4 Convolution

Recommended Problems

Discussion 2 problems 4.1, 4.2 & 4.3 (if Time allows)

P4.1

This problem is a simple example of the use of superposition. Suppose that a discrete-time linear system has outputs $y[n]$ for the given inputs $x[n]$ as shown in Figure P4.1-1.

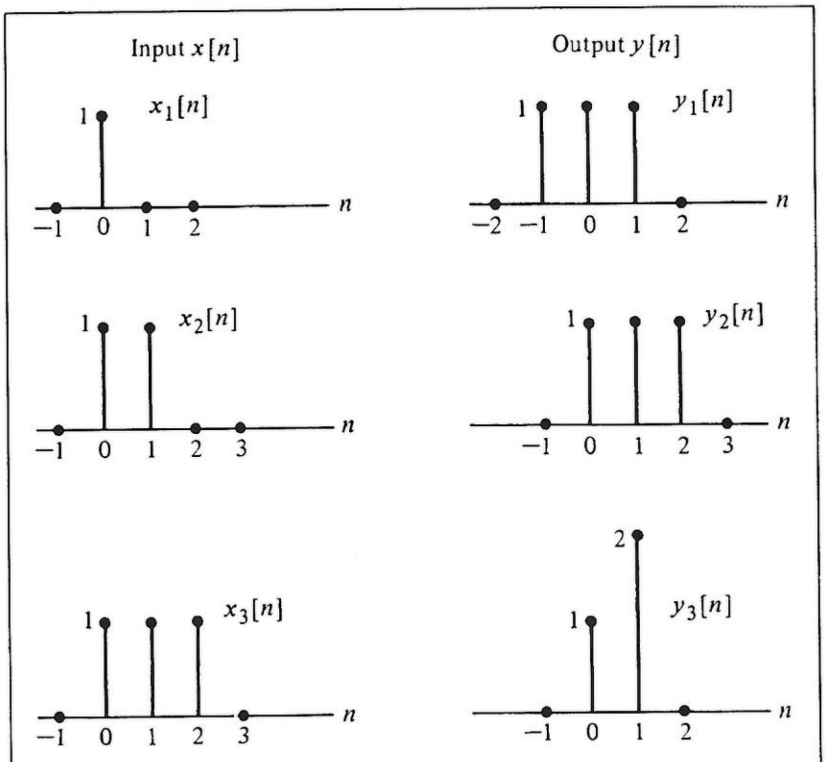

Figure P4.1-1

Determine the response $y_4[n]$ when the input is as shown in Figure P4.1-2.

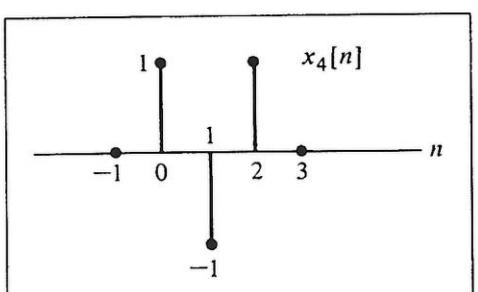

Figure P4.1-2

(a) Express $x_4[n]$ as a linear combination of $x_1[n]$, $x_2[n]$, and $x_3[n]$.

(b) Using the fact that the system is linear, determine $y_4[n]$, the response to $x_4[n]$.

(c) From the input-output pairs in Figure P4.1-1, determine whether the system is time-invariant.

P4.2

Determine the discrete-time convolution of $x[n]$ and $h[n]$ for the following two cases.

(a)

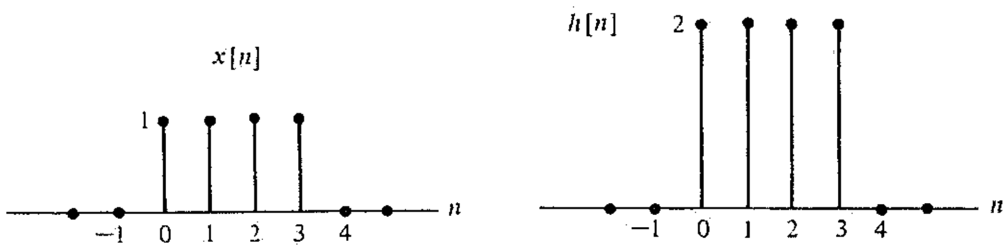

Figure P4.2-1

(b)

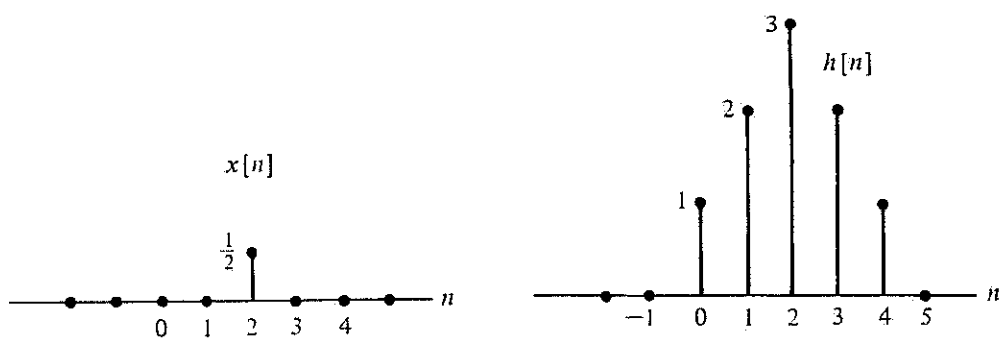

Figure P4.2-2

P4.3

Determine the continuous-time convolution of $x(t)$ and $h(t)$ for the following three cases:

(a)

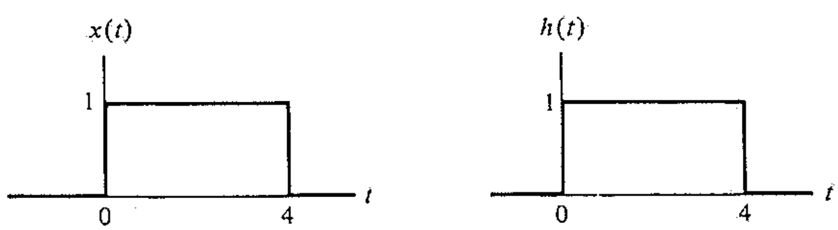

Figure P4.3-1

(b)

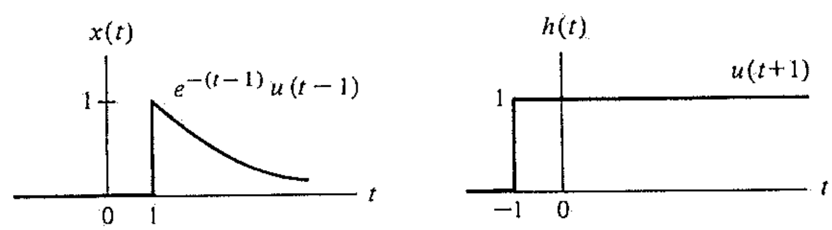

Figure P4.3-2

(c)

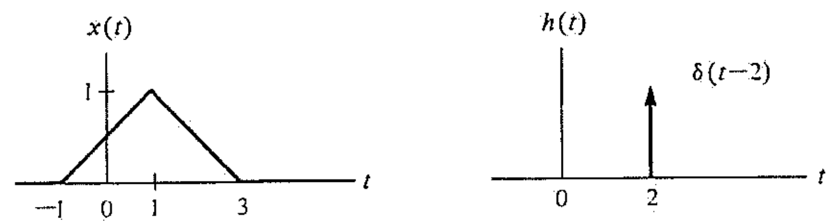

Figure P4.3-3

P4.4

Consider a discrete-time, linear, shift-invariant system that has unit sample response $h[n]$ and input $x[n]$.

(a) Sketch the response of this system if $x[n] = \delta[n - n_0]$, for some $n_0 > 0$, and $h[n] = (\frac{1}{2})^n u[n]$.

(b) Evaluate and sketch the output of the system if $h[n] = (\frac{1}{2})^n u[n]$ and $x[n] = u[n]$.

(c) Consider reversing the role of the input and system response in part (b). That is,

$$h[n] = u[n],$$
$$x[n] = (\tfrac{1}{2})^n u[n]$$

Evaluate the system output $y[n]$ and sketch.

P4.5

(a) Using convolution, determine and sketch the responses of a linear, time-invariant system with impulse response $h(t) = e^{-t/2}\,u(t)$ to each of the two inputs $x_1(t)$, $x_2(t)$ shown in Figures P4.5-1 and P4.5-2. Use $y_1(t)$ to denote the response to $x_1(t)$ and use $y_2(t)$ to denote the response to $x_2(t)$.