Homework 6 - Due October 22

Homework instructions: Complete the assigned problems on your own paper. Once you are finished, scan or photograph your work and upload it to Gradescope. When prompted, tell Gradescope where to find each problem.
You are allowed (and in fact encouraged) to work with other students on homework assignments. If you do that, please indicate on each problem who you worked with. If you use sources other than your notes, the textbook, and any resources on Canvas for your homework, you must indicate the source on each problem. You are not permitted to view, request, or look for solutions to any of the homework problems from solutions manuals, homework help websites, online forums, other students, or any other sources.

Textbook Problems:

- §5.2: 9, 16, 24
- §5.3: 8, 11, 14, 18

Additional Problems:

1. This problem will walk you through finding the general solution of the differential equation

$$y^{(7)} - 2y^{(6)} + 9y^{(5)} - 16y^{(4)} + 24y^{(3)} - 32y'' + 16y' = 0$$

   (a) Write the characteristic equation for this differential equation. Factor out the common factor of r.

   (b) Check that 1 is a root of the remaining polynomial. This means that $(r-1)$ is a factor, so use polynomial long division to factor it out. Repeat until 1 is no longer a factor of the remaining polynomial.

   (c) The remaining polynomial should be of the form ar^4+br^2+c. Make the substitution $x = r^2$ and factor the quadratic $ax^2 + bx + c$.

   (d) Substitute back $x = r^2$ and find the roots of whatever remains.

   (e) List all the roots of the characteristic polynomial and their multiplicities. Use this list to write down the general solution. Since this differential equation is of order 7, your general solution should have 7 terms.