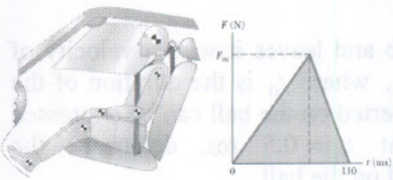
HW#4

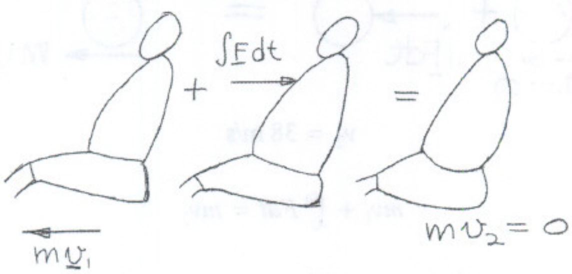
PROBLEM 13.144



An estimate of the expected load on over-the-shoulder seat belts is made before designing prototype belts that will be evaluated in automobile crash tests. Assuming that an automobile traveling at 72 km/h is brought to a stop in 110 ms, determine (*a*) the average impulsive force exerted by a 100-kg man on the belt, (*b*) the maximum force F_m exerted on the belt if the force-time diagram has the shape shown.

SOLUTION

(*a*) Force on the belt is opposite to the direction shown.



$$v_1 = 72 \text{ km/h} = 20 \text{ m/s}, \qquad m = 100 \text{ kg}$$

$$mv_1 - \int F dt = mv_2 \qquad \int F dt = F_{\text{ave}} \Delta t$$

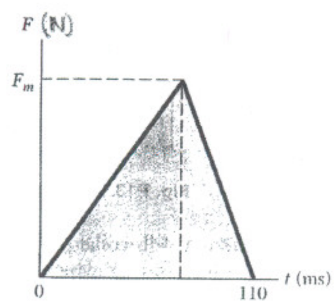
$$(100 \text{ kg})(20 \text{ m/s}) - F_{\text{ave}}(0.110 \text{ s}) = 0 \qquad \Delta t = 0.110 \text{ s}$$

$$F_{\text{ave}} = \frac{(100)(20)}{(0.110)} = 18182 \text{ N}$$

$F_{\text{ave}} = 18.18 \text{ kN}$ ◀

(*b*) Impulse = area under $F - t$ diagram

$$= \frac{1}{2} F_m (0.110 \text{ s})$$



From (*a*) Impulse = $F_{\text{ave}} \Delta t$

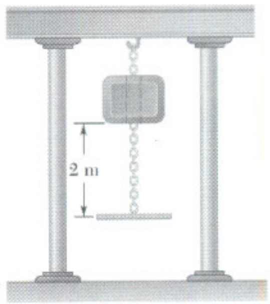
$$= (18182 \text{ N})(0.110 \text{ s})$$

$$\frac{1}{2} F_m (0.110) = 18182(0.110)$$

$F_m = 36.4 \text{ kN}$ ◀

HW#4

PROBLEM 13.152



In order to test the resistance of a chain to impact, the chain is suspended from a 120-kg rigid beam supported by two columns. A rod attached to the last link is then hit by a 30-kg block dropped from a 2-m height. Determine the initial impulse exerted on the chain and the energy absorbed by the chain, assuming that the block does not rebound from the rod and that the columns supporting the beam are (*a*) perfectly rigid, (*b*) equivalent to two perfectly elastic springs.

SOLUTION

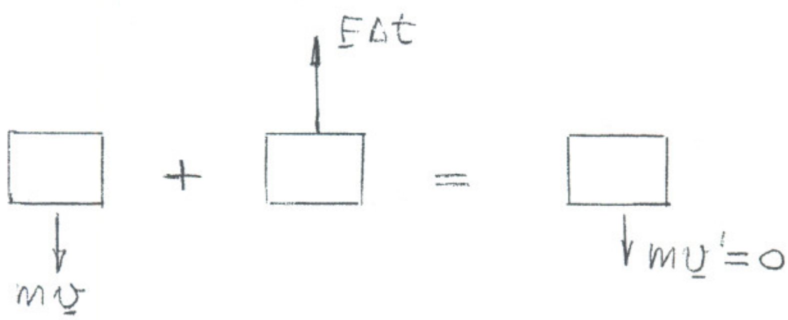
Before impact

$$T_1 = 0,\ V_1 = mgh = (30\text{ kg})(9.81\text{ m/s}^2)(2\text{ m}) = 588.6\text{ N}$$

$$T_2 = \frac{1}{2}mv^2,\ V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2:\quad 588.6 = \frac{1}{2}(30)v^2 \Rightarrow v = 6.2642\text{ m/s}$$

(*a*) Rigid columns



$$+\uparrow\ -mv + F\Delta t = 0$$

$$30(6.2642) = F\Delta t$$

$$F\Delta t = 187.93\text{ N}\cdot\text{s}\uparrow\text{ on the block}$$

$F\Delta t = 187.9\text{ N}\cdot\text{s}$ ◀

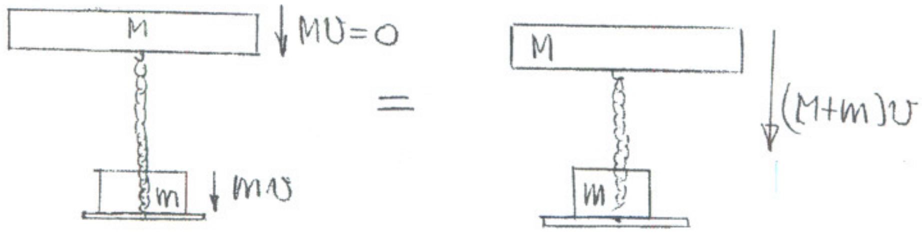
All of the kinetic energy of the block is absorbed by the chain.

$$T = \frac{1}{2}(30)(6.2642)^2 = 588.6\text{ J}$$

$E = 589\text{ J}$ ◀

PROBLEM 13.152 CONTINUED

(*b*) Elastic columns



Momentum of system of block and beam is conserved

$$mv = (M + m)v' \qquad v' = -\frac{m}{m + M}v = \frac{30}{150}(6.2642) = 1.2528 \text{ m/s}$$

Referring to figure in Part (*a*) $\qquad -mv + F\Delta t = -mv'$

$$F\Delta t = m(v - v') = 30(6.2642 - 1.2528) = 150.34$$

$F\Delta t = 150.3 \text{ N}\cdot\text{s}$ ◀

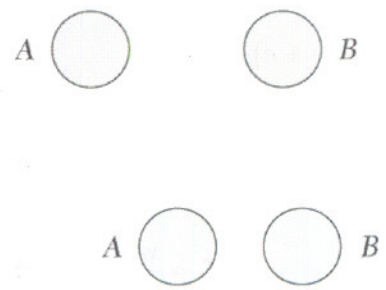
$$E = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 = \frac{30}{2}\left[(6.2642)^2 - (1.2528)^2\right] - \frac{120}{2}(1.2528)^2$$

$$= 565.06 - 94.170 = 470.89$$

$E = 471\text{J}$ ◀

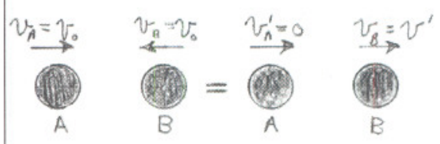
HW#4

PROBLEM 13.162



Two disks sliding on a frictionless horizontal plane with opposite velocities of the same magnitude v_0 hit each other squarely. Disk A is known to have a mass of 6 kg and is observed to have zero velocity after impact. Determine (*a*) the mass of disk B, knowing that the coefficient of restitution between the two disks is 0.5, (*b*) the range of possible values of the mass of disk B if the coefficient of restitution between the two disks is unknown.

SOLUTION



(*a*) Total momentum conserved

$$\xrightarrow{+} \quad m_A v_A + m_B v_B = m_A v_A' + m_B v'$$

$$(6\text{ kg})v_0 + m_B(-v_0) = 0 + m_B v' \Rightarrow v' = \left(\frac{6}{m_B} - 1\right)v_0 \qquad (1)$$

Relative velocities

$$\xrightarrow{+} \quad (v_A - v_B)e = v_B' - v_A' \Rightarrow v' = 2v_0 e \qquad (2)$$

From equations (1) and (2)

$$2v_0 e = \left(\frac{6}{m_B} - 1\right)v_0 \Rightarrow 2v_0(0.5) = \left(\frac{6}{m_B} - 1\right)v_0$$

$m_B = 3\text{ kg}$ ◀

(*b*) Using

$$2v_0 e = \left(\frac{6}{m_B} - 1\right)v_0$$

Gives,

$$2e + 1 = \frac{6}{m_B} \Rightarrow m_B = \frac{6}{2e + 1}$$

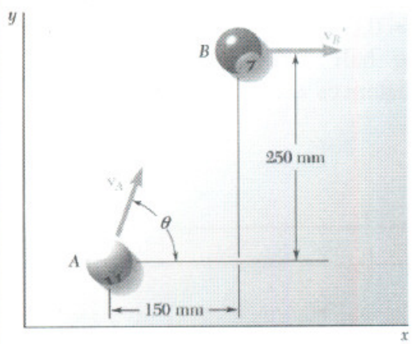
$$e = 0, \; m_B = 6\text{ kg}$$

$$e = 1, \; m_B = 2\text{ kg}$$

$2\text{ kg} \le m_B \le 6\text{ kg}$ ◀

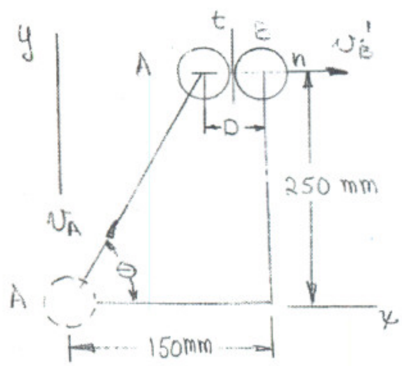
HW#4

PROBLEM 13.171



The coefficient of restitution is 0.9 between the two 60-mm-diameter billiard balls *A* and *B*. Ball *A* is moving in the direction shown with a velocity of 1 m/s when it strikes ball *B*, which is at rest. Knowing that after impact *B* is moving in the *x* direction, determine (*a*) the angle θ, (*b*) the velocity of *B* after impact.

SOLUTION



(*a*) Since v_B' is in the *x*-direction and (assuming no friction), the common tangent between *A* and *B* at impact must be parallel to the *y*-axis

Thus $$\tan\theta = \frac{250}{150 - D}$$

$$\theta = \tan^{-1}\frac{250}{150 - 60} = 70.20°$$

$\theta = 70.2°$ ◀

(*b*) Conservation of momentum in *x*(*n*) direction

$$mv_A\cos\theta + m(v_B)_n = m(v_A')_n + mv_B'$$

$$(1)\cos(70.20) + 0 = (v_A')_n + v_B'$$

$$0.3387 = (v_A')_n + (v_B') \quad (1)$$

Relative velocities in the *n* direction

$$e = 0.9 \qquad (v_A\cos\theta - (v_B)_n)e = v_B' - (v_A')_n$$

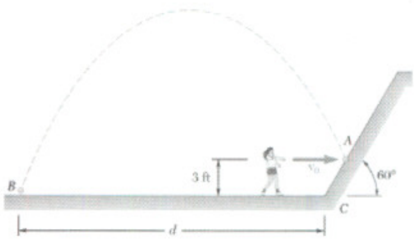
$$(0.3387 - 0)(0.9) = v_B' - (v_A')_n \quad (2)$$

(1) + (2)

$$2v_B' = 0.3387(1.9) \qquad v_B' = 0.322 \text{ m/s}$$ ◀

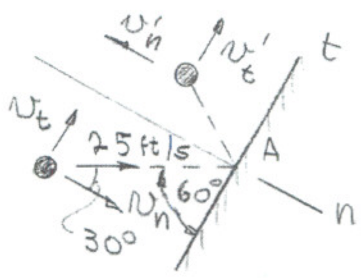
HW#4

PROBLEM 13.175



A girl throws a ball at an inclined wall from a height of 3 ft, hitting the wall at A with a horizontal velocity $\mathbf{v}_0$ of magnitude 25 ft/s. Knowing that the coefficient of restitution between the ball and the wall is 0.9 and neglecting friction, determine the distance d from the foot of the wall to the point B where the ball will hit the ground after bouncing off the wall.

SOLUTION



Momentum in t direction is conserved

$$mv\sin 30° = mv_t'$$

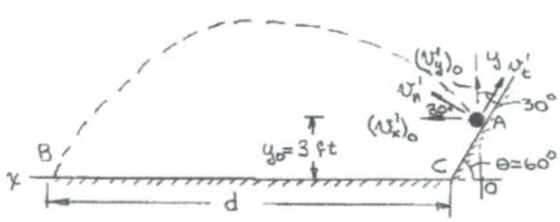
$$(25)(\sin 30°) = v_t'$$

$$v_t' = 12.5 \text{ ft/s}$$

Coefficient of restitution in n-direction

$$(v\cos 30°)e = v_n'$$

$$(25)(\cos 30°)(0.9) = v_n' \qquad v_n' = 19.49 \text{ ft/s}$$



Write v' in terms of x and y components

$$(v_x')_0 = v_n'(\cos 30°) - v_t'(\sin 30°) = 19.49(\cos 30°) - 12.5(\sin 30°)$$

$$= 10.63 \text{ ft/s}$$

$$(v_y')_0 = v_n'(\sin 30°) + v_t'(\cos 30°) = 19.49(\sin 30°) + 12.5(\cos 30°)$$

$$= 20.57 \text{ ft/s}$$

HW#4

PROBLEM 13.175 CONTINUED

Projectile motion

$$y = y_0 + \left(v'_y\right)_0 t - \frac{1}{2}gt^2 = 3\text{ ft} + (20.57\text{ ft/s})t - \left(32.2\text{ ft/s}^2\right)\frac{t^2}{2}$$

At B,

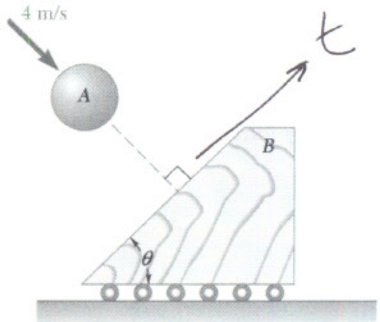
$$y = 0 = 3 + 20.57t_B - 16.1t_B^2;\; t_B = 1.4098\text{ s}$$

$$x_B = x_0 + \left(v'_x\right)_0 t_B = 0 + 10.63(1.4098);\; x_B = 14.986\text{ ft}$$

$$d = x_B - 3\cos 60° = (14.986\text{ ft}) - (3\text{ ft})\cot 60° = 13.254\text{ ft}$$

$d = 13.25$ ft ◀

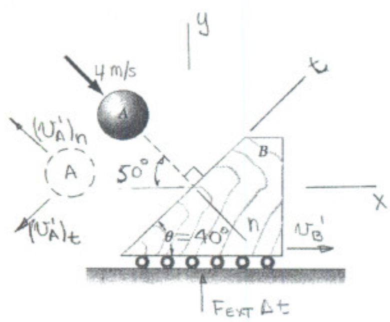
HW#4



PROBLEM 13.188

A 2-kg sphere A strikes the frictionless inclined surface of a 6-kg wedge B at a 90° angle with a velocity of magnitude 4 m/s. The wedge can roll freely on the ground and is initially at rest. Knowing that the coefficient of restitution between the wedge and the sphere is 0.50 and that the inclined surface of the wedge forms an angle $\theta = 40°$ with the horizontal, determine (a) the velocities of the sphere and of the wedge immediately after impact, (b) the energy lost due to the impact.

SOLUTION



(a) Momentum of the sphere A alone is conserved in the t-direction.

$$m_A(v_A)_t = m_A(v'_A)_t \qquad (v_A)_t = 0$$

$$(v'_A)_t = 0 \qquad (v'_A)_n = v'_A \searrow 50°$$

Total momentum is conserved in the x-direction.

$$m_A v_A \cos 50° + m_B v_B = m_A(-v'_A)\cos 50° + m_B v'_B$$

$$v_B = 0 \qquad v_A = 4 \text{ m/s}$$

$$2(4)\cos 50° + 0 = 2(-v'_A)\cos 50° + 6v'_B$$

$$5.1423 = -1.2855v'_A + 6v'_B \tag{1}$$

Relative velocities in the n-direction

$$(v_A - v_B)e = (v'_B \cos 50° + v'_A); \;\; v_B = 0, \; v_A = 4 \text{ m/s}$$

$$4(0.5) = 0.6428v'_B + v'_A; \;\; 2 = 0.6428v'_B + v'_A \tag{2}$$

Solving Equation (1) and Equation (2) simultaneously

$$v'_A = 1.2736 \text{ m/s}; \;\; v'_B = 1.1299 \text{ m/s}$$

$v'_A = 1.274$ m/s $\searrow$ 50° ◀

$v'_B = 1.130$ m/s ⟶ ◀

(b)

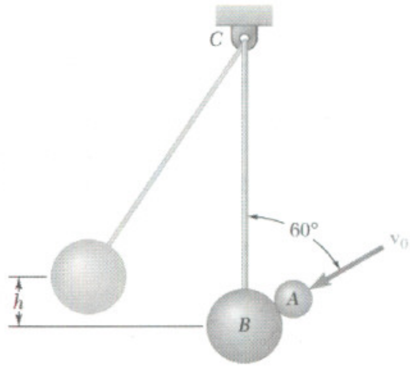
$$T_{\text{lost}} = \frac{1}{2}m_A v_A^2 - \frac{1}{2}\left[m_A(v'_A)^2 + m_B(v'_B)^2\right]$$

$$= \frac{1}{2}\Big[(2 \text{ kg})(4 \text{ m/s})^2 - (2 \text{ kg})(1.274 \text{ m/s})^2$$

$$- (6 \text{ kg})(1.130 \text{ m/s})^2\Big] = 10.546 \text{ J}$$

$T_{\text{lost}} = 10.55$ J ◀

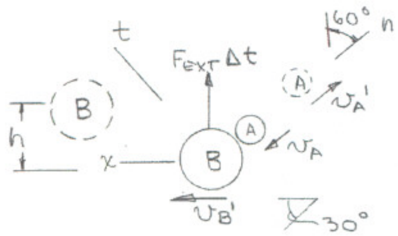
HW#4



PROBLEM 13.189

A 340-g ball B is hanging from an inextensible cord attached to a support C. A 170-g ball A strikes B with a velocity $\mathbf{v}_0$ of magnitude 1.5 m/s at an angle of 60° with the vertical. Assuming perfectly elastic impact ($e = 1$) and no friction, determine the height h reached by ball B.

SOLUTION



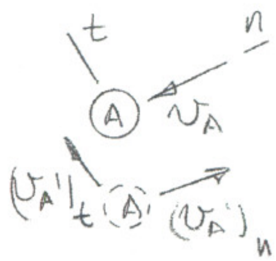
Ball A alone

Momentum in t-direction conserved

$$m_A(v_A)_t = m_A(v'_A)_t$$

$$(v_A)_t = 0 = (v'_A)_t$$

Thus $$(v'_A)_n = v'_A \angle 60°$$



Total momentum in the x-direction is conserved.

$$m_A v_A \sin 60° + m_B (v_B)_x = m_A(-v'_A)\sin 60 + m_B v'_B$$

$$v_A = v_0 = 1.5 \text{ m/s} \qquad (v_B)_x = 0$$

$$0.17(1.5)(\sin 60°) + 0 = -(0.17)(v'_A)(\sin 60°) + (0.34)v'_B$$

$$0.2208 = -0.1472 v'_A + 0.34 v'_B \tag{1}$$

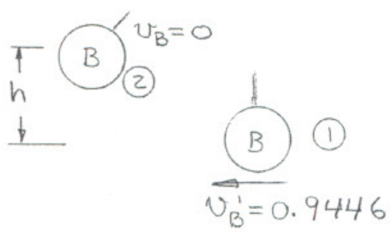
Relative velocity in the n-direction

$$\left[-v_A - (v_B)_n\right]e = -v'_B \cos 30° - v'_A;$$

$$(-1.5 - 0)(1) = -0.866 v'_B - v'_A \tag{2}$$

Solving Equations (1) *and* (2) simultaneously

$$v'_B = 0.9446 \text{ m/s}, \; v'_A = 0.6820 \text{ m/s}$$



Conservation of energy ball B

$$T_1 = \frac{1}{2} m_B (v'_B)^2$$

$$T_1 = \frac{1}{2}\frac{W_B}{g}(3.0232)^2 \qquad T_2 = 0$$

HW#4

PROBLEM 13.189 CONTINUED

$$V_1 = 0 \qquad V_2 = W_B h$$

$$T_1 + V_1 = T_2 + V_2; \quad \frac{1}{2}\frac{W_B}{g}(0.9446)^2 = 0 + W_B h;$$

$$h = \frac{(0.9446)^2}{(2)(9.81)} = 0.0455 \text{ m}$$

$h = 45.5$ mm ◀