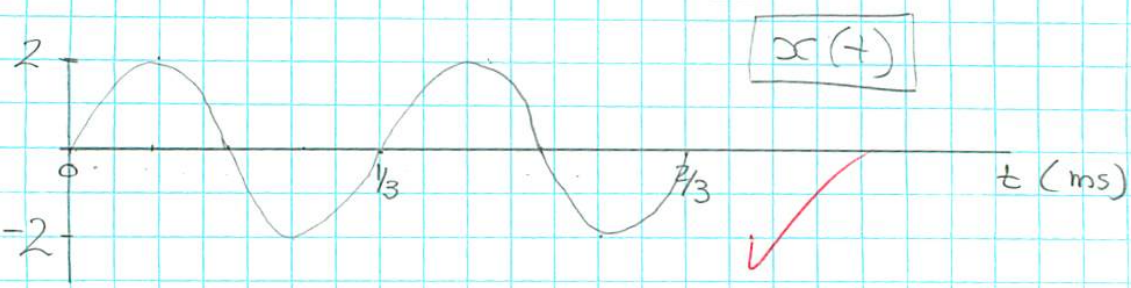
$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$
$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
$\sin(\alpha)\cos(\beta) = (1/2)\,[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$
$\sin(\alpha)\sin(\beta) = (1/2)\,[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos(\alpha)\cos(\beta) = (1/2)\,[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\cos(\alpha)\sin(\beta) = (1/2)\,[-\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

$\hat{x}(t) = 2\sin(2\pi\, 3000t - \frac{\pi}{2})$

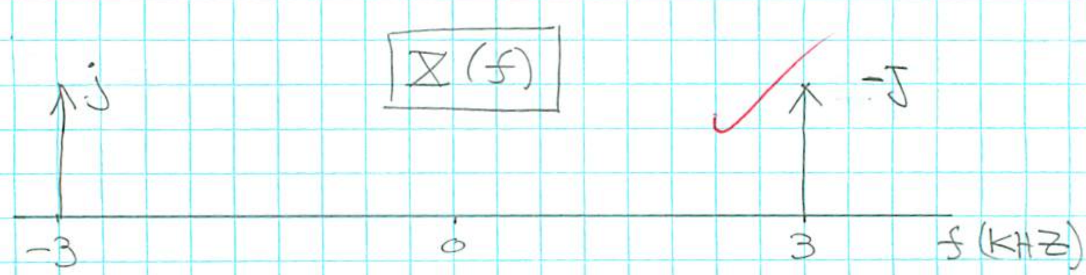
(1)

$x(t) = 2\sin(2\pi\, 3000 t) \qquad f_c = 30{,}000\ \text{Hz}$

a) $f_m = 3000\ \text{Hz}$. so $T_0 = \frac{1}{3000} = \boxed{\frac{1}{3}\ \text{ms}}$.

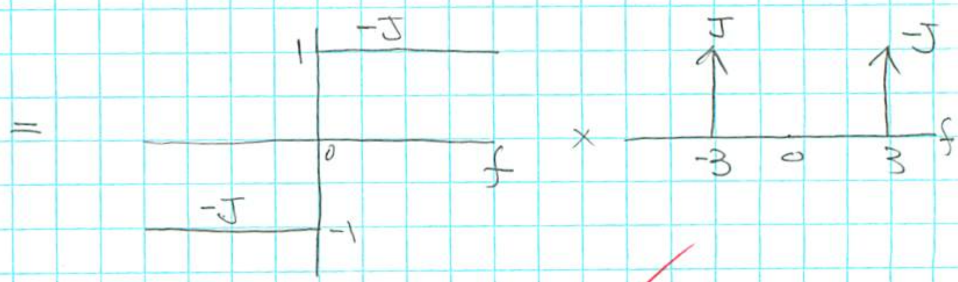


(b)



(c) $\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau$.

or $\hat{X}(f) = -J\ \text{sgn}(f)\ X(f)$.

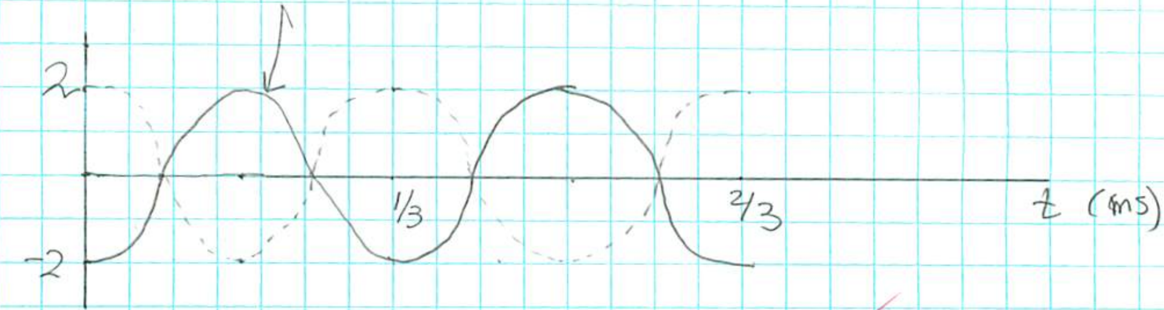


so $\hat{x}(t) = \boxed{-2\cos(2\pi\, 3000 t)}$

(2)

Plot $\hat{x}(t)$ for 2 periods

$$\hat{x}(t) = -2\cos(2\pi\, 3000 t)$$



(d) Plot $\hat{X}(t)$

I allready did this:

(e) waveform at (1) is $\boxed{x(t)\cos w_c t}$

$$s_1(t) = 2\sin(\underbrace{2\pi\, 3000 t}_{\alpha})\cdot\cos(\underbrace{2\pi\, 30000 t}_{\beta})$$

$$= 2\left[\tfrac{1}{2}\left[\sin(-27000 t\, 2\pi) + \sin(33000 t\, 2\pi)\right]\right]$$

$$\boxed{s_1(t) = -\sin(2\pi\, 27000 t) + \sin(33000 t\, 2\pi)}$$

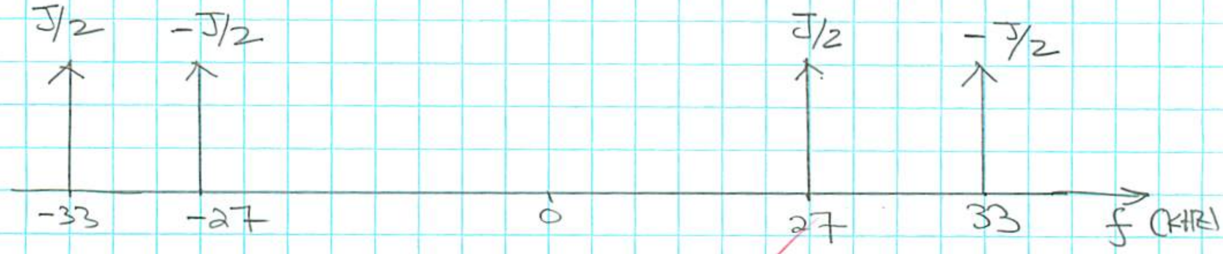
plot in time domain next page.

so $S_1(f) = -\left[-j/2\, \delta(f-27000) + \frac{j}{2}\delta(f+27000)\right]$

$$+ \left[-j/2\, \delta(f-33000) + \frac{j}{2}\delta(f+33000)\right]$$

$$= j/2\, \delta(f-27^k) - j/2\, \delta(f+27^k) - j/2\, \delta(f-33^k) + j/2\, \delta(f+33^k)$$

$$S_1(f) = \frac{j}{2}\left[\delta(f-27000) - \delta(f+27000) - \delta(f-33000) + \delta(f+33000)\right]$$ 3

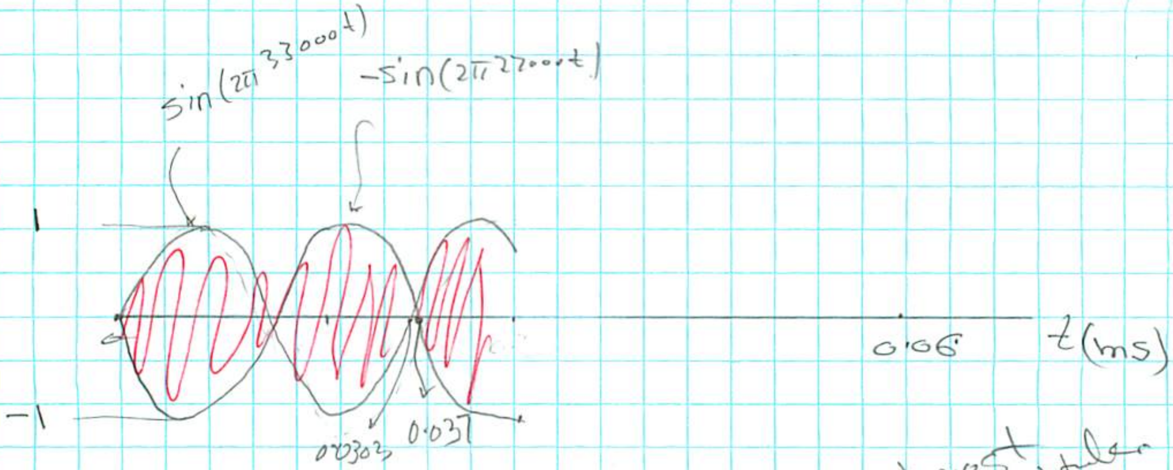


Plot $s_1(t)$ in time domain:

$$s_1(t) = -\sin(2\pi\, 27000t) + \sin(33000t\, 2\pi)$$

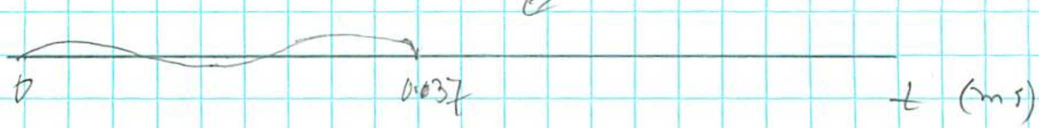
$f=27000 \Rightarrow T = \frac{1}{27000} = 0.037$ ms for period

$f=33000 \Rightarrow T = \frac{1}{33000} = 0.0303$ ms for period.



notice almost zero amplitude

So add



4

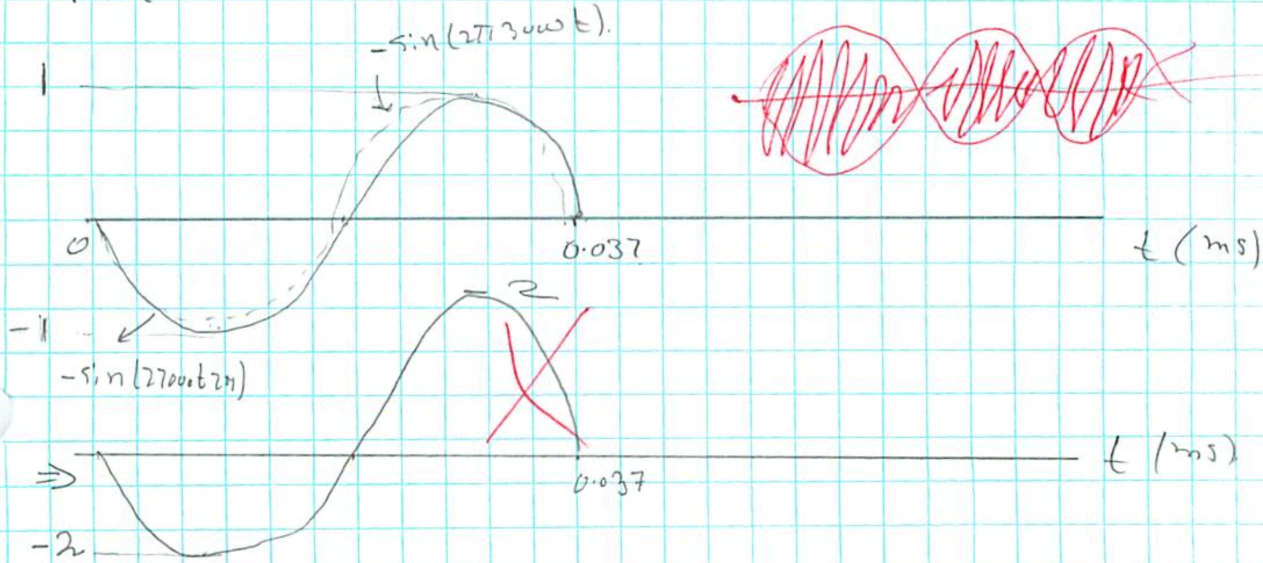
(g) $s_2(t) = \hat{x}(t)\sin(2\pi f_c t)$

$$= -2\cos(2\pi \overset{\alpha}{3000} t)\,\sin(2\pi \overset{\beta}{30000} t)$$

$$= -2\left[\frac{1}{2}\left(-\sin(-27000 t\, 2\pi) + \sin(2\pi\, 33000 t)\right)\right]$$

$$= -\sin(27000 t\, 2\pi) - \sin(2\pi\, 33000 t)$$

Plot in time domain.



(h) Spectrum is

$$S_2(f) = -\left[\frac{-j}{2}\delta(f-27000) + \frac{j}{2}\delta(f+27000)\right] - \left[\frac{-j}{2}\delta(f-33000) + \frac{j}{2}\delta(f+33000)\right]$$

$$= \frac{j}{2}\delta(f-27000) - \frac{j}{2}\delta(f+27000) + \frac{j}{2}\delta(f-33000) - \frac{j}{2}\delta(f+33000)$$

So spectrum

(1) 5

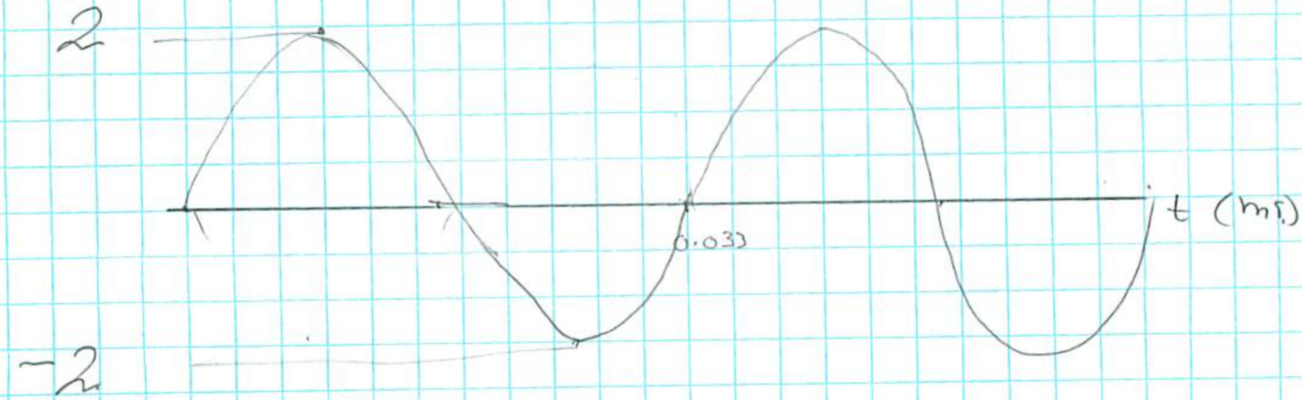
Since S_{USSB}, then need to subtract $S_1(f) - S_2(f)$ to find $S_3(f)$

first, in time domain

$$S_3(t) = x(t)\cos\omega_c t - \hat{x}(t)\sin\omega_c t$$
$$= S_1(t) - S_2(t)$$
$$= \left(-\sin(2\pi\, 27000t) + \sin(33000t\, 2\pi)\right) - \left(-\sin(2\pi\, 27000t) - \sin(2\pi\, 33000t)\right)$$
$$= -\sin(2\pi\, 27000t) + \sin(33000t\, 2\pi) + \sin(2\pi\, 27000t) + \sin(2\pi\, 33\ldots$$
$$S_3(t) = 2\sin(33000t\, 2\pi)$$

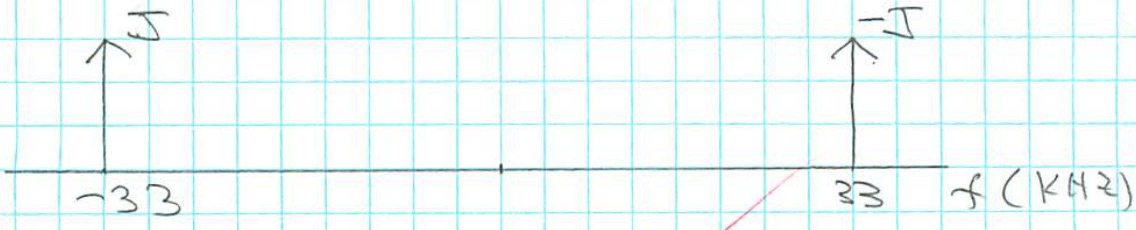
plot in time domain:

$2\sin(33000t)$



(j) spectrum is 6

since $s_3(t) = 2\sin(2\pi\, 33000t)$ then
spectrum is



$$S_3(f) = -J\delta(f-33000) + J\delta(f+33000)$$

which is also $S_1(f) - S_2(f)$ OK verified

2.4 Quizz 4

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ECE 409 QUIZ #4 20 POINTS

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1. The loop filter of the second-order analog PLL is given by $F(s) = \frac{1+T_1 s}{s}$. Assume that K = 0.32, $T_1 = 15/4$.

(a) Find the open loop transfer function G(s) = F(s)K/s.

(b) Find the closed-loop transfer function $H(s) = \frac{\hat{\Phi}(s)}{\Phi(s)}$.

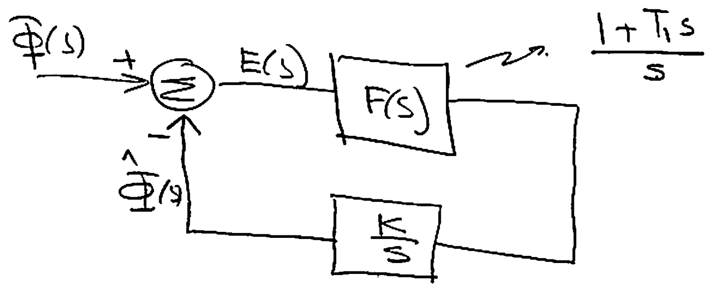
(c) Find and plot the impulse response h(t) of this APLL.

(d) Find and plot the step response $h_s(t)$ of this APLL.

(e) Find and plot the error response $h_e(t)$ to step input of this APLL.

1

1



a $G(s) = F(s)\frac{K}{s} = \left(\frac{1+T_1 s}{s}\right)\left(\frac{k}{s}\right)$

b $H(s) = \frac{\hat{\Phi}(s)}{\Phi(s)} = \frac{E(s)\,G(s)}{\Phi(s)} = \frac{[\Phi(s) - \hat{\Phi}(s)]\,G(s)}{\Phi(s)}$

$$= \frac{\Phi(s)\,G(s) - \hat{\Phi}(s)G(s)}{\Phi(s)} = G(s) - \frac{\hat{\Phi}(s)}{\Phi(s)}\,G(s)$$

$H(s) = G(s) - H(s)\,G(s)$

Solve for $H(s)$

$$\boxed{H(s) = \frac{G(s)}{1+G(s)}}$$

here $H(s) = \frac{\frac{1+T_1 s}{s}\,\frac{k}{s}}{1 + \frac{1+T_1 s}{s}\,\frac{k}{s}} = \frac{(1+T_1 s)\,k}{s^2 + (1+T_1 s)k}$

$$\boxed{H(s) = \frac{K + KT_1 s}{s^2 + T_1 k s + K}}$$

→

$$\therefore H(s) = \frac{K + K T_1 s}{s^2 + T_1 K s + K}.$$

②

$$K = \frac{32}{100}, \quad T_1 = \frac{15}{4}.$$

$$H(s) = \frac{\frac{32}{100} + \frac{32}{100}\frac{15}{4} s}{s^2 + \frac{15}{4}\frac{32}{100} s + \frac{32}{100}} = \frac{.32 + 1.2 s}{s^2 + 1.2 s + .32}$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1.2}{2} \pm \frac{1}{2}\sqrt{1.2^2 - 4(.32)}$$

$$= -0.6 \pm \frac{1}{2}\sqrt{1.44 - 1.28} = -0.6 \pm \frac{1}{2}\sqrt{0.16}$$

$$= -0.6 \pm \frac{1}{2}(0.4) = -0.6 + .2$$

$$= -0.6 - 0.2$$

$$\therefore \boxed{s_1 = -0.4, \quad s_2 = -0.8}$$

note the poles < 0 hence stable system

$$\therefore \boxed{H(s) = \frac{0.32 + 1.2 s}{(s + 0.4)(s + 0.8)}} = \frac{A}{s + 0.4} + \frac{B}{s + 0.8}$$

$$A = \left.\frac{0.3 + 1.2 s}{s + 0.8}\right|_{s=-0.4} = \frac{(0.3) + 1.2(-0.4)}{(-0.4) + (0.8)} = \frac{-0.18}{0.4}$$

$$= \boxed{-0.\cancel{45}}$$

$$B = \left.\frac{0.32 + 1.2 s}{s + 0.4}\right|_{s=-0.8} = \frac{(0.32) + 1.2(-0.8)}{-0.8 + 0.4} = \frac{.32 - 0.96}{-0.4}$$

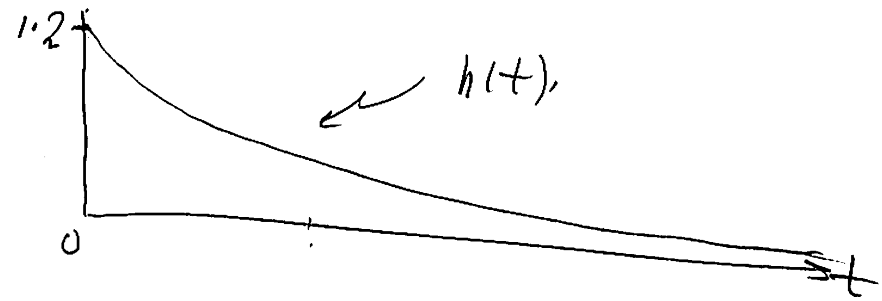
$$= 1.6$$

-0.4

$$\therefore \boxed{H(s) = \frac{-0.45}{s + 0.4} + \frac{1.6}{s + 0.8}} \longrightarrow$$

so $\boxed{h(t) = \left[-0.45\, e^{-0.4t} + 1.6\, e^{-0.8t}\right] u(t)}$ 3

so



4 step response is when $\Phi(t) = u(t)$.

so $\boxed{\Phi(s) = \frac{1}{s}}$

hence $\hat{\Phi}(s) = \Phi(s)\, H(s)$

$$= \frac{1}{s} \frac{0.32 + 1.2s}{s^2 + 1.2s + 0.32}$$

$$= \frac{1}{s} \frac{0.32 + 1.2s}{(s+0.4)(s+0.8)}$$

so $\hat{\Phi}(s) = \frac{A}{s} + \frac{B}{s+0.4} + \frac{C}{s+0.8}$

$$A = \left.\frac{0.32+1.2s}{(s+0.4)(s+0.8)}\right|_{s=0} = \frac{0.32}{(0.4)(0.8)} = \underline{1}$$

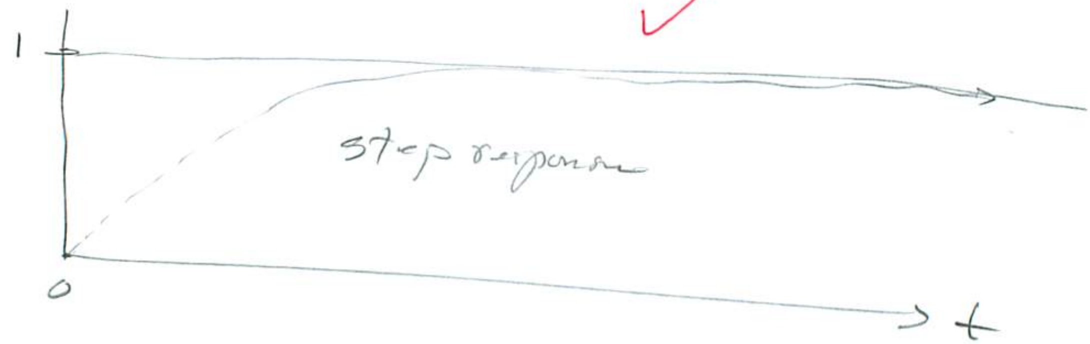
$$B = \left.\frac{0.32+1.2s}{s(s+0.8)}\right|_{s=-0.4} = \frac{(0.32)+(1.2(-0.4))}{(-0.4)(-0.4+0.8)} = \frac{-0.16}{-0.16} = 1$$

$$C = \left.\frac{0.32+1.2s}{s(s+0.4)}\right|_{s=-0.8} = \frac{(.32)-(1.2)(.8)}{(-.8)(-.8+0.4)} = \frac{-.64}{-.32} = 2$$

so $\hat{\Phi}(s) = \frac{1}{s} + \frac{1}{s+0.4} - \frac{2}{s+0.8}$ 2

so $\hat{\Phi}(t) = u(t) + e^{-0.4t}u(t) - 2e^{-0.8t}u(t)$

$= \left(1 + e^{-0.4t} - 2e^{-0.8t}\right) u(t)$



at $t=0$, $1+1-2 = 0$

at $t \to \infty \to 1$

at $t=1 \to$ $1 + .67 - 2(.449) = .772$.

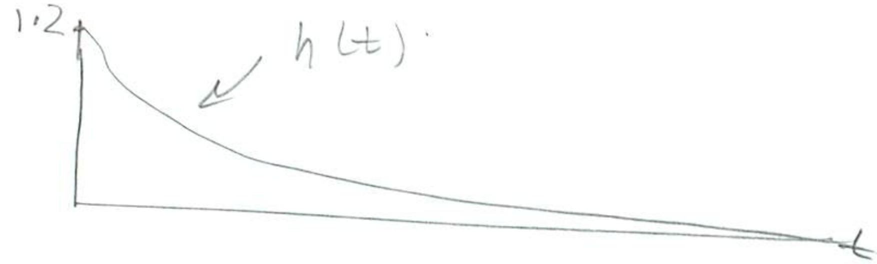
$\longrightarrow$

<u>$h_e(t)$</u>

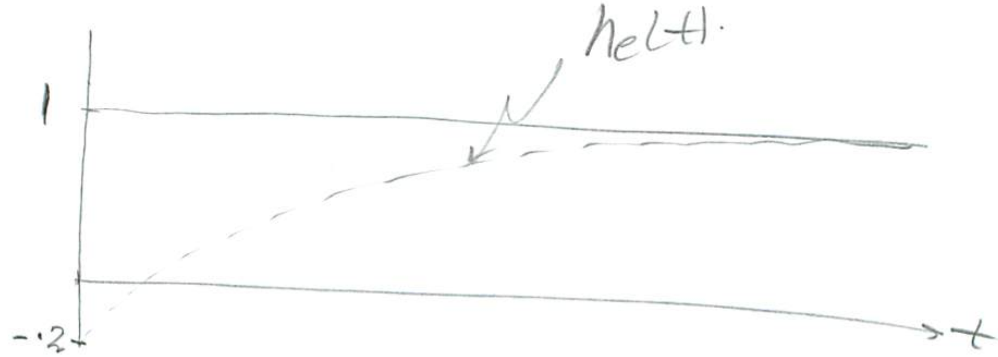
since $H_e(s) = 1 - H(s)$

then $h_e(t) = 1 - h(t)$ $\longrightarrow$

5



so $1-h(t)$ is



at $t=0$, $1-1.2=-.2$

at $t=1$, $1-.8=.2$

at $t\to\infty$ $h_e(t)=1-0=1$

$$h_e(t)=\left[\cancel{1}+\cancel{0.4e^{-0.4t}}-\cancel{1.6}e^{-0.8t}\right]u(t)$$

$$\left(2e^{-0.8t}-e^{-0.4t}\right)u(t)$$

2.5 Quizz 5

Nasser M. Abbasi

20/20

ECE 405 QUIZ #5 20 POINTS

1. A message m(t) = 5 cos(2π×3000t) frequency modulates a carrier of frequency 30 MHz. Let $k_f = 2\pi \times 3000$ rad/s/V and let the amplitude of the modulated wave be 0.2 V. Determine the output signal-to-noise ratio in dB if the one-sided noise power spectral density of the receiver is $N_o = 10^{-6}$ W/Hz (no deemphasis circuit is used).

2. A message m(t) = 5 cos(2π×3000t) frequency modulates a carrier of frequency 30 MHz. Let $k_f = 2\pi \times 3000$ rad/s/V and let the amplitude of the modulated wave be 0.2 V. Determine the output signal-to-noise ratio in dB if the one-sided noise power spectral density of the receiver is $N_o = 10^{-6}$ W/Hz and deemphasis is used at the receiver with $\omega_a = \pi f_m$ and $B = f_m$.

3. Determine the minimum value for the carrier amplitude A_c if the output signal-to-noise ratio is 25 dB, and the one-sided noise power spectral density of the receiver is $N_o = 10^{-7}$ W/Hz, $\beta = 5$, $f_m = 15$ kHz.

1

$m(t) = 5\cos(2\pi\, 3000t)$ $\qquad A_m = 5,\ f_m = 3000\ \text{Hz}.$

$K_f = 2\pi\, 3000$ rad/s/v.

$A_c = 0.2\ \text{V}.$

this is FM. so

$$(SNR)_i \xrightarrow[P_i]{S_i} \boxed{\text{FM demodulator}} \xrightarrow[P_o]{S_o} (SNR)_o.$$

$$(SNR)_o = \frac{3}{4}\frac{A_c^2\beta^2}{N_o B} \quad B=f_m$$

$$\text{But } \beta = \frac{K_f A_m}{2\pi f_m} = \frac{(2\pi\, 3000)(5)}{2\pi(3000)} = \boxed{5}$$

$$\text{so } (SNR)_o = \frac{3}{4}\frac{(0.2)^2(5)^2}{(10^{-5})(3000)} = 250.$$

$$\text{so } (SNR)_{o,db} = 10\log_{10}(250) = \boxed{23.979}\ \text{db}$$

2

#2

$m(t) = 5\cos(2\pi 3000t)$

$k_f = 2\pi\ 3000$ rad/s/V.

$A_c = 0.2V$.

$$(SNR)_o = \frac{\cancel{2}\pi A_c^2 k_f^2 \overline{m^2(t)}}{\cancel{2} N_o \omega_a^2 \left[2\pi B - \omega_a \tan^{-1}\left(\frac{2\pi B}{\omega_a}\right)\right]}$$

$\overline{m^2(t)} = \frac{A_m^2}{2} = \frac{25}{2} = \boxed{12.5 \text{ watt}}$

$\omega_a = \pi f_m = \pi(3000)$

$A_c = 0.2$

hence $(SNR)_o = \dfrac{\pi (0.2)^2 (2\pi 3000)^2 (12.5)}{(10^{-6})(\pi 3000)^2 \left[2\pi(3000) - \pi 3000 \tan^{-1}\left(\frac{2\pi 3000}{\pi 3000}\right)\right]}$

$$= \frac{\pi (0.2)^2 (12.5)(4)}{(10^{-6})\left[2\pi(3000) - \pi 3000 \tan^{-1}(2)\right]}$$

$\boxed{\tan^{-1}(2) = 1.1071}$ → radians

so $(SNR)_o = \dfrac{6.28318}{(10^{-6})[18849.55 - 10434.17]} = \dfrac{6.28318}{0.008415} = 746.63$

so $(SNR)_{o,\text{db}} = 10\log_{10}(746.63) = \boxed{28.73}$

#3

(3)

$$(SNR)_o = \frac{3}{4}\frac{A_c^2 \beta^2}{N_o B}$$

so $10 \log_{10}\left(\frac{3}{4}\frac{A_c^2 \beta^2}{N_o B}\right) = 25$

But $\beta = 5$, $B = 15 \times 10^3$ hz, $N_o = 10^{-7}$.

so $\log_{10}\left(\frac{3}{4}\frac{A_c^2 (5)^2}{(10^{-7})(15\times 10^3)}\right) = 2.5$

so $\log_{10}\left(12500\, A_c^2\right) = 2.5$

so $12500\, A_c^2 = 10^{2.5}$

so $A_c^2 = \frac{10^{2.5}}{12500} \Rightarrow A_c = \sqrt{\frac{10^{2.5}}{12500}}$

so $\boxed{A_c = 0.159 \text{ Volt}}$

so min $A_c = 0.159$ V.

2.6 Quizz 6

Nasser Abbasi

ECE 405 QUIZ #6 20 POINTS

20/20

1. A sinusoidal message $x(t) = 2\cos(2\pi 2000t)$ is sampled at a rate of 10000 samples per second ($f_s = 10000$, $T_s = 1/10000$). The sampling signal $p(t)$ is a rectangular pulse train with period 1/10000 seconds, amplitude $h = 1V$, and duty cycle $d = 1/3$.

(a) Plot $x(t)$ in the time domain for two periods.
(b) Plot the spectrum $X(f)$ in the frequency domain.
(c) Find the expression of $P(f)$ and plot the spectrum $P(f)$ of the pulse train in the frequency domain.
(d) Find expression of the sampled waveform $x_s(t)$, and plot $x_s(t)$ in the time domain for two periods of $x(t)$.
(e) Find the expression of the spectrum $X_s(f)$ of the sampled waveform, and plot $X_s(f)$ in the frequency domain for $-6f_s \le f \le 6f_s$.
(f) The sampled signal is applied to an ideal lowpass filter with bandwidth $f_s/2$. Find the expression of the spectrum $Y(f)$ of the output signal, and plot $Y(f)$ in the frequency domain.
(g) The sampled signal is applied to an ideal lowpass filter with bandwidth $f_s/2$. Find the expression $y(t)$ of the output signal, and plot $y(t)$ in the time domain for two periods.

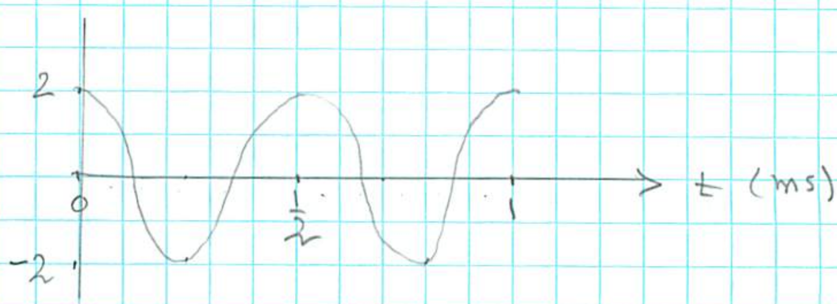
$$\left(\frac{t}{\tau} - n\frac{T_s}{\tau}\right) = \left(\frac{t}{\frac{1}{30{,}000}} - 3n\right) = (30{,}000t - 3n)$$

1

$x(t) = 2\cos(2\pi\, 2000 t) \Rightarrow f_1 = 2000\,\text{hz},\ T_1 = \frac{1}{2}\,\text{ms}.$

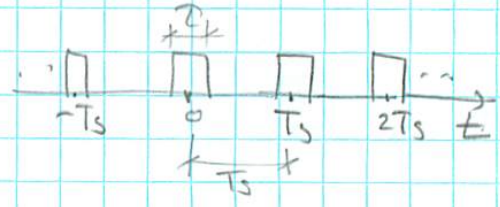
$f_s = 10{,}000\ \text{HZ}.$

(a)



(b)

(c) $p(t) = \sum_{n=-\infty}^{\infty} h\, \text{rect}\left(\frac{t-nT_s}{\tau}\right)$



$= \sum \text{rect}\left(\frac{t-nT_s}{\tau}\right)$

but $\frac{\tau}{T_s} = d = \frac{1}{3}$, and $T_s = \frac{1}{10000}$ so $10{,}000\,\tau = \frac{1}{3}$ $\therefore$ $\tau = \frac{1}{3}\frac{1}{10{,}000}$

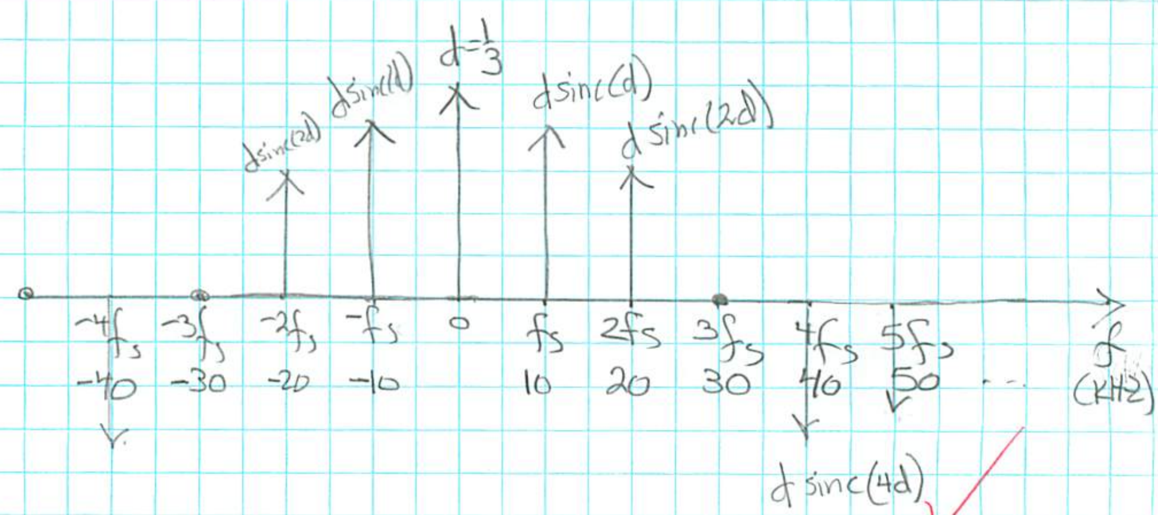
so $\tilde{p}(t) = \sum_{n=-\infty}^{\infty} \frac{\tau}{T_s} \text{sinc}(nd)\, e^{j\frac{2\pi}{T_s}nt}.$

so $\boxed{P(f) = \frac{\tau}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(nd)\, \delta(f - nf_s)}$

or $\boxed{P(f) = \frac{1}{3} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{n}{3}\right) \delta(f - n\,10000)}$

(2)

plot $P(f)$:

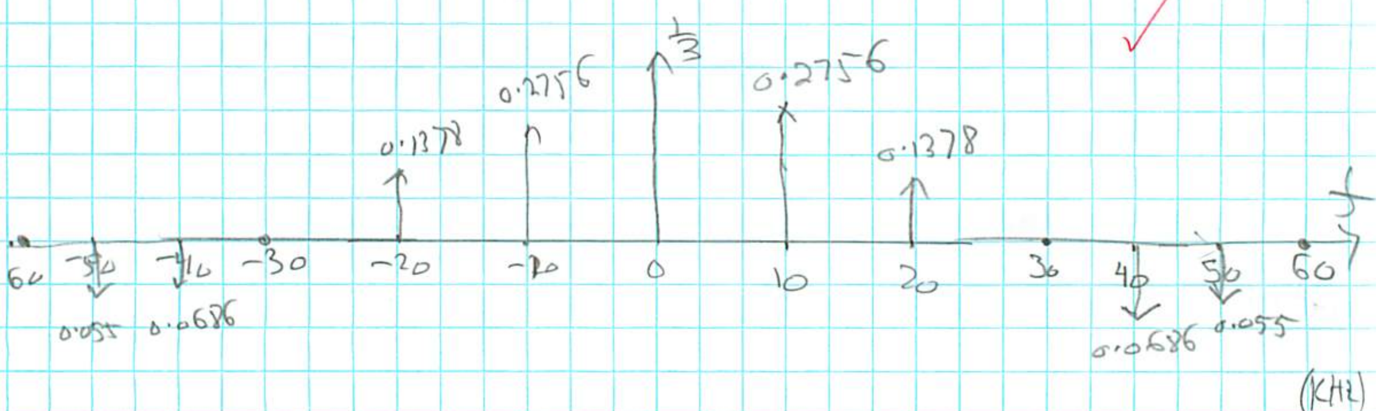


$$\text{sinc}(d) = \frac{\sin(\pi \frac{1}{3})}{\pi \frac{1}{3}} = 0.8269 \left(\tfrac{1}{3}\right) = 0.2756$$

$$\text{sinc}(2d) = \frac{\sin(\pi \frac{2}{3})}{\pi \frac{2}{3}} = 0.4134 \left(\tfrac{1}{3}\right) = 0.1378$$

$$\text{sinc}(4d) = \frac{\sin(\pi \cdot \frac{4}{3})}{\pi \frac{4}{3}} = -0.206 \left(\tfrac{1}{3}\right) = -0.0686$$

$$\text{sinc}(5d) = \frac{\sin(\pi \frac{5}{3})}{\pi \frac{5}{3}} = -0.165 \left(\tfrac{1}{3}\right) = -0.055$$



(3)

(d) $$x_s(t) = x(t)\left(\sum h\, \text{rect}\left(\frac{t-nT_s}{\tau}\right)\right)$$

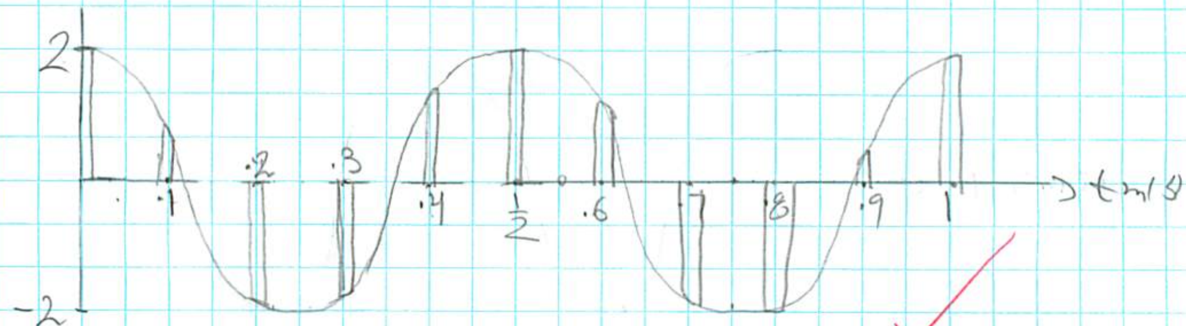
$$= x(t)\left(d\sum \text{sinc}(nd)\, e^{j\frac{2\pi}{T_s}nt}\right)$$

$$= x(t)\left(\frac{1}{3}\sum \text{sinc}\left(\frac{n}{3}\right) e^{j2\pi(10{,}000)nt}\right)$$

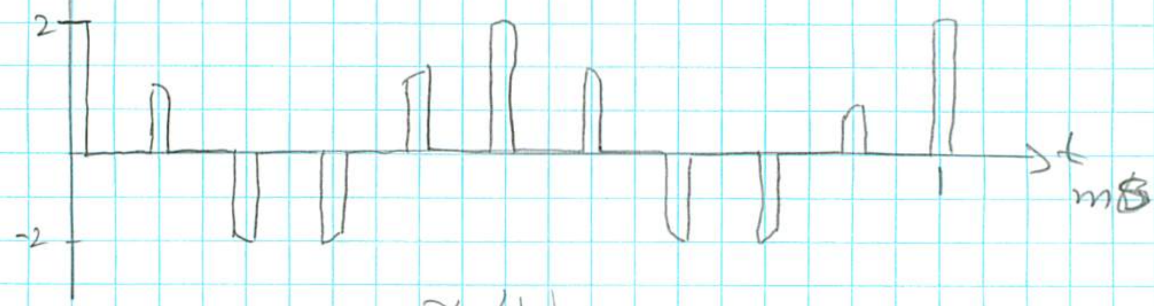
or $$x_s(t) = \frac{1}{3}\sum 2\cos(2\pi 2000t)\,\text{sinc}\left(\frac{n}{3}\right) e^{j2\pi 10000nt}$$

$$\boxed{x_s(t) = \frac{2}{3}\sum \cos(2\pi\, 2000\, nT_s)\,\text{sinc}\left(\frac{n}{3}\right) e^{j2\pi 10000nt}}$$

$T_s = 0.1$ (ms) , $T_1 = \frac{1}{2}$ ms , $d = \frac{1}{3} = \frac{\tau}{T_s}$ so $\tau = \frac{1}{3}T_s$.



⇓



$x_s(t)$.

→

4

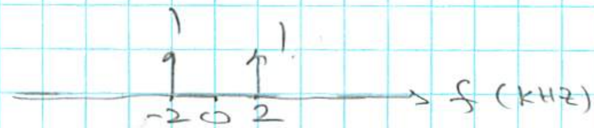
e

spectrum of $x(t)$ — spectrum of practical puls train

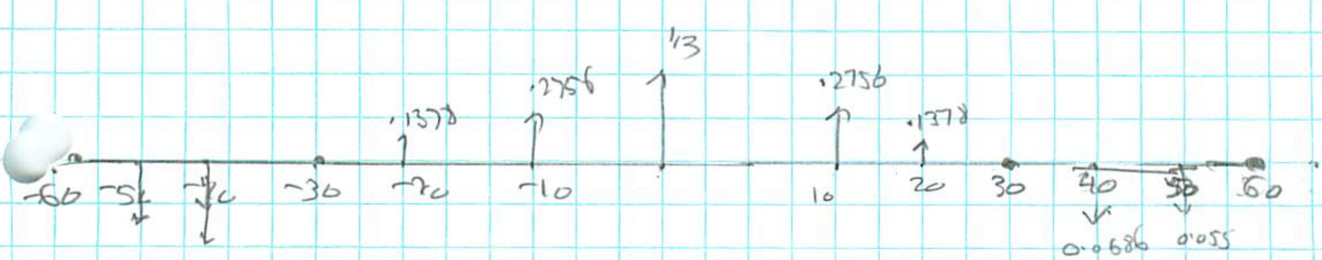
$$X_s(f) = X(f) \circledast G(f)$$

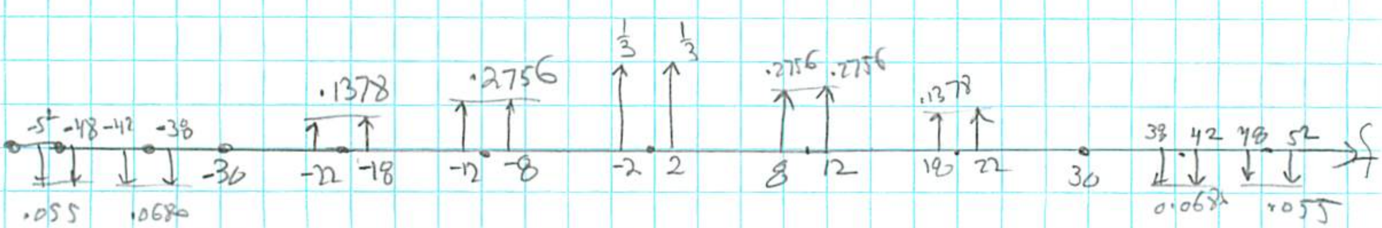
$$= X(f) \circledast d \sum \text{sinc}(nd)\, \delta(f - nf_s)$$

$$\boxed{X_s(f) = d \sum \text{sinc}(nd)\, X(f - nf_s)}$$



$\circledast$

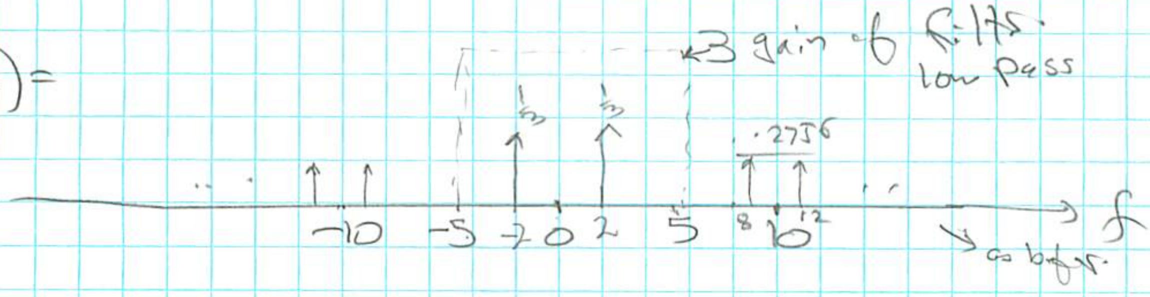




$X_s(f)$

5

$Y(f) =$



$$Y(f) = X_s(f) \cdot H(f)$$

where $H(f) = \frac{1}{d}\,\text{Rect}\left(\frac{f}{2B}\right)$

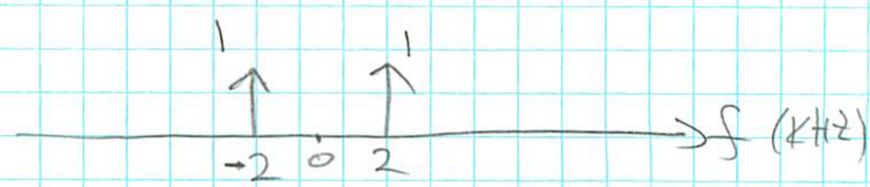
where $d = \frac{1}{3}$, $B = 2\,\text{KHZ}$.

so $\boxed{H(f) = 3\,\text{Rect}\left(\frac{f}{4000}\right)}$

$$\text{so } Y(f) = \left(d \sum \text{sinc}(nd)\, X(f - nf_s)\right) \frac{1}{d}\text{Rect}\left(\frac{f}{2B}\right)$$

$$\boxed{Y(f) = \sum_{n=-\infty}^{\infty} \text{Rect}\left(\frac{f}{4000}\right) \text{sinc}\left(\frac{n}{3}\right) X(f - n6000)}$$

$Y(f) =$
after filter applied.

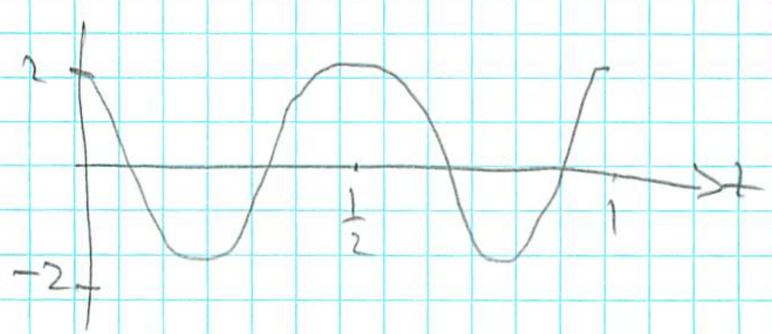


6

9 so $\boxed{y(t) = 2\cos(2\pi\, 2000 t)}$

$y(t)$ is same as $x(t)$!

so same plot as a !



Chapter 3

Handouts

Local contents

3.1 Mathematical indentities

Mathematical Tables

Trigonometric Identities

$\tan(\alpha) = [\sin(\alpha)]/\cos(\alpha)$
$\mathrm{cosec}\,(\alpha) = 1/\sin(\alpha)$
$\sec(\alpha) = 1/\cos(\alpha)$
$\cot(\alpha) = 1/\tan(\alpha)$
$\sin(\alpha) = \cos(90^o - \alpha) = \sin(180^o - \alpha)$
$\cos(\alpha) = \sin(90^o - \alpha) = - \cos(180^o - \alpha)$
$\tan(\alpha) = \cot(90^o - \alpha) = - \tan(180^o - \alpha)$

$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$
$\sin(\alpha - \beta) = \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$
$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$
$\cos(\alpha - \beta) = \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$
$\tan(\alpha + \beta) = [\tan(\alpha) + \tan(\beta)]/[1 - \tan(\alpha) \tan(\beta)]$
$\tan(\alpha - \beta) = [\tan(\alpha) - \tan(\beta)]/[1 + \tan(\alpha) \tan(\beta)]$

$\sin(\alpha) \cos(\beta) = (1/2) [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$
$\sin(\alpha) \sin(\beta) = (1/2) [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos(\alpha) \cos(\beta) = (1/2) [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$
$\cos(\alpha) \sin(\beta) = (1/2) [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

$\sin (2\alpha) = 2 \sin(\alpha) \cos(\alpha) = [2 \tan(\alpha)]/[1 + \tan^2(\alpha)]$
$\cos(2\alpha) = 2 \cos^2(\alpha) - 1 = 1 - 2 \sin^2(\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$
$\quad = [1 - \tan^2(\alpha)]/[1 + \tan^2(\alpha)]$
$\tan(2\alpha) = [2 \tan(\alpha)]/[1 - \tan^2(\alpha)]$
$\sin(3\alpha) = 3 \sin(\alpha) - 4 \sin^3(\alpha)$
$\cos(3\alpha) = 4 \cos^3(\alpha) - 3 \cos(\alpha)$
$\tan(3\alpha) = [3 \tan(\alpha) - \tan^3(\alpha)]/[1 - 3 \tan^2(\alpha)]$
$\sin(4\alpha) = 4 \sin(\alpha) \cos(\alpha) - 8 \sin^3(\alpha) \cos(\alpha)$
$\cos(4\alpha) = 8 \cos^4(\alpha) - 8 \cos^2(\alpha) + 1$
$\tan(4\alpha) = [4 \tan(\alpha) - 4 \tan^3(\alpha)]/[1 - 6 \tan^2(\alpha) + \tan^4(\alpha)]$

$\sin^2(\alpha) = (1/2) [1 - \cos(2\alpha)] = 1 - \cos^2(\alpha)$
$\cos^2(\alpha) = (1/2) [1 + \cos(2\alpha)] = 1 - \sin^2(\alpha)$
$\tan^2(\alpha) = [1 - \cos(2\alpha)]/[1 + \cos(2\alpha)]$
$\sin^3(\alpha) = (1/4) [3 \sin(\alpha) - \sin(3\alpha)]$
$\cos^3(\alpha) = (1/4) [3 \cos(\alpha) + \cos(3\alpha)]$
$\sin^4(\alpha) = (1/8) [3 - 4 \cos(2\alpha) + \cos(4\alpha)]$
$\cos^4(\alpha) = (1/8) [3 + 4 \cos(2\alpha) + \cos(4\alpha)]$

$\cos(\alpha) = [e^{j\alpha} + e^{-j\alpha}]/2$

1

$\sin(\alpha) = [e^{j\alpha} - e^{-j\alpha}]/2j$
$\tan(\alpha) = (-j)\,[e^{j\alpha} - e^{-j\alpha}]/[e^{j\alpha} + e^{-j\alpha}]$
$e^{j\alpha} = \cos(\alpha) + j\sin(\alpha)$
$e^{-j\alpha} = \cos(\alpha) - j\sin(\alpha)$

$\sin^2(\alpha) + \cos^2(\alpha) = 1$
$1 + \tan^2(\alpha) = \sec^2(\alpha)$
$1 + \cot^2(\alpha) = \mathrm{cosec}^2(\alpha)$

$\sin(\alpha) + \sin(\beta) = 2\sin[(1/2)(\alpha + \beta)]\cos[(1/2)(\alpha - \beta)]$
$\sin(\alpha) - \sin(\beta) = 2\cos[(1/2)(\alpha + \beta)]\sin[(1/2)(\alpha - \beta)]$
$\cos(\alpha) + \cos(\beta) = 2\cos[(1/2)(\alpha + \beta)]\cos[(1/2)(\alpha - \beta)]$
$\cos(\alpha) - \cos(\beta) = -2\sin[(1/2)(\alpha + \beta)]\sin[(1/2)(\alpha - \beta)]$
$\tan(\alpha) + \tan(\beta) = [\sin(\alpha + \beta)]/[\cos(\alpha)\cos(\beta)]$
$\tan(\alpha) - \tan(\beta) = [\sin(\alpha - \beta)]/[\cos(\alpha)\cos(\beta)]$

Indefinite Integrals

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax)$$

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \cos^2(ax)dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int \sin(ax)\cos(ax)dx = \frac{1}{2a}\sin^2(ax)$$

$$\int x\sin(ax)dx = \frac{1}{a^2}[\sin(ax) - ax\cos(ax)]$$

$$\int x\cos(ax)dx = \frac{1}{a^2}[\cos(ax) + ax\sin(ax)]$$

$$\int x^2\sin(ax)dx = \frac{1}{a^3}[2ax\sin(ax) + 2\cos(ax) - a^2x^2\cos(ax)]$$

$$\int x^2\cos(ax)dx = \frac{1}{a^3}[2ax\cos(ax) - 2\sin(ax) + a^2x^2\sin(ax)]$$

$$\int e^{ax}dx=\frac{1}{a}e^{ax}$$

$$\int xe^{ax}dx=\frac{1}{a^2}e^{ax}(ax-1)$$

$$\int x^2e^{ax}dx=\frac{1}{a^3}e^{ax}(a^2x^2-2ax+2)$$

$$\int x^ne^{ax}dx=\frac{x^n}{a}e^{ax}-\frac{n}{a}\int x^{n-1}e^{ax}dx$$

$$\int e^{ax}sin(bx)dx=\frac{1}{a^2+b^2}e^{ax}[asin(bx)-bcos(bx)]$$

$$\int e^{ax}cos(bx)dx=\frac{1}{a^2+b^2}e^{ax}[acos(bx)+bsin(bx)]$$

$$\int\frac{dx}{a^2+b^2x^2}=\frac{1}{ab}tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int\frac{x^2dx}{a^2+b^2x^2}=\frac{x}{b^2}-\frac{a}{b^3}tan^{-1}\left(\frac{bx}{a}\right)$$

Sums of Powers of the First n Integers

$$\sum_{k=1}^{n} k = 1+2+3+\dots\dots\dots+n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+\dots\dots\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^{n} k^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$$

$$\sum_{k=1}^{n} k^5 = \frac{n^2}{12}(n+1)^2(2n^2+2n-1)$$

If

$$\sum_{k=1}^{n} k^p = a_1n^{p+1}+a_2n^p+a_3n^{p-1}+\dots\dots\dots+a_{p+1}n$$

then

$$\sum_{k=1}^{n} k^{p+1} = \frac{p+1}{p+2} a_1 n^{p+2} + \frac{p+1}{p+1} a_2 n^{p+1} + \frac{p+1}{p} a_3 n^{p} + \ldots\ldots\ldots + \frac{p+1}{2} a_{p+1} n^{2}$$
$$+ \left[1 - (p+1) \sum_{k=1}^{p+1} \frac{a_k}{(p+3-k)} \right] n$$

Series Expansion

$$tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \ldots\ldots\ldots$$

$$sin^{-1}(x) = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}\frac{x^9}{9} + \ldots\ldots\ldots$$

$$cos^{-1}(x) = \frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3x^5}{40} - \ldots\ldots\ldots$$

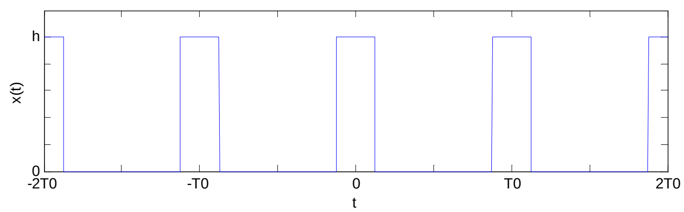
$$sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots\ldots\ldots$$

$$cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots\ldots\ldots$$

3.2 Fourier series represenation of common signals

FOURIER SERIES REPRESENTATION OF COMMON SIGNALS

Rectangular Pulse Train



τ = pulse width ($-\tau/2$ to $\tau/2$)
d = duty cycle = τ/T_0.
ω_0 = fundamental frequency = $2\pi/T_0$
sinc(x) = sin(πx)/(πx)

$$X_n = \frac{h\tau}{T_0}\operatorname{sinc}(nd) = h\,d\,\operatorname{sinc}(nd) = \begin{cases} hd, & n = 0 \\ h\dfrac{\sin(n\pi d)}{n\pi}, & n \neq 0 \end{cases}$$

$$x(t) = hd + \sum_{n=1}^{\infty} 2hd\,\operatorname{sinc}(nd)\cos(n\omega_0 t)$$

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$$c_0 = hd = \frac{h\tau}{T_0}, \quad c_n = \left|2hd\,\operatorname{sinc}(nd)\right|, \quad \theta_n = \begin{cases} \pi, & 2hd\,\operatorname{sinc}(nd) < 0 \\ 0, & otherwise \end{cases}$$

If $\tau = T_0/2$, d = 1/2, and the equations given above becomes

$$X_n = \frac{h}{2}\operatorname{sinc}\left(\frac{n}{2}\right) = \begin{cases} \dfrac{h}{2}, & n = 0 \\ h\dfrac{\sin\left(\dfrac{n\pi}{2}\right)}{n\pi}, & n \neq 0 \end{cases}$$

$$x(t) = \frac{h}{2} + \sum_{n=1}^{\infty} h\,\operatorname{sinc}\left(\frac{n}{2}\right)\cos(n\omega_0 t)$$

1

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

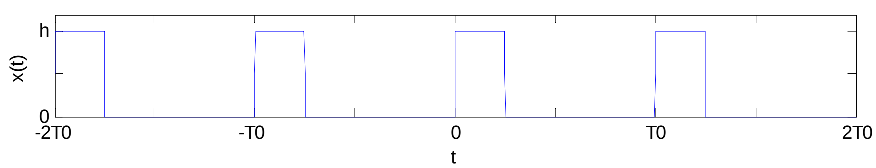
$$c_0 = \frac{h}{2}, \quad c_n = \left| h\,\mathrm{sinc}\left(\frac{\mathrm{n}}{2}\right) \right|, \quad \theta_n = \begin{cases} \pi, & h\,\mathrm{sinc}\left(\frac{\mathrm{n}}{2}\right) < 0 \\ 0, & otherwise \end{cases}$$

Let y(t) = x(t – T_0/2). Then,

$$Y_n = X_n e^{-jn\frac{2\pi}{T_0}\frac{T_0}{2}} = X_n e^{-jn\pi} = X_n \cos(n\pi) = \begin{cases} \frac{h}{2}, & n = 0 \\ h\frac{(-1)^{\mathrm{n}} \sin\left(\frac{\mathrm{n}\pi}{2}\right)}{n\pi}, & n \neq 0 \end{cases}$$

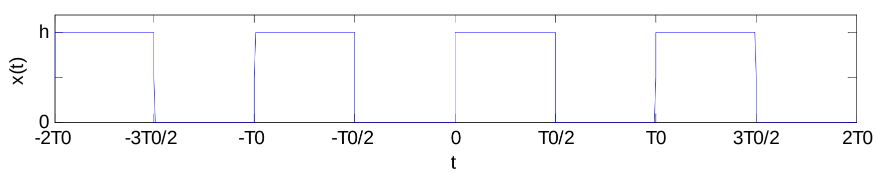
Rectangular Pulse Train with Time Shifting

$t_o = \tau/2$.



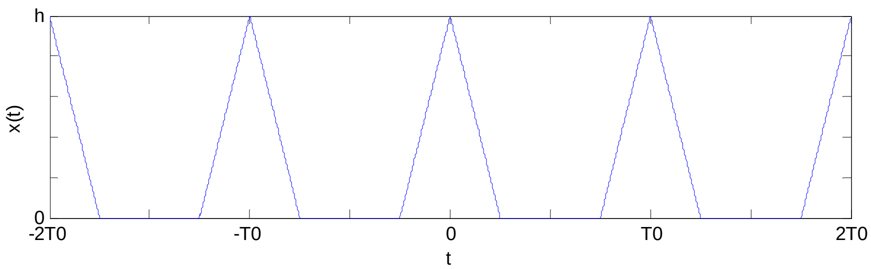
$$X_n = \frac{h\tau}{T_0}\mathrm{sinc}(nd)e^{-jn\frac{2\pi}{T_o}t_o} = h\,d\,\mathrm{sinc}(nd)e^{-jn\frac{2\pi}{T_o}t_o} = h\,d\,\mathrm{sinc}(nd)e^{-jn\frac{2\pi}{T_o}\frac{\tau}{2}} = h\,d\,\mathrm{sinc}(nd)e^{-jn\pi\frac{\tau}{T_o}}$$

If $\tau = T_o/2$, we have



$$X_n = \frac{h}{2}\mathrm{sinc}\left(\frac{n}{2}\right)e^{-jn\pi} = \frac{h}{2}\mathrm{sinc}\left(\frac{n}{2}\right)e^{-jn\pi} = \frac{h}{2}\mathrm{sinc}\left(\frac{n}{2}\right)\cos(n\pi)$$

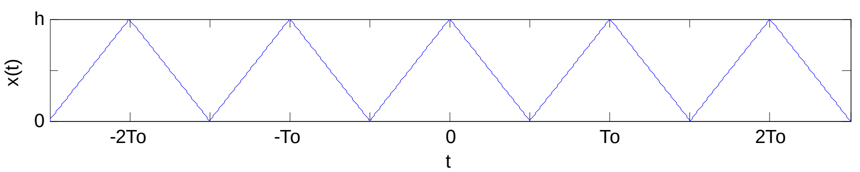
Triangular Pulse Train



τ = half of the base of the triangle ($-\tau \leq t \leq \tau$)
d = duty cycle = τ/T_0.
ω_0 = fundamental frequency = $2\pi/T_0$

$$X_n = hd\,\text{sinc}^2(nd) = \frac{h\tau}{T_0}\text{sinc}^2\left(\frac{n\omega_0\tau}{2\pi}\right)$$

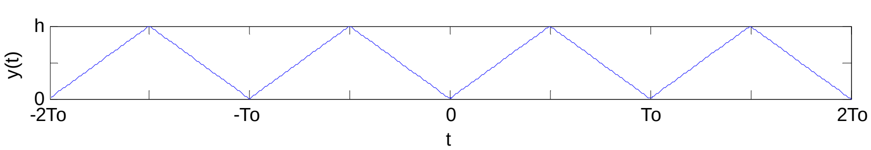
If $\tau = T_0/2$, then the pulse train looks like



and

$$X_n = \frac{h}{2}\text{sinc}^2\left(\frac{n}{2}\right) = \begin{cases} \dfrac{h}{2}, & n = 0 \\ 0, & n = even \\ \dfrac{2h}{n^2\pi^2}, & n = odd \end{cases}$$

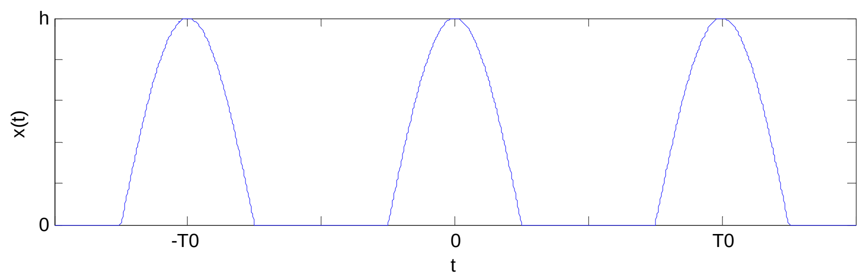
Let $y(t) = x(t - T_0/2)$.



Then,

$$Y_n = X_n e^{-jn\frac{2\pi}{T_0}\frac{T_0}{2}} = X_n e^{-jn\pi} = X_n \cos(n\pi) = \begin{cases} \frac{h}{2}, & n = 0 \\ 0, & n = even \\ \frac{-2h}{n^2\pi^2}, & n = odd \end{cases}$$

Half-Wave Rectified Cosine

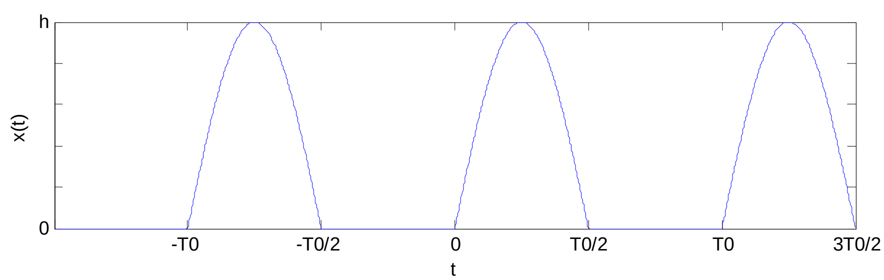


$$X_1 = \frac{h}{4}$$

$$X_{-1} = \frac{h}{4}$$

$$X_n = \frac{h}{\pi} \frac{\cos\left(\frac{n\pi}{2}\right)}{1 - n^2}, \quad n \neq \pm 1$$

Half-Wave Rectified Sine



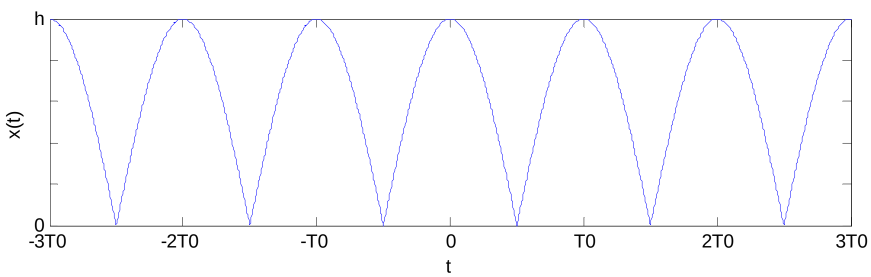
h = amplitude, $j = \sqrt{-1}$

$$X_1 = \frac{-jh}{4}$$

$$X_{-1} = \frac{jh}{4}$$

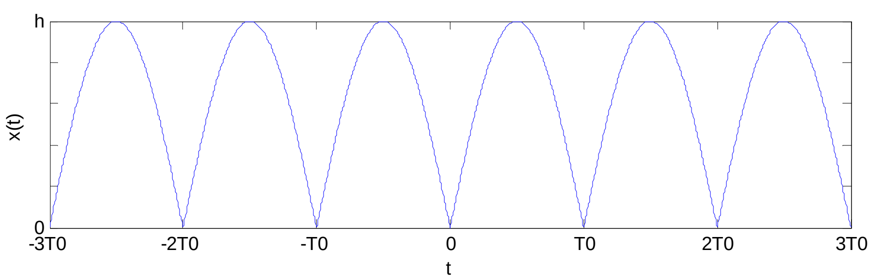
$$X_n = \frac{h}{\pi} \frac{\cos^2\left(\frac{n\pi}{2}\right)}{1-n^2} = \begin{cases} \frac{h}{\pi}, & n = 0 \\ 0, & n = \pm 3, \pm 5, \pm 7, ... \\ \frac{h}{\pi}\frac{1}{1-n^2}, & n = \pm 2, \pm 4, \pm 6, ... \end{cases}$$

Full-Wave Rectified Cosine



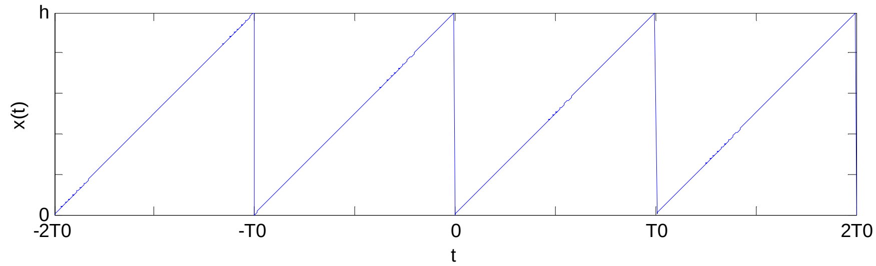
$$X_n = \frac{2h}{\pi} \frac{\cos(n\pi)}{1-4n^2} = \frac{2h}{\pi} \frac{(-1)^n}{1-4n^2}$$

Full-Wave Rectified Sine



$$X_n = \frac{2h}{\pi} \frac{\cos^2(n\pi)}{1-4n^2} = \frac{2h}{\pi} \frac{1}{1-4n^2}$$

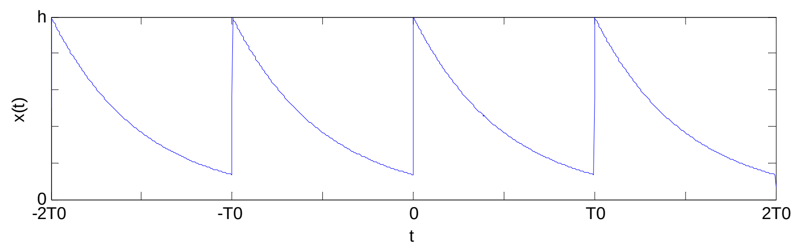
Sawtooth



$$X_0 = \frac{h}{2}$$

$$X_n = \frac{jh}{2\pi n}, \quad n \neq 0, \quad j = \sqrt{-1}$$

Exponential Decay



$$X_n = \frac{h}{T_0} \frac{1 - e^{-aT_0}}{a + jn\frac{2\pi}{T_0}}$$

3.3 Properties of Fourier transform

Properties of the Fourier Transform

Property	$f(t)$	$F(\omega)$
Linearity (Superposition)	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Time Shifting	$f(t - t_d)$	$e^{-j\omega t_d} F(\omega)$
Time Scaling	$f(ct)$	$\frac{1}{\lvert c\rvert} F\left(\frac{\omega}{c}\right)$
Symmetry (Duality)	$F(t)$	$2\pi f(-\omega)$
Time Reversal	$f(-t)$	$F(-\omega)$
Frequency Scaling	$f(t)e^{j\omega_c t}$	$F(\omega - \omega_c)$
Modulation	$f(t)cos(\omega_c t)$	$\frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c)$
Time Differentiation	$\frac{df(t)}{dt}$	$j\omega F(\omega)$
Frequency Differentiation	$tf(t)$	$j\frac{dF(\omega)}{d\omega}$
Conjugate	$f^*(t)$	$F^*(-\omega)$
Integration	$\int_{-\infty}^{t} f(\lambda)d\lambda$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
Convolution	$\int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$	$H(\omega)X(\omega)$
Multiplication	$f_1(t)f_2(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(v)F_2(\omega - v)dv$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert f(t)\rvert^2 dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2 d\omega$

Table 16.2 Fourier Transform Pairs (a > 0)

$f(t)$	$F(\omega)$	$F(f)$
$\Pi\left(\frac{t}{a}\right) = rect\left(\frac{t}{a}\right)$	$a\,\mathrm{sinc}\left(\frac{\omega a}{2\pi}\right)$	$a\,\mathrm{sinc}(fa)$
$\Lambda\left(\frac{t}{a}\right) = tri\left(\frac{t}{a}\right)$	$a\,\mathrm{sinc}^2\left(\frac{\omega a}{2\pi}\right)$	$a\,\mathrm{sinc}^2(fa)$
$e^{-at}u(t)$	$\frac{1}{j\omega + a}$	$\frac{1}{j2\pi f + a}$
$e^{at}u(-t)$	$\frac{1}{-j\omega + a}$	$\frac{1}{-j2\pi f + a}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{\omega^2 + a^2}$	$\frac{2a}{4\pi^2 f^2 + a^2}$
$e^{-at}u(t) - e^{at}u(-t)$	$\frac{-2j\omega}{\omega^2 + a^2}$	$\frac{-j4\pi f}{4\pi^2 f^2 + a^2}$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$tu(t)$	$\frac{\pi}{j\omega}\delta(\omega) + \frac{1}{(j\omega)^2}$	$\frac{1}{j4\pi f}\delta(f) + \frac{1}{(j2\pi f)^2}$
$te^{-at}u(t)$	$\frac{1}{(j\omega + a)^2}$	$\frac{1}{(j2\pi f + a)^2}$
$cos(\omega_c t) = cos(2\pi f_c t)$	$\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$

$sin(\omega_c t) = sin(2\pi f_c t)$	$-j\pi[\delta(\omega-\omega_c)-\delta(\omega+\omega_c)]$	$\frac{-j}{2}[\delta(f-f_c)-\delta(f+f_c)]$
$e^{-at}u(t)cos(\omega_c t)$	$\frac{j\omega+a}{(j\omega+a)^2+\omega_c^2}$	$\frac{j2\pi f+a}{(j2\pi f+a)^2+(2\pi f_c)^2}$
$e^{-at}u(t)sin(\omega_c t)$	$\frac{\omega_c}{(j\omega+a)^2+\omega_c^2}$	$\frac{2\pi f_c}{(j2\pi f+a)^2+(2\pi f_c)^2}$
$sgn(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$\mathrm{sinc}(ct)$	$\frac{1}{c}rect\left(\frac{\omega}{2\pi c}\right)$	$\frac{1}{c}rect\left(\frac{f}{c}\right)$
$\mathrm{sinc}^2(ct)$	$\frac{1}{c}tri\left(\frac{\omega}{2\pi c}\right)$	$\frac{1}{c}tri\left(\frac{f}{c}\right)$
$cos\left(\frac{\pi t}{a}\right)rect\left(\frac{t}{a}\right)$	$\frac{2a}{\pi}\frac{cos\left(\frac{\omega a}{2}\right)}{1-\left(\frac{\omega a}{\pi}\right)^2}$	$\frac{2a}{\pi}\frac{cos(\pi af)}{1-(2af)^2}$
$\frac{1}{2}\left[1+cos\left(\frac{\pi t}{a}\right)\right]rect\left(\frac{t}{2a}\right)$	$a\frac{sin(\omega a)}{\omega a\left[1-\left(\frac{\omega a}{\pi}\right)^2\right]}$	$a\frac{sin(2\pi fa)}{2\pi fa\left[1-(2af)^2\right]}$

Chapter 4

HW's, and computer assignments

Local contents

4.1 HW's

Nasser M. Abbasi

HW1, ECE 405

By Nasser M. Abbasi
Cal Poly Pomona, ECE 405, first session, summer 2010.

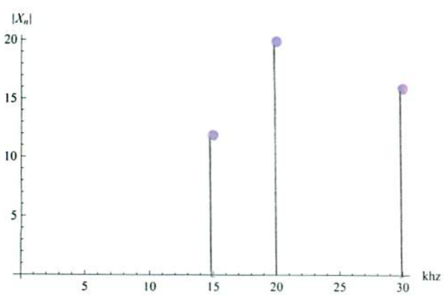
Problem 1

- **part(a)**

$$x[t\_] := 12\,\text{Cos}\left[2\,\pi\,15\,000\,t - \frac{60}{180}\,\pi\right] - 20\,\text{Cos}\left[2\,\pi\,20\,000\,t + \frac{30}{180}\,\pi\right] - 16\,\text{Cos}\left[2\,\pi\,30\,000\,t - \frac{70}{180}\,\pi\right];$$

one sided magnitude spectrum

```
data = {{15, 12}, {20, -20}, {30, 16}};
ListPlot[Abs[data], Filling → Axis, AxesOrigin → {0, 0},
 PlotMarkers → {Automatic, 12}, AxesLabel → {"khz", "|Xn|"}]
```



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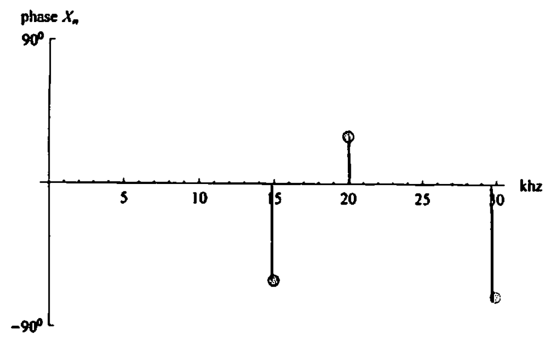
- **part(b)**

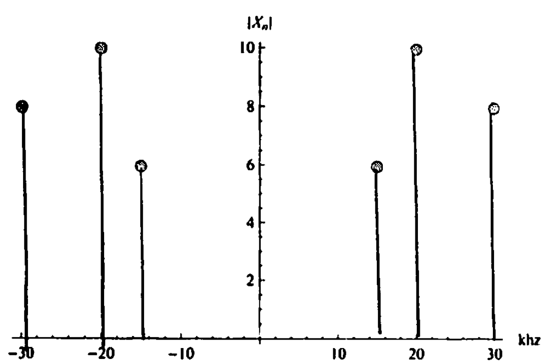
```
data = {{15, -60}, {20, 30}, {30, -70}};
ListPlot[data, Filling → Axis, AxesOrigin → {0, 0},
 Ticks → {Automatic, {{-90, "-90°"}, {90, "90°"}}}, PlotMarkers → {Automatic, 12},
 AxesLabel → {"khz", "phase Xn"}, PlotRange → {Automatic, {-90, 90}}]
```



question: As Dr Kang why key solution has angles summed in different way

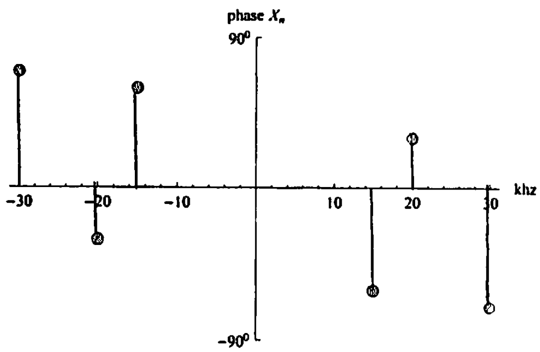
- **part(c)**

```
data = {{15, 6}, {20, 10}, {30, 8}, {-15, 6}, {-20, 10}, {-30, 8}};
ListPlot[data, Filling → Axis, AxesOrigin → {0, 0},
 PlotMarkers → {Automatic, 12}, AxesLabel → {"khz", "|Xn|"}]
```



- **part(d)**

```
data = {{15, -60}, {20, 30}, {30, -70}, {-15, 60}, {-20, -30}, {-30, 70}};
ListPlot[data, Filling → Axis, AxesOrigin → {0, 0},
 Ticks → {Automatic, {{-90, "-90°"}, {90, "90°"}}}, PlotMarkers → {Automatic, 12},
 AxesLabel → {"khz", "phase Xn"}, PlotRange → {Automatic, {-90, 90}}]
```



Problem 2

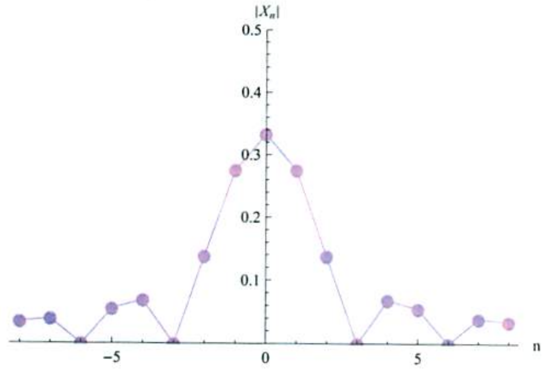
- **part(a)**

```
ClearAll["Global`*"];
xn[n_] := h d Sinc[Pi n d] Exp[-I 2 Pi f0 n t0];
parameters = {h → 1, d → 1 / 3, f0 -> 1 / (3 * 10^-3), t0 → 10^-3};
xn[n] /. parameters
```

$$\frac{1}{3} e^{-\frac{2}{3} i n \pi} \text{Sinc}\left[\frac{n \pi}{3}\right]$$

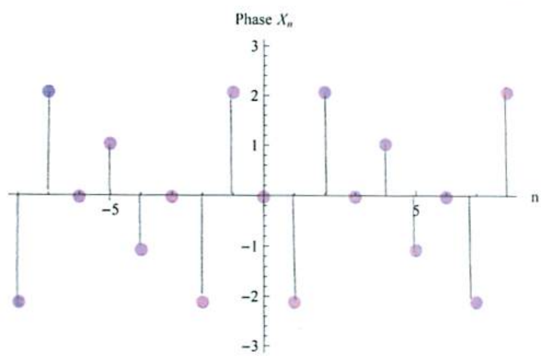
- **part(b)**

```
data = Table[{n, Abs[xn[n] /. parameters]}, {n, -8, 8}];
Show[ListPlot[data, Filling → Axis, PlotRange → {Automatic, {0, .5}},
  PlotMarkers → {Automatic, 12}, AxesLabel → {"n", "|Xn|"}], ListPlot[data, Joined → True]]
```



- **part(c)**

```
data = Table[{n, Arg[xn[n] /. parameters]}, {n, -8, 8}];
ListPlot[data, Filling → Axis, PlotRange → {Automatic, {-Pi, Pi}},
 PlotMarkers → {Automatic, 12}, AxesLabel → {"n", "Phase Xn"}]
```



- **part(d)**

Power in the n^{th} harmonic is $2\,|\,X_n\,|^2$ where we multiply by 2 to take care of both sides of the spectrum. Hence for $n = 2$

```
Abs[xn[2] /. parameters];
Row[{N[2 * %^2], " watt"}]
```

```
0.0379954 watt
```

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- **part(e)**

Fourier serices of x(t) is $\sum_{n=-\infty}^{\infty} x_n$ Exp[I 2 π f0 n t]
at n=0

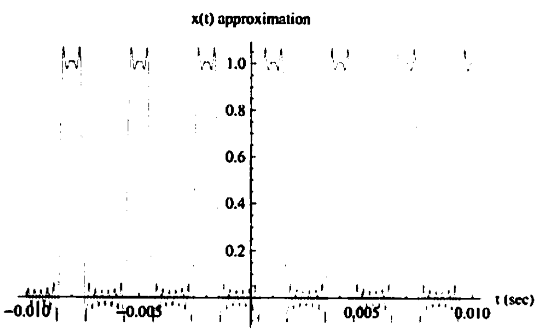
```
xn[0] /. parameters
```

$\frac{1}{3}$

substituting values, we obtain $x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{3} e^{-\frac{2}{3} i n \pi + \frac{2000}{3} i n \pi t} \text{Sinc}\left[\frac{n\pi}{3}\right]$

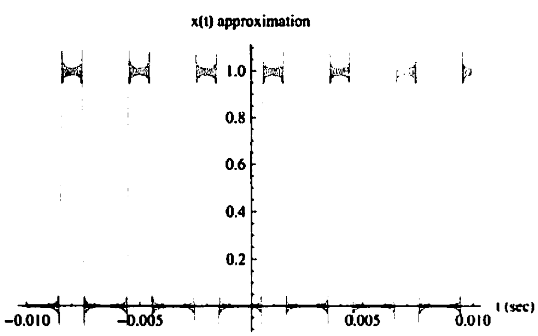
To verify, here is a plot of x(t) for n=10 terms for t=-10 ms to t=10 ms. Notice the delay which is 1 ms

```
fourier = Sum[xn[n] Exp[I 2 π f0 n t], {n, -10, 10}];
Plot[fourier /. parameters, {t, -0.01, .01}, AxesLabel -> {"t (sec)", "x(t) approximation"}]
```



Changine for numbers for terms to say n=30, we obtain

```
fourier = Sum[xn[n] Exp[I 2 π f0 n t], {n, -30, 30}];
Plot[fourier /. parameters, {t, -0.01, .01}, AxesLabel -> {"t (sec)", "x(t) approximation"}]
```



The approximation is better. But notice Gibbs phenomena at the corners.

part(f)

y(t)=x(t)*cos(2π 30000t). To find y(t), convolve the fourier transform of x(t) with the fourier transform of cos(2π 30000t). The fourier transform of cos(2π 30000t) is $\frac{1}{2}$ times impulse at frequency -30khz and at frequency 30khz. The effect of convolving the fourier transform of x(t) with these 2 impluse is to shift the fourier transform of x(t) and center it over the impulses. Hence $Y_n = \frac{1}{2} X_{n-m} + \frac{1}{2} X_{n+m}$ where m is amount of shift needed to center X_n over 30khz and -30khz

The amount of shift is given by $m = \frac{30\text{ khz}}{\frac{1}{3}\text{khz}} = 90$, hence 90 spectral lines are needed to shift X_n , hence

$Y_n = \frac{1}{2} X_{n-90} + \frac{1}{2} X_{n+90}$

$Y_n = \frac{1}{2}\, h\, d$ Sinc[$(n-90)\,d$] Exp[$-I$ 2 Pi f0 $(n-90)$ t0]+$\frac{1}{2}\, h\, d$ Sinc[$(n+90)\,d$] Exp[$-I$ 2 Pi f0 $(n+90)$ t0]

simplify, the above becomes

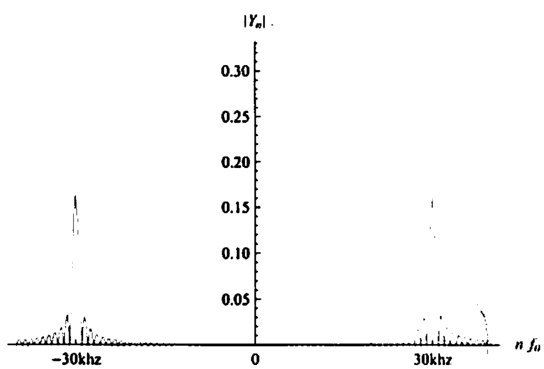
$$\text{yn[n\_]} := \frac{1}{6} e^{-\frac{2}{3} i (-90+n) \pi} \text{Sinc}\left[\text{Pi} \frac{1}{3} (-90+n)\right] + \frac{1}{6} e^{-\frac{2}{3} i (90+n) \pi} \text{Sinc}\left[\text{Pi} \frac{90+n}{3}\right]$$

```
Simplify[yn[n]]
```

$$\frac{1}{6} e^{-\frac{2}{3} i n \pi} \left(\text{Sinc}\left[\frac{1}{3}(-90+n)\pi\right] + \text{Sinc}\left[\frac{1}{3}(90+n)\pi\right]\right)$$

Here is a plot of the magnitude and phase of Y_n

```
data = Table[{n, Abs[yn[n]]}, {n, -120, 120, 1}];
ListPlot[data, Joined → True, AxesLabel → {Style["n f0", Italic], "|Yn|"},
 PlotRange → {Automatic, {0, 2 / 6}}, Ticks → {{{-90, "-30khz"}, 0, {90, "30khz"}}, Automatic}]
```



Now plot the phase

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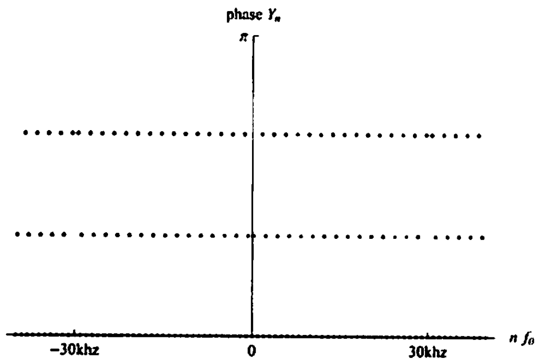
```
data = Table[{n, Arg[yn[n]]}, {n, -120, 120, 1}];
ListPlot[data, Joined → False, Filling → Axis,
 AxesLabel → {Style["n f0", Italic], "phase Yn"}, PlotRange → {Automatic, {0, Pi}},
 Ticks → {{{-90, "-30khz"}, 0, {90, "30khz"}}, {0, {Pi, Pi}}}]
```



- **part(g)**

Applying the filter to y(t) in the frequency domain: The filter has width of 5khz, hence 2.5khz on each side of the center of the filter. The filter is centered at 30khz, hence frequencies of 30+2.5 khz and 30-2.5 kz will be allowed through. Since each f_0 is $\frac{1}{3}$ khz, then the number of spectral lines that will be allowed through is

```
5 / (1 / 3)
15
```

Hence there will be 7 spectral lines on each side of the center of the filter. Hence n will run from 90 to 97, and also run from 83 to 89. Let $w(t)$ be the signal whose spectrum is those spectral lines obtained from the filter. Hence we write $w(t) = \sum_{n=83}^{97} Y_n\, e^{-i 2\pi f_0 t}$ where $f_0 = \frac{1}{3}$ khz and $Y_n = \frac{1}{6}\, e^{-\frac{2}{3} i(-90+n)\pi}\, \text{Sinc}\left[\text{Pi}\, \frac{1}{3}(-90+n)\right] + \frac{1}{6}\, e^{-\frac{2}{3} i(90+n)\pi}\, \text{Sinc}\left[\text{Pi}\, \frac{90+n}{3}\right]$
Using the second form of the fourier series, compute to obtain

$$\text{w1}[\text{n\_}] := \frac{1}{3}\,\text{Sinc}\left[\frac{\text{Pi n}}{3}\right] \text{Cos}\left[2\,\pi\,(30\,000 + \text{n}\,333)\ \text{t} - \text{n}\,\frac{2}{3}\,\pi\right]$$

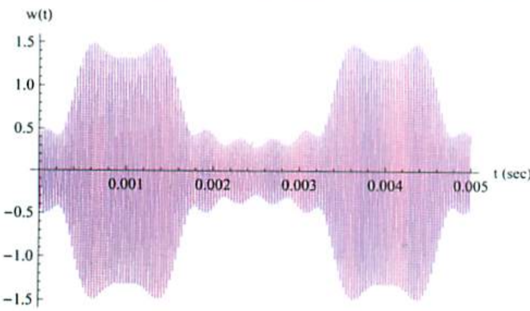
$$\text{w2}[\text{n\_}] := \frac{1}{3}\,\text{Sinc}\left[\frac{\text{Pi n}}{3}\right] \text{Cos}\left[2\,\pi\,(30\,000 - \text{n}\,333)\ \text{t} + \text{n}\,\frac{2}{3}\,\pi\right]$$

$$\text{filterOutput} = \left(\sum_{n=1}^{7} \text{w1}[\text{n}] + \sum_{n=-7}^{-1} \text{w2}[\text{n}] + \text{w1}[0] + \text{w2}[0]\right) \ //\ \text{N}$$

```
0.666667 Cos[188 496. t] - 0.0787613 Sin[0.523599 - 203 142. t] +
 0.137832 Sin[0.523599 - 196 865. t] - 0.551329 Sin[0.523599 - 190 588. t] -
 0.275664 Sin[0.523599 + 192 680. t] + 0.110266 Sin[0.523599 + 198 957. t]
```

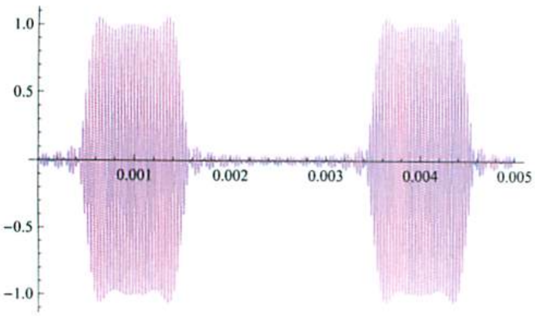
plot w(t)

```
Plot[filterOutput, {t, 0, .005}, PlotRange → All, AxesLabel → {"t (sec)", "w(t)"}]
```



compare w(t), the signal from the bandpass filter, with the original signal y(t) to see the effect of the filtering

```
Plot[(fourier /. parameters) * Cos[2 π * 30 000 t], {t, 0, .005}, PlotRange → All]
```



- **part(h)**

using the shifting property, $Z_n = Y_n\, e^{-\mathrm{I} 2 \pi f_0 t_0 n}$ where f_0 is the fundamental frequency of y(t) and t_0 is the delay amout

```
parameters = {h → 1, d → 1 / 3, f0 -> 1 / (3 * 10^-3), t0 → 10^-3/4};
zn[n_] := yn[n] Exp[-I 2 π f0 t0 n] /. parameters
zn[n] // Simplify
```

$$\frac{1}{6} e^{-\frac{1}{6} i n \pi} \left(\mathrm{Sinc}\left[\frac{1}{3} (-90 + n) \pi\right] + \mathrm{Sinc}\left[\frac{1}{3} (90 + n) \pi\right]\right)$$

Problem 3

- **part(a)**

```
width = 0.25 * 10^-3; period = 10^-3; h = 1; f0 = 1000;
```

X_n for triangular pulse the width term used to find the duty cycle is taken as 1/2 of the width of the base of the triangle.

```
d = width / period
0.25
```

Hence X_n =h d (Sinc[Pi n d])^2=$\frac{1}{2}$ (Sinc[Pi $\frac{n}{2}$])^2

```
xn[n_] := 1/2 (Sinc[Pi n/2])^2
```

- **part(b)**

The fourier series approximation is given by $\sum_{n=-\infty}^{\infty} X_n\, e^{-I\,2\,\pi\,f_0\,n\,t}$ where $f_0 = 1$ khz in this example. Hence

$x(t) = \frac{1}{2} \sum_{n=-\infty}^{\infty} (\text{Sinc}[\text{Pi}\, \frac{n}{2}])^2\, e^{-I\,2\,\pi\, 1000\, n\, t}$

- **part(c)**

$H(s) = \frac{\frac{1}{RC}}{S+\frac{1}{RC}} = \frac{1}{RCS+1}$ where R=1000 ohm, $C = 10^{-6}$, hence RC = 10^{-3}, then H(s)=$\frac{1}{10^{-3}S+1} = \frac{1000}{S+1000}$,

hence H(j ω) = $\frac{1000}{j\omega+1000}$

$H(j\omega) = |H(j\omega)|$ Arg(H(jω).

Now, y(t)=H(jω) x(t) , hence in terms of the fourier coefficients, we write

$Y_n = H(j\omega_0 n) X_n$=(| H(jω) | Arg(H(jω))* (| X_n | Arg(X_n))

hence

Y_n = | $H(j\omega_0 n)$ | * | X_n | (Arg($H(j\omega_0 n)$) + (Arg(X_n)))

Hence | Y_n | = | $H(j\omega_0 n)$ |* | X_n |
and
Arg(Y_n) = Arg($H(j\omega_0 n)$) + Arg(X_n)

But | $H(j\omega_0 n)$ | = $\frac{1000}{\sqrt{\omega_0^2 n^2+1000^2}}$
and
Arg($H(j\omega_0 n)$)) = $-\arctan\left(\frac{n\omega_0}{1000}\right)$
Now, | X_n | = $\frac{1}{2}$ (Sinc[Pi $\frac{n}{2}$])^2 and Arg[X_n] = 0 since there is no delay term.

Hence $Y_n = \frac{1000}{\sqrt{\omega_0^2 n^2+1000^2}} \frac{1}{2}$ (Sinc[Pi $\frac{n}{2}$])^2 Exp[-j arctan ($\frac{n\omega_0}{1000}$)]
Now, $\omega_0 = 2\pi f_0 = 2\pi$ 1000, hence
$Y_n = \frac{1000}{\sqrt{(2\pi\, 1000\, n)^2+1000^2}} \frac{1}{2}$ (Sinc[Pi $\frac{n}{2}$])^2 Exp[-j arctan (n 2 π)]

```
yn[n_] := 1/2 (Sinc[Pi n/2])^2 Exp[-I ArcTan [n 2 π]] 1000/Sqrt[(2 π 1000 n)^2 + 1000^2]
```

Now that Y_n is found, we can find

y(t)=$\sum_{n=-\infty}^{\infty} Y_n$ Exp[$I\, 2\pi f_0 n t$]

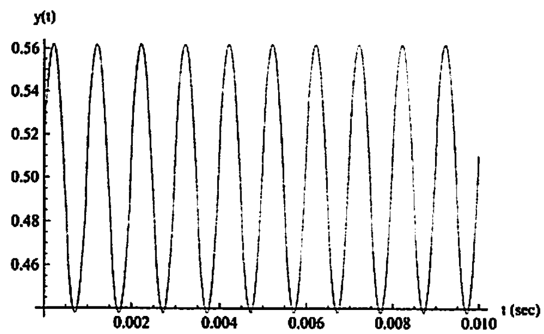
where f_0 = 1 khz, hence to plot y(t) using say 10 terms in foruier series and compare to x(t)

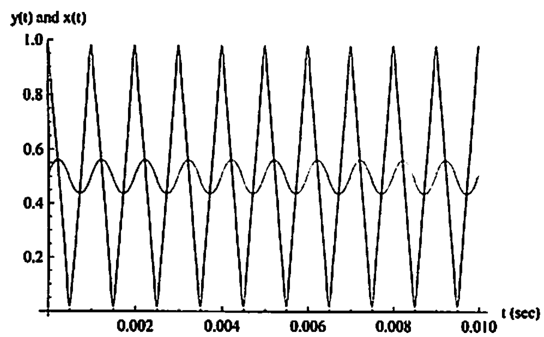
```
y[t_] := Sum[yn[n] Exp[I 2 π 1000 n t], {n, -10, 10}]
```

$$x[t\_] := \frac{1}{2}\sum_{n=-10}^{10}\left(\text{Sinc}\left[\text{Pi}\ \frac{n}{2}\right]\right)^2\ \text{Exp}[-\text{I}\ 2\ \pi\ 1000\ n\ t]$$

```
Plot[y[t], {t, 0, .01}, AxesLabel → {"t (sec)", "y(t)"}]

Plot[{y[t], x[t]}, {t, 0, .01}, AxesLabel → {"t (sec)", "y(t) and x(t)"}]
```





problem 4

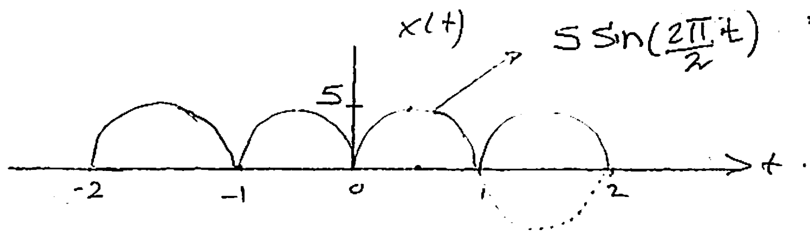
$$5\int_0^1 \text{Sin}[\text{Pi}\ t]\ \text{Exp}[\text{I}\ 2\ \text{Pi}\ n\ t]\ dt$$

$$\frac{5\left(1+e^{2\,i\,n\,\pi}\right)}{\pi - 4\,n^2\,\pi}$$

(1)

4

$= 5\sin(\pi t)$



$T_0 = 1$ sec, $f_0 = 1$ Hz, $h = 5$, $\omega_0 = 2\pi$

Since $x(t)$ is even, it will only have a_n terms. so using.

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\omega_0 n t)$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{1}\int_0^1 5\sin(\pi t)\,dt = 5\left[\frac{-\cos(\pi t)}{\pi}\right]_0^1$$

$$= -\frac{5}{\pi}\left[\cos\pi - \cos 0\right] = -\frac{5}{\pi}\left[-1-1\right] = \boxed{\frac{10}{\pi}}$$

$$a_n = \frac{2}{T}\int_T x(t)\, e^{j\frac{2\pi}{1}nt}\,dt = 2\int_0^1 5\sin(\pi t)\, e^{j2\pi n t}\,dt.$$

$$= 10\int_0^1 \sin(\pi t)\, e^{j2\pi n t}\,dt \quad \text{integration by parts. gives}$$

$$= \frac{10\cdot(1+e^{j2n\pi})}{\pi - 4n^2\pi}$$

for n integer, $e^{j2n\pi} = \cos 2n\pi + j\,\underbrace{\cancel{\sin 2n\pi}}_{=0} = 1.$

so $$a_n = \frac{10(2)}{\pi(1-4n^2)} = \frac{20}{\pi(1-4n^2)}$$

so $$\boxed{x(t) = \frac{10}{\pi} + \sum_{n=1}^{\infty} \frac{20}{\pi(1-4n^2)}\cos(2\pi n t)}$$

(5) $F[x(t)] = X(\omega)$. (3)

(a) $2\,x(t+5) \longrightarrow 2\,F[x(t+5)] = 2\,X(\omega)e^{j\omega 5}$

delay

(b) $10\,x[(t-7)/3] \rightarrow 10\,F\left[x\left(\frac{t-7}{3}\right)\right] = 30\,X(3\omega)e^{-j\omega 7}$

Using property that $F[x(\frac{t}{a})] = a\,X(a\omega)$

(c) $t\,x\left(\frac{t+2}{5}\right) \otimes \frac{d}{dt}x(t)$ Conv.

using property that $F[\frac{d}{dt}x(t)] = j\omega X(\omega)$

and $F\left[x\left(\frac{t+2}{5}\right)\right] = 5X(5\omega)e^{j2\omega}$

and $F[t\,x(t)] = j\frac{d}{d\omega}X(\omega)$

Then $F\left[t\,x\left(\frac{t+2}{5}\right)\right] = j\frac{d}{d\omega}\left(5X(5\omega)e^{j2\omega}\right)$

so $F\left[(\ )\otimes\frac{d}{dt}x(t)\right] = F[(\ )]\ F\left[\frac{d}{dt}x(t)\right]$

i.e. $F[\text{convolution}] \Rightarrow$ multiplication.

So $F\left[t\,x\left(\frac{t+2}{5}\right)\otimes\frac{d}{dt}x(t)\right] = j\frac{d}{d\omega}\left(5X(5\omega)e^{j2\omega}\right)\, j\omega X(\omega)$

$$= \boxed{-\frac{d}{d\omega}\left(5X(5\omega)e^{j2\omega}\right)\omega X(\omega)}$$

(d) $t\, x^*(8t)$ → complex conjugate (4)

$F[t\, x^*(8t)]$.

Using property $F[x^*(t)] = X^*(-\omega)$.

Then $F[x^*(8t)] = \frac{1}{8} X^*\left(-\frac{\omega}{8}\right)$.

Using property $F[t\, x(t)] = j \frac{d}{d\omega} X(\omega)$, then

$$F[t\, x^*(8t)] = j \frac{d}{d\omega}\left(\frac{1}{8} X^*\left(\frac{-\omega}{8}\right)\right) = \boxed{\frac{j}{8}\frac{d}{d\omega}\left(X^*\left(\frac{-\omega}{8}\right)\right)}$$

(e) Find Fourier transform of

$$-x\left(-\frac{(t+20)}{12}\right) e^{j1000t}$$

Using property that $F[x(t)\, e^{j\alpha t}] = X(\omega - \alpha)$

Then $F\left[-x\left(-\frac{(t+20)}{12}\right) e^{j1000t}\right]$

$$= +\,12\, X(-12\omega)\, e^{j\omega 20} \Big|_{\omega = \omega - 1000}$$

$$= \boxed{12 X\left(-12(\omega - 1000)\right) e^{j(\omega-1000)20}}$$

Problem 5

Part (f). $\frac{d}{dt}\left(x^*(-2t)\right)\cos 500t$.

Use Property $F(x(at)) = \frac{1}{|a|}X\left(\frac{\omega}{a}\right)$ —— (1)

and Property $F\left(\frac{d}{dt}x(t)\right) = j\omega X(\omega)$ —— (2)

and Property $F\left(x(t)\cos(bt)\right) = \frac{1}{2}\left[X(\omega-a) + X(\omega+a)\right]$ —— (3)

Use Property $F\left(x^*(t)\right) = X^*(-\omega)$ —— (4)

Then we obtain:

$$F[x(-2t)] = \frac{1}{|2|}X\left(\frac{\omega}{-2}\right) \qquad \text{Rule (1)}$$

$$F[x^*(-2t)] = \frac{1}{2}X^*\left(\frac{\omega}{2}\right) \qquad \text{rule (4)}$$

$$F\left[\frac{d}{dt}x^*(-2t)\right] = \frac{j\omega}{2}X^*\left(\frac{\omega}{2}\right) \qquad \text{rule (2)}$$

$\downarrow$ rule (3)

$$\frac{j(\omega-500)}{(2)(2)}\left[X^*\left(\frac{\omega-500}{2}\right) + X^*\left(\frac{\omega+500}{2}\right)\right]$$

$$= \frac{j(\omega-500)}{4}\left[X^*\left(\frac{\omega-500}{2}\right) + X^*\left(\frac{\omega+500}{2}\right)\right]$$

Problem 5.

Part (g)

using rule $F[x_1(t)\,x_2(t)] = [X_1(\omega) \otimes X_2(\omega)]\,\frac{1}{2\pi}$

Then

$F[x^2(t)] = F[x(t)\,x(t)] = \frac{1}{2\pi} X(\omega) \otimes X(\omega)$

$F[x^3(t)] = F[x(t)\,x(t)\,x(t)] = \frac{1}{(2\pi)^3} X(\omega) \otimes X(\omega) \otimes X(\omega)$.

$F[\text{constant}] = c \cdot \delta(\omega)$.

So the result is

$$\frac{1}{(2\pi)^3} X(\omega) \otimes X(\omega) \otimes X(\omega) + \frac{3}{2\pi} X(\omega) \otimes X(\omega) - 9\,X(\omega) + 15\,\delta(\omega)$$

Part h

$(t-5)\,x(3-t)$.

Using rule $F[x(-t)] = X(-\omega)$.

using rule $F[t\,x(t)] = j\frac{d}{d\omega} X(\omega)$

using rule $F[x(t-a)] = X(\omega)\,e^{-j\omega a}$

Then we obtain

$$F[(t-5)\,x(3-t)] = F[t\,x(-(t+3)) - 5\,x(-(t-3))]$$

$$= F[t\,x(-(t-3))] - 5\,F[x(-(t-3))]$$

$$= j\frac{d}{d\omega}[X(-\omega)e^{-j\omega 3}] - 5\,X(-\omega)e^{-j3\omega}$$

Problem 5, Part (1)

$x(\frac{t}{2}) \circledast X(-5t)$.

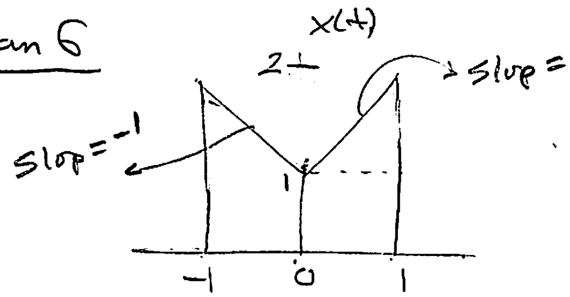
using property $F[x_1(t) \circledast x_2(t)] = X_1(\omega)\, X_2(\omega)$

and using property $F(x(\frac{t}{a})) = F(x(\frac{1}{a}t)) = |a|\, X(a\omega)$

and property $F(X(t)) = 2\pi\, x(-\omega)$

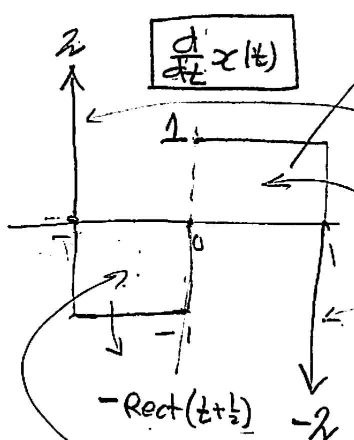
$$\left(2X(2\omega)\right)\left(\frac{2\pi}{5}\, x\left(\frac{\omega}{5}\right)\right) = \frac{4}{5}\pi\, X(2\omega)\, x\left(\frac{\omega}{5}\right)$$

Problem 6



The time differentiation property is $F[\frac{d}{dt}x(t)] = j\omega\, X(\omega)$

make a Function whose is the derivation of the above:



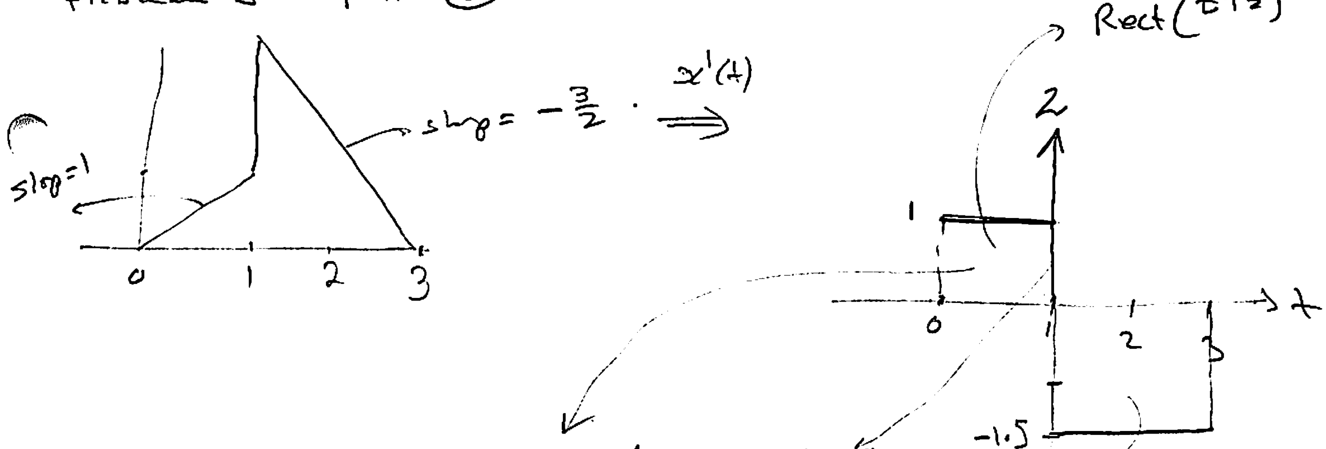
but this has Fourier Transform -6

$$2e^{j\omega} - 2e^{-j\omega} - \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)e^{j\omega\frac{1}{2}} + \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)e^{j\omega\frac{1}{2}}$$

...ne.

$$j\omega\, X(\omega) = [\text{the above}] \Rightarrow X(\omega) = \frac{1}{j\omega}[\;]$$

Problem 6 Part (b)



$$\text{so } F(x'(t)) = \operatorname{sinc}\left(\frac{\omega}{2\pi}\right)e^{-j\frac{\omega}{2}} + 2e^{-j\omega} - 1.5(2)\operatorname{sinc}\left(\frac{\omega}{2\pi}\right)e^{-2j\omega}$$

$$= \operatorname{sinc}\left(\frac{\omega}{2\pi}\right)e^{-j\frac{\omega}{2}} - 3\operatorname{sinc}\left(\frac{\omega}{2\pi}\right)e^{-2j\omega} + 2e^{-j\omega}$$

problem 7

(a) $x(t) = t e^{-at} u(t)$

use rule $F[t\ x(t)] = j \frac{d}{d\omega} X(\omega)$.

but $F[e^{-at} u(t)] = \frac{1}{a+j\omega}$

so $j \frac{d}{d\omega}\left(\frac{1}{a+j\omega}\right) = j \frac{d}{d\omega}\left((a+j\omega)^{-1}\right) = j\left[-(a+j\omega)^{-2} \cdot j\right]$

$$= j\left(\frac{-j}{(a+j\omega)^2}\right) = \boxed{\frac{1}{(a+j\omega)^2}}$$

(b) $x(t) = t e^{-at} u(t) \cos(\omega_c t)$.

$$= \frac{1}{(a+j\omega)^2} \circledast \frac{1}{2}\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

$$= \frac{1}{2} \frac{1}{\left(a+j(\omega-\omega_c)\right)^2} + \frac{1}{2} \frac{1}{\left(a+j(\omega+\omega_c)\right)^2}$$

(c) $10 e^{-10|t|} \cos(200t)$.

Use rule $F\left[e^{-a|t|}\right] = \frac{2a}{a^2+\omega^2}$, $\Rightarrow 10 F\left[e^{-10|t|}\right] = \frac{200}{100+\omega^2}$

Then $\frac{200}{(10)^2+\omega^2} \circledast \frac{1}{2}\left[\delta(\omega-200) + \delta(\omega+200)\right]$

$$= \frac{1}{2} \frac{200}{100+(\omega-200)^2} + \frac{1}{2} \frac{200}{100+(\omega+200)^2} = \boxed{\frac{100}{100+(\omega-200)^2} + \frac{100}{100+(\omega+200)^2}}$$

Problem 7. . part (d)

$$x(t) = \text{rect}\left(\frac{t}{12}\right) e^{j200t}.$$

Use property $F[x(t)e^{jat}] = X(\omega - a)$

Use property $F[\text{rect}(\frac{t}{a})] = a\,\text{sinc}\left(a\frac{\omega}{2\pi}\right)$

so $12\,\text{sinc}\left(\frac{12\omega}{2\pi}\right)\Big|_{\omega=\omega-200}$

$$= \boxed{12\,\text{sinc}\left(\frac{6(\omega-200)}{\pi}\right)}$$

Part e $x(t) = \text{sgn}(t)\cos(100t)$.

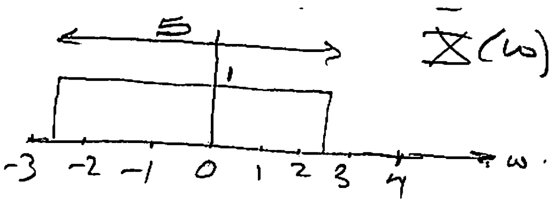
Use property $F[\text{sgn}(t)] = \frac{2}{j\omega}$.

Then $\frac{2}{j\omega} \circledast \frac{1}{2}[\delta(\omega-100) + \delta(\omega+100)]$

$$= \boxed{\frac{1}{j(\omega-100)} + \frac{1}{j(\omega+100)}}$$

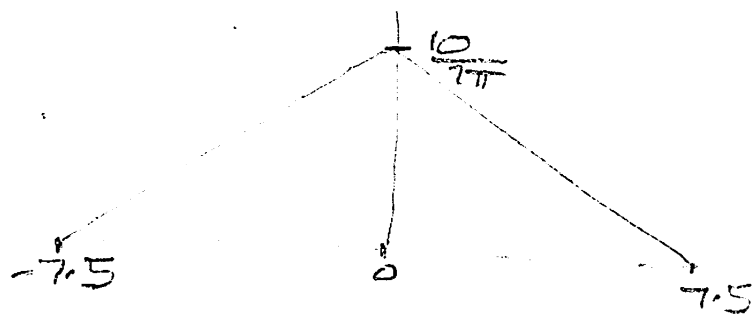
Problem 8

$F[x(t)] = \text{rect}\left(\frac{\omega}{5}\right)$



$F[x^2(t)] = \frac{1}{2\pi} X(\omega) \circledast X(\omega)$.

So need to convolve 2 rect. This gives a triangle. extent is from -7.5 to $+7.5$ max is at $\omega=0$, hight is $\frac{10}{2\pi}$ (total area when both rects are on top of each other).



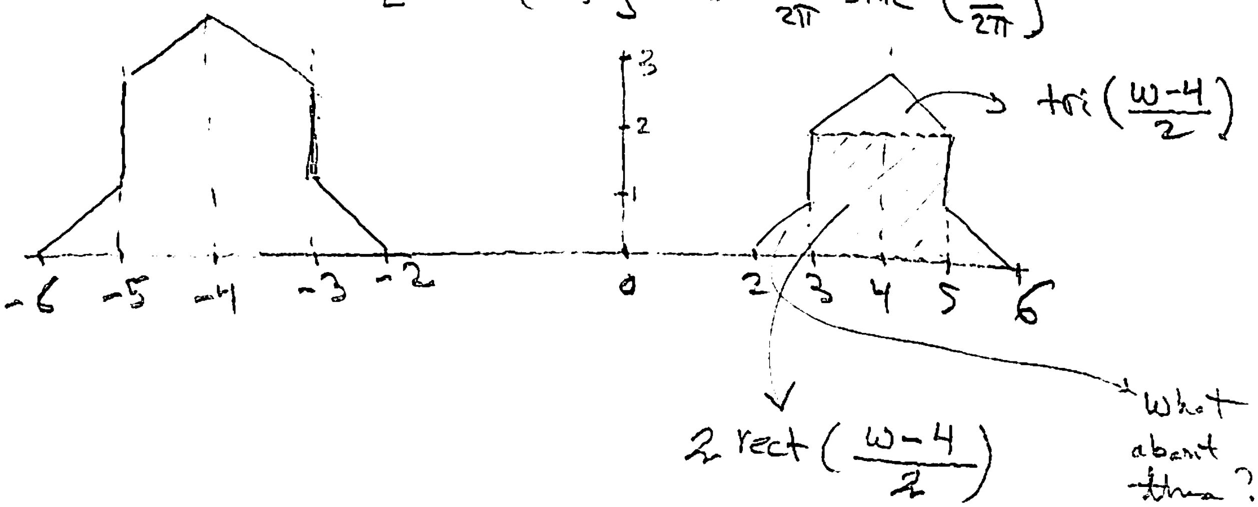
Problem 9

Use property $F^{-1}\left[\text{tri}\left(\frac{\omega}{2\pi}\right)\right] = \text{sinc}^2(t)$.

and $F^{-1}\left[\text{rect}\left(\frac{\omega}{2\pi}\right)\right] = \text{sinc}(t)$

then $F^{-1}\left[\text{rect}(\omega)\right] = \frac{1}{2\pi}\text{sinc}\left(\frac{t}{2\pi}\right)$

and. $F^{-1}\left[\text{tri}(\omega)\right] = \frac{1}{2\pi}\text{sinc}^2\left(\frac{t}{2\pi}\right)$



Need to check this more.

Problem 10

(a) $$\int_{-\infty}^{\infty} \frac{\sin\left(\frac{\pi t}{2}\right)}{5t} e^{-2t}\, \delta(t-1)\, dt. \quad \leftarrow \text{ at } \underline{\underline{t=1}} \quad \delta(t-1) = 1.$$

$$= \int_{-\infty}^{\infty} \frac{\sin\left(\frac{\pi}{2}\right)}{5} e^{-2} \delta(t) dt = \frac{\sin\left(\frac{\pi}{2}\right)}{5} e^{-2} \underbrace{\int_{-\infty} \delta(t)\, dt}_{=1}.$$

$$= \frac{\sin\left(\frac{\pi}{2}\right)}{5} e^{-2} = \frac{1}{5} e^{-2} = 0.027067.$$

(b) $$\int_{-\infty}^{\infty} e^{j2\pi t} \delta\left(t-\frac{1}{8}\right) dt.$$

note at $\boxed{t=\frac{1}{8}}$ $\delta\left(t-\frac{1}{8}\right)$ is not zero. hence

$$\int_{-\infty}^{\infty} e^{j2\pi\frac{1}{8}} \delta(t)\, dt = e^{j\frac{\pi}{4}}$$

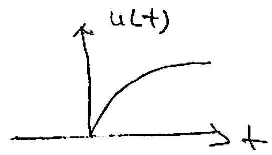
$$= \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = 0.707 + j\, 0.707$$

Problem 11

scaling the input causes scaling in the output.
delay in the input causes delay in the output.

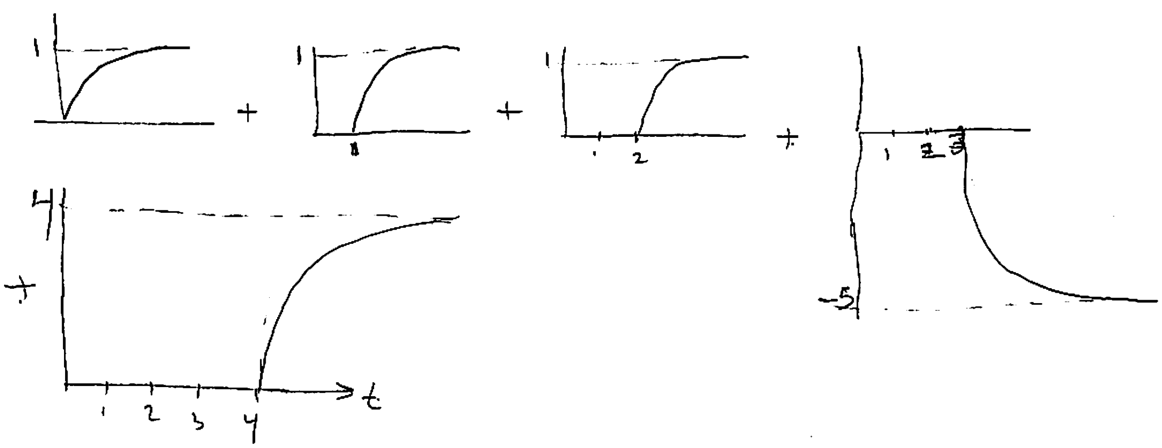
so $y(t) = u(t) + u(t-1) + u(t-2) - 5u(t-3) + 3u(t-4) - u(t-5)$

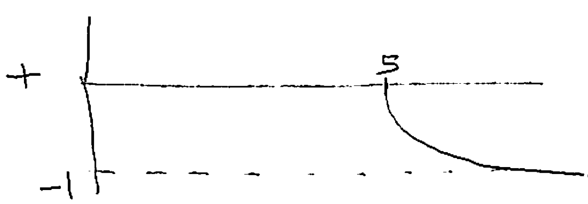
so, assume $u(t) =$



then.

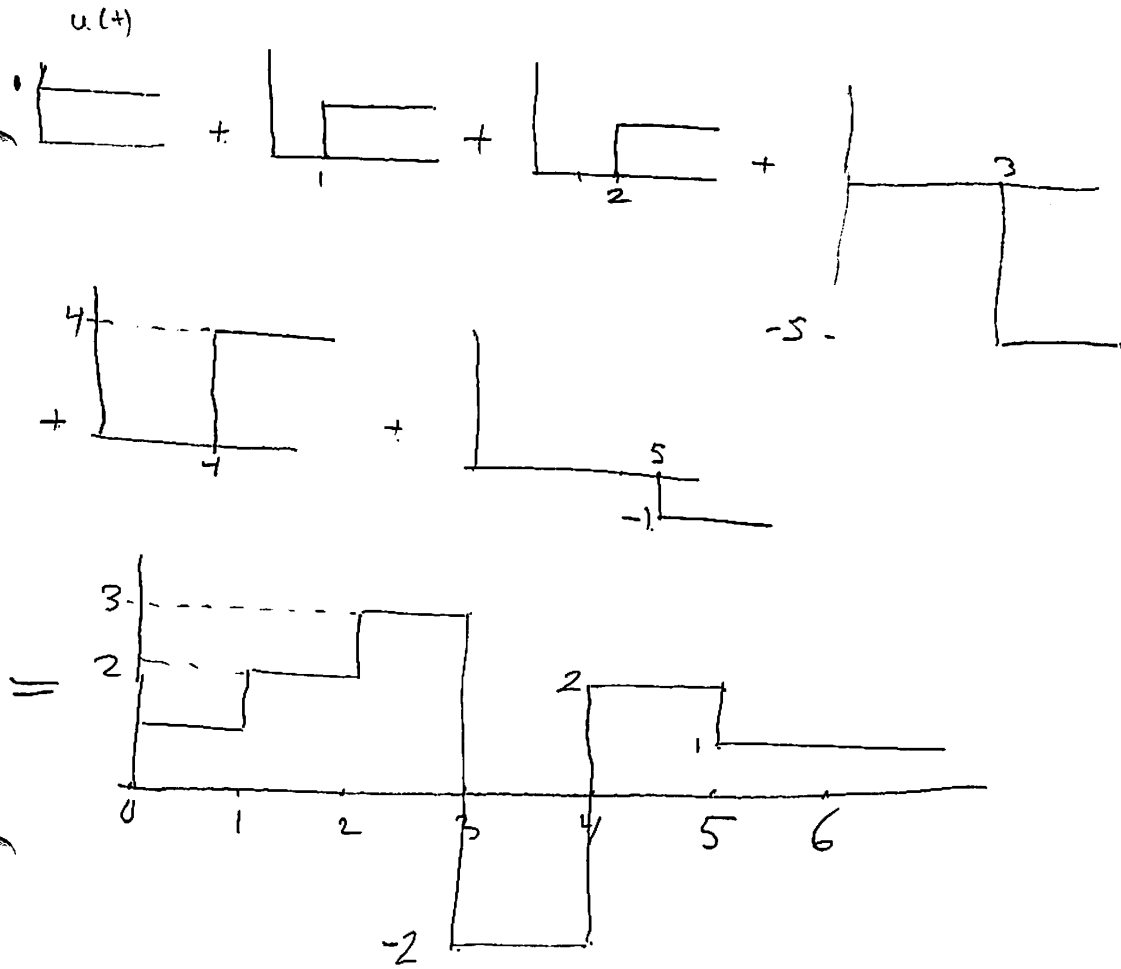
$y(t) =$





so need to add all the above together to see the final result.

if u(t) is the unit step, then see next page →

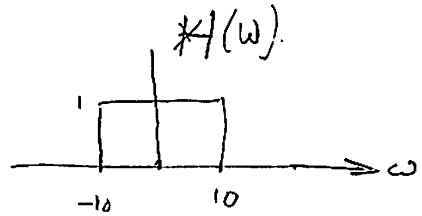

u(t)
1
+
1
+
2
+
3
-5
4
+
4
+
5
-1
=
3
2
2
1
0
1
2
3
4
5
6
-2

(13) $x(t)$ is band limited to B Hz.
What is bandwidth of $x^3(t)$?

from convolution property: $F\left(x^3(t)\right) = \frac{1}{2\pi} X(\omega) \circledast X(\omega) \circledast X(\omega)$.

when convolving, the bandwidth become to sum of the bandwidth of the $X(\omega)$. hence final bandwidth is $\boxed{3B}$

(14) $H(\omega) = rect\left(\frac{\omega}{40\pi}\right)$
$= rect\left(\frac{\omega}{20(2\pi)}\right)$



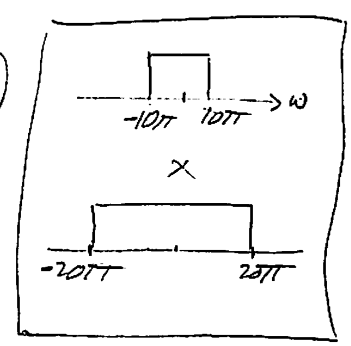
when $\delta(t)$ is applied, the the output is the $F^{-1}[H(\omega)]$

Which is $\boxed{20 \operatorname{sinc}(20t)}$

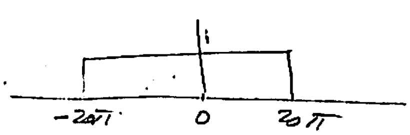
(b) if input is $10 \operatorname{sinc}(10t)$, then output is

$F[10 \operatorname{sinc}(10t)] \quad H(\omega)$

$= rect\left(\frac{\omega}{10(2\pi)}\right) \quad rect\left(\frac{\omega}{20(2\pi)}\right)$



so result is (in ω domain)



so the $F^{-1}[\quad]$ is a sinc.

so $\boxed{y(t) = 20 \operatorname{sinc}(20t)}$

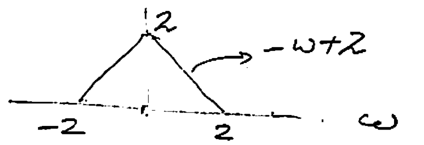
problem 15

$|X(\omega)| = 2\,\text{tri}\left(\frac{\omega}{2}\right)$. Find energy of $x(t)$.

from parsavell, energy in ω domain = energy in time domain.

so energy of $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$

the triangle is



so $2 \cdot \frac{1}{2\pi}\int_0^2 (-\omega+2)^2\, d\omega$.

this 2 since adding both sides.

so $\frac{1}{\pi}\int_0^2 \omega^2+4-4\omega\, d\omega = \frac{1}{\pi}\left(\left[\frac{\omega^3}{3}\right]_0^2 + 4\left[\omega\right]_0^2 - 4\left[\frac{\omega^2}{2}\right]_0^2\right)$

$= \frac{1}{\pi}\left(\frac{1}{3}(8) + 4(2) - 2(4)\right)$

$= \frac{1}{\pi}\left(\frac{8}{3} + 8 - 8\right) = \boxed{\frac{1}{\pi}\left(\frac{8}{3}\right)}$

HW # 2
Nasser M. Abbasi

Problem 1

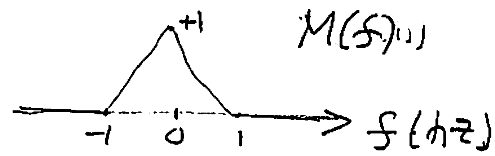
$m(t) = \text{sinc}^2(t)$.

$c(t) = 2\cos(2\pi 10 t)$.

(a) $F(m(t)) = F(\text{sinc}(t)) \circledast F(\text{sinc}(t))$

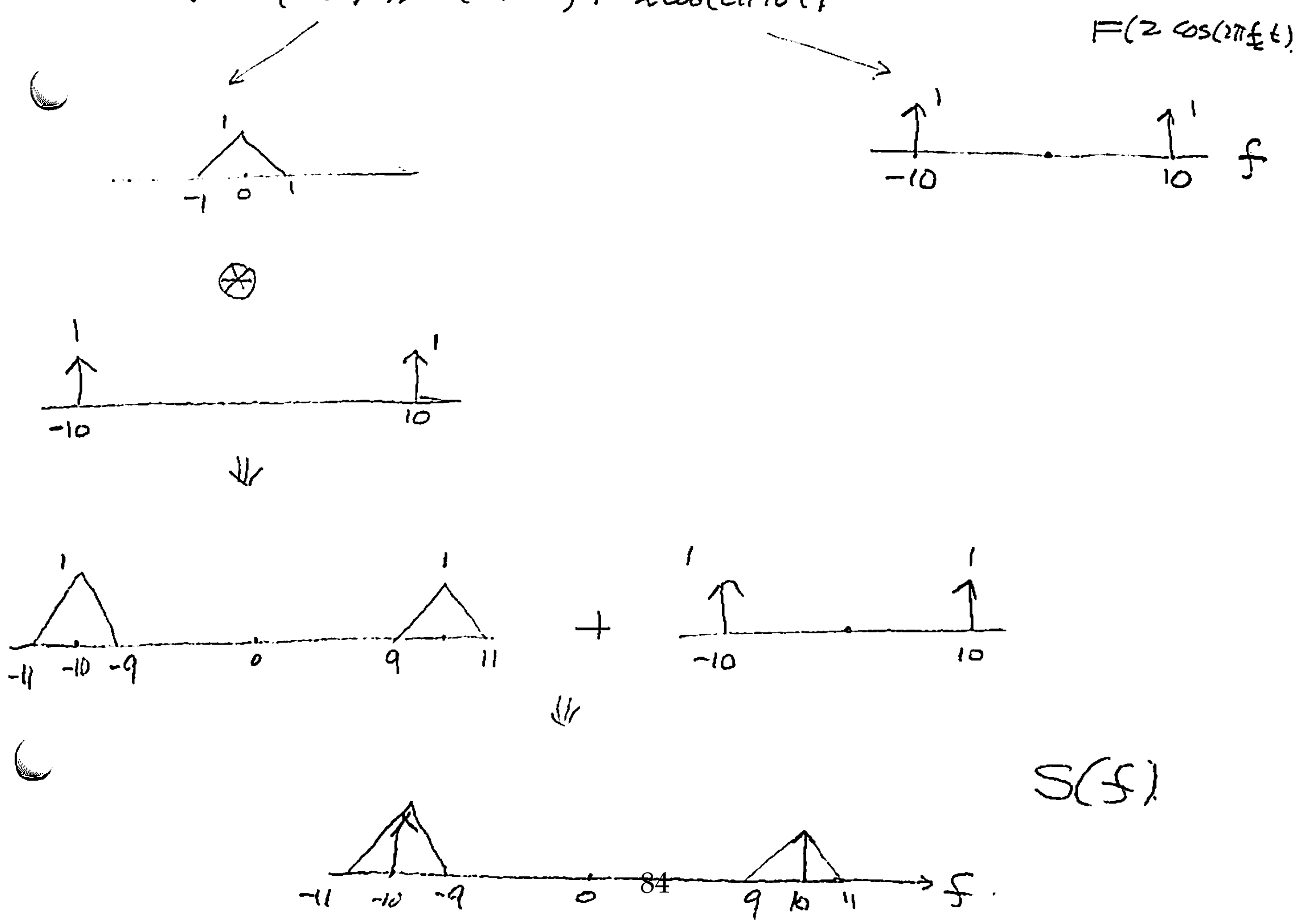
$= \text{rect}(f) \circledast \text{rect}(f)$

$= \text{tri}(f)$



(b) AM:

$s(t) = \left(m(t) \cdot 2\cos(2\pi 10 t)\right) + 2\cos(2\pi 10 t)$



2

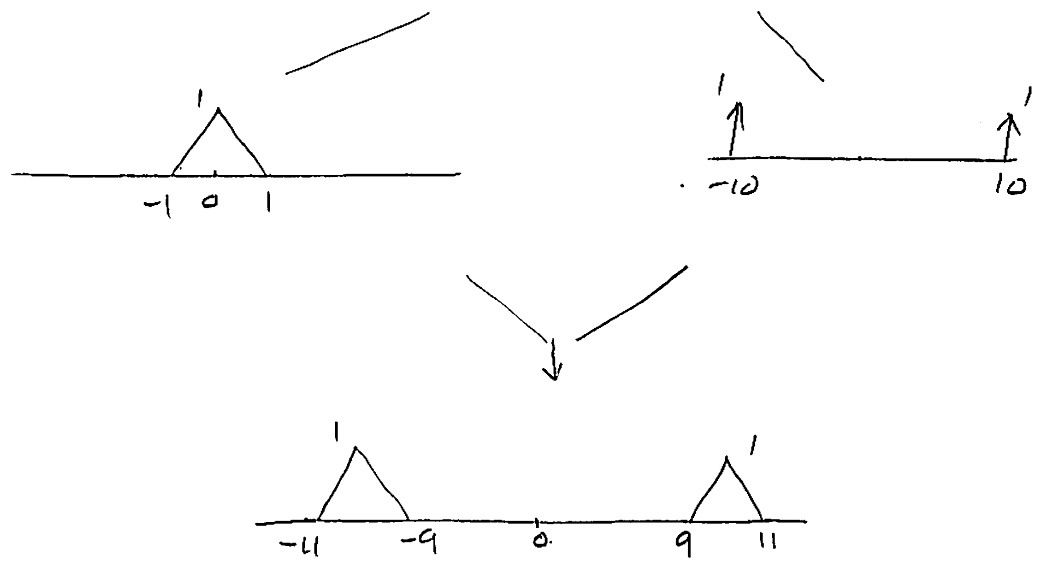
Problem 1.

(b) 2 DSB-SC.

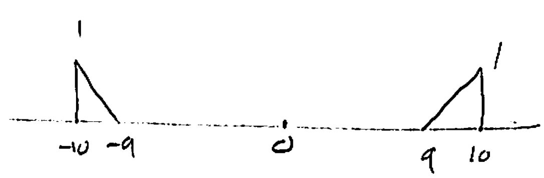
for DSB-SC, the modulated carrier is given by:

$$s(t) = m(t)\cdot c(t) = \mathrm{sinc}^2(t)\cdot 2\cos(2\pi 10t).$$

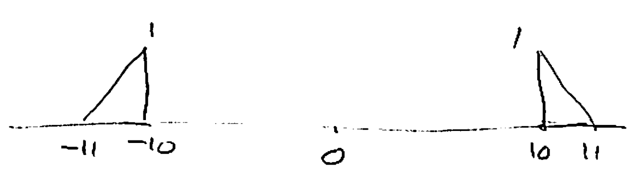
$$\text{so } F(s(t)) = F(\mathrm{sinc}^2(t)) \circledast F(2\cos(2\pi 10t))$$



3 LSB. here, we remove USB from DSB-SC to obtain:



3 VSB. remove LSB to obtain:



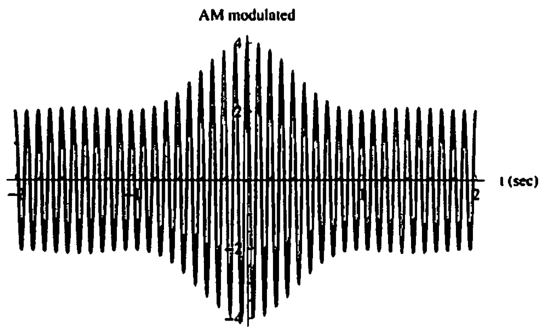
3

Problem 1. Part c

In[18]:=
```
s = Sinc[Pi t]^2 2 Cos[2 π 10 t] + 2 Cos[2 π 10 t]
Plot[s, {t, -2, 2}, PlotRange → All,
 AxesLabel → {"t (sec)", "AM modulated"}, PlotStyle → Thick]
```

Out[18]= $2\,\mathrm{Cos}[20\,\pi\,t] + 2\,\mathrm{Cos}[20\,\pi\,t]\,\mathrm{Sinc}[\pi\,t]^2$

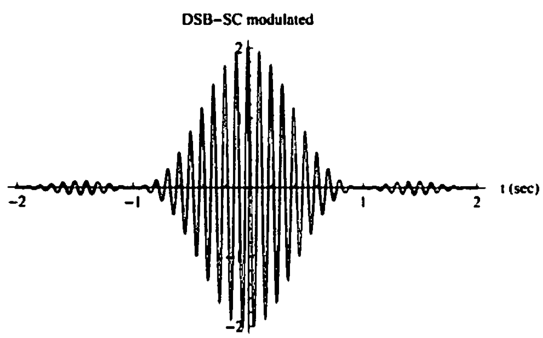
Out[19]=



In[20]:=
```
s = Sinc[Pi t]^2 2 Cos[2 π 10 t]
Plot[s, {t, -2, 2}, PlotRange → All,
 AxesLabel → {"t (sec)", "DSB-SC modulated"}, PlotStyle → Thick]
```

Out[20]= $2\,\mathrm{Cos}[20\,\pi\,t]\,\mathrm{Sinc}[\pi\,t]^2$

Out[21]=



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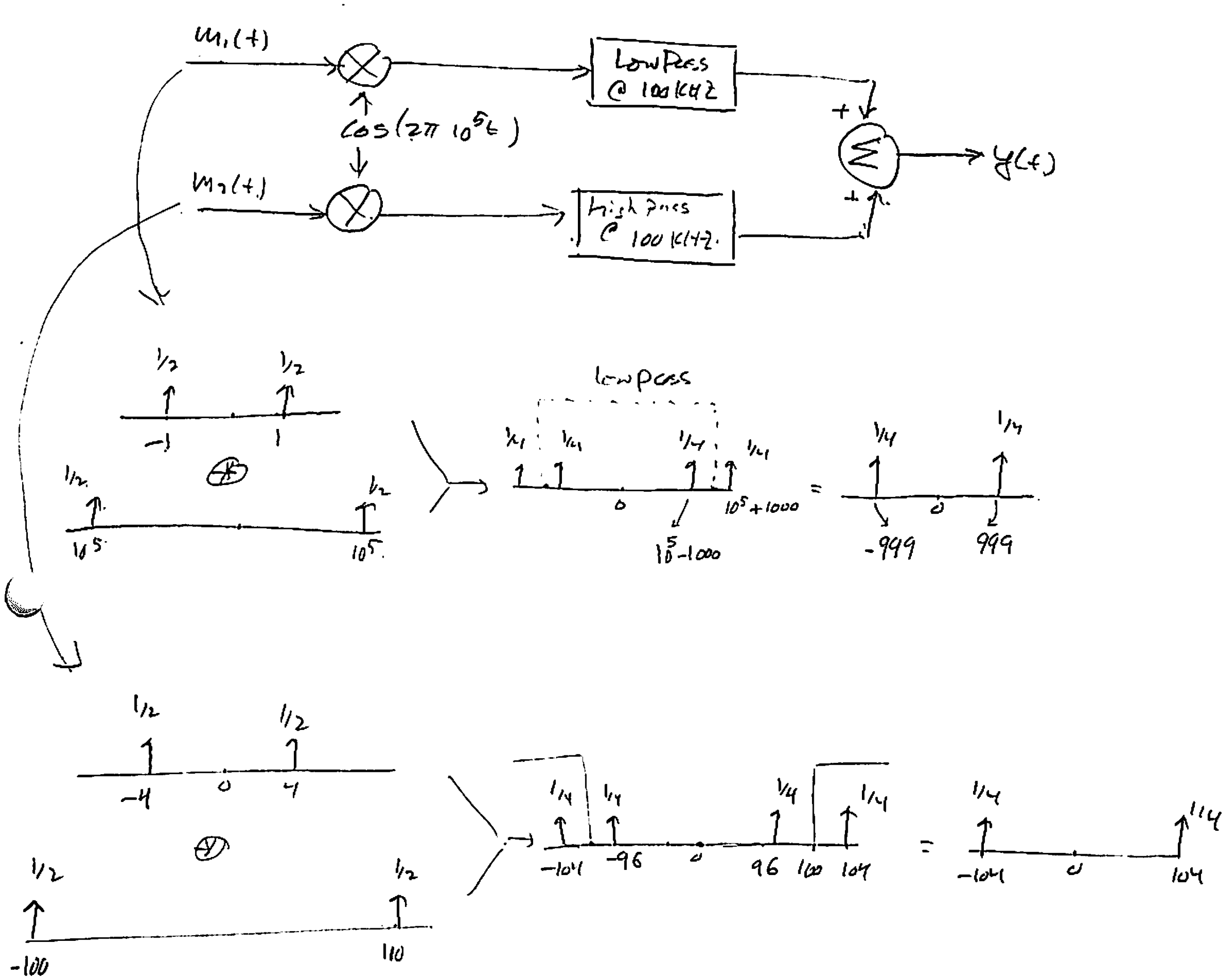
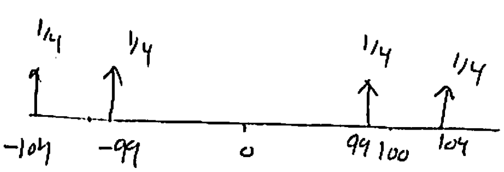
Problem 5 4

(a) $m_1(t) = \cos(2\pi\, 1000 t)$, $m_2(t) = \cos(2\pi\, 4000 t)$.
Find $y(t)$.

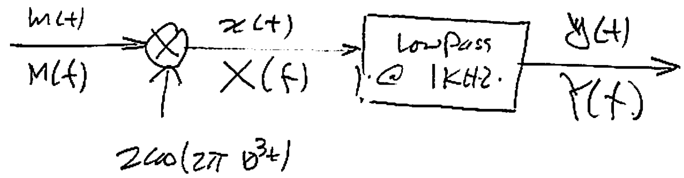


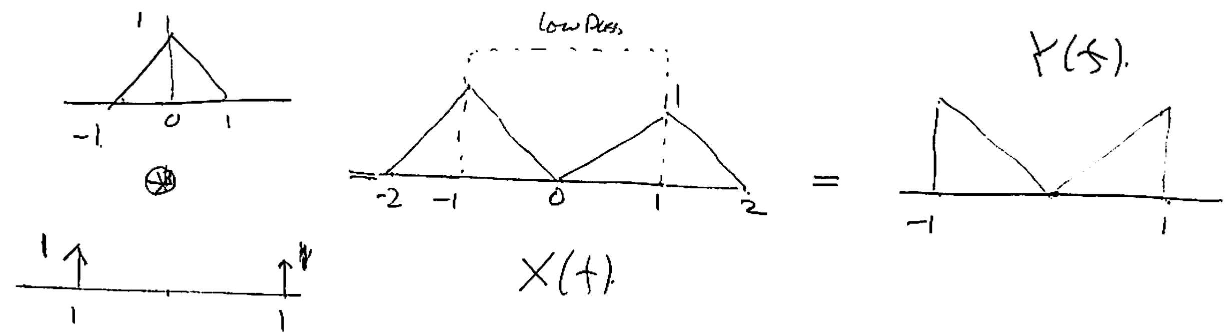
so in frequency, $Y(f) =$



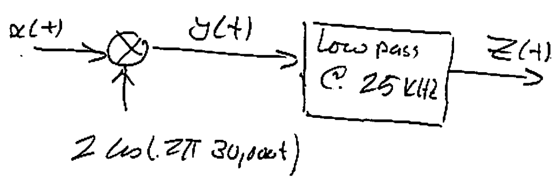
so in time domain $\boxed{y(t) = \frac{1}{2}\cos(2\pi\, 99000 t) + \frac{1}{2}\cos(2\pi\, 104000 t)}$

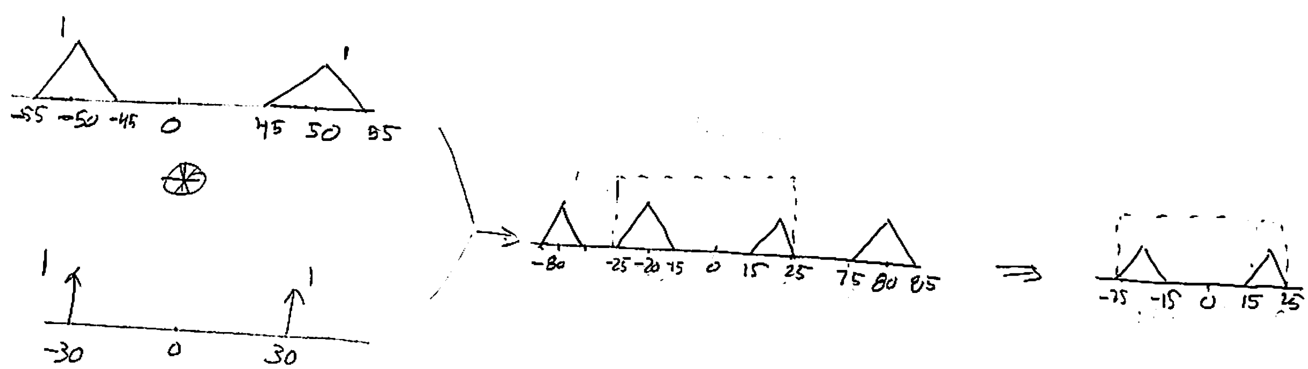
(3) Sketch the spectrum $X(f)$ and $Y(f)$. in the system below (6)



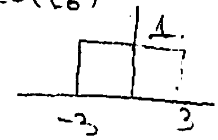


(4) Sketch the pectrum





$\mathrm{rect}\left(\frac{t}{6}\right)$



HW # 3

problem ①

$$m(t) = 10\,\mathrm{rect}\left(\frac{t}{6}\right).$$

ⓐ Find $\hat{m}(t)$

$$\hat{m}(t) = \frac{10}{\pi}\int_{-\infty}^{\infty} m(\tau)\frac{1}{(t-\tau)}\,d\tau = \frac{10}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{rect}\left(\frac{\tau}{6}\right)}{t-\tau}\,d\tau.$$

$$= \frac{10}{\pi}\left[\int_{-3}^{3}\frac{1}{t-\tau}\,d\tau\right] = \frac{10}{\pi}\quad \ln(t-\tau)\Big|_{-3}^{3} = \frac{10}{\pi}\left[\ln(t-3) - \ln(t+3)\right]$$

$$= \boxed{\frac{10}{\pi}\ln\left(\frac{t-3}{t+3}\right)}.$$

ⓑ $m_{USSB}(t) = m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t.$

$$= \boxed{10\,\mathrm{rect}\left(\frac{t}{6}\right)\cos(2\pi\,1000t) - \frac{10}{\pi}\ln\left(\frac{t-3}{t+3}\right)\sin(2\pi\,1000t)}$$

② $m(t) = 4\sin(12t)\cos(7t)\cos(3t)$

$$= 4\left[\frac{1}{2}\left(\sin 5t + \sin 19t\right)\right]\cos(3t)$$

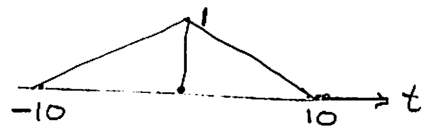
$$= 2\left[\sin 5t\cos 3t + \sin 19t\cos 3t\right]$$

$$= \sin 2t + \sin 8t + \sin 16t + \sin 22t.$$

$$\hat{m}(t) = \sin\left(2t - \frac{\pi}{2}\right) + \sin\left(8t - \frac{\pi}{2}\right) + \sin\left(16t - \frac{\pi}{2}\right) + \sin\left(22t - \frac{\pi}{2}\right)$$

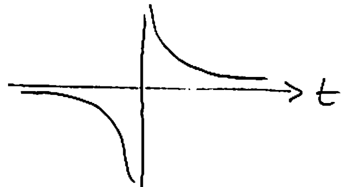
$$= -\cos 2t - \cos 8t - \cos 16t - \cos 22t.$$

③ $m(t) = tri\left(\frac{t}{10}\right)$.



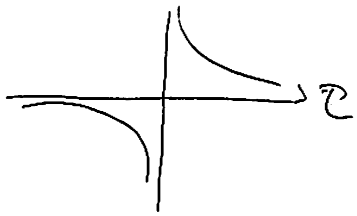
$\hat{m}(t) = m(t) \circledast \frac{1}{\pi t}$.

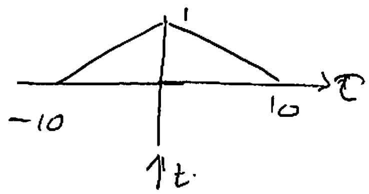
$\frac{1}{\pi t} =$



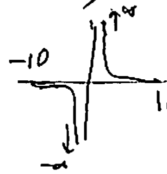
. so need to convolve with.

flip the tri function (easier). stay the same;

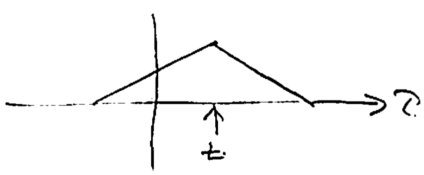




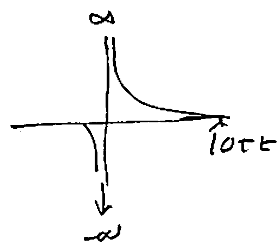
at $t=0$, multiply and integrate



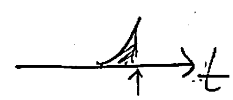
so area = 0 (areas cancel) so at $t=0 \Rightarrow 0$



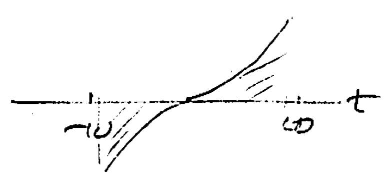
at $t>0$,



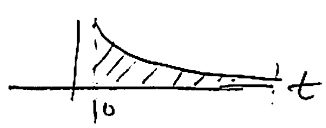
$\Rightarrow$



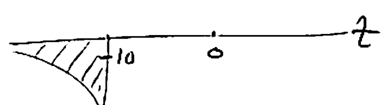
So for $-10 \le t \le 10$ we have



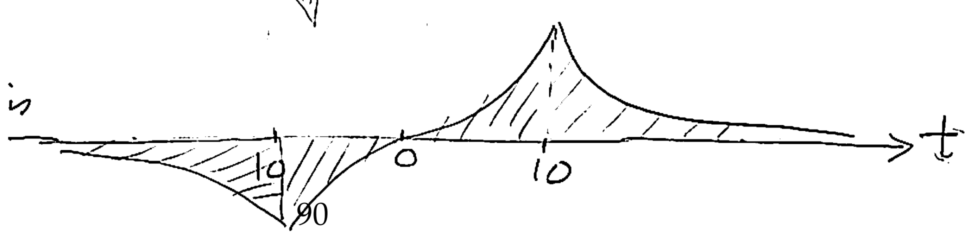
for $t>10$, we get



for $t<10$, we get



so final answer is



(5) $m(t) = 2\sin(\overset{\alpha}{5000t})\sin(\overset{\beta}{7000t})$.

$c(t) = \cos(2\pi 10^7 t)$

(a) Find $m_{LSSB}(t)$

$$m_{LSSB}(t) = m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t.$$

$$m(t) = 2\left[\tfrac{1}{2}\cos(-2000t) - \tfrac{1}{2}\cos(12000t)\right]$$

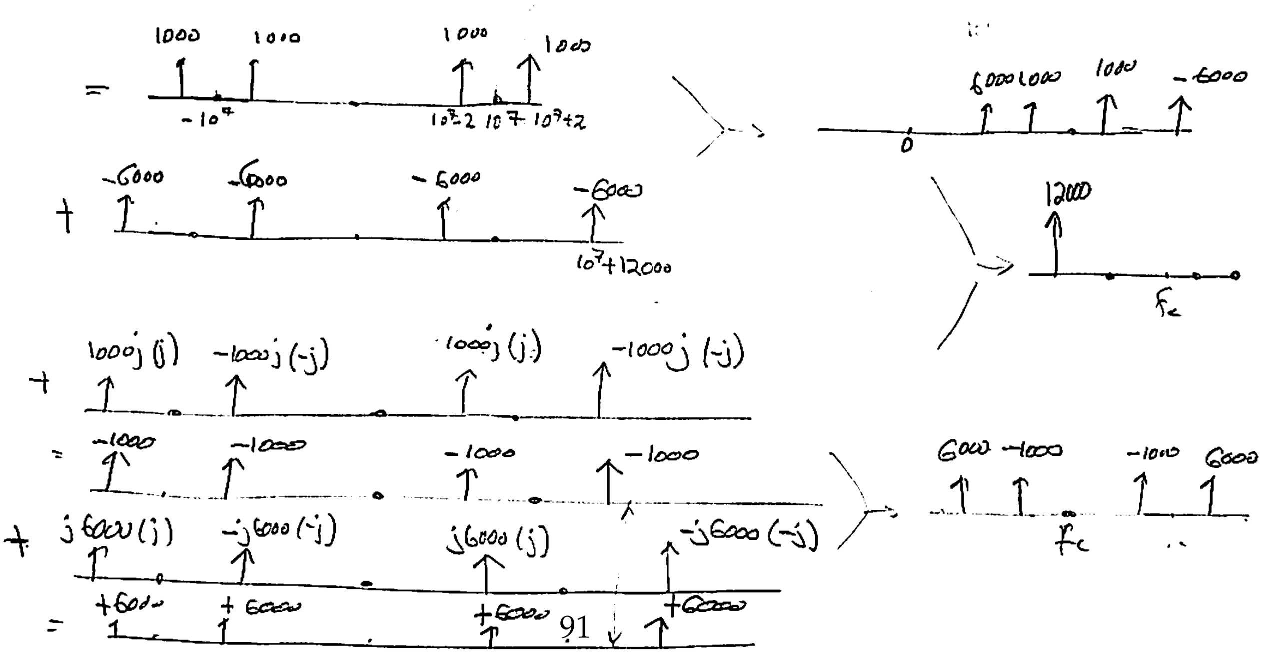
$$= \boxed{\cos(2000t) - \cos(12000t)}$$

so $\hat{m}(t) = \cos\left(2000t - \frac{\pi}{2}\right) - \cos\left(12000t - \frac{\pi}{2}\right)$

$$= \boxed{\sin(2000t) - \sin(12000t)}$$

so $m_{LSSB}(t) = \left[\cos(2000t) - \cos(12000t)\right]\cos\omega_c t + \left[\sin(2000t) - \sin(12000t)\right]\sin\omega_c t$

$$= \cos 2000t\cos\omega_c t - \cos 12000t\cos\omega_c t + \sin 2000t\sin\omega_c t - \sin(12000t)\sin\omega_c t.$$



(6) $m(t) = 24\cos(2\pi\, 100t) + 16\cos(2\pi\, 200t)$

$c(t) = 2\cos(2\pi\, 1000t)$.

let $m(t) = 24\cos(\alpha t) + 16\cos(\beta t)$

$c(t) = 2\cos(w_c t)$.

so $x(t) = 48\cos(\alpha t)\cos w_c t + 32\cos(\beta t)\cos w_c t$.

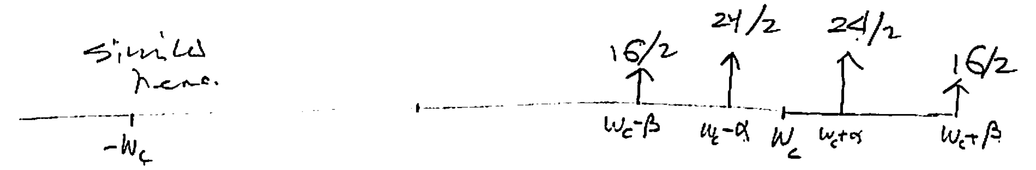
$$= 48\left[\tfrac{1}{2}\cos((w_c-\alpha)t) + \tfrac{1}{2}\cos(w_c+\alpha)t\right]$$

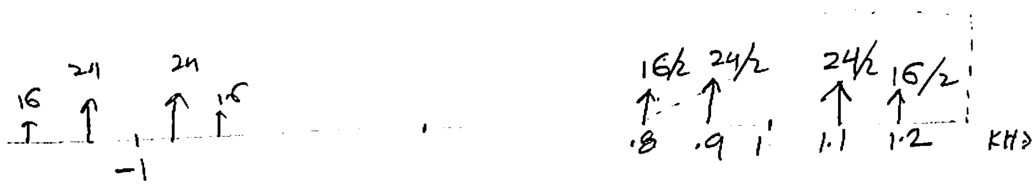
$$+ 32\left[\tfrac{1}{2}\cos(w_c-\beta)t + \tfrac{1}{2}\cos(w_c+\beta)t\right]$$

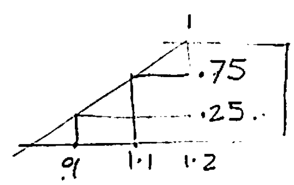
$$= 24\cos(w_c-\alpha)t + 24\cos((w_c+\alpha)t)$$

$$+ 16\cos(w_c-\beta)t + 16\cos(w_c+\beta)t.$$

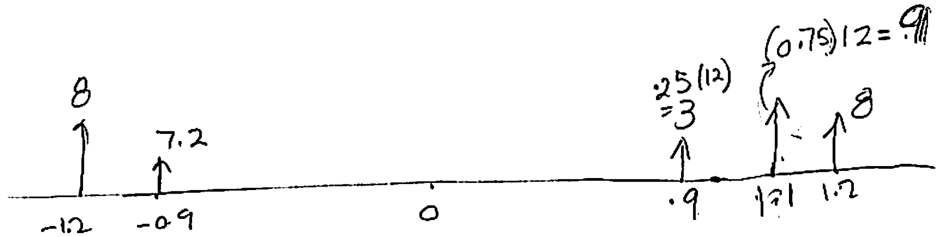
so $X(f) =$





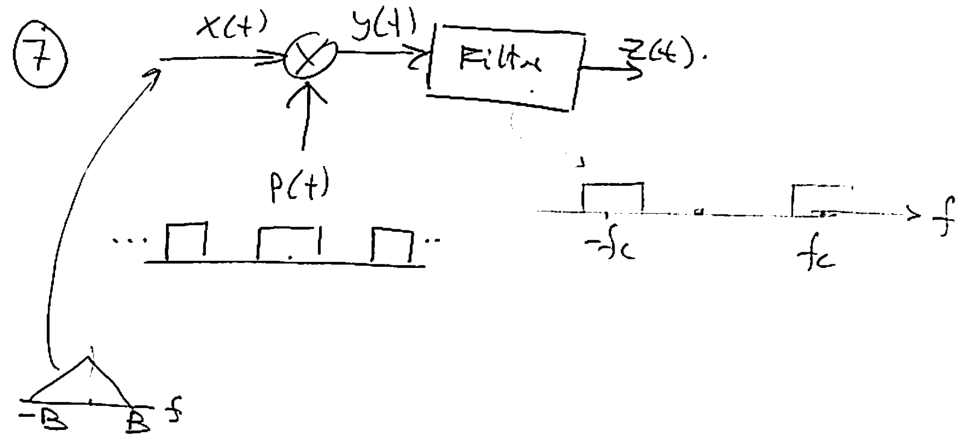


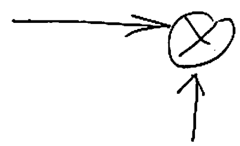
So $Y(f)$



so $\boxed{y(t) = 16\cos(1200t) + 18\cos(1100t) + 6\cos(900t)}$

(7)

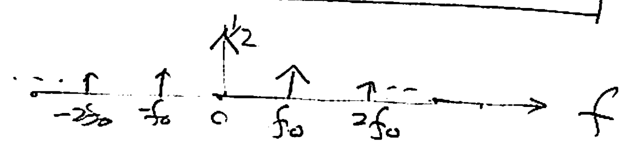




$$P(t) = \sum \tfrac{1}{2}\operatorname{sinc}\left(\tfrac{n}{2}\right) e^{j\frac{2\pi}{T_c}nt}$$

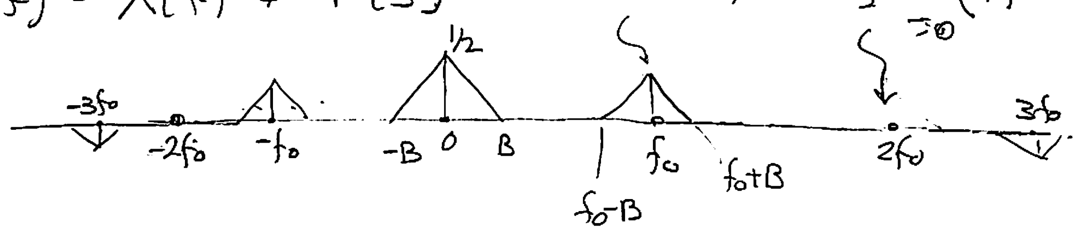
$h = 1$, $d = \frac{1}{2}$ so $\boxed{P(f) = \frac{1}{2}\operatorname{sinc}\left(\frac{n}{2}\right) \sum \delta(f - nf_0)}$

so $P(f) =$

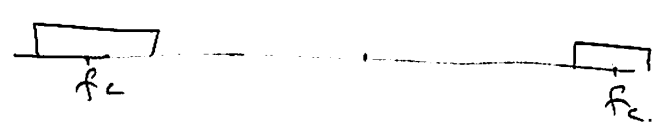


So $y(t) = x(t) \cdot p(t)$

or $Y(f) = X(f) \circledast P(f)$



so when we multiply the above with $H(f)$ which is

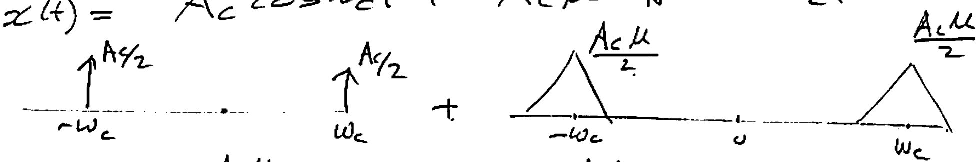


we will get, in the frequency domain $X(f_c - f) \times \underbrace{\frac{1}{2}\operatorname{sinc}(f - f_c)}_{\text{scale factor}}$

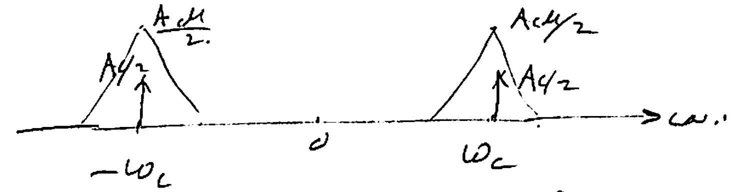
(b) if $x(t) = A_c[1+\mu m_n(t)]\cos(\omega_c t)$.

Then $x(t) = A_c\cos\omega_c t + A_c\mu\, m_N(t)\cos\omega_c t$.

$X(f) =$

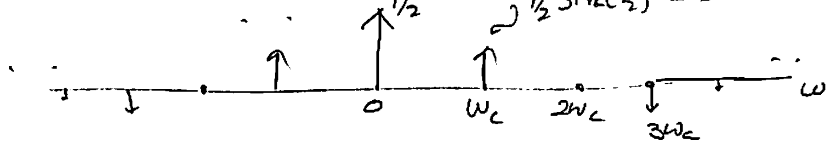


$=$



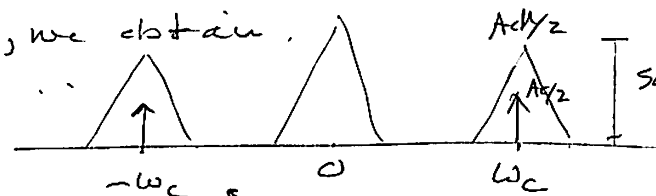
Convolve the above with $P(f)$, which is a puls train.

$P(f) =$



so, at ω_c, we obtain.

$Y(f) =$



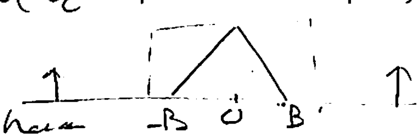
scaled by $\frac{1}{2}\text{sinc}(\frac{1}{2})$

so $Z(f)$ will only contain these bandpass regions for band pass.

hence
$$Z(f) = \tfrac{1}{2}\text{sinc}(\tfrac{1}{2})\left[\frac{A_c}{2}\delta(f-f_c) + \frac{A_c\mu}{2}X(f-f_c)\right] + \tfrac{1}{2}\text{sinc}(\tfrac{1}{2})\left[\frac{A_c}{2}\delta(f+f_c) + \frac{A_c\mu}{2}X(f+f_c)\right]$$

so $z(t) = \frac{1}{2}\text{sinc}(\frac{1}{2})\left[A_c\cos(\omega_c t) + A_c\mu\, m_N(t)\right]$

If low Pass, then



only this will be used. here

$$Z(f) = \tfrac{1}{2}\text{sinc}(\tfrac{1}{2})\left[X(f)\right] = \tfrac{1}{2}\text{sinc}(\tfrac{1}{2})\left[A_c\mu M(f)\right]$$

$$\text{so } z(t) = \tfrac{1}{2}\text{sinc}(\tfrac{1}{2})A_c\mu\, m_N(t) = \tfrac{1}{2}\frac{\sin(\pi/2)}{\pi/2}A_c\mu\, m_N(t) = \boxed{\frac{A_c}{\pi}\mu\, m_N(t)}$$

(8.)

$$v_i(t) = \frac{5}{1000}\cos(2\pi\, 600t) + \frac{1}{1000}\cos(2\pi\, 1000t).$$

$$v_o(t) = a_1 v_1(t) + a_2 v_2^2(t)$$

$$V_o(t) = a_1\left[\frac{5}{1000}\cos(2\pi 600t) + \frac{1}{1000}\cos(2\pi 1000t)\right] + a_2\left[\frac{5}{1000}\cos(2\pi 600t) + \frac{1}{1000}\cos(2\pi 1000t)\right]^2$$

$$= a_1\frac{5}{1000}\cos(2\pi 600t) + a_1\frac{1}{1000}\cos(2\pi 1000t) + a_2\left[\left(\frac{5}{1000}\right)^2\cos^2(2\pi 600t) + \left(\frac{1}{1000}\right)^2\cos^2(2\pi 1000t) + 2\left(\frac{5}{1000}\right)\left(\frac{1}{1000}\right)\cos(2\pi 600t)\cos(2\pi 1000t)\right]$$

expand, use trig identity, look for coeff. of $\cos(2\pi 1000t)$ and $\cos(2\pi 400t)$. equate to 1 and 0.001 to solve for a_1, a_2.

(9.) $m_{Am}(t) = 10\left[1 + 0.8\cos(2\pi\, 2000t)\right]\cos(2\pi 10^6 t)$

(a) Find modulation index. μ.

~~the~~ rewrite as $m_{Am}(t) = A_c\left[1 + \mu\, m_N(t)\right]\cos(2\pi\omega_c t)$

so $\boxed{\mu = 0.8}$

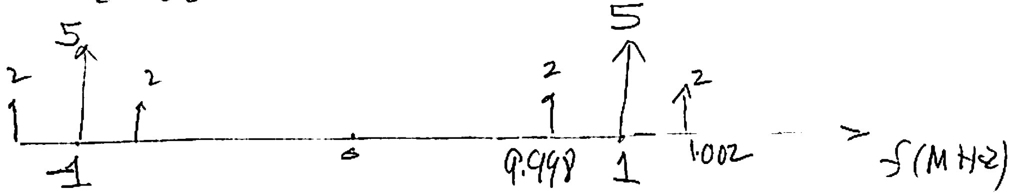
(b) transmission bandwidth. $2B$. when $B = 2$ kHz.

hence $B_T = \boxed{4\text{ KHZ}}$

(c) efficiency = $\dfrac{\text{Power in signal}}{\text{Total power}} = \dfrac{\frac{A_c^2\mu^2}{2}}{A_c^2 + \frac{A_c^2\mu^2}{2}} = \dfrac{\mu^2}{2+\mu^2}$

$$= \frac{0.8^2}{2+0.8^2} = 24.24\%$$

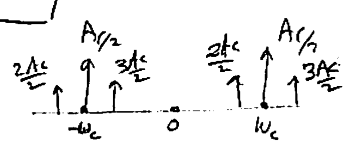
(d)



(9) $m(t) = -6\cos(2\pi 10t) - 4\cos(2\pi 30t)$.
$c(t) = A_c \cos(2\pi 100t)$.
$\mu = 0.8$.

(a). $$AM(t) = m(t)\cos W_c t + A_c \cos W_c t$$
$$= [A_c + m(t)]\cos W_c t$$
$$= A_c\left[1 + \frac{1}{A_c} m(t)\right]\cos W_c t$$
$$= A_c\left[1 + \frac{1}{A_c}\left(-6\cos(2\pi 10t) - 4\cos(2\pi 30t)\right)\right]\cos W_c t$$
$$= A_c\left[1 + \underbrace{\left(\frac{\max(m(t))}{A_c}\right)}_{\mu = 0.8} \cdot \frac{m(t)}{\max(m(t))}\right]\cos W_c t$$
$$AM(t) = \boxed{A_c\left[1 + \mu\, m_N(t)\right]\cos W_c t}$$
so $$\boxed{m_N(t) = \frac{-6}{10}\cos(2\pi 10t) - \frac{4}{10}\cos(2\pi 30t)}$$

(b) $$\overline{m_N^2(t)} = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} m_N^2(t)\,dt$$



$\Downarrow$ Can also Find power from $|H(t)|$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{T/2} \frac{36}{100}\cos^2(2\pi 10t) + \frac{16}{100}\cos^2(2\pi 30t)$$
$$+ \frac{80}{100}\cos(2\pi 10t)\cos(2\pi 30t)\;dt.$$
$$= \frac{36}{100}\left(\frac{1}{2}\right) + \frac{16}{100}\left(\frac{1}{2}\right) + \frac{80}{100}\int \underbrace{\cancel{}}_{=0}\,dt$$
$$= \boxed{0.26}$$

(c) $$E_{ff} = \frac{\text{Av. Power in Signal}}{\text{Total. av. power}} = \frac{\frac{1}{2}A_c^2\mu^2\,\overline{m_N^2(t)}}{\frac{A_c^2}{2} + \frac{1}{2}A_c^2\mu^2\,\overline{m_N^2(t)}}$$
$$= \frac{0.8^2(0.26)}{1 + 0.8^2(0.26)} = 14.27\%$$

HW#4

(1) $M_{EM}(t) = 50 \cos\left[2\pi\, 10^7 t + 6 \sin(2\pi\, 5000 t)\right]$

(a) For PM, $M_{EM}(t) = 50 \cos\left[2\pi f_c t + k_p\, m(t)\right]$.

$k_p = 3$, hence $M_{EM}(t) = 50 \cos\left[2\pi f_c t + 3\, m(t)\right]$.

hence $3\, m(t) = 6 \sin(2\pi\, 5000 t)$

hence $\boxed{m(t) = 2 \sin(2\pi\, 5000 t)}$

(b) for FM, $M_{EM}(t) = 50 \cos\left[2\pi f_c t + k_f \int m(\lambda)\, d\lambda\right]$

for $k_f = 2\pi\, 3000$ rad/s/V, then

$M_{EM}(t) = 50 \cos\left[2\pi f_c t + 2\pi\, 3000 \int m(\lambda)\, d\lambda\right]$.

hence. $2\pi\, 3000 \int m(\lambda)\, d\lambda = 6 \sin(2\pi\, 5000 t)$

or $\int m(\lambda)\, d\lambda = \frac{6}{2\pi\, 3000} \sin(2\pi\, 5000 t)$

or $m(t) = \frac{6}{2\pi\, 3000} \frac{d}{dt} \sin(2\pi\, 5000 t)$

$= \frac{6}{\cancel{2\pi}\, 3000} \cdot \cancel{2\pi}\, 5000 \cdot \cos(2\pi\, 5000 t)$

$\boxed{m(t) = 10 \cos(2\pi\, 5000 t)}$

HW 4

(2) BW for FM = 220 KHZ.
using Carson Rule.

(a) $\Delta f = 80$ KHZ. (max freq. deviation)

$$(B_T)_{carson} = 2(B+\Delta f) = 2(B + 80 \text{ KHZ}).$$

So $220 \text{ KHZ} = 2B + 160 \text{ KHZ}$.

So $B = \frac{60}{2} = \boxed{30 \text{ KHZ}}$

(b) $(B_T)_{carson} = 2(B+\Delta f)$.

Given $B = 20$ KHZ, then

$$220 \text{ KHZ} = 2(20+\Delta f)$$

$$\frac{220-40}{2} = \Delta f$$

So $\boxed{\Delta f = 90 \text{ KHZ}}$

(3)

$$FM(t) = \cos\left(2\pi \cdot 30 \cdot 10^6 t + k_f \int m(\lambda)\, d\lambda\right)$$

$$\omega_i(t) = 2\pi\, 30 \cdot 10^6 + k_f\, m(t)$$

$$\Delta\omega = k_f \max|m(t)|$$

$$\boxed{\Delta\omega = k_f\ (6)} \Rightarrow \Delta f = 6\ (2000) = 12 \text{ KHZ}.$$

So Bandwidth is $(B_T)_{carson} = 2(f_m + \Delta f) = 2(6+12) = 36$ KHZ

HW 4

(4) $m(t) = 6\cos(2\pi\, 1000\, t)$

$f_c = 50$ KHZ.

$\beta = 9$.

Unmodulated Carrier Power = 32 Watt.

(a) Find frequency deviation Constant K_f.

$A_m = 6$

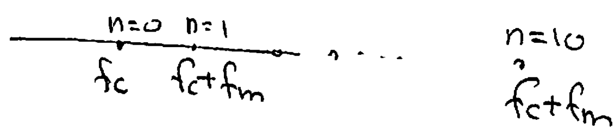
$f_m = 1000$ hz.

$$\beta = \frac{K_f A_m}{\omega_m}$$

so $K_f = \dfrac{\beta \omega_m}{A_m} = \dfrac{(9)(2\pi\ 1000)}{6} = \cancel{2}\pi\ 3000$ rad/s/V.

(b) 60 KHZ is 10 KHZ away from Carrier

so



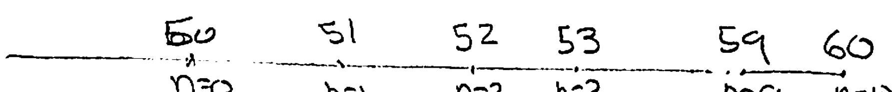
So for one side band, power is $\dfrac{A_c^2}{2} J_{10}^2(\beta=9)$

$= \dfrac{A_c^2}{2}(0.1247)^2$ watt..

so 2 sided power is $\boxed{0.1247^2} A_c^2$

but $\dfrac{A_c^2}{2} = 32 \Rightarrow A_c^2 = 64$.

So 2 sided power $= (0.1247)^2\, 64$



HW 4

(5). $m_c(t) = 50 \cos[2\pi 10^6 t + 20t + 5 \sin(2\pi 10t)]$

$$\omega_i(t) = \frac{d}{dt}\theta_i(t) = 2\pi 10^6 + 20 + 5(2\pi \times 10)\cos(2\pi 10t)$$

(6) $m_{FM}(t) = 50 \cos[2\pi 10^7 t + 8 \cos(2\pi 2000 t)]$

what is max freq. deviation?

$$\text{frequency deviation} = \frac{d}{dt}\phi(t) = \frac{d}{dt}[8 \cos(2\pi 2000 t)]$$

$$= -8 \sin(2\pi 2000 t)\,(2\pi 2000)$$

so max is $8(2\pi 2000) = 2\pi 16000$

$= \boxed{16 \text{ KHZ}}$

(7) 30 MHZ carrier.

single tone. $f_m = 11$ KHZ.

max freq. deviation $= 99$ KHZ.

(a) 1% side bandwidth.

$$\cos(\omega_c t + \beta \sin \omega_m t)$$

$$\beta = \frac{\Delta f}{f_m} = \frac{99}{11} = \boxed{9}$$

From table, we find $\boxed{n = 13}$

so $(B_T)_{1\%} = 2 f_m n_{max} = 2(11)(13) = 286$ KHZ.

(b) $(B_T)_{Carson} = 2 f_m (1+\beta) = 2(11)(1+9) = 220$ KHZ.

(c) $(B_T)_{Carson} = 2(f_m + \Delta f)$

so $(B_T)_{Carson} \longrightarrow 2(3 f_m + 3\Delta f) = 2(3 \times 11 + 3 \times 99) = 660$ kHz

HW 4

(a) $m(t) = 60\cos(2\pi 1000t) + 20\cos(2\pi 3000t)$.

$c(t) = \cos(2\pi 10^5 t)$

$K_f = 2\pi 100$ rad/s/V.

(a) $$M_{FM}(t) = \cos\left(2\pi 10^5 t + K_f \int 60\cos(2\pi 1000\lambda) + 20\cos(2\pi 3000\lambda)\,d\lambda\right)$$

$$= \cos\left(2\pi 10^5 t + K_f\left[\frac{60\sin(2\pi 1000t)}{2\pi 1000} + \frac{20\sin(2\pi 3000t)}{2\pi 3000}\right]\right)$$

$$= \cos\left(2\pi 10^5 t + 6\sin(2\pi 1000t) + \frac{2}{3}\sin(2\pi 3000t)\right)$$

so $A_{m_1} = 6$, $f_{m_1} = 1000$ hz

$A_{m_2} = \frac{2}{3}$, $f_{m_2} = 3000$ hz.

$$(B_T)_{Carson} = 2(f_m + \Delta f)$$

$\Delta f = K_f \max|(m(t))|$.

but $\max|m(t)| = 60 + 20 = 80$.

so $\Delta f = 2\pi 100 (80) = 2\pi 8000$ rad/sec. $= \boxed{8\text{ kHz}}$

so $(B_T)_{Carson} = 2(3 + 8) = 22$ kHz

we take the larger of the frequencies

HW# 4

(12) FM transmitter $\cdot A_m$, f_m

$m(t) = 8\cos(2\pi\, 200t)$, $\beta = 6$.

unmodulated power is 12 Watt. across 50Ω.

(a) find frequency deviation k_f.

$$FM(t) = A_c \cos\left(\omega_c t + k_f \int m(\lambda)\,d\lambda\right)$$

$$= A_c \cos\left(\omega_c t + 8 k_f \frac{\sin(2\pi\, 200t)}{2\pi\, 200}\right)$$

Compare to canonical form

$$FM(t) = \cos(\omega_c t + \beta \sin(\omega_m t))$$

so $\boxed{\beta = \frac{8 k_f}{2\pi\, 200}}$

so $k_f = \frac{(6)(2\pi\, 200)}{8} = \boxed{942.477}$ rad/s/Volt

(b) to Find A_c, use Power specifications

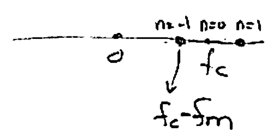
since $\text{Power} = \frac{1}{\Omega}\frac{A_c^2}{2}$

Then $12 = \frac{1}{50}\frac{A_c^2}{2}$. solve for $A_c = \sqrt{(12)(50)(2)} = 34.64$ Volt.

Peak amplitude at $f_c - 200$ hence $\boxed{n = -1}$ since $f_m = 200$.

$$S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

From tables $\boxed{J_1(6) = -0.2767}$

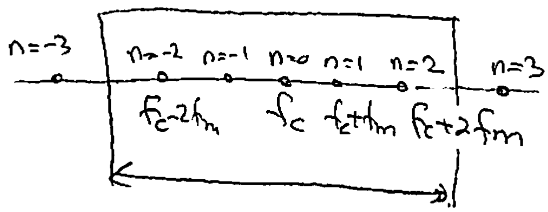


so for $n = +1$, we have

$$\text{Amplitude} = \underbrace{A_c\,(J_1(6))}\cos(\omega_c + \omega_m)t$$

$$= (34.64)(-0.2767) = \boxed{9.584 \text{ Volt}}$$

(10) $\beta = 1$,



$J_0(1) = 0.7652$

$J_1(1) = 0.4401$

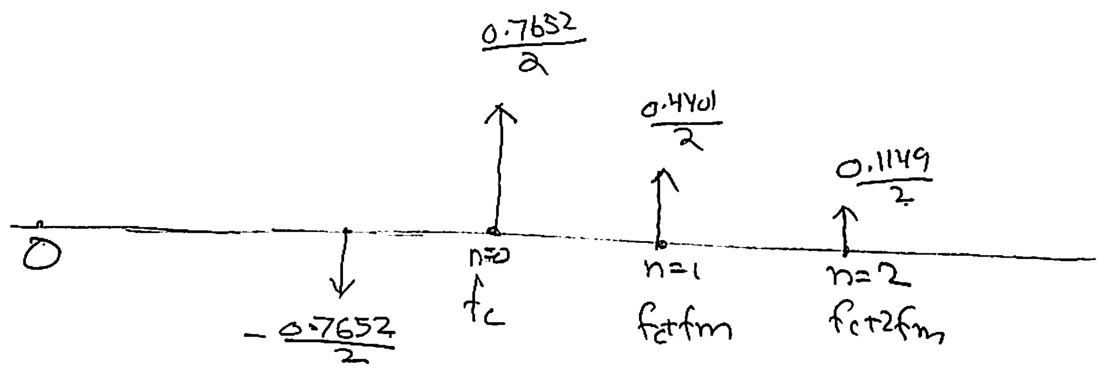
$J_2(2) = 0.1149$

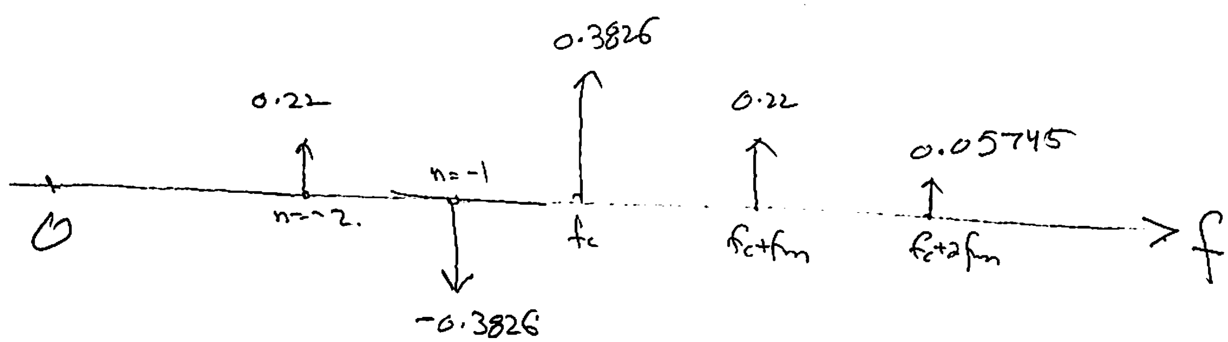
$$FM(t) = \cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right).$$

so output $FM(f)$ is

$$1 = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta\left(f - (f_c + f_m n)\right) + \delta\left(f + (f_c + n f_m)\right)\right]$$

need to go to $n=2$ on right side.





HW#4

#11

$m_{FM}(t) = 25\cos(2\pi 10^6 t + 8\sin(2\pi 3000t)]$

(a) Total average power $= \left(\frac{25^2}{2}\right)/50 = 6.25$ Watt.

(b) The Fourier Series of the above is

$$\hat{m}_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m t)$$

where $\omega_c = 2\pi 10^6$ rad/sec $\boxed{\beta = 8}$
$\omega_m = 2\pi 3000$ rad/sec.

so at $n=0$, we have $A_c J_0(8)\cos(2\pi 10^6 t)$

so the power of this is $\frac{[A_c J_0(8)]^2}{2} = \frac{[(25)(0.1717)]^2}{2}$

$= 9.212$

so over $50\,\Omega \longrightarrow \frac{9.212}{50} = 0.18425$ Watt.

so % is $\frac{0.18425}{6.25} \times 100 = 2.9\,\%$.

(c) to Find Peak freq. deviation:

Frequency deviation $= \frac{d}{dt}$ Phase deviation

$= \frac{d}{dt}\left(8\sin(2\pi 3000t)\right)$

$= 8\cos(2\pi 3000t)\,(2\pi 3000)$

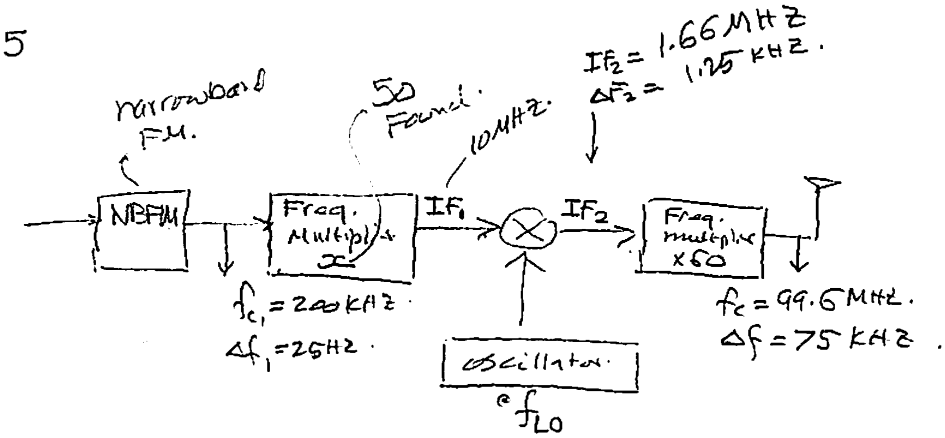
so max of the above is $(8)(2\pi 3000) = 150{,}796$ rad/sec.
$= \boxed{24 \text{ KHZ}}$

(d) $n_{max} = 11$

so $(B_T)_{1\%} = 2 f_m n_{max} = 2(3\text{KHZ})(11) = 66$ KHZ.

HW 5

(1)



we need $IF_1 = \dfrac{99.6 \text{ MHZ}}{60} = 1.66$ MHZ.

Oscillator do not affect Δf.

hence $(\Delta f_1) x (60) = \Delta f$

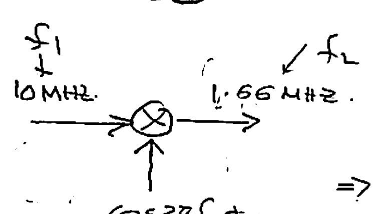
so $(25) x (60) = 75{,}000$.

hence $x = \dfrac{75{,}000}{(25)(60)} = \boxed{50}$ so multiplication factor = 50

now we use this to find the rest.

$IF_1 = (200 \text{ KHZ})(50) = 10$ MHZ.

$IF_2 = \dfrac{99.6 \text{ MHZ}}{60} = 1.66$ MHZ.



$\Rightarrow \frac{1}{2}\cos(2\pi(f_{LO}+f_1)t + \frac{1}{2}\cos(2\pi(f_{LO}-f_1)t)$

need $f_{LO} + f_1 = 1.66$ MHZ.

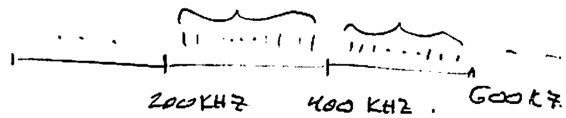
or $f_{LO} - f_1 = 1.66$ MHZ.

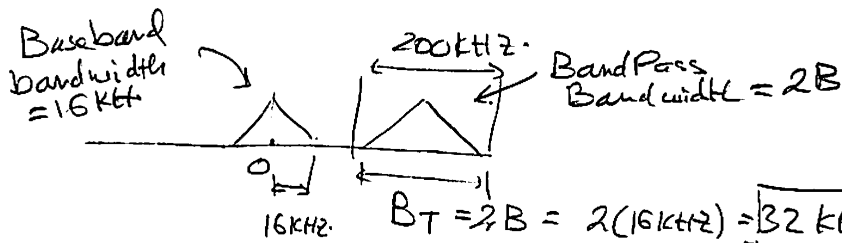
so $f_{LO} = 10 \text{ MHZ} - 1.66 \text{ MHZ} = \boxed{8.34 \text{ MHZ}}$

HW5

3

12 audio @ 16KHz.





$B_T = 2B = 2(16\,KHz) = \boxed{32\,KHz}$

so $200\,KHz \geq 32\,KHz\,(1+\beta)$ ← ask about this.

$= \beta \leq 5.25$

(b) $(B_T) = 2(B + \Delta f) = 2(16\,kHz + 2\,MHz)$

carson.

$B \rightarrow 16\,KHz$

$= 4.032\ MHz$.

(Why answer 9MHz)?

HW#5

9

FM

$$\longrightarrow \boxed{} \longrightarrow (SNR)_o = \frac{3}{4}\frac{A_c^2\beta^2}{N_o f_m} = \frac{3}{4}\frac{A_{FM}^2\beta^2}{N_o f_m}$$

for AM

$$A_m(t) = A_c \cos w_c t + m(t)\cos w_c t$$

$$= A_c(1 + \mu\, m(t))\cos w_c t$$

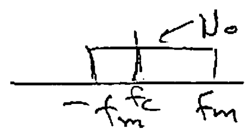
let $\mu = 1$

$$= A_c(1 + m(t))\cos w_c t$$

$$= A_c(1 + A_m \cos(2\pi f_m t))\cos w_c t$$

$$(S_o)_{Am} = \frac{A_m^2}{2}$$

$$(P_o)_{Am} = 2 N_o f_m$$



so $(SNR)_{o,Am} = \boxed{\dfrac{A_m^2}{4 N_o f_m}}$

so need

$$\frac{A_m^2}{4 N_o f_m} = \frac{3}{4}\frac{A_{FM}^2 \beta^2}{N_o f_m}$$

so $\dfrac{A_{Am}}{A_{FM}} = \sqrt{3\beta^2}$ but $\beta = 9$.

$$= \sqrt{3\,9^2} = \sqrt{3}\;9 = \boxed{15.58}$$

HW#5

10

$m(t) = 8\cos(2\pi 5000t)$

$K_f = 2\pi\, 4000$ rad/s/V.

$A_c = 0.1$ V.

$$(SNR)_o = \frac{3}{4}\frac{A_c^2\beta^2}{N_o f_m} = \frac{3}{4}\frac{(0.1)^2\beta^2}{(10^{-6})(5000)}. \quad \text{—— } (1)$$

$$\text{but } \beta = \frac{K_f A_m}{2\pi f_m} = \frac{(2\pi 4000)(8)}{2\pi\,(5000)}$$

plug all this into (1) and Calculate

$$(SNR)_o = 614.4$$

$$\text{so in db} = 10\log_{10}(614.4) = 27.88 \text{ db}$$

HW5

11

FM, $(SNR)_i = 30$ db.

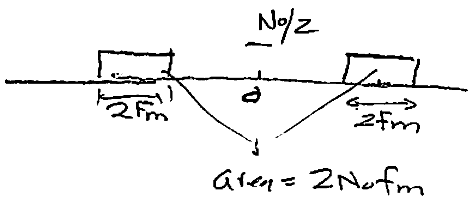
$(SNR)_o = 48.$ db..

$(SNR)_i \;\; \frac{S_i}{P_i} \longrightarrow \boxed{\text{FM receiver}} \longrightarrow (SNR)_o$

$S_i = \frac{A_c^2}{2}$.

$P_i = 2 N_o f_m$

$(SNR)_o = \frac{3}{4} \frac{A_c^2 \beta^2}{N_o f_m}$



here. $10 \log \left(\frac{A_c^2}{2 N_o f_m} \right) = 30$

$10 \log \left(\frac{3}{4} \frac{A_c^2 \beta^2}{N_o f_m} \right) = 48$

so $\frac{A_c^2}{2 N_o f_m} = 10^3$

so $10 \log \left(\frac{3}{2} \beta^2 (10^3) \right) = 48$.

so $\frac{3}{2} \beta^2 (10^3) = 10^{4.8}$

so $\beta^2 = \frac{\left(\frac{2}{3} 10^{4.8}\right)}{10^3} \Rightarrow \boxed{\beta = 6.485}$

HW#5

7

Power in signal after filter =

$$\int_{-2,000}^{2,000} S_m(f)\,df = (2\,\text{kHz})(10^{-2}) + (1\,\text{kHz})(10^{-2})$$

$$= (3,000)(10^{-2}) = 30 \text{ Watt}$$

Power in noise after filter = $\displaystyle\int_{-2,000}^{2,000} \frac{1}{2}\times 10^{-6}\,df$

$$= \left(\frac{1}{2}\times 10^{-6}\right)(4,000) = (10^{-6})(2,000) = 2\times 10^{-3} \text{ Watt}$$

so $\text{SNR} = \dfrac{30}{2\times 10^{-3}} = 1.5\times 10^{4}$

so in db,

$$(\text{SNR})_{db} = 10 \log_{10}(1.5\times 10^{4})$$

$$= 10\left[\log_{10} 1.5 + \log_{10} 10^{4}\right]$$

$$= 10\left(\log_{10} 1.5 + 4\right)$$

$$= 40 + 10\log_{10} 1.5$$

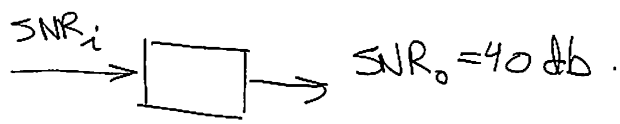
$$= \boxed{41.76}$$

HW#5

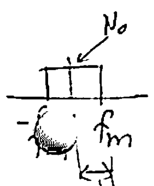
(8) $m(t) = 2\cos(2\pi\, 5000\, t)$

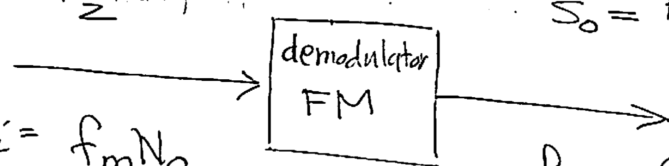
$c(t) \Rightarrow \omega_c = 2\pi\, 30\times10^3$

$k_f = 2\pi\, 15000$ rad/sec per Volt.



$S_i = \dfrac{A_c^2}{2}$ $\qquad$ $S_o = k_f^2 \dfrac{A_m^2}{2}$





$P_i = f_m N_o$ $\qquad$ $P_o = \dfrac{8\pi^2 N_o f_m^3}{3A_c^2}$

Bandwidth at Baseband $= f_m$

so $(SNR)_i = \dfrac{A_c^2}{2 f_m N_o}$ $\qquad$ $(SNR)_o = \dfrac{3}{4}\dfrac{A_c^2 \beta^2}{N_o f_m}$

hence we see that $\boxed{(SNR_o) = \frac{3}{2}\beta^2 (SNR)_i}$

so in db, $40 = 10\log_{10}\left(\frac{3}{2}\beta^2\right) + (SNR)_{i,db}$

so $(SNR)_{i,db} = 40 - 10\log_{10}\left(\frac{3}{2}\beta^2\right)$.

But $\beta = \dfrac{k_f A_m}{2\pi f_m} = \dfrac{\cancel{2\pi}(15000)(2)}{\cancel{2\pi}(5000)} = \dfrac{30{,}000}{5{,}000} = \boxed{6}$

so $(SNR_i)_{db} = 40 - 10\log_{10}\left(\frac{3}{2}6^2\right) = \boxed{22.6761}$

HW 5

12. AM: $A_c \cos \omega_c t + m(t) \cos \omega_c t$.

FM: $A_c \cos(\omega_c t + k_f \int m(\lambda) d\lambda)$

(a) B_T for AM $= 2 f_m$.

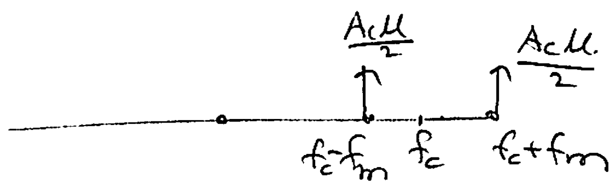
Frequency deviation for FM is $\frac{d}{dt}\left(k_f \int m(\lambda) d\lambda\right) = k_f\, m(t)$.

so max of this $= k_f |m(t)|_{max} = k_f A_m \equiv \Delta f$

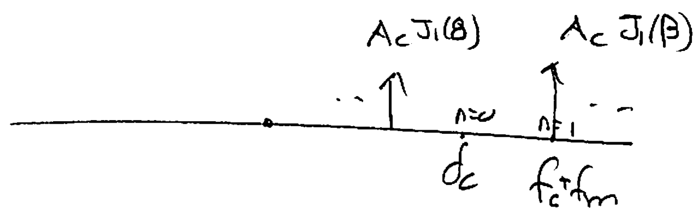
so $\overset{\Delta f}{(k_f A_m)} = 4(2 f_m)$

$\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m} = \frac{8 f_m}{f_m} = \boxed{8}$

(b) For AM



For FM



So

$$A_c J_1(8) = \frac{A_c \mu}{2}$$

so

$$\mu = 2 J_1(8) = \boxed{0.469}$$

HW 6, Problem 1

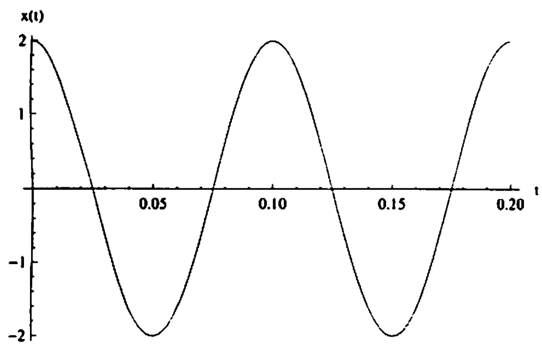
by Nasser M. Abbasi

```
In[281]:= << dsp`
```

- **part (a)**

```
In[282]:= Clear[w, t];
          fm = 10;
          period = 1 / fm;
          x[t_] := 2 Cos[2 Pi fm t]
          Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}]
```

Out[286]=



- **part(b)**

```
In[287]:= ft = FourierTransform[x[t], t, f, FourierParameters → {0, -2 Pi}];
          Print["Fourier Transform of x(t) is"];
          ft
```

```
Fourier Transform of x(t) is
```

```
Out[289]= DiracDelta[-10 + f] + DiracDelta[10 + f]
```

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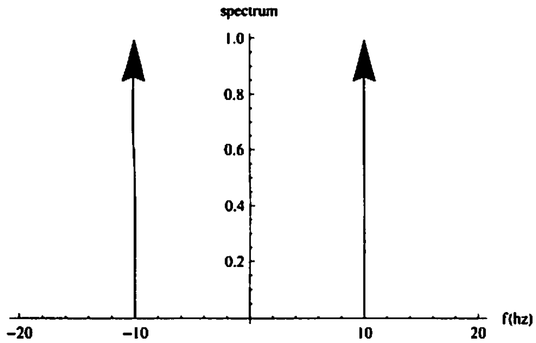
2 | *prob1.nb*

In[290]:=
```
dsp`plotFourierTransform[ft, f, -2 fm, 2 fm, 0, .5, Large];
Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

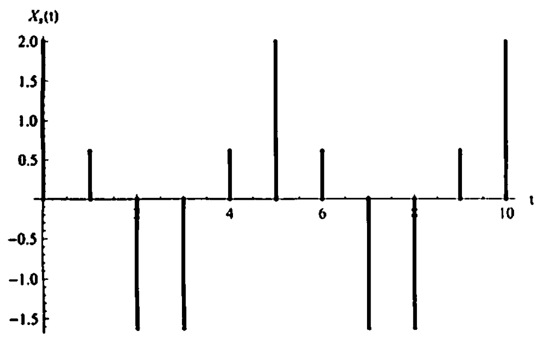
Out[291]=



- **part(c)**

In[292]:=
```
Ts = 0.02;
nSamples = 2 * period / Ts;
data = Table[{n, x[n Ts]}, {n, 0, nSamples}];
ListPlot[data, Filling → Axis, FillingStyle → Thick, AxesLabel → {"t", "X_s(t)"}]
```

Out[295]=



- **part(d)**

In[296]:=
```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

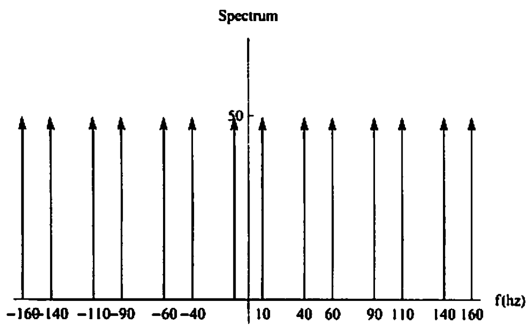
```
Sampling frequency = 50. hz
```

In[390]:=
```
spectrum = Expand[fs Sum[ft /. f → (f - n * fs), {n, -3, 3}]];
Print["spectrum=", spectrum];
```

```
spectrum=50. DiracDelta[-160. + f] + 50. DiracDelta[-140. + f] + 50. DiracDelta[-110. + f] +
 50. DiracDelta[-90. + f] + 50. DiracDelta[-60. + f] + 50. DiracDelta[-40. + f] +
 50. DiracDelta[-10 + f] + 50. DiracDelta[10 + f] + 50. DiracDelta[40. + f] + 50. DiracDelta[60. + f] +
 50. DiracDelta[90. + f] + 50. DiracDelta[110. + f] + 50. DiracDelta[140. + f] + 50. DiracDelta[160. + f]
```

```
In[352]:= Show[First@dsp`plotFourierTransform[spectrum, f, -3 * fs, 3 * fs, -.1 fs, 1.4 fs, Small],
   AxesLabel → {"f(hz)", "Spectrum"},
   Ticks → {{-160, -140, -110, -90, -60, -40, -10, 10, 40, 60, 90, 110, 140, 160}, {50}}]
```

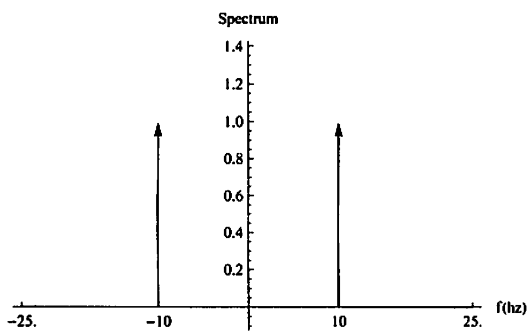
Out[352]=



- **part(e)**

```
In[374]:= bandwidth = 0.5 fs;
   Print["bandwidth = ", bandwidth, " hz"];

bandwidth = 25. hz

In[345]:= gain = Ts;
   Print["Gain=", gain];

Gain=0.02

In[396]:= spectrum = fs DiracDelta[-10 + f] + fs DiracDelta[10 + f];
   Show[First@dsp`plotFourierTransform[
      Expand[gain * spectrum], f, -bandwidth, bandwidth, -.1 , 1.4 , Small],
    AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-bandwidth, -10, 0, 10, bandwidth}}]
```

Out[397]=



- **part(f)**

From the output above, we conclude that $y(t) = 2\,\mathrm{Cos}[2\,\pi\,10\,t]$

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HW 6, Problem 2

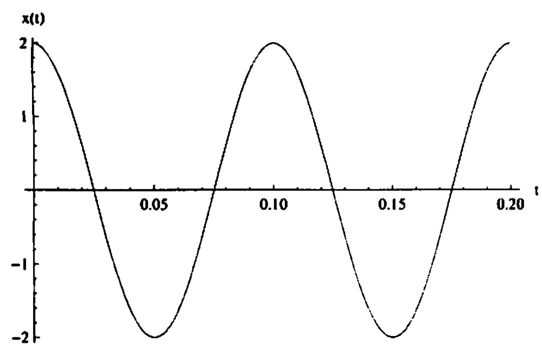
by Nasser M. Abbasi

```
In[96]:= << dsp`
```

- **part (a)**

```
In[144]:= Clear[w, t, t, f];
          fm = 10;
          period = 1 / fm;
          Print["Period of message = ", N@period , " seconds"];

Period of message = 0.1 seconds

In[148]:= x[t_] := 2 Cos[2 Pi fm t]
          Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}]
```

Out[149]=

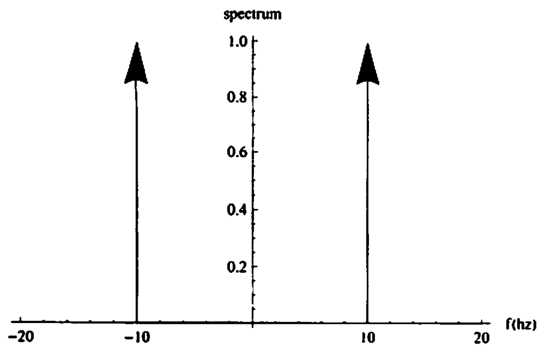


- **part(b)**

```
In[150]:= ft = FourierTransform[x[t], t, f, FourierParameters -> {0, -2 Pi}];
          Print["Fourier Transform of x(t) is", ft];

Fourier Transform of x(t) isDiracDelta[-10 + f] + DiracDelta[10 + f]
```

In[152]:=
```
dsp`plotFourierTransform[ft, f, -2 fm, 2 fm, 0, .5, Large];
Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

Out[153]=



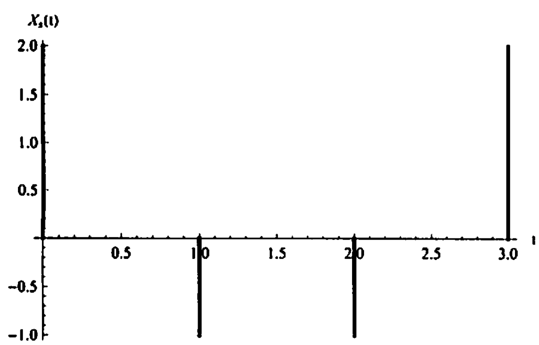
- **part(c)**

In[156]:=
```
Ts = 1 / 15;
Print[" samping period = ", N@Ts, " seconds"];
```

```
samping period = 0.0666667 seconds
```

In[158]:=
```
nSamples = 2 * period / Ts;
data = Table[{n, x[n Ts]}, {n, 0, nSamples}];
ListPlot[data, Filling → Axis, FillingStyle → Thick, AxesLabel → {"t", "Xs(t)"}]
```

Out[160]=



- **part(d)**

In[161]:=
```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

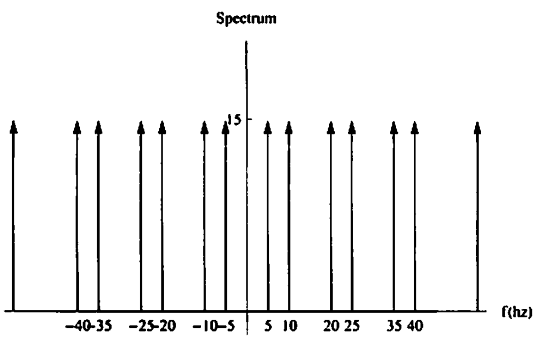
```
Sampling frequency = 15 hz
```

In[164]:=
```
spectrum = Expand[fs Sum[ft /. f → (f - n * fs), {n, -3, 3}]];
Print["spectrum=", spectrum];
```

```
spectrum=15 DiracDelta[-55 + f] + 15 DiracDelta[-40 + f] + 15 DiracDelta[-35 + f] +
  15 DiracDelta[-25 + f] + 15 DiracDelta[-20 + f] + 15 DiracDelta[-10 + f] +
  15 DiracDelta[-5 + f] + 15 DiracDelta[5 + f] + 15 DiracDelta[10 + f] + 15 DiracDelta[20 + f] +
  15 DiracDelta[25 + f] + 15 DiracDelta[35 + f] + 15 DiracDelta[40 + f] + 15 DiracDelta[55 + f]
```

In[166]:=
```
Show[First@dsp`plotFourierTransform[spectrum, f, -3 * fs, 3 * fs, -.1 fs, 1.4 fs, Small],
  AxesLabel → {"f(hz)", "Spectrum"},
  Ticks → {{-40, -35, -25, -20, -10, -5, 5, 10, 20, 25, 35, 40}, {fs}}]
```

Out[166]=



- **part(e)**

In[167]:=
```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
```

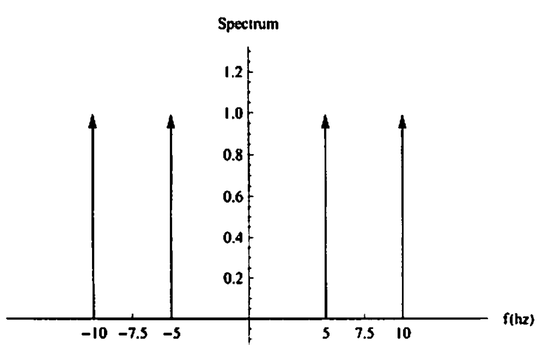
```
bandwidth = 7.5 hz
```

In[169]:=
```
gain = Ts;
Print["Gain=", N@gain];
```

```
Gain=0.0666667
```

In[173]:=
```
spectrum =
  (15 DiracDelta[-10 + f] + 15 DiracDelta[-5 + f] + 15 DiracDelta[5 + f] + 15 DiracDelta[10 + f]);
Show[First@dsp`plotFourierTransform[Expand[gain * spectrum],
   f, -2 * bandwidth, 2 * bandwidth, -.1 , 1.3 , Small],
 AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-bandwidth, -10, -5, 0, 5, 10, bandwidth}}]
```

Out[174]=



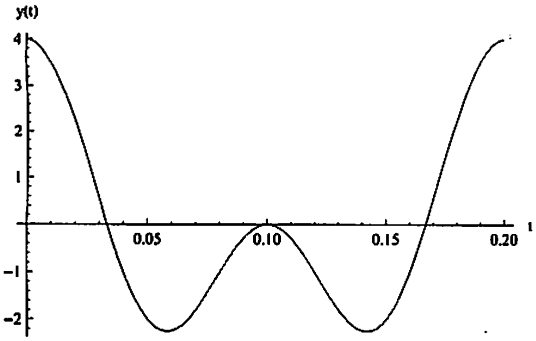
- **part(f)**

From the output above, we conclude that $y(t)$ = 2 Cos[2 π 10 t]+2Cos[2 π 5 t]

4 | *prob2.nb*

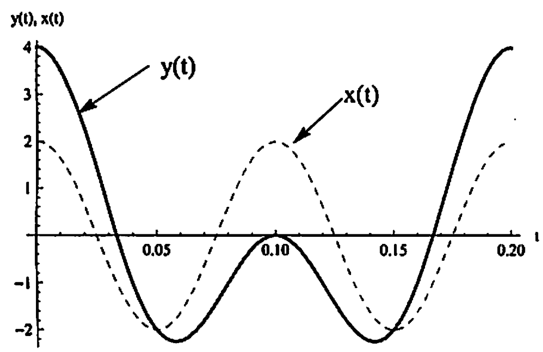
```
In[175]:= y[t_] := 2 Cos[2 Pi 10 t] + 2 Cos[2 Pi 5 t];
          Plot[y[t], {t, 0, 2 period}, AxesLabel → {"t", "y(t)"}]
```

Out[176]=



Compare this with the original signal x(t) to see aliasing

```
In[178]:= Plot[{x[t], y[t]}, {t, 0, 2 period},
            AxesLabel → {"t", "y(t), x(t)"}, PlotStyle → {Dashed, Thick}]
```



HW 6, Problem 3

by Nasser M. Abbasi

```
<< dsp`
```

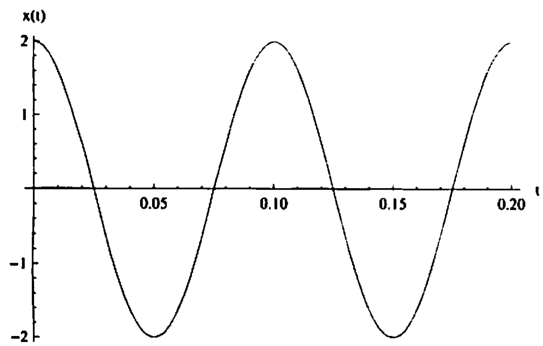
- **part (a)**

```
In[468]:= Clear[w, t, t, f];
          fm = 10;
          period = 1 / fm;
          Print["Period of message = ", N@period , " seconds"];
```

```
Period of message = 0.1 seconds
```

```
In[472]:= x[t_] := 2 Cos[2 Pi fm t]
          Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}]
```

Out[473]=



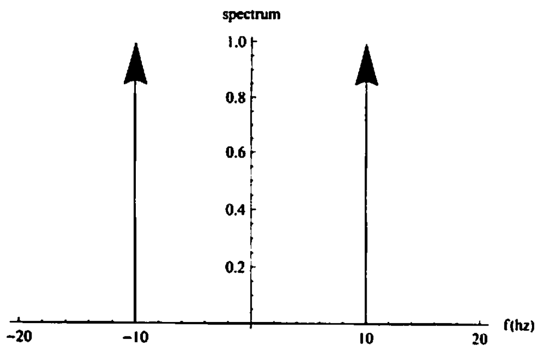
- **part(b)**

```
In[485]:= ft = FourierTransform[x[t], t, f, FourierParameters → {0, -2 Pi}];
          Print["Fourier Transform of x(t) = ", ft];
```

Fourier Transform of x(t) = $\delta(f-10)+\delta(f+10)$

```
In[476]:= dsp`plotFourierTransform[ft, f, -2 fm, 2 fm, 0, .5, Large];
          Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

Out[477]=



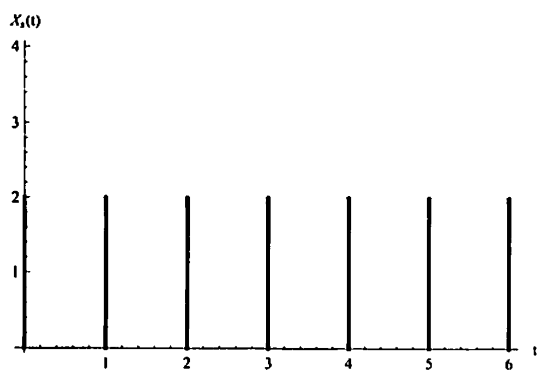
- **part(c)**

```
In[478]:= Ts = 1 / 10;
          Print[" samping period = ", N@Ts, " seconds"];
```

```
 samping period = 0.1 seconds
```

```
In[480]:= nSamples = 6 * period / Ts;
          data = Table[{n, x[n Ts]}, {n, 0, nSamples}];
          ListPlot[data, Filling → Axis, FillingStyle → Thick, AxesLabel → {"t", "X_s(t)"}]
```

Out[482]=

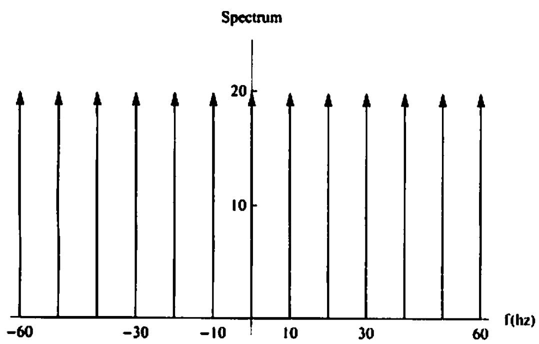


- **part(d)**

```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

```
Sampling frequency = 10 hz
```

```
spectrum = fs Sum[ft /. f → (f - n * fs), {n, -7, 7}];
spectrum = Expand[spectrum];
spectrum = 20 DiracDelta[-60 + f] + 20 DiracDelta[-50 + f] + 20 DiracDelta[-40 + f] +
   20 DiracDelta[-30 + f] + 20 DiracDelta[-20 + f] + 20 DiracDelta[-10 + f] +
   20 DiracDelta[f] + 20 DiracDelta[10 + f] + 20 DiracDelta[20 + f] + 20 DiracDelta[30 + f] +
   20 DiracDelta[40 + f] + 20 DiracDelta[50 + f] + 20 DiracDelta[60 + f];

Show[First@dsp`plotFourierTransform[spectrum, f, -6 * fs, 6 * fs, -.1 fs, 2.4 fs, Small],
 AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-60, -30, -10, 10, 30, 60}, {fs, 2 * fs}}]
```



- **part(e)**

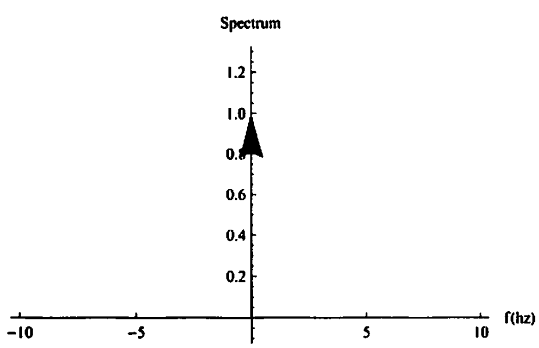
```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
```

```
bandwidth = 5. hz
```

```
gain = Ts;
Print["Gain=", N@gain];
```

```
Gain=0.1
```

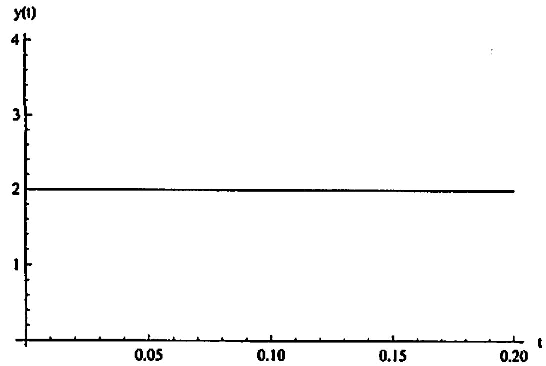
```
spectrum = ( 20 DiracDelta[f]);
Show[First@dsp`plotFourierTransform[
   Expand[gain * spectrum], f, -2 * bandwidth, 2 * bandwidth, -.1 , 1.3 , Large],
 AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-10, -5, 0, 5, 10}}]
```



- **part(f)**

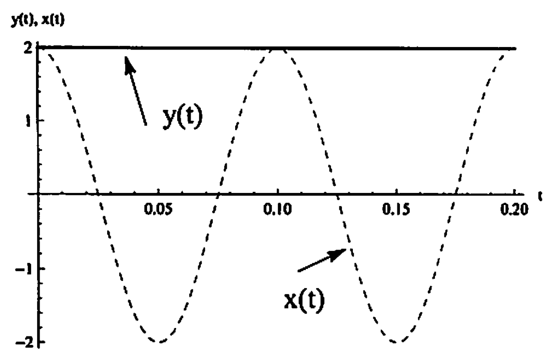
From the output above, we conclude that $y(t) = 2\,\mathrm{Cos}[2\,\pi\,0\,t]$=2

```
y[t_] := 2 ;
Plot[y[t], {t, 0, 2 period}, AxesLabel → {"t", "y(t)"}]
```



Compare this with the original signal x(t)

```
Plot[{x[t], y[t]}, {t, 0, 2 period},
 AxesLabel → {"t", "y(t), x(t)"}, PlotStyle → {Dashed, Thick}]
```



HW 6, Problem 4

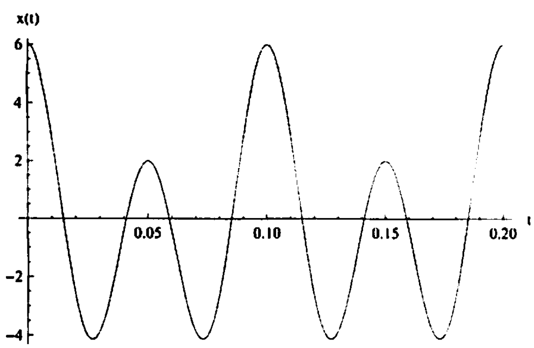
by Nasser M. Abbasi

```
In[266]:= << dsp`
```

- **part (a)**

```
In[269]:= Clear[w, t, t, f];
          f1 = 20;
          f2 = 10;
          period1 = 1 / f1;
          period2 = 1 / f2;

          Print["Period of message 1 = ", N@period1 , " seconds"];

Period of message 1 = 0.05 seconds

In[275]:= Print["Period of message 2 = ", N@period2 , " seconds"];

Period of message 2 = 0.1 seconds

In[302]:= x[t_] := 4 Cos[2 Pi f1 t] + 2 Cos[2 Pi f2 t];
          p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}]
```

Out[303]=



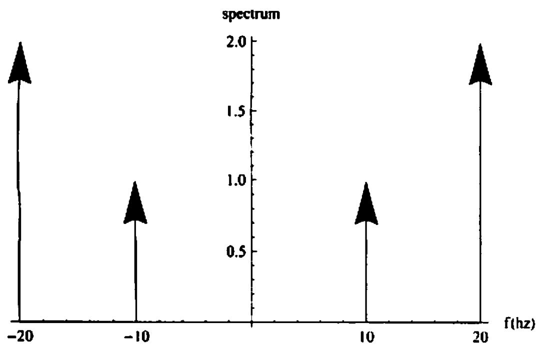
- **part(b)**

```
In[338]:= ft = FourierTransform[x[t], t, f, FourierParameters → {0, -2 Pi}];
          Print["Fourier Transform of x(t) = ", ft];

Fourier Transform of x(t) =
 2 DiracDelta[-20 + f] + DiracDelta[-10 + f] + DiracDelta[10 + f] + 2 DiracDelta[20 + f]
```

2 | *prob4.nb*

In[340]:=
```
dsp`plotFourierTransform[ft, f, -2 f2, 2 f2, 0, .5, Large];
Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

Out[341]=



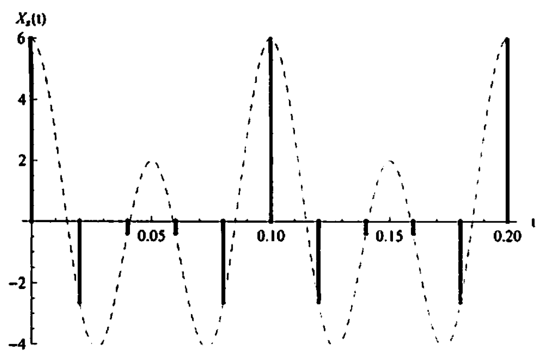
- **part(c)**

In[282]:=
```
Ts = 1 / 50;
Print[" samping period = ", N@Ts, " seconds"];
```

```
samping period = 0.02 seconds
```

In[322]:=
```
nSamples = 2 * period2 / Ts;
p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}, PlotStyle → Dashed];
data = Table[{n * Ts, x[n Ts]}, {n, 0, nSamples}];
Show[{ListPlot[data, Filling → Axis, FillingStyle → Thick,
   AxesLabel → {"t", "X_s(t)"}, PlotRange → {Automatic, {-4, 6}}], p1}]
```

Out[325]=



- **part(d)**

In[326]:=
```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

```
Sampling frequency = 50 hz
```

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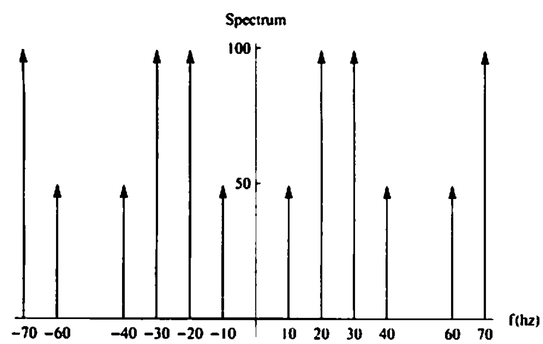
In[365]:=
```
spectrum = fs Sum[ft /. f → (f - n * fs), {n, -1, 1}];
spectrum = Expand[spectrum]
```

Out[366]=
```
100 DiracDelta[-70 + f] + 50 DiracDelta[-60 + f] + 50 DiracDelta[-40 + f] +
 100 DiracDelta[-30 + f] + 100 DiracDelta[-20 + f] + 50 DiracDelta[-10 + f] +
 50 DiracDelta[10 + f] + 100 DiracDelta[20 + f] + 100 DiracDelta[30 + f] +
 50 DiracDelta[40 + f] + 50 DiracDelta[60 + f] + 100 DiracDelta[70 + f]
```

In[370]:=
```
Show[First@dsp`plotFourierTransform[spectrum, f, -fs, fs, -.1 fs, 2 fs, Small],
 AxesLabel → {"f(hz)", "Spectrum"},
 Ticks → {{-70, -60, -40, -30, -20, -10, 10, 20, 30, 40, 60, 70}, {fs, 2 fs}}]
```

Out[370]=



- **part(c)**

In[371]:=
```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
```

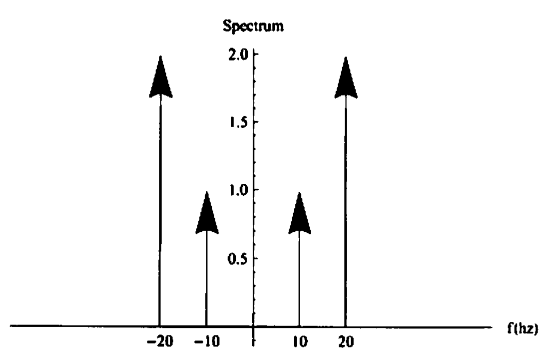
```
bandwidth = 25. hz
```

In[373]:=
```
gain = Ts;
Print["Gain=", N@gain];
```

```
Gain=0.02
```

In[381]:=
```
spectrum = ( 100 DiracDelta[-20 + f] +
    50 DiracDelta[-10 + f] + 50 DiracDelta[10 + f] + 100 DiracDelta[20 + f]);
Show[First@dsp`plotFourierTransform[Expand[gain * spectrum], f,
   -2 * bandwidth, 2 * bandwidth, -.1 , 2 , Large],
 AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-20, -10, 0, 10, 20}}]
```

Out[382]=

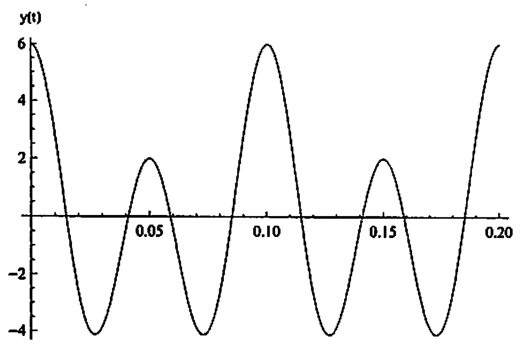


part(f)

From the output above, we conclude that $y(t)$ = 2 Cos[2 π 10 t]+4Cos[2π 20t]

```
In[383]:= y[t_] := 2 Cos[2 Pi 10 t] + 4 Cos[2 π 20 t];
          Plot[y[t], {t, 0, 2 period}, AxesLabel → {"t", "y(t)"}]
```

Out[384]=



which is the same as orginal x(t)

HW 6, Problem 5

by Nasser M. Abbasi

In[385] := `<< dsp``

- **part (a)**

In[498] :=
```
Clear[w, t, t, f];
f1 = 20;
f2 = 10;
period1 = 1/f1;
period2 = 1/f2;
Print["Period of message 1 = ", N[period1], " seconds"];
```

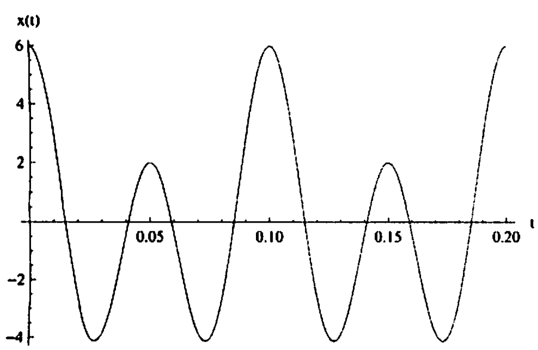
Period of message 1 = 0.05 seconds

In[504] :=
```
Print["Period of message 2 = ", N@period2 , " seconds"];
```

Period of message 2 = 0.1 seconds

In[505] :=
```
x[t_] := 4 Cos[2 Pi f1 t] + 2 Cos[2 Pi f2 t];
p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel -> {"t", "x(t)"}]
```

Out[506]=



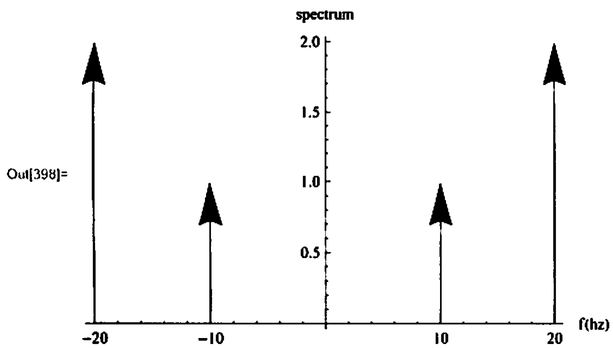
- **part(b)**

In[507] :=
```
ft = FourierTransform[x[t], t, f, FourierParameters -> {0, -2 Pi}];
Print["Fourier Transform of x(t) = ", ft];
```

Fourier Transform of x(t) = $2\,\delta(f-20)+\delta(f-10)+\delta(f+10)+2\,\delta(f+20)$

In[397]:=
```
dsp`plotFourierTransform[ft, f, -2 f2, 2 f2, 0, .5, Large];
Show[%, PlotRange → All, AxesLabel → {"f(hz)", "spectrum"}]
```

Out[398]=



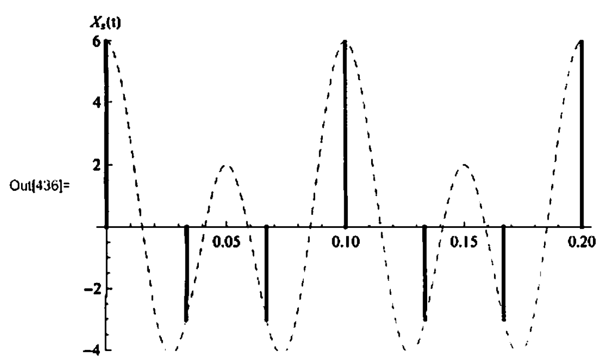
- **part(c)**

In[431]:=
```
Ts = 1 / 30;
Print[" samping period = ", N@Ts, " seconds"];
```

```
samping period = 0.0333333 seconds
```

In[433]:=
```
nSamples = 2 * period2 / Ts;
p1 = Plot[x[t], {t, 0, 2 period}, AxesLabel → {"t", "x(t)"}, PlotStyle → Dashed];
data = Table[{n * Ts, x[n Ts]}, {n, 0, nSamples}];
Show[{ListPlot[data, Filling → Axis, FillingStyle → Thick,
   AxesLabel → {"t", "X_s(t)"}, PlotRange → {Automatic, {-4, 6}}], p1}]
```

Out[436]=



- **part(d)**

In[509]:=
```
Clear[n, f];
fs = 1 / Ts;
Print["Sampling frequency = ", fs, " hz"];
```

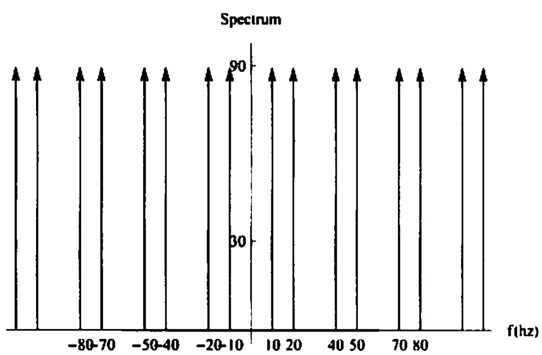
Sampling frequency = 10 hz

In[515]:=
```
spectrum = fs Sum[ft /. f → (f - n * fs), {n, -4, 4}];
spectrum = Expand[spectrum];
spectrum = 90 DiracDelta[-110 + f] + 90 DiracDelta[-100 + f] + 90 DiracDelta[-80 + f] +
   90 DiracDelta[-70 + f] + 90 DiracDelta[-50 + f] + 90 DiracDelta[-40 + f] +
   90 DiracDelta[-20 + f] + 90 DiracDelta[-10 + f] + 90 DiracDelta[10 + f] +
   90 DiracDelta[20 + f] + 90 DiracDelta[40 + f] + 90 DiracDelta[50 + f] + 90 DiracDelta[70 + f] +
   90 DiracDelta[80 + f] + 90 DiracDelta[100 + f] + 90 DiracDelta[110 + f]
```

Out[517]= $90\,\delta(f-110)+90\,\delta(f-100)+90\,\delta(f-80)+90\,\delta(f-70)+90\,\delta(f-50)+90\,\delta(f-40)+90\,\delta(f-20)+90\,\delta(f-10)+90\,\delta(f+10)+90\,\delta(f+20)+90\,\delta(f+40)+90\,\delta(f+50)+90\,\delta(f+70)+90\,\delta(f+80)+90\,\delta(f+100)+90\,\delta(f+110)$

In[454]:=
```
Show[First@dsp`plotFourierTransform[spectrum, f, -2 fs, 2 fs, -.1 fs, 3.2 fs, Small],
 AxesLabel → {"f(hz)", "Spectrum"},
 Ticks → {{-80, -70, -50, -40, -20, -10, 10, 20, 40, 50, 70, 80}, {fs, 3 fs}}]
```

Out[454]=



- **part(e)**

In[455]:=
```
bandwidth = 0.5 fs;
Print["bandwidth = ", bandwidth, " hz"];
```

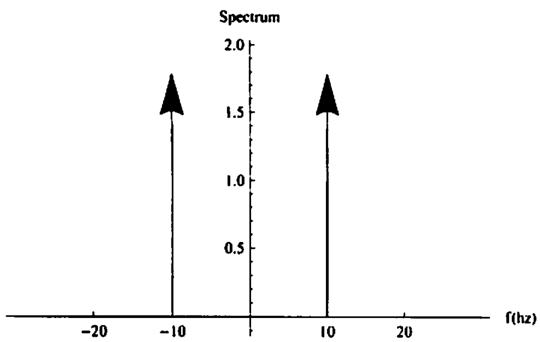
```
bandwidth = 15. hz
```

In[373]:=
```
gain = Ts;
Print["Gain=", N@gain];
```

```
Gain=0.02
```

In[457]:=
```
spectrum = (90 DiracDelta[-10 + f] + 90 DiracDelta[10 + f]);
Show[First@dsp`plotFourierTransform[
    Expand[gain * spectrum], f, -2 * bandwidth, 2 * bandwidth, -.1 , 2 , Large],
  AxesLabel → {"f(hz)", "Spectrum"}, Ticks → {{-20, -10, 0, 10, 20}}]
```

Out[458]=



In[460]:=
```
h = 0.02 * 90;
v = 2 * 1.8
```

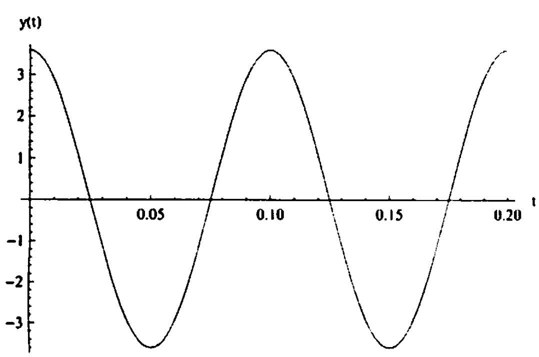
Out[461]= 3.6

- **part(f)**

From the output above, we conclude that $y(t) = 3.6 \, \text{Cos}[2\,\pi\, 10\, t]$

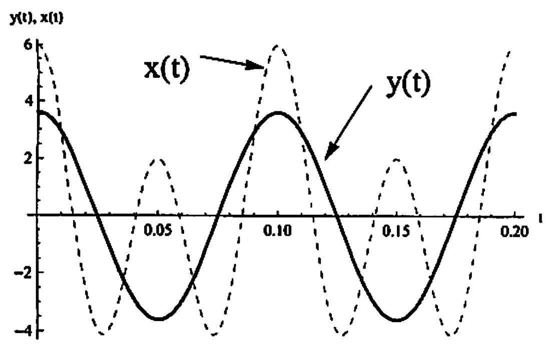
In[462]:=
```
y[t_] := 3.6 Cos[2 Pi 10 t];
Plot[y[t], {t, 0, 2 period}, AxesLabel → {"t", "y(t)"}]
```

Out[463]=



compare to orginal x(t)

In[464]:=
```
Plot[{x[t], y[t]}, {t, 0, 2 period},
  AxesLabel → {"t", "y(t), x(t)"}, PlotStyle → {Dashed, Thick}]
```


y(t), x(t)
x(t)
y(t)
6
4
2
-2
-4
0.05
0.10
0.15
0.20
t

HW#6

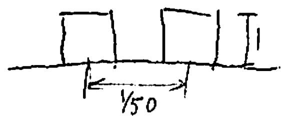
6

$T = 0.1$ sec.

$\frac{\tau}{T_s} = \frac{1}{4}$ so $\boxed{\tau = \frac{T_s}{4}}$

$x(t) = 2\cos(2\pi\, 10 t)$.

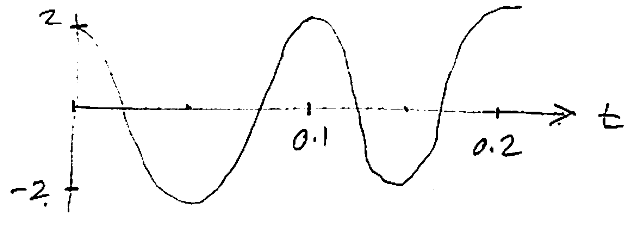
sampled uniformly by



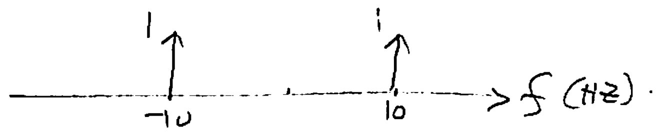
dutycycle $= \frac{1}{4}$, $T_s = \frac{1}{50}$.

$= \frac{1}{5}\left(\frac{1}{10}\right)$ sec

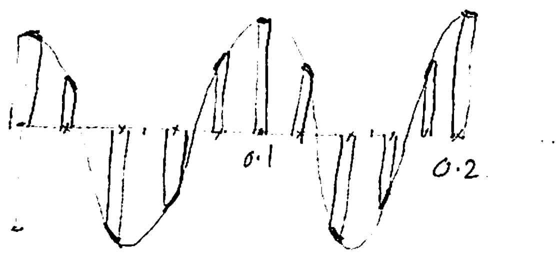
(a)



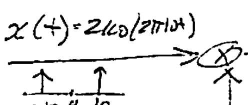
(b)



(c)



(d)



$X_s(t) = x(t) g(t) = x(t) \sum h\, \text{rect}\left(\frac{t - nT_s}{\tau}\right)$

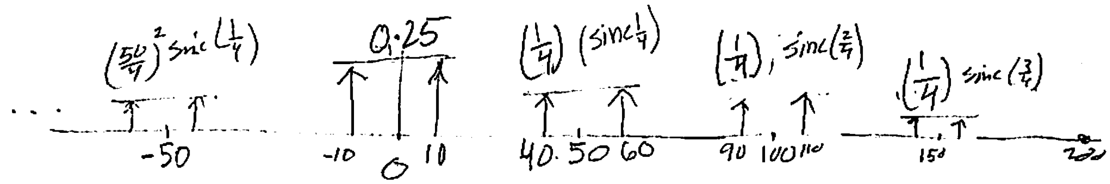
$\downarrow$ F.T

$X_s(f) = X(f) \circledast f_s h \sum \tau\, \text{sinc}\left(\frac{n\tau}{T_s}\right) \delta(f - nf_s)$

$= f_s h \tau \sum \text{sinc}\left(\frac{n\tau}{T_s}\right) X(f - nf_s)$

$g(t) = \sum h\, \text{rect}\left(\frac{t - nT_s}{\tau}\right)$

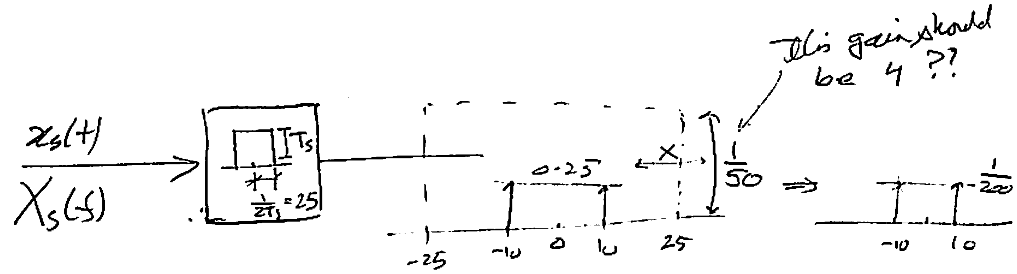
$G(f) = f_s \sum h\tau\, \text{sinc}\left(n\frac{\tau}{T_s}\right)\delta(f - nf_s)$ $= (50)\left(\frac{1}{4}\cdot\frac{1}{50}\right) \sum \text{sinc}\left(\frac{n}{4}\right) X(f - nf_s)$ ^{50}



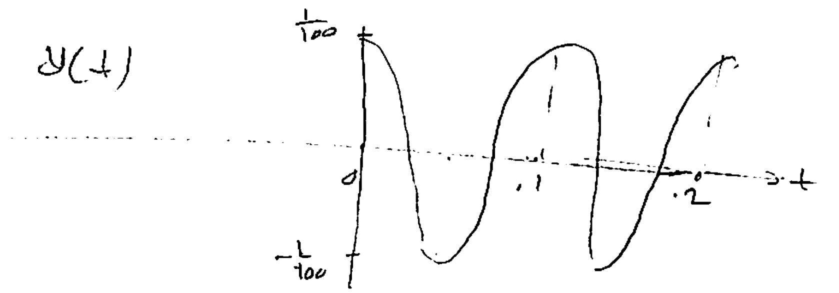
HW6

6.

(e)



(f) we see that $\boxed{y(t) = \frac{1}{100}\cos(2\pi 10 t)}$

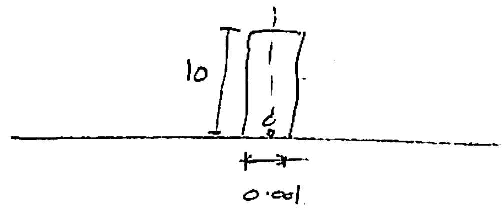


note amplitude is different, because gain we are asked to use is T_s which is not what normally used for pulse rect train which should have been (d).

HW6

(7) determine Nyquist rate for

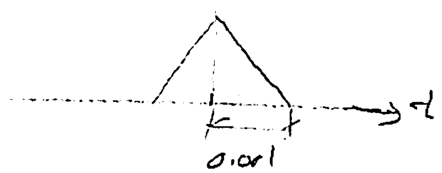
(a) $x(t) = 10 \, \text{rect}\left(\frac{t}{0.001}\right)$.



so $\tau = 0.001$

this signal is not periodic. hence its bandwidth is ∞. hence require ∞ sampling frequency.

(b) $x(t) = \text{tri}\left(\frac{t}{0.001}\right)$



for similar reasoning as (a). this is not periodic, hence ∞ bandwidth $\Rightarrow$ ∞ sampling freq.

(c) $\text{sinc}(1000t) = \frac{\sin(\pi 1000 t)}{\pi 1000}$

so $f_m = 500$ Hz.

hence Nyquist $= 1000$ Hz.

(d) $\text{sinc}(2000t) + \text{sinc}^2(1100t)$

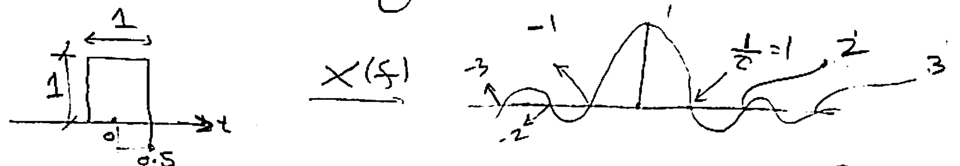
$= \frac{\sin(2\pi 1000 t)}{\pi 2000} + \left[\frac{\sin(2\pi 550 t)}{2\pi 550}\right]^2$

$= \frac{\sin 2\pi 1000 t}{\pi 2000} + \left(\frac{1}{2\pi 550}\right)^2 \left[\frac{1}{2} - \frac{1}{2}\cos(2\pi 1100 t)\right] \Rightarrow$ Nyquist $=$ 2200 H

HW 6

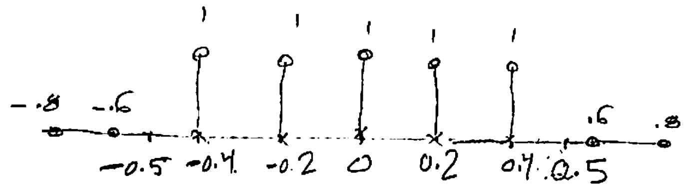
8. $x(t) = \mathrm{rect}(t)$ is ideally sampled at rate 5 samples/s.

(a)



Nyquist rate $= \infty$. see problem 7, part (a).

(b) $T_s = \frac{1}{5} = 0.2$



(c) need to print spectrum of $\hat{x}$.

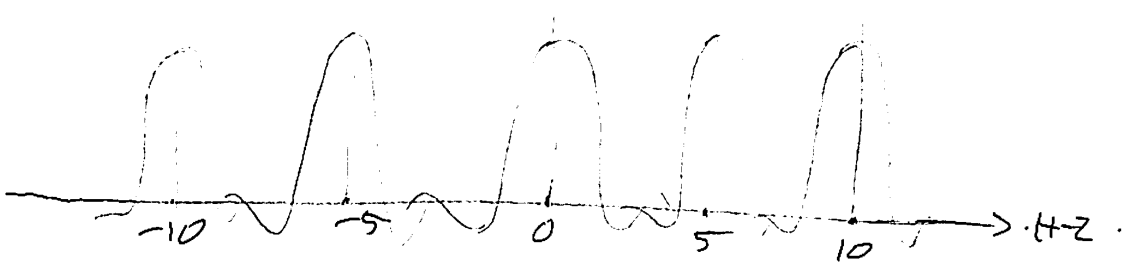
$x_s(t) = \delta(t) + \delta(t-0.2) + \delta(t-0.4) + \delta(t+0.2) + \delta(t+0.4)$.

so $F[x_s(t)] = X_s(f) = f_s\left[X(f) + X(f-f_s) + X(f-2f_s) + X(f+f_s) + X(f+2f_s)\right]$

but $X(f) = 1 \cdot \mathrm{sinc}(f)$

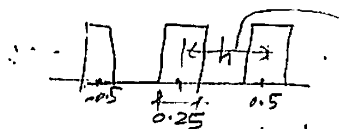
so $X_s(f) = 5\left[\mathrm{sinc}(f) + \mathrm{sinc}(f-5) + \mathrm{sinc}(f-10) + \mathrm{sinc}(f+5) + \mathrm{sinc}(f+10)\right]$

so, place a sinc function of height $= 5$ at $f = 0, \pm 5, \pm 10$.



(d) it is not possible to recover, since sinc function extent for $\pm\infty$. so there will always be a loss.

HW6



9.

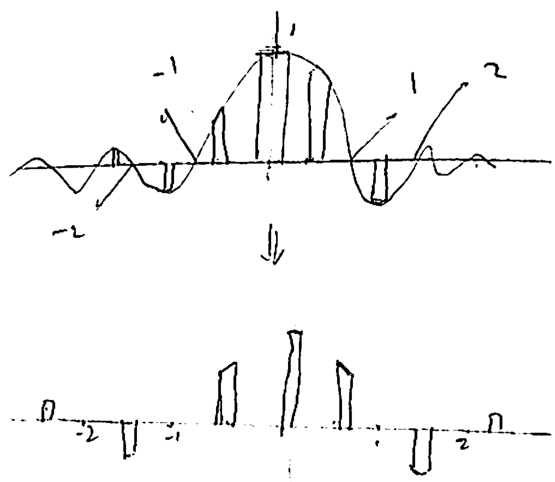
$x(t) = \text{sinc}(t)$. sampled by rect train, $h=1$, $\tau=0.25$ s,

$T = 0.5$ sec.

(a) Nyquist rate for $x(t)$ is ∞ since it is a sinc function with ∞ bandwidth?. but answer says 1 Hz? I do not understand.

if we apply this to the rect pulse, then I set $(2)(f_m) = 2\left(\frac{1}{T}\right) = 2\left(\frac{1}{0.5}\right) = 2(2) = 4$ Hz.

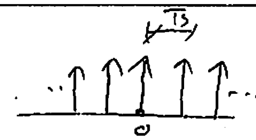
(b)



HW6

(10) $x(t) = 2\cos(2\pi\, 10t) + \cos(2\pi\, 20t)$.

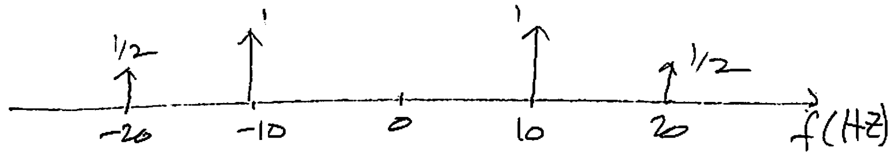
$f_s = 40\text{ HZ}$.



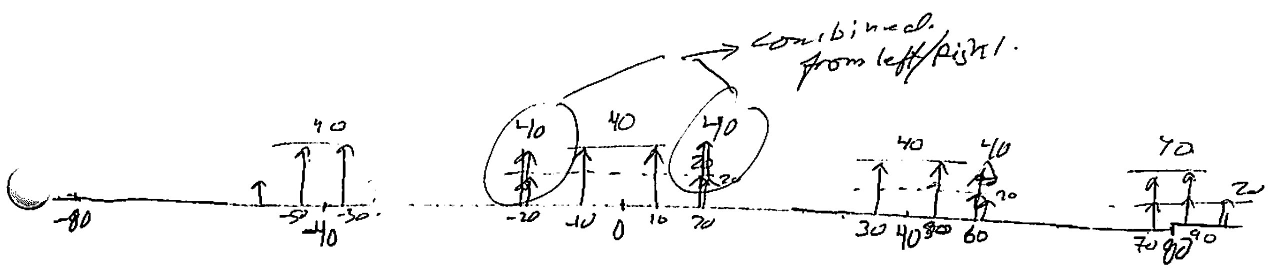
$T_s = 0.025$ sec

(a) see plot next page.

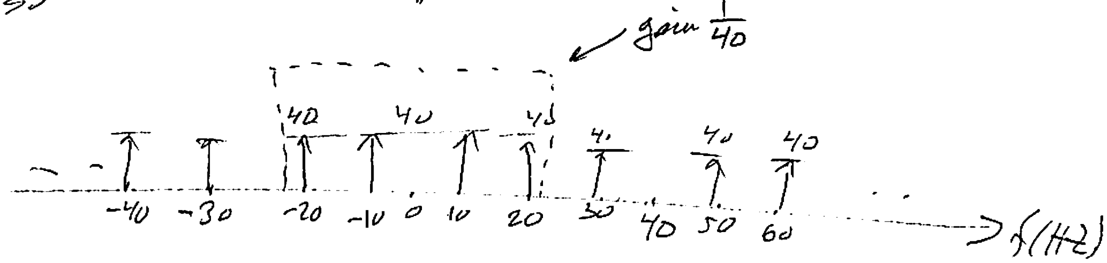
$X(f)$.



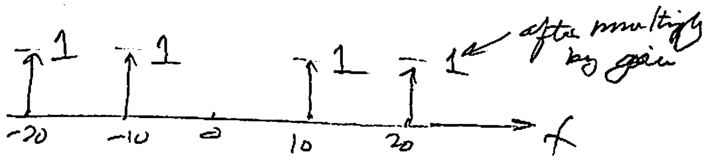
(b) $X_s(f) = f_s \sum X(f - nf_s)$.



so



d) after Sampling:

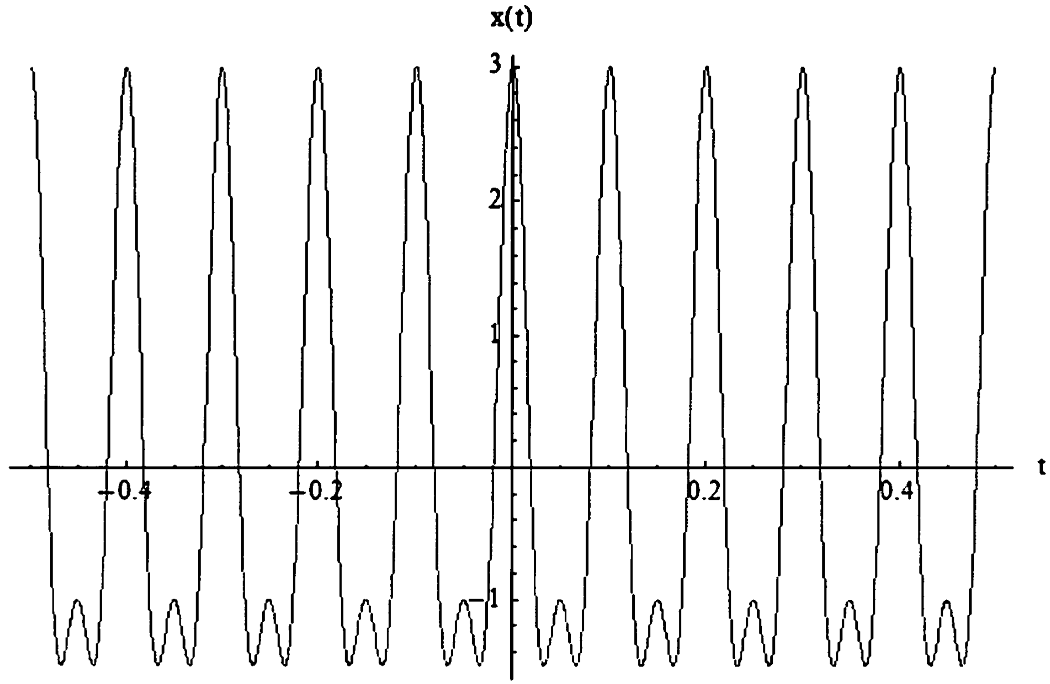


so time domain is $\boxed{2\cos(2\pi\, 10t) + 2\cos(2\pi\, 20t)}$

(e) $\boxed{2\cos(2\pi\, 10\, t)}$

```
In[15]:= Plot[2 Cos[2 Pi 10 t] + Cos[2 Pi 20 t], {t, -.5, .5}, AxesLabel → {"t", "x(t)"}]
```

Out[15]=

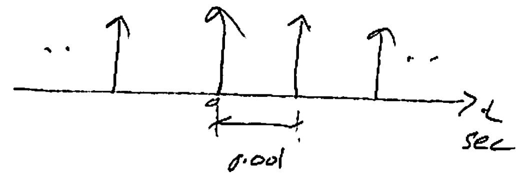


HW6

12

$m_1(t) = 500 \ \text{sinc}^2(500t)$.

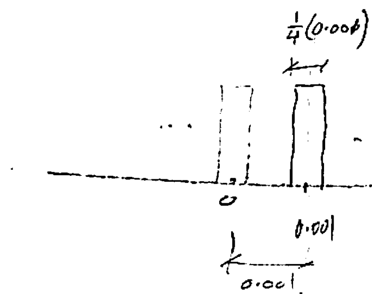
$P_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - 0.001k)$.



$P_2(t) = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t - 0.001k}{0.00025}\right)$

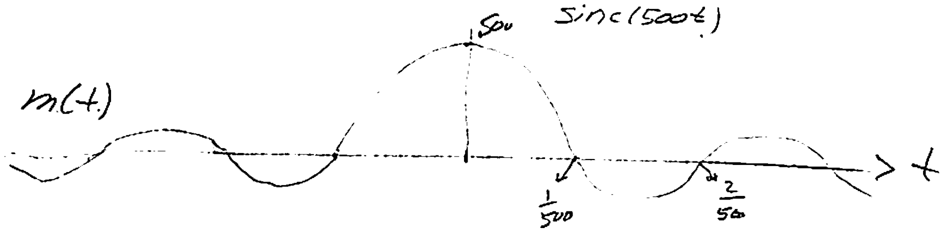
delay

width

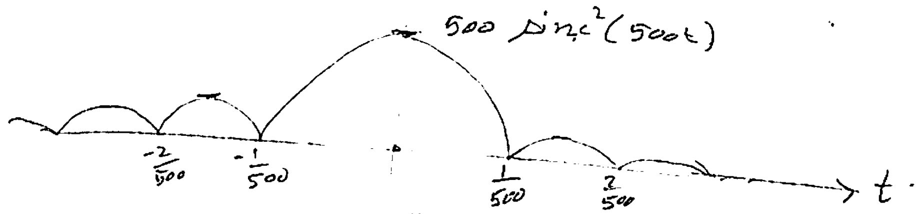


$\boxed{d = \frac{1}{4}}$

(a) $x_1(t) = m_1(t)\, P_1(t)$.



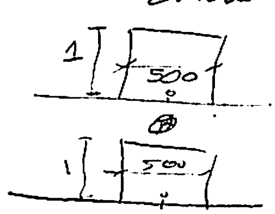
$\text{sinc}^2(500t)$



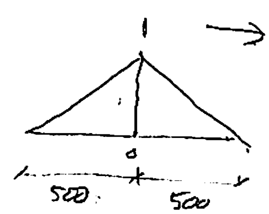
to find spectrum of ↑ we can use relation.

$$F[\text{sinc}^2] = F[(\text{sinc})(\text{sinc})]$$
$$= F(\text{sinc}) \circledast F(\text{sinc})$$

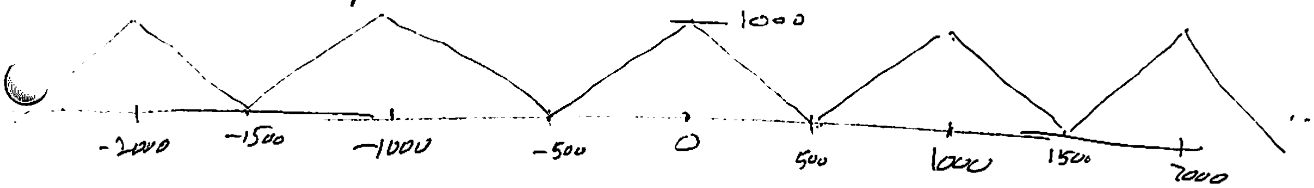
so need to convolve 2 rect.



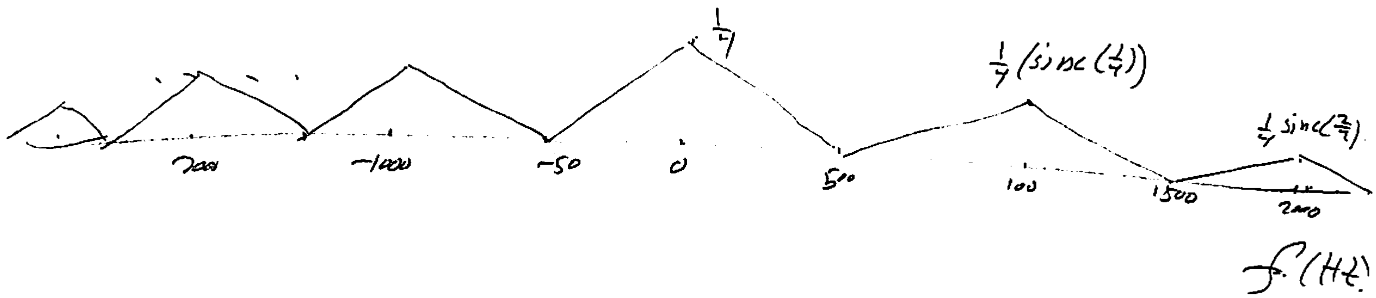
$\circledast \Longrightarrow \text{tri}\left(\frac{f}{500}\right)$



so (a) has spectrum (note $f_s = \frac{1}{0.001} = 1000$.

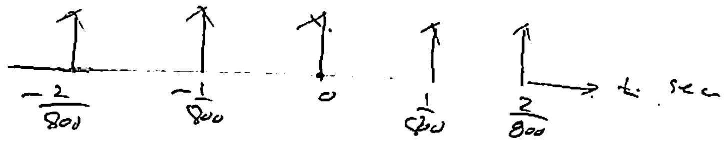


(b). note that $d = \frac{1}{4} \cdot 50$,

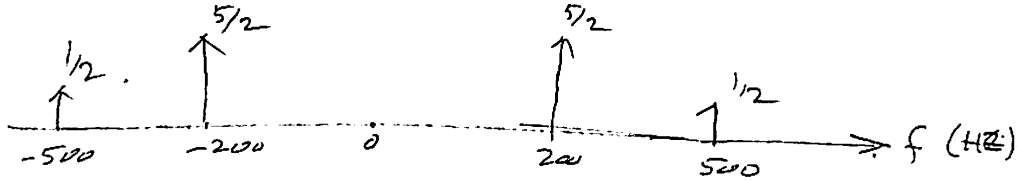


HW 6

13 $m(t) = 5\cos(2\pi \underbrace{200}_{f_1} t) + \cos(2\pi \underbrace{500}_{f_2} t)$. $\Rightarrow f_s = \frac{1}{T_s} = 800$ Hz

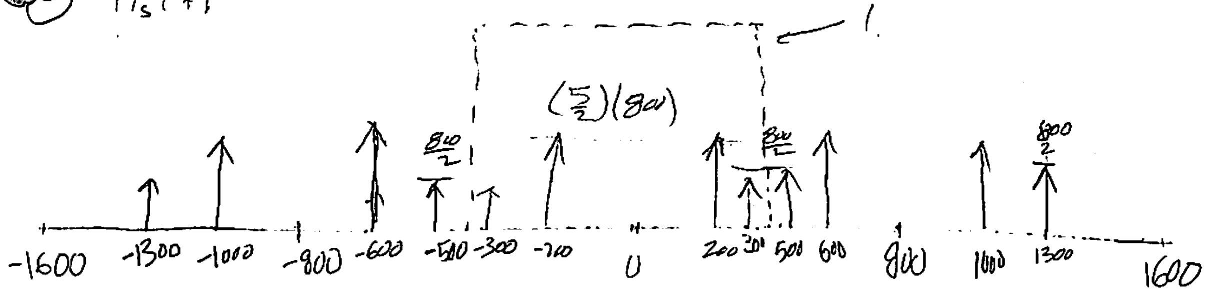


(a) $\mathcal{F}(m(t)) = \frac{5}{2}\left(\delta(f-f_1)+\delta(f+f_1)\right) + \frac{1}{2}\left(\delta(f-f_2)+\delta(f+f_2)\right)$.

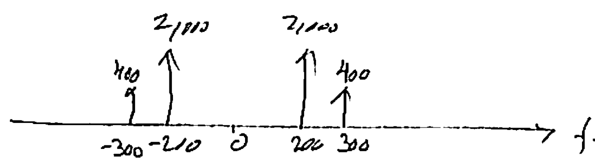


(b) $B = 500$, hence $2B = 1000$ HZ $\Rightarrow \boxed{1 \text{KHz}}$

(c) $M_s(f)$



(d) so $Y(f)$



so $y(t) = 4000\cos(2\pi 200t) + 800\cos(2\pi 300t)$.

HW7

(1) $mp = 16V$.
$x = -8.7V$.

(a) $N = 8$ bits. Find ~~2'~~ offset binary code.

$\Delta = \frac{16}{2^7} = \boxed{0.125}$

quantization level $= \left(\frac{Abs(x)}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$

Since x is negative, then Code $= 2^7 - 70 = 128 - 70 = \boxed{58}$

in binary this is $\boxed{0011\ 1010}$

(b) sign/magnitude..

since $x < 0$ Then Code $= 2^7 + 70 = 128 + 70 = 198$

which in binary is $\boxed{1100\ 0110}$

(c) 2's Complement.

Since $x < -\frac{\Delta}{2}$ then Code. $= 2^8 - 70 = 256 - 70$
$= 186$

which in binary is $\boxed{1011\ 1010}$

(d) 1's complement.

since $x < 0$ Then

code $(2^8 - 1) - 70 = 255 - 70 = 185$

which in binary is $\boxed{1011\ 1001}$

extra: to illustrate this more, this is the calculations assuming $x = +8.7V$.

offset binary

$$\Delta = \frac{16}{2^7} = 0.125$$

$$\text{Level} = \text{round}\left(\frac{8.7}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$$

Since $x>0$ then code $(70)_2 = 0100\ 0110$

Sign magnitude

Since $x>0$ then code $(70)_2 = 0100\ 0110$

2's complement

Since $x>0$ then code $= (70)_2 = 0100\ 0110$

1's complement

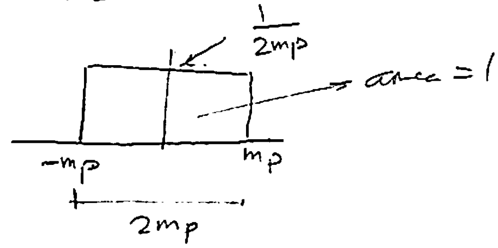
Since $x>0$, then code $= (70)_2 = 0100\ 0110$.

HW 7

(3) $m_p = 16$.

$N = 11$ bits.

$m(t)$ Pdf



(a) $$SNR = \frac{\text{signal power}}{\text{noise power}} = \frac{\overline{m^2(t)}}{\frac{1}{12}S^2}$$

When $S = \frac{m_p}{2^{N-1}}$ hence Noise $\boxed{P_{av} = \frac{1}{12}\left(\frac{m_p}{2^{10}}\right)^2}$

and since this is a Random message, then

$$\overline{m^2(t)} = E(m^2(t)) = \int x^2 f_x \, dx$$

$$= \int_{-m_p}^{m_p} \left(\frac{1}{2m_p}\right) x^2 dx$$

$$= \frac{1}{2m_p}\left[\frac{x^3}{3}\right]_{-m_p}^{m_p} = \frac{1}{6m_p}\left[m_p^3 - (-m_p)^3\right]$$

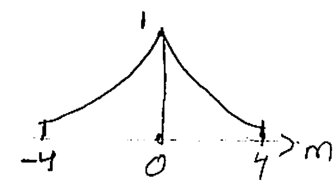
$$= \frac{1}{6m_p}\left[m_p^3 + m_p^3\right] = \boxed{\frac{1}{3}m_p^2}$$

So $$SNR = \frac{\frac{1}{3}m_p^2}{\frac{1}{12}\left(\frac{m_p}{2^{10}}\right)^2} = \frac{(12)(2^{20})}{3} = (4)(2)^{20};$$

$$= \boxed{66.226 \text{ db}}$$

HW 7

(6) $f(m) = \begin{cases} k e^{-|m|} & -4 < m < 4 \\ 0 & \text{o.w.} \end{cases}$



(a) Find k.

$$\int_{-4}^{4} k e^{-|m|}\, dm = 1 \Rightarrow \int_{-4}^{0} k e^{m}\, dm + \int_{0}^{4} k e^{-m}\, dm$$

$$= k\left[\left[e^{m}\right]_{-4}^{0} + \frac{\left[e^{-m}\right]_{0}^{4}}{-1} \right]$$

$$= k\left[e^{0} - e^{-4} + \frac{e^{-4} - e^{0}}{-1} \right] = k\left[1 - e^{-4} + \frac{e^{-4} - 1}{-1} \right]$$

$$= k\left[(1 - e^{-4}) + (1 - e^{-4}) \right] = k\left(2 - 2e^{-4} \right)$$

$$= \boxed{2k\left(1 - e^{-4}\right) = 1}$$

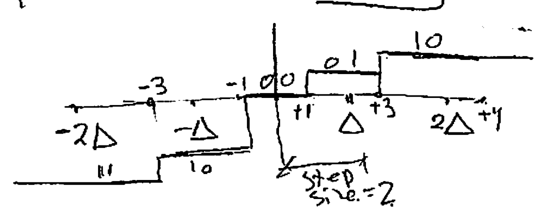
so $k = \dfrac{1}{2(1 - e^{-4})} = \boxed{0.50932}$

(b) $m_p = 4$. so $\Delta = \dfrac{m_p}{2^{N-1}}$ where N is number of bits. or $\Delta = \dfrac{8}{2^N}$

but 2^N = number of levels.

but we are told to use 4 levels. hence $\boxed{N=2}$

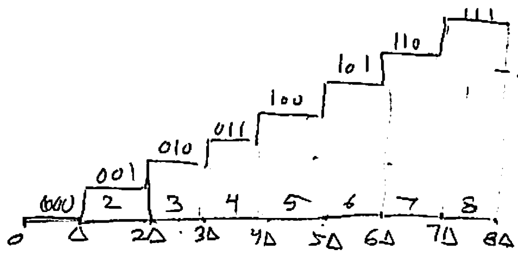
so $\Delta = \dfrac{8}{2^2} = \dfrac{8}{4} = \boxed{2}$



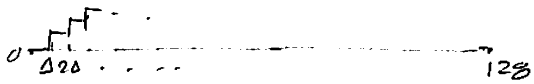
HW 7

$\boxed{7}$

1st channel:



2nd channel:



$m_p = 2$ Volt.

noise power $= \frac{1}{12} S^2$ where S is step size.

for Channel 1, $S_1 = \frac{m_p}{8} = \frac{2}{8}$.

for channel 2, $S_2 = \frac{m_p}{128} = \frac{2}{128}$.

so for channel 1, $\overline{e_1^2} = \frac{1}{12}\left(\frac{2}{8}\right)^2 = \frac{4}{(12)(64)}$.

for channel 2, $\overline{e_2^2} = \frac{1}{12}\left(\frac{2}{128}\right)^2$

so $\left(\frac{\overline{e_1^2}}{\overline{e_2^2}}\right)_{db} = 10 \log_{10} \frac{\overline{e_1^2}}{\overline{e_2^2}} = 10 \left[\log_{10} \overline{e_1^2} - \log \overline{e_2^2}\right]$

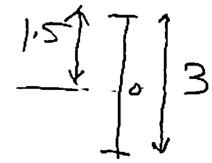
$= 10\left[\log_{10} \frac{4}{(12)(64)} - \log_{10} \frac{1}{12}\frac{4}{128^2}\right] =$

$= 10\left[-2.2833 + 4.6915\right] = \boxed{24.082 \text{ db}}$

HW7.

$\boxed{8}$ $m_p = 3V$.

levels = 64.



RMS of noise = $\sqrt{\overline{e^2}} = \sqrt{\frac{1}{12}s^2}$

but $s = \frac{3}{64}$, so RMS $= \sqrt{\frac{1}{12}\left(\frac{3}{64}\right)^2} = 0.01353$

so peak signal to RMS ratio is

$$\frac{1.5}{0.01353} = 110.85$$

$\boxed{9.}$ number of levels = 512

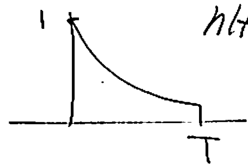
$B = 4.2\text{ MHz}$. so $2B = 8.4 \times 10^6$ Hz.

hence $N = 9$. ($2^9 = 512$).

so binary pulses $= \underbrace{(9)}_{\text{bits per sample}}\underbrace{(8.4 \times 10^6)}_{\text{samples/sec}} = \boxed{75600}$ binary pulses/sec.

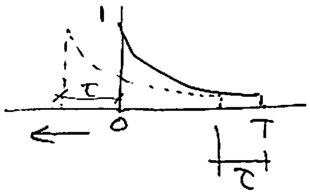
HW 8

#1.



$$h(t) = e^{-at}u(t).$$

Case 1



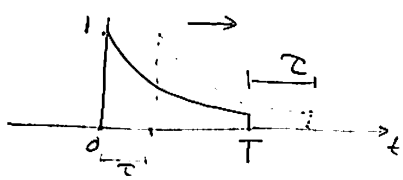
$$0 \leq \tau \leq T.$$

$$R(\tau) = \int_0^{T-\tau} h(t)\,h(t+\tau)\,dt = \int_0^{T-\tau} e^{-at} e^{-a(t+\tau)}\,dt$$

$$= e^{-a\tau}\int_0^{T-\tau} e^{-2at}\,dt = \frac{e^{-a\tau}}{-2a}\left[e^{-2at}\right]_0^{T-\tau}$$

$$= \frac{e^{-a\tau}}{-2a}\left[e^{-2a(T-\tau)} - 1\right] = \boxed{\frac{e^{-a\tau}}{2a}\left(1 - e^{-2a(T-\tau)}\right)}$$

Case 2



$$0 \leq \tau \leq T$$

$$R(\tau) = \int_\tau^T h(t)\,h(t-\tau)\,dt = \int_\tau^T e^{-at} e^{-a(t-\tau)}\,dt$$

$$= e^{a\tau}\int_\tau^T e^{-2at}\,dt = \frac{e^{a\tau}}{-2a}\left[e^{-2at}\right]_\tau^T$$

$$= \frac{e^{a\tau}}{-2a}\left[e^{-2aT} - e^{-2a\tau}\right] = \frac{e^{a\tau}}{2a}\left[e^{-2a\tau} - e^{-2aT}\right]$$

$$= \frac{1}{2a}\left[e^{-a\tau} - e^{-2aT+a\tau}\right] = \frac{e^{-a\tau}}{2a}\left[1 - e^{-2aT+2a\tau}\right]$$

$$= \boxed{\frac{e^{-a\tau}}{2a}\left[1 - e^{-2a(T-\tau)}\right]}$$

We see that $R(\tau) = R_+(\tau)$ as expected.

so $R(\tau) = \begin{cases} \frac{e^{-a\tau}}{2a}\left[1 - e^{-2a(T-\tau)}\right] & |\tau| \leq T \\ 0 & o.w. \end{cases}$

so $R_X(\tau) = \frac{1}{T} R(\tau)$

now $S_X(f) = F.T\left[R_X(\tau)\right] \cdot F.T\left[R_X(\tau)\right]^*$

or $S_X(f) = \left| F.T\left[R_X(\tau)\right] \right|^2$

so need to find F.T. of $R_X(\tau)$ first.

$$F.T\left(R_X(\tau)\right) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f\tau} d\tau$$

$$= \int_{-T}^{T} \frac{e^{-a|\tau|}}{2a}\left(1 - e^{-2a(T-|\tau|)}\right) e^{-j2\pi f\tau} d\tau$$

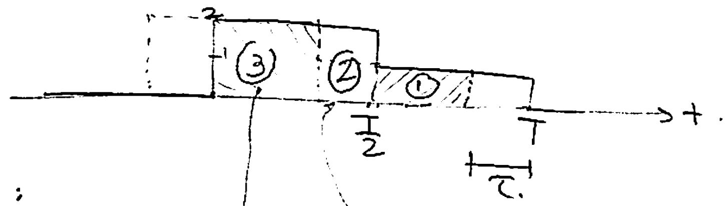
$$H(\omega) \longrightarrow \frac{1 - e^{-aT}\left[\cos(2\pi f T) - j\sin(2\pi f T)\right]}{a + j\omega}$$

so $S_X(f) = H(\omega) H^*(\omega)$

$$\Rightarrow \frac{1}{T} \frac{1}{a^2 + (2\pi f)^2}\left(1 - 2e^{-aT}\cos(\omega T) + e^{-2aT}\right)$$

HW 8

(2)



(a) $0 < \tau < \frac{T}{2}$:

3 regions as above.

$$R_p(\tau) = \int_0^{\frac{T}{2}-\tau} (2)(2)\,d\lambda + \int_{\frac{T}{2}-\tau}^{\frac{T}{2}} (1)(2)\,d\lambda + \int_{\frac{T}{2}}^{T-\tau} (1)(1)\,d\lambda.$$

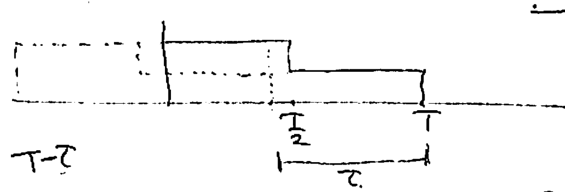
$$= 4\left[\lambda\right]_0^{\frac{T}{2}-\tau} + 2\left[\lambda\right]_{\frac{T}{2}-\tau}^{\frac{T}{2}} + 1\left[\lambda\right]_{\frac{T}{2}}^{T-\tau}.$$

$$= 4\left(\frac{T}{2}-\tau\right) + 2\left(\frac{T}{2}-\frac{T}{2}+\tau\right) + \left(T-\tau-\frac{T}{2}\right)$$

$$= 4\frac{T}{2} - 4\tau + 2\tau + \frac{T}{2} - \tau = \frac{5}{2}T - 3\tau$$

$$= \boxed{\frac{5}{2}T\left(1-\frac{6}{5}\tau\right)}$$

$\frac{T}{2} < \tau \le T$

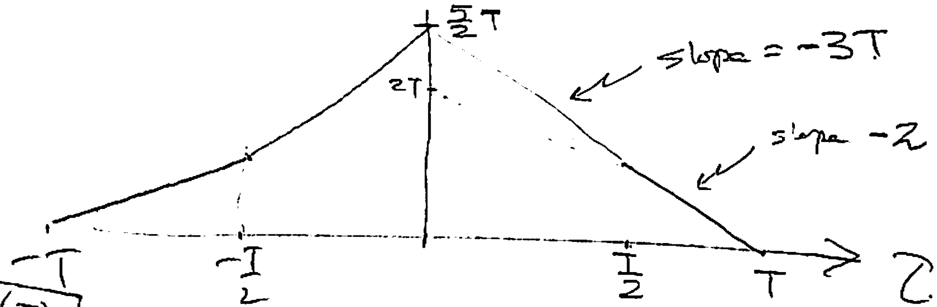


$$R_p(\tau) = \int_0^{T-\tau} (1)(2)\,d\lambda = 2\left[\lambda\right]_0^{T-\tau} = \boxed{2(T-\tau)}$$

hence
$$R_p(\tau) = \begin{cases} \frac{5}{2}T\left(1-\frac{6}{5}|\tau|\right) & 0<|\tau|<\frac{T}{2} \\ 2(T-|\tau|) & \frac{T}{2}\le|\tau|\le T \\ 0 & \text{o.w.} \end{cases}$$

$\longrightarrow$

B. plot



so $\boxed{R_X = \frac{1}{T} R_p(\tau)}$

to find. F.T. of this, use tri function.

so $H(f) \longrightarrow$ sinc.

and $S_X(f) = |H(f)|^2$

HW7

(1) $m_p = 16V$.
$x = -8.7V$.

(a) $N = 8$ bits. Find ~~offset~~ binary code.

$\Delta = \frac{16}{2^7} = \boxed{0.125}$

quantization level $= \left(\frac{\text{Abs}(x)}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$

Since x is negative, then $\text{Code} = 2^7 - 70 = 128 - 70 = \boxed{58}$

in binary this is $\boxed{0011\ 1010}$

(b) sign/magnitude..

since $x < 0$ Then $\text{Code} = 2^7 + 70 = 128 + 70 = 198$

Which in binary is $\boxed{1100\ 0110}$

(c) 2's Complement.

since $x < -\frac{\Delta}{2}$ then $\text{Code.} = 2^8 - 70 = 256 - 70 = 186$

which in binary is $\boxed{1011\ 1010}$

(d) 1's complement.

since $x < 0$ Then

$\text{Code}\ (2^8 - 1) - 70 = 255 - 70 = 185$

Which in binary is $\boxed{1011\ 1001}$

extra: to illustrate this more, this is the Calculations assuming $x = +8.7V$.

offset binary

$$\Delta = \frac{16}{2^7} = 0.125$$

$$\text{Level} = \text{round}\left(\frac{8.7}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$$

Since $x > 0$ Then code $(70)_2 = 0100\ 0110$

Sign magnitude

since $x > 0$ Then code $(70)_2 = 0100\ 0110$

2's Complement

Since $x > 0$ then code $= (70)_2 = 0100\ 0110$

1's Complement

since $x > 0$, then code $= (70)_2 = 0100\ 0110$.

4.2 Computer assignments

Computer Assignment #1

ECE 405, Summer session 1, Cal Poly Pomona, CA
By Nasser M. Abbasi

PART(1) LOW PASS

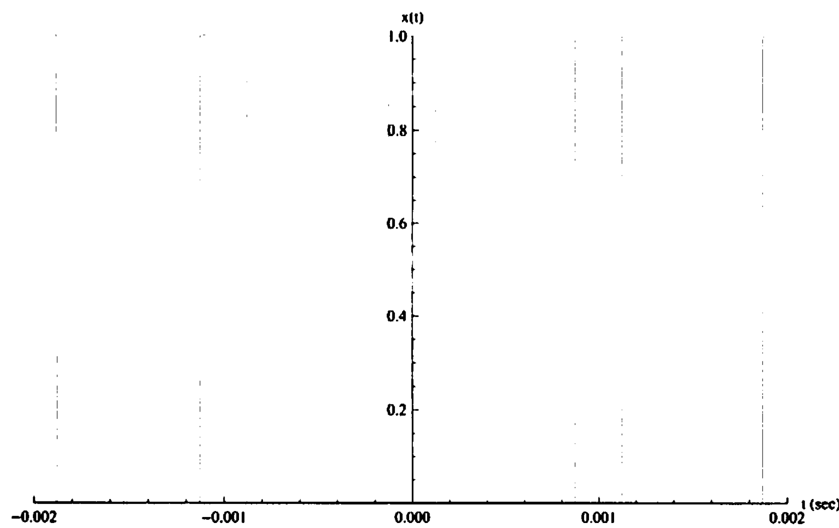
- **Load my DSP functions that I wrote for this course**

```
In[2]:= << dsp`
```

- **Plot the pulse train**

In[95] =
```
delay = 0;
period = 1 * 10^-3;
range = 2 * 10^-3;
tao = .25 * 10^-3;
h = 1;
w0 = 2 Pi / period; (*rad/sec*)
f0 = 1 / period; (*hz*)
dutyCycle = tao / period;
numberOfCoeff = 20;
currentPulses = dsp`makePulseTrain[delay, period, range, tao, h];

Plot[0, {x, -range, range},
   PlotRange → {{-range, range}, {0, h}}, AxesLabel → {"t (sec)", "x(t)"},
   Epilog → {Thin, Red, currentPulses}
  ]
```

Out[105]=



- **Find the fourier series coefficients of the above pulse train**

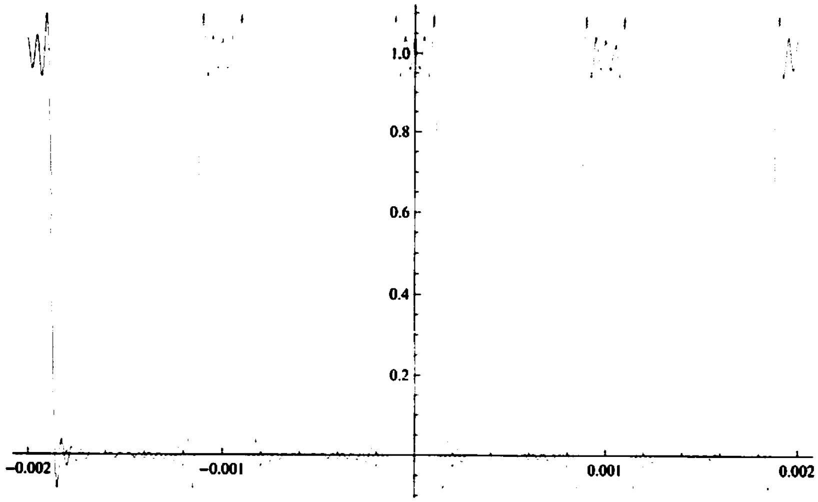
In[106] =
```
xn = getFourierCoeffPulseTrain[h, tao, period, numberOfCoeff]
```

Out[106]=
```
{0.25, 0.225079, 0.159155, 0.0750264, 9.74543 × 10^-18, -0.0450158, -0.0530516,
 -0.0321542, -9.74543 × 10^-18, 0.0250088, 0.031831, 0.0204617, 9.74543 × 10^-18,
 -0.0173138, -0.0227364, -0.0150053, -9.74543 × 10^-18, 0.0132399, 0.0176839, 0.0118463}
```

Plot fourier series approximation to the above pulse on top of it to compare

In[107]:=
```
Plot[getFourierApproximation[t, xn, period], {t, -range, range}]
```

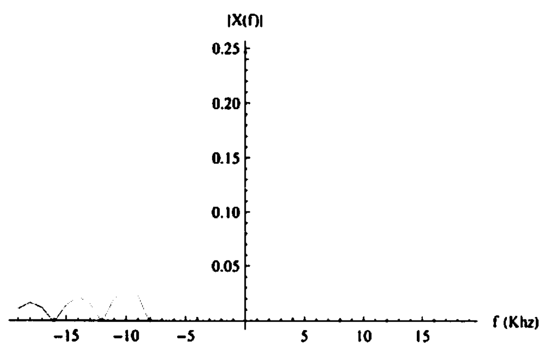
Out[107]=



- **Plot the spectrum of the pulse**

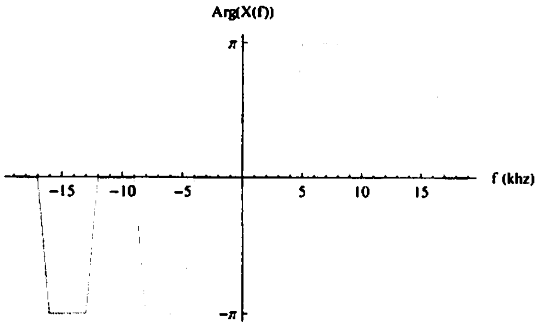
In[108]:=
```
data = getMagnitudeOfPulseTrainFourierCoeff[delay, period, range, dutyCycle, numberOfCoeff];
ListPlot[data, Joined → True, AxesLabel → {"f (Khz)", "|X(f)|"}]
```

Out[109]=



In[110]:=
```
data = getPhaseOfPulseTrainFourierCoeff [delay, period, range, dutyCycle, numberOfCoeff];
ListPlot[data, Joined → True,
 AxesLabel → {"f (khz)", "Arg(X(f))"}, Ticks → {Automatic, {-Pi, Pi}}]
```

Out[111]=



- **Generate normalized low pass butterworth of order 4**

In[112]:=
```
Clear[s, form];
order = 4;
cuttoff = 1;
{poles, hs} = dsp`getButterworthPolynomial [order, cuttoff, s];
TraditionalForm@hs
```

Out[116]//TraditionalForm=

$$\frac{1.}{s^4 + 2.61313\, s^3 + 3.41421\, s^2 + 2.61313\, s + 1.}$$

- **convert the above to low pass butterworth with specified cuttoff**

In[117]:=
```
newHs = dsp`butterToLowPass [hs, 2 Pi/tao, s];
TraditionalForm@newHs
```

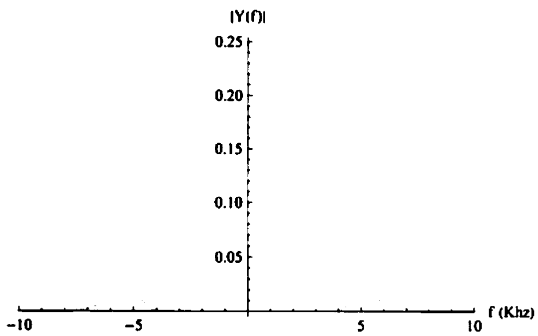
Out[118]//TraditionalForm=

$$\frac{1.}{2.50634\times 10^{-18}\, s^4 + 1.64604\times 10^{-13}\, s^3 + 5.40519\times 10^{-9}\, s^2 + 0.000103973\, s + 1.}$$

- **Multiply H(jwn) with Pulse fourier series y(n), and plot Y(f)**

In[119]:=
```
Clear[w];
xnFourier[n_] := h dutyCycle Sinc[Pi n dutyCycle]

tf[n_, w0_] := newHs /. s → (I w0 n)

yn[n_, w0_] := xnFourier[n] * tf[n, w0]

y[t_, w0_, numberOfCoeff_] := Sum[If[n == 0, yn[n, w0] * Exp[I w0 n t ],
   (yn[n, w0] * Exp[I w0 n t ] + yn[-n, w0] * Exp[-I w0 n t ])], {n, 0, numberOfCoeff}]

data = Table[{m, Abs[yn[m, w0]]}, {m, -20, 20}];
ListPlot[data, Joined → True,
 PlotRange → {{-10, 10}, All}, AxesLabel → {"f  (Khz)", "|Y(f)|"}]
```

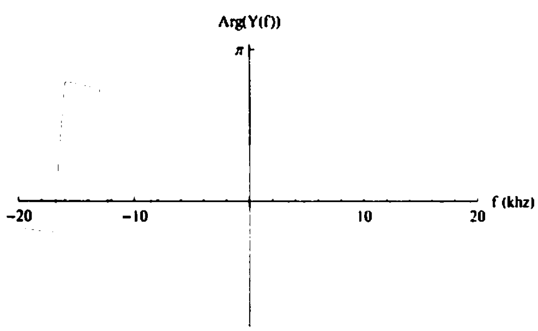
Out[125]=



- **Plot the phase spectrum**

In[126]:=
```
data = Table[{m, Arg[yn[m, w0]]}, {m, -20, 20}];
ListPlot[data, Joined → True, PlotRange → {{-20, 20}, All},
 AxesLabel → {"f  (khz)", "Arg(Y(f))"}, Ticks → {Automatic, {-Pi, Pi}}]
```

Out[127]=

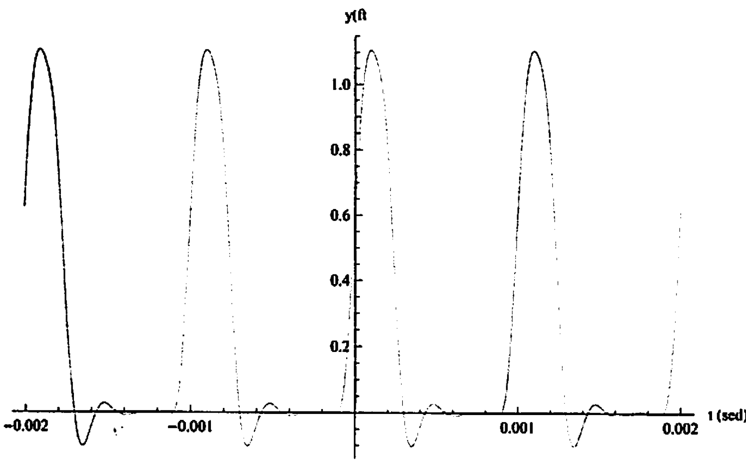


6 | ca1_v2.nb

- **Plot y(t)**

```
In[128]:= Plot[y[t, w0, 10], {t, -range, range}, PlotRange → All, AxesLabel → {"t (sed)", "y(ft"}]
```

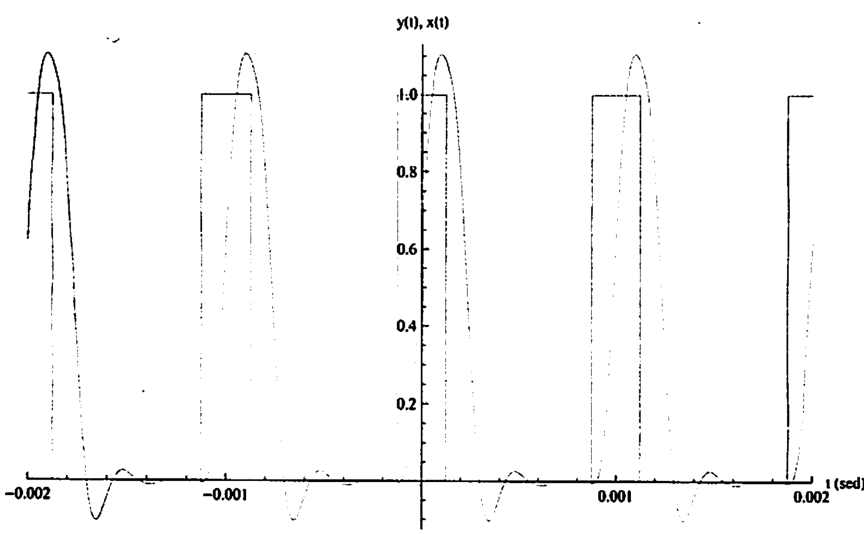
Out[128]=



- **Plot y(t) and x(t) on same plot to compare**

```
In[129]:= Plot[y[t, w0, 10], {t, -range, range},
    PlotRange → {{-range, range}, All}, AxesLabel → {"t (sed)", "y(t), x(t)"},
      Epilog → {Thin, Red, currentPulses}
    ]
```

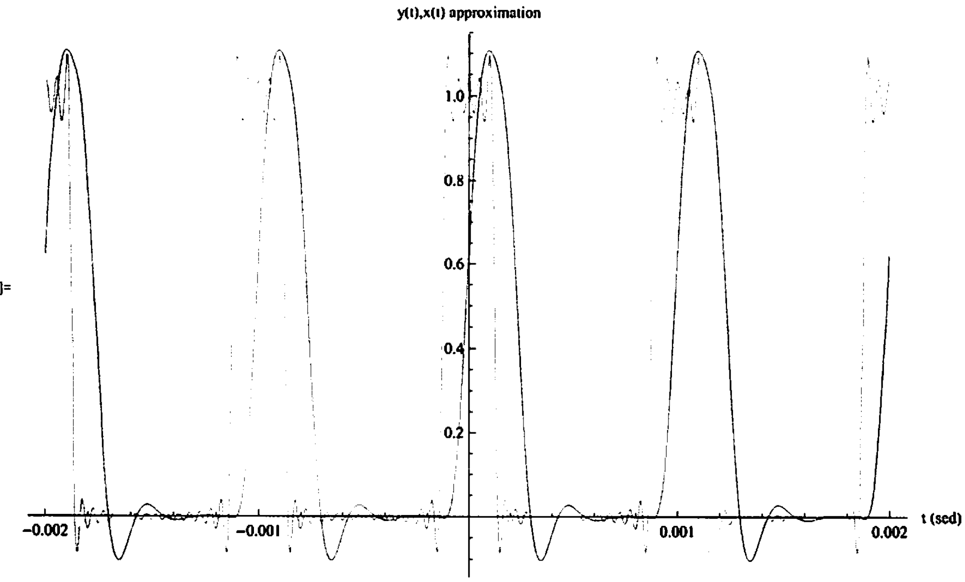
Out[129]=



- **Plot y(t) on top of approximation of x(n) used**

In[50]:=
```
Plot[{getFourierApproximation[t, xn, period], y[t, w0, 10]}, {t, -range, range},
 PlotRange → All, AxesLabel → {"t (sed)", "y(t),x(t) approximation"}]
```

Out[50]=



Part (2) High Pass

- **convert normalized butterworth to high pass butterworth**

In[282]:=
```
newHs = dsp`butterToHighPass[hs, 2 Pi/tao, s];
newHs = Numerator[newHs] / Together[Denominator[newHs]];
TraditionalForm@newHs
```

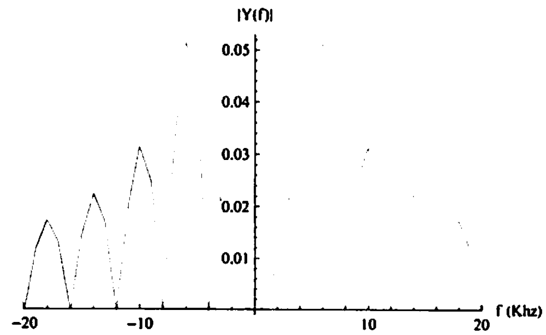
Out[284]//TraditionalForm=

$$\frac{1.\, s^4}{1.\, s^4 + 65675.\, s^3 + 2.1566\times 10^9 s^2 + 4.14839\times 10^{13} s + 3.98988\times 10^{17}}$$

- **Multiply H(jwn) with Pulse fourier series y(n), and plot Y(f)**

In[285]:=
```
Clear[w];
tf[n_, w0_] := newHs /. s → (I w0 n)

yn[n_, w0_] := dsp`fcPulseTrain[n, h, tao, period] * tf[n, w0]

y[t_, w0_, numberOfCoeff_] := Sum[If[n == 0, yn[n, w0] * Exp[I w0 n t ],
    (yn[n, w0] * Exp[I w0 n t ] + yn[-n, w0] * Exp[-I w0 n t ])], {n, 0, numberOfCoeff}]

data = Table[{m, Abs[yn[m, w0]]}, {m, -40, 40}];
ListPlot[data, Joined → True,
 PlotRange → {{-20, 20}, All}, AxesLabel → {"f (Khz)", "|Y(f)|"}]
```

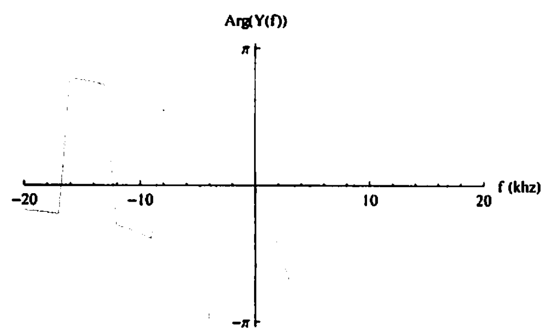
Out[290]=



- **Plot the phase spectrum**

In[227]:=
```
data = Table[{m, Arg[yn[m, w0]]}, {m, -20, 20}];
ListPlot[data, Joined → True, PlotRange → {{-20, 20}, All},
 AxesLabel → {"f (khz)", "Arg(Y(f))"}, Ticks → {Automatic, {-Pi, Pi}}]
```

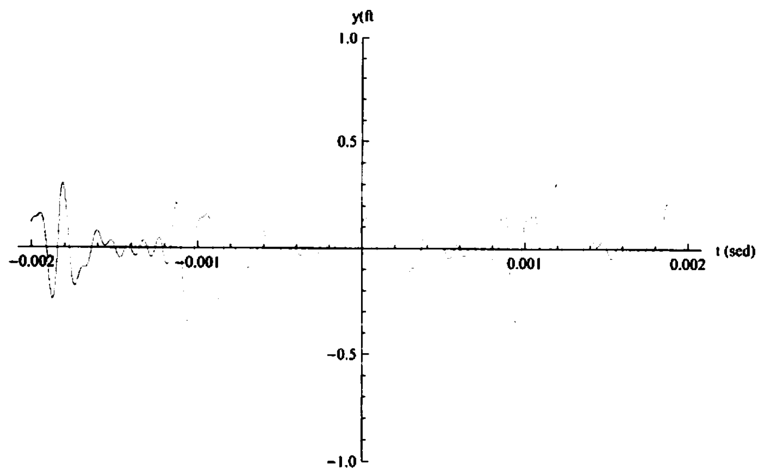
Out[228]=



- **Plot y(t)**

In[229]:=
```
Plot[y[t, w0, 10], {t, -range, range},
  PlotRange → {Automatic, {-h, h}}, AxesLabel → {"t (sed)", "y(ft"}]
```

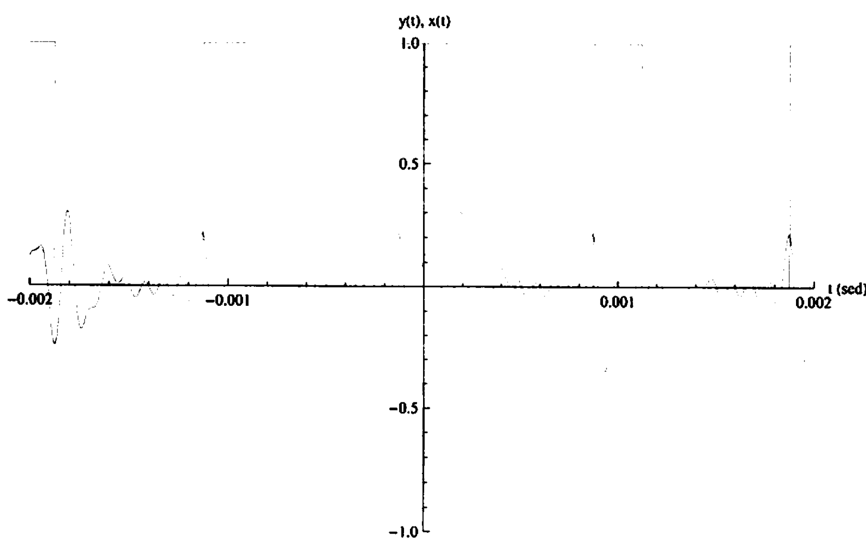
Out[229]=



- **Plot y(t) and x(t) on same plot to compare**

In[230]:=
```
Plot[y[t, w0, 10], {t, -range, range},
  PlotRange → {{-range, range}, {-h, h}}, AxesLabel → {"t (sed)", "y(t), x(t)"},
    Epilog → {Thin, Red, currentPulses}
  ]
```

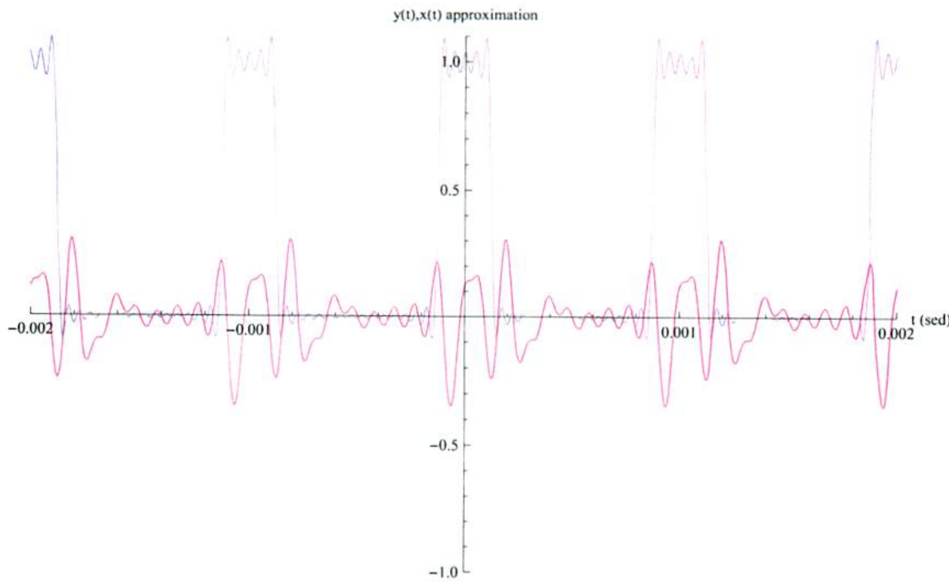
Out[230]=



Plot y(t) on top of approximation of x(n) used

In[232]:=
```
Plot[{getFourierApproximation[t, xn, period], y[t, w0, 10]},
 {t, -range, range}, PlotRange → {{-range, range}, {-h, 1.1 h}},
 AxesLabel → {"t (sed)", "y(t),x(t) approximation"}]
```

Out[232]=



Part (3) BandPass filter

- **convert normalized butterworth to band pass butterworth**

In[273]:=
```
newHs = dsp`butterToBandPass[hs, 2 Pi/tao, 4 Pi/tao, s];
newHs = Numerator[newHs] / Together[Denominator[newHs]];
TraditionalForm@newHs
```

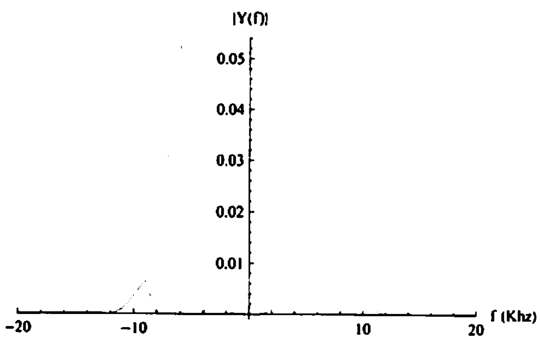
Out[275]//TraditionalForm=

$$(1.\, s^4)/(2.50634\times10^{-18}\, s^8 + 1.64604\times10^{-13}\, s^7 + 1.80703\times10^{-8}\, s^6 + 0.000727811\, s^5 + 38.6569\, s^4 + 919450.\, s^3 + 2.88394\times10^{10}\, s^2 + 3.31871\times10^{14}\, s + 6.3838\times10^{18})$$

- **Multiply H(jwn) with Pulse fourier series y(n), and plot Y(f)**

In[276]:=
```
Clear[w];
tf[n_, w0_] := newHs /. s → (I w0 n)

yn[n_, w0_] := dsp`fcPulseTrain[n, h, tao, period] * tf[n, w0]

y[t_, w0_, numberOfCoeff_] := Sum[If[n == 0, yn[n, w0] * Exp[I w0 n t ],
    (yn[n, w0] * Exp[I w0 n t ] + yn[-n, w0] * Exp[-I w0 n t ])], {n, 0, numberOfCoeff}]

data = Table[{m, Abs[yn[m, w0]]}, {m, -40, 40}];
ListPlot[data, Joined → True,
 PlotRange → {{-20, 20}, All}, AxesLabel → {"f  (Khz)", "|Y(f)|"}]
```

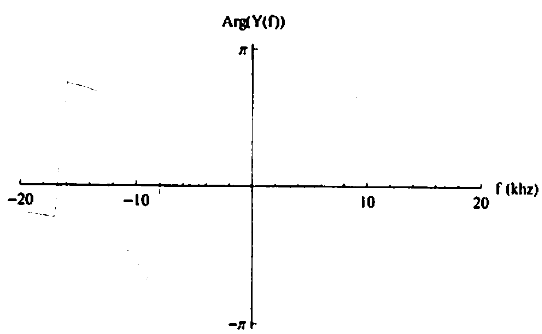
Out[281]=



- **Plot the phase spectrum**

In[242]:=
```
data = Table[{m, Arg[yn[m, w0]]}, {m, -20, 20}];
ListPlot[data, Joined → True, PlotRange → {{-20, 20}, All},
 AxesLabel → {"f  (khz)", "Arg(Y(f))"}, Ticks → {Automatic, {-Pi, Pi}}]
```

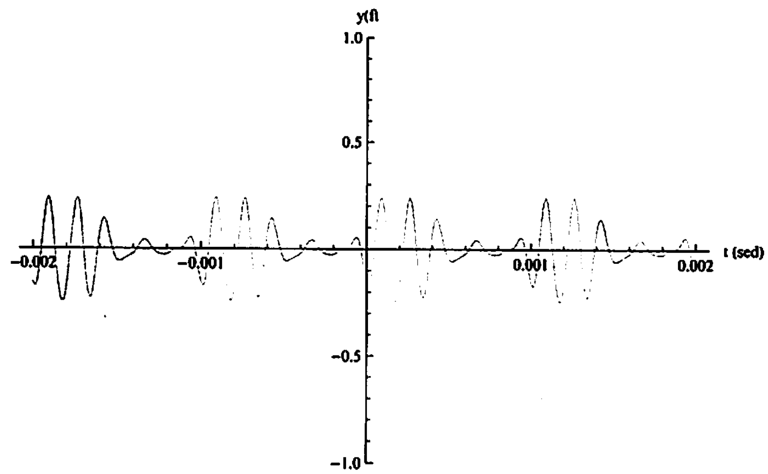
Out[243]=



- **Plot y(t)**

In[244]:=
```
Plot[y[t, w0, 10], {t, -range, range},
  PlotRange → {Automatic, {-h, h}}, AxesLabel → {"t (sed)", "y(ft")}]
```

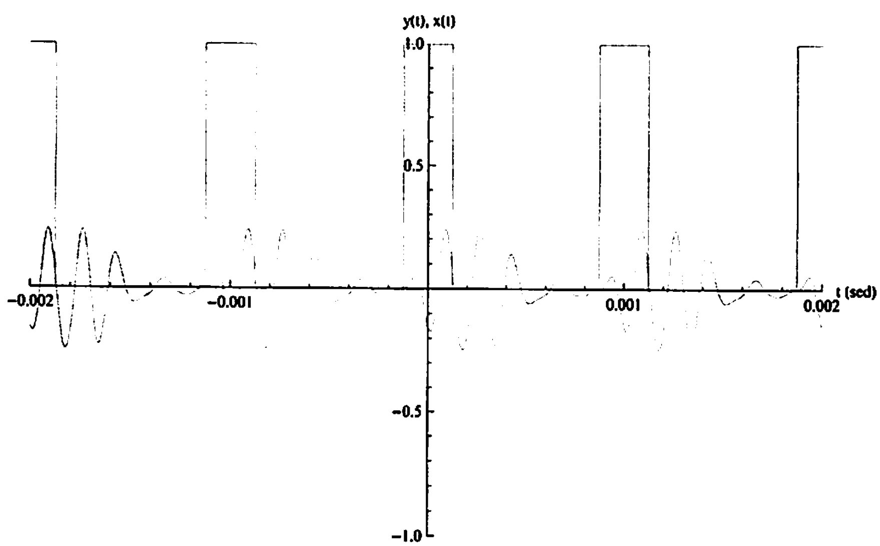
Out[244]=



- **Plot y(t) and x(t) on same plot to compare**

In[245]:=
```
Plot[y[t, w0, 10], {t, -range, range},
  PlotRange → {{-range, range}, {-h, h}}, AxesLabel → {"t (sed)", "y(t), x(t)"},
    Epilog → {Thin, Red, currentPulses}
  ]
```

Out[245]=

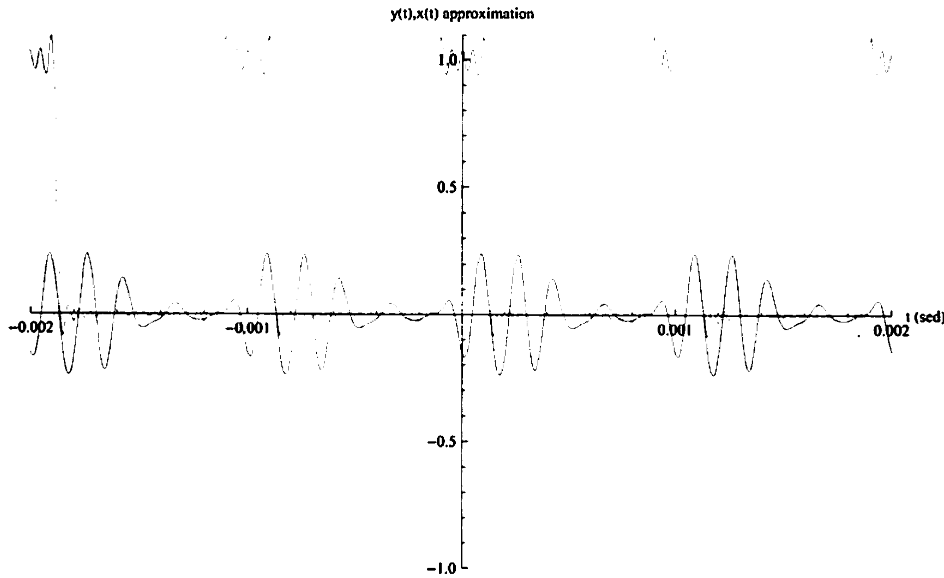


Plot y(t) on top of approximation of x(n) used

In[246]:=

```
Plot[{getFourierApproximation[t, xn, period], y[t, w0, 10]},
 {t, -range, range}, PlotRange → {{-range, range}, {-h, 1.1 h}},
 AxesLabel → {"t (sed)", "y(t),x(t) approximation"}]
```

Out[246]=



Part (4) BandStop filter

- **convert normalized butterworth to band stop butterworth**

In[293]:=

```
newHs = dsp`butterToBandStop[hs, 2 Pi/tao, 4 Pi/tao, s];
newHs = Numerator[newHs] / Together[Denominator[newHs]];
TraditionalForm@newHs
```

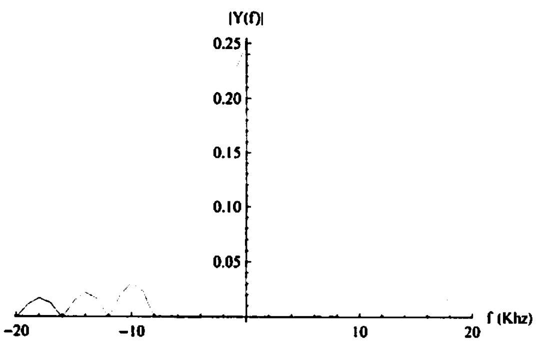
Out[295]//TraditionalForm=

$$(1.\,(s^2 + 1.26331\times10^9)^4)/(1.\,s^8 + 65675.\,s^7 + 7.20984\times10^9 s^6 + 2.90388\times10^{14} s^5 + 1.54236\times10^{19} s^4 + 3.66849\times10^{23} s^3 + 1.15066\times10^{28} s^2 + 1.32413\times10^{32} s + 2.54706\times10^{36})$$

- **Multiply H(jwn) with Pulse fourier series y(n), and plot Y(f)**

In[296]:=
```
Clear[w];
tf[n_, w0_] := newHs /. s → (I w0 n)

yn[n_, w0_] := dsp`fcPulseTrain[n, h, tao, period] * tf[n, w0]

y[t_, w0_, numberOfCoeff_] := Sum[If[n == 0, yn[n, w0] * Exp[I w0 n t ],
    (yn[n, w0] * Exp[I w0 n t ] + yn[-n, w0] * Exp[-I w0 n t ])], {n, 0, numberOfCoeff}]

data = Table[{m, Abs[yn[m, w0]]}, {m, -40, 40}];
ListPlot[data, Joined → True,
 PlotRange → {{-20, 20}, All}, AxesLabel → {"f (Khz)", "|Y(f)|"}]
```

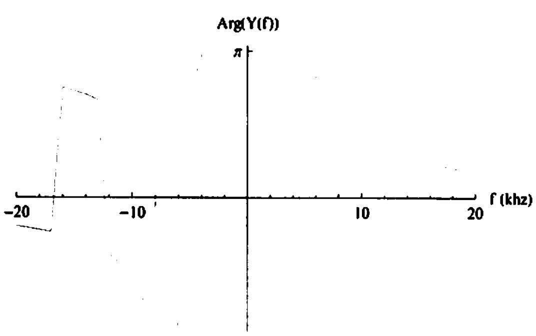
Out[301]=



- **Plot the phase spectrum**

In[302]:=
```
data = Table[{m, Arg[yn[m, w0]]}, {m, -20, 20}];
ListPlot[data, Joined → True, PlotRange → {{-20, 20}, All},
 AxesLabel → {"f (khz)", "Arg(Y(f))"}, Ticks → {Automatic, {-Pi, Pi}}]
```

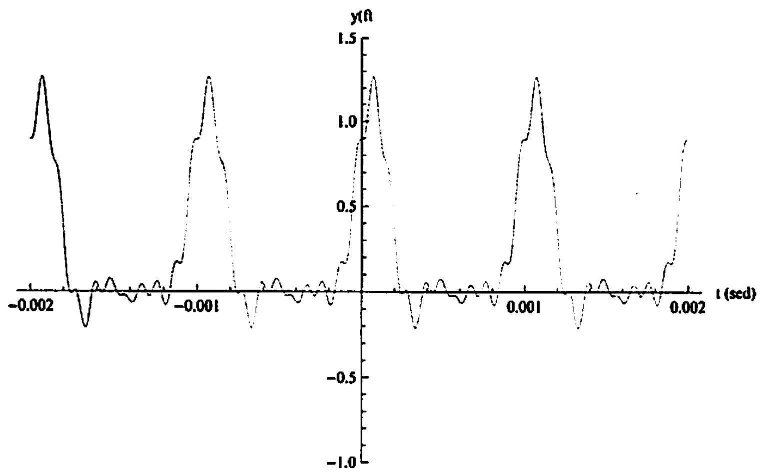
Out[303]=



- **Plot y(t)**

In[306]:=
```
Plot[y[t, w0, 10], {t, -range, range},
    PlotRange → {Automatic, {-h, 1.5 h}}, AxesLabel → {"t (sed)", "y(ft"}]
```

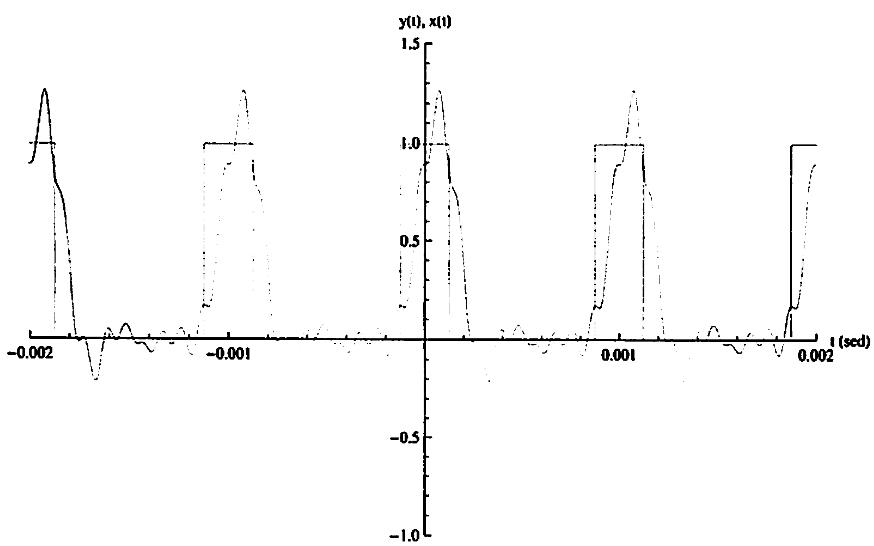
Out[306]=



- **Plot y(t) and x(t) on same plot to compare**

In[307]:=
```
Plot[y[t, w0, 10], {t, -range, range},
    PlotRange → {{-range, range}, {-h, 1.5 h}}, AxesLabel → {"t (sed)", "y(t), x(t)"},
      Epilog → {Thin, Red, currentPulses}
   ]
```

Out[307]=



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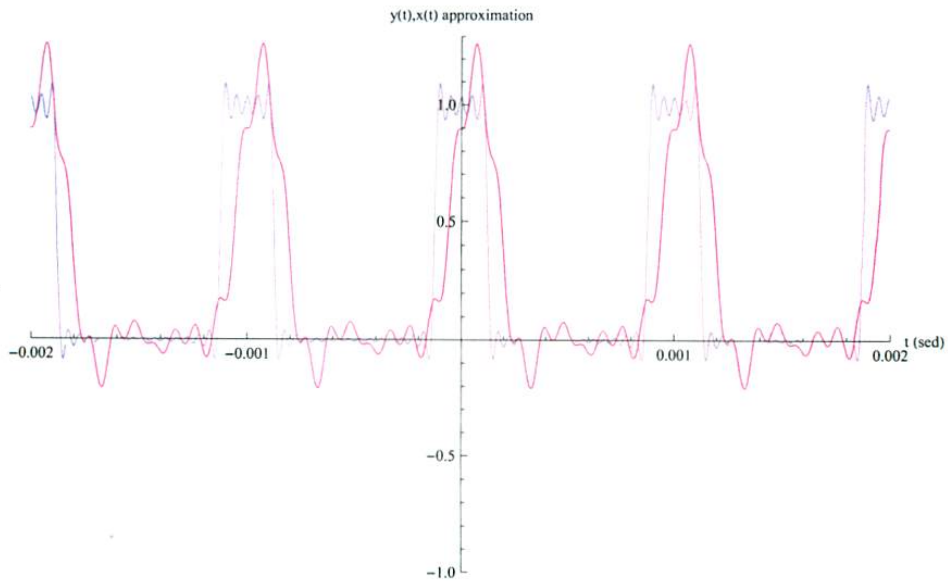
Plot y(t) on top of approximation of x(n) used

In[308]:=

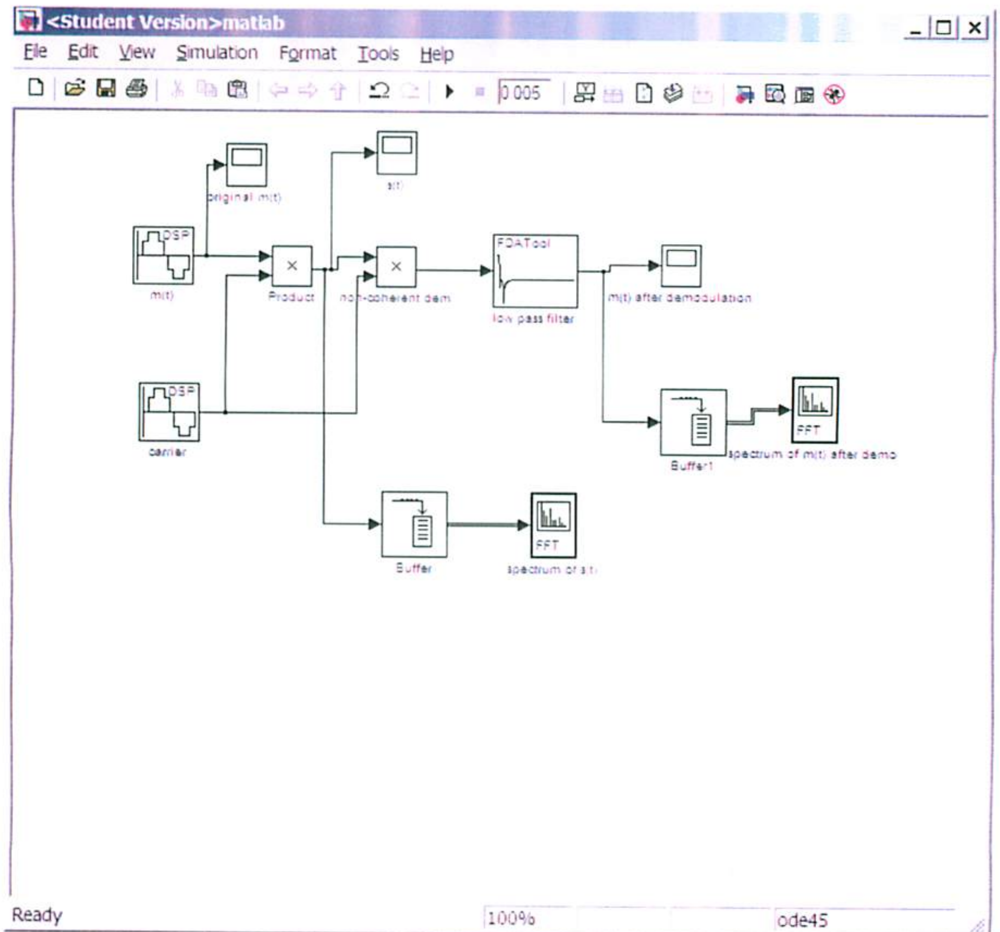
```
Plot[{getFourierApproximation[t, xn, period], y[t, w0, 10]},
 {t, -range, range}, PlotRange → {{-range, range}, {-h, 1.3 h}},
 AxesLabel → {"t (sed)", "y(t),x(t) approximation"}]
```

Out[308]=



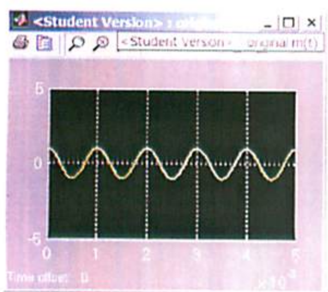
Computer Assignment #2
by Nasser M. Abbasi
ECE 405, summer session 1, Cal Poly Pomona

Simulink setup

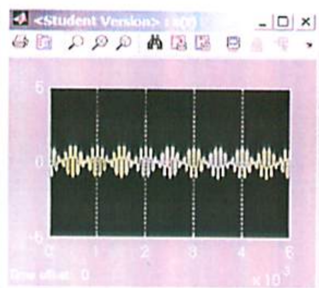


Part (1) *non-coherent demodulation*

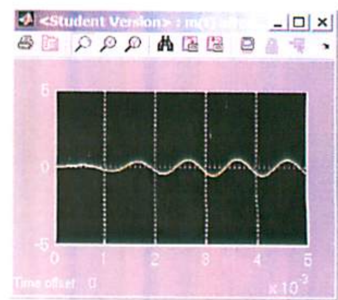
After run of the simulation, the following are the outputs time scope:



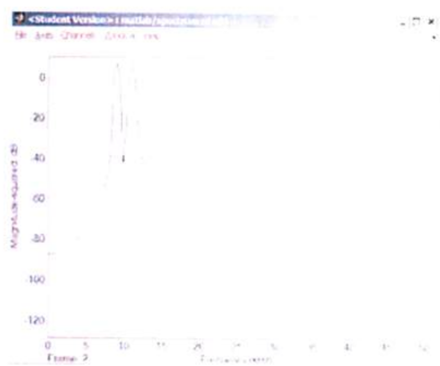
Original m(t)



S(t)=m(t)*cos(2 pi fc t)



Demodulated m(t)

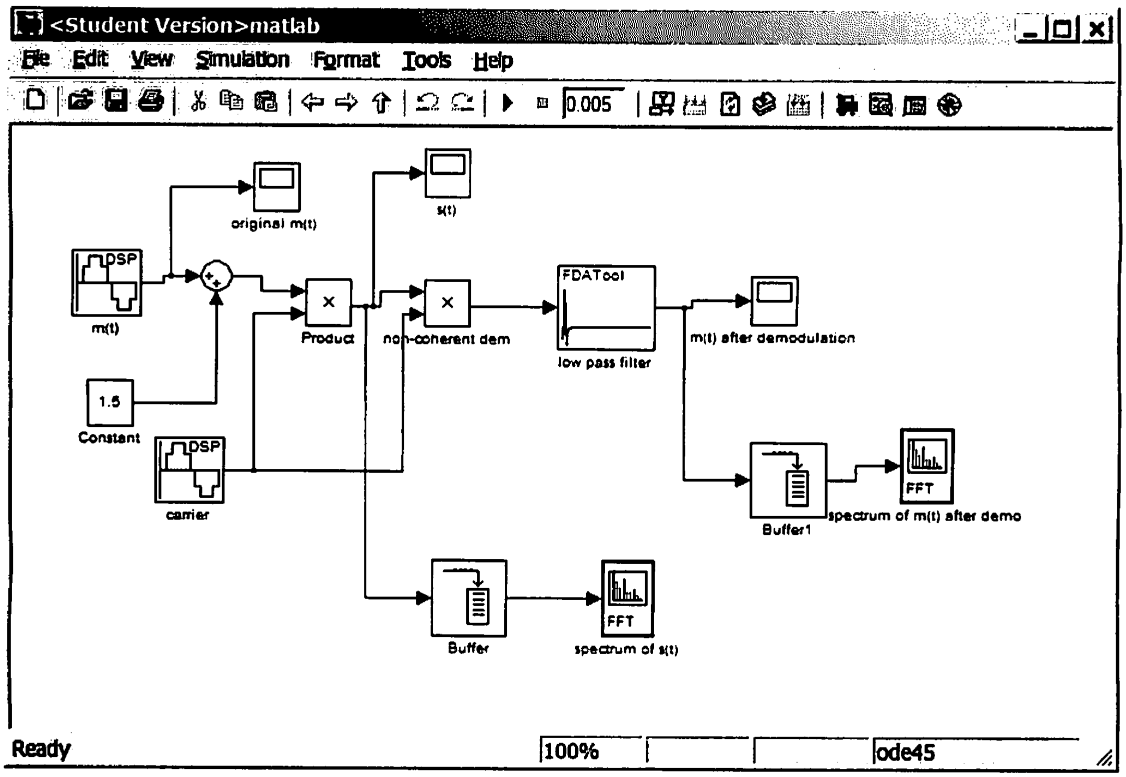


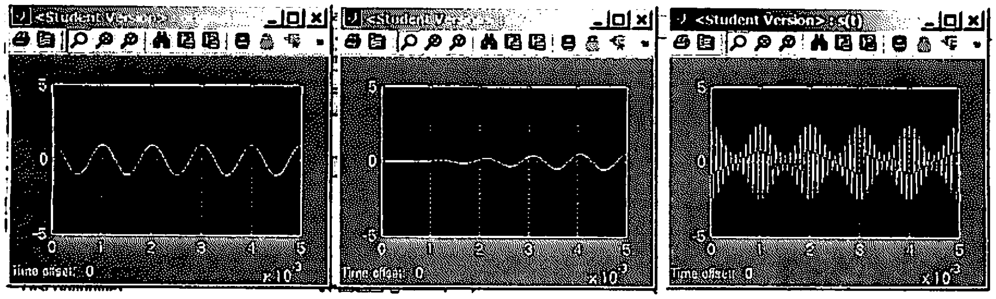
Magnitude Spectrum of s(t)

Magnitude Spectrum of m(t) after demodulation, notice fm=1000 hz

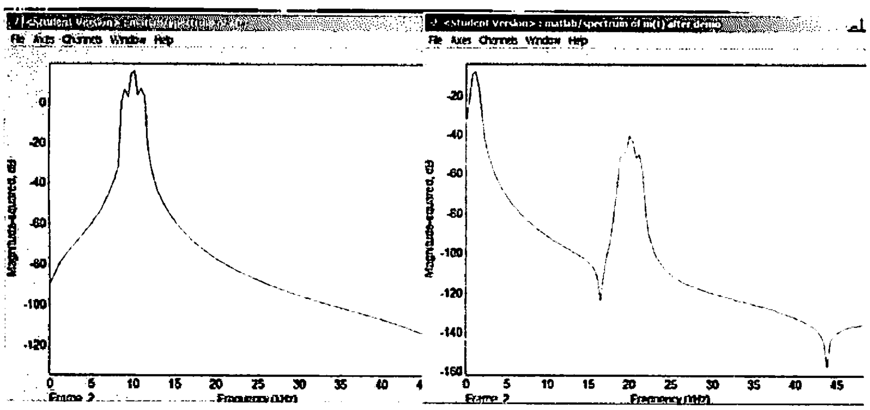
Computer Assignment #3
by Nasser M. Abbasi
ECE 405, summer session 1, Cal Poly Pomona

Simulink setup



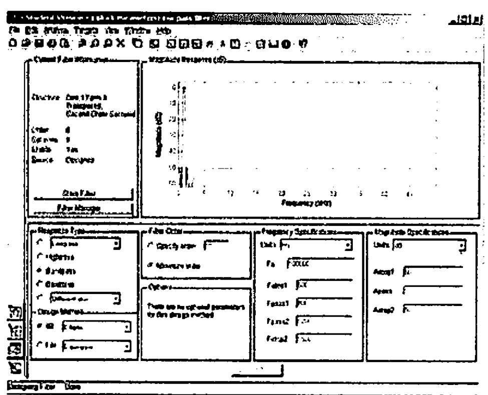


Original m(t) Demodulated m(t) S(t)=(Ac+cos(2 pi fm t))*cos(2 pi fc t)



Magnitude Spectrum of s(t)

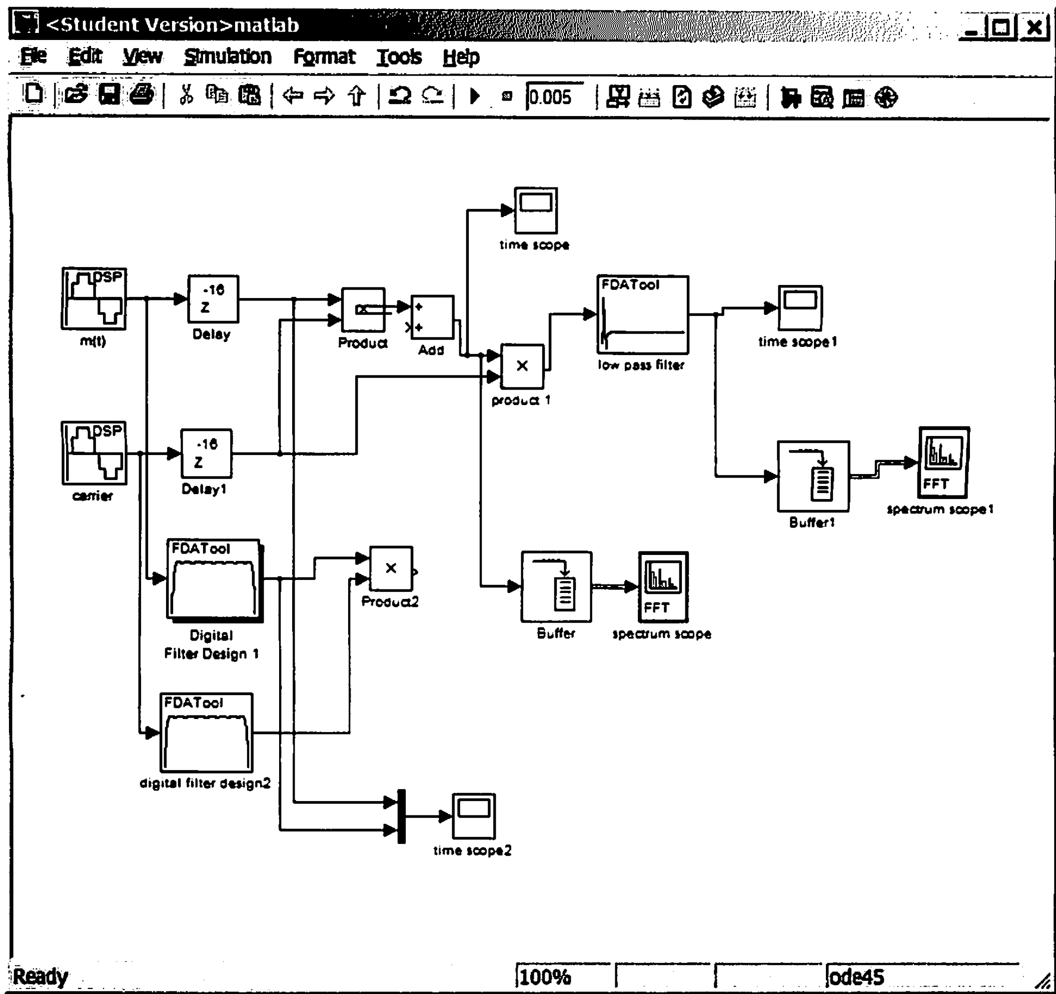
Magnitude Spectrum of m(t) after demodulation, notice fm=1000 hz

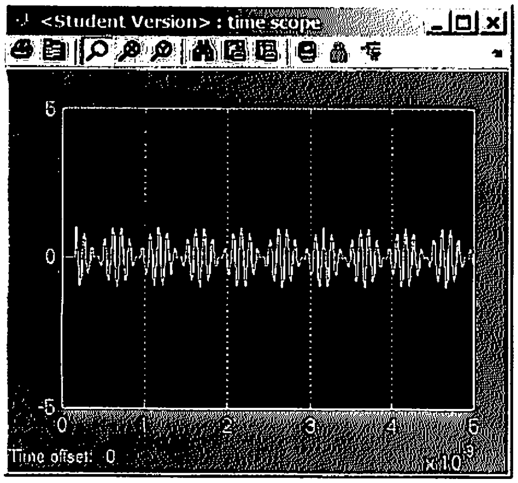


Computer #3 Part (1)

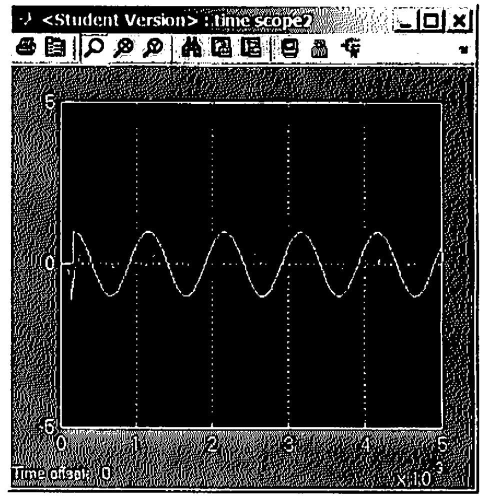
Computer Assignment #4
by Nasser M. Abbasi
ECE 405, summer session 1, Cal Poly Pomona

Simulink setup

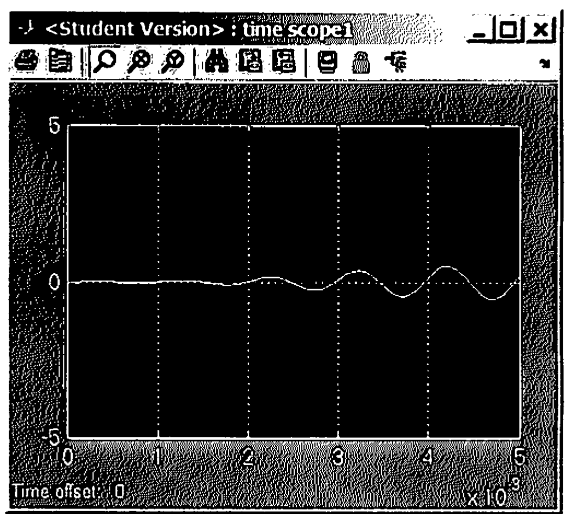




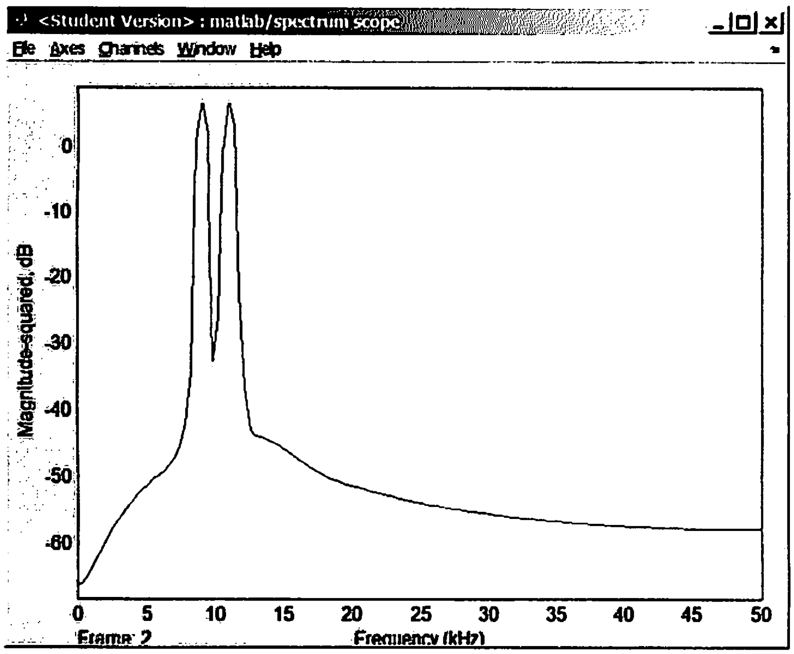
Time scope output



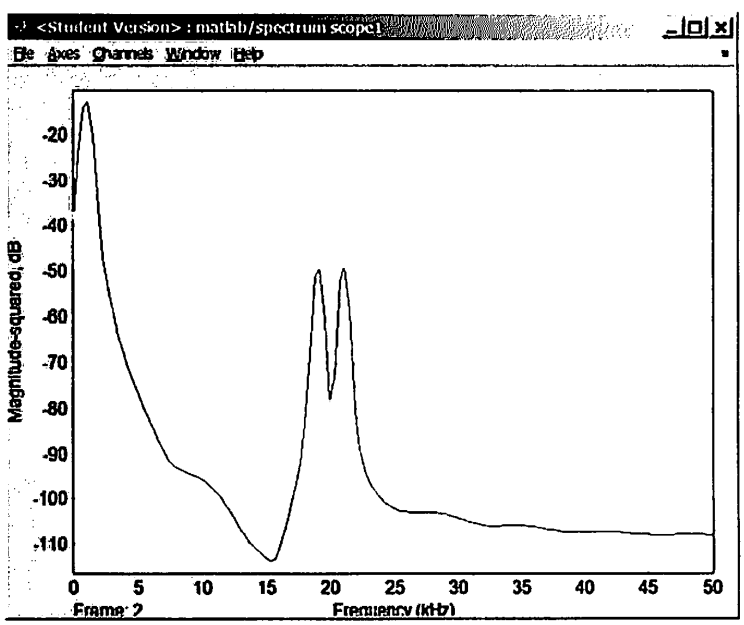
Time scope 2 output



Time scope 1 output



Spectrum scope output

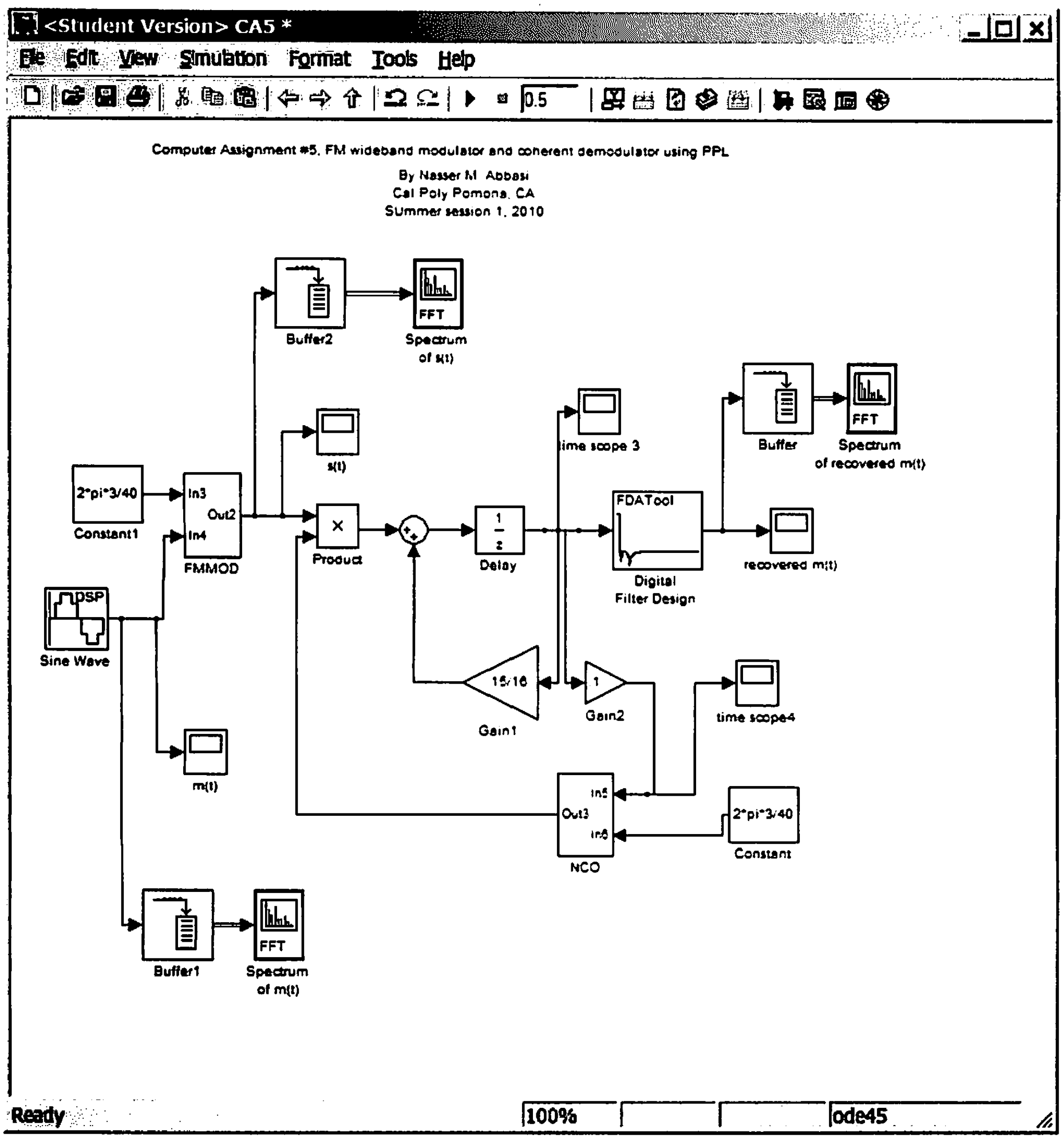


Spectrum scope 1 output

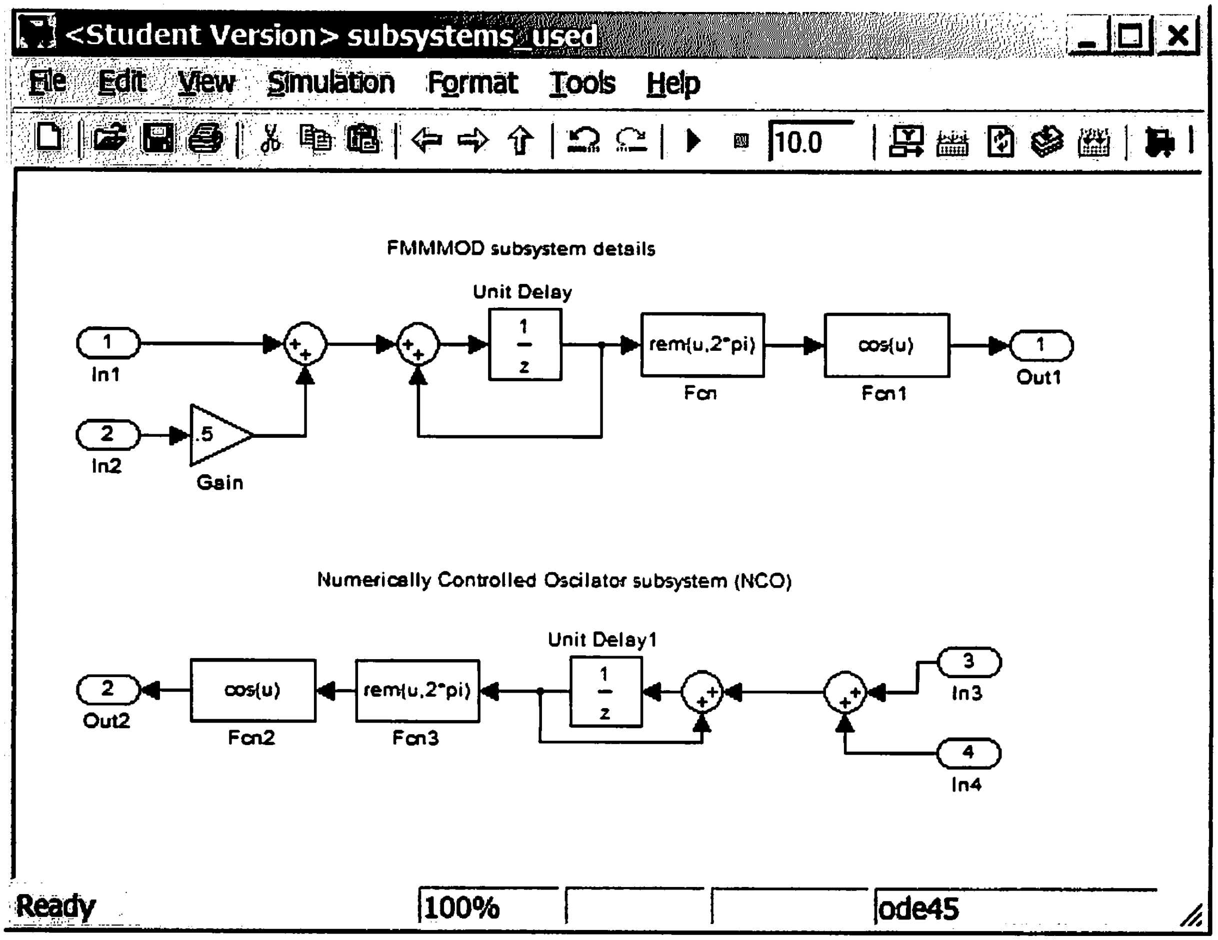
Computer Assignment #5, FM wideband modulator and coherent demodulator using PPL
Nasser M. Abbasi
Cal Poly Pomona, CA
SUmmer session 1, 2010

Simulink model

model is here

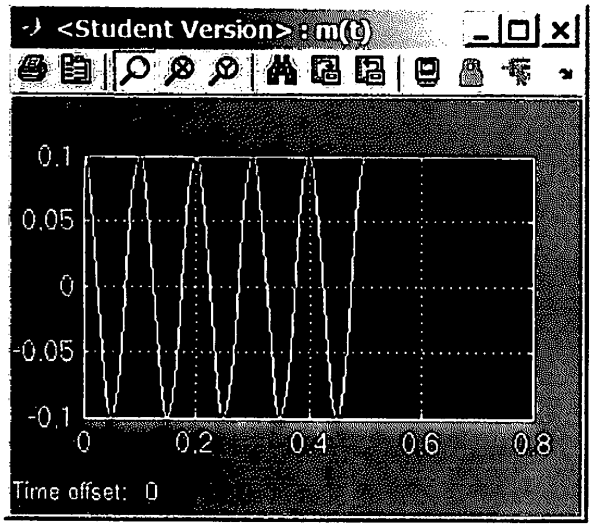


Models of subsystems

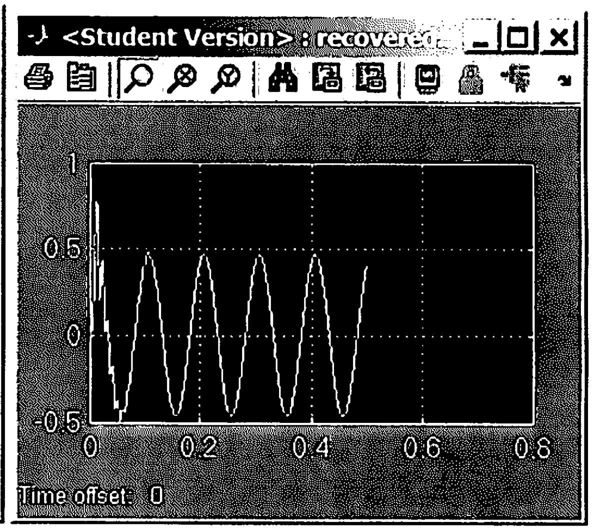


model file of the above is here

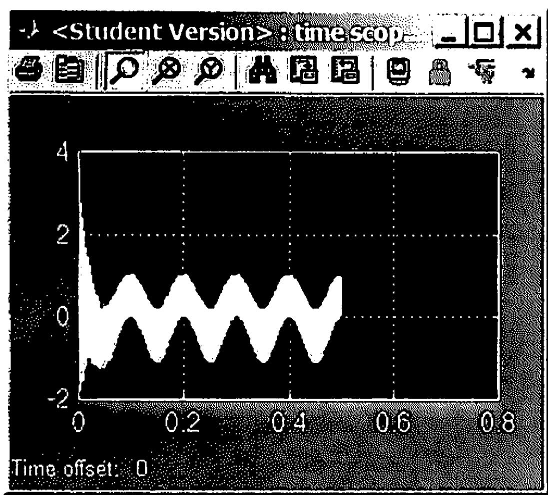
output



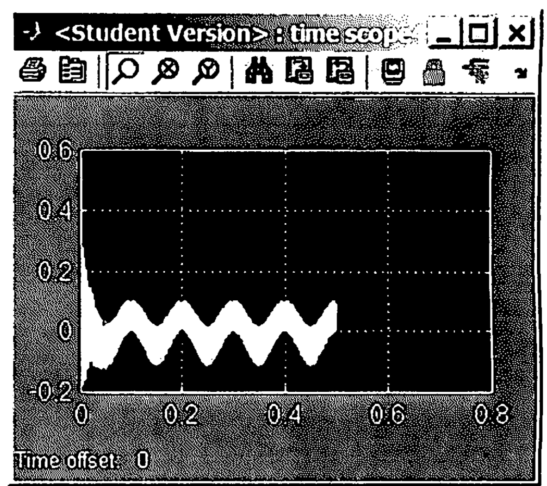
Original message m(t)



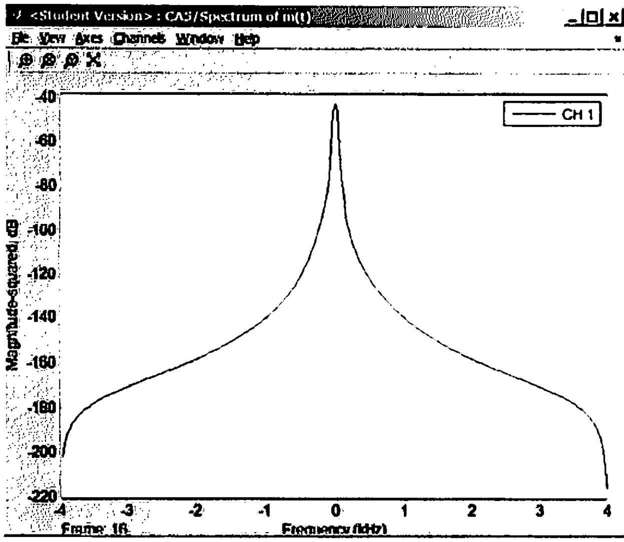
Recovered (demodulated) message m(t)



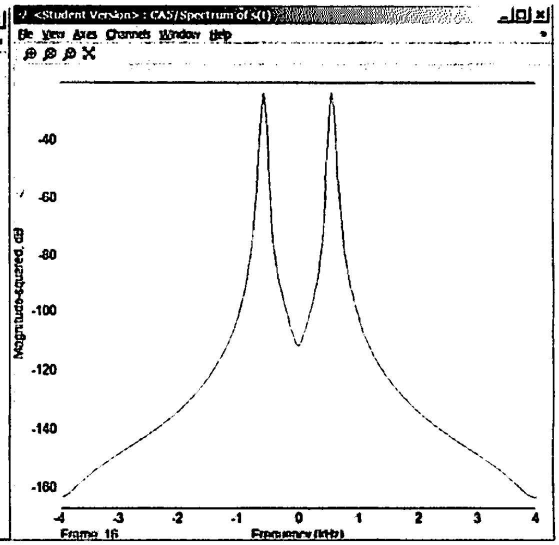
Time scope (3) in the model



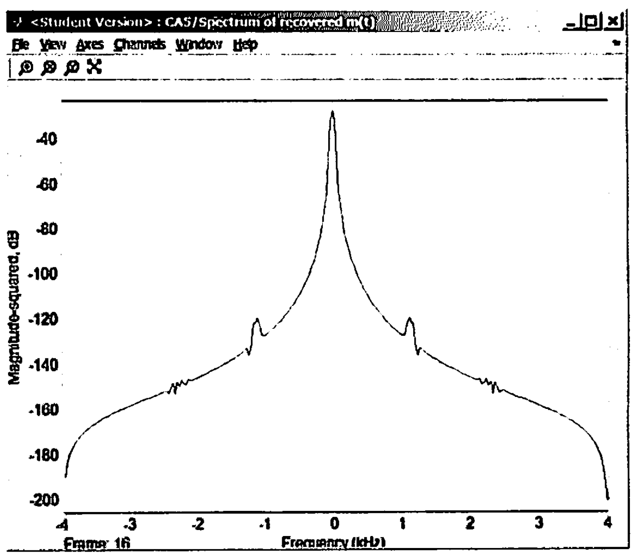
Time scope (4) in the model



Spectrum of original message m(t)



Spectrum of s(t), the modulated carrier (FM)



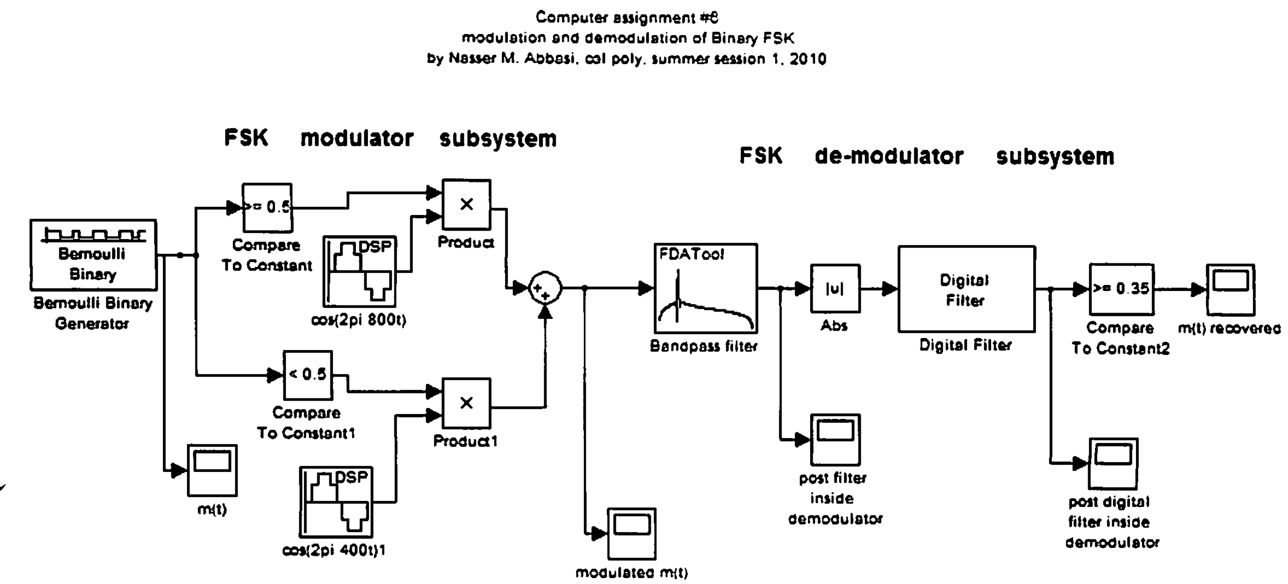
Spectrum of recovered message m(t) (compare to spectrum of original m(t))

Computer Assignment #6
Nasser M. Abbasi
Cal Poly Pomona, CA
SUmmer session 1, 2010

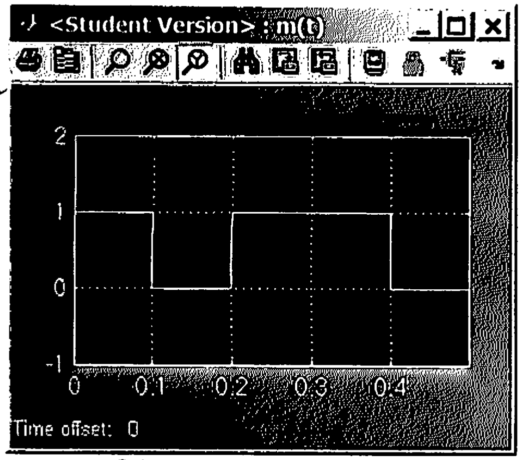
problem description is here

Simulink model

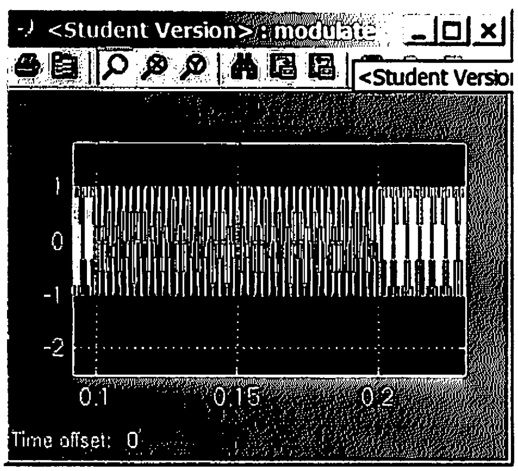
model is here



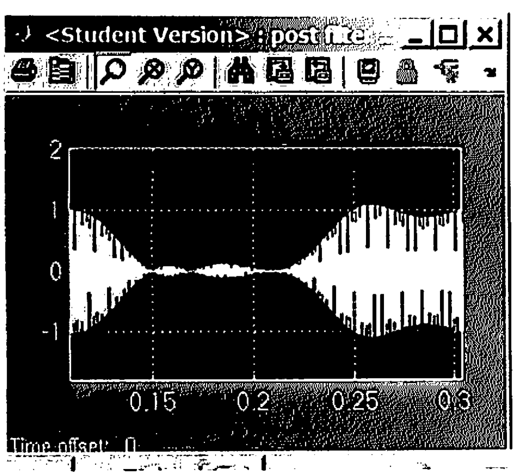
Output and result



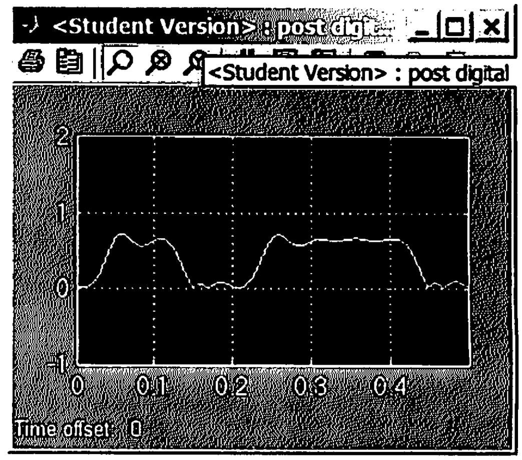
Original message m(t)



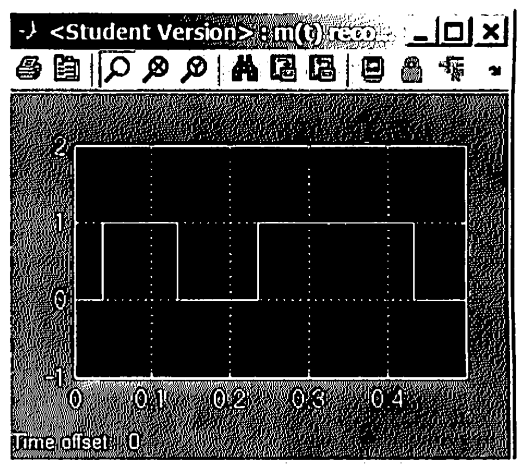
S(t), the modulated carrier message



Output of digital filter inside the demodulator



Output of bandpass filter in the demodulator



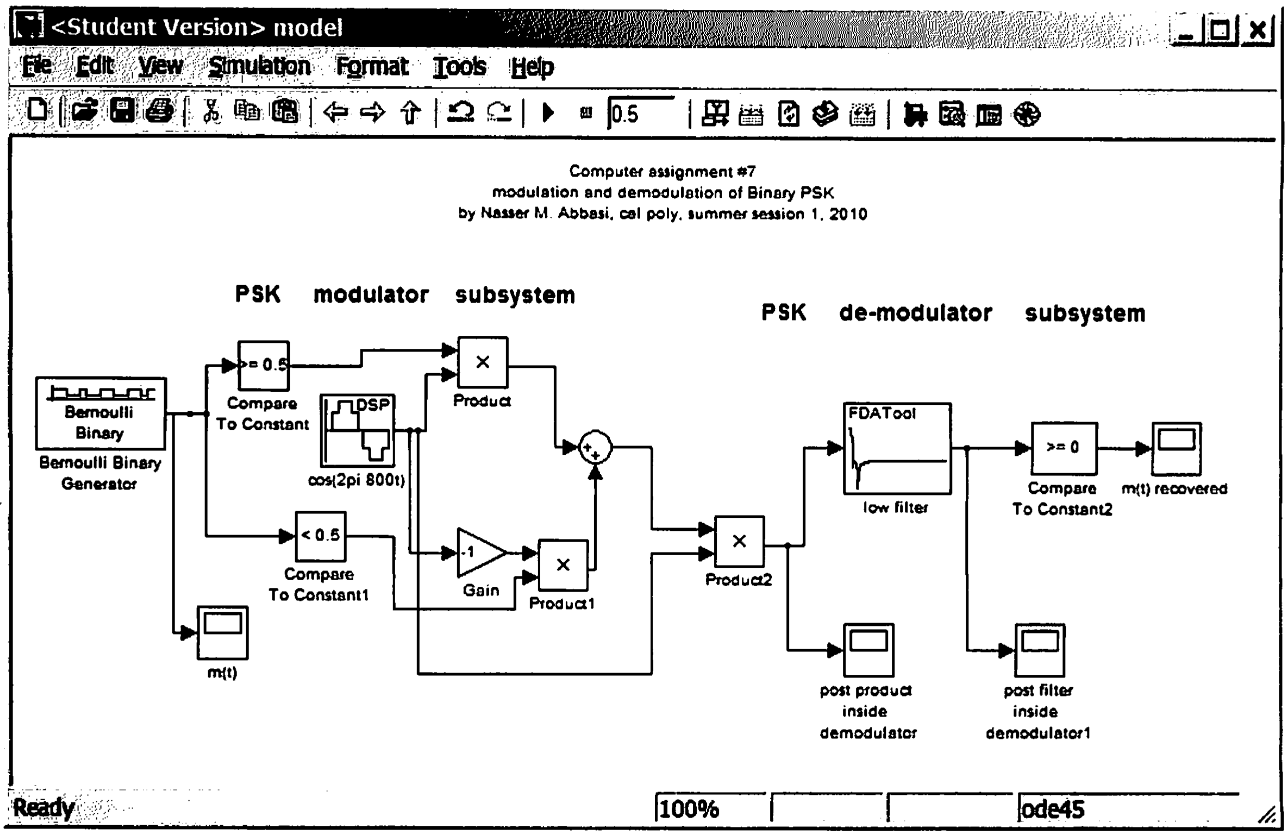
demodulated message m(t). Notice some delay at the start compared to original m(t)

Computer Assignment #7
Nasser M. Abbasi
Cal Poly Pomona, CA
SUmmer session 1, 2010

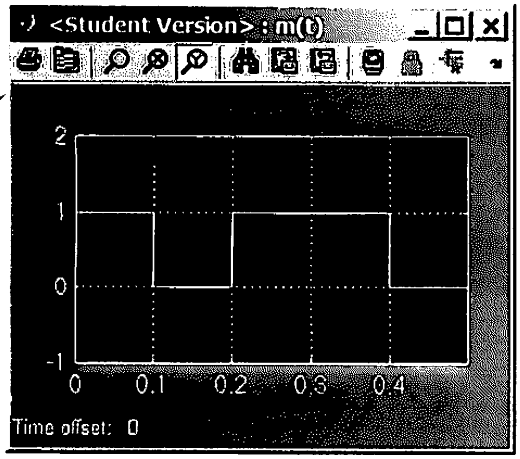
problem description is here

Simulink model

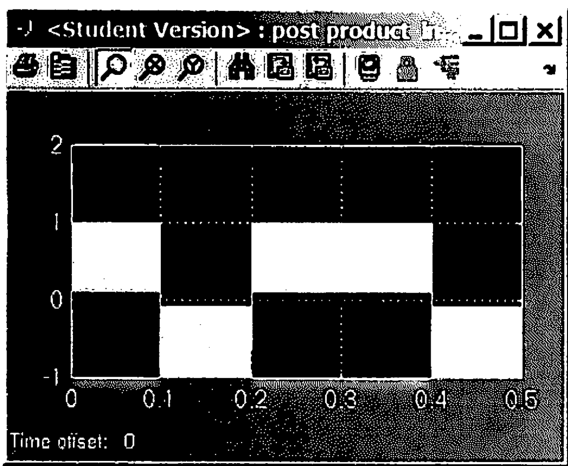
model is here



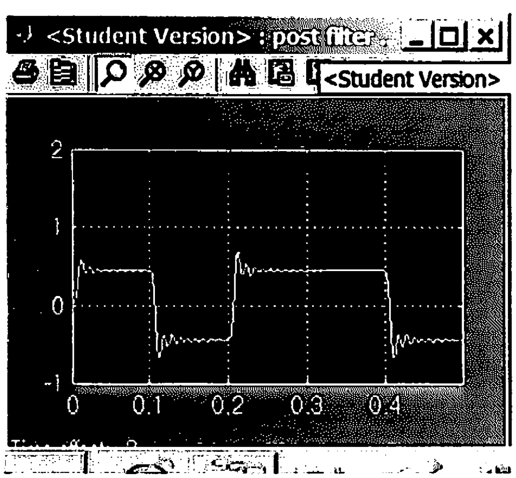
Output and result



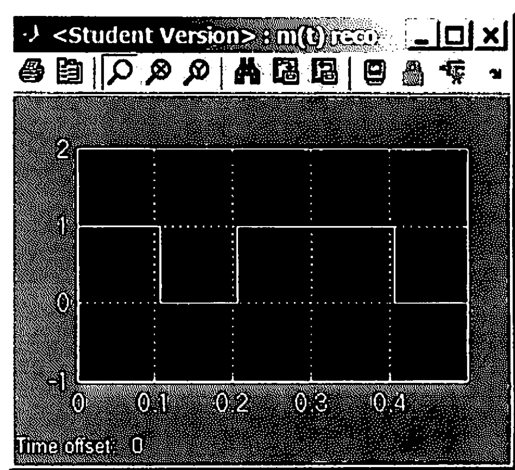
Original binary message m(t)



Modulated carrier s(t)



Message after passing the low pass filter, before compare



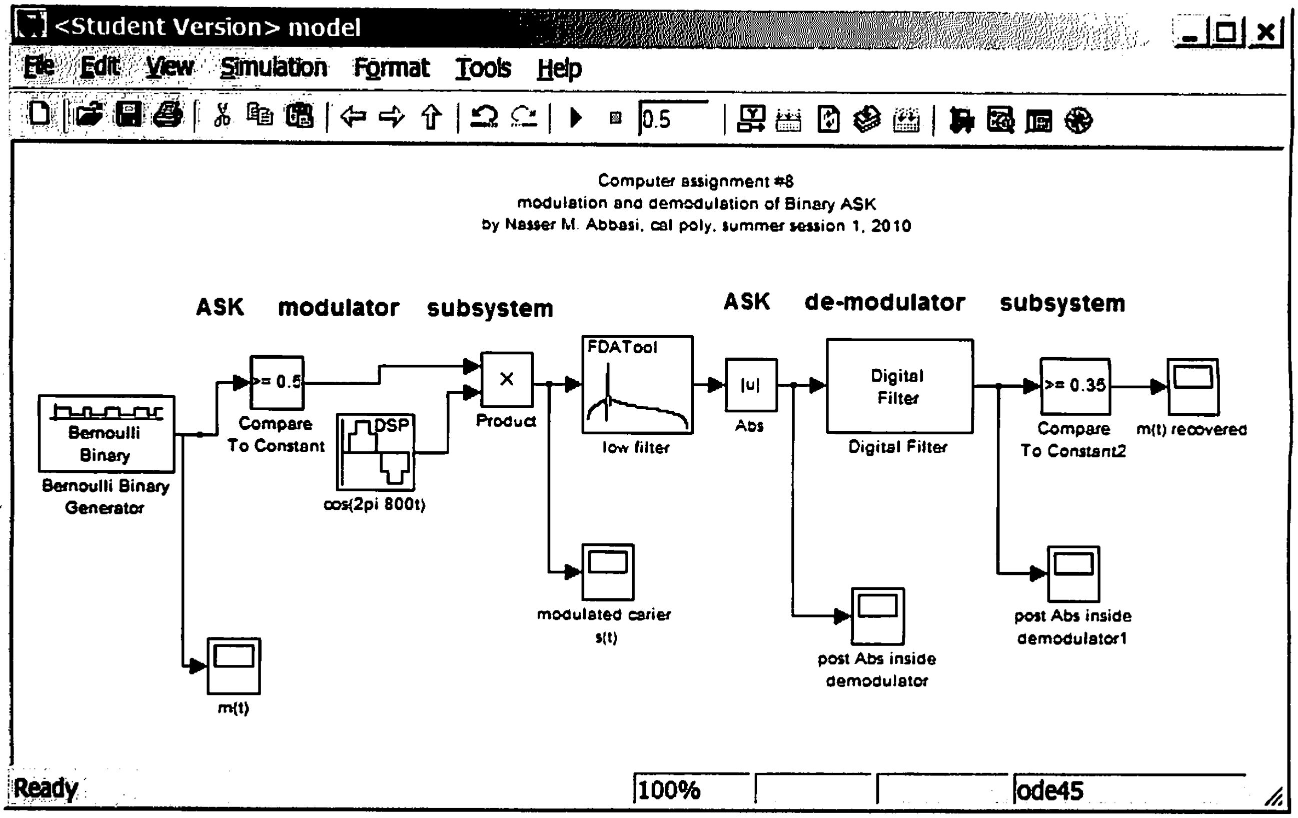
Recovered (demodulated) binary message m(t)

Computer Assignment #8
Nasser M. Abbasi
Cal Poly Pomona, CA
SUmmer session 1, 2010

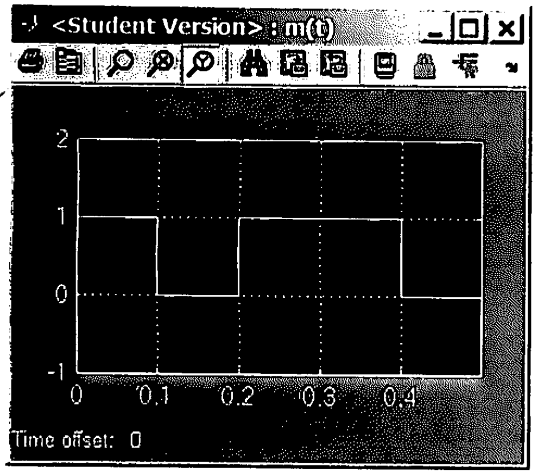
problem description is here

Simulink model

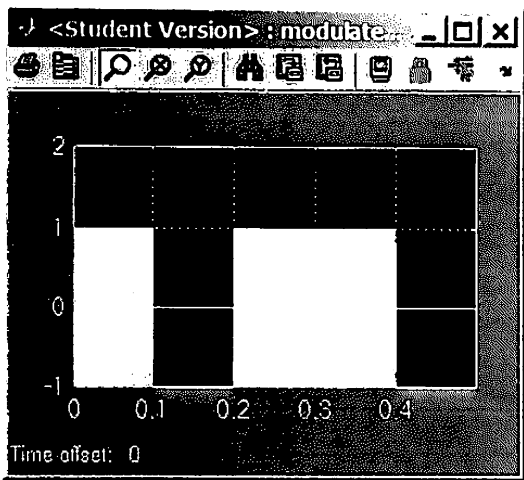
model is here



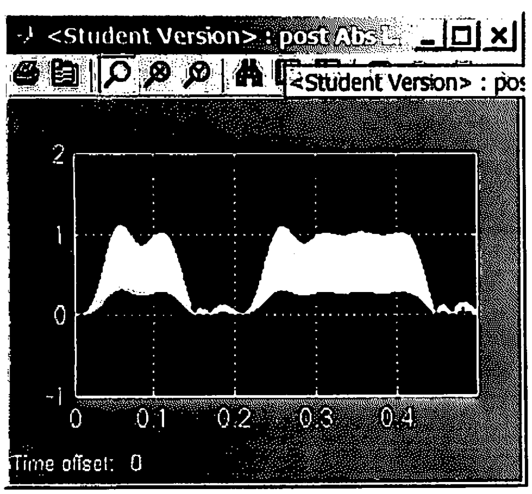
Output and result



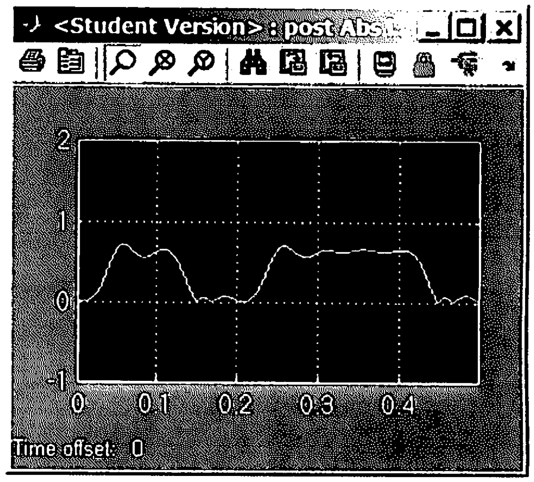
Original message m(t)



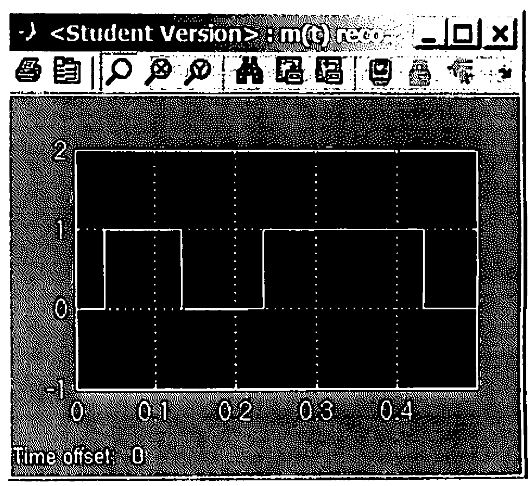
Modulated carier s(t)



After check on Abs value



After the digital filter



Demodulated message
m(t) recovered. Notice
delay at the start
compared to original
message m(t)

Chapter 5

Study notes, cheat sheets

Local contents

5.1 sheet 1

Single tone FM modulation

$$x_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

$\downarrow$

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - (f_c + nf_m)) + \delta(f + (f_c + nf_m)) \right]$$

for one sided

$$= A_c \sum_{n=0}^{\infty} J_n(\beta) \left[\delta(f - (f_c + nf_m) - \cdots \right]$$

$$(BT)_{carson} = 2(f_m + \Delta f)$$

$$= 2 f_m (\underset{f_m}{\beta} + 1)$$

$$(BT)_{1\%} = 2\, n_{max} f_m$$

$$\text{Power in Carrier} = \frac{A_c^2}{2} J_0^2(\beta)$$

$$P_{Total} = \frac{A_c^2}{2} \left[\underbrace{J_0^2(\beta) + 2 J_1^2(\beta) + \cdots}_{=1} \right]$$

$$J_{-n}(\beta) = J_n(\beta) \quad n \text{ even}$$

$$J_{-n}(\beta) = -J_n(\beta) \quad n \text{ odd.}$$

single tone

$$FM(t) = A_c \cos\Big(\underbrace{\omega_c t + \overbrace{\frac{k_f A_m}{\omega_m}\sin(\omega_m t)}^{\phi(t)\ \text{(phase deviation)}}}_{\theta_i(t)}\Big)$$

freq deviation $\dfrac{d\phi(t)}{dt} = k_f\, m(t)$

so $\Delta\omega$ = max freq. deviation = $\boxed{k_f A_m}$

$$\boxed{\beta = \frac{k_f A_m}{\omega_m}} = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$$

$k_f = 2\pi f_d$ called deviation constant

$$\boxed{\beta = \frac{f_d A_m}{f_m}}$$

HW7

(1) $m_p = 16V$.
$x = -8.7V$.

(a) $N = 8$ bits. Find ~~offset~~ offset binary code.

$\Delta = \frac{16}{2^7} = \boxed{0.125}$

quantization level $= \left(\frac{\text{Abs}(x)}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$

Since x is negative, then $\text{Code} = 2^7 - 70 = 128 - 70 = \boxed{58}$

in binary, this is $\boxed{0011\ 1010}$

(b) sign/magnitude..

since $x < 0$ Then $\text{Code} = 2^7 + 70 = 128 + 70 = 198$

Which in binary is $\boxed{1100\ 0110}$

(c) 2's Complement.

since $x < -\frac{\Delta}{2}$ then $\text{Code.} = 2^8 - 70 = 256 - 70$
$= 186$

which in binary is $\boxed{1011\ 1010}$

(d) 1's complement.

since $x < 0$ Then

code $(2^8 - 1) - 70 = 255 - 70 = 185$

Which in binary is $\boxed{1011\ 1001}$

extra: to illustrate this more, this is the Calculations assuming $x = +8.7V$.

offset binary

$\Delta = \frac{16}{2^7} = 0.125$

Level $=$ round $\left(\frac{8.7}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$

Since $x > 0$ Then code $(70)_2 = 0100\ 0110$

Sign magnitude

Since $x > 0$ Then code $(70)_2 = 0100\ 0110$

2's Complement

Since $x > 0$ then code $= (70)_2 = 0100\ 0110$

1's complement

Since $x > 0$, then code $= (70)_2 = 0100\ 0110$.

$$SNR = \frac{\overline{m^2(t)}}{\text{noise Pow}} = \frac{E(m^2)}{\frac{m_p^2}{(3)(2^{2N})}} \rightarrow \int_{-m_p}^{m_p} m^2 f_X(x)\,dx$$

USE if m_p f(Noise)

Handout 7/6/2010. ECE 405

			β		
n	11.0000	12.0000	13.0000	14.0000	15.0000
0	-0.1712	0.0477	0.2069	0.1711	-0.0142
1.0000	-0.1768	-0.2234	-0.0703	0.1334	0.2051
2.0000	0.1390	-0.0849	-0.2177	-0.1520	0.0416
3.0000	0.2273	0.1951	0.0033	-0.1768	-0.1940
4.0000	-0.0150	0.1825	0.2193	0.0762	-0.1192
5.0000	-0.2383	-0.0735	0.1316	0.2204	0.1305
6.0000	-0.2016	-0.2437	-0.1180	0.0812	0.2061
7.0000	0.0184	-0.1703	-0.2406	-0.1508	0.0345
8.0000	0.2250	0.0451	-0.1410	-0.2320	-0.1740
9.0000	0.3089	0.2304	0.0670	-0.1143	-0.2200
10.0000	0.2804	0.3005	0.2338	0.0850	-0.0901
11.0000	0.2010	0.2704	0.2927	0.2357	0.1000
12.0000	0.1216	0.1953	0.2615	0.2855	0.2367
13.0000	0.0643	0.1201	0.1901	0.2536	0.2787
14.0000	0.0304	0.0650	0.1188	0.1855	0.2464
15.0000	0.0130	0.0316	0.0656	0.1174	0.1813
16.0000	0.0051	0.0140	0.0327	0.0661	0.1162
17.0000	0.0019	0.0057	0.0149	0.0337	0.0665
18.0000	0.0006	0.0022	0.0063	0.0158	0.0346
19.0000	0.0002	0.0008	0.0025	0.0068	0.0166
20.0000	0.0001	0.0003	0.0009	0.0028	0.0074
21.0000	0.0000	0.0001	0.0003	0.0010	0.0031
22.0000	0.0000	0.0000	0.0001	0.0004	0.0012
23.0000	0.0000	0.0000	0.0000	0.0001	0.0004
24.0000	0.0000	0.0000	0.0000	0.0000	0.0002
25.0000	0.0000	0.0000	0.0000	0.0000	0.0001

			β		
n	16.0000	17.0000	18.0000	19.0000	20.0000
0	-0.1749	-0.1699	-0.0134	0.1466	0.1670
1.0000	0.0904	-0.0977	-0.1880	-0.1057	0.0668
2.0000	0.1862	0.1584	-0.0075	-0.1578	-0.1603
3.0000	-0.0438	0.1349	0.1863	0.0725	-0.0989
4.0000	-0.2026	-0.1107	0.0696	0.1806	0.1307
5.0000	-0.0575	-0.1870	-0.1554	0.0036	0.1512
6.0000	0.1667	0.0007	-0.1560	-0.1788	-0.0551
7.0000	0.1825	0.1875	0.0514	-0.1165	-0.1842
8.0000	-0.0070	0.1537	0.1959	0.0929	-0.0739
9.0000	-0.1895	-0.0429	0.1228	0.1947	0.1251
10.0000	-0.2062	-0.1991	-0.0732	0.0916	0.1865
11.0000	-0.0682	-0.1914	-0.2041	-0.0984	0.0614
12.0000	0.1124	-0.0486	-0.1762	-0.2055	-0.1190
13.0000	0.2368	0.1228	-0.0309	-0.1612	-0.2041
14.0000	0.2724	0.2364	0.1316	-0.0151	-0.1464
15.0000	0.2399	0.2666	0.2356	0.1389	-0.0008
16.0000	0.1775	0.2340	0.2611	0.2345	0.1452
17.0000	0.1150	0.1739	0.2286	0.2559	0.2331
18.0000	0.0668	0.1138	0.1706	0.2235	0.2511
19.0000	0.0354	0.0671	0.1127	0.1676	0.2189
20.0000	0.0173	0.0362	0.0673	0.1116	0.1647
21.0000	0.0079	0.0180	0.0369	0.0675	0.1106
22.0000	0.0034	0.0084	0.0187	0.0375	0.0676
23.0000	0.0013	0.0037	0.0089	0.0193	0.0380
24.0000	0.0005	0.0015	0.0039	0.0093	0.0199
25.0000	0.0002	0.0006	0.0017	0.0042	0.0098
26.0000	0.0001	0.0002	0.0007	0.0018	0.0045
27.0000	0.0000	0.0001	0.0003	0.0007	0.0020
28.0000	0.0000	0.0000	0.0001	0.0003	0.0008
29.0000	0.0000	0.0000	0.0000	0.0001	0.0003
30.0000	0.0000	0.0000	0.0000	0.0000	0.0001

2

Bessel Function Table Dr. James S. Kang, Professor, Cal Poly Pomona

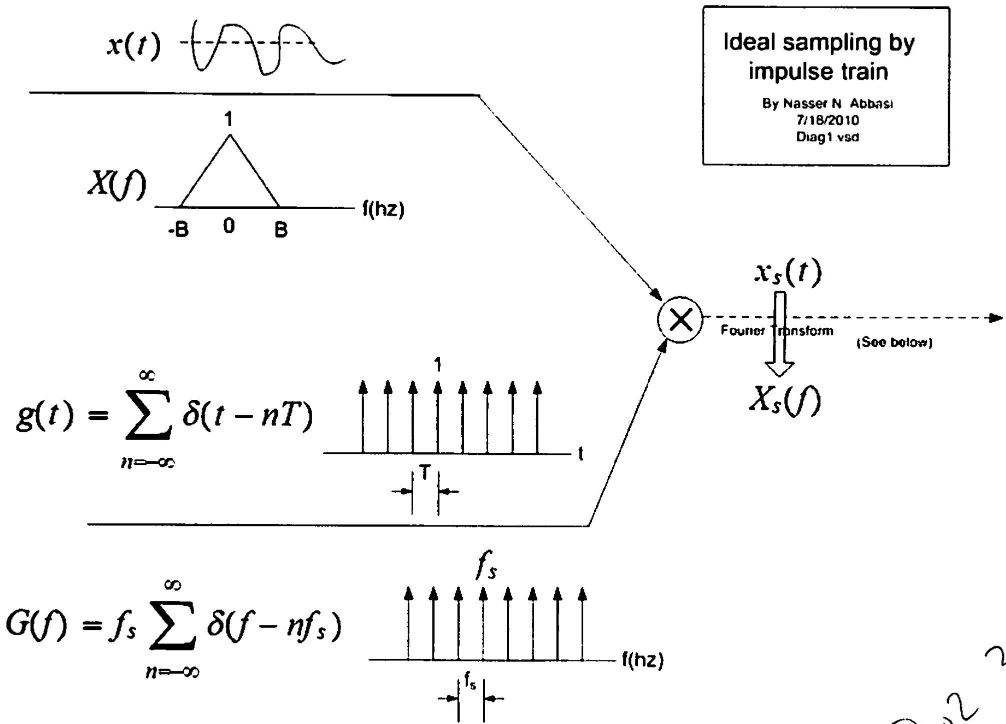
n	β = 0.1000	0.2000	0.3000	0.4000	0.5000	0.6000
0	0.9975	0.9900	0.9776	0.9604	0.9385	0.9120
1.0000	0.0499	0.0995	0.1483	0.1960	0.2423	0.2867
2.0000	0.0012	0.0050	0.0112	0.0197	0.0306	0.0437
3.0000	0.0000	0.0002	0.0006	0.0013	0.0026	0.0044
4.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003
5.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

n	β = 0.7000	0.8000	0.9000	1.0000	1.1000	1.2000
0	0.8812	0.8463	0.8075	0.7652	0.7196	0.6711
1.0000	0.3290	0.3688	0.4059	0.4401	0.4709	0.4983
2.0000	0.0588	0.0758	0.0946	0.1149	0.1366	0.1593
3.0000	0.0069	0.0102	0.0144	0.0196	0.0257	0.0329
4.0000	0.0006	0.0010	0.0016	0.0025	0.0036	0.0050
5.0000	0.0000	0.0001	0.0001	0.0002	0.0004	0.0006
6.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001

n	β = 1.0000	2.0000	3.0000	4.0000	5.0000
0	0.7652	0.2239	-0.2601	-0.3971	-0.1776
1.0000	0.4401	0.5767	0.3391	-0.0660	-0.3276
2.0000	0.1149	0.3528	0.4861	0.3641	0.0466
3.0000	0.0196	0.1289	0.3091	0.4302	0.3648
4.0000	0.0025	0.0340	0.1320	0.2811	0.3912
5.0000	0.0002	0.0070	0.0430	0.1321	0.2611
6.0000	0.0000	0.0012	0.0114	0.0491	0.1310
7.0000	0.0000	0.0002	0.0025	0.0152	0.0534
8.0000	0.0000	0.0000	0.0005	0.0040	0.0184
9.0000	0.0000	0.0000	0.0001	0.0009	0.0055
10.0000	0.0000	0.0000	0.0000	0.0002	0.0015
11.0000	0.0000	0.0000	0.0000	0.0000	0.0004
12.0000	0.0000	0.0000	0.0000	0.0000	0.0001

n	β = 6.0000	7.0000	8.0000	9.0000	10.0000
0	0.1506	0.3001	0.1717	-0.0903	-0.2459
1.0000	-0.2767	-0.0047	0.2346	0.2453	0.0435
2.0000	-0.2429	-0.3014	-0.1130	0.1448	0.2546
3.0000	0.1148	-0.1676	-0.2911	-0.1809	0.0584
4.0000	0.3576	0.1578	-0.1054	-0.2655	-0.2196
5.0000	0.3621	0.3479	0.1858	-0.0550	-0.2341
6.0000	0.2458	0.3392	0.3376	0.2043	-0.0145
7.0000	0.1296	0.2336	0.3206	0.3275	0.2167
8.0000	0.0565	0.1280	0.2235	0.3051	0.3179
9.0000	0.0212	0.0589	0.1263	0.2149	0.2919
10.0000	0.0070	0.0235	0.0608	0.1247	0.2075
11.0000	0.0020	0.0083	0.0256	0.0622	0.1231
12.0000	0.0005	0.0027	0.0096	0.0274	0.0634
13.0000	0.0001	0.0008	0.0033	0.0108	0.0290
14.0000	0.0000	0.0002	0.0010	0.0039	0.0120
15.0000	0.0000	0.0001	0.0003	0.0013	0.0045
16.0000	0.0000	0.0000	0.0001	0.0004	0.0016
17.0000	0.0000	0.0000	0.0000	0.0001	0.0005
18.0000	0.0000	0.0000	0.0000	0.0000	0.0002

1



$$x_s(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT) = \overbrace{\sum_{n=-\infty}^{\infty}x(t)\delta(t-nT) = \sum_{n=-\infty}^{\infty}x(nT)\delta(t-nT)}^{\text{Alternative way to write the sampled signal x(t)}}$$

Fourier series approx

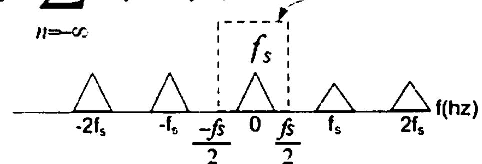
$$x_s(t) \approx x(t)\left(f_s\sum_{n=-\infty}^{\infty}e^{j\frac{2\pi}{T}nt}\right)$$

Fourier series approximation of the pulse train

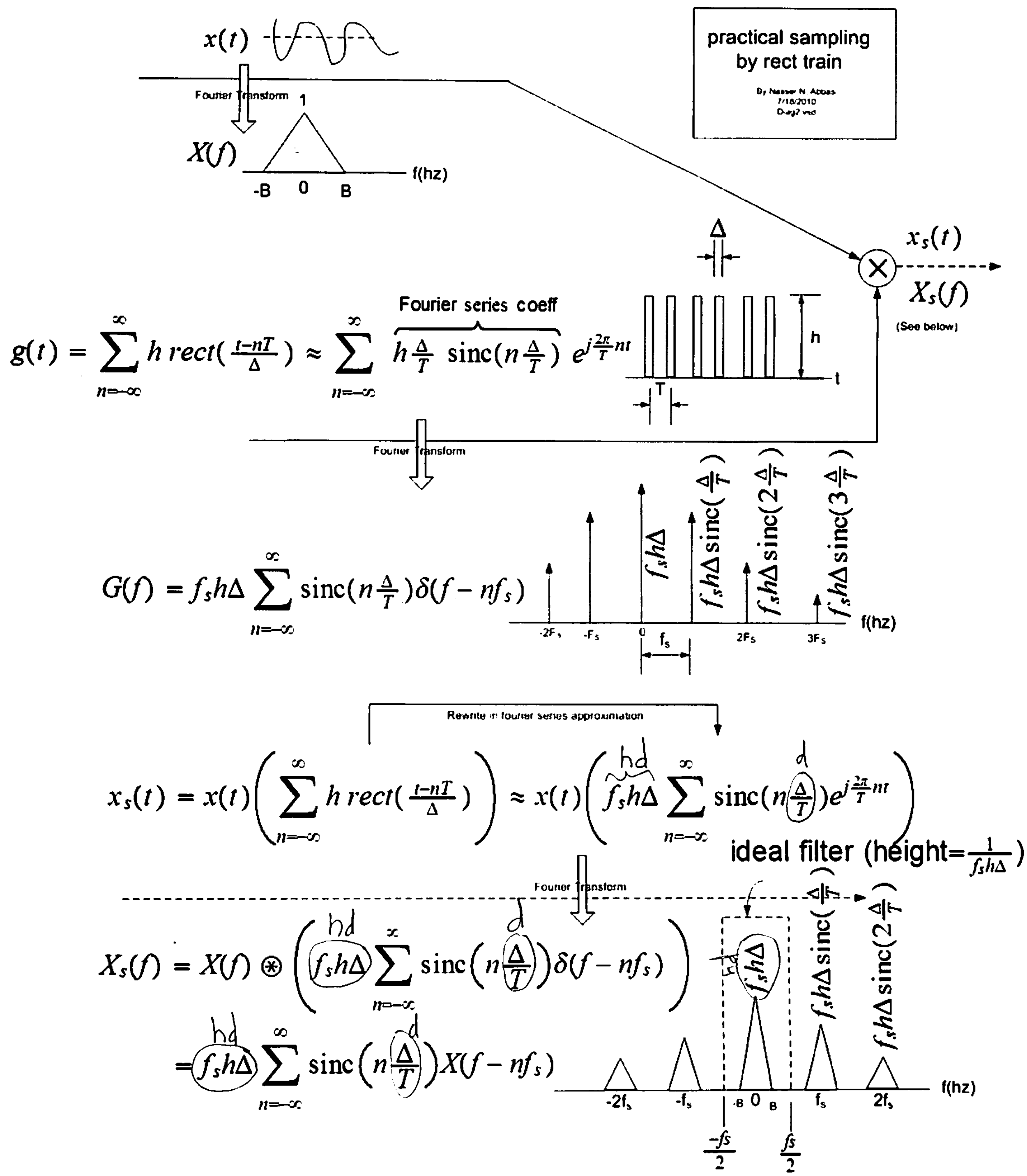
Fourier Transform

$$X_s(f) = X(f)\circledast\underbrace{\left(f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)\right)}_{G(f)} = f_s\sum_{n=-\infty}^{\infty}X(f-nf_s)$$

ideal filter (height=$\frac{1}{f_s}$)



2 Practical sampling



$$g(t)=\sum_{n=-\infty}^{\infty} h\, rect\left(\frac{t-nT}{\Delta}\right)\approx\sum_{n=-\infty}^{\infty}\overbrace{h\frac{\Delta}{T}\,\mathrm{sinc}\left(n\frac{\Delta}{T}\right)}^{\text{Fourier series coeff}} e^{j\frac{2\pi}{T}nt}$$

$$G(f)=f_s h\Delta\sum_{n=-\infty}^{\infty}\mathrm{sinc}\left(n\frac{\Delta}{T}\right)\delta(f-nf_s)$$

$f_s h\Delta$, $f_s h\Delta\,\mathrm{sinc}(\frac{\Delta}{T})$, $f_s h\Delta\,\mathrm{sinc}(2\frac{\Delta}{T})$, $f_s h\Delta\,\mathrm{sinc}(3\frac{\Delta}{T})$

$$x_s(t)=x(t)\left(\sum_{n=-\infty}^{\infty} h\, rect\left(\frac{t-nT}{\Delta}\right)\right)\approx x(t)\left(\underbrace{f_s h\Delta}_{hd}\sum_{n=-\infty}^{\infty}\mathrm{sinc}\left(n\frac{\Delta}{T}\right)e^{j\frac{2\pi}{T}nt}\right)$$

$$X_s(f)=X(f)\circledast\left(f_s h\Delta\sum_{n=-\infty}^{\infty}\mathrm{sinc}\left(n\frac{\Delta}{T}\right)\delta(f-nf_s)\right)$$

$$=f_s h\Delta\sum_{n=-\infty}^{\infty}\mathrm{sinc}\left(n\frac{\Delta}{T}\right)X(f-nf_s)$$

Welcome to Vibration Data Laplace Transform Table

Laplace transforms are used to solve differential equations.
As an example, Laplace transforms are used to determine the response of a harmonic oscillator to an input signal.

By Tom Irvine Email: tomirvine@aol.com

Operation Transforms

N	F(s)	f (t) , t > 0
1.1	$Y(s) = \int_0^\infty \exp(-st)\,y(t)\,dt$	definition of a Laplace transform y(t)
1.2	Y(s)	inversion formula $y(t) = \frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty} \exp(st)\,Y(s)\,ds$
1.3	sY(s) - y(0)	first derivative $y'(t)$
1.4	$s^2\,Y(s) - s\,y(0) - y'(0)$	second derivative $y''(t)$
1.5	$s^n\,Y(s) - s^{n-1}\,[y(0)]$ $-s^{n-2}\,[y'(0)] - \ldots - s\left[y^{(n-2)}(0)\right]$ $-[y^{(n-1)}(0)]$	nth derivative $y^{(n)}(t)$
1.6	(1/s) F(s)	integration $\int_0^t Y(\tau)\,d\tau$
1.7	F(s)G(s)	convolution integral $\int_0^t f(t-\tau)g(\tau)\,d\tau$
1.8	$\frac{1}{\alpha}F\left(\frac{s}{\alpha}\right)$	f (at)
1.9	F(s - a)	shifting in the s-plane

Table of Laplace Transforms

http://www.vibrationdata.com/Laplace.htm

		exp(-at) f(t)
1.10	$\frac{1}{1-\exp(-sT)}\int_0^T \exp(-st)f(t)\,dt$	f(t) has period T, such that f(t + T) = f (t)
1.11	$\frac{1}{1+\exp(-sT)}\int_0^T \exp(-st)g(t)\,dt$	g(t) has period T, such that g(t + T) = - g(t)

Function Transforms

N	F(s)	$f(t), t > 0$
2.1	1	d(t) unit impulse at t = 0
2.2	s	$\frac{d}{dt}\delta(t)$ double impulse at t = 0
2.3	$\exp(-\alpha s),\quad \alpha \geq 0$	d(t-a)
2.4a	1/s	unit step u(t)
2.4b	$\frac{1}{s}\left[\exp(-as)-\exp(-bs)\right]$	0 $t < a$ 1 $a < t < b$ 0 $t > b$
2.5	$\frac{1}{s}\exp(-\alpha s)$	u(t-a)
2.6	$\frac{1}{s^2}$	t
2.7a	$\frac{1}{s^n},\quad n = 1, 2, 3, \ldots$	$\frac{t^{n-1}}{(n-1)!}$
2.7b	$\frac{n!}{s^{n+1}}$, n=1, 2, 3,?.	t^n
2.8	$\frac{1}{s^k}$, k is any real number > 0	

		$\dfrac{t^{k-1}}{\Gamma(k)}$ the Gamma function is given in Appendix A
2.9	$\dfrac{1}{s+\alpha}$	exp(-at)
2.10	$\dfrac{1}{(s+\alpha)^2}$	t exp(-at)

2.11	$\dfrac{1}{(s+\alpha)^n}$; $n = 1, 2, 3, \ldots$	$\left[\dfrac{t^{n-1}}{(n-1)!}\right]\exp(-\alpha t)$
2.12	$\dfrac{\alpha}{s(s+\alpha)}$	1 - exp(-at)
2.13	$\dfrac{1}{(s+\alpha)(s+\beta)}$, $\beta \neq \alpha$	$\dfrac{1}{(\beta-\alpha)}[\exp(-\alpha t) - \exp(-\beta t)]$
2.14	$\dfrac{1}{s(s+\alpha)(s+\beta)}$, $\beta \neq \alpha$	$\dfrac{1}{\alpha\beta} + \dfrac{\exp(-\alpha t)}{\alpha(\alpha-\beta)} + \dfrac{\exp(-\beta t)}{\beta(\beta-\alpha)}$
2.15	$\dfrac{s}{(s+\alpha)(s+\beta)}$, $\beta \neq \alpha$	$\dfrac{1}{(\alpha-\beta)}[\,\alpha\exp(-\alpha t) - \beta\exp(-\beta t)\,]$
2.16a	$\dfrac{\alpha}{s^2+\alpha^2}$	sin(at)
2.16b	$\dfrac{[\sin(\phi)]\,s + \alpha[\cos(\phi)]}{s^2+\alpha^2}$	sin(at + f)
2.17	$\dfrac{s}{s^2+\alpha^2}$	cos(at)
2.18	$\dfrac{s^2-\alpha^2}{\left[s^2+\alpha^2\right]^2}$	t cos(at)

Table of Laplace Transforms http://www.vibrationdata.com/Laplace.htm

2.19	$\frac{1}{s(s^2+\alpha^2)}$	$\frac{1}{\alpha^2}[1-\cos(\alpha t)]$
2.20	$\frac{1}{(s^2+\alpha^2)^2}$	$\frac{1}{2\alpha^3}[\sin(\alpha t)-\alpha t\ \cos(\alpha t)]$
2.21	$\frac{s}{(s^2+\alpha^2)^2}$	$\frac{1}{2\alpha}[t\sin(\alpha t)]$
2.22	$\frac{s^2}{(s^2+\alpha^2)^2}$	$\frac{1}{2\alpha}[\sin(\alpha t)+\alpha t\cos(\alpha t)]$

2.23	$\frac{1}{(s^2+\omega^2)(s^2+\alpha^2)}, \quad \alpha \neq \omega$	$\left\{\frac{1}{\omega^2-\alpha^2}\right\}\left\{\frac{1}{\alpha}\sin(\alpha t)-\frac{1}{\omega}\sin(\omega t)\right\}$
2.24	$\frac{\alpha}{s^2(s+\alpha)}$	$t-\frac{1}{\alpha}[1-\exp(-\alpha t)]$
2.25	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	exp(-at)sin(bt)
2.26	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$	exp(-at)cos(bt)
2.27	$\frac{s+\lambda}{(s+\alpha)^2+\beta^2}$	$\exp(-\alpha t)\left\{\cos(\beta t)+\left[\frac{\lambda-\alpha}{\beta}\right]\sin(\beta t)\right\}$
2.28	$\frac{s+\alpha}{s^2+\beta^2}$	$\frac{\sqrt{\alpha^2+\beta^2}}{\beta}\sin(\beta t+\phi), \quad \phi=\arctan\left(\frac{\beta}{\alpha}\right)$
2.29	$\frac{1}{s^2-\alpha^2}$	$\frac{1}{\alpha}\sinh(\alpha t)$
2.30	$\frac{s}{s^2-\alpha^2}$	cosh(at)
2.31	$\arctan\left(\frac{\alpha}{s}\right)$	$\frac{1}{t}\sin(\alpha t)$
2.32	$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$

2.33	$\frac{1}{\sqrt{s+\alpha}}$	$\frac{1}{\sqrt{\pi t}}\exp[-\alpha t]$

2.34	$\frac{1}{\sqrt{s^3}}$	$2\sqrt{\frac{t}{\pi}}$
2.35	$\frac{1}{\sqrt{s^2+\alpha^2}}$	$J_o(\alpha t)$ Bessel function given in Appendix A
2.36	$\frac{1}{(s^2+\alpha^2)^{3/2}}$	$\left(\frac{t}{\alpha}\right)J_1(\alpha t)$
2.37	$\frac{1}{\sqrt{s^2-\alpha^2}}$	$I_o(\alpha t)$ Modified Bessel function given in Appendix A
2.38	$\frac{1}{(s^2-\alpha^2)^{3/2}}$	$\left(\frac{t}{\alpha}\right)I_1(\alpha t)$
2.39	$\sqrt{s-\alpha}-\sqrt{s-\beta}$	$\frac{1}{2t\sqrt{\pi t}}[\exp(\beta t)-\exp(\alpha t)]$

Examples of the Laplace Transform as a Solution for Mechanical Shock and Vibration Problems:

Free Vibration of a Single-Degree-of-Freedom System: free.pdf

Response of a Single-degree-of-freedom System Subjected to a Unit Step Displacement: unit_step.pdf

Response of a Single-degree-of-freedom System Subjected to a Classical Pulse Base Excitation: sbase.pdf

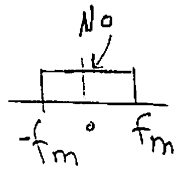
Partial Fractions in Shock and Vibration Analysis: partial.pdf

References

1. Jan Tuma, Engineering Mathematics Handbook, McGraw-Hill, New York, 1979.

2. F. Oberhettinger and L. Badii, Table of Laplace Transforms, Springer-Verlag, N.Y., 1972.

FM SNR



$S_i = \frac{A_c^2}{2}$ watt

$S_o = K_f^2 \frac{A_m^2}{2}$

$\longrightarrow \boxed{\text{FM demod}} \longrightarrow$

$P_i = N_0 f_m$

$P_o = \frac{8\pi^2 N_0 f_m^3}{3 A_c^2}$

$(SNR)_i = \frac{A_c^2}{2 N_0 f_m}$

$SNR_o = \frac{3}{4} \frac{A_c^2 \beta^2}{N_0 f_m}$

$$\boxed{\beta = \frac{K_f A_m}{2\pi f_m}}$$

AM, $\mu = 1$

$S_i = \frac{A_m^2}{2}$

$S_o = \frac{A_c^2}{2}$

$\longrightarrow \boxed{\text{AM demodulator}} \longrightarrow$

$P_i = N_0 f_m$

$P_o = 2 N_0 f_m$

$(SNR)_{o,AM} = \frac{A_c^2}{4 N_0 f_m}$

5.2 sheet 2

6 What is the relation between variance and power for a random signal $x(t)$?

Variance is the sum of the total average normalized power and the DC power.

$$\sigma_x^2 = \overbrace{E\left[x^2(t)\right]}^{\text{total Power}} + \overbrace{E\left[x(t)\right]^2}^{\text{DC power}}$$

For the a signal whose mean is zero,

$$\sigma_x^2 = \overbrace{E\left[x^2(t)\right]}^{\text{total Power}}$$

How to find average, power, PEP, effective value (or the RMS) of a periodic function?

Let $x(t)$ be a periodic function, of period T, then

$$\text{average of } x(t) = \langle x(t)\rangle = \frac{1}{T}\int_0^T x(t)\,dt$$

The average power is

$$p_{av} = \langle x^2(t)\rangle = \frac{1}{T}\int_0^T |x(t)|^2\,dt$$

Effective value, or the RMS value is

$$x_{rms}(t) = \sqrt{\langle x^2(t)\rangle} = \sqrt{p_{av}} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

For example, for $x(t) = \cos(x), \langle x(t)\rangle = 0, P_{av} = \frac{1}{2}, x_{rms}(t) = 0.707$

To find PEP (which is the peak envelope power), find the complex envelope $\tilde{x}(t)$, then find the average power of it. i.e.

$$PEP = \frac{1}{2}\tilde{x}_{\max}^2(t)$$

7 How to derive the Phase and Frequency modulation signals?

For any bandpass signal, we can write it as

$$x(t) = \operatorname{Re}\left(\tilde{x}(t)\, e^{j\omega_c t}\right)$$

Where $\tilde{x}(t)$ is the complex envelope of $x(t)$. For PM and FM, the baseband modulated signal, $\tilde{x}(t)$ has the form $A_c e^{j\theta(t)}$ Hence the above becomes

$$\begin{aligned} x(t) &= \operatorname{Re}\left(A_c e^{j\theta(t)} e^{j\omega_c t}\right) \\ &= A_c\left(\cos\omega_c t\cos\theta(t) - \sin\omega_c t\sin\theta(t)\right) \end{aligned}$$

4

But $\cos(A+B) = \cos A \cos B - \sin A \sin B$, hence the above becomes

$$x(t) = \cos(\omega_c t + \theta(t)) \tag{1}$$

The above is the general form for PM and FM. Now, for PM, $\theta(t) = k_p m(t)$ and for FM, $\theta(t) = k_f \int_0^t m(t_1)\, dt_1$. Hence, substituting in (1) we obtain

$$x_{FM}(t) = \cos\left(\omega_c t + k_f \int_0^t m(t_1)\, dt_1\right)$$

and

$$x_{PM}(t) = \cos(\omega_c t + k_p m(t))$$

I) Amplitude Modulation   Short sheet 443 < SUF

a) • AM wave $s_{AM}(t) = A_c [1 + K_a m(t)] \cos w_c t$

• modulation index $\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$, where A_{max} is the max. of envelope

b) DSB-SC $s(t) = A_c m(t) \cos w_c t$

c) SSB $s(t) = \frac{A_c}{2} m(t) \cos w_c t \mp \frac{A_c}{2} \hat{m}(t) \sin w_c t$

where (−) for USB and (+) for LSB

$\hat{m}(t) = H.T[m(t)] = m(t) \circledast \frac{1}{\pi t}$ or

$\hat{M}(f) = -j \operatorname{sgn}(f) M(f)$

II) PM wave;

$s(t) = A_c \cos(w_c t + K_p m(t))$

III) FM wave:

• $s(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int_0^t m(x)\, dx\right]$ (1)

• If $m(t)$ is a sine or cosine wave for example if $m(t) = A_m \cos W_m t$ then eq (1) becomes single tone modulated signal;

• $s(t) = A_c \cos[2\pi f_c t + \beta \sin W_m t]$, where

• $\beta = \frac{\Delta f}{f_m} = \frac{K_f . A_m}{f_m}$, β is modulation index

• $\Delta f = K_f A_m$ is the freq. deviation

• $f_i(t) = f_c + K_f m(t)$ inst. freq.

• $\theta_i(t) = 2\pi \int_0^t f_i(t_1)\, dt_1$ or $f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$

$\theta_i(t)$ is the inst. phase.

IV) Narrow band noise $n(t)$: Note if $E\{n(t)\} = 0 \Rightarrow E\{n_I\} = E\{n_Q\} = 0$

• $n(t) = n_I(t) \cos w_c t - n_Q(t) \sin w_c t$

• $S_{N_I}(f) = S_{N_Q}(f) = [S_N(f - f_c) + S_N(f + f_c)] \operatorname{rect}\left(\frac{f}{2B}\right)$

where these are p.s.d of the narrowband noise and its in-phase and quadrature components.

• The envelope of $n(t)$ is $a(t) = \sqrt{n_I^2 + n_Q^2}$

Table A11.1 Summary of Properties of the Fourier Transform

Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons \underline{G(f)\exp(-j2\pi f t_0)}$
5. Frequency shifting	$\exp(j2\pi f_c t)g(t) \rightleftharpoons \overline{G(f - f_c)}$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\, dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\, df$
8. Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\, d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f - \lambda)\, d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t - \tau)\, d\tau \rightleftharpoons G_1(f)G_2(f)$

instant. phase

$A_c \cos(\Theta_i(t))$ → phase deviation

$\Theta_i(t) = \omega_c t + \Phi(t)$

$\frac{d\Theta_i}{dt} = \omega_i(t) = \omega_c + \frac{d\Phi}{dt}$

instant. Frequency

Frequency deviation

so inst. Phase $= \omega_c t +$ Phase deviation

inst. Freq $= \omega_c +$ freq. deviation

$\sin(t - \frac{\pi}{2}) = -\cos t$

$\cos(t - \frac{\pi}{2}) = \sin(t)$

Table A11.4 Trigonometric Identities

$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$

$\cos\theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$

$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - \sin^2\theta = \cos(2\theta)$

$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)]$

$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$

$2\sin\theta\cos\theta = \sin(2\theta)$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

$\cos\alpha\sin\beta = -\frac{1}{2}\sin(\alpha - \beta) + \frac{1}{2}\sin(\alpha + \beta)$

$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -J & -1 & J \\ 1 & -1 & 1 & -1 \\ 1 & J & -1 & -J \end{bmatrix}$; $W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}-\frac{i\sqrt{3}}{2} & -\frac{1}{2}+\frac{i\sqrt{3}}{2} \\ 1 & -\frac{1}{2}+\frac{i\sqrt{3}}{2} & -\frac{1}{2}-\frac{i\sqrt{3}}{2} \end{bmatrix}$; $W_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. So if $x[n] = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$, $X(k) = x[n] W_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$

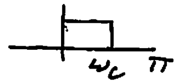
To find IDFT, do $x[n] = \frac{W^*}{4} X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & J & -1 & -J \\ 1 & -1 & 1 & -1 \\ 1 & -J & -1 & J \end{bmatrix} \begin{bmatrix} \ \end{bmatrix}$

Properties of DFT $X(k)$ periodic, period $= N$, Linear: $a_1 x_1 + a_2 x_2 \xleftrightarrow{DFT} a_1 X_1 + a_2 X_2$

$x_p(n) = \sum_{\ell=-\infty}^{\infty} x[n-\ell N]$. $\boxed{X_1 X_2 = \text{circular convolution of } x_1, x_2}$

$\boxed{W_N^k} = e^{-J\frac{2\pi k}{N}}$

ideal low pass



$\rightarrow h(n) = \begin{cases} \frac{\omega_c}{\pi} & n=0 \\ \frac{\omega_c}{\pi} \frac{\sin \omega_c n}{\omega_c n} & n \neq 0 \end{cases}$

LTI is causal if specified by difference equation $y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$

$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$. To do circular convolution, do same as linear convolution, just shift right!

$\Omega = 2\pi F \rightarrow$ cycle/sec. $\qquad$ input signal $\qquad \omega = 2\pi f n$

continuous $x(t)$ $\qquad$ energy $\qquad E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(F)|^2 dF$ $\qquad$ discrete $x[n]$

Power

periodic(T): CTFS (Fourier Series)

$c(k) = \frac{1}{T}\int_T x(t) e^{-jw_0 kt} dt$

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jw_0 kt}$

$\boxed{P_x = \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k|^2}$

aperiodic (Transform)

$X(\Omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\Omega t} dt$

$x(t) = \frac{1}{2\pi}\int_{\Omega=-\infty}^{\infty} X(\Omega) e^{j\omega t} d\Omega$

$\boxed{\text{if } x(t) \text{ real then } c(-k) = c^*(k)}$

periodic T. DTFS

$C[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}n}$

$x[n] = \sum_{k=0}^{N-1} c[k] e^{j\frac{2\pi k}{N}n}$

$E_N = \sum_{n=0}^{N-1} |x[n]|^2 = N\sum_{k=0}^{N-1} |c_k|^2$

aperiodic

$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega) e^{j\omega n} d\omega$

$E_x = \sum |x[n]|^2 = \frac{1}{2\pi}\int |X(\omega)|^2 d\omega$

$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}}$

DFT

$$\boxed{\begin{aligned} X[k] &= \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}n} \qquad k=0,1,\dots N-1 \\ x[n] &= \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\left(\frac{2\pi k}{N}\right)n} \qquad n=0,1,\dots N-1 \end{aligned}}$$

properties of Z transform

$u(n) \xleftrightarrow{z} \frac{1}{1-z^{-1}}$ ROC $|z|>1$

$u(-n) \xleftrightarrow{z} \frac{1}{1-z}$ ROC $|z|<1$

$n x(n) \xleftrightarrow{z} -z\frac{dX(z)}{dz}$

$a^n u(n) \xleftrightarrow{z} \frac{1}{1-az^{-1}}$

$a^n x(n) \xleftrightarrow{z} X(a^{-1}z)$

$\boxed{x(-n) = X(z^{-1})}$ so find $X(z)$ for $x(n)$ and replace z by z^{-1}

$x^*(n) \xleftrightarrow{z} X^*(z^*)$

If $y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$

Then $H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$

Correlation

$r_{x_1x_2}(\ell) = \sum_{n=-\infty}^{\infty} x_1(n) x_2(n-\ell) \xleftrightarrow{z} X_1(z) X_2(z^{-1})$

$r_{x_1x_2}(\ell) = x_1(\ell) * x_2(-\ell)$ (so correlation is convolution but without flipping)

Time		Frequency
Real, even	$\longleftrightarrow$	Real, even
Real, odd	$\longleftrightarrow$	Imaginary, odd
Im, even	$\longleftrightarrow$	Imaginary, even
Im, odd	$\longleftrightarrow$	Real, odd

$\delta(n) \xleftrightarrow{z} 1$ All z

$u(n) \xleftrightarrow{z} \frac{1}{1-z^{-1}}$ $|z|>1$

$a^n u(n) \xleftrightarrow{z} \frac{1}{1-az^{-1}}$ $|z|>a$

$\sum_{n=0}^{\infty} (a)^n = \frac{1}{1-a}$ $\quad |a|<1$

$\sum_{n=0}^{N} (a)^n = \frac{1-a^{N+1}}{1-a}$ or $\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$ $\quad a \neq 1$

Symmetry relationships for $X(\omega)$

$x^*(n) \longleftrightarrow X^*(-\omega)$

$x^*(-n) \longleftrightarrow X^*(\omega)$

If $x[n]$ is real then:

$X(\omega) = X^*(-\omega)$

$|X(\omega)| = |X(-\omega)|$

convolution

$y(n) = \sum_{k=-\infty}^{k=\infty} x(k) h(n-k)$

IIR $\Rightarrow y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$

$H(z) = \sum_{k=0}^{M} b_k z^{-k} \Big/ 1+\sum_{k=1}^{N} a_k z^{-k}$

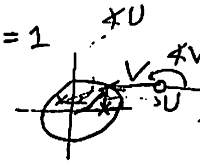
FIR = all zero $\Rightarrow y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$

$H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$

$$H(z)=\frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = b_0 \frac{\prod_{k=1}^{M}(1-z_k z^{-1})}{\prod_{k=1}^{N}(1-p_k z^{-1})} = \frac{(z-z_1)(z-z_2)\cdots}{(z-p_1)(z-p_2)\cdots}$$

b_0 selected such that $|H(\omega_0)|=1$

$$|H(\omega)| = |b_0| \frac{|V_1(\omega)||V_2(\omega)|\cdots|V_M(\omega)|}{|U_1(\omega)||U_2(\omega)|\cdots|U_N(\omega)|}$$



$\boxed{\measuredangle H(\omega) = \measuredangle b_0 + \omega(N-M) + \measuredangle V_1 + \measuredangle V_2 + \cdots - [\measuredangle U_1 + \measuredangle U_2 + \cdots]}$

$$|H(\omega)|^2 = H(\omega)H^*(\omega) = H(\omega)H(-\omega) = H(z)H(z^{-1})\Big|_{z=e^{j\omega}}$$

Real a_k, b_k.

Properties of DFT $X(N-k)=X^*(k)=X(-k)$ for real $x[n]$, $|X[N-k]| = |X[k]|$, $\measuredangle X[N-k] = -\measuredangle X[k]$

if $x[n]$ real & even, then $X[k]=\sum_{0}^{N-1} x[n]\cos\frac{2\pi k}{N}n$, $x[n]=\frac{1}{N}\sum X(k)\cos\frac{2\pi k}{N}n$

if $x[n]$ is real & odd, then $X[k]=\sum_{0}^{N-1}\left(-j\sin\frac{2\pi k}{N}n\right)$, $x[n]=j\frac{1}{N}\sum X(k)\sin\frac{2\pi k}{N}n$

$x[N-n]=X[N-k]$, $x((n-\ell))_N \longleftrightarrow X(k)e^{-j\frac{2\pi k}{N}\ell}$; $x[n]e^{j\frac{2\pi \ell}{N}n} \longleftrightarrow X((k-\ell))_N$

$x^*[n] = X^*[N-k]$

$\tilde{r}_{xy}(\ell) = x(\ell) \circledN y^*(-\ell)$ ← circular correlation $\qquad x_1 x_2 \longleftrightarrow \frac{1}{N} X_1 \circledN X_2$

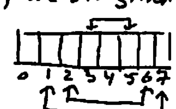
given $x[n]$ of length L, $h[n]$ of length M, How to know Length of DFT? $N=L+M-1$

so if x has $L=4$, length $h[n]=3$, then we need $N=6$

note we can $X_1 X_2$ to find X_3, Then IDFT to find response of system.
like Linear convolution, as long as we padd sequences.

$$\int_0^\infty e^{-t(1+j2\pi F)}dt = \frac{1}{1+j2\pi F}$$

if we are given some points of DFT, we can find rest using properties $\boxed{X(N-K)=X^*(K)=X(-K)}$



$N=8$. $X(6)=X(8-2)=X^*(2)$

for DFT when input is sin/cos, use

$$\sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(k-1)} = N\delta(k-1) \qquad \cos\alpha = \frac{e^{j\alpha}+e^{-j\alpha}}{2}$$

$$\sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(k+1)} = N\delta(k+1) \qquad \sin\alpha = \frac{e^{j\alpha}-e^{-j\alpha}}{2j}$$

so if $x(n)=\cos\frac{2\pi}{N}n \to DFT = \frac{N}{2}[\delta(k-1)+\delta(k+1)]$

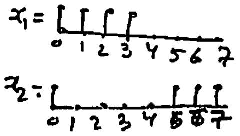
$\ell(n)=\sin\frac{2\pi}{N}n \to DFT=\frac{N}{2j}[\delta(k-1)+\delta(k+1)]$

if $X(k)=\frac{N^2}{4j}[\delta(k-1)+\delta(k+1)] \Rightarrow x(n)=\frac{N}{2}\sin\left(\frac{2\pi}{N}n\right)$

DFT if given sequence $x(n)$ and asked to find its Energy do

$E=\sum_{n=0}^{N-1} x(n)x^*(n)$. Example. $x(n)=\cos\frac{2\pi k}{N}n \Rightarrow x(n)x^*(n)=\frac{1}{4}\left(2+e^{+j\frac{4\pi k n}{N}}+e^{-j\frac{4\pi k n}{N}}\right)$

so $E=\sum_{n=0}^{N-1} x(n)x^*(n) = \frac{1}{4}2N = \frac{N}{2}$.



$x_2 = x_1((n-5))_8 \Rightarrow X_2 = X_1 e^{-j\frac{2\pi 5k}{N}}$ delay

const space between.

Least square method for design
IIR & FIR

Properties of the Fourier Transform

Property	$f(t)$	$F(\omega)$
Linearity (Superposition)	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Time Shifting	$f(t - t_d)$	$e^{-j\omega t_d} F(\omega)$
Time Scaling	$f(ct)$	$\frac{1}{\lvert c\rvert} F\left(\frac{\omega}{c}\right)$
Symmetry (Duality)	$F(t)$	$2\pi f(-\omega)$
Time Reversal	$f(-t)$	$F(-\omega)$
Frequency Scaling	$f(t)e^{j\omega_c t}$	$F(\omega - \omega_c)$
Modulation	$f(t)cos(\omega_c t)$	$\frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c)$
Time Differentiation	$\frac{df(t)}{dt}$	$j\omega F(\omega)$
Frequency Differentiation	$tf(t)$	$j\frac{dF(\omega)}{d\omega}$
Conjugate	$f^*(t)$	$F^*(-\omega)$
Integration	$\int_{-\infty}^{t} f(\lambda)d\lambda$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
Convolution	$\int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$	$H(\omega)X(\omega)$
Multiplication	$f_1(t)f_2(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(v)F_2(\omega - v)dv$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert f(t)\rvert^2 dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2 d\omega$

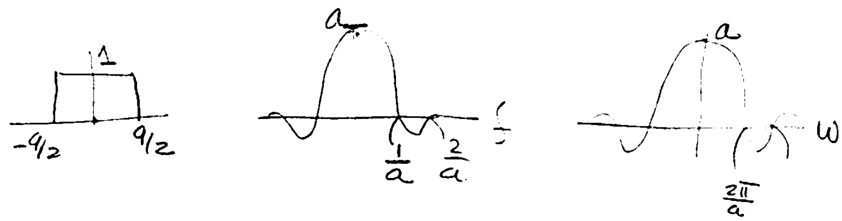
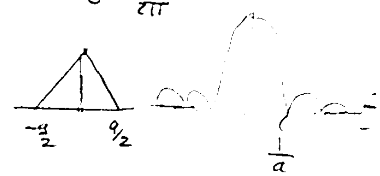


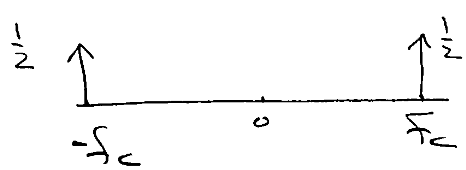
Table 16.2 Fourier Transform Pairs (a > 0)

$\omega = 2\pi f$

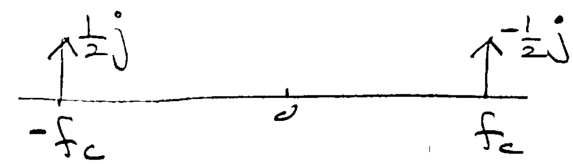
$f = \frac{\omega}{2\pi}$

$f(t)$	$F(\omega)$	$F(f)$
$\Pi\left(\frac{t}{a}\right) = rect\left(\frac{t}{a}\right)$	$a\,\text{sinc}\left(\frac{\omega a}{2\pi}\right)$	$a\,\text{sinc}(fa)$
$\Lambda\left(\frac{t}{a}\right) = tri\left(\frac{t}{a}\right)$	$a\,\text{sinc}^2\left(\frac{\omega a}{2\pi}\right)$	$a\,\text{sinc}^2(fa)$
$e^{-at}u(t)$	$\frac{1}{j\omega + a}$	$\frac{1}{j2\pi f + a}$
$e^{at}u(-t)$	$\frac{1}{-j\omega + a}$	$\frac{1}{-j2\pi f + a}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{\omega^2 + a^2}$	$\frac{2a}{4\pi^2 f^2 + a^2}$
$e^{-at}u(t) - e^{at}u(-t)$	$\frac{-2j\omega}{\omega^2 + a^2}$	$\frac{-j4\pi f}{4\pi^2 f^2 + a^2}$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$tu(t)$	$\frac{\pi}{j\omega}\delta(\omega) + \frac{1}{(j\omega)^2}$	$\frac{1}{j4\pi f}\delta(f) + \frac{1}{(j2\pi f)^2}$
$te^{-at}u(t)$	$\frac{1}{(j\omega + a)^2}$	$\frac{1}{(j2\pi f + a)^2}$
$\cos(\omega_c t) = \cos(2\pi f_c t)$	$\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$

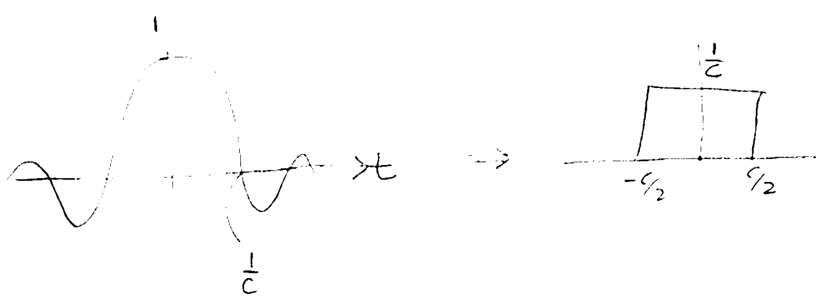




$\sin 2\pi f_c t$



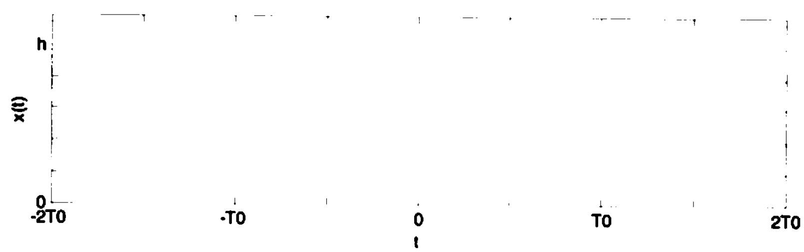
$\sin(\omega_c t) = \sin(2\pi f_c t)$	$-j\pi[\delta(\omega-\omega_c)-\delta(\omega+\omega_c)]$	$\frac{-j}{2}[\delta(f-f_c)-\delta(f+f_c)]$
$e^{-at}u(t)\cos(\omega_c t)$	$\frac{j\omega+a}{(j\omega+a)^2+\omega_c^2}$	$\frac{j2\pi f+a}{(j2\pi f+a)^2+(2\pi f_c)^2}$
$e^{-at}u(t)\sin(\omega_c t)$	$\frac{\omega_c}{(j\omega+a)^2+\omega_c^2}$	$\frac{2\pi f_c}{(j2\pi f+a)^2+(2\pi f_c)^2}$
$sgn(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$\text{sinc}(ct)$	$\frac{1}{c}rect\left(\frac{\omega}{2\pi c}\right)$	$\frac{1}{c}rect\left(\frac{f}{c}\right)$
$\text{sinc}^2(ct)$	$\frac{1}{c}tri\left(\frac{\omega}{2\pi c}\right)$	$\frac{1}{c}tri\left(\frac{f}{c}\right)$
$\cos\left(\frac{\pi t}{a}\right)rect\left(\frac{t}{a}\right)$	$\frac{2a}{\pi}\frac{\cos\left(\frac{\omega a}{2}\right)}{1-\left(\frac{\omega a}{\pi}\right)^2}$	$\frac{2a}{\pi}\frac{\cos(\pi af)}{1-(2af)^2}$
$\frac{1}{2}\left[1+\cos\left(\frac{\pi t}{a}\right)\right]rect\left(\frac{t}{2a}\right)$	$a\frac{\sin(\omega a)}{\omega a\left[1-\left(\frac{\omega a}{\pi}\right)^2\right]}$	$a\frac{\sin(2\pi fa)}{2\pi fa\left[1-(2af)^2\right]}$



$\sin(\pi ct) = \pi K$
$t = \frac{k}{c}$

FOURIER SERIES REPRESENTATION OF COMMON SIGNALS

Rectangular Pulse Train



τ = pulse width ($-\tau/2$ to $\tau/2$)
d = duty cycle = τ/T_0.
ω_0 = fundamental frequency = $2\pi/T_0$
sinc(x) = $\sin(\pi x)/(\pi x)$

$$X_n = \frac{ha}{T_0}\operatorname{sinc}(nd) = h\,d\operatorname{sinc}(nd) = \begin{cases} hd, & n=0 \\ h\dfrac{\sin(n\pi d)}{n\pi}, & n \neq 0 \end{cases}$$

$$x(t) = hd + \sum_{n=1}^{\infty} 2hd\operatorname{sinc}(nd)\cos(n\omega_0 t)$$

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$$c_0 = hd = \frac{h\tau}{T_0}, \quad c_n = |2hd\operatorname{sinc}(nd)|, \quad \theta_n = \begin{cases} \pi, & 2hd\operatorname{sinc}(nd) < 0 \\ 0, & otherwise \end{cases}$$

If $\tau = T_0/2$, d = 1/2, and the equations given above becomes

$$X_n = \frac{h}{2}\operatorname{sinc}\left(\frac{n}{2}\right) = \begin{cases} \dfrac{h}{2}, & n=0 \\ h\dfrac{\sin\left(\dfrac{n\pi}{2}\right)}{n\pi}, & n \neq 0 \end{cases}$$

$$x(t) = \frac{h}{2} + \sum_{n=1}^{\infty} h\operatorname{sinc}\left(\frac{n}{2}\right)\cos(n\omega_0 t)$$

1

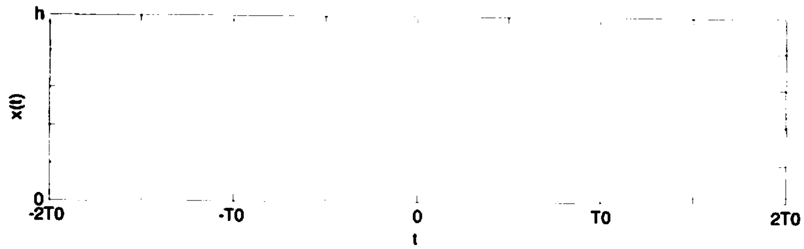
$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$$c_0 = \frac{h}{2}, \quad c_n = \left| h \operatorname{sinc}\left(\frac{n}{2}\right) \right|, \quad \theta_n = \begin{cases} \pi, & h \operatorname{sinc}\left(\frac{n}{2}\right) < 0 \\ 0, & \textit{otherwise} \end{cases}$$

Let $y(t) = x(t - T_0/2)$. Then,

$$Y_n = X_n e^{-jn\frac{2\pi}{T_0}\frac{T_0}{2}} = X_n e^{-jn\pi} = X_n \cos(n\pi) = \begin{cases} \frac{h}{2}, & n = 0 \\ h\frac{(-1)^n \sin\left(\frac{n\pi}{2}\right)}{n\pi}, & n \neq 0 \end{cases}$$

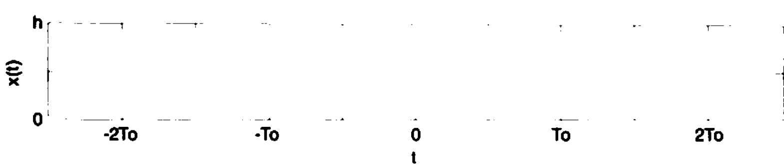
Triangular Pulse Train



τ = half of the base of the triangle ($-\tau \le t \le \tau$)
d = duty cycle = τ/T_0.
ω_0 = fundamental frequency = $2\pi/T_0$

$$X_n = hd \operatorname{sinc}^2(nd) = \frac{h\tau}{T_0} \operatorname{sinc}^2\left(\frac{n\omega_0 \tau}{2\pi}\right)$$

If $\tau = T_0/2$, then the pulse train looks like

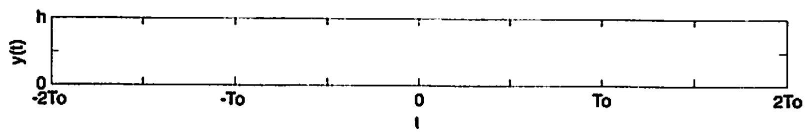


and

2

$$X_n = \frac{h}{2}\text{sinc}^2\left(\frac{n}{2}\right) = \begin{cases} \frac{h}{2}, & n = 0 \\ 0, & n = even \\ \frac{2h}{n^2\pi^2}, & n = odd \end{cases}$$

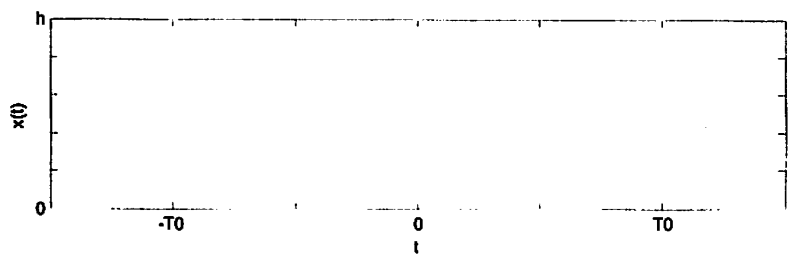
Let y(t) = x(t − T_0/2).



Then,

$$Y_n = X_n e^{-jn\frac{2\pi}{T_0}\frac{T_0}{2}} = X_n e^{-jn\pi} = X_n \cos(n\pi) = \begin{cases} \frac{h}{2}, & n = 0 \\ 0, & n = even \\ \frac{-2h}{n^2\pi^2}, & n = odd \end{cases}$$

Half-Wave Rectified Cosine



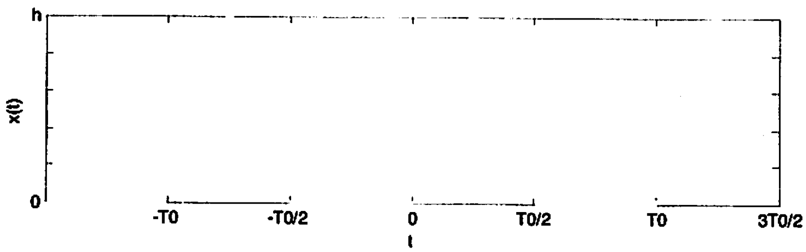
$$X_1 = \frac{h}{4}$$

$$X_{-1} = \frac{h}{4}$$

$$X_n = \frac{h}{\pi}\frac{\cos\left(\frac{n\pi}{2}\right)}{1-n^2}, \quad n \neq \pm 1$$

Half-Wave Rectified Sine

3



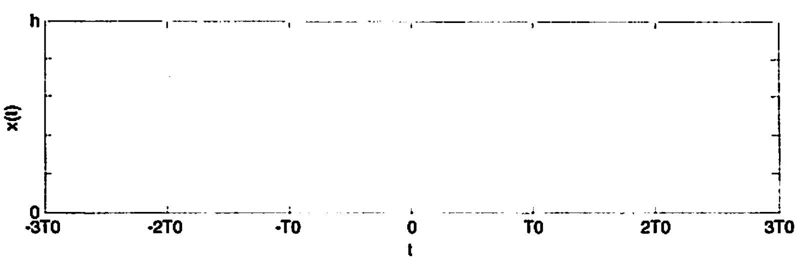
h = amplitude, $j=\sqrt{-1}$

$$X_1=\frac{-jh}{4}$$

$$X_{-1}=\frac{jh}{4}$$

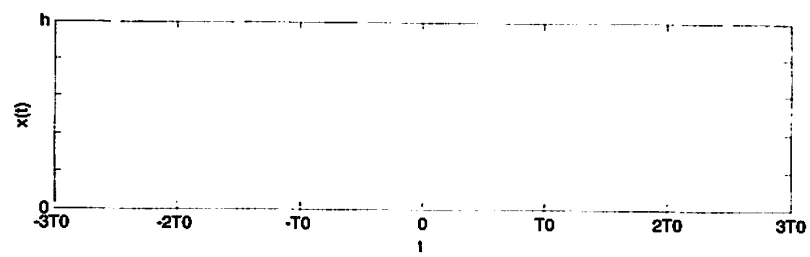
$$X_n=\frac{h}{\pi}\frac{\cos^2\left(\frac{n\pi}{2}\right)}{1-n^2}=\begin{cases}\frac{h}{\pi}, & n=0\\ 0, & n=\pm3,\pm5,\pm7,..........\\ \frac{h}{\pi}\frac{1}{1-n^2}, & n=\pm2,\pm4,\pm6,..........\end{cases}$$

Full-Wave Rectified Cosine



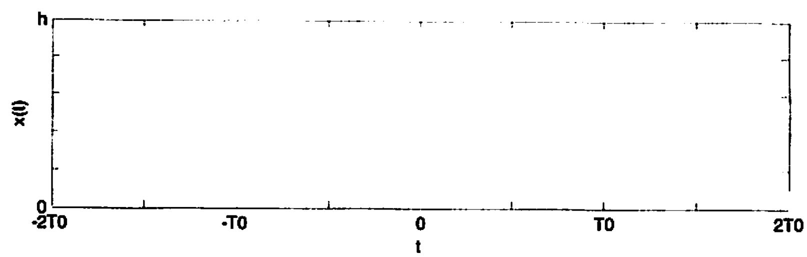
$$X_n=\frac{2h}{\pi}\frac{\cos(n\pi)}{1-4n^2}=\frac{2h}{\pi}\frac{(-1)^n}{1-4n^2}$$

Full-Wave Rectified Sine



$$X_n = \frac{2h}{\pi}\frac{\cos^2(n\pi)}{1-4n^2} = \frac{2h}{\pi}\frac{1}{1-4n^2}$$

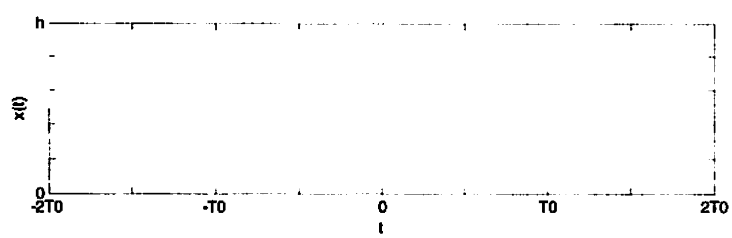
Sawtooth



$$X_0 = \frac{h}{2}$$

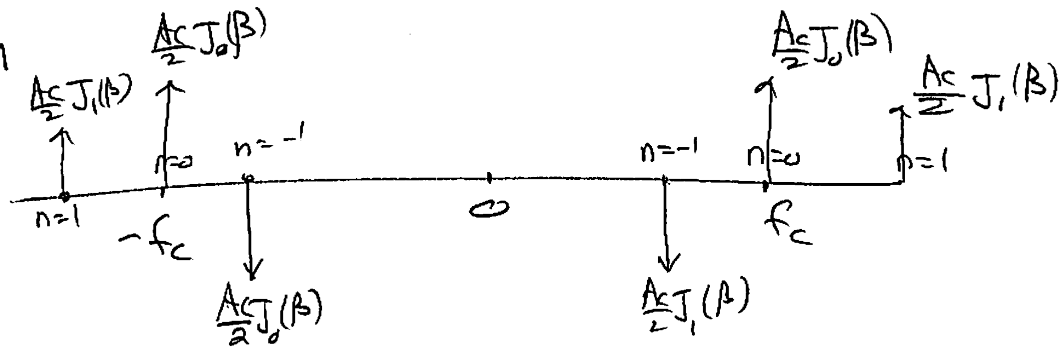
$$X_n = \frac{jh}{2\pi n}, \quad n \neq 0, \quad j = \sqrt{-1}$$

Exponential Decay



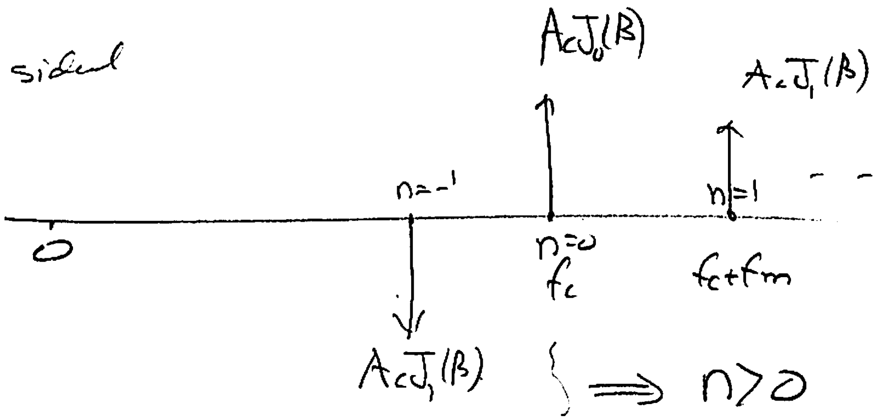
$$X_n = \frac{h}{T_0}\frac{1-e^{-aT_0}}{a+jn\frac{2\pi}{T_0}}$$

FM



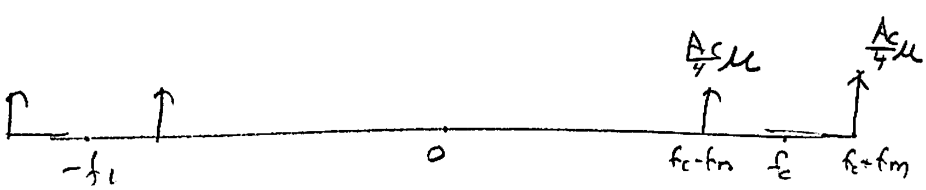
$\Rightarrow$ 2 sided.

one sided

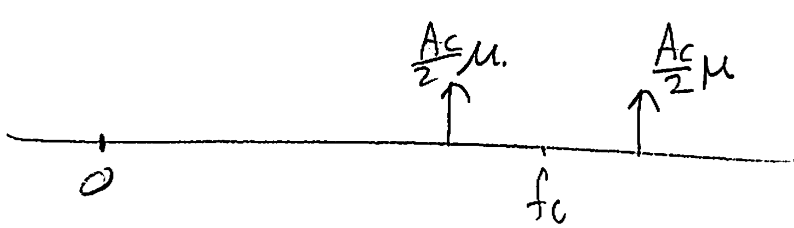


AM

2 sided.



one sided



FM

$$FM(t) = A_c \cos\Big(\overbrace{\omega_c t + \underbrace{k_f \int m(\lambda)\,d\lambda}_{\phi(t)\ \text{phase deviation}}}^{\theta_i(t)}\Big)$$

Frequency deviation $= \frac{d}{dt}\phi(t)$

$= k_f\, m(t)$

max frequency deviation $= \max |k_f m(t)|$

$\Delta\omega = k_f \max |m(t)|$

$\omega_i(t)$ = instant. frequency

$$= \frac{d}{dt}\theta_i(t) = \omega_c + \overbrace{k_f m(t)}^{\phi'(t)}$$

PM

$$PM(t) = A_c \cos\Big(\overbrace{\omega_c t + \underbrace{k_p m(t)}_{\phi(t)}}^{\theta_i(t)}\Big)$$

$\phi(t)$ = phase deviation

Frequency deviation: $\frac{d}{dt}\phi(t) = k_p \frac{d}{dt} m(t)$.

max frequency deviation $= k_p \max \left|\frac{dm}{dt}\right|$

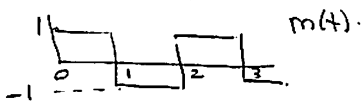
$$\omega_i(t) = \frac{d}{dt}\theta_i(t) = \omega_c + k_p \frac{dm}{dt}$$

---

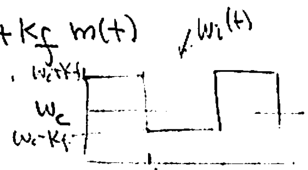
to plot FM(t), do

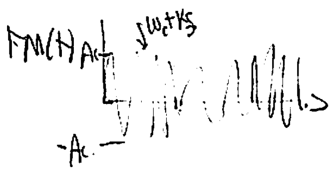
Find $\omega_i(t) = \omega_c + k_f\, m(t)$.

example



$\omega_i(t) = \omega_c + k_f\, m(t)$

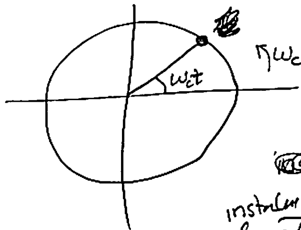




5.3 sheet 3

PM modulation

time = t

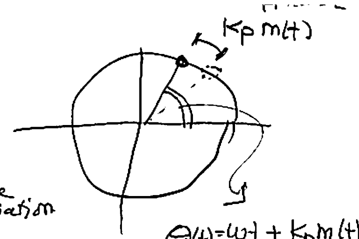


perturbe angle $\Rightarrow$

$\omega_c t \Rightarrow \omega_c t + \underbrace{k_p m(t)}_{\text{phase deviation}}$

so radial frequency of the generalized sinusoid is

instant freq $\frac{d}{dt}\theta_g(t) = \omega_c + \underbrace{k_p \dot{m}(t)}_{\text{frequency deviation for PM}}$



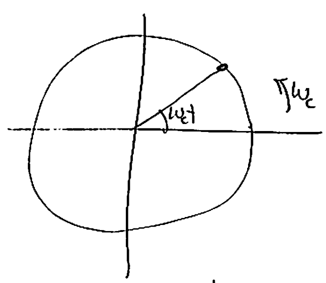
$\theta_g(t) = \omega_c t + k_p m(t)$

so $\varphi(t) = \cos \theta_g(t)$

$= \cos(\omega_c t + \ldots)$

FM

time = t

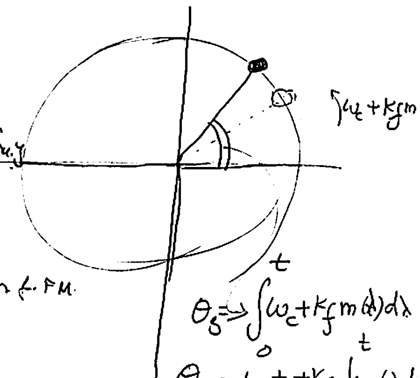


perturbe ω_c

$\longrightarrow$ instantious freq

$\omega_c \rightarrow \omega_c + \underbrace{k_f m(t)}_{\text{called frequency deviation for FM}}$

time = t



$\theta_g \Rightarrow \int_0^t \omega_c + k_f m(\lambda) d\lambda$

$\theta_g = \omega_c t + k_f \int_0^t m(\lambda) d\lambda$

hence the radial freq of generalized sinusoidal is

$\omega_c + k_f m(t)$

and the phase is

$\omega_c t + k_f \int m(\lambda) d\lambda$

instantous freq