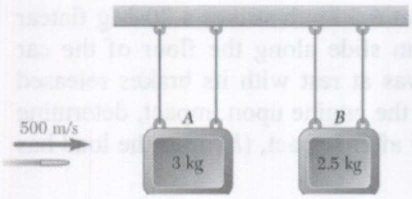
HW # 5 7th

PROBLEM 14.5



A bullet is fired with a horizontal velocity of 500 m/s through a 3-kg block *A* and becomes embedded in a 2.5-kg block *B*. Knowing that blocks *A* and *B* start moving with velocities of 3 m/s and 5 m/s, respectively determine (*a*) the mass of the bullet, (*b*) its velocity as it travels from block *A* to block *B*.

SOLUTION

The masses are m for the bullet and m_A and m_B for the blocks.

(*a*) The bullet passes through block *A* and embeds in block *B*. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) + m_B(0) = mv_0$

Final momentum: $mv_B + m_Av_A + m_Bv_B$

Equating, $mv_0 = mv_B + m_Av_A + m_Bv_B$

$$m = \frac{m_Av_A + m_Bv_B}{v_0 - v_B} = \frac{(3)(3) + (2.5)(5)}{500 - 5} = 43.434 \times 10^{-3} \text{ kg}$$

$m = 43.4 \text{ g}$ ◀

(*b*) The bullet passes through block *A*. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) = mv_0$

Final momentum: $mv_1 + m_Av_A$

Equating, $mv_0 = mv_1 + m_Av_A$

$$v_1 = \frac{mv_0 - m_Av_A}{m} = \frac{(43.434 \times 10^{-3})(500) - (3)(3)}{43.434 \times 10^{-3}} = 292.79 \text{ m/s}$$

$v_1 = 293 \text{ m/s} \longrightarrow$ ◀

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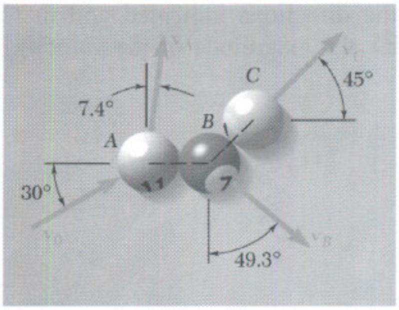
PROBLEM 14.21



In a game of pool, ball A is traveling with a velocity $\mathbf{v}_0$ when it strikes balls B and C which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and that $v_0 = 4$ m/s and $v_C = 2.1$ m/s, determine the magnitude of the velocity of (*a*) ball A, (*b*) ball B.

SOLUTION

Velocity vectors:

$$\mathbf{v}_0 = v_0(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$$
$$\mathbf{v}_A = v_A(\sin 7.4°\mathbf{i} + \cos 7.4°\mathbf{j})$$
$$\mathbf{v}_B = v_B(\sin 49.3°\mathbf{i} - \cos 49.3°\mathbf{j})$$
$$\mathbf{v}_C = v_C(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$$

Conservation of momentum:

$$m_A\mathbf{v}_0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

Divide by $m_A = m_B = m_C$ and substitute data.

$$4(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = v_A(\sin 7.4°\mathbf{i} + \cos 7.4°\mathbf{j}) + v_B(\sin 49.3°\mathbf{i} - \cos 49.3°\mathbf{j}) + 2.1(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$$

Resolve into components and rearrange.

$$\mathbf{i}: \quad (\sin 7.4°)v_A + (\sin 49.3°)v_B = 4\cos 30° - 2.1\cos 45°$$
$$\mathbf{j}: \quad (\cos 7.4°)v_A - (\cos 49.3°)v_B = 4\sin 30° - 2.1\sin 45°$$

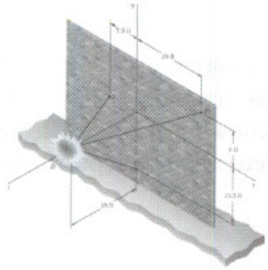
Solving simultaneously,

(*a*) $v_A = 2.01$ m/s ◀

(*b*) $v_B = 2.27$ m/s ◀

HW#5 7th

PROBLEM 14.26



An 18-lb shell moving with a velocity $\mathbf{v}_0 = (60 \text{ ft/s})\mathbf{i} - (45 \text{ ft/s})\mathbf{j} - (1800 \text{ ft/s})\mathbf{k}$ explodes at point D into three fragments A, B, and C weighing, respectively, 6 lb, 4 lb, and 8 lb. Knowing that the fragments hit the vertical wall at the points indicated, determine the speed of each fragment immediately after the explosion.

SOLUTION

Position vectors (ft):

$$\mathbf{r}_D = 18\mathbf{k}$$

$$\mathbf{r}_A = -7.5\mathbf{i} \qquad \mathbf{r}_{A/D} = -7.5\mathbf{i} - 18\mathbf{k} \qquad r_{A/D} = 19.5$$

$$\mathbf{r}_B = 18\mathbf{i} + 9\mathbf{j} \qquad \mathbf{r}_{B/D} = 18\mathbf{i} + 9\mathbf{j} - 18\mathbf{k} \qquad r_{B/D} = 27$$

$$\mathbf{r}_C = -13.5\mathbf{j} \qquad \mathbf{r}_{C/D} = -13.5\mathbf{j} - 18\mathbf{k} \qquad r_{C/D} = 22.5$$

Unit vectors:

Along $\mathbf{r}_{A/D}$, $$\lambda_A = \frac{1}{19.5}(-7.5\mathbf{i} - 18\mathbf{k})$$

Along $\mathbf{r}_{B/D}$, $$\lambda_B = \frac{1}{27}(18\mathbf{i} + 9\mathbf{j} - 18\mathbf{k})$$

Along $\mathbf{r}_{C/D}$, $$\lambda_C = \frac{1}{22.5}(-13.5\mathbf{j} - 18\mathbf{k})$$

Assume that elevation changes due to gravity may be neglected. Then, the velocity vectors after the explosition have the directions of the unit vectors.

$$\mathbf{v}_A = v_A\lambda_A \qquad \mathbf{v}_B = v_B\lambda_B \qquad \mathbf{v}_C = v_C\lambda_C$$

Conservation of momentum:

$$m\mathbf{v}_0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$\frac{18}{g}(60\mathbf{i} - 45\mathbf{j} - 1800\mathbf{k}) = \frac{6}{g}\left(\frac{v_A}{19.5}\right)(-7.5\mathbf{i} - 18\mathbf{k}) + \frac{4}{g}\left(\frac{v_B}{27}\right)(18\mathbf{i} + 9\mathbf{j} - 18\mathbf{k}) + \frac{8}{g}\left(\frac{v_C}{22.5}\right)(-13.5\mathbf{j} - 18\mathbf{k})$$

Multiply by g and resolve into components.

$$1080 = -45\left(\frac{v_A}{19.5}\right) + 72\left(\frac{v_B}{27}\right)$$

$$-810 = 36\left(\frac{v_B}{27}\right) - 108\left(\frac{v_C}{22.5}\right)$$

$$-32400 = -108\left(\frac{v_A}{19.5}\right) - 72\left(\frac{v_B}{27}\right) - 144\left(\frac{v_C}{22.5}\right)$$

Solving,

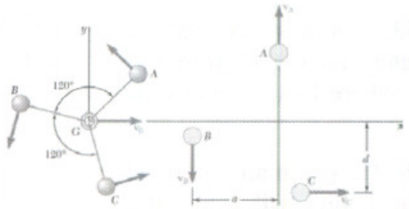
$$\frac{v_A}{19.5} = 161.311 \qquad v_A = 3150 \text{ ft/s} \blacktriangleleft$$

$$\frac{v_B}{27} = 115.820 \qquad v_B = 3130 \text{ ft/s} \blacktriangleleft$$

$$\frac{v_C}{22.5} = 46.106 \qquad v_C = 1037 \text{ ft/s} \blacktriangleleft$$

HW#5 7th

PROBLEM 14.55



Three small identical spheres A, B, and C, which can slide on a horizontal, frictionless surface, are attached to three strings of length l which are tied to a ring G. Initially the spheres rotate about the ring which moves along the x axis with a velocity $\mathbf{v}_0$. Suddenly the ring breaks and the three spheres move freely in the xy plane. Knowing that $\mathbf{v}_A = (8.66 \text{ ft/s})\mathbf{j}$, $\mathbf{v}_C = (15 \text{ ft/s})\mathbf{i}$, $a = 0.866$ ft, and $d = 0.5$ ft, determine (a) the initial velocity of the ring, (b) the length l of the strings, (c) the rate in rad/s at which the spheres were rotating about G.

SOLUTION

Use a frame of reference that is translating with the mass center G of the system. Let $\mathbf{v}_0$ be its velocity.

$$\mathbf{v}_0 = v_0\mathbf{i}$$

The initial velocities in this system are $(\mathbf{v}'_A)_0$, $(\mathbf{v}'_B)_0$ and $(\mathbf{v}'_C)_0$, each having a magnitude of $l\omega$. They are directed 120° apart. Thus,

$$(\mathbf{v}'_A)_0 + (\mathbf{v}'_B)_0 + (\mathbf{v}'_C)_0 = 0$$

(a) Conservation of linear momentum:

$$m(\mathbf{v}'_A)_0 + m(\mathbf{v}'_B)_0 + m(\mathbf{v}'_C)_0 = m(\mathbf{v}_A - \mathbf{v}_0) + m(\mathbf{v}_B - \mathbf{v}_0) + m(\mathbf{v}_C - \mathbf{v}_0)$$

$$0 = (v_A\mathbf{j} - v_0\mathbf{i}) + (-v_B\mathbf{j} - v_0\mathbf{i}) + (v_C\mathbf{i} - v_0\mathbf{i})$$

Resolve into components.

i: $v_C - 3v_0 = 0 \qquad v_0 = \frac{1}{3}v_C = \frac{1}{3}(15)$ $\qquad \mathbf{v}_0 = 5.00 \text{ ft/s} \longrightarrow$ ◀

j: $v_A - v_B = 0 \qquad v_B = v_A = 8.66 \text{ ft/s}$

Conservation of angular momentum about G:

$$+\circlearrowleft \; \mathbf{H}_G = 3ml^2\omega\mathbf{k} = \mathbf{r}_A \times m(\mathbf{v}_A - \mathbf{v}_0) + r_B \times (\mathbf{v}_B - \mathbf{v}_0) + \mathbf{r}_C \times (\mathbf{v}_C - \mathbf{v}_0)$$

$$3l^2\omega\mathbf{k} = (\mathbf{r}_A - \mathbf{r}_B) \times (v_A\mathbf{j}) + \mathbf{r}_C \times (v_C\mathbf{i}) - (\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)(v_0\mathbf{i})$$

$$= a\mathbf{i} \times (v_A\mathbf{j}) + (-d\,\mathbf{j}) \times (v_C\,\mathbf{i}) = (av_A + dv_C)\mathbf{k}$$

$$l^2\omega = \frac{1}{3}[(0.866)(8.66) + (0.5)(15)] = 5.00 \text{ ft/s}$$

Conservation of energy: $\qquad T_1 = 3\frac{1}{2}(ml^2\omega^2) = \frac{3}{2}ml^2\omega^2$

$$\mathbf{v}_A - \mathbf{v}_0 = 8.66\mathbf{j} - 5.00\mathbf{i} \qquad |\mathbf{v}_A - \mathbf{v}_0| = 10 \text{ ft/s}$$

$$\mathbf{v}_B - \mathbf{v}_0 = -8.66\mathbf{j} - 5.00\mathbf{i} \qquad |\mathbf{v}_B - \mathbf{v}_0| = 10 \text{ ft/s}$$

$$\mathbf{v}_C - \mathbf{v}_0 = 15\mathbf{i} - 5.00\mathbf{i} \qquad |\mathbf{v}_C - \mathbf{v}_0| = 10 \text{ ft/s}$$

HW#5 7th

PROBLEM 14.55 CONTINUED

$$T_2 = \frac{1}{2}m(\mathbf{v}_A - \mathbf{v}_0)^2 + \frac{1}{2}m(\mathbf{v}_B - \mathbf{v}_0)^2 + \frac{1}{2}m(\mathbf{v}_C - \mathbf{v}_0)^2$$

$$T_1 = T_2$$

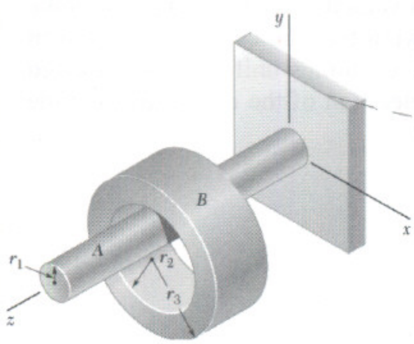
$$\frac{3}{2}ml^2\omega^2 = \frac{1}{2}m(10)^2 + \frac{1}{2}m(10)^2 + \frac{1}{2}m(10)^2$$

$$l\omega = 10 \text{ ft/s}$$

(*b*) $$l = \frac{l^2\omega}{l\omega} = \frac{5 \text{ ft}^2\text{/s}}{10 \text{ ft/s}}$$ $l = 0.500$ ft ◀

(*c*) $$\omega = \frac{l\omega}{l} = \frac{10 \text{ ft/s}}{0.5 \text{ ft}}$$ $\omega = 20.0$ rad/s ◀

HW # 5 7th



PROBLEM 15.26

Ring B has an inner radius r_2 and hangs from the horizontal shaft A as shown. Shaft A rotates with a constant angular velocity of 25 rad/s and no slipping occurs. Knowing that $r_1 = 0.5$ in., $r_2 = 2.5$ in., and $r_3 = 3.5$ in., determine (*a*) the angular velocity of ring B, (*b*) the acceleration of the points of shaft A and ring B which are in contact, (*c*) the magnitude of the acceleration of a point on the outside surface of ring B.

SOLUTION

(*a*) Let point C be the point of contact between the shaft and the ring.

$$v_C = r_1\omega_A = (0.5)(25) = 12.5 \text{ in./s}$$

$$\omega_B = \frac{v_C}{r_2} = \frac{12.5}{2.5} = 5.0 \text{ rad/s}$$

$\omega_B = 5.00$ rad/s ◀

(*b*) On shaft A:

$$a_A = r_1\omega_A^2 = (0.5)(25)^2$$

$$= 312.5 \text{ in./s}^2,$$

$\mathbf{a}_A = 26.0 \text{ ft/s}^2 \downarrow$ ◀

On ring B:

$$a_B = r_2\omega_B^2 = (2.5)(5.0)^2$$

$$= 62.5 \text{ in./s}^2,$$

$\mathbf{a}_B = 5.21 \text{ ft/s}^2 \downarrow$ ◀

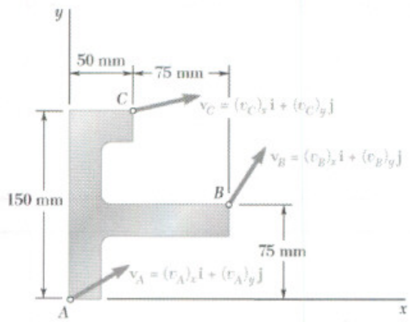
(*c*) At a point on the outside of the ring, $r = r_3 = 3.5$ in.

$$a = r\omega_B^2 = (3.5)(5.0)^2 = 87.5 \text{ in./s}^2$$

$a = 7.29 \text{ ft/s}^2$ ◀

HW#5 7th

PROBLEM 15.45



The sheet metal form shown moves in the xy plane. Knowing that $(v_A)_x = 100$ mm/s, $(v_B)_y = -75$ mm/s, and $(v_C)_x = 400$ mm/s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of point *A*.

SOLUTION

In units of mm/s,

$$\mathbf{v}_{B/A} = \omega\mathbf{k} \times \mathbf{r}_{B/A} = \omega\mathbf{k} \times (125\mathbf{i} + 75\mathbf{j}) = -75\omega\mathbf{i} + 125\omega\mathbf{j}$$

$$\mathbf{v}_{C/A} = \omega\mathbf{k} \times \mathbf{r}_{C/A} = \omega\mathbf{k} \times (50\mathbf{i} + 150\mathbf{j}) = -150\omega\mathbf{i} + 50\omega\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$(v_B)_x\mathbf{i} - 75\mathbf{j} = 100\mathbf{i} + (v_A)_y\mathbf{j} - 75\omega\mathbf{i} + 125\omega\mathbf{j}$$

Components.

$$\mathbf{i}:\ (v_B)_x = 100 - 75\omega \quad (1)$$

$$\mathbf{j}:\ -75 = (v_A)_y + 125\omega \quad (2)$$

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$$

$$400\mathbf{i} + (v_C)_y\mathbf{j} = 100\mathbf{i} + (v_A)_y\mathbf{j} - 150\omega\mathbf{i} + 50\omega\mathbf{j}$$

Components.

$$\mathbf{i}:\ 400 = 100 - 150\omega \quad (3)$$

$$\mathbf{j}:\ (v_C)_y = (v_A)_y + 125\omega \quad (4)$$

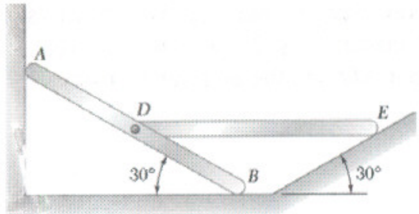
(*a*) From (3), $\omega = -2$ rad/s $\qquad \boldsymbol{\omega} = -(2\ \text{rad/s})\mathbf{k}$ ◄

(*b*) From (2), $(v_A)_y = -75 - 125\omega = -75 - 125(-2) = 175$ mm/s

$$\mathbf{v}_A = (100.0\ \text{mm/s})\mathbf{i} + (175.0\ \text{mm/s})\mathbf{j}$$ ◄

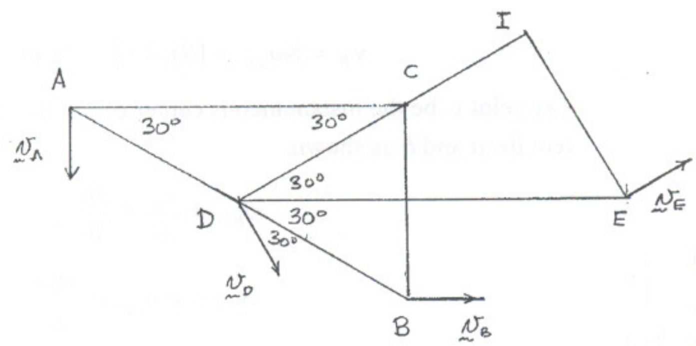
HW#5 7th

PROBLEM 15.95



Two 20-in. rods *AB* and *DE* are connected as shown. Point *D* is the midpoint of rod *AB*, and at the instant shown rod *DE* is horizontal. Knowing that the velocity of point *A* is 1 ft/s downward, determine (*a*) the angular velocity of rod *DE*, (*b*) the velocity of point *E*.

SOLUTION



$$\mathbf{v}_A = 12 \text{ in./s} \downarrow$$

$$\mathbf{v}_B = v_B \longrightarrow$$

Point *C* is the instantaneous center of bar *AB*.

$$\omega_{AB} = \frac{v_B}{AC} = \frac{12}{20\cos 30°}$$

$$= 0.69282 \text{ rad/s}$$

$$CD = 10 \text{ in.}$$

$$v_D = (CD)\omega_{AB} = (10)(0.69282) = 6.9282 \text{ in./s}$$

$$\mathbf{v}_D = 6.9282 \text{ in./s} \searrow 30°$$

$$\mathbf{v}_E = v_E \measuredangle 30°$$

Point *I* is the instantaneous center of bar *DE*.

$$DI = 20\cos 30°$$

(*a*) $\omega_{DE} = \frac{v_D}{DI} = \frac{6.9282}{20\cos 30°} = 0.4 \text{ rad/s}$ $\qquad \omega_{DE} = 0.400 \text{ rad/s}$ ◀

(*b*) $v_E = (EI)\omega_{DE} = (20\sin 30°)(0.4) = 4 \text{ in./s}$ $\qquad \mathbf{v}_E = 0.333 \text{ ft/s} \measuredangle 30°$ ◀