HW 9

EE 3015
Signals and Systems

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Contents

1 Problem 10.2

Consider the signal

$$x[n] = \left(\frac{1}{5}\right)^n u[n-3]$$

Use eq. (10.3)

$$X[z] = \sum_{n=-\infty}^{n=\infty} x[n]\, z^{-n} \tag{10.3}$$

to evaluate the Z-transform of this signal, and specify the corresponding region of convergence.

solution

$$X[z] = \sum_{n=-\infty}^{n=\infty} \left(\frac{1}{5}\right)^n u[n-3]\, z^{-n}$$

But $u[n-3]$ is zero for $n < 3$ and 1 otherwise. Hence the above becomes

$$X[z] = \sum_{n=3}^{n=\infty} \left(\frac{1}{5}\right)^n z^{-n}$$

Let $m = n - 3$. When $n = 3, m = 0$ therefore the above can be written as

$$\begin{aligned} X[z] &= \sum_{m=0}^{m=\infty} \left(\frac{1}{5}\right)^{m+3} z^{-(m+3)} \\ &= \left(\frac{z^{-1}}{5}\right)^3 \sum_{m=0}^{m=\infty} \left(\frac{1}{5}\right)^m z^{-m} \\ &= \frac{z^{-3}}{125} \sum_{m=0}^{m=\infty} \left(\frac{1}{5}\right)^m z^{-m} \end{aligned}$$

Renaming back to n

$$X[z] = \frac{z^{-3}}{125} \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n z^{-n} \tag{1}$$

Now, looking at $\sum_{n=0}^{n=\infty} \left(\frac{1}{5z}\right)^n$ then assuming $|5z| > 1$ and using the formula $\sum_{n=0}^{n=\infty} a^n = \frac{1}{1-a}$, where $a = \frac{1}{5z}$ in this case gives

$$\sum_{n=0}^{n=\infty} \left(\frac{1}{5z}\right)^n = \frac{1}{1 - \frac{1}{5} z^{-1}}$$

Hence (1) becomes

$$X[z] = \frac{z^{-3}}{125} \left(\frac{1}{1 - \frac{1}{5} z^{-1}}\right)$$

The above shows a pole at $\frac{1}{5}z^{-1} = 1$ or $z = \frac{1}{5}$ and a pole at $z = 0$. Since this is right handed signal, then the ROC is outside the outer most pole. Therefore ROC is

$$|z| > \frac{1}{5}$$

Which means the region is outside a circle of radius $\frac{1}{5}$. Since this ROC includes the unit circle, meaning a DTFT exist, it shows that this is a stable signal.

2 Problem 10.9

Using partial-fraction expansion and the fact that

$$a^n u[n] \overset{Z}{\longleftrightarrow} \frac{1}{1-az^{-1}} \qquad |z|>|a|$$

Find the inverse Z-transform of

$$X(z)=\frac{1-\frac{1}{3}z^{-1}}{\left(1-z^{-1}\right)\left(1+2z^{-1}\right)} \qquad |z|>2$$

solution

Let

$$\frac{1-\frac{1}{3}z^{-1}}{\left(1-z^{-1}\right)\left(1+2z^{-1}\right)}=\frac{A}{\left(1-z^{-1}\right)}+\frac{B}{\left(1+2z^{-1}\right)}$$

Hence $A=\left(\frac{1-\frac{1}{3}z^{-1}}{1+2z^{-1}}\right)_{z^{-1}=1}=\frac{1-\frac{1}{3}}{1+2}=\frac{2}{9}$ and $B=\left(\frac{1-\frac{1}{3}z^{-1}}{\left(1-z^{-1}\right)}\right)_{z^{-1}=-\frac{1}{2}}=\frac{1-\frac{1}{3}\left(-\frac{1}{2}\right)}{\left(1-\left(-\frac{1}{2}\right)\right)}=\frac{7}{9}$ Therefore the above becomes

$$X(z)=\frac{2}{9}\frac{1}{\left(1-z^{-1}\right)}+\frac{7}{9}\frac{1}{\left(1+2z^{-1}\right)}$$

The pole of first term at $z^{-1}=1$ or $z=1$ and the pole for second term is $2z^{-1}=-1$ or $z=-2$. Since the ROC is outside the out most pole, then this is right handed signal. Hence

$$\begin{aligned} x[n] &= \frac{2}{9}u[n]+\frac{7}{9}(-2)^n u[n] \\ &= \left(\frac{2}{9}+\frac{7}{9}(-2)^n\right)u[n] \end{aligned}$$

Which is valid when $X(z)$ defined for $|z|>2$ since this is the common region for $|z|>1$ and $|z|>2$ at the same time. We notice the ROC does not include the unit circle and hence it is not stable signal. This is confirmed by looking at the term $(-2)^n$ which grows with n with no limit.

3 Problem 10.26

Consider a left-sided sequence $x[n]$ with z-transform

$$X(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-z^{-1}\right)}$$

a Write $X(z)$ as a ratio of polynomials in z instead of z^{-1}

b Using a partial-fraction expression, express $X(z)$ as a sum of terms, where each term represents a pole from your answer in part (a).

c Determine $x[n]$

solution

3.1 Part a

$$\begin{aligned} X(z) &= \frac{z}{z\left(1-\frac{1}{2}z^{-1}\right)\left(1-z^{-1}\right)} \\ &= \frac{z}{\left(z-\frac{1}{2}\right)\left(1-z^{-1}\right)} \\ &= \frac{z^2}{z\left(z-\frac{1}{2}\right)\left(1-z^{-1}\right)} \\ &= \frac{z^2}{\left(z-\frac{1}{2}\right)(z-1)} \\ &= \frac{z^2}{z^2-\frac{3}{2}z+\frac{1}{2}} \end{aligned}$$

One pols at $z=\frac{1}{2}$ and one pole at $z=1$.

3.2 Part b

$$X(z) = \frac{z^2}{\left(z-\frac{1}{2}\right)(z-1)}$$

To do partial fractions, the degree in numerator must be smaller than in the denominator, which is not the case here. Hence we start by factoring out a z which gives

$$\begin{aligned} X(z) &= z^2\left(\frac{1}{\left(z-\frac{1}{2}\right)(z-1)}\right) \\ &= z^2\left(\frac{A}{z-\frac{1}{2}}+\frac{B}{z-1}\right) \end{aligned}$$

Hence

$$\frac{1}{\left(z-\frac{1}{2}\right)(z-1)} = \frac{A}{z-\frac{1}{2}}+\frac{B}{z-1}$$

Therefore $A=\left(\frac{1}{(z-1)}\right)_{z=\frac{1}{2}}=\frac{1}{\left(\frac{1}{2}-1\right)}=-2$ and $B=\left(\frac{1}{z-\frac{1}{2}}\right)_{z=1}=\frac{1}{1-\frac{1}{2}}=2$. Hence the above becomes

$$\begin{aligned} X(z) &= z^2\left(-\frac{2}{z-\frac{1}{2}}+\frac{2}{z-1}\right) \\ &= 2z^2\left(-\frac{1}{z-\frac{1}{2}}+\frac{1}{z-1}\right) \end{aligned}$$

Pole at $z=\frac{1}{2}$ and one at $z=1$.

3.3 Part c

Writing the above as

$$X(z) = 2z^2 X_1(z)$$

Where $x_1[n] \iff X_1(z)$ where ROC for $X_1(z)$ is inside the inner most pole (since left sided). Hence ROC for $X_1(z)$ is $|z|<\frac{1}{2}$. What is left is to find $x_1[n]$ which is the inverse Z transform of $\frac{-1}{z-\frac{1}{2}}+\frac{1}{z-1}$. We want to use $a^n u[n] \overset{Z}{\longleftrightarrow} \frac{1}{1-az^{-1}}$ so rewriting this as

$$\begin{aligned} X_1(z) &= \frac{-1}{z-\frac{1}{2}}+\frac{1}{z-1} \\ &= \frac{-z^{-1}}{1-\frac{1}{2}z^{-1}}+\frac{z^{-1}}{1-z^{-1}} \end{aligned}$$

Hence

$$\begin{aligned} X(z) &= 2z^2 X_1(z) \\ &= 2z^2\left(\frac{-z^{-1}}{1-\frac{1}{2}z^{-1}}+\frac{z^{-1}}{1-z^{-1}}\right) \\ &= 2z\left(\frac{-1}{1-\frac{1}{2}z^{-1}}+\frac{1}{1-z^{-1}}\right) \qquad (1) \end{aligned}$$

Then (since left handed) then $\frac{-1}{1-\frac{1}{2}z^{-1}} \longleftrightarrow \left(\frac{1}{2}\right)^n u[-n-1]$. Similarly for $\frac{1}{1-z^{-1}} \longleftrightarrow -u[-n-1]$. Hence

$$x[n] = \left(\frac{1}{2}\right)^n u[-n-1] - u[-n-1]$$

Substituting the above in (1) gives

$$x[n] = 2\left(\left(\frac{1}{2}\right)^n u[-n-2] - u[-n-2]\right)$$

Where $u[-n-1]$ is changed to $u[-n-2]$ because of the extra z in (1) outside, which causes extra shift and same for $u[-n-1]$ changed to $u[-n-2]$. Therefore the final answer is

$$x[n] = 2\left(\frac{1}{2}\right)^n u[-n-2] - 2u[-n-2]$$

4 Problem 10.34

A causal LTI system is described by the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

a Find the system function $H(z) = \frac{Y(z)}{X(z)}$ for this system. Plot the poles and zeros of $H(z)$ and indicate the region of convergence.

b Find the unit sample response of the system.

c You should have found the system to be unstable. Find a stable (non causal) unit sample response that satisfies the difference equation.

solution

4.1 Part a

Taking the Z transform of the difference equation gives

$$\begin{aligned} Y(z) &= z^{-1}Y(z) + z^{-2}Y(z) + z^{-1}X(z) \\ Y(z)\left(1 - z^{-1} - z^{-2}\right) &= z^{-1}X(z) \\ \frac{Y(z)}{X(z)} &= \frac{z^{-1}}{1 - z^{-1} - z^{-2}} \\ &= \frac{z}{z^2 - z - 1} \\ &= \frac{z}{\left(z - \left(\frac{1}{2}\sqrt{5} + \frac{1}{2}\right)\right)\left(z - \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)\right)} \end{aligned}$$

Hence a pole at $z = \frac{1}{2}\sqrt{5} + \frac{1}{2} = 1.618$ and a pole at $z = \left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right) = -0.618$ and zero at $z = 0$

Since this is a causal $H(z)$ then ROC is always to the right of the right most pole. Hence ROC is

$$|z| > \frac{1}{2}\sqrt{5} + \frac{1}{2} = 1.618$$

Here is a plot of the poles and zeros. The ROC is all the region to the right of 1.618 pole.

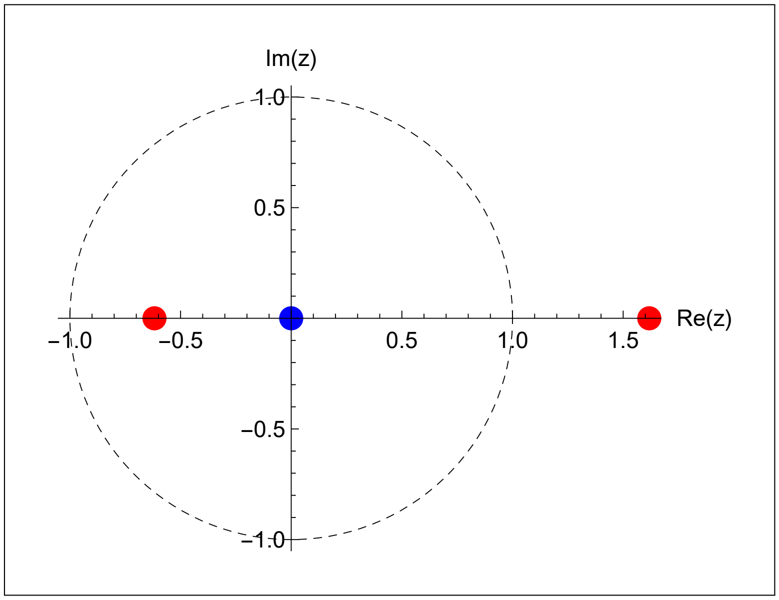


Figure 1: $H(z)$ Pole Zero plot. Red points are poles. Blue is zeros

```
p = Graphics[
        {
            {Dashed, Circle[{0, 0}, 1]},
            {PointSize[.04], {Red, Point[{-0.618, 0}]},
                {Red, Point[{1.618, 0}]}, {Blue, Point[{0, 0}]}}
        }, Axes → True, AxesLabel → {"Re(z)", "Im(z)"}, BaseStyle → 12];
```

Figure 2: Code used for the above

4.2 Part b

If the input $x[n] = \delta[n]$ then the difference equation is now

$$y[n] = y[n-1] + y[n-2] + \delta[n-1]$$

Hence taking the Z transform gives

$$\begin{aligned} Y(z) &= z^{-1}Y(z) + z^{-2}Y(z) + z^{-1} \\ Y(z)\left(1 - z^{-2} - z^{-1}\right) &= z^{-1} \\ Y(z) &= \frac{z^{-1}}{1 - z^{-1} - z^{-2}} \\ &= \frac{-z^{-1}}{z^{-2} + z^{-1} - 1} \\ &= \frac{-z^{-1}}{\left(z^{-1} - \left(-\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)\right)\left(z^{-1} - \left(-\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)\right)} \end{aligned} \tag{1}$$

Applying partial fractions gives

$$\frac{-z^{-1}}{\left(z^{-1}-\left(-\frac{1}{2}+\frac{1}{2}\sqrt{5}\right)\right)\left(z^{-1}-\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)\right)}=\frac{A}{z^{-1}-\left(-\frac{1}{2}+\frac{1}{2}\sqrt{5}\right)}+\frac{B}{z^{-1}-\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)}$$

Hence

$$A=\left(\frac{-z^{-1}}{\left(z^{-1}-\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)\right)}\right)_{z^{-1}=\left(-\frac{1}{2}+\frac{1}{2}\sqrt{5}\right)}=\frac{-\left(-\frac{1}{2}+\frac{1}{2}\sqrt{5}\right)}{\left(-\frac{1}{2}+\frac{1}{2}\sqrt{5}\right)-\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)}=\frac{1}{10}\sqrt{5}-\frac{1}{2}$$

And

$$B=\left(\frac{-z^{-1}}{z^{-1}-\left(-\frac{1}{2}+\frac{1}{2}\sqrt{5}\right)}\right)_{z=\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)}=\frac{-\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)}{\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)-\left(-\frac{1}{2}+\frac{1}{2}\sqrt{5}\right)}=-\frac{1}{10}\sqrt{5}-\frac{1}{2}$$

Therefore (1) becomes

$$\begin{aligned}
Y(z)&=\left(\frac{1}{10}\sqrt{5}-\frac{1}{2}\right)\frac{1}{z^{-1}-\left(-\frac{1}{2}+\frac{1}{2}\sqrt{5}\right)}-\left(\frac{1}{10}\sqrt{5}+\frac{1}{2}\right)\frac{1}{z^{-1}-\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)}\\
&=\frac{\left(\frac{1}{10}\sqrt{5}-\frac{1}{2}\right)}{-\frac{1}{2}+\frac{1}{2}\sqrt{5}}\frac{1}{\frac{1}{-\frac{1}{2}+\frac{1}{2}\sqrt{5}}z^{-1}-1}-\frac{\left(\frac{1}{10}\sqrt{5}+\frac{1}{2}\right)}{\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)}\frac{1}{\frac{1}{\left(-\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)}z^{-1}-1}\\
&=\frac{1}{5}\sqrt{5}\frac{1}{1-\left(\frac{2}{-1+\sqrt{5}}\right)z^{-1}}-\frac{1}{5}\sqrt{5}\frac{1}{1-\frac{2}{\left(-1-\sqrt{5}\right)}z^{-1}}\\
&=\frac{1}{5}\sqrt{5}\frac{1}{1-\left(\frac{1}{2}\sqrt{5}+\frac{1}{2}\right)z^{-1}}-\frac{1}{5}\sqrt{5}\frac{1}{1-\left(\frac{1}{2}-\frac{1}{2}\sqrt{5}\right)z^{-1}}
\end{aligned}$$

Now we can use the table $\frac{1}{1-az^{-1}}\to a^{n}u[n]$ for $|z|>a$. Taking the inverse Z transform of the above gives

$$\begin{aligned}
y[n]&=-\left(\frac{1}{5}\sqrt{5}\right)\left(\frac{1+\sqrt{5}}{2}\right)^{n}u[n]+\left(\frac{1}{5}\sqrt{5}\right)\left(\frac{1-\sqrt{5}}{2}\right)^{n}u[n]\\
&=\left(-(0.44721)(1.618)^{n}+(0.44721)(-0.618)^{n}\right)u[n]
\end{aligned}$$

This is <u>unstable</u> response $y[n]$ due to the term $(1.618)^{n}$ which grows with no limit as $n\to\infty$.

4.3 Part c

Using the ROC where $0.618<|z|<1.618$ instead of $|z|>1.618$, then

$$\begin{aligned}
y[n]&=\left(\frac{1}{5}\sqrt{5}\right)\left(\frac{1+\sqrt{5}}{2}\right)^{n}u[-n-1]+\left(\frac{1}{5}\sqrt{5}\right)\left(\frac{1-\sqrt{5}}{2}\right)^{n}u[n]\\
&=\left((0.44721)(1.618)^{n}u[-n-1]+(0.44721)(-0.618)^{n}\right)u[n]
\end{aligned}$$

which is now stable since the index on 1.618^n run is <u>negative</u> instead of positive.

5 Problem 10.36

Consider the linear, discrete-time, shift-invariant system with input $x[n]$ and output $y[n]$ for which

$$y[n-1]-\frac{10}{3}y[n]+y[n+1]=x[n]$$

is stable. Determine the unit sample response.

solution

Taking the Z transform of the difference equation gives

$$\begin{aligned}z^{-1}Y(z)-\frac{10}{3}Y(z)+zY(z)&=X(z)\\ Y(z)\left(z^{-1}-\frac{10}{3}+z\right)&=X(z)\end{aligned}$$

Hence the unit sample is when $x[n]=\delta[n]$. Hence $X(z)=1$. Therefore the impulse response is

$$\begin{aligned}H(z)&=\frac{1}{z^{-1}-\frac{10}{3}+z}\\ &=\frac{z^{-1}}{z^{-2}-\frac{10}{3}z^{-1}+1}\\ &=\frac{z^{-1}}{\left(z^{-1}-3\right)\left(z^{-1}-\frac{1}{3}\right)}\end{aligned}$$

Applying partial fractions

$$H(z)=\frac{A}{\left(z^{-1}-3\right)}+\frac{B}{\left(z^{-1}-\frac{1}{3}\right)}$$

Hence $A=\left(\frac{z^{-1}}{\left(z^{-1}-\frac{1}{3}\right)}\right)_{z^{-1}=3}=\frac{3}{\left(3-\frac{1}{3}\right)}=\frac{9}{8}$ and $B=\left(\frac{z^{-1}}{\left(z^{-1}-3\right)}\right)_{z^{-1}=\frac{1}{3}}=\frac{\frac{1}{3}}{\left(\frac{1}{3}-3\right)}=-\frac{1}{8}$. Therefore

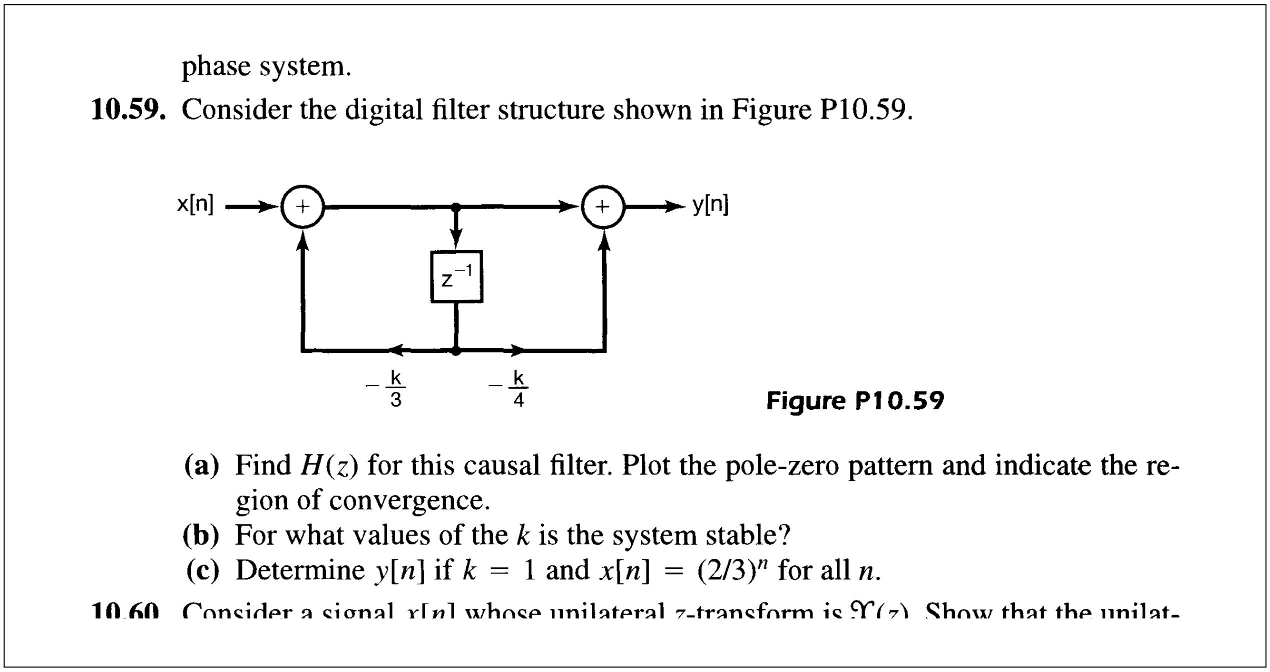
$$\begin{aligned}H(z)&=\frac{9}{8}\frac{1}{\left(z^{-1}-3\right)}-\frac{1}{8}\frac{1}{\left(z^{-1}-\frac{1}{3}\right)}\\ &=\frac{3}{8}\frac{1}{\left(\frac{1}{3}z^{-1}-1\right)}-\frac{3}{8}\frac{1}{\left(3z^{-1}-1\right)}\\ &=\frac{3}{8}\frac{1}{1-3z^{-1}}-\frac{3}{8}\frac{1}{1-\frac{1}{3}z^{-1}} \qquad (1)\end{aligned}$$

We see a pole at $z=3$ and a pole at $z=\frac{1}{3}$.

For $\frac{1}{1-3z^{-1}}$, this is stable only for a left sided signal, this is because a which is 3 here is larger than 1. Hence its inverse Z transform is of this is $x_1[n] = -\frac{3}{8}3^n u[-n-1]$ and for the second term $\frac{1}{1-\frac{1}{3}z^{-1}}$ is stable for right sided signal, since $\frac{1}{3} < 1$. Hence its inverse Z transform is $-\frac{3}{8}\left(\frac{1}{3}\right)^n u[n]$. Therefore

$$h[n] = -\frac{3}{8}(3)^n u[-n-1] - \frac{3}{8}\left(\frac{1}{3}\right)^n u[n]$$

6 Problem 10.59


phase system.

10.59. Consider the digital filter structure shown in Figure P10.59.

(a) Find $H(z)$ for this causal filter. Plot the pole-zero pattern and indicate the region of convergence.

(b) For what values of the k is the system stable?

(c) Determine $y[n]$ if $k = 1$ and $x[n] = (2/3)^n$ for all n.

10.60. Consider a signal $x[n]$ whose unilateral z-transform is $\mathcal{X}(z)$. Show that the unilat-

Figure 3: Problem description

solution

6.1 Part (a)

Let the value at the branch just to the right of $x[n]$ summation sign be called $A[z]$.

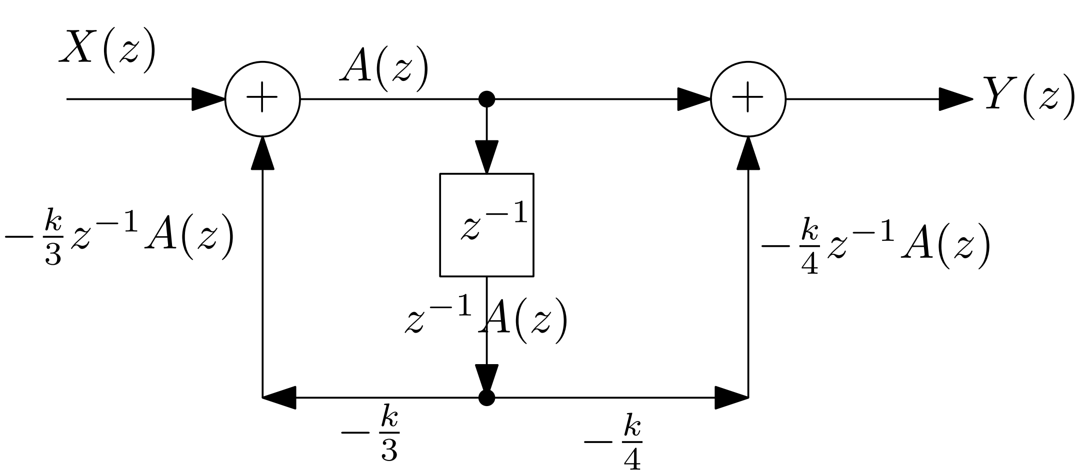


Figure 4: Filter diagram

Then we see that

$$Y(z) = A(z) - \frac{k}{4} z^{-1} A(z)$$

We just need to find $A(z)$. We see that $A(z) = X(z) - \frac{k}{3} z^{-1} A(z)$. Hence $A(z)\left(1 + \frac{k}{3} z^{-1}\right) = X(z)$ or $A(z) = \frac{X(z)}{1+\frac{k}{3}z^{-1}}$. Therefore the above becomes

$$\begin{aligned} Y(z) &= \frac{X(z)}{1 + \frac{k}{3} z^{-1}} - \frac{k}{4} z^{-1} \frac{X(z)}{1 + \frac{k}{3} z^{-1}} \\ &= X(z) \left(\frac{1}{1 + \frac{k}{3} z^{-1}} - \frac{k}{4} \frac{z^{-1}}{1 + \frac{k}{3} z^{-1}} \right) \end{aligned}$$

Hence

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} \\ &= \frac{1}{1 + \frac{k}{3} z^{-1}} - \frac{k}{4} \frac{z^{-1}}{1 + \frac{k}{3} z^{-1}} \\ &= \frac{1 - \frac{k}{4} z^{-1}}{1 + \frac{k}{3} z^{-1}} \end{aligned}$$

The pole is when $\frac{k}{3} z^{-1} = -1$ or $z = -\frac{k}{3}$. Zero is when $1 - kz^{-1} = 0$ or $kz^{-1} = 1$ or $z = k$. Since this causal system, then the ROC is to the right of the most right pole. Hence $|z| > \frac{|k|}{3}$ is the ROC.

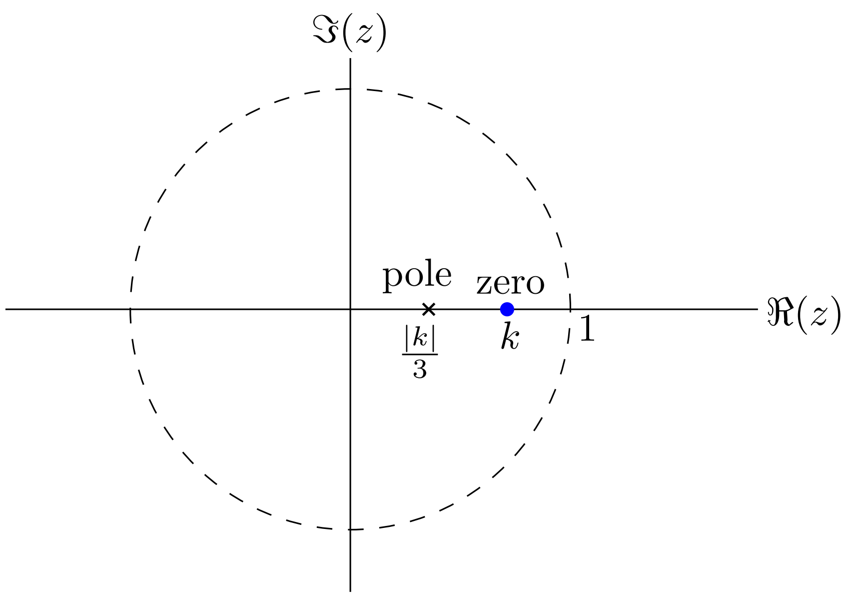


Figure 5: Pole zero polt. ROC is $|z| >]frac|k|3$

6.2 Part (b)

System is stable if it has a Discrete time Fourier transform. This implies the ROC must include the unit circle. Hence $\frac{|k|}{3} < 1$ or $|k| < 3$.

6.3 Part (c)

From part (a), the unit sample response is $H(z) = \frac{1-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}$. When $k = 1$ this becomes $H(z) = \frac{1-\frac{1}{4}z^{-1}}{1+\frac{1}{3}z^{-1}}$

Since $x[n] = \left(\frac{2}{3}\right)^n$ for all n and this is casual system, then this means $x[n] = \left(\frac{2}{3}\right)^n u[n]$. Therefore

$$X(z) = \frac{1}{1-\frac{2}{3}z^{-1}}$$

Hence from part (a)

$$\begin{aligned} Y(z) &= H(z)X(z) \\ &= \frac{1-\frac{1}{4}z^{-1}}{1+\frac{1}{3}z^{-1}}\frac{1}{1-\frac{2}{3}z^{-1}} \\ &= \frac{1-\frac{1}{4}z^{-1}}{\left(1+\frac{1}{3}z^{-1}\right)\left(1-\frac{2}{3}z^{-1}\right)} \\ &= \frac{A}{1+\frac{1}{3}z^{-1}} + \frac{B}{1-\frac{2}{3}z^{-1}} \end{aligned}$$

Therefore $A = \left(\frac{1-\frac{1}{4}z^{-1}}{\left(1-\frac{2}{3}z^{-1}\right)}\right)_{z^{-1}=-3} = \frac{1-\frac{1}{4}(-3)}{\left(1-\frac{2}{3}(-3)\right)} = \frac{7}{12}$ and $B = \left(\frac{1-\frac{1}{4}z^{-1}}{\left(1+\frac{1}{3}z^{-1}\right)}\right)_{z^{-1}=\frac{3}{2}} = \left(\frac{1-\frac{1}{4}\left(\frac{3}{2}\right)}{\left(1+\frac{1}{3}\left(\frac{3}{2}\right)\right)}\right) = \frac{5}{12}$. Hence

$$Y(z) = \frac{7}{12}\frac{1}{1+\frac{1}{3}z^{-1}} + \frac{5}{12}\frac{1}{1-\frac{2}{3}z^{-1}}$$

Therefore

$$y[n] = \frac{7}{12}\left(-\frac{1}{3}\right)^n u[n] + \frac{5}{12}\left(\frac{2}{3}\right)^n u[n]$$

The following is a plot of the solution

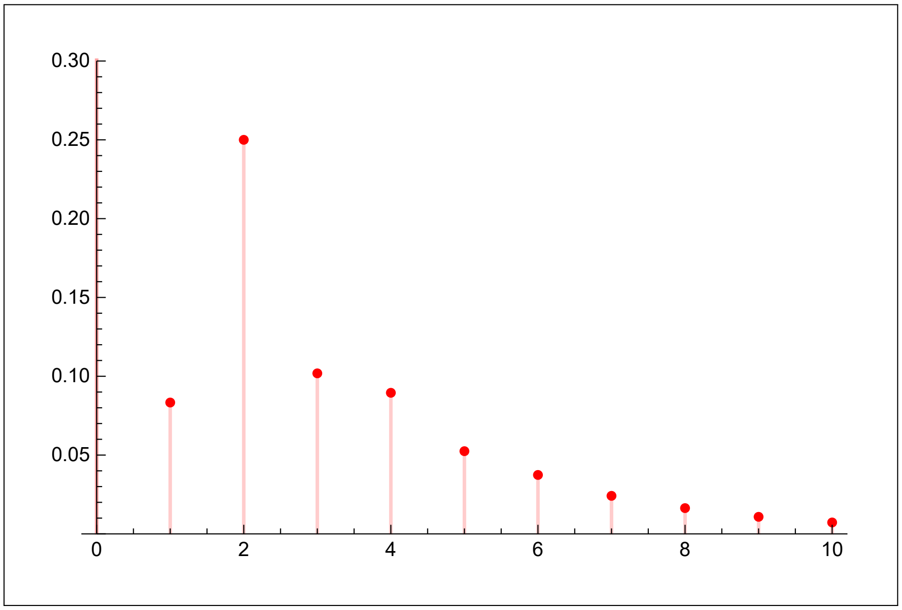


Figure 6: Plot of $y[n]$

```
mySol = 7/12 (-1/3)^n UnitStep[n] + 5/12 (2/3)^n UnitStep[n];
p =
    DiscretePlot[mySol, {n, 0, 10}, PlotRange → {Automatic, {0, 0.3}}, PlotStyle → Red];
```

Figure 7: Code used