HW 3

Math 2243
Linear Algebra and Differential Equations

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Contents

1 Problem 3 section 3.7

In each of Problems 1–10, $n+1$ data points are given. Find the n^{th} degree polynomial $y=f(x)$ that fits these points.

$$\left(x,y\right)=\{(0,3),(1,1),(2,-5)\}$$

Solution

Since $n+1=3$, then $n=2$. Therefore we need degree 2 polynomial

$$f(x)=A+Bx+Cx^2$$

From the data given, we obtain the following three equations

$$\begin{aligned}3&=A\\1&=A+B+C\\-5&=A+2B+4C\end{aligned}$$

This gives the system

$$\begin{bmatrix}1&0&0\\1&1&1\\1&2&4\end{bmatrix}\begin{bmatrix}A\\B\\C\end{bmatrix}=\begin{bmatrix}3\\1\\-5\end{bmatrix}$$

Augmented matrix is

$$\begin{bmatrix}1&0&0&3\\1&1&1&1\\1&2&4&-5\end{bmatrix}$$

$R_2\to -R_1+R_2$ gives

$$\begin{bmatrix}1&0&0&3\\0&1&1&-2\\1&2&4&-5\end{bmatrix}$$

$R_3\to -R_1+R_3$ gives

$$\begin{bmatrix}1&0&0&3\\0&1&1&-2\\0&2&4&-8\end{bmatrix}$$

$R_3\to -2R_2+R_3$ gives

$$\begin{bmatrix}1&0&0&3\\0&1&1&-2\\0&0&2&-4\end{bmatrix}$$

Hence we obtain the system in Echelon form as

$$\begin{bmatrix}1&0&0\\0&1&1\\0&0&2\end{bmatrix}\begin{bmatrix}A\\B\\C\end{bmatrix}=\begin{bmatrix}3\\-2\\-4\end{bmatrix}$$

Back substitution: Last row gives $2C=-4$ or $C=-2$. Second row gives $B-C=-2$ or $B=0$. First row gives $A=3$. Therefore the solution is

$$\begin{bmatrix}A\\B\\C\end{bmatrix}=\begin{bmatrix}3\\0\\-2\end{bmatrix}$$

The polynomial is

$$\begin{aligned}f(x)&=A+Bx+Cx^2\\&=3-2x^2\end{aligned}$$

Here is plot of the solution fitted on the points

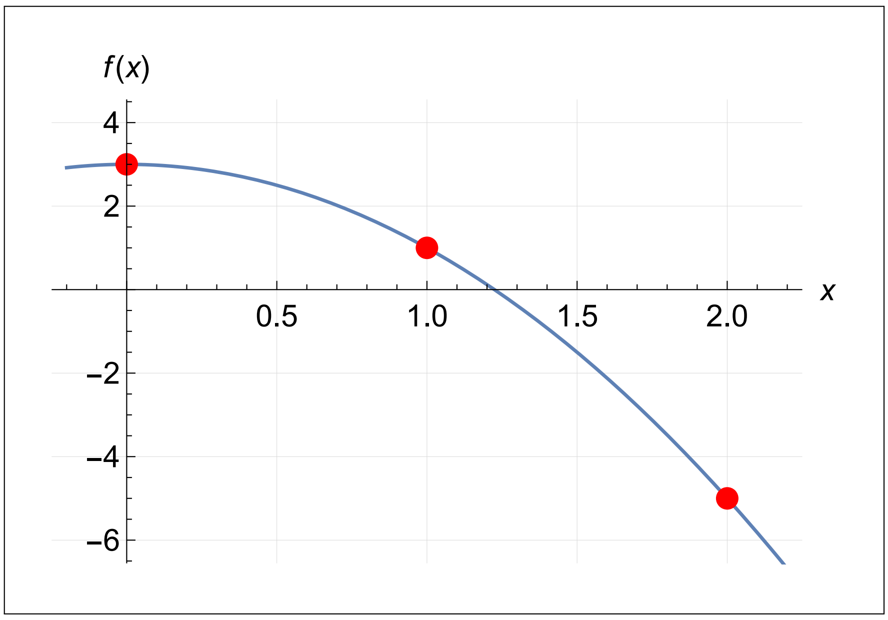

Figure 1: Fitted polynomial plot

```
p1 = ListPlot[{{0, 3}, {1, 1}, {2, -5}}, PlotStyle → {PointSize[.03], Red}];
p2 = Plot[3 + 0 x - 2 x^2, {x, -.2, 2.2}, AxesLabel → {x, f[x]},
   BaseStyle → 14, GridLines → Automatic, GridLinesStyle → LightGray];
p = Show[p2, p1, PlotRange → {Automatic, {-6, 4}}];
```

Figure 2: Code used for the above plot

2 Problem 1 section 4.1

In Problems 1–4, find $\left|\vec{a}-\vec{b}\right|, 2\vec{a}+\vec{b}, 3\vec{a}-4\vec{b}$

$$\vec{a}=(2,5,-4),\vec{b}=(1,-2,-3)$$

<u>Solution</u>

$$\begin{aligned}\left|\vec{a}-\vec{b}\right| &= |(2,5,-4)-(1,-2,-3)|\\ &= |(2-1,5+2,-4+3)|\\ &= |(1,7,-1)|\\ &= \sqrt{1+49+1}\\ &= \sqrt{51}\end{aligned}$$

And

$$\begin{aligned}2\vec{a}+\vec{b} &= 2(2,5,-4)+(1,-2,-3)\\ &= (4,10,-8)+(1,-2,-3)\\ &= (4+1,10-2,-8-3)\\ &= (5,8,-11)\end{aligned}$$

And

$$\begin{aligned}3\vec{a}-4\vec{b} &= 3(2,5,-4)-4(1,-2,-3)\\ &= (6,15,-12)-(4,-8,-12)\\ &= (6-4,15+8,-12+12)\\ &= (2,23,0)\end{aligned}$$

3 Problem 19 section 4.1

In Problems 19–24, use the method of Example 3 to determine whether the given vectors $\vec{u},\vec{v}$, and $\vec{w}$ are linearly independent or dependent. If they are linearly dependent, find scalars a,b, and c not all zero such that $a\vec{u}+b\vec{v}+c\vec{w}=\vec{0}$

$$\begin{aligned}\vec{u}&=(2,0,1)\\ \vec{v}&=(-3,1,-1)\\ \vec{w}&=(0,-2,-1)\end{aligned}$$

Solution

We set up $Ax=0$ and solve for x where x here is (a,b,c) vector. If x is the trivial solution, then the vectors are linearly independent. If we find non-trivial solution, then the vectors are linearly dependent.

$$a\vec{u}+b\vec{v}+c\vec{w}=\vec{0}$$
$$a\begin{bmatrix}2\\0\\1\end{bmatrix}+b\begin{bmatrix}-3\\1\\-1\end{bmatrix}+c\begin{bmatrix}0\\-2\\-1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$
$$\begin{bmatrix}2&-3&0\\0&1&-2\\1&-1&-1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix}2&-3&0\\0&1&-2\\1&-1&-1\end{bmatrix}$$

$R_3\rightarrow-\frac{1}{2}R_1+R_3$ gives

$$\begin{bmatrix}2&-3&0\\0&1&-2\\0&\frac{1}{2}&-1\end{bmatrix}$$

$R_3\rightarrow\frac{-1}{2}R_2+R_3$ gives

$$\begin{bmatrix}2&-3&0\\0&1&-2\\0&0&0\end{bmatrix}$$

Hence the system becomes

$$\begin{bmatrix}2&-3&0\\0&1&-2\\0&0&0\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

a,b are leading variables and c is free variable. Let $c=t$ which can be any value. Then $b=2t$ and $2a-3b=0$ or $a=3t$. Hence solution is

$$\begin{bmatrix}a\\b\\c\end{bmatrix}=t\begin{bmatrix}3\\2\\1\end{bmatrix}$$

There are infinite solutions. We need only one non-zero solution to show that the vectors are linearly dependent. Let $t=1$

$$\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}3\\2\\1\end{bmatrix}$$

Hence vectors are linearly dependent

$$3\vec{u}+2\vec{v}+\vec{w}=\vec{0}$$

4 Problem 23 section 4.1

In Problems 19–24, use the method of Example 3 to determine whether the given vectors $\vec{u}, \vec{v}$, and $\vec{w}$ are linearly independent or dependent. If they are linearly dependent, find scalars a, b, and c not all zero such that $a\vec{u} + b\vec{v} + c\vec{w} = \vec{0}$

$$\begin{aligned}\vec{u} &= (2,0,3)\\ \vec{v} &= (5,4,-2)\\ \vec{w} &= (2,-1,1)\end{aligned}$$

<u>Solution</u>

We set up $Ax = 0$ and solve for x where x here is (a, b, c) vector. If x is the trivial solution, then the vectors are linearly independent. If we find non-trivial solution, then the vectors are linearly dependent.

$$a\vec{u} + b\vec{v} + c\vec{w} = \vec{0}$$

$$a\begin{bmatrix}2\\0\\3\end{bmatrix} + b\begin{bmatrix}5\\4\\-2\end{bmatrix} + c\begin{bmatrix}2\\-1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}2 & 5 & 2\\0 & 4 & -1\\3 & -2 & 1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix}2 & 5 & 2\\0 & 4 & -1\\3 & -2 & 1\end{bmatrix}$$

$R_3 \to -3R_1 + 2R_3$ gives

$$\begin{bmatrix}2 & 5 & 2\\0 & 4 & -1\\0 & -19 & -4\end{bmatrix}$$

$R_3 \to -19R_2 + 4R_3$ gives

$$\begin{bmatrix}2 & 5 & 2\\0 & 4 & -1\\0 & 0 & 3\end{bmatrix}$$

Hence the system in Echelon form becomes

$$\begin{bmatrix}2 & 5 & 2\\0 & 4 & -1\\0 & 0 & 3\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

Last row gives $c = 0$. Second row gives $4b = 0$ or $b = 0$. First row gives $2a = 0$ or $a = 0$.

Therefore the vectors are <u>linearly independent</u> because only the trivial solution exist.

5 Problem 27 section 4.1

In Problems 25–28, express the vector $\vec{t}$ as a linear combination of the vectors $\vec{u}, \vec{v}$, and $\vec{w}$.

$$\vec{t} = (0,0,19), \vec{u} = (1,4,3), \vec{v} = (-1,-2,2), \vec{w} = (4,4,1)$$

Solution

In system form we are looking for

$$a\vec{u} + b\vec{v} + c\vec{w} = \vec{t}$$
$$a\begin{bmatrix}1\\4\\3\end{bmatrix} + b\begin{bmatrix}-1\\-2\\2\end{bmatrix} + c\begin{bmatrix}4\\4\\1\end{bmatrix} = \begin{bmatrix}0\\0\\19\end{bmatrix}$$
$$\begin{bmatrix}1 & -1 & 4\\4 & -2 & 4\\3 & 2 & 1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}0\\0\\19\end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix}1 & -1 & 4 & 0\\4 & -2 & 4 & 0\\3 & 2 & 1 & 19\end{bmatrix}$$

$R_2 \rightarrow -4R_1 + R_2$ gives

$$\begin{bmatrix}1 & -1 & 4 & 0\\0 & 2 & -12 & 0\\3 & 2 & 1 & 19\end{bmatrix}$$

$R_3 \rightarrow -3R_1 + R_3$ gives

$$\begin{bmatrix}1 & -1 & 4 & 0\\0 & 2 & -12 & 0\\0 & 5 & -11 & 19\end{bmatrix}$$

$R_3 \rightarrow -\frac{5}{2}R_2 + R_3$ gives

$$\begin{bmatrix}1 & -1 & 4 & 0\\0 & 2 & -12 & 0\\0 & 0 & 19 & 19\end{bmatrix}$$

The above is Echelon form. Hence the system is

$$\begin{bmatrix}1 & -1 & 4\\0 & 2 & -12\\0 & 0 & 19\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}0\\0\\19\end{bmatrix}$$

Last row gives $19c = 19$ or $c = 1$. Second row gives $2b - 12c = 0$ or $b = 6$. First row gives $a - b + 4c = 0$ or $a = b - 4c$ or $a = 6 - 4 = 2$. Hence

$$a\vec{u} + b\vec{v} + c\vec{w} = \vec{t}$$
$$2\vec{u} + 6\vec{v} + \vec{w} = \vec{t}$$

6 Problem 2 section 4.2

Apply Theorem 1 to determine whether or not W is a subspace of $\mathbb{R}^n$.

W is the set of all vectors in $\mathbb{R}^3$ such that $x_1 = 5x_2$

Solution

Theorem 1 at page 225 gives conditions for subspace:

The non empty subset W of the vector space V is a subspace of V if and only if it satisfies the following two conditions:

1. If $\vec{u}$ and $\vec{v}$ are vectors in W , then $\vec{u} + \vec{v}$ is also in W.
2. If $\vec{u}$ is in W and c is a scalar, then the vector $c\vec{u}$ is also in W.

Let $\vec{u} = (x_1, x_2, x_3)$ and $\vec{v} = \left(y_1, y_2, y_3\right)$ where $x_1 = 5x_2$ and $y_1 = 5y_2$. then (1) becomes

$$\begin{aligned} \vec{u} + \vec{v} &= (x_1, x_2, x_3) + \left(y_1, y_2, y_3\right) \\ &= \left(x_1 + y_1, x_2 + y_2, x_3 + y_3\right) \end{aligned}$$

Then

$$\begin{aligned} x_1 + y_1 &= 5x_2 + 5y_2 \\ &= 5\left(x_2 + y_2\right) \end{aligned}$$

Hence closed under addition. Condition (2) says

$$\begin{aligned} c\vec{u} &= c(x_1, x_2, x_3) \\ &= (cx_1, cx_2, cx_3) \end{aligned}$$

Hence $cx_1 = c(5x_2) = 5(cx_2)$. Therefore closed under scalar multiplication as well. Therefore this is a subspace.

7 Problem 4 section 4.2

Apply Theorem 1 to determine whether or not W is a subspace of $\mathbb{R}^n$.

W is the set of all vectors in $\mathbb{R}^3$ such that $x_1 + x_2 + x_3 = 1$

Solution

Theorem 1 at page 225 gives conditions for subspace:

The non empty subset W of the vector space V is a subspace of V if and only if it satisfies the following two conditions:

1. If $\vec{u}$ and $\vec{v}$ are vectors in W , then $\vec{u} + \vec{v}$ is also in W.
2. If $\vec{u}$ is in W and c is a scalar, then the vector $c\vec{u}$ is also in W.

Let $\vec{u} = (x_1, x_2, x_3)$ and $\vec{v} = (y_1, y_2, y_3)$ where $x_1 + x_2 + x_3 = 1$ and $y_1 + y_2 + y_3 = 1$. then (1) becomes

$$\begin{aligned}\vec{u} + \vec{v} &= (x_1, x_2, x_3) + (y_1, y_2, y_3) \\ &= (x_1 + y_1, x_2 + y_2, x_3 + y_3)\end{aligned}$$

Then

$$\begin{aligned}x_1 + y_1 + x_2 + y_2 + x_3 + y_3 &= (x_1 + x_2 + x_3) + (y_1 + y_2 + y_3) \\ &= 1 + 1 \\ &= 2\end{aligned}$$

Therefore this is not closed under addition since $\vec{u}+\vec{v}$ does not satisfy (1). Hence not a subspace.

8 Problem 17 section 4.2

In Problems 15–18, apply the method of Example 5 to find two solution vectors $\vec{u}$ and $\vec{v}$ such that the solution space is the set

of all linear combinations of the form $s\vec{u} + t\vec{v}$

$$\begin{aligned} x_1 + 3x_2 + 8x_3 - x_4 &= 0 \\ x_1 - 3x_2 - 10x_3 + 5x_4 &= 0 \\ x_1 + 4x_2 + 11x_3 - 2x_4 &= 0 \end{aligned}$$

(notice: typo in book. Last term in second equation is $5x_5$ in book, but it should be $5x_4$).

<u>Solution</u>

System is

$$\begin{bmatrix} 1 & 3 & 8 & -1 \\ 1 & -3 & -10 & 5 \\ 1 & 4 & 11 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 1 & 3 & 8 & -1 \\ 1 & -3 & -10 & 5 \\ 1 & 4 & 11 & -2 \end{bmatrix}$$

$R_2 \to -R_1 + R_2$ gives

$$\begin{bmatrix} 1 & 3 & 8 & -1 \\ 0 & -6 & -18 & 6 \\ 1 & 4 & 11 & -2 \end{bmatrix}$$

$R_3 \to -R_1 + R_3$ gives

$$\begin{bmatrix} 1 & 3 & 8 & -1 \\ 0 & -6 & -18 & 6 \\ 0 & 1 & 3 & -1 \end{bmatrix}$$

$R_3 \to \frac{1}{6}R_2 + R_3$ gives

$$\begin{bmatrix} 1 & 3 & 8 & -1 \\ 0 & -6 & -18 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Leading variables are x_1, x_2. Free variables are x_3, x_4. Let $x_4 = t, x_3 = s$. The system becomes

$$\begin{bmatrix} 1 & 3 & 8 & -1 \\ 0 & -6 & -18 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

From second row, $-6x_2 - 18s + 6t = 0$ or $x_2 = -\frac{18s-6t}{6} = -3s + t$.

From first row, $x_1 + 3x_2 + 8s - t = 0$. Hence $x_1 = -3x_2 - 8s + t$ or $x_1 = -3(-3s + t) - 8s + t$ or

$x_1 = s - 2t$. Therefore the solution is

$$\begin{aligned}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} &= \begin{bmatrix} s - 2t \\ -3s + t \\ s \\ t \end{bmatrix} \\ &= s\begin{bmatrix} 1 \\ -3 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \\ &= s\vec{u} + t\vec{v}\end{aligned}$$

Therefore he solution space is the set of all linear combinations of the form $s\vec{u} + t\vec{v}$

9 Problem 21 section 4.2

In Problems 19–22, reduce the given system to echelon form to find a single solution vector $\vec{u}$ such that the solution space is

the set of all scalar multiples of $\vec{u}$.

$$\begin{aligned} x_1 + 7x_2 + 2x_3 - 3x_4 &= 0 \\ 2x_1 + 7x_2 + x_3 - 4x_4 &= 0 \\ 3x_1 + 5x_2 - x_3 - 5x_4 &= 0 \end{aligned}$$

Solution

System is

$$\begin{bmatrix} 1 & 7 & 2 & -3 \\ 2 & 7 & 1 & -4 \\ 3 & 5 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 1 & 7 & 2 & -3 \\ 2 & 7 & 1 & -4 \\ 3 & 5 & -1 & -5 \end{bmatrix}$$

$R_2 \to -2R_1 + R_2$ gives

$$\begin{bmatrix} 1 & 7 & 2 & -3 \\ 0 & -7 & -3 & 2 \\ 3 & 5 & -1 & -5 \end{bmatrix}$$

$R_3 \to -3R_1 + R_3$ gives

$$\begin{bmatrix} 1 & 7 & 2 & -3 \\ 0 & -7 & -3 & 2 \\ 0 & -16 & -7 & 4 \end{bmatrix}$$

$R_3 \to \frac{-16}{7} R_2 + R_3$ gives

$$\begin{bmatrix} 1 & 7 & 2 & -3 \\ 0 & -7 & -3 & 2 \\ 0 & 0 & -\frac{1}{7} & -\frac{4}{7} \end{bmatrix}$$

Hence the system in Echelon form is

$$\begin{bmatrix} 1 & 7 & 2 & -3 \\ 0 & -7 & -3 & 2 \\ 0 & 0 & -\frac{1}{7} & -\frac{4}{7} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Leading variables are x_1, x_2, x_3. Free variable is $x_4 = t$. Last row gives $-\frac{1}{7}x_3 - \frac{4}{7}t = 0$. Hence $x_3 = -4t$. Second row gives $-7x_2 - 3x_3 + 2x_4 = 0$ or $-7x_2 = 3x_3 - 2x_4$ or $-7x_2 = 3(-4t) - 2(t)$. Hence $-7x_2 = -14t$ or $x_2 = 2t$.

First row gives $x_1 + 7x_2 + 2x_3 - 3x_4 = 0$ or $x_1 = -7(2t) - 2(-4t) + 3(t)$. Hence $x_1 = -3t$. Therefore the solution is

$$\begin{aligned}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} &= \begin{bmatrix} -3t \\ 2t \\ -4t \\ t \end{bmatrix} \\ &= t\begin{bmatrix} -3 \\ 2 \\ -4 \\ 1 \end{bmatrix} \\ &= t\vec{u}\end{aligned}$$

The solution space is the set of all scalar multiples of $\vec{u}$.

10 Additional problem 1

My fictional company Linear Algebra Inc had a stock price of \$10 on day 1, \$15 on day 2, and \$10 on day 3. Interpolate this data with a quadratic polynomial $f(t) = a + bt + ct^2$,where t is the day and $f(t)$ is the price on day t. Is it a good idea to use $f(t)$ to predict the stock price of Linear Algebra Inc on day 4?

Solution

Data is $(1,10),(2,15),(3,10)$. Therefore we obtain 3 equations using $f(t) = a + bt + ct^2$ as

$$\begin{aligned}10 &= a + b + c\\15 &= a + 2b + 4c\\10 &= a + 3b + 9c\end{aligned}$$

Which gives the system

$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & 4\\1 & 3 & 9\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}10\\15\\10\end{bmatrix}$$

The augmented matrix is

$$\begin{bmatrix}1 & 1 & 1 & 10\\1 & 2 & 4 & 15\\1 & 3 & 9 & 10\end{bmatrix}$$

$R_2 \to -R_1 + R_2$ gives

$$\begin{bmatrix}1 & 1 & 1 & 10\\0 & 1 & 3 & 5\\1 & 3 & 9 & 10\end{bmatrix}$$

$R_3 \to -R_1 + R_3$ gives

$$\begin{bmatrix}1 & 1 & 1 & 10\\0 & 1 & 3 & 5\\0 & 2 & 8 & 0\end{bmatrix}$$

$R_3 \to -2R_2 + R_3$ gives

$$\begin{bmatrix}1 & 1 & 1 & 10\\0 & 1 & 3 & 5\\0 & 0 & 2 & -10\end{bmatrix}$$

Hence the system in Echelon form is

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\0 & 0 & 2\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}10\\5\\-10\end{bmatrix}$$

Leading variables are a, b, c. There are no free variables. From last row $2c = -10$ hence $c = -5$. From second row $b + 3c = 5$ or $b = 5 - 3c$ or $b = 5 - 3(-5)$ or $b = 20$. From first row $a + b + c = 10$. Hence $a = 10 - b - c$ or $a = 10 - 20 + 5$ or $a = -5$. The solution is

$$\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}-5\\20\\-5\end{bmatrix}$$

Therefore, the interpolation polynomial is

$$f(t) = a + bt + ct^2$$

Or

$$\boxed{f(t) = -5 + 20t - 5t^2}$$

It is not good idea to use $f(t)$ to predict the price outside the range of interpolation, which is $t = 1 \cdots 3$. Doing so is extrapolation and can produce wrong prediction. For example, using $t = 4$ gives $f(4) = -5$ dollars as stock price, which is not possible. The lowest value a stock can have is zero dollars, which is when the company go bankrupt.

Here is plot of the solution fitted on the points

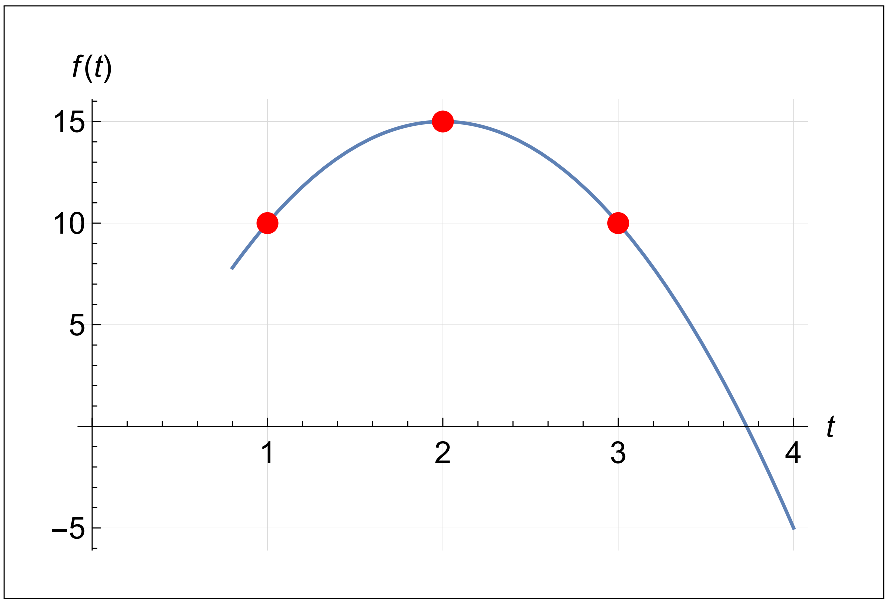

Figure 3: Fitted polynomial plot

```
p1 = ListPlot[{{1, 10}, {2, 15}, {3, 10}}, PlotStyle → {PointSize[.03], Red}];
p2 = Plot[-5 + 20 t - 5 t^2, {t, 0.8, 4}, AxesLabel → {t, f[t]},
    BaseStyle → 14, GridLines → Automatic, GridLinesStyle → LightGray];
p = Show[p2, p1, AxesOrigin → {0, 0}, PlotRange → All];
```

Figure 4: Code used for the above plot

11 Additional problem 2

Geometrically, what do subspaces of $\mathbb{R}^2$ look like?

<u>Solution</u>

A Subspace of $\mathbb{R}^2$ is all straight lines that pass through the origin. So each straight lines that pass through the origin is a subspace. This shows there are infinite number of subspaces.

Another subspace of $\mathbb{R}^2$ is just the origin $\vec{0}$. And $\mathbb{R}^2$ itself is subspace of itself.

12 Additional problem 3

Let A be an $n \times n$ matrix and consider the linear system $A\vec{x} = \vec{b}$. If I know that the solution set to this linear system is a subspace of $\mathbb{R}^n$, what can you say about $\vec{b}$?

Solution

The vector $\vec{b}$ is the zero vector. This is by theorem 2, page 226 in the textbook.