discussion week 2

EE 3015
Signals and Systems

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University of Minnesota, Twin Cities

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Contents

1 Questions

4 Convolution

Recommended Problems Discussion 2 problems 4.1, 4.2 & 4.3 (if Time allows)

P4.1

This problem is a simple example of the use of superposition. Suppose that a discrete-time linear system has outputs $y[n]$ for the given inputs $x[n]$ as shown in Figure P4.1-1.

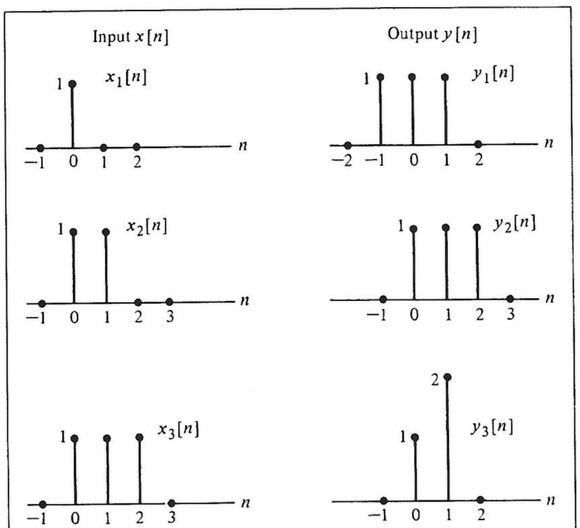

Figure P4.1-1

Determine the response $y_4[n]$ when the input is as shown in Figure P4.1-2.

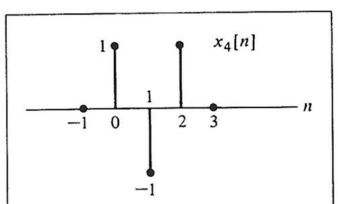

Figure P4.1-2

(a) Express $x_4[n]$ as a linear combination of $x_1[n]$, $x_2[n]$, and $x_3[n]$.

(b) Using the fact that the system is linear, determine $y_4[n]$, the response to $x_4[n]$.

(c) From the input-output pairs in Figure P4.1-1, determine whether the system is time-invariant.

P4.2

Determine the discrete-time convolution of $x[n]$ and $h[n]$ for the following two cases.

(a)

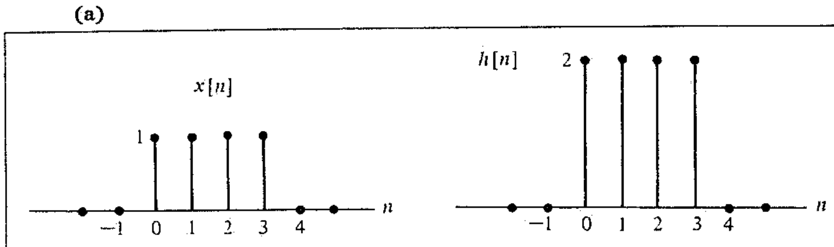

Figure P4.2-1

(b)

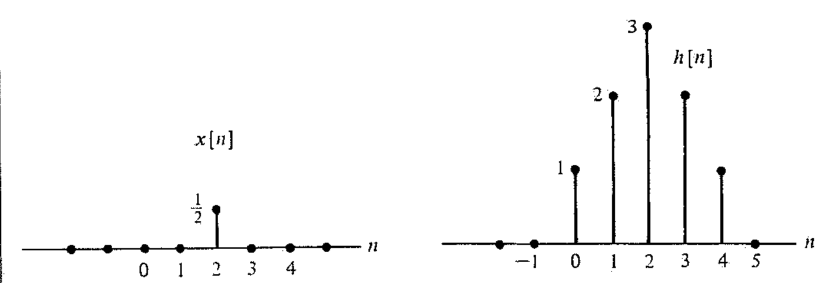

Figure P4.2-2

P4.3

Determine the continuous-time convolution of $x(t)$ and $h(t)$ for the following three cases:

(a)

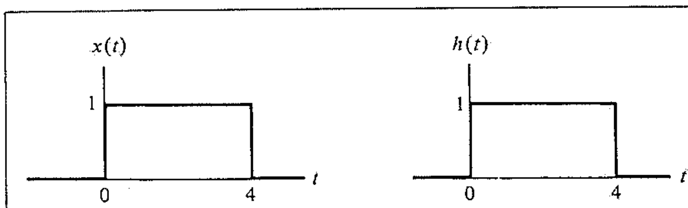

Figure P4.3-1

(b)

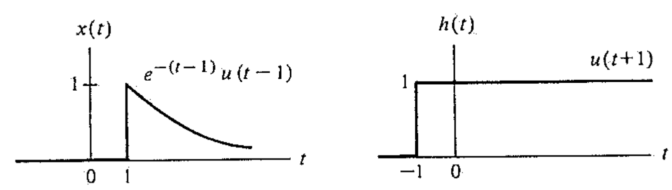

Figure P4.3-2

(c)

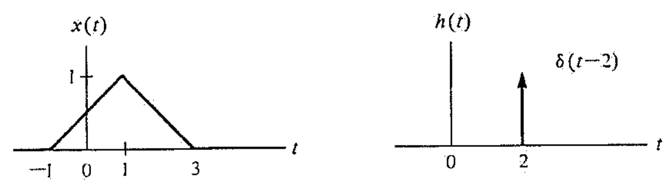

Figure P4.3-3

P4.4

Consider a discrete-time, linear, shift-invariant system that has unit sample response $h[n]$ and input $x[n]$.

(a) Sketch the response of this system if $x[n] = \delta[n - n_0]$, for some $n_0 > 0$, and $h[n] = (\frac{1}{2})^n u[n]$.

(b) Evaluate and sketch the output of the system if $h[n] = (\frac{1}{2})^n u[n]$ and $x[n] = u[n]$.

(c) Consider reversing the role of the input and system response in part (b). That is,

$$h[n] = u[n],$$
$$x[n] = (\tfrac{1}{2})^n u[n]$$

Evaluate the system output $y[n]$ and sketch.

P4.5

(a) Using convolution, determine and sketch the responses of a linear, time-invariant system with impulse response $h(t) = e^{-t/2}u(t)$ to each of the two inputs $x_1(t)$, $x_2(t)$ shown in Figures P4.5-1 and P4.5-2. Use $y_1(t)$ to denote the response to $x_1(t)$ and use $y_2(t)$ to denote the response to $x_2(t)$.

2 Problem 4.1

Solution

2.1 part a

We need to find linear combination of $x_1[n], x_2[n], x_3[n]$ which gives $x_4[n]$. In other words, looking at samples at $n = 0, 1, 2$ and adding corresponding samples gives

$$\begin{aligned} a + b + c &= 1 \\ b + c &= -1 \\ c &= 1 \end{aligned}$$

But from second equation $b = -1 - 1 = -2$ and from first equation $a = 1 - b - c = 1 + 2 - 1 = 2$. Hence

$$2x_1[n] - 2x_2[n] + x_3[n] = x_4[n]$$

2.2 part b

Therefore by linearity

$$2y_1[n] - 2y_2[n] + y_3[n] = y_4[n]$$

Hence

$$y_4[n] = 2\delta[n+1] + \delta[n] + 2\delta[n-1] - 2\delta[n-2]$$

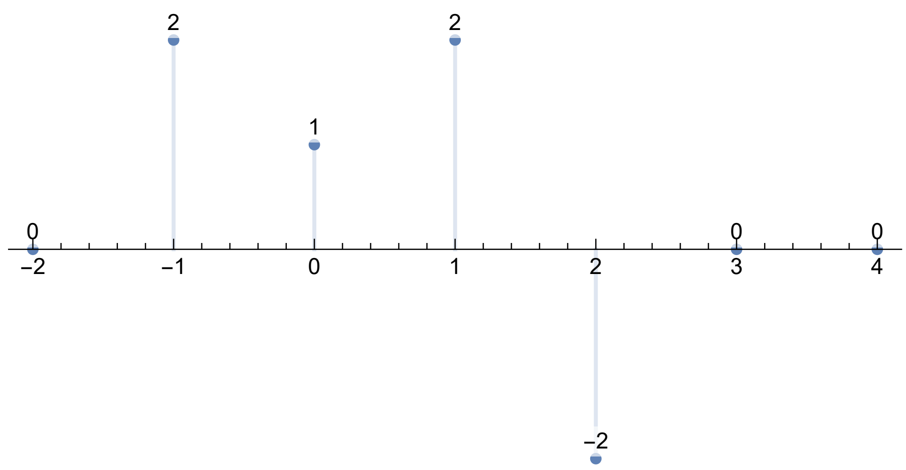

Figure 1: Plot of $y[n]$

2.3 part c

System is time invariant if shifted input gives same output but also shifted by the same amount as the input is shifted by. Let us consider $x_1[n]$. By shifting it to the right by one, then the output should $y_1[n]$ but shifted to the right by one which is $y_1[n-1]$

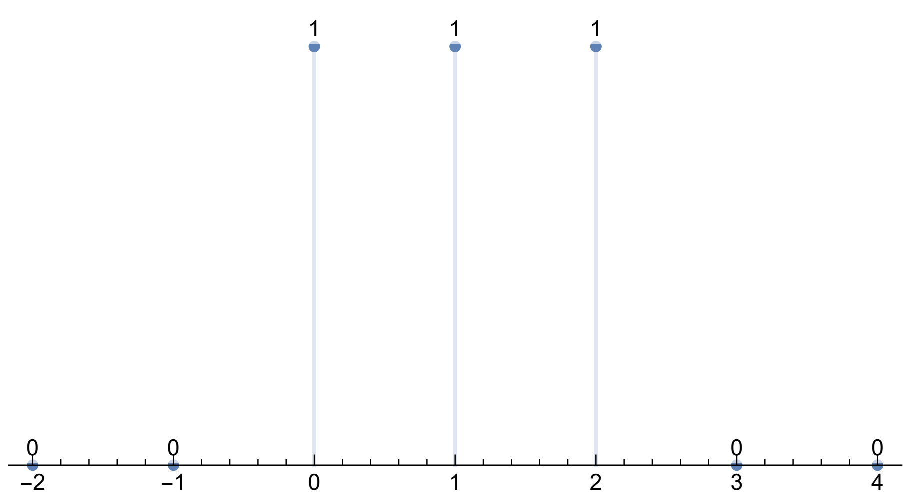

Figure 2: Plot of $y_1[n-1]$

Shifting $x_1[n]$ by 2 now the output should be $y_1[n-2]$

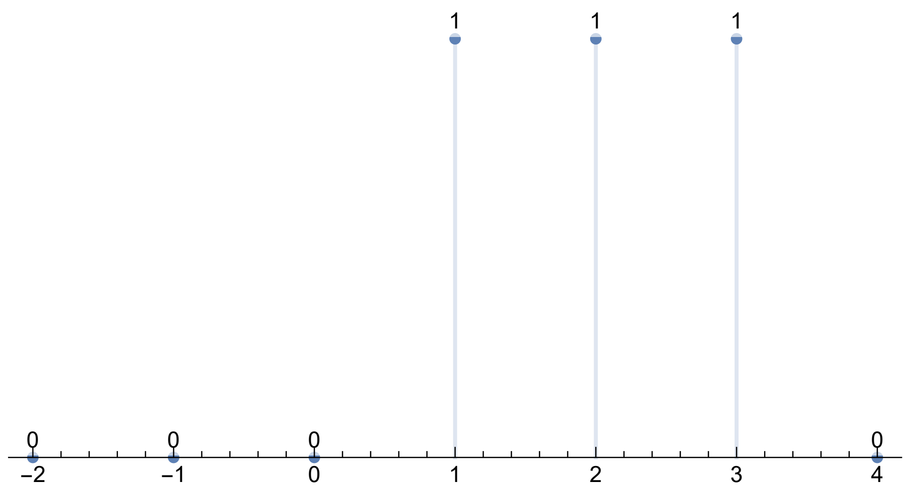

Figure 3: Plot of $y_1[n-2]$

But adding $x_1[n] + x_1[n-1] + x_1[n-2]$ gives $x_3[n]$. Which has the output shown. Let us now add $y_1[n] + y_1[n-1] + y_1[n-2]$ and see if this gives same as $y_3[n]$

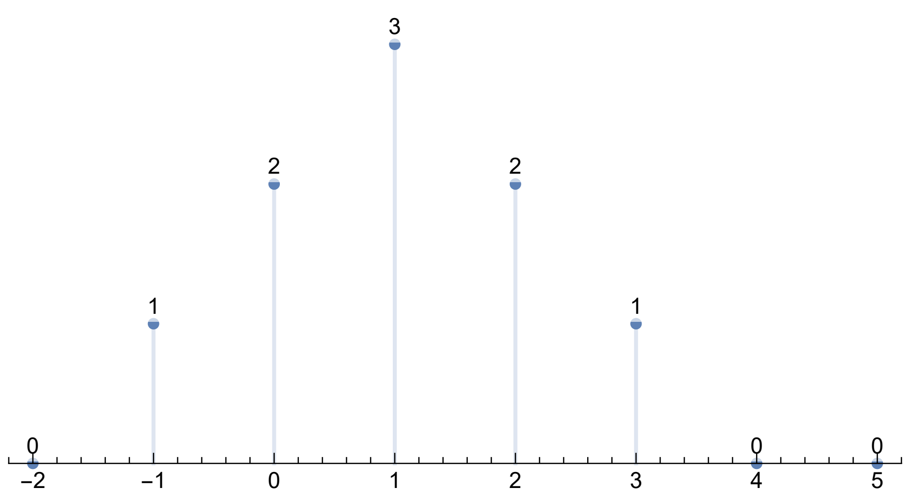

Figure 4: Plot of all shifted inputs of $x_1[x]$

Since the above is not the same as $y_3[n]$ then the system is not time invariant.

3 Problem 4.2

Solution

3.1 Part a

By folding $x[n]$ and shifting to the right, we see that $y[0] = 2, y[1] = 2 + 2 = 4, y[2] = 2 + 2 + 2 = 6, y[3] = 8, y[4] = 6, y[5] = 4, y[6] = 2, y[7] = 0$ and $y[n] = 0$ for all other values.

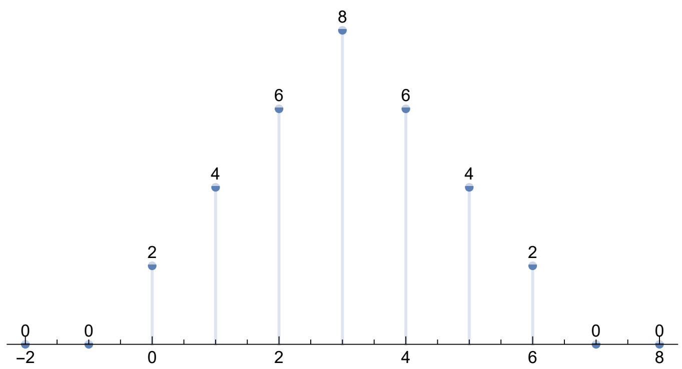

Figure 5: $y[n]$

3.2 Part b

By folding $x[n]$ and shifting to the right, we see that $y[0] = 0, y[1] = 0, y[2] = 0.5, y[3] = 1, y[4] = 1.5, y[5] = 1, y[6] = 0.5, y[7] = 0$ and $y[n] = 0$ for all other values.

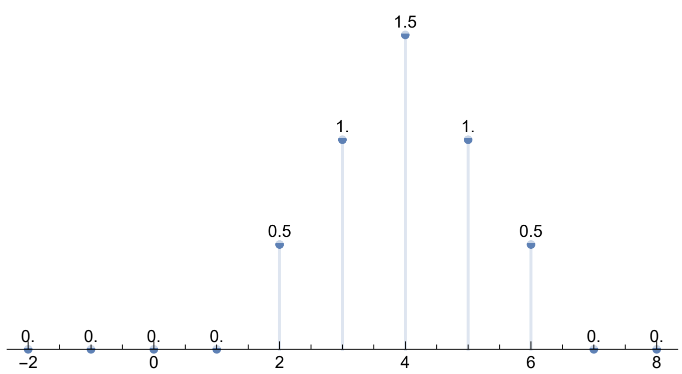

Figure 6: $y[n]$

4 Problem 4.3

Solution

4.1 Part a

By folding $x(t)$ and shifting, we see that for $t < 0$ that $y(t) = 0$. And for $0 < t < 4$ the integral becomes

$$\begin{aligned} y(t) &= \int_0^t h(\tau)\, d\tau \qquad 0 < t < 4 \\ &= \int_0^t 1 d\tau \\ &= t \end{aligned}$$

And for $0 < t-4 < 4$ or $4 < t < 8$

$$\begin{aligned} y(t) &= \int_{t-4}^{4} h(\tau)\,d\tau \qquad 4 < t < 8 \\ &= \int_{t-4}^{4} 1 d\tau \\ &= 4-(t-4) \\ &= 8-t \end{aligned}$$

And for $4 < t-4$ or $t > 8$

$$y(t) = 0$$

Hence $y(t)$ is

$$y(t) = \begin{cases} 0 & t < 0 \\ t & 0 < t < 4 \\ 8-t & 4 < t < 8 \\ 0 & t > 8 \end{cases}$$

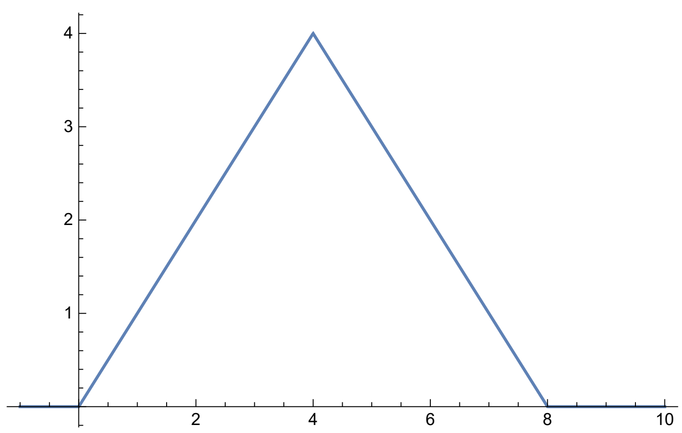

Figure 7: $y(t)$

4.2 Part b

By folding $h(t)$ and shifting, we see that for $t < 0$ that $y(t) = 0$. And for $t > 0$ the integral becomes

$$\begin{aligned} y(t) &= \int_{1}^{1+t} h(\tau)\,d\tau \qquad t > 0 \\ &= \int_{1}^{1+t} e^{-(\tau-1)} d\tau \\ &= \left[\frac{e^{-(\tau-1)}}{-1}\right]_{1}^{1+t} \\ &= -\left[e^{-(\tau-1)}\right]_{1}^{1+t} \\ &= -\left[e^{-((1+t)-1)} - e^{-(1-1)}\right] \\ &= -\left[e^{-t} - 1\right] \\ &= 1 - e^{-t} \end{aligned}$$

Hence $y(t)$ is

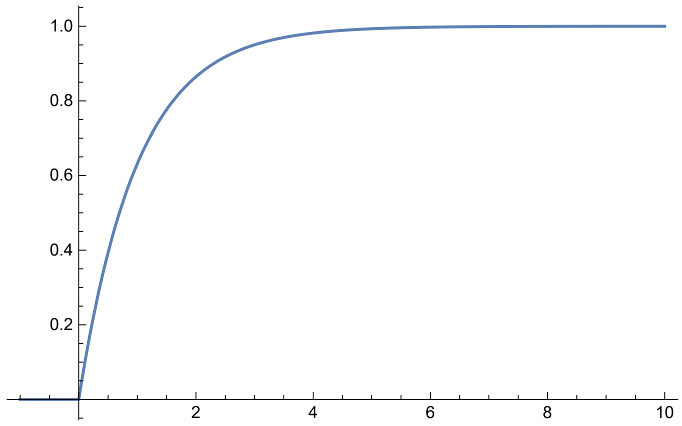

Figure 8: $y(t)$

4.3 Part c

By folding $h(t)$ and shifting, we see that for $-2+t<-1$ or $t<1$ that $y(t)=0$. And for $-1<-2+t<3$ or $1<t<5$ the integral becomes $x(t)$ itself (i.e. original $x(t)$ but shifted to right by 2). And for $3<-2+t$ or $t>5$ then $y(t)=0$. Hence

$$y(t)=\begin{cases} 0 & t<1 \\ x(t-2) & 1<t<5 \\ 0 & t>5 \end{cases}$$

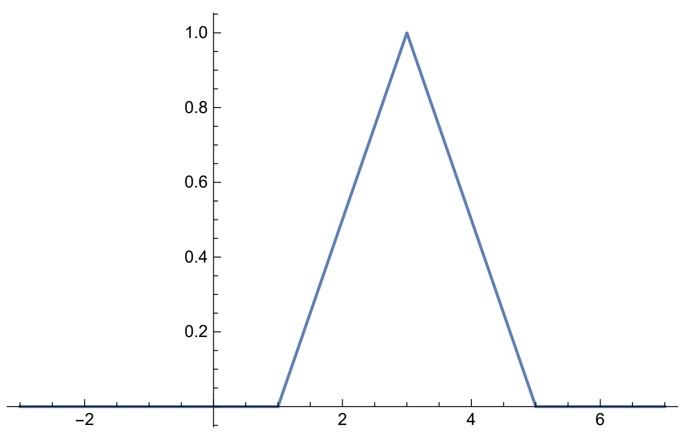

Figure 9: $y(t)$