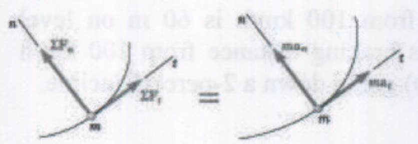
PROBLEM 12.9

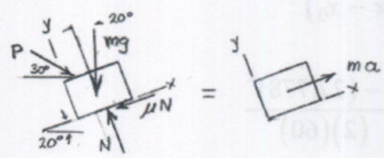


A 40-kg package is at rest on an incline when a force **P** is applied to it. Determine the magnitude of **P** if 4 s is required for the package to travel 10 m up the incline. The static and kinetic coefficients of friction between the package and the incline are 0.30 and 0.25, respectively.

SOLUTION

Kinematics: Uniformly accelerated motion. $(x_0 = 0,\ v_0 = 0)$

$$x = x_0 + v_0 t + \frac{1}{2}at^2, \qquad \text{or} \qquad a = \frac{2x}{t^2} = \frac{(2)(10)}{(4)^2} = 1.25 \text{ m/s}^2$$



$$+\nwarrow \Sigma F_y = 0: \quad N - P\sin 50° - mg\cos 20° = 0$$

$$N = P\sin 50° + mg\cos 20°$$

$$+\nearrow \Sigma F_x = ma: \quad P\cos 50° - mg\sin 20° - \mu N = ma$$

$$\text{or} \quad P\cos 50° - mg\sin 20° - \mu\left(P\sin 50° + mg\cos 20°\right) = ma$$

$$P = \frac{ma + mg\left(\sin 20° + \mu\cos 20°\right)}{\cos 50° - \mu\sin 50°}$$

For motion impending, set $a = 0$ and $\mu = \mu_s = 0.30$.

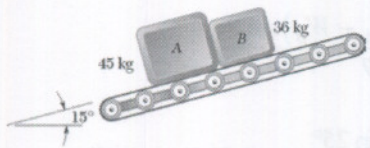
$$P = \frac{(40)(0) + (40)(9.81)(\sin 20° + 0.30\cos 20°)}{\cos 50° - 0.30\sin 50°} = 593 \text{ N}$$

For motion with $a = 1.25 \text{ m/s}^2$, use $\mu = \mu_k = 0.25$.

$$P = \frac{(40)(1.25) + (40)(9.81)(\sin 20° + 0.25\cos 20°)}{\cos 50° - 0.25\sin 50°}$$

$P = 612$ N ◀

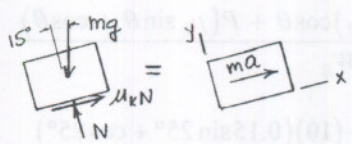
PROBLEM 12.17



Boxes A and B are at rest on a conveyor belt that is initially at rest. The belt is suddenly started in an upward direction so that slipping occurs between the belt and the boxes. Knowing that the coefficients of kinetic friction between the belt and the boxes are $(\mu_k)_A = 0.30$ and $(\mu_k)_B = 0.32$, determine the initial acceleration of each box.

SOLUTION

Assume $\mathbf{a}_B > \mathbf{a}_A$ so that the boxes separate. Boxes are slipping.



$$\mu = \mu_k$$

$$\Sigma F_y = 0: \quad N - mg\cos 15° = 0$$

$$N = mg\cos 15°$$

$$\Sigma F_x = ma: \quad \mu_k N - mg\sin 15° = ma$$

$$\mu_k mg\cos 15° - mg\sin 15° = ma$$

$$a = g(\mu_k \cos 15° - \sin 15°), \quad \text{independent of } m.$$

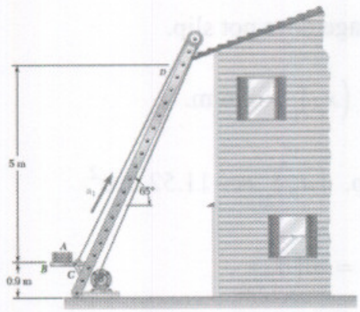
For box A, $\mu_k = 0.30$

$$a_A = 9.81(0.30\cos 15° - \sin 15°) \qquad \text{or } \mathbf{a}_A = 0.304 \text{ m/s}^2 \measuredangle 15° \blacktriangleleft$$

For box B, $\mu_k = 0.32$

$$a_B = 9.81(0.32\cos 15° - \sin 15°) \qquad \text{or } \mathbf{a}_B = 0.493 \text{ m/s}^2 \measuredangle 15° \blacktriangleleft$$

PROBLEM 12.22

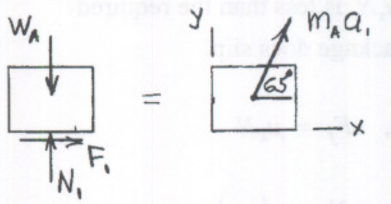


To transport a series of bundles of shingles A to a roof, a contractor uses a motor-driven lift consisting of a horizontal platform BC which rides on rails attached to the sides of a ladder. The lift starts from rest and initially moves with a constant acceleration $\mathbf{a}_1$ as shown. The lift then decelerates at a constant rate $\mathbf{a}_2$ and comes to rest at D, near the top of the ladder. Knowing that the coefficient of static friction between the bundle of shingles and the horizontal platform is 0.30, determine the largest allowable acceleration $\mathbf{a}_1$ and the largest allowable deceleration $\mathbf{a}_2$ if the bundle is not to slide on the platform.

SOLUTION

Acceleration $\mathbf{a}_1$: Impending slip. $\quad F_1 = \mu_s N_1 = 0.30\,N_1$

$$\Sigma F_y = m_A a_y: \quad N_1 - W_A = m_A a_1 \sin 65^\circ$$



$$N_1 = W_A + m_A a_1 \sin 65^\circ$$

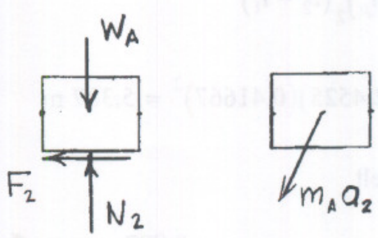
$$= m_A\left(g + a_1 \sin 65^\circ\right)$$

$$\xrightarrow{+} \Sigma F_x = m_A a_x: \quad F_1 = m_A a_1 \cos 65^\circ$$

$$F_1 = \mu_s N \quad \text{or} \quad m_A a_1 \cos 65^\circ = 0.30 m_A\left(g + a_1 \sin 65^\circ\right)$$

$$a_1 = \frac{0.30g}{\cos 65^\circ - 0.30 \sin 65^\circ} = (1.990)(9.81) = 19.53 \text{ m/s}^2$$

$$\mathbf{a}_1 = 19.53 \text{ m/s}^2 \measuredangle 65^\circ \blacktriangleleft$$



Deceleration $\mathbf{a}_2$: Impending slip. $\quad F_2 = \mu_S N_2 = 0.30\,N_2$

$$\Sigma F_y = m a_y: \quad N_1 - W_A = -m_A a_2 \sin 65^\circ$$

$$N_1 = W_A - m_A a_2 \sin 65^\circ$$

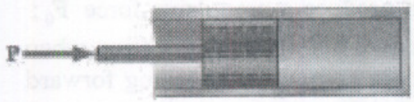
$$\xleftarrow{+} \Sigma F_x = m a_x: \quad F_2 = m_A a_2 \cos 65^\circ$$

$$F_2 = \mu_S N_2 \qquad \text{or} \qquad m_A a_2 \cos 65^\circ = 0.30 m_A\left(g - a_2 \cos 65^\circ\right)$$

$$a_2 = \frac{0.30g}{\cos 65^\circ + 0.30 \sin 65^\circ} = (0.432)(9.81) = 4.24 \text{ m/s}^2$$

$$\mathbf{a}_2 = 4.24 \text{ m/s}^2 \nearrow 65^\circ \blacktriangleleft$$

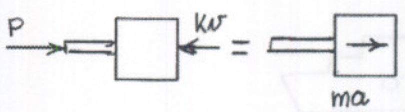
PROBLEM 12.25



A constant force **P** is applied to a piston and rod of total mass m to make them move in a cylinder filled with oil. As the piston moves, the oil is forced through orifices in the piston and exerts on the piston a force of magnitude kv in a direction opposite to the motion of the piston. Knowing that the piston starts form rest at $t = 0$ and $x = 0$, show that the equation relating x, v, and t, where x is the distance traveled by the piston and v is the speed of the piston, is linear in each of the variables.

SOLUTION

$$\xrightarrow{+} \Sigma F = ma: \quad P - kv = ma$$



$$\frac{dv}{dt} = a = \frac{P - kv}{m}$$

$$\int_0^t dt = \int_0^v \frac{m\,dv}{P - kv} = -\frac{m}{k}\ln(P - kv)\Big|_0^v = -\frac{m}{k}\left[\ln(P - kv) - \ln P\right]$$

$$t = -\frac{m}{k}\ln\frac{P - kv}{P} \quad \text{or} \quad \ln\frac{P - kv}{m} = -\frac{kt}{m}$$

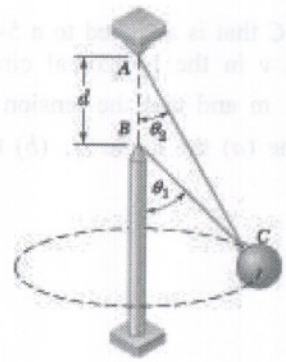
$$\frac{P - kv}{m} = e^{-kt/m} \quad \text{or} \quad v = \frac{P}{k}\left(1 - e^{-kt/m}\right)$$

$$x = \int_0^t v\,dt = \frac{Pt}{k}\Big|_0^t - \frac{P}{k}\left(-\frac{k}{m}e^{-kt/m}\right)\Big|_0^t$$

$$= \frac{Pt}{k} + \frac{P}{m}\left(e^{-kt/m} - 1\right) = \frac{Pt}{k} - \frac{P}{m}\left(1 - e^{-kt/m}\right)$$

$$x = \frac{Pt}{k} - \frac{kv}{m}, \quad \text{which is linear.}$$ ◀

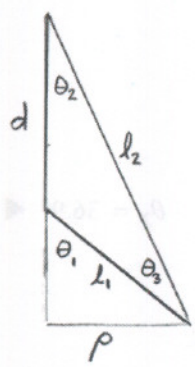
PROBLEM 12.36



Two wires AC and BC are tied to a 15-lb sphere which revolves at a constant speed v in the horizontal circle shown. Knowing that $\theta_1 = 50°$ and $\theta_2 = 25°$ and that $d = 4$ ft, determine the range of values of v for which both wires are taut.

SOLUTION

$$\theta_3 = \theta_1 - \theta_2 = 50° - 25° = 25°$$



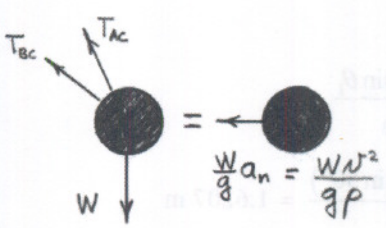
$$\frac{\ell_1}{\sin\theta_2} = \frac{d}{\sin\theta_3} \quad \text{or} \quad \ell_1 = \frac{d\sin\theta_2}{\sin\theta_3}$$

$$\rho = \ell_1 \sin\theta_1 = \frac{d\sin\theta_2\sin\theta_1}{\sin\theta_3}$$

$$= \frac{(4)(\sin 25°)(\sin 50°)}{\sin 25°} = 3.0642 \text{ ft}$$

$$\Sigma F_y = 0: \quad T_{AC}\cos\theta_2 + T_{BC}\cos\theta_1 - W = 0 \qquad (1)$$

$$\xleftarrow{+} \Sigma F_x = ma_x: \quad T_{AC}\sin\theta_2 + T_{BC}\sin\theta_1 = \frac{Wv^2}{g\rho} \qquad (2)$$



Case 1: $T_{BC} = 0$.

$$T_{AC}\cos\theta_2 - W = 0 \quad \text{or} \quad T_{AC} = \frac{W}{\cos\theta_2}$$

$$T_{AC}\sin\theta_2 = W\tan\theta_2 = \frac{Wv^2}{g\rho}$$

$$v^2 = g\rho\tan\theta_2 = (32.2)(3.0642)\tan 25°$$

$$= 46.01 \text{ ft}^2/\text{s}^2$$

$$v = 6.78 \text{ ft/s}$$

Case 2: $T_{AC} = 0$. $\quad T_{BC}\cos\theta_1 - W = 0 \quad \text{or} \quad T_{BC} = \dfrac{W}{\cos\theta_1}$

$$T_{BC}\sin\theta_1 = W\tan\theta_1 = \frac{Wv^2}{g\rho}$$

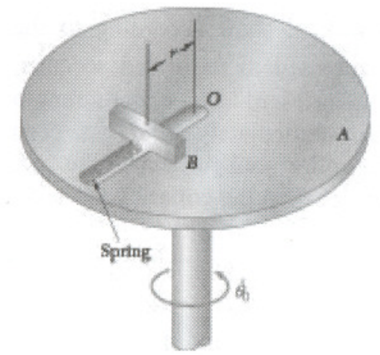
PROBLEM 12.36 CONTINUED

$$v^2 = g\rho\tan\theta_1 = (9.81)(3.0642)\tan 50° = 117.59\ \text{ft}^2/\text{s}^2$$

$$v = 10.84\ \text{ft/s}$$

$$6.78\ \text{ft/s} \le v \le 10.84\ \text{ft/s}\ ◀$$

PROBLEM 12.70

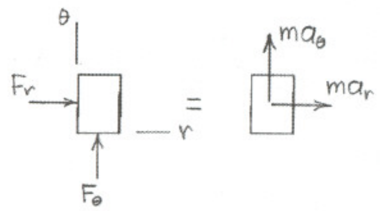


Disk A rotates in a horizontal plane about a vertical axis at the constant rate of $\dot{\theta}_0 = 15$ rad/s. Slider B has a mass of 230 g and moves in a frictionless slot cut in the disk. The slider is attached to a spring of constant $k = 60$ N/m, which is undeformed when $r = 0$. Knowing that at a given instant the acceleration of the slider relative to the disk is $\ddot{r} = -12$ m/s^2 and that the horizontal force exerted on the slider by the disk is 9 N, determine at that instant (*a*) the distance r, (*b*) the radial component of the velocity of the slider.

SOLUTION

$$\dot{\theta} = 15 \text{ rad/s}, \ m = 230 \text{ g} = 0.230 \text{ kg}, \ \ddot{\theta} = 0, \ F_\theta = 9 \text{ N}, \ \ddot{r} = -12 \text{ m/s}^2$$

Due to the spring, $F_r = -kr, \quad k = 60$ N/m



$$\xrightarrow{+} \Sigma F_r = F_r = ma_r: \ -kr = m\left(\ddot{r} - r\dot{\theta}^2\right)$$

$$\left(k - m\dot{\theta}^2\right)r = -m\ddot{r}$$

(*a*) Radial coordinate.

$$r = -\frac{m\ddot{r}}{k - m\dot{\theta}^2} = -\frac{(0.230)(-12)}{60 - (0.230)(15)^2}$$

$$= 0.33455 \text{ m} \qquad\qquad r = 335 \text{ mm}$$

$$\Sigma F_\theta = ma_\theta: \ F_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

$$2\dot{r}\dot{\theta} = \frac{F_\theta}{m} - r\ddot{\theta}$$

$$\dot{r} = \frac{F_\theta - mr\ddot{\theta}}{2m\dot{\theta}} = \frac{9 - 0}{(2)(0.230)(15)} = 1.304 \text{ m/s}$$

(*b*) Radial component of velocity. $\quad v_r = \dot{r} \qquad v_r = 1.304$ m/s ◀