Math 2243 - Section 002 - Midterm 1 - Cover Page

Instructions:

You are required to be present on Zoom with your camera on from 6pm until your exam is submitted. While taking the exam, you must remain in the frame of the camera at all times. You may not ask for or receive help from notes, textbooks, online resources, or other people during the exam. You may use a calculator, but as always in this class you are expected to show all work.

There is an honor statement at the bottom of this page. You are required to copy the statement in your own handwriting, sign it, and submit it with your exam. Unless we have made prior arrangements, failure to comply with these requirements may result in a grade of 0 on the exam.

If you have questions during the exam or encounter technical difficulties, you can ask me using the Zoom chat or by email. The exam is written to take approximately 60 minutes. This file has been made available at 6pm and you will have until 7:15pm to submit your solutions on Gradescope. It is your responsibility to ensure that you have sufficient time to scan and upload your work before the deadline. It is your responsibility to ensure that your scan is legible and includes all of your work.

Honor Statement:

Please copy the following in your own handwriting and sign it. Your exam will not be accepted without a signed honor statement.

I certify that I have not accepted, sought, or received help from any source on this exam. I have not used any notes, textbooks, or online resources and I have not spoken to any person. I understand that signing this statement untruthfully will constitute a violation of the University of Minnesota's Academic Honesty policies.

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Problem 1 (10 points): Suppose that $\mathbf{A}$ is a matrix such that

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -2 \\ 1 & a & b \end{bmatrix} \qquad \mathbf{A}^{-1} = \begin{bmatrix} c & -1 & -1 \\ -2 & 1 & d \\ -1 & 0 & 1 \end{bmatrix}$$

Find a, b, c and d.

Problem 2 (5 points): Find a matrix $\mathbf{X}$ such that $\mathbf{AX} = \mathbf{B}$, given that

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -4 & 0 & 5 & 1 & 2 \\ 7 & 1 & 3 & 1 & -3 \end{bmatrix}$$

Problem 3 (10 points): Let k be any real number and let $\mathbf{A}$ be the matrix

$$\mathbf{A} = \begin{bmatrix} k+2 & 6 & 1 \\ 1 & 3 & -2 \\ 2 & 6 & k \end{bmatrix}$$

(a) Compute $\det \mathbf{A}$ (your answer should be in terms of the constant k).

(b) For what values of k is it true that the linear system $\mathbf{A}\vec{x} = \vec{0}$ has exactly one solution?

Problem 4 (8 points): Let $\vec{v}_1 = (1, -3, 3), \vec{v}_2 = (-1, -4, 2)$, $\vec{v}_3 = (7, -7, 11), \vec{v}_4 = (2, 1, 1)$ and let $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$. Consider the subspace $W = \operatorname{Span} S$ of $\mathbb{R}^3$.

Find a subset of S which is a basis for W. Your answer should explain why you know the vectors you have chosen are a basis for W.

Problem 5 (7 points): Let $\mathcal{P}_2$ denote the vector space of all polynomials of degree at most 2. For which values of the real constant k do the polynomials

$$1 + x, x + x^2, kx + 3x^2$$

form a basis for $\mathcal{P}_2$?

Problem 6 (10 points): For the following matrix $\mathbf{A}$, the dimension of the solution space of $\mathbf{A}\vec{x} = \vec{0}$ depends on the constant k.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 3k+2 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & k & 2 \end{bmatrix}$$

(a) What are the possible dimensions of the solution space $\mathbf{A}\vec{x} = \vec{0}$? What values of k do they correspond to?

(b) Find a basis for the solution space of of $\mathbf{A}\vec{x} = \vec{0}$ for each of the possibilities found in part (a).