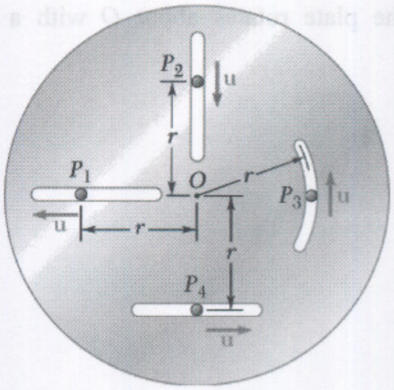
HW #6

PROBLEM 15.156



Four pins slide in four separate slots cut in a circular plate as shown. When the plate is at rest, each pin has a velocity directed as shown and of the same constant magnitude u. If each pin maintains the same velocity relative to the plate when the plate rotates about O with a constant counterclockwise angular velocity ω, determine the acceleration of each pin.

SOLUTION

For each pin: $$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_c$$

Acceleration of the coinciding point P′ of the plate.

For each pin, $\mathbf{a}_{P'} = r\omega^2$ towards the center O.

Acceleration of the pin relative to the plate.

For pins P_1, P_2, and P_4, $$\mathbf{a}_{P/F} = 0$$

For pin P_3, $$\mathbf{a}_{P/F} = \frac{u^2}{r} \longleftarrow$$

Coriolis acceleration $\mathbf{a}_c$.

For each pin $a_c = 2\omega u$ with $\mathbf{a}_c$ in a direction obtained by rotating $\mathbf{u}$ through $90°$ in the sense of ω, i.e. ↺.

Then, $$\mathbf{a}_1 = \left[r\omega^2 \longrightarrow\right] + \left[2\omega u \downarrow\right] \qquad \mathbf{a}_1 = r\omega^2\mathbf{i} - 2\omega u\mathbf{j} \blacktriangleleft$$

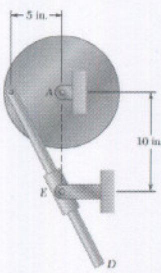
$$\mathbf{a}_2 = \left[r\omega^2 \downarrow\right] + \left[2\omega u \longrightarrow\right] \qquad \mathbf{a}_2 = 2\omega u\mathbf{i} - r\omega^2\mathbf{j} \blacktriangleleft$$

$$\mathbf{a}_3 = \left[r\omega^2 \longleftarrow\right] + \left[\frac{u^2}{r} \longleftarrow\right] + \left[2\omega u \longleftarrow\right] \qquad \mathbf{a}_3 = -\left(r\omega^2 + \frac{u^2}{r} + 2\omega u\right)\mathbf{i} \blacktriangleleft$$

$$\mathbf{a}_4 = \left[r\omega^2 \uparrow\right] + \left[2\omega u \uparrow\right] \qquad \mathbf{a}_4 = \left(r\omega^2 + 2\omega u\right)\mathbf{j} \blacktriangleleft$$

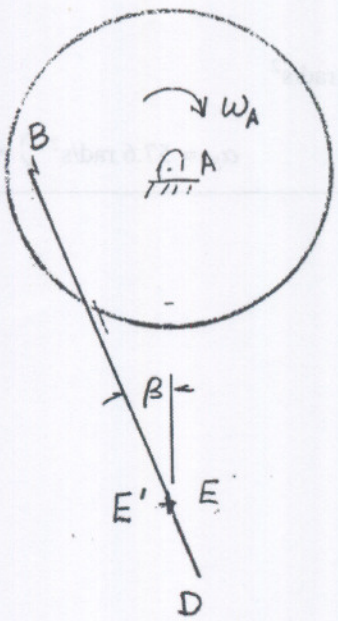
HW #6

PROBLEM 15.179



The disk shown rotates with a constant clockwise angular velocity of 12 rad/s. At the instant shown, determine (*a*) the angular velocity and angular acceleration of rod *BD*, (*b*) the velocity and acceleration of the point of the rod coinciding with *E*.

SOLUTION



Geometry. $\tan\beta = \frac{5}{10}, \qquad \beta = 26.565°$

$$l_{AE} = \frac{10}{\cos\beta} = 11.1803 \text{ in.}$$

Velocity analysis. $\boldsymbol{\omega}_{AB} = 12 \text{ rad/s} \curvearrowright, \quad \boldsymbol{\omega}_{BD} = \omega_{BD} \curvearrowright$

$$\mathbf{v}_B = (AB)\omega_{AB} = (5)(12) = 60 \text{ in./s} \uparrow$$

$$\mathbf{v}_{E'} = \mathbf{v}_B + (BE)\omega_{BD} \nearrow \beta$$

$$= [60 \uparrow] + [11.1803\omega_{BD} \nearrow \beta]$$

$$\mathbf{v}_{E/BD} = [u \nwarrow \beta], \qquad \mathbf{v}_E = 0$$

Use $\mathbf{v}_E = \mathbf{v}_{E'} + \mathbf{v}_{E/BD}$ and resolve into components.

$$+\nearrow \beta: \quad 0 = -60\sin\beta + 11.1803\omega_{BD}, \qquad \omega_{BD} = 2.400 \text{ rad/s}$$

$$+\nwarrow \beta: \quad 0 = 60\cos\beta - u, \qquad u = 53.666 \text{ m/s}$$

$$\mathbf{v}_{E'} = [60 \uparrow] + [(11.1803)(2.400) \nearrow \beta] = 53.7 \text{ in./s} \searrow 63.4°$$

Acceleration analysis.

$$\mathbf{a}_B = (AB)\omega_{AB}^2 = (5)(12)^2 = 720 \text{ in./s}^2 \longrightarrow$$

$$\mathbf{a}_{E'} = \mathbf{a}_B + [(BE)\alpha_{BD} \nearrow \beta] + [(BE)\omega_{BD}^2 \nwarrow \beta]$$

$$= [720 \longrightarrow] + [11.1803\alpha_{BD} \nearrow \beta] + [64.399 \nwarrow \beta]$$

$$\mathbf{a}_{E/BD} = [\dot{u} \nwarrow \beta] \qquad \mathbf{a}_E = 0$$

Coriolis acceleration.

$$2\omega_{BD}u = (2)(2.400)(53.666) = [257.60 \nearrow \beta]$$

PROBLEM 15.179 CONTINUED

Use $\mathbf{a}_E = \mathbf{a}_{E'} + \mathbf{a}_{E/BD} + \left[2\omega_{BD} u \nearrow \beta\right]$ and resolve into components.

$$+\nearrow \beta: \quad 0 = -720\cos\beta + 11.1803\alpha_{BD} + 257.60$$

$$\alpha_{BD} = 34.56 \text{ rad/s}^2$$

$$+\searrow \beta: \quad 0 = -720\sin\beta + 64.399 - \dot{u}, \qquad \dot{u} = -257.59 \text{ in/s}^2$$

$$\mathbf{a}_{E'} = \left[720 \longrightarrow\right] + \left[(11.1803)(34.56) \nearrow \beta\right] + \left[64.399 \searrow \beta\right]$$

$$= \left[720 \longrightarrow\right] + \left[386.39 \nearrow \beta\right] + \left[64.399 \searrow \beta\right]$$

$$= 365 \text{ in./s}^2 \nwarrow 18.4^\circ$$

Summary:

(*a*) $\omega_{BD} = 2.40 \text{ rad/s}$ ↻, $\alpha_{BD} = 34.6 \text{ rad/s}^2$ ↻ ◀

(*b*) $\mathbf{v}_{E'} = 53.7 \text{ in./s} \searrow 63.4^\circ$, $\mathbf{a}_{E'} = 365 \text{ in./s}^2 \nwarrow 18.4^\circ$ ◀