10.2. Using eq. (10.3),

$$
\begin{aligned}
X(z) &= \sum_{n=-\infty}^{\infty} \left(\frac{1}{5}\right)^n u[n-3] z^{-n} \\
&= \sum_{n=3}^{\infty} \left(\frac{1}{5}\right)^n z^{-n} \\
&= \left[\frac{z^{-3}}{125}\right] \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n z^{-n} \\
&= \left[\frac{z^{-3}}{125}\right] \frac{1}{1 - \frac{1}{5} z^{-1}} \qquad |z| > \frac{1}{5}
\end{aligned}
$$

10.9. Using partial-fraction expansion,

$$X(z) = \frac{2/9}{1 - z^{-1}} + \frac{7/9}{1 + 2z^{-1}}, \qquad |z| > 2.$$

Taking the inverse z-transform,

$$x[n] = \frac{2}{9} u[n] + \frac{7}{9} (-2)^n u[n].$$

3.26) $X(z) = \dfrac{1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}$

a) write as ratio of polynomials

$$X(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})} \cdot \frac{z^2}{z^2} = \frac{z^2}{(z-\frac{1}{2})(z-1)}$$

b) Use partial fraction expansion to express $X(z)$ as sum of terms.

$$X(z) = z^2\left(\frac{1}{(z-\frac{1}{2})(z-1)}\right)$$

$$\frac{1}{(z-\frac{1}{2})(z-1)} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-1}$$

$$A = \frac{1}{z-1}\Big|_{z=\frac{1}{2}} = -2$$

$$B = \frac{1}{z-\frac{1}{2}}\Big|_{z=1} = 2$$

$$\Rightarrow X(z) = z^2\left(\frac{-2}{z-\frac{1}{2}} + \frac{2}{z-1}\right) = \frac{-2z^2}{z-\frac{1}{2}} + \frac{2z^2}{z-1}$$

c) rewrite $X(z)$ as $2z\left[\dfrac{z}{z-1} - \dfrac{z}{z-\frac{1}{2}}\right]$

$$= 2z\left[\frac{1}{1-z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}\right]$$

we've been told $x[n]$ is left sided, so

$$\left[\frac{1}{1-z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}\right] \longleftrightarrow u[-n-1] - (\tfrac{1}{2})^n u[-n-1]$$

factoring in the scaling & time shift gives

$$x[n] = 2u[-n-2] - 2(\tfrac{1}{2})^{n+1} u[-n-2].$$

Scanned with CamScanner

10.34. (a) Taking the z-transform of both sides of the given difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}.$$

The poles of $H(z)$ are at $z = (1/2) \pm (\sqrt{5}/2)$. $H(z)$ has a zero at $z = 0$. The pole-zero plot for $H(z)$ is as shown in Figure S10.34. Since $h[n]$ is causal, the ROC for $H(z)$ has to be $|z| > (1/2) + (\sqrt{5}/2)$.

(b) The partial fraction expansion of $H(z)$ is

$$H(z) = -\frac{1/\sqrt{5}}{1 - (\frac{1+\sqrt{5}}{2})z^{-1}} + \frac{1/\sqrt{5}}{1 - (\frac{1-\sqrt{5}}{2})z^{-1}}.$$

Therefore,

$$h[n] = -\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[n] + \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n].$$

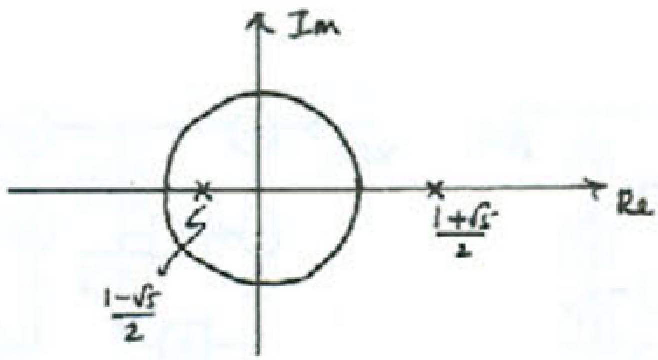

Figure S10.34

(c) Now assuming that the ROC is $(\sqrt{5}/2) - (1/2) < |z| < (1/2) + (\sqrt{5}/2)$, we get

$$h[n] = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[-n-1] + \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n].$$

10.36. Taking the z-transform of both sides of the given difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} - \frac{10}{3} + z} = \frac{z^{-1}}{1 - \frac{10}{3}z^{-1} + z^{-2}}.$$

The partial fraction expansion of $H(z)$ is

$$H(z) = -\frac{3/8}{1 - \frac{1}{3}z^{-1}} + \frac{3/8}{1 - 3z^{-1}}.$$

Since $H(z)$ corresponds to a stable system, the ROC has to be $(1/3) < |z| < 3$. Therefore,

$$h[n] = -\frac{3}{8}\left(\frac{1}{3}\right)^n u[n] - \frac{3}{8}(3)^n u[-n-1].$$

0.59. (a) From Figure S10.59, we have

$$W_1(z) = X(z) - \frac{k}{3}z^{-1}W_1(z) \quad \Rightarrow \quad W_1(z) = X(z)\frac{1}{1+\frac{k}{3}z^{-1}}.$$

Also,

$$W_2(z) = -\frac{k}{4}z^{-1}W_1(z) = -X(z)\frac{\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}.$$

Therefore, $Y(z) = W_1(z) + W_2(z)$ will be

$$Y(z) = X(z)\frac{1}{1+\frac{k}{3}z^{-1}} - X(z)\frac{\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}.$$

Finally,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}.$$

Since $H(z)$ corresponds to a causal filter, the ROC will be $|z| > |k|/3$.

(b) For the system to be stable, the ROC of $H(z)$ must include the unit circle. This is possible only if $|k|/3 < 1$. This implies that $|k|$ has to be less than 3.

59 c) for $K=1$ $H(Z) = \frac{1-\frac{1}{4}Z^{-1}}{1+K/3\,Z^{-1}}$

determine response $y[n]$ to $x[n] = (2/3)^n$.

we know that

$$y[n] = x[n] * h[n]$$

$$= \sum_{K=-\infty}^{\infty} x[n-K]\, h[K]$$

$$= \sum_{K=-\infty}^{\infty} (2/3)^{n-K}\, h[K]$$

$$= (2/3)^n \sum_{K=-\infty}^{\infty} (2/3)^{-K}\, h[K]$$

but we know that

$$H(Z) = \sum_{n=-\infty}^{\infty} Z^{-n}\, h[n]$$

so we have $y[n] = (2/3)^n\, H(2/3)$

Scanned with CamScanner