PROBLEM 11.5

The motion of a particle is defined by the relation $x = 5t^4 - 4t^3 + 3t - 2$, where x and t are expressed in feet and seconds, respectively. Determine the position, the velocity, and the acceleration of the particle when $t = 2$ s.

SOLUTION

Position: $x = 5t^4 - 4t^3 + 3t - 2$ ft

Velocity: $v = \dfrac{dx}{dt} = 20t^3 - 12t^2 + 3$ ft/s

Acceleration: $a = \dfrac{dv}{dt} = 60t^2 - 24t$ ft/s^2

When $t = 2$ s,

$$x = (5)(2)^4 - (4)(2)^3 - (3)(2) - 2$$ $x = 52$ ft ◀

$$v = (20)(2)^3 - (12)(2)^2 + 3$$ $v = 115$ ft/s ◀

$$a = (60)(2)^2 - (24)(2)$$ $a = 192$ ft/s ◀

HW#1

PROBLEM 11.20

The acceleration of a particle is defined by the relation $a = k\left(1 - e^{-x}\right)$, where k is a constant. Knowing that the velocity of the particle is $v = +9$ m/s when $x = -3$ m and that the particle comes to rest at the origin, determine (*a*) the value of k, (*b*) the velocity of the particle when $x = -2$ m.

SOLUTION

Note that a is a function of x. $\qquad a = k\left(1 - e^{-x}\right)$

Use $v\,dv = a\,dx = k\left(1 - e^{-x}\right)dx$ with the limits $v = 9$ m/s when $x = -3$ m, and $v = 0$ when $x = 0$.

$$\int_9^0 v\,dv = \int_{-3}^0 k\left(1 - e^{-x}\right)dx$$

$$\left[\frac{v^2}{2}\right]_9^0 = k\left[x + e^{-x}\right]_{-3}^0$$

$$0 - \frac{9^2}{2} = k\left[0 + 1 - (-3) - e^3\right] = -16.0855k$$

(*a*) $\qquad k = 2.5178 \qquad\qquad k = 2.52\ \text{m/s}^2$ ◀

Use $v\,dv = a\,dx = k\left(1 - e^{-x}\right)dx = 2.5178\left(1 - e^{-x}\right)dx$ with the limit $v = 0$ when $x = 0$.

$$\int_0^v v\,dv = \int_0^x 2.5178\left(1 - e^{-x}\right)dx$$

$$\frac{v^2}{2} = \left[2.5178\left(x + e^{-x}\right)\right]_0^x = 2.5178\left(x + e^{-x} - 1\right)$$

$$v^2 = 5.0356\left(x + e^{-x} - 1\right) \qquad v = \pm 2.2440\left(x + e^{-x} - 1\right)^{1/2}$$

Letting $x = -2$ m,

$$v = \pm 2.2440\left(-2 + e^2 - 1\right)^{1/2} = \pm 4.70\ \text{m/s}$$

Since x begins at $x = -2$ m and ends at $x = 0$, $\quad v > 0$.

Reject the minus sign.

$v = 4.70$ m/s ◀

HW#1

PROBLEM 11.21

The acceleration of a particle is defined by the relation $a = -k\sqrt{v}$, where k is a constant. Knowing that $x = 0$ and $v = 25$ ft/s at $t = 0$, and that $v = 12$ ft/s when $x = 6$ ft, determine (*a*) the velocity of the particle when $x = 8$ ft, (*b*) the time required for the particle to come to rest.

SOLUTION

$$v\,dv = a\,dx = -k\sqrt{v}dx, \qquad x_0 = 0, \qquad v_0 = 25 \text{ ft/s}$$

$$dx = -\frac{1}{k}v^{1/2}dv$$

$$\int_{x_0}^{x} dx = -\frac{1}{k}\int_{v_0}^{v}\sqrt{v}dv = -\frac{2}{3k}v^{3/2}\Big]_{v_0}^{v}$$

$$x - x_0 = \frac{2}{3k}\left(v_0^{3/2} - v^{3/2}\right) \qquad \text{or} \qquad x = \frac{2}{3k}\left[(25)^{3/2} - v^{3/2}\right] = \frac{2}{3k}\left[125 - v^{3/2}\right]$$

Noting that $x = 6$ ft when $v = 12$ ft/s,

$$6 = \frac{2}{3k}\left[125 - 12^{3/2}\right] = \frac{55.62}{k} \qquad \text{or} \qquad k = 9.27\sqrt{\text{m/s}}$$

Then,

$$x = \frac{2}{(3)(9.27)}\left[125 - v^{3/2}\right] = 0.071916\left(125 - v^{3/2}\right)$$

$$v^{3/2} = 125 - 13.905x$$

(*a*) When $x = 8$ ft,

$$v^{3/2} = 125 - (13.905)(8) = 13.759 \text{ (ft/s)}^{3/2}$$

$v = 5.74$ ft/s ◀

(*b*)

$$dv = a\,dt = -k\sqrt{v}dt$$

$$dt = -\frac{1}{k}\frac{dv}{v^{1/2}}$$

$$t = -\frac{1}{k}\cdot 2\left[v^{1/2}\right]_{v_0}^{v} = \frac{2}{k}\left(v_0^{1/2} - v^{1/2}\right)$$

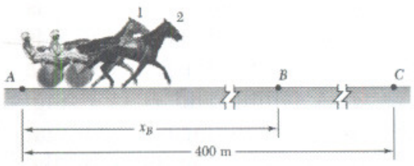
At rest, $v = 0$

$$t = \frac{2v_0^{1/2}}{k} = \frac{(2)(25)^{1/2}}{9.27}$$

$t = 1.079$ s ◀

HW #1

PROBLEM 11.39



In a close harness race, horse *2* passes horse *1* at point *A*, where the two velocities are $v_2 = 7$ m/s and $v_1 = 6.8$ m/s. Horse *1* later passes horse *2* at point *B* and goes on to win the race at point *C*, 400 m from *A*. The elapsed times from *A* to *C* for horse *1* and horse *2* are $t_1 = 61.5$ s and $t_2 = 62.0$ s, respectively. Assuming uniform accelerations for both horses between *A* and *C*, determine (*a*) the distance from *A* to *B*, (*b*) the position of horse *1* relative to horse *2* when horse *1* reaches the finish line *C*.

SOLUTION

① First Find a_1, a_2 using initial and final data

Constant acceleration $(a_1$ and $a_2)$ for horses *1* and *2*.

Let $x = 0$ and $t = 0$ when the horses are at point *A*.

Then, $$x = v_0 t + \frac{1}{2}at^2$$

Solving for *a*, $$a = \frac{2(x - v_0 t)}{t^2}$$

Using $x = 400$ m and the initial velocities and elapsed times for each horse,

$$a_1 = \frac{x - v_1 t_1}{t_1^2} = \frac{2\left[400 - (6.8)(61.5)\right]}{(61.5)^2} = -9.6239 \times 10^{-3} \text{ m/s}^2$$

$$a_2 = \frac{x - v_2 t_2}{t_2^2} = \frac{2\left[400 - (7.0)(62.0)\right]}{(62.0)^2} = -17.6899 \times 10^{-3} \text{ m/s}^2$$

once a_1, a_2 is found find $x_1 - x_2$ as function of time

Calculating $x_1 - x_2$, $$x_1 - x_2 = (v_1 - v_2)t + \frac{1}{2}(a_1 - a_2)t^2$$

$$x_1 - x_2 = (6.8 - 7.0)t + \frac{1}{2}\left[\left(-9.6239 \times 10^{-3}\right) - \left(-17.6899 \times 10^{-3}\right)\right]t^2$$
$$= -0.2t + 8.066 \times 10^{-3}t^2$$

At point *B*, $$x_1 - x_2 = 0 \qquad -0.2t_B + 4.033 \times 10^{-3}t_B^2 = 0$$

$$t_B = \frac{0.2}{4.033 \times 10^{-3}} = 49.59 \text{ s}$$

Calculating x_B using data for either horse,

Horse *1*: $$x_B = (6.8)(49.59) + \frac{1}{2}\left(-9.6239 \times 10^{-3}\right)(49.59)^2$$ $x_B = 325$ m ◀

Horse *2*: $$x_B = (7.0)(49.59) + \frac{1}{2}\left(-17.6899 \times 10^{-3}\right)(49.59)^2 = 325 \text{ m}$$

When horse *1* crosses the finish line at $t = 61.5$ s,

$$x_1 - x_2 = -(0.2)(61.5) + \left(4.033 \times 10^{-3}\right)(61.5)^2$$ $\Delta x = 2.95$ m ◀

HW #1

PROBLEM 11.45

Two automobiles A and B traveling in the same direction in adjacent lanes are stopped at a traffic signal. As the signal turns green, automobile A accelerates at a constant rate of 6.5 ft/s^2. Two seconds later, automobile B starts and accelerates at a constant rate of 11.7 ft/s^2. Determine (*a*) when and where B will overtake A, (*b*) the speed of each automobile at that time.

SOLUTION

For $t > 0$,

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2 = 0 + 0 + \frac{1}{2}(6.5)t^2 \quad \text{or} \quad x_A = 3.25t^2$$

For $t > 2$ s,

$$x_B = (x_B)_0 + (v_B)_0(t-2) + \frac{1}{2}a_B(t-5)^2 = 0 + 0 + \frac{1}{2}(11.7)(t-2)^2$$

or

$$x_B = 5.85(t-2)^2 = 5.85t^2 - 23.4t + 23.4$$

For $x_A = x_B$,

$$3.25t^2 = 5.85t^2 - 23.4t + 23.4,$$

or

$$2.60t^2 - 23.4t + 23.4 = 0$$

Solving the quadratic equation, $t = 1.1459$ and $t = 7.8541$ s

Reject the smaller value since it is less than 5 s.

(*a*) $\qquad t = 7.85 \text{ s}$ ◀

$$x_A = x_B = (3.25)(7.8541)^2 \qquad x = 200 \text{ ft}$$ ◀

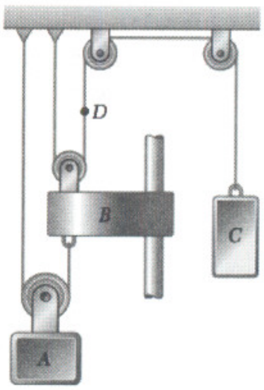
(*b*)

$$v_A = (v_A)_0 + a_A t = 0 + (6.5)(7.8541) \qquad v_A = 51.1 \text{ ft/s}$$ ◀

$$v_B = (v_B)_0 + a_B(t-2) = 0 + (11.7)(7.8541 - 2) \qquad v_B = 68.5 \text{ ft/s}$$ ◀

HW#1

PROBLEM 11.48



Block C starts from rest and moves down with a constant acceleration. Knowing that after block A has moved 1.5 ft its velocity is 0.6 ft/s, determine (*a*) the accelerations of A and C, (*b*) the velocity and the change in position of block B after 2 s.

SOLUTION

Let x be positive downward for all blocks.

Constraint of cable supporting A: $x_A + (x_A - x_B) = \text{constant}$

$$2v_A - v_B = 0 \quad \text{or} \quad v_B = 2v_A \quad \text{and} \quad a_B = 2a_A$$

Constraint of cable supporting B: $2x_B + x_C = \text{constant}$

$$2v_B + v_C = 0, \quad \text{or} \quad v_C = -2v_B, \quad \text{and} \quad a_C = -2a_B = -4a_A$$

Since v_C and a_C are down, v_A and a_A are up, i.e. negative.

$$v_A^2 - (v_A)_0^2 = 2a_A\left[x_A - (x_A)_0\right]$$

(*a*) $a_A = \dfrac{v_A^2 - (v_A)_0^2}{2\left[x_A - (x_A)_0\right]} = \dfrac{(0.6)^2 - 0}{(2)(-1.5)} = -0.12 \text{ ft/s}^2$ $\qquad \mathbf{a}_A = 0.12 \text{ ft/s}^2 \uparrow$ ◀

$$a_C = -4a_A$$ $\qquad \mathbf{a}_C = 0.48 \text{ ft/s}^2 \downarrow$ ◀

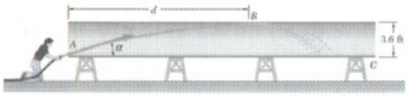
(*b*) $a_B = 2a_A = (2)(-0.12) = -0.24 \text{ ft/s}^2$

$$\Delta v_B = a_B t = (-0.24)(2) = -0.48 \text{ ft/s}$$ $\qquad \Delta v_B = 0.48 \text{ ft/s} \uparrow$ ◀

$$\Delta x_B = \frac{1}{2}a_B t^2 = \frac{1}{2}(-0.24)(2)^2 = -0.48 \text{ ft}$$ $\qquad \Delta x_B = 0.48 \text{ ft} \uparrow$ ◀

HW #1

PROBLEM 11.114



A worker uses high-pressure water to clean the inside of a long drainpipe. If the water is discharged with an initial velocity $\mathbf{v}_0$ of 35 ft/s, determine (*a*) the distance *d* to the farthest point *B* on the top of the pipe that the water can wash from his position at *A*, (*b*) the corresponding angle α.

SOLUTION

Vertical motion: $a_y = -g$ with $v_y = 0$ at point *B*.

assume $v = 0$ vertical at top

$$v_y^2 - (v_y)_0^2 = 2a(y - y_0) \quad \text{or} \quad (v_y)_0^2 = 2g(y_B - y_0)$$

$$(v_y)_0^2 = (2)(32.2)(3.6) = 231.84 \text{ ft}^2/\text{s}^2 \quad \text{or} \quad (v_y)_0 = 15.226 \text{ ft/s}$$

$$v_y = (v_y)_0 - gt = \quad \text{or} \quad t_B = \frac{(v_y)_0}{g} = 0.47287 \text{ s}$$

$$\sin\alpha = \frac{(v_y)_0}{v_0} = \frac{15.226}{35} = 0.43504$$

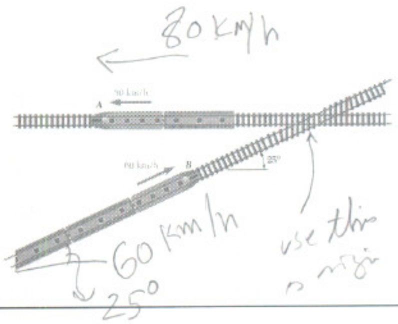
$$\alpha = 25.79°$$

Horizontal motion: $x = (v_0 \cos\alpha)t$

(*a*) $x_B = (35\cos 25.79)(0.47287)$ $x_B = 14.90$ ft ◀

(*b*) From above, $\alpha = 25.8°$ ◀

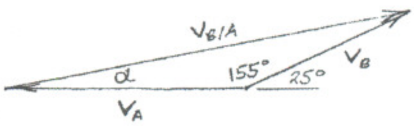
PROBLEM 11.121



The velocities of commuter trains A and B are as shown. Knowing that the speed of each train is constant and that B reaches the crossing 10 min after A passed through the same crossing, determine (a) the relative velocity of B with respect to A, (b) the distance between the fronts of the engines 3 min after A passed through the crossing.

SOLUTION

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$



Sketch the vector addition as shown in the velocity diagram.

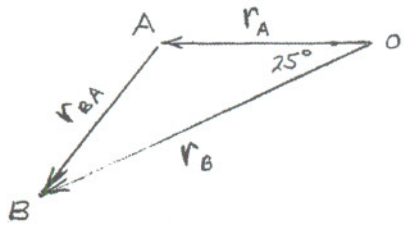
By law of cosines:

$$v_{B/A}^2 = v_A^2 + v_B^2 - 2v_A v_B \cos 155°$$

$$= 80^2 + 60^2 - (2)(80)(60)\cos 155°$$

$$= 18.7005 \times 10^3 \,(\text{km/h})^2$$

$$v_{B/A} = 136.7 \text{ km/h}$$



Law of sines:

$$\frac{\sin\alpha}{v_B} = \frac{\sin 155°}{v_{B/A}}$$

$$\sin\alpha = \frac{60\sin 155°}{136.7} = 0.18543$$

$$\alpha = 10.69°$$

(a) $\mathbf{v}_{B/A} = 136.7 \text{ km/h} \angle 10.69°$ ◀

Determine positions relative to the crossing.

$$\mathbf{r}_A = \mathbf{v}_A t = 80\frac{3}{60} = 4 \text{ km} \leftarrow$$

$$\mathbf{r}_B = (\mathbf{r}_B)_0 + \mathbf{v}_B t = 60\left(\frac{10}{60}\right) \swarrow + 60\left(\frac{3}{60}\right) \nearrow = 7 \text{ km} \measuredangle\, 25°$$

$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$ Sketch the vector addition as shown.

By law of cosines :

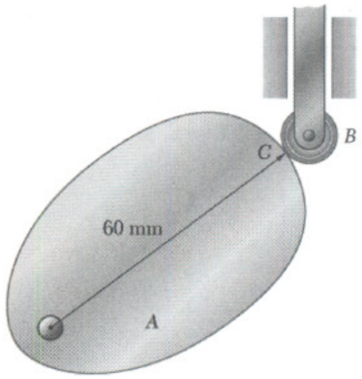
$$r_{B/A}^2 = r_A^2 + r_B^2 - 2r_A r_B \cos 25°$$

$$= 4^2 + 7^2 - (2)(4)(7)\cos 25° = 14.25 \text{ km}^2$$

(b) $d = r_{B/A}$ $\qquad d = 3.77$ km ◀

HW#1

PROBLEM 11.136



As cam A rotates, follower wheel B rolls without slipping on the face of the cam. Knowing that the normal components of the acceleration of the points of contact at C of the cam A and the wheel B are 0.66 m/s^2 and 6.8 m/s^2, respectively, determine the diameter of the follower wheel.

SOLUTION

$$\left[(a_c)_n\right]_A = \frac{v_c^2}{\rho_A}, \quad \left[(a_c)_n\right]_A = \frac{v_c^2}{\rho_A}$$

$$v_c^2 = \rho_A\left[(a_c)_n\right]_A = \rho_B\left[(a_c)_n\right]_B$$

$$\frac{\rho_B}{\rho_A} = \frac{\left[(a_c)_n\right]_A}{\left[(a_c)_n\right]_B} = \frac{0.66}{6.8} = 0.09706$$

$$\rho_B = 0.09706\rho_A = (0.09706)(60) = 5.8235 \text{ mm}$$

$$d_B = 2\rho_B = 11.65 \text{ mm} \blacktriangleleft$$

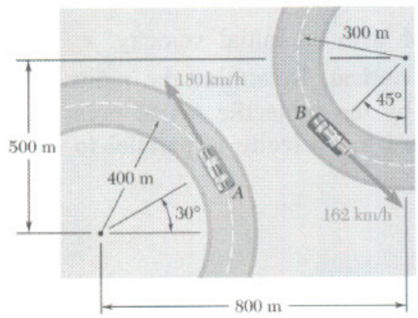
$$\underline{a} = \frac{dv_c}{dt}\bar{e}_t + \frac{v_c^2}{\rho}\bar{e}_n$$

the idea is that v_c is the same at contact point of Both.

equate Normal components.

HW#1

PROBLEM 11.142



Racing cars A and B are traveling on circular portions of a race track. At the instant shown, the speed of A is decreasing at the rate of 8 m/s^2, and the speed of B is increasing at the rate of 3 m/s^2. For the positions shown, determine (*a*) the velocity of B relative to A, (*b*) the acceleration of B relative to A.

SOLUTION

(*a*)

$$\mathbf{v}_A = 180\text{ km/h} = 50\text{ m/s} \searrow 30°, \quad \mathbf{v}_B = 162\text{ km/h} = 45\text{ m/s} \searrow 45°$$

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A = 45(\cos 45°\mathbf{i} - \sin 45°\mathbf{j}) - 50(\cos 120°\mathbf{i} + \sin 120°\mathbf{j})$$

$$= 56.82\mathbf{i} - 75.12\mathbf{j} = 94.2\text{ m/s} \searrow 52.9°$$

$$v_{B/A} = 339\text{ km/h} \searrow 52.9° \blacktriangleleft$$

(*b*)

$$(\mathbf{a}_A)_t = 8\text{ m/s}^2 \searrow 60°, \quad (\mathbf{a}_B)_t = 3\text{ m/s}^2 \searrow 45°$$

$$(\mathbf{a}_A)_n = \frac{v_A^2}{\rho_A} = \frac{(50)^2}{400} = 6.25\text{ m/s}^2 \swarrow 30°$$

$$(\mathbf{a}_B)_n = \frac{v_B^2}{\rho_B} = \frac{(45)^2}{300} = 6.75\text{ m/s}^2 \nearrow 45°$$

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n - (\mathbf{a}_A)_t - (\mathbf{a}_A)_n$$

$$= 3(\cos 45°\mathbf{i} - \sin 45°\mathbf{j}) + 6.75(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$$

$$-8(\cos 60°\mathbf{i} - \sin 60°\mathbf{j}) - 6.25(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$$

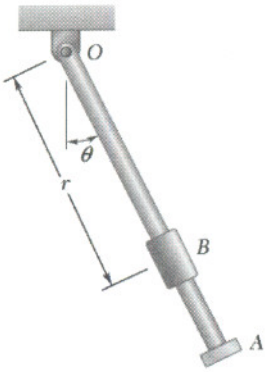
$$= 8.31\mathbf{i} + 12.70\mathbf{j}\text{ m/s}^2,$$

or

$$\mathbf{a}_{B/A} = 15.18\text{ m/s}^2 \nearrow 56.8° \blacktriangleleft$$

$$\underline{a} = \frac{dv}{dt}\overline{e_t} + \frac{v^2}{\rho}\overline{e_n}$$

HW#1



PROBLEM 11.162

The oscillation of rod OA about O is defined by the relation $\theta = (4/\pi)(\sin \pi t)$, where θ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = 10/(t+6)$, where r and t are expressed in mm and seconds, respectively. When $t = 1$ s, determine (*a*) the velocity of the collar, (*b*) the total acceleration of the collar, (*c*) the acceleration of the collar relative to the rod.

SOLUTION

Differentiate the expressions for r and θ with respect to time.

$$r = \frac{10}{t+6}\text{ mm},\qquad \dot{r} = -\frac{10}{(t+6)^2}\text{ mm/s},\qquad \ddot{r} = \frac{20}{(t+6)^3}\text{ mm/s}^2$$

$$\theta = \frac{4}{\pi}\sin \pi t\text{ rad},\qquad \dot{\theta} = 4\cos \pi t\text{ rad/s}\qquad \ddot{\theta} = 4\pi \sin \pi t\text{ rad/s}^2$$

At $t = 1$ s,

$$r = \frac{10}{7}\text{ mm};\qquad \dot{r} = -\frac{10}{49}\text{ mm/s},\qquad \ddot{r} = \frac{20}{343}\text{ mm/s}^2$$

$$\theta = 0,\qquad \dot{\theta} = -4\text{ rad/s},\qquad \ddot{\theta} = 0$$

(*a*) Velocity of the collar.

$$v_r = \dot{r} = 0.204\text{ mm/s},\qquad v_\theta = r\dot{\theta} = -5.71\text{ mm/s}$$

$$\mathbf{v}_B = (0.204\text{ mm/s})\mathbf{e}_r - (5.71\text{ mm/s})\mathbf{e}_\theta \blacktriangleleft$$

(*b*) Acceleration of the collar.

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{20}{343} - \left(\frac{10}{7}\right)(-4)^2 = -22.8\text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \left(\frac{10}{7}\right)(0) + (2)\left(-\frac{10}{49}\right)(-4) = 1.633\text{ mm/s}^2$$

$$\mathbf{a}_B = -\left(22.8\text{ mm/s}^2\right)\mathbf{e}_r + \left(1.633\text{ mm/s}^2\right)\mathbf{e}_\theta \blacktriangleleft$$

(*c*) Acceleration of the collar relative to the rod.

$$\mathbf{a}_{B/OA} = \ddot{r}\mathbf{e}_r = \frac{20}{343}\mathbf{e}_r \qquad \mathbf{a}_{B/OA} = (0.0583\text{ mm/s})\mathbf{e}_r \blacktriangleleft$$

$\dot{e}_r = e_\theta$

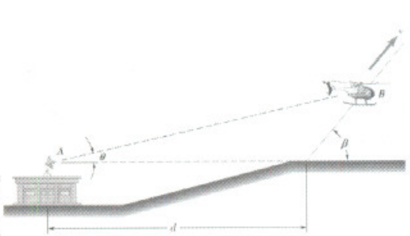
$\dot{e}_\theta = -e_r$

just differentia r, θ twice w.r.t. time.
and use polars $\bar{v} = \dot{r}\bar{e}_r + r\dot{\theta}\bar{e}_\theta$

$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\bar{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\bar{e}_\theta$

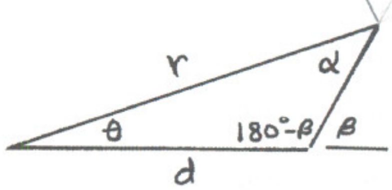
HW#1

PROBLEM 11.169



After taking off, a helicopter climbs in a straight line at a constant angle β. Its flight is tracted by radar from point A. Determine the speed of the helicopter in terms of d, β, θ, and $\dot{\theta}$.

SOLUTION



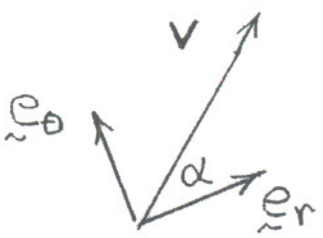
Sketch the geometry.

$$\theta + (180° - \beta) + \alpha = 180°$$

$$\alpha = \beta - \theta$$

$$\frac{r}{\sin(180° - \beta)} = \frac{d}{\sin\alpha}$$

$$r = \frac{d \sin\beta}{\sin\alpha}$$



Sketch the velocity vectors.

$$v_\theta = \mathbf{v} \cdot \mathbf{e}_\theta = v\cos(90° - \alpha)$$

The Thick.

$$= v\sin\alpha$$

But $v_\theta = r\dot{\theta}$ or $v\sin\alpha = \dfrac{d\sin\beta}{\sin\alpha}\dot{\theta}$,

or $v = \dfrac{d\sin\beta}{\sin^2\alpha}\dot{\theta}$ $\qquad v = \dfrac{d\sin\beta}{\sin^2(\beta - \theta)}\dot{\theta}$ ◀

$$\sin(180 - \beta) = \sin\beta$$

$$\bar{v} = \dot{r}\,\bar{e}_r + r\dot{\theta}\,\bar{e}_\theta$$

$$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\,\bar{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\bar{e}_\theta$$

Polar