# 4 Convolution

## Solutions to Recommended Problems

Discussion 2 Solution

### S4.1

The given input in Figure S4.1-1 can be expressed as linear combinations of $x_1[n]$, $x_2[n]$, $x_3[n]$.

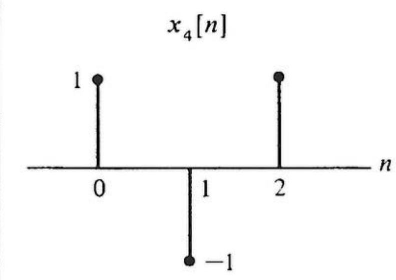


**Figure S4.1-1**

**(a)** $x_4[n] = 2x_1[n] - 2x_2[n] + x_3[n]$

**(b)** Using superposition, $y_4[n] = 2y_1[n] - 2y_2[n] + y_3[n]$, shown in Figure S4.1-2.

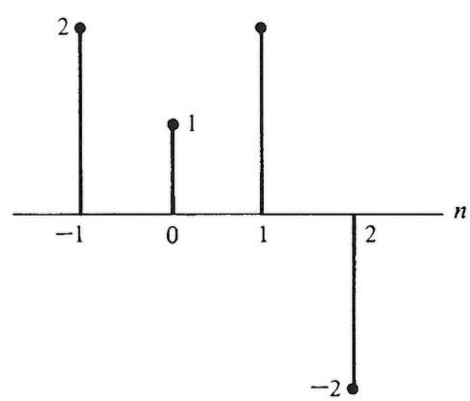


**Figure S4.1-2**

**(c)** The system is not time-invariant because an input $x_1[n] + x_1[n-1]$ does not produce an output $y_1[n] + y_1[n-1]$. The input $x_1[n] + x_1[n-1]$ is $x_1[n] + x_1[n-1] = x_2[n]$ (shown in Figure S4.1-3), which we are told produces $y_2[n]$. Since $y_2[n] \neq y_1[n] + y_1[n-1]$, this system is not time-invariant.

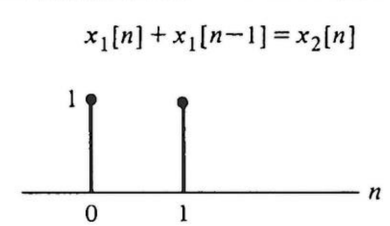


**Figure S4.1-3**

## S4.2

The required convolutions are most easily done graphically by reflecting $x[n]$ about the origin and shifting the reflected signal.

**(a)** By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure S4.2-1.

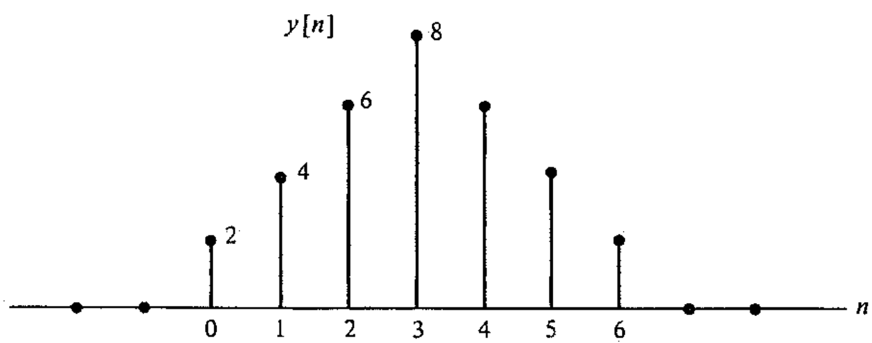


**Figure S4.2-1**

**(b)** By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure S4.2-2.

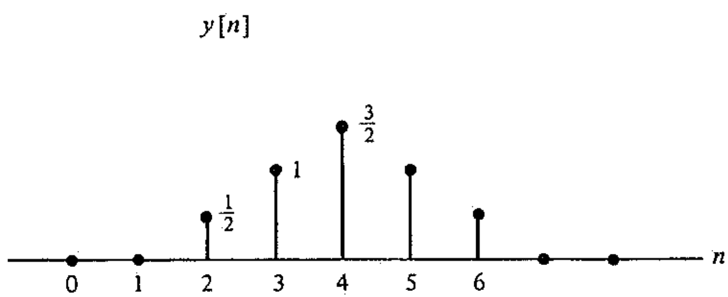


**Figure S4.2-2**

Notice that $y[n]$ is a shifted and scaled version of $h[n]$.

## S4.3

**(a)** It is easiest to perform this convolution graphically. The result is shown in Figure S4.3-1.

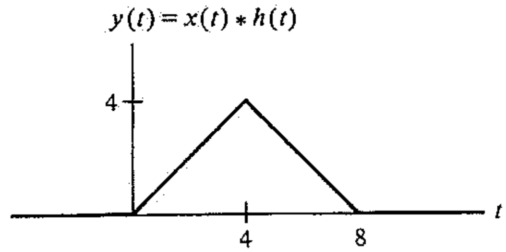


**Figure S4.3-1**

**(b)** The convolution can be evaluated by using the convolution formula. The limits can be verified by graphically visualizing the convolution.

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} e^{-(\tau-1)}u(\tau-1)u(t-\tau+1)d\tau \\ &= \begin{cases} \int_{1}^{t+1} e^{-(\tau-1)}d\tau, & t > 0, \\ 0, & t < 0, \end{cases} \end{aligned}$$

Let $\tau' = \tau - 1$. Then

$$y(t) = \begin{cases} \int_{0}^{t} e^{-\tau'}d\tau' \\ 0 \end{cases} = \begin{cases} 1 - e^{-t}, & t > 0, \\ 0, & t < 0 \end{cases}$$

**(c)** The convolution can be evaluated graphically or by using the convolution formula.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau-2)\, d\tau = x(t-2)$$

So $y(t)$ is a shifted version of $x(t)$.

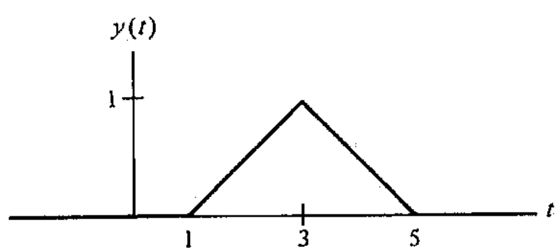


**Figure S4.3-2**