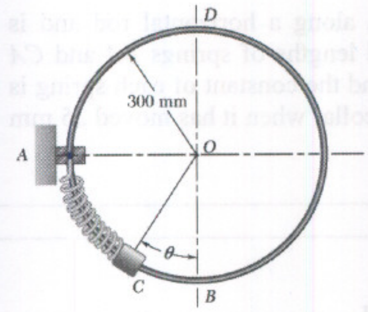
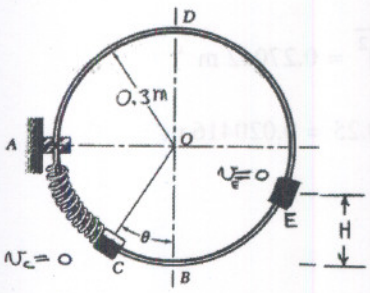
HW #3

PROBLEM 13.61



A thin circular rod is supported in a *vertical plane* by a bracket at A. Attached to the bracket and loosely wound around the rod is a spring of constant $k = 40$ N/m and undeformed length equal to the arc of circle AB. A 200-g collar C, not attached to the spring, can slide without friction along the rod. Knowing that the collar is released from rest when $\theta = 30°$, determine (*a*) the maximum height above point B reached by the collar, (*b*) the maximum velocity of the collar.

SOLUTION



(*a*) Maximum height

Above B is reached when the velocity at E is zero

$$T_C = 0$$

$$T_E = 0$$

$$V = V_e + V_g$$

Point C

$$\theta = 30° = \frac{\pi}{6}\text{ rad}$$

$$R = 0.3\text{ m}$$

$$\Delta L_{BC} = (0.3\text{ m})\left(\frac{\pi}{6}\text{ rad}\right)$$

$$\Delta L_{BC} = \frac{\pi}{20}\text{ m}$$

$$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2 = \frac{1}{2}(40\text{ N/m})\left(\frac{\pi}{20}\text{ m}\right)^2 = 0.4935\text{ J}$$

$$(V_C)_g = WR(1-\cos\theta) = (0.2\text{ kg}\times 9.81\text{ m/s}^2)(0.3\text{ m})(1-\cos 30°)$$

$$(V_C)_g = 0.07886\text{ J}$$

$$(V_E)_e = 0 \qquad \text{(spring is unattached)}$$

$$(V_E)_g = WH = (0.2\times 9.81)(H) = 1.962H\text{ (J)}$$

$$T_C + V_C = T_E + V_E$$

$$0 + 0.4935 + 0.07886 = 0 + 0 + 1.962H$$

$$H = 0.292\text{ m} \blacktriangleleft$$

HW #3

PROBLEM 13.61 CONTINUED

(*b*) The maximum velocity is at *B* where the potential energy is zero, $v_B = v_{max}$

$$T_C = 0 \qquad V_C = 0.4935 + 0.07886 = 0.5724 \text{ J}$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.2 \text{ kg})v_{max}^2$$

$$T_B = 0.1v_{max}^2$$

$$V_B = 0$$

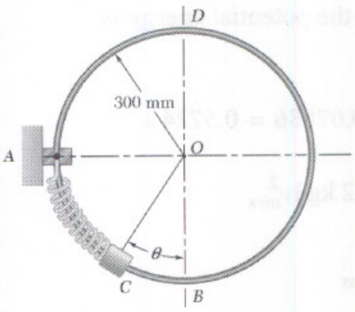
$$T_C + V_C = T_B + V_B \qquad 0 + 0.5724 = (0.1)v_{max}^2$$

$$v_{max}^2 = 5.72 \text{ m}^2/\text{s}^2$$

$$v_{max} = 2.39 \text{ m/s} \blacktriangleleft$$

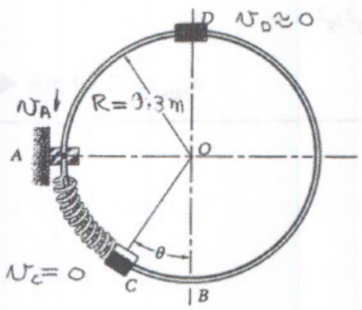
HW#3

PROBLEM 13.62



A thin circular rod is supported in a *vertical plane* by a bracket at *A*. Attached to the bracket and loosely wound around the rod is a spring of constant $k = 40$ N/m and undeformed length equal to the arc of circle *AB*. A 200-g collar *C*, not attached to the spring, can slide without friction along the rod. Knowing that the collar is released from rest at an angle θ with respect to the vertical, determine (*a*) the smallest value of θ for which the collar will pass through *D* and reach point *A*, (*b*) the velocity of the collar as it reaches point *A*.

SOLUTION



$$R = 0.3 \text{ m}$$

$$W = (0.2 \text{ kg}) \times (9.81 \text{ m/s}^2)$$
$$= 1.962 \text{ N}$$

(*a*) *Smallest angle* θ occurs when the *velocity at D is close to zero*

$$v_C = 0 \qquad v_D = 0$$
$$T_C = 0 \qquad T_D = 0$$
$$V = V_e + V_g$$

Point C

$$\Delta L_{BC} = (0.3 \text{ m})\theta = 0.3\theta \text{ m}$$
$$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2$$
$$(V_C)_e = 1.8\theta^2$$
$$(V_C)_g = WR(1-\cos\theta)$$
$$(V_C)_g = (1.962 \text{ N})(0.3 \text{ m})(1-\cos\theta)$$
$$V_C = (V_C)_e + (V_C)_g = 1.8\theta^2 + 0.5886(1-\cos\theta)$$

Point D

$$(V_D)_e = 0 \qquad \text{(spring is unattached)}$$
$$(V_D)_g = W(2R) = (2)(1.962 \text{ N})(0.3 \text{ m}) = 1.1772 \text{ J}$$

$$T_C + V_C = T_D + V_D; \qquad 0 + 1.8\theta^2 + 0.5586(1-\cos\theta) = 1.1772 \text{ J}$$
$$(1.8)\theta^2 - (0.5886)\cos\theta = 0.5886$$

By trial $\qquad \theta = 0.7522$ rad

$\theta = 43.1°$ ◀

HW #3

PROBLEM 13.62 CONTINUED

(b) *Velocity at A*

Point D

$$V_D = 0 \quad T_D = 0 \quad V_D = 1.1772 \text{ J (see Part } (a))$$

Point A

$$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.2 \text{ kg})v_A^2$$

$$T_A = 0.1v_A^2$$

$$V_A = (V_A)_g = W(R) = (1.962 \text{ N})(0.3 \text{ m}) = 0.5886 \text{ J}$$

$$T_A + V_A = T_D + V_D$$

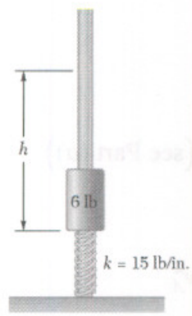
$$0.1v_A^2 + 0.5886 = 0 + 1.1772$$

$$v_A^2 = 5.886 \text{ m}^2/\text{s}^2$$

$$v_A = 2.43 \text{ m/s} \blacktriangleleft$$

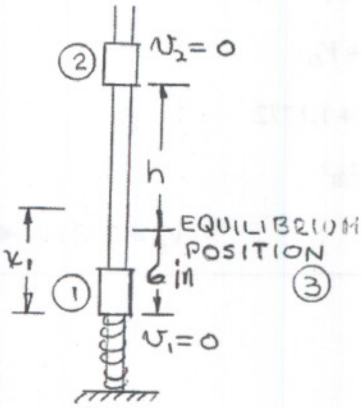
HW#3

PROBLEM 13.63



A 6-lb collar can slide without friction on a vertical rod and is resting in equilibrium on a spring. It is pushed down, compressing the spring 6 in., and released. Knowing that the spring constant is $k = 15$ lb/in., determine (*a*) the maximum height h reached by the collar above its equilibrium position, (*b*) the maximum velocity of the collar.

SOLUTION



(*a*) Maximum height when $v_2 = 0$

$$\therefore T_1 = T_2 = 0$$

$$V = V_g + V_e$$

Position ① $(V_g)_1 = 0$

$$x_1 = \frac{6 \text{ lb}}{15 \text{ lb/in.}} + 6 \text{ in.} = 0.4 + 6 = 6.4 \text{ in.}$$

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(15 \text{ lb/in.})(6.4 \text{ in.})^2$$

$$= 307.2 \text{ lb}\cdot\text{in.} = 25.6 \text{ lb}\cdot\text{ft}$$

Position ② $(V_g)_2 = mg\left(\frac{6}{12} + h\right) = 6(0.5 + h)$

$$(V_e)_2 = 0$$

$$T_1 + V_1 = T_2 + V_2: \ (V_g)_1 + (V_e)_1 = (V_g)_2 + (V_e)_2$$

$$25.6 = 6(0.5 + h)$$

$h = 3.767$ ft $h = 45.2$ in. ◀

(*b*) Maximum velocity occurs when acceleration is 0, equilibrium position

$$T_3 = \frac{1}{2}mv_3^2 = \frac{1}{2}\left(\frac{6}{32.2}\right)v_3^2 = 0.093167v_3^2$$

$$V_3 = (V_g)_3 + (V_e)_3 = 6(6) + \frac{1}{2}k(x_1 - 6)^2 = 36 + 7.5(6.4 - 6)^2$$

$$= 37.2 \text{ lb}\cdot\text{in.} = 3.1 \text{ lb}\cdot\text{ft}$$

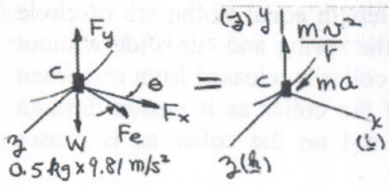
$$T_1 + V_1 = T_3 + V_3: \ 25.6 = 0.093167v_3^2 + 3.1$$

$v_{max} = 15.54$ ft/s ◀

PROBLEM 13.69 CONTINUED

(*b*) Force of rod on collar *AC*

$$F_z = 0 \quad \text{(no friction)}$$



$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

$$\theta = \tan^{-1}\frac{75}{300} = 14.04°$$

$$\mathbf{F_e} = (k\Delta L_{AC})(\cos\theta\mathbf{i} + \sin\theta\mathbf{k})$$

$$\mathbf{F_e} = (320)(0.10923)(\cos 14.04°\mathbf{i} + \sin 14.04°\mathbf{k})$$

$$\mathbf{F_e} = 33.909\mathbf{i} + 8.4797\mathbf{k}\ \text{(N)}$$

$$\Sigma\mathbf{F} = (F_x + 33.909)\mathbf{i} + (F_y - 4.905)\mathbf{j} + 8.4797\mathbf{k} = \frac{mv^2}{r}\mathbf{j} + mg\mathbf{k}$$

$$F_x + 33.909\ \text{N} = 0 \qquad F_y = 4.905\ \text{N} + (0.5)\frac{(8.5212\ \text{m}^2/\text{s}^2)}{0.15\ \text{m}}$$

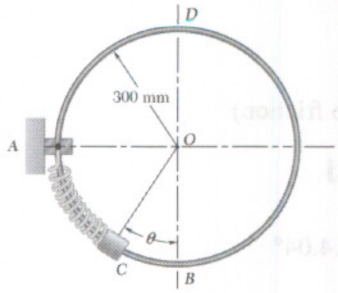
$$F_x = -33.909\ \text{N}$$

$$F_y = 33.309\ \text{N}$$

$$\mathbf{F} = -33.9\ \text{N}\,\mathbf{i} + 33.3\ \text{N}\,\mathbf{j}$$ ◀

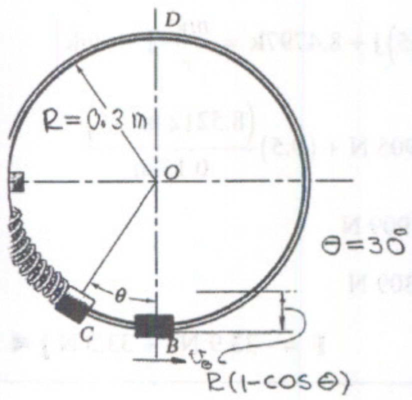
HW #3

PROBLEM 13.70



A thin circular rod is supported in a *vertical plane* by a bracket at *A*. Attached to the bracket and loosely wound around the rod is a spring of constant $k = 40$ N/m and undeformed length equal to the arc of circle *AB*. A 200-g collar *C* is unattached to the spring and can slide without friction along the rod. Knowing that the collar is released from rest when $\theta = 30°$, determine (*a*) the velocity of the collar as it passes through point *B*, (*b*) the force exerted by the rod on the collar as is passes through *B*.

SOLUTION



(*a*)

$$v_C = 0, \qquad T_C = 0$$

$$T_B = \frac{1}{2}mv_B^2$$

$$T_B = \frac{1}{2}(0.2\text{ kg})v_B^2$$

$$T_B = 0.1v_B^2 \qquad V_C = (V_C)_e + (V_C)_g$$

$$\text{arc } BC = \Delta L_{BC} = R\theta$$

$$\Delta L_{BC} = (0.3\text{ m})(30°)\frac{(\pi)}{180°}$$

$$\Delta L_{BC} = 0.15708\text{ m}$$

$$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2 = \frac{1}{2}(40\text{ N/m})(0.15708\text{ m})^2 = 0.49348\text{ J}$$

$$(V_C)_g = WR(1-\cos\theta) = (0.2\text{ kg})(9.81\text{ m/s}^2)(0.3\text{ m})(1-\cos 30°)$$

$$(V_C)_g = 0.078857\text{ J}$$

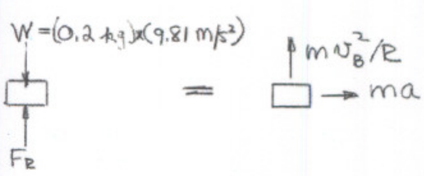
$$V_C = (V_C)_e + (V_C)_g = 0.49348\text{ J} + 0.078857\text{ J} = 0.57234\text{ J}$$

$$V_B = (V_B)_e + (V_B)_g = 0 + 0 = 0$$

$$T_C + V_C = T_B + V_B; \qquad 0 + 0.57234 = 0.1v_B^2$$

$$v_B^2 = 5.7234\text{ m}^2/\text{s}^2 \qquad v_B = 2.39\text{ m/s} \blacktriangleleft$$

(*b*)

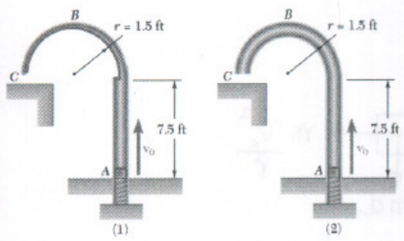


$$+\uparrow \Sigma F = F_R - W = \frac{mv_B^2}{R}$$

$$F_R = 1.962\text{ N} + (0.2\text{ kg})\frac{(5.7234\text{ m}^2/\text{s}^2)}{(0.3\text{ m})}$$

$$F_R = 1.962\text{ N} + 3.8156\text{ N} = 5.7776\text{ N} \qquad F_R = 5.78\text{ N} \blacktriangleleft$$

HW #3



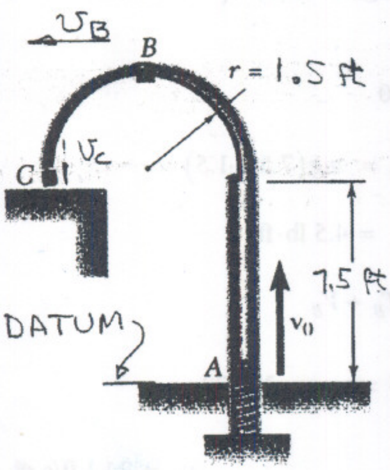
PROBLEM 13.75

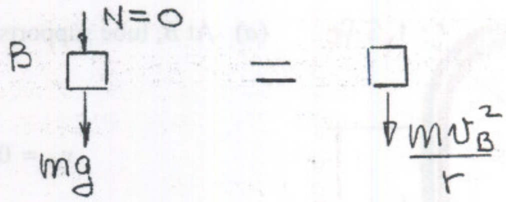
An 8-oz package is projected upward with a velocity $\mathbf{v}_0$ by a spring at A; it moves around a frictionless loop and is deposited at C. For each of the two loops shown, determine (*a*) the smallest velocity $\mathbf{v}_0$ for which the package will reach C, (*b*) the corresponding force exerted by the package on the loop just before the package leaves the loop at C.

SOLUTION

Loop 1

(*a*) The smallest velocity at B will occur when the force exerted by the tube on the package is zero.





$$+\downarrow \Sigma F = 0 + mg = \frac{mv_B^2}{r}$$

$$v_B^2 = rg = 1.5\text{ ft}\left(32.2\text{ ft/s}^2\right)$$

$$v_B^2 = 48.30$$

At A $\quad T_A = \frac{1}{2}m\,v_0^2$

$$V_A = 0 \quad \left(8\text{ oz} = 0.5\text{ lb} \Rightarrow = \frac{0.5}{32.2} = 0.01553\right)$$

At B $\quad T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(48.30) = 24.15\text{ m}$

$$V_B = mg(7.5 + 1.5) = 9mg = 9(0.5) = 4.5\text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_B + V_B:\quad \frac{1}{2}(0.01553)v_0^2 = 24.15(0.01553) + 4.5$$

$$v_0^2 = 627.82 \qquad v_0 = 25.056 \qquad v_0 = 25.1\text{ ft/s} \blacktriangleleft$$

At C

$$T_C = \frac{1}{2}mv_C^2 = 0.007765v_C^2 \qquad V_C = 7.5mg = 7.5(0.5) = 3.75$$

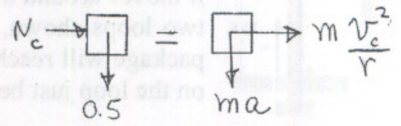
$$T_A + V_A = T_C + V_C:\quad 0.007765v_0^2 = 0.007765v_C^2 + 3.75$$

$$0.007765(25.056)^2 - 3.75 = 0.007765v_C^2$$

$$v_C^2 = 144.87$$

PROBLEM 13.75 CONTINUED

(*b*)

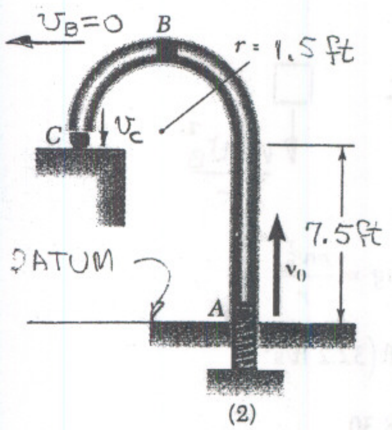


$$\xrightarrow{+} \Sigma F = ma_n: \quad N = 0.01553\frac{(144.87)}{1.5}$$

$$N = 1.49989$$

{Package in tube} $N_C = 1.500$ lb ⟵ ◀

Loop 2



(*a*) At *B*, tube supports the package so,

$$v_B \approx 0$$

$$v_B = 0,\ T_B = 0 \qquad V_B = mg(7.5 + 1.5)$$

$$= 4.5 \text{ lb} \cdot \text{ft}$$

$$T_A + V_A = T_B + V_B$$

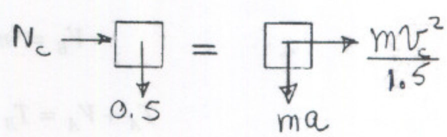
$$\frac{1}{2}(0.01553)v_A^2 = 4.5 \Rightarrow v_A = 24.073$$

$v_A = 24.1$ ft/s ◀

(*b*) At *C* $\qquad T_C = 0.007765v_C^2,\ V_C = 7.5mg = 3.75$

$$T_A + V_A = T_C + V_C: \quad 0.007765(24.073)^2 = 0.007765v_C^2 + 3.75$$

$$v_C^2 = 96.573$$

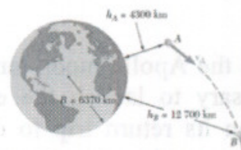


$$N_C = 0.01553\left(\frac{96.573}{1.5}\right) = 0.99985$$

{Package on tube} $N_C = 1.000$ lb ◀

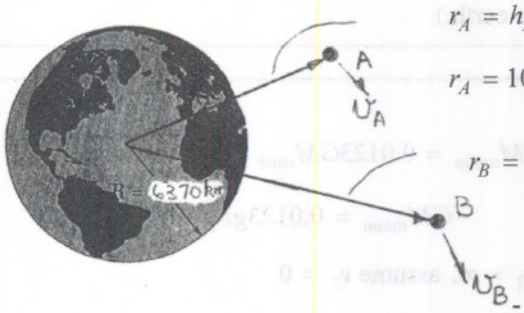
HW #3

PROBLEM 13.83



Knowing that the velocity of an experimental space probe fired from the earth has a magnitude $v_A = 32.5$ Mm/h at point A, determine the velocity of the probe as it passes through point B.

SOLUTION



$$r_A = h_A + R = 4.3Mm + 6.37Mm$$

$$r_A = 10.67Mm$$

$$r_B = h_B + R = 72.7Mm + 6.37Mm$$

$$r_B = 19.07Mm$$

At A,

$$v_A = 32.5\text{ Mm/h} = 9028\text{ m/s}$$

$$T_A = \frac{1}{2}m(9028\text{ m/s})^2 = 40.752 \times 10^6\text{ m}$$

$$V_A = -\frac{GMm}{r_A} = \frac{-gR^2m}{r_A}$$

$$r_A = 10.67\text{ Mm} = 10.67 \times 10^6\text{ m}$$

$$R = 6370\text{ km} = 6.37 \times 10^6\text{ m}$$

$$V_A = -\frac{(9.81\text{ m/s}^2)(6.37 \times 10^6\text{ m})^2}{(10.67 \times 10^6\text{ m})}m = -37.306 \times 10^6 m$$

At B

$$T_B = \frac{1}{2}mv_B^2;\ V_B = -\frac{GMm}{r_B} = \frac{-gR^2m}{r_B}$$

$$r_B = 19.07\text{ Mm} = 19.07 \times 10^6\text{ m}$$

$$V_B = -\frac{(9.81\text{ m/s}^2)(6.37 \times 10^6\text{ m})^2\ m}{(19.07 \times 10^6\text{ m})} = -20.874 \times 10^6 m$$

$$T_A + V_A = T_B + V_B;\ 40.752 \times 10^6 m - 37.306 \times 10^6 m = \frac{1}{2}mv_B^2 - 20.874 \times 10^6 m$$

$$v_B^2 = 2\left[40.752 \times 10^6 - 37.306 \times 10^6 + 20.874 \times 10^6\right]$$

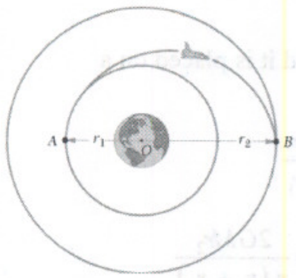
$$v_B^2 = 48.64 \times 10^6\text{ m}^2/\text{s}^2$$

$$v_B = 6.9742 \times 10^3\text{ m/s} = 25.107\text{ Mm/h}$$

$v_B = 25.1$ Mm/h ◀

HW#3

PROBLEM 13.116



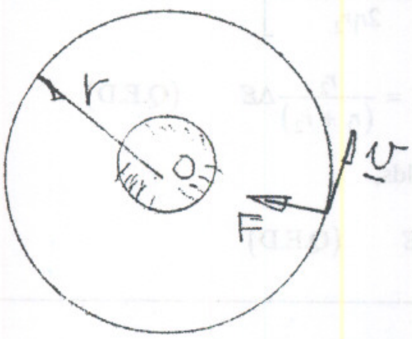
A spacecraft of mass m describes a circular orbit of radius r_1 around the earth. (*a*) Show that the additional energy ΔE which must be imparted to the spacecraft to transfer it to a circular orbit of larger radius r_2 is

$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2}$$

where M is the mass of the earth. (*b*) Further show that if the transfer from one circular orbit to the other circular orbit is executed by placing the spacecraft on a transitional semielliptic path AB, the amounts of energy ΔE_A and ΔE_B which must be imparted at A and B are, respectively, proportional to r_1 and r_2:

$$\Delta E_A = \frac{r_2}{r_1 + r_2}\Delta E \qquad \Delta E_B = \frac{r_1}{r_1 + r_2}\Delta E$$

SOLUTION



(*a*) For a *circular* orbit of radius r

$$F = ma_n: \quad \frac{GMm}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM}{r}$$

$$E = T + V = \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r} \qquad (1)$$

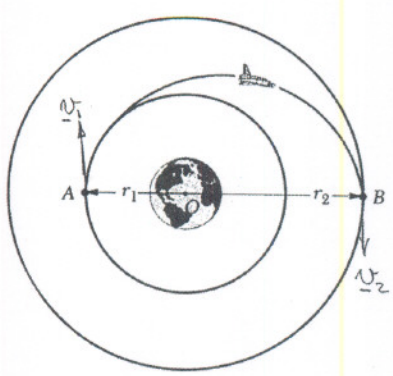
Thus ΔE required to pass from circular orbit of radius r_1 to circular orbit of radius r_2 is

$$\Delta E = E_1 - E_2 = -\frac{1}{2}\frac{GMm}{r_1} + \frac{1}{2}\frac{GMm}{r_2}$$

$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2} \qquad (2) \text{ (Q.E.D.)}$$

(*b*) For an *elliptic orbit* we recall Equation (3) derived in Problem 13.113 (with $v_P = v_1$)

$$v_1^2 = \frac{2Gm}{(r_1 + r_2)}\frac{r_2}{r_1}$$



At point A: Initially spacecraft is in a circular orbit of radius r_1

$$v_{circ}^2 = \frac{GM}{r_1}$$

$$T_{circ} = \frac{1}{2}mv_{circ}^2 = \frac{1}{2}m\frac{GM}{r_1}$$

PROBLEM 13.116 CONTINUED

After the spacecraft engines are fired and it is placed on a semi-elliptic path AB, we recall

$$v_1^2 = \frac{2GM}{(r_1 + r_2)} \cdot \frac{r_2}{r_1}$$

And $$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1 + r_2)}$$

At point A, the increase in energy is

$$\Delta E_A = T_1 - T_{\text{circ}} = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1 + r_2)} - \frac{1}{2}m\frac{GM}{r_1}$$

$$\Delta E_A = \frac{GMm(2r_2 - r_1 - r_2)}{2r_1(r_1 + r_2)} = \frac{GMm(r_2 - r_1)}{2r_1(r_1 + r_2)}$$

$$\Delta E_A = \frac{r_2}{r_1 + r_2}\left[\frac{GMm(r_2 - r_1)}{2r_1r_2}\right]$$

Recall Equation (2): $$\Delta E_A = \frac{r_2}{(r_1 + r_2)}\Delta E \qquad \text{(Q.E.D)}$$

A similar derivation at point B yields,

$$\Delta E_B = \frac{r_1}{(r_1 + r_2)}\Delta E \qquad \text{(Q.E.D)}$$