University Course

MAE 106
Mechanical Systems lab

University Of California, Irvine (UCI)
Winter 2005

My Class Notes

Nasser M. Abbasi

Winter 2005

Contents

Chapter 1

Introduction

Local contents

I took this course in winter 2005 at Univ. Of California, Irvine (UCI). This was an undergraduate course in the mechanical engineering dept.

The course is mainly a hands on course in designing basic control system using basic electrical and mechanical components. The lab was a lot of work, many times we spent most of the time trying to get the circuit to work after building it. Knowing how use the oscilliscope is really usefull for this course.

1.1 Links

1. Course web page `http://www.eng.uci.edu/~dreinken/MAE106/mae106home.htm`

1.2 Syllabus

Required for Mechanical & Aerospace Engineering

MAE106: Mechanical Systems Laboratory
Winter Quarter 2005

Catalog Data: MAE106 Mechanical Systems Laboratory Units: 4
Experiments in linear systems, including op-amp circuits, vibrations, and control systems. Introduction to digital sampling concepts. Emphasis on demonstrating that mathematical models are useful tools for analysis and design of electro-mechanical systems.
Prerequisites: MAE140 or MAE147; ECE72
Course Overlap: MAE170 provides control theory useful for this course
Cross Listed Course(s): none
Restrictions: none
(Design Units: 2)
Lecture Location: PSCB 120, Tues Thurs 3:30-4:50 Lab Location: EG2102

Textbook: Modern Control Engineering, Fourth Edition, Katsuhiko Ogata, Prentice Hall, 2002

References: Supplemental course notes will be at the Engineering copy center ET203. Course Web Site: http://www.eng.uci.edu/~dreinken/MAE106/mae106home.htm

Coordinator: Professor David J. Reinkensmeyer
Department of Mechanical and Aerospace Engineering
Office: EG3225, 824-5218, dreinken@uci.edu
Office Hours: Tuesday 2-3 PM or by appointment
TA's: (Office hours to be announced)
Daisuke Aoyagi daoyagi@uci.edu
Jiayin Liu jiayinl@uci.edu
Sadegh Dabiri sdabiris@uci.edu

Goals: This course covers theory and experiments on motor control systems, electrical filters, amplifiers, structural resonance and vibration. These topics are important for building robots, mechatronic devices, and structures. These systems will be described by linear, ordinary, differential equations. A key goal of the class is to use these equations to predict, understand, and control the behavior of machines.

Prerequisites by Topics: Introduction to Engineering Analysis II (MAE140)
Vibrations (MAE147)
Network Theory and Operational Amplifiers (ECE72)

Lecture Topics:
Week 1 (1/6): No lab scheduled
Lecture 1: Overview, Design Exercise, Review of Circuit Analysis
Reading: Section 3-8
Week 2 (1/11): Lab 1: Laboratory Tools and Motor Control
Lecture 2: Time and Frequency Domains
Lecture 3: First-Order Systems: DC Motors and Electrical Filters
Reading: Chapter 2, Sections 3-1, 3-2, 5-1, 5-2
Week 3 (1/18): Lab 2: Electrical Filters and First-Order Systems
Lecture 4: Lab 1 Quiz; Introduction to Control Theory
Lecture 5: Example of Feedback Control: P-type Velocity Control of a Motor
Reading: Chapter 1, Section 3-3
Week 4 (1/25): Lab 3: Feedback I: P-type Velocity Control of a Motor
Lecture 6: Lab 2 Quiz; Second Order Systems: Time domain
Lecture 7: Second Order systems: Frequency domain
Reading: Sections 5-3, 8-1, 8-2
Week 5 (2/1): Lab 4: Vibration I: Lightly Damped Second Order Systems
Lecture 8: Lab 3 Quiz and Midterm
Lecture 9: PD Motor Control
Reading: Section 5-8
Week 6 (2/8): Lab 5: Feedback II: P and PD Motor Position Control

Lecture 10: Lab 4 Quiz; Systems with Two Modes of Vibration
Lecture 11: Design of a Vibration Isolator
Reading: Class Notes
Week 7 (2/15): Lab 6: Vibration II: System with Two Masses
Lecture 12: Lab 5 Quiz; Advanced Control
Lecture 13: Advanced Control
Reading: Class Notes
Week 8 (2/22): Lab 7: Advanced Control
Lecture 14: Lab 6 Quiz and Design Exam
Lecture 15: Design Exam Review
Week 9 (3/1): No Experiment This Week
Lecture 16: Lab 7 Quiz/ Final Project Discussion
Lecture 17: No Class
Week 10 (3/8): Lecture-free week for working on final projects
Week 11 (3/15): Finals Week – final project contest on day of scheduled final

Computer Usage: For laboratory write-ups and data acquisition.

Laboratory Projects: **Laboratory Location:** Engineering Gateway 2102

Laboratory times:
Section A: Tues 11:00-01:50
Section B: Tues 06:00-08:50P
Section C: Wed 04:00-06:50P
Section D: Thurs 06:00-08:50P
Section E: Friday 10:00-12:50

Laboratory Exercises: Handouts that describe the experiments will be made available on the course web site, along with their solutions. You should work through the lab, referring to the solution. The solution is provided to relieve time pressure and to act as a "consultant" if you get stuck. You can also ask the TA for help if you are confused. Be creative, explore, and have fun in the lab. This is your opportunity to build things that move and see how they work.

Lab Pre-Quizzes: There will be a brief quiz at the beginning of each lab testing whether you have read the experiment handout before coming to laboratory.

Lab Write-Up: Each student will be required to turn in a brief write-up for the lab. The write-up must be typed. You must use a computer graphing program (e.g. Microsoft Exel or Matlab) for all graphs. Zero credit if you don't do this!

Lab Post-Quizzes: There will be a 30-minute quiz in lecture the Tuesday following each laboratory.

Final Project There will be a final project competition involving the design and head-to-head testing of a robotic device. The final project tournament will take place on the day of the scheduled final exam, and will replace the final exam. There will be a write-up due on the day of the final project.

Design Content Description: This course requires solution of design problems related to control and vibration, as well as design and construction of a robotic device for the final project.

Grading Criteria: The grading scale will be:
Lab Pre-Quizzes: 7%
Lab Post-Quizzes: 14%
Lab Write-Ups: 14%
Mid-term exam: 20%
Design exam: 20%
Final project: 25%

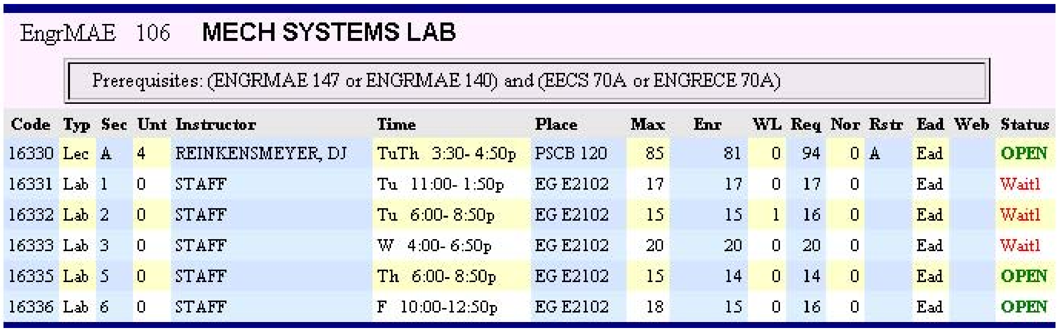
Estimated ABET Category Content:
Engineering Science: 2 credits or 50%
Engineering Design: 2 credits or 50%

Prepared by: Prof. David Reinkensmeyer **Date:** 1/6/05

1.3 Instructor

`https://engineering.uci.edu/users/david-reinkensmeyer`

1.4 Course and Text book



EngrMAE 106 **MECH SYSTEMS LAB**

Prerequisites: (ENGRMAE 147 or ENGRMAE 140) and (EECS 70A or ENGRECE 70A)

Code	Typ	Sec	Unt	Instructor	Time	Place	Max	Enr	WL	Req	Nor	Rstr	Ead	Web	Status
16330	Lec	A	4	REINKENSMEYER, DJ	TuTh 3:30- 4:50p	PSCB 120	85	81	0	94	0	A	Ead		OPEN
16331	Lab	1	0	STAFF	Tu 11:00- 1:50p	EG E2102	17	17	0	17	0		Ead		Waitl
16332	Lab	2	0	STAFF	Tu 6:00- 8:50p	EG E2102	15	15	1	16	0		Ead		Waitl
16333	Lab	3	0	STAFF	W 4:00- 6:50p	EG E2102	20	20	0	20	0		Ead		Waitl
16335	Lab	5	0	STAFF	Th 6:00- 8:50p	EG E2102	15	14	0	14	0		Ead		OPEN
16336	Lab	6	0	STAFF	F 10:00-12:50p	EG E2102	18	15	0	16	0		Ead		OPEN

Figure 1.1: course schedule

Figure 1.2: textbook

Chapter 2

Lab reports

Local contents

2.1 Lab 1

Local contents

2.1.1 questions

MAE 106 Laboratory Exercise #1
Laboratory Tools and Control of a Motor

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Introduction

There are two parts to this lab exercise. In the first part, you will learn how to use the oscilloscope, function generator, breadboard, ohmmeter and potentiometer. In the second part, you will learn how to use a low power signal and a power transistor to control the speed of a motor. There are 4 practical exams problems for which you will have to demonstrate something to the TA. There is also a brief write-up (read the last page now to see what it is!!).

Note: When making electrical circuits in lab, a mistake in your wiring may result in a component getting "fried." If you smell something burning, immediately turn off your proto-board and "debug" your circuit.

PART 1: Laboratory Tools

The oscilloscope and function generator are useful tools for making measurements and debugging machines. The solderless breadboard is useful for building circuits. Potentiometers are a very common circuit element for controlling a voltage or sensing a rotation.

REQUIRED PARTS:

Qty	Parts	Equipment
1	50K Potentiometer	Trainer Kit (XK-550)
1	150Ω Resistor	Oscilloscope with scope probe
Var.	22 gauge wire	

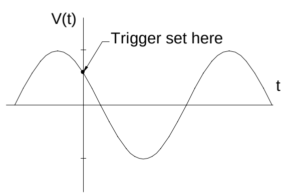


Figure 1 – Trigger and Sweep Rate

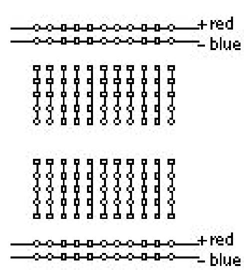


Figure 2 – Solderless Breadboard

1 The Oscilloscope and Function Generator

The oscilloscope is used to measure and view voltage as a function of time. A voltage waveform such as $v(t)=a \sin\omega t$ will appear on the scope's cathode ray tube much like you would plot it on a piece of paper. The voltage at which the trace begins is adjusted with the *trigger level*. The duration of the waveform that appears on the screen is determined by the *sweep rate* (or time scale) of the scope. You can refer to the HP oscilloscope user's guide to better understand its basic operation.

A *function generator* is a device that produces voltage waveforms such as sine, square, and triangle waves, all with variable *amplitude, frequency,* and *offset.* A function generator is often used to provide an input signal to the oscilloscope. For example, the function generator can produce a voltage with the form

$$v(t) = v_{0ffset} + a \sin\omega t$$

where the amplitude a, the frequency ω, and the offset v_{0ffset} are all adjustable.

Connect the oscilloscope channel 1 input scope probe to the trainer kit function generator output (Freq.), with the scope alligator clip to GND (Note: A BNC connector is a common type of connector with a bayonet coupling mechanism. BNC stands for (Bayonet Neill Concelman), because the connector was invented by and named after Amphenol Engineer Carl Concelman and Bell Labs Engineer Paul Neill. It was developed in the late 1940's.

Set the function generator to output a sine wave at about 100 Hz. Press the **Auto-scale** button on the scope. When you push the button, the scope measures the maximum and minimum values of the current signal, and sets the screen scaling to match these values. Get the scope to display the peak-to-peak voltage of the sine wave by pressing **Voltage** and then **Vp-p** (make sure the **Source** option for the **Voltage** function measurement is set at **Line 1** – this will use channel 1 as the input line). Now adjust the function generator to output a 2 Vpp (volts peak-to-peak) sine wave at 100 Hz with no offset (**DC offset** button out). On the scope, make sure the **Probe** setting is 1 (so voltage is multiplied by 1; as explained, some probes divide the voltage by 10, and if you are using those probes rather than BNC cables, the Probe setting should be 10X). Make sure the **coupling** is set to DC (under channel **1)**.

P1. Plot the trace on the scope. (You should see the sine wave clearly now). Label plot with the voltage and time scales.

Q1. Using the utilities on the scope, obtain and record Vmax, Vmin, Vp-p, Vavg, Period, frequency, and Duty Cycle (under **Voltage** and **Time**). Notice that these functions won't work unless at least one period of the whole voltage signal appears on the screen.

Q2. Turn the **Volts/Div**, **Time/Div**, and **position** dials on the scope. Does the **Volts/Div** dial change the amplitude of the sine wave? Does the **Time/Div** dial change the frequency of the sine wave? Does the **position** dial add a constant voltage to the signal? What do these dials do?

P2. Adjust the sine wave amplitude to 1Vpp at 200 Hz. Plot (on the same plot as for P1) the trace on scope using the same voltage/time scales as before.

Q3. Make the grid turn off and on (under **Display**). Change trigger (under **Source**) from 1 to 2. What happened to the sine wave? Change it back to 1. Why does it do this? Hint: think about what the scope must do to make a sine wave appear without moving (rolling) across the screen.

Q4. Offset the sine wave by pushing the **DC offset** button on the function generator and adjusting the dial. What does the trace do? On the scope, set the coupling to AC (under channel **1**). Now what does the trace do when adjusting the DC offset? What is the purpose of using AC coupling?

P3. Remove the DC offset (**DC offset** button out) and get the function generator to output square and then triangle waves. Plot the traces in **one** plot.

Practical Exam 1: Ask the TA to come by your station and set the voltage and frequency of a sine wave on the frequency generator. Demonstrate to the TA that you can measure the amplitude and frequency of the sine wave. If the TA is busy with another group, you can go ahead with the lab and ask the TA to come by later.

2 Solderless (Breadboards)

The electronic breadboard (solderless breadboard) is used to wire up temporary circuits. Electronic components and wires (use solid wires at 22-gauge thickness) are inserted into the numerous sockets (holes) on the board. The sockets (dots) are connected internally (lines) as shown in Figure 2. A good method for wiring complicated circuits is to connect the source voltage (+5V, ±15 V) and ground terminals from the trainer kit to the long narrow horizontal strips (Figure 2). Electronic chips now have ready access to power through short wires to sockets along the long strips.

After wiring your circuit to the solderless board, you may use the oscilloscope to measure voltages at various points on the circuit using the scope probe. Note that the scope probes will usually divide the voltage they read by 10, so you must compensate for this by setting **Probe** (under channel **1**, or appropriate channel number) on the scope to 10 (this will multiply the voltage value by 10).

3 Potentiometer and Voltage Divider Circuits

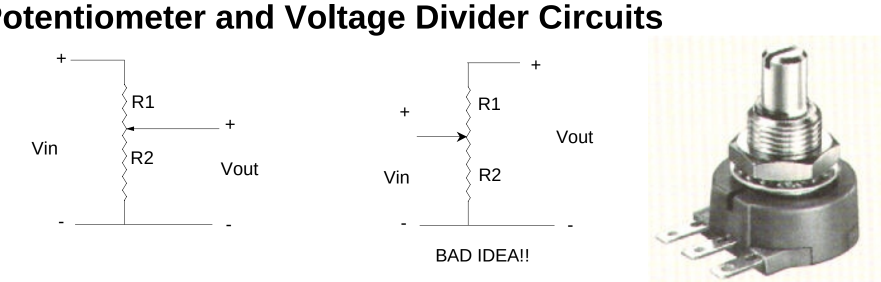


Figure 3A & 3B – Potentiometer circuits, and actual potentiometer. In the circuit diagrams, the wiper is the wire with an arrow on it. For the actual pot, unless otherwise labeled, the wiper is usually the middle connector (how could you check this with an ohmmeter?).

A potentiometer (also called *pot*) is a device that can provide a variable resistance between 2 of its leads. As you turn the knob of a pot, the wiper moves along a resistive element. Look at a broken pot in lab and see if you can figure out how it works. Figure 3A shows a pot being used to produce a variable output voltage.

Q5. Derive V_{out} as a function of V_{in} ,R_1, and R_2 for Figure 3A.

Q6. Explain why you should never use the circuit in Figure 3B.

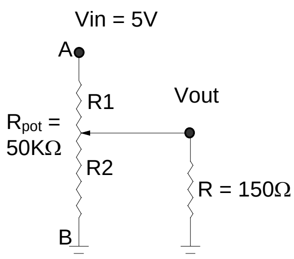


Figure 4. Circuit for question 9

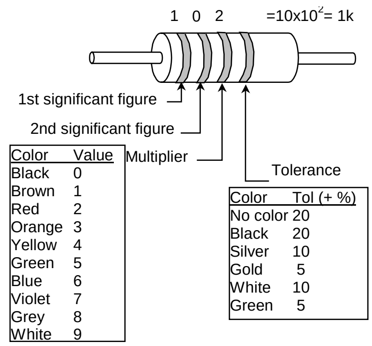


Figure 5. Resistor Color Code

Using the color code at the bottom of the page, pick out a 150 ohm resistor. Confirm the resistor value by measuring it with an ohmmeter. Then wire the circuit shown in Figure 4 on the breadboard. With R = ∞ (i.e. an open circuit), measure V_{out} in five approximately equally spaced potentiometer positions (i.e. full clockwise, full counter-clockwise, the center, etc.). Repeat the test with R = 150Ω.

P4. Sketch V_{out} vs. pot angle for both cases.

Q7. One plot is linear and one is not. Using your knowledge of circuit theory, show mathematically that this is the case by deriving the equation for V_{out} as a function of θ (don't plug actual values in until you derive the equation!). Note: first derive the equations that relate R1 and R2 to pot angle ($0 \le \theta \le \theta_{max}$). Assume that R1 varies linearly with θ. Under what conditions would a pot be a good way of making an adjustable voltage source? Brainstorm two possible uses for potentiometers on your final project.

Practical Exam 2: Show the TA that you can calculate the value of a resistor using the color code, and that you can measure it using the Ohmmeter. Explain to the TA why you will never wire a potentiometer as shown in Figure 3B. Explain to the TA how you might use a potentiometer for you final project.

PART 2: Control of an Electric Motor

REQUIRED PARTS:

Qty	Parts	Equipment
1	N-type Power MOSFET (IRF510 or NTE2382)	Trainer Kit (XK-550)
1	LM324 Quad Op-amp chip	Oscilloscope with scope probe
1	50kΩ potentiometer	Small DC motor
1	470Ω resistor	Integrated circuit puller
1	1kΩ resistor	Grounding wrist strap
1	47K Resistor	
1	100Ω resistor, 2 Watt (with smaller ga. wire leads)	
var	22 gauge (AWG) wire	

1 Introduction

Engineers use electric motors for a variety of applications requiring mechanical movement (robots, automation equipment, disk drives, etc.). A motor is only useful, however, if we know how to control it. Sometimes we want to control the motor's position (computer disk drives, CD players, plotters), sometimes its speed (cruise control on autos, CD players), and sometimes its torque (robots, some heavy machinery). In this lab, we will investigate controlling the voltage across a motor, which, assuming there are not external forces acting on the motor, will control the speed of the motor. In other words, if the motor load is just inertial, then the steady-state speed of the motor is proportional to the voltage across its terminals.

In this part of the lab, we will investigate two circuits that can control the voltage across a small electric motor. Each circuit will involve the use of a MOSFET and/or an operational amplifier (or op-amp). The op amp circuit will utilize "feedback" to control the motor voltage. So, in this experiment you will gain insight into how DC brushed motors behave, how to control the power supplied to a motor with a MOSFET, and how to regulate the behavior of the motor with an op-amp controller.

2 Voltage Follower for Voltage Control

Op amps are often used in analog circuits. They take 2 input voltages at their inputs (the inverting input (V_-) and the non-inverting input (V_+)) and produce an output voltage

$$Vout = K(V_+ - V_-), \qquad \text{where } K \approx 1x10^5. \qquad (3)$$

They also have a very high input resistance, so for practical purposes they draw no current at their inputs. These characteristics of op amps allow them to serve many circuit functions, such as voltage addition and subtraction, feedback control, and buffering. The op amps used in this lab can output only tens of mA of current; you will calculate the exact value in the lab.

In Figure 2, an op amp is used in a voltage follower circuit. In this circuit, the op-amp attempts to adjust the output voltage (Vout) so that it "follows" (makes it equal to) Vin. This circuit is also called a buffer or isolation amplifier because the output current does not affect the input voltage (Vin). The load resistance (R_L) draws current from the op-amp. A small value of R_L is considered a “large load.” A large value of R_L is a “small load.” Note that large loads can cause a drop in voltage in a voltage source if the voltage source cannot supply sufficient current.

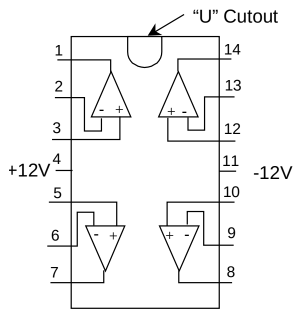


Figure 1 – LM324 Quad Op-Amp Chip

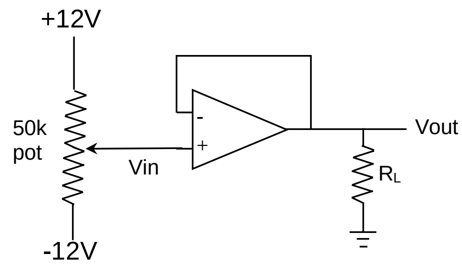


Figure 2 – Voltage Follower Circuit.

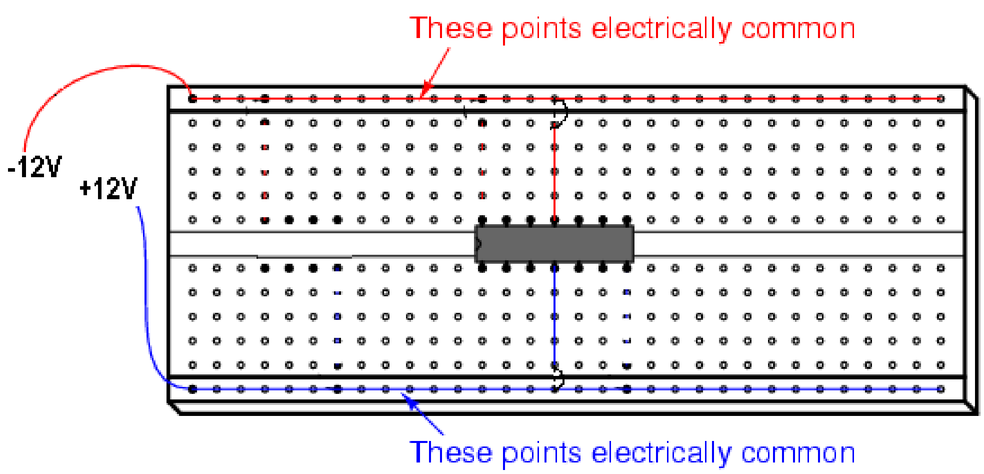


Figure 3 – Suggested Layout of Circuit on Solderless Breadboard

Construct the circuit shown in Figure 2 (suggested layout is in Figure 3). We will use the LM324 chip (Figure 1), which has 4 op-amps built on the chip. Power must be supplied at pin 4 (positive voltage) and pin 11 (negative voltage), else the chip will burn out.

Q1 Using the op-amp equation above, derive the equation of Vout for the buffer circuit.

Q2 There are limitations in the buffer circuit in Figure 2. That is, there are conditions when the op-amp would not be able to make Vout = Vin. Discuss 2 limitations.

P1 Measure Vout and Vin across the full range of pot angle positions, for several values of the pot angle. Do this with R_L = 1 kΩ and with R_L = 470Ω. Sketch Vout vs. Vin for both R_L values on one plot.

Q3 Which plot follows Vin more closely, particularly at the voltage extremes of Vin? Explain. Using your results, calculate the maximum current that your op amp can supply. Is the 470Ω resistor close to blowing up (consider its power rating)?

Suppose you replace R_L with a motor whose resistance is 30Ω, and you hold the shaft of the motor still. Would you be able to control the amount of torque that the motor can generate? Note: When to motor shaft is held still, the amount of torque that a DC motor generates is proportional to the current going through it.

Q4 THOUGHT EXPERIMENT: If you changed the power input to the LM324 chip to +5V (pin 4) and 0V (pin 11), and adjusted Vin over the +12V to –12V range, do you think Vout would follow Vin? Explain. NOTE: DO NOT ATTEMPT THIS; IT CAN BLOW THE OP AMP AND POT!

3 Open-Loop MOSFET Voltage Control Circuit

A MOSFET is a type of transistor that restricts or allows current flow through the *source* and *drain* leads based on voltage applied at its *gate* with respect to the source (V_{GS}). It can be thought of as a variable resistor whose resistance value is determined by V_{GS}. For the MOSFET's used in class, the effective resistance between source and drain (R_{DS}) varies between infinity (with V_{GS} < 3 or 4 volts) and about ½ ohms (with V_{GS} ~5-6 V). These characteristics allow MOSFETS (and other transistors) to be used as either current amplifying devices (power MOSFETS for motors, etc.) or switches (low power MOSFETS in computers).

Construct the circuit shown in Figure 4 (make load resistance R_L = 100Ω at 2 Watt rating). In this circuit, we directly control the MOSFET gate voltage (V_G) by turning the potentiometer (recall that V_{GS} will vary linearly with the pot angle), which ultimately controls the motor voltage (V_{motor}). In this section, we will study how V_{motor} varies as we vary V_{GS}.

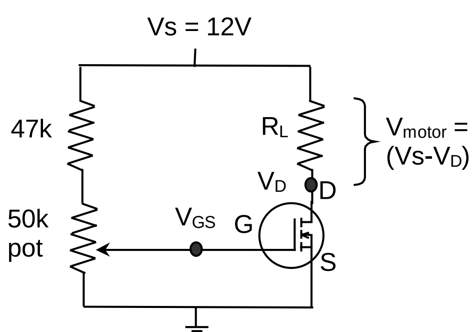


Figure 4 – MOSFET voltage control circuit

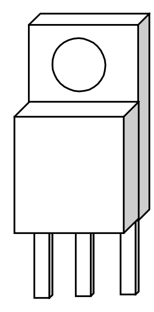


Figure 5 – MOSFET leads

Q5 Derive the following formula relating the MOSFET resistance (R_{DS}) to Vs, V_D, and R_L.

$$R_{DS} = V_D R_L/(Vs – V_D)$$

Q6 Consider the circuit in Figure 4 and assume that R_L is a motor. Explain how turning the pot (which we control) ultimately controls motor voltage (V_{motor}). Include in your explanation the role of the pot and the MOSFET.

P2 Measure V_{GS}, and V_D at enough potentiometer positions to produce a nice plot of R_{DS} vs. V_{GS} and V_{motor} vs. V_{GS} (on same plot, not V_D vs. V_{GS}). Take more measurements in the area of pot positions where V_D changes quickly. Use the equation above to compute R_{DS}. Plot V_{motor} with respect to V_{GS} (i.e. the input voltage). Is this relationship linear?

Q7 Replace the load resistor (R_L) with a small DC motor. Slowly increase V_{GS} from a value of zero and visually observe the resulting changes in motor speed. Try to make the motor shaft rotate at approximately once per second. Is it difficult? Does the speed of the motor relate linearly to the V_{GS}? Explain why this is so. Would you say that you are controlling motor voltage well?

Q8 Measure the DC resistance of the motor with an ohmmeter and report the value. You have just measured R for the equation 1. Disconnect your motor from the circuit and connect it to an oscilloscope. What is the maximum voltage you can generate by hand? Describe how B_1 (in equation 1) can be measured experimentally. Be precise in describing the experiment you would perform.

Practical Exam 3: Demonstrate to the TA that you can control the speed of your motor with the potentiometer.

4 Closed-Loop Voltage Control Circuit

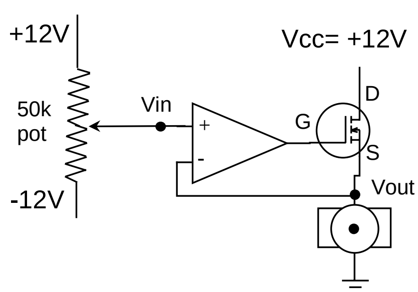


Figure 6 – Closed-Loop Voltage Control Circuit. ***(Note: Power connections to Op Amp not shown!)***

In Part 2, we learned that an op-amp might not provide sufficient current to run a motor. MOSFET's may be necessary to control higher levels of current required by devices such as heaters and electric motors, but they are highly non-linear to the input voltage. Now we will add "feedback" to our controller. A controller that uses feedback takes information about some state in our system (voltage, position, velocity, temperature, etc.) that is measured by a sensor (voltmeter, pot, tachometer, thermocouple, etc.), and uses it to compute a "control" (an input to the system that we control, often denoted "u"). The control is designed to make the state of the system be what we want it to be.

Construct the circuit shown in Figure 6.

Q9 Explain how turning the pot (which we control) ultimately controls motor voltage (Vout). Include in your explanation the role of the pot, op-amp, and the MOSFET.

P3 Make about five measurements of v_{in} and v_{out} as v_{in} varies between 0 and 13 volts. For each pot setting of v_{in}, measure v_{out} (let the motor shaft spin freely). Plot v_{out} versus v_{in} on the same plot as P2.

Q10 Try again to make the motor rotate at approximately one cycle per second. Explain why it is easier than before. Explain why this is a better voltage control circuit than the previous one.

Practical Exam 4: Demonstrate to the TA that you can control the speed of your motor with the potentiometer. You should be able to control it more precisely than in Practical Exam 3 – Explain why to the TA.

Q11 In this experiment, you focused on controlling the motor voltage, which controls the motor speed when there are no disturbances on the motor shaft (e.g. when you are not holding the shaft). In the real world there are often unexpected disturbances that produce undesired changes in the motor speed (imagine an electric car that has to go up a hill, or, try stopping the motor with your hand). Suggest an improved method for controlling motor velocity (hint: consider how you might design a cruise control system a car using a tachometer).

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words!!
- include your name and laboratory time on the write-up
- the write-up must be type-written
- Graphs for the lab write-up must be generated using Excel or Matlab, and must include labels on the axes, voltage and time scales used on the scope, and a legend for multiple-line plots.
- Page limit = 2 pages, including graph

1. Briefly explain uses for an:
    a. oscilloscope
    b. function generator
    c. solderless breadboard
    d. potentiometer
2. Briefly explain how a low-power signal and a power MOSFET can be used to control the speed of a motor.
3. Briefly explain why the operational amplifier made it easier to control the speed of the motor
4. Turn in the graph for P2 and P3 from PART 2 of the laboratory exercise

2.1.2 key solution

MAE 106 Laboratory Exercise # 1 - Solution
Laboratory Tools and Control of a Motor

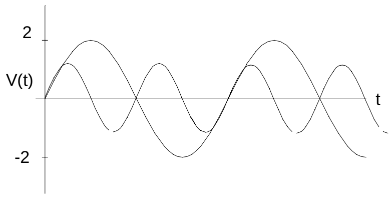
Part 1: Laboratory Tools

P1 See below P2 below

Q1 Vmax = 1V | Vavg = 0V | Duty cycle = 50%
Vmin = -1V | Frequency = 100 Hz
Vpp = 2V | Period = .01s

Q2 They do NOT adjust the amplitude, frequency, and DC offset, respectively. Rather, they adjust the voltage scale, time scale, and vertical position for the scope trace, respectively.

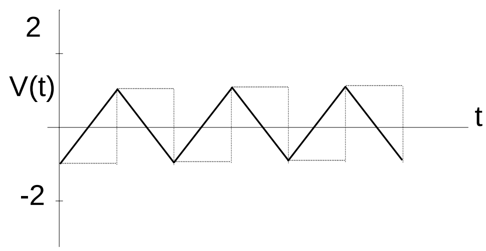
P2



Q3 The trigger is responsible for making the trace "stay still" on the screen, by capturing the signal for a fixed duration of time whenever it crosses the trigger voltage, then displaying the captured signal with a predetermined temporal offset. If the trigger channel is set to 2, than signals on channel 1 will not be triggered properly because the scope is trying to align the signal based on the (floating) voltage being input into channel 2.

Q4 AC coupling removes the DC offset from the signal.

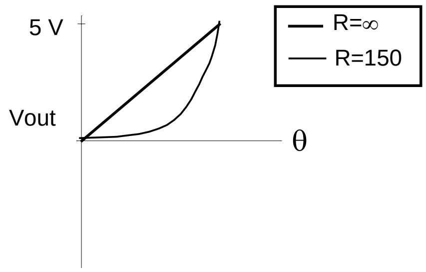
P3



Q5 Vout = {R2/(R1 + R2)} Vin

Q6 If the pot is turned to the extreme end, there is no resistance to current flow and the pot will be burned out.

P4 In **one** plot, show V_{out} vs. pot angle for both cases.



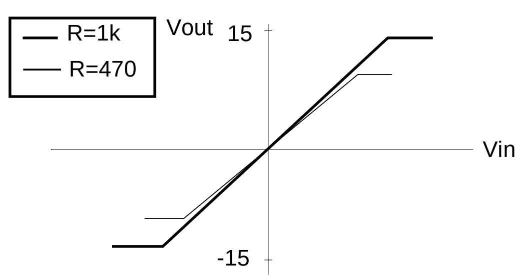
Q7 R1 = kθ; R2 = Rpot - kθ; with k=(50K/θmax), Rpot = 50K
Vout = Vin {(Rp)/(R1 + Rp)}, with Rp = R2*R/(R+R2); voltage divider rule
Vout = Vin [1/{1 + (R1/R) + (R1/R2)}]
Vout = Vin [1/{1 + (kθ/R) + (kθ/(Rpot-kθ))}]
Vout = Vin[(-k)θ/{(-k^2/R)θ^2 + (Rpot/R)θ + (Rpot)}]
(linear divided by quadratic)
A pot is a good way of making an adjustable voltage source if its output is connected to a high resistance. On your final project, you could use one pot to specify the desired angle of the motor (i.e. the "control knob" you hold in your hand), and another to sense the actual angle of the motor (i.e. the "sensor" that senses motor angle).

Part 2: Control of an Electric Motor

Q1 Vout = K(V_+ - V_-), but Vout = V_-, so Vout = K(V_+ - Vout), so Vout = (K/(1+K)) Vin, and for K >> 1, Vout ≈ Vin.

Q2 R_L cannot be so small as to draw more current than the op-amp can supply (> 20mA usually)
Vin cannot be greater than the voltage supplied to the op-amp chip.

P1



Q3 With large R_L, the plot follows better. For large R_L, Vout can go all the way to 15V (minus some loss in the op-amp) since Iout = 15V/1k = 15mA, which is less than the maximum current the op amp can supply. The small R_L draws at a maximum Iout = (observed peak voltage = 9.4V)/470Ω = 20mA peak, which is the maximum current the op-amp can supply (or "source"). Note that different op amps may have slightly different peak currents. Put another way, small R_L draws too much current for the op-amp to maintain Vout. The power drawn by the 470Ω resistor is I^2R = 0.19 W, which is less than the 0.25 W rating, but near it, so you should feel the resistor warming up.

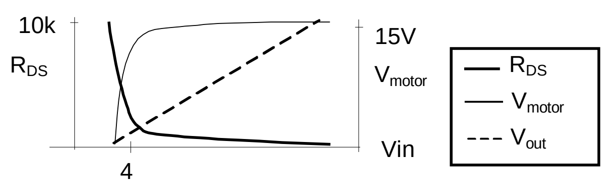
A motor with a stationary shaft acts like a resistor. A motor with R=30 ohms would initially attempt to draw way too much current (15V/30Ω = 500mA) and the output voltage from the op amp would be very small (Vout=20mA*30Ω=0.6V). You could only control the torque of the motor across a small range of currents corresponding to 0-20 mA (see Equation 2).

Q4 The op-amp can only output ~0 – 5V (minus a small voltage loss) if powered between 0V and 5V.

Q5 $R_{DS} = V_D R_L/(Vs - V_D)$ (ans: use voltage divider rule)

Q6 We control the pot angle. The pot allows us to adjust V_G. The MOSFET allows, through a corresponding change in R_{DS} the adjustment of V_D. Motor voltage is $(Vs - V_D)$, where Vs is a constant, so we ultimately control V_{motor}.

P2 V_{motor} is not linear with respect to V_G



Q7 It is difficult to make the motor turn at 1 Hz. The motor speed is not linear to V_{GS}. This results from non-linear input/output characteristics of the MOSFET (i.e., R_{DS} is not linear to V_{GS}). Since we are unable to control motor speed easily, and motor speed correlates with motor voltage in the steady state, it seems that we are able to control the voltage, but not easily.

Q8 The voltage V_D increases when you stop the motor shaft since the back EMF term no longer contributes to the voltage across the motor. Specifically, when the motor is allowed to accelerate to its no load speed, the back EMF builds up proportionally to angular velocity until $i \approx 0$ (i never goes completely to zero because some torque is needed to overcome the motor's friction). If $i \approx 0$, then $V_D \approx 0$. If you stop the motor from turning, the back EMF term goes to zero, and more current is allowed to flow through the MOSFET, increasing V_D.

Q9 We control pot angle, so we control Vin. The op-amp, having negative feedback, takes Vin and adjusts V_{GS} so that Vout = Vin. That is, V_{GS} is altered in such a way as to make the MOSFET's R_{DS} change so that Vout=Vin.

P3 See **P2** above.

Q10 It is easier due to feedback. The op-amp, since it has negative feedback, will try to adjust V_{GS} so that Vout is equal to Vin. In this way, the op-amp can "correct" for nonlinearities in the MOSFET and makes Vout linear with respect to Vin. Since we are able to get Vout to more closely parallel Vin, this is a better control system.

Q11 An improved method for controlling motor velocity would be to actually sense the motor's velocity (for example, with a tachometer), then to make feedback adjustments to the current supplied to the motor based on the difference between the actual and desired velocity. We will explore this approach in a subsequent lab.

2.1.3 Lab post quizz solution

SOLUTION

MAE 106 Post-Laboratory Quiz

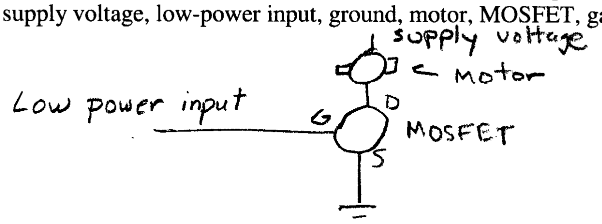
Laboratory Exercise #1: Laboratory Tools and Control of a Motor

In Lab 1, you learned how to use the oscilloscope, function generator, breadboard, ohmmeter and potentiometer. You also learned how to use a low power signal, power transistor, and operational amplifier to control the speed of a motor.

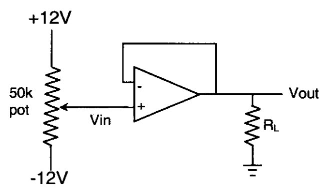
25 pts 1. Match the instrument with its use.

5 a. Oscilloscope	c	1. Building circuits
5 b. Function generator	e	2. Controlling a voltage (b is ok.)
5 c. Breadboard	d	3. Measuring a resistance
5 d. Ohmmeter	a	4. Measuring voltage as a function of time
5 e. Potentiometer	b	5. Producing sine, square, and triangle waves

25 pts 2. Draw a circuit for controlling power to a motor, using a low-power input and a MOSFET. Make sure to label supply voltage, low-power input, ground, motor, MOSFET, gate, drain, source, motor.



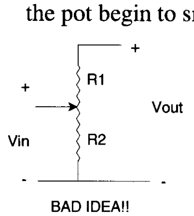
25 pts 3. Prove that $V_- = V_+$ for the following voltage follower circuit, using the fact that $V_o = K(V_+ - V_-)$ for an op amp, with K = 10,000.



$$V_{out} = K(V_+ - V_-) = V_-$$
$$KV_+ = (1+K)V_-$$
$$V_- = \frac{K}{1+K} V_+ \approx V_+ \text{ because } K \text{ is big}$$

also OK:
$$V_o = K(V_+ - V_-)$$
$$V_+ - V_- = \frac{V_o}{K} \approx 0$$
$$\Rightarrow V_+ \approx V_-$$

25 pts 4. Shown below is a 10 KΩ potentiometer wired incorrectly. Assume the potentiometer has a ¼ watt power rating (remember power = IV). Assume that V_{in} = 10 Volts. When the pot's shaft angle = 0°, then R1 = 0 and R2 = 10 KΩ. When the pot's shaft angle = 180° then R2 = 0 and R1 = 10 KΩ. Assume that you start with the pot angle = 0°, and slowly turn it toward 180°. At what angle will the pot begin to smoke?



$$P = \frac{V^2}{R2} = \frac{1}{4}\text{ watt} = \frac{(10V)^2}{R2} = \frac{100}{R2} \Rightarrow R2 = 400\,\Omega,$$ when power rating is exceeded

At what angle is $R2 = 400\,\Omega$?

$$\frac{\theta}{180} = \frac{9{,}600}{10{,}000} \Rightarrow \theta = \frac{(180)(9600)}{10{,}000} = 1.8 \times 96$$

$$\boxed{\theta = 172.8°}$$

96 × 1.8 = 768 + 96 = 172.8

2.1.4 my solution

LAB #1 report. MAE 106. UCI. Winter 2005

Nasser Abbasi, LAB time: Tuesday 1/11/2005 11:00 AM-1:50 PM

January 18, 2005

1 Answer 1.

1. Oscilloscope: This is a device to allow one to analyze and display the electric signal in the circuit. One can use it to display the electric signal trace on the screen and to measure different properties about the signal. One can use it to display different properties about the voltage, such as the max/min, vpp. In addition it is used to measure the frequency properties of the signal.
2. Function generator: This device is attached to the training kit, and was used to generate electric signals of different time-domain shapes, such as square, triangular and sinusoidal signals. One can also adjust the frequency, amplitude and phase offset at which these signals are generated.
3. Solderless breadboard: This makes it convenient to quickly build and connect simple circuits since it eliminates the need to make soldering to connect different parts of the circuits together.
4. potentiometer: Also called 'pot'. This allows one to adjust the voltage entering one branch of the circuit by allowing one to adjust the resistance by turning a knob. It is a Voltage divider.

2 Answer 2.

The MOSFET has 3 ports. G, D, S. We control the voltage supplied to the gate G by using a pot. When V_G changes, this causes voltage at port D to change (V_D) as well. But voltage across the motor depends on V_D hence we can control the voltage across the motor.

By controlling the voltage across the motor, we control the torque generated by the motor.

Note that the change between V_G and V_D is not linear. As V_G changes, the internal MOSFET resistance R_{DS} changes, and this causes V_D to change.

The voltage across the motor depends on V_D by the relation $V_{motor} = V_s - V_D$ where V_s is the fixed source voltage.

By using MOSFET only to control voltage to the motor, it acted as an approximation to an on/off switch. This is because a small increment in V_G caused a sudden large increase in V_m to appear. However, as V_G continued to increase, V_m did not continue to increase as well, but remained steady. See plot of V_m vs. V_G. This shows that the nonlinearity of MOSFET makes it hard to use to control the speed of the motor.

On the other hand, a small voltage at the gate caused a large voltage to appear across the motor, so this shows that MOSFET acted as a device that can be use to supply power to other devices.

3 Answer 3.

When we used just the MOSFET to control the speed of the motor, it was hard to slow down or speed up the motor shaft spin. The motor will either spin or stop by changing the pot dial across the range of the dial. This is due to the nonlinearity of the MOSFET. So, to use MOSFET to supply power to the motor, we need to be able to better control the voltage it generates, and to do this, we use an Op-Amp.

By using an OpAmp, using negative feedback, we feed the voltage output from MOSFET back into the opAmp. This causes Voltage at the gate V_G to adjust so that voltage output from MOSFET follows voltage input to the opAmp.

So, by changing the input voltage to the OpAmp via the use of the pot, and having negative feedback, the voltage output from MOSFET follows the input voltage more closely. Since output voltage from MOSFET is linearly related to the speed of the motor, we are now able to better control the speed of the motor. This circuit is shown in figure 6 in LAB1 handout.

4 ANSWER 4

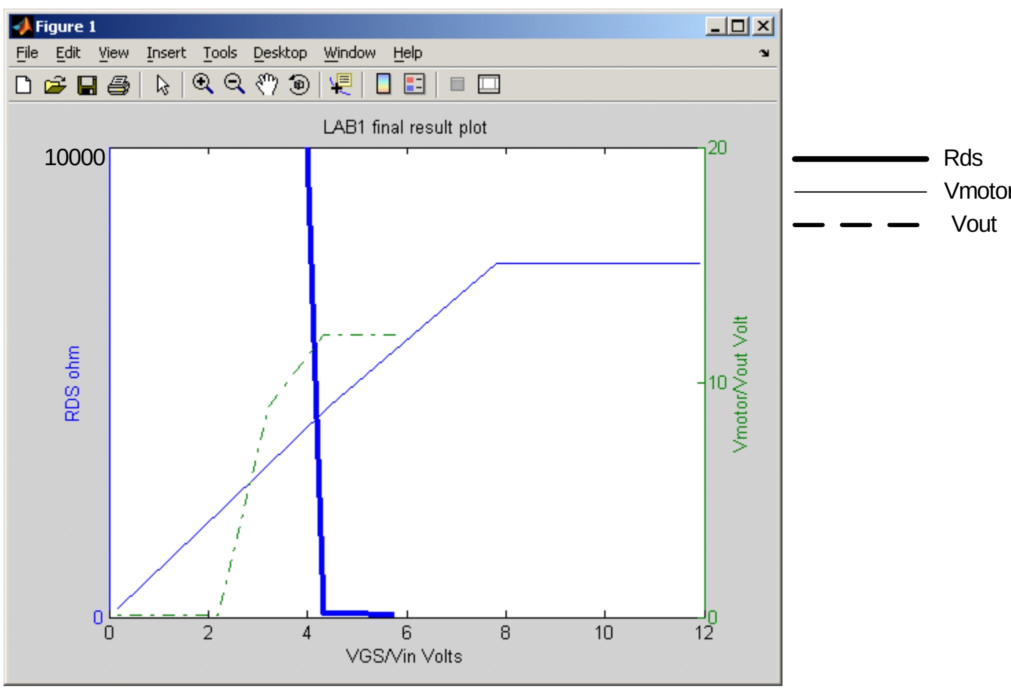
I have written a simple program to generate the diagrams required from the data collected in the Lab. This is the final plot output. First, this is the data collected:

Out[46]//TableForm=

	VGS (Volt)	RDS (Ohm)	Vmotor (Volt)
1	5.77	0.0525276	11.9937
2	5.23	0.0592017	11.9929
3	4.33	0.0817334	11.9902
4	3.21	33.3333	9.
5	2.21	10809.1	0.11
6	1.18	10809.1	0.11
7	0.1	10809.1	0.11

Out[47]//TableForm=

	Vin (Volt)	Vout (volt)
1	11.9	7.52
2	10.82	7.52
3	7.8	7.52
4	4.45	4.5
5	2.4	2.4
6	0.155	0.155



2.2 Lab 2

Local contents

2.2.1 questions

MAE 106 Laboratory Exercise #2

Electrical Filters and First Order Systems

University of California, Irvine
Department of Mechanical and Aerospace Engineering

REQUIRED PARTS:

Qty	Parts	Equipment
		Breadboard
2	1 kΩ resistor	Oscilloscope with scope probe
1	1μF capacitor	LabJack
Var.	Cables	Small DC motor
Var.	Wires	BNC T-Adapter
		2 BNC cables
		Female BNC-to-Alligator clip breakout
		Digital Multimeter, 1 or 2/lab section
		Two scope probes
		IC puller

1 Introduction

Filters are an important part of electrical signal processing. They are often found in stereos (cross-overs, graphic equalizers, etc.), control systems (to clean up sensor readings from strain gauges, tachometers, potentiometers, etc.), and many other applications. In control systems, filters can help remove unwanted high-frequency noise that may adversely affect the controller.

Filters are important conceptually because we can view any system as a filter. For example, you can view the steering system of a new car in terms of how it responds to low, medium, and high frequency inputs. Understanding how systems respond to different input frequencies requires understanding how filters work.

In this lab, we will study the RC circuit, which can be used as a low-pass or high-pass filter. Low-pass filters attenuate high frequency signals (i.e. reduce them in amplitude), but leave low frequency signals relatively unchanged. High pass filters attenuate low frequency signals.

Low pass filters are often useful for filtering out high-frequency noise (due to electromagnetic interference from the lights or radio signals, for example.) Also, many objects and systems in the world act like low-pass filters. High pass filters are used to filter low frequency noise from signals.

This lab also provides a chance for you to understand how first-order systems behave in the time and frequency domains. The RC circuit is a first-order system (i.e. it is described by a first-order differential equation).

2 Low-Pass Filter

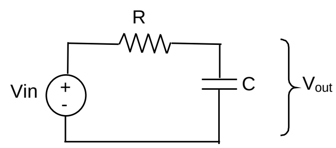


Figure 1 – RC Circuit Used As a Low-Pass Filter

Consider the response of the circuit in Figure 1 to a square wave input. The capacitor acts as a charge bucket, which is alternately charged (filled) and discharged. The capacitor (C) is charged by V_{in} (input voltage) until the voltage across the capacitor matches that of V_{in}. If V_{in} is then switched off (or to a lower voltage value) C begins to discharge through R so that V_{out} heads toward V_{in} again. It takes time, however, for the capacitor to charge and discharge through the resistor. That is, the capacitor has dynamics that slow down V_{out} and prevent it from exactly following V_{in}.

Q1 Write down the differential equation relating V_{out} and its derivatives to V_{in}. Assume V_{in} is a constant input and that $V_c(0) = 0$, and solve this equation for V_{out} as a function of time. Given $C = 1\ \mu F$ and $R = 1K\Omega$, what is the time constant (τ) of the system?

P1 Draw the theoretical response (i.e., solve the differential equation above) to a square wave input to this circuit. Assume the period of the square wave is large compared to τ.

Construct the circuit in Figure 1 on the breadboard. Use $R = 1K\Omega$ and $C = 1.0\ \mu F$.

Q2 Use the function generator to apply a square wave input (4Vpeak-to=peak-p) and measure the filter's response to the square wave. Display both V_{in} and V_{out} on the oscilloscope. Adjust the input frequency and the voltage and time scales on the scope so that a nice waveform is displayed and record these values. Measure and record the time constant of the system by using the cursors on scope to find the time it takes for V_{out} to change 63% from its initial value to its final value. Explain using the equation from Q1 why the time constant corresponds to the time at which the output has gone 63% to its final value.

P2 Record the waveform that you see on the screen using a LabJack.

Q3 Changing the amplitude of the square wave you use as the input affects the amplitude of the output. Does it affect the shape? This is a property of a linear system.

Q4 **I**ncrease the frequency of the square wave and observe the amplitude of V_{out}. Explain briefly why this is a low-pass filter.

P3 Now lets consider how such a filter might be useful in cleaning up the signal from a sensor. Replace the function generator with the output of a DC motor, which will serve as a makeshift tachometer (velocity sensor). Spin the tachometer by hand to produce a voltage and display both V_{in} and V_{out} from the filter on the scope. Why does spinning the tachometer produce a voltage (remember the motor voltage equation?). What effect does the low pass filter have on the tachometer? Sketch and label the difference between the input and output voltage signals.

PRACTICAL EXAM 1: DEMONSTRATE TO THE TA THAT YOU CAN FILTER THE TACHOMETER OUTPUT.

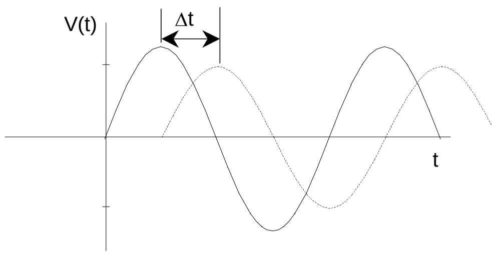


Figure 2 – Phase Shift Between 2 Sine Waves

So far we have been looking at the time response of systems. *Frequency response* is the response of a system to a sine wave input over a range of frequencies. It is an important measure of how a system behaves, especially with respect to how it filters signals. For a linear system, the response to a sinusoidal input at some frequency is always a sinusoidal output at the same frequency, but with a different amplitude and a phase shift. In other words, linear systems don't create new frequencies in the output, they only scale and phase shift their input frequencies.

We will now characterize the frequency response of the low pass filter by inputting a sine wave over a range of different frequencies. On the oscilloscope, we will directly measure time shift (Δt) by placing the cursors (t_1 & t_2) on 2 corresponding points on the 2 signals (Figure 2). Phase shift can then be computed using the following formula, where T is the period (sec.), f is the frequency (Hz), and ϕ is phase shift (rad.).

$$\Delta t/T = \phi/2\pi \text{ and } f = 1/T$$

Q5 Switch the function generator from square wave to sine wave. Record the ratio of the output amplitude to the input amplitude and the phase lag of the output for input frequencies of 100, 200, 500, 1k, 2k, and 4k Hz.

P4 Plot the amplitude ratio vs. frequency. Plot the phase lag vs. frequency. On a third graph, plot the log(amplitude ratio) vs. log(frequency) – This log-log plot is called a Bode plot, and is a common way to display frequency responses. Which of the two amplitude ratio plots makes it easier to see why this circuit is a low-pass filter and why?

P5 Now, read the output voltage across the resistor instead of the capacitor, as shown in Figure 3. Record the ratio of the output amplitude to the input amplitude and the phase lag of the output for input frequencies of 100, 200, 500, 1k, 2k, and 4k Hz. Repeat the plots that you did for P4. What type of filter is this?

Q6 What happens if you provide a triangle wave input into the circuit in Figure 3? Explain based on the transfer function for the circuit.

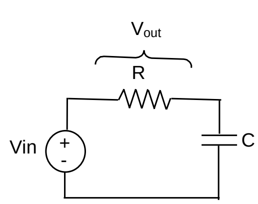


Figure 3 – RC Circuit Used As Another Type of Filter

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words!!
- include your name and laboratory time on the write-up
- the write-up must be type-written
- Graphs for the lab write-up must be generated using Excel or Matlab, and must include labels on the axes, voltage and time scales used on the scope, and a legend for multiple-line plots.
- Page limit = 2 pages, including graphs
    1. Briefly explain what the time constant of a first-order system is.
    2. Briefly explain what the cut-off frequency of a first-order low-pass filter is.
    3. Explain why a mass (i.e. a rock, a ball, a robot arm) acts like a low pass filter, if force is considered its input and position its output.
    4. Turn in the graph for P2, with an overlay of the theoretical step response. Label the time constant.
    5. Turn in the graphs for P4 and P5, with an overlay of the theoretical frequency responses. Hint: To plot the theoretical frequency responses, you must derive the transfer functions $H(s) = V_{out}(s)/V_{in}(s)$ for the circuits shown in Figure 1 and Figure 3. Find the magnitude and phase of the transfer function evaluated at $s = j\omega$. Plug in C = 1μF and all R = 1KΩ to get the actual values. Label the cut-off frequency on the graph.

How to read - Capacitor codes

This is for the reading of smaller capacitor codes, for example when you are confronted with a capacitor marked 103J etc. Some capacitors just have a **two** digit number printed on them, e.g. 47 printed on a small disc would usually be 47pF. The majority have a **three** digit number, this works in a similar manner to the resistor colour coding sytem, the first two numbers refering to the value in **pF** with the third digit being the multiplier or how many 0's follow. e.g. **104F** is **10** with **4** more zeros **100000**pF which is **.1**μF The letter refers to the tolerance value of the capacitor, so in the example above,104**F** is +/- 1%

first 2 digits	third digit	multiplier		Letter symbol	tolerance
XX	0	1		D	+/- 0.5pF
XX	1	10		F	+/- 1%
XX	2	100		G	+/- 2%
XX	3	1000		H	+/- 3%
XX	4	10000		J	+/- 5%
XX	5	100000		K	+/- 10%
XX	6	not used		M	+/- 20%
XX	7	not used		P	+100%, -0%
XX	8	.01		Z	+ 80%, -20%
XX	9	.1			

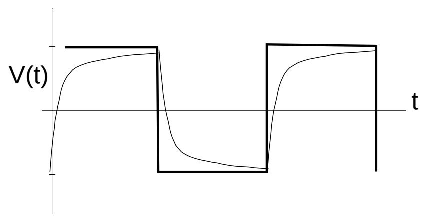
2.2.2 key solution

MAE 106 Laboratory Exercise #2 (Solution)

Electrical Filters and First Order Systems

Q1 Vout = Vin($1-e^{-t/RC}$)
τ = RC = (1μF)(1kΩ) = .001 sec

P1



Q2 $\tau \approx$ RC $\approx$ (1μF)(1kΩ) $\approx$ .001 sec; when t = τ then Vout = A($1-e^{-1}$) = 0.63A

P2 Record the waveform that you see on the screen. **Ans**: Same as P1.

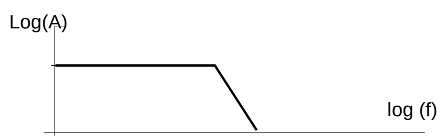
Q3 No – for a linear system, scaling the input just scales the output. Avin ⇒ AVout

Q4 The output amplitude decreases as we increase frequency—a low pass filter.

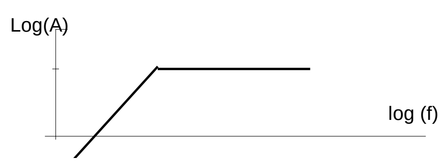
P3 You get a noisy and a less-noisy trace.

Q5 Output should decrease in amplitude at higher frequencies.

P4



The bode plot makes it easier because the curves become lines.



P5

Q6 G(s) = RCs/(RCs + 1)
At low frequencies, RCs << 1, so G(s) → RCs (a differentiator). The derivative of the triangle wave is a square wave.

1

2.2.3 Lab post quizz solution

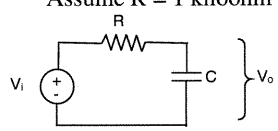
SOLUTION VERSION "A"

MAE 106 Post-Laboratory Quiz

Laboratory Exercise #2: Electrical Filters and First-Order Systems

In Lab 1, you learned how simple RC low-pass and high-pass filter work, and how first-order systems behave in the time and frequency domains.

50 pts 1. Sketch V_o if V_i is a 1 Volt square wave input at 0.5 Hz for the low-pass filter circuit shown below. Assume R = 1 kiloohm and C=1milliFarad. Assume that square wave turns on at t = 0.0 sec.

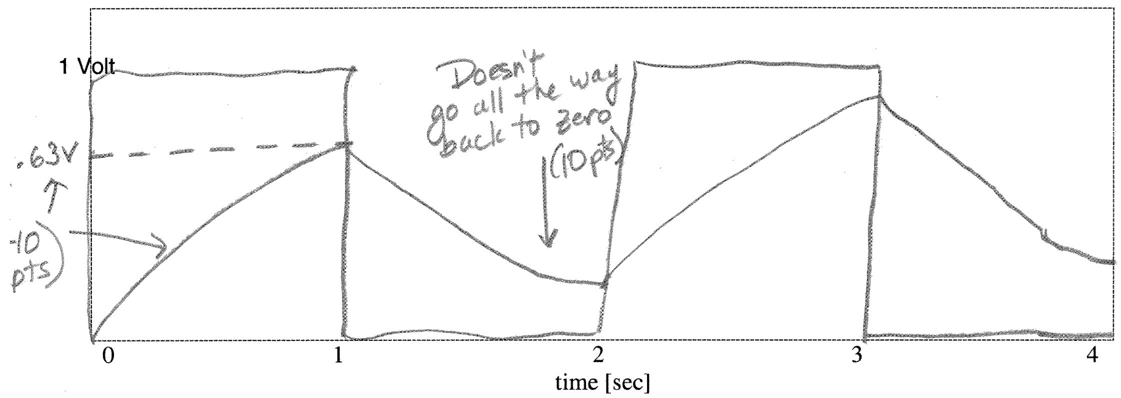


$\tau = RC = (1\times10^{3})(1\times10^{-3}) = 1$ sec

$-V_i + Ri + V_o = 0$

$-V_i + RC\frac{dV_o}{dt} + V_o = 0$

$V_o = V_i(1 - e^{-t/\tau})$ ← SOLN



50 pts 2. How does the circuit filter a low frequency input? Specifically, find what the resulting scaling and phase-shift would be for an input sinusoid with a frequency of $\frac{1}{2\pi}$ = 0.16 Hz?

Scaling $= |H(j\omega)| = \frac{1}{\sqrt{1+(\omega\tau)^2}} = \frac{1}{\sqrt{1+1}}$ $\omega = 2\pi f = (2\pi)\frac{1}{2\pi} = 1$ rad/sec

phase $= \phi_H(j\omega) = -\tan^{-1}\frac{1}{1}$

(25) Scaling: $= \frac{1}{\sqrt{2}} = .707$

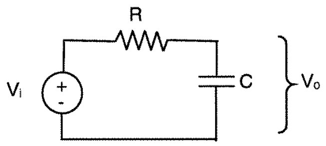
(25) Phase Shift $= -\tan^{-1} 1 = -45^\circ = .78$ rad $= \frac{\pi}{4}$ rad

SOLUTION VERSION "B"

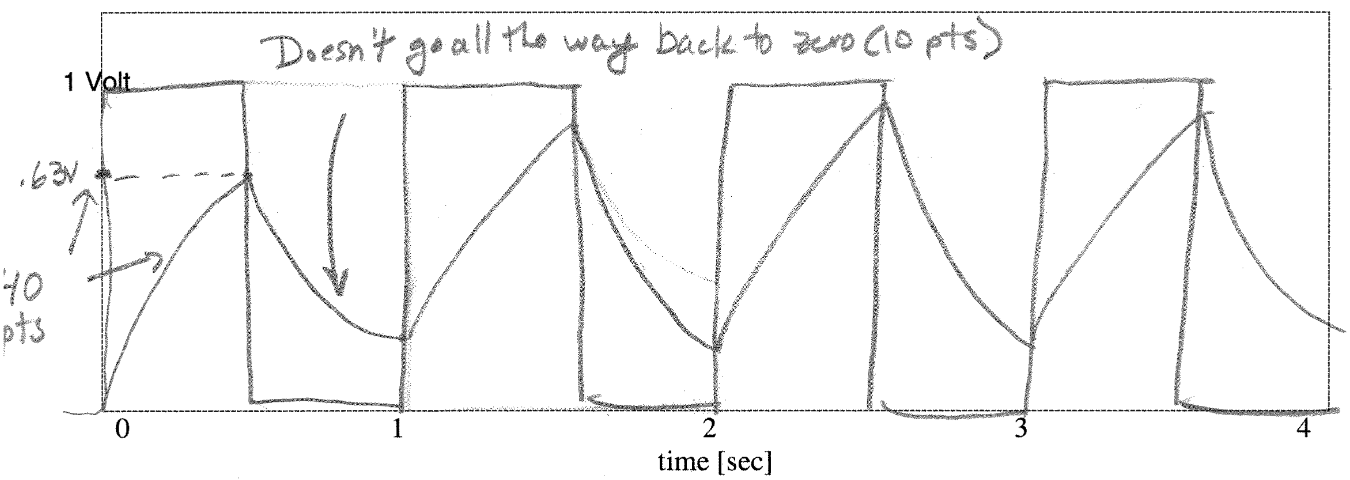
MAE 106 Post-Laboratory Quiz
Laboratory Exercise #2: Electrical Filters and First-Order Systems

In Lab 1, you learned how simple RC low-pass and high-pass filter work, and how first-order systems behave in the time and frequency domains.

50 3. Sketch V_o if V_i is a 1 Volt square wave input at 1.0 Hz for the low-pass filter circuit shown below. Assume R = 0.5 kilo-ohm and C=1milliFarad. Assume that square wave turns on at t = 0.0 sec.



$\tau = RC = .5\times10^{3}\times 1\times10^{-3} = .5$ sec



50 4. How does the circuit filter a low frequency input? Specifically, find what the resulting scaling and phase-shift would be for an input sinusoid with a frequency of $\frac{1}{4\pi}$ = 0.08 Hz?

$H(s) = \frac{1}{1+\tau s}$

$\omega = 2\pi f = 2\pi \frac{1}{4\pi} = \frac{1}{2}$

scaling $= |H(j\omega)| = \left|\frac{1}{1+\tau j\omega}\right| = \frac{1}{\sqrt{1+(\omega\tau)^2}} = \frac{1}{\sqrt{1+((\frac{1}{2})(\frac{1}{2}))^2}} = \frac{1}{\sqrt{1+\frac{1}{8}}} = \frac{1}{\sqrt{1.125}}$

phase shift $= \phi_H(j\omega) = 0^\circ - \tan^{-1}\omega\tau$
$= 0 - \tan^{-1}\frac{1}{4}$

25 Scaling: $\frac{1}{\sqrt{1.125}} = .94$

25 Phase Shift: $-\tan^{-1} .25 = -14^\circ$ or $-.245$ rad

2.2.4 my solution

LAB #2 report. MAE 106. UCI. Winter 2005

Nasser Abbasi, LAB time: Thursday 1/20/2005 6 PM

March 27, 2005

1 Answer 1.

Explain time constant: This is the time the output will take to reach 63% of its final value.

2 Answer 2.

Cut-off frequency is the frequency at which the output amplitude (Such as voltage) is 70.1% of the input. This corresponds to a drop of $-3db$ from the input. It is also the frequency at which the output power is 50% that of the input. This frequency is usually taken as the boundary frequency between the highpass region and the low pass region of the frequency response plot.

3 Answer 3

A heavy object has large inertia. Which means it will take time for it to accelerate when subjected to the same force as compared to an object of low mass. When the force fluctuates very quickly, the object will be slow to react due to its high mass, and by the time it starts to move in response to the force, the force will change its direction quickly, and the object will have to start to reverse its direction again in the direction of the force, and will again be slow in doing this new movement. So this mean the object will have a small motion amplitude of the same frequency as the input force. This is a low pass filter.

4 Answer 4.

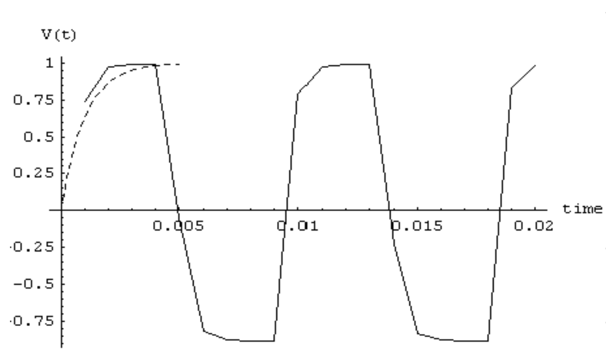
I collected the data using LabJack. This below shows the first few lines of the text file collected:

```
1/20/05
8:14 PM

channelA, SE 0, G=1, y=v;
channelB, SE 1, G=1, y=v;
channelC, SE 0, G=1, y=v;
channelD, SE 0, G=1, y=v;


Time   V1        V2     V3        V4    channelA    channelB    channelC    channelD    IO
0.0000 1.518555         1.308594        1.699219    1.728516    1.518555    1.308594    1.699219    1.728516    0
0.0033 1.748047         1.728516        1.752930    1.762695    1.748047    1.728516    1.752930    1.762695    0
0.0067 1.757812         1.757812        1.762695    1.762695    1.757812    1.757812    1.762695    1.762695    0
0.0100 1.762695         1.757812        1.757812    1.762695    1.762695    1.757812    1.757812    1.762695    0
```

Now I wrote a small script to plot the data and the theoretical response $V_{in}\left(1-e^{-\frac{t}{RC}}\right)$ on the same plot. this is the result. I normalized both amplitudes to 1.



5 Answer 5.

For the circuit in figure 1, the ODE is $V_o = V_i - CR\frac{dV_o}{dt}$. Take Laplace transform, we get $V_o(s) = V_i(s) - CRsV_o(s) \Rightarrow V_o(s)[1+CRs] = V_i(s) \Rightarrow \boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{1+CRs}}$ hence $\boxed{H(j\omega) = \frac{1}{1+jCR\omega}}$ hence, $|H| = \frac{1}{\sqrt{1+(CR\omega)^2}}$ and phase is $\phi(H) = -\tan^{-1}(\omega CR) = -\tan^{-1}(\omega CR)$

Hence for $C = 10^{-6}F$, and $R = 10^3$ we get $|H| = \frac{1}{\sqrt{1+\omega^2(10^{-6}10^3)^2}} = \boxed{\frac{1}{\sqrt{1+\omega^2 1.0\times10^{-6}}}}$

and $\phi(H) = \tan^{-1}(\omega 10^{-6}\times 10^3) = \boxed{-\tan^{-1}(0.001\,\omega)}$

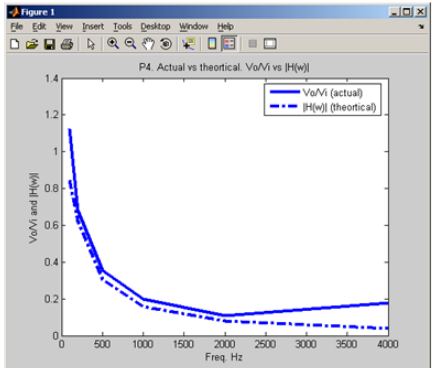
The following is the data collected for **P4,** and the theoretical data based on above Laplace transform. This is a **low pass filter.**

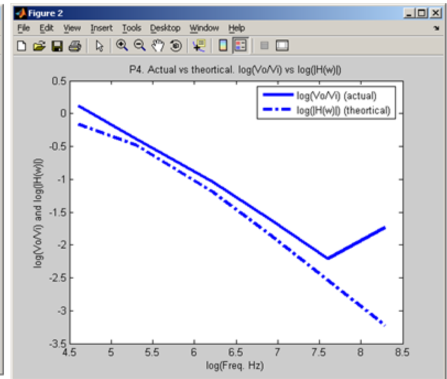
Input Freq. (Hz)	Vpp input	Vpp output	Δt	T	$\Phi = \frac{\Delta t}{T}2\pi\ \frac{180}{\pi}$(degrees)
100	590 mV	662 mV	1.04 ms	10 ms	$\frac{-1.04}{10}2\pi\ \frac{180}{\pi}$ = -37.44^0
200	1.7 V	1.156 V	740 μs	4.9 ms	$\frac{-740\times10^{-6}}{4.9\times10^{-3}}2\pi\ \frac{180}{\pi}$ = -54.367
500	1.59 V	562 mV	390 μs	1.99 ms	$\frac{-390\times10^{-6}}{1.99\times10^{-3}}2\pi\ \frac{180}{\pi} = -70.553$
1 K	1.562 V	312 mV	244 μs	990 μs	$\frac{-244}{990}2\pi\ \frac{180}{\pi} = -88.727$
2 K	1.56 V	171 mV	126 μs	515 μs	$\frac{-126}{515}2\pi\ \frac{180}{\pi} = -88.078$
4 K	2.8 V	500 mV	65 μs	242 μs	$\frac{-65}{242}2\pi\frac{180}{\pi} = -96.694$

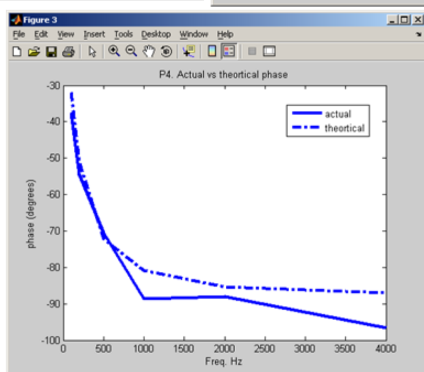
Theortical:

Input Freq. (Hz)	$\vert H\vert$	$\phi(H)$ (degrees)
100	$\frac{1}{\sqrt{1+(2\pi\times100)^2 1.0\times10^{-6}}} = 0.84673$	-tan$^{-1}(0.001\times2\pi\times100)\ \frac{180}{\pi} = -32.14^0$
200	$\frac{1}{\sqrt{1+(2\pi\times200)^2 1.0\times10^{-6}}} = 0.62268$	-tan$^{-1}(0.001\times2\pi\times200)\ \frac{180}{\pi} = -51.488$
500	$\frac{1}{\sqrt{1+(2\pi\times500)^2 1.0\times10^{-6}}} = 0.30331$	-tan$^{-1}(0.001\times2\pi\times500)\ \frac{180}{\pi}$ = -72.343
1 K	$\frac{1}{\sqrt{1+(2\pi\times1000)^2 1.0\times10^{-6}}} = 0.15718$	-tan$^{-1}(0.001\times2\pi\times1000)\ \frac{180}{\pi} = -80.957$
2 K	$\frac{1}{\sqrt{1+(2\pi\times2000)^2 1.0\times10^{-6}}} = 7.9327\times10^{-2}$	$-\tan^{-1}(0.001\times2\pi\times2000)\ \frac{180}{\pi} = -85.45$
4 K	$\frac{1}{\sqrt{1+(2\pi\times4000)^2 1.0\times10^{-6}}} = 3.9757\times10^{-2}$	$-\tan^{-1}(0.001\times2\pi\times4000)\ \frac{180}{\pi} = -87.721$

I now wrote a script to plot the needed plots as required by P4. This is the result. This shows that the amplitude plot involving the log is more clear and it shows the low pass filter, this is because when using the log scaling, the becomes straight lines.







The following is the data collected for **P5.**

Since $q = CV_c$ for the capacitor, and for the circuit in figure 3, we get $V_{in} - V_o - \frac{q}{C} = 0$. Take derivative, we get $\frac{dV_i}{dt} - \frac{dV_o}{dt} - \frac{1}{c}\frac{dq}{dt} = 0$, but $\frac{dq}{dt} = i = \frac{V_o}{R}$, hence ODE becomes $\frac{dV_i}{dt} - \frac{dV_o}{dt} - \frac{1}{C}\frac{V_o}{R} = 0$, take Laplace transform we get $sV_i(s) - sV_o(s) - \frac{V_o(s)}{RC} \Rightarrow sV_i(s) = V_o(s)\left[s + \frac{1}{RC}\right] \Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{s}{s+\frac{1}{RC}} = \boxed{\frac{sRC}{sRC+1}}$

Let $s = j\omega$, $\Rightarrow H(j\omega) = \frac{j\omega RC}{j\omega RC+1} \Rightarrow |H| = \frac{\omega RC}{\sqrt{(\omega RC)^2+1}}$ and $\phi(H) = \frac{\pi}{2} - \left(\tan^{-1}\omega RC\right)$

Hence for $C = 10^{-6}F$, and $R = 10^3$ we get $|H| = \frac{0.001\omega}{\sqrt{(0.001\omega)^2+1}} = \boxed{\frac{0.001\omega}{\sqrt{1+\omega^2 1.0\times 10^{-6}}}}$

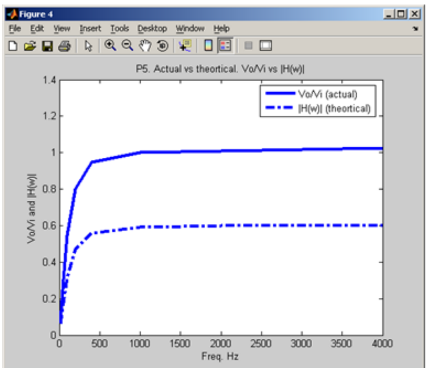
and $\phi(H) = \boxed{\frac{\pi}{2} - \left(\tan^{-1} 0.001\omega\right)}$

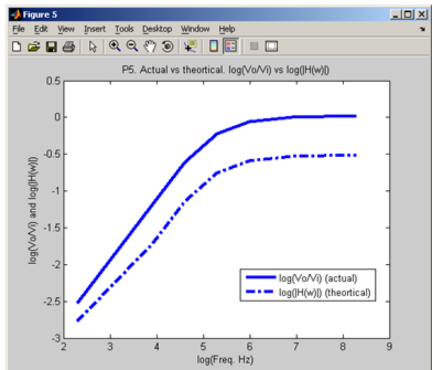
This is a **high pass filter**

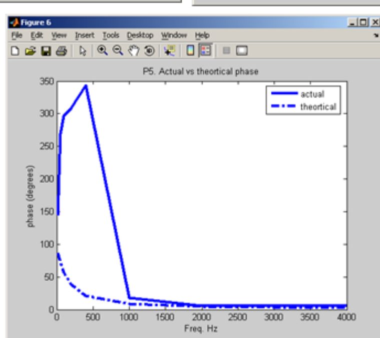
Input Freq. (Hz)	Vpp output	Vpp input	Δt	T	$\Phi = \frac{\Delta t}{T} 2\pi \frac{180}{\pi}$ (degrees)
10	800 mV	10 V	38 ms	97 ms	$\frac{38}{95} 2\pi \frac{180}{\pi} = 144.0$
50	3 V	9.8 V	15 ms	20 ms	$\frac{15}{20} 2\pi \frac{180}{\pi} = 270.0$
100	5 V	9.18 V	8.24 ms	10 ms	$\frac{8.24}{10} 2\pi \frac{180}{\pi} = 296.64$
200	6.6 V	8.25 V	4.34 ms	5.07 ms	$\frac{4.34}{5.07} 2\pi \frac{180}{\pi} = 308.17$
400	7.3 V	7.7 V	2.4 ms	2.52 ms	$\frac{2.4}{2.52} 2\pi \frac{180}{\pi} = 342.86$
1000	7.47 V	7.47 V	48 μs	989 μs	$\frac{48}{989} 2\pi \frac{180}{\pi} = 17.472$
2000	7.57 V	7.47 V	8 μs	500 μs	$\frac{8}{500} 2\pi \frac{180}{\pi} = 5.76$
4000	7.5 V	7.3 V	4 μs	247 μs	$\frac{4}{247} 2\pi \frac{180}{\pi} = 5.8300$

Theortical:

Input Freq. (Hz)	$\|H\|$	$\phi(H) = \frac{\pi}{2} - \left(\tan^{-1} 0.001\omega\right)$
10	$\frac{0.001\times 2\pi\times 10}{\sqrt{1+(2\pi\times 10)^2 1.0\times 10^{-6}}} = 6.2708 \times 10^{-2}$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 10)\right)\frac{180}{\pi} = 86.405^0$
50	$\frac{0.0012\pi\times 50}{\sqrt{1+(2\pi\times 50)^2 1.0\times 10^{-6}}} = 0.17983$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 50)\right)\frac{180}{\pi} = 72.559$
100	$\frac{0.0012\pi\times 100}{\sqrt{1+(2\pi\times 100)^2 1.0\times 10^{-6}}} = 0.31921$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 100)\right)\frac{180}{\pi} = 57.858$
200	$\frac{0.0012\pi\times 200}{\sqrt{1+(2\pi\times 200)^2 1.0\times 10^{-6}}} = 0.46949$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 200)\right)\frac{180}{\pi} = 38.512$
400	$\frac{0.0012\pi\times 400}{\sqrt{1+(2\pi\times 400)^2 1.0\times 10^{-6}}} = 0.55749$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 400)\right)\frac{180}{\pi} = 21.697$
1000	$\frac{0.0012\pi\times 1000}{\sqrt{1+(2\pi\times 1000)^2 1.0\times 10^{-6}}} = 0.59254$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 1000)\right)\frac{180}{\pi} = 9.0431$
2000	$\frac{0.0012\pi\times 2000}{\sqrt{1+(2\pi\times 2000)^2 1.0\times 10^{-6}}} = 0.59811$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 2000)\right)\frac{180}{\pi} = 4.5499$
4000	$\frac{0.0012\pi\times 4000}{\sqrt{1+(2\pi\times 4000)^2 1.0\times 10^{-6}}} = 0.59953$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 4000)\right)\frac{180}{\pi} = 2.2785$







2.3 Lab 3

Local contents

2.3.1 questions

MAE 106 Laboratory Exercise #3
Feedback I: P-type Velocity Control of a Motor

University of California, Irvine
Department of Mechanical and Aerospace Engineering

REQUIRED PARTS:

Qty	Parts	Equipment
3	1KΩ resistor	Breadboard
1	10KΩ resistor	Oscilloscope
1	LM324 Quad op-amp	DC Power Supply
2	100KΩ resistor	Function generator
4	Banana –to-alligator-clip cable	Motor-tachometer-amplifier combo
2	Banana-to-banana cable	IC puller
var.	Wires	wrist grounding strap
		scope probe

1 Introduction

Engineers sometimes want to control the speed of a motor (for example, auto cruise control). In this lab, we will construct a circuit that will control the speed of a DC (direct current) motor using P-type (or proportional) velocity control. The resulting control system will be described by first order dynamics (i.e a first order differential equation). In the frequency domain, the system will behave like a first-order, low-pass filter (where desired velocity is the input and actual velocity the output). Thus, this lab not only shows you how to control the speed of the motor with feedback, it also shows you in general how a first-order control system respond.

The velocity "control law" for the motor is:

$$u = - K (\omega_{actual} - \omega_d) \quad (1)$$

where

u = the control (our input to the system)
K = feedback gain
ω_{actual} = actual motor speed
ω_d = desired motor speed

A control law is an equation that computes an input (u, something we control that influences the system, sometimes called a "control") to the system. In our circuit implementation of this controller (Figure 2), the motor speed variables are represented as voltages. ω_d is represented by a voltage produced by the function generator. ω_{actual} is represented by a voltage produced by a tachometer. Since this controller determines the input to the controlled system using sensed information about some state in our system, measured by a sensor (the tach), it is said to use "feedback." Many types of sensors produce a voltage that is proportional to the variable that they sense.

Notice how equation 1 looks much like $F=k\Delta x$, the equation for a spring. Think of this controller as a spring, only in the motor velocity world instead of position world. Just as a spring constantly tries to return to its unstretched length, this controller drives the motor towards some desired speed. A bigger K is like having a stiffer spring. Given the appropriate system dynamics, this controller can drive ω_{actual} to ω_d, which is what we want!

1

2 Proportional Velocity Control System

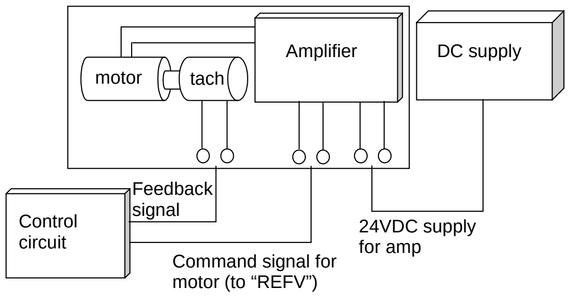


Figure 1 Motor-Amplifier set up

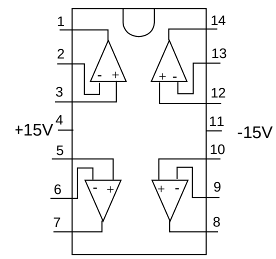


Figure 2 – Top view of LM324 Quad-Op Amp

The motor in this experiment is attached to a built-in tachometer, which is really just another motor that is turned by the powered motor. The tachometer measures motor speed by producing a voltage proportional to rotational velocity (remember the back EMF term in the motor equation). The motor is connected to a relatively expensive amplifier (the motor was about \$20 and the amplifier about \$300) that is set up in "torque mode" which means that the amplifier takes an input voltage and passes a proportional current through the motor windings. The amplifier has an internal feedback loop that makes the current proportional to the input voltage. The resulting motor torque is proportional to the input voltage to the amplifier, independent of the motor's speed $\tau = \alpha v_i$, where α = BC , with B = the torque constant of the motor and C = calibration constant of the amplifier. Assuming that the motor drives an inertial load (i.e. the inertia of its own shaft), its dynamics are: $\tau = J\dot{\omega}$ where J is the shaft inertia. The amplifier and motor thus together implement the equation: $J\dot{\omega} = \alpha v_i$. By taking the Laplace Transform of this equation, we can find the transfer function of the amplifier/motor system: $\frac{\omega(s)}{v_i(s)} = G(s) = \frac{\alpha}{Js} = \frac{K_m}{s}$ Where K_m is a constant. Remember, the transfer function relates the input (voltage) to the output (motor speed) in the complex frequency domain. We can now use this transfer function to draw a block diagram of the feedback system:

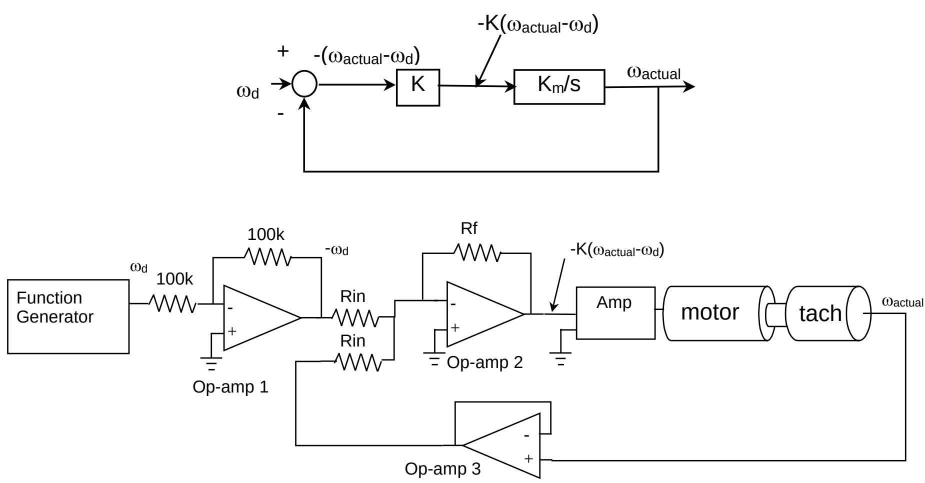


Figure 3 – Block diagram and circuit implementation of P-type velocity control

The transfer function of the *closed loop system* shown in Figure 2 is

$$\omega_{actual}(s)/\ \omega_d(s) = KK_m/(s + KK_m)$$

where ω_{actual} is the measured angular velocity of the shaft, and ω_d is the desired (or "reference") angular velocity that we input to the controller. The transfer function has the form of a first-order low-pass filter with a time constant of $\tau = 1/KK_m$

Q1 Figure 3 shows a control circuit that can implement the block diagram for the controller. The key thing to remember is that variables such as velocity and desired velocity are represented as voltages in the circuit. Describe the specific purpose of each op-amp. Find the equivalent K from the circuit diagram in terms of the resistor values. What physical features of the motor/amplifier does the K_m/s term represent? (Hint: what does a 1/s term represent in the time domain and what is K_m equal to?)

Construct the circuit in Figure 3. Copy the pin numbers for the LM324 op amp chip onto the Figure 3 diagram. *Note:* The motor/amplifier boards have room for a position sensing potentiometer to be coupled to the motor shaft: you do not need this pot for this lab – make sure it is not connected as you an easily break it. *Important: wire your circuit neatly! A neat circuit requires little extra time to wire, and it's easier to debug. In general, circuits take much more time to debug than they do to initially wire!*

With $R_{in} = 1K\Omega$ and $R_f = 10K\Omega$, set the function generator to pass a 2Vpp (1V amplitude) 10 Hz sine wave to the system and to the oscilloscope. Also capture the tachometer output on the scope. If all is well, the motor's velocity should follow the sine wave. If the motor doesn't turn at all, the power supply to the amplifier may not be set up to provide enough current to the motor. Make sure the current switch on the power supply is set on "high," and adjust the current knob to provide "enough" current to the motor. If the motor is running at full speed, something may be wrong with your circuit (for example, you may have implemented positive feedback instead of negative feedback). Turn off the power to the motor. Debug your circuit systematically, considering what each voltage should be. Verify that op-amp 1 is inverting and op-amp 3 is following. Verify that the output of op-amp 2 changes as you adjust ω_d with ω_{actual} constant.

Practical Exam 1: Demonstrate to the TA that your motor follows the sine wave.

Q2 Changing R_f changes the feedback gain of the system. Try to get intuition about how the gain affects system performance by experimentally determining the following information. Report the results in a table such as this:

		R_f = 10 KΩ	R_f = 1 KΩ
1	f_c		
2	$\tau = 1/\omega_c$		
3	τ		
4	e_{ss}		
5	K_m		
6	Open loop gain		

1) The "cut-off" frequency. At this frequency, the output amplitude is .707 of the input amplitude and the output waveform lags the input by 45 degrees. frequency at the -3db point (ω_{-3db}). The cutoff frequency is used by engineers to

describe when the frequency at which the control system performance starts to degrade. Sometimes this frequency is called the 3dB point. dB stands for "decibels" and is a unit commonly used by engineers to describe frequency responses. Amplitude in dB = $20\log_{10}$(amplitude). The amplitude is decreased by 3dB at the cut-off frequency, since $20\log_{10}(0.707)=-3.0$dB.

2) Compute time constant ($\tau = 1/\omega_c$) where ω_c (in rad/sec) is obtained from part 1. In other words, infer τ from the measured cut-off frequency. Memorize the fact that ω_c [in radians] = $2\pi f_c$ [in Hertz]
3) The time constant (τ) measured from the time response of the system. Display both the input square wave and output exponential response on the scope. Measure τ on the scope using the cursors.
4) The steady state error. Input 10V DC from the function generator and record the difference between the input and output voltages.
5) The predicted value of K_m (Knowing K and knowing τ you can compute what K_m ought to be).
6) The magnitude of the open loop gain of the system = $K*K_m/s$

P1 Using the Labjack, record the input voltage for the function generator (which represents the "desired motor velocity") and the output voltage from the tachometer (which is the "actual motor velocity") at three input sinusoid frequencies: .1* ω_c, ω_c, and 10* ω_c.

Q3 Get the function generator to pass a small constant (DC) voltage to the system and try to move the motor shaft by hand. Why is it difficult to change its rotation? Power down the system and make R_f = 1KΩ. Turn the system back on. Can you stop the motor from turning now? Explain the difference.

Q4 How does the system behavior change as you changed R_f in question Q3?

Q5 What do you think caused the steady state error measured above? Integral control is one way to get rid of steady state error.

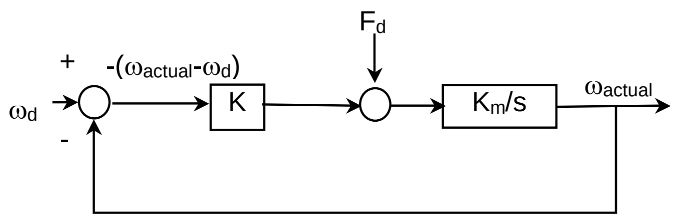
Q6 In a previous lab, you implemented a voltage controller for a motor. In this lab, you implemented a velocity controller. Explain why the two control circuits achieve the same thing (controlling velocity) if the motor load is constant. If you want to design a rotating sign that you can adjust the speed (e.g. UNOCAL 76 rotating ball), which circuit would you use? Consider the relative costs of the two circuits, and which circuit would be better for an indoor sign versus an outdoor sign where there are gusts of wind.

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words!!
- include your name and laboratory time on the write-up
- the write-up must be type-written
- Graphs for the lab write-up must be generated using Excel or Matlab, and must include labels on the axes, voltage and time scales used on the scope, and a legend for multiple-line plots.

- Page limit = 2 pages, including graphs

1. What type of filter does the motor velocity control system act like?
2. Turn in a graph for P1, overlaying the desired velocity and the actual velocity at the three input frequencies for several cycles of the input.
3. One of the major benefits of feedback is its ability to cancel the effects of unmodeled "disturbances". Disturbances are outside influences that affect the output of the system you are trying to control, keeping it from achieving the desired behavior. In the case of the system you built, the desired behavior was for the motor to track a desired reference velocity. Sometimes you tried to prevent the shaft from rotating at the desired velocity by using your hand. The forces applied by your hand can be viewed as a disturbance F_d, and can be incorporated into the block diagram in the following way:



Derive an expression that relates ω_{actual} to ω_d and F_d. Explain why high-gain feedback (i.e. big K) was able to cancel the effects of the "hand disturbance" on the motor shaft.

2.3.2 key solution

MAE 106 Laboratory Exercise #3 (Solution)
Feedback I: P-type Velocity Control of a Motor

Q1 Op-amp 1 inverts ω_d to $-\omega_d$. This is necessary to get the correct sign for ω_d on the controller. Op-amp 2 adds ω_{actual} and $-\omega_d$ and multiplies by a gain. Op-amp 3 is a buffer, necessary so that little current is drawn from the tachometer. K = Rf/Rin. The Km/s term represents the dynamics of the amplifier and motor, and relates input voltage to the motor to output speed of the motor. The amplifier produces a current proportional to its input voltage I = Av, and the motor produces a torque proportional to its input current, τ = BI, and the motor accelerates proportional to its torque $\alpha = \tau/J = B/J*I = B/J*A*v \equiv K_m v$. Taking the Laplace transform gives $\omega/v = K_m/s$.The 1/s term is an integration term, and represents the fact that the motor integrates torque to get angular velocity.

Q2

	$R_f = 10$ KΩ	$R_f = 1$ KΩ
f_{3db}	~30-40 Hz	~2-4 Hz
$\tau = 1/\omega_{3db}$	~5 ms	~50 ms
τ	~5-10 ms	~50-60 ms
e_{ss}	~0	~1.3V
K_m	~10-20	~10-20
Gain	~100-200	~10-20

$K_m = 1/\tau K$
Gain = K K_m

Q3 The small DC voltage has commanded the motor to move slowly. When you try to move the motor shaft by hand, you create a disturbance, which the controller tries to compensate for. At a lower feedback gain (Rf = 1k), the controller is less sensitive to disturbance, so you are able to disturb the shaft more easily.

Q4 The lower feedback gain resulted in a more sluggish controller: the –3dB point is difficult to find, as the motor does not follow the desired velocity profile well. The time constant is larger for small gain, and the steady state error is larger.

Q5 The steady state error was caused by coulomb friction.

Q6 The voltage controller controls velocity because motor speed is proportional to velocity for an inertially loaded motor. The velocity controller explicitly tries to control velocity based on sensing from the tachometer. The voltage controller is cheaper because you do not need a tachometer, but would not perform well if there are external disturbances such as the wind since it does not sense velocity changes. (i.e. as long as the motor voltage is at the desired value, the circuit is “happy” and doesn’t try to change anything).

1

2.3.3 Lab post quizz solution

MAE 106 Lab 3 Quiz and Midterm Exam
Winter 2005 Lab 3 Quiz = 100 pts

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Part 1: Lab 3 Quiz

In Lab 3, you built a motor velocity controller, using a proportional feedback control.

15 1. Write the velocity control law that you used for the motor in the box, where:

u = the control input into the motor
K = feedback gain
ω_{actual} = actual motor speed
ω_d = desired motor speed

$u = -K(\omega_{actual} - \omega_d)$

$u = K(\omega_d - \omega_{actual})$ $u = K(\omega_{actual} - \omega_d)$ → OK ALSO

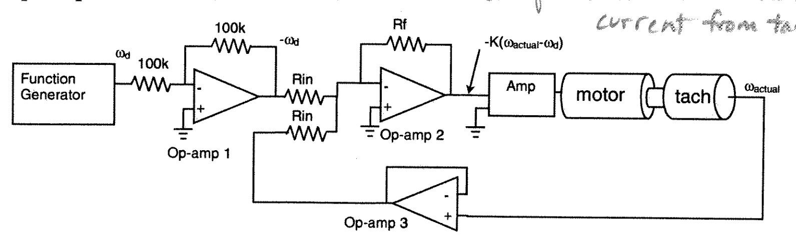
10 2. For the Lab 3 motor amplifier, the motor torque was proportional to the input.

24 3. Below is the control circuit that you used to implement P-type velocity control. Briefly explain the purpose of each op-amp.

Op Amp 1: Multiplies ω_d by −1 (inverts ω_d)

Op Amp 2: Implements control law $u = -K(\omega_{actual} - \omega_d)$

Op Amp 3: Buffers tachometer so that rest of circuit does not draw current from tach.



24 4. Fill out the below chart based on your experience in lab:

Increasing R_f will (circle one)		
Increase or decrease (circled: Increase)	f_{3db}	Cutoff frequency
Increase or decrease (circled: decrease)	$\tau = 1/\omega_{3db}$	Time constant
Increase or decrease (circled: decrease)	e_{ss}	Steady state error

15 5. The controlled motor behaved like what kind of filter? Low pass filter

12 6. At the cutoff frequency of the motor, the output amplitude was .707 of the input amplitude and the output waveform lagged the input by 45 degrees.

Part 2: Midterm

Problem 1 (10 Pts Extra Credit)

An oscilloscope is used to measure this:
2 Answer b a) resistance b) voltage c) current d) power

The time constant of a first-order system tells when the output has gotten how far along the way to its final value?

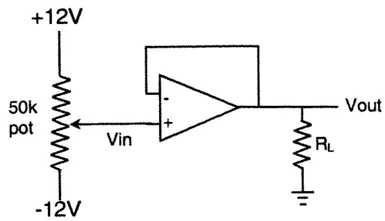
2 Answer c a) 37% b) 10% c) 63% d) 90%

If you put a sine wave into a linear system, you get the following out
2 Answer d
a) square wave
b) sine wave at different frequency
c) triangle wave
d) sine wave at same frequency, scaled and shifted

A filter scales a sinusoidal input. The amount of scaling is determined by:
2 Answer a
a) the magnitude of the transfer function, evaluated at s=jω
b) the magnitude of the transfer function, evaluated at s = ω
c) the phase of the transfer function, evaluated at s=jω

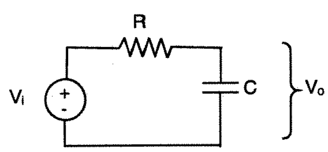
A low pass filter attenuates
2 Answer b
a) low frequencies
b) high frequencies
c) a band of frequencies

Problem 2 (25 pts)
How close is V_{out} to V_{in} for the following voltage follower circuit, if the op-amp gain is 1,000? (Hint, use the fact that $V_o = K(V_+ - V_-)$ for the op amp)



$$V_{out} = K(V_+ - V_-)$$
$$= K(V_{in} - V_{out})$$
$$V_{out}(1+K) = K V_{in}$$
$$V_{out} = \frac{K}{1+K} V_{in} = \frac{1000}{1001} V_{in}$$

Problem 3 (25 pts)
How does the following circuit filter a low frequency input? Specifically, find what the resulting scaling and phase-shift would be for an input sinusoid with a frequency of $\frac{1}{2\pi} = 0.16$ Hz. Assume R = 1 kiloohm and C=1milliFarad.



Transfer function using impedances:

$$V_o = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_i = \frac{1}{1+RCs} V_i$$

$$H(s) = \frac{1}{1+RCs}$$

5 pts → $H(j\omega) = \frac{1}{1+RCj\omega}$ $\qquad \omega = 2\pi f = (2\pi)(\frac{1}{2\pi}) = 1$ rad/sec

$$\text{Scaling} = |H(j\omega)| = \frac{1}{\sqrt{1+(RC\omega)^2}}$$ $\qquad RC = 1$ sec

$$= \boxed{\frac{1}{\sqrt{2}} = .707}$$ ← 10 pts

$$\text{phase} = 0 - \tan^{-1}\frac{RC\omega}{1} = -\tan^{-1} RC\omega = -\tan^{-1} 1$$

$$\boxed{= 45^\circ \text{ or } \frac{\pi}{4}}$$

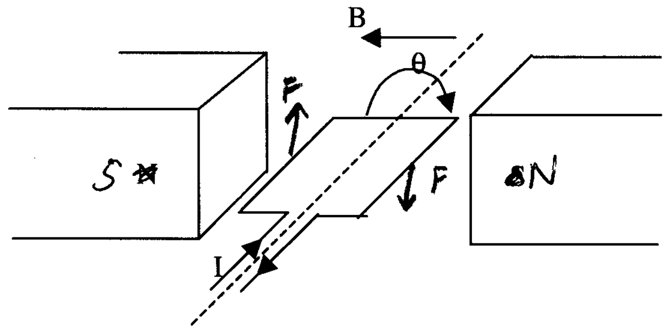
10 pts →

Problem 4: 25 pts

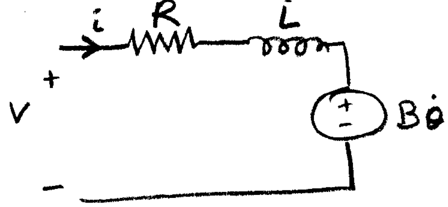
5 a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at $\theta = 0°$ as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.

$F = i\vec{\ell} \times \vec{B}$

$\theta = 90°$



5 b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:



$V = Ri + L\frac{di}{dt} + B\dot{\theta}$

15 b. Solve this differential equation for the current through the motor as a function of time when:

- the shaft of the motor is held fixed
- a constant voltage v is applied across the motor at time = 0
- the initial current i(t = 0) through the inductor is zero

Shaft fixed $\Rightarrow \dot{\theta} = 0$

$L\frac{di}{dt} + Ri = V$

Soln to homog. eqn:

$\frac{di}{dt} = -\frac{R}{L}i \Rightarrow i = Ae^{-t/\tau}$ $\tau = \frac{L}{R}$

Part. Soln:

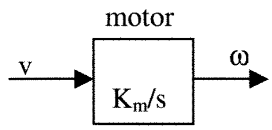
$i = \frac{V}{R}$

Total Soln: $i = Ae^{-t/\tau} + \frac{V}{R}$ but $i(0) = 0$

$\Rightarrow$ $\boxed{i = \frac{V}{R}(1 - e^{-t/\tau}) \quad \tau = \frac{L}{R}}$

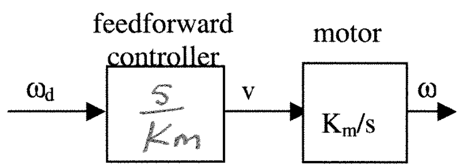
Problem 5: 25 pts

1) You want to control the speed of a motor. You are using a current amplifier with the motor, so the speed is related to the input voltage to the current amplifier by the following transfer function:

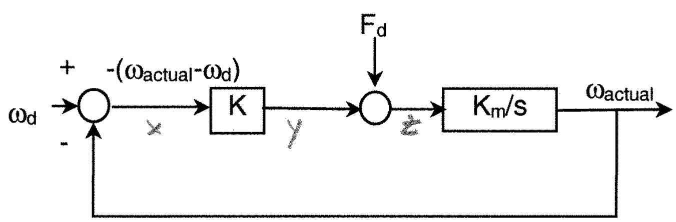


where v is the voltage input to the motor and ω is the angular velocity of the shaft and K_m is a constant.

10 pts a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What transfer function should the controller box have to make the output equal the desired output? Write this function controller box.



15 pts b) One of the major benefits of feedback is its ability to cancel the effects of unmodeled “disturbances”. Assume you build a feedback controller, but there is a disturbance force F_d affecting the motor:



Derive an expression that relates ω_{actual} to ω_d and F_d, then prove that the disturbance is cancelled if K is large enough.

$x = w_d - w_{actual}$

$y = Kx = K(w_d - w_{actual})$

$z = y + F_d$

$w_{actual} = \frac{K_m}{s} z = \frac{K_m}{s}(K(w_d - w_{actual}) + F_d)$

$s\, w_{actual} = K_m K w_d - K_m K w_{actual} + K_m F_d$

$\rightarrow w_{actual}(s + K_m K) = K_m K w_d + K_m F_d$

10 pts $w_{actual} = \frac{K_m K}{s + K_m K} w_d + \frac{K_m}{s + K_m K} F_d$

as $K \to \infty$
$w_{actual} \to w_d$ 5 pts

2.3.4 my solution

LAB #3 report. MAE 106. UCI. Winter 2005

Nasser Abbasi, LAB time: Thursday 1/27/2005 6 PM

March 27, 2005

1 Answer 1.

The motor velocity control system acts as a low pass filter.

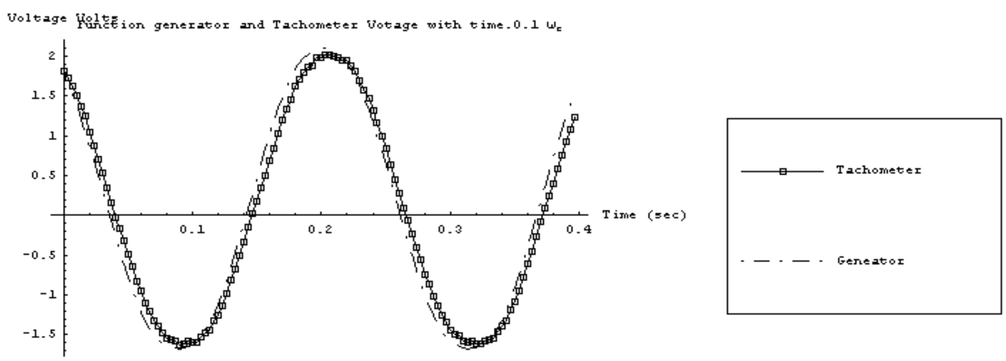
2 Answer 2.

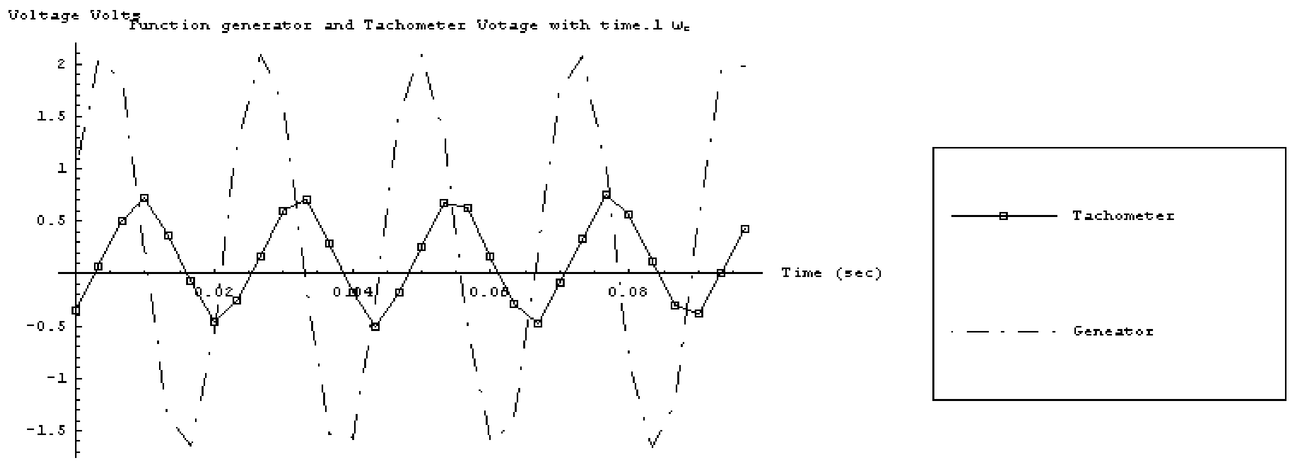
The cutoff frequency used was 44 Hz

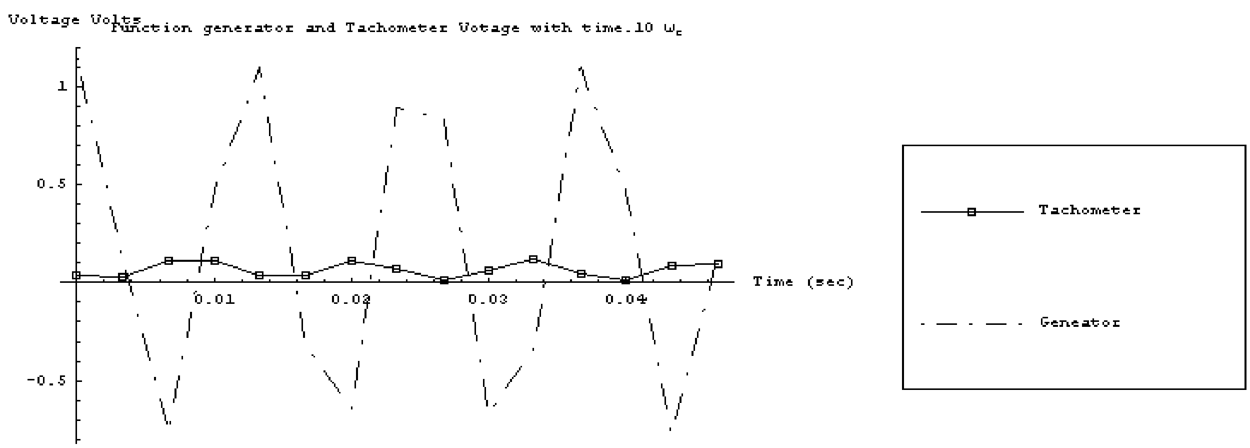
From the 3 data files, I generated 3 plots. One for $0.1\omega_c$ and one for ω_c and one for $10\omega_c$.

From looking at the 3 plots, I see that the output of the tachmeter shows the amplitude is decreasing as the input (function generator) frequency is increasing. This means the controller acts a a low pass filter.

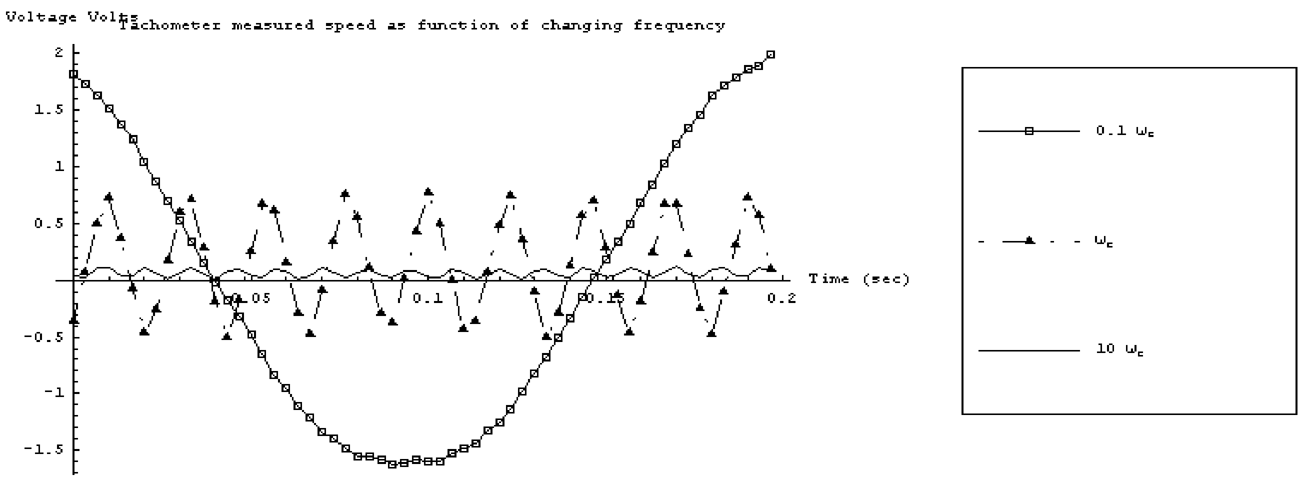
Below are the 3 plots geneated showing on each the actual and the velocity.







To make more clear, I also plot on the same plot, how the actual velocity changes as the input frequency changes. This is the result.



3 Answer 3

$$
\begin{aligned}
\left[k\left(\omega_d-\omega\right)+F_d\right]\frac{k_m}{s} &= \omega \\
\left[k\omega_d-k\omega+F_d\right]\frac{k_m}{s} &= \omega \\
\frac{k\ k_m}{s}\omega_d-\frac{k\ k_m}{s}\omega+\frac{F_d\ k_m}{s} &= \omega \\
\omega\left(1+\frac{k\ k_m}{s}\right) &= \frac{k\ k_m}{s}\omega_d+\frac{F_d\ k_m}{s}
\end{aligned}
$$

Divide by $\left(1+\frac{k\ k_m}{s}\right)$

$$
\omega = \frac{\frac{k\ k_m}{s}\omega_d}{\left(1+\frac{k\ k_m}{s}\right)}+\frac{\frac{F_d\ k_m}{s}}{\left(1+\frac{k\ k_m}{s}\right)}
$$

for $k \gg 1$, $\left(1+\frac{k\ k_m}{s}\right) \approx \frac{k\ k_m}{s}$, hence we get

$$
\begin{aligned}
\omega &= \frac{\frac{k\ k_m}{s}\omega_d}{\frac{k\ k_m}{s}}+\frac{\frac{F_d\ k_m}{s}}{\frac{k\ k_m}{s}} \\
\omega &= \omega_d+\frac{F_d\ k_m}{k\ k_m} \\
\omega &= \omega_d+\frac{F_d}{k}
\end{aligned}
$$

But for $k \gg 1$, $\frac{F_d}{k} \rightarrow 0$
hence

$$
\omega \rightarrow \omega_d
$$

Hence this shows that by using feeback, and by using very large gain k we can eliminate the effect of the disturbances.

2.4 Lab 4

Local contents

2.4.1 questions

MAE 106 Laboratory Exercise #4

Vibration I: Lightly Damped Second Order Systems

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Required Parts:

QTY	PART	
1	50kΩ Potentiometer	Oscilloscope
		Breadboard
	EQUIPMENT	Vibrating beam experiment fixture
	BNC to Alligator Clip Breakout	Accelerometer
	BNC Cable	Accelerometer Amplifier
	Scope Probe	24V DC Power Supply
	Strobe Light	

1 Introduction

In this laboratory exercise you will look at the dynamic response of a cantilevered beam that supports a motor with an unbalanced load attached at its end. If you use your imagination, this system represents many typical problems in vibrations. For instance, a large rotating machine attached to a building floor can be analyzed in the same manner as this experiment, with the floor taking the place of the cantilevered beam. Alternatively, a building shaking during an earthquake can also be represented by the same equations, with the building acting as the beam itself.

This lab is also the first lab that deals with a second order system. In other words the differential equation that describes the system has second derivatives, and the transfer function has s^2 terms. Many mechanical systems behave as second order systems because of Newton's second law (F = ma) because acceleration is the second derivative of position. In fact, you can view a vast number of mechanical and control systems as second order linear mass, spring, damper systems. Thus, developing intuition about how second-order systems behave is very important. One key difference between second and first order systems, as you will see in this lab, is that second order systems can oscillate. First order systems cannot oscillate.

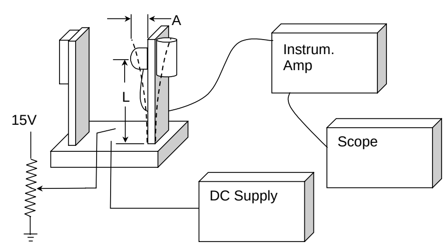


Figure 1 - Vibrating Beam Fixture: Be gentle with the beam fixture. The accelerometer can be easily damaged by impulsive forces. Also, place the beam fixture on the floor, being careful to not to pull any short wires.

1

2 Time Domain Analysis (Transient Response)

In this part of the lab, you will measure how the beam responds to an impulsive input. This is known as the “transient response”, and more specifically, as the “impulse response” of the beam, and is a typical way to look at the time-domain response.

An accelerometer is mounted at approximately the center of mass of the vibrating load at the beam end. For all the subsequent analysis, assume that the length of the beam is from the clamped end to the center of the accelerometer. The following analysis also assumes that the *acceleration* of the beam is a good measure of its *position*. The reason for this assumption is that the equation of motion of the unforced system has the form

$$m\ddot{x} + c\dot{x} + kx = 0,$$

and if c ≈ 0, then $\ddot{x} = (-k/m)x$.

Q1 Compute the theoretical natural frequency of the system. The motor weighs about 2.25 lb., the beam is made of carbon steel of dimensions 0.125 in. by 2.0 in. in cross-section. You need to measure the length of your beam as the distance from the base to the center of the accelerometer. (You can do the calculations at home but be sure to measure the length of your beam, since they are all different!)

Connect the accelerometer to the instrumentation amplifier. Attach the amplifier output to the oscilloscope and set the amplifier gain to x5.

Q2 You need to calibrate the accelerometer to make sense of its output. **“**Calibrating” a sensor refers to the process of measuring what voltage corresponds to what level of the measured variable. For the accelerometer, you need to know how the output voltage and acceleration correspond. You can use gravity as your known acceleration, and measure the voltage output corresponding to gravity. Set the zero voltage adjustment on the instrumentation amplifier to give zero volts on the oscilloscope. Then rotate the entire apparatus on its side so that the accelerometer reads the acceleration of gravity (1 g). Report the accelerometer output voltage corresponding to 1 g. You are now able to calculate actual acceleration by measuring the accelerometer voltage. Give an example of how you would do this. What assumptions are you making about the accelerometer and amplifier?

Q3 Twang the beam with your hand and set the oscilloscope so that a nice periodic waveform appears on the screen. Report the frequency of vibration (both in rad/sec and Hz). Use the **stop** button on the scope for this.

Q4 Estimate the damping using the logarithmic decrement method. Use the **stop** button on the scope and a slow sweep rate to obtain a good scope trace. After twanging the structure, measure the initial amplitude, the number of cycles, and the final amplitude. Repeat this process a few times to be certain of your measurement. Hint: use “Roll” (horizontal mode) and storage mode of the scope. Report you estimated value of ζ using both the exact and approximate formulas in Equation 3 of the notes.

P1 Using the LabJack, record the impulse response of the beam. You will turn this plot in.

3 Frequency Domain Analysis (Forced Response)

In this part of the lab, you will determine how the beam responds when you apply sinusoidal forces to it at different frequencies. A key phenomenon that you will observe is resonance. Resonance is the increase in amplitude of oscillation of an electric or mechanical system exposed to a periodic force whose frequency is equal or very close to the undamped natural frequency of the system.

You will use a motor with an off-balance load to apply the sinusoidal forces to the beam. The motor is driven by a high gain, *velocity control system*. The voltage into the amplifier and controller produces an angular velocity of the motor with a gain of about 300 rpm/volt (but you need to measure this to find the exact value). So if you input zero volts (a short) to the amplifier, you should get zero rpm out, while a 2 volt input would give 600 rpm out. Use a potentiometer in a voltage divider circuit on the breadboard, or the function generator, to provide a variable voltage input to the motor velocity control system.

Q5 It is important to know the relation of the motor's velocity to input voltage accurately for the system (i.e. to calibrate the motor). Using the strobe light and an input voltage of 2 volts, determine the actual rpm of the motor. Be sure to hold the beam so that it does not vibrate much. Repeat the measurement with an input voltage of 4 volts. Based on these measurements, what is your estimate of the gain that relates voltage to velocity? State your answer in rpm/volt and in (rad/sec)/volt. Now that you've calibrated the system, you can estimate the motor velocity by measuring input voltage. Conversely, you can adjust motor velocity by adjusting the input voltage.

Q6 Try to estimate the natural frequency, ω_n, for the system by getting it to resonate. The motor angular velocity corresponding to the maximum amplitude response gives the resonant frequency ($\omega_r = \omega_n$). Try not to let the system shake too badly, i.e. do not let the system stay in resonance too long. Record the accelerometer voltage amplitude at resonance. What is the corresponding maximum acceleration? Also, estimate the amplitude of the tip beam motion in inches at resonance with a ruler as shown in Figure 1.

Q7 Increase the voltage to the motor controller so that you are exciting the system well past its resonant frequency, but not to the point where new, higher frequency resonance is occurring (you can hear other things begin to shake). Record this input voltage. As above, record the accelerometer amplitude voltage and estimate the amplitude of the beam tip motion, A_{high}, with a ruler by eye.

PRACTICAL EXAM: Demonstrate to the TA that you can drive your beam into resonance.

3 Problems to Consider at Home

You are now done with the experimental part of the lab. The rest is analytical.

Q8 It was shown in lecture that the forcing function on the mass has the form

$$f(t) = a\omega^2 \sin(\omega t).$$

Therefore, the output must be of the form

$$a\omega^2 |G(j\omega)| \sin(\omega t + \phi), \quad (1)$$

where

$$G(s) = \frac{w_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

When the beam is vibrating near resonance, $|G(j\omega_n)| = 1/(2\zeta)$, and the amplitude of the output wave (that you measured with a ruler) is

$$A_{res} = \frac{a\omega_n^2}{2\zeta}.$$

In Q12, below you will prove that at higher frequencies, as $\omega \to \infty$,

$$a\omega^2|G(j\omega)| \to a\omega_n^2.$$

Therefore, the amplitude of the output wave (also measured with a ruler) is $A_{high} = a\omega_n^2$.

From these two facts, the damping ratio ζ can be estimated as

$$\zeta \approx \frac{A_{high}}{2A_{res}}. \quad (2)$$

Using this formula and the amplitudes measured by eye in Q6 and Q7, estimate the damping ratio, ζ.

Q9 Prove that the resonant frequency ω_r which is the frequency of maximum output vibration amplitude, is given by

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}, \quad \zeta \le 0.707.$$

Q10 Show that

$$|G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}.$$

Q11 Show that as $\zeta \to 0$, $\omega_r \to \omega_n$, and that

$$|G(j\omega)| \to |G(j\omega_n)| = \frac{1}{2\zeta}.$$

Q12 Show that as $\omega \to \infty$, $\omega^2|G(j\omega)| \to \omega_n^2$.

Q13 Make a table of the estimates of the natural frequency ω_n that you computed using the three methods (Q1, Q3, and Q6) and compare them. Which do you think is the most accurate? Why?

Q14 Make a table of the estimates of the damping ratio ζ that you computed using the methods of Q4, and Q8 and compare them. Which do you think is the most accurate? Why?

Q15 Using your measured values of ζ, ω_n, and m, estimate the viscous damping constant c from the notes. Be sure to state your units.

Q16 Show that if you know the natural frequency, ω_1, for one beam of length l_1, you can estimate the natural frequency for any other beam of length l_2 with the equation

$$\omega_2^2 = \left(\frac{l_1}{l_2}\right)^3 \omega_1^2.$$

Use this formula to estimate the natural frequency of a 15-in. long the beam with the same properties. Note: To use this equation, you must assume that tip mass is constant or that the beam is massless.

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words!!
- include your name and laboratory time on the write-up
- the write-up must be type-written
- Graphs for the lab write-up must be generated using Excel or Matlab, and must include labels on the axes, voltage and time scales used on the scope, and a legend for multiple-line plots.
- Page limit = 2 pages, including graphs

1. Plot the impulse response of the beam for several cycles (Plot P1).

A schematic representation of the accelerometer used in the experiments is shown in Figure 2.

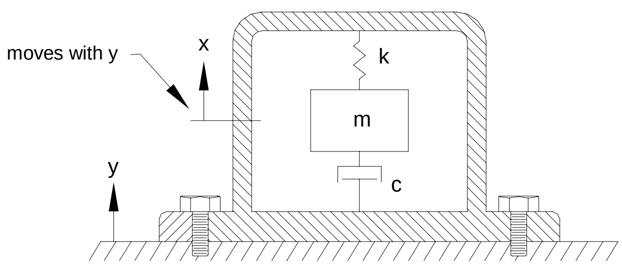
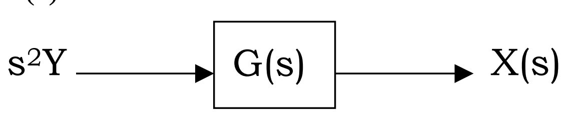


Figure 2 Schematic of accelerometer. Note the way in which the x-axis moves with the y-axis, i.e. if the entire accelerometer moves, the mass must move with it.

The variable y is the position of the housing relative to a fixed reference frame, and x is the position of the internal mass relative to the housing. The position of the internal mass with respect to a fixed reference frame is, therefore $x + y + constant$. The acceleration $\ddot{y}$ causes the internal spring to deform some distance x. The spring deformation x is measured electrically with a capacitive distance sensor, and this voltage is output to the instrumentation amplifier.

2. Derive the differential equation of motion for the accelerometer in terms of x, y, and their derivatives.
3. These acceleration measurement dynamics can be represented by a transfer function with acceleration, s^2Y as the input and spring deformation x as the output, as shown. Determine the transfer function $G(s)$.



4. The manufacturer's data sheet for the accelerometer is attached. The model we used is the (+/-)2-g nominal range unit. Determine the values of k/m and c/m for this accelerometer from the data sheet.

SETRA SYSTEMS, INC.

HIGH OUTPUT LINEAR ACCELEROMETER

Model 141

FOR VIBRATION, SHOCK, IMPACT

Ranges from: ± 2g to ± 600g

With External R_{cal} Calibration

Features

- Excellent static and dynamic response
- Temperature-insensitive gas damping (0.7 critical)
- High output signal
- High overload capability, (2000g static)
- Low transverse sensitivity (0.005 g/g)
- Wide-range R_{cal} type calibration
- Easy-to-replace cable attachment
- Compact, lightweight

Description

The Model 141 is a linear accelerometer that produces a high level instantaneous DC output signal proportional to sensed accelerations (ranging from static acceleration up to 3000 Hz as reported below).

Setra accelerometers are unique in their ability to withstand exceedingly high g overload without damage. The Model 141 incorporates the super-rugged Setra capacitance-type sensor and a new miniaturized electronic circuit.

Its excellent dynamic response is maintained by air damping, which varies with temperature approximately one-tenth as much as the best fluid damping.

The electrical characteristics are compatible with conventional strain-gage type signal conditioning, including the use of shunt R_{cal} over any selected range up to 100% full scale.

The stainless steel case is O-ring sealed, has a well defined base plane, is quite insensitive to mounting strain.

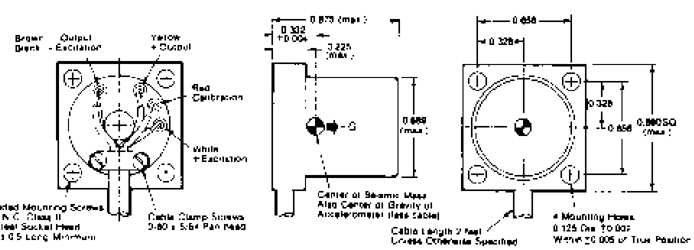
Cross axis interference is exceedingly low. The external easy-to-replace cable attachment facilitates installation and service.

Full Scale Ranges

For each of the available g ranges, the linearity is characterized by this range chart: (Non-linearity as % full range, best straight line).

Nominal Range	Non-Linearity ± 0.5%	± 1%	± 3%	Natural Frequency (Nominal)	Flat Response (± 3 db) 0 Hz to:
± 2g	± 1.5g	± 2g	± 2.5g	275 Hz	200 Hz
± 4g	± 3g	± 4g	± 5g	330 Hz	260 Hz
± 8g	± 6g	± 8g	± 10g	350 Hz	300 Hz
± 15g	± 10g	± 15g	± 20g	600 Hz	400 Hz
± 30g	± 20g	± 30g	± 40g	1150 Hz	700 Hz
± 60g	± 40g	± 60g	± 80g	1600 Hz	1000 Hz
± 150g	± 100g	± 150g	± 200g	2600 Hz	1600 Hz
± 600g	± 400g	± 600g	± 800g	5000 Hz	3000 Hz

Outline Drawing



Note: All dimensions are in inches.

setra

45 Nagog Park, Acton, Massachusetts 01720 / Telephone: (617) 263-1400

Model 141 Specifications

Ranges, Non-Linearity, Frequency Data.	Please refer to chart on front page.
Other Accuracy Data	
Hysteresis	<±0.1%
Non-Repeatability	<±0.05% Nominal range
Transverse Acceleration Response	<±0.005 g/g
Damping	Approximates second order system with 0.7 critical damping. The frequency band for all ranges is flat from static to approximately 60% of the natural frequency. Damping is gas squeeze-film, 0.7 ±0.2 of critical at 77°F. Damping ratio increases approximately 0.15%/°F.
Resolution	Infinite, limited only by output noise level.
Thermal Effects	Operating temperature -10°F to 150°F Zero shift <±0.02% Nominal Range/°F Sensitivity shift <±0.02% Nominal Range/°F Slightly higher thermal effects when 141A is operated at excitation voltage below 10VDC. **Model 141A (special order) −65°F to 220°F**
Zero G Output	<±25 mv (factory calibrated at designated excitation)
Noise Level	<±0.01% Nominal Range (RMS, in-band)
Calibration Data	Each unit is supplied with a full scale continuous plot of output vs. acceleration (centrifuge), at a designated excitation voltage. Sensitivity is reported at Nominal Range. Model 141A calibrated at 10VDC excitation. Model 141B calibrated at 24VDC excitation.
Electrical Data	
Electrical Circuit	Three-terminal equivalent, common -excitation and -output signal. Circuit is capacitively isolated from case, greater than 100 megohm isolation. Power applied to output, or shorted output, will not damage unit. No reverse excitation protection. Operates at internal frequency approximately 20 MHz. Model 141B operable on regulated 28 VDC aircraft power, (recommend high voltage transient protection to prevent damage by emergency power conditions as defined in MIL-STD-704A, and voltage regulation to attain highest accuracy).
Calibration Signal (R_{cal})	Available up to 100% Nominal Range by shunting external calibration resistor from calibration lead to -signal lead.
Voltages and Currents	Two versions are available, offering your choice of units for different excitation voltages. Output is proportional to excitation voltage. Output impedance 9K ohms (nominal).

Typical performance for nominal G range:

Model	Excitation Range	At Excitation Voltage of:	Excitation Current	Output (open circuit)
141A	5VDC–15VDC	10V	5 milliamperes	± 500 millivolts
141B	10VDC–28VDC	24V	10 milliamperes	± 1000 millivolts

Cable, Weight, Case	
Electrical Connection	2 foot multiconductor cable
Weight	30 grams (not including cable)
Case	Stainless steel. O-ring sealed

Ordering Information

Specify:	Model 141A or Model 141B
Specify G Range:	Nominal Range (±specific g)
Specify:	Excitation voltage for calibration (If non-standard, at extra charge)

Specifications subject to change without notice

Setra Systems

45 Nagog Park, Acton, Massachusetts 01720 / Telephone: (617) 263-1400

2.4.2 key solution

MAE 106 Laboratory Exercise #4
Solution

Q1: For a beam with a length of 8 inches:

$$
\begin{aligned}
I &= \frac{1}{12}bh^3 \\
&= \frac{(2in)(.125in)^3}{12} \\
&= 3.26 \times 10^{-4} in^4 \\
k &= \frac{3EI}{l^3} \\
&= \frac{(3)(30 \times 10^6 \frac{lb}{in^2})(3.26 \times 10^{-4} in^4)}{(8in)^3} \\
&= 57.2\frac{lb}{in} = 686.6\frac{lb}{ft} \\
\rho &= .283\frac{lb}{in^3} \\
m &= (.283)(2)(.125)(8) \\
&= .566lb \\
M_{total} &= 2.25lb + .283(.566lb) \\
&= 2.41lb \\
&= \frac{2.41}{32.2}\frac{lb \cdot s^2}{ft} \\
&= 0.075\frac{lb \cdot s^2}{ft} \\
\omega_n &= \sqrt{\frac{k}{m}} \\
&= \sqrt{\frac{686.6\frac{lb}{ft}}{0.075\frac{lb \cdot s^2}{ft}}} \\
&= 95.7\frac{rad}{sec} = 15.2Hz
\end{aligned}
$$

These computations are slightly easier when using SI units. When you use english units, you must be very careful to make sure that you get the correct units (namely $\frac{1}{sec}$) for ω_n. The most common problem is to forget to convert from pounds (which is a unit of force) to a suitable unit of mass or to convert from pounds when it is not necessary. It is also not a good idea to call the unit of mass a "slug", instead call it a $\frac{lb \cdot s^2}{ft}$ so that you can be assured of cancelling the correct units as I did in the above problem.

Q2: Typical accelerometer voltages are in the 100 to 300 mV range. This measurement, plus the assumption that the relationship between output voltage and acceleration is linear allows us to compute acceleration. For example, if the accelerometer read 250 mV at 1 gravity, then we use the following formula to compute acceleration:

$$a = 32.2\frac{ft}{sec^2}\left(\frac{V}{.25V}\right)$$

where V is the output reading (in Volts) of the accelerometer.

Q3: Typical damped natural frequencies are in the 7 to 20 Hz range.

Q4: Run at least two tests and find a_0, a_n and n for each test. Example:
Test 1: a_0 = 306 mV, a_n = 140.6 mV, n = 15
Test 2: a_0 = 222 mV, a_n = 97 mV, n = 15
Use this data and the formula:

$$\delta = ln\left(\frac{a_0}{a_n}\right)$$

$$\xi = \frac{\delta}{\sqrt{\delta^2 + 4\pi^2 n^2}} \approx \frac{\delta}{2\pi n}$$

to find exact and approximate ξ values for each test run. Then average these values to obtain the best estimate.
A sample solution:

$$\xi = \frac{.78}{.78^2 + 4\pi^2(15)^2} = .0083$$

Q5: This is different for each motor, but typical values are in the range of 450 to 500 revolutions per minute for a 2 V input signal and 950 to 1000 revolutions per minute for a 4 V input signal. We can combine these data points in order to develop a formula that relates the speed of the motor to the input voltage.

Example data: 495 rpm at 2 V, 968 rpm at 4 V. Converting rpm to radians per second:

$$495\frac{revolutions}{min}\left(\frac{1min}{60sec}\right)\left(\frac{2\pi\ radians}{revolution}\right) = 51\frac{rad}{sec}$$

$$968\frac{revolutions}{min}\left(\frac{1min}{60sec}\right)\left(\frac{2\pi\ radians}{revolution}\right) = 101\frac{rad}{sec}$$

From this data, we see that the gain is approximately $25\frac{rad}{sec \times Volt}$. We can multiply this gain by the input voltage to obtain the frequency of the forcing function in radians per second.

Q6: We asked you to record the input voltage at resonance. Typical values are in the 1.8 to 2 Volt range for the input voltage. You can now convert this using the results of Q5 to obtain the frequency of the forcing function at resonance. You can also get the frequency by examining the frequency of the accelerometer output trace on the scope.

The amplitude of the accelerometer output, combined with the formula in **Q2** will yield the maximum accelerations. If you measured the peak to peak voltage of the accelerometer output, you need to divide by 2 before using the formula in **Q2**.

The estimated tip motion could be anywhere from 1 inch (very small) to 4 or more inches depending upon how close you got to the actual resonance value. This estimated tip motion was denoted A_{res} in the lab.

Q7: Exciting the system at a frequency well past resonance should result in very small tip motions. A typical value is $\frac{1}{16}$ inch. This tip motion was denoted A_{high} in the lab.

Q8: Combine the estimated tip motions from **Q6** and **Q7** to find an estimate of ξ using

$$\xi = \frac{A_{high}}{2A_{res}}$$

A typical result might be

$$\xi = \frac{0.0625in}{2(2in)} = 0.0156$$

Q9 The transfer function is given by

$$\begin{aligned} G(s) &= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \\ G(j\omega) &= \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n j\omega + \omega_n^2} \\ |G(j\omega)| &= \frac{\omega_n^2}{\sqrt{((\omega_n^2 - \omega^2)^2 + (2\xi\omega_n\omega)^2)}} \end{aligned}$$

Resonance occurs when the output is the maximum. This occurs when the denomenator of $|G(j\omega)|$ is minimum:

$$\frac{d}{d\omega}\left((\omega_n^2 - \omega^2)^2 + (2\xi\omega_n\omega)^2\right) = -4(\omega_n^2 - \omega^2)\omega + 8\xi^2\omega_n^2\omega = 0$$

Solving this equation for ω yields the resonance frequency:

$$\omega_r = \omega_n\sqrt{1 - 2\xi^2}$$

ξ must be less than .707 to insure that the square root does not yield an imaginary number.

Q10

$$\begin{aligned} G(s) &= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \\ G(j\omega_r) &= \frac{\omega_n^2}{-\omega_r^2 + 2\xi\omega_n j\omega_r + \omega_n^2} \\ G(j\omega_r) &= \frac{\omega_n^2}{-\omega_n^2(1 - 2\xi^2) + 2\xi\omega_n^2 j\sqrt{1 - 2\xi^2} + \omega_n^2} \\ &= \frac{1}{2\xi j\sqrt{1 - 2\xi^2} + 2\xi^2} \\ |G(j\omega_r)| &= \frac{1}{\sqrt{4\xi^4 + 4\xi^2(1 - 2\xi^2)}} \\ &= \frac{1}{\sqrt{4\xi^2 - 4\xi^4}} \\ &= \frac{1}{2\xi\sqrt{1 - \xi^2}} \end{aligned}$$

Q11 It is fairly obvious that as $\xi \to 0$, $\omega_r \to \omega_n$ using the result of **Q9**.
From above we have

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n j\omega + \omega_n^2}$$

Now, as $\omega \to \omega_n$, we have:

$$\begin{aligned} G(j\omega_n) &= \frac{\omega_n^2}{-\omega_n^2 + 2\xi\omega_n^2 j + \omega_n^2} \\ &= \frac{1}{2\xi j} \\ |G(j\omega_n)| &= \frac{1}{2\xi} \end{aligned}$$

Q12 As $\omega \to \infty$,

$$\begin{aligned} G(s) &= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \\ G(j\omega) &= \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n j\omega + \omega_n^2} \\ \omega^2 G(j\omega) &= \frac{\omega^2\omega_n^2}{(\omega_n^2 - \omega^2) + 2\xi\omega_n j\omega} \\ &= \frac{\omega_n^2}{(\frac{\omega_n^2}{\omega^2} - 1) + \frac{2\xi\omega_n j}{\omega}} \end{aligned}$$

As $\omega \to \infty$, any term with ω in the denomenator becomes zero, so

$$\begin{aligned} G(j\omega) &= \frac{\omega_n^2}{(0-1)+0} \\ &= -\omega_n^2 \\ |G(j\omega)| &= \omega_n^2 \end{aligned}$$

Q13 Normally the **Q6** values are the most accurate, since **Q1** contained several assumptions about the mass distribution. **Q3** values can also be as accurate as the **Q6** values depending on how well you read the scope. I accepted any reasonable answers since several groups had problems or bad data associated with some tests.

Q14 For this problem, the values from **Q4** are most accurate. The values from **Q8** involved an "eyeball" measurement of the tip motion which is nearly impossible to obtain with great precision.

Q15 Use the formula:

$$c = 2\xi M\omega_n$$

where M is the total mass of the system.

Example solution:

- $M = 0.075\frac{lb \cdot s^2}{ft}$ from **Q2**
- $\xi = .0083$ from **Q4**
- ω_n from your best run (either **Q3** or **Q6**)

$$c = 2(.0083)(0.075\frac{lb \cdot s^2}{ft})(98\frac{rad}{sec} = .122\frac{lb \cdot sec}{ft}$$

Note that the units on c match the notes.

Q16 From **Q1**, we know that for a beam of length l_1:

$$\omega_1 = \sqrt{\frac{3EI}{M_1 l_1^3}}$$

For another beam of length l_2 we have

$$\omega_2 = \sqrt{\frac{3EI}{M_2 l_2^3}}$$

Combining these two equations gives:

$$\frac{\omega_2^2}{\omega_1^2} = \frac{3EI(M_1 l_1^3)}{3EI(M_2 l_2^3)}$$

For a beam of similar properties, we know that E and I are the same for each beam. If we also assume that $M_1 \approx M_2$ we have

$$\frac{\omega_2^2}{\omega_1^2} = \frac{l_1^3}{l_2^3}$$

We can now plug in the length of our beam from **Q1** and the natural frequency obtained from **Q13** to find the natural frequency of a beam with a length of 15 inches.

$$\omega_2^2 = (98\frac{rad}{sec})\left(\frac{8in}{15in}\right)^3$$

2.4.3 Lab post quizz solution

SOLUTION

MAE106 Mechanical Systems Laboratory

Quiz for Laboratory Exercise 4: Vibration I - Lightly Damped Second Order Systems

In lab 4, you measured the impulse and frequency responses of a beam.

10 1. Write in the box the differential equation that describes the beam dynamics, where:
m is the mass
c is the damping
k is the stiffness
f is the input force
x is the beam deflection

$$m\ddot{x} + c\dot{x} + kx = f$$

10 2. What is the transfer function for the beam in terms of m, c, and k, given force as input and position as output?

$$\frac{x}{f} = H(s) = \frac{1}{ms^2 + cs + k} \qquad (ms^2 + cs + k)x = f$$

10 3. How would you calculate the undamped natural frequency of the beam, given m, c, and k?

$$\omega_n = \sqrt{\frac{k}{m}}$$

10 4. How did you measure the natural frequency of the beam in lab?

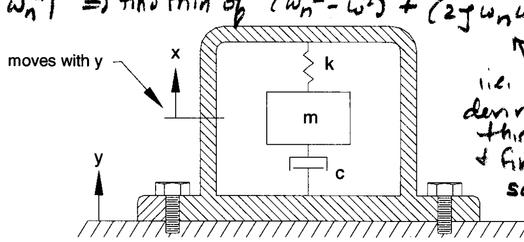
Either answer is OK:
- Measured period between oscillations for impulse response $f_n = \frac{1}{\text{period}}$
- changed motor speed (i.e. forcing frequency) until beam resonated

10 5. Was the beam overdamped, critically damped, or underdamped? (circle one) — **underdamped** (circled)

10 6. Given the equation of the transfer function for the beam, how would you mathematically solve for the exact resonant frequency ω_r for the beam? (You don't need to do it, just briefly explain how).

at ω_r, scaling factor $= |G(j\omega)|$ is maximal $\Rightarrow$ denominator of $|G(j\omega)|$ is minimal
$\Rightarrow$ find minimum of denominator of transfer fcn. evaluated at $s = j\omega$
$\Rightarrow$ find minimum of $|(j\omega)^2 + 2\zeta\omega_n\omega j + \omega_n^2|^2 \Rightarrow$ find min of $(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2$
↑ i.e. take derivative of this eqn & find ω so derivative $= 0$

10 7. A schematic representation of the accelerometer used in the experiments is shown in the figure.



You found the transfer function of the accelerometer to be: $G(s) = \frac{-m}{ms^2 + cs + k}$

What are the input and output for the transfer function?

Input: acceleration

Output: displacement of the spring ($\frac{1}{2}$ credit for "voltage")

10 8. How is the output measured inside the accelerometer?

A capacitive sensor measures displacement of the spring

10 9. The accelerometer does not provide an accurate output signal for ___high___ input frequencies because it acts like a ___low pass filter___.

10 10. In lab you calibrated the accelerometer by finding the calibration coefficient. The calibration coefficient relates ___acceleration___ input to ___voltage___ output.

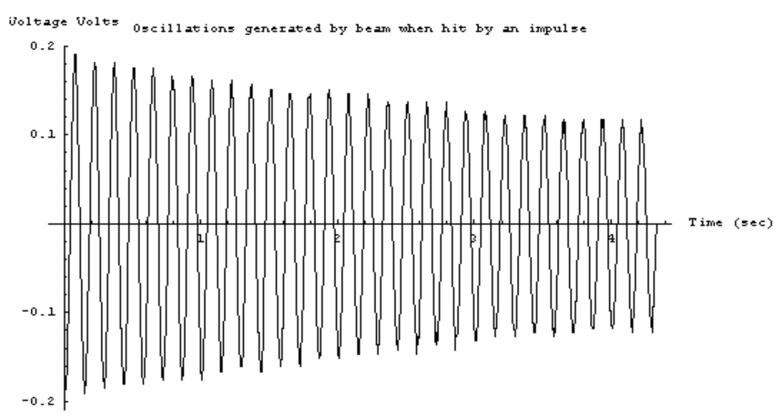
2.4.4 my solution

LAB #4 report. MAE 106. UCI. Winter 2005

Nasser Abbasi, LAB time: Thursday 2/3/2005 6 PM

February 4, 2005

1 Answer 1.



2 Answer 2.

Using Newton's law $F = Ma$, then we get, when the origin of the coordinates systems is taken as the center of the mass, and taking the upwards motions and forces as positive and downwards forces as negative

$$-Mg + k\ x + c\frac{dx}{dt} = -M\frac{d^2x}{dt^2}$$

Where Mg is the weight of the mass. This is the force that causes the mass to be displaced from its initial position. Let me call this force as F (which is constant in this case)

When we take the origin of the coordinates system as the inertial reference of frame, whose origin is distant y from the center of the mass, then we get

$$F = M\frac{d^2(x+y)}{dt^2} + c\frac{dx}{dt} + \ k\ x$$

where $\frac{d^2(x+y)}{dt^2} = \frac{d}{dt}\left(\frac{d}{dt}(x+y)\right) = \frac{d}{dt}(\dot{x}+\dot{y}) = \ddot{x}+\ddot{y}$

Hence the equation of motion becomes

$$F = M(\ddot{x}+\ddot{y}) + c\frac{dx}{dt} + \ k\ x$$

3 Answer 3

Apply Laplace transform to the above ODE, we get

$$
\begin{aligned}
F(s) &= M\left(s^2 X(s) + s^2 Y(s)\right) + csX(s) + kX(s) \\
F(s) &= Ms^2 X(s) + Ms^2 Y(s) + csX(s) + kX(s) \\
F(s) &= X(s)\left[\ Ms^2 + cs + k\right] + Ms^2 Y(s)
\end{aligned}
$$

So, to find the transfer function between $Y(s)$ and $X(s)$, set $F(s) = 0$ we get

$$\frac{Y(s)}{X(s)} = -\frac{Ms^2 + Cs + k}{Ms^2} = -\frac{s^2 + \frac{C}{M}s + \frac{k}{M}}{s^2}$$

4 Answer 4

For the 0.2 g nominal range, the specification sheet says that the natural frequency $\omega_n = 275$ Hz and $\xi = 0.7$

Hence since $\omega_n = \sqrt{\frac{k}{m}}$ then

$$\frac{k}{m} = 275^2 = 75\,625$$

and

$$
\begin{aligned}
\frac{c}{m} &= 2\xi\omega_n \\
&= 2 \times .7 \times 275 \\
&= 385
\end{aligned}
$$

2.5 Lab 5

Local contents

2.5.1 questions

MAE 106 Laboratory Exercise #5
PD Control of Motor Position

University of California, Irvine
Department of Mechanical and Aerospace Engineering

REQUIRED PARTS:

Qty	Parts	Equipment
2	1kΩ resistor, ¼ W (brown/black/red)	Breadboard
2	10kΩ resistor, ¼ W (brown/black/orange)	Oscilloscope
2	100kΩ resistor, ¼ W (brown/black/yellow)	Function Generator
1	LM 324 quad op amp chip	Motor-Amp-Tach Console
4	1μF capacitors	Position-sensing "pot"
1	BNC cable	IC puller
1	breakout (BNC to alligator clips)	wrist grounding strap
2	banana-to-banana cable (1 black, 1 red)	multimeter
2	banana-to-alligator clip cable (1 black, 1 red)	scope probe
var	wire, 22AWG	
1	∅1/4" shaft coupling (c.1/2"lg)	

1 Introduction

In this lab you will build a control system to make a motor shaft move to a position that you command. Controlling motor position is a common goal in automation (e.g. multi-joint robot arms, radars, numerically controlled milling machines, manufacturing systems). In addition, you will need a position controller for your final project.

The controller that you will build is called a "Proportional Plus Derivative (PD) Position Feedback System," and is the most common controller found in industry. The PD control law is:

$$\tau = -K_p(\theta - \theta_d) - K_d\dot{\theta} \qquad (1)$$

Where θ = actual motor angular position
θ_d = desired motor angular position
$\dot{\theta}$ = actual motor angular velocity
K_p = position error gain
K_d = derivative gain
τ = desired motor torque

Note that the controller has two terms – one proportional to the position error (the "P" part), and one proportional to the derivative of position (i.e. velocity, the "D" part). Thus, it is called a "PD" controller.

1

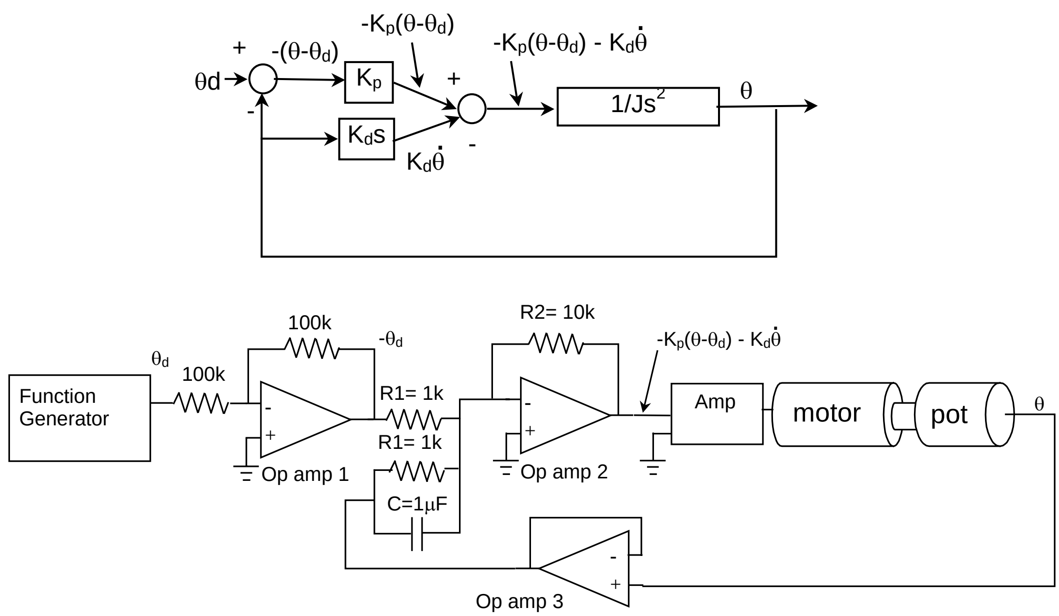


Figure 1 – PD Motor Position Control System (Block Diagram and Circuit)

Figure 1 shows the block diagram and op-amp circuit that you will implement to make the PD control law for the motor. J is the inertia of the motor shaft.

The lab has the following four parts. You can do Parts 1 and 2 before coming to lab.

Part 1: What is the theoretical behavior of the controlled system?
The key point to understand here is that the controlled system obeys the same differential equation as a mass-spring-damper system. Thus, the controlled system acts dynamically like a mass-spring-damper system. The control gains K_p and K_d determine the equivalent stiffness and damping of the system. The desired angular position of the motor (θ_d) is equivalent to the rest length of the spring.

Part 2: How can a circuit implement the control law?
Op-amp circuits (adder, gain, inverter derivative circuits) can be used to implement the control law. The resistors and capacitors set the control gains K_p and K_d.

Part 3: What is the step response of the actual system?
One way to characterize the system behavior is to measure how it performs when it is commanded to move rapidly from one position to another (i.e. to follow a step function input). You will find that the motor will overshoot and oscillate if the damping is too small.

Part 4: What is the frequency response of the actual system?
Another common way to characterize the system behavior is to measure how it performs when it is commanded to follow a sinusoidal position. You will find that the controller acts like a low pass filter. It tracks low input frequencies well, and high frequencies poorly. Also, if the controller has low enough damping, it will resonate just like the spring-mass-damper system you experimented with in Lab 4.

2 What is the theoretical behavior of the controlled system?

In this section, you will derive the theoretical behavior the PD position controlled motor. In the time domain, the theoretical behavior is described by a differential equation. In the frequency domain, the theoretical behavior is described by the frequency response.

Q1 Derive the dynamical equation that describes how θ evolves with time when the controller is attached to the motor. Assume θ_d is the input.

Q2 Derive the differential equation for a spring-mass-damper system (assume force is the input and position is the output). The differential equation for Q1 should be similar to the equation for Q2. This means that the PD position control system has the same dynamics as a spring-mass-damper system; i.e. it follows the same equations of motion. Thus, you can use your intuition about how the spring-mass-damper system works to design the PD controller. Explain what the mass (m), spring (k), damper (c) in the mechanical system correspond to in the PD system.

Q3 Derive the closed-loop transfer function, G(s), for the controlled system (the input is θ_d, the output is θ). Use either block diagram algebra (applied to the block diagram from Figure 1) or take the Laplace Transform of the differential equation that you derived in Q1.

Q4 Express the damping ratio and natural frequency of the system in terms of the control gains and motor inertia. The damping ratio is important because it determines whether the system oscillates. The natural frequency determines the frequency at which it oscillates.

P1 Plot the predicted response of the system to a step change in θ_d from 0 to 1 radians, for damping ratios of 0.1, 1.0, and 2.0.

P2 Plot the predicted frequency response (both scaling and phase shift) for damping ratios of 0.1, 1.0, and 2.0. Do this on a Bode plot by plotting {20log(output amplitude/input amplitude)} vs. {input frequency on a log scale}, and {phase shift} vs. {input frequency on a log scale}.

3 How can a circuit implement the control law?

To implement the PD control law, you need to build the circuit shown in Figure 1.

Q5 By applying the op-amp golden rules, show that the input to the motor amplifier is:

$$Vout = -\frac{R_2}{R_1}(\theta - \theta_d) - R_2 C\dot{\theta}$$

The derivation will be easier if you substitute the impedance 1/sC for the capacitor then treat it as a resistor in the frequency domain, then transform back to the time domain.

Q6 Compare this equation with the control law of equation (1). What are K_P and K_d in terms of the electronic components (resistor and capacitor values)? How would you increase the damping of the system?.

Q7 Briefly describe the specific purpose for each of the op-amps in Figure 1.

Part 3 What is the step response of the actual system?

Construct the circuit in Figure 1. *Important: wire your circuit neatly! A neat circuit requires little extra time to wire, and it's easier to debug. Circuits often take much more time to debug than they do to initially wire!* Make sure to hook up the potentiometer correctly! The wiper of the pot should **not** be connected to the power supply!

If your circuit works correctly, the motor position should follow whatever input signal you provide with the function generator (square wave, sinusoid, constant voltage…).

IMPORTANT: Sometimes the circuit will not work and the motor will run uncontrollably at a very high speed. This wrecks the pots. If this happens, turn the motor off right away by turning of the DC supply to the motor. First, try to fix the instability problem by reversing the polarity across the sensing pot (you may have positive feedback instead of negative feedback). If this doesn't work, debug your circuit. Do not try to debug your circuit with the motor running! Debug your circuit systematically.

Here are some debugging hints:

- Compare your wiring diagram to your circuit to make sure all of the connections are correct.
- Make sure you don't have any loose connections.
- Verify that your output pot is working properly by connecting the scope to the wiper and moving the motor shaft by hand. The scope trace should move up and down.
- Verify that op-amp 1 is inverting and op-amp 3 is following.
- Verify that the output of op-amp 2 changes as you adjust θ_d with θ constant. You can adjust θ_d using the function generator or another pot.

Q8 Provide a step-input by using the function generator (4V peak-peak, 1 Hz square wave). Is the system underdamped or overdamped? Does the observed response agree with the theoretical one? Why is it different? What is the frequency at which is oscillates (the damped natural frequency, ω_{damped}).

PRACTICAL EXAM: Demonstrate to the TA that your motor is following the step input.

P3 Suppose you didn't want your motor to oscillate so much. **This is an important issue!** PD controllers are used in many applications such as NC milling machines, plotters, etc. You usually want your motor to go to a desired value quickly and accurately without oscillating! Which variable would you change in your differential equation for the PD system to increase damping? Increase the total capacitance to 2μF by adding another capacitor (recall that capacitors add in parallel). Observe the response to a step input. Repeat this for a total capacitance of 3 μF and 4 μF. Record the step response using the LabJack for C = 1, 2, 3, and 4 μF.

P4 In a mechanical system, if you wanted to the system to respond more quickly, you would increase the natural frequency (ω_n) by picking a stiffer spring (higher k). Double ω_n for the PD system by changing the appropriate resistor value. Keep C = 4 μF. Record the step response of the system using the LabJack, and plot it on the same plot as P3.

Part 4: What is the frequency response of the actual system?

The goal of this last part of the lab is to characterize the frequency response of the system. In particular, you will explore how well the system tracks the desired input position when the input is a sinusoid, across a range of frequencies. Remember, you can view linear systems such as this one as "filters". The PD controller acts like a low-pass filter, although it has a resonant peak if the damping is not great enough.

Q9 Change R1 to 1 KΩ, R2 to 10 KΩ, and C = 1 μF. Now input a 1.0V amplitude sine wave. Start at a low frequency. Keep the voltage scale on the scope the same for both input and output. What is the output amplitude and phase shift at 1, 2, 4, 8, 16, 32 Hz? Does the system have a resonant frequency? The amplitude should increase dramatically at this point. What is the resonant frequency and the output amplitude at resonance? Do you notice any high frequency oscillations in your output signal? What do you think might be causing those? **Note:** Don't let the motor oscillate for a long time at resonance.

P5 Make a Bode plot of the frequency response of the system. Plot {20log(output amplitude/input amplitude)} vs. {input frequency on a log scale}, and {phase shift} vs. {input frequency on a log scale}.

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words and to type the write-up!!
- include your name and laboratory time on the write-up
- Graphs for the lab write-up must be generated using Excel or Matlab, and must include labels on the axes, voltage and time scales used on the scope, and a legend for multiple-line plots.

Page limit = 2 pages, including graphs

1. A controller that performs a little better than the PD controller used in this lab is the following:

$$\tau = -K_p(\theta - \theta_d) - K_d(\dot{\theta} - \dot{\theta}_d)$$

   a. Derive the closed-loop transfer function for this controller.
   b. Provide a reason why this controller performs better in tracking a changing desired position input.
2. Step Response: Turn in the plots for P1, P3 and P4
3. Frequency Response: Turn in the plot for P2 and P5.

2.5.2 key solution

MAE 106 Laboratory Exercise #5 Solution
PD Control of Motor Position

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Q1 Dynamics of motor and shaft: $J\ddot{\theta} = \tau$

Dynamics of controller system: $\tau = J\ddot{\theta} = -K_p(\theta - \theta_d) - K_d\dot{\theta}$

Re-writing to make input and output clear: $J\ddot{\theta} + K_d\dot{\theta} + K_p\theta = K_p\theta_d$

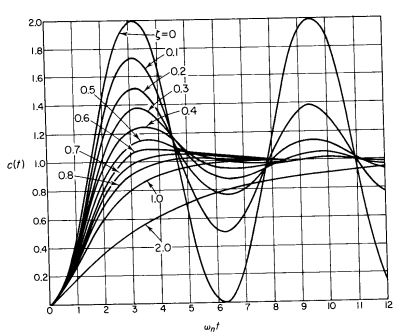
Q2 $M\ddot{\theta} + B\dot{\theta} + K\theta = F$

Mass = motor inertia (m = J), spring = Proportional control term (k = K_p) damper = Derivative control term (c = K_d)

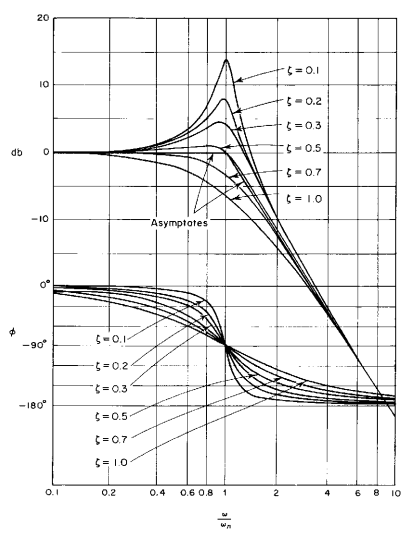
Q3 $G(s) = \dfrac{K_p}{Js^2 + K_d s + K_p}$

Q4 $\omega_n = \sqrt{\dfrac{K_p}{J}}$ $\zeta = \dfrac{K_v}{2\sqrt{K_p J}}$

P1



P2



Q5 $\frac{-\theta_d}{R_1} + \frac{\theta}{Z} = \frac{-V_{out}}{R_2}$ $\quad Z = \frac{R_1 \frac{1}{sc}}{R_1 + \frac{1}{sc}} = \frac{R_1}{1 + R_1 cs}$

$$Vout = -\frac{R_2}{R_1}(\theta - \theta_d) - R_2 Cs\theta$$

Converting back to time domain:

$$Vout = -\frac{R_2}{R_1}(\theta - \theta_d) - R_2 C\dot{\theta}$$

Q6 $K_P = R_2/R_1$ $K_d = R_2C$; Increase damping by increasing R_2 or C

Q7 Op amp 1 creates $-\theta_d$ for use in the control law
Op amp 2 implements the control law equation as in Q5
Op amp 3 is a buffer so that the motor potentiometer is not loaded by the rest of the circuit

Q8 Underdamped: $f_{damped} \approx 24$ Hz $\quad \omega_{damped}=2\pi f_{damped}$

P3 As you add capacitors, the damping increases and the oscillations decreases. With 2 C's you should have about 2 oscillations, 3 C' gives 1.5 oscillations, 4C's gives about 1 oscillation.

P4 Rise time and peak time should be much faster now.

Q9 Resonant frequency should be around 24 Hz, output amplitude should be about 1.5 times input amplitude at resonance.
Possible cause of the high frequency oscillations: compliance in the coupling between the potentiometer and motor.

P5 See plot on P1.

2.5.3 Lab post quizz solution

MAE 106 Laboratory Exercise #5 Post-Quiz 100 pts possible
PD Motor Position Control System

In this lab you built a control system to make a motor shaft move to a commanded position.

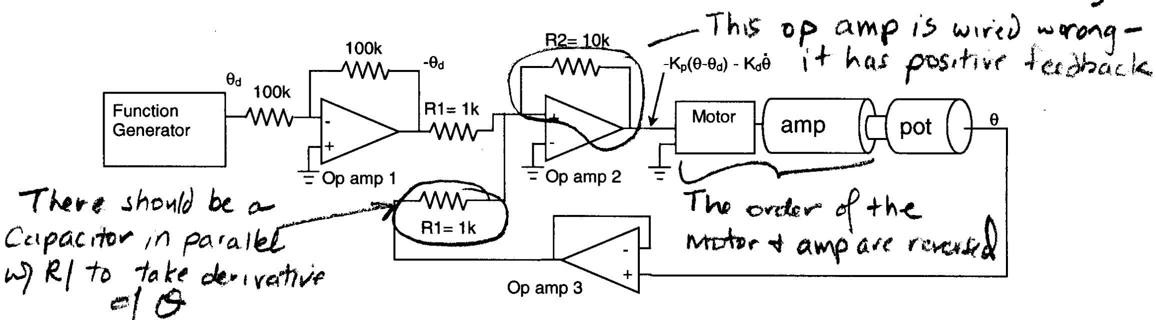
10 1. What do "P" and "D" stand for in "PD Motor Position Control"?

Proportional Derivative

10 2. Write a PD control law in the box, where
Where θ = actual motor angular position
θ_d = desired motor angular position
$\dot{\theta}$ = actual motor angular velocity
K_p = position error gain (K_p>0)
K_d = derivative gain (K_d>0)
τ = desired motor torque

$\tau = -K_p(\theta-\theta_d) - K_d\dot{\theta}$
or
$\tau = -K_p(\theta-\theta_d) - K_d(\dot{\theta}-\dot{\theta}_d)$
or $\tau = K_p(\theta_d-\theta) - K_d\dot{\theta}$
or $\tau = K_p(\theta_d-\theta) + K_d(\dot{\theta}_d-\dot{\theta})$

15 3. Identify three errors in this attempt at a PD control circuit:



10 4. A key point of the lab was that the controlled system acted dynamically like what kind of a mechanical system?

Mass-spring-damper system

10 5. The gains K_p and K_d determined the equivalent stiffness and damping of the mechanical system.

10 6. The desired angular position of the motor (θ_d) is equivalent to the spring rest length

10 7. In the lab, you measured the step (or time) response and the frequency (or sinusoidal) response of the system.

5 8. In lab, what type of circuit element did you change to increase the damping of the system?

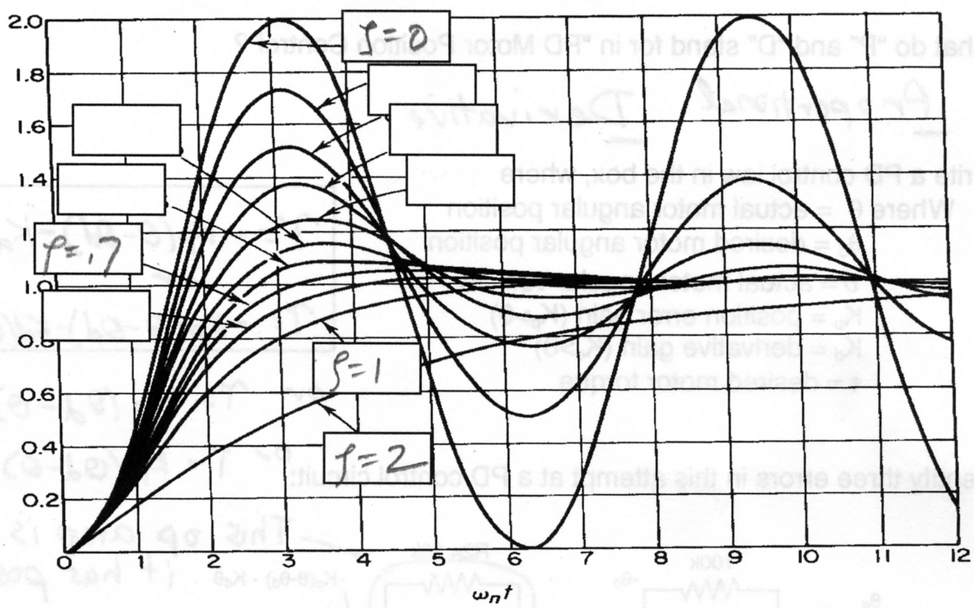
Capacitor

1

10

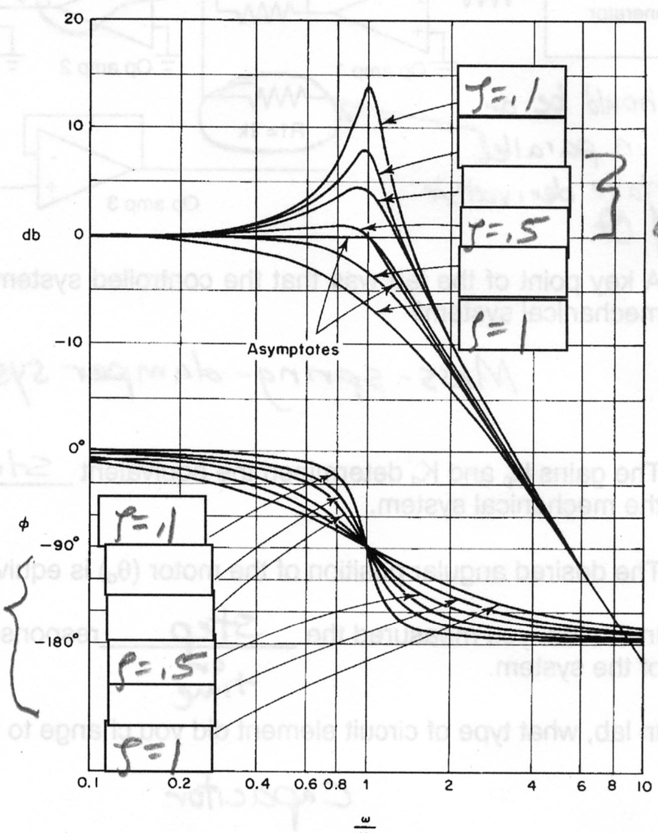
9. Shown below is the predicted response of the system to a step change in θ_d. In four of the boxes provided, label which response corresponds to $\zeta = 0$, $\zeta = 0.7$, $\zeta = 1$, $\zeta = 2$.

Any of these three is OK for ζ=.7



10

10. Shown to the right is the predicted scaling and phase shift for a sinusoidal input, for $0.1 \leq \zeta \leq 1$. In three of the boxes provided, label which response corresponds to $\zeta = 0.1$, $\zeta = 0.5$, $\zeta = 1$.



2.5.4 my solution

LAB #5 report. MAE 106. UCI. Winter 2005

Nasser Abbasi, LAB time: Thursday 2/10/2005 6 PM

February 14, 2005

1 Answer 1.

a) $\tau = K_p(\theta_d - \theta) + K_d\left(\dot{\theta}_d - \dot{\theta}\right)$

The output is θ and the input is θ_d

But $\tau = J\ddot{\theta}$

Take Laplace transform we get

$$\begin{aligned} Js^2\theta(s) &= K_p\theta_d(s) - K_p\theta(s) + K_d(s\theta_d(s) - s\theta(s)) \\ Js^2\theta(s) + K_p\theta(s) + K_d s\theta(s) &= K_p\theta_d(s) + K_d s\theta_d(s) \\ \theta(s)\left[Js^2 + sK_d + K_p\right] &= \theta_d(s)\left[K_p + K_d s\right] \end{aligned}$$

Hence the transfer function

$$G(s) = \frac{\theta(s)}{\theta_d(s)} = \frac{K_p + K_d s}{Js^2 + sK_d + K_p}$$

Compare this transfer function with

$$G_1(s) = \frac{\theta(s)}{\theta_d(s)} = \frac{K_p}{Js^2 + sK_d + K_p}$$

the one we used in the Lab. We see that new $G(s)$ has a zero at $s = -\frac{K_p}{K_d}$ while $G_1(s)$ has no zero. This controller will perform better as it tracks speed error as well as position error. This will make it more sensitive to changes.

2 Answer 2.

For p1, we are asked to plot the predicted response of $G(s) = \frac{K_p}{Js^2+K_d s+K_p}$ for a step input for $\zeta = 0.1, 1, 2$

Write the equation in standard form, we get $G(s) = \frac{\frac{K_p}{J}}{s^2+\frac{K_d}{J}s+\frac{K_p}{J}} = \frac{\omega_n^2}{s^2+2\xi\omega_n\ s+\omega_n^2}$

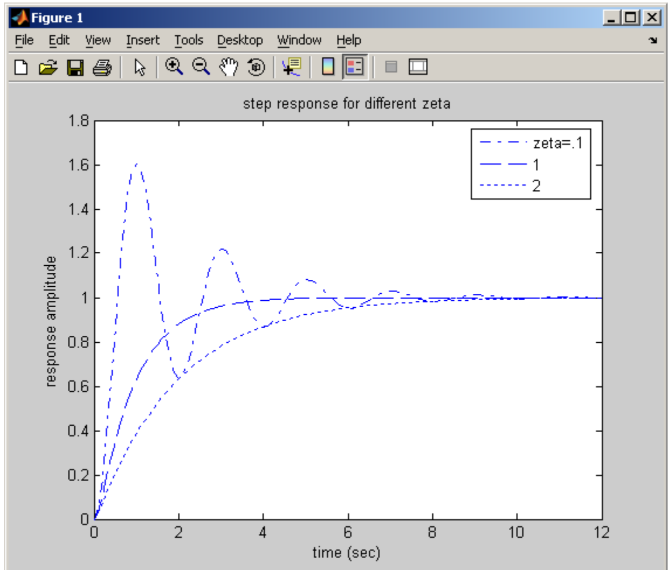
Hence $\omega_n = \sqrt{\frac{K_p}{J}}$ and $\xi = \frac{K_d}{2\sqrt{JK_p}}$

Where $K_p = \frac{R_2}{R_1} = \frac{10k}{1k} = 10$ and $K_d = R_2 C = 10 \times 10^3 \times 1 \times 10^{-3} = 10$

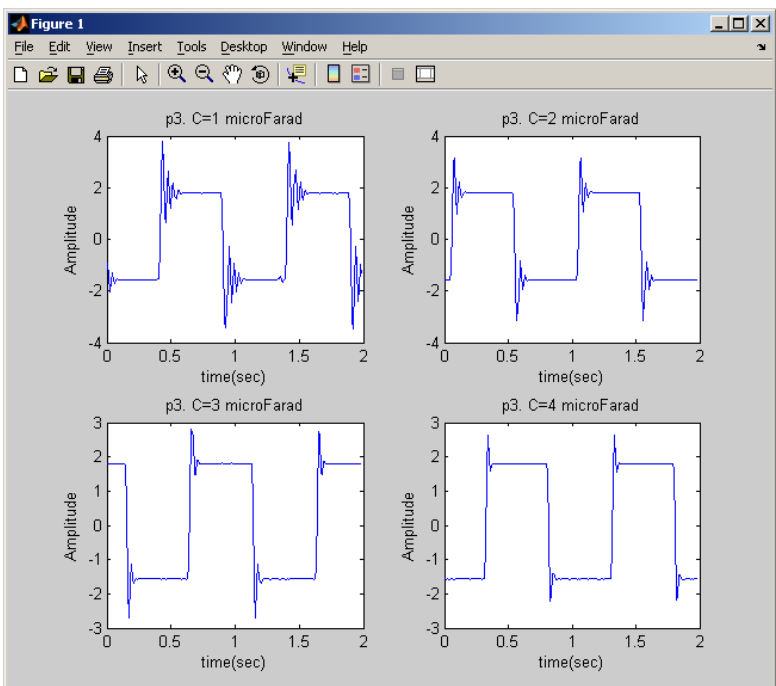
I will use $J = 1$ hence the transfer function becomes

$$G(s) = \frac{10}{s^2 + 20\ \xi\ s + 10}$$

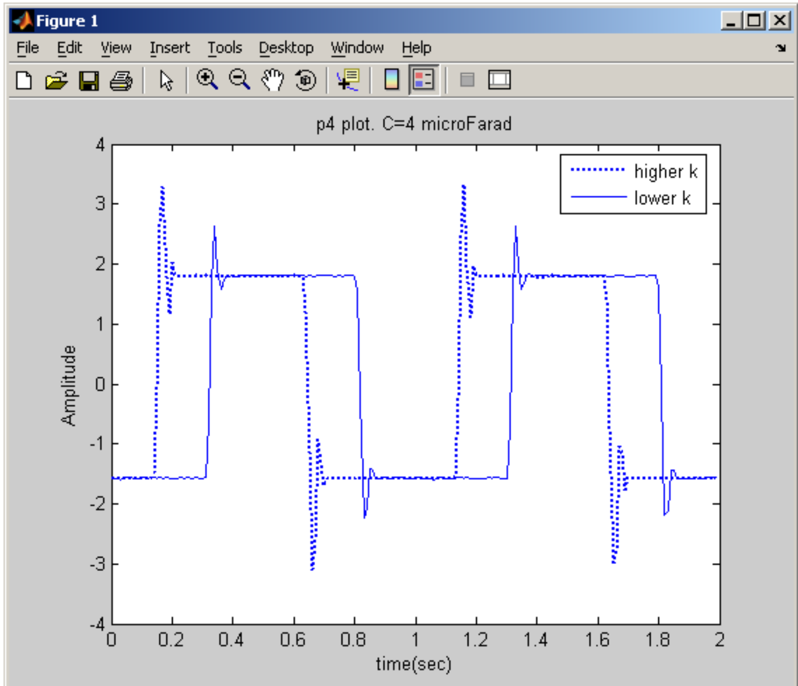
The following are the plots generated by a small program



For p3, we are asked to show plots for step input response for $C = 1, 2, 3, 4\mu F$ These are plots:



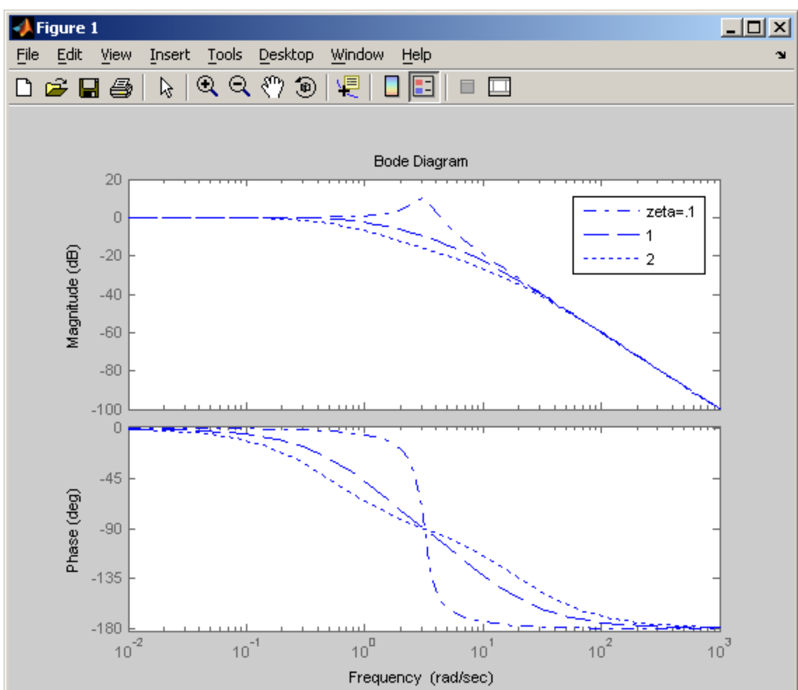
for p4, we are asked to plot the step response with higher k on top of the step response using original gain. this is the result



We see that with higher k, then the system responded more quickly.

3 Answer 3

Here we are asked to plot the frequency response for the predicted response. p2:



This is the result for p5

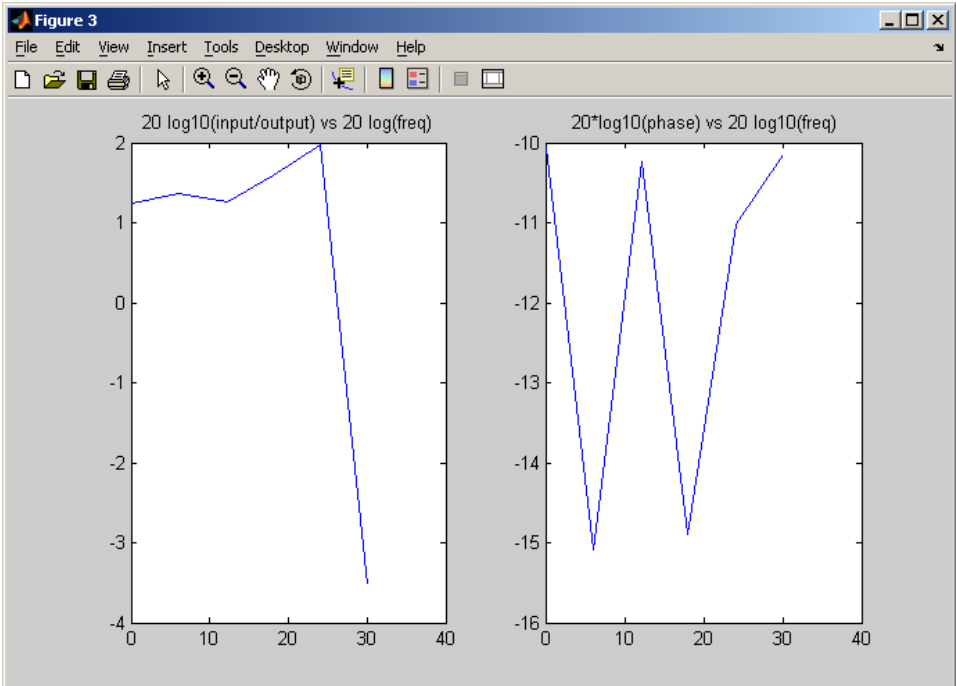


Figure 3
File Edit View Insert Tools Desktop Window Help
20 log10(input/output) vs 20 log(freq)
20*log10(phase) vs 20 log10(freq)
2
1
0
-1
-2
-3
-4
-10
-11
-12
-13
-14
-15
-16
0
10
20
30
40

2.6 Lab 6

Local contents

2.6.1 questions

MAE 106 Laboratory Exercise #6
Vibration II: System with Two Masses

University of California, Irvine
Department of Mechanical and Aerospace Engineering

REQUIRED PARTS

QTY	PARTS	
		Oscilloscope
1	50kΩ Potentiometer	Breadboard
		Vibrating beam experiment fixture
	EQUIPMENT	Accelerometer
	BNC to Alligator Clip Breakout	Accelerometer Amplifier
	BNC Cable	24V DC Power Supply
	Scope Probe	Strobe Light
		Spring

1 Introduction

The purpose of this laboratory exercise is to observe the free and forced dynamic response of a system of two, vibrating, cantilever beams connected by a spring. This experiment uses the same apparatus as the first vibration laboratory with a spring connecting the two vibrating masses. Our goal is to demonstrate that, although the mathematics that represent the vibrations of the system may appear complicated (4^{th} order system, as opposed to the 1^{st} and 2^{nd} order systems that we have been examining), the non-intuitive, predicted results are very real and extremely useful for machine design. You will observe that the system has two dominant modes of vibration, and that there is a frequency at which forced vibrations of one mass can be completely eliminated. Many mechanical systems, including your washing machine, use the type of vibration isolation demonstrated in this experiment. Mathematically, the vibration isolation frequency corresponds to a zero in the transfer function.

2 What are the Theoretical Resonant and Vibration Isolation Modes for the Beam System?

You can do this part before coming to lab

Q1 Derive the transfer function for the beam system, in matrix format. Note that this system has two inputs (the external forces applied to each beam), and two outputs (the position of each beam).

Q2 Derive the equations for the 2 resonant frequencies, ω_1 and ω_2. Show your work clearly. Then plug in numbers to obtain their values (in Hz). Assume that the second mass weighs 2.5 lb. and the spring constant of the center spring is 14 lb./in. You must also measure the beam lengths for the calculations.

Q3 Derive the vibration isolation frequency, ω_o (in Hz), corresponding the zero in the transfer function. Show your work clearly.

3 What are the Experimentally Measured Resonant and Vibration Isolation Modes for the Beam System?

You will now use a motor attached to one of the beams to create a sinusoidal forcing function. You will vary the frequency of the forcing function much as you did in the last vibration experiment. You will try to find the three frequencies of interest: the resonant frequency corresponding to the first mode, the frequency at which vibration isolation occurs, and the resonant frequency corresponding to the second mode of vibration.

Q4 Calibrate the accelerometer. Set the zero voltage adjustment on the instrumentation amplifier to give zero volts on the scope. Then rotate the entire apparatus on its side so that the accelerometer reads the acceleration of gravity (1 g). Report the accelerometer output voltage corresponding to 1 g. Explain the purpose of doing this task.

Q5 Using the strobe light and an input voltage of 2 volts from the function generator (with zero amplitude), determine the actual rpm of the motor. Be sure to hold the beam so that it does not vibrate much. Repeat the measurement for an input of 4 volts. Based on these measurements, what is your estimate of the average gain? State your answer in hertz/volt. What is the purpose of doing this task?

Q6 Measure the resonance corresponding to the first mode by slowly increasing the speed of the motor from rest. Report the motor voltage at this frequency and the corresponding frequency (in hertz).

Q7 Determine and report the vibration isolation frequency (in Hz) by increasing the frequency of the forcing function beyond the first resonance. Describe the behavior of the system at this frequency and estimate the frequency of at which this behavior occurs.

Q8 While leaving the system operating at the vibration isolation frequency, what happens when you hold the second mass from vibrating with your hand? Explain why this behavior occurs.

Q9 Determine and report (in Hz) the second resonance by slowly increasing the motor speed.

Q10 There is a third mode of vibration not predicted by the mathematics. Find it experimentally and describe it in words.

Q11 In a table, report the two resonant frequencies and vibration isolation frequency as derived in your theoretical analysis and as measured from the forced response. Explain what might cause any observed differences in the experimental and theoretical values.

PRACTICAL EXAM: Demonstrate to the TA the first two resonant modes, the vibration isolation phenomenon, and the third mode of vibration not predicted by the mathematics.

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words and to type the write-up!!
- include your name and laboratory time on the write-up

1. A more realistic model of the system has some damping represented for the beams. Suppose that the two dampers are placed in parallel to the springs that represent the beams k_1 and k_3 in the notes. Denote them as c_1 and c_3 respectively.

a. Draw a schematic of the system and the necessary free-body diagrams. Derive the modified equations of motion for the system.

b. Determine the transfer function $\frac{X_1(s)}{F(s)}$, where x_1 is the displacement of the forced mass.

c. Show that it is no longer possible to get perfect vibration isolation of x_1, but that if the damping is small as in our case, the amplitude of vibration of x_1 can be made small.

2. A planar model of an automotive suspension is shown in Figure 1. The position of the center of mass, x, is zero when the springs are not deformed. The pitch motion, θ, is positive in the clockwise direction and is zero in the undeformed spring position. You can forget about gravity in the following questions since its effect only adds constant offset displacements to the equilibrium positions.

a. Derive the equations of motion for this system, assuming that the pitch motion is small.

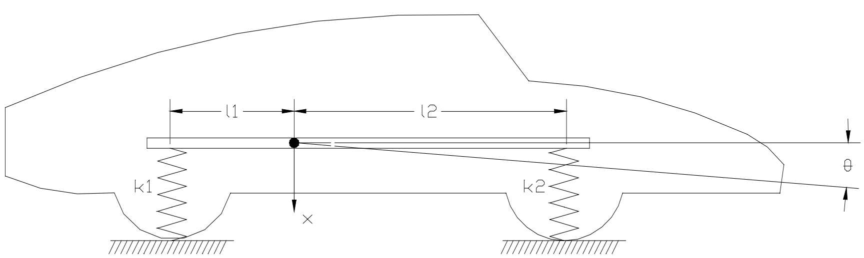


Figure 1 A planar model of an automotive

Denote the mass of the car as m and the inertia about the center of mass as J. (Hint: Assume x and θ are both positive and draw a free-body diagram.)

b. Set up the eigenvalue problem for this system, i.e. write the equations in the form

$$\left[M^{-1}K - \omega^2 I\right]\begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

c. Determine the natural frequencies and their corresponding mode shapes for a 2500 lb. car with

$k_1 = k_2 = 4000$ lb/ft
$l_1 = 4$ ft.
$l_2 = 5$ ft.
inertia about the center of mass, $J = mr^2$ (m is the mass of the car)
radius of gyration, $r = 3$ ft.

2.6.2 key solution

MAE 106 Laboratory Exercise #6 Solution
Vibration II: System with Two Masses

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Q1

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$ where f_1 = forcing function, $f_2 = 0$ for our system

$M\ddot{\bar{x}} + K\bar{x} = f$ in matrix/vector notation; taking Laplace transform:

$(Ms^2 + K)X = F$

$X = (Ms^2 + K)^{-1}F$ the transfer function relates the input F to the output X

$X = H(s)F \quad H(s) = (Ms^2 + K)^{-1}$

$$H(s) = \frac{1}{m_1 m_2 s^4 + (m_2(k_1+k_2) + m_1(k_2+k_3))s^2 + k_1k_2 + k_1k_3 + k_2k_3}\begin{bmatrix} m_2 s^2 + k_2 + k_3 & k_2 \\ k_2 & m_1 s^2 + k_1 + k_2 \end{bmatrix}$$

Q2 Resonant frequencies occur where H(jω) blows up (i.e. denominator ⇒ 0).
Note that the denominator of H(jω) is of the form:

$a\omega^4 + b\omega^2 + c$

with

$a = m_1 m_2 \quad b = -(m_2(k_1+k_2) + m_1(k_2+k_3)) \quad c = k_1k_2 + k_1k_3 + k_2k_3$

So we can use the quadratic equation to find where the denominator ⇒ 0 :

$$\omega_1^2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \omega_2^2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

Q3 The vibration isolation happens when the transfer function relating f_1 to position x_1 has a zero. I.e. H_{11}(jω)=0 where $H_{11}(j\omega) = -m_2\omega^2 + k_2 + k_3 = 0 \Rightarrow \omega_o = \sqrt{\frac{k_2+k_3}{m_2}}$

Q4 The purpose of calibrating the accelerometer is so that you know what voltage corresponds to what acceleration.

Q5 The purpose of this task is to determine what motor voltage corresponds to what forcing function frequency.

Q6 At the first resonant frequency, the beams' vibrations should get large, and the beams should move in phase with each other.

Q7 At the isolation frequency, the beam with the motor on it will stop moving while the other beam moves a lot. Imagine the beam with the motor on it is the casing for your washing machine.

Q8 The beam with the motor on it starts moving again because you are removing the vibration isolation.

1

Q9 At the second resonant frequency, the beams' vibration should again get large, and the beams should move out of phase with each other.

Q10 The spring has mass so you can get the spring's mass to start resonating at higher frequencies.

Q11 Differences in the experimental and theoretical values could be caused by errors in calibrating the motor or accelerometer, by errors in estimating the beam parameters, and/or by no including some dynamics, such as friction and damping, in your model.

2.6.3 Lab post quizz solution

Solution

MAE 106 Laboratory Exercise #6 Quiz
Modes of Vibration of a System with Two Masses

In this laboratory exercise you observed the free and forced dynamic response of a system of two, vibrating, cantilever beams connected by a spring. You observed that the system had two dominant modes of vibration, and that there was a frequency at which forced vibrations of one mass could be completely eliminated. This vibration isolation phenomenon is used in washing machines and other vibrating machinery.

15 pts 1. The vibration isolation frequency corresponded to a zero in the transfer function.

15 2. The resonant frequencies corresponded to poles in the transfer function.

15 3. At the first resonant frequency, the beams oscillated (in phase) or out of phase with each other (circle one).

15 4. How would quadrupling the stiffness of the center spring alter the vibration isolation frequency, assuming the stiffness of the beams is relatively small?

$$\omega_0 = \sqrt{\frac{k_2+k_3}{m_2}} = \sqrt{\frac{4k_2+k_3}{m_2}} \approx \sqrt{\frac{4k_2}{m_2}} = 2\sqrt{\frac{k_2}{m_2}}$$

It would double the isolation frequency

15 5. When you included some damping in the model of the system, the magnitude of the numerator of the transfer function was $\sqrt{(k_2+k_3-m_2\omega^2)^2+(c_2\omega)^2}$, where c_2 is the damping coefficient of the unforced beam. Using this information, explain why you could never get perfect vibration isolation with a damped beam.

Because $(c_2\omega)^2$ is always positive, and thus the magnitude of the numerator never equals zero.

25 6. Derive the transfer function for the beam system, in matrix format. Assume that this system has two inputs (the external forces applied to each beam), and two outputs (the position of each beam). You do not need to perform the matrix inversion, but define what each matrix is before the inversion.

Assume damping = 0

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$M\,\ddot{\underline{x}} + K\,\underline{x} = \underline{F}$$

$$M\ddot{\underline{x}} + K\underline{x} = \underline{F}$$

↓ Laplace

$$(Ms^2+K)\underline{x} = \underline{F}$$

$$\underline{x} = \underbrace{(Ms^2+K)^{-1}}_{\text{Transfer Function}}\underline{F}$$

1

2.6.4 my solution

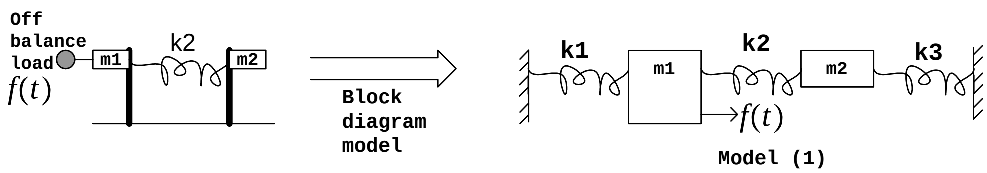
LAB #6 report. MAE 106. UCI. Winter 2005

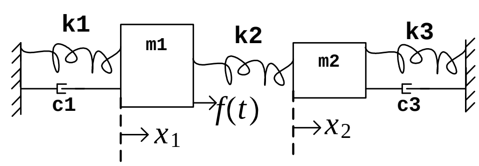
Nasser Abbasi, LAB time: Thursday 2/17/2005 6 PM

February 23, 2005

1 Answer 1.

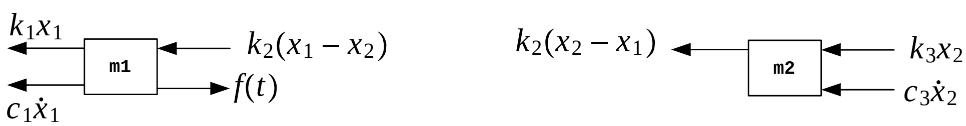
1.1 part(a)





Model (2) more realistic

Free diagram for model (2) is the following (assuming m1 is moving to right faster than m2)



Now derive equations. Take right to be positive.
For m1:

$$\begin{aligned}
F &= ma \\
-k_1 x_1 - c_1\dot{x}_1 - k_2(x_1 - x_2) + f(t) &= m_1\ddot{x}_1 \\
m_1\ddot{x}_1 + \ k_1 x_1 + c_1\dot{x}_1 + k_2(x_1 - x_2) &= f(t) \\
m_1\ddot{x}_1 + \ k_1 x_1 + c_1\dot{x}_1 + k_2 x_1 - k_2 x_2 &= f(t) \\
m_1\ddot{x}_1 + \ x_1(k_1 + k_2) + c_1\dot{x}_1 - k_2 x_2 &= f(t)
\end{aligned}$$

For m2:

$$\begin{aligned}
F &= ma \\
-k_2(x_2 - x_1) - k_3 x_2 - c_3\dot{x}_2 &= m_2\ddot{x}_2 \\
m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_3 x_2 + c_3\dot{x}_2 &= 0 \\
m_2\ddot{x}_2 + x_2(k_2 + k_3) - k_2 x_1 \ + c_3\dot{x}_2 &= 0
\end{aligned}$$

1.2 part(b)

determine transfer function $\frac{X_1(s)}{F(s)}$

Write the dynamic equations in matrix form, we get
$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_3 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix}$

Above can be written as

$$M\ddot{X} + C\dot{X} + KX = F$$

Hence , taking laplace transform we get

$$\begin{aligned} Ms^2X(s) + CsX(s) + KX(s) &= F(s) \\ X(s)\left[Ms^2 + Cs + K\right] &= F(s) \\ X(s) &= \left(Ms^2 + Cs + K\right)^{-1} F(s) \end{aligned}$$

Now, $Ms^2 = \begin{bmatrix} m_1 s^2 & 0 \\ 0 & m_2 s^2 \end{bmatrix}$

$Cs = \begin{bmatrix} c_1 s & 0 \\ 0 & c_3 s \end{bmatrix}$

$K = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}$

Hence

$$\begin{aligned} \left(Ms^2 + Cs + K\right)^{-1} &= \left(\begin{bmatrix} m_1 s^2 & 0 \\ 0 & m_2 s^2 \end{bmatrix} + \begin{bmatrix} c_1 s & 0 \\ 0 & c_3 s \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \right)^{-1} \\ &= \begin{bmatrix} m_1 s^2 + c_1 s + k_1 + k_2 & -k_2 \\ -k_2 & m_2 s^2 + c_3 s + k_2 + k_3 \end{bmatrix}^{-1} \end{aligned}$$

Now $A^{-1} = \frac{adj(A)}{\det(A)}$

But for the above,

$$\begin{aligned} \det(A) &= \left(m_1 s^2 + c_1 s + k_1 + k_2\right)\left(m_2 s^2 + c_3 s + k_2 + k_3\right) - \left(-k_2 \times (-k_2)\right) \\ &= \left(m_1 s^2 + c_1 s + k_1 + k_2\right)\left(m_2 s^2 + c_3 s + k_2 + k_3\right) - k_2^2 \end{aligned}$$

$$adj(A) = \begin{bmatrix} m_2 s^2 + c_3 s + k_2 + k_3 & k_2 \\ k_2 & m_1 s^2 + c_1 s + k_1 + k_2 \end{bmatrix}$$

Hence

$$\begin{aligned} \frac{X(s)}{F(s)} &= \left(Ms^2 + Cs + K\right)^{-1} \\ &= \frac{\begin{bmatrix} m_2 s^2 + c_3 s + k_2 + k_3 & k_2 \\ k_2 & m_1 s^2 + c_1 s + k_1 + k_2 \end{bmatrix}}{(m_1 s^2 + c_1 s + k_1 + k_2)(m_2 s^2 + c_3 s + k_2 + k_3) - k_2^2} \end{aligned}$$

i.e.

$$\frac{X_{1(s)}}{F(s)} = \frac{m_2 s^2 + c_3 s + k_2 + k_3}{(m_1 s^2 + c_1 s + k_1 + k_2)(m_2 s^2 + c_3 s + k_2 + k_3) - k_2^2}$$

and

$$\frac{X_{2(s)}}{F(s)} = \frac{k_2}{(m_1 s^2 + c_1 s + k_1 + k_2)(m_2 s^2 + c_3 s + k_2 + k_3) - k_2^2}$$

1.3 part(c)

Let $s = j\omega$ hence

$$\frac{X_{1(s)}}{F(s)} = \frac{-m_2\omega^2 + jc_3\omega + k_2 + k_3}{(-m_1\omega^2 + jc_1\omega + k_1 + k_2)(-m_2\omega^2 + jc_3\omega + k_2 + k_3) - k_2^2}$$

x_1 will not move when

$$\left|\frac{X_{1(s)}}{F(s)}\right| = 0 \Rightarrow \left|-m_2\omega^2 + jc_3\omega + k_2 + k_3\right| = 0$$

but $\left|-m_2\omega^2 + jc_3\omega + k_2 + k_3\right| = 0$ implies $\sqrt{(-m_2\omega^2 + k_2 + k_3)^2 + (c_3\omega)^2} = 0$. i.e. $\left(-m_2\omega^2 + k_2 + k_3\right)^2 + (c_3\omega)^2 = 0$. But this is the sum of 2 positive quantities. So it is only possible to sum to zero only when each quantity itself is zero. i.e.

$$c_3\omega = 0$$

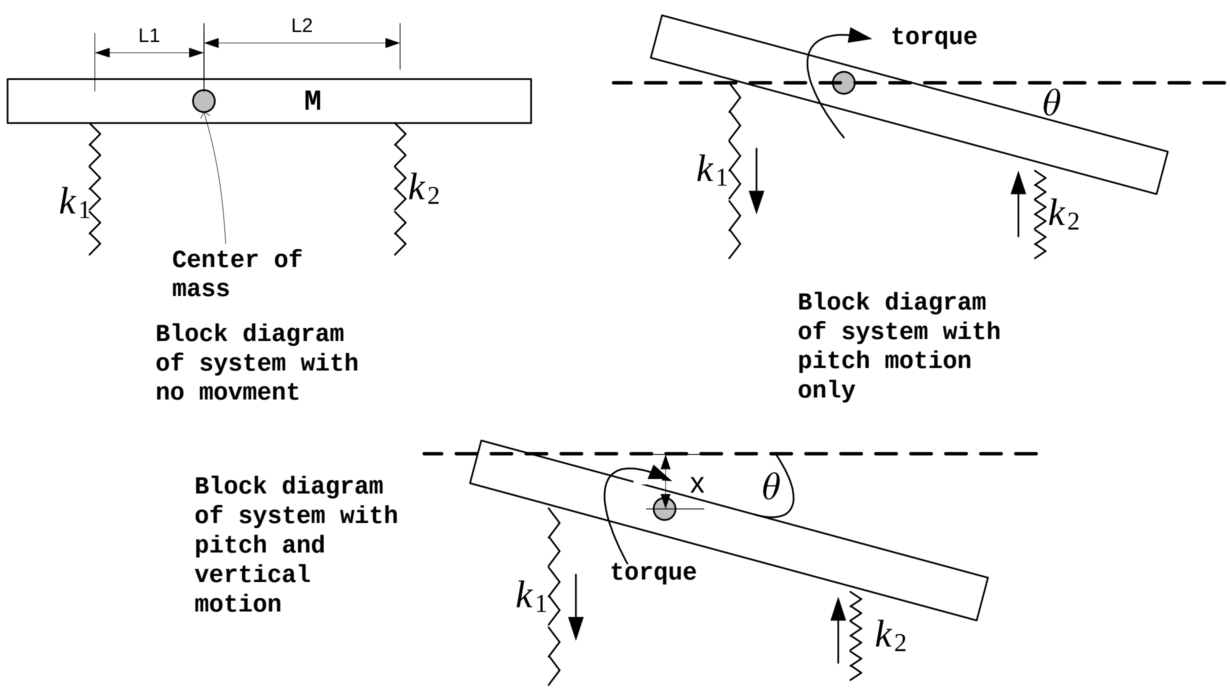
But for non zero ω this means that $c_3 = 0$. But c_3 (the samping) is not zero, since we do have damping in the systems, hence it is not possible that $\left|\frac{X_{1(s)}}{F(s)}\right| = 0$. In otherwords, there will not be an isolation fequency, and x_1 will always be non-zero.

But if c_3 is very small, then $c_3\omega = 0$ and in this case $\left|\frac{X_{1(s)}}{F(s)}\right| = 0$ when $-m_2\omega^2 + k_2 + k_3 = 0$ or when $\omega = \sqrt{\frac{k_2+k_3}{m_2}}$

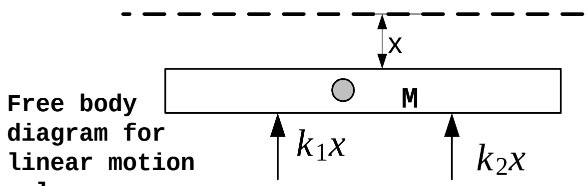
2 Answer 2.

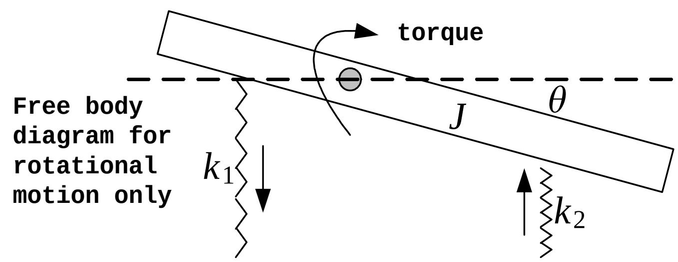
2.1 part(a)

Need to derive a mathematical model. First step is to make a block diagram as follows.



There are 2 motions. One rotational about the center of mass, and one translation, up and down. Free body diagrams are





Now the equation of motion for the rotational motion is

$$\tau = J\ddot{\theta}$$

But $\tau = k_1 L_1 \sin\theta - k_2 L_2 \sin\theta$
Hence we get for small θ, using $\sin\theta \approx \theta$

$$\begin{aligned} k_1 L_1\ \theta - k_2 L_2\ \theta &= J\ddot{\theta} \\ \theta\left(k_1 L_1 - k_2 L_2\right) &= J\ddot{\theta} \\ J\ddot{\theta} + \theta\left(k_2 L_2 - k_1 L_1\right) &= 0 \end{aligned}$$

For the translation motion, $F = ma$
hence

$$\begin{aligned} -k_1 x - k_2 x &= m\ddot{x} \\ x\left(-k_1 - k_2\right) &= m\ddot{x} \\ m\ddot{x} + x\left(k_1 + k_2\right) &= 0 \end{aligned}$$

2.2 part (b)

Write the above in matrix form, we get

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k_2 + k_1 & 0 \\ 0 & k_2 L_2 - k_1 L_1 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Take laplace transform we get

Let $M = \begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}$

$A = \begin{bmatrix} x \\ \theta \end{bmatrix}$

$K = \begin{bmatrix} k_2 + k_1 & 0 \\ 0 & k_2 L_2 - k_1 L_1 \end{bmatrix}$

Hence above matrix equation can be written as
$M\ddot{A} + KA = 0$
Take laplace transform, we get
$Ms^2 A + KA = 0$
$\left[Ms^2 - IK\right] A = 0$ where I is the 2×2 identity matrix.
let $s = j\omega$ we get
$\left[-\omega^2 M - IK\right] A = 0$
multiply both side by M^{-1} we get
$\left[-\omega^2 I - KM^{-1}\right] A = 0$
i.e.

$\left[-\omega^2 I - KM^{-1}\right] \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Which is what we are required to show.

2.3 Part (c)

$k_1 = k_2 = 4000lb/ft$

$L_1 = 4ft$

$L_2 = 5ft$

$m = 2500lb$

$J = mr^2 = 25000 \times 3^2 = 2.25 \times 10^5$

$\omega_0 = \sqrt{\frac{K}{M}} = \sqrt{KM^{-1}}$

$$M^{-1} = \begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}^{-1} = \begin{bmatrix} 2500 & 0 \\ 0 & 2.25 \times 10^5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{2.25\times10^5}{2500\times2.25\times10^5} & 0 \\ 0 & \frac{2500}{2500\times2.25\times10^5} \end{bmatrix} = \begin{bmatrix} 0.0004 & 0 \\ 0 & 4444 \times 10^{-6} \end{bmatrix}$$

$$K = \begin{bmatrix} k_2 + k_1 & 0 \\ 0 & k_2L_2 - k_1L_1 \end{bmatrix} = \begin{bmatrix} 8000 & 0 \\ 0 & 4000 \times 5 - 4000 \times 4 \end{bmatrix} = \begin{bmatrix} 8000 & 0 \\ 0 & 4000.0 \end{bmatrix}$$

Hence $\omega_0 = \sqrt{\begin{bmatrix} 8000 & 0 \\ 0 & 4000.0 \end{bmatrix} \begin{bmatrix} 0.0004 & 0 \\ 0 & 4444 \times 10^{-6} \end{bmatrix}} = \sqrt{\begin{bmatrix} 3.2 & 0 \\ 0 & 17.776 \end{bmatrix}} = \begin{bmatrix} 1.7889 & 0 \\ 0 & 4.2162 \end{bmatrix}$

Hence the natural frequency for the linear (translation) motion is $\boxed{\text{1.7889 rad/sec}}$, and for the rotational motion it is $\boxed{\text{4.2162 rad/sec}}$.

2.7 Lab 7

Local contents

2.7.1 questions

MAE 106 Laboratory Exercise #7
Computer Control of a Motor

University of California, Irvine
Department of Mechanical and Aerospace Engineering

REQUIRED PARTS

QTY	PARTS
	Switching current amplifier
	Breadboard
	Small Motor
	Labjack
	Wires

1 Introduction

The purpose of this laboratory exercise is to learn how to use a computer to control a motor. In the previous labs, you used operational amplifiers to control motors. Op amps are inexpensive and respond quickly, but require changing resistors and capacitors to change the control gains. Another common way to implement a control law is to use a computer. One of the major benefits of using a computer is that you can express the control law and control gains in software. Then, changing the control law just involves typing in new software code.

The basic idea is to:
1. electronically measure the system performance (e.g. the motor shaft's position with a pot)
2. read this measurement into the computer using an analog-to-digital (AD) converter
3. calculate an appropriate control law in a program running on the computer.
4. generate a voltage output to the motor with a digital-to-analog (DA) convertor

For this lab, you will use the LabJack and a PC for the analog-to-digital conversion (reading the motor shaft position in), and for the digital-to-analog conversion (sending a voltage command to the current amplifier/motor). The LabJack is a low-cost ($90) USB-based device that contains a chip in it that does A-to-D and D-to-A conversion. The LabJack has eight analog input channels (labeled AI0 through AI7), which can read in voltages between -10 and 10 volts, with 12 bit resolution (2^{12} = 4096 discrete levels). It also has two analog output channels (labeled AO0 and AO1), which can output voltages between 0 and 5 volts, with 10 bit resolution (2^{10} = 1024 voltage levels). It also has four digital input-output (labeled IO0 through iO3) channels that can either read in or put out "high" (5 volts) or "low" (0 volts) signals.

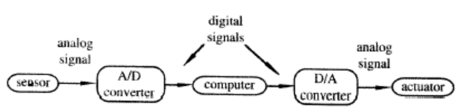


Figure 8.13 Computer control hardware.

Picture from: Ch. 8 Data Acquisition, Introduction to Mechatronics and Measurement Systems, 2nd Edition, David G. Alciatore and Michael B. Histand, McGraw-Hill 2003

1

2 Analog-to-Digital Conversion

In the first part of the lab, you will learn how to use the Labjack to read in a voltage waveform. Wire the function generator from the trainer kit to generate an approximately –2 to +2V, 2 Hz sinusoid into the AI0 input on the Labjack (use the osilloscope to check it – the amplitude and frequency don't need to be exact). To read in the voltage, you need to run the Matlab program readvoltage.m, which can be found in the directory C:\mae106\lab7. You are going to modify this program, so make a new directory with your name under C:\mae106 (i.e. create C:\mae106\yourname), and copy readvoltage.m into that directory. Here is a copy of the program:

```
% readvoltage.m
% MAE106 Laboratory Exercise #7
% created by Prof. David Reinkensmeyer, 2-21-2005

%LabJack Parameters
Idnum = 0;  % which Labjack you are using -- default = 0
demo = 0; % if > 0, puts software in "demo mode" -- can call functions without a Labjack
ADchannel = 0; % which analog-to-digital channel to read
channelgain = 0; % 0 = gain of 1

numsamps = 100; % how many samples to read
v = zeros(numsamps,1);  % initializes an array to hold the voltage samples
t0 = clock; % read the system clock
for i = 1:numsamps
   % read the voltage from the Labjack AD converter
    [voltage OverVoltage errorcode Idnum] = EAnalogIn(Idnum, demo, ADchannel, channelgain);
    v(i)= voltage; % save the voltage in the array
end;
tf = etime(clock,t0); % read the system clock again
samprate = numsamps/tf;  % calculate how long it took on average to read each sample
disp(['Samprate = ' num2str(samprate) ' Hz']);
```

Note that the "%" in Matlab denotes a comment. This program reads in 100 samples of the voltage on AI0 and stores the values in the array v. It also calls the PC's clock to calculate how many samples per second it read.

The program reads in data from the Labjack using the function `EanalogIn`. The documentation for the `EanalogIn` function is appended to the end of this lab so you can see what the its input and output parameters are. This function was written by the engineers at the company that makes the Labjack, and makes low level calls to the Labjack through the USB port.

To run the program, start Matlab by clicking the Matlab icon on the desktop. Change the Matlab working directory to be the directory that you just created. You can do this by entering the following command into the Matlab command prompt:

```
>> cd('c:\mae106\yourname')
```

Now, run the program by typing:

```
>> readvoltage
```

If the program is typed in correctly, it should run without an error, show you the sampling rate for the data you just took (i.e. how many samples per second the Labjack is returning). To see the data that you just collected, type:

```
>> plot(v,'.-')
```

This will show a graph of the sine wave that you read into the computer. The '.-' tells Matlab to put a dot at each sample and to connect the dots.

Q1 The labjack sampling rate is limited because it takes time to process data in the AD converter, the USB port, and the Matlab program. What sampling rate did the readvoltage.m program calculate? The sampling rate is an important parameter because it determines what the maximum frequency is that can be read accurately. For example, imagine that you are reading data from an accelerometer that is measuring ground movement due to an earthquake. If you read in the data too slowly (say at 1 Hz), you would miss some of the oscillations of the earthquake. With your Labjack setup, what is the fastest sine wave that you can read in from the function generator and still get a reasonable representation when you plot the voltage? How does this frequency compare to the Nyquist frequency?

3 Digital-to-Analog Conversion

The Labjack can also output a voltage between 0 and 5 volts on its DA channels (AO0 or AO1). You can make a call to the DA channel using the function `EanalogOut`. The documentation for this function is included at the end of the lab. Remove the function generator input to AI0, and wire the AO0 channel so that is the input to AI0. Now, you are going to use the computer to generate a sine wave voltage on DA0, and read it into AI0. Modify the "for" loop in your program so that it contains a call to the DA converter by inserting the following lines in your program. The required three NEW LINES are marked; you don't need to type in the comments.

```
t0 = clock; % read the system clock
f = 2; % NEW LINE 1 determines how many oscillations of the sinusoid to put out
for i = 1:numsamps
    analogOut0 = 1+sin(2*pi*f*i/numsamps); % NEW LINE 2 create a sine wave on AD0
    analogOut1 = 0; % NEW LINE 3 put 0 volts out on AD1
    [Errorcode Idnum] = EAnalogOut(Idnum, demo, analogOut0, analogOut1); % NEW LINE
    % read the voltage from the Labjack AD converter
     [voltage OverVoltage errorcode Idnum] = EAnalogIn(Idnum, demo, ADchannel, channelgain);
     v(i)= voltage; % save the voltage in the array
end;
```

Save the program, run it, and plot the voltage that you measured.

Q2: Why did the sampling rate change when you included a call to the DA convertor?

PRACTICAL EXAM 1: Demonstrate to the TA that you can read in a sinusoid with five oscillations (you will have to modify the code slightly to read exactly five oscillations)

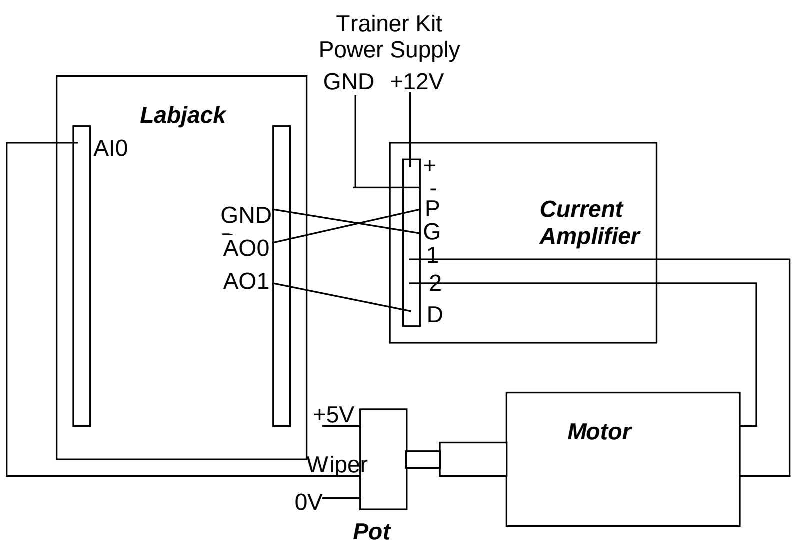
4 Computer Control of the Motor

For the final part of this lab, you will use a Matlab program to implement a feedback controller for the motor. To provide power to the motor, you will use a low-cost current amplifier. This current amplifier is conceptually very similar to the one you used previously. You apply a low-power voltage (-5 to 5 volts) to the amplifier, and it provides a high-power current (0-3 amps, 12-55 Volts depending on the supply voltage) for the motor that is proportional to the voltage.

There are a couple of details about the current amplifier that are different, however. First, the amplifier is a switching type amplifier. This means that it creates the desired current by rapidly switching on and off at an appropriate rate. The switching rate for switching amplifiers is typically more than 10 KHz. The advantage of a switching type amplifier is that it consumes less power because the power transistors inside it don't heat up much when they are fully off (so I = 0 and Power = IV = 0), or when they are fully on (so R = 0 and $P=I^2R=0$). The other common type of amplifier that you used previously, a linear amplifier, biases power transistors between fully “off” and fully “on” (0<R<inifinity), and thus the transistors heat up. The disadvantage of a switching amplifier is that the rapid switching adds electrical noise (and sometimes audible noise – try to listen to see if you can hear the amplifier switching frequency).

Second, the amplifier has the option to provide a unipolar input between 0 and 5 volts (instead of a bipolar one between –5 and 5 volts). If you do this, then you must provide a second input – the Direction input (which is either 0 or 5 Volts). The Direction input tells the motor which way to spin (i.e. it changes the direction of the current provided by the amplifier). This feature allows the amplifier to be controlled with two unipolar voltages, instead of one bipolar voltage. This is useful for this laboratory exercise because the Labjack DA output can only vary between 0 and 5 volts. We will use the second Labjack DA output to provide the Direction input to tell the motor which way to spin (you could also use a digital output from the Labjack).

Wire up your motor, amplifier, Labjack, and power supply as follows:



The current amplifier terminals are as follows:
\+ = Positive power supply (connect to +12 V on trainer kit)
\- = Negative power supply (connect to Ground on trainer kit)
P = Input into current amplifier (connect to Labjack AO0)
G = Ground (connect to any Labjack ground terminal)
1,2 = leads through which current is generated (connect to motor)
D = Direction input (connect to Labjack AO1)

The following Matlab program is called controlmotor.m and can be found in C:\mae106\lab7***. Copy it over to your directory,*** because you will need to modify it. This program implements a PD position feedback controller. It reads in the pot signal, calculates the control law, and then sends out the control signal to the motor amplifier. Notice that the derivative of the error is calculated by taking the difference between the current error and the previous error.

```
% controlmotor.m
% MAE106 Laboratory Exercise #7
% This program implements a PD controller using the Labjack
% Written by Prof. David Reinkensmeyer, 2/21/2005

%LabJack Parameters
Idnum = 0;  % which Labjack you are using -- default = 0
demo = 0; % if > 0, puts software in "demo mode" -- can call functions without a Labjack
ADchannel = 0; % which analog-to-digital channel to read
channelgain = 0; % 0 = gain of 1
analogOut0 = 1; %//        analogOut0 - Voltage from 0.0 to 5.0 for AO0
analogOut1 = 1; %//        analogOut1 - Voltage from 0.0 to 5.0 for AO1

% Control parameters
desiredangle0 = 2.5; % can range from 0 to 5 volts, for 0 to angular range of potentiometer
desiredfrequency = 0; % frequency of desired sinusoid to track
desiredamplitude = 1; % amplitude of sinusoid to track

pgain = .4; % proportional gain
dgain = .2; % derivative gain
numsamps = 60; % number of samples to read
samplingrate = 30; % nominal sampling rate in Hz.

% initialize data arrays and other variables to zero
cv = zeros(numsamps,1);
pv = zeros(numsamps,1);
dv = zeros(numsamps,1);
da = zeros(numsamps,1);
poserror= 0;
lastposerror = 0;
poserrordot = 0;
t0 = clock; % read the system clock

for i =1:numsamps
    % calculate the desired angle to move the motor
    desiredangle= desiredangle0+desiredamplitude*sin(2*pi*desiredfrequency*i/samplingrate);
    % read in the pot voltage
    [voltage OverVoltage errorcode Idnum] = EAnalogIn(Idnum, demo, ADchannel, channelgain);
    % update control variables
    lastposerror = poserror;
    poserror = voltage-desiredangle;
    poserrordot = poserror-lastposerror;
    % calculate PD control law
    controlvoltage = -pgain*poserror-dgain*poserrordot;
    % save data in arrays
    cv(i)=controlvoltage;
    pv(i)=voltage;
    dv(i)=dgain*poserrordot;
    da(i) = desiredangle;
    % Calculate direction input for current amplifier
    if controlvoltage > 0
        analogOut0 = controlvoltage;
        analogOut1 = 5;
    else
        analogOut0 = -controlvoltage;
        analogOut1 = 0;
    end
    % create the desired output voltages on the DA channels
    [Errorcode Idnum] = EAnalogOut(Idnum, demo, analogOut0, analogOut1);
end

% calculate sampling rate
tf = etime(clock,t0);
samprate = numsamps/tf;
disp(['Samprate = ' num2str(samprate) ' Hz']);

% zero motor ouput
analogOut0 = 0;
analogOut1 = 0;
[Errorcode Idnum] = EAnalogOut(Idnum, demo, analogOut0, analogOut1);

% plot results
```

```
figure(1); clf;
subplot(221);
plot(pv);
hold on;
plot(da,'r');
ylabel('Pot Voltage');
a = axis;
legend('actual','desired');
%line(a(1:2), [desiredangle desiredangle]);
ylim([0 5]);
xlim([0 numsamps]);
xlabel('Sample Number');

subplot(222);
ylabel('Control Voltage');
plot(cv)
hold on;
plot(dv,'r');
legend('total voltage','voltage due to d-term');
ylim([-5 5]);
xlim([0 numsamps]);
xlabel('Sample Number');
ylabel('Control voltages')
hold on;
shg; % show the graph
```

Type “controlmotor” at the Matlab command prompt (or press F5 from the Matlab editor window to run the program), and the motor should move so that the pot is in its middle position (corresponding to an output voltage of 2.5 volts). The program should then plot the response of the motor. If the motor moves all the way to one extreme, then you probably have implemented positive feedback. You can correct this by changing the sign of the gains in the program, or by switching the 1 and 2 leads from the current amplifier to the motor.

To measure the step response of the motor, manually turn the shaft away from the desired position, then start the program.

PRACTICAL EXAM 2: Demonstrate to the TA that your motor is under control.

Q3: Experiment with different proportional and derivative gains and see how they affect the step response of the controller. Can you make the system behave like an overdamped system? What happens when you decrease the P gain? What happens when you increase the P gain? What happens when you decrease the D gain? What happens when you increase the D gain?

Modify the code so that the desired pot angle is a 2 Volt peak-to-peak sine wave at 1.0 Hz. (Hint: you can just change the `desiredfrequency` parameter in the program).

Q4: Tune the proportional and derivative gains of the controller to get the best tracking you can for the 1.0 Hz sine wave. Record data showing how well your controller performs. You can use the Matlab command “save” to save your data to a file (type help save in the Matlab prompt for documentation).

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words and to type the write-up!!
- include your name and laboratory time on the write-up

1. Plot the best tracking performance of the 1 Hz sinusoid that you achieved. The plot should show the desired angle of the motor, and the actual angle for several cycles of the sinusoid. State the proportional and derivative gains that achieved this performance.
2. If you increased your gains too much, the motor went unstable. This result is not predicted by the continuous time model of the system (i.e. increasing Kp and Kd should not make the second order system go unstable). Provide an explanation. Hint: Model the effect of the Labjack as adding an e^{-sT} in the forward loop of the controller (i.e. a pure delay). Approximate the e^{-St} as $1-e^{-St}$ (1^{st} order Taylor's expansion), and calculate the closed loop transfer function. Show that the system goes unstable if the gains are too large.

Documentation for EAnalogIn and EAnalogOut

```
%/*****************************************************************************
**************/
%// EAnalogIn
%//
%// Easy function.  This is a simplified version of AISample. Reads the voltage from
1 analog input.
%// Calling this function turns/leaves the status LED on.  Execution time for this
function is 50 ms
%// or less.
%//
%// MATLAB Syntax:
%//         [voltage OverVoltage errorcode Idnum] = EAnalogIn(Idnum, demo, channel,
gain)
%//
% // Inputs:
%//         idnum - Local Id, serial number, or -1 for first LJ found
%//         demo - 0 for normal operation, >0 for demo mode.  Demo mode allows this
function to be called
%//                without a LabJack.
%//         channel - Channel command is 0-7 for single-ended, or 8-11 for
differential
%//         gain - Gain command is 0=1, 1=2, ..., 7=20.
%//
%// Outputs:
%//         voltage - Returns the voltage reading.
%//         OverVoltage - If >0 over voltage has been detected on one of the selected
analog inputs
%//         errorcode - LabJack error codes or 0 for no error.
%//         idnum - Local ID number of Labjack, or -1 if no LabJack is found.


/*****************************************************************************
*************/
// EAnalogOut
//
// Easy function.  This is a simplified version of AOUpdate. Sets the voltage of both
analog outputs.
// Execution time for this function is 50 milliseconds or less.
//
// MATLAB Syntax:
//          [Errorcode Idnum] = EAnlaogOut(Idnum, demo, analogOut0, analogOut1)
//
// Inputs:
//          Idnum - Local Id, serial number, or -1 for first LJ found
//          demo - 0 for normal operation, >0 for demo mode.  Demo mode allows this
function to be
//          called without a LabJack.
//          analogOut0 - Voltage from 0.0 to 5.0 for AO0
//          analogOut1 - Voltage from 0.0 to 5.0 for AO1
//
// Outputs:
//          idnum - Local ID number of Labjack, or -1 if no LabJack is found.
//          errorcode - LabJack error codes or 0 for no error.
```

2.7.2 key solution

MAE 106 Laboratory Exercise #7
Computer Control of a Motor
SOLUTION

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Q1 The sampling rate for sampling a single channel from Matlab using the Labjack is about 63 Hz., but depends on the computer that you are using. The fastest input sinusoid that still looks like a sinusoid when you plot it is about a 10 Hz sinusoid. The sampling theorem states that to read a sinusoid of *f* Hz accurately, we need to sample at more than the Nyquist Frequency which is 2**f* Hz. Since the Labjack samples at about 63 Hz, then we should be able to read in a 31.5 Hz sinusoid accurately. However, the Labjack sampling rate is not fixed – it varies from sample to sample because of the interference of the Windows operating system. So, in practice, a 10 Hz sinusoid is about the fastest sinusoid that we can accurately sample using Matlab function calls to the Labjack.

Q2 The sampling rate halved (to about 31 Hz) because each call to the Labjack takes time. You are now making two calls, one for the DA and one for the AD conversion.

Q3 One way to try to make the system act like an overdamped system is to increase the D gain. However, the system goes unstable before it becomes overdamped because of the time delay associated with sampling. Another way to make the system act like an overdamped system is to decrease the P gain, and make the D gain small or zero. The system will have a steady-state error if the P gain is small, due to the friction in the motor.

Increasing the P gain makes the system respond more quickly with more overshoot. Increasing the P gain too much causes instability due to the sampling delay. Increasing the D gain provides a limited amount of damping of the oscillations before it causes time-delay related instability.

2.7.3 Lab post quizz solution

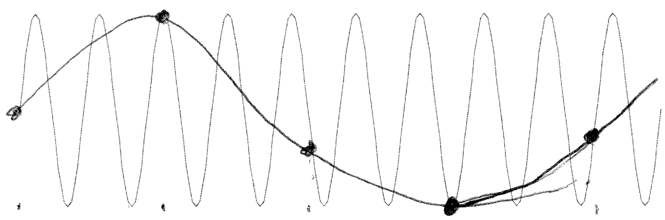
MAE 106 Laboratory Exercise #7 Post-Quiz
Computer Control of a Motor

In this laboratory exercise you used a computer and the Labjack to control a motor. You changed the gains of a PD controller in software and observed their effect on position tracking.

20 pts 1. Assume you are using a 10 bit A-D converter to reads in a voltage between –5 and 5 volts. What is the smallest voltage change you can detect? $2^{10} = 1024$ levels $\frac{10 \text{ volts}}{1024 \text{ levels}} = .98 \text{ mV} \approx 1 \text{ mV}$

20 2. In order to accurately reproduce a sine wave at 10 Hz, the Sampling Theorem states that you must sample the sine wave at at least 20 Hz.

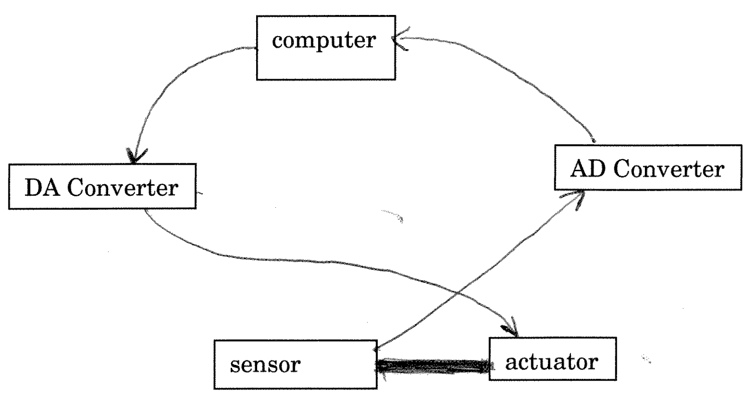
20 3. Below is a sine wave. Illustrate how a sampled version of the sine wave can be interpreted as arising from a slower sine wave (i.e. aliasing), if the sampling rate is too slow.



Samples should be evenly spaced and connected w/ a sinusoid

20 4. In the first part of the lab, you read in a sine wave generated by the function generator through the AD converter. In the second part of the lab, you generated a sine wave with the DA converter and read it into the AD converter. Why was the sampling rate slower when you used the DA converter to generate the sine wave?
Matlab had to make two software calls to the Labjack instead of one

20 5. Below are the parts you need to make a computer-controlled feedback system. Connect the parts with arrows that show the flow of information. Draw a thick line to indicate the connection that is typically made by the laws of physics rather than by a wire.



2.7.4 my solution

LAB #7 report. MAE 106. UCI. Winter 2005

Nasser Abbasi, LAB time: Friday 2/25/2005 10 AM

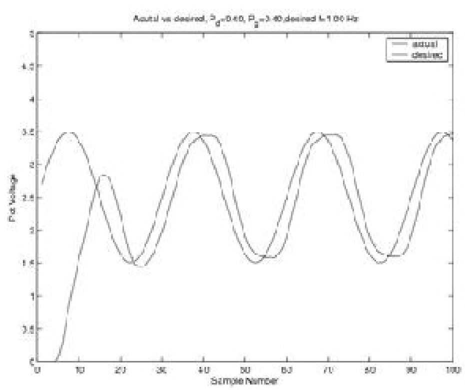
March 2, 2005

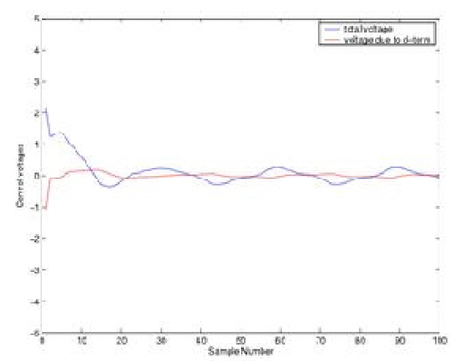
1 Answer 1.

I have tried a number of runs with different proportional and derivative gain constants running at 1 Hz. This plot below shows few of the tests I've run, and below them the one I think achieved the best tracking.

Derivative gain=0.4

Proportional gain=0.4



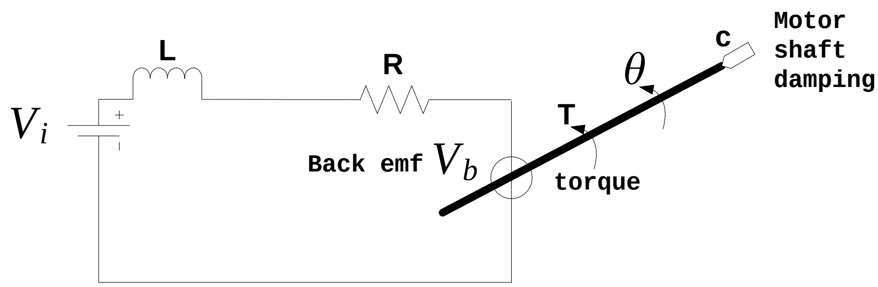


2 Answer 2

First I need to derive the transfer function. I can decide to control the speed of the motor shaft, or its angular position. I need to decide on this since this affect what the transfer function will be. i.e. wither I will select the position or the speed to be the output. In both cases I will take the motor voltage supply as the input.

I selected to use position as the controller variable.

First, I show the model of the motor itself, then the block diagram. Next I show the block diagram with a delay element added in the feedforward path, and the compare the transfer functions with and without delay and show that with delay, it is possible for the output to become unstable.



From the diagram below, using Kirchoff law around the motor circle, we get

$$V_i = L\frac{di}{dt} + Ri + V_b$$

Take Laplace transform we get

$$V_i(s) = sL + RI(s) + V_b(s) \tag{1}$$

Now, we know that the backemf voltage V_b produced is proportional to the angular speed of the shaft. Let this proportionality constant be called B_b then we write

$$V_b = B\frac{d\theta}{dt}$$

Take Laplace transform of the above, we get

$$V_b(s) = B_b s\theta(s) \tag{2}$$

Substitute equation (2) into (1) we get

$$V_i(s) = sL + RI(s) + B_b s\theta(s) \tag{3}$$

Now consider the dynamic equation for the motor shaft, we get

$$T - c\frac{d\theta}{dt} = J\frac{d^2\theta}{dt^2}$$

Where J is the moment of inertial of the motor shaft around its axis of rotation. Take Laplace transform of the above we get

$$T - cs\theta(s) = Js^2\theta(s) \tag{4}$$

We also know that the torque produced is proportional to the current in the motor. Lets call the proportionality constant B_t hence we write

$$T = B_t i \tag{5}$$

Take Laplace transform of (5) we get

$$T = B_t I(s) \tag{6}$$

Substitute (6) into (4) we get

$$\begin{aligned} B_t I(s) - cs\theta(s) &= Js^2\theta(s) \\ I(s) &= \frac{Js^2\theta(s) + cs\theta(s)}{B_t} \\ &= \frac{\theta(s)(Js^2 + cs)}{B_t} \end{aligned} \tag{7}$$

Now substitute (7) into (3) we get

$$\begin{aligned} V_i(s) &= sL + R\frac{\theta(s)(Js^2+cs)}{B_t} + B_b s\theta(s) \\ &= sL + \theta(s)\left[\frac{R(Js^2+cs)}{B_t} + sB_b\right] \end{aligned} \tag{8}$$

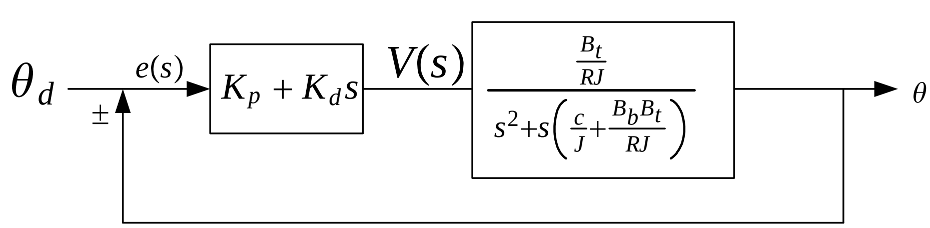
Now L is usually very small compare to R so equation (8) can be written as

$$V_i(s) = \theta(s)\left[\frac{R(Js^2+cs)}{B_t} + sB_b\right]$$

Hence the transfer function between V_i and θ is

$$\begin{aligned} \frac{\theta(s)}{V_i(s)} &= \frac{1}{\frac{R(Js^2+cs)}{B_t} + sB_b} = \frac{B_t}{R(Js^2+cs) + sB_bB_t} \\ &= \frac{\frac{B_t}{RJ}}{s^2 + s\left(\frac{c}{J} + \frac{B_bB_t}{RJ}\right)} \end{aligned}$$

This transfer function is in the standard form. It is a second order system. The above is the transfer function of the plant itself. Now I put the above into the loopback block diagram, assuming the controller we used is PD controller of the form $K_p + k_d s$ we get this



Now we do block simplification to obtain the closed loop transfer function. Let $P(s) = \frac{\frac{B_t}{RJ}}{s^2+s\left(\frac{c}{J}+\frac{B_bB_t}{RJ}\right)}$, hence the feedforward transfer function is $G_o(s) = (K_p + K_d)P$ hence the closed loop transfer function is $G_c(s) = \frac{G_o}{1+G_o} = \frac{(K_p+K_d)P}{1+(K_p+K_d)P}$

Let $K_p + K_d = K$ then we write $G_c(s) = \frac{KP}{1+KP}$

The charaterstic equation is $1 + KP = 0$. The closed loop poles are the roots of this equation.

Replace P above to be able to solve for the roots, we get

$$
\begin{aligned}
1 + K\frac{\frac{B_t}{RJ}}{s^2 + s\left(\frac{c}{J} + \frac{B_bB_t}{RJ}\right)} &= 0 \\
s^2 + s\left(\frac{c}{J} + \frac{B_bB_t}{RJ}\right) + K\frac{B_t}{RJ} &= 0
\end{aligned}
$$

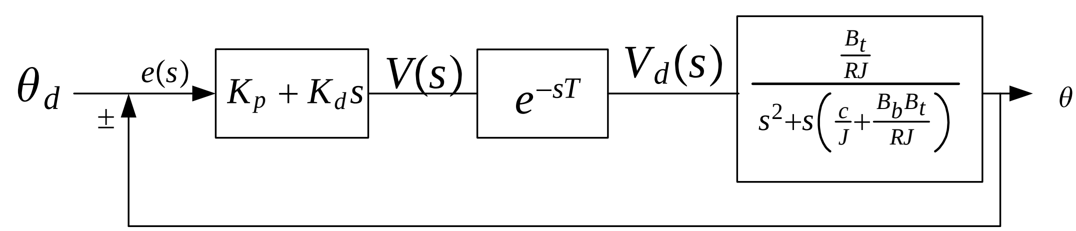
the roots are

$$
s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\left(\frac{c}{J} + \frac{B_bB_t}{RJ}\right)}{2} \pm \frac{\sqrt{\left(\frac{c}{J} + \frac{B_bB_t}{RJ}\right)^2 - 4K\frac{B_t}{RJ}}}{2}
$$

We see from the above, that independent of the values under the $\sqrt{}$sign, the system will have its poles in the left hand side. This is because the quantity $\frac{c}{J} + \frac{B_bB_t}{RJ}$ is positive.

Hence the system is always stable no matter how large the gain K is.

Now let see what happens when we add the effect of Labjack into the system, model this effect as a time delay, which in the Laplace transform becomes e^{-sT} where T is the time it takes Labjack to sample one data point, i.e. T is the sampling period. Hence now the block diagram becomes



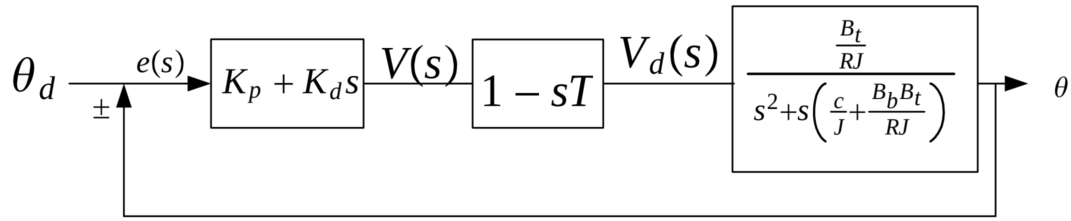
Where I wrote V_d as the output from the labjack. (delayed voltage).

Now, since $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} - \cdots$

Then $e^{-sT} = 1 + (-sT) + \frac{(-sT)^2}{2} + \frac{(-sT)^3}{3!} - \cdots = 1 - sT + \frac{s^2T^2}{2} - \frac{s^3T^3}{3!} - \cdots$

Now for very small T , all terms with s^n for $n > 1$ can be ignored. Hence we get an approximation $\boxed{e^{-sT} = 1 - sT}$

Hence the above system becomes



Now obtain the closed loop transfer function.

The open loop transfer function is $G_o(s) = (K_p + K_d)(1 - sT)P(s)$

As before, let $(K_p + K_d) = K$, hence we get $G_o(s) = K(1 - sT)P(s)$

Then the closed loop transfer function is

$$G_c(s) = \frac{G_o}{1 + G_o} = \frac{K(1 - sT)P(s)}{1 + K(1 - sT)P(s)}$$

The characteristic equation is

$$\begin{aligned}
1 + K(1 - sT)P(s) &= 0\\
1 + K(1 - sT)\frac{\frac{B_t}{RJ}}{s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right)} &= 0\\
s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right) + K(1 - sT)\frac{B_t}{RJ} &= 0\\
s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right) + K\frac{B_t}{RJ} - KsT &= 0\\
s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ} - KT\right) + K\frac{B_t}{RJ} &= 0
\end{aligned}$$

The roots of this equation (i.e. the poles of the closed loop) now can be found as

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\left(\frac{c}{J} + \frac{B_b B_t}{RJ} - KT\right)}{2} \pm \frac{\sqrt{\left(\frac{c}{J} + \frac{B_b B_t}{RJ} - KT\right)^2 - 4K\frac{B_t}{RJ}}}{2}$$

Now we clearly see the effect of the delay of the closed loop poles.

We see that the real part of the pole can occur at the positive side of the s plane, and this will happen when $\frac{c}{J} + \frac{B_b B_t}{RJ} - KT < 0$ or when $KT > \frac{c}{J} + \frac{B_b B_t}{RJ}$

hence we see that as K is increased, the closed loop pole will move to the right until it will cross the imaginary axes making the system unstable. In addition, for a fixed gain K, as T is increased the system can become stable. An increase in T implies that the sampling frequency becomes smaller, since $f = \frac{1}{T}$.

This is what we are asked to show.

Chapter 3

Exams

Local contents

3.1 Exam 1

Local contents

3.1.1 Review sheet for midterm

University of California at Irvine
MAE106 Midterm Review Sheet

The Midterm is scheduled for Tuesday, February 1, 2005. There are several practice midterms on the course web site: **http://www.eng.uci.edu/~dreinken/MAE106/mae106home.htm**

Circuit Analysis
- Charge, voltage, current
- Kirchoff's current and voltage laws
- Power
- Resistors
  - Ohm's law
  - Parallel and series resistances
  - Voltage Divider Circuit
  - Potentiometers
- Operational Amplifiers
  - Input/output relationship (i.e high-gain differential amplifier)
  - Golden rules (true only if op-amp has appropriately-connected negative feedback and is not saturated)
  - Design of op-amp circuits for amplifier, inverter, buffer, addition, subtraction, filters
- Capacitors, Inductors
- RC circuits, time constants, cut-off frequency for low-pass and high-pass filters

Solving a Linear Differential Equations
- Generals: Finding general solution to homogeneous equation
- Particular: Solving particular equation
- Initial Condition: Finding total solution by solving for initial conditions
- Doing the above steps for a 1^{st} order differential equation

DC Brush Motors
- How they work (Lorentz force law, commutation)
- Torque/ Current Relationship
- Mathematical model, back EMF, use as tachometer
- Torque versus velocity relationship, stall torque, no load speed, mechanical power

Power Control
- MOSFETS – (n-type) voltage controlled resistor characteristic; gate resistance
- Use as a voltage-controlled switch
- Physical structure, basic description of how it works

Control Theory
- Block Diagrams
- Basic concepts of feedforward control and feedback control
- Using negative feedback for disturbance rejection and to compensate for plant variations
- Positive feedback/instability

Time Domain Analysis
- Time constant of a first-order system

Frequency Domain Analysis
- Basic idea of frequency response; sine wave in $\Rightarrow$ sine wave out, amplitude scaled and phase shifted
- 1^{st} order low-pass and high-pass filter characteristic, corner frequency and relationship to time constant
- Complex variables
- Laplace Transform (of step function, exponential, sinusoid, derivative, integral)
- Transfer Functions (what are they how do you find them?)
- Impedances (of resistors, capacitors, and inductors)

You should also review your lecture notes and laboratory exercises 1- 3.

3.1.2 my exam

Part 2: Midterm

90 + 100

Problem 1 (10 Pts Extra Credit)

An oscilloscope is used to measure this:
Answer _b_ a) resistance b) voltage c) current d) power

The time constant of a first-order system tells when the output has gotten how far along the way to its final value?

Answer _c_ a) 37% b) 10% c) 63% d) 90%

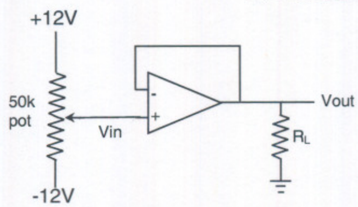
If you put a sine wave into a linear system, you get the following out
Answer _d_
a) square wave
b) sine wave at different frequency
c) triangle wave
d) sine wave at same frequency, scaled and shifted

A filter scales a sinusoidal input. The amount of scaling is determined by:
Answer _a_
a) the magnitude of the transfer function, evaluated at s=jω
b) the magnitude of the transfer function, evaluated at s = ω
c) the phase of the transfer function, evaluated at s=jω

A low pass filter attenuates
Answer _b_
a) low frequencies
b) high frequencies
c) a band of frequencies

Problem 2 (25 pts)

How close is V_{out} to V_{in} for the following voltage follower circuit, if the op-amp gain is 1,000? (Hint, use the fact that $V_o = K(V_+ - V_-)$ for the op amp)



$V_o = K(V_+ - V_-)$

since negative feed back

$V_o = V_-$

so $V_o = K(V_+ - V_o) \Rightarrow V_o = KV_+ - KV_o$

$\Rightarrow V_o(1+K) = KV_+$

i.e. $V_o(1+K) = KV_{in}$

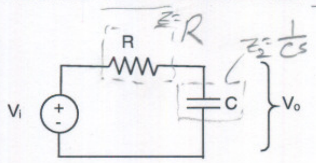
so $\frac{V_o}{V_{in}} = \frac{K}{1+K} = \frac{1000}{1001} \approx \boxed{1}$

so V_o almost same as V_{in}.

Problem 3 (25 pts)

How does the following circuit filter a low frequency input? Specifically, find what the resulting scaling and phase-shift would be for an input sinusoid with a frequency of $\frac{1}{2\pi} = 0.16$ Hz.

Assume R = 1 kiloohm and C=1milliFarad.



$V = Ri$, $i = \frac{V}{R}$

$q = CV$

$\frac{dq}{dt} = i = C\frac{dv}{dt}$

$\boxed{I = Cs\,V(s)}$

so $\boxed{V(s) = I\frac{1}{Cs}}$

$V_i - Ri - V_o = 0$

$V_i - R\left(C\frac{dV_o}{dt}\right) - V_o = 0$

$RC\frac{dV_o}{dt} + V_o = V_i \Rightarrow \frac{dV_o}{dt} + V_o\frac{1}{RC} = \frac{V_i}{RC}$

-10 $\rightarrow$ $\boxed{V_o = V_i\left(1 - e^{-\frac{t}{\tau}}\right)}$ where $\tau = RC$

To find transfer function use impedance

$V_i(s) - Z_1 I(s) - V_o(s) = 0$

but $V_o(s) = \frac{1}{Cs}I(s) \Rightarrow I(s) = V_o(s)Cs$

so $V_i(s) - Z_1 V_o(s)Cs - V_o(s) = 0 \Rightarrow V_i(s) = V_o(s)(1 + RCs)$

$\frac{V_o}{V_{in}} = \frac{1}{1+RCs} \Rightarrow \boxed{H(s) = \frac{1}{1+RCs}} \Rightarrow H(j\omega) = \frac{1}{1+RCj\omega}$

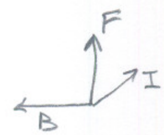
as $\omega \to \infty$, $|H(j\omega)| \to 0$

as $\omega \to 0$, $|H(j\omega)| \to 1$ so $\boxed{\text{Low pass}}$ when $\omega = \frac{1}{2\pi}$

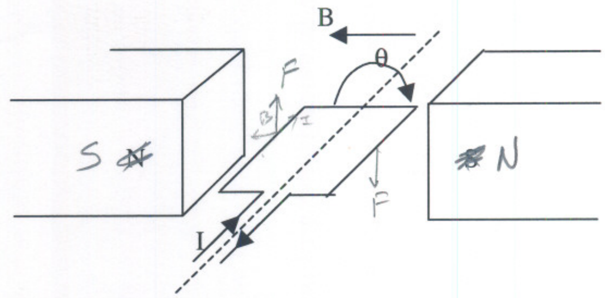
$|H(j\omega)| = \frac{1}{\sqrt{1+(RC\omega)^2}} = \frac{1}{\sqrt{1+\left(\frac{1}{2\pi}\right)^2}} = \boxed{\frac{1}{\sqrt{1+\frac{1}{4\pi^2}}}} \approx 1$

$\angle H(j\omega) = 0 - \tan^{-1} RC\omega = \boxed{-\tan^{-1}\frac{1}{2\pi}}$

Problem 4: 25 pts

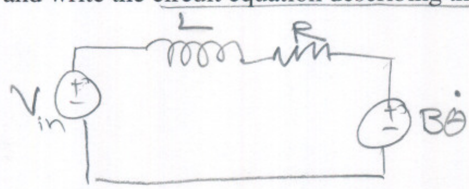


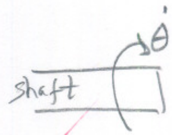
a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at θ = 0° as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.



$\theta = 90^\circ$ ✓

b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:





$\tau = Bi$

$i = \frac{\tau}{B}$

motor side

$V_{in} - L\frac{di}{dt} - Ri - B\dot{\theta} = 0$

$i = \frac{\tau}{B}$ → mechanical side

b. Solve this differential equation for the current through the motor as a function of time when:

- the shaft of the motor is held fixed
- a constant voltage v is applied across the motor at time = 0
- the initial current i(t = 0) through the inductor is zero

when shaft fixed ⇒ $\dot{\theta} = 0$

so $V_{in} - L\frac{di}{dt} - Ri = 0$

so $L\frac{di}{dt} + Ri = V_{in} \Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{V_{in}}{L}$

homogeneous: $i(t) = Ae^{-\frac{R}{L}t}$, particular $i = \frac{V_{in}}{R}$

so $i(t) = Ae^{-\frac{R}{L}t} + \frac{V_{in}}{R}$ at $t = 0$, $i(t) = 0 \Rightarrow 0 = A + \frac{V_{in}}{R} \Rightarrow A = -\frac{V_{in}}{R}$

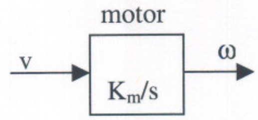
so $i(t) = V_{in}\left(\frac{1}{R} - \frac{1}{R}e^{-\frac{R}{L}t}\right) = \frac{V_{in}}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

so $i(t) = \frac{V_{in}}{R}\left(1 - e^{-\frac{R}{L}t}\right)$ ✓ so $\tau = \frac{B}{R}V_{in}\left(1 - e^{-\frac{R}{L}t}\right)$

at steady state $\tau = \frac{B}{R}V_{in}$

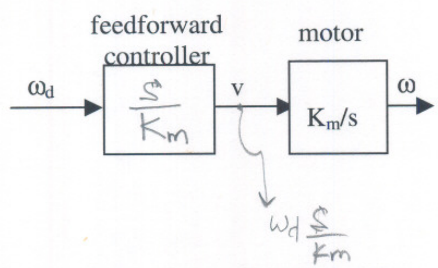
Problem 5: 25 pts

1) You want to control the speed of a motor. You are using a current amplifier with the motor, so the speed is related to the input voltage to the current amplifier by the following transfer function:



where v is the voltage input to the motor and ω is the angular velocity of the shaft and K_m is a constant.

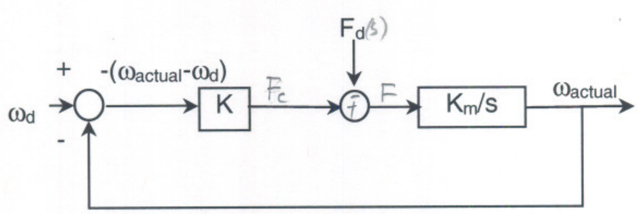
a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What transfer function should the controller box have to make the output equal the desired output? Write this function controller box.



since $V = \omega_d \frac{s}{K_m}$

so $\omega = V \frac{K_m}{s} = \omega_d \frac{s}{K_m} \cdot \frac{K_m}{s} = \omega_d$

b) One of the major benefits of feedback is its ability to cancel the effects of unmodeled "disturbances". Assume you build a feedback controller, but there is a disturbance force F_d affecting the motor:



Derive an expression that relates ω_{actual} to ω_d and F_d, then prove that the disturbance is cancelled if K is large enough.

$F_c = K(\omega_d - \omega_a)$, $F = F_c - F_d = K(\omega_d - \omega_a) - F_d$

$\omega_{actual} = F \frac{K_m}{s} = [K(\omega_d - \omega_a) - F_d] \frac{K_m}{s}$

$\omega_a = [K\omega_d - K\omega_a - F_d] \frac{K_m}{s} = \frac{KK_m}{s}\omega_d - \frac{KK_m}{s}\omega_a - \frac{K_m}{s} F_d$

$\omega_a \left[1 + \frac{KK_m}{s}\right] = \frac{KK_m}{s}\omega_d + \frac{K_m}{s} F_d$

when $K \gg K_m$ Then $\frac{KK_m}{s} \gg \frac{K_m}{s}$

So $\omega_a \approx \omega_d$

since $\frac{K_m}{s}$ is small in comparison to weights on ω_a and ω_d

so effect of F_d is minimized.

3.2 final exam (design)

Local contents

3.2.1 Question

MAE 106 Mechanical Systems Laboratory
Winter 2005 Design Exam

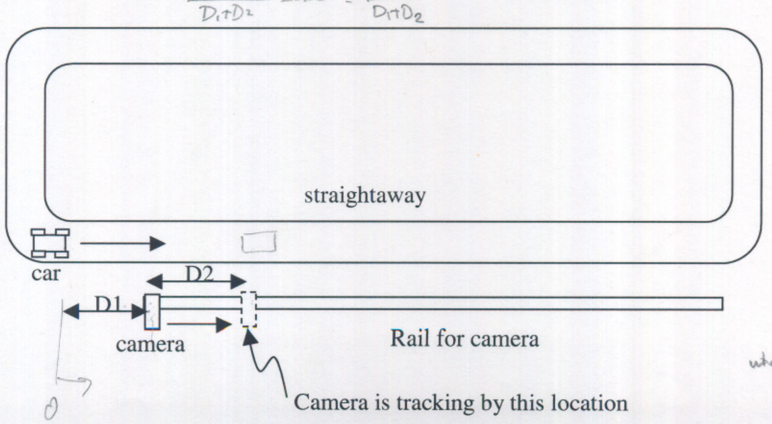
You have been hired as a control engineering consultant for NASCAR, the car racing organization. Your job is to design a control circuit for a robotic camera that will ride on a rail along the straightaway (i.e. the straight part of the racetrack). The camera control system will receive a measurement of the velocity from a selected car. The velocity information will come from the car's own speedometer, via a wireless link. Your job is to make the camera track the car, so that television viewers worldwide can "ride along" next to the driver on the straightaway. You are given the following information:

- The motor that drives the robotic camera along its track is a DC brushed motor with a current amplifier. It produces a force on the camera in response to a voltage input with a calibration coefficient of C_1 N/volt.
- The robotic camera has a mass of M.
- There is a tachometer on the robotic camera, which returns the linear velocity of the camera as a voltage with a calibration coefficient of C_2 volts/meter/sec
- The robotic rail system has some static friction, which can be modeled as constant force F_f. Dynamic friction is negligible.
- The wireless link begins working when the car is D_1 meters away from the camera. The robot should be moving at the right speed, right next to the car, after it moves D_2 meters.
- You may assume that the car moves at a constant velocity in the straightaway, and the maximum velocity that it moves at is S.
- You are to design the controller with the lowest gains possible, because NASCAR plans to update your controller next year using a computer-based system. They believe the computer-based system will work better if the controller you design has as low-as-possible gains, because of possible time delays in sampling with the planned computer-based system.

Design an analog circuit to control the robotic camera. You will get full credit if you:

1) Show your control law, in MKS units.
2) Provide appropriate control gains with units.
3) Show your control law, in units of volts
4) Draw an op-amp circuit that can implement your controller. Label what the inputs and outputs of your circuit should be connected to.
5) Choose appropriate values for the resistors and capacitors in your circuit.

PLEASE ENTER YOUR ANSWERS ON THE ANSWER SHEET

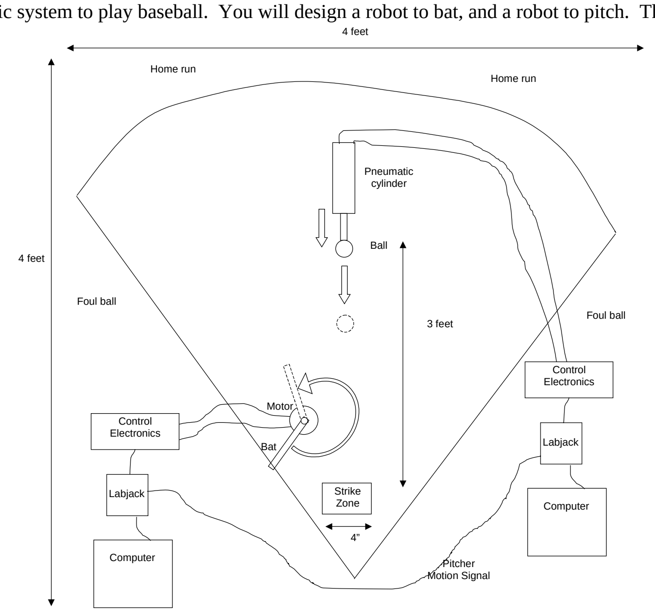


3.2.2 Detailed description

MAE 106 Mechanical Systems Laboratory
Final Project Details 2005

HOME RUN DERBY

You will design a robotic system to play baseball. You will design a robot to bat, and a robot to pitch. The setup will be:



The pitching robot will be a pre-built pneumatic cylinder with an integrated, with a 6" stroke length, and a linear potentiometer to sense position. The air into the cylinder will be controlled with a pneumatic servovalve. You don't have to build the pitching robot, but you must write a Matlab computer program to control the servovalve, allowing air into the cylinder so that the pitching robot moves and pitches the ball. The interface to the cylinder will be through a Labjack. You will not be allowed to take the pneumatic cylinder, valve, or Labjack home with you. There will be several pratice fields set up in the lab for testing.

You will make the batting robot using a DC brushed motor and other items in a starter kit. You can control the batting robot using an op-amp circuit, using the Labjack, or using both – it's up to you. You can use a computer algorithm to control the batting robot, or teleoperate it with a potentiometer – it's up to you. To help time the swing of the batting robot, it will receive a copy of the signal from the linear potentiometer on the pitching robot. You will receive a small current amplifier to use with the batting robot (the same one that you are using in Lab7).
Teams: You will work in teams of two or three. If you do not have a team, email Prof. Reinkensmeyer at dreinken@uci.edu and he will assign you to a team.

Competition Time and Place: On the day of the scheduled final (Tuesday, March 22, 4-6 PM) there will be a tournament testing your robotic baseball player. The tournament will be held in the MAE106 Laboratory (EG 2102).

Lab Hours: The lab will be open from 8-12 A.M. and 1-5 P.M. on school days starting February 28th until the contest. If the lab door is locked, you must sign-in with Dave Hartwig in room EG2118 before the lab will be opened. For safety reasons, there must always be at least two students in lab for the door to be opened for you, so bring a partner. While you are in lab, do not leave the room unattended. You are responsible for all laboratory equipment while you are in the lab. Sign-out with Dave Hartwig when you leave.

Contest Rules:
1) The contest will be a single-elimination, head-to-head tournament. Two teams will take turns batting and pitching, each getting 10 pitches. The team with the most homeruns will advance to the next round.
2) The approximate dimensions of the course are shown in the figure.

3) You may not modify the mechanical structure of the pitching robot, only its software.
4) The only actuator allowed on your batting robot is the DC motor provided in your kit.
5) You will be given three minutes to set-up your robot.
6) The strike zone will be 4" wide.
7) There will be no wall in the outfield.
8) There will be a connector that can quickly plug the Labjack into your protoboard on the trainer kit. The connector will provide 2 DA channels, 2 AD channels, ground, and the pitching signal.
9) You will receive a specification sheet describing the pneumatic cylinder and valve.
10) The batting robot must be stationary when the "pitch" signal is given (i.e. it can't spin continuously)

Starter Kits: Starter kits may be purchased from David Hartwig in EG2118 for $20 beginning Monday February 28, 8 AM – 12 PM, 1 PM – 5 PM. You may pick up the motor today. ***IMPORTANT!: You must return the kit in order to receive your final grade. If the motor or trainer kit is carelessly damaged, you will be required to pay for them (~$200).***

Other Parts: You may wish to purchase other components for your project. Suggested vendors are:

- Radio Shack: 4716 Barranca Parkway, Irvine (949)552-1091 (and other locations)
- Marvac Electronics: 2001 Harbor Blvd., Costa Mesa (949)650-2001
- Fry's Electronics: 10800 Kalama River Ave., Fountain Valley (714)378-4400 (and other locations)
- C&H Sales (Pasadena), Ultimate Hobbies, Hobby Shack, Gyro Hobbies, Wright Hardware
- Digikey: www.digikey.com
- Newark Electronics: www.newark.com

Grading: The final project is worth 25% of your grade. You can score a maximum of 120 pts on your final project. Your points will be based on:

1. The performance of your robot on the day of the contest
    - +10 pts if you have a plausible circuit and robot, but it doesn't work
    - +20 pts if your robot works
    - +30 pts if you finish in the final four
    - +40 pts if you win the competition
2. A written final project report (80 pts maximum)
    - The goal of the written final project report is to describe your design as clearly as possible, and the effort you put into building and testing the robot.
    - One write-up should be turned in per project group.
    - Your final project write-up should have the following sections:

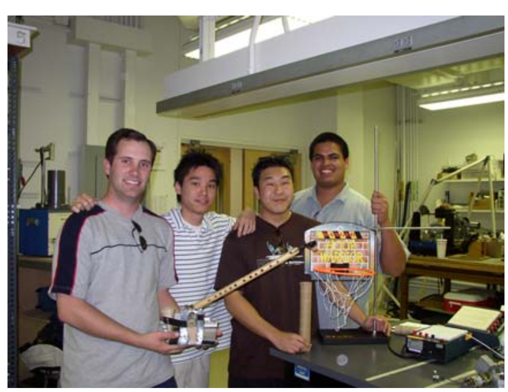
1. Overview (brief summary of the design project and your controller design) (10 pts)
2. Block diagram of the controlled system (10 pts)
3. Circuit diagrams for the circuits that you built (10 pts)
4. Equations relating circuit and block diagram to controller (10 pts)
5. Methodology for choosing controller gain values (10 pts)
6. Mechanical design features (10 pts)
7. Parts list (10 pts)
8. Testing (Any tests that you performed to calibrate/verify/improve performance -- with graphs) (10 pts)

INTRODUCTION by Matt Traum

There it sits, glistening in the morning sunlight. Representing weeks of tears and tedium, a P-controlled car capable of maintaining constant velocity sits upon the drywall track. Every battery has been charged in full, every MOSFET has been tested and retested, and even Op-Amp has a particular gleam to it. The gauntlet was tossed down ten weeks before, and now this tiny warrior is ready to meet the challenge: steady velocity control.

The signal is given. The little racer is off! With calculated precision it accelerates to its predetermined velocity. It passes the first photo gate as it's tires meet the edge of the cliff, the 30 degree sheer incline that must be traversed. The motor screams with all its will as the small warrior climbs the mountain before it. Like the Little Engine that Could, the car puffs its way to the summit.

At the top there is a brief flat rest and beyond, the treacherous downhill. The car begins its decent, nearly slipping on the slick drywall surface. At the bottom, it manages to clear the second photo gate and is free to head for home. Like the finish of a marathon, the termination of a long journey, or the closing scene of a romance novel, the little fighter drives to victory. Its motor purrs with the satisfaction of a task completed and well done. The end of the track is centimeters away; almost within reach.... WHAM!!!



3.2.3 key solution

MAE 106 Mechanical Systems Laboratory
Winter 2005 Design Exam

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3) Show your control law, in units of volts
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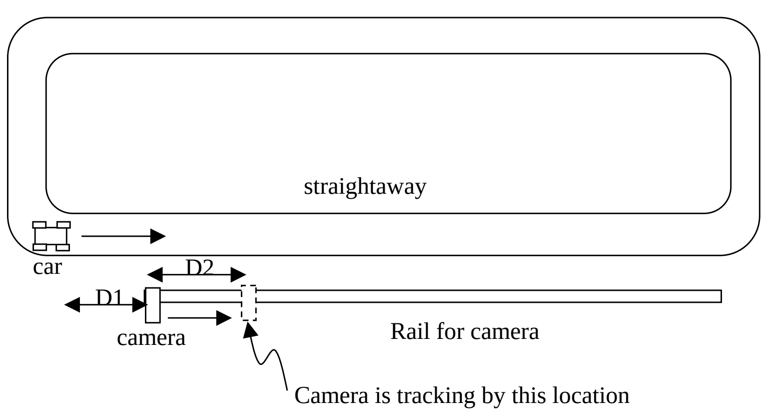


velocity is the signal that is available. Implementing position control would require a position sensor or an integrator.

Answer Sheet **NAME:** SOLUTION

$-K_p(v-v_d)-K_d\dot v = m\dot v$

$-K_p(v-v_d)=(m+K_d)\dot v$ 1st order

1) Control law; MKS units (20 pts)

$F=-K_p(v-v_d)-K_I\int(v-v_d)dt$

Basic idea: Use proportional velocity feedback (15 pts)

- Adding an integral term eliminates steady state error from friction (5 pts)
- Derivative term is not necessary - does not increase order of system or improve performance

To find control gains, find closed loop dynamics

2) Control gains (with units) (20 pts)

$K_I = \frac{9mS^2}{D^2}\left[\frac{N}{m}\right]$ or $K_p = \frac{3mS}{D}\left[\frac{Ns}{m}\right]$ if just P-control

$K_p = \frac{6mS}{D}\left[\frac{Ns}{m}\right]$

$D = D_1 + D_2$

$m\dot v = F_f - K_p(v-v_d) - K_I\int(v-v_d)dt \Big\} \frac{d}{dt}$

$m\ddot v + K_p\dot v + K_I v = K_I v_d$ 2nd order system

$\frac{V}{V_d} = \frac{K_I}{ms^2+K_ps+K_I} = \frac{\frac{K_I}{m}}{s^2+\frac{K_p}{m}s+\frac{K_I}{m}} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

If just P ctrl: $-K_p(v-v_d) = m\dot v$; step input

$V = V_d(1-e^{-t/\tau})$ $\tau = \frac{m}{K_p}$

want $3\tau = T = \frac{D}{S}$ $D = D_1 + D_2$

$\tau = \frac{T}{3} = \frac{m}{K_p} \Rightarrow K_p = \frac{3m}{T}$

$D = D_1 + D_2$

System needs to reach V_d after $T = \frac{D}{S}$ seconds

Use 5% criterion $3\tau = \frac{3}{\omega_n\zeta}$ (we'll set $\zeta=1$) $\Rightarrow \frac{3}{\omega_n} = T$ $\omega_n = \frac{3}{T} = \frac{3S}{D}$

$\omega_n^2 = \frac{K_I}{m} \Rightarrow K_I = m\omega_n^2 = \frac{9mS^2}{D^2}\left[\frac{N}{m}\right]$

$\zeta = \frac{K_p}{2\sqrt{K_I m}} \Rightarrow K_p = 2\sqrt{K_I m}\,\zeta = 2\sqrt{\frac{9m^2S^2}{D^2}} = \frac{6mS}{D}\left[\frac{Ns}{m}\right]$

Note: Input is a step input since transmitter suddenly starts working

3) Control Law; Volts as units (20 pts)

$V_m = \underbrace{\frac{-K_p}{C_1C_2}}_{\bar K_p}(V_T - V_{dT}) - \underbrace{\frac{K_I}{C_1C_2}}_{\bar K_I}\int(V_T - V_{dT})dt$

Express in units of volts

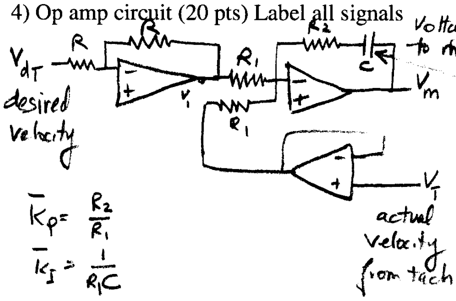
$F = C_1V_m$ V_m = motor voltage

$V_T = C_2V$ V_T = tach voltage

$C_1V_m = -K_p\left(\frac{V_T}{C_2} - \frac{V_{dT}}{C_2}\right)$

$V_m = -\frac{K_p}{C_1C_2}(V_T - V_{dT})$

4) Op amp circuit (20 pts) Label all signals



$\bar K_p = \frac{R_2}{R_1}$

$\bar K_I = \frac{1}{R_1C}$

$\frac{V_{dT}}{R} = -\frac{V_1}{R} \Rightarrow V_1 = -V_{dT}$

this capacitor adds an I term: $\frac{V_1}{R_1} + \frac{V_T}{R_1} = \frac{-V_m}{R_2 + \frac{1}{sC}}$

$\frac{1}{R_1}(V_T - V_{dT}) = \frac{-V_m sC}{R_2Cs+1}$

$V_m = -\frac{(R_2Cs+1)}{R_1sC}(V_T - V_{dT})$

$V_m = -\left(\frac{R_2}{R_1} + \frac{1}{R_1Cs}\right)(V_T - V_{dT})$

$= -\left(\bar K_p + \bar K_I\frac{1}{s}\right)(V_T - V_{dT})$

5) Resistor and Capacitor Values (20 pts)

$R_1 = \frac{C_1C_2}{K_IC} = \frac{C_1C_2D^2}{9mS^2C}$ choose C arbitrarily

$R_2 = \frac{R_1K_p}{C_1C_2} = \frac{R_1 6mS}{C_1C_2D}$

$\bar K_I = \frac{1}{R_1C} \Rightarrow \boxed{R_1 = \frac{1}{\bar K_I C}} = \frac{C_1C_2}{K_IC}$ C arbitrary

$\bar K_p = \frac{R_2}{R_1} \Rightarrow \boxed{R_2 = R_1\bar K_p} = \frac{R_1K_p}{C_1C_2}$

3.2.4 my solution

Home Run Derby.

Final project report.
MAE 106. UCI. Winter 2005.

By Nasser Abbasi
March 22, 2005

Table of contents

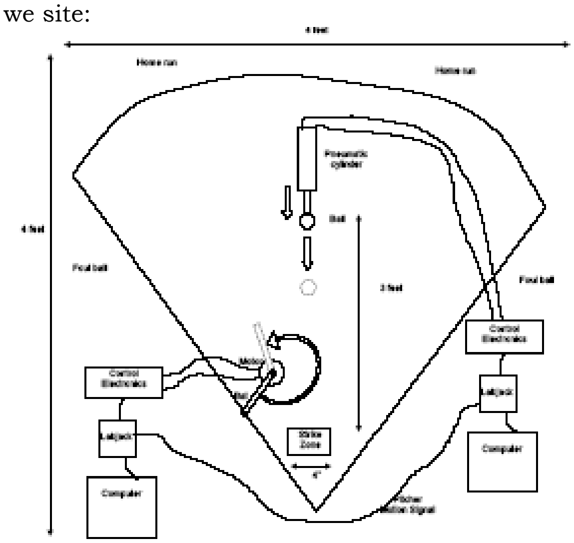
1 Overview

This is the report for the final project for MAE 106 course, UCI, winter 2005.

The project objective is to design a robotic system to play baseball for the final project for MAE 106. The project description and rules can be found at professor's Reinkensmeyer web site

http://www.eng.uci.edu/~dreinken/MAE106/mae106home.htm

Below is the general setup diagram taken from the project description document found in the above we site:



There are 2 main components to the project: the ***pitcher*** and the ***batter***.

For both parts we used Labjack with Matlab to communicate between the PC and the mechanical components of the project (Piston and Motor). We used Matlab as the main tool to implement the control laws.

The basic design criteria we wanted to achieve are a fully automated and controlled batter with no manual nor visual control. This implied the restriction on the use of a manual controller to control the motor. We have started with the assumption that one is now allowed to use the keyboard once the game starts.

Due to this restriction we imposed on our design, the final solution consisted of software program where the controller was written in Matlab with heavy reliance on the LabJack Matlab interface calls. This solution produced some technical problems on its own, since this controller in essence in an ***open loop controller***, we needed to have a good timing analysis done earlier and impede these timing estimates in the controller software. In addition, the sampling rate of Labjack was not fast enough to allow us to obtain more readings of the ball position as it is being hit by the piston; this resulted in less than optimal estimates being made on the position of the ball. Since it is not possible to track the position of the ball itself, implementing a feedback control system based on tracking the ball position is not possible (this would have been the optimal solution).

On the pitcher side, we controlled the speed of the piston by adjusting the voltage delivered to the piston actuator.

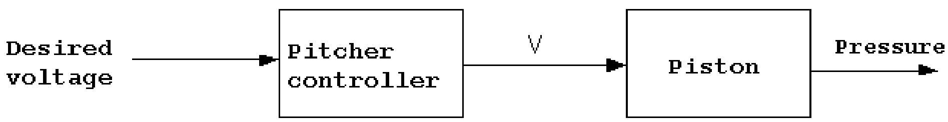
On the batter side (which was the more complicated controller), we detect when the ball was hit and at what speed, this in turn correlated to the amount of time delay before we issue the signal to start the motor to swing the bat. We also added logic to the bat controller to avoid attempting to hit balls with low enough speed which we calculated earlier (by running many tests) to be at such low speeds that the ball will most likely not be arriving in a straight line and hence will not hit the strike zone.

In addition, on the batter side, we added a mechanical stop mounted on the motor to be able to set the initial position of the bat to be at a fixed location before the start of each swing. This was critical to the mechanical design of our batter, since the bat controller is an open loop; hence having accurate timings is the most important aspect (by definition, an open loop contains no feedback and must rely on initial accurate timings). Hence, being able to set the bat at a fixed location before each swing allowed us to achieve this objective.

2 Block diagrams of the controlled system

2.1 Pitcher controller block diagram

The pitcher is an open loop proportional controller with a gain of one. The desired voltage is supplied, and the controller simply interfaces with Labjack to send this voltage to the piston (the plant). We add logic to the controller to add some delay and to move the piston back and forth before the voltage signal is send to the piston (the plant) to try to confuse the batter controller, but the final voltage send to the piston is the same as the desired voltage.



Open Loop Pitcher controller

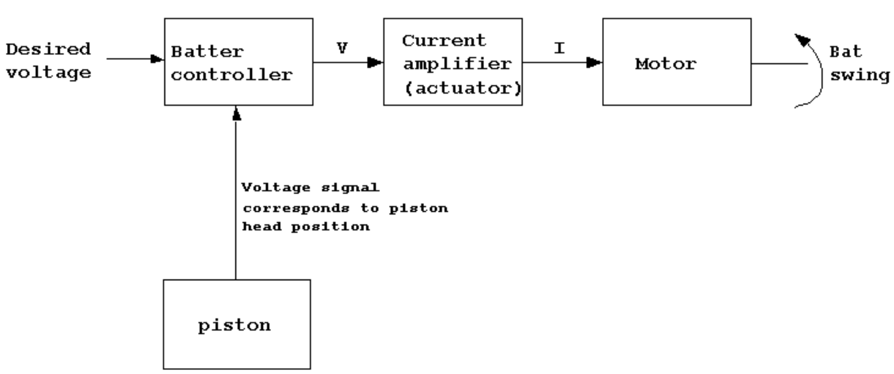
2.2 Batter controller block diagram

The batter is also an open loop proportional controller with a gain of one. The input to the batter controller is fixed at 5 volts (which is the maximum voltage labjack can send). This voltage moves the motor head at the maximum speed.

Initially we wanted the controller to control the speed of the motor. For fastballs, we wanted to speed up the motor, and for slow balls we wanted to slow down the motor.

However, after some testing, we discovered that the current amplifier we were given to use on this project was not designed for this purpose, it was only able to generate a constant current to run the motor at the same speed regardless of the voltage that was sent to it. (Actually the motor would run at a fixed low speed for voltages below 4 volts, and at fixed high speed for voltages over 4, but the point is that it was not possible to change the speed of the motor in direct proportion of the voltage being sent to the current amplifier we used)

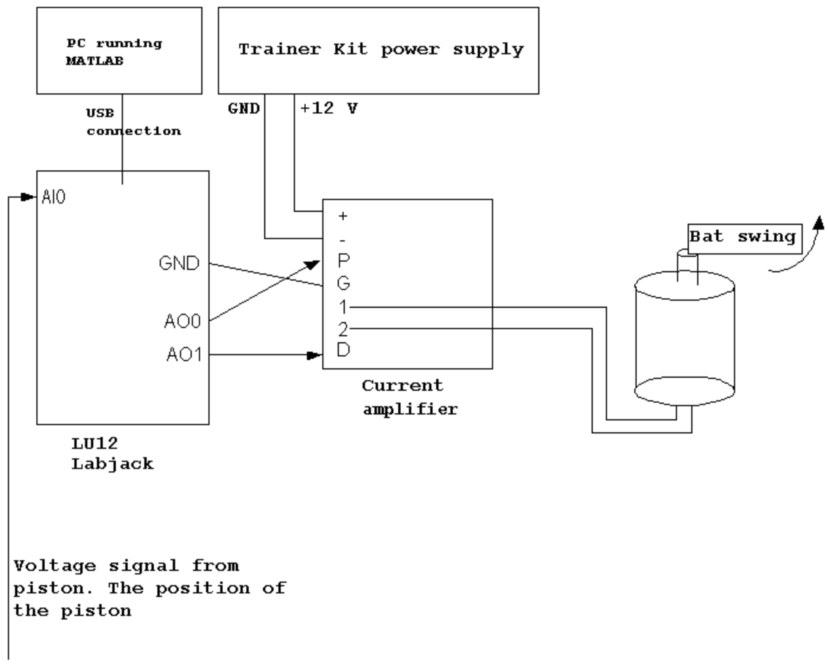
For this reason, we designed the controller to delay more for slow balls and to delay less for fast ones, but to run the motor at the same speed for both cases. The block diagram for this open loop controller is shown below. In this diagram we show how the controller reads the piston head position signal from the piston in order to decide when to issue the command to the motor to start.



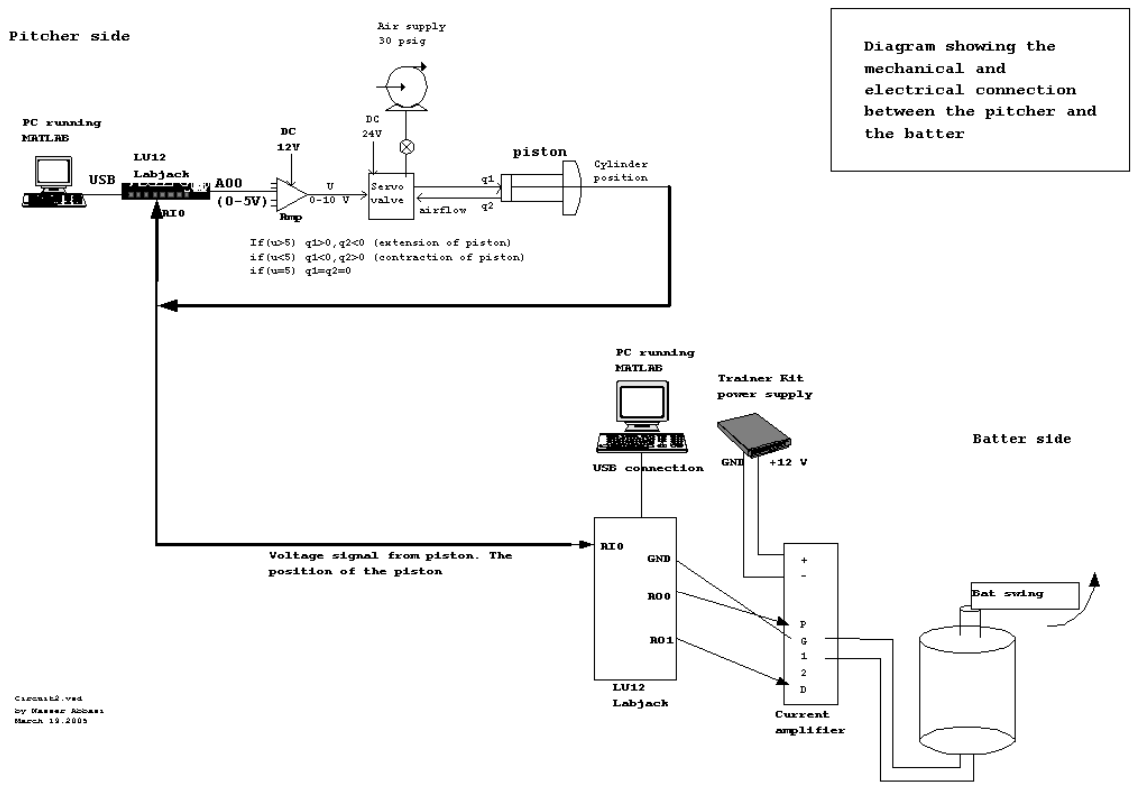
Open Loop Batter controller

3 Circuit diagrams

The following diagram shows the full hardware configuration used with all the connections needed at the batter side



The batter reads the piston position signal as it arrives at the AI0 port of labjack. The following diagram combines both the pitcher and the batter circuits to better illustrate how the connection is achieved.



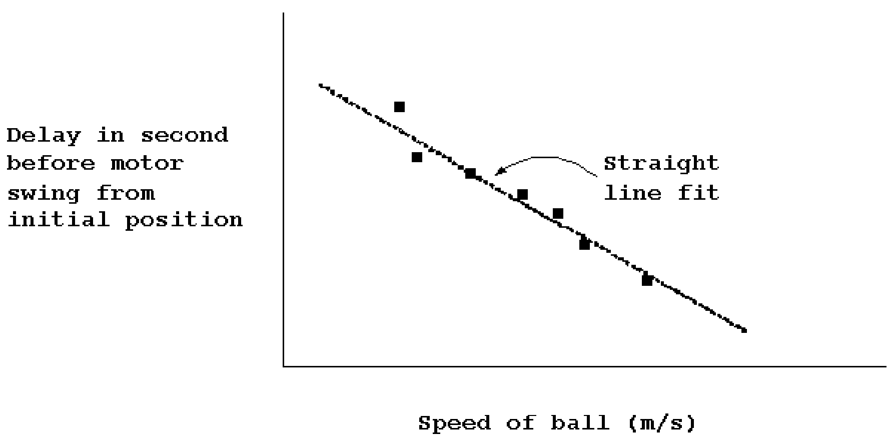
4 Equations used

Since the controller we used is a proportional controller with fixed gain of one, the control law is obviously very simple. However, the complexity came about in determining the amount of delay and in the calculation of the ball speed and in forcing labjack to use a fixed sampling rate so that our calibration will remain valid across different play stations.

So in what follows we describe how we performed the calibration.

We used a linear relation between the speed of the ball when it was hit and the amount of delay required before we swing the bat. We performed many experiments, hitting the ball at different speeds and determined the amount of delay needed to obtain a good swing for this speed (Please see the testing section for additional data on this).

The result of this testing is a number of data points. We then did a linear straight-line fit of the data and obtained the equation of the line. This line equation was used by the controller to determine the time delay needed based on the ball speed.



THE LINE EQUATION

$$y = m + cx$$

Used in the controller is

$$delay = .525 - \left(\frac{0.03}{6}\right)(speed - 51)$$

In the above we assumed that the ball speed does not change greatly from the time the controller detected that the ball was hit until the time the ball arrives to the strike zone. Since the distance between the pitcher and the batter is small (three feets), and the ball is fairly smooth, this assumption is reasonable. In addition, the actual speed calculation itself is not very accurate due to the low sampling rate used by Labjack, which did not allow us to obtain more accurate estimates of the ball speed.

The speed of the ball was calculated as follows. The batter controller monitors the position of the piston head continuously by reading the piston head position signal which can vary from 0 volts upto 10 volts. The ball is situated half way between a fully extended and fully contracted piston. Hence a voltage reading of around 5 volts will indicate that the piston has hit the ball.

The batter controller keeps track of the last 2 positions read from the piston and the amount of time elapsed between these 2 readings.

When it detects that the piston position has passed the half way mark, indicating that the ball was hit, it will calculate the speed using the equation

$$speed = \frac{currentPosition - LastPosition}{timeElapsed}$$

This value is then substituted in the delay equation discussed above to give the amount of delay in seconds needed before starting the motor.

In addition to the above, the batter controller will check if the speed of the ball is below a certain threshold. If so, the batter will not turn the motor in this case. The reason for this is that we found by experimentations that a slow ball will most likely not remain in a straight line and will not arrive to the strike zone. Hence we did not want to take the chance on trying to hit a ball that is going out anyway and losing a point. This will maximize our chances of winning. This threshold is currently set at 52 m/s.

5 Methodology used

We will consider the pitcher side and the batter side separately.

5.1 Pitcher controller logic

The logic of the pitcher controller is simple. We apply voltage, which causes the piston head to move. A voltage over 2.5 will move the head forward, and voltage below 2.5 will move the head backwards. We varied the speed we hit the ball at, and also moved the piston back and forth before hitting the ball in order to try to confuse the batter controller.

5.2 Batter controller logic

We have build 2 mountings on top of the motor mounting. One mounting to attach the bat, and another to attach a mechanical stop.

Controller starts by swinging the bat handle to the initial position, this will cause the bat handle to hit the mechanical stop mounting attached to the motor mounting. Immediately, the motor will stall, drawing current but not moving.

The controller will next turn the voltage off the motor and waits for the ball to be hit by the pitcher. The Batter controller continuously monitors the ball position signal coming from the pitcher.

The batter controller monitors the position of the piston head and calculates its speed after each sample reading using Labjack Matlab function calls.

When the batter controller detects that the position of the piston head has hit the ball (recall that the ball is located at a fixed position), then the speed of the ball will be known at that moment. It will be the speed of the piston head.

From the speed of the ball, the batter controller decides when to start the bat swing. The controller will always swing the bat at the maximum (5 volts) level motor speed.

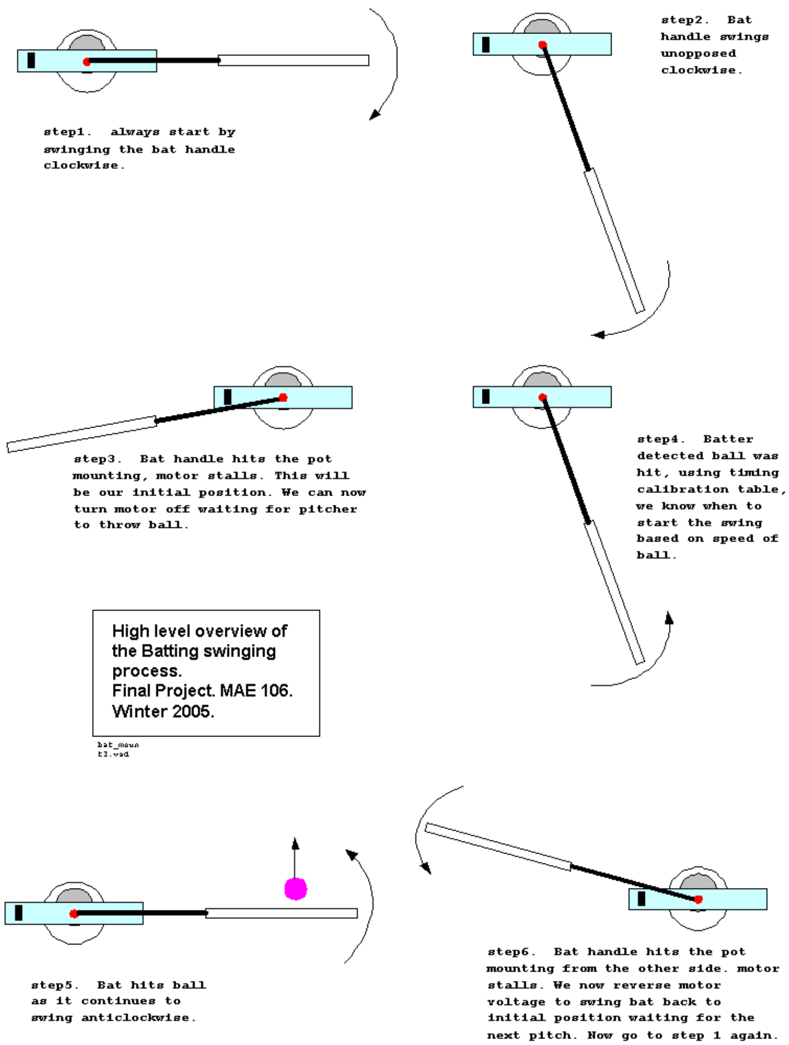
The controller knows when to start the swing by a calibration process that we have performed prior to the start of the tournament. During calibration we did a number of runs and derived a line equation that allowed the controller to determine the delay needed given the speed of the ball.

Since the batter always starts from the same position (due to the use of the mechanical stop), this process resulted in a reasonable accurate swings designed to hit the ball at 90 degrees to the direction of the ball to obtain the maximum force on the ball.

At the end of each swing the bat handle will hit the mechanical stop on the other side, causing the motor to stall again. Controller will switch the voltage sign again, causing the bat to quickly swing back, hitting the mechanical stop again from the other side, and resting there until the next pitch.

This process will continue automatically. To help illustrate this process, the following diagram shows the main steps for the batter controller.

5.2.1 Batter controller steps diagram



5.2.2 Batter controller algorithm

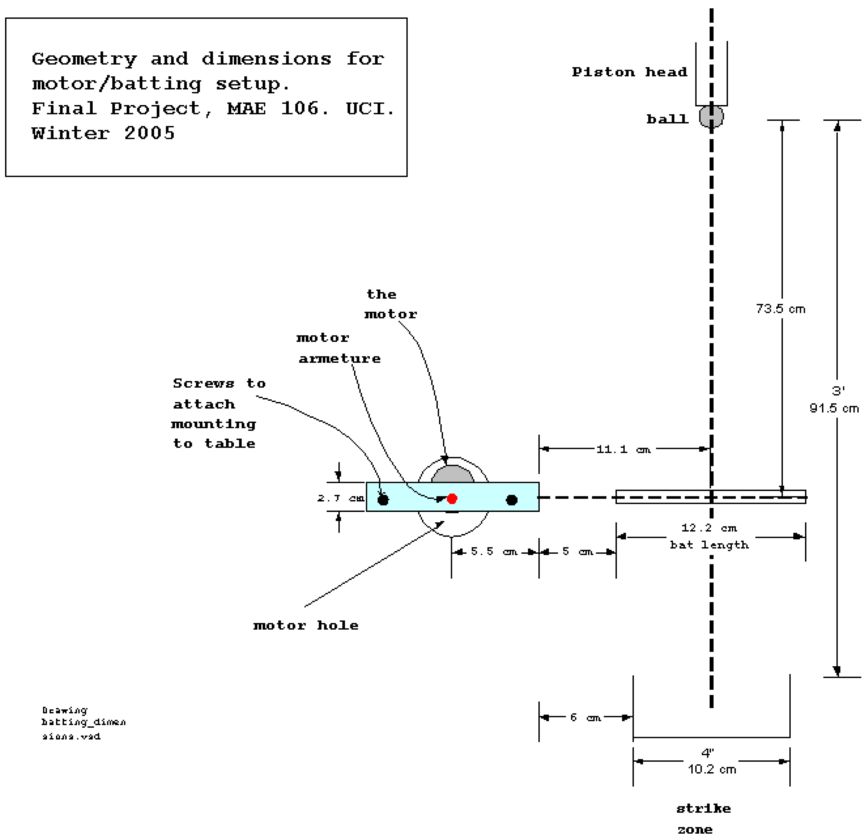
We have used Matlab to implement the batter controller logic. The following is the algorithm for this controller.

```
LOOP
    reverse motor voltage to swing clockwise to hit the mechanical
stop.

    start reading the piston signal -- use LABKACK Matlab call.

    Calculate the position and speed of the piston head.

    WHILE  piston has not hit the ball
           read next sample from the piston signal
           update position and speed of the piston.
    END WHILE

    determine delay needed based on ball speed. (use linear fitting)

    pause the delay needed.

    issue a control signal to the motor to swing at maximum speed.

    delay for known amount of time (calibrated) to allow bat to swing
    fully.

 END LOOP
```

6 Mechanical design features

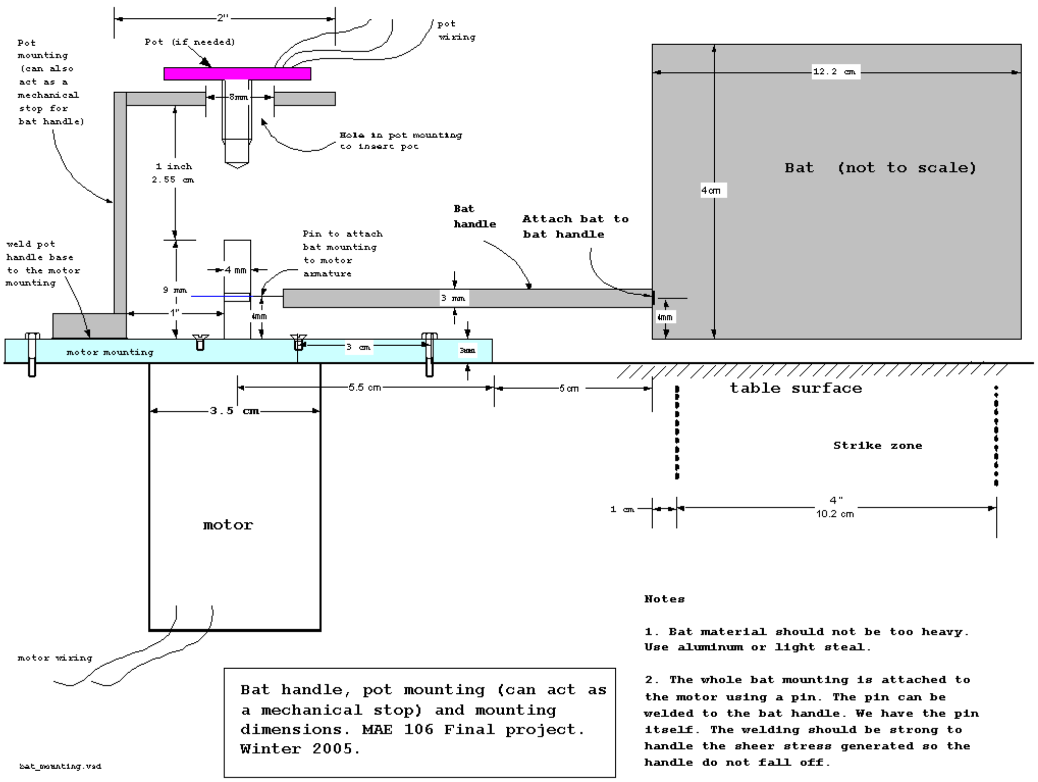
6.1 Geometry and dimensions of the playing field

To be able to build the mounting for the bat handle and the bat itself, we needed exact dimensions of the playing field.



6.2 Geometry and dimensions of the batting mounting

The dimensions of the batting and mechanical stop mounting are shown below. We initially designed the mounting to allow us to add a potentiometer if we had to.



7 Parts list

- Pittman DC gearhead motor P/N GM8714F560
- Series SRX stainless Steel body air cylinders with position feedback.
- Motor mounting (The size and dimensions are provided to us by the instructor).
- Mechanical stop mounting L-shaped.
- One mechanical bat arm to hit the ball with.
- 2 LU12 labjack's with USB connection.
- 2 PC's running windows and Matlab
- One current amplifier.
- A number of wires to connect current amplifier to motor and to labjack
- 2 screws and bolts to attach the motor mounting to the table.
- Power supply for the current amplifier (we used +12V DC provided by the training kit).

8 Testing with graphs

Since we implemented an open loop controller for the batter, most of the testing concentrated on calibration of the bat controller. We needed to able to estimate the amount of delay before a swing based on the speed of the ball. To accomplish this, as we discussed above in the methodology section, we ran many tests and collected many data points. This is a partial listing of data collected for calibration.

Voltage to piston (Volts)	Calculated speed of ball (speed units)	Amount of delay needed (milliseconds)
4	51.04	79
	49.97	81
	55.60	75
	61.52	69
	56.07	74
3.9	50.26	80
	52.03	78
	49.25	81
	52.92	78
	52.76	82
3.8	53.08	83
	47.52	84
	56.00	83

For each piston voltage, the average of the speed was taken and the average of the amount of the delay needed was also taken. This resulted in a number of (x, y) data points that we fitted a straight line through to obtain the line equation.

We also needed to fix the sampling rate of Labjack, and for this purpose we used one Labjack Matlab call that allowed us to specify the sampling rate to be used. Instead of using the standard call EAnalogIn() to read the piston position signal from AI0, we used the call AIBurst that also can read AI0, but in addition allowed us to specify a scan rate from 400 up to 8192 hz. We used 8192 Hz to be able to avoid the possibility of having one labjack running at different sampling rates at different play stations.

8.1 Example output of one test run

When the bat controller runs, it continuously displays basic results showing the speed of the ball being hit and the amount of delay. It will also show if the ball we is too slow, which in this case the batter will not attempt to hit. These messages made it easier to debug and calibrate the program. Below is such example output

```
>> nma_motor
detected ball was hit.....
elapsedTime=62.000000 ms
last position=2.153320, currentPosition=5.410156
speed=52.529612
delay=0.517352
**************************************
detected ball was hit.....
elapsedTime=63.000000 ms
last position=2.211914, currentPosition=5.048828
speed=45.030382
delay=0.554848
ball too slow, will not try to hit...
**************************************
detected ball was hit.....
elapsedTime=64.000000 ms
last position=2.192383, currentPosition=5.668945
speed=54.321289
delay=0.508394
**************************************
detected ball was hit.....
elapsedTime=67.000000 ms
last position=4.619141, currentPosition=8.447266
speed=57.136194
delay=0.494319
**************************************
```

9 Appendix

In the appendix we show the Matlab code used for the bat and pitcher controller.

9.1 Batter controller Matlab code

For the batter it was not clear if the rules of the game allow one to touch the keyboard once the game starts.

For this reason, we have 2 versions of the bat program. One is called nma_motor_auto.m, and the other is called nma_motor_manual.m.

The first program above is the one discussed in details in this report, since it is the fully automated version and requires no manual intervension once it starts. The second version is a modified version that requires the user to hit the keyboard to start the motor running resulting is the bat moving.

Below is the code for both version of the controller.

9.1.1 nma_motor_auto.m

```
%
% MAE Final project. UCI, winter 2005
% motor controller script.
% by Nasser Abbasi
%
% This script implements the open loop bat controller
% See the project report for background information
% about this project.
%

% define some constants used in the program
LEFT=5;
RIGHT=0;
HIGH_V=5;
LOW_V=0;
DEBUG=1;
TRUE=1;
FALSE=0;
BALL_POSITION=5;
SAMPLE_RATE=8192; %Hz
LOW_SPEED_THRESHOLD=0;
SLOPE=(0.25-0.12)/(65.33-51);

pistonLastPosition=0;  %set the piston last position to be fully retracted
```

```
while TRUE

    EAnalogOut(0,0,HIGH_V,RIGHT);  %set the handle
    pause(0.1);                    %wait to reach mechanical stop.
    EAnalogOut(0,0,LOW_V,RIGHT);   %turn off motor

    BallHit=FALSE;
    [count tnow err id]=ECount(0,0,0);

    while ~BallHit

        % replace the call below by AIBurst. To allow better control
        % on the sampling rate.
        % [currentPosition  o err id]=EAnalogIn(0,0,0,0);

        tlast=tnow;
        [pistonCurrentPosition  s scans over err id]=…
                   AIBurst(0,0,0,0,0,1,0,0,SAMPLE_RATE,0,0,0,1,1,0);
        if err~=0
            fprintf('Failed AIBusrt call. error is %s\n',GetErrorString(err));
            return;
        end
        [count tnow err id]=ECount(0,0,0);

        %if DEBUG fprintf('currentPosition=%f\n',currentPosition); end;
        %if DEBUG fprintf('tnow=%f, tlast=%f\n',tnow,tlast); end;

        % if(DEBUG)
        %    if(currentPosition>2) fprintf('currentPosition=%f\n',currentPosition); end
        % end

        if(pistonCurrentPosition>BALL_POSITION)
            if pistonLastPosition<=BALL_POSITION
                if DEBUG fprintf('detected ball was hit by piston.....\n');  end;

                elapsedTime=tnow-tlast;
                fprintf('elapsedTime=%f ms\n',elapsedTime);

                speed=(pistonCurrentPosition-pistonLastPosition)*1000/elapsedTime;

                if DEBUG fprintf('ball speed=%f\n',speed); end;

                fprintf('piston last position=%f, piston currentPosition=%f\n',...
                         pistonLastPosition,pistonCurrentPosition);

                if speed>LOW_SPEED_THRESHOLD
                   BallHit=TRUE;
                else
                   fprintf('ball too slow, will not try to hit...\n*********************\n');
                end
            end
        end
        pistonLastPosition=pistonCurrentPosition;

    end %while waiting for ball to be hit

    if(speed<51)
        delay=0.25+SLOPE*(51-speed);
    else
        delay=0.25-SLOPE*(speed-51);
    end

    if DEBUG fprintf('motor delay=%f\n',delay); end;

    pause(delay);

    EAnalogOut(0,0,HIGH_V,LEFT);  %HIT the ball.
    pause(0.5); %wait for bat to reach mechanical stop.
    EAnalogOut(0,0,LOW_V,LEFT); %stop the motor
    fprintf('**************************************\n');
end
```

9.1.2 nma_motor_manual.m

```
%
% MAE Final project. UCI, winter 2005
% motor controller script.
% by Nasser Abbasi
%
% This script implements the open loop bat controller
% for the manual version of the batter,
%
% See the project report for background information
% about this project.
%

% define some constants used in the program
LEFT=5;
RIGHT=0;
HIGH_V=5;
LOW_V=0;
DEBUG=1;
TRUE=1;
FALSE=0;

while TRUE
  EAnalogOut(0,0,HIGH_V,RIGHT);  %set the handle
  pause(0.1);                    %wait to reach mechanical stop.
  EAnalogOut(0,0,LOW_V,RIGHT);   %turn off motor

  fprintf('hit any key to turn bat.....\n');

  pause;

  EAnalogOut(0,0,HIGH_V,LEFT);  %HIT the ball.
  pause(0.5); %wait for bat to reach mechanical stop
  EAnalogOut(0,0,LOW_V,LEFT); %stop the motor
End
```

9.2 Pitcher controller Matlab code

```
EAnalogOut(0,0,5,0);
EAnalogOut(0,0,0,0);
```

Attached floppy disk contains more examples of the pitcher code and the code for the batter. This is the content of the flopy disk attached showing all the matlab .m files used.

Name	Size	Type	Date Modified
fastball.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:58 AM
kanauto2.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:44 AM
kanauto.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:33 AM
kanautoSMOOTH.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:48 AM
kanautoSMOOTHbeg.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:53 AM
kanhittwice2.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:57 AM
kanhittwice.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:57 AM
kanpitch1.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 8:56 AM
kanpitch2.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:15 AM
kanpitchslow1.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 8:58 AM
kanrapidfire2.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:28 AM
kanrapidfire.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:26 AM
nma_motor_auto.m	4 KB	Mathematica 4.1 Pa...	3/19/2005 6:07 PM
nma_motor_man.m	1 KB	Mathematica 4.1 Pa...	3/19/2005 6:34 PM
slowball.m	1 KB	Mathematica 4.1 Pa...	3/21/2005 9:59 AM

Chapter 4

Other class material

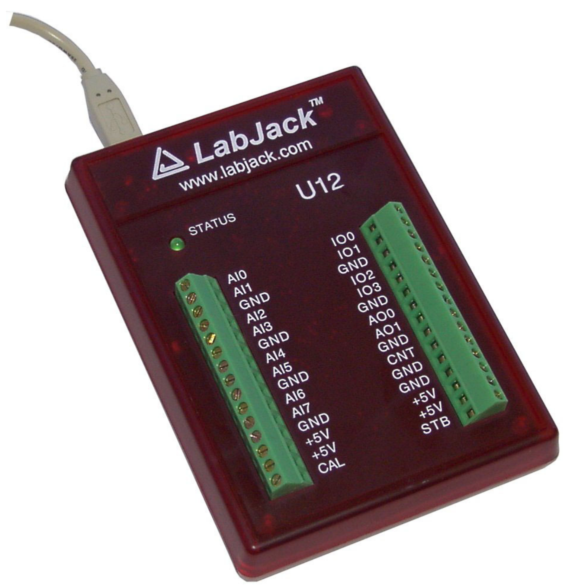
Local contents

4.1 Lab jack U21 user guide

LabJack U12 User's Guide

Revision 1.09
12/22/2004

LabJack Corporation
www.labjack.com
support@labjack.com



For the latest version of the user's guide, the quickstart guide, or software, go to www.labjack.com.

The LabJack U12 is a measurement and automation peripheral that enables the connection of a PC to the real-world. Although the LabJack U12 has various redundant protection mechanisms, it is possible, in the case of improper and/or unreasonable use, to damage the LabJack and even the PC to which it is connected. LabJack Corporation will not be liable for any such damage.

The LabJack U12 and associated products are not designed to be a critical component in life support or systems where malfunction can reasonably be expected to result in personal injury. Customers using these products in such applications do so at their own risk and agree to fully indemnify LabJack Corporation for any damages resulting from such applications.

LabJack assumes no liability for applications assistance or customer product design. Customers are responsible for their applications using LabJack products. To minimize the risks associated with customer applications, customers should provide adequate design and operating safeguards.

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Table Of Figures

1. Installation

The LabJack U12 requires a PC running Windows 98SE, ME, 2000, or XP. To determine your operating system version, go to

Start => Settings => Control Panel => System => General

and make sure the version number is 4.10.2222 or higher (Win98SE=4.10.2222, WinME=4.90.3000, Win2000=5.0.2195, WinXP=5.1.XXXX).

It does not matter if the hardware or software is installed first.

If you experience installation problems on Windows 98 Second Edition, before contacting us, please go to the downloads page at labjack.com and download “win98sehid.zip” (see the readme file for more information).

1.1 Hardware Installation

With the PC on and using the included cable, connect the LabJack U12 to the USB port on the PC or USB hub. The USB cable provides power and communication for the LabJack U12. The status LED should immediately blink 4 times (at about 4 Hz), and then stay off while the LabJack enumerates.

Enumeration is the process where the PC’s operating system gathers information from a USB device that describes it and it’s capabilities. The low-level drivers for the LabJack U12 come with Windows and enumeration will proceed automatically. The first time a device is enumerated on a particular PC, it can take a minute or two, and Windows might prompt you about installing drivers. Accept all the defaults at the Windows prompts, and reboot the PC if asked to do so. The Windows Installation CD might also be needed at this point. Make sure a CD with the correct version of Windows is provided. Enumeration occurs whenever the USB cable is connected, and only takes a few seconds after the first time.

When enumeration is complete, the LED will blink twice and remain on. This means Windows has enumerated the LabJack properly.

If the LabJack fails to enumerate:

- Make sure you are running Windows OS version 4.10.2222 or higher,
- Try connecting the LabJack to another PC,
- Try connecting a different USB device to the PC,
- Check our online forum and/or contact LabJack.

1.2 Software Installation

Although, the low-level USB drivers for the LabJack are included with Windows, high-level drivers are needed to send and receive data. The included LabJack CD installs the high-level drivers, example source code, and example applications.

Close all open applications, especially LabJack related software, and insert the LabJack CD. If autorun is enabled, the installation program should start automatically. If the installation does not start, you will have to manually double-click on LabJackVXXX.exe.

When the LabJack installation is finished, it will start the National Instruments LabVIEW Run-Time Engine (LVRTE) setup. The LVRTE is required for the example applications such as LJtest. If prompted to reboot after this installation, go ahead and do so. Virus scanners can often interfere with the installation of the LVRTE. If you have trouble running the example applications, repeat the LabJack software installation to make sure the LVRTE is installed.

To test the installation, start LJtest by selecting

Start => Programs => LabJack => LJtest.

Make sure “Test Fixture Installed” and “Continuous” are not selected, and press the “Run” button. LJtest will step through 8 separate tests and all should pass.

2. Hardware Description

The external features of the LabJack U12 are:

- USB connector,
- DB25 digital I/O connector,
- Status LED,
- 30 screw terminals.

The USB connection provides power and communication. No external power supply is needed. The +5 volt connections available at various locations are outputs, do not connect a power supply.

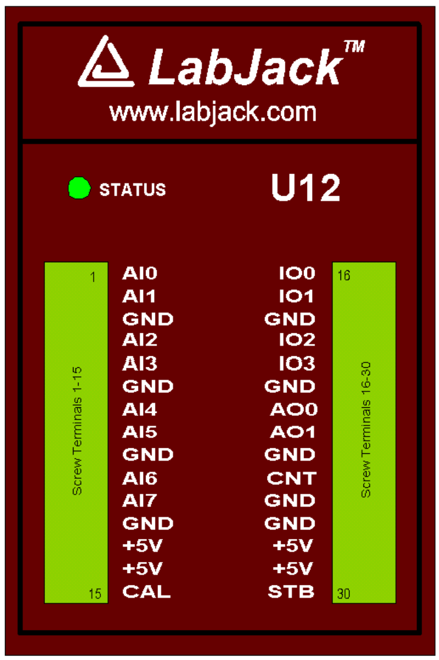


Figure 2-1. LabJack U12 top surface.

Figure 2-1 shows the top surface of the LabJack U12. Not shown is the USB and DB25 connector, which are both on the top edge. The DB25 connector provides connections for 16 digital I/O lines, called D0-D15. It also has connections for ground and +5 volts. All connections besides D0-D15, are provided by the 30 screw terminals shown in Figure 1. Each individual screw terminal has a label, AI0 through STB.

The status LED blinks 4 times at power-up, and then blinks once and stays on after enumeration (recognition of the LabJack U12 by the PC operating system). The LED also blinks during burst and stream operations, unless disabled. The LED can be enabled/disabled through software using the functions AISample, AIBurst, or AIStreamStart. Since the LED uses 4-5 mA of current, some users might wish to disable it for power-sensitive applications.

2.1 AI0 – AI7

Hardware

The LabJack U12 has 8 screw terminals for analog input signals. These can be configured individually and on-the-fly as 8 single-ended channels, 4 differential channels, or combinations in between. Each input has a 12-bit resolution and an input bias current of ±90 μA.

- Single-Ended: The input range for a single-ended measurement is ±10 volts.
- Differential channels can make use of the low noise precision PGA to provide gains up to 20. In differential mode, the voltage of each AI with respect to ground must be between +20 and -10 volts, but the range of voltage difference between the 2 AI is a function of gain (G) as follows:

G=1	±20 volts
G=2	±10 volts
G=4	±5 volts
G=5	±4 volts
G=8	±2.5 volts
G=10	±2 volts
G=16	±1.25 volts
G=20	±1 volt

The reason the range is ±20 volts at G=1 is that, for example, AI0 could be +10 volts and AI1 could be -10 volts giving a difference of +20 volts, or AI0 could be -10 volts and AI1 could be +10 volts giving a difference of -20 volts.

The PGA (programmable gain amplifier, available on differential channels only) amplifies the AI voltage before it is digitized by the A/D converter. The high level drivers then divide the reading by the gain and return the actual measured voltage.

Figure 2-2 shows a typical single-ended connection measuring the voltage of a battery. This same measurement could also be performed with a differential connection to allow the use of the PGA. In general, any single-ended measurement can be performed using a differential channel by connecting the voltage to an even-numbered analog input, and grounding the associated odd-numbered analog input (as shown by the dashed connection to AI1 in Figure 2-2).

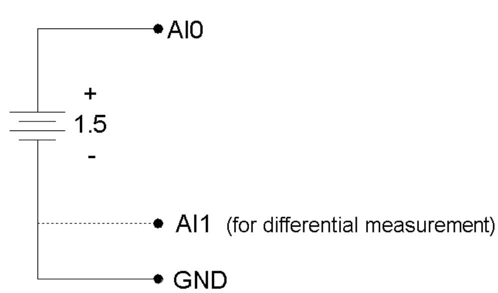


Figure 2-2. Single-ended measurement.

Figure 2-3 shows a typical differential connection measuring the voltage across a current shunt. A differential connection is required when neither leg of the shunt is at ground potential. Make sure that the voltage of both AI0 an AI1 with respect to ground is within ±10 volts. For instance, if the source (Vs) shown in Figure 2-3 is 120 VAC, the difference between AI0 and AI1 might be small, but the voltage from both AI0 and AI1 to ground will have a maximum value near 170 volts, and will seriously damage the LabJack.

Whether or not the ground (GND) connection is needed (Figure 2-3) will depend on the nature of Vs.

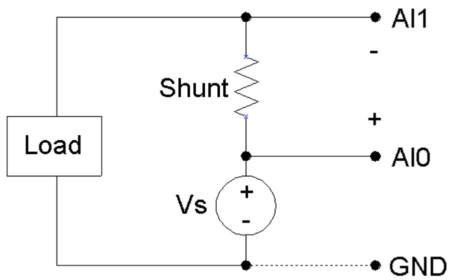


Figure 2-3. Differential measurement.

Figure 2-4 shows a single-ended connection used to measure the output voltage of a typical voltage-divider circuit. The voltage divider circuit is a simple way to convert a varying resistance (thermistor, photoresistor, potentiometer, etc.) to a varying voltage. With nothing connected to Va, the value of the unknown resistance, R2, can be calculated as:

R2 = Va*R1 / (Vs-Va),

where Vs is the supply voltage (+5V in Figure 2-4).

When Va is connected to AI0, as shown in Figure 2-4, the input bias current of the LabJack affects the voltage divider circuit, and if the resistance of R1 and R2 is too large, this effect must be accounted for or eliminated. This is true for any signal with too high of a source impedance.

All measuring devices have maximum analog input bias currents that very from picoamps to milliamps. The input bias current of the LabJack U12's analog inputs varies from +70 to -94 microamps (μA). This is similar to an input impedance of about 100 kΩ, but because the current is nonzero at 0 volts, it is better to model the analog input as a current sink obeying the following rule:

Iin = 8.181*Va – 11.67 μA

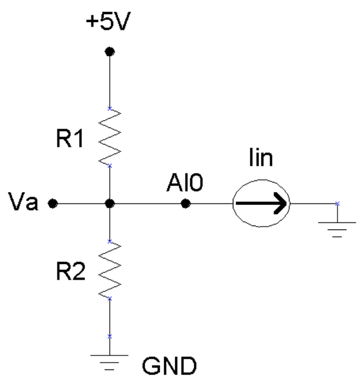


Figure 2-4. Single-ended measurement with voltage divider circuit.

Because the input bias current is known, as a function of input voltage, the simple voltage divider equation can be modified as follows to account for input bias current:

R2 = Va / [((Vs-Va)/R1) – (8.181μ * Va) + 11.67μ]

As an alternative to the equation above, Va can be buffered by a single-supply rail-to-rail operational amplifier, and the original simple voltage divider equation can be used. This solution works for any single-ended signal which stays between 0 and +5 volts. Some op-amp choices are:

- TLV2462
- LMC6482
- MAX4166

Software

Readings from the analog inputs are returned by the functions EAnalogIn, AISample, AIBurst, and AIStreamRead.

EAnalogIn is a simplified (E is for easy) function that returns a single reading from 1 analog input channel. Execution time is up to 20 ms.

AISample returns a single reading of 1-4 channels, and takes up to 20 ms to execute, providing a maximum date rate of about 50 Hz per channel.

AIBurst acquires multiple samples of 1-4 channels at a hardware-timed sample rate of 400-8192 Hz. The acquisition can be triggered based on a change of state on IO0 or IO1. This function also returns the states of the IO pins (which are read every 4 samples).

Internally, the actual number of samples collected and transferred by the LabJack during an AIBurst call is the smallest power of 2, from 64 to 4096, which is at least as big as numSamples. The execution time of this function, in milliseconds, can be estimated as:

Turbo (default) => 30+(1000*numSamplesActual/sampleRate)+(0.4*numSamplesActual)
Normal => 30+(1000*numSamplesActual/sampleRate)+(2.5*numSamplesActual)

numSamples = numScans * numChannels
sampleRate = scanRate * numChannels

AIStreamRead is called periodically during a stream acquisition started by AIStreamStart. Each call retrieves multiple samples of 1-4 channels from the LabJack stream buffer, along with the states of the IO pins (read every 4 samples). Hardware-timed sample rates of 200-1200 Hz are available. If any function besides AIStreamRead is called while a stream is in progress, the stream will be stopped.

2.2 AO0 & AO1

The LabJack U12 has 2 screw terminals for analog output voltages. Each analog output can be set to a voltage between 0 and the supply voltage (+5 volts nominal) with 10-bits of resolution.

The output voltage is ratiometric with the +5 volt supply (+5V), which is generally accurate to ±5% (see Appendix A). If an output voltage of 5 volts is specified, the resulting output will be 100% of the supply voltage. Similarly, specifying 2.5 volts actually gives 50% of the supply voltage. The maximum output voltage is almost 100% of +5V at no-load, and decreases with load. See the specifications in Appendix A relating to maximum output voltage. Also note that loading either analog output will cause an IR drop through the source impedance of each.

If improved accuracy is needed, measure the +5 volt supply with an analog input channel, and the actual output voltage can be calculated. For instance, if an analog output of 2.5 volts is specified and a measurement of +5V returns 5.10 volts, the actual output voltage is 2.55 volts (at no-load). Alternatively (and preferably), the analog output can itself be measured with an analog input.

There is a 1st order low-pass filter on each analog output with a 3dB frequency around 22 Hz.

The analog outputs are initialized to 0.0 volts on power-up or reset.

The analog outputs can withstand a continuous short-circuit to ground, even when set at maximum output.

Voltage should never be applied to the analog outputs, as they are voltage sources themselves. In the event that a voltage is accidentally applied to either analog output, they do have protection against transient overvoltages such as ESD (electrostatic discharge) and continuous overvoltage of a couple volts. An applied voltage that exceeds the capability of this protection will most likely damage the resistor R63 (AO0) or R62 (AO1) on the LabJack U12 PCB. The symptom of such a failure would be reduced voltage from the analog outputs, particularly at load, and could be verified by measuring the resistance of R62/R63 (should be less than 50 ohms but a damaged resistor will measure higher). A simple repair for such damage is to remove the damaged resistor and simply make a short with a blob of solder.

Software

The analog outputs are set using the function EAnalogOut (easy function) or AOUpdate, which take up to 20 ms to execute, providing a maximum update rate of about 50 Hz per channel. AOUpdate also controls/reads all 20 digital I/O and the counter.

2.3 IO0 – IO3

Connections to 4 of the LabJack's 20 digital I/O are made at the screw terminals, and are referred to as IO0-IO3. Each pin can individually be set to input, output high, or output low. These 4 channels include a 1.5 k Ω series resistor that provides overvoltage/short-circuit protection. Each channel also has a 1 MΩ resistor connected to ground.

All digital I/O are set to input on power-up or reset.

One common use of a digital input is for measuring the state of a switch as shown in Figure 2-5. If the switch is open, IO0 reads FALSE. If the switch is closed, IO0 reads TRUE.

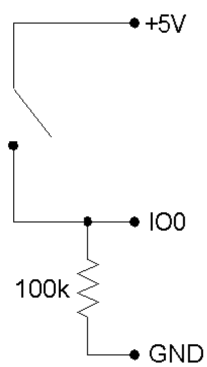


Figure 2-5. IO used to detect the state of a switch.

While providing overvoltage/short-circuit protection, the 1.5 k Ω series resistor on each IO pin also limits the output current capability. For instance, with an output current of 1 mA, the series resistor will drop 1.5 volts, resulting in an output voltage of about 3.5 volts.

Software

The easy functions EDigitalIn or EDigitalOut are used to read or set the state of one digital line, and both take up to 20 ms to execute.

The functions AOUpdate and DigitalIO are used to set the direction, set the state, and/or read the state, of each IO pin. Both of these functions take up to 20 ms to execute, providing a maximum update rate of about 50 Hz per pin.

The function AISample can set/read the state of each IO, but setting the state will have no effect unless the IO have been configured as outputs using another function. The function Counter reads the state of each IO.

The functions AIBurst and AIStreamRead, take a reading of the IO states and return it with the analog data. The states of the 4 IO are read simultaneously every 4 samples, providing a data rate of up to 2048 Hz per pin for burst mode, or 300 Hz per pin for stream mode. For 1 or 2 channel scans, duplicate data (4x or 2x) will be added to the read array such that the size is numScans.

2.4 D0 – D15

Connections to 16 of the LabJack's 20 digital I/O are made at the DB25 connector, and are referred to as D0-D15. These 16 lines have no overvoltage/short-circuit protection, and can sink or source up to 25 mA each (total sink or source current of 200 mA max for all 16). This allows the D pins to be used to directly control some relays. All digital I/O are CMOS output and TTL input except for D13-D15, which are Schmitt trigger input. Each D pin has a 1 MΩ resistor connected to ground.

All digital I/O are set to input on power-up or reset.

DB25 Pinouts:

1: D0	6: D5	11: +5V	16: GND	21: D11
2: D1	7: D6	12: +5V	17: GND	22: D12
3: D2	8: D7	13: +5V	18: D8	23: D13
4: D3	9: NC	14: GND	19: D9	24: D14
5: D4	10: +5V	15: GND	20: D10	25: D15

These digital I/O can detect the state of a switch using the same circuit shown in Figure 2-5.

Because the D pins have no overvoltage/short-circuit protection, the user must be careful to avoid damage. A series resistor can provide substantial protection for these pins (see the CB25 datasheet). The following are examples of things that could damage a D pin and/or the entire LabJack:

- Shorting a high output to ground (or any potential other than +5V).
- Shorting a low output to a nonzero voltage (such as +5V).
- Exceeding the voltage limits specified in Appendix A.

Software

The easy functions EDigitalIn or EDigitalOut are used to read or set the state of one digital line, and both take up to 20 ms to execute.

The functions AOUpdate and DigitalIO are used to set the direction, set the state, and/or read the state, of each D pin. In addition, DigitalIO also returns the current state of the direction and output registers. Both of these functions take up to 20 ms to execute, providing a maximum update rate of about 50 Hz per pin.

2.5 CNT

The input connection to the 32-bit counter is made at screw-terminal CNT. The counter is incremented when it detects a falling edge followed by a rising edge. This means that if you reset the counter while your signal is low, you will not get the first count until it goes high-low-high. In situations where this first count is important, you should simply substract the initial count from the final count, rather than doing a reset.

Software

The functions ECount (easy function), AOUpdate, and Counter are used to reset or read the counter. If a reset is specified, the counter is read first. All of these functions take up to 20 ms to execute, providing a maximum update rate of about 50 Hz.

Counter readings can also be returned in stream mode (AIStreamRead) at up to 300 Hz.

2.6 CAL – STB

These terminals are used during testing and calibration. CAL is a precision 2.5 volt reference, and can be used during normal operation, but care should be taken to observe the current limits specified in Appendix A. The CAL pin is protected from ESD and overvoltage, but severe overvoltage (steady-state or transient) can damage CAL, and result in the failure of all analog inputs.

2.7 +5V

The LabJack has a nominal +5 volt internal power supply. Power can be drawn from this power supply by connecting to the +5V screw-terminals, or the +5V pins on the DB25 connector. The total amount of current that can be drawn from the +5V pins, analog outputs, and digital outputs, is 450 mA for most desktop computers and self-powered USB hubs. Some notebook computers and bus-powered hubs will limit this available current to about 50 mA.

The USB specification requires all hosts and hubs to have overcurrent protection. If the user puts too large a load on +5V (including a short circuit of +5V to GND) of the LabJack U12 (a USB device), the host or hub is responsible for limiting the current.

2.8 GND

The GND connections available at the screw-terminals and DB25 connector provide a common ground for all LabJack functions. They are all the same.

Caution should be used whenever making connections with systems that have their own power source. It is normal to connect U12 ground to other grounds to create a common reference, but the risk is that the U12 ground will become the preferred ground for the other systems and they could try to send high currents into the U12. To prevent this it is often a good idea to put a 10-100 ohm resistor (or even a fuse) in series with GND on the U12 and any grounds from active systems.

2.9 OEM Versions

The LabJack U12 is also available in 2 OEM (original equipment manufacturer) versions:

- **LJU12-PH:** This is a populated LabJack U12 PCB with pin-headers installed (on the component side of the PCB) instead of screw-terminals. Also, the LED is installed on the component side of the PCB, so nothing is installed on the solder side.
- **LJU12-NTH:** This is a populated LabJack U12 PCB with no through-hole components (DB25 connector, USB connector, LED, screw-terminals). This board is meant for OEMs who solder connections directly to the PCB, or wish to install only certain connectors.

Dimensional drawings are available from the downloads page at labjack.com.

Normally, nothing ships with these OEM LabJacks except for the populated PCB. All software is of course available online at labjack.com.

3. Example Applications

The LabJack U12 CD installs 9 example applications: LJconfig, LJcounter, LJfg, LJlogger, LJscope, LJstream, Ljtest, LJSHT, and LJSHTmulti.

- LJconfig: Lists all LabJacks connected to the USB and allows the local ID to be set on each.
- LJcounter: Reads the LabJack counter and provides the current frequency or count.
- LJfg (Function Generator): Outputs basic waveforms on AO0 (analog output zero).
- LJlogger: Saves data to disk, writes data to an HTML page on the Internet, and performs various actions (including email) on trigger events.
- LJscope: Simulates an oscilloscope by reading data from 2 AI channels in burst mode.
- LJstream: Uses stream mode to read, graph, and write to file, 4 AI channels.
- LJtest: Runs a sequence of tests on the LabJack itself.
- LJSHT: Retrieves and records data from 1 or 2 EI-1050 digital temperature/humidity probes.
- LJSHTmulti: Displays data from up to 20 EI-1050 digital temperature/humidity probes.

The LabVIEW source code for most of these applications is installed in the examples directory.

3.1 LJconfig

Every LabJack has a local ID and serial number. The local ID is a value between 0 and 255 that can be changed by the user. The serial number is a value between 256 and 2,147,483,647 that is unique among all LabJacks and cannot be changed by the user. LJconfig is used to set the local ID of a particular LabJack. When using multiple U12s, each should be assigned a unique local ID.

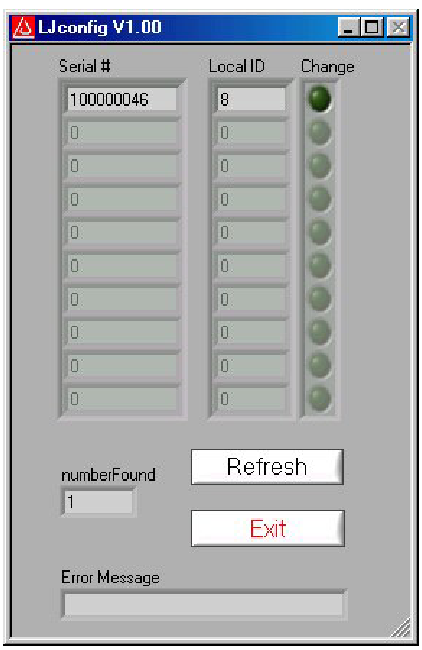


Figure 3-1. LJconfig

Figure 3-1 shows the window that opens when LJconfig is run. Each time the "Refresh" button is pushed, LJconfig will scan the USB for all LabJacks. To change the local ID of a particular LabJack, push the "Change" button next to that LabJack, and the window shown in Figure 3-2 will appear.

Figure 3-2. LJconfig Change Local ID

Enter a new local ID between 0 and 255 and push the "Change" button. The new local ID will be written and the LabJack will be forced to re-enumerate.

3.2 LJcounter

Reads the LabJack counter and provides the current frequency or count.

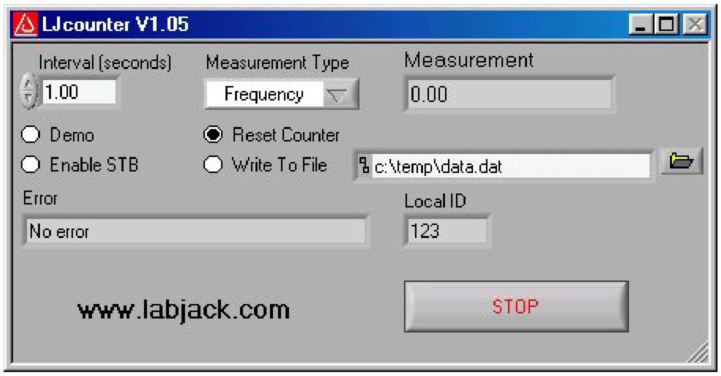


Figure 3-3. LJcounter

Figure 3-3 shows the LJcounter window:

- **Interval (seconds):** Specifies the interval, in seconds, between calls to the DLL function "Counter".
- **Measurement Type:** If set to "Frequency", this application divides the count by the interval to determine frequency in Hertz, and automatically resets the counter every read. If set to "Count", the measurement is simply the current reading from the counter.
- **Measurement:** Displays frequency or count, depending on "Measurement Type".

3.3 LJfg

This application allows the LabJack U12 to be used as a simple function generator. The DLL function "AOUpdate" is called every 25 milliseconds providing an update rate of 40 Hz, and thus a maximum reasonable signal frequency of a few Hertz.

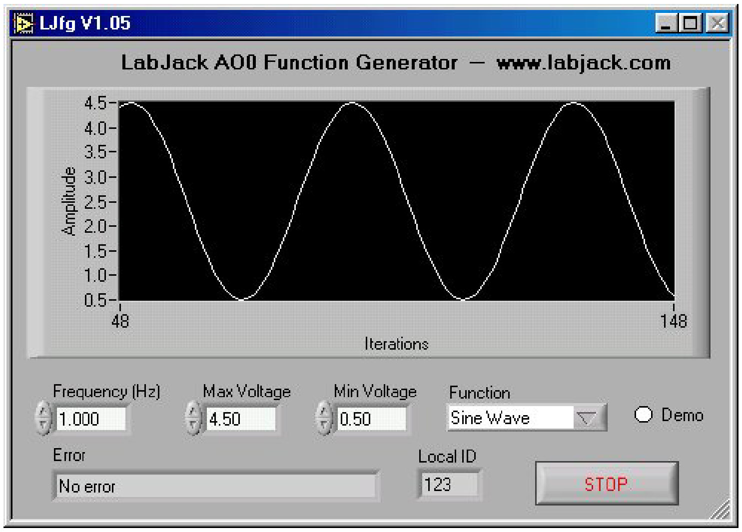


Figure 3-4. LJfg

3.4 LJlogger

LJlogger sends and receives data in command/response mode by making 2 DLL calls to “AISample” and 1 DLL call to “AOUpdate”. It is capable of saving data to disk (2 Hz max) and performing various actions on trigger events.

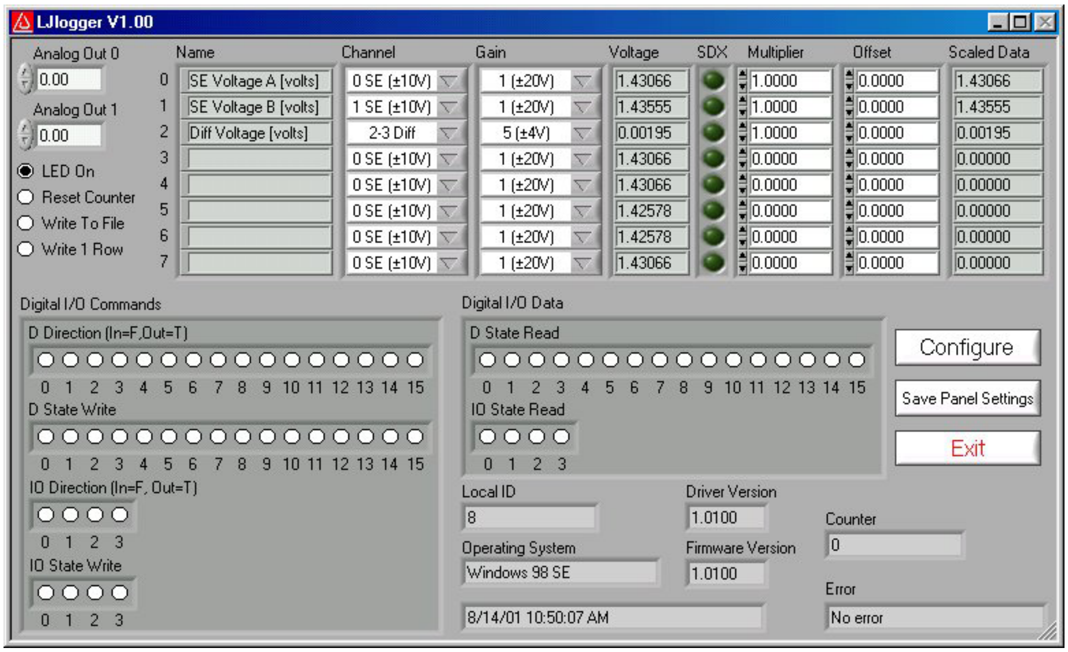


Figure 3-5. LJlogger

The main window for LJlogger is shown in Figure 3-5. The white colored items and the “SDX” buttons are controls to be edited/selected by the user. The grey colored items are indicators which display various information about the LabJack. Clicking the button labeled “Save Panel Settings” will save the current values of the controls as the default values.

If SDX is activated for a given analog input, the corresponding SDX DLL will be used to determine the scaled data. Users can make their own SDX DLLs (see the source code for more information), to provide more complex scaling or scaling that depends on other analog inputs.

Clicking the “Configure” button shown in Figure 3-5 brings up the window shown in Figure 3-6:

- **Working Directory:** This is the directory where data and configuration files will be written.
- **Data File Name:** Determines the name of the data file to which data will be written. New data is appended to the end of this file.
- **Data File Write Interval:** Determines the interval at which a new row of data will be written to the data file. Minimum of 0.5 seconds.
- **HTML Write Interval:** Determines the interval at which the HTML file is rewritten.

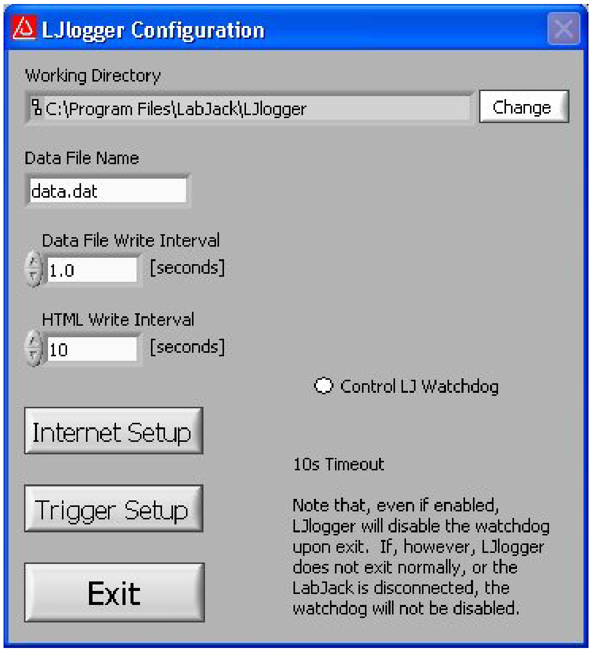


Figure 3-6. LJlogger Configuration

Clicking on the "Internet Setup" button in Figure 3-6 brings up the Internet configuration window shown in Figure 3-7. Basic customization of the HTML file can be done by clicking on "Advanced HTML Configuration" which brings up Figure 3-8.

Clicking on the "Trigger Setup" button in Figure 3-6 brings up the window shown in Figure 3-9.

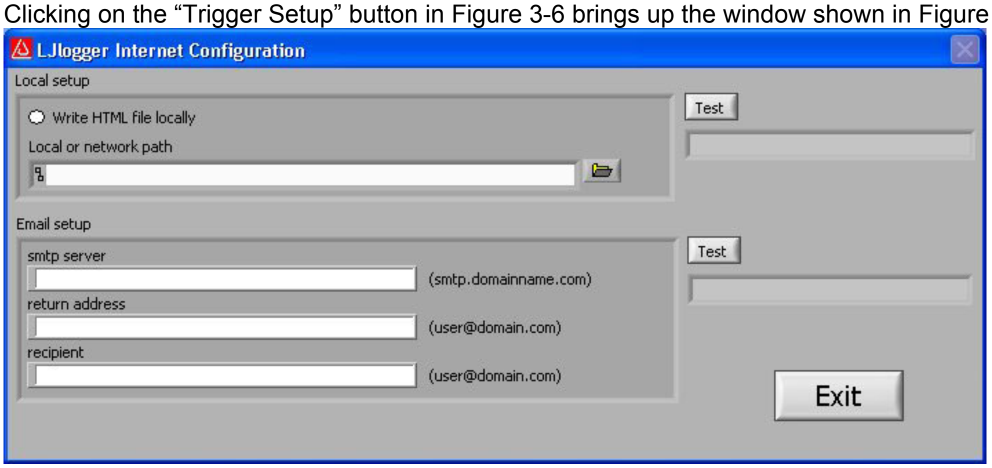


Figure 3-7. LJlogger Internet Configuration

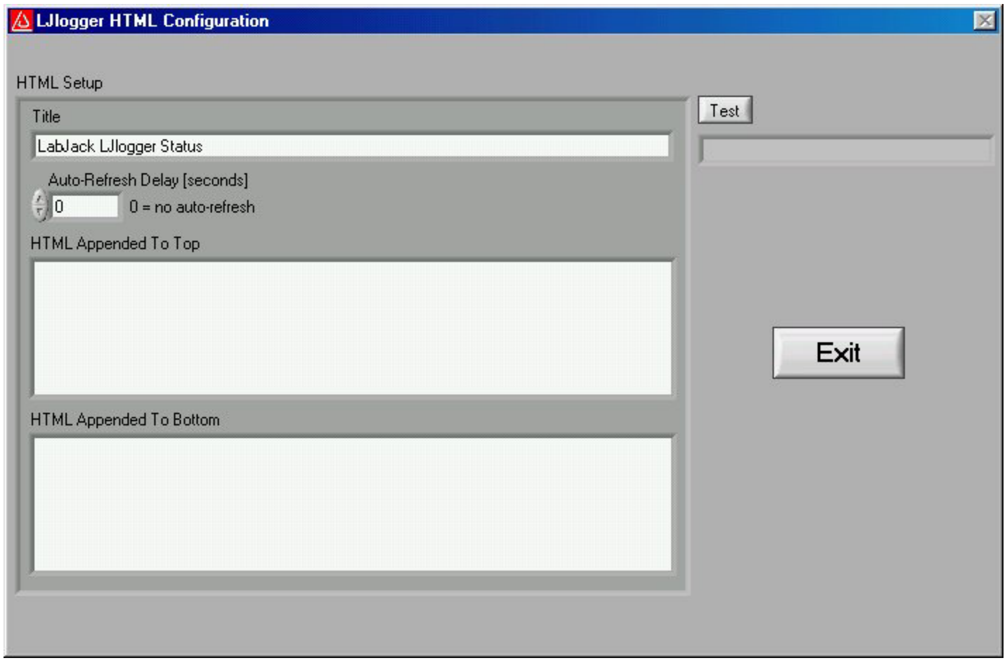


Figure 3-8. LJlogger HTML Configuration

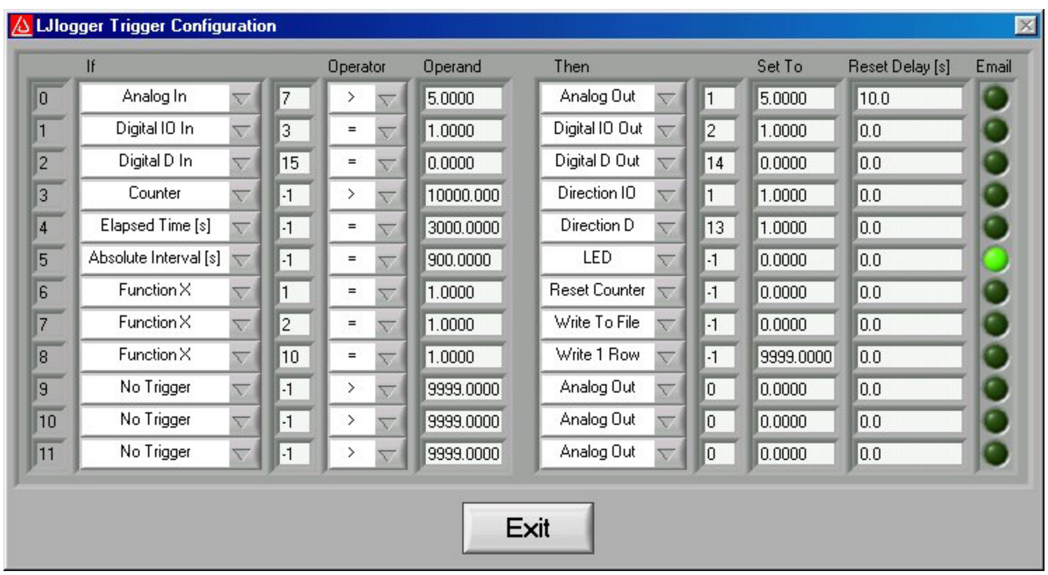


Figure 3-9. LJlogger Trigger Configuration

Figure 3-9 shows 9 example triggers:

- **Trigger #0:** If the scaled data from analog input row 7 (Figure 3-5) is greater than 5, then set AO1 to 5 volts. Once triggered, there is a 10 second delay before it can be triggered again.

16

- **Trigger #1:** If IO3 is high, set IO2 high. Reset delay is zero so this trigger can occur every iteration (every 0.1 seconds) if IO3 is high.
- **Trigger #2:** If D15 is low, set D14 low.
- **Trigger #3:** If the count is greater than 10,000, set IO1 to an output.
- **Trigger #4:** If it has been 3000 seconds since LJlogger started, set D13 to an output.
- **Trigger #5:** When the PC's clock is at 15 minute intervals, the status LED will be turned off and an email will be sent.
- **Trigger #6:** Calls FunctionX from function1.dll. If the function returns True, reset the counter. Users can make their own FunctionX DLLs. See the source code for more information.
- **Trigger #7:** Calls FunctionX from function2.dll. If the function returns True, stop writing data to file.
- **Trigger #8:** Calls FunctionX from function10.dll. If the function returns True, write 1 row to the data file.

3.5 LJscope

LJscope simulates an oscilloscope by reading data from 2 analog input channels in burst mode.

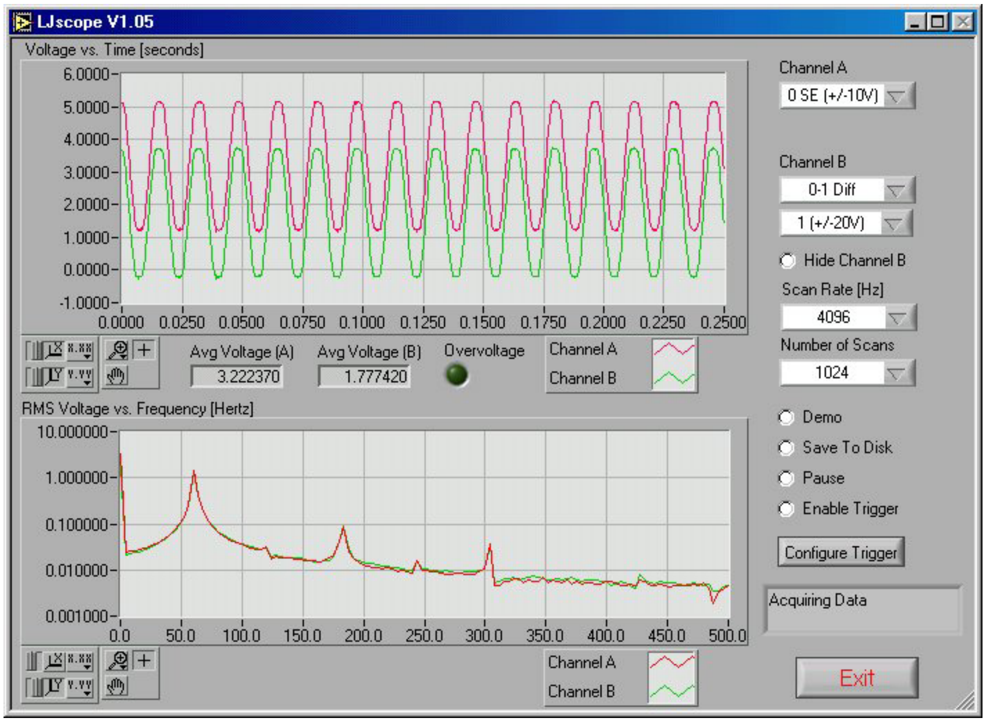
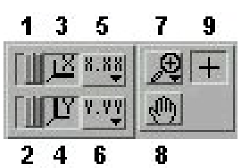


Figure 3-10. LJscope

There are two graphs on the LJscope main window (Figure 3-10), which show voltage versus time and voltage versus frequency. Both graphs have a palette to control various features such as autoscaling and zooming:



1. When you press this button it locks button 3 on (autoscale) position.
2. When you press this button it locks button 4 on (autoscale) position.
3. Pressing this button autoscales the x-axis.
4. Pressing this button autoscales the y-axis.
5. Miscellaneous x-axis formatting options.
6. Miscellaneous y-axis formatting options.
7. Zooming tool. Press this button to see different options for zooming. When collecting data, zooming will not work well unless autoscaling is off.
8. Panning tool. Allows you to drag and scroll around the graph.
9. Not applicable.

Other LJscope controls include:

- **Channel A/B:** Select the two AI channels that will be acquired. If a differential channel, is selected, the gain selection control will appear.
- **Hide Channel B:** When selected channel B will not be shown on the graph.
- **Scan Rate [Hz]:** (256 to 4096) Determines the scans/second for both channels.
- **Number of Scans:** (32 to 2048) Determines the number of scans that will be collected, and thus the total acquisition period. For example, if 1024 scans are collected at 4096 Hz, a quarter second of data will be collected (as shown in Figure 3-10).
- **Demo:** Calls “AIBurst” in demo mode so timing and data is simulated.
- **Save To Disk:** If selected a prompt will appear for a filename, and the current burst of data is saved to a tab-delimited file (time, channel A, channel B).
- **Pause:** Pauses data acquisition.
- **Enable Trigger:** Enables the IO trigger.
- **Configure Trigger:** Brings up the window shown in Figure 3-11. Choose the IO line to trigger on and whether to trigger when it is high or low. Also set the timeout period so the application will continue (with an error) if the trigger is not detected.

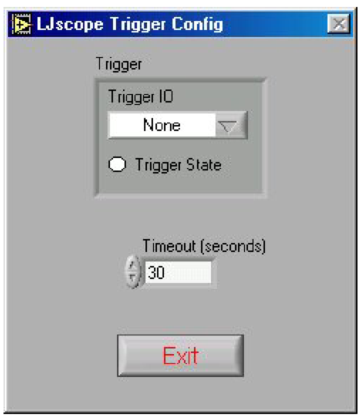


Figure 3-11. LJscope

3.6 LJstream

Uses stream mode to read, graph, and write to file, 4 AI channels. For more information, read about the stream functions (AIStreamStart, AIStreamRead, and AIStreamClear).

- **Enable Stream:** Starts and stops the stream acquisition.
- **Scan Rate:** Determines the scans/second (50 to 300).
- **Number of Scans:** Determines the number of scans that will be collected each iteration, and thus determines how fast this application iterates.
- **Demo:** Calls the “AIStream” functions in demo mode so timing and data is simulated.
- **Read Counter:** Collects 1 analog input and the counter if selected.
- **Configure Channels:** Click this button to bring up the channel configuration window shown in Figure 3-13.
- **Save Current Settings:** Saves all the current settings, including channel configuration.
- **Graph History:** Determines how much past history appears on the graph.

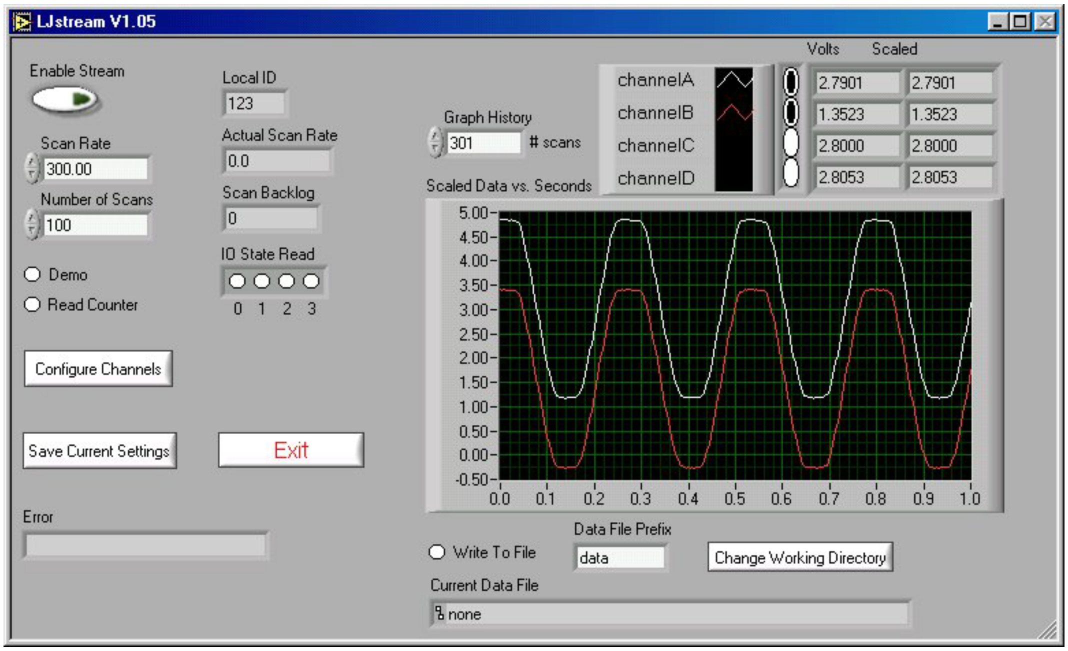


Figure 3-12. LJstream

Figure 3-13 shows the LJstream channel configuration window. Here you can select analog inputs and gains and enter scaling equations. Use "Test Data" to see the effect of the scaling equations ("v" column is the measured voltage and the "y" column is the output of the scaling equations). "Manual/Sampled" determines where the "Test Data" in the "v" column originates.

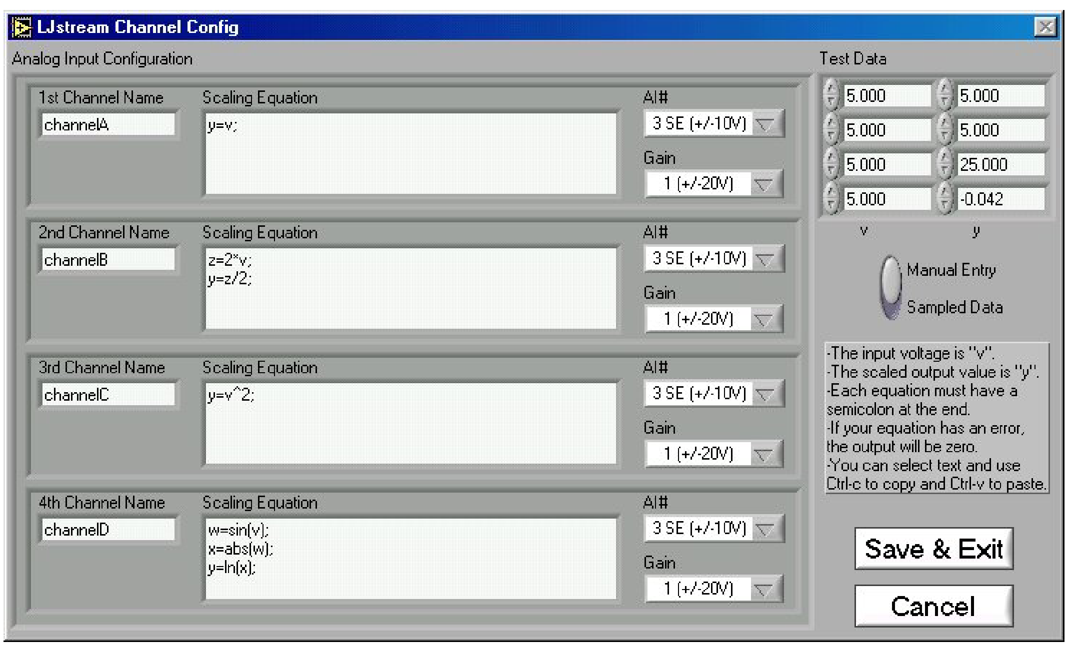


Figure 3-13. LJstream Channel Configuration

3.7 LJtest

LJtest runs a sequence of tests on the LabJack itself. Users will generally leave “Test Fixture Installed” unselected and execute the tests **with nothing connected** to the LabJack (except the USB of course).

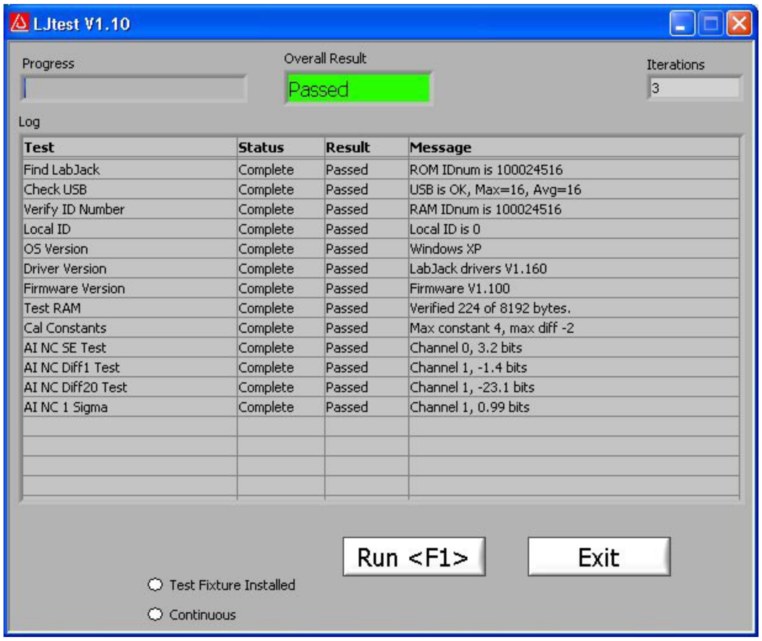


Figure 3-14. LJtest

If all tests fail except “OS Version” and “Driver Version”, it generally indicates LJtest does not detect a LabJack U12 at all. Check for proper blinking of status LED upon power-up, and if using Windows 98 SE check out the related file (Win98sehid.zip) from the downloads page at labjack.com.

If “Find LabJack” is the only failure, it is often because more than 1 LabJack U12 is connected.

“Check USB” performs some basic tests to detect any obvious problems with the Universal Serial Bus. Proper LabJack U12 communication is required for this test.

“Local ID” will show a yellow warning if the Local ID has been changed from the factory default of 0.

Failures from “Test RAM” or any of the “AI …” tests could indicate damage to the unit. Make sure there are no connections to the LabJack U12 (except for the USB cable), and contact LabJack support if the failures continue. Yellow warnings on any of the “AI …” tests (make sure nothing is connected to the AI channels) could indicate that a self-calibration needs to be performed (see below).

A yellow warning from the “Cal Constants” test is usually because the constants have all been set to zero. Most often this is due to selecting “Test Fixture Installed” and running LJtest without the proper connections. Follow the below procedure to correct this issue.

To write new calibration data, a self-calibration should be performed using LJtest and 12 small (1.5" will work) jumper wires:

1) Make the following initial connections:
    - AI0 <=> AI2 <=> AI4 <=> AI6 <=> +5V
    - AI1 <=> AI3 <=> AI5 <=> AI7 <=> +5V
    - IO0 <=> IO1
    - IO2 <=> AO0
    - IO3 <=> AO1
    - CNT <=> STB

2) Start LJtest and select "Test Fixture Installed" and "Prompt During Cal", and then click on the "Run" button.

3) LJtest will step through various tests and then prompt to connect GND to all 8 AI channels (AI6<=>GND and AI7<=>GND), then to connect CAL to the even channels (AI6<=>CAL), then to connect CAL to all 8 AI channels (AI7<=>CAL), and finally to connect GND to the even channels (AI6<=>GND).

4) When finished, remove all wires and unplug the USB cable. Reconnect the USB cable and the new calibration constants will be loaded at power-up. Run LJtest again with “Test Fixture Installed” unselected, to make sure the unit passes the normal self-test.

3.8 LJSHT

Reads and records data from one or two EI-1050 digital temperature/humidity probes.

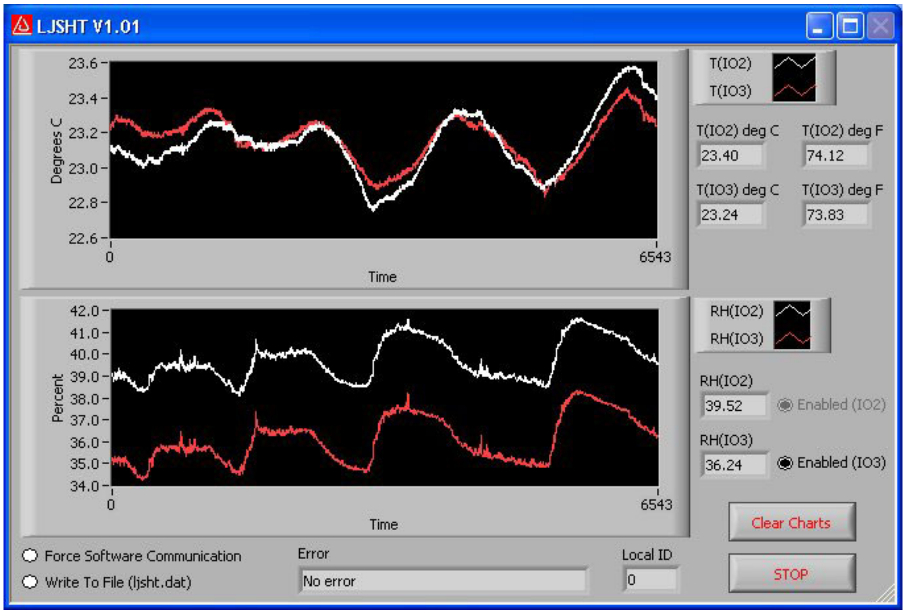


Figure 3-15. LJSHT

Figure 3-15 shows the LJSHT window:

- **Enabled (IO2/IO3):** At least one EI-1050 must be connected. IO2 will be controlled as the enable line for this probe. If two probes are connected, then enable control of IO3.
- **Force Software Communication:** Forces software based SHT1X communication, even if the LabJack U12 firmware is V1.10 or higher.
- **Write To File:** Appends data to a tab-delimited ASCII file called ljsht.dat in the current directory. Data is written as seconds since 1904, followed by tempC/tempF/RH for each probe.

3.9 LJSHTmulti

Displays readings from up to 20 EI-1050 digital temperature/humidity probes connected to a single LabJack U12.

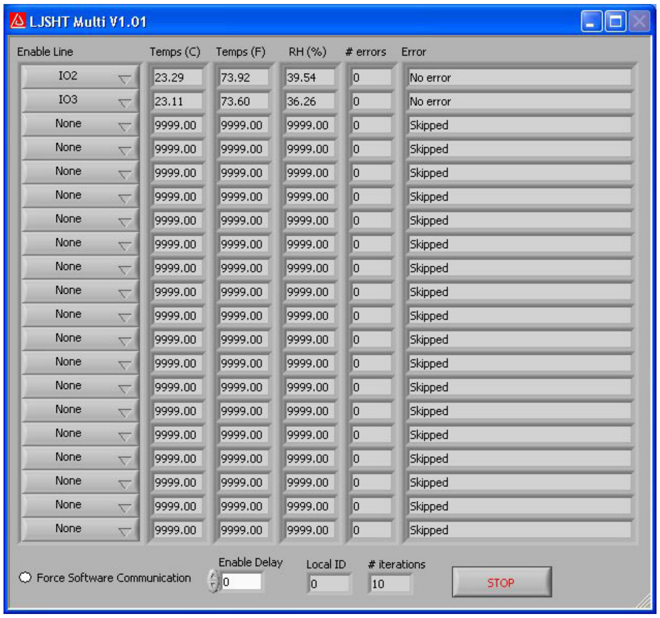


Figure 3-16. LJSHTmulti

Figure 3-16 shows the LJSHTmulti window:

- **Enable Line:** Choose the LabJack U12 output used to control the enable line on each EI-1050 probe.
- **Force Software Communication:** Forces software based SHT1X communication, even if the LabJack U12 firmware is V1.10 or higher.
- **Enable Delay:** Adds a delay between each reading for testing purposes.

4. Programming Reference

The LabJack U12 CD installs high-level drivers (ljackuw.dll), an ActiveX interface to the high-level drivers (ljackuwx.ocx), and LabVIEW6 (or higher) VIs which call all the DLL functions. The DLL and OCX are installed in the Windows System directory. If the installation program can determine the LabVIEW directory, it copies the LabVIEW VIs into that directory (\vi.lib\addons\) so they show up on the function palette. Otherwise, the LabVIEW drivers are copied into the LabJack installation directory (c:\Program Files\LabJack)\drivers\labview, and can manually be transferred to the LabVIEW directory. LabVIEW5 VIs are also installed and can be found in the \LabJack\examples directory.

There are 38 functions exported by the LabJack DLL, and matching functions in the OCX and LabVIEW VIs. There are two additional support functions in the OCX, provided due to the limitations of ActiveX. All functions are command/response except for AIBurst and AIStreamStart/Read/Clear.

There are 2 parameters that are used by most functions:

- **errorcode –** A LabJack specific numeric error code. 0 means no error and 2 means no LabJacks were found. Use the function “GetErrorString” to get a description of the error, or see the list in Section 4.24 of this document.
- **idnum –** Functions with this input take either a local ID, serial number, or -1. A local ID or serial number will specify a specific LabJack, while –1 means the first found LabJack. Every LabJack has a local ID and serial number. The local ID is a value between 0 and 255 that can be changed by the user. The serial number is a value between 256 and 2,147,483,647 that is unique among all LabJacks and cannot be changed by the user. When using multiple U12s, each should be assigned a unique local ID.

To maintain compatibility with as many languages as possible, the every attempt has been made to keep the parameter types very basic. The declarations that follow, are written in C. If there are any differences in the ActiveX version of a function, they are described.

When a parameter name is preceded by a “*”, it means that it is a pointer. In most cases, this means that the parameter is an input and/or output, whereas a non-pointer parameter is input only. In some cases a pointer points to an array of values rather than a single value, and this will be noted in the parameter description.

Some of the digital I/O parameters contain the information for each bit of I/O in one value, where each bit of I/O corresponds to the same bit in the parameter (i.e. the direction of D0 is set in bit 0 of parameter trisD). For instance, in the function DigitalIO, the parameter *trisD is a pointer to a single memory location that sets/reads the direction of each of the 16 D lines:

- if *trisD points to 0, all D lines are input,
- if *trisD points to 1 (2^0), D0 is output, D1-D15 are input,
- if *trisD points to 5 (2^0 + 2^2), D0 and D2 are output, all other D lines are input,
- if *trisD points to 65535 (2^0 + … + 2^15), D0-D15 are output.

The range of the value pointed to by *trisD is 0 to 65535. When calling DigitalIO, if updateDigital is >1, the D lines will be set to input or output based on the value pointed to by *trisD. When DigitalIO returns, the value pointed to by *trisD will have been set to reflect the status of the direction register in the LabJack U12.

4.1 EAnalogIn

Easy function. This is a simplified version of AISample. Reads the voltage from 1 analog input. Calling this function turns/leaves the status LED on.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Declaration:

```
long EAnalogIn (    long *idnum,
                    long demo,
                    long channel,
                    long gain,
                    long *overVoltage,
                    float *voltage )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **channel –** Channel command is 0-7 for single-ended, or 8-11 for differential.
- **gain –** Gain command is 0=1, 1=2, …, 7=20. This amplification is only available for differential channels.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***overVoltage –** If >0, an overvoltage has been detected on one of the selected analog inputs.
- ***voltage –** Returns the voltage reading.

4.2 EAnalogOut

Easy function. This is a simplified version of AOUpdate. Sets the voltage of both analog outputs.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

If either passed voltage is less than zero, the DLL uses the last set voltage. This provides a way to update 1 output without changing the other. Note that when the DLL is first loaded, it does not know if the analog outputs have been set, and assumes they are both the default of 0.0 volts. Similarly, there are situations where the LabJack could reset without the knowledge of the DLL, and thus the DLL could think the analog outputs are set to a non-zero voltage when in fact they have been reinitialized to 0.0 volts.

Declaration:

```
long EAnalogOut (   long *idnum,
                    long demo,
                    float analogOut0,
                    float analogOut1 )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.

- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **analogOut0 –** Voltage from 0.0 to 5.0 for AO0.
- **analogOut1 –** Voltage from 0.0 to 5.0 for AO1.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.3 ECount

Easy function. This is a simplified version of Counter. Reads & resets the counter (CNT). Calling this function disables STB (which is the default anyway).

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Declaration:

```
long ECount(  long *idnum,
              long demo,
              long resetCounter,
              double *count,
              double *ms )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **resetCounter –** If >0, the counter is reset to zero after being read.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***count –** Current count, before reset.
- ***ms –** Value of Window's millisecond timer at the time of the counter read (within a few ms). Note that the millisecond timer rolls over about every 50 days. In general, the millisecond timer starts counting from zero whenever the computer reboots.

4.4 EDigitalIn

Easy function. This is a simplified version of DigitalIO that reads the state of one digital input. Also configures the requested pin to input and leaves it that way.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Note that this is a simplified version of the lower level function DigitalIO, which operates on all 20 digital lines. The DLL (ljackuw) attempts to keep track of the current direction and output state of all lines, so that this easy function can operate on a single line without changing the others. When the DLL is first loaded, though, it does not know the direction and state of the lines and assumes all directions are input and output states are low.

Declaration:

```
long EDigitalIn (    long *idnum,
                     long demo,
                     long channel,
                     long readD,
```

long *state)

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **channel –** Line to read. 0-3 for IO or 0-15 for D.
- **readD –** If >0, a D line is read as opposed to an IO line.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***state –** The selected line is TRUE/Set if >0. FALSE/Clear if 0.

4.5 EDigitalOut

Easy function. This is a simplified version of DigitalIO that sets/clears the state of one digital output. Also configures the requested pin to output and leaves it that way.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Note that this is a simplified version of the lower level function DigitalIO, which operates on all 20 digital lines. The DLL (ljackuw) attempts to keep track of the current direction and output state of all lines, so that this easy function can operate on a single line without changing the others. When the DLL is first loaded, though, it does not know the direction and state of the lines and assumes all directions are input and output states are low.

Declaration:
```
long EDigitalOut (    long *idnum,
                      long demo,
                      long channel,
                      long writeD,
                      long state )
```

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **channel –** Line to read. 0-3 for IO or 0-15 for D.
- **writeD –** If >0, a D line is written as opposed to an IO line.
- **state –** If >0, the line is set, otherwise the line is cleared.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.6 AISample

Reads the voltages from 1,2, or 4 analog inputs. Also controls/reads the 4 IO ports.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Declaration:
```
long AISample (    long *idnum,
```

```
long demo,
long *stateIO,
long updateIO,
long ledOn,
long numChannels,
long *channels,
long *gains,
long disableCal,
long *overVoltage,
float *voltages )
```

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- ***stateIO –** Output states for IO0-IO3. Has no effect if IO are configured as inputs, so a different function must be used to configure as output.
- **updateIO –** If >0, state values will be written. Otherwise, just a read is performed.
- **ledOn –** If >0, the LabJack LED is turned on.
- **numChannels –** Number of analog input channels to read (1,2, or 4).
- ***channels –** Pointer to an array of channel commands with at least numChannels elements. Each channel command is 0-7 for single-ended, or 8-11 for differential.
- ***gains –** Pointer to an array of gain commands with at least numChannels elements. Gain commands are 0=1, 1=2, …, 7=20. This amplification is only available for differential channels.
- **disableCal –** If >0, voltages returned will be raw readings that are not corrected using calibration constants.
- ***voltages –** Pointer to an array where voltage readings are returned. Send a 4-element array of zeros.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***stateIO –** Returns input states of IO0-IO3.
- ***overVoltage –** If >0, an overvoltage has been detected on one of the selected analog inputs.
- ***voltages –** Pointer to an array where numChannels voltage readings are returned.

ActiveX Function Differences:
The “channels” and “gains” arrays are replaced with “channelsPacked” and “gainsPacked”. The OCX has a function “FourPack” which will convert 4 elements to a packed value. The packed value is determined as: element[0] + (element[1] * 2^8) + (element[2] * 2^16) + (element[3] * 2^24).

The “voltages” array is replaced with 4 individual parameters.

Declaration (ActiveX):

```
long AISampleX (    long FAR* idnum,
                    long demo,
                    long FAR* stateIO,
```

```
long updateIO,
long ledOn,
long numChannels,
long channelsPacked,
long gainsPacked,
long disableCal,
long FAR* overVoltage,
float FAR* voltageA,
float FAR* voltageB,
float FAR* voltageC,
float FAR* voltageD )
```

4.7 AIBurst

Reads a specified number of scans (up to 4096) at a specified scan rate (up to 8192 Hz) from 1,2, or 4 analog inputs. First, data is acquired and stored in the LabJack's 4096 sample RAM buffer. Then, the data is transferred to the PC.

If the LED is enabled (ledOn>0), it will blink at about 4 Hz while waiting for a trigger, turn off during acquisition, blink rapidly while transferring data to the PC, and turn on when done.

The execution time of this function, in milliseconds, depends on transfermode and can be estimated with the below formulas. The actual number of samples collected and transferred by the LabJack is the smallest power of 2 from 64 to 4096 which is at least as big as numScans*numChannels. This is represented below as numSamplesActual.

Normal => 30+(1000*numSamplesActual/sampleRate)+(2.5*numSamplesActual)

Turbo => 30+(1000*numSamplesActual/sampleRate)+(0.4*numSamplesActual)

Declaration:

```
long AIBurst ( long *idnum,
               long demo,
               long stateIOin,
               long updateIO,
               long ledOn,
               long numChannels,
               long *channels,
               long *gains,
               float *scanRate,
               long disableCal,
               long triggerIO,
               long triggerState,
               long numScans,
               long timeout,
               float (*voltages)[4],
               long *stateIOout,
               long *overVoltage,
               long transferMode )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.

- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **stateIOin –** Output states for IO0-IO3. Has no effect if IO are configured as inputs, so a different function must be used to configure as output.
- **updateIO –** If >0, state values will be written. Otherwise, just a read is performed.
- **ledOn –** If >0, the LabJack LED is turned on.
- **numChannels –** Number of analog input channels to read (1,2, or 4).
- ***channels –** Pointer to an array of channel commands with at least numChannels elements. Each channel command is 0-7 for single-ended, or 8-11 for differential.
- ***gains –** Pointer to an array of gain commands with at least numChannels elements. Gain commands are 0=1, 1=2, …, 7=20. This amplification is only available for differential channels.
- ***scanRate –** Scans acquired per second. A scan is a reading from every channel (1,2, or 4). The sample rate (scanRate * numChannels) must be between 400 and 8192.
- **disableCal –** If >0, voltages returned will be raw readings that are not corrected using calibration constants.
- **triggerIO –** The IO port to trigger on (0=none, 1=IO0, or 2=IO1).
- **triggerState –** If >0, the acquisition will be triggered when the selected IO port reads high.
- **numScans –** Number of scans which will be returned. Minimum is 1. Maximum numSamples is 4096, where numSamples is numScans * numChannels.
- **timeout –** This function will return immediately with a timeout error if it does not receive a scan within this number of seconds. Timeouts of 3 seconds or less are generally recommended to keep the U12 in turbo transfer mode.
- ***voltages –** Pointer to a 4096 by 4 array where voltage readings are returned. Send filled with zeros.
- ***stateIOout –** Pointer to a 4096 element array where IO states are returned. Send filled with zeros.
- **transferMode –** 0=auto,1=normal, 2=turbo. If auto, turbo mode is used unless timeout is >= 4, or numScans/scanRate >=4.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***scanRate –** Returns the actual scan rate, which due to clock resolution is not always exactly the same as the desired scan rate.
- ***voltages –** Pointer to a 4096 by 4 array where voltage readings are returned. Unused locations are filled with 9999.0.
- ***stateIOout –** Pointer to a 4096 element array where IO states are returned. Unused locations are filled with 9999.0.
- ***overVoltage –** If >0, an overvoltage has been detected on at least one sample of one of the selected analog inputs.

<u>ActiveX Function Differences:</u>
The “channels” and “gains” arrays are replaced with “channelsPacked” and “gainsPacked”. The OCX has a function “FourPack” (4.39) which will convert 4 elements to a packed value. The packed value is determined as: element[0] + (element[1] * 2^8) + (element[2] * 2^16) + (element[3] * 2^24).

The parameters “demo”, “ledOn”, “disableCal”, “transferMode”, “updateIO”, and “stateIOin”, are replaced by an “optionBits” parameter. Call the OCX function “BuildOptionBits” (4.38) to determine this parameter.

The “voltages” and “stateIOout” arrays are represented as strings. Floating point data is returned as 13 characters per number (XXXX.XXXXXXXX) and integers are returned as 10 characters per number (XXXXXXXXXX). Zeros are used for padding where necessary. The total number of bytes in the “voltages” string is 13*numSamples. The total number of bytes in the “stateIOout” string is 10*numScans. Note that to avoid a memory leak, these strings should be emptied (set to “”) after each call to AIBurstX.

Declaration (ActiveX):

```
long AIBurstX (      long FAR* idnum,
                     long numChannels,
                     long channelsPacked,
                     long gainsPacked,
                     float FAR* scanRate,
                     long triggerIO,
                     long triggerState,
                     long numScans,
                     long timeout,
                     BSTR FAR* voltages,
                     BSTR FAR* stateIOout,
                     long FAR* overVoltage,
                     long optionBits)
```

4.8 AIStreamStart

Starts a hardware timed continuous acquisition where data is sampled and stored in the LabJack RAM buffer, and can be simultaneously transferred out of the RAM buffer to the PC application. A call to this function should be followed by periodic calls to AIStreamRead, and eventually a call to AIStreamClear. Note that while streaming the LabJack U12 is too busy to do anything else. If any function besides AIStreamRead is called while a stream is in progress, the stream will be stopped.

Execution time for this function is 30 milliseconds or less (typically 24 milliseconds in Windows).

If the LED is enabled (ledOn>0), it will toggle every 40 samples during acquisition and turn on when the stream operation stops.

Declaration:

```
long AIStreamStart ( long *idnum,
                     long demo,
                     long stateIOin,
                     long updateIO,
                     long ledOn,
                     long numChannels,
                     long *channels,
                     long *gains,
                     float *scanRate,
                     long disableCal,
                     long reserved1,
                     long readCount )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **stateIOin –** Output states for IO0-IO3.
- **updateIO –** If >0, state values will be written. Otherwise, just a read is performed.
- **ledOn –** If >0, the LabJack LED is turned on.
- **numChannels –** Number of analog input channels to read (1,2, or 4). If readCount is >0, numChannels should be 4.
- ***channels –** Pointer to an array of channel commands with at least numChannels elements. Each channel command is 0-7 for single-ended, or 8-11 for differential.
- ***gains –** Pointer to an array of gain commands with at least numChannels elements. Gain commands are 0=1, 1=2, …, 7=20. This amplification is only available for differential channels.
- ***scanRate –** Scans acquired per second. A scan is a reading from every channel (1,2, or 4). The sample rate (scanRate * numChannels) must be between 200 and 1200.
- **disableCal –** If >0, voltages returned will be raw readings that are not corrected using calibration constants.
- **reserved1 –** Reserved for future use. Send 0.
- **readCount –** If >0, the current count (CNT) is returned instead of the 2nd, 3rd, and 4th analog input channels. 2nd channel is bits 0-11. 3rd channel is bits 12-23. 4th channel is bits 24-31. This feature was added to the LabJack U12 starting with firmware version 1.03, and this input has no effect with earlier firmware versions.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***scanRate –** Returns the actual scan rate, which due to clock resolution is not always exactly the same as the desired scan rate.

ActiveX Function Differences:

The “channels” and “gains” arrays are replaced with “channelsPacked” and “gainsPacked”. The OCX has a function “FourPack” (4.39) which will convert 4 elements to a packed value. The packed value is determined as: element[0] + (element[1] * 2^8) + (element[2] * 2^16) + (element[3] * 2^24).

The parameters “demo”, “ledOn”, “disableCal”, “updateIO”, and “stateIOin”, are replaced by an “optionBits” parameter. Call the OCX function “BuildOptionBits” (4.38) to determine this parameter.

Declaration (ActiveX):

```
long AIStreamStartX (    long FAR* idnum,
                         long numChannels,
                         long channelsPacked,
                         long gainsPacked,
                         float FAR* scanRate,
                         long optionBits,
                         long readCount)
```

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4.9 AIStreamRead

Waits for a specified number of scans to be available and reads them. AIStreamStart should be called before this function and AIStreamClear should be called when finished with the stream. Note that while streaming the LabJack U12 is too busy to do anything else. If any function besides AIStreamRead is called while a stream is in progress, the stream will be stopped.

Note that you must pass the actual local ID to this function, not the idnum parameter used for most functions. Usually you simply pass the value returned by the idnum parameter in AIStreamStart.

Declaration:

```
long AIStreamRead ( long localID,
                    long numScans,
                    long timeout,
                    float (*voltages)[4],
                    long *stateIOout,
                    long *reserved,
                    long *ljScanBacklog,
                    long *overVoltage )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- **localID –** Send the local ID from AIStreamStart.
- **numScans –** Function will wait until this number of scans is available. Minimum is 1. Maximum numSamples is 4096, where numSamples is numScans * numChannels. Internally this function gets data from the LabJack in blocks of 64 samples, so it is recommended that numSamples be at least 64.
- **timeout –** Function timeout value in seconds. 1 is usually a good value.
- ***voltages –** Pointer to a 4096 by 4 array where voltage readings are returned. Send filled with zeros.
- ***stateIOout –** Pointer to a 4096 element array where IO states are returned. Send filled with zeros.

Outputs:

- ***voltages –** Pointer to a 4096 by 4 array where voltage readings are returned. Unused locations are filled with 9999.0.
- ***stateIOout –** Pointer to a 4096 element array where IO states are returned. Unused locations are filled with 9999.0.
- ***reserved –** Reserved for future use. Send a pointer to a 0.
- ***ljScanBacklog –** Returns the scan backlog of the LabJack RAM buffer. This is the number of scans remaining in the U12 buffer after this read. If this value is growing from read to read, data is not being read fast enough and the buffer will eventually overflow. In normal operation this will return 0 almost all the time. The size of the buffer in terms of scans is 4096/numChannels.
- ***overVoltage –** If >0, an overvoltage has been detected on at least one sample of one of the selected analog inputs.

ActiveX Function Differences:

The “voltages” and “stateIOout” arrays are represented as strings. Floating point data is returned as 13 characters per number (XXXX.XXXXXXXX) and integers are returned as 10 characters per number (XXXXXXXXXX). Zeros are used for padding where necessary. . The total number of bytes in the “voltages” string is 13*numSamples. The total number of bytes in the "stateIOout" string is 10*numScans. Note that to avoid a memory leak, these strings should be emptied (set to "") after each call to AIStreamReadX.

Declaration (ActiveX):

```
long AIStreamReadX (     long localID,
                         long numScans,
                         long timeout,
                         BSTR FAR* voltages,
                         BSTR FAR* stateIOout,
                         long FAR* ljScanBacklog,
                         long FAR* overVoltage)
```

4.10 AIStreamClear

This function stops the continuous acquisition. It should be called once when finished with the stream. The sequence of calls for a typical stream operation is: AIStreamStart, AIStreamRead, AIStreamRead, AIStreamRead, …, AIStreamClear.

Note that you must pass the actual localID to this function, not the idnum parameter used for most functions. Usually you simply pass the value returned by the idnum parameter in AIStreamStart.

Declaration:

```
long AIStreamClear (   long localID )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Input:

- **localID –** Send the local ID from AIStreamStart/Read.

4.11 AOUpdate

Sets the voltages of the analog outputs. Also controls/reads all 20 digital I/O and the counter.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

If either passed voltage is less than zero, the DLL uses the last set voltage. This provides a way to update 1 output without changing the other. Note that when the DLL is first loaded, it does not know if the analog outputs have been set, and assumes they are both the default of 0.0 volts. Similarly, there are situations where the LabJack could reset without the knowledge of the DLL, and thus the DLL could think the analog outputs are set to a non-zero voltage when in fact they have been reinitialized to 0.0 volts.

Declaration:

```
long AOUpdate (     long *idnum,
                    long demo,
                    long trisD,
                    long trisIO,
                    long *stateD,
                    long *stateIO,
                    long updateDigital,
                    long resetCounter,
                    unsigned long *count,
                    float analogOut0,
                    float analogOut1)
```

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **trisD –** Directions for D0-D15. 0=Input, 1=Output.
- **trisIO –** Directions for IO0-IO3. 0=Input, 1=Output.
- ***stateD –** Output states for D0-D15.
- ***stateIO –** Output states for IO0-IO3.
- **updateDigital –** If >0, tris and state values will be written. Otherwise, just a read is performed.
- **resetCounter –** If >0, the counter is reset to zero after being read.
- **analogOut0 –** Voltage from 0.0 to 5.0 for AO0.
- **analogOut1 –** Voltage from 0.0 to 5.0 for AO1.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***stateD –** States of D0-D15.
- ***stateIO –** States of IO0-IO3.
- ***count –** Current value of the 32-bit counter (CNT). This value is read before the counter is reset.

ActiveX Function Differences:
The counter read is returned as a double precision float, instead of an unsigned long.

Declaration (ActiveX):

```
long AOUpdateX (    long FAR* idnum,
                    long demo,
                    long trisD,
                    long trisIO,
                    long FAR* stateD,
                    long FAR* stateIO,
                    long updateDigital,
                    long resetCounter,
                    double FAR* count,
                    float analogOut0,
                    float analogOut1)
```

4.12 AsynchConfig

Requires firmware V1.1 or higher. This function writes to the asynch registers and sets the direction of the D lines (input/output) as needed.

Execution time for this function is 60 milliseconds or less (typically 48 milliseconds in Windows).

The actual 1-bit time is about 1.833 plus a "full" delay (us).

The actual 1/2-bit time is about 1.0 plus a "half" delay (us).

full/half delay = 0.833 + 0.833C + 0.667BC + 0.5ABC (us)

Common baud rates (full A,B,C; half A,B,C):

1	55,153,232 ; 114,255,34
10	63,111,28 ; 34,123,23
100	51,191,2 ; 33,97,3
300	71,23,4 ; 84,39,1
600	183,3,6 ; 236,7,1
1000	33,29,2 ; 123,8,1
1200	23,17,4 ; 14,54,1
2400	21,37,1 ; 44,3,3
4800	10,18,2 ; 1,87,1
7200	134,2,1 ; 6,9,2
9600	200,1,1 ; 48,2,1
10000	63,3,1 ; 46,2,1
19200	96,1,1 ; 22,2,1
38400	3,5,2 ; 9,2,1
57600	3,3,2 ; 11,1,1
100000	3,3,1 ; 1,2,1
115200	9,1,1 ; 2,1,1 or 1,1,1

When using data rates over 38.4 kbps, the following conditions need to be considered:

- When reading the first byte, the start bit is first tested about 11.5 us after the start of the tx stop bit.
- When reading bytes after the first, the start bit is first tested about "full" + 11 us after the previous bit 8 read, which occurs near the middle of bit 8.

When enabled, STB does the following to aid in debugging asynchronous reads:

- STB is set about 6 us after the start of the last tx stop bit, or about "full" + 6 us after the previous bit 8 read.
- STB is cleared about 0-2 us after the rx start bit is detected.
- STB is set after about "half".
- STB is cleared after about "full".
- Bit 0 is read about 1 us later.
- STB is set about 1 us after the bit 0 read.
- STB is cleared after about "full".
- Bit 1 is read about 1 us later.
- STB is set about 1 us after the bit 1 read.
- STB is cleared after about "full".
- This continues for all 8 data bits and the stop bit, after which STB remains low.

Declaration:

```
long AsynchConfig(  long *idnum,
                    long demo,
                    long timeoutMult,
                    long configA,
                    long configB,
                    long configTE,
                    long fullA,
                    long fullB,
                    long fullC,
                    long halfA,
                    long halfB,
                    long halfC)
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **timeoutMult –** If enabled, read timeout is about 100 milliseconds times this value (0-255).
- **configA –** If >0, D8 is set to output-high and D9 is set to input.
- **configB –** If >0, D10 is set to output-high and D11 is set to input.
- **configTE –** If >0, D12 is set to output-low.
- **fullA/B/C –** Time constants for a "full" delay (1-255).
- **halfA/B/C –** Time constants for a "half" delay (1-255).

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.13 Asynch

Requires firmware V1.1 or higher. This function writes and then reads half-duplex asynchronous data on 1 of two pairs of D lines (8,n,1). Call AsynchConfig to set the baud rate. Similar to RS232, except that logic is normal CMOS/TTL (0=low=GND, 1=high=+5V, idle state of transmit line is high). Connection to a normal RS232 device will probably require a converter chip such as the MAX233.

Execution time for this function is about 20 milliseconds to write and/or read up to 4 bytes, plus about 20 milliseconds for each additional 4 bytes written or read. Slow baud rates can result in longer execution time.

PortA => TX is D8 and RX is D9
PortB => TX is D10 and RX is D11
Transmit Enable is D12

Up to 18 bytes can be written and read. If more than 4 bytes are written or read, this function uses calls to WriteMem/ReadMem to load/read the LabJack's data buffer.

Declaration:

```
long Asynch(  long *idnum,
              long demo,
              long portB,
              long enableTE,
              long enableTO,
              long enableDel,
              long baudrate,
              long numWrite,
              long numRead,
              long *data)
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **portB –** If >0, asynch PortB is used instead of PortA.

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- **enableTE –** If >0, D12 (Transmit Enable) is set high during transmit and low during receive.
- **enableTO –** If >0, timeout is enabled for the receive phase (per byte).
- **enableDel –** If >0, a 1 bit delay is inserted between each transmit byte.
- **baudrate –** This is the bps as set by AsynchConfig. Asynch needs this so it has an idea how long the transfer should take.
- **numWrite –** Number of bytes to write (0-18).
- **numRead –** Number of bytes to read (0-18).
- ***data –** Serial data buffer. Send an 18 element array. Fill unused locations with zeros.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***data –** Serial data buffer. Returns any serial read data. Unused locations are filled with 9999s.

ActiveX Function Differences:
The maximum number of bytes to read and/or write is limited to 6 (numWrite and numRead should be 0-6). The data array is replaced with pointers to 6 individual data bytes.

Declaration (ActiveX):

```
long AsynchX(long FAR* idnum,
             long demo,
             long portB,
             long enableTE,
             long enableTO,
             long enableDel,
             long baudrate,
             long numWrite,
             long numRead,
             long FAR* data0,
             long FAR* data1,
             long FAR* data2,
             long FAR* data3,
             long FAR* data4,
             long FAR* data5)
```

4.14 BitsToVolts

Converts a 12-bit (0-4095) binary value into a LabJack voltage. No hardware communication is involved.

Volts=((2*Bits*Vmax/4096)-Vmax)/Gain where Vmax=10 for SE, 20 for Diff.

Declaration:

```
long BitsToVolts (   long chnum,
                     long chgain,
                     long bits,
                     float *volts )
```

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- **chnum –** Channel index. 0-7=SE, 8-11=Diff.

- **chgain –** Gain index. 0=1, 1=2, …, 7=20.
- **bits –** Binary value from 0-4095.

Outputs:

- ***volts –** Voltage.

4.15 VoltsToBits

Converts a voltage to it's 12-bit (0-4095) binary representation. No hardware communication is involved.

Bits=(4096*((Volts*Gain)+Vmax))/(2*Vmax) where Vmax=10 for SE, 20 for Diff.

Declaration:

```
long VoltsToBits (   long chnum,
                     long chgain,
                     float volts,
                     long *bits )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- **chnum –** Channel index. 0-7=SE, 8-11=Diff.
- **chgain –** Gain index. 0=1, 1=2, …, 7=20.
- **volts –** Voltage.

Outputs:

- ***bits –** Binary value from 0-4095.

4.16 Counter

Controls and reads the counter. The counter is disabled if the watchdog timer is enabled.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Declaration:

```
long Counter ( long *idnum,
               long demo,
               long *stateD,
               long *stateIO,
               long resetCounter,
               long enableSTB,
               unsigned long *count )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **resetCounter –** If >0, the counter is reset to zero after being read.
- **enableSTB –** If >0, STB is enabled. Used for testing and calibration. (This input has no effect with firmware V1.02 or earlier, in which case STB is always enabled)

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

- ***stateD –** States of D0-D15.
- ***stateIO –** States of IO0-IO3.
- ***count –** Current value of the 32-bit counter (CNT). This value is read before the counter is reset.

<u>ActiveX Function Differences:</u>
The counter read is returned as a double precision float, instead of an unsigned long.

<u>Declaration (ActiveX):</u>

```
long CounterX(long FAR* idnum,
              long demo,
              long FAR* stateD,
              long FAR* stateIO,
              long resetCounter,
              long enableSTB,
              double FAR* count)
```

4.17 DigitalIO

Reads and writes to all 20 digital I/O. The order of execution within the U12 is:

-Set D states
-Set D directions
-Set IO states
-Set IO directions
-Read D states
-Read IO states

Even more detail of the execution order with the approximate time between each step:

-Set D7-D0 states
-1 us
-Set D12-D8 states
-2 us
-Set D15-D13 states
-0.5 us
-Set D7-D0 directions
-1 us
-Set D12-D8 directions
-1 us
-Set D15-D13 directions
-16 us
-Set IO states
-16 us
-Set IO directions
-2 us
-Read D7-D0 states
-0.3 us
-Read D12-D8 states
-0.7 us
-Read D15-D13 states
-10 us
-Read IO states

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Declaration:

```
long DigitalIO (long *idnum,
                long demo,
                long *trisD,
                long trisIO,
                long *stateD,
                long *stateIO,
                long updateDigital,
                long *outputD )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- ***trisD –** Directions for D0-D15. 0=Input, 1=Output.
- **trisIO –** Directions for IO0-IO3. 0=Input, 1=Output.
- ***stateD –** Output states for D0-D15.
- ***stateIO –** Output states for IO0-IO3.
- **updateDigital –** If >0, tris and state values will be written. Otherwise, just a read is performed.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***trisD –** Returns a read of the direction registers for D0-D15.
- ***stateD –** States of D0-D15.
- ***stateIO –** States of IO0-IO3.
- ***outputD –** Returns a read of the output registers for D0-D15.

4.18 GetDriverVersion

Returns the version number of ljackuw.dll. No hardware communication is involved.

Declaration:

float GetDriverVersion (void)

Parameter Description:

Returns: Version number of ljackuw.dll.

ActiveX Function Differences:

Uses parameters to return DLL and OCX version.

Declaration (ActiveX):

```
void GetDriverVersionX (    float FAR* dllVersion,
                            float FAR* ocxVersion)
```

4.19 GetErrorString

Converts a LabJack errorcode, returned by another function, into a string describing the error. No hardware communication is involved.

Declaration:
void GetErrorString (long errorcode,
char *errorString)

Parameter Description:
Returns: Nothing.
Inputs:

- **errorcode –** LabJack errorcode.
- ***errorString –** Pointer to a 50 element array of characters.

Outputs:

- ***errorString –** Pointer to a sequence of characters describing the error. Unused locations are filled with 0x00.

4.20 GetFirmwareVersion

Retrieves the firmware version from the LabJack's processor.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Declaration:
float GetFirmwareVersion (long *idnum)

Parameter Description:
Returns: Version number of the LabJack firmware or 0 for error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found. If error, returns 512 plus a normal LabJack errorcode.

4.21 GetWinVersion

Uses a Windows API function to get the OS version.

Declaration:
long GetWinVersion (unsigned long *majorVersion,
unsigned long *minorVersion,
unsigned long *buildNumber,
unsigned long *platformID,
unsigned long *servicePackMajor,
unsigned long *servicePackMinor)

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.

Outputs:

	Platform	Major	Minor	Build
Windows 3.1	0	-	-	-
Windows 95	1	4	0	950
Windows 95 OSR2	1	4	0	1111
Windows 98	1	4	10	1998
Windows 98SE	1	4	10	2222
Windows Me	1	4	90	3000
Windows NT 3.51	2	3	51	-

Windows NT 4.0	2	4	0	1381
Windows 2000	2	5	0	2195
Windows XP	2	5	1	-

ActiveX Function Differences:
All unsigned long parameters are changed to double precision float.

4.22 ListAll

Searches the USB for all LabJacks, and returns the serial number and local ID for each.

Declaration:
```
long ListAll (  long *productIDList,
                long *serialnumList,
                long *localIDList,
                long *powerList,
                long (*calMatrix)[20],
                long *numberFound,
                long *reserved1,
                long *reserved2 )
```

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***productIDList –** Pointer to a 127 element array. Send filled with zeros.
- ***serialnumList –** Pointer to a 127 element array. Send filled with zeros.
- ***localIDList –** Pointer to a 127 element array. Send filled with zeros.
- ***powerList –** Pointer to a 127 element array. Send filled with zeros.
- ***calMatrix –** Pointer to a 127 by 20 element array. Send filled with zeros.

Outputs:

- ***serialnumList –** Pointer to a 127 element array where serial numbers are returned. Unused locations are filled with 9999.0.
- ***localIDList –** Pointer to a 127 element array where local ID numbers are returned. Unused locations are filled with 9999.0.
- ***numberFound –** Number of LabJacks found on the USB.

ActiveX Function Differences:
The arrays are represented as strings with 10 characters per number (XXXXXXXXXX). Zeros are used for padding where necessary.

Declaration (ActiveX):
```
long ListAllX (      BSTR FAR* productIDList,
                     BSTR FAR* serialnumList,
                     BSTR FAR* localIDList,
                     BSTR FAR* powerList,
                     BSTR FAR* calMatrix,
                     long FAR* numberFound,
                     long FAR* reserved1,
                     long FAR* reserved2)
```

4.23 LocalID

Changes the local ID of a specified LabJack. Changes will not take effect until the LabJack is re-enumerated or reset, either manually by disconnecting and reconnecting the USB cable or by calling ReEnum or Reset.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Declaration:
long LocalID (long *idnum,
long localID)

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **localID –** New local ID.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.24 NoThread

This function is needed when interfacing TestPoint to the LabJack DLL on Windows 98/ME (see ljackuw.h for more information). Call this function to disable/enable thread creation for other functions. Normally, thread creation should be enabled, but it must be disabled for LabJack functions to work when called from TestPoint. One other situation where disabling thread creation might be useful, is when running a time-critical application in the Visual C debugger. Slow thread creation is a known problem with the Visual C debugger.

Execution time for this function is about 80 milliseconds.

If the read thread is disabled, the "timeout" specified in AIBurst and AIStreamRead is also disabled.

Declaration:
long NoThread (long *idnum,
long noThread)

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **noThread –** If >0, the thread will not be used.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.25 PulseOut

Requires firmware V1.1 or higher. This command creates pulses on any/all of D0-D7. The desired D lines must be set to output using another function (DigitalIO or AOUpdate). All selected lines are pulsed at the same time, at the same rate, for the same number of pulses.

Execution time for this function is about 20 milliseconds plus pulse output time.

This function commands the time for the first half cycle of each pulse, and the second half cycle of each pulse. Each time is commanded by sending a value B & C, where the time is,

1st half-cycle microseconds = ~17 + 0.83*C + 20.17*B*C,
2nd half-cycle microseconds = ~12 + 0.83*C + 20.17*B*C,

which can be approximated as,

microseconds = 20*B*C.

For best accuracy when using the approximation, minimize C. B and C must be between 1 and 255, so each half cycle can vary from about 38/33 microseconds to just over 1.3 seconds.

If you have enabled the LabJack Watchdog function, make sure it's timeout is longer than the time it takes to output all pulses.

The timeout of this function, in milliseconds, is set to:
5000+numPulses*((B1*C1*0.02)+(B2*C2*0.02))

Declaration:

```
long PulseOut(long *idnum,
              long demo,
              long lowFirst,
              long bitSelect,
              long numPulses,
              long timeB1,
              long timeC1,
              long timeB2,
              long timeC2)
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **lowFirst –** If >0, each line is set low then high, otherwise the lines are set high then low.
- **bitSelect –** Set bits 0 to 7 to enable pulsing on each of D0-D7 (0-255).
- **numPulses –** Number of pulses for all lines (1-32767).
- **timeB1 –** B value for first half cycle (1-255).
- **timeC1 –** C value for first half cycle (1-255).
- **timeB2 –** B value for second half cycle (1-255).
- **timeC2 –** C value for second half cycle (1-255).

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.26 PulseOutStart

Requires firmware V1.1 or higher. PulseOutStart and PulseOutFinish are used as an alternative to PulseOut (See PulseOut for more information). PulseOutStart starts the pulse output and returns without waiting for the finish. PulseOutFinish waits for the LabJack's response which signifies the end of the pulse output.

Execution time for this function about 10 milliseconds.

If anything besides PulseOutFinish is called after PulseOutStart, the pulse output will be terminated and the LabJack will execute the new command. Calling PulseOutStart repeatedly, before the previous pulse output has finished, provides a pretty good approximation of continuous pulse output.

Note that due to boot-up tests on the LabJack U12, if PulseOutStart is the first command sent to the LabJack after reset or power-up, there will be no response for PulseOutFinish. In practice, even if no precautions were taken, this would probably never happen, since before calling PulseOutStart, a call is needed to set the desired D lines to output.

Also note that PulseOutFinish must be called before the LabJack completes the pulse output to read the response. If PulseOutFinish is not called until after the LabJack sends it's response, the function will never receive the response and will timeout.

Declaration:

```
long PulseOutStart(  long *idnum,
                     long demo,
                     long lowFirst,
                     long bitSelect,
                     long numPulses,
                     long timeB1,
                     long timeC1,
                     long timeB2,
                     long timeC2)
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **lowFirst –** If >0, each line is set low then high, otherwise the lines are set high then low.
- **bitSelect –** Set bits 0 to 7 to enable pulsing on each of D0-D7 (0-255).
- **numPulses –** Number of pulses for all lines (1-32767).
- **timeB1 –** B value for first half cycle (1-255).
- **timeC1 –** C value for first half cycle (1-255).
- **timeB2 –** B value for second half cycle (1-255).
- **timeC2 –** C value for second half cycle (1-255).

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.27 PulseOutFinish

Requires firmware V1.1 or higher. See PulseOutStart for more information.

Declaration:

```
long PulseOutFinish( long *idnum,
                     long demo,
                     long timeoutMS)
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **timeoutMS –** Amount of time, in milliseconds, that this function will wait for the PulseOutStart response.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.28 PulseOutCalc

Requires firmware V1.1 or higher. This function can be used to calculate the cycle times for PulseOut or PulseOutStart.

Declaration:
```
long PulseOutCalc(   float *frequency,
                     long *timeB,
                     long *timeC)
```

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***frequency –** Desired frequency in Hz.

Outputs:

- ***frequency –** Actual best frequency found in Hz.
- ***timeB –** B value for first and second half cycle.
- ***timeC –** C value for first and second half cycle.

4.29 ReEnum

Causes the LabJack to electrically detach from and re-attach to the USB so it will re-enumerate. The local ID and calibration constants are updated at this time.

Declaration:
```
long ReEnum ( long *idnum )
```

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.30 Reset (or ResetLJ)

Causes the LabJack to reset after about 2 seconds. After resetting the LabJack will re-enumerate. Reset and ResetLJ are identical.

Declaration:
```
long Reset ( long *idnum )
```

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.31 SHT1X

This function retrieves temperature and/or humidity readings from an SHT1X sensor. Data rate is about 2 kbps with firmware V1.1 or higher (hardware communication). If firmware is less than V1.1, or TRUE is passed for softComm, data rate is about 20 bps.

DATA = IO0
SCK = IO1

The EI-1050 has an extra enable line that allows multiple probes to be connected at the same time using only the one line for DATA and one line for SCK. This function does not control the enable line.

This function automatically configures IO0 has an input and IO1 as an output.

Note that internally this function operates on the state and direction of IO0 and IO1, and to operate on any of the IO lines the LabJack must operate on all 4. The DLL keeps track of the current direction and output state of all lines, so that this function can operate on IO0 and IO1 without changing IO2 and IO3. When the DLL is first loaded, though, it does not know the direction and state of the lines and assumes all directions are input and output states are low.

Declaration:

```
long SHT1X(  long *idnum,
             long demo,
             long softComm,
             long mode,
             long statusReg,
             float *tempC,
             float *tempF,
             float *rh)
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **softComm –** If >0, forces software based communication. Otherwise software communication is only used if the LabJack U12 firmware version is less than V1.1.
- **mode –** 0=temp and RH,1=temp only,2=RH only. If mode is 2, the current temperature must be passed in for the RH corrections using *tempC.
- **statusReg –** Current value of the SHT1X status register. The value of the status register is 0 unless you have used advanced functions to write to the status register (enabled heater, low resolution, or no reload from OTP).

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***tempC –** Returns temperature in degrees C. If mode is 2, the current temperature must be passed in for the RH corrections.
- ***tempF –** Returns temperature in degrees F.

- ***rh –** Returns RH in percent.

4.32 SHTComm

Low-level public function to send and receive up to 4 bytes to from an SHT1X sensor. Data rate is about 2 kbps with firmware V1.1 or higher (hardware communication). If firmware is less than V1.1, or TRUE is passed for softComm, data rate is about 20 bps.

DATA = IO0
SCK = IO1

The EI-1050 has an extra enable line that allows multiple probes to be connected at the same time using only the one line for DATA and one line for SCK. This function does not control the enable line.

This function automatically configures IO0 has an input and IO1 as an output.

Note that internally this function operates on the state and direction of IO0 and IO1, and to operate on any of the IO lines the LabJack must operate on all 4. The DLL keeps track of the current direction and output state of all lines, so that this function can operate on IO0 and IO1 without changing IO2 and IO3. When the DLL is first loaded, though, it does not know the direction and state of the lines and assumes all directions are input and output states are low.

Declaration:

```
long SHTComm(   long *idnum,
                long softComm,
                long waitMeas,
                long serialReset,
                long dataRate,
                long numWrite,
                long numRead,
                unsigned char *datatx,
                unsigned char *datarx)
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **softComm –** If >0, forces software based communication. Otherwise software communication is only used if the LabJack U12 firmware version is less than V1.1.
- **waitMeas –** If >0, this is a T or RH measurement request.
- **serialReset –** If >0, a serial reset is issued before sending and receiving bytes.
- **dataRate –** 0=no extra delay (default), 1=medium delay, 2=max delay.
- **numWrite –** Number of bytes to write (0-4).
- **numRead –** Number of bytes to read (0-4).
- ***datatx –** Array of 0-4 bytes to send. Make sure you pass at least numWrite number of bytes.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***datarx –** Returns 0-4 read bytes as determined by numRead.

ActiveX Function Differences:

Transmit and receive data arrays are replaced with 4 individual parameters which each pass 1 transmit byte and return 1 read byte.

Declaration (ActiveX):

```
long SHTCommX(   long FAR* idnum,
                 long softComm,
                 long waitMeas,
                 long serialReset,
                 long dataRate,
                 long numWrite,
                 long numRead,
                 long FAR* data0,
                 long FAR* data1,
                 long FAR* data2,
                 long FAR* data3)
```

4.33 SHTCRC

Checks the CRC on an SHT1X communication. Last byte of datarx is the CRC. Returns 0 if CRC is good, or SHT1X_CRC_ERROR_LJ if CRC is bad.

Declaration:

```
long SHTCRC(long statusReg,
            long numWrite,
            long numRead,
            unsigned char *datatx,
            unsigned char *datarx)
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- **statusReg –** Current value of the SHT1X status register.
- **numWrite –** Number of bytes that were written (0-4).
- **numRead –** Number of bytes that were read (1-4).
- ***datatx –** Array of 0-4 bytes that were sent.
- ***datarx –** Array of 1-4 bytes that were read.

ActiveX Function Differences:

Transmit and receive data arrays are each replaced with 4 individual parameters.

Declaration (ActiveX):

```
long SHTCRCX(    long statusReg,
                 long numWrite,
                 long numRead,
                 long datatx0,
                 long datatx1,
                 long datatx2,
                 long datatx3,
                 long datarx0,
                 long datarx1,
                 long datarx2,
                 long datarx3)
```

4.34 Synch

Requires firmware V1.1 or higher. This function performs SPI communication. Data rate is about 160 kbps with no extra delay, although delays of 100 us or 1 ms per bit can be enabled.

Execution time for this function is about 20 milliseconds to write and/or read up to 4 bytes, plus about 20 milliseconds for each additional 4 bytes written or read. Extra 20 milliseconds if configD is true, and extra time if delays are enabled.

Control of CS (chip select) can be enabled in this function for D0-D7 or handled externally via any digital output.

MOSI is D13
MISO is D14
SCK is D15

If using the CB25, the protection resistors might need to be shorted on all SPI connections (MOSI, MISO, SCK, CS).

The initial state of SCK is set properly (CPOL), by this function, before !CS is brought low (final state is also set properly before !CS is brought high again). If chip-select is being handled manually, outside of this function, care must be taken to make sure SCK is initially set to CPOL.

All modes supported (A, B, C, and D).

Mode A: CPHA=1, CPOL=1
Mode B: CPHA=1, CPOL=0
Mode C: CPHA=0, CPOL=1
Mode D: CPHA=0, CPOL=0

If Clock Phase (CPHA) is 1, data is valid on the edge going to CPOL. If CPHA is 0, data is valid on the edge going away from CPOL. Clock Polarity (CPOL) determines the idle state of SCK.

Up to 18 bytes can be written/read. Communication is full duplex so 1 byte is read at the same time each byte is written. If more than 4 bytes are written or read, this function uses calls to WriteMem/ReadMem to load/read the LabJack's data buffer.

This function has the option (configD) to automatically configure default state and direction for MOSI (D13 Output), MISO (D14 Input), SCK (D15 Output CPOL), and CS (D0-D7 Output High for !CS). This function uses a call to DigitalIO to do this. Similar to EDigitalIn and EDigitalOut, the DLL keeps track of the current direction and output state of all lines, so that these 4 D lines can be configured without affecting other digital lines. When the DLL is first loaded, though, it does not know the direction and state of the lines and assumes all directions are input and output states are low.

<u>Declaration:</u>

```
long Synch(   long *idnum,
              long demo,
              long mode,
              long msDelay,
              long husDelay,
              long controlCS,
              long csLine,
              long csState,
```

long configD,
long numWriteRead,
long *data)

Parameter Description:
Returns: LabJack errorcodes or 0 for no error.
Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **mode –** Specify SPI mode as: 0=A,1=B,2=C,3=D (0-3).
- **msDelay –** If >0, a 1 ms delay is added between each bit.
- **husDelay –** If >0, a hundred us delay is added between each bit.
- **controlCS –** If >0, D0-D7 is automatically controlled as CS. The state and direction of CS is only tested if control is enabled.
- **csLine –** D line to use as CS if enabled (0-7).
- **csState –** Active state for CS line. This would be 0 for the normal !CS, or >0 for the less common CS.
- **configD –** If >0, state and tris are configured for D13, D14, D15, and !CS.
- **numWriteRead –** Number of bytes to write and read (1-18).
- ***data –** Serial data buffer. Send an 18 element array of bytes. Fill unused locations with zeros.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***data –** Serial data buffer. Returns any serial read data. Unused locations are filled with 9999s.

ActiveX Function Differences:
The maximum number of bytes to write/read is limited to 5 (numWriteRead should be 1-5). The data array is replaced with pointers to 5 individual data bytes.

Declaration (ActiveX):
long SynchX(long FAR* idnum,
long demo,
long mode,
long msDelay,
long husDelay,
long controlCS,
long csLine,
long csState,
long configD,
long numWriteRead,
long FAR* data0,
long FAR* data1,
long FAR* data2,
long FAR* data3,
long FAR* data4,
long FAR* data5)

4.35 Watchdog

Controls the LabJack watchdog function. When activated, the watchdog can change the states of digital I/O if the LabJack does not successfully communicate with the PC within a specified timeout period. This function could be used to reboot the PC allowing for reliable unattended operation. The 32-bit counter (CNT) is disabled when the watchdog is enabled.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

If you set the watchdog to reset the LabJack, and choose too small of a timeout period, it might be difficult to make the device stop resetting. To disable the watchdog, reset the LabJack with IO0 shorted to STB, and then reset again without the short.

Declaration:

```
long Watchdog (   long *idnum,
                  long demo,
                  long active,
                  long timeout,
                  long reset,
                  long activeD0,
                  long activeD1,
                  long activeD8,
                  long stateD0,
                  long stateD1,
                  long stateD8 )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **active –** Enables the LabJack watchdog function. If enabled, the 32-bit counter is disabled.
- **timeout –** Timer reset value in seconds (1-715).
- **reset –** If >0, the LabJack will reset on timeout.
- **activeDn –** If >0, Dn will be set to stateDn upon timeout.
- **stateDn –** Timeout state of Dn, 0=low, >0=high.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.36 ReadMem

Reads 4 bytes from a specified address in the LabJack's nonvolatile memory.

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Declaration:

```
long ReadMem (    long *idnum,
                  long address,
                  long *data3,
                  long *data2,
                  long *data1,
                  long *data0 )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **address –** Starting address of data to read (0-8188).

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.
- ***data3 –** Byte at address.
- ***data2 –** Byte at address+1.
- ***data1 –** Byte at address+2.
- ***data0 –** Byte at address+3.

4.37 WriteMem

Writes 4 bytes to the LabJack's 8,192 byte nonvolatile memory at a specified address. The data is read back and verified after the write. Memory 0-511 is reserved for configuration and calibration data. Memory from 512-1023 is unused by the LabJack and available for the user (this corresponds to starting addresses from 512-1020). Memory 1024-8191 is used as a data buffer in hardware timed AI modes (burst and stream).

Execution time for this function is 20 milliseconds or less (typically 16 milliseconds in Windows).

Declaration:

```
long WriteMem (      long *idnum,
                     long unlocked,
                     long address,
                     long data3,
                     long data2,
                     long data1,
                     long data0 )
```

Parameter Description:

Returns: LabJack errorcodes or 0 for no error.

Inputs:

- ***idnum –** Local ID, serial number, or -1 for first found.
- **unlocked –** If >0, addresses 0-511 are unlocked for writing.
- **address –** Starting address for writing (0-8188).
- **data3 –** Byte for address.
- **data2 –** Byte for address+1.
- **data1 –** Byte for address+2.
- **data0 –** Byte for address+3.

Outputs:

- ***idnum –** Returns the local ID or –1 if no LabJack is found.

4.38 BuildOptionBits (ActiveX only)

This function is only in the OCX, and is used to build the optionBits parameter for AIBurst and AIStreamStart.

The parameter optionBits is made up of the following bits and can often just be set to 2 (normal operation with the LED on):

- bit 0 => demo
- bit 1 => ledOn
- bit 2 => disableCal
- bits 3,4 => transferMode

- bit 5 => updateIO
- bit 6 => stateIOin(0)
- bit 7 => stateIOin(1)
- bit 8 => stateIOin(2)
- bit 9 => stateIOin(3)

Declaration:

```
long BuildOptionBits (    long demo,
                          long ledOn,
                          long disableCal,
                          long transferMode,
                          long updateIO,
                          long stateIOin )
```

Parameter Description:
Returns: optionBits
Inputs:

- **demo –** Send 0 for normal operation, >0 for demo mode. Demo mode allows this function to be called without a LabJack.
- **ledOn –** If >0, the LabJack LED is turned on.
- **disableCal –** If >0, voltages returned will be raw readings that are not corrected using calibration constants.
- **transferMode –** Set to 0 (automatic).
- **updateIO –** If >0, state values will be written. Otherwise, just a read is performed.
- ***stateIOin –** Output states for IO0-IO3.

4.39 FourPack (ActiveX only)

This function is only in the OCX, and is used to convert a 4 element array into an integer. The packed value is determined as: valueA + (valueB * 2^8) + (valueC * 2^16) + (valueD * 2^24).

Declaration:

```
long FourPack (    long valueA,
                   long valueB,
                   long valueC,
                   long valueD )
```

Parameter Description:
Returns: Packed representation of a 4 element array.
Inputs:

- **valueA –** Element 0 of the array to be converted.
- **valueB –** Element 1 of the array to be converted.
- **valueC –** Element 2 of the array to be converted.
- **valueD –** Element 3 of the array to be converted.

4.40 Description of errorcodes.

It is recommended that the function GetErrorString be used to interpret errorcodes, but this list is provided as a convenience.

- **0 –** No error.
- **1 –** Unknown error.

- **2 –** No LabJacks found.
- **3 –** LabJack n not found.
- **4 –** Set USB buffer error.
- **5 –** Open handle error.
- **6 –** Close handle error.
- **7 –** Invalid ID.
- **8 –** Invalid array size or value.
- **9 –** Invalid power index.
- **10 –** FCDD size too big.
- **11 –** HVC size too big.
- **12 –** Read error.
- **13 –** Read timeout error.
- **14 –** Write error.
- **15 –** Turbo error.
- **16 –** Illegal channel index.
- **17 –** Illegal gain index.
- **18 –** Illegal AI command.
- **19 –** Illegal AO command.
- **20 –** Bits out of range.
- **21 –** Illegal number of channels.
- **22 –** Illegal scan rate.
- **23 –** Illegal number of samples.
- **24 –** AI response error.
- **25 –** LabJack RAM checksum error.
- **26 –** AI sequence error.
- **27 –** Maximum number of streams.
- **28 –** AI stream start error.
- **29 –** PC buffer overflow.
- **30 –** LabJack buffer overflow.
- **31 –** Stream read timeout.
- **32 –** Illegal number of scans.
- **33 –** No stream was found.
- **40 –** Illegal input.
- **41 –** Echo error.
- **42 –** Data echo error.
- **43 –** Response error.
- **44 –** Asynch read timeout error.
- **45 –** Asynch read start bit error.
- **46 –** Asynch read framing error.
- **47 –** Asynch DIO config error.
- **48 –** Caps error.
- **49 –** Caps error.
- **50 –** Caps error.
- **51 –** HID number caps error.
- **52 –** HID get attributes warning.
- **57 –** Wrong firmware version error.
- **58 –** DIO config error.
- **64 –** Could not claim all LabJacks.
- **65 –** Error releasing all LabJacks.
- **66 –** Could not claim LabJack.
- **67 –** Error releasing LabJack.

- **68 –** Claimed abandoned LabJack.
- **69 –** Local ID –1 thread stopped.
- **70 –** Stop thread timeout.
- **71 –** Thread termination failed.
- **72 –** Feature handle creation error.
- **73 –** Create mutex error.
- **80 –** Synchronous CS state or direction error.
- **81 –** Synchronous SCK direction error.
- **82 –** Synchronous MISO direction error.
- **83 –** Synchronous MOSI direction error.
- **89 –** SHT1X CRC error.
- **90 –** SHT1X measurement ready error.
- **91 –** SHT1X ack error.
- **92 –** SHT1X serial reset error.

If bit 8 is set, the error occurred in the stream thread. Bit 10 is set for Windows API errors.

A. Specifications

Parameter	Conditions	Min	Typical	Max	Units
General					
USB Cable Length				3	meters
User Connection(s) Length	CE compliance			3	meters
Supply Current (1)			20		mA
Operating Temperature		-40		85	°C
Clock Error	~ 25 °C			±30	ppm
	0 to 70 °C			±50	ppm
	-40 to 85 °C			±100	ppm
+5 Volt Power Supply (+5V)					
Voltage (Vs) (2)	Self-Powered	4.5		5.25	volts
	Bus-Powered	4.1		5.25	volts
Output Current (2) (3)	Self-Powered	450		500	mA
	Bus-Powered	50		100	mA
Analog Inputs (AI0 - AI7)					
Input Range For Linear Operation	AIx to GND, SE	-10		10	volts
	Aix to GND, Diff.	-10		20	volts
Maximum Input Range	AIx to GND	-40		40	volts
Input Current (4)	Vin = +10 volts		70.1		μA
	Vin = 0 volts		-11.7		μA
	Vin = -10 volts		-93.5		μA
Resolution (No Missing Codes)	C/R and Stream		12		bits
	Burst Diff. (5)		12		bits
	Burst SE (5)		11		bits
Offset	G = 1 to 20		±1 * G		bits
Absolute Accuracy	SE		±0.2		% FS
	Diff.		±1		% FS
Noise	C/R and Stream		±1		bits
Integral Linearity Error			±1		bits
Differential Linearity Error			±0.5		bits
Repeatability			±1		bits
CAL Accuracy	CAL = 2.5 volts		±0.05	±0.25	%
CAL Current	Source			1	mA
	Sink	20	100		μA
Trigger Latency	Burst	25		50	μs
Trigger Pulse Width	Burst	40			μs
Analog Outputs (AO0 & AO1)					
Maximum Voltage (6)	No Load		Vs		volts
	At 1 mA		0.99 * Vs		volts
	At 5 mA		0.96 * Vs		volts
Source Impedance			20		Ω
Output Current	Each AO			30	mA

Parameter	Conditions	Min	Typical	Max	Units
IO					
Low Level Input Voltage				0.8	volts
High Level Input Voltage		3		15	volts
Input Leakage Current (7)			±1		µA
Output Short-Circuit Current (8)	Output High		3.3		mA
Output Voltage (8)	No Load	Vs - 0.4	Vs		volts
	At 1 mA		Vs - 1.5		volts
D					
Low Level Input Voltage (9)	D0 - D12			0.8	volts
	D13 - D15			1	volts
High Level Input Voltage (9)	D0 - D12	2		Vs + 0.3	volts
	D13 - D15	4		Vs + 0.3	volts
Input Leakage Current (7)			±1		µA
Output Current (9)	Per Line			25	mA
	Total D0 - D15			200	mA
Output Low Voltage				0.6	volts
Output High Voltage		Vs - 0.7			volts
CNT					
Low Voltage (10)		GND		1	volts
High Voltage (10)		4		15	volts
Schmitt Trigger Hysteresis			20-100		mV
Input Leakage Current (7)			±1		µA
Minimum High Time				500	ns
Minimum Low Time				500	ns
Maximum Input Frequency		1			MHz

(1) Current drawn by the LabJack through the USB. The status LED is responsible for 4-5 mA of this current.

(2) Self-powered would apply to USB hubs with a power supply, all known desktop computer USB hosts, and some notebook computer USB hosts. Bus-powered would apply to USB hubs without a power supply and some notebook computer USB hosts.

(3) This is the total current that can be sourced by +5V, analog outputs, and digital outputs.

(4) The input current at each analog input is a function of the voltage at that input (Vin) with respect to ground and can be calculated as: (8.181*Vin - 11.67) µA.

(5) Single-ended burst mode only returns even binary codes, and thus has a net resolution of 11 bits. In addition, extra noise in burst mode can reduce the effective resolution.

(6) Maximum analog output voltage is equal to the supply voltage at no load.

(7) Must also consider current due to 1 MΩ resistor to ground.

(8) The IO lines each have a 1500 ohm series resistor.

(9) These lines have no series resistor. It is up to the user to make sure the maximum voltages and currents are not exceeded.

(10) CNT is a Schmitt Trigger input.

4.2 Professor Reinkensmeyer lecture notes given during class

University of California at Irvine
MAE106 Mechanical Systems Laboratory: Lecture 1

Part 1: Overview of the Class

If you work hard, you will leave this class with knowledge and practical experience in three interrelated areas:

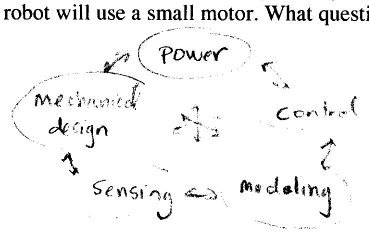
1. Physical intuition about how 1st and 2nd order linear, dynamical systems behave
   - You will be exposed to examples of common electrical, vibration, and robotic systems
   - Basic Idea: The dynamics of a wide variety of physical objects obey 1st and 2nd order linear, differential equations. These systems respond exponentially, sinusoidally, and exposoidally (OK, that's not a real word, but try to get the idea) in the time domain.
   - You will learn how to think about their behavior in both the time and frequency domains
2. Basic understanding of how feedback control works
   - Feedback is a common way to make cars, planes, robots, etc. respond like we want them too
   - You will learn about proportional feedback control (and derivative and integral control)
   - Basic idea: Measure error and try to reduce by changing the input to the controlled object
3. Familiarity with the components and tools for building mechatronic and robotic systems
   - Motors, potentiometers, tachometers, analog computational circuits (op-amps), electrical filters, power amplifiers, data acquisition systems, oscilloscopes, protoboards, ohmmeters

Part 2: Design Exercise

Final Project Competition: Build a robotic soccer player that can do two things:

1. kick a penalty kick
2. goal tend to block a penalty kick by another robot

Your robot will use a small motor. What questions do you need to know the answer to in order to build this robot?



Part 3: Review of Circuit Theory

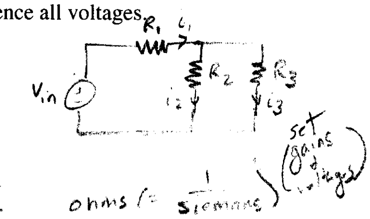
3.1 Linear circuit elements

Current: think of it as the flow of charge through a circuit element (such as a wire or resistor) Units: amps=coulombs/sec

Voltage: think of it as the electrical pressure that can cause charge carriers to flow

Current is always measured through something at a point; voltage is always measured between two points

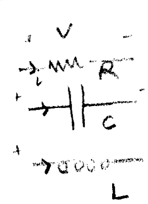
For this class, "ground" is an arbitrarily defined point on a circuit to which we reference all voltages



Toolbox for circuit analysis

- Kirchoff's Current Law: $\sum$ current in = $\sum$ current out
- Kirchoff's Voltage Law: $\sum_{loop}$ voltage = 0 $-V_i + V_1 + V_2 = 0$
- Power $P = Vi$ (stored or dissipated)
- Triad of linear circuit elements:

	i	$\frac{di}{dt}$
v	R	L
$\frac{dv}{dt}$	C	"Flux Capacitor"



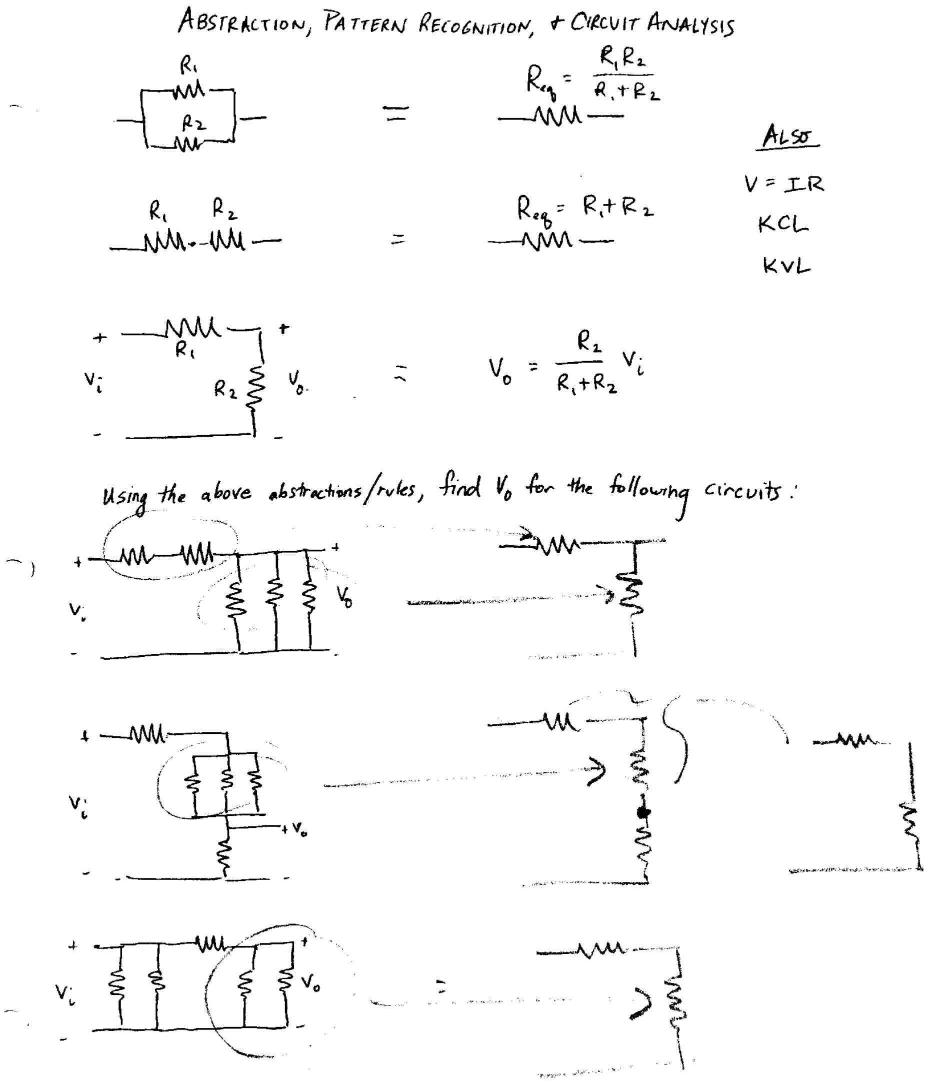
$V = L\frac{di}{dt}$

ohms (= 1/siemens)

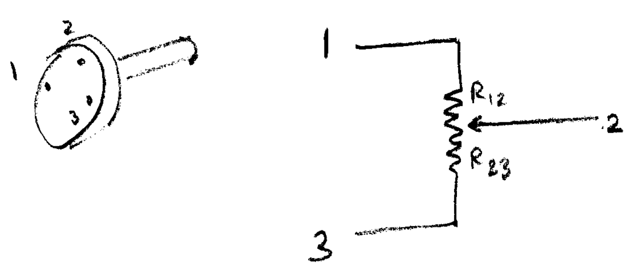
farads (filters)

henrys (motors, used to ignite spark plug in automobile engine)

Resistor Analysis Exercise:
Abstraction: the act of considering something as a general quality or characteristic apart from any concrete realities, specific object, or actual instance. It's the idea of a "black box"



Potentiometers:
Typically used as voltage dividers. The two resistor values are changed by turning the pot.

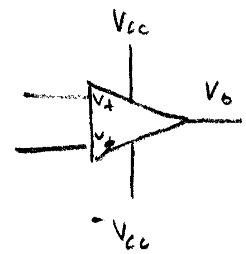


$R = R_{12} + R_{23} = $ [50 KΩ, for example]

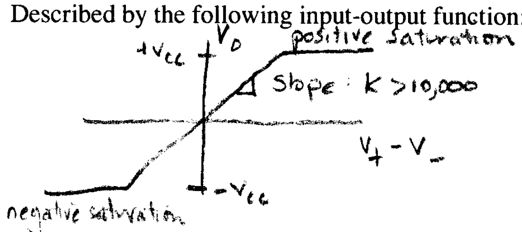
$V_{out} = \frac{R_{23}}{R} V_{in}$

3.2 Operational Amplifiers

- important building blocks for circuits; easy to use, cheap
- used to build filters, amplifiers, feedback controllers, computational circuits
- the "brains" in the analog control circuits that you will build for the class
- What are they? High gain, differential, linear voltage amplifiers
- Made of > 20 transistors plus resistors and capacitors
- Two input terminals, one output, two power supply lines (five pins total)
- Typically operate over a wide range of supply voltages
- By design, they have a high input resistance and a low output resistance



Described by the following input-output function:



in linear region

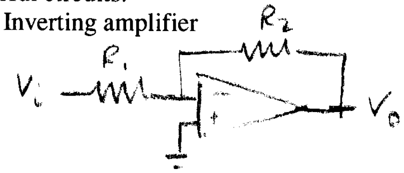
$V_0 = K(V_+ - V_-)$

Golden Rules of Op-amp Circuit design:

1. Input currents are zero (op amps are designed to have a high input resistance)
2. Input voltages are equal (If operating in linear region, and connected with negative feedback)

Four useful circuits:

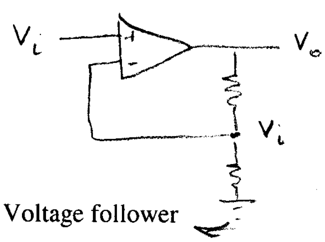
1. Inverting amplifier



What is V_o as a function of V_i?

KCL: $\frac{V_i}{R_1} + \frac{V_o}{R_2} = 0 \qquad V_o = \frac{-R_2}{R_1} V_i$

2. Non-inverting amplifier

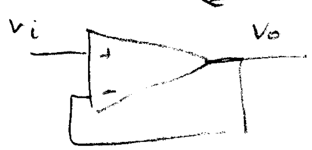


$\frac{V_o - V_i}{R_1} = \frac{V_i}{R_2} \Rightarrow V_o = \frac{R_1}{R_2} V_i + V_i$

$V_o = \left(\frac{R_1 + R_2}{R_2}\right) V_i$

as $R_2 \to \infty$ what does this circuit do?

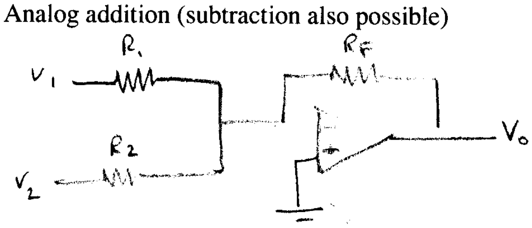
3. Voltage follower



$V_o = V_i$

Why? – high input impedance lets us connect circuit modules without altering their performance

4. Analog addition (subtraction also possible)



$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_o}{R_F} = 0$

$V_o = -R_F\left(\frac{1}{R_1}V_1 + \frac{1}{R_2}V_2\right)$

If $R_1 = R_2 = R$

$V_o = \frac{-R_F}{R}(V_1 + V_2)$

Note: feedback is always to V_- (negative feedback)
$|V_o| < |V_{cc}|$ and $i_{out} < i_{max}$ else op-amp saturates

3.3 Controlling power needed for devices like motors, light bulbs, etc.

Often we want to control a device that requires a lot of power (e.g. a motor) with signals that have very low power (e.g. an op amp or computer).

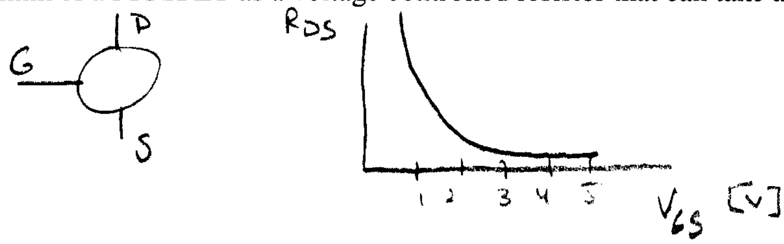
Small DC brushed motor: $V = 10\text{ V}$, $R = 2\ \Omega$ $i = \frac{V}{R} = 5$ amps

Typical Op-amp: $V = \pm 15\text{ V}$, $i_{max} < 20$ mamp

Solutions?

1. Power op-amp
2. Power transistor (e.g power MOSFET – simple and cheap)

Can think of a MOSFET as a voltage controlled resistor that can take a lot of current



Notes: Input resistance is very high (therefore effectively no current goes into gate)

Low-power MOSFETS are the "switches" used in computers (what is a switch?)

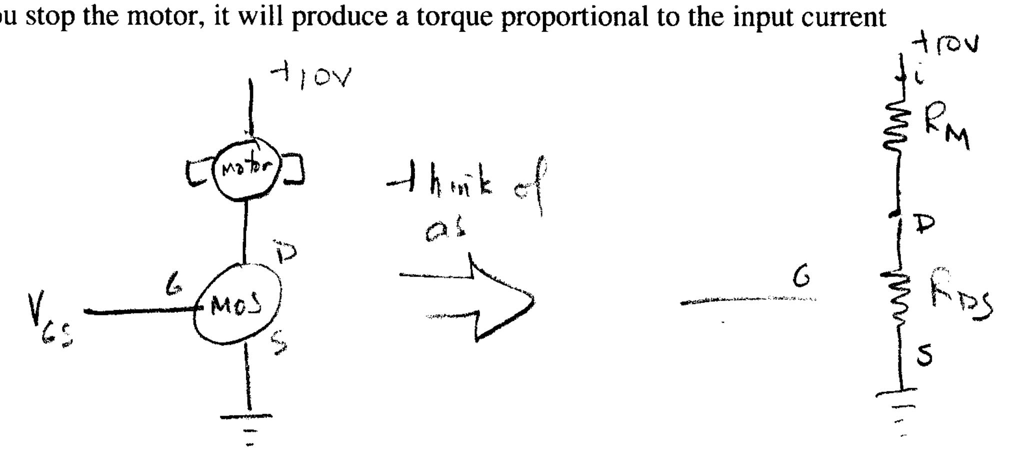
MOSFETS are very sensitive to static electricity – use a grounding strap when you handle them in lab

Example: use a power transistor to control a motor with a low-power computer output

Hints about motors:

A DC brushed motor spins at a speed proportional to the input voltage, if it is just turning an inertial load.

If you stop the motor, it will produce a torque proportional to the input current



If R_{DS} is big, no current flows

If R_{DS} is small (because V_{GS} is small) current flows

Thus, by controlling V_{GS}, we can make the motor turn or not turn

(V_{GS}: low power, no current)

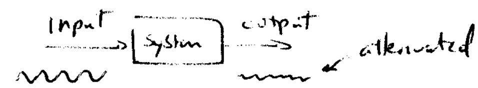
MAE106 Mechanical Systems Laboratory: Time and Frequency Domain Notes

1. Why do engineers analyze systems in both the time and frequency domain?

Why the time domain? We live in the time domain.

Typical questions:
- How does the system respond to a step input? Example: 0-60 mph
- How does the system respond to a impulse input? Example: bump suspension
- How fast does the system respond? (Useful #: time constant)
- Does it overshoot ?
- Does it oscillate ?

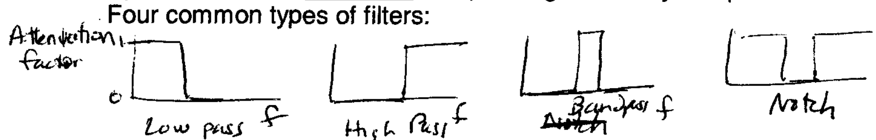
Why the frequency domain?



a. Intuition

Systems act like filters, responding differently to inputs at different frequencies

Four common types of filters:



b. Ease – sometimes its easier to solve differential equations in the frequency domain (Laplace Transform)

2. What is a transfer function and what is a frequency response?

A linear differential equation in the time domain becomes a transfer function in the freq. domain.

To see this take the Laplace transform of a differential equation:

Recall $\mathcal{L}\left(\frac{dx}{dt}\right) = s\,\mathcal{L}(x(t))$

$\mathcal{L}(x_1 + x_2) = \mathcal{L}(x_1) + \mathcal{L}(x_2)$

$$\frac{dx}{dt} + ax = u$$

Assume initial conditions are zero

$$sX(s) + aX(s) = U(s) \qquad X(s) = \frac{1}{s+a}U(s) = H(s)\,U(s)$$

FACT: The transfer function tells how a system responds to any input in the frequency domain. The output is just the input multiplied by the transfer function. $U(s) \rightarrow \boxed{H(s)} \rightarrow X(s) = H(s)U(s)$

The transfer function also tells how a system responds to a sinusoidal input.

FACT: Using Laplace Transforms, it is possible to prove that: sine wave in ⇒ sine wave out (scaled, shifted)

The transfer function tells how much an input sine wave is scaled and shifted as a function of its frequency. $\sin(\omega t) \rightarrow \boxed{G(s)} \rightarrow |G(j\omega)|\sin(\omega t + \phi_{G(j\omega)})$

Note $G(j\omega)$ is a complex variable, so it has a magnitude & phase

Knowing these two things means you know the frequency response of the system, which is characterized by the Magnitude and the phase responses

These facts are very useful when combined with two other facts:

FACT: Any signal can be represented as the sum of sinusoids. Fourier analysis

FACT: The response of a linear system to the sum of two inputs is the sum of the individual outputs.

$x_1 \rightarrow y_1$

$x_2 \rightarrow y_2$

$x_1 + x_2 \rightarrow y_1 + y_2$

<u>THESE FACTS LET US THINK OF LINEAR SYSTEMS AS FILTERS.</u>

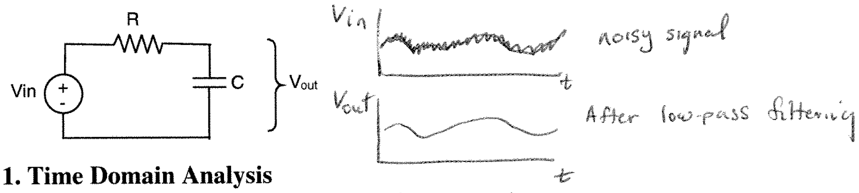
Linear System

any signal → Σ sines → $\boxed{G(s)}$ → Σ scaled shifted sines → output signal

Mechanical Systems Laboratory: Lecture 3

Analysis of a 1st-order, Low-Pass Filter Circuit in the Time and Frequency Domains

The following circuit is a low-pass filter. It is useful to clean up signals with high frequency noise on it:



1. Time Domain Analysis

Let's analyze the response of this circuit to a step input

We'll use the method of undetermined coefficients to solve the differential equation. You can remember this very useful technique for linear, ordinary, differential equations using the following mnemonic:

1. Generals: set the forcing function = 0 and find the general solution to homogenous equation (don't evaluate it's coefficient yet)
2. are Particular: find the particular solution (assume particular soln is same form as forcing function)
3. about Initial Conditions: sum the homogenous and particular solutions and solve for the coefficient to the homogenous equation that satisfies the initial conditions.

KVL: $-V_i + iR + V_o = 0$

$i = C\frac{dV_o}{dt}$

$RC\frac{dV_o}{dt} + V_o = V_i$

$* \quad \frac{dV_o}{dt} = -\frac{1}{RC}V_o + \frac{1}{RC}V_i$

Assume: $V_o(0) = 0$, $V_i = \begin{cases} 0 & t<0 \\ \text{constant} & t>0 \end{cases}$

Homog. $V_h = Ae^{-t/\tau}$ $\quad \tau = RC$

Part: $V_p = V_i$ (assume V_i is a constant)

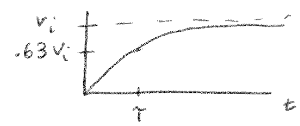
Total: $V_o = Ae^{-t/\tau} + V_i$ but $V(0) = 0 = A + V_i$

$\Rightarrow A = -V_i$

$V_o = V_i(1 - e^{-t/\tau})$

at $t = \tau$

$V_o = V_i(1 - e^{-1}) = .63V_i$



Summary of important concepts:

- Method of undetermined coefficients for solving a differential equation.
- Time constant: a 1st order system has gone 63% of the way to its final value after one time constant – standard engineering technique for quantifying "how fast" a system responds.

2. Frequency Domain Analysis

Let's analyze how this system responds to a sinusoidal input. Remember: sine in ⇒ sine out (scaled and shifted), for a linear system. We will use three methods to find the scaling and shifting.

Method 1. Solve differential equation using method of undetermined coefficients (difficult) Assume $V_i = \sin \omega t$

Homogenous solution: $V_h = Ae^{-t/\tau}$ Note: as $t \to \infty$ $V_h \to 0$ ("Transient")

Particular solution: try $V_p = a\sin(\omega t + \phi)$ where a + ϕ are unknown

subst into $*$ = $a\omega\cos(\omega t + \phi) = -\frac{a}{RC}\sin(\omega t + \phi) + \frac{1}{RC}\sin \omega t$

Useful trig. identity: $A\cos(\theta) + B\sin(\theta) = \sqrt{A^2 + B^2}\sin(\theta + \tan^{-1}(\frac{A}{B}))$

$a\omega\cos(\omega t + \phi) + \frac{a}{RC}\sin(\omega t + \phi) = \frac{1}{RC}\sin\omega t = \sqrt{(a\omega)^2 + (\frac{a}{RC})^2}\sin(\omega t + \phi + \tan^{-1}\omega RC)$

for the right-most equality to hold

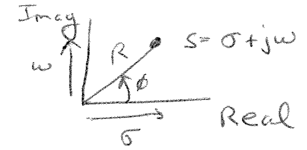
$\frac{1}{RC} = \sqrt{(a\omega)^2 + (\frac{a}{RC})^2} \Rightarrow a = \frac{1}{\sqrt{1 + (\omega RC)^2}}$ scaling

$\phi = -\tan^{-1}\omega RC$ phase shift

$V_o = a\sin(\omega t + \phi)$ with

Method 2: Take Laplace Transform of differential equation that describes circuit, find the transfer function, and solve for frequency response (easier than Method 1)

Brief review of complex variables:
Complex variables keep track of two pieces of information, real and imaginary part, or magnitude and phase



Can think of complex variables as a point in the complex plane.

Can write point in Cartesian or polar coordinates. $s = \sigma + j\omega$ $\quad s = Re^{j\phi}$

To find the magnitude in Cartesian form: $|s| = \sqrt{\sigma^2+\omega^2} = R$

To find the phase in Cartesian form: $\phi_s = \tan^{-1}\frac{\omega}{\sigma} = \phi$

$s = \sigma + j\omega = R\cos\phi + jR\sin\phi = Re^{j\phi}$

Magnitude of two complex variables divided by each other: $\left|\frac{R_1 e^{j\phi_1}}{R_2 e^{j\phi_2}}\right| = \frac{|R_1|}{|R_2|}$

Phase of two complex variables divided by each other: $= \frac{R_1}{R_2} e^{j(\phi_1-\phi_2)}$ $\quad \phi = \phi_1 - \phi_2$

Euler's law $e^{j\phi} = \cos\phi + j\sin\phi$ (can derive by Taylor's expansion)

Now, find the transfer function and frequency response:

$\frac{dV_o}{dt} = \frac{-1}{RC}V_o + \frac{1}{RC}V_i \qquad sV_o = \frac{1}{RC}V_o + \frac{1}{RC}V_i$

$\frac{V_o}{V_i} = \frac{\frac{1}{RC}}{s+\frac{1}{RC}} = \frac{1}{1+RCs} = G(s)$

$|G(s)|_{s=j\omega} = \frac{|1|}{|1+RCj\omega|} = \frac{1}{\sqrt{1+(RC\omega)^2}}$

$\angle\phi(s)|_{s=j\omega} = \tan^{-1}\frac{0}{1} - \tan^{-1}\frac{RC\omega}{1} = -\tan^{-1}RC\omega$

Method 3: Use "impedances" to find transfer function (easiest)

Circuit element	Time domain	Frequency domain	Impedance
Resistor	V(t) = RI(t)	V(s) = R I(s)	R
Capacitor	$V(t) = \frac{1}{C}\int i(t)$	$V(s) = \frac{1}{sC}I(s)$	$\frac{1}{sC}$
Inductor	V(t) = L di/dt	$V(s) = sL\,I(s)$	sL

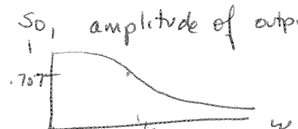
Note: All the usual circuit rules till hold in the frequency domain because of superposition (KVL, KCL, Op amp rules, voltage divider…). So, treat impedances like (frequency dependent) resistors in finding a circuit's transfer function.

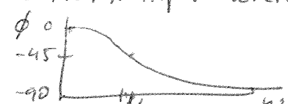
$V_o = \frac{\frac{1}{sC}}{R+\frac{1}{sC}}V_i = \frac{1}{1+RCs}V_i \qquad \frac{V_o}{V_i} = G(s) = \frac{1}{1+RCs}$

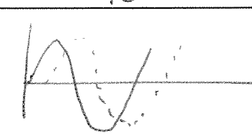
What do the magnitude response (i.e. scaling or attenuation factor) and phase shift response actually look like? Fill in the following chart:

	Magnitude or Scaling	Phase
Small ω	1	0
ω = 1/RC = 1/τ	$\sqrt{\frac{1}{2}} = .707$	−45°
ω ⇒ infinity	0	−90°

So, amplitude of output is .707 x input when $\omega = \frac{1}{\tau}$







The frequency 1/τ is called the "corner frequency" or "bandwidth" of the system. For this low-pass filter, input sinusoids with a frequency higher than the bandwidth are "filtered" or "attenuated".
Summary of important concepts:

- How to find a transfer function and the frequency response
- Impedances
- Corner frequency

Mechanical Systems Laboratory: Lecture 4
How DC Brushed Motors Work (Another example of a first-order system)

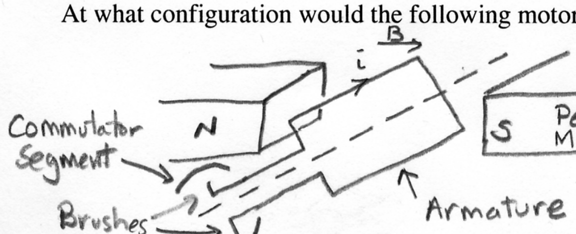
1. Introduction to DC Brushed Motors

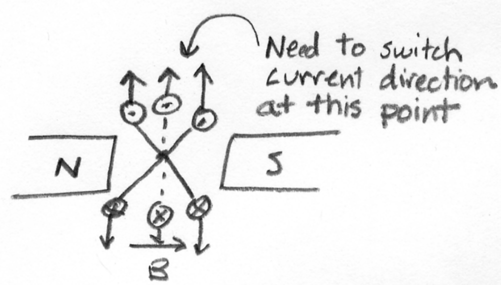
- very common for small jobs (toys, some appliances, robots)
- invented by Michael Faraday in the 1850's
- Operating principle:
    - o apply voltage, motor spins
    - o Polarity of voltage determines motor direction
    - o Amplitude of voltage determines motor speed
- Other motor types: AC motors (washing machine), DC brushless motors, DC stepper motors

2. Physics of Operation

a. Makes use of Lorentz Force Law: $\vec{F} = i\vec{\ell} \times \vec{B}$
where F = force, l = unit vector in direction of current flow, B = magnetic flux, i = current into motor
i.e. current-carrying conductors placed in magnetic fields create forces

At what configuration would the following motor get stuck?





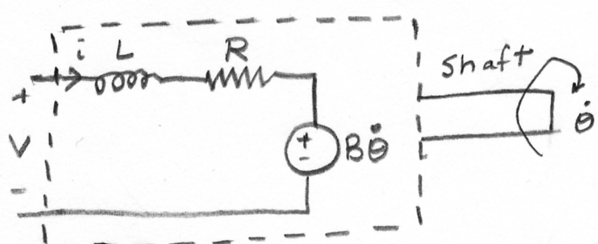
Use "commutation" to reverse current direction and keep motor turning
Adding enough commutator segments gives: τ = Bi, where B = "torque constant"

b. Back EMF

- back EMF" (electromotive force or 'voltage')
- the voltage produced by motor as a result of its speed
- voltage is proportional to speed $V = B\dot{\theta}$
- physical basis: armature windings are an inductor
- as motor spins, get di/dt in armature
- V = Ldi/dt ∝ angular velocity
- Can use a motor as a velocity sensor (i.e. a "tachometer") by measuring voltage across terminals
- This is also the principle used by generators.
- Real tachometers have many armature coils to reduce voltage ripple

3. Mathematical Model of a DC Brushed Motor

A motor has a resistance and inductance associated with its coils.



Electrical Side
$V = L\frac{di}{dt} + Ri + B\dot{\theta}$

Mechanical Side
$\tau = Bi$

To see how the model predicts the motor behavior, consider two cases:

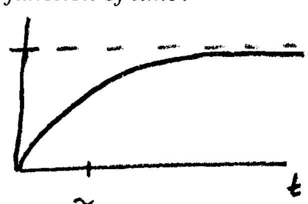
Case 1) Hold shaft fixed, apply constant voltage. What is the motor torque as a function of time?

Assume $i(0)=0$; shaft fixed $\Rightarrow \dot{\theta}=0$

$V = L\frac{di}{dt} + Ri + B\dot{\theta}$ ($B\dot{\theta} \to 0$)

Solve 1st order DEQ

Stall torque ↗ $\frac{BV}{R}$



1. General soln: $\frac{di}{dt} = -\frac{R}{L}i \Rightarrow i = Ke^{-t/\tau_c}$, $\tau_c = \frac{L}{R}$ = time constant

2. Part soln: Assume $i = \text{constant} \equiv A$

$L\frac{dA}{dt} + RA = V \Rightarrow A = \frac{V}{R}$

3. Total: $i = \frac{V}{R} + Ke^{-t/\tau_c}$

Find $K \Rightarrow i(0)=0 = \frac{V}{R} + K \Rightarrow K = -\frac{V}{R}$

Soln: $i = \frac{V}{R}(1 - e^{-t/\tau_c})$

Torque: $\tau = Bi$

$\tau = \frac{BV}{R}(1 - e^{-t/\tau_c})$

Observations:
The stall torque = the torque you feel if you hold the motor shaft fixed
It takes time for a motor to develop torque (describable with a time constant)
After the transient response, the motor acts like a resistor

Case 2) Allow shaft to spin freely, apply constant voltage. What is the motor speed as a function of time?

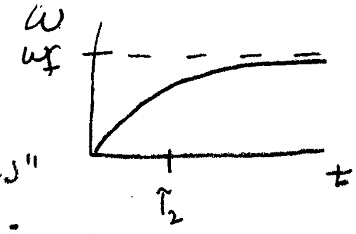
Assume shaft has inertia $\tau = J\ddot{\theta}$, assume $\frac{di}{dt} \approx 0$ (current reaches steady state)

$V = L\frac{di}{dt} + Ri + B\dot{\theta} \Rightarrow V = Ri + B\dot{\theta}$ recall $\tau = Bi \Rightarrow i = \frac{\tau}{B} = \frac{J\ddot{\theta}}{B}$

$\frac{RJ}{B}\ddot{\theta} + B\dot{\theta} = V$ let $\omega = \dot{\theta}$

$\frac{RJ}{B}\dot{\omega} + B\omega = V$ SOLUTION: $\omega = \frac{V}{B}(1 - e^{-t/\tau_2})$ $\tau_2 = \frac{RJ}{B^2}$

as $t \to \infty$, $\omega \to \omega_f = \frac{V}{B}$ "No Load Speed"



Observations:
No load speed is independent of inertia and proportional to voltage
Time constant of speed increase depends on inertia
Motor requires no power at no load speed (actually does because of friction)

at $\omega = \dot{\theta} = \omega_f$
$V = Ri + B\dot{\theta}$
$V = Ri + B\frac{V}{B} \Rightarrow i = 0$

Summary: Torque-speed curve for a DC brushed motor

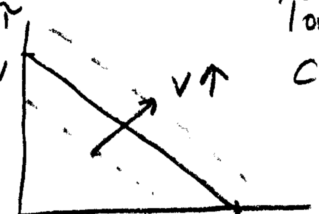
For V = constant ($\&\ \frac{di}{dt} = 0$) you get $V = Ri + B\dot{\theta}$, $i = \frac{\tau}{B}$

$V = \frac{R}{B}\tau + B\dot{\theta}$ → can be written in eqn of line

$\tau = -\frac{B^2}{R}\dot{\theta} + \frac{B}{R}V$

$y = mx + b$

Torque-Speed Characteristic



Important Ideas:

- Lorentz force law
- Commutation
- Back EMF
- Mathematical model of motor (inductor, resistor, back EMF)
- Exponential increase in torque if shaft is fixed; in speed if shaft is free to spin
- Torque-speed curve (no-load speed, stall torque)

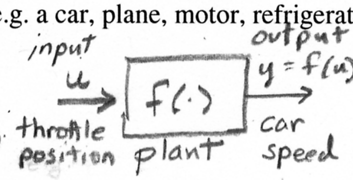
Mechanical Systems Laboratory: Lecture 5

Basic Control Concepts; Example of Feedback Control of Motor Velocity

1. Basic Control Concepts

a. The problem of automatic control

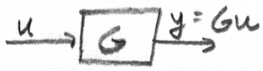
Given a system with inputs and outputs (the "plant" – e.g. a car, plane, motor, refrigerator, …)
And a desired output y_d
Find an input u to give you the desired output y_d



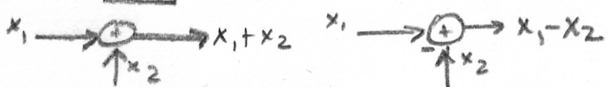
b. Block diagrams

Useful notation for visualizing control systems.
Two common blocks:

1) Gain block



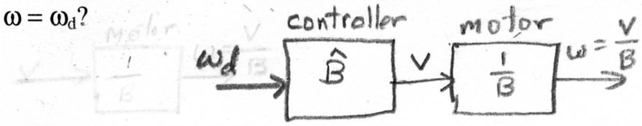
2) Summer block



c. Two general approaches to designing the input "u"

Approach 1: Feedforward Control (or "Open Loop" Control)

Choose input to plant based on knowledge of plant (i.e. based on an "inverse model" of the plant).
Example: Control a motor's steady state velocity ω using voltage v as the input. What should v be such that $\omega = \omega_d$?



$V = \hat{B}\omega_d$

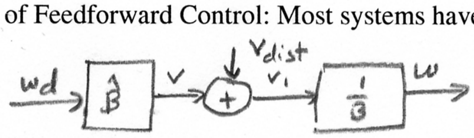
$\omega = \frac{V}{B} = \frac{\hat{B}}{B}\omega_d$ ^ = an estimate

If $\hat{B} = B$, $\omega = \omega_d$

Shortcoming 1 of Feedforward Control: Need to have an accurate model of the plant

If $\hat{B} = .5B$, then $\omega = .5\omega_d$

Shortcoming 2 of Feedforward Control: Most systems have unpredicted "disturbances" that affect the output



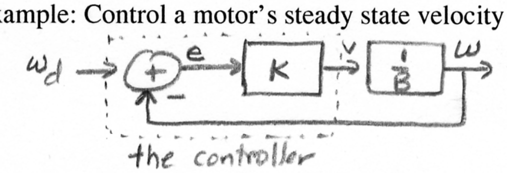
$\omega = \frac{V_1}{B} = \frac{V + V_{dist}}{B}$

$\omega = \frac{\hat{B}}{B}\omega_d + \frac{1}{B}V_{dist}$

If V_{dist} is big, $\omega \neq \omega_d$

Approach 2: Feedback Control (or "Closed Loop" Control)

Refers to measuring a system's output and "feeding it back" to change the input so $y = y_d$.
Also called "closed-loop control" because you "close the control loop" by "feeding back" the sensed output.
Usually, you subtract the desired output from the actual output to get an error signal, then apply an input to the system proportional to the error in the direction that reduces the error (negative feedback).
Example: Control a motor's steady state velocity ω using voltage v as the input.



$e = \omega_d - \omega$

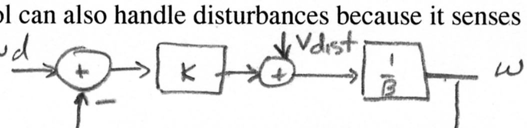
$V = Ke = K(\omega_d - \omega)$

$\omega = \frac{1}{B}V = \frac{K}{B}(\omega_d - \omega)$

$\rightarrow \omega\left(1 + \frac{K}{B}\right) = \frac{K}{B}\omega_d$

$$\boxed{\omega = \frac{K}{B+K}\omega_d}$$

For feedback control, you don't need an accurate model of the plant (you just need to know which way to "push" the plant to reduce error). Feedback control can also handle disturbances because it senses their effects on the output. If $K >> B$, then $\omega = \omega_d$

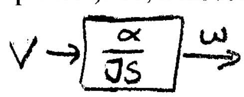


solve: $\omega = \frac{K}{K+B}\omega_d + \frac{1}{K+B}V_{dist}$ if $K >> B$, $\omega \approx \omega_d$

What are two limitations of feedback control?
1) an error has to develop before controller acts
2) delay causes instability
3) requires a sensor
Many control systems use feedforward and feedback control together.

2. Example: Feedback Control of Motor Velocity (Lab 3)

In lab, you will build an op-amp circuit for controlling velocity of a motor using proportional feedback.
To place this lab in a context, imagine that you are designing a control system for "Robbie the Rescue Robot".
The amplifiers that you use in lab give an output current (and thus motor torque) that is proportional to input voltage ("current amplifier"). So, a model of the motor and amplifier together is:



$\tau = J\dot{\omega}$ (inertial load)
$\tau = \alpha V$ (current amplifier)
V: voltage to amplifier

$J\dot{\omega} = \alpha V$
$Js\omega = \alpha V$
$\frac{\omega}{V} = \frac{\alpha}{Js} = G(s)$

Problem 1: Design a feedback control law relating V, ω, and ω_d, and verify that it works.

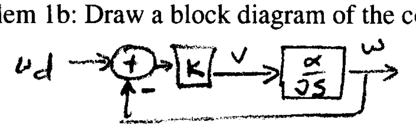
$$V = -K(\omega - \omega_d)$$

Intuitively: If motor turns too fast ($\omega > \omega_d$), apply a negative torque to slow it down. If motor turns too slowly ($\omega < \omega_d$), apply a positive torque to speed it up. Called a "proportional" or "P" control system since motor torque is proportional to error.

Problem 1a: Write the differential equation that describes the behavior of the controlled system

$$V = -K(\omega - \omega_d) = \frac{J}{\alpha}\dot{\omega}$$

Problem 1b: Draw a block diagram of the controlled system



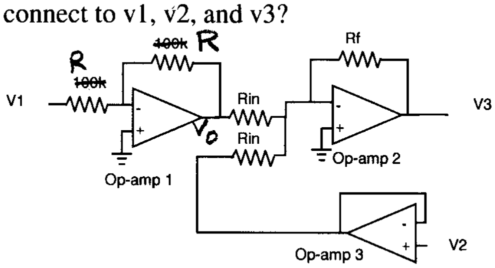
Problem 1c: Find the transfer function for this system

$$\frac{J}{\alpha}s\omega + K\omega = K\omega_d \qquad \omega\left(K + \frac{Js}{\alpha}\right) = K\omega_d \qquad \frac{\omega}{\omega_d} = \frac{K}{K + \frac{J}{\alpha}s} = G(s) = \frac{1}{1 + \tau_1 s}$$

$$\tau_1 = \frac{J}{K\alpha}$$

Problem 1d: Discus the system's behavior in the time and frequency domains
time domain → 1st order system $\omega = \omega_d(1 - e^{-t/\tau})$
frequency domain → 1st order low pass filter

Problem 1e: You will use the following circuit to implement P-control of the motor. What hardware would you connect to v1, v2, and v3?



$$\frac{V1}{R} = \frac{-V_0}{R} \Rightarrow V_0 = -V1$$

$$\frac{V_0}{R_{in}} + \frac{V_2}{R_{in}} = \frac{-V_3}{R_f} \Rightarrow V_3 = -\frac{R_f}{R_{in}}(V_2 - V_1)$$

V_3: input to amplifier; $\frac{R_f}{R_{in}}$: feedback gain; V_2: ω tach; V_1: ω_d signal generator

Important Ideas: feedforward and feedback (basic idea and limitations), how to implement feedback control

Mechanical Systems Laboratory: Lecture 6
Integral Control; Introduction to Second Order Systems

1. Integral Control

In lab this week you are building an op-amp circuit for controlling velocity of a motor using proportional feedback. To place this lab in a context, we imagined in the last lecture that you are designing a velocity control system for "Robbie the Rescue Robot". We created a Proportional-type controller for Robbie, and found that the controlled system dynamics were as follows:

$$\underbrace{J\dot{\omega}}_{\text{dynamics}} = \underbrace{\tau = \alpha V}_{\text{current amplifier/motor}} \qquad \underbrace{V = -K(\omega - \omega_d)}_{\text{FB controller}} \implies \frac{J}{\alpha}\dot{\omega} + K\omega = K\omega_d$$

where v= voltage input to current amplifier that powers Robbies motors, ω = actual angular velocity of wheels, sensed with a tachometer, ω_d = desired angular velocity, K = proportional feedback gain, alpha = proportionality constant relating v (i.e. current amplifier input) to torque output from motor.

Note that the steady-state error for this system is zero:

$$\dot{\omega} = 0 \text{ (steady state)} \rightarrow \omega = \omega_d$$

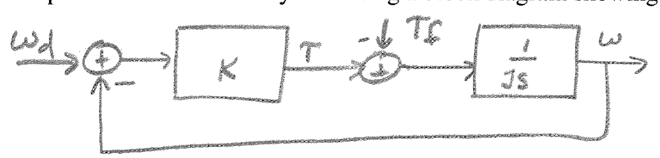
A more realistic model of Robbie's dynamics would include some friction in Robbie's wheels:

$$J\dot{\omega} = \tau - \tau_f \qquad \text{Assume } \tau_f = \text{constant (stiction)}$$

Let's assume that we control the torque to the motor directly, and express our control law in terms of torque. (Note, we actually control the current into the motor, but this is proportional to torque).

$$\tau = -K(\omega - \omega_d)$$

We can represent the combined system using a block diagram showing friction as disturbance.



Problem: Show that there is a steady-state error in velocity due to the friction.

$$\tau = -Ke \quad e = \omega - \omega_d$$

$$J\dot{\omega} = -Ke - \tau_f \qquad \text{In steady state } \dot{\omega} = 0 \implies e = \frac{-\tau_f}{K}$$

KEY IDEA: We can get rid of this steady-state error by using a proportional plus integral (PI) controller:

$$\tau = -K_p e - K_I \int e\,dt$$

$$J\dot{\omega} = \tau - \tau_f = -K_p e - K_I \int e\,dt - \tau_f$$

$$\frac{d}{dt}: \quad J\ddot{\omega} = -K_p\dot{e} - K_I e \qquad \text{Not } \ddot{\omega} = \ddot{e} \text{ if } \omega_d = \text{constant}$$

$$\rightarrow J\ddot{e} + K_p\dot{e} + K_I e = 0 \qquad e = \omega - \omega_d$$

$$\text{steady-state: } \dot{e} = \ddot{e} = 0 \qquad \dot{e} = \dot{\omega}$$

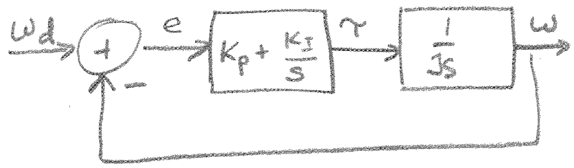
$$\implies e = 0 \qquad \ddot{e} = \ddot{\omega}$$

How does I control work? (try to explain it to your neighbor in words).

Integral control works in the following way:

If error e(t) does not equal zero, then $\int e(t)dt$ increases with time, and eventually the torque (which is proportional to this integral) becomes high enough to overcome friction.

The block diagram for a P-I compensator is:



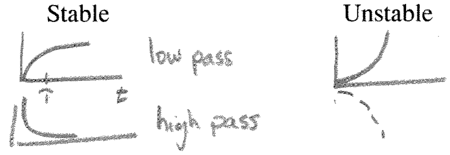
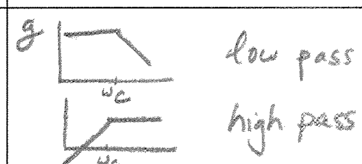
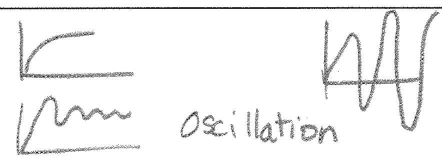
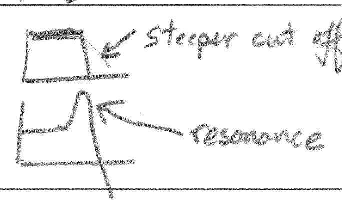
What is the transfer function for this system?

$$e = \omega_d - \omega$$
$$\tau = \left(K_p + \frac{K_I}{s}\right)e$$
$$\omega = \frac{1}{Js}\tau = \frac{1}{Js}\left(K_p + \frac{K_I}{s}\right)(\omega_d - \omega)$$

→ Solve for ω

$$\omega = \frac{K_p s + K_I}{Js^2 + K_p s + K_I}\,\omega_d$$

This is an example of second order system, which behaves differently than a first order system.

	Typical behaviors in time domain (step response)	Typical behaviors in frequency domain
First order system	Stable / Unstable 	
Second order system		

Important Ideas: integral control can help remove steady state error. However, I-control adds dynamics to the system, which can lead to non-1st-order phenomena such as oscillation and resonance.

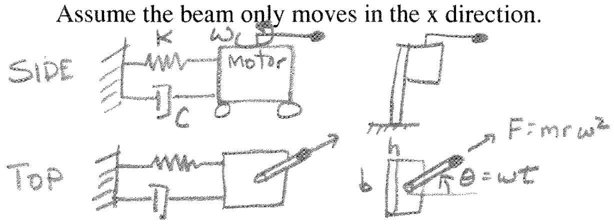
Mechanical Systems Laboratory: Lecture 7
Time and Frequency Response of Second Order Systems

1. A Common Second-Order System: A Mass-Spring-Damper System

In lab next week you will measure how a vibrating beam behaves in the time and frequency domains. The vibrating beam is an example of a system with a mass, some springiness, and some damping. Many physical systems have a mass, some springiness, and some damping, in different proportions. We can describe their behavior with a second order differential equation, and solve the equation to predict responses.

Modeling the Vibrating Beam

Assume the beam only moves in the x direction.



The force caused by the unbalanced load m in the x direction is: $F = mr\omega^2 \sin\theta = mr\omega^2 \sin(\omega t)$

So, we can use the unbalanced load to provide a sinusoidal force input into the beam.

What is K for the beam?

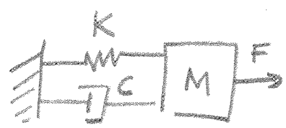
The load-deflection relationship for the beam (from any strength of materials book) is: $x = \frac{F\ell^3}{3EI}$

Where:

F = applied load
X = deflections of beam
E = modulus of elasticity
I = area moment of inertia of beam

For a spring: F=Kx, so K = $\frac{3EI}{\ell^3}$

So, a simple model is:



$M = M_{motor} + m + .236 M_{beam}$ (m = mass of unbalanced load; $.236 M_{beam}$ because beam has mass; can find using energy methods of vibration analysis)

$F = A \sin\omega t$ $\quad A = mr\omega^2$

ω = speed of motor

$C \approx 0$

The differential equation describing this system is:

$$M\ddot{x} = -kx - c\dot{x} + F$$

The transfer function for this system is:

$$(Ms^2 + cs + k)x = F \qquad \frac{x}{F} = H(s) = \frac{1}{Ms^2 + cs + k}$$

Note, for second order systems, we can write the denominator of the transfer function in a general form:

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

ω_n = "undamped natural frequency"
ζ = damping ratio

For the mass-spring-damper system, find the damping ratio and natural frequency

$$H(s) = \frac{\frac{1}{m}}{s^2 + \frac{c}{m}s + \frac{k}{m}} \Rightarrow \omega_n = \sqrt{\frac{k}{m}}$$

$$2\zeta\omega_n = \frac{c}{m} \Rightarrow \zeta = \frac{c}{2\sqrt{KM}}$$

(c ← actual damping; $2\sqrt{KM}$ ← "critical" damping)

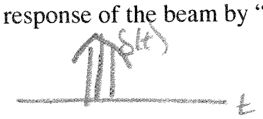
We will see if $\zeta < 1 \Rightarrow$ system is "underdamped" + oscillates

2. How does this system behave in the time domain?

In lab you will measure the transient response of the beam by "twanging" it. How does a system behave when you hit it with an impulse input?

Define $\delta(t) = 0$ for $t \neq 0$

$\int_{-\epsilon}^{\epsilon} \delta(t)\,dt = 1$ for $\epsilon > 0$



$\int_{-\infty}^{\infty} f(t)\delta(t)\,dt = f(0) \Rightarrow$ "sampling property of impulse"

$\delta(t) \longrightarrow$ [DEQ] $\longrightarrow y(t)$

$1 \longrightarrow$ [$H(s)$] $\longrightarrow Y(s) = H(s) \times 1 = H(s)$

L.T. of Impulse $\int_0^{\infty} \delta(t) e^{-st}\,dt = e^{-s(0)} = 1$

Thus, the inverse Laplace transform of the transfer function it the impulse response.

What is the impulse response of the vibrating beam?

Use a partial fraction expansion to find the inverse Laplace transform. Basic idea: write the transfer function as the sum of factors that we know how to take the Laplace transform of. Trick to find numerators: multiply by factor, chose s to set factor to zero.

$$H(s) = \frac{\frac{1}{m}}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\frac{1}{m}}{(s-p_1)(s-p_2)} = \frac{A}{s-p_1} + \frac{B}{s-p_2} \Rightarrow h(t) = Ae^{p_1 t} + Be^{p_2 t}$$

The poles of the transfer function are the zeros of the denominator, and they tell us a lot about the way the system behaves, because they became the exponents of exponentials in the time domain.

What are the poles of the vibrating beam?

quadratic eqn $\quad p_i = \dfrac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} \Rightarrow \begin{array}{l} p_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} \\ p_2 = \zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1} \end{array}$

$\zeta^2 < 1 \Rightarrow$ poles are imaginary $\Rightarrow$ oscillation

Use partial fraction expansion to find A and B:

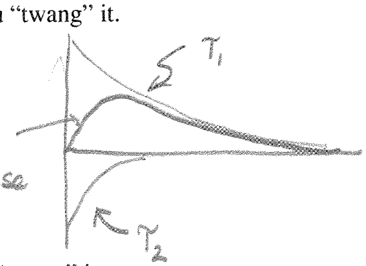
$$\left.\frac{\frac{1}{m}(s-p_1)}{(s-p_1)(s-p_2)} = \frac{A(s-p_1)}{(s-p_1)} + \frac{B(s-p_1)}{(s-p_2)}\right|_{s=p_1} \Rightarrow A = \frac{\frac{1}{m}}{p_1 - p_2}$$

Likewise: $B = \dfrac{\frac{1}{m}}{p_2 - p_1} = -A$

For $\zeta^2 > 1$ the poles are real, and the system does not oscillate when you "twang" it.

$$h(t) = Ae^{p_1 t} + Be^{p_2 t} = A\left(e^{-t/\tau_1} - e^{-t/\tau_2}\right)$$

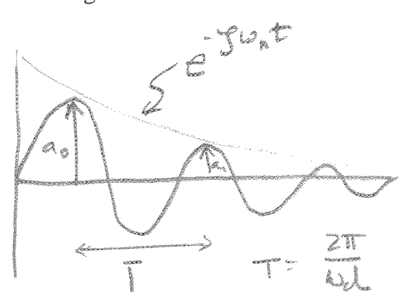
$\tau_1 = -\frac{1}{p_1} \quad \tau_2 = \frac{-1}{p_2}$



For $\zeta^2 < 1$ the poles are imaginary, and the system oscillates when you "twang" it.

$$p_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} = \zeta\omega_n + \underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}\, j$$

$$h(t) = \frac{1}{m\omega_d} e^{-\zeta\omega_n t} \sin \omega_d t$$



How do you measure damping given the impulse response? One way that you can estimate the damping is by using the "logarithmic damping method"

$$\frac{a_0}{a_n} = e^{-\zeta\omega_n T n}$$

$$\hat{\delta} \equiv \ln \frac{a_0}{a_n} \Rightarrow \text{solve for } \zeta = \frac{\hat{\delta}}{\sqrt{\hat{\delta}^2 + 4\pi^2 n^2}}$$

calculate $\hat{\delta} = \ln \frac{a_0}{a_n}$ ← measure (both a_0 and a_n)

n = # of peaks (start counting from zero!)

3. Frequency response of the beam

How does the system behave if you apply a sinusoidal force input to it?

$\sin \omega t \rightarrow \boxed{H(s)} \rightarrow |H(j\omega)| \sin(\omega t + \phi_H(j\omega))$

Scaling:

Assume $c = 0$

$M\ddot{x} + Kx = F \xrightarrow{\mathcal{L}} (Ms^2 + K)X = F$

$$H(s) = \frac{X}{F} = \frac{1}{Ms^2+K} = \frac{1/m}{s^2 + K/m} = \frac{\frac{1}{m}}{s^2+\omega_n^2} \qquad \omega_n = \sqrt{\frac{K}{m}}$$

$$H(j\omega) = \frac{\frac{1}{m}}{(j\omega)^2 + \omega_n^2} = \frac{\frac{1}{m}}{\omega_n^2 - \omega^2}$$

Phase shift:

$$\phi_H(j\omega) = 0 - \tan^{-1}\frac{0}{\omega_n^2 - \omega^2} = \begin{cases} 0^\circ & \omega^2 < \omega_n^2 \\ 180^\circ & \omega^2 > \omega_n^2 \end{cases}$$

$$|H(j\omega)| = \frac{\frac{1}{m}}{|\omega_n^2 - \omega^2|}$$

Plot on a log-log scale (makes curves into lines)

a) Asymptote 1: for $\omega \ll \omega_n$ $|H(j\omega)| \approx \frac{\frac{1}{m}}{\omega_n^2}$ = constant

b) Asymptote 2: for $\omega \gg \omega_n$ $|H(j\omega)| \approx \frac{\frac{1}{m}}{\omega^2} \Rightarrow 20\log|H(j\omega)| = 20\log\frac{1}{m} - 20\log\omega^2$

$$= 20\log\frac{1}{m} - 40\log\omega$$

$$y = b + mx$$

c) $\omega = \omega_n \Rightarrow |H(j\omega)| = \infty$

Resonance

What are some uses of resonance?

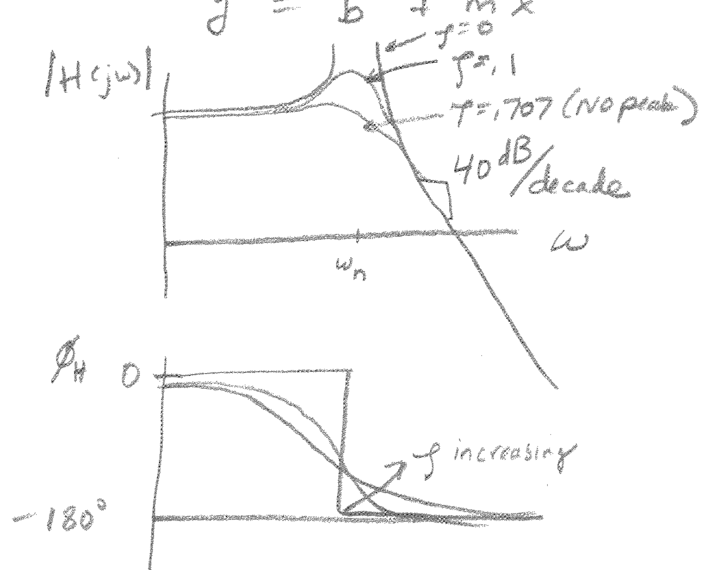
the military (star wars)

communication (detecting 1 frequency)

entertainment (musical instruments)

medicine (shattering a kidney stone w/ ultrasound)

the playground (swing set)



Mechanical Systems Laboratory: Lecture 8
Brief Review of Stability; PD Position Control of a Robot Arm

1. Brief Review of Stability

Stability refers to the concept of whether a system's performance "blows up" or converges to some value.
What are some applications in which stability analysis is very important?

Airplanes, Spaceships, Active suspensions for Cars, Surgery Robots

Three types of stability are:

stable, unstable, marginally stable (sustained oscillations)

The location of the poles of the transfer function determine the type of stability.
Why? Consider a second-order system:

$$H(s) = \frac{1}{s^2+as+b} = \frac{1}{(s-p_1)(s-p_2)} = \frac{A}{s-p_1} + \frac{B}{s-p_2} \qquad p_i = \text{"poles"}$$

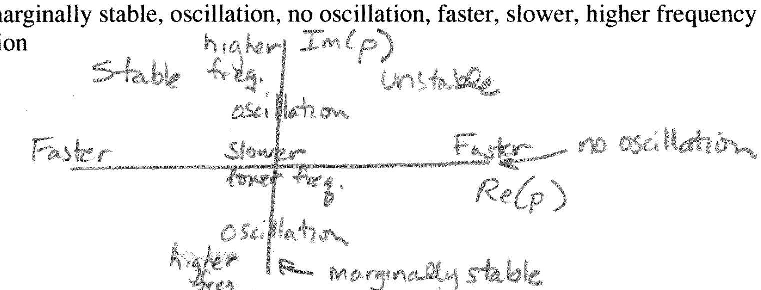
Remember, the inverse Laplace transform of the transfer function is the impulse response:

$$h(t) = Ae^{p_1 t} + Be^{p_2 t}$$

Case 1: $\mathrm{Re}\{p_i\} < 0$	stable	exponential decay; if $\lvert \mathrm{Re}(p_2)\rvert > \lvert \mathrm{Re}(p_1)\rvert$, $e^{p_2 t}$ decays more quickly
Case 2: $\mathrm{Re}\{p_i\} = 0$	marg. stable	if $\mathrm{Im}\{p_i\} \neq 0 \Rightarrow$ sustained oscillation
Case 3: $\mathrm{Re}\{p_i\} > 0$	unstable	exponential blow-up (w/ oscillation if $\mathrm{Im}\{p_i\} \neq 0$)

So the location of the poles in the complex plane determines the type of response of the system.

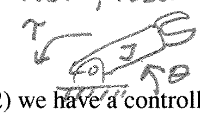
Exercise 1: Label the complex plane with the following words:
stable, unstable, marginally stable, oscillation, no oscillation, faster, slower, higher frequency oscillation, lower frequency oscillation



2. PD Position Control of a Robot Arm (P = proportional, D = Derivative)

Position control – most common industrial control system
Can you think of some applications? radar, robot arm, NC milling maching, manufacturing

Consider a one-joint robot arm:



Assume: 1) no friction or gravity; 2) we have a controller that can apply any torque that we want; 3) we can sense θ (for example, with a potentiometer)

Exercise 2: Design a proportional feedback controller to position the robot arm at $\theta=\theta_d$, find its transfer function, and analyze its stability

P-Control $\tau = -K_p(\theta-\theta_d)$

Robot Dynamics $\tau = J\ddot{\theta}$

Overall System Dynamics $J\ddot{\theta} = -K_p(\theta-\theta_d)$

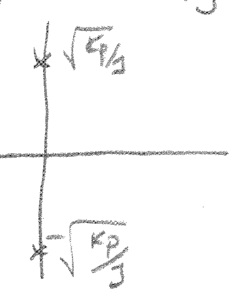
$$J\ddot{\theta} + K_p\theta = K_p\theta_d$$

Transfer Fun: $\frac{\theta(s)}{\theta_d(s)} = \frac{K_p}{Js^2+K_p} = \frac{K_p/J}{s^2+K_p/J}$

→ Poles: $s^2 = \frac{-K_p}{J} \Rightarrow s = \pm\sqrt{\frac{K_p}{J}}\, j$

Same form as undamped mass-spring system

Robot would oscillate forever



Exercise 3: Design a way to fix the problem. What kind of hardware would you need?

Add damping w/ control law: $\tau = -K_p(\theta - \theta_d) - K_v\dot{\theta}$] Proportional-Derivative Control

Two approaches to sensing angular velocity:
1) use a tachometer
2) differentiate position (e.g. using an op-amp circuit)

What are the dynamics and transfer function of the robot with the new controller?

$$J\ddot{\theta} = -K_p(\theta - \theta_d) - K_v\dot{\theta}$$
$$J\ddot{\theta} + K_v\dot{\theta} + K_p\theta = K_p\theta_d$$
$$\frac{\theta(s)}{\theta_d(s)} = H(s) = \frac{K_p}{Js^2 + K_v s + K_p}$$

We choose K_p + K_v when we design our controller!

How are the gains K_p and K_v related to the natural frequency and damping ratio?

$$H(s) = \frac{K_p/J}{s^2 + \frac{K_v}{J}s + \frac{K_p}{J}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n = \sqrt{\frac{K_p}{J}}$ K_p determines ω_n

$\zeta = \frac{K_v}{2\sqrt{K_p J}}$ After choosing K_p based on desired ω_n, can set damping ratio w/ K_v

What is the step response of the system?

$\zeta = 1$ $\quad \theta(t) = 1 - \cos(\omega_n t)$

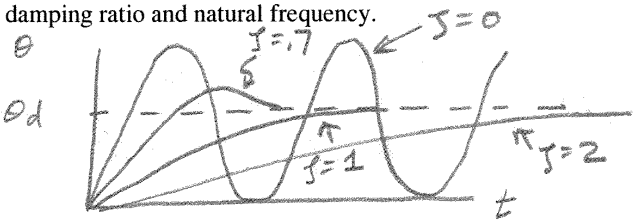
$0 < \zeta < 1$ $\quad \theta(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta})$ $\quad \omega_d = \omega_n\sqrt{1-\zeta^2}$

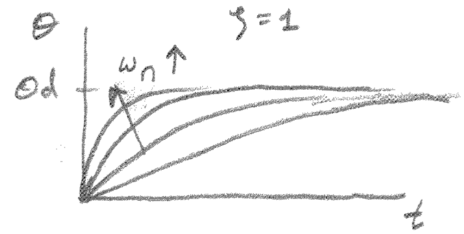
$\zeta = 1$ $\quad \theta(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$

$\zeta > 1$ $\quad$ sum of two exponentials – see book; can neglect one exponential if $\zeta > 2$

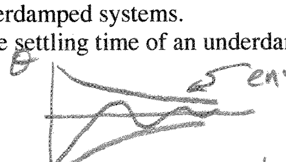
Note: the damping ratio determines whether the system oscillates (i.e. whether the poles have an imaginary part)

Exercise 4: Plot the step response of the system of the system would look like for different values of the damping ratio and natural frequency.





Notes: Overdamped systems are "sluggish"
Among systems responding without overshoot, critically damped systems exhibit the fastest response.
Underdamped systems with $0.5 < \zeta < .8$ get close to final value more rapidly than critically damped or overdamped systems.
The settling time of an underdamped (or critically damped) system is:



$e^{-t/\tau}$ is w/in 5% of final value after 3τ secs

$t_s\big|_{5\% \text{ criterion}} \approx 3\tau = \frac{3}{\zeta\omega_n}$ (minimized at $\zeta = .68$)

Exercise 5: Given a one-joint robot arm (no friction, no gravity) with J = 1 kgm². Design a PD position controller such that the robot finishes 95% of a commanded step-function movement in .5 seconds, with no overshoot.

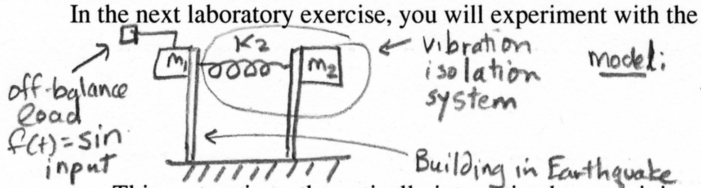
$t_s = \frac{3}{\zeta\omega_n} = .5$ $\quad \zeta = 1$ (no overshoot) $\Rightarrow \omega_n = \frac{3}{.5} = 6.0$ rad/sec

$\omega_n = \sqrt{\frac{K_p}{J}} \Rightarrow K_p = \omega_n^2 J = 36 \frac{kg\,m^2}{s^2}$ $\quad \zeta = \frac{K_v}{2\omega_n J} \Rightarrow K_v = 2\omega_n J\zeta = 12 \frac{kg\,m^2}{sec}$

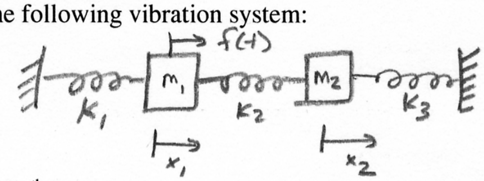
Mechanical Systems Laboratory: Lecture 9
Systems with Two Modes of Vibration/Design of a Vibration Isolator

1. Experimental Apparatus and Relationship to Vibration Isolation

In the next laboratory exercise, you will experiment with the following vibration system:

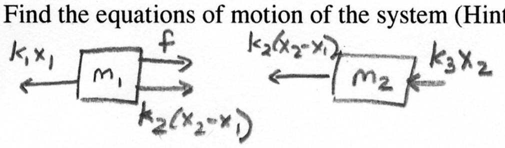


model:



- This system is mathematically interesting because it is a 4[th] order system, and because it has a zero in the transfer function.
- This system is practically interesting because it works much the same way that real vibration isolation systems do, such as ones used in washing machines or to stabilize a building in an earthquake. The key observation is that m1 doesn't move, even though it is being forced with a sinusoidal force, if m2 and k2 are chosen appropriately. The sinusoidal force could represent an off-balance load in a washing machine, or the forces from an earthquake on a building. Appropriate choice of m2 and k2 can stop the washing machine or building from shaking.

Find the equations of motion of the system (Hint: Use a free-body diagram and assume x2>x2>0)



$$m_1\ddot{x}_1 = f + k_2(x_2-x_1) - k_1x_1$$
$$m_2\ddot{x}_2 = -k_2(x_2-x_1) - k_3x_2$$

We can express these equations in matrix format:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix} \quad \text{or} \quad M\ddot{\underline{x}} + K\underline{x} = \underline{f}$$

(M, K: 2x2 Matrices)

The transfer function of the system is then expressed as follows:

To find x(s), we need to find $(ms^2+K)^{-1}$

$$(Ms^2+K)\underline{X} = \underline{f} \Rightarrow \underline{x} = (Ms^2+K)^{-1}\underline{f}$$

Fact: if $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, $A^{-1} = \frac{1}{\det A}\begin{bmatrix} c & -b \\ -b & a \end{bmatrix}$, $\det A = ac - b^2$

so

$$[Ms^2+K]^{-1} = \underbrace{\frac{1}{(m_1s^2+k_1+k_2)(m_2s^2+k_2+k_3)-k_2^2}}_{\text{"DEN"}}\begin{bmatrix} m_2s^2+k_2+k_3 & k_2 \\ k_2 & m_1s^2+k_1+k_2 \end{bmatrix}$$

Now, suppose f(t) = sin ωt and we want to find $x_1(t)$. To do this, we need to find the frequency response:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} f \\ 0 \end{bmatrix} \Rightarrow x_1 = m_{11}f \Rightarrow \left.\frac{x_1}{f}\right|_{s=j\omega} = \frac{-m_2\omega^2+k_2+k_3}{(-m_1\omega^2+k_1+k_2)(-m_2\omega^2+k_2+k_3)-k_2^2}$$

At what input frequency will x1 not move?

when $-m_2\omega^2+k_2+k_3=0 \Rightarrow \frac{x_1}{f}=0$ (zero of transfer fcn) $\Rightarrow \omega_0 = \sqrt{\frac{k_2+k_3}{m_2}}$

What happens to m2 in this situation? $x_2 = m_{21}f = \frac{k_2}{DEN}f \Rightarrow m_2$ shakes, m_1 stays still

Assume k3 = 0 (like in a washing machine vibration isolation system). What is another name for the frequency at which the vibration isolation is achieved?

$\omega_0 = \sqrt{\frac{k_2}{m_2}} \Rightarrow$ resonant frequency of $\{k_2, m_2\}$

The system also has two resonant frequencies. Find the resonant frequencies.

"DEN" has form $a\omega^4 + b\omega^2 + c$

so $\omega_1^2, \omega_2^2 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = m_1 m_2$

$b = -(m_2(k_1+k_2) + m_1(k_2+k_3))$

$c = k_1 k_2 + k_1 k_3 + k_2 k_3$

How do the masses move at the resonant frequencies? We can gain insight by considering the case of free vibrations (i.e. f(t) = 0).

$M\ddot{\underline{x}} + K\underline{x} = 0 \quad (1)$

Assume $\underline{x} = \underline{x}_0 \sin(\omega t + \phi)$ $\quad \underline{x}_0 = \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix}$

plugging into (1):

$(-M\omega^2 + K)\underline{x}_0 = 0 \Rightarrow \underbrace{(M^{-1}K - \omega^2 I)}_{B}\underline{x}_0 = 0$

→ If B is invertible, only soln is $x_0 = B^{-1}0 = 0$

∴ for $\underline{x}_0 \neq 0$ to exist B must be singular

Note: this is an eigenvalue problem:

$(M^{-1}K)x_0 = \omega^2 x_0$

eigenvector (x_0) — eigenvalue (ω^2)

For what values of ω is B singular?

$$B = M^{-1}K - \omega^2 I = \begin{bmatrix} \frac{k_1+k_2}{m_1} - \omega^2 & \frac{-k_2}{m_1} \\ \frac{-k_2}{m_2} & \frac{k_2+k_3}{m_2} - \omega^2 \end{bmatrix}$$

B singular ⇒ det B = 0

$$\Rightarrow \left(\frac{k_1+k_2}{m_1} - \omega^2\right)\left(\frac{k_2+k_3}{m_2} - \omega^2\right) - \frac{k_2^2}{m_1 m_2} = 0$$

$$\Rightarrow \underbrace{m_1 m_2 \omega^4 - (m_2(k_1+k_2) + m_1(k_2+k_3))\omega^2 + k_1k_2 + k_1k_3 + k_2k_3 = 0}_{\text{"Characteristic Eqn"}}$$

Notes:

det(B) = 0 ⇔ denominator of transfer function = 0

roots of characteristic equation ⇔ poles of transfer function

free response frequencies ⇔ resonant frequencies

at resonant frequencies it is possible to have free response of non-zero amplitudes

At the resonant frequencies, what is x0? (the amplitude of the free response)

$$B\underline{x}_0 = 0 = \begin{bmatrix} \frac{k_1+k_2}{m_1} - \omega^2 & \frac{-k_2}{m_1} \\ -\frac{k_2}{m_2} & \frac{k_2+k_3}{m_2} - \omega^2 \end{bmatrix}\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix}$$

Top Row says:

$$\left(\frac{k_1+k_2}{m_1} - \omega^2\right)x_{10} - \frac{k_2}{m_1}x_{20} = 0$$

$$\Rightarrow x_{10} = \frac{\frac{k_2}{m_1}}{\frac{k_1+k_2}{m_1} - \omega^2}x_{20}$$

Bottom Row says:

$$x_{20} = \frac{\frac{k_2}{m_2}}{\frac{k_2+k_3}{m_2} - \omega^2}x_{10}$$

Example: $m_1 = m_2 = k_1 = k_2 = k_3 = 1$

char eqn: $\omega^4 - 4\omega^2 + 3 = 0$

roots: $\omega^2 = \dfrac{4 \pm \sqrt{16-12}}{2} = 2 \pm 1 = 1 \text{ or } 3$

at $\omega_1^2 = 1$ $\quad x_{10} = \dfrac{1}{2-1}x_{20} = x_{20}$ IN phase

at $\omega_2^2 = 3$ $\quad x_{10} = \dfrac{1}{2-3}x_{20} = -x_{20}$ 180° out of Phase

Isolation Freq: $\omega_0 = \sqrt{\dfrac{k_2+k_3}{m_2}} = \sqrt{2}$

$\omega_1 < \omega_0 < \omega_2$

General free vibration contains both modes.

Mechanical Systems Laboratory: Lecture 10
Data Acquisition; Computer-Based Feedback Control

Note: These notes are derived from Ch. 8 Data Acquisition, Introduction to Mechatronics and Measurement Systems, 2nd Edition, David G. Alciatore and Michael B. Histand, McGraw-Hill 2003

1. Experimental Apparatus

For the next laboratory exercise, you will use a computer to control a motor. Up until now, you have used op-amp circuits as analog computers to implement the computations you need for feedback control. Another common way to implement controllers is digitally by using computers. A common set-up is:

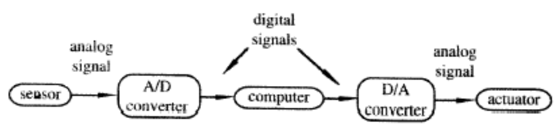


Figure 8.13 Computer control hardware.

The computer could be:

- a PC with a data acquisition card installed. A data acquisition card is sort of like a "video card", except it inputs and outputs arbitrary analog signals instead of a video signals. The Labjack is essentially a data acquisition card that communicates with the computer through the USB port.
- a microcontroller, which is a computer on a single chip. A digital signal processing chip is similar to a microcontroller.
- a programmable logic controller (PLC), which is a specialized industrial device for interfacing to analog and digital devices. PLC's are typically programmed with ladder logic, which is a graphical language for connecting inputs, outputs, and logic.
- Digital circuits, made with logic gates (e.g. AND, OR, NOT gates), or programmable logic arrays, which allow you to set-up arrays of logic gates.

2. Sampling, the Nyquist Frequency, and Aliasing

Many types of sensors (e.g. potentiometers, tachometers, accelerometers, force transducers) provide analog (i.e. continuous) voltage outputs, and many types of actuators (e.g. dc brushed motors) require analog inputs. Computers represent numbers using sequences of digital voltages (i.e. sequences of "bits"). Digital voltages (or "bits") can take only two discrete values, logical 0 (typically corresponding to 0 volts) and logical 1 (typically corresponding to 5 volts). Getting analog signals into digital forms usable by computers requires two processes: sampling and quantization.

Sampling refers to evaluating an analog signal at discrete instants in time. The sampling frequency (or sampling "rate") is how many times per second the signal is sampled.

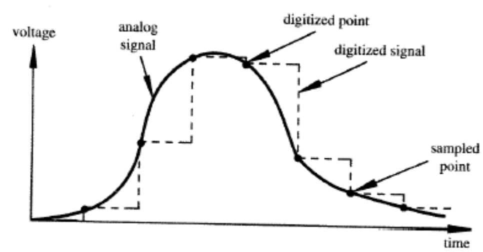


Figure 8.1 Analog signal and sampled equivalent.

The sampling theorem states that you must sample a signal at a frequency that is twice the maximum frequency in the signal (i.e. at the "Nyquist Frequency"), in order to preserve all of the information in the signal. If a signal is sampled at less than this frequency, "aliasing" happens. The result of aliasing is that a high frequency signal looks like a lower frequency signal.

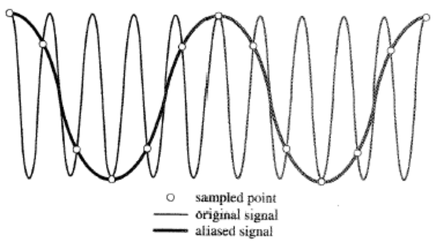
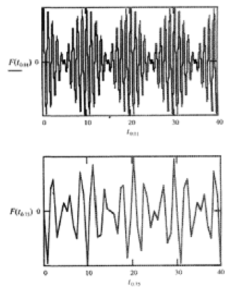
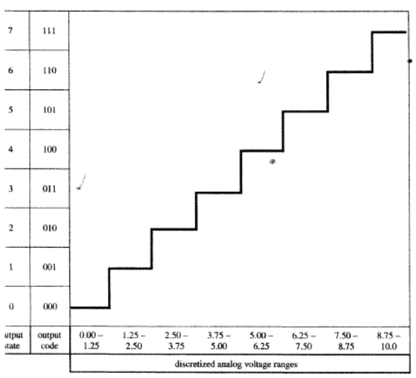


Figure 8.2 Aliasing.



3. Quantizing Theory

Quantizing transforms a continuous, analog input into a set of discrete output states. Coding is the assignment of a digital code word or number to each output state.



4. Analog-to-Digital Conversion (A/D)

An A/D converter quantizes an analog signal at some sampling rate, which is determined by a "trigger signal" from the computer. The resolution of the A/D converter is the number of bits that is uses to represent the analog value of the input. The number of possible states N is equal to the number of bit combinations that can be output from the converter: $N=2^n$. Most commercial A/D converters are 8, 10, or 12 bit devices that resolve 256, 1024, and 4096 output states, respectively. Here is a flash AD converter:

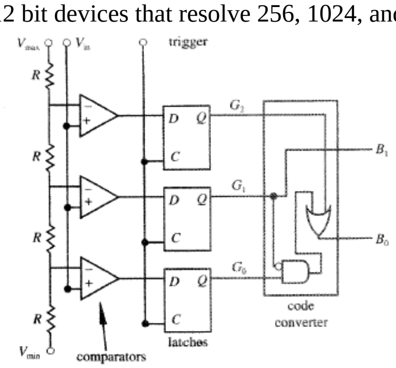


Figure 8.10 A/D flash converter.

Table 8.1 2-bit flash converter output

State	Code ($G_2G_1G_0$)	Binary (B_1B_0)	Voltage range
0	000	00	0–1
1	001	01	1–2
2	011	10	2–3
3	111	11	3–4

5. Digital-to-Analog (D/A) Conversion

A D/A converter takes the binary representation of a signal and converts it into an analog output signal.
A ladder D/A Converter works like this:

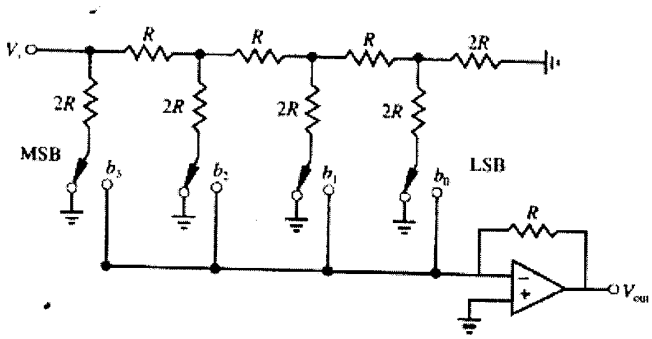


Figure 8.11 4-bit resistor ladder D/A converter.

7. Effect of Sampling Rate on Control Stability

Sampling introduces delays into a control system. If the sampling rate is high enough, the delay is negligible. But if sampling rate is low (e.g. < 100Hz for a robot), then the associated delay can make the control system unstable, especially for large feedback gains. Delay essentially causes "the right information" to be delivered at the wrong time. As an example, consider a proportional feedback control of a first-order system (such as the motor velocity control lab that you did). When there is no delay in this system, the system is stable for all positive values of the gain. What happens when we add delay?

Model of Motor

$M = J\dot{\omega} + B\omega$ (M: torque; J: inertia; B: friction; ω: angular velocity) $\Longrightarrow$ Transfer fcn

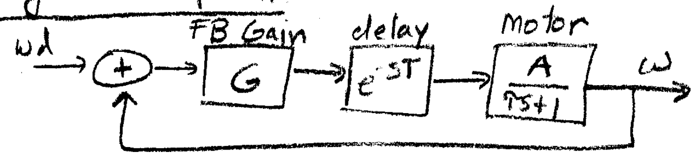
$$\frac{\omega}{M} = \frac{1}{Js+B} = \frac{\frac{1}{B}}{\frac{J}{B}s+1} = \frac{A}{\tau s+1} \qquad \tau = \frac{J}{B} \text{ (time constant)}$$

$\therefore$ motor acts like a low pass filter

How to Model Delay due to Sampling?

Note: $\mathcal{L}(x(t-T)) = e^{-sT}\mathcal{L}(x(t))$ (delay by T)

Velocity Control System:



Solve for T.F. $\Rightarrow \omega = \frac{A}{\tau s+1} e^{-sT} G(\omega_d - \omega)$

Note: $e^{-sT} = 1 - sT + \frac{1}{2}(sT)^2 + \ldots$ } Taylor's series

$\approx 1 - sT$ if T small

$$\Rightarrow \omega = \frac{A}{\tau s+1}(1-sT)G(\omega_d - \omega)$$

$$\Downarrow$$

$$\frac{\omega}{\omega_d} = \frac{AG(1-sT)}{1+AG+(\tau - AGT)s}$$

1st order system

Pole at $s = \frac{-(1+AG)}{\tau - AGT}$

Pole stays in left half plane (& system remains stable) if

$\tau - AGT > 0 \Rightarrow G < \frac{\tau}{AT}$ Gain is limited!

SCIENCE AND TECHNOLOGY

Robomoths

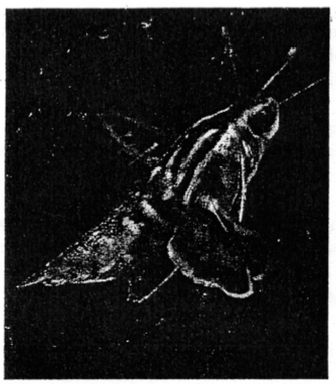
INSECTS are not nearly as biddable as dogs or horses. Although they can perform amazing feats of strength and dexterity on their own scale, that scale is so much smaller than humanity's that it is not surprising they have been overlooked. With rare exceptions, such as bees and silkworms, the insect world is a source of pests rather than of pets or pack animals.

In an age of miniaturisation, however, a few researchers are wondering if more insects might be harnessed to the service of man. One is John Hildebrand, a neurobiologist at the University of Arizona. As part of a project run by America's Defence Advanced Research Projects Agency (DARPA), he and his colleagues have been working with the giant sphinx moth to create a "biobot"—an animal that can be controlled electronically by a human. They have designed a radio transmitter small enough to attach to a sphinx moth without impairing its ability to fly. The next stage is to add a receiver to tell the moth where to go.

Moths may not be that bright, but Dr Hildebrand believes they can be manipulated in rather the same way as a donkey is by dangling a carrot in front of its head. The "carrot" he proposes is a sex pheromone—a mixture of chemicals that female sphinx moths give off to attract males. It is potent stuff. Previous research has shown that a few molecules are enough to attract a male's attention, and that, given a favourable wind, an amorous male can find a mate who is several kilometres away.

One of the team's ideas is to fit male sphinx moths with small, radio-controlled pheromone dispensers. A moth's pheromone-detectors are its antennae. It can work out where pheromone molecules are coming from by comparing the signals from each antenna, in the same way that a person works out the direction of a sound by comparing signals from each ear. A moth's senses could be subverted by puffing suitable molecules from a dispenser to steer it towards a chosen target.

That is a rather crude approach. Dr Hildebrand hopes to be more subtle. He has spent much of his career examining how a moth's nervous system responds to the pheromone, and he thinks he knows enough to steer a moth directly, without the need for the chemicals themselves. He plans to do it by attaching electrodes to the nerves involved and stimulating them appropriately—turning the moth into a genuine, radio-controlled biobot.



Now, where's my backpack?

That would be an interesting demonstration of mankind's powers over nature. Could it also be useful? Brian Smith and colleagues at Ohio State University have recently shown that sphinx moths can be manipulated like dogs as well as donkeys. Pavlov's early experiments on reflexes trained dogs to salivate at the sound of a bell, by ringing one every time they were fed with meat. Dr Smith's team has mimicked Pavlov by training moths to stick their tongues out in response to a chemical called cyclohexanone, which was puffed at them while they were fed sugared water.

The reason that this trick might be useful—and the reason for DARPA's interest—is that cyclohexanone is a volatile component of TNT, an explosive often used in landmines. By releasing a swarm of trained moths over a minefield, and observing where they stuck their tongues out, it should be possible to locate mines without risk either to people or to expensive mine-detecting machinery.

It is hard to see if a moth is sticking its tongue out at a range of several hundred metres. But Dr Smith has thought of a way round that. He can sense when a moth is blowing a raspberry by attaching a wire to the muscle that controls the insect's tongue, and using it to transmit a signal via one of Dr Hildebrand's tiny electronic backpacks.

If lepidopteran mine detectors work they could be the start of a new indust The rate at which video cameras are bein miniaturised means that they, too, may soon be light enough for insects to carry. That would have obvious military applications, even if one countermeasure is obvious, too: surrounding sensitive installations with giant candles.

4.3 Professor Bobrow lecture notes given to use as reference

University of California, Irvine
Department of Mechanical and Aerospace Engineering
ME106 Course Notes from Prof. J Bobrow

Background Notes

This class has a combination of mathematical analysis and experimentation. It is important for you to understand the mathematical analysis if you wish to improve upon an existing design. Most designs have weaknesses that can be corrected if one understands both the theoretical and practical aspects of their operation. It is also important for you to be able to experimentally test a given design, otherwise, you would not understand its practical limitations. We begin with the most important mathematical concepts which will be used throughout the quarter.

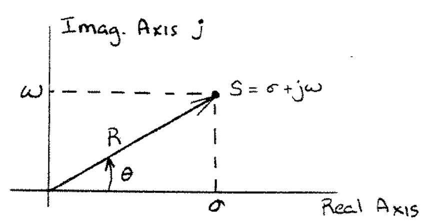
1 Complex Variables

Much of the analysis in this class will be in the *frequency domain* rather than *time domain.* A function $f(t)$ is transformed from time to frequency domain using the Laplace transform. In frequency domain analysis, the independent variable is complex, and has the form

$$s = \sigma + jw, \tag{1}$$

where both σ and ω are real numbers, and $j \equiv \sqrt{-1}$ or $j^2 = -1$. We see that s has a real part, σ, and an imaginary part ω. Often we write $\text{Re}(s) = \sigma$ and $\text{Im}(s) = \omega$. A function of a complex variable $F(s)$ is also complex, and we often need its real and imaginary parts $\text{Re}(F(s))$, and $\text{Im}(F(s))$.

It is helpful to think of a complex variable as a two dimensional vector in a plane, or *s-plane* as shown in figure below.



If we say that R is the length of the vector s, then $R^2 = \sigma^2 + \omega^2$, or equivalently, $R = |s|$. The beauty of this representation is that we can define the same point as

$$s = Re^{j\theta}, \text{ where } \theta = \tan^{-1}\left(\frac{\omega}{\sigma}\right). \tag{2}$$

To show that this representation works, recall the definition

$$e^{j\theta} = \cos\theta + j\sin\theta. \tag{3}$$

Hence, $s = Re^{j\theta} = R\cos\theta + jR\sin\theta = \sigma + j\omega$ from geometry shown in the figure. Sometimes we also write $s = R\angle\theta$ for shorthand. Also note that $\text{Re}(s) = \sigma$ and $\text{Im}(s) = \omega$ are both non-imaginary real numbers.

1

1.1 Multiplication and Division

Suppose we have two complex numbers s_1 and s_2, that we would like to multiply or divide. Using the above notation,

$$s_1 = R_1 e^{j\theta_1} = R_1 \angle \theta_1$$

$$s_2 = R_2 e^{j\theta_2} = R_2 \angle \theta_2,$$

then

$$s_1 s_2 = R_1 e^{j\theta_1} R_2 e^{j\theta_2} = R_1 R_1 e^{j(\theta_1 + \theta_2)}$$

or

$$s_1 s_2 = R_1 \angle \theta_1 R_2 \angle \theta_2 = R_1 R_2 \angle (\theta_1 + \theta_2).$$

Similarly,

$$\frac{s_1}{s_2} = \frac{R_1 \angle \theta_1}{R_2 \angle \theta_2} = \frac{R_1}{R_2} \angle (\theta_1 - \theta_2)$$

since,

$$\frac{R_1 e^{j\theta_1}}{R_2 e^{j\theta_2}} = \frac{R_1}{R_2} e^{j(\theta_1 - \theta_2)}.$$

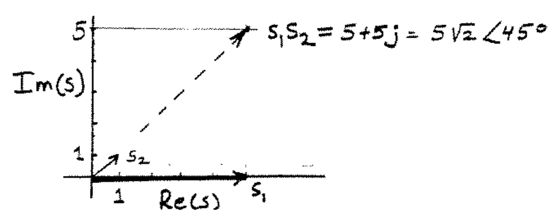
Note that the hard way is to multiply components as

$$\begin{aligned} s_1 s_2 &= (\sigma_1 + j\omega_1)(\sigma_2 + j\omega_2) \\ &= \sigma_1\sigma_2 + j\omega_2\sigma_1 + j\omega_1\sigma_2 + j2\omega_1\omega_2 \\ &= (\sigma_1\sigma_2 - \omega_1\omega_2) + j(\omega_2\sigma_1 + \omega_1\sigma_2). \end{aligned}$$

Then the real part, $\text{Re}(s_1 s_2) = \sigma_1\sigma_2 - \omega_1\omega_2 = R_1 R_2 \cos(\theta_1 + \theta_2)$, and the imaginary part, $\text{Im}(s_1 s_2) = \omega_2\sigma_1 + \omega_1\sigma_2 = R_1 R_2 \sin(\theta_1 + \theta_2)$.

Example 1 multiplication $s_1 s_2$

Suppose $s_1 = 5 = 5\angle 0$, and $s_2 = 1 + j = \sqrt{1^2 + 1^2}\angle 45^o$. This can be interpreted as multiplication of a vector by a scalar, so the direction should not change. Now, $s_1 s_2 = 5(1 + j) = 5 + 5j$. Also, $s_1 s_2 = (5\angle 0)(\sqrt{2}\angle 45^o) = 5\sqrt{2}\angle 45^o$. The sketch below shows that they are the same numbers.



Example 2

Suppose $s_1 = 3j = 3\angle 90^o$, and $s_2 = -3j$. First note that $s_2 = 3\angle -90^o = 3\angle 270^o$, but that $s_2 \neq -3\angle 90^o$ since the coefficient of the exponent must always be a positive number (why?). Then, $s_1 s_2 = (3j)(-3j) = 9$, or

$$s_1 s_2 = (3\angle 90^o)(3\angle -90^o) = 9\angle 0 = 9\angle 360^o = 9.$$

1.2 Evaluating Functions of s Using Poles and Zeros (Skip this section on first read)

The most important use of the vector representation is for the evaluation of functions of s at certain points. A function of s, say $G(s)$, will also have an imaginary and a real part which will need to be computed for a given value of s. We will do this many times in later experiments. For instance, a second order rational polynomial in s might have the form

$$G(s) = \frac{as+b}{s^2+cs+d},$$

which can also be written in factored form as

$$G(s) = \frac{(s-s_o)k}{(s-s_1)(s-s_2)}$$

where $s_o = -\frac{b}{a}$, $k = a$, and $s_{1,2} = \frac{-c\pm\sqrt{c^2-4d}}{2}$.

The value s_o is called a *zero* of $G(s)$ since $|G(s_o)| = 0$. The values s_1 and s_2 are called *poles* of $G(s)$ since $|G(s_1)| = |G(s_2)| = \infty$. If we plot the points s_o, s_1, and s_2 on the s plane, we can evaluate $G(s)$ graphically.

For example, suppose

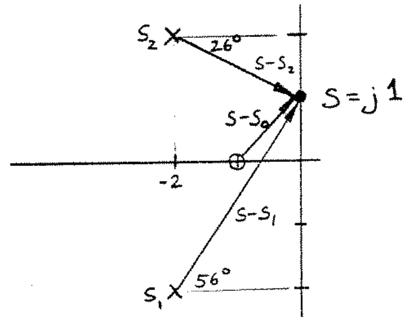
$$G(s) = \frac{s+1}{(s+2+2j)(s+2-2j)} = \frac{s+1}{(s+2)^2+4} = \frac{s+1}{s^2+4s+8},$$

which is of form

$$G(s) = \frac{s-s_o}{(s-s_1)(s-s_2)},$$

where $s_o = -1$, $s_1 = -2-2j$, $s_2 = -2+2j$.

Say we want to evaluate $G(j1)$, i.e. the value of G at $s = j$. We can either plug $s = j$ into $G(s)$ and do some algebra, or we can use the poles and zeros of $G(s)$, and graphically compute $G(j)$ from a *pole-zero* plot:



The figure shows that $s-s_o$ can also be thought of as a vector whose tip is at s and tail is at s_o. Then, at $s = j$, $s-s_o = j - -1 = 1+j$, $s-s_1 = j-(-2-2j) = 2+3j$, and $s-s_2 = j-(-2+2j) = 2-j$. Now using the exponential form of each factor we have:

$$G(s) = \frac{|s-s_o|\angle(s-s_o}{(|s-s_1|\angle(s-s_1)(|s-s_2|\angle(s-s_2)}$$

$$
\begin{aligned}
G(s=j) &= \frac{\sqrt{2}\langle 45^\circ}{\left(\sqrt{13}\langle \underbrace{\tan^{-1}\frac{3}{2}}_{56.31^\circ}\right)\left(\sqrt{5}\langle -\underbrace{\tan^{-1}\frac{1}{2}}_{26.56^\circ}\right)} \\
&= \sqrt{\frac{2}{13\cdot 5}}\langle 45^\circ - 56.31^\circ + 26.56^\circ \\
&= \sqrt{\frac{2}{65}}\langle 15.25^\circ
\end{aligned}
$$

Finally, we could have alternatively evaluated $G(s=j)$ algebraically as

$$
\begin{aligned}
G(j) = \left.\frac{s+1}{s^2+4s+8}\right|_{s=j} &= \frac{1+j}{-1+4j+8} = \frac{1+j}{7+4j} \\
&= \frac{1+j}{7+4j}\left(\frac{7-4j}{7-4j}\right) = \frac{7-4j+7j+4}{49+16} \\
&= \frac{11}{65}+\frac{3j}{65} = 0.169 + j0.046
\end{aligned}
$$

Note that the two methods are completely identical, since

$$
\sqrt{\frac{2}{65}}\langle 15.25^\circ = 0.1754\,(\cos\ 15.25 + j\sin\ 15.25) = 0.169 + j0.046.
$$

The most appropriate method to use to evaluate functions $G(s)$ at given points s will usually be clear from the problem.

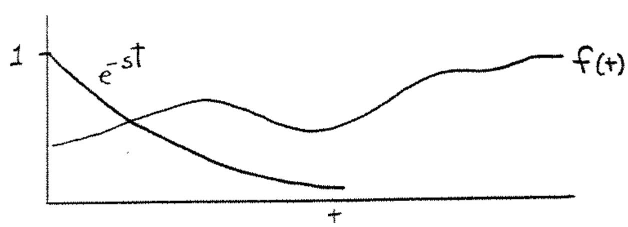
2 Laplace Transforms

A function $f(t)$ can be transformed to the frequency domain using the Laplace Transform if the following integral exists for *some* value of s.

$$
F(s) = L[f(t)] \equiv \int_0^\infty f(t)e^{-st}dt \quad \text{(an improper integral).} \tag{4}
$$

$$
= \lim_{T\to\infty}\int_0^T f(t)e^{-st}dt \tag{5}
$$

For instance, if $\mathrm{Re}(s) > 0$, e^{-st} and $f(t)$ might look like



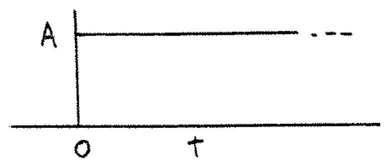
For $f(t)$ to be transformable, the product $f(t)e^{-st}$ must go to zero as $t \to \infty$ for the integral to exist. The mathematical condition is $|f(t)| < Me^{ct}$ for some

$M > 0$, $c > 0$ and all $t > 0$. For instance, if $f(t) = e^{t^2}$,

$$\int_0^\infty e^{t^2} e^{-st} dt = \infty \Rightarrow \text{no Laplace Transform.}$$

A few important examples of transformable functions are given below.

Step Function.



Let $f(t) = Au(t)$, where $u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases}$.

$$L[Au(t)] = \int_0^\infty Ae^{-st} dt = \frac{-A}{s} e^{-st} |_0^\infty = -\frac{A}{s}\left[e^{-s\infty} - e^{-s0}\right] = \underline{\underline{\frac{A}{s}}} \tag{6}$$

Note that t is integrated out of the function, and we are left with a new function of s only.

Exponential e^{-at}.

$$\begin{aligned} L\left[e^{-at}\right] &= \int_0^\infty e^{-at}e^{-st}dt = \int_0^\infty e^{-(s+a)t}dt \\ &= -\frac{1}{s+a}e^{-(s+a)t}|_0^\infty = -\frac{1}{s+a}\left[e^{-\infty} - e^0\right] \\ &= \underline{\underline{\frac{1}{s+a}}} \leftarrow \text{you will see this one a lot!} \end{aligned} \tag{7}$$

Sinusoid $\sin \omega t$.

First note that

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}, \tag{8}$$

which can be verified using the fact that $e^{j\theta} = \cos\theta + j\sin\theta$. Also note that

$$\begin{aligned} L[f_1(t) + f_2(t)] &= \int_0^\infty (f_1(t) + f_2(t))e^{-st}dt = \int_0^\infty f_1(t)e^{-st}dt + \int_0^\infty f_2(t)e^{-st}dt \\ &= L[f_1(t)] + L[f_2(t)], \end{aligned}$$

so, using the transform of the exponential in (7),

$$\begin{aligned} L[\sin\omega t] &= \frac{1}{2j}\left(L\left[e^{j\omega t}\right] - L\left[e^{-j\omega t}\right]\right) \\ &= \frac{1}{2j}\left(\frac{1}{s - jw} - \frac{1}{s + jw}\right) = \frac{1}{2j}\left(\frac{(s + jw) - (s - jw)}{(s - jw)(s + jw)}\right) \\ &= \underline{\underline{\frac{\omega}{s^2 + \omega^2}}}. \end{aligned} \tag{9}$$

These transforms and other common ones encountered in linear system theory are tabulated in the attached tables from Ogata "Modern Control Engineering."

2.1 Differentiation using Laplace Transforms

The most important property of Laplace Transforms is the relationship between $L\left[\frac{df}{dt}\right]$ and $L\left[f(t)\right]$, where $f(t)$ is *any* differentiable function of time. It is:

$$L\left[\frac{df}{dt}\right] = sL\left[f(t)\right] - f(0) = sF(s) - f(0). \tag{10}$$

To show this, use the definition of the transform (4) on the derivative $L\left[\frac{df}{dt}\right] = \int_0^\infty \frac{df}{dt}e^{-st}dt$, and integrate this expression by parts: $\int udv = uv - \int vdu$. To do this, choose $u = e^{-st}$, and $dv = \frac{df}{dt}dt = df$. Then

$$\begin{aligned}
L\left[\frac{df}{dt}\right] &= \int_0^\infty \frac{df}{dt}\, e^{-st}dt \\
&= e^{-st}f(t)|_0^\infty - \int_0^\infty f(t)(-se^{-st})dt \\
&= e^{-s\infty}f(\infty) - f(0) + s\int_0^\infty f(t)\, e^{-st}dt \\
L\left[\frac{df}{dt}\right] &= sL\left[f(t)\right] - f(0) \qquad (11)
\end{aligned}$$

We will see this many times as $L[\dot{x}] = sX(s) - x(0)$, where $x(t)$ is used in place of $f(t)$, and $\dot{x}$ was used for $\frac{df}{dt}$. Note that we can obtain $L\left[\ddot{x}\right]$ recursively as follows. Let $\dot{x} = v$, so $\dot{v} = \ddot{x}$. Then

$$L\left[\ddot{x}\right] = L[\dot{v}] = sV(s) - v(0),$$

but

$$V(s) = L\left[v\right] = L\left[\dot{x}\right] = sX(s) - x(0)$$

so

$$\underline{L\left[\ddot{x}\right] = s^2X(s) - sx(0) - \dot{x}(0)} \tag{12}$$

Example

If we are given that the derivative of $\sin\omega t$ is $\omega\cos\omega t$, we can use (10) to derive $L\left[\cos\omega t\right]$ from $L\left[\sin\omega t\right]$. Recall that for $f(t) = \sin\omega t$, its transform is $F(s) = \frac{\omega}{s^2+\omega^2}$ as derived before. Then because $\dot{f}(t) = \omega\cos\omega t$, we have

$$\begin{aligned}
L[\cos\omega t] &= \frac{1}{\omega}L[\dot{f}] \\
&= \frac{1}{\omega}\left(sF(s) - f(0)\right) \\
&= \frac{1}{\omega}s\left(\frac{\omega}{s^2+\omega^2}\right) \\
&= \underline{\underline{\frac{s}{s^2+w^2}}}, \qquad (13)
\end{aligned}$$

where we have used the fact that $f(0) = \sin\omega 0 = 0$.

In a manner similar to finding $L[\frac{df}{dt}]$, we can derive $L\left[\int f(t)\right]$, which is

$$L\left[\int f(t)dt\right] = \frac{F(s)}{s} + \frac{1}{s}\int f\,dt\,|_{t=0}$$

2.2 Solving Differential Equations

One big use of Laplace Transforms is to manipulate and solve differential equations. A very common first order example is to solve

$$\dot{x} = ax + b,$$

where a, b are constants and $x(0) = x_o$. First take the Laplace Transform of both sides of the differential equation

$$sX - x_0 = aX + \frac{b}{s},$$

where we have assumed that b is applied at $t = 0$ (so its really $bu(t)$). Now solve for the unknown X

$$X(s-a) = \frac{b}{s} + x_0 = \frac{b + x_0 s}{s}$$

or

$$X = \frac{b + x_0 s}{s(s-a)}.$$

The final step is to determine $x(t)$ given $X(s)$. Since there is no expression of this form in our table of transform pairs, we can use a *partial fraction* expansion to find an equivalent form. That is,

$$X(s) = \frac{b + x_0 s}{s(s-a)} = \frac{A}{s} + \frac{B}{s-a}, \tag{14}$$

where A and B are unknowns that we can determine. Once we know A and B, the terms on the right hand side are in the transform table, and hence their time domain counterparts are known. To determine A and B, note that this equation is true for any s, so multiply both sides of (14) by s and let $s = 0$ to get A.

$$\left(\frac{b + x_o s}{s(s-a)}\right)s = A + \left(\frac{B}{s-a}\right)s$$

$$A = \left(\frac{b + x_o s}{s(s-a)}\right)s\,|_{s=0} = -\frac{b}{a.}$$

Similarly, multiply both sides of (14) by $s - a$ and set $s = a$ to find B

$$B = \left(\frac{b + x_o s}{s(s-a)}\right)(s-a)\,|_{s=a} = \frac{b + x_0 a}{a},$$

so

$$X(s) = \frac{-b/a}{s} + \frac{(b + x_0 a)/a}{s-a}.$$

Table 1–1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1,2,3, \ldots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1,2,3, \ldots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1,2,3, \ldots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1,2,3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

7A

Table 1–1 Cont'd

18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
21	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
24	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
25	$1 - \cos \omega t$	$\frac{\omega^2}{s(s^2 + \omega^2)}$
26	$\omega t - \sin \omega t$	$\frac{\omega^3}{s^2(s^2 + \omega^2)}$
27	$\sin \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2 + \omega^2)^2}$
28	$\frac{1}{2\omega} t \sin \omega t$	$\frac{s}{(s^2 + \omega^2)^2}$
29	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
30	$\frac{1}{\omega_2^2 - \omega_1^2}(\cos \omega_1 t - \cos \omega_2 t) \qquad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$
31	$\frac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$	$\frac{s^2}{(s^2 + \omega^2)^2}$

Table 1–2 Properties of Laplace Transforms

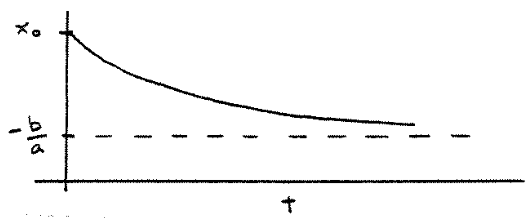
1	$\mathcal{L}[Af(t)] = AF(s)$
2	$\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathcal{L}_{\pm}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
4	$\mathcal{L}_{\pm}\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$
5	$\mathcal{L}_{\pm}\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k} \overset{(k-1)}{f}(0\pm)$ where $\overset{(k-1)}{f}(t) = \frac{d^{k-1}}{dt^{k-1}}f(t)$
6	$\mathcal{L}_{\pm}\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \frac{\left[\int f(t)\,dt\right]_{t=0\pm}}{s}$
7	$\mathcal{L}_{\pm}\left[\int\int f(t)\,dt\,dt\right] = \frac{F(s)}{s^2} + \frac{\left[\int f(t)\,dt\right]_{t=0\pm}}{s^2} + \frac{\left[\int\int f(t)\,dt\,dt\right]_{t=0\pm}}{s}$
8	$\mathcal{L}_{\pm}\left[\int\cdots\int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^{n}\frac{1}{s^{n-k+1}}\left[\int\cdots\int f(t)(dt)^k\right]_{t=0\pm}$
9	$\mathcal{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$
10	$\int_0^\infty f(t)\,dt = \lim_{s\to 0} F(s)$ if $\int_0^\infty f(t)\,dt$ exists
11	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$
12	$\mathcal{L}[f(t-\alpha)1(t-\alpha)] = e^{-\alpha s}F(s) \qquad \alpha \geq 0$
13	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$
14	$\mathcal{L}[t^2f(t)] = \frac{d^2}{ds^2}F(s)$
15	$\mathcal{L}[t^nf(t)] = (-1)^n\frac{d^n}{ds^n}F(s) \qquad n = 1,2,3,\ldots$
16	$\mathcal{L}\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s)\,ds$
17	$\mathcal{L}\left[f\left(\frac{t}{a}\right)\right] = aF(as)$

7c

Now, using the table,

$$x(t) = -\frac{b}{a}u(t) + \frac{b + x_o a}{a}e^{at}.$$

Note that the initial conditions are satisfied, $x(0) = -\frac{b}{a} + \frac{b+x_0 a}{a} = x_0$, and if $a < 0$, the final value of x is $x(\infty) = -\frac{b}{a}$ (See the figure). If $a > 0$, $x(\infty) = \infty$ since $e^{at} \to \infty$ in this case.

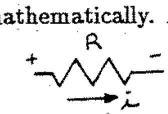


Also note that the poles of this system are $s = 0$, and $s = a$. They characterize the response of the system since the two time dependent terms in the inverse transform, $u(t)$ and e^{at}, arise because of these poles. That is why the expression that determines the roots of the denominator of $X(s)$, i.e. $s(s-a) = 0$ is called the *characteristic equation.*

3 Electronic Elements, Impedance, and Block Diagrams

3.1 Resistor

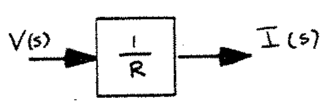
Our first experiment uses capacitors and resistors, which have dynamic properties that we need to understand mathematically. A *resistor* has the symbol



the voltage across the resistor is proportional to current, or

$$v = Ri. \tag{15}$$

The Laplace Transform of this expression is $V(s) = RI(s)$. A block diagram can be used to describe the same relation. For this case it is



The block diagram shows $\frac{L[output]}{L[input]} = \frac{I(s)}{V(s)} = \frac{1}{R}$ in the box, where all the initial conditions are assumed to be zero (for the case of a resistor the initial conditions do not matter). For a general system a rational polynomial will be in the box called the *transfer function.* Multiplication of the input by the transfer function gives the output.

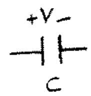
Again, a

$$\textbf{Transfer Function} \equiv \frac{L[\text{output}]}{L[\text{input}]}, \tag{16}$$

assuming zero initial conditions. It is up to you to choose what the input and the output of the system are. The choice is usually clear from the problem that is being solved.

3.2 Capacitor

A *capacitor* has the symbol

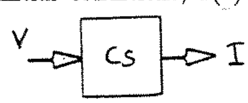


with the voltage across the capacitor proportional to the charge on its plates, or $q = \int i dt = Cv$. Differentiating this expression with respect to time gives

$$i = C\frac{dv}{dt}. \tag{17}$$

Taking the Laplace Transform of both sides with zero initial conditions, $I(s) = CsV(s)$ or

$$V(s) = \frac{1}{Cs}I(s). \quad \equiv$$



The coefficient of $I(s)$ is called the *impedance.* For a capacitor the impedance is $\frac{1}{Cs}$, for a resistor it is just R. The impedance is often labeled as $Z(s)$, so

$$V(s) = Z(s)I(s). \tag{18}$$

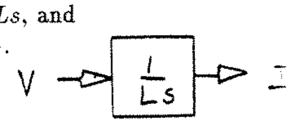
3.3 Inductor

An *inductor* has the symbol

with the voltage across it proportional to the change in current through it, or

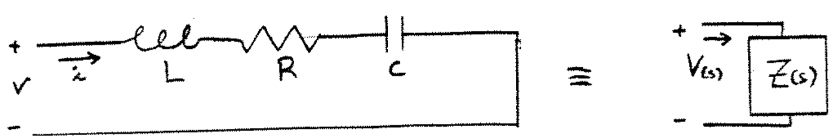
$$v = L\frac{di}{dt}. \tag{19}$$

Taking the Laplace Transform gives $V(s) = LsI(s)$, so the impedance is Ls, and if we assume voltage is the input, the transfer function for an inductor is $\frac{1}{Ls}$.



3.4 Combinations of Elements

To analyze a series LRC circuit as shown,



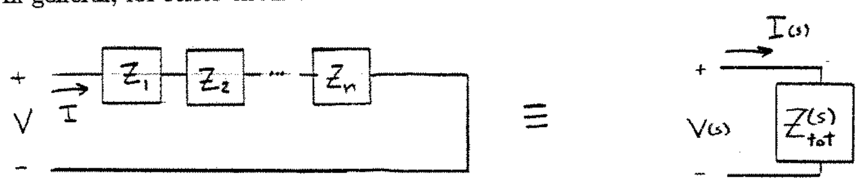
sum the transformed voltages around the loop to obtain

$$V = RI + LsI + \frac{I}{Cs} = (R + Ls + \frac{1}{Cs})I.$$

We see that the coefficient of I, or the impedance is

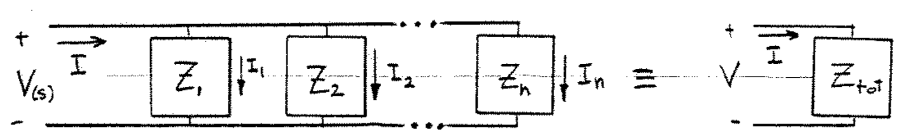
$$Z(s) = (R + Ls + \frac{1}{Cs}).$$

In general, for series circuits



$$Z_{tot} = \sum_{i=1}^{n} Z_i.$$

For parallel circuits,



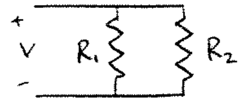
$V(s) = Z_{tot}I(s)$ and $I_i = \frac{V}{Z_i}$ for each branch.

$$I = \sum_{i=1}^{n} I_i = \sum_{i=1}^{n} \frac{V}{Z_i} = V \sum_{i=1}^{n} \frac{1}{Z_i},$$

so $V = \frac{1}{\sum_{i=1}^{n} \frac{1}{Z_i}} I$ and $Z_{tot} = \frac{1}{\sum_{i=1}^{n} \frac{1}{Z_i}}$.

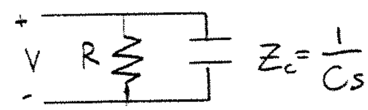
Example 1

Two resistors in parallel:



$$Z_{tot} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2+R_1}{R_1R_2}} = \frac{R_1R_2}{R_1 + R_2}.$$

Example 2
A resistor and capacitor in parallel:



$$Z_{tot} = \frac{R\frac{1}{cs}}{R + \frac{1}{cs}} = \frac{R}{1 + Rcs}$$

3.5 Semiconductors, the Diode and the MOSFET

The following notes were taken from Radio Shack's *Getting Started in Electronics.* You may need to purchase a MOSFET for use with two experiments, and possibly your project. For more detailed information on these elements, refer to Horowitz and Hill: *The Art of Electronics,* or Millman and Halkias: *Integrated Electronics.*

11

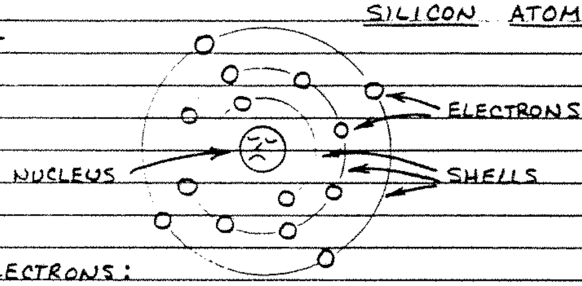
3. SEMICONDUCTORS

THE MOST EXCITING AND IMPORTANT ELECTRONIC COMPONENTS ARE MADE FROM CRYSTALS CALLED SEMICONDUCTORS. DEPENDING ON CERTAIN CONDITIONS, A SEMICONDUCTOR CAN ACT LIKE A CONDUCTOR OR AN INSULATOR.

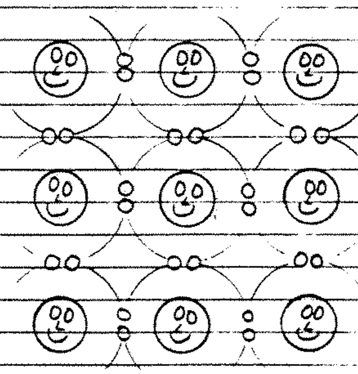
SILICON

THERE ARE MANY DIFFERENT SEMICONDUCTING MATERIALS, BUT SILICON, THE MAIN INGREDIENT OF SAND, IS THE MOST POPULAR.

SILICON ATOM



A SILICON ATOM HAS BUT FOUR ELECTRONS IN ITS OUTERMOST SHELL, BUT IT WOULD LIKE TO HAVE EIGHT. THEREFORE, A SILICON ATOM WILL LINK UP WITH FOUR OF ITS NEIGHBORS TO SHARE ELECTRONS:

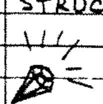


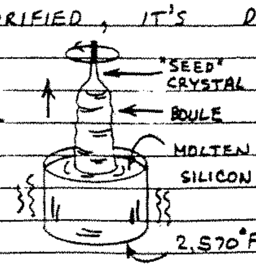
A CLUSTER OF SILICON ATOMS SHARING OUTER ELECTRONS FORMS A REGULAR ARRANGEMENT CALLED A CRYSTAL.

← THIS IS A MAGNIFIED VIEW OF A SILICON CRYSTAL. TO KEEP THINGS SIMPLE, ONLY THE OUTER ELECTRONS OF EACH ATOM ARE SHOWN.

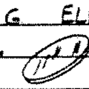
SILICON FORMS 27.7 % OF THE EARTH'S CRUST! ONLY OXYGEN IS MORE COMMON. IT'S NEVER FOUND IN THE PURE STATE. WHEN PURIFIED, IT'S DARK GRAY IN COLOR.

SILICON AND DIAMOND SHARE THE SAME CRYSTAL STRUCTURE AND OTHER PROPERTIES. BUT SILICON IS NOT TRANSPARENT.

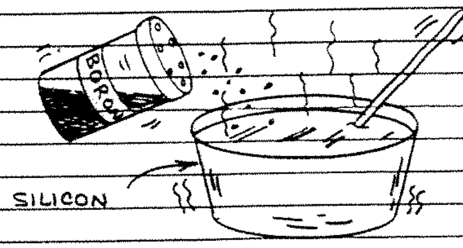




SILICON CAN BE GROWN INTO BIG CRYSTALS. IT'S CUT INTO WAFERS FOR MAKING ELECTRONIC PARTS.



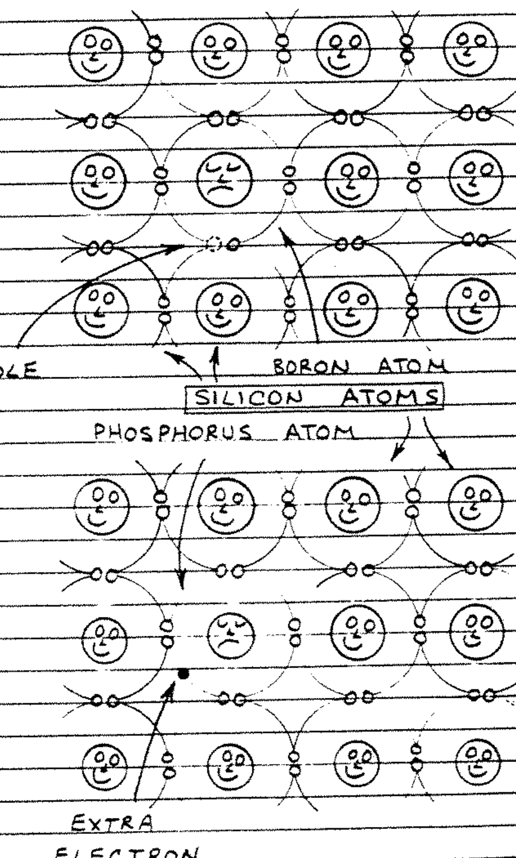
☐ SILICON RECIPES — PURE SILICON ISN'T VERY USEFUL. THAT'S WHY SILICON MAKERS SPICE UP THEIR SILICON RECIPES WITH A DASH OF PHOSPHORUS, BORON OR OTHER GOODIES. THIS IS CALLED DOPING THE SILICON. WHEN GROWN INTO CRYSTALS, DOPED SILICON HAS VERY USEFUL ELECTRONIC PROPERTIES!



☐ P & N SPICED SILICON LOAF — BORON, PHOSPHORUS AND CERTAIN OTHER ATOMS CAN JOIN WITH SILICON ATOMS TO FORM CRYSTALS. HERS'S THE CATCH: A BORON ATOM HAS ONLY THREE ELECTRON IN ITS OUTER SHELL. AND A PHOSPHORUS ATOM HAS FIVE ELECTRONS IN ITS OUTER SHELL. SILICON WITH EXTRA PHOSPHORUS ELECTRONS IS CALLED N-TYPE SILICON (N = NEGATIVE). SILICON WITH ELECTRON DEFICIENT BORON ATOMS IS CALLED P-TYPE SILICON (P = POSITIVE).

☐ P-TYPE SILICON — A BORON ATOM IN A CLUSTER OF SILICON ATOMS LEAVES A VACANT ELECTRON OPENING CALLED A HOLE. IT'S POSSIBLE FOR AN ELECTRON FROM A NEARBY ATOM TO "FALL" INTO THE HOLE. THEREFORE, THE HOLE HAS MOVED TO A NEW LOCATION. REMEMBER, HOLES CAN MOVE THROUGH SILICON (JUST AS BUBBLES MOVE THROUGH WATER).

☐ N-TYPE SILICON — A PHOSPHORUS ATOM IN A CLUSTER OF SILICON ATOMS DONATES AN EXTRA ELECTRON. THIS EXTRA ELECTRON CAN MOVE THROUGH THE CRYSTAL WITH COMPARATIVE EASE. IN OTHER WORDS, N-TYPE SILICON CAN CARRY AN ELECTRICAL CURRENT. BUT SO CAN P-TYPE SILICON! HOLES "CARRY" THE CURRENT.



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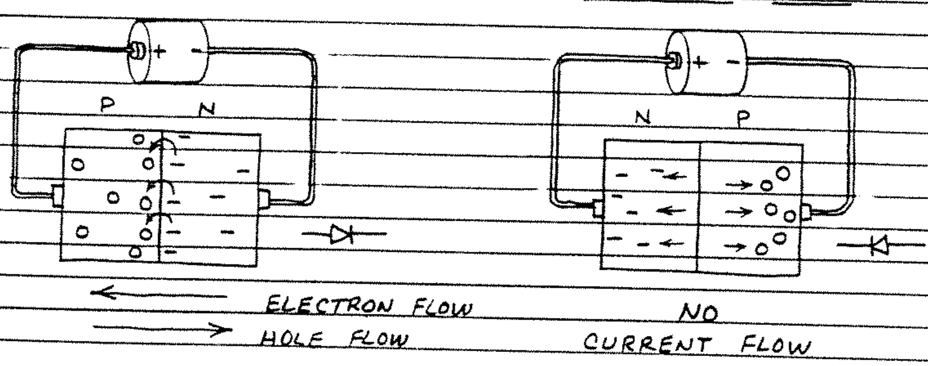
THE DIODE

BOTH P-TYPE AND N-TYPE SILICON CONDUCT ELECTRICITY. THE RESISTANCE OF BOTH TYPES IS DETERMINED BY THE PROPORTION OF HOLES OR SURPLUS ELECTRONS. THEREFORE BOTH TYPES CAN FUNCTION AS RESISTORS. AND THEY WILL CONDUCT ELECTRICITY IN ANY DIRECTION.

BY FORMING SOME P-TYPE SILICON IN A CHIP OF N-TYPE SILICON, ELECTRONS WILL FLOW THROUGH THE SILICON IN ONLY ONE DIRECTION. THIS IS THE PRINCIPLE OF THE DIODE. THE P-N INTERFACE IS CALLED THE PN JUNCTION.

☐ HOW THE DIODE WORKS — HERE'S A SIMPLIFIED EXPLANATION OF HOW A DIODE CONDUCTS ELECTRICITY IN ONE DIRECTION (FORWARD) WHILE BLOCKING THE FLOW OF CURRENT IN THE OPPOSITE DIRECTION (REVERSE).

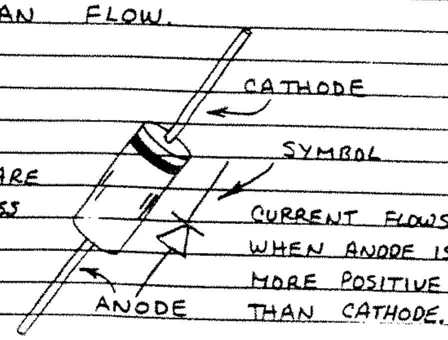
FORWARD BIAS — REVERSE BIAS



HERE THE CHARGE FROM THE BATTERY REPELS HOLES AND ELECTRONS TOWARD THE JUNCTION. IF THE VOLTAGE EXCEEDS 0.6-VOLT (SILICON), THEN ELECTRONS WILL CROSS THE JUNCTION AND COMBINE WITH HOLES. A CURRENT THEN FLOWS.

HERE THE CHARGE FROM THE BATTERY ATTRACTS HOLES AND ELECTRONS AWAY FROM THE JUNCTION. THEREFORE, NO CURRENT CAN FLOW.

☐ A TYPICAL DIODE — DIODES ARE COMMONLY ENCLOSED IN SMALL GLASS CYLINDERS. A DARK BAND MARKS THE CATHODE TERMINAL. THE OPPOSITE TERMINAL IS THE ANODE



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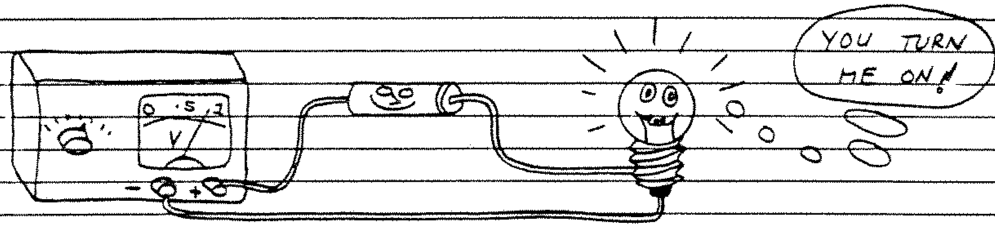
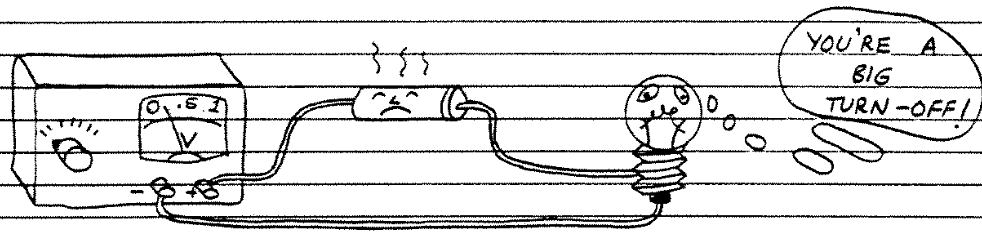
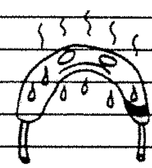
☐ DIODE OPERATION — YOU ALREADY KNOW A DIODE IS LIKE AN ELECTRONIC ONE-WAY VALVE. IT'S IMPORTANT TO UNDERSTAND SOME ADDITIONAL ASPECTS OF DIODE OPERATION. HERE ARE SOME KEY ONES:

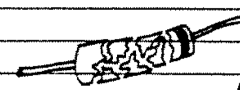
1. A DIODE WILL NOT CONDUCT UNTIL THE FORWARD VOLTAGE REACHES A CERTAIN THRESHOLD POINT. FOR SILICON DIODES THIS VOLTAGE IS ABOUT 0.6-VOLT.



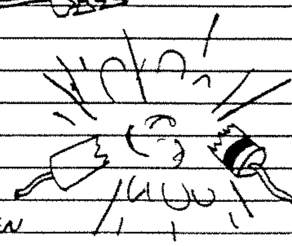


2. IF THE FORWARD CURRENT BECOMES EXCESSIVE, THE SEMICONDUCTOR CHIP MAY CRACK OR MELT! AND THE CONTACTS MAY SEPARATE. IF THE CHIP MELTS, THE DIODE MAY SUDDENLY CONDUCT IN BOTH DIRECTIONS. THE RESULTING HEAT MAY VAPORIZE THE CHIP!



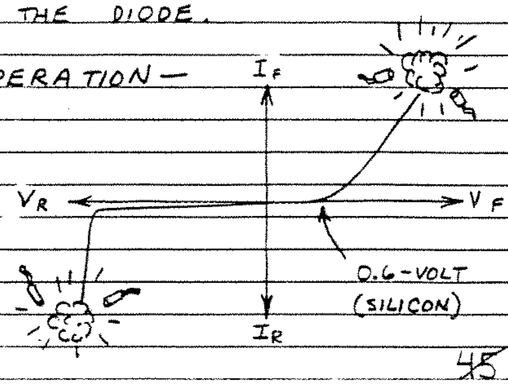


3. TOO MUCH REVERSE VOLTAGE WILL CAUSE A DIODE TO CONDUCT IN THE WRONG DIRECTION. SINCE THIS VOLTAGE IS FAIRLY HIGH, THE SUDDEN CURRENT SURGE MAY ZAP THE DIODE.



☐ SUMMING UP DIODE OPERATION — THIS GRAPH SUMS UP DIODE OPERATION. (IT'S APPROXIMATE.)

V_F = FORWARD VOLTAGE
V_R = REVERSE VOLTAGE
I_F = FORWARD CURRENT
I_R = REVERSE CURRENT



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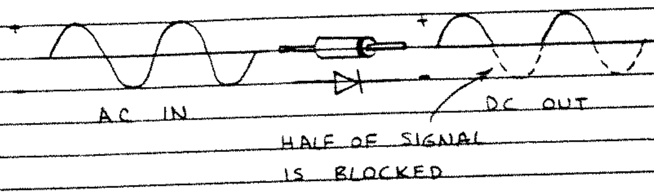
HOW DIODES ARE USED

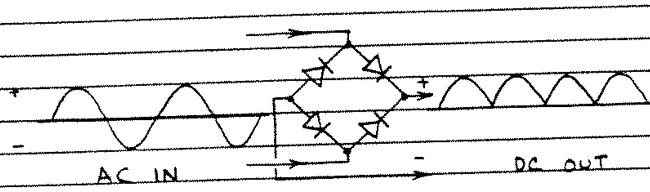
IN CHAPTER 9 YOU'LL SEE HOW VARIOUS TYPES OF DIODES ARE USED IN MANY APPLICATIONS. FOR NOW HERE ARE TWO OF THE MOST IMPORTANT ROLES FOR SMALL SIGNAL DIODES AND RECTIFIERS:

☐ HALF-WAVE RECTIFIER



AN UNDULATING (AC) SIGNAL (OR VOLTAGE) IS RECTIFIED INTO A SINGLE POLARITY (DC) SIGNAL (OR VOLTAGE).

☐ FULL-WAVE RECTIFIER P. 101



THIS 4-DIODE "NETWORK" (OR BRIDGE RECTIFIER) RECTIFIES BOTH HALVES OF AN AC SIGNAL.

MORE ABOUT THE DIRECTION OF CURRENT FLOW

AN ELECTRICAL CURRENT IS THE MOVEMENT OF ELECTRONS THROUGH A CONDUCTOR OR SEMICONDUCTOR. SINCE ELECTRONS MOVE FROM A NEGATIVELY CHARGED TO A POSITIVELY CHARGED REGION, WHY DOES THE ARROWHEAD IN A DIODE SYMBOL POINT IN THE OPPOSITE DIRECTION? THERE ARE TWO REASONS:

1. BEGINNING WITH BENJAMIN FRANKLIN, IT WAS TRADITIONALLY ASSUMED ELECTRICITY FLOWS FROM A POSITIVELY CHARGED TO A NEGATIVELY CHARGED REGION. THE DISCOVERY OF THE ELECTRON CORRECTED THAT. (BUT MOST ELECTRICAL CIRCUIT DIAGRAMS TODAY STILL FOLLOW THE OLD TRADITION IN WHICH THE POSITIVE POWER SUPPLY CONNECTION IS PLACED ABOVE THE NEGATIVE CONNECTION AS IF GRAVITY SOMEHOW INFLUENCES THE FLOW OF A CURRENT.)

2. IN A SEMICONDUCTOR, AS SHOWN ON PAGE 44, HOLES FLOW IN THE DIRECTION OPPOSITE THAT OF ELECTRON FLOW. IT'S THEREFORE COMMON TO REFER TO POSITIVE CURRENT FLOW IN SEMICONDUCTORS.

FOR ACCURACY, IN THIS BOOK "CURRENT FLOW" REFERS TO ELECTRON FLOW. BUT WE'RE STUCK WITH SYMBOLS THAT INDICATE HOLE FLOW.

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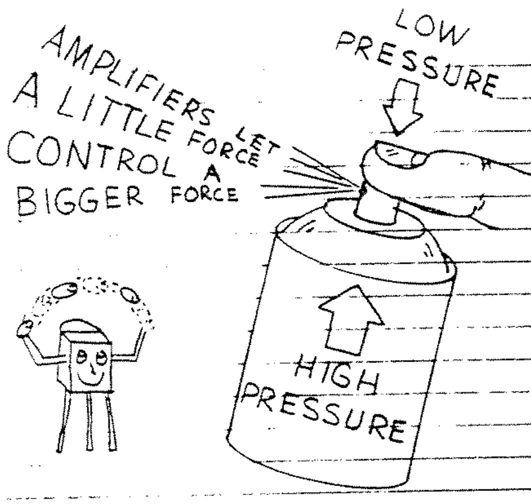
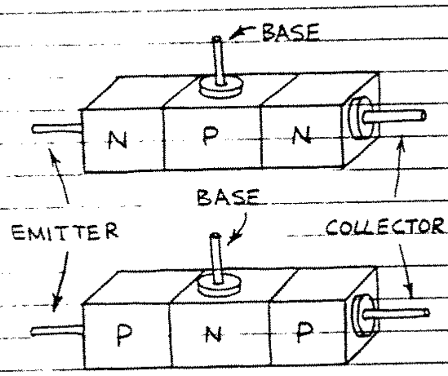
THE TRANSISTOR

TRANSISTORS ARE SEMICONDUCTOR DEVICES WITH THREE LEADS. A VERY SMALL CURRENT OR VOLTAGE AT ONE LEAD CAN CONTROL A MUCH LARGER CURRENT FLOWING THROUGH THE OTHER TWO LEADS. THIS MEANS TRANSISTORS CAN BE USED AS AMPLIFIERS AND SWITCHES. THERE ARE TWO MAIN FAMILIES OF TRANSISTORS: BIPOLAR AND FIELD-EFFECT.

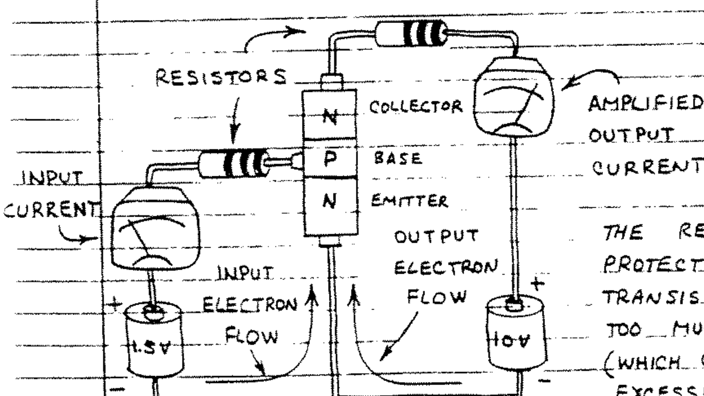


BIPOLAR TRANSISTORS

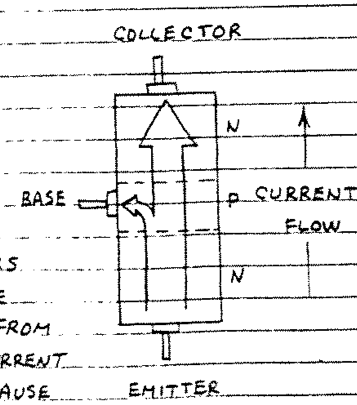
ADD A SECOND JUNCTION TO A PN JUNCTION DIODE AND YOU GET A 3-LAYER SILICON SANDWICH. THE SANDWICH CAN BE EITHER NPN OR PNP. EITHER WAY, THE MIDDLE LAYER ACTS LIKE A FAUCET OR GATE THAT CONTROLS THE CURRENT MOVING THROUGH THE THREE LAYERS.



☐ BIPOLAR TRANSISTOR OPERATION — THE THREE LAYERS OF A BIPOLAR TRANSISTOR ARE THE EMITTER, BASE AND COLLECTOR. THE BASE IS VERY THIN AND HAS FEWER DOPING ATOMS THAN THE EMITTER AND COLLECTOR. THEREFORE A VERY SMALL EMITTER-BASE CURRENT WILL CAUSE A MUCH LARGER EMITTER-COLLECTOR CURRENT TO FLOW.



THE RESISTORS PROTECT THE TRANSISTOR FROM TOO MUCH CURRENT (WHICH CAN CAUSE EXCESSIVE HEAT).



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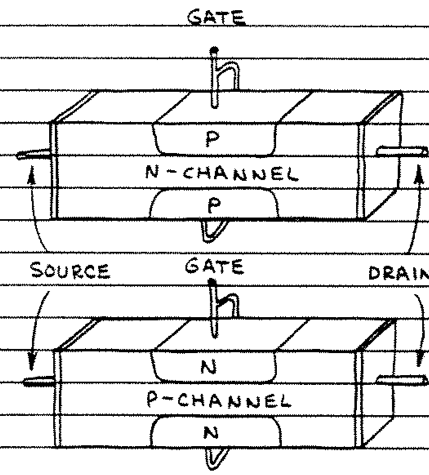
FIELD-EFFECT TRANSISTORS

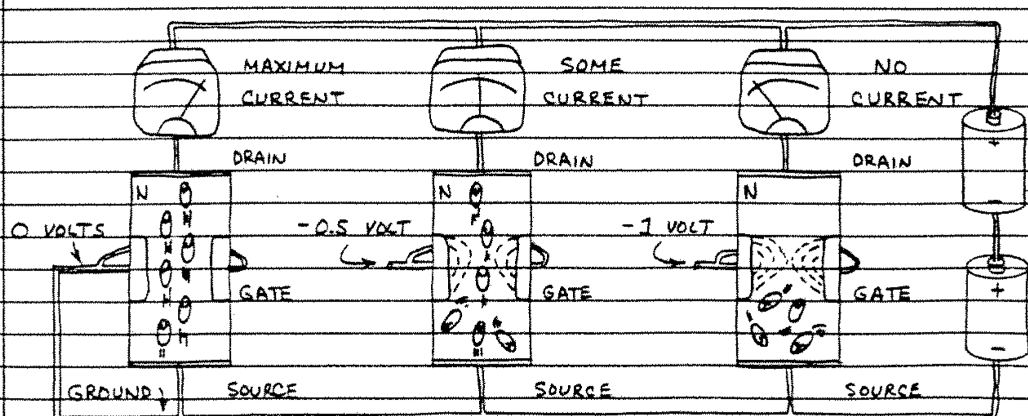
FIELD-EFFECT TRANSISTORS (OR FETs) HAVE BECOME MORE IMPORTANT THAN BIPOLAR TRANSISTORS. THEY ARE EASY TO MAKE AND REQUIRE LESS SILICON. THERE ARE TWO MAJOR FET FAMILIES, JUNCTION AND METAL-OXIDE-SEMICONDUCTOR. IN BOTH KINDS AN OUTPUT CURRENT IS CONTROLLED BY A SMALL INPUT VOLTAGE AND PRACTICALLY NO INPUT CURRENT!

JUNCTION FETs

THE TWO MAIN KINDS OF FETs ARE N-CHANNEL AND P-CHANNEL. THE CHANNEL IS LIKE A SILICON RESISTOR THAT CONDUCTS CURRENT MOVING FROM THE SOURCE TO THE DRAIN. A VOLTAGE AT THE GATE INCREASES THE CHANNEL RESISTANCE AND REDUCES THE DRAIN-SOURCE CURRENT. THEREFORE THE FET CAN BE USED AS AN AMPLIFIER OR A SWITCH.



☐ JUNCTION FET OPERATION — THE ARRANGEMENT BELOW SHOWS HOW AN N-CHANNEL FET WORKS. A NEGATIVE GATE VOLTAGE CREATES TWO HIGH RESISTANCE REGIONS (THE FIELD) IN THE CHANNEL ADJACENT TO THE P-TYPE SILICON. MORE GATE VOLTAGE WILL CAUSE THE FIELDS TO MERGE TOGETHER AND COMPLETELY BLOCK THE CURRENT. THE GATE-CHANNEL RESISTANCE IS VERY HIGH.



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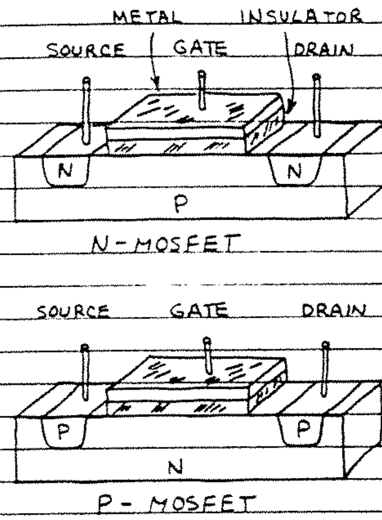
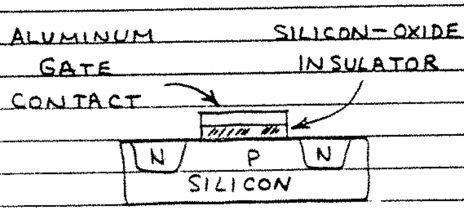
METAL-OXIDE-SEMICONDUCTOR FETs

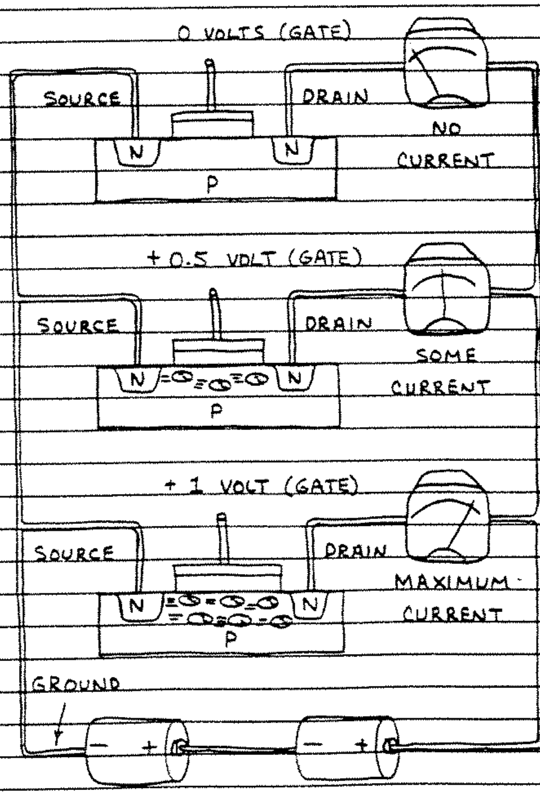
THE METAL-OXIDE-SEMICONDUCTOR FET (OR MOSFET) HAS BECOME THE MOST IMPORTANT TRANSISTOR. MOST MICROCOMPUTER AND MEMORY INTEGRATED CIRCUITS ARE ARRAYS OF THOUSANDS OF MOSFETS ON A SMALL SLIVER OF SILICON. WHY? MOSFETS ARE EASY TO MAKE, THEY CAN BE VERY SMALL, AND SOME MOSFET CIRCUITS CONSUME NEGLIGIBLE POWER. NEW KINDS OF POWER MOSFETS ARE ALSO VERY USEFUL.



□ MOSFET OPERATION— ALL MOSFETS ARE N-TYPE OR P-TYPE. UNLIKE THE JUNCTION FET, THE GATE OF A MOSFET HAS NO ELECTRICAL CONTACT WITH THE SOURCE AND DRAIN. A GLASS-LIKE LAYER OF SILICON-DIOXIDE (AN INSULATOR) SEPARATES THE GATE'S METAL CONTACT FROM THE REST OF THE TRANSISTOR.



A POSITIVE GATE VOLTAGE ATTRACTS ELECTRONS TO THE REGION BELOW THE GATE. THIS CREATES A THIN N-TYPE CHANNEL IN THE P-TYPE SILICON BETWEEN THE SOURCE AND DRAIN. CURRENT CAN THEN FLOW THROUGH THE CHANNEL. THE GATE VOLTAGE DETERMINES THE RESISTANCE OF THE CHANNEL.



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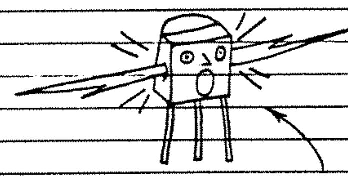
☐ MORE ABOUT MOSFETS — THE INPUT RESISTANCE OF THE MOSFET IS THE HIGHEST OF ANY TRANSISTOR. THIS AND OTHER FACTORS GIVE MOSFETS IMPORTANT ADVANTAGES:

1. THE GATE-CHANNEL RESISTANCE IS ALMOST INFINITE (TYPICALLY 1,000,000,000,000,000-OHMS). THIS MEANS THE GATE PULLS NO CURRENT FROM EXTERNAL CIRCUITS. (WELL, IT MAY BORROW A FEW TRILLIONTHS OF AN AMPERE.)

2. MOSFETS CAN FUNCTION AS VOLTAGE-CONTROLLED VARIABLE RESISTORS. THE GATE VOLTAGE CONTROLS CHANNEL RESISTANCE.

3. NEW KINDS OF MOSFETS CAN SWITCH VERY HIGH CURRENTS IN A FEW BILLIONTHS OF A SECOND.

☐ CAUTION — BECAUSE THE GLASS-LIKE SILICON OXIDE LAYER BELOW THE GATE IS SO THIN, IT CAN BE PIERCED BY TOO MUCH VOLTAGE OR EVEN STATIC ELECTRICITY. EVEN THE STATIC CHARGE GENERATED BY CLOTHING OR A CELLOPHANE WRAPPER CAN ZAP THE GATE OF A MOSFET!

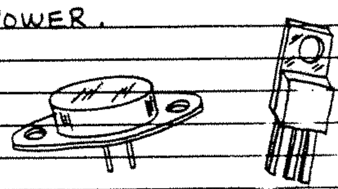


ZAPPED MOSFET

CAUTION SYMBOL

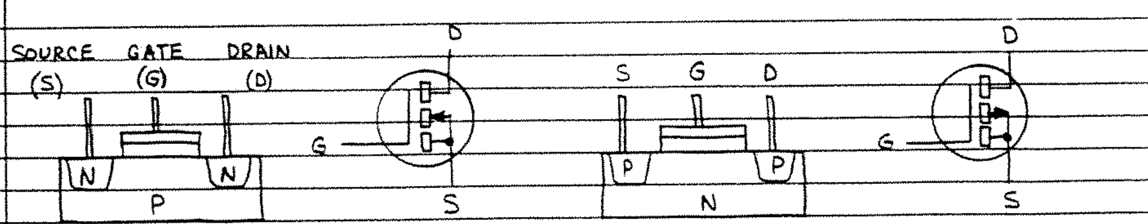
☐ KINDS OF MOSFETS — LIKE JFETS, MOSFETS INSTALLED IN SMALL METAL OR PLASTIC PACKAGES ARE USED TO GIVE AMPLIFIERS AN ULTRA-HIGH INPUT RESISTANCE. THEY ARE ALSO USED AS VOLTAGE CONTROLLED RESISTORS AND SWITCHES. THE MOST IMPORTANT CATEGORY HAS BECOME:

POWER.



POWER MOSFETS ALLOW A FEW VOLTS TO SWITCH OR AMPLIFY MANY AMPERES AT VERY FAST SPEEDS.

☐ MOSFET SYMBOLS — THESE ARE THE MOST COMMON.



N-MOSFET P-MOSFET

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Circuit Analysis Review

Main Electrical Quantities:

1) Charge — 1 Coulomb (C) = 6.25×10^{18} electrons
   1 electron (e) = 1.6×10^{-19} C

2) Current — $I = dQ/dt$
   1 Amp(A) = 1 C/sec

3) Voltage — V = energy/charge ; potential difference
   1 Volt(V) = 1 Joule(J)/C

4) Power — P = Work/time
   1 Watt(W) = 1 J/sec

5) Resistance (Impedance (Z) for AC) — $R = V/I$
   1 Ohm (Ω) = 1 V/A

6) Conductance (Admittance (Y) for AC) — $G = 1/R$
   1 Mho ($\mho$) = 1 A/V
   (1 Siemen = 1 Mho)

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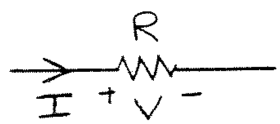
Ohm's Law

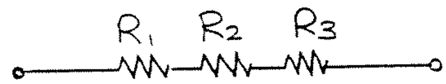
$$V = IR$$
$$I = V/R$$
$$R = V/I$$

Power Equations $P = VI = I^2 R = V^2/R$

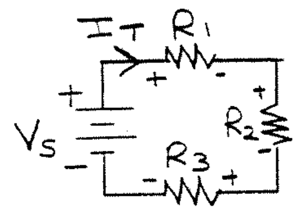
Conventional Current Polarity



Series Resistive Circuits



$$R_T = R_1 + R_2 + R_3$$
$$P_T = P_1 + P_2 + P_3$$



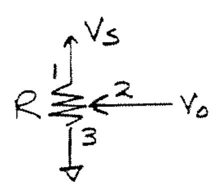
$$I_T = I_{R1} = I_{R2} = I_{R3} = V_S/R_T$$

$$V_{R1} = \underbrace{I_T R_1}_{\text{Ohm's Law}} = \underbrace{V_S\left(\frac{R_1}{R_T}\right)}_{\text{Voltage Divider Eq.}}$$

Kirchhoff's Voltage Law: Σ voltages in closed loop = 0
(KVL) or Net Voltage supplied = Σ voltage drops

$$V_S = V_{R1} + V_{R2} + V_{R3}$$

Potentiometer:



$$R = R_{12} + R_{23}$$
$$V_o = V_S\left(\frac{R_{23}}{R}\right)$$

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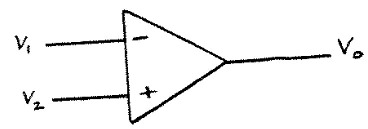
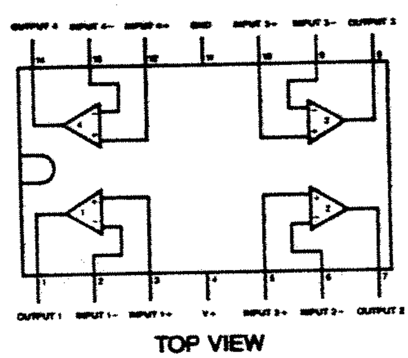
1 Operational-Amplifiers

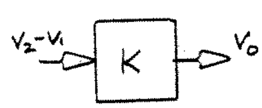
Operational amplifiers are the most common elements of analog circuits. Nearly all electronic controllers use some op-amp circuit to provide the desired signal conditioning. We will use these devices for many of our experiments. The schematic symbol is



You can buy electronic chips with four op-amps on each chip for about $1.25 each. We use a such a quad op-amp chip in the lab called an LM324. The four amplifiers are connected to the output leads as:



Power must be input to the chip through a positive supply voltage at pin 4, and a negative voltage or ground at pin 11. Circuits that use op-amps are sometimes called *active filters* because power may be added to the circuit by the amplifier. Circuits that don't use op-amps, such as R-C networks are called *passive filters* because they can only absorb power. The voltages v_1, v_2, and v_0 in the schematic diagram are related by the block diagram



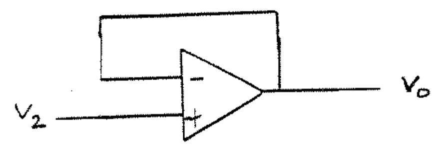
or

$$v_0 = K(v_2 - v_1), \tag{1}$$

where K is a large gain ($K \approx 1 \times 10^5$). In reality, the op-amp has some dynamics which means that K is a function of frequency, or $K = K(s)$, but we assume it acts as a pure gain, so the dynamics are neglected.

1.1 A Voltage Follower

Suppose we hook v_0 up to v_1



hence $v_1 = v_0$ and, using (1)

$$v_0 = K(v_2 - v_0)$$

or

$$v_0 = \frac{K}{(1+K)} v_2 \approx v_2,$$

since $\frac{K}{(1+K)} \approx 1$ for large values of K. This is called a *voltage follower* circuit. With this circuit $v_0(t) = v_2(t)$ for all ~~but~~ voltages except very quickly time varying signals. As explained below, this simple circuit is used in almost every electronic instrument because the output $v_2(t)$ draws no current from the input $v_0(t)$. For fast signal variations, the dynamics of the amplifier gain $K(s)$ can not be ignored. You will test this in Experiment 2.

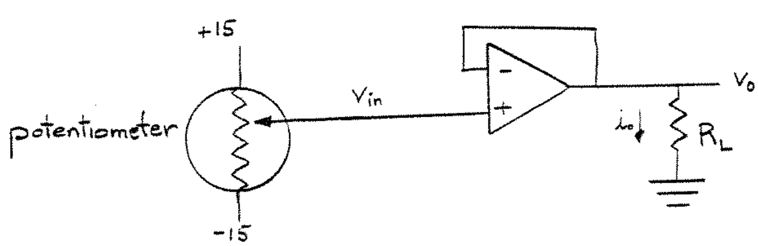
The inputs to the op-amp v_1 and v_2 are applied to the base of internal field effect transistors (FET) which require no current at their base to operate, they only use the field produced by the electric potential. Hence, no current flows into the + and - inputs. An analysis similar to what we used for the voltage follower will show that whenever there is any feedback path from v_0 to v_1, the amplifier forces v_1 to equal v_2. The feedback path can be through any network. Thus, to analyze op-amp circuits that have negative feedback (a path from v_0 to v_1), we always make the following basic assumptions.

1.2 Three Op-Amp Assumptions.

Use these assumptions whenever negative feedback is present in an op-amp circuit.

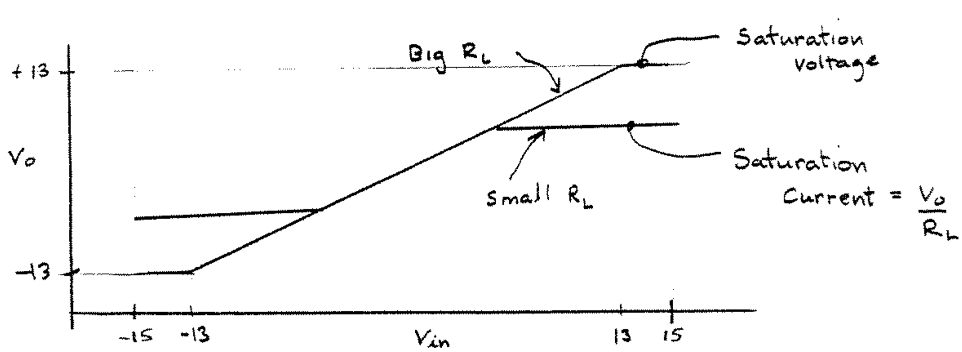
1. $v_1 = v_2$.
2. No current flows into the + or - inputs.
3. The op-amp is an ideal gain, or $K(s) = K$, a constant

For the lab experiment we make the circuit:



If the op-amp is working correctly $v_o = v_{in}$ regardless of R_L. The output current is $i_o = \frac{v_o}{R_L}$. Our op-amps can only output about $i_o = 0.020$ amp before the internal transistors *saturate*, or reach their current limit. This means that for small loads (high R_L) everything works fine, and for high loads (low R_L) the operational amplifier no longer amplifies as it should. You will find the saturation current in the second experiment.

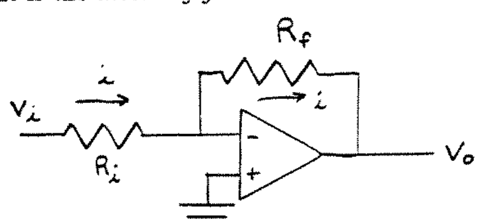
A plot of v_{out} versus v_{in} for some load R_L might look like



The voltage follower circuit acts like an ***ideal voltage source*** as long as i_o is below the saturation current. Note that the input voltage v_{in} is not affected by changes in i_o and R_L, because no current can flow into the op-amp. That is why the voltage follower is also called a *buffer* or an *isolation amplifier.*

1.3 An Inverting Gain

Another useful circuit is the *inverting gain.*



Using op-amp assumptions 1 and 2, we see that

$$\frac{v_i}{R_i} = i = \frac{(0 - v_o)}{R_f}$$

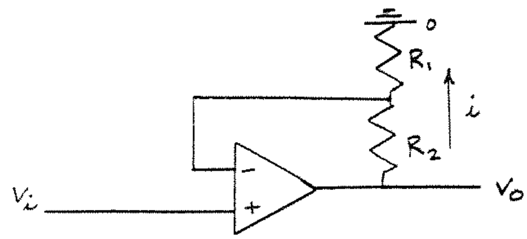
or

$$v_o = -v_i \frac{R_f}{R_i}$$

In reality this equation remains true for any v_i that causes v_o to be less than some *saturation voltage* that is determined by the voltage used to provide power to the op-amps. The operational amplifier can not do its job of maintaining a linear input-output voltage relationship if the input voltage is trying to produce an output voltage is beyond the saturation level.

1.4 A Noninverting Gain.

A circuit performs the same function as the last, but with no minus sign is the *noninverting gain.*



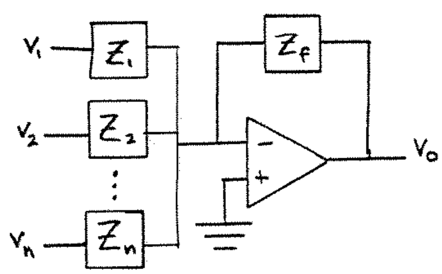
The voltage at the minus input is the voltage between the two resistors, which is $v_o\frac{R_1}{(R_1+R_2)}$. Then, using assumption 1,

$$v_o = \frac{(R_1 + R_2)}{R_1} v_i.$$

Note that the *gain* of this circuit is always greater than one, while for the previous circuit this is not the case.

1.5 A General Circuit.

One general op-amp circuit that is commonly used has the form



where the $Z_i(s)$ are general impedances of networks of capacitors and/or inductors. Using assumptions 1 and 2 we have

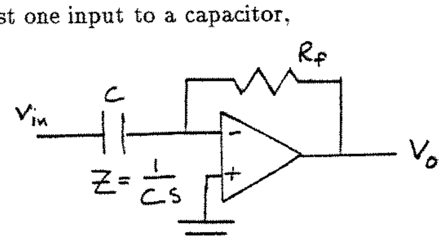
$$-\frac{V_o(s)}{Z_f(s)} = \frac{V_1}{Z_1} + \frac{V_2}{Z_2} + \cdots + \frac{V_n(s)}{Z_n(s)}$$

or

$$V_o(s) = -Z_f \sum_{i=1}^{n} \frac{V_i(s)}{Z_i(s)}. \tag{2}$$

1.6 Example: A Differentiator.

For instance, if there is just one input to a capacitor,



then (2) gives,

$$V_o(s) = -R_f C s V_{in}(s).$$

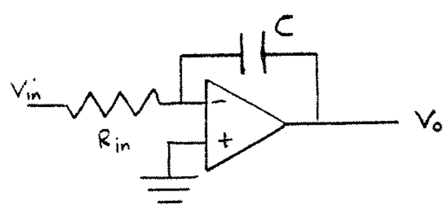
Note that the above equation is in the frequency domain. In the time domain, the same equation is

$$v_o = -R_f C \frac{dv_{in}}{dt}. \quad (3)$$

Hence, this circuit takes the time derivative of a *any* input voltage as a function of time, provided that it is differentiable. If it is not differentiable, then noise is output.

1.7 Example: An Integrator.

If we swap the resistor and the capacitor of the above circuit we get an integrator.



(2) gives,

$$V_o(s) = -\frac{1}{Cs}\frac{V_{in}(s)}{R_{in}}.$$

In the time domain, the same equation is

$$v_o = -\frac{1}{R_{in}C}\int v_{in}\,dt.$$

Because of the integration property of this circuit, if we input a square wave to this system, we get out a triangle wave.

1.8 The Concept of Lead and Lag

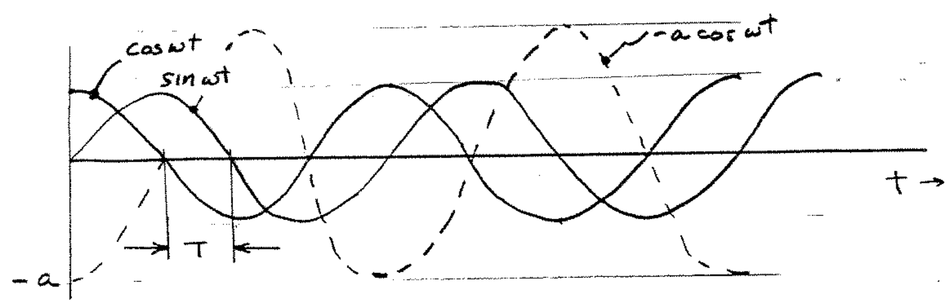
Suppose a sin wave is input from a function generator into the above differentiator circuit. That is,

$$v_{in} = a \sin \omega t,$$

then, the above analysis in (3) shows that

$$v_o = -R_f C a \omega \cos \omega t$$

We see that, although the output has the same frequency as the input, the amplitude of the output depends on the frequency, and the *phase* of the output is different than the input. To understand the phase relationship, consider the three waves sketched below.



The function $\cos \omega t$ is said to *lead* $\sin \omega t$ by 90^o. The sin wave is behind (or *lags*) the cos wave by the time corresponding to $\omega T = \pi/2 (= 90^o)$. If it were shifted ahead in time by T seconds it is said to be *in phase* with the cos wave. This is stated mathematically as

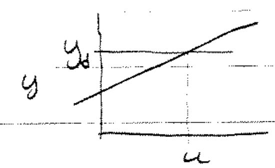
$$\cos \omega t = \sin(\omega t + \frac{\pi}{2}).$$

The wave coming out of the circuit above is a negative cos wave, which lags the input sin wave by 90^o. Finally, note that it is also correct to say that this wave *leads* the input by 270^o, since $\sin \theta = \sin(\theta \pm 2\pi)$.

The concept of feedback

Feedback is a measurement of a system's output. You can use this feedback to change the input to the system to make it do what you want it to. For instance, suppose

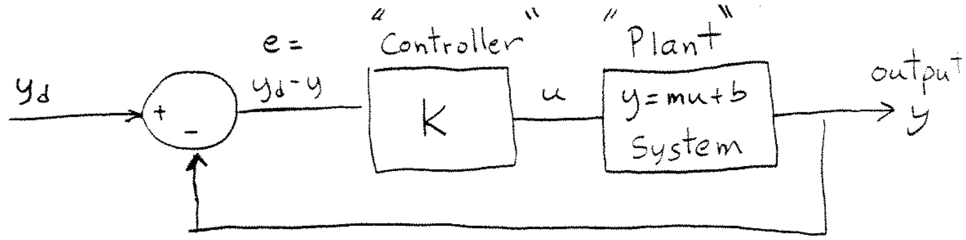
(1) $y \cong mu + b$



where y is the output of some system, and u is the input. Let m and b be highly uncertain constants. They might change with time or temperature. What u makes the output y close to y_d (d means desired)?

Feedback can be used to make $y \to y_d$ for any m or b!

Block diagram:



Let $u = K(y_d - y)$.

From (1)

$$y = mK(y_d - y) + b$$

$$y(1 + mK) = mK y_d + mK b$$

(2) $$y = \frac{mk\, y_d}{(1+mk)} + \frac{mk\, b}{(1+mk)}.$$

By letting $K \to$ big, $y \to y_d$. In the limit

$$\lim_{K \to \infty} y = y_d$$

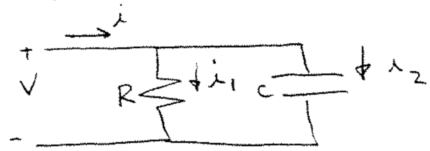
So you don't even need to know m or b!

Notes For Lab 2

The impedance, and transfer funtions are usually used to solve for the <u>forced</u> response of the system. Recall initial conditions are set to zero.

For the transient response, or free response, we need to include initial conditions.

In Exp 2, we have



We need $V(t)$, given $i=0$ and $V(0)=V_0$.

$i = i_1 + i_2 \qquad i_1 = \frac{V}{R} \qquad \int i_2\,dt = CV \Rightarrow i_2 = C\frac{dV}{dt}$

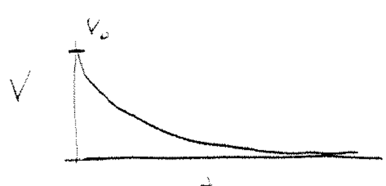
$0 = \frac{V}{R} + C\frac{dV}{dt}$

$\frac{dV}{dt} = -\frac{1}{RC}V$

This is like $\dot{x} = ax + b$ with $b=0$ $a = -\frac{1}{RC}$

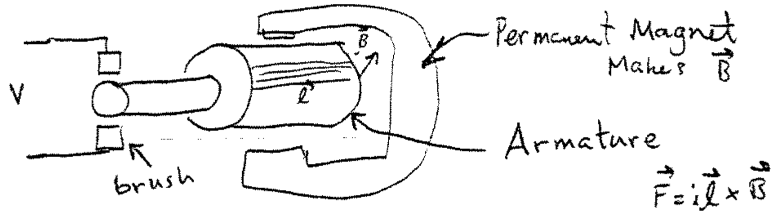
Solution $x(t) = e^{at}x(0)$

$V(t) = e^{-\frac{t}{RC}}V(0)$



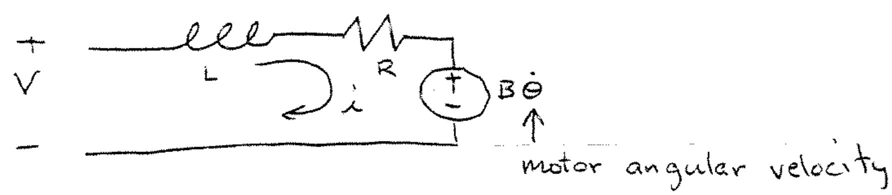
$\frac{1}{RC}$ is the <u>time constant</u>

Notes for Lab 3



DC Motors

We use DC motors in many experiments and projects; the math model is



So $V = L\frac{di}{dt} + Ri + B\dot{\theta}$. Also, Torque = ki for all $\dot{\theta}$.

- If you hold the motor shaft fixed $\dot{\theta} = 0$, and apply a constant voltage $\frac{di}{dt} \to 0$ (Why?),

so the motor acts like a pure resistor, with $V = Ri$

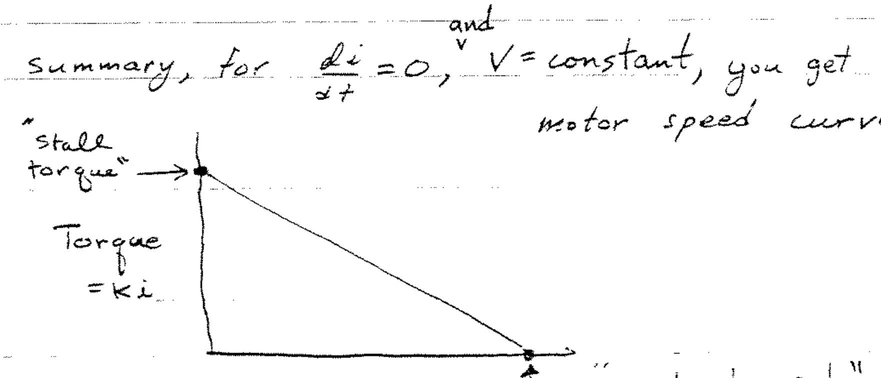
The torque you feel is called the "stall torque" and $T_{stall} = k\, i_{stall} = k\frac{V}{R}$

- If the shaft can spin freely, then Torque = small, and i = small, so the motor speeds up until

$i \cong 0$ $\frac{di}{dt} = 0$ and

$V \cong B\dot{\theta}$ ← called the "no load speed"

In summary, for $\frac{di}{dt} = 0$, and V = constant, you get the motor speed curve:



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Notes for Lab 4

First Order Systems

Many systems have the block diagram

which corresponds to the differential equation:

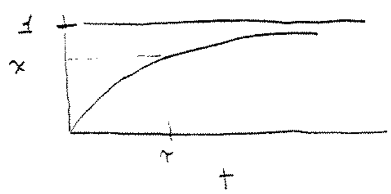
$$\dot{x} = -\frac{1}{\tau}x + \frac{u(t)}{\tau}.$$

This is a first order system with forcing function $u(t)$ and time constant τ. If we apply a unit step function $U(s)=\frac{1}{s}$ to the system with zero initial conditions,

$$X = \frac{1}{s(\tau s+1)} = \frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}$$

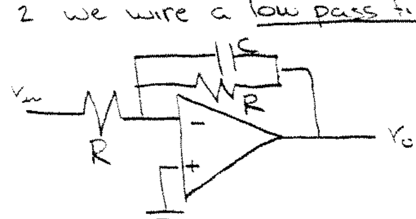
or,

$$x(t) = 1\,u(t) - e^{-\frac{t}{\tau}}.$$



when $t=\tau$ seconds, the output $x(\tau) = 1 - e^{-1} = .632$, which is 63% of the way to the final value $x(\infty)=1$. Hence τ seconds is a measure of how fast the system responds to changes in the input. τ can be many hours for thermal systems or micro seconds for electical system

In experiment 2 we wire a low pass filter as



$$Z_F = \frac{R}{1+RCs}$$

$$Z_{in} = R$$

$$V_o = -\frac{Z_F}{Z_{in}} V_{in} = \frac{-1}{1+RCs} V_{in}$$

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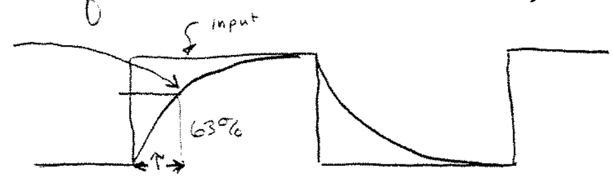
If we input a square wave to the circuit, the output will look like

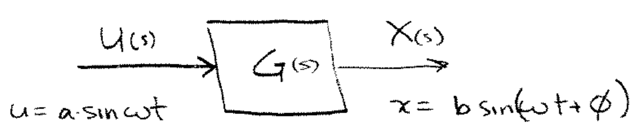


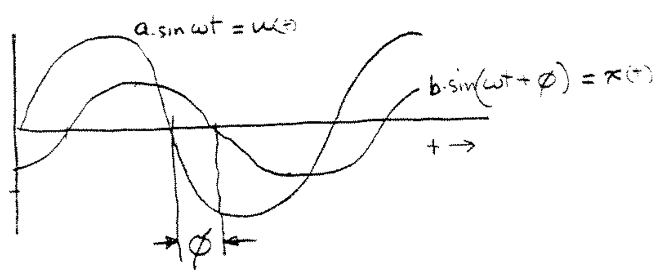
so we can measure τ from the output.

By making this measurement we have identified the system using time-domain measurements. We can also use frequency domain methods as follows.

Frequency Response

The most important concept of this class is the notion of frequency response of a dynamic system. If we input a sine wave to a stable linear system, the output will be a sine wave of the same frequency with a different amplitude and phase.





For the signals shown $x(t)$ lags $u(t)$ by ϕ°; and this means that $\phi < 0$.

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We can predict the output amplitude b and phase ϕ if we have a mathematical model of the system. Consider a general n^{th} order linear system

$$\frac{d^n x}{dt^n} + a_{n-1}\frac{d^{n-1} x}{dt^{n-1}} + \cdots a_0 x = b_{n-1}\frac{d^{n-1} u}{dt^{n-1}} + b_{n-2}\frac{d^{n-2} u}{dt^{n-2}} \cdots b_0 u$$

where the a_i and b_i are constant coefficients. Now take the Laplace Transform

$$X\underbrace{\left(s^n + a_{n-1}s^{n-1} + \cdots a_0\right)}_{\overset{\shortparallel}{A(s)}} = U\underbrace{\left(b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdots b_0\right)}_{\overset{\shortparallel}{B(s)}} + IC(s),$$

where $IC(s)$ is a polynomial in s that arises from the initial conditions. Solve of $X(s)$,

$$X = \frac{B(s)}{A(s)}U + \frac{IC(s)}{A(s)}$$

$A(s)$ is called the characteristic polynomial of the system. If we solve for the n roots of $A(s)$, $s_1, s_2 \cdots s_n$, we can write it in factored form as

$$A(s) = (s-s_1)(s-s_2)\cdots(s-s_n).$$

A partial fraction expansion of $X(s)$ with $U(s)=0$ will then be a function of the form

$$x(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t} \cdots c_n e^{s_n t} \qquad \left(\mathcal{L}\left[e^{-at}\right] = \frac{1}{s+a}\right)$$

So if $Re(s_i) < 0$ the output $x(t)$ decays exponetially when

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The system $\frac{B(s)}{A(s)}$ is said to be stable. Equivalently, if all the roots of the characteristic polynomial $A(s)$ lie in the left half s plane the system is stable.

Now, assume we have a stable system, and we apply the input

$$u(t) = a \cdot \sin \omega t$$

or

$$U(s) = \frac{a\omega}{s^2+\omega^2}.$$

Then

$$X(s) = \frac{B(s)}{A(s)} \frac{a\omega}{s^2+\omega^2} + \frac{IC(s)}{A(s)}$$

($\frac{IC(s)}{A(s)}$ decays to zero)

For any input, the contribution of the response due to the initial conditions decays to zero since $A(s)$ is stable.
Now look at the forced part of the response

$$X(s) = \frac{B(s)}{A(s)} \frac{a\omega}{s^2+\omega^2} = \frac{K_1}{(s+j\omega)} + \frac{K_2}{(s-j\omega)} + \frac{K_3}{(s-s_1)} + \frac{K_4}{(s-s_2)} + \cdots$$

($s+j\omega$, $s-j\omega$: roots of $s^2+\omega^2$; $s-s_1$, $s-s_2$: roots of $A(s)$)

Take the inverse Laplace Transform

$$x(t) = K_1 e^{-j\omega t} + K_2 e^{j\omega t} + K_3 e^{s_1 t} + K_4 e^{s_2 t} + \cdots$$

($K_3 e^{s_1 t} \to 0$, $K_4 e^{s_2 t} \to 0$)

The terms $e^{s_i t}$ decay to zero since $Re(s_i) < 0$.

$$K_1 = \frac{A(s)}{B(s)} \frac{a\omega}{s^2+\omega^2} (s+j\omega) \Big|_{s=-j\omega} = \frac{A(-j\omega)}{B(-j\omega)} \frac{a\omega}{-2j\omega} = -\frac{a}{2j} G(-j\omega)$$

where $G(s) = \frac{B(s)}{A(s)}$.

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Similarly,

$$K_2 = \frac{a}{2j} G(j\omega)$$

Now express $G(j\omega) = |G(j\omega)| e^{j\phi}$, and note that $G(-j\omega)$ is the complex conjugate of $G(j\omega)$ or $G(-j\omega) = |G(j\omega)| e^{-j\phi}$

So

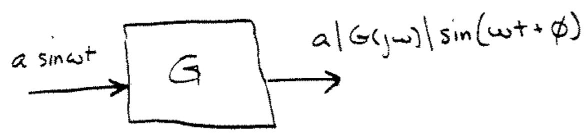
$$x(t) = \underbrace{-\frac{a}{2j}|G|e^{-j\phi}}_{K_1} e^{-j\omega t} + \underbrace{\frac{a}{2j}|G|e^{j\phi}}_{K_2} e^{j\omega t}$$

$$= \frac{a}{2j}|G|\left(-e^{-j(\omega + \phi)t} + e^{j(\omega+\phi)t}\right).$$

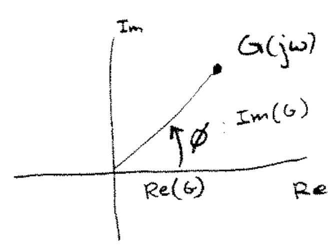
but $\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$, so

$$x(t) = a\,|G(j\omega)|\,\sin(\omega t + \phi).$$

In summary,



where $\phi = \angle G(j\omega) = \tan^{-1}\left(\frac{\text{Im } G(j\omega)}{\text{Re } G(j\omega)}\right)$.



Hence $|G(j\omega)| = \frac{\text{amplitude of output}}{\text{amplitude of input}} \equiv$ Amplitude ratio.

Consider the system of experiment 2.

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$$G(j\omega) = \frac{1}{\tau j\omega + 1} = \frac{1 \angle 0}{\sqrt{1+(\tau\omega)^2} \angle \tan^{-1} \tau\omega}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\tau\omega^2}} \qquad \angle G(j\omega) = 0 - \angle \tan^{-1} \tau\omega$$

At low frequencies $\omega \ll \frac{1}{\tau}$

$$|G| \cong 1 \qquad \phi \cong 0$$ (hence the term low pass filter)

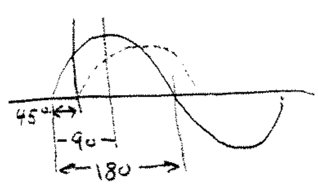
At high frequencies $\omega \gg \frac{1}{\tau}$

$$|G| \to 0 \qquad \phi \to -90^\circ$$

When $\omega = \frac{1}{\tau}$

$$|G(\tfrac{j}{\tau})| = \frac{1}{\sqrt{2}} \qquad \phi = -\tan^{-1}(1) = -45^\circ .$$

So the amplitude of the output wave when $\omega = \frac{1}{\tau}$ is .707 x (amplitude of the input wave), and the output lags the input by 45°.



The frequency $\omega = \frac{1}{\tau}$ is called the corner frequency or bandwidth of this system.

Input frequencies higher than the bandwidth are filtered or attenuated.

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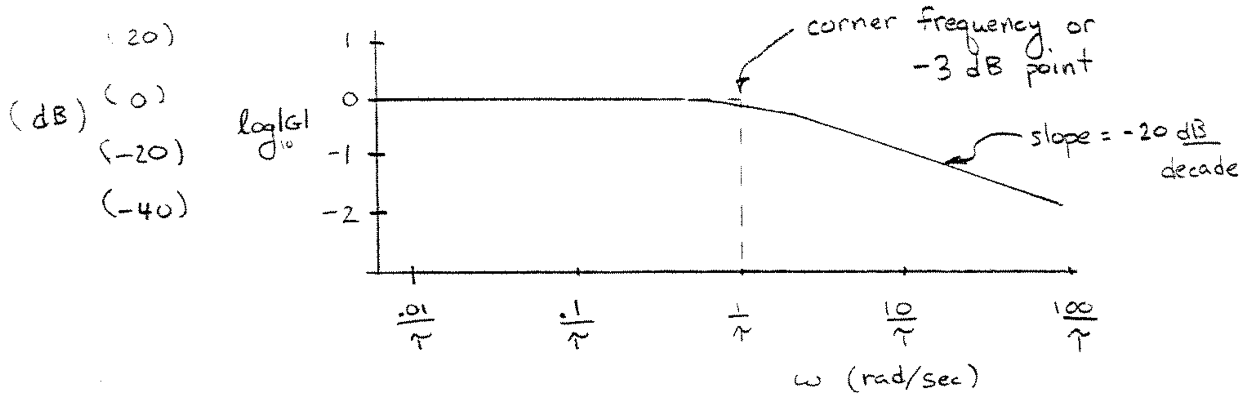
We can also plot $|G(j\omega)|$ and ϕ on a <u>Bode Plot</u>. This is a log-log plot of $\log_{10}|G(j\omega)|$ versus ω. For our example, the magnitude plot is:



Bode Plot of $G = \frac{1}{\tau s + 1}$

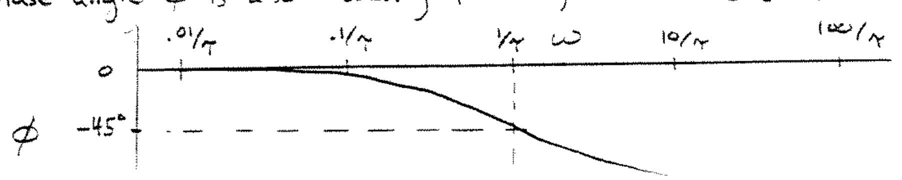
Sometimes the units of <u>decibels</u> (dB) are used for the $\log|G|$ axis. The number of dB can be computed for $|G|$ by

$$\# \text{ dB} \equiv 20 \log_{10} |G(j\omega)|.$$

At the corner frequency $\omega = \frac{1}{\tau}$, so $20 \log|G| = 20 \log(.707) = -3$ dB so this frequency is refered to as the <u>-3dB point</u>

A decibel is a unit used originally to mesure the intensity of sound named after Alexander G. Bell. (decibel $\equiv$ 10 * Bell, - Bell $\equiv \log_{10} v^2$ the log of the power)

The phase angle ϕ is also usually plotted, for our case it is:



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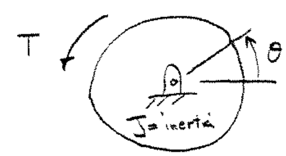
Motor Velocity Control

An interesting system with a transfer function of $\frac{1}{\tau s+1}$ is a motor velocity control system. The relation between angular velocity and motor torque is



$T = J\ddot{\theta}$, where T is the torque applied to the motor by the magnetic field, J is mass moment of inertia of the rotating portion of the motor and all attached gears and linkage, and $\ddot{\theta}$ is the angular acceleration.

Suppose you want to control the angular velocity $\dot{\theta} \equiv \Omega$ very accurately, and you can apply any torque you want T to the motor shaft. A good <u>control</u> <u>law</u> is

$$T = -K(\Omega - \Omega_d)$$

where K is a gain (constant) and Ω_d is the desired angular velocity. Intuitively if $\Omega > \Omega_d$ the motor is turning too fast, so we apply a negative torque to slow it down. If $\Omega < \Omega_d$ we apply a positive torque to speed it up.

This control law works very well for this case, and is called a proportional or "P" control system, since the motor torque is proportional to the angular velocity error $(\Omega - \Omega_d)$.

We can show why this control law works mathematically as follows.

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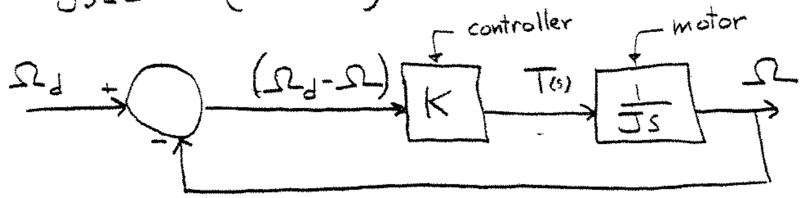
Motor dynamics : $T = J\dot{\Omega}$ $\quad (\dot{\Omega} = \ddot{\theta})$

Controller : $T = K(\Omega - \Omega_d)$ (also called the "<u>compensator</u>")

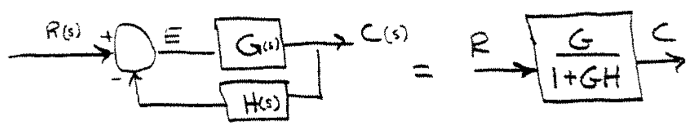
equation of combined system ; $J\dot{\Omega} = K(\Omega - \Omega_d)$.

Laplace Transform : $Js\Omega = K(\Omega - \Omega_d)$.

Block Diagram :



Or, using block diagram algebra,



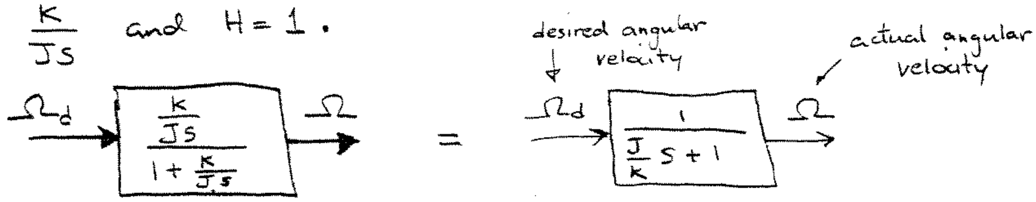
since,

$$\left.\begin{array}{l} E = R - CH \\ C = EG \end{array}\right\} C = (R - CH)G \Rightarrow C(1+GH) = RG \text{ or } \frac{C}{R} = \frac{G}{1+GH}$$

$\frac{G}{1+GH}$ is called the <u>closed loop system</u>

For our case, the closed loop system is found by setting

$G = \frac{K}{Js}$ and $H = 1$.



So the time constant for the closed loop system is $\tau = \frac{J}{K}$ seconds.

To reduce τ, just increase the gain K.

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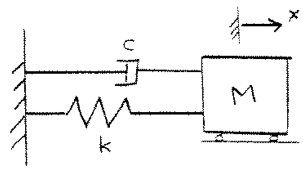
Notes for Lab 5

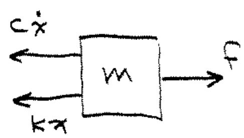
Second Order Systems

- A very important differential equation in engineering is second order with constant coefficients. This equation arises all the time in dynamics, for instance



Units
f : lb
k : lb/in
c : $\frac{\text{lb-sec}}{\text{in}}$
m : $\frac{\text{weight}}{g} = \frac{\text{lb}}{386.4}$

where x is the displacement of the mass from the undeformed spring position, and $f(t)$ is an applied force. We find the equation of motion for this system by drawing a free-body diagram of the forces acting on m, assuming the system is not at rest ($x \neq 0$, $\dot{x} \neq 0$). If we assume $x > 0$, $\dot{x} > 0$ then the positive forces are labeled as



so ΣF_x gives

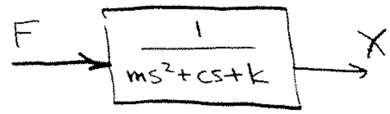
$$m\ddot{x} = f - c\dot{x} - kx \quad , \text{ or } \quad \underline{m\ddot{x} + c\dot{x} + kx = f}$$

The Laplace transform of this equation with zero initial conditions is

$$X(ms^2 + cs + k) = F(s),$$

or, the block diagram is

$$\frac{X}{F} = \frac{1}{ms^2 + cs + k}$$



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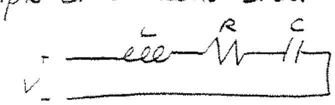
Another example of a second order system is an RLC circuit



where we saw in the first lecture that

$$V = \left[R + Ls + \frac{1}{Cs}\right] I \quad \text{or,} \quad \frac{I}{V} = \frac{Cs}{LCs^2 + RCs + 1}$$

$$V \longrightarrow \boxed{\frac{Cs}{LCs^2 + RCs + 1}} \longrightarrow I \,.$$

Both of these systems denominators can be expressed in the form

(1) $$s^2 + 2\zeta\omega_n s + \omega_n^2 \,.$$

For the mass-spring oscillator, divide the numerator and denominator by m to make the coefficient of s^2 be 1.

Hence

$$2\zeta\omega_n = \frac{c}{m} \quad \text{and} \quad \omega_n^2 = \frac{k}{m},$$

ω_n is called the <u>undamped natural frequency</u>

ζ is called the <u>damping ratio</u>

The roots of (1) are

$$s_i = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \,.$$

The interesting situation is when $\zeta < 1$. If $\zeta \geq 1$ we have two real roots, and the response $x(t)$ will be the sum

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Step Response

If a step force is applied to the mass-spring system $f(t) = A\,u(t)$, $F(s) = \frac{A}{s}$, the response is

$$X = \frac{A/m}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}.$$

From the transform table we know if $\zeta < 1$:

$$F(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \Rightarrow f(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n\sqrt{1-\zeta^2}\,t + \phi)$$

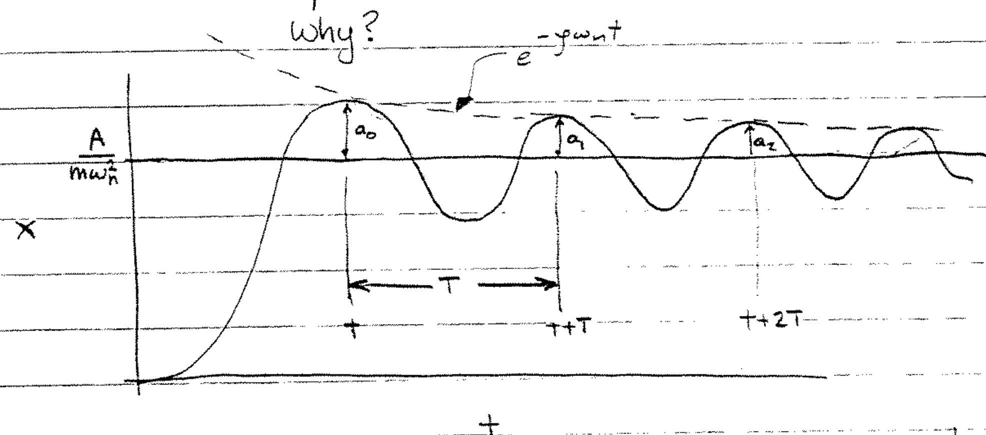
$$\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

(You will derive this relation for the lab work!)

So for our system

$$(2) \qquad x(t) = \frac{A}{m\omega_n^2}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}\,t + \phi)\right]$$

↑ why?



We see that the response decays exponentially with $e^{-\zeta\omega_n t}$ and $\tau = \frac{1}{\zeta\omega_n}$ is a measure of how quickly the system reaches 44

3 4

The quantity $\omega_n\sqrt{1-\zeta^2} \equiv \omega_d$ is called the damped natural frequency. For small damping $\zeta = \frac{c}{2\omega_n m}$ is small.

say $\zeta < .1$, so $\omega_n \cong \omega_d$.

In experiment 3, you determine ζ and ω_n experimentally. One method is to look at the free, unforced, response of the system on the oscilloscope. The form of the solution is the same as (2) with no steady-state term (the 1). If you measure the amplitude of succesive peaks, $a_0, a_1, a_2 \cdots a_n$ you can estimate ζ as follows.

The period between peaks, T, is the time it takes $\sin(\omega_d t + \phi)$ to oscillate once, so

$$\left[\omega_d(t+T)+\phi\right] - \left[\omega_d t + \phi\right] = 2\pi$$

$$\omega_d T = 2\pi$$

$$T = \frac{2\pi}{\omega_d}$$

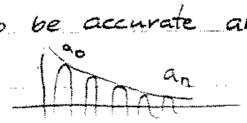
The ratio $\frac{a_0}{a_1}$, which you can measure, is

$$\frac{a_0}{a_1} = \frac{e^{-\zeta\omega_n t}}{e^{-\zeta\omega_n(t+T)}} = e^{\zeta\omega_n T}$$

called the logarithmic decrem

$$\text{so} \quad \ln\frac{a_0}{a_1} = \zeta\omega_n T = \zeta\omega_n\frac{2\pi}{\omega_d} = \zeta\omega_n\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\zeta 2\pi}{\sqrt{1-\zeta^2}} \equiv \delta$$

If you want to be accurate about it, measure $\frac{a_0}{a_n}$



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Then $\frac{a_0}{a_n} = e^{\zeta \omega_n T n}$

$$\hat{\delta} \equiv \ln \frac{a_0}{a_n} = \frac{\zeta 2\pi n}{\sqrt{1-\zeta^2}},$$ or, solving for ζ,

$$(3) \qquad \zeta = \frac{\hat{\delta}}{\sqrt{4\pi^2 n^2 + \hat{\delta}^2}} \approx \frac{\hat{\delta}}{2\pi n}$$

So to compute ζ, measure a_0, n, a_n, then solve for $\hat{\delta}$ and use (3).

Frequency Response

Another important concept is the frequency response of a second order system. Suppose we apply a sine wave to the system $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$. If the input is

$$a \sin \omega t,$$

then the output is

$$a\,|G(j\omega)|\sin(\omega t + \phi) \qquad \phi = \angle G(j\omega).$$

Now write $G(j\omega)$ as

$$G(j\omega) = \frac{1}{\frac{(j\omega)^2}{\omega_n^2} + \frac{2\zeta j\omega}{\omega_n} + 1}$$

$$= \frac{1}{\ldots}$$

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so

$$|G(j\omega)| = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

and

$$\phi = 0 - \tan^{-1}\left[\frac{\frac{2\zeta\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

The maximum amplitude response occurs when

(4) $$\omega_r = \omega_n\sqrt{1-2\zeta^2} \qquad \zeta \le .707,$$

which is called the <u>resonant frequency</u>.

(5) $$|G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$ is the amplitude ratio

at the resonant frequency. You will derive (4) and (5) for the lab write up. In general $\omega_n \neq \omega_d \neq \omega_r$ but they are usually very close.

Equation (5) shows that as $\zeta \to 0$ $|G(j\omega_r)| \to \infty$ and (4) shows $\omega_r \to \omega_n$.

Physically this means that an <u>undamped</u> system will blow up (break) when forced at its natural frequency!

A Bode Plot of $20 \log|G(j\omega)|$ vs. $\frac{\omega}{\omega_n}$ is shown for typical ζ on the next page.

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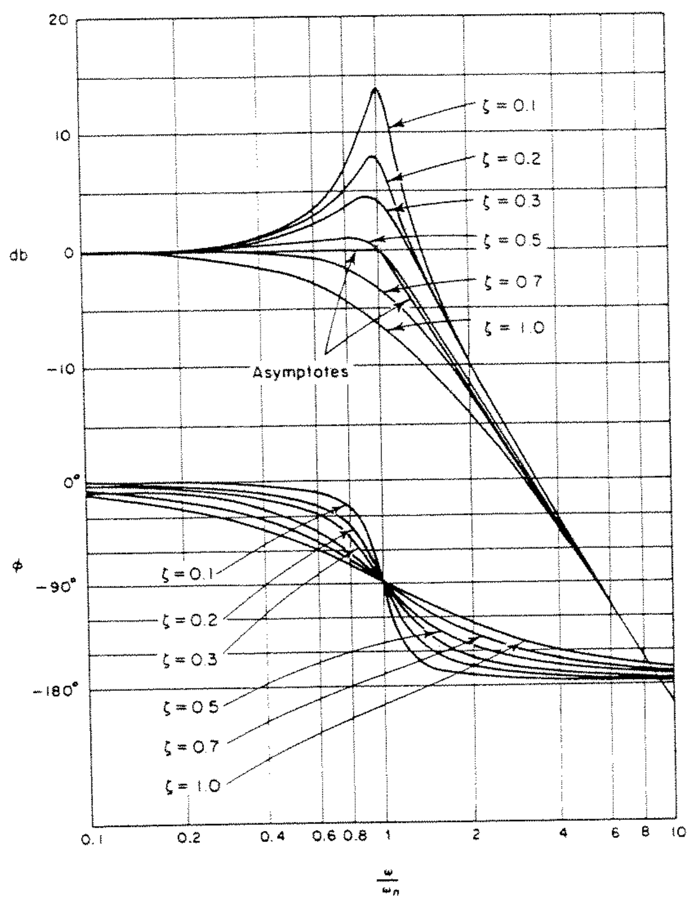

20
10
db 0
-10
0°
φ
-90°
-180°
ζ = 0.1
ζ = 0.2
ζ = 0.3
ζ = 0.5
ζ = 0.7
ζ = 1.0
Asymptotes
0.1
0.2
0.4
0.6 0.8
1
2
4
6
8
10
ω/ωn

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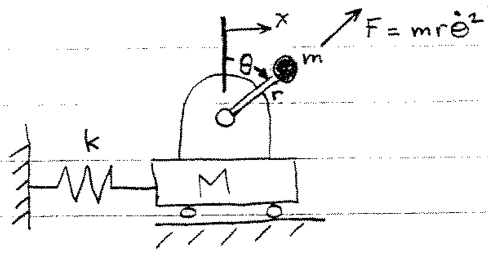
A common vibration problem is due to rotating machinery with unbalanced loads. Consider the model system

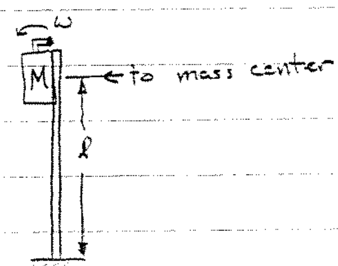


where m is an unbalanced mass rotating with constant angular velocity $\dot{\theta} = \omega$. The force that m applies on M is radially outward of magnitude $F = mr\omega^2$. The component of this force in the x direction is $mr\omega^2 \sin\theta$. For constant $\dot{\theta}$, $\theta = \omega t$, so the forcing function is

$$f = mr\omega^2 \sin\omega t .$$

This sine wave produces a frequency response of the machine as just described, but note that the amplitude of the input wave is a function of ω^2, so it increases with motor velocity.

The spring k could represent structural rigidity. In experiment 3, we have a motor with an unbalanced load mounted on a cantilevered beam.



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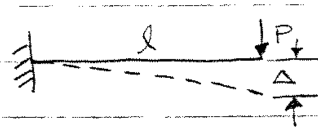
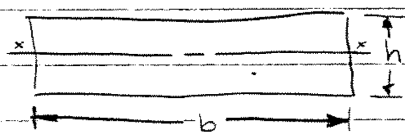
The spring constant of the structure can be found in any strength of materials book from the load-deflection relation:



$$\Delta = \frac{P\ell^3}{3EI}$$

Where P is an applied load, Δ is the deflection at the beam tip, E is the modulus of elasticity, and I is the area moment of inertia of the beam cross-section. For our beam



$$I_{xx} = \frac{1}{12} b h^3$$

The spring constant is $k = \frac{F}{x} = \frac{P}{\Delta} = \frac{3EI}{\ell^3}$

Because the beam itself has mass, M should be increased. An approximate amount to increase M by is .236 m, where m is the total mass of the beam. This number can be found using energy methods of vibration analysis. The estimated natural frequency is

$$\omega_n^2 \cong \frac{k}{M_{tot}} = \frac{3EI}{\ell^3(M+.236m)}.$$

This is only an approximation, but a good one. There are also higher harmonics, or other natural frequencies of the beam as you will see in experiment 7.

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Second Order Control Systems

Although the ~~above~~ discussion on page 29 on motor velocity control indicates that $\omega(t) \to \omega_d(t)$ with time constant τ, it actually does not work so well because friction has been ignored. If you touch the rotating shaft in experiment 4 you will introduce an error. One common method used to eliminate the error is to add integral control action as follows.

The motor dynamics with friction are

$$T - T_f = J\dot{\omega},$$

where T_f is the torque due to friction. The form of a proportional controller is

$$T = -k_p e,$$

where $e = (\omega - \omega_d)$, the error of the actual angular velocity from the desired value ω_d. The form of a proportional plus integral control is

$$T = -k_p e - k_I \int_0^t e\,dt, \quad \text{or} \quad T(s) = -E(s)\left(K_p + \frac{K_I}{s}\right)$$

where K_I is the integral gain. Now, if $e(t) \neq 0$ say it is some constant ϵ, then $\int e\,dt$ increases with time and eventually the torque becomes high enough to overcome friction.

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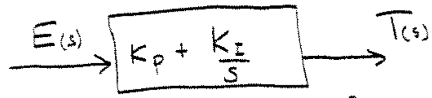
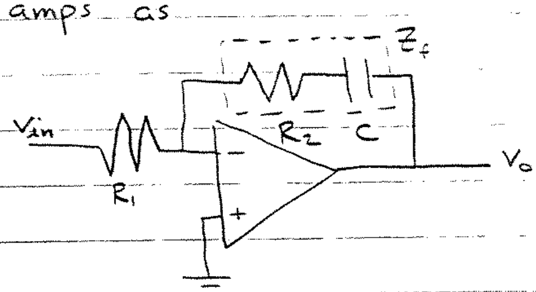
The block diagram of a P-I compensator is



A PI control circuit can be implemented with op amps as



$$V_o = -\frac{Z_f}{Z_{in}} V_{in} \qquad Z_{in} = R_1$$

$$Z_f = R_2 + \frac{1}{Cs} \text{ (two impedances in series)}$$

$$V_o = -\frac{1}{R_1}\left(R_2 + \frac{1}{Cs}\right) V_{in}$$

$$T \Rightarrow V_o = -\left[\frac{R_2}{R_1} + \frac{1}{R_1 C s}\right] V_{in} \Rightarrow E$$

$$\uparrow \qquad \uparrow$$

$$K_p = \frac{R_2}{R_1} \qquad K_I = \frac{1}{R_1 C}$$

To show how the whole system works mathematically, look at the closed loop system. For ω_d = constant, $\dot{e} = \dot{\omega}$, hence

$$T - T_f = J\dot{e}$$
$$-k_p e - k_I \int e\,dt - T_f = J\dot{e}$$

or,

$$J\dot{e} + k_p e + k_I \int e\,dt = -T_f .$$

Now take the derivative of this equation with respect to time and assume T_f = constant to give

$$J\ddot{e} + k_p\dot{e} + k_I e = 0 .$$

This is an unforced second order system that will converge to $e(\infty) = 0$ as long as the roots of the characteristic equation are in the left half s plane. This means that $k_p > 0$ and $k_I > 0$

The response can be found from

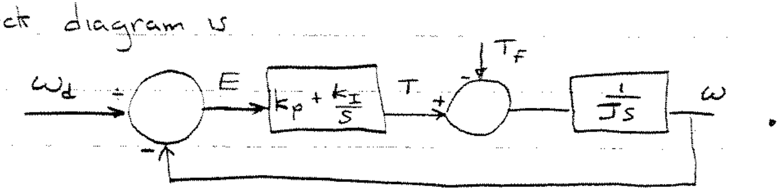
initial conditions

$$E(s)\left(Js^2 + k_p s + k_I\right) = IC(s)$$

$$E(s) = \frac{IC(s)}{Js^2 + k_p s + k_I} = \frac{IC/J}{s^2 + k_p/J\, s + k_I/J}$$

so $\omega_n^2 = \frac{k_I}{J}$ and $2\zeta\omega_n = \frac{k_p}{J}$, and these numbers can be set to be any values just by adjusting two gains, k_p and k_I.

The block diagram is



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You should confirm that this matches the above differential equations.

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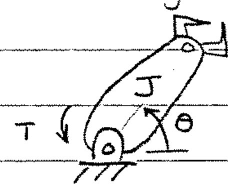
Notes for Lab 6

A PD motor position control system.

By far, the most common control system used in industry is a motor position controller. Examples are robot arm controllers, radar tracking systems, numerically controlled milling machines and lathes, remote controlled toys, ...

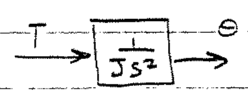
Consider a one jointed robot arm,



First suppose there is no friction and no gravity. We have the ability to apply any torque T we want to the arm from a DC motor. How can we design a controller which will quickly position the arm to any desired angle θ_d ?

We could first try a proportional controller as in experiment 2.

Dynamic Model $T = J\ddot{\theta}$ or $T = s^2 J \Theta(s)$



Controller

$T = -k_p(\theta - \theta_d)$ ⟶ This is like a torsional spring

↑ need to measure, say with a potentiometer.

Closed loop system

$J\ddot{\theta} = -k_p(\theta - \theta_d)$ 54

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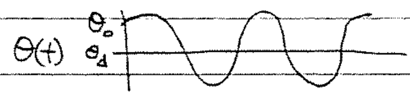
Define $e = \theta - \theta_d$. If θ_d = constant, $\ddot{\theta} = \ddot{e}$ and the closed loop system is

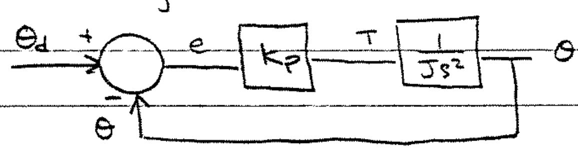
$$J\ddot{e} + k_p e = 0.$$

This is the same form as an undamped mass spring system (i.e. $m\ddot{x} + kx = 0$). The system is marginally stable, so the

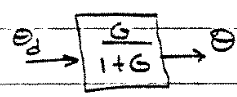


robot arm would oscillate forever about the unstretched "spring" position $\theta = \theta_d$.

In terms of block diagrams:



The closed loop transfer function is:



$$\frac{G}{1+G} = \frac{\frac{k_p}{Js^2}}{1 + \frac{k_p}{Js^2}} = \frac{k_p}{Js^2 + k_p},$$

hence $\zeta = 0$ and there are 2 roots on the imaginary axis.

To fix the problem, we need to somehow add damping to the system. If a tachometer signal is available, the following control law works

$$T = -k_p(\theta - \theta_d) - k_v\dot{\theta}$$

($\dot{\theta}$ from tachometer)

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The closed loop system is

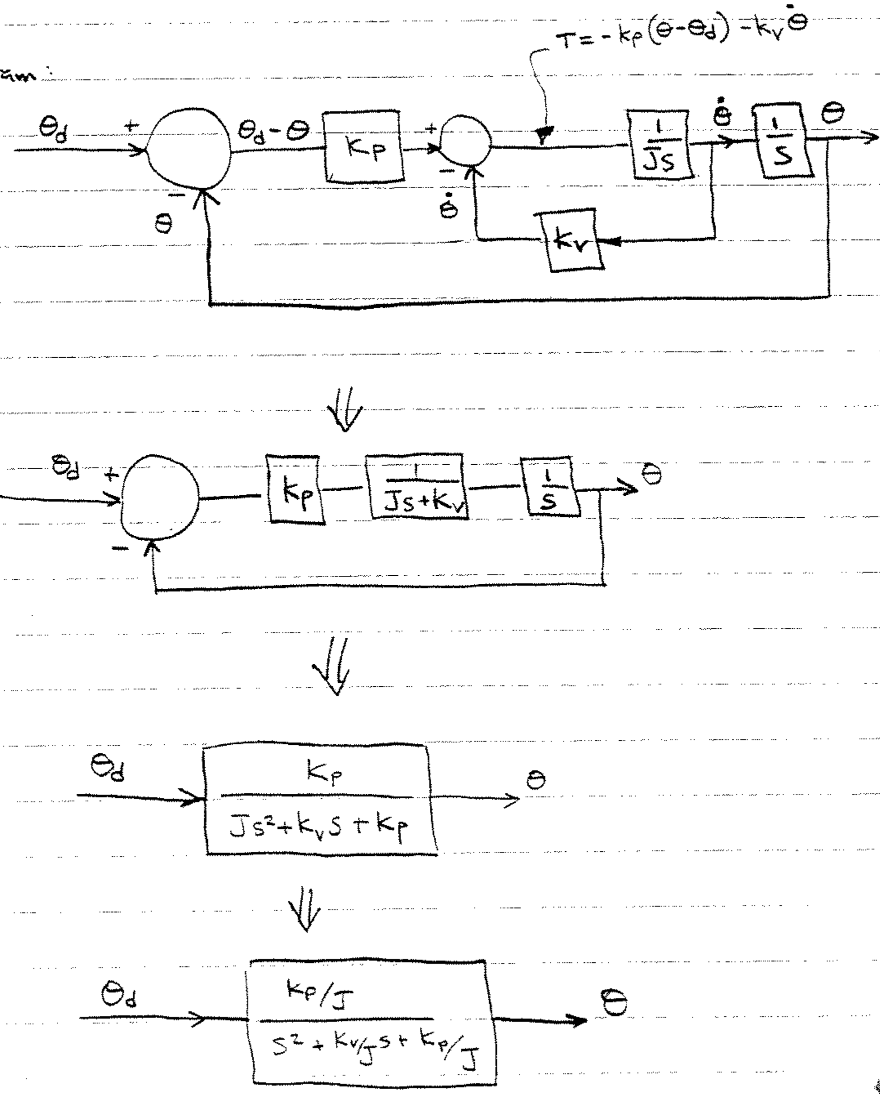
$$J\ddot{\theta} = -k_p(\theta - \theta_d) - k_v\dot{\theta}$$

or

$$J\ddot{e} + k_v\dot{e} + k_p e = 0,$$

so we now have the required damping term. The system is stable for any $k_v > 0$ and $k_p > 0$.

Block Diagram:



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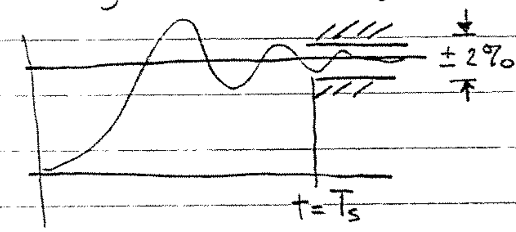
Typically control system designers care very much about performance. That is, they want $e(t) = \theta - \theta_d$ to go to zero quickly. We said in the last lecture that the time constant for a second order system is $\tau = \frac{1}{\zeta\omega_n}$. It can be shown that if you wait $t = 4\tau$ seconds after an input is applied, the output will be within 2% of its final value. This is called the <u>settling</u> <u>time</u> $T_s = 4/\zeta\omega_n$.



So a motor control design specification may call for a settling time of .1 second with a ζ of .7. It is easy to do this electrically using the gains k_p and k_v. Since

$$(1)\quad 2\zeta\omega_n = \frac{k_v}{J} \quad \text{and} \quad (2)\quad \omega_n^2 = \frac{k_p}{J},$$

$$T_s = \frac{4}{\zeta\omega_n} \overset{(1)}{=} \frac{8J}{k_v} \Rightarrow k_v = \frac{8J}{T_s}$$

$$(2)\quad k_p = J\omega_n^2 \overset{(1)}{=} J\left(\frac{k_v}{2\zeta J}\right)^2 = \left(\frac{k_v}{2\zeta}\right)^2 \frac{1}{J}.$$

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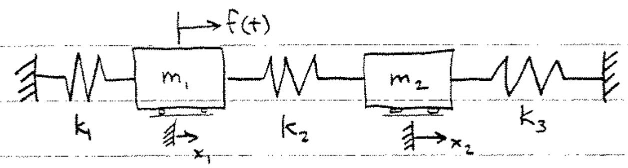
Notes for Lab 7

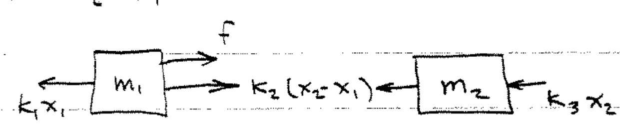
Systems with Two Modes of Vibration

No structure can ever be modelled exactly using just the simple mass-spring oscillator. In reality there are always higher frequency modes of vibration present in any structure. Even in Experiment 3, you probably noticed oscillations at higher frequencies than the fundamental (lowest one) present in the accelerometer output signal.

Consider a model for Experiment 5,



f is a force applied to m_1, and x_1, x_2 are the positions of m_1 and m_2 relative to a fixed reference frame. Draw a free body diagram to obtain the equations of motion. Assume $x_2 > x_1 > 0$:



$$m_1\ddot{x}_1 = f + k_2(x_2 - x_1) - k_1 x_1$$
$$m_2\ddot{x}_2 = -k_2(x_2 - x_1) - k_3 x_2$$

or,

$$m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2 x_2 = f$$
$$m_2\ddot{x}_2 - k_2 x_1 + (k_2+k_3)x_2 = 0$$

or in matrix form,

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix}. \quad (2)$$

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Symbolically we often write these as

(1) $$M\ddot{\underline{x}} + K\underline{x} = \underline{f}$$, where

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad K = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \quad \underline{f} = \begin{pmatrix} f \\ 0 \end{pmatrix}.$$

Equation (1) is a very general form that works for systems with any number of masses, $m_3, m_4 \cdots$ etc. It can be shown using energy methods that M and K are always symmetric, which means $M = M^T$, $K = K^T$ (T denotes transpose).

To solve for the motion $\underline{x}(t)$ given $f(t)$ we can still use Laplace transforms on (1)

$$\underline{F} = \begin{pmatrix} F(s) \\ 0 \end{pmatrix}$$

$$M s^2 \underline{X} + K\underline{X} = \underline{F} + \underline{I}(s), \quad \underline{X} = \begin{pmatrix} X_1(s) \\ X_2(s) \end{pmatrix}, \text{ and}$$

$\underline{I}(s)$ is a vector arising from the initial conditions. Solving for $\underline{X}(s)$ we have

$$[Ms^2 + K]\underline{X} = \underline{F} + \underline{I}(s)$$

(2) $$\underline{X} = [Ms^2 + K]^{-1}(\underline{F} + \underline{I})$$

In this case it is easy to invert a 2×2 matrix because

$$\text{if} \quad A = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \quad A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} c & -b \\ -b & a \end{bmatrix}; \quad \det(A) = ac - b^2$$

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So $Ms^2+K = \begin{bmatrix} m_1 s^2 + k_1 + k_2 & -k_2 \\ -k_2 & m_2 s^2 + k_2 + k_3 \end{bmatrix}$ and the inverse formula gives

$$[Ms^2+K]^{-1} = \frac{1}{(m_1 s^2 + k_1 + k_2)(m_2 s^2 + k_2 + k_3) - k_2^2} \begin{bmatrix} m_2 s^2 + k_2 + k_3 & k_2 \\ k_2 & m_1 s^2 + k_1 + k_2 \end{bmatrix}.$$

Plugging this into (2) and looking at the first component of X gives:

$$(3) \quad X_1(s) = \frac{F(s)(m_2 s^2 + k_2 + k_3)}{(m_1 s^2 + k_1 + k_2)(m_2 s^2 + k_2 + k_3) - k_2^2} \quad (\text{with } I(s) = 0).$$

You could also relate $X_1(s)$ to $F(s)$ by solving 2 equations and 2 unknowns without explicitly inverting $[Ms^2+K]$.

Now suppose $f(t) = \sin\omega t$ and we want $x_1(t)$. From (3) the transfer function X_1/F is known. Hence

$$\frac{X_1}{F}(j\omega) = \frac{(-m_2\omega^2 + k_2 + k_3)}{(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2 + k_3) - k_2^2}.$$

or, expanding the denominator,

$$(4) \quad \frac{X_1}{F}(j\omega) = \frac{(-m_2\omega^2 + k_2 + k_3)}{m_1 m_2 \omega^4 - \omega^2(m_2(k_1+k_2) + m_1(k_2+k_3)) + (k_1k_2 + k_3k_1 + k_2k}$$

Note that the denominator is quadratic in ω^2, so the roots (poles) can be found with the quadratic formula. 60

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In some cases you might want to stop $x_1(t)$ from vibrating even though a oscillating force is applied to m_1 with some frequency $\omega = \omega_0$.

Equation (4) shows that

$$\left| \frac{X_1}{F}(j\omega_0) \right| = 0 \quad \text{when}$$

we choose $\underline{\frac{k_2+k_3}{m_2} = \omega_0^2}$ to make the numerator of (4) zero. If you choose m_2, k_2, k_3 according to the above formula, $x_1(t) = 0$, when $\omega = \omega_0$. That is, mass m_1 will not move if $f = \sin\omega_0 t$. This is a common method for vibration isolation. You will find ω_0 for a given k_2, k_3, m_2 in Experiment 5.

Equation (4) also shows that

$$\left| \frac{X_1}{F}(j\omega) \right| \to \infty \quad \text{when}$$

ω approaches a root of the denominator. This occurs at two distinct frequencies, say ω_1 and ω_2. Usually,

$$\omega_1 < \omega_0 < \omega_2.$$

You will determine all three frequencies experimentally in Exp 5.

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Modes of Vibration

More insight is gained regarding the motion of the system if we consider the case of free vibrations, i.e. $f(t) \equiv 0$.

Assume that $\underline{x}(t)$ has the form $\underline{x}_0 \sin(\omega t + \phi)$, so that

(5) $$\underline{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} x_{10} \sin(\omega t + \phi) \\ x_{20} \sin(\omega t + \phi) \end{pmatrix} = \underline{x}_0 (\sin \omega t + \phi).$$

The vector $\underline{x}_0 = \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix}$ is the amplitude of vibration for each mass. Now substitute (5) into

$$M\ddot{\underline{x}} + Kx = 0$$

to obtain

$$M -\omega^2 \underline{x}_0 \sin(\omega t + \phi) + K \underline{x}_0 \sin(\omega t + \phi) = 0$$

or

(6) $$\left[-\omega^2 M + K\right]\underline{x}_0 = 0$$ (note ω^2 is a scalar).

Now multiply (6) by $M^{-1} = \begin{bmatrix} \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix}$ to obtain

(7) $$\left[-\omega^2 I + M^{-1} K\right]\underline{x}_0 = 0$$

Now, (7) has the general form $[A - \lambda I]\underline{x}_0 = 0$ which is called an eigenvalue problem. The question is, are there any values of $\underline{x}_0 \neq 0$ that satisfy (7)? For $\underline{x}_0 \neq 0$ to exist $[-\omega^2 I + M^{-1}K]$ must be singular, i.e. not invertible. If it were invertible, the only solution to (7) would be $\underline{x}_0 = 0$.

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$$M^{-1}K = \begin{bmatrix} \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \begin{bmatrix} k_1+k_2 & k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} = \begin{bmatrix} (k_1+k_2)/m_1 & -k_2/m_1 \\ -k_2/m_2 & k_2+k_3/m_2 \end{bmatrix}$$

$$(8) \quad \left[-\omega^2 I + M^{-1}K\right] = \begin{bmatrix} (k_1+k_2)/m_1 - \omega^2 & -k_2/m_1 \\ -k_2/m_2 & (k_2+k_3)/m_2 - \omega^2 \end{bmatrix}$$

For $\left[-\omega^2 I + M^{-1}K\right]$ to be singular,

$$\det\left(-\omega^2 I + M^{-1}K\right) = 0 \quad \text{or}$$

$$\left(\frac{k_1+k_2}{m_1} - \omega^2\right)\left(\frac{k_2+k_3}{m_2} - \omega^2\right) - \frac{k_2^2}{m_1 m_2} = 0 \quad \text{or}$$

$$(9) \quad (k_1+k_2 - m_1\omega^2)(k_2+k_3 - m_2\omega^2) - k_2^2 = 0 .$$

The value of ω^2 that satisfies this equation is also a pole of the transfer function in (4). It is called an eigenvalue of M^{-1}**K**. Suppose we solve for the two values of ω^2 that satisfy (9) and denote them as ω_1^2 and ω_2^2, as before, so that

$$\det\left(\left[-\omega_1^2 I + M^{-1}K\right]\right) = \det\left(\left[-\omega_2^2 I + M^{-1}K\right]\right) = 0.$$

At these frequencies, it is possible to have nonzero amplitudes of vibration, so the amplitudes $\underline{x}_0 = \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix}$

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that satisfy (7) for $\omega=\omega_1$ or $\omega=\omega_2$ can be found. Note that there is no solution (other than $x_0=0$) possible for $\underline{x}_0$ at any frequency other than $\omega=\omega_1$ or $\omega=\omega_2$. Why? The form of (7) is

$$A\underline{x}_0 = 0, \quad \text{and} \quad \det(A)=0 \quad \text{for}$$

$\omega=\omega_1, \omega_2$. Hence, the rows of A are linearly dependent on one another at $\omega=\omega_1$ or $\omega=\omega_2$. Say

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then let } \underline{x}_0 = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix},$$

$$A\underline{x}_0 = \begin{pmatrix} a x_{10} + b x_{20} \\ c x_{10} + d x_{20} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad \left(A = \left[-\omega^2 I + M^{-1}K\right]\right) \text{ (8}$$

The 2 equations contain the same information. We see that the top equation shows

$$x_{10} = -\frac{b}{a} x_{20}, \qquad (10)$$

where $b = -k_2/m_1$ and $a = \left[(k_1+k_2)/m_1 - \omega^2\right]$ for $\omega=\omega_1$ or $\omega=\omega_2$. Equation (10) is very helpful for visualizing the motion. That is, the amplitudes must be in exactly the ratio $\frac{x_{10}}{x_{20}} = \frac{-b}{a}$ in order for free vibration to be possible at all.

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For example, consider the case when $m_1=m_2=k_1=k_2=k_3=1$.

54 Eq (8) gives

$$[-\omega^2 I - M^{-1}K] = \begin{bmatrix} 2-\omega^2 & -1 \\ -1 & 2-\omega^2 \end{bmatrix} \equiv A .$$

The characteristic equation (9) is $|A| = 0$:

$$(2-\omega^2)(2-\omega^2) - 1 = 0 \quad \text{or}$$

$$\omega^4 - 4\omega^2 + 3 = 0 . \quad \text{(Also see the denominator of (4))}$$

The roots are

$$\omega^2 = \frac{4 \pm \sqrt{16-12}}{2} = 2 \pm 1 = \underline{\underline{1 \text{ or } 3}}.$$

The amplitudes that correspond to the root $\omega_1^2 = 1$, with $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, satisfy (10)

$$x_{10} = -\left[\frac{-1}{1}\right] x_{20} = x_{20}$$

Hence $\underline{x}_o$ has the form

$$\underline{x}_o^1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \alpha ,$$

where α is a scalar amplitude for both masses (and the superscript means the first mode). Hence, in the first mode of vibration for the system, both masses move to the left and right together in exactly the same phase.

The amplitudes that correspond to the root $\omega_2^2 = 3$ satisfy

$$x_{10} = -\left[\frac{-1}{(2-3)}\right] x_{22} = -x_{20} .$$

For the second mode then $\underline{x}_o$ has the form

$$\underline{x}_o^2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \alpha .$$

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directions, or 180° out of phase. They do this at a higher frequency. Note that the center spring deforms in this mode whereas it remains undeformed in the first mode, and this requires more energy during the vibration.

A general free vibration can be expressed as

$$x(t) = \underline{X}_0^1 \sin(\omega_1 t + \phi_1) + \underline{X}_0^2 \sin(\omega_2 t + \phi_2)$$

because this solution satisfies (1) for both $\underline{x}_0^1$ and $\underline{x}_0^2$. Hence a general motion is not usually all in one mode or the other, but a linear combination of the two.

Finally, note that the vibration isolation frequency for m_1 in this example is given by

$$\omega_0^2 = \frac{k_2 + k_3}{m_2} = 2$$

so

$$\omega_1^2 = 1 < \omega_0^2 = 2 < \omega_2^2 = 3 .$$

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4.4 OLd exam to practice from

These are old exams given to us to practice on

Local contents

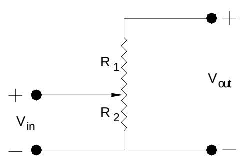
4.4.1 midterm 1999

MAE 106 Midterm Exam
Winter 1999

University of California, Irvine
Department of Mechanical and Aerospace Engineering

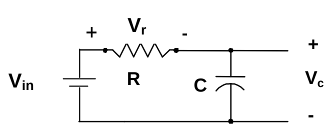
1) Based on your experience in laboratory and lecture, provide brief answers to the following questions:

a) Why is it wrong to use a potentiometer in the following configuration?

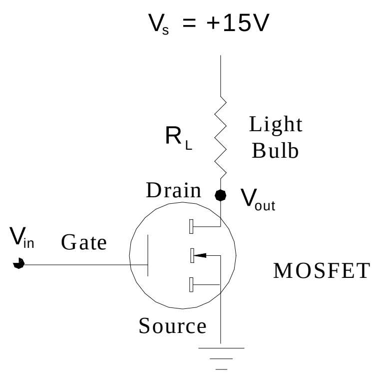


b) A motor can also be used as what kind of sensor?

c) What kind of filter is the following circuit? (Assume V_c is the output)



2) A circuit for controlling a light bulb with a MOSFET is shown below. The light bulb is modeled as a resistor. By changing the input gate voltage to the MOSFET, the light bulb can be turned on and off. A table of the MOSFET's drain/source resistance (like the one you generated in lab) is also shown.



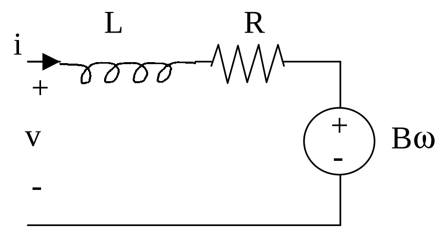
V_{in} (volts)	R_{DS} (K□ □
0	100
1	100
2	90
3	80
3.4	75
3.6	41
3.9	4
4.2	0.1
5.5	0.01

a) To turn the light bulb on, should the input gate voltage (V_{in}) to the MOSFET be high or low?

b) Assume R_L = 1KΩ. Calculate V_{out} corresponding to an input gate voltage of 3.9 volts.

c) How many watts of power will the light bulb consume with the input gate voltage equal to 3.9 volts?

3) A circuit model of a DC Brush motor is:



where ω is the angular velocity of the motor shaft.

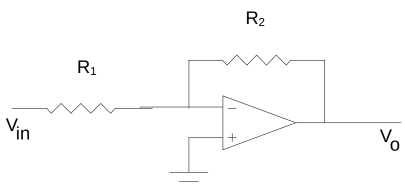
a. Write the differential equation that describes the relationship between input voltage and current for this circuit. (Hint: use Kirchoff's voltage law).

b. Solve this differential equation for the current through the motor as a function of time when:

- the shaft of the motor is held fixed
- a constant voltage v is applied across the motor at time = 0
- the initial current i(t = 0) through the inductor is zero

c. Plot the torque that the motor generates as a function of time for the conditions described in part b. Label the axes, the final value of the torque, and the time at which the torque has reached 63% of its final value. Assume the motor's torque constant is some constant B.

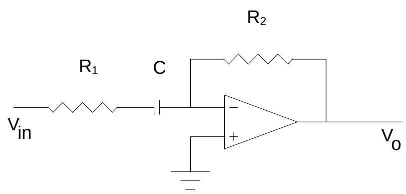
4) Consider the following op-amp circuit:



a) Derive the relationship of the output to the input:

b) What function does this circuit perform if $R_2 > R_1$?

Problem 4 continued: If we add a capacitor, the circuit becomes a filter:



c) Derive the transfer function of the filter using the impedance of the capacitor, Kirchoff's laws, and your knowledge of how the Op-Amp works.

d) What are the time constant and corner frequency of this filter?

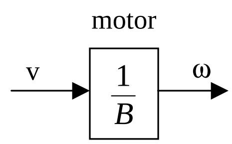
e) Find the filter's gain (i.e. the magnitude of the transfer function):

f) Assume we input a sinusoid to this filter. For frequencies much greater than the corner frequency, what is the filter's gain?

g) For frequencies much less than the corner frequency, what is the filter's gain?

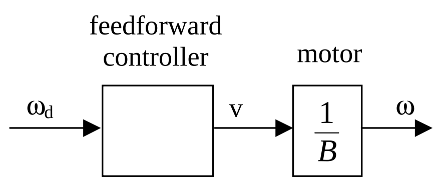
h) What kind of filter is this?

5) Consider the problem of controlling the velocity of a motor. A simple model of the motor is given by the following block diagram:



where v is the voltage input to the motor and ω is the angular velocity of the shaft.

a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What gain value should the controller box have to make the output equal the desired output? Write the gain in the controller box.



b) What is a disadvantage of an open-loop controller like this one?

c) Draw a block diagram of a feedback controller for the motor, label all arrows, including the error signal.

d) What hardware (beside the motor) that we have discussed in class could you use to implement this feedback controller?

4.4.2 Design Exam 2004 Solution

MAE 106 Mechanical Systems Laboratory
Winter 2004 Design Exam

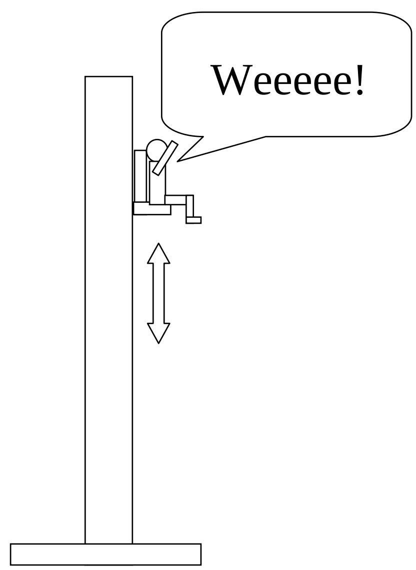
You have been hired as a control engineering consultant for a new ride for Disney's California Adventure Theme Park, called "Bouncing Over California". In this ride, the rider will feel like they are bungee jumping off of different bridges in California. Your job is to design a control circuit for the ride. You are given the following information:

- The ride is in a 100 foot tower that has a linear track. The rider sits in a harness which slides up and down the track. The track is driven by a motor.
- The track has substantial friction, which can be modeled as viscous friction with a damping coefficient of B (i.e. friction force = F=-Bv = damping coefficient*velocity).
- The motor that drives the track is a DC brushed motor with a current amplifier. It produces a force on the harness in response to a voltage input with a calibration coefficient of C_1 N/volt.
- There is a position sensor on the track that measures the position of the harness. The sensor has a calibration coefficient of C_2 volts/meter, and gives a reading of zero volts at the top of the track, where the person enters the harness.
- A typical bungee cord acts like a linear spring with a stiffness of K_b.
- The Disney people would like the system to behave like an undamped spring.

Design an analog circuit to control the security robot. You will get full credit if you:

1) Show your control law, in MKS units.
2) Provide appropriate control gains with units.
3) Show your control law, in units of volts
4) Draw an op-amp circuit that can implement your controller. Label what the inputs and outputs of your circuit should be connected to.
5) Choose appropriate values for the resistors and capacitors in your circuit.

PLEASE ENTER YOUR ANSWERS ON THE ANSWER SHEET



Solutions to MAE106 Winter 2004 Design Exam

1) Answer: (20 pts)
$F = -K_p(x - x_d) - K_d\dot{x}$

2) Answer: (20 pts)
Approach 1: $K_p = K_b$ N/m or kg/s^2
$K_d = -B$ Ns/m or kg/s

Approach 2: $K_p = 100M\omega_s^2$ N/m
$K_d = 2\sqrt{K_pM} - B$ Ns/m

3) Answer: (20 pts)
Note: $F=C_1v_o = v_o$ = motor voltage
v_s = sensor voltage = C_2*x
v_d = input voltage = C_2*x_d

$$C_1v_o = -K_p/C_2(v_s - v_d) - K_d\dot{v}_s/C_2$$
$$v_o = -1/C_1(K_p/C_2(v_s - v_d) - K_d\dot{v}_s/C_2)$$

Overview: There are two ways to do this design. In both approaches, we will use a PD position controller to make the controlled system behave like an undamped spring with $K=K_b$. The PD position controller equation is:

$$F = -K_p(x - x_d) - K_d\dot{x}$$

Approach 1: Make the controller have $K_p = K_b$, and cancel the friction with K_d = -B. Then the controlled dynamics are:

$$M\ddot{x} + B\dot{x} = -K_p(x - x_d) - K_d\dot{x} = -K_b(x - x_d) + B\dot{x} \rightarrow M\ddot{x} = -K_b(x - x_d)$$

which are the dynamics of an undamped spring with $K=K_b$.
In this approach, we want the rest length of the spring to be half way down the track x_d=50ft =16.5 meters

Approach 2: Make the controller track the motion of an undamped spring with $K=K_b$. In this case, we let x_d=16 ($\cos(\omega_s t)$-1) meters, so x_d = 0, max(x_d) = 33 m = 100ft. The frequency of the sinusoid ω_s should be the frequency of an undamped spring with $K=K_b$ and a rider of mass M $\omega_s = \sqrt{K_b/M}$.

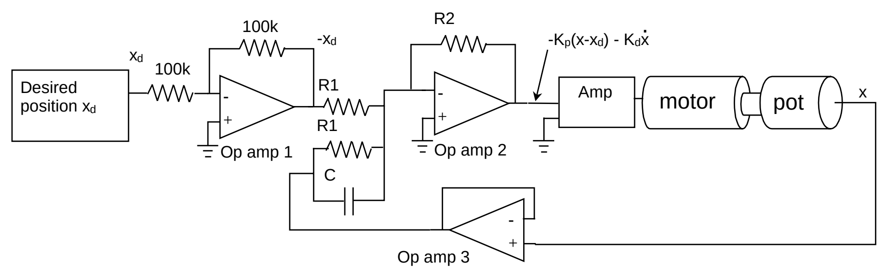
Then, we choose K_p and K_v so that the tracking is good. We choose the bandwidth of the controlled system ω_n = 10 ω_s (the factor 10 is arbitrary; the point is that we want the bandwidth significantly wider than ω_s for good tracking)

$$\omega_n = \sqrt{K_p/M} = 10\omega_s \rightarrow K_p = 100M\omega_s^2 = 100K_b$$

And choose ζ = 1 for good tracking without resonance and find K_d

$$\zeta = (B + K_d)/(2\sqrt{K_pM}) = 1 \rightarrow K_d = 2\sqrt{K_pM} - B$$

4) Answer: (20 pts) The circuit from Lab 6 will work for Approach 2:



For Approach 1, we need to invert the voltage into the capacitor with an op-amp inverter circuit (similar to Op amp 1 circuit), in order to get a negative K_d

5) Answer: (20 pts)

$K_P = R_2/R_1$
$K_d = R_2C$
So choose:
$R_2 = K_d / C$ $R_1 = R_2/K_P$

4.4.3 Design Exam Solutions 2002 2003

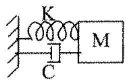
MAE 106 Mechanical Systems Laboratory
Winter 2003 Design Exam

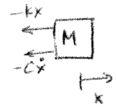
Answer Sheet

Part 1: In-Class Problems – NOTE: You must show your work to get full credit. You will not full credit if you just write the answer – you need to show how you got the answer.

30/30

1. Write the differential equation that describes the following system:

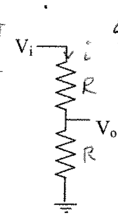




$M\ddot{x} = -kx - c\dot{x}$

$\boxed{M\ddot{x} + c\dot{x} + kx = 0}$ 5

2. Using Ohm's and Kirchoff's laws, find Vo for this circuit (Assume both resistors have value R). Note that you will **NOT** get credit if you simply write the answer – you must derive the answer.



$i = \frac{V_i}{2R} \quad V_o = iR = \frac{R}{2R} V_i = \boxed{\frac{1}{2} V_i}$ 5

3. Given $\dot{x} + Ax = A \quad x(0) = 0$ Find x(t).

H: $\dot{x} = -Ax \quad x = Ce^{-At}$

P: $x_p = 1 \quad Ax_p = A$

T: $x = Ce^{-At} + 1 \quad x(0) = 0 \Rightarrow C = -1 \Rightarrow \boxed{x = (1 - e^{-At})}$ 5

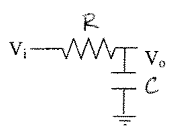
4. What is s1s2 if s1 = 2+j, s2= 2-j?

$(2+j)(2-j)$

$4 + 2j - 2j - j^2$

$\boxed{5}$ 5

5. Give the amount by which a 100 Hz sinusoid input at V_i would be scaled and phase shifted in creating the output Vo for the following circuit. Assume R = 1KΩ and C = 1 μF.



$V_o = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_i \qquad RC = (10^3)(10^{-6}) = 10^{-3}$

$\frac{V_o}{V_i} = \frac{1}{1 + RCs} = G(s)$

6

$\omega = 2\pi f = (2\pi)(100\text{Hz}) \quad \omega = 628 \frac{\text{rad}}{\text{sec}}$

$G(j\omega) = \frac{1}{1 + RCj\omega} \qquad \frac{1}{\sqrt{1 + [(10^{-3})(628)]^2}}$

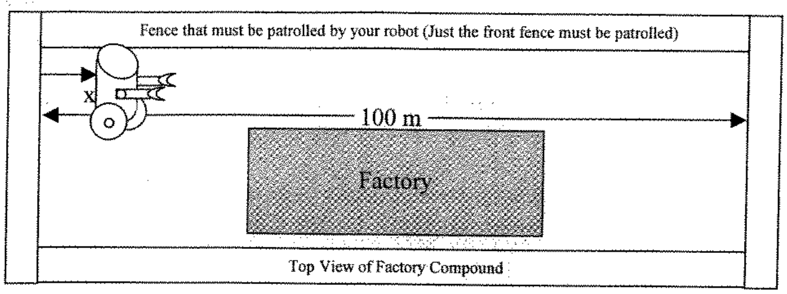
2 $|G(j\omega)| = \frac{1}{\sqrt{1 + (RC\omega)^2}} = ~~.8469~~ \ \boxed{.85}$

2 $\phi_{G(j\omega)} = 0 - \tan^{-1} RC\omega = -\tan^{-1} RC\omega$

$= -\tan^{-1}(10^{-3})(628)$

$= \boxed{-.56 \text{ rad}}$

Part 2: Control Design Exam
You are a control engineer working for the security department of a weapons factory. Your task is to design a mobile robot that can patrol the fence in front of the factory. The robot should also be able to move quickly to confront any intruders that are attempting to penetrate the fence.



You are given the following information and equipment to work with:

- The fence that must be patrolled is 100 m wide.
- The robot behaves as a mass M = 100 kg with some damping B = 1 ~~N/s~~. Ns/m
- The robot is controlled by a DC brushed motor and a linear current amplifier. A one volt input into the current amplifier gives 1 Nm of motor torque.
- You have access to the voltage signal from a potentiometer that measures the rotation of the robot wheels, and thus the position (x) of the robot along the fence. The pot voltage is zero at x = 0 m, and 1 volt at x = 100 m.
- The robot must be able to operate in two modes with the same controller: patrol mode and attack mode.
- In patrol mode, the robot must move back and forth along the fence at 0.1 Hz. The controller receives a sinusoidal command voltage from the command center in the factor. The voltage varies between 0 and 1 volt at 0.1 Hz telling it to patrol the fence in this periodic fashion.
- In attack mode, the robot must move within 1 sec to the position of an identified intruder. The intruder is identified by a camera, which radios a command voltage to the robot. A command voltage of 0 corresponds to an intruder at left end of the fence, and a command voltage of 1 corresponds to the right end of the fence. Voltages between 0 and 1 correspond to other locations along the fence. When an intruder is identified, the command voltage changes in step function fashion.
- You must implement your controller using op-amps, resistors, and capacitors.
- The radius r of the wheels of the robot is 0.1 m.

Design an analog circuit to control the security robot. You will get full credit if you:

1) Show a control law for the robotic conveyor belt that relates the motor torque to the desired (x_d) and measured robot position along the fence (x).
2) Write the differential equation that describes the dynamics of the *controlled* robotic system.
3) Choose gain values for your control law. State units. Make sure to consider both patrol and attack modes, and remember that one set of gain values must work for both modes.
4) Briefly explain why you chose the gains you did.
5) Draw an op-amp circuit that can implement your controller. Label what the inputs and outputs of your circuit should be connected to.
6) Choose appropriate values for the resistors and capacitors in your circuit.

PLEASE ENTER YOUR ANSWERS ON THE ANSWER SHEET

Part 2: Answer Sheet – Put your answers in the boxes, and show your work to the right of the boxes.

100 pts

20 — **1) Answer:**
$\tau = -K_p(x-x_d) - K_D\dot{x}$ or
$\tau = -r K_p(x-x_d) - r K_D \dot{x}$

sign error = -5, τ

20 — **2) Answer:**
$M\ddot{x} + c\dot{x} + Kx = Kx_d$
$c = B + \frac{K_D}{r}$ $K = \frac{K_p}{r}$

+10 if just dynamics
+5 if show controlled system closed

$M\ddot{x} + B\dot{x} = \frac{\tau}{r} = -\frac{K_p}{r}(x-x_d) - \frac{K_D}{r}\dot{x}$

$M\ddot{x} + (B + \frac{K_D}{r})\dot{x} + \frac{K_p}{r}x = \frac{K_p}{r}x_d$ (c, K, K)

20 — **3) Answer:**

ω_n	ζ	K_p	K_D
6.3	1	397 kg/s²	126 kg/s
4	1	160 kg/s²	79.9 kg/s

-2 units, -2 units

Patrol Mode: $\frac{x}{x_d} = \frac{K}{ms^2+cs+K} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ $\omega_n = \sqrt{\frac{K}{M}} = 63$ $= \sqrt{\frac{K_p/r}{M}}$

set $\omega_n = 2\pi f$, $(2\pi)(.1)\times 10 = (.63) = 6.3$ rad/s (cut-off frequency)

set $\zeta = 1 = \frac{C}{2\sqrt{MK}} = \frac{B + \frac{K_D}{r}}{2\sqrt{100\frac{K_p}{r}}} = \frac{1\frac{Ns}{m} + \frac{K_D}{.1m}}{2\sqrt{100kg\,\frac{397\,kgm}{.1m\,s^2}}}$

$= \sqrt{\frac{K_p}{.1m \cdot 100kg}}$, $63\frac{rad}{sec} = \sqrt{\frac{K_p}{10\,kgm}}$, $K_p = 397\frac{kg\,m}{s^2}$

check $N = \frac{kgm}{sec^2}$, $Nm = \frac{kgm^2}{s^2}$, $\frac{Nm}{m} = N \cdot \frac{kgm}{s^2}$ ✓

$= \frac{1\frac{Ns}{m} + 10\frac{K_D}{m}}{2\sqrt{3970\,kg^2/s^2}}$

$1 = \frac{1\,Ns/m + 10\,K_D/m}{126\,kg/s}$

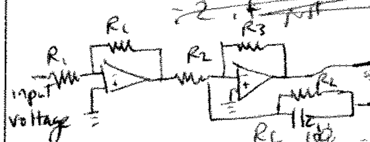
$126\,kg/s - 1\,Ns/m = 10\,K_D\frac{1}{m}$

$125\,kg/s = 10\,K_D$

$K_D = 12.5\frac{kgm}{s}$

20 — **4) Answer:** $\zeta = 1$ desirable for no overshoot/resonance
Attack mode, $t_{s2\%} = 1$ sec $\Rightarrow \omega_n = 4$ rad/sec
Patrol mode $\omega_n = (10\times)(2\pi \cdot .1Hz) \Rightarrow \omega_n = 6.3$ rad/sec
choose bandwidth to be 10x patrol freq
-10 if no consideration of patrol mode

10 — **5) Answer:**
-2 if motor instead of motor amplifier?
~~-2 if ...~~



$K_D = R_F C\,100$

$\frac{R_f}{R_i} = 100 K_p = 160$ $C = \frac{K_D}{100R_F}$

$\frac{R_f}{R_i} = 1.6$ $R_f = 1.6 R_i$

$\frac{4}{\zeta\omega_n} = \frac{4}{\frac{c}{2m}} = \frac{8M}{C}$

-3 if scaling? $\frac{160}{.01} = \frac{R_f}{R_i}$ $R_f = 16000 R_i = (1.6\times10^4)R_i$
let $R_i = 100\Omega = 10^2\Omega$
$R_f = 1.6\times10^6\Omega = 1.6M\Omega$

10 — **6) Answer:**
$R_i = 100\Omega$
$R_f = 1.6M\Omega$
$C = 5mF$

$K_D = R_F C A$, $C = \frac{K_D}{R_F A} = \frac{80}{(1.6e6)(.01)}$

$1v = 100m$
$1v = 1Nm$

$K_p = \frac{R_f}{R_i}A$ — does $\tau = K_p(x-x_d) - K_D\dot{x}$

$K_D = R_f C A$

$\tau = \frac{A}{100}(-K_p(x-x_d) - K_D\dot{x})$, 1 volt = 100 m, 1 volt corresponds to 100 m, should have changed ... 100 V

$\tau = 100(-K_p(x-x_d) - K_D\dot{x})$

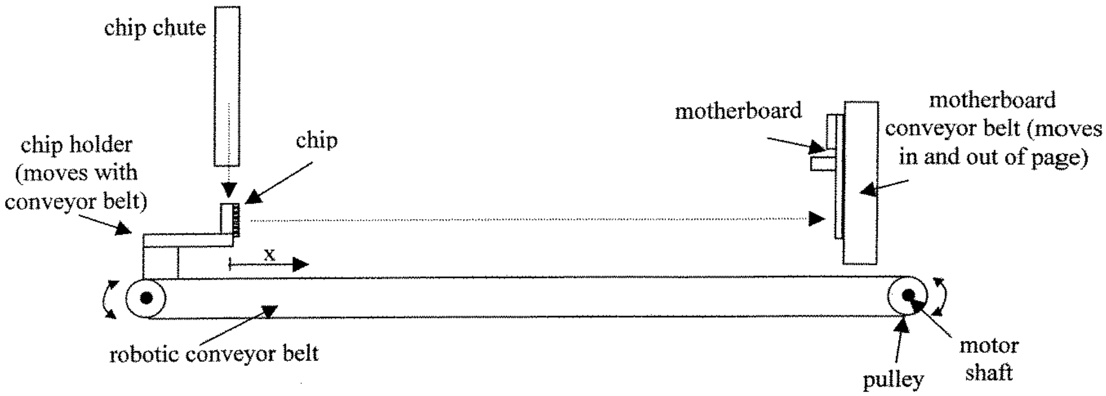
let $R_f = 1.6M\Omega$, $R_i =$, $80 = K_D = \frac{1.6\times10^6}{C} .01$, 1.6×10^6

Attack mode: choose let $\zeta = 1$, $t_s|_{2\%} = \frac{4}{\zeta\omega_n} = \frac{4}{\omega_n} = 1$ sec $\Rightarrow \omega_n = 4$ rad/sec

for Patrol mode, we chose $\omega_n = 6.3$ rad/sec, which is more stringent and makes K_p bigger $\Rightarrow$ gains for Patrol mode will work for attack mode

MAE 106 Mechanical Systems Laboratory
Winter 2002 Design Exam

You are a control engineer working for a mass-manufacturer of computers. Your task is to design a robotic conveyor belt for the motherboard assembly line. The conveyor belt should catch a microprocessor chip that falls from a chute, and plug it into a motherboard that arrives on another conveyor belt:



You are given the following information and equipment to work with:

- the chip chute is at x = 0 m and motherboard is at x = 1 m
- chips are ejected out of the chip chute onto the chip holder once per second. Likewise, a new motherboard arrives on the motherboard conveyor belt once per second. The delay between arrival of a new chip and arrival of a new motherboard is 0.5 seconds. The chip holder must hold stationary for 0.1 seconds at the chip chute to catch the chip, and at the motherboard to insert the chip. Thus, your robotic conveyor belt must move between the chip chute and the motherboard conveyor belt in 0.4 seconds.
- the conveyor belt behaves like a linear mass (M = 1 kg) and a linear damper (B = 1 N/s).
- the conveyor belt is controlled by a DC brushed motor with a linear current amplifier. A one volt input into the current amplifier gives 1 Nm of motor torque.
- you have access to the voltage signal from a potentiometer that measures the rotation of the conveyor belt pulleys, and thus the position (x) of the chip holder along the conveyor belt. The pot voltage is zero at x = 0 m, and 1 volt at x = 1 m.
- your controller receives a 0-1 V square wave voltage input telling it to move from the chip chute to the motherboard. When the input voltage is 0 V the conveyor belt should move to the chip chute. When the input voltage is 1 V the conveyor belt should move to the motherboard.
- you must implement your controller using op-amps, resistors, and capacitors.
- the radius of the pulley that drives the conveyor belt is r = .1 m

Design an analog circuit to control the robotic conveyor. You will get full credit if you:

1) Show a control law for the robotic conveyor belt that relates the motor torque to the desired (x_d) and measured chip holder position (x).
2) Choose gain values for your control law. State units.
3) Explain why you chose the gains you did.
4) Draw an op-amp circuit that can implement your controller. Label what the inputs and outputs of your circuit should be connected to.
5) Choose appropriate values for the resistors and capacitors in your circuit.

Mean 54
Std Dev 18

MAE106 2002
Design Exam Solution

The basic idea of this problem is to design a proportional/derivative (PD) position controller that is critically damped and moves to its final value in about 0.4 seconds in response to a step input.

1. (20 pts) Control law: $\tau = -k_p(x - x_d) - k_d\dot{x}$

2. (20 pts) Conveyor dynamics: $M\ddot{x} + B\dot{x} = \frac{\tau}{r}$

   Controlled dynamics: $M\ddot{x} + B\dot{x} = \frac{-k_p}{r}(x - x_d) - \frac{k_d}{r}\dot{x}$

   Rewrite in standard from: $M\ddot{x} + (B + \frac{k_d}{r})\dot{x} + \frac{k_p}{r}x = \frac{k_p}{r}x_d$

   i.e. $M\ddot{x} + C\dot{x} + Kx = Kx_d$ where $C = B + \frac{k_d}{r}$ $K = \frac{k_p}{r}$

   -- design system to be critically damped to push chip into board without overshoot $\zeta = 1$

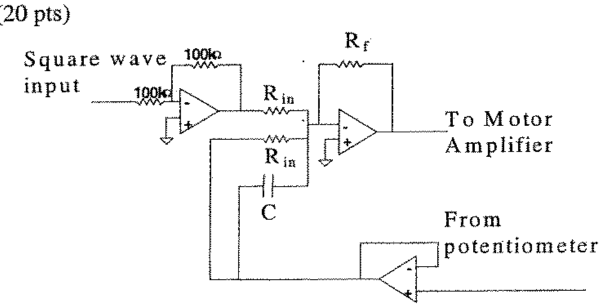
   -- design system so that it is within 2% of its final value in 0.4 sec $t_s = \frac{4}{\zeta\omega_n} = .4s$

   thus, ω_n=10 rad/s = $\sqrt{\frac{K}{M}}$ $\Rightarrow$ K=100 kg/s^2 = k_p/r $\Rightarrow$ **k_p= 10 kg m/ s^2**

   now, $\zeta = \frac{C}{2\sqrt{KM}} = 1 \Rightarrow C = B + \frac{k_d}{r} = 2\sqrt{(100)(1)} \Rightarrow$ **k_d = 1.9 kg m/s**

3. (20 pts) I choose the damping ratio to be 1 (critically damped) so the system would not overshoot. I choose the settling time so that the system was within 2% of its final value at 0.4s.

4. (20 pts)



5. (20 pts) Choosing the values is made easy because 1 volt = 1 m on the inputs, and 1 volt = 1 Nm at the output to the motor amplifier (i.e. the calibration factors are equal to 1, and circuit gain values should just equal the controller gain values from the dynamics)
   $k_p = R_f/R_{in} \Rightarrow$ So if I choose R_f = 100KΩ, then **R_{in} = 10 KΩ**
   $k_d = R_fC \Rightarrow$ **C = 1.9/100KΩ = .19 μF**

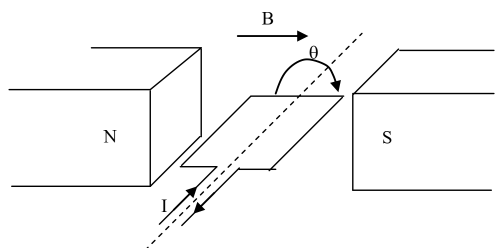
4.4.4 MAE 106 Midterm Exam 2001

MAE 106 Midterm Exam
Winter 2001

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Problem 1: Motors (25 pts)

a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at θ = 0° as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.



b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:

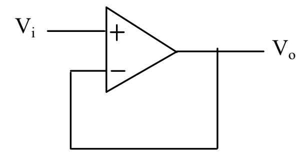
c. Derive and plot the torque versus velocity relationship for the motor. Plot torque on the y-axis and velocity on the x-axis. Label the x-axis and y-axis intercepts with the appropriate terminology. Assume that:

- the motor receives a constant voltage input of V
- the inertia of the motor shaft equals J
- the motor's torque constant is B and is equal to the back-EMF constant
- the current into the motor does not change.

d. For what torque load does the motor produce maximum power?

e. Assume you have a low-power control signal from a computer, and that you would like the unloaded motor shaft to spin at ω_d when the control signal = +5V, and to stop spinning when the control signal is 0 V. Design a circuit using only a MOSFET, a power supply, and resistors to achieve this control. Label the gate, drain, and source of the MOSFET, and the control input from the computer. Specify the voltage of the power supply assuming the motor's torque constant is B and is equal to the back-EMF constant.

Problem 2: Op Amps and Feedback (25 pts)



a. Write the name of this circuit:

b. Briefly describe a practical situation in which you might want to use the circuit in a)

c. Assume that $V_o = K(V_+ - V_-)$ for the op amp. How big would K have to be such that V_o is within 1% of V_i?

d. For a real op amp, the output takes some time to respond to the inputs due to delays in the op-amp's internal circuitry. We can model these dynamics with a differential equation:

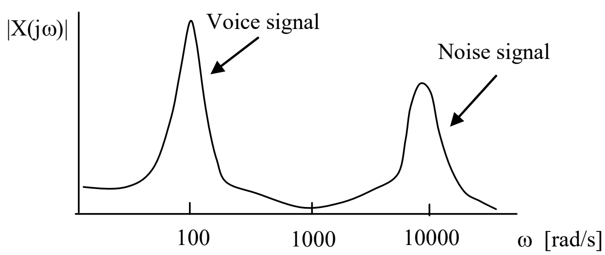
$$\dot{V}_o = -AV_o + AK(V_+ - V_-)$$

where A > 0 is a constant. Given these dynamics, plot the voltage response of the circuit in a) to a unit step input in V_i versus time. Label the time at which the output is 63% of its final value.

e. Assume that you misconnect the circuit in a) such that the feedback goes to the non-inverting input instead of the inverting input (i.e. positive feedback instead of negative feedback), and the input goes to the inverting input. Assuming the op-amp has the dynamics in d), for what values of K will the circuit be unstable (i.e. for what values of K will the output go to infinity as time goes to infinity)?

Problem 3: Filters and Signal Processing (25 pts)

You are hired to improve a wire-tapping signal for the FBI. Shown below is a Bode plot of the typical frequency content of the signal that the FBI's currently-used system generates:



a) Sketch (very roughly) what the signal would like in the time domain. Include a time scale.

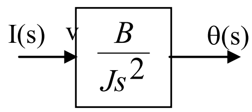
b) What type of filter should you apply to clean up the signal?

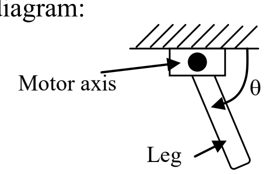
c) You have only a few resistors and capacitors in your toolbox, as shown below. Design an RC circuit to filter the unwanted noise. Draw the circuit, with the input and outputs labeled, and identify the values of R and C that you would use.

Toolbox Contents: R = 1Ω, 10Ω, 100Ω, 1 KΩ C = 1 pF, 0.1 μF, 1 μF

d) By what factor would your filter attenuate the noise in the signal at 10000 rad/s?

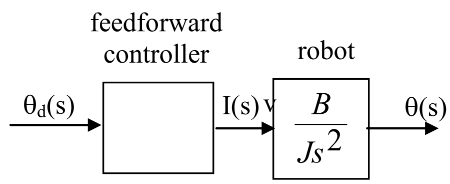
Problem 4: Control Theory and Laplace Transforms (25 pts)
Consider the problem of controlling the angle of a one degree-of-freedom robotic leg. The transfer function of the robot arm is given by the following block diagram:





where I(s) is the current input to the robot's motor, J is the inertia of the motor, B is the motor torque constant, and θ is the angular position of the robot.

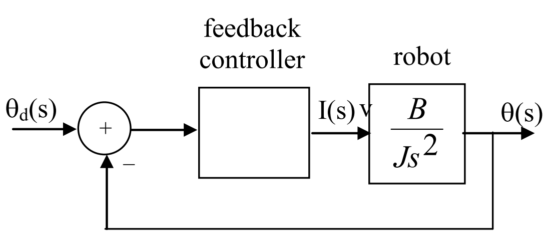
a) Shown to the side is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where θ_d is the desired angular position of the robot. What transfer function should the controller box have to make the output equal the desired output? Write the transfer function the controller box.



b) In the *time domain*, write the relationship between I and θ_d

c) What is a disadvantage of an open-loop controller like this one?

d) A proportional-derivative (PD) feedback controller is a common type of controller that provides a control signal to the plant that is the sum of a signal proportional (by a gain K_p) to the output error and a signal proportional by (a gain K_d) to the derivative of the output error. In the following diagram, fill in the appropriate transfer function such that the robot would be controlled by a PD controller.



e) Find the overall transfer function for the PD-feedback controlled robot system in d)

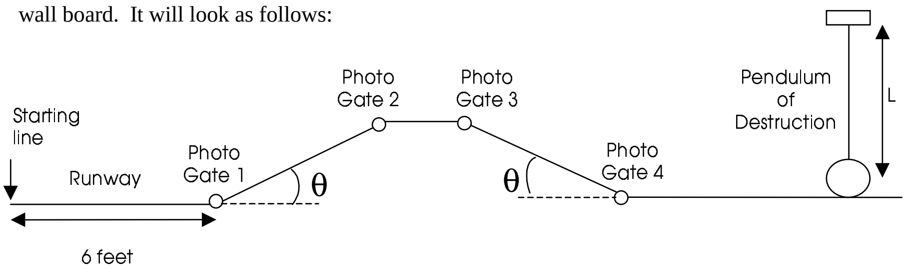
4.4.5 MAE106 1999 Final Project Description

Department of Mechanical and Aerospace Engineering
University of California, Irvine
MAE 106 Mechanical Systems Laboratory
Winter 1999

Final Project Details:
Cruise Control Crash Course

Project Summary:
Your final project involves the design of a battery-powered car that is capable of cruising at a constant velocity. The car should maintain its velocity whether it is travelling uphill, downhill, or on a level surface. On the day of the scheduled final (March 18, 8-10 AM in EG2110), there will be a contest testing your car. The car will be required to traverse a course that contains a hill, then pass through a swinging pendulum. The car will have to move at the appropriate speed so that the pendulum does not squash it.

Course Details:
The course will be four feet wide and approximately 35 feet long. It will be made out of white wall board. It will look as follows:



Notes:
1) There is a 6 feet-long runway for your car to get up to constant speed.
2) The time T_{12} it takes your car to pass between Photo Gate 1 and 2 will be measured, and compared to the time T_{34} it takes to pass between Photo Gates 3 and 4. The goal is for your car to make $T_{12} - T_{34} = 0$. Cars will be rank-ordered based on how close the time difference is to zero.
3) The angle theta will be less than 30 degrees.
4) The diameter of the mass at the end of the pendulum will be less than 6 inches.
5) *On the day of the contest,* you will be given the pendulum length L and the surface distance along the track from the starting line to the pendulum. The pendulum will be dropped from a starting angle of 15 degrees away from vertical. Based on these course parameters, you will have to select a suitable speed for your car. You should design your car with an adjustable, calibrated speed control, and you should do the math ahead of time to predict the desired speed for your car based on the unknown parameters.

For extra credit, you can incorporate automatic steering into your car, and the ability to track black electrical tape laid in a curved pattern on top of the wallboard.

Grading:

The final project is worth 30% of your grade. You grading will be based on:

1. the performance of your car the day of the contest
2. a written final project report

The goal of the written final project report is to describe your design as clearly as possible, and the effort you put into building and testing the car. One write-up should be turned in per project group.

Format for Final Project Write-Up (3-5 pages):

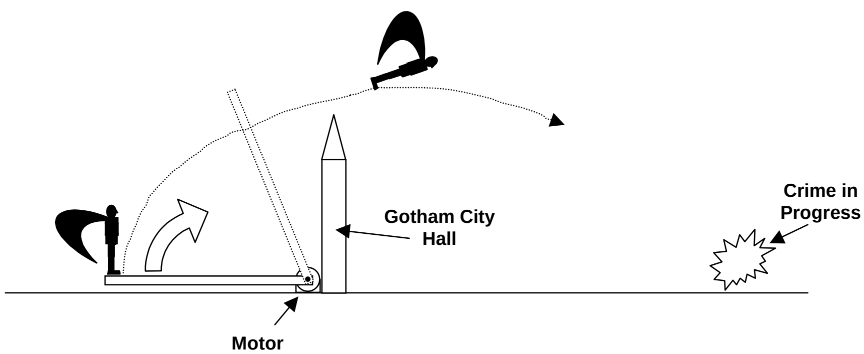
1. Summary (200 words or less)
2. Design
   - A. Controller Design, including:
     - block diagram
     - brief summary of how the controller works
     - dynamical equations and solution
     - circuit diagrams/equations
     - how you chose controller gain values
   - B. Mechanical Design
3. Construction/Testing
   - A. How you built the controller (include a parts list, including where you bought/procured the parts and how much they cost)
   - B. Any tests you performed to calibrate/verify/improve performance (with graphs)

4.4.6 MAE106 2000 Final Project Description

MAE 106 Mechanical Systems Laboratory
Final Project 2000

BATAPULT

For your final project you will design and build a motorized catapult that is capable of hurling a Batman action figure over a scale representation of Gotham City Hall in order to “fight crime”:



The goal is to make Batman fly as far as possible without the catapult knocking over Gotham City Hall. No mechanical stops will be allowed to halt the catapult.

On the day of the scheduled final, there will be a contest testing your catapult.

You will be provided with a motor as a starting point, and will work in teams of two or three.

Discussion Question: *What are some of the subproblems that you will need to solve in order to successfully complete your project?*

4.4.7 MAE106 2001 Detailed Final Project Description

Department of Mechanical and Aerospace Engineering
University of California, Irvine
MAE 106 Mechanical Systems Laboratory
Winter 2000

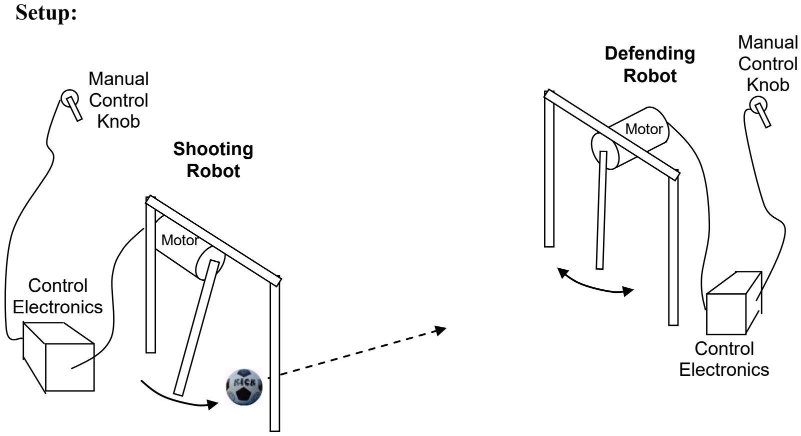
Final Project Details:
PELE′ 2001

Project Summary:
For your final project you will design and build a teleoperated robotic soccer player capable of both kicking a ball into a goal and defending the goal. You are expected to work in project groups of 2 or 3. On the day of the scheduled final (Tuesday, March 20, 4-6 PM) there will be a tournament testing your robotic soccer player. The tournament will be held in the MAE106 Lab.

Lab Hours:
The lab will be open from 8-12 A.M. and 1-5 P.M from February 26th until the contest. If the lab door is locked, you must sign-in with Dwayne Gates in room EG2118 before the lab will be opened. For safety reasons, there must always be at least two students in lab for the door to be opened for you, so bring a partner. While you are in lab, do not leave the room unattended. You are responsible for all laboratory equipment while you are in the lab. Sign-out with Dwayne when you leave.

There will be no lecture March 13th or 15th to allow time to work on your project. Prof. Reinkensmeyer will be available in lab during the regular lecture time on those days for consulting. In addition, your TA's will be available in the lab during your regular lab time that week.

Setup:



Contest Rules:
The contest will be a single-elimination tournament similar to a soccer "shoot-out". Two teams will be selected. The referee will toss a coin and the team that wins the toss will take the first kick. The kicks will be taken alternately by the teams. Subject to the conditions explained below, both teams will take five kicks. The kicks will be taken alternately by the teams. If, before both teams have taken five kicks, one has scored more goals than the other could score, even if it were to complete its five kicks, no more kicks will be taken. If, after both teams have taken five kicks, both have scored the same number of goals, or have not scored any goals, kicks will continue to be taken in the same order until one team has scored a goal more than the other from the same number of kicks.

Notes:
1) The distance between goals will be 31¼".
2) The width of the goals will be 11".
3) The robot motor will be required to rest on a ring stand.
4) The height of the ring stands will be 14".
5) The diameter of the rings on the ring stands will be 4".
6) Two practice fields will be available in lab.
7) The same robot and control system must be used for both kicking and defending.
8) You will be allowed to adjust the yaw angle of the robot on the ring stand before kicking or defending.
9) The leg of the robot must be able to fit through a paper-towel core (at all times).
10) The ball will be a ping-pong ball.
11) You will be provided with a power supply for your motor (the same ones used in the labs).
12) You are not allowed to store energy in your system *before* the kick.
13) Your robot must be teleoperated using a potentiometer to control leg angle.
14) Your robot must use a feedback controller.

Starter Kits:
You will be provided with a starter kit comprised of:
- 1 Pittman DC Gearhead Motor
- 1 Butyrate Tube (1.5")
- 1 Vinyl Tube (1.5")
- 1 Garolite Collar (.5")
- 2 Spring Pins

The tubing, collar, and pins are useful for coupling the motor shaft to a lever and to a potentiometer. The kit may be checked out from Duane Gates in EG2118 beginning Feb 28th, 8 AM – 12 PM, 1 PM – 5 PM. ***IMPORTANT!!!!!: You must return the kit in order to receive your final grade. If the motor is not in working order, you will be required to pay for it ($23.50).***

Other Required Parts:
You will need to purchase other components for your project, such as a protoboard, potentiometers, resistors, wire, op-amps, and MOSFETS. The total cost of your project should be under $20. Suggested vendors are:

- Radio Shack: 4716 Barranca Parkway, Irvine (949)552-1091 (and other locations)
- Marvac Electronics: 2001 Harbor Blvd., Costa Mesa (949)650-2001
- Fry's Electronics: 10800 Kalama River Ave., Fountain Valley (714)378-4400 (and other locations)

You can also purchase parts from bulk vendors, such as:

- Digikey: www.digikey.com
- Newark Electronics: www.newark.com

To avoid a service charge for small orders, you may want to partner with several groups when ordering from these vendors.

Grading:

The final project is worth 30% of your grade. Your grade will be based on:

1. the performance of your catapult on the day of the contest
2. a written final project report

The goal of the written final project report is to describe your design as clearly as possible, and the effort you put into building and testing the catapult. One write-up should be turned in per project group.

Format for Final Project Write-Up (<= 5 pages):

1. Summary (200 words or less)
2. Design
   A. Controller Design, including:
      block diagram
      brief summary of how the controller works
      dynamical equations and solution
      circuit diagrams/equations
      how you chose controller gain values
   B. Mechanical Design, including:
      any mathematical analysis you did in the design
      a parts list, including where you bought/procured the parts and how much they
3. Testing: Any tests that you performed to calibrate/verify/improve performance (with graphs)

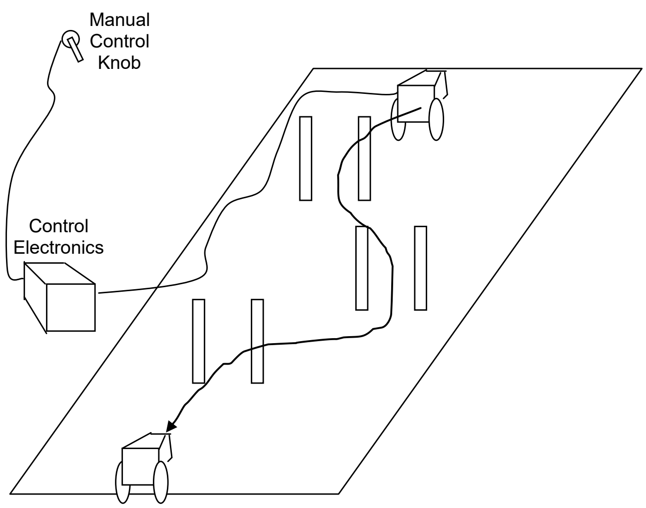
Projects with quantitative analysis and detailed experimental testing will score higher.

4.4.8 MAE106 2002 Final Project Description

MAE 106 Mechanical Systems Laboratory
Final Project 2002

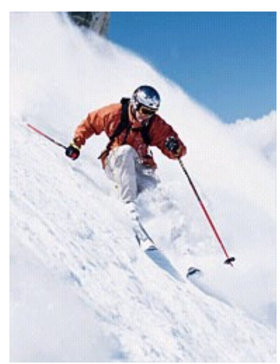
SKIBOT 2002

In honor of the 2002 Winter Olympics, you will design a teleoperated robotic vehicle capable of "skiing" down a slalom course for your final project.



The goal is to steer the gravity-powered vehicle using a manual control knob. You will be provided with a motor for steering as a starting point, and will work in teams of two or three. On the day of the scheduled final, there will be a contest testing your skibot.

Discussion Question: *What are some of the subproblems that you will need to solve in order to successfully complete your project?*

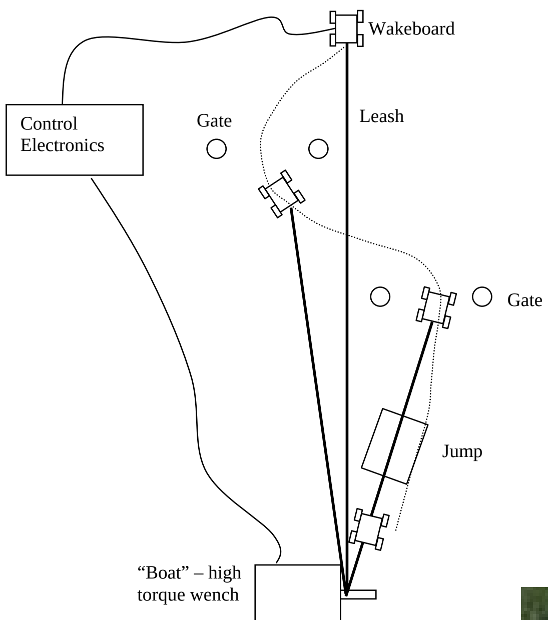


4.4.9 MAE106 2004 Final Project Description

MAE 106 Mechanical Systems Laboratory
Final Project 2004

WAKEBOT 2004

You will design a teleoperated robotic vehicle capable of wakeboarding through a slalom course for your final project. A high-power motor will pull on a string (the "leash") attached to your robot.



The goal is to control the speed of the "boat" and to steer your wakeboard using a manual control knob. You will be provided with a motor for steering as a starting point, and will work in teams of two or three. On the day of the scheduled final, there will be a contest testing your wakebot.

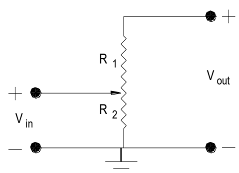
Discussion Question: *What are some of the subproblems that you will need to solve in order to successfully complete your project?*

4.4.10 Midterm Exam 2002

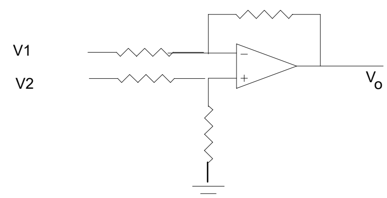
MAE 106 Midterm Exam
Winter 2002
University of California, Irvine
Department of Mechanical and Aerospace Engineering

Problem 1: Circuits (25 pts)

a) Shown below is a 10Kohm potentiometer wired incorrectly. Assume Vin is 10 V and is provided by a 100 Watt power supply. Assume the potentiometer is rated at 1 Watt. Assume the shaft can rotate 180 degrees, and define 0 degrees rotation as the shaft angle when the resistance between the wiper and the ground is 10Kohm. At what shaft angle do you expect to smell smoke?



b) What does the following circuit do? (give proof) Assume the four resistors have equal values.



c) Why are op amps such as the ones used in lab unsatisfactory for powering most motors?

d) Assume that you have a low-power control signal from a computer, and that you would like to make a motor spin when the control signal is +5 v, and to stop spinning when the control signal is 0 V. Design a circuit using a MOSFET to achieve this control.

Problem 2: Motors (25 pts)

A DC brushed motor has an internal resistance of 10 ohms. You hold the motor shaft fixed and apply 10 V, and find that the stall torque is 0.1 Nm, and that it takes 1.5 s to generate 0.063 Nm of torque. Assume the motor's torque constant equals the back EMF constant.

a) How fast will the unloaded motor shaft turn if you let it go?

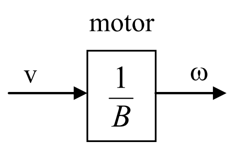
b) What is the inductance of the motor windings?

c) The motor takes 5 seconds to reach its no-load speed. If you double the shaft inertia, how much longer will the motor take to reach 99% of it's no-load speed?

d) Why does a DC brushed motor have brushes? (Explain in 15 words or less).

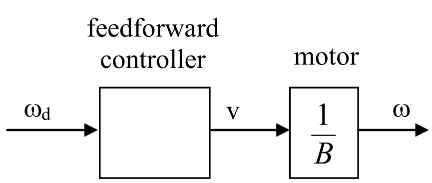
Problem 3: Control Theory (25 pts)

1) Consider the problem of controlling the velocity of a motor. A simple model of the motor is given by the following block diagram:



where v is the voltage input to the motor and ω is the angular velocity of the shaft.

a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What gain value should the controller box have to make the output equal the desired output? Write the gain in the controller box.



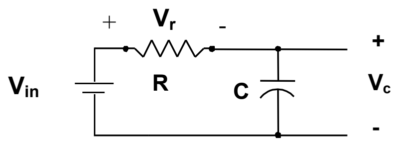
b) Name *two* disadvantages and *two* advantages of an open-loop controller like this one.

c) Draw a block diagram of a feedback controller for the motor, label all arrows, including the error signal.

d) Prove that your feedback controller cancels an additive disturbance V_d to the voltage input to the motor, given a high enough feedback gain.

Problem 4: Signal Processing, Differential Equations, and Frequency Analysis (25 pts)

a) Describe a practical situation in which the following circuit would be useful:



b) Assume V_{in} is a step input at time zero and $V_c(0) = 0$. Find $V_c(t)$.

c) Find the transfer function for the above circuit.

d) Find the frequency response of the above circuit. Be sure to provide equations for how the circuit scales and phase shifts a sinusoidal input.

e) Assume R = 100 ohm, C = .01 F. How much more attenuated will a 1000 Hz sinusoidal input signal be than a 100 Hz input signal? (Provide proof).

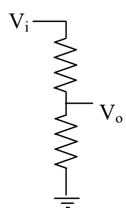
4.4.11 Midterm Exam 2003

MAE 106 Midterm Exam
Winter 2003

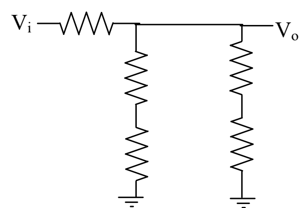
University of California, Irvine
Department of Mechanical and Aerospace Engineering

Problem 1 Circuits (25 pts)

a) Find V_o for the following two circuits. Assume all resistors values = R.

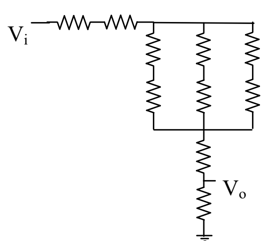


Answer:



Answer:

b) For the following circuit, draw conceptual sketches to show how you would simplify the network to solve for V_o. You do not need to find V_o, just illustrate the steps.



c) What are the two “golden rules” of op-amp analysis?

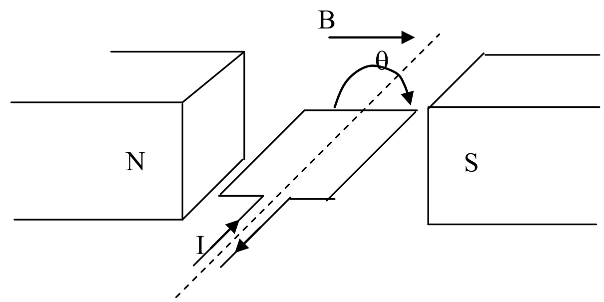
d) What two conditions must be true for these golden rules to apply?

e) Using an op-amp and resistors, design a circuit to amplify an input signal by a factor of 20. Show mathematically that your design works.

f) Assume that you have a low-power control signal from a computer, and that you would like to make a motor spin when the control signal is +5 v, and to stop spinning when the control signal is 0 V. Design a circuit using a MOSFET to achieve this control.

Problem 2: Motors (25 pts)

a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest?
Assume the armature is initially at θ = 0° as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.



b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:

b. Solve this differential equation for the current through the motor as a function of time when:
- the shaft of the motor is held fixed
- a constant voltage v is applied across the motor at time = 0
- the initial current i(t = 0) through the inductor is zero

c. Plot the torque that the motor generates as a function of time for the conditions described in part b. Label the axes, the final value of the torque, and the time at which the torque has reached 63% of its final value. Assume the motor's torque constant is some constant B, equal to the back EMF constant.

d. What is the term for the maximum torque a motor can produce when its shaft is held constant?

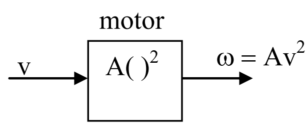
e. What is the term for the maximum speed that a motor free to spin will reach?

f. What happens to an unloaded motor spinning at steady-state speed ω when you double its input voltage?

g. At what speed does an ideal DC brushed motor produce maximum power and why?

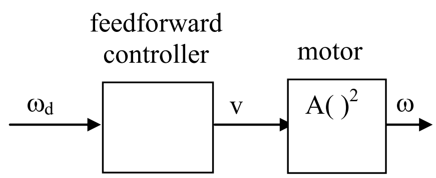
Problem 3: Control Theory (25 pts)

1) You design a new type of motor for which the speed is proportional to the input voltage squared:



where v is the voltage input to the motor and ω is the angular velocity of the shaft and A is a constant.

a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What function should the controller box compute to make the output equal the desired output? Write this function controller box.



b) If the estimate of A used by the feedforward controller is too small by 10%, how much faster will the actual speed be than the desired speed?

c) Draw a block diagram of a feedback controller for the motor, label all arrows, including the error signal.

d) Prove that, the actual velocity equals the desired velocity as the feedback gain gets large.

Problem 4: Frequency Analysis: Motivation, Theory, and Practice (25 pts)

a) Give two reasons for studying frequency domain analysis:

g) If you input a sinusoidal input $u(t) = a\sin(\omega t)$ into a linear, time-invariant system, what is the output $x(t)$? Express your answer in terms of an equation <u>and</u> in words.

h) Below is a proof of the correct answer to part b). On the left are the equations for each step of the proof. On the right is a description of what is happening in the proof. Fill in all blanks to complete the proof.

Step	Description	Equation
1	An n^{th} order linear system with output x(t) and input u(t) can be described by a differential equation like this:	
2		X(s)=B(s)/A(s)U(s)+IC(s)/A(s)
3	Assume the system is stable, then in the steady state Equation 2 can be simplified to:	
4	The Laplace transform of the sinusoidal input is:	
5	Thus, the steady state output of the system in the frequency domain is:	
6		$K_1/(s+jw)+K_2/(s-jw)+K_3/(s-s1)+K_4(s-s2)+\ldots$
7	But the system is stable, so the K3 and K4 terms go to zero with time, and the output in the time domain is thus Equation 7, with K_1=-a/(2j)G(-jw), and K_2 = a/(2j)G(jw), G(s)=B(s)/A(s)	
8	Simplifying Equation 7 using the fact $\sin(\theta) = 1/2j(e^{j\theta}-e^{-j\theta})$ gives:	
	PROOF COMPLETE	PROOF COMPLETE

i) What two pieces of information do you need to describe the frequency response of a system?

j) Draw an example of the magnitude response of the following types of filters. Make sure to label the axes.

Low Pass Filter

High Pass Filter

Notch Filter

Band-pass filter

k) Prove that a mass M acts like a low-pass filter, if force f is considered its input and position x its output

4.4.12 Midterm Solution 2004

MAE 106 Midterm Exam: Closed Book/Notes Section
Winter 2004 Mean= 37/50

$\sigma = 8$

University of California, Irvine
Department of Mechanical and Aerospace Engineering

4 1. What is Ohm's Law?

$V = IR$

3 2. What is Kirchoff's current law?

$\sum_{node} i = 0$

3 3. What is Kirchoff's voltage law?

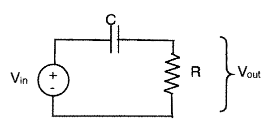
$\sum_{loop} V = 0$

4 4. If you input a sinusoidal input u(t) = asin(ωt) into a linear, time-invariant system, with a transfer function H(s), what is the output x(t)?

$x(t) = a\,|H(j\omega)|\sin(\omega t + \phi_H(j\omega))$

8 5. Now, apply these concepts to find the frequency response of the following circuit. Provide both the magnitude and phase response.

−2 low pass but right method



Transfer function:

$V_{out}(s) = \frac{R}{R + \frac{1}{sC}} V_{in}(s)$ or ODE 2pts

$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{RCs}{1+RCs}$

Magnitude response $= |H(j\omega)| = \left|\frac{RCj\omega}{1+RC\omega j}\right| = \frac{RC\omega}{\sqrt{1+(RC\omega)^2}}$ 3

Phase response $= \phi_H(j\omega) = \tan^{-1}\frac{RC\omega}{0} - \tan^{-1}\frac{RC\omega}{1}$ 3

$= 90^\circ - \tan^{-1} RC\omega$

4 6. What type of filter is this? Provide mathematical proof.

if $\omega \to 0$ $|H(j\omega)| = \frac{0}{\sqrt{1+0}} = 0$ attenuates low frequencies

3 if $\omega >> \frac{1}{RC}$ $|H(j\omega)| \approx \frac{RC\omega}{RC\omega} = 1$ passes high frequencies

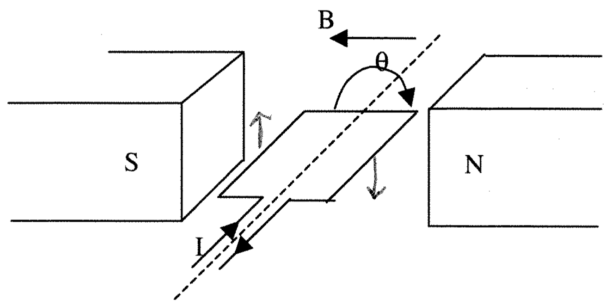
1 ∴ High pass filter

26 Low pass but right method −1

4 7. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at θ = 0° as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.

$\vec{F} = i\vec{l} \times \vec{B}$

$\theta = 90^{0}$



8 8. Assume that the commutation is working correctly, and the motor's torque constant is B. Find the torque that the motor produces as a function of time, when:

- The shaft of the motor is held fixed
- A constant voltage is applied across the motor at time = 0
- The initial current through the motor is zero

2 Motor eqn: $L\frac{di}{dt} + B\dot{\theta} + iR = V$ $B\dot{\theta} = 0$ because shaft is fixed

$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$

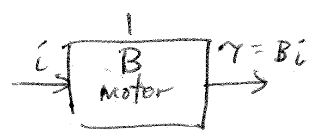
2 Homog. soln: $i = Ae^{-t/\tau_c}$ $\tau_c = \frac{L}{R}$

2 Part soln: $i = V/R$

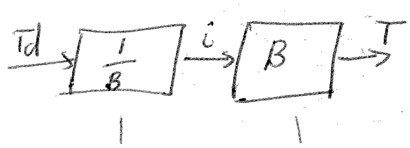
2 Tot soln: $i = V/R + Ae^{-t/\tau_c}$ but $i(0) = 0$ so $i = \frac{V}{R}(1 - e^{-t/\tau_c})$ $\tau_c = \frac{L}{R}$

$\tau = Bi = \frac{BV}{R}(1 - e^{-t/\tau_c})$

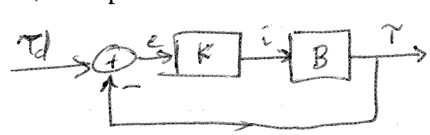
2 9. Draw a block diagram of the motor, assuming that the input is the current i and the output is torque τ.



4 10. Draw a block diagram of an open-loop (i.e. feedforward) controller for the plant of part (9), where the input to the controller is τ_d, the desired torque output of the motor.



4 11. Draw a block diagram of a feedback controller for the motor, label all arrows, including the error signal. Again, the input to the controller should be τ_d, the desired torque output of the motor.



2 12. What type of sensor do you need to make this feedback controller work?

torque sensor

24

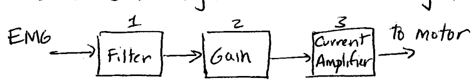
MAE 106 Midterm Exam: Open Book/Notes Section
Winter 2004

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Your goal is to design a power-assist for a motorized bicycle. The device will measure the small voltage produced by leg muscles of the rider, using small electrodes taped over the muscles. This measurement is called the electromyogram, or "EMG". The device will low-pass filter the EMG voltage signal with a cutoff frequency of 10 Hz in order to get a smooth control signal, and generate a motor torque proportional to the low-pass-filtered EMG signal. The proportionality constant should be adjustable between two values by flipping a switch, from 10 (workout mode) to 100 (cruise mode). Thus, when the rider pedals, he or she will also activate the motor attached to the bicycle, giving a power assist.

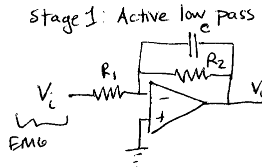
Design a circuit using operational amplifiers that can implement the power-assist controller for the bicycle. You may assume that you have a power operational amplifier capable of generating the current needed to directly power the motor. Hint: In order to control the current through the motor, you can control the voltage across a resistor that is in series with the motor, using the power op amp.

I choose to design a circuit with 3 stages:



Mean = 28/50
σ = 10

Stage 1: Active low pass filter (could also use passive filter w/ a buffer)



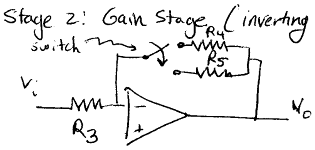
$\frac{V_o}{V_i} = \frac{-R_2}{R_1}\left(\frac{1}{1+R_2 C s}\right)$ (gain) (low pass filter)

cutoff frequency $\omega_c = \frac{1}{R_2 C} = 2\pi(10\,Hz) = 63$ rad/sec.

choose $C = 100\mu F \Rightarrow R_2 = 630\,K\Omega$

Choose $R_1 = 630\,K\Omega$ so Gain = −1

Stage 2: Gain Stage (inverting amplifier; could also use non-inverting amplifier)



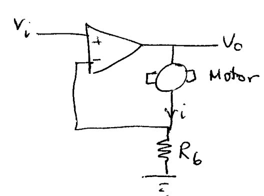
By flipping the switch, you can change the gain, for cruise or workout mode

$\frac{V_o}{V_i} = \frac{-R_4}{R_3}$

Cruise Mode: Switch up, gain = 100
choose $R_3 = 1\,K\Omega$, $R_4 = 100\,K\Omega$

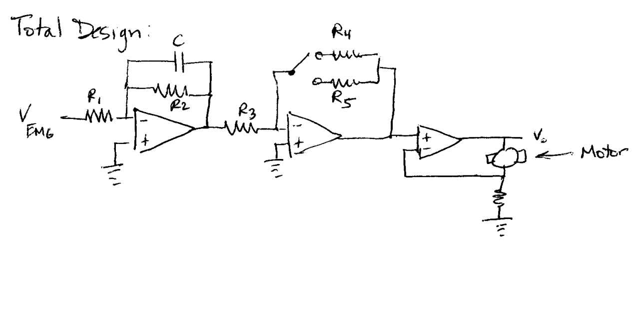
Workout Mode: Switch down, gain = 10
$\Rightarrow R_4 = 10\,K\Omega$

Stage 3: Current Amplifier



Note: $i = \frac{V_i}{R_6} \Rightarrow$ current through motor is proportional to input voltage. Choose R_6 to give appropriate levels of current for motor. R_6 must be a power resistor, + this op-amp must be a power op-amp, both capable of large currents

Total Design:



Have fun riding!

Grading:
1) 10 pts Filter circuit
2) 10 pts R, C values for ω_c
3) 5 pts Buffering OK
4) 10 pts Gain stage
5) 15 pts Current amplifier

4.4.13 midterm 2000

MAE 106 Midterm Exam
Winter 2000

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Problem 1: Motors

a. What type of motor did we analyze in Lecture and did you use in Lab 2?

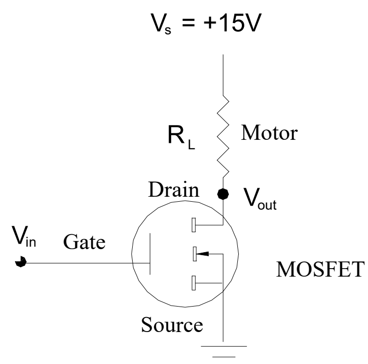
b. Draw the circuit model of such a motor, as presented in class:

c. Assume that the motor shaft is allowed to spin freely. At t = 0 a constant voltage V is applied to the motor. Derive an expression for the angular velocity ω of the motor as a function of time. Assume that the inertia of the motor shaft equals J, the motor's torque constant is B, and the current into the motor does not change. (Hint: Based on the model you drew in part b and your knowledge of physics, write a differential equation in terms of ω and solve it).

Problem 2: MOSFETS and Power Control

A circuit for controlling a motor with a MOSFET is shown below. The motor is modeled simply as a resistor. By changing the input gate voltage to the MOSFET, the motor can be made to spin. A table of the MOSFET's drain/source resistance (like the one you generated in lab) is also shown.

V_{in} (volts)	R_{DS} (KΩ)
0	100
1	100
2	90
3	80
3.4	75
3.6	41
3.9	4
4.2	0.1
5.5	0.01



a) To make the motor spin, should the input gate voltage (V_{in}) to the MOSFET be high or low and why?

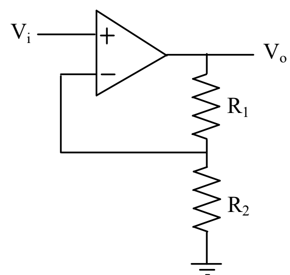
b) Assume R_L = 1KΩ. Calculate V_{out} corresponding to an input gate voltage of 3.9 volts.

c) How many watts of power will the motor consume with the input gate voltage equal to 3.9 volts?

Problem 3: Operational Amplifiers

a) Why are op-amps such as the ones used in lab unsatisfactory for powering most motors?

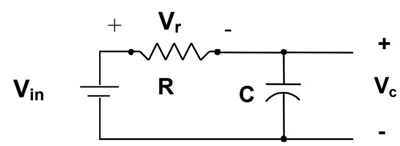
b) Find the relationship between V_o and V_i for the following non-inverting amplifier circuit:



c) Using an op-amp and resistors, design a circuit to amplify *and invert* an input signal by a factor of -100. Show mathematically that your design works.

Problem 4: Filters and Frequency Response

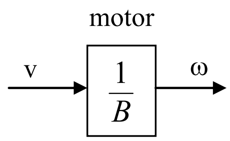
a) Find the transfer function relating V_c to V_{in} for the following filter:



b) What kind of filter is this? Show mathematically.

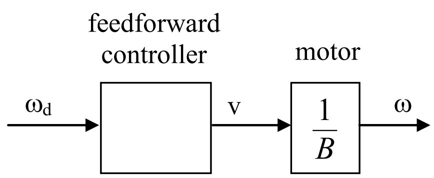
c) Suppose $V_{in} = \sin t$, $R = 1\ \Omega$ and $C = 1$ Farad. What is $V_c(t)$ Assume $V_c(0) = 0$.

Problem 5: Control Theory
Consider the problem of controlling the velocity of a motor. A simple model of the motor is given by the following block diagram:



where v is the voltage input to the motor and ω is the angular velocity of the shaft.

a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What gain value should the controller box have to make the output equal the desired output? Write the gain in the controller box.



b) What is a disadvantage of an open-loop controller like this one?

c) Draw a block diagram of a feedback controller for the motor. Label all arrows, including the error signal.

d) Assume that the output velocity of the motor is affected by a disturbance such that $\omega = (1/B)v + \omega_e$ where ω_e is the (constant) error in velocity introduced by the disturbance. Prove that your feedback controller, with high enough gain, can cancel the disturbance and make the motor move at the desired velocity ω_d.

4.4.14 Solutions to Old Midterms

Key for Grading

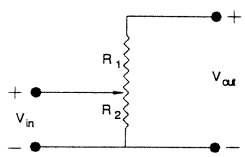
MAE 106 Midterm Exam
Winter 1999

100 pts Total

University of California, Irvine
Department of Mechanical and Aerospace Engineering

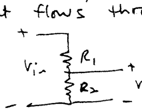
(15 pts) 1) Based on your experience in laboratory and lecture, provide brief answers to the following questions:

(5) a) Why is it wrong to use a potentiometer in the following configuration?



(5) When $R_2 \to 0$, have a short circuit & will burn out the pot. (Or, w/ no load across V_{OUT}, no current flows through R_1, $V_{OUT} = V_{in}$)

(3) Go from



, $\frac{V_{in}}{R_1+R_2} = \frac{V_o}{R_2}$ ⇒ $V_o = \frac{R_2}{R_1+R_2} V_{in}$

(2) A load is acting on the potentiometer | Doesn't take resistance into consideration correctly

implies that for this circuit, $V_o = \frac{R_1+R_2}{R_2} V_{in}$, and as $R_2 \to 0$, $V_o \to \infty$, not possible

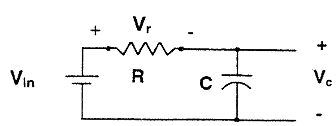
(5) W/ no load across V_{out}, will get no current through R_1,

(1) Gibberish, mostly wrong

(5) b) A motor can also be used as what kind of sensor?

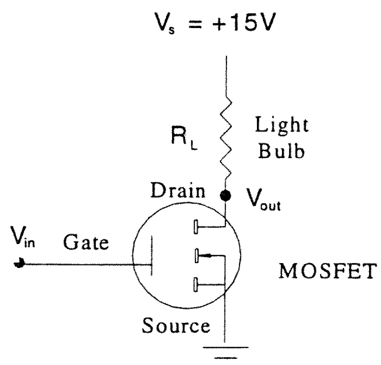
(5) Tachometer
(4) Speed or velocity sensor
(5) Angular velocity sensor
(3) Voltage output sensor
(2) Feedback sensor monitoring output, fn. of speed
(1) Power output

(5) c) What kind of filter is the following circuit? (Assume V_c is the output)



(1) passive
(4) low-pass filter

(20 pts) 2) A circuit for controlling a light bulb with a MOSFET is shown below. The light bulb is modeled as a resistor. By changing the input gate voltage to the MOSFET, the light bulb can be turned on and off. A table of the MOSFET's drain/source resistance (like the one you generated in lab) is also shown.



V_{in} (volts)	R_{DS} (KΩ)
0	100
1	100
2	90
3	80
3.4	75
3.6	41
3.9	4
4.2	0.1
5.5	0.01

(5) a) To turn the light bulb on, should the input gate voltage (V_{in}) to the MOSFET be high or low?

(5) **High**

$$\frac{V_S}{R_L + R_{DS}} = \frac{V_{OUT}}{R_{DS}} \qquad V_{OUT} = \frac{R_{DS}}{R_L + R_{DS}} V_S$$

Light bulbs need a certain amount of voltage drop to turn on. That means V_{OUT} should not be near +15 V, so V_{in} should be high.

(8) b) Assume $R_L = 1K\Omega$. Calculate V_{out} corresponding to an input gate voltage of 3.9 volts.

$V_{in} = 3.9$ V ⇒ $R_{DS} = 4$ kΩ

R_{DS} right (1), R_L right (1), V_s right (1)

$$V_{OUT} = \left(\frac{4\ k\Omega}{1\ k\Omega + 4\ k\Omega}\right) 15\ V = \left(\frac{4}{5}\right)(15) = 12\ V$$

Some valid expression, (4)

Right arithmetic, (1) $V_{OUT} = 12$ V

Neg. sign on answer, (−1)

(7) c) How many watts of power will the light bulb consume with the input gate voltage equal to 3.9 volts?

Have one of these, (2) → $P = VI = I^2R = \frac{V^2}{R}$ (Can use any of these.)

Algebra right, (1), units

$$I = \frac{V_S}{R_L + R_{DS}} = \frac{15\ V}{5\ k\Omega} = 3\ mA = .003\ Amps$$

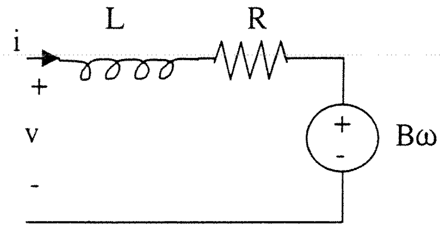
Need two of three
V right (2)
I (2)
R (2)

$$P = VI = (3\ V)(.003\ A) = .009\ Watts$$

$$= I^2R = (3 \times 10^{-3})^2 (1 \times 10^3\ \Omega) = 9 \times 10^{-3}\ Watts$$

$$P = \frac{V^2}{R} = \left(\frac{9\ V}{1\ k\Omega}\right) = .009\ Watts$$

(20 pts) 3) A circuit model of a DC Brush motor is:



where ω is the angular velocity of the motor shaft.

(5) a. Write the differential equation that describes the relationship between input voltage and current for this circuit. (Hint: use Kirchoff's voltage law).

$V = V_L + V_R + V_{B\omega}$

(5) $-V + L\frac{di}{dt} + Ri + B\omega = 0$

(1) (1) (1) (1) (1) - signs

Missing term = -2; $\dot{\theta}$ = -1

(8) b. Solve this differential equation for the current through the motor as a function of time when:

+0 - voltage, not current

- the shaft of the motor is held fixed
- a constant voltage v is applied across the motor at time = 0
- the initial current i(t = 0) through the inductor is zero

$V(0) = \text{constant}, \quad \omega = 0, \quad i(0) = 0$

(4) Homogeneous soln:

$L\frac{di}{dt} + Ri = 0$

$\int \frac{di}{i} = \int -\frac{R}{L}\, dt$

$\ln i = -\frac{R}{L}t + c$

$i_H = Ce^{(-R/L)t}$

(2) Particular soln

sub. in & get:

$L\frac{di_p}{dt}^{\,0} + Ri_p = V$

$i_p = \frac{V}{R}$

i_p = constant

(2) Init. Cond

$(i_H + i_p)\big|_{t=0} = ($

$c_1 e^{-Rt/L} + \frac{V}{R} =$

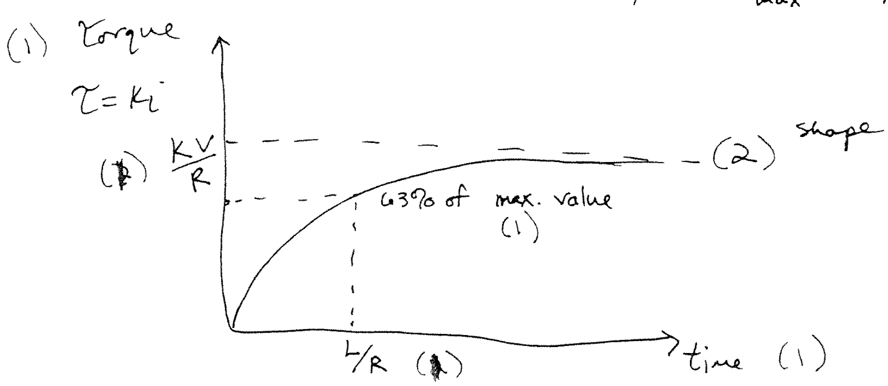
$c_1 = -$

Naive integration +3

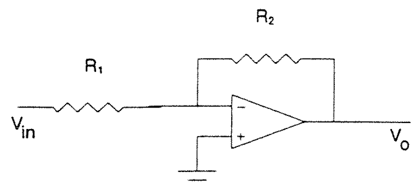
$$i_{tot} = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

(7) c. Plot the torque that the motor generates as a function of time for the conditions described in part b. Label the axes, the final value of the torque, and the time at which the torque has reached 63% of its final value. Assume the motor's torque constant is some constant B.

$\tau = Ki$, so $\tau_{max} = K\left(\frac{V}{R}\right)$



(25 pts) 4) Consider the following op-amp circuit:



(3) a) Derive the relationship of the output to the input:

(3) $V_o = -\frac{R_2}{R_1} V_{in}$ (from op-amp assumptions when neg. feedback is present)

(2) $V_o = \frac{R_2}{R_1} V_{in}$

(3) b) What function does this circuit perform if $R_2 > R_1$?

(1) inverting (recall the − sign!)

(2) amplifier, by factor of $\frac{R_2}{R_1}$ (will take $V_o > V_{in}$)

~~(−1) say something silly~~

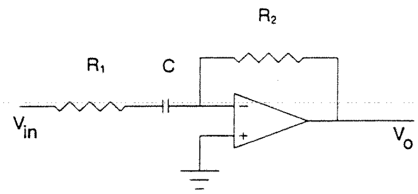
(1) negative feedback

(1) passive

(0) Voltage follower

(0) Decrease the Vout

Problem 4 continued: If we add a capacitor, the circuit becomes a filter:



(4) c) Derive the transfer function of the filter using the impedance of the capacitor, Kirchoff's laws, and your knowledge of how the Op-Amp works.

$$V_o = -\frac{R_2}{R_1 + \frac{1}{Cs}} V_{in} = -\frac{R_2 Cs}{R_1 Cs + 1} V_{in}$$

(1) Right sign
(3) Right answer

$\frac{V_o}{V_i} = -\frac{R_2}{R_1}(1 + R_1 sC) \Rightarrow (-2)$

(3) d) What are the time constant and corner frequency of this filter?

time constant $= R_1 C$ (2) (0) for RC, corner frequency $= \frac{1}{R_1 C}$ (1) (will take $\omega_c = \frac{1}{\tau}$)

(3) e) Find the filter's gain (i.e. the magnitude of the transfer function):

$$G(j\omega) = -\frac{R_2 C j\omega}{R_1 C j\omega + 1} \Rightarrow \quad \leftarrow +3$$

$$|G(j\omega)| = \frac{R_2 C\omega}{\sqrt{(R_1 C\omega)^2 + 1}}$$

1
1, 1

(3) f) Assume we input a sinusoid to this filter. For frequencies much greater than the corner frequency, what is the filter's gain?

$\omega >> \frac{1}{\tau} = \frac{1}{R_1 C}$, $|G| \cong \frac{R_2 C\omega}{\sqrt{(R_1 C\omega)^2}} \cong \frac{R_2}{R_1}$ Gain = 1 (1)

$-\frac{R_2}{R_1}$ ⇐ take it...

(3) g) For frequencies much less than the corner frequency, what is the filter's gain?

$\omega << \frac{1}{R_1 C}$, $|G| \cong R_2 C\omega$ Gain = 0 (1)

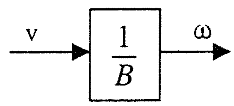
$\frac{R_2}{R_1}$ (0)

As freq. drops, gain gets smaller (1)

(3) h) What kind of filter is this?

(1) Inverting (2) High-pass filter
(1) Active

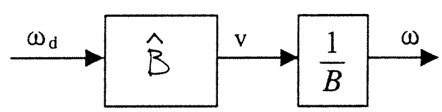
(20 pts) 5) Consider the problem of controlling the velocity of a motor. A simple model of the motor is given by the following block diagram:



where v is the voltage input to the motor and ω is the angular velocity of the shaft.

(5) a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What gain value should the controller box have to make the output equal the desired output? Write the gain in the controller box.

(5)



$\frac{\omega}{\omega_d} B$ (3) R (0) (1) $\frac{v}{\omega_d}$

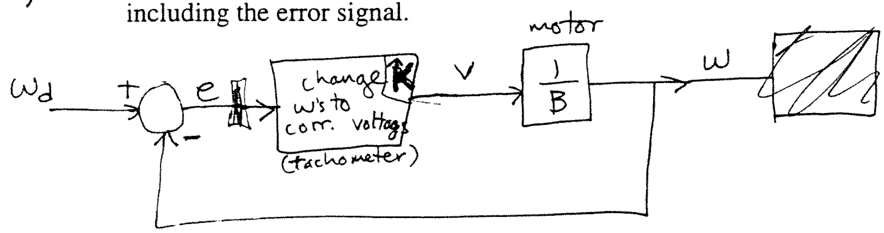
(4) b) What is a disadvantage of an open-loop controller like this one?

(4) { i) doesn't resist disturbances
or
ii) need a good estimate of B, which is changing

(3) {more or less implied

(1) gibberish, but not out 2 out wrong

(7) c) Draw a block diagram of a feedback controller for the motor, label all arrows, including the error signal.



where
$e = \omega_d - \omega$

(4) d) What hardware (beside the motor) that we have discussed in class could you use to implement this feedback controller?

(4) { tachometer (2)
op-amp (2) summer (1)

MOSFET (ok., but not req'd) (1)

Mean = 71.3

SD = 17.1

jvanida@eng.uci.edu

MAE 106 Midterm Exam
Winter 2000

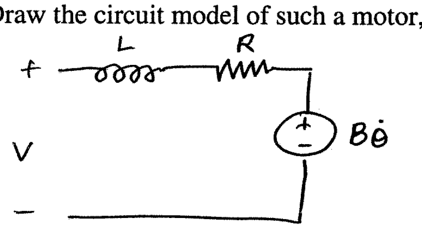
University of California, Irvine
Department of Mechanical and Aerospace Engineering

Problem 1: Motors

a. What type of motor did we analyze in Lecture and did you use in Lab 2?

DC Brush Motor

b. Draw the circuit model of such a motor, as presented in class:



c. Assume that the motor shaft is allowed to spin freely. At t = 0 a constant voltage V is applied to the motor. Derive an expression for the angular velocity ω of the motor as a function of time. Assume that the inertia of the motor shaft equals J, the motor's torque constant is B, and the current into the motor does not change. (Hint: Based on the model you drew in part b and your knowledge of physics, write a differential equation in terms of ω and solve it).

$V = L\frac{di}{dt} + Ri + B\dot{\theta}$ (with $L\frac{di}{dt}$ crossed out, $\to 0$) $\qquad \tau = Bi; \quad \tau = J\ddot{\theta}$

$\hookrightarrow i = \frac{\tau}{B} = \frac{J}{B}\ddot{\theta} = \frac{J}{B}\dot{\omega}$

$\frac{RJ}{B}\dot{\omega} + B\omega = V$

$\dot{\omega} = \frac{-B^2}{RJ}\omega + \frac{B}{RJ}V$

Homogeneous: $\omega_h = Ae^{-t/\tau} \qquad \tau = \frac{RJ}{B^2}$

Particular: $\omega_p = C \Rightarrow \quad 0 = \frac{-B^2}{RJ}C + \frac{B}{RJ}V \Rightarrow C = \frac{1}{B}V$

Total: $\omega = Ae^{-t/\tau} + \frac{V}{B}$

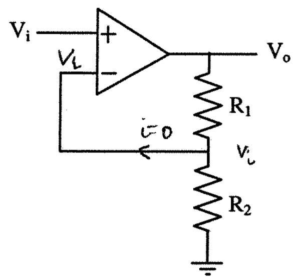
$\omega(0) = 0 \Rightarrow A + \frac{V}{B} = 0 \quad A = -\frac{V}{B}$

$\omega = \frac{V}{B}\left(1 - e^{-t/\tau}\right) \quad \tau = \frac{RJ}{B^2}$

Problem 3: Operational Amplifiers

a) Why are op-amps such as the ones used in lab unsatisfactory for powering most motors?

b) Find the relationship between V_o and V_i for the following non-inverting amplifier circuit:

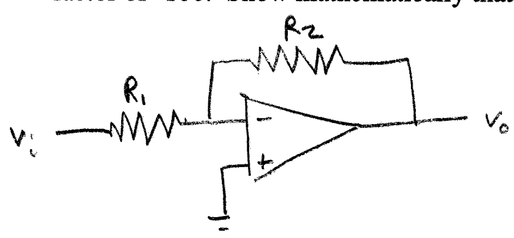


$$\frac{V_o - V_i}{R_1} = \frac{V_i}{R_2}$$

$$\frac{V_o}{R_1} = V_i\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V_i\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$

$$\frac{V_o}{V_i} = \frac{R_1 + R_2}{R_2}$$

c) Using an op-amp and resistors, design a circuit to amplify *and invert* an input signal by a factor of -100. Show mathematically that your design works.



$$\frac{V_i}{R_1} + \frac{V_o}{R_2} = 0$$

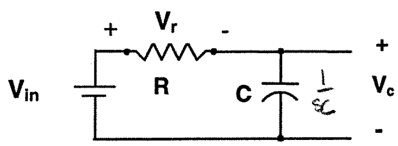
$$V_o = \frac{-R_2}{R_1} V_1$$

$R_1 = 10\,\Omega$ $\Rightarrow V_o = -100\,V_i$

$R_2 = 1000\,\Omega$

Problem 4: Filters and Frequency Response

a) Find the transfer function relating V_c to V_{in} for the following filter:



$$V_c = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_i = \frac{1}{RCs+1} V_i$$

$$H(s) = \frac{1}{RCs+1}$$

b) What kind of filter is this? Show mathematically.

low-pass $\quad |H(j\omega)| = \frac{1}{\sqrt{(RC\omega)^2+1}}$

ω small $\rightarrow |H(j\omega)| \approx 1$ pass low freq.

ω big $\rightarrow |H(j\omega)| \approx 0$ attenuate high freq

c) Suppose $V_{in} = \sin t$, $R = 1\ \Omega$ and $C = 1$ Farad. What is $V_c(t)$ (the force response) ~~Assume $V_c(0) = 0$~~.

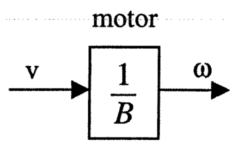
$$V_c(t) = |H(j\omega)| \sin(t + \phi_{H(j\omega)}) \Big|_{\omega=1}$$

$$= \frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$$

$$\phi_{H(j\omega)} = 0 - \tan^{-1}(RC\omega) = -\tan^{-1}(1) = -45^\circ = \frac{\pi}{4}$$

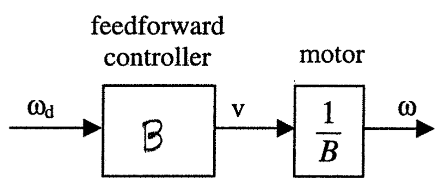
Problem 5: Control Theory

Consider the problem of controlling the velocity of a motor. A simple model of the motor is given by the following block diagram:



where v is the voltage input to the motor and ω is the angular velocity of the shaft.

a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What gain value should the controller box have to make the output equal the desired output? Write the gain in the controller box.



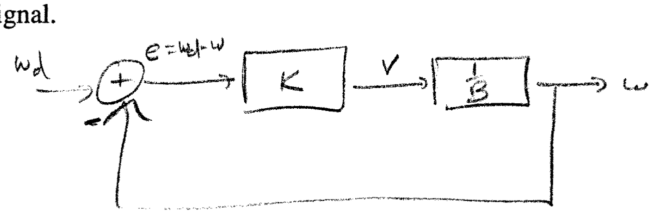
$V = B\omega_d$

$\omega = \frac{1}{B} V = \frac{1}{B}(B\omega_d) = \omega_d$

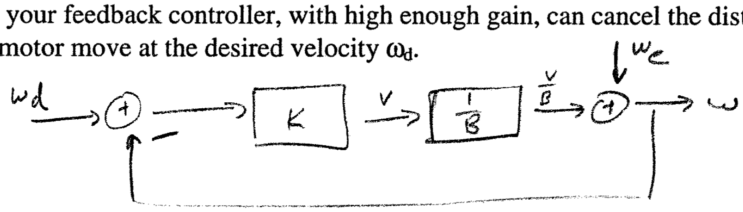
b) What is a disadvantage of an open-loop controller like this one?

can't reject disturbances, need a good model of motor

c) Draw a block diagram of a feedback controller for the motor. Label all arrows, including the error signal.



d) Assume that the output velocity of the motor is affected by a disturbance such that $\omega = (1/B)v + \omega_e$ where ω_e is the (constant) error in velocity introduced by the disturbance. Prove that your feedback controller, with high enough gain, can cancel the disturbance and make the motor move at the desired velocity ω_d.



$$\omega = \frac{V}{B} + \omega_e = \frac{K}{B}(\omega_d - \omega) + \omega_e$$

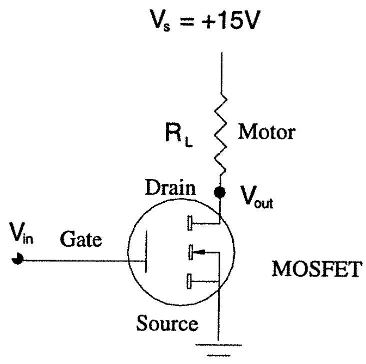
$$\omega\left(1 + \frac{K}{B}\right) = \frac{K}{B}\omega_d + \omega_e$$

$$\omega = \frac{\frac{K}{B}\omega_d}{1 + \frac{K}{B}} + \frac{1}{\left(1 + \frac{K}{B}\right)}\omega_e \quad \text{as } K \to \infty \quad \omega = \omega_d$$

Problem 2: MOSFETS and Power Control

A circuit for controlling a motor with a MOSFET is shown below. The motor is modeled simply as a resistor. By changing the input gate voltage to the MOSFET, the motor can be made to spin. A table of the MOSFET's drain/source resistance (like the one you generated in lab) is also shown.

V_{in} (volts)	R_{DS} (KΩ)
0	100
1	100
2	90
3	80
3.4	75
3.6	41
3.9	4
4.2	0.1
5.5	0.01

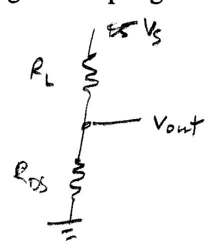


a) To make the motor spin, should the input gate voltage (V_{in}) to the MOSFET be high or low and why?

High ⇒ lower the MOSFET resistance, allow current to flow thru motor

b) Assume R_L = 1KΩ. Calculate V_{out} corresponding to an input gate voltage of 3.9 volts.

$V_{in} = 3.9 \Rightarrow R_{DS} = 4\ K\Omega$



$$V_{out} = \left(\frac{R_{DS}}{R_{DS} + R_L}\right) V_s = \left(\frac{4}{4+1}\right)(15) = 12\ V$$

c) How many watts of power will the motor consume with the input gate voltage equal to 3.9 volts?

$$P = \frac{V^2}{R} = \frac{(15-12)^2}{1K\Omega} = \frac{9}{1000} = .009 \text{ watts}$$

Mean = 59
SD = 13

SOLUTION KEY

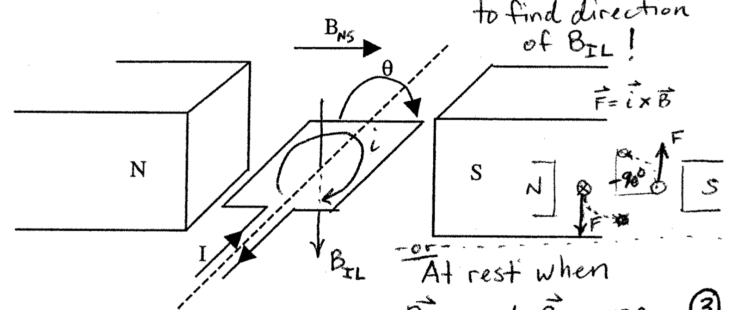
MAE 106 Midterm Exam
Winter 2001

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Problem 1: Motors (25 pts)

3pts

a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at θ = 0° as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.

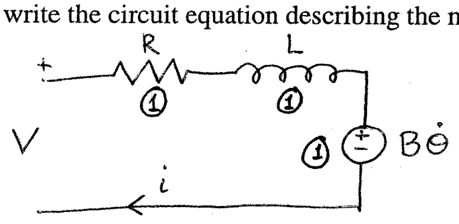


* Use RH rule to find direction of B_{IL}!

—or— At rest when $\vec{B}_{NS}$ and $\vec{B}_{IL}$ are aligned. ⇒ $\theta = -90°$ or $270°$ (3)

5pts

b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:



$$V = iR + L\frac{di}{dt} + B\dot{\theta} \quad (2)$$

6pts

c. Derive and plot the torque versus velocity relationship for the motor. Plot torque on the y-axis and velocity on the x-axis. Label the x-axis and y-axis intercepts with the appropriate terminology. Assume that:

- the motor receives a constant voltage input of V
- ~~the inertia of the motor shaft equals J~~ → not needed!
- the motor's torque constant is B and is equal to the back-EMF constant
- the current into the motor does not change.

$$V = iR + L\frac{di}{dt}^{\,0} + B\dot{\theta}$$

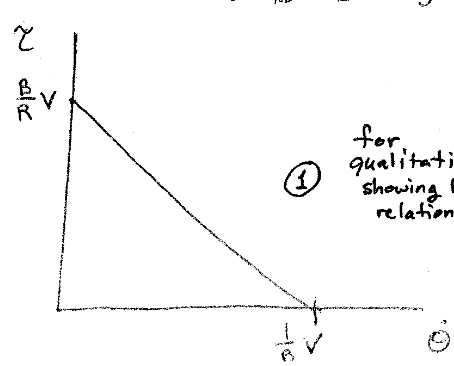
$$\tau = Bi, \quad i = \frac{\tau}{B} \text{ (substitute)}$$

$$V = \frac{\tau}{B}R + B\dot{\theta}$$

solve for τ,

$$\tau = -\frac{B^2}{R}\dot{\theta} + \frac{B}{R}V \quad (3)$$

(1) at stall $\dot{\theta} = 0$, $\tau_s = \frac{B}{R}V$
(1) at no load $\tau = 0$, $\dot{\theta}_{NL} = \frac{1}{B}V$
} linear intercepts



(1) for qualitatively showing linear relationship

d. For what torque load does the motor produce maximum power?

6pts

$P = \tau \dot{\theta}$ (Mechanical Power) ②

need $\dot{\theta}(\tau)$: $\tau = -\frac{B^2}{R}\dot{\theta} + \frac{B}{R}V$ (from (c))

$$\Rightarrow \dot{\theta} = -\frac{R}{B^2}\tau + \frac{1}{B}V$$

$$P = \tau \dot{\theta}$$

$$P(\tau) = \tau\left(-\frac{R}{B^2}\tau + \frac{1}{B}V\right) = \frac{-R}{B^2}\tau^2 + \frac{V}{B}\tau \quad ①$$

Maximum Power?

$$\left.\frac{\partial P}{\partial \tau}\right|_{\tau=\tau_{MP}} = 0 = -2\frac{R}{B^2}\tau_{MP} + \frac{V}{B} \Rightarrow \boxed{\tau_{MP} = \frac{B}{2R}V} = \frac{\tau_{stall}}{2} \quad ②$$

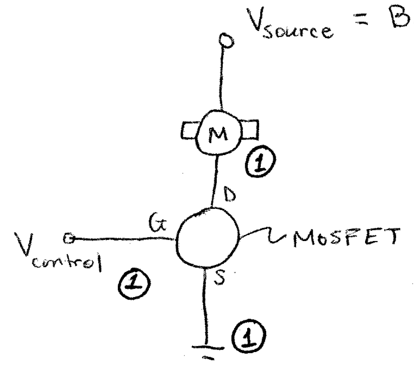
①

$\frac{\partial^2 P}{\partial \tau^2} < 0$, so max (not min)

e. Assume you have a low-power control signal from a computer, and that you would like the unloaded motor shaft to spin at ω_d when the control signal = +5V, and to stop spinning when the control signal is 0 V. Design a circuit using only a MOSFET, a power supply, and resistors to achieve this control. Label the gate, drain, and source of the MOSFET, and the control input from the computer. Specify the voltage of the power supply assuming the motor's torque constant is B and is equal to the back-EMF constant.

5pts

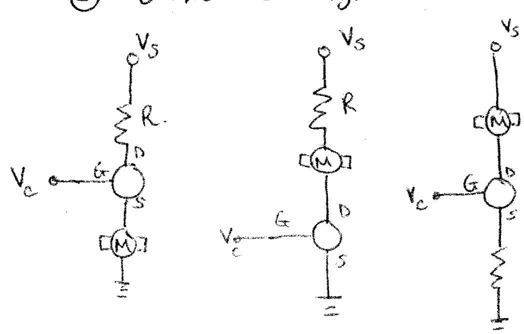
$V_{source} = B\omega_d$ ②



Notes:

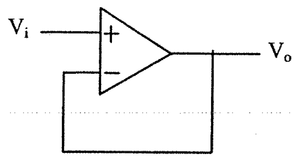
① V_{source} found from taking no load speed: $\dot{\theta}_{NL} = \frac{V}{B}$, (recall $i \approx 0$ at no load) and solving for correct V.

② Other configurations acceptable:



* Note that V_{source} will be different due to voltage division.

Problem 2: Op Amps and Feedback (25 pts)



a. Write the name of this circuit:

4pts Voltage Follower, Buffer, etc.

b. Briefly describe a practical situation in which you might want to use the circuit in a)

5pts e.g. Isolating sensitive circuitry (V_i) from high current loads at V_o. (V_i is a low current power source.)

Applying a signal (V_o) to a circuit without loading the circuit that generates the signal ($V_i = V_o$)

c. Assume that $V_o = K(V_+ - V_-)$ for the op amp. How big would K have to be such that V_o is within 1% of V_i?

5pts

$V_o = K(V_i - V_o)$ ①

$V_o = \frac{K}{(1+K)} V_i$ ②

$\rightarrow \frac{K}{1+K} = 0.99 \Rightarrow \boxed{K = 99}$ ②

d. For a real op amp, the output takes some time to respond to the inputs due to delays in the op-amp's internal circuitry. We can model these dynamics with a differential equation:

$$\dot{V}_o = -AV_o + AK(V_+ - V_-)$$

pts

where A > 0 is a constant. Given these dynamics, plot the voltage response of the circuit in a) to a unit step input in V_i versus time. Label the time at which the output is 63% of its final value.

$\dot{V}_o = -AV_o + AK(V_i - V_o)$

$\dot{V}_o + A(1+K)V_o = AKV_i$ (1st order ODE)

method: integrating factor —

$\frac{d}{dt}\left(e^{A(1+K)t} V_o\right) = AKe^{A(1+K)t} V_i$

$e^{A(1+K)t} V_o = \int AKe^{A(1+K)t} V_i \, dt + C_1$

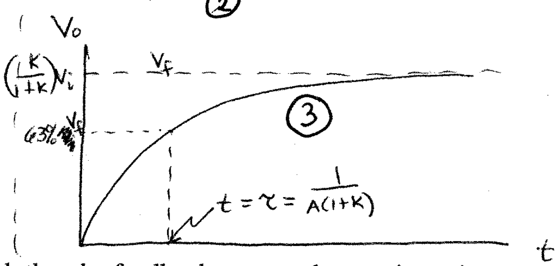
$= \frac{K}{(1+K)} e^{A(1+K)t} V_i + C_1$

$V_o = \frac{K}{1+K} V_i + C_1 e^{-A(1+K)t}$

assume $V_o = 0$, at $t = 0$:

$\boxed{V_o = \left(\frac{K}{1+K} V_i\right)\left(1 - e^{-A(1+K)t}\right)}$ ①

$\tau = 1/A(1+K)$: time-constant ②



e. Assume that you misconnect the circuit in a) such that the feedback goes to the non-inverting input instead of the inverting input (i.e. positive feedback instead of negative feedback), and the input goes to the inverting input. Assuming the op-amp has the dynamics in d), for what values of K will the circuit be unstable (i.e. for what values of K will the output go to infinity as time goes to infinity)?

5pts

switch op-amp leads equivalent to substituting K = −K in Solution to diff eq in (d):

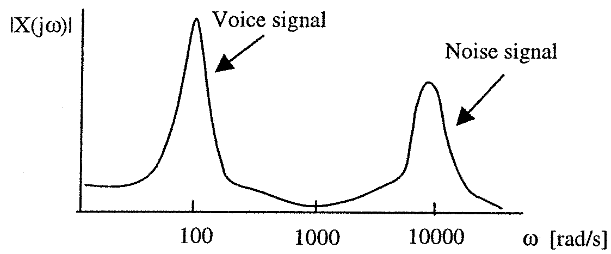
$V_o = \left(\frac{-K}{1-K} V_i\right)\left(1 - e^{-A(1-K)t}\right)$

→ exponential term decays unless K > 1. Therefore

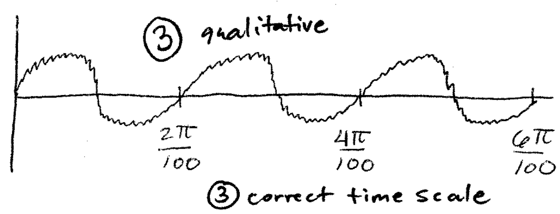
for K > 1, output will go to infinity

Problem 3: Filters and Signal Processing (25 pts)

You are hired to improve a wire-tapping signal for the FBI. Shown below is a Bode plot of the typical frequency content of the signal that the FBI's currently-used system generates:



a) Sketch (very roughly) what the signal would like in the time domain. Include a time scale.



Base frequency $= 100 \frac{rad}{s}$ with high freq. noise on top

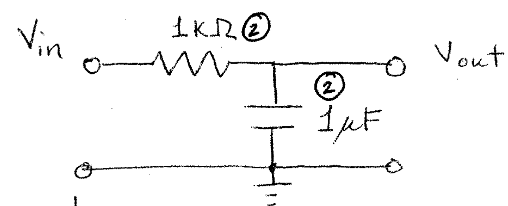
Base period $= \frac{1}{f} = \frac{1}{100/2\pi} = \frac{2\pi}{100}$

b) What type of filter should you apply to clean up the signal?

Low Pass Filter ⑤

c) You have only a few resistors and capacitors in your toolbox, as shown below. Design an RC circuit to filter the unwanted noise. Draw the circuit, with the input and outputs labeled, and identify the values of R and C that you would use.

Toolbox Contents:
R = 1Ω, 10Ω, 100Ω, (1 KΩ)
C = 1 pF, 0.1 μF, (1 μF)



Want $\omega_c \geq 1000 \frac{rad}{s}$ $\quad \omega_c = \frac{1}{RC} = \frac{1}{(1k\Omega)(1\mu F)} = 1000 \frac{rad}{s}$ ③

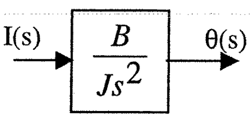
single R, single C (All other combinations give $\omega_c < 1000$ thereby infringing on "good" voice signal)

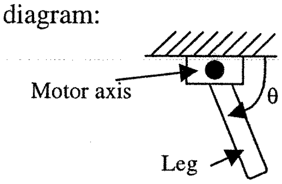
d) By what factor would your filter attenuate the noise in the signal at 10000 rad/s?

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{10000}{1000}\right)^2}} = 0.0995$$ ④ ③

$$\boxed{\left|\frac{V_{out}}{V_{in}}\right| \approx 0.1}$$ attenuation factor

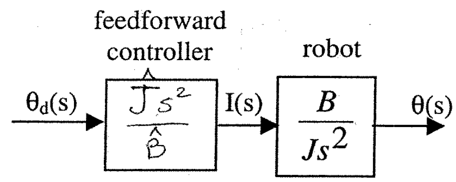
Problem 4: Control Theory and Laplace Transforms (25 pts)
Consider the problem of controlling the angle of a one degree-of-freedom robotic leg. The transfer function of the robot arm is given by the following block diagram:





where I(s) is the current input to the robot's motor, J is the inertia of the motor, B is the motor torque constant, and θ is the angular position of the robot.

5 pts a) Shown to the side is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where θ_d is the desired angular position of the robot. What transfer function should the controller box have to make the output equal the desired output? Write the transfer function the controller box.



where the "^" indicates approximation.

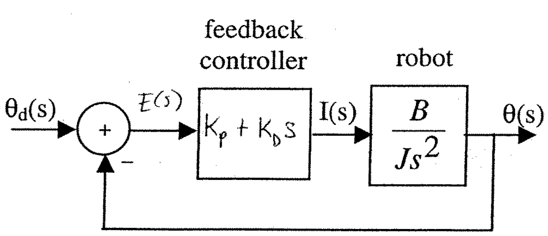
6 pts b) In the *time domain*, write the relationship between I and θ_d

Laplace domain: $I(s) = \frac{J}{B} s^2 \theta_d(s)$ $\Longrightarrow$ time domain: $i(t) = \frac{J}{B}\frac{d^2}{dt^2}\theta_d(t)$ or $\boxed{i(t) = \frac{J}{B}\ddot{\theta}_d(t)}$

6 pts c) What is a disadvantage of an open-loop controller like this one?

- Don't know J and B exactly.
- Can't reject disturbances

4 pts d) A proportional-derivative (PD) feedback controller is a common type of controller that provides a control signal to the plant that is the sum of a signal proportional (by a gain K_p) to the output error and a signal proportional by (a gain K_d) to the derivative of the output error. In the following diagram, fill in the appropriate transfer function such that the robot would be controlled by a PD controller.



3 pts e) Find the overall transfer function for the PD-feedback controlled robot system in d)

Let $(k_p + k_D s)\left(\frac{B}{Js^2}\right) \Rightarrow G(s)$

$\therefore \theta(s) = G(s)E(s)$, where $E(s) = \theta_d(s) - \theta(s)$

$= G(s)\left[\theta_d(s) - \theta(s)\right]$

$\theta(s)\left[1 + G(s)\right] = G(s)\theta_d(s)$

$$\frac{\theta(s)}{\theta_d(s)} = \frac{G(s)}{1+G(s)}$$

$$\frac{\theta}{\theta_d} = \frac{(k_p + k_D s)\left(\frac{B}{Js^2}\right)}{1 + (k_p + k_D s)\left(\frac{B}{Js^2}\right)}$$

$$\boxed{\frac{\theta(s)}{\theta_d(s)} = \frac{B(k_p + k_D s)}{Js^2 + B(k_p + k_D s)}}$$

Solution

Mean 61
SD 11

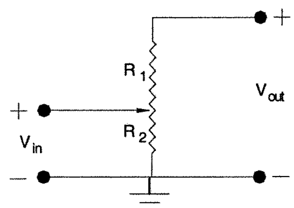
MAE 106 Midterm Exam
Winter 2002

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Problem 1: Circuits (25 pts)

7 a) Shown below is a 10Kohm potentiometer wired incorrectly. Assume Vin is 10 V and is provided by a 100 Watt power supply. Assume the potentiometer is rated at 1 Watt. Assume the shaft can rotate 180 degrees, and define 0 degrees rotation as the shaft angle when the resistance between the wiper and the ground is 10Kohm. At what shaft angle do you expect to smell smoke?

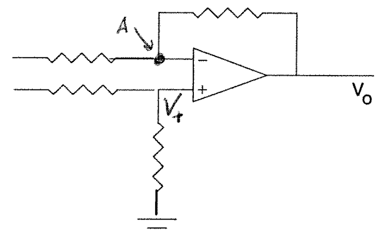
$P = 1V \quad I = \frac{V}{R} \rightarrow P = \frac{V^2}{R_{smoke}} = 1\,W = \frac{100\,V^2}{R_{smoke}} \rightarrow R_{smoke} = 100\,\Omega$



$180^\circ - \frac{R_{smoke}}{R_{total}}(180^\circ) = \theta \rightarrow 180^\circ - \frac{100}{10000}(180^\circ) = 180^\circ - 1.8^\circ$

$\boxed{\theta = 178.2^\circ}$

7 b) What does the following circuit do? (give proof) Assume the four resistors have equal values.



V_+ is just a voltage divider so

$V_+ = \frac{R}{2R} V_2 = \frac{1}{2} V_2$

At node A:

$V_A = V_+$

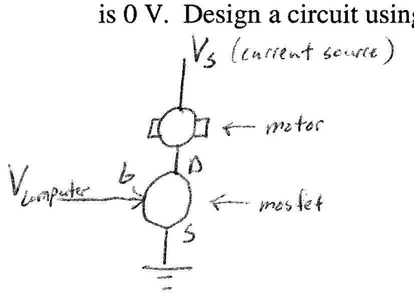
$\frac{V_1 - \frac{1}{2}V_2}{R} = \frac{\frac{1}{2}V_2 - V_o}{R} \rightarrow \boxed{V_o = V_2 - V_1}$

This circuit is a subtractor

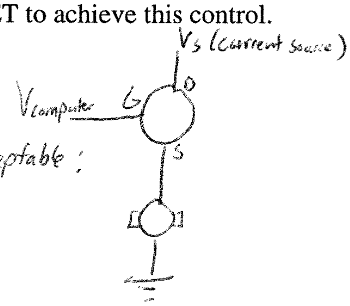
5 c) Why are op amps such as the ones used in lab unsatisfactory for powering most motors?

They cannot provide enough current or power for most motors.

6 d) Assume that you have a low-power control signal from a computer, and that you would like to make a motor spin when the control signal is +5 v, and to stop spinning when the control signal is 0 V. Design a circuit using a MOSFET to achieve this control.



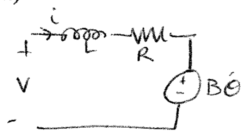
also acceptable:



Problem 2: Motors and Power Control (25 pts)

A DC brushed motor has an internal resistance of 10 ohms. You hold the motor shaft fixed and apply 10 V, and find that the stall torque is 0.1 Nm, and that it takes 1.5 s to generate 0.063 Nm of torque. Assume the motor's torque constant equals the back EMF constant.

7 a) How fast will the unloaded motor shaft turn if you let it go?



$V = L\frac{di}{dt} + iR + B\dot{\theta}$

motor model

shaft fixed, steady-state $V = iR \Rightarrow 10\,V = i\,10\,\Omega \Rightarrow i = 1$ amp

now find torque constant $\tau = Bi \Rightarrow B = \frac{.1\,Nm}{1\,a} = .1\,\frac{Nm}{a}$

Now assume shaft is free to spin: ($\frac{di}{dt} \approx 0$)

then $B\omega + iR = V$, $\tau = Bi = J\ddot{\theta} \Rightarrow i = \frac{J}{B}\ddot{\theta} = \frac{J}{B}\dot{\omega}$, so

$B\omega + \frac{RJ}{B}\dot{\omega} = V \Rightarrow$ in steady state $\omega = \frac{V}{B}$ = no-load speed

$= \frac{10\,V}{.1\,Nm/a} = 100\,\frac{Watt}{Nm} = 100\,\frac{Nm/s}{Nm}$ $\boxed{= 100\,\frac{rad}{sec}}$

7 b) What is the inductance of the motor windings?

time constant depends on L when shaft is fixed:

$L\frac{di}{dt} + iR = V$

$\frac{di}{dt} = -\frac{R}{L}i + V \Rightarrow$ time constant of this 1st order DEQ $\tau_c^{fixed} = \frac{L}{R}$

$.063\,Nm = 63\%$ of $.1\,Nm \Rightarrow 1$ time constant elapsed $\Rightarrow \tau_c^{fixed} = \frac{L}{R} = 1.5\,s$

$L = 1.5\,s \times 10\,\Omega = \boxed{15\,H}$

7 c) The motor takes 5 seconds to reach its no-load speed. If you double the shaft inertia, how much longer will the motor take to reach 99% of it's no-load speed?

shaft free $\frac{RJ}{B}\dot{\omega} + B\omega = V$ (from (a))

$\dot{\omega} = \frac{-B^2}{RJ}\omega + V \Rightarrow \tau_c^{free} = \frac{RJ}{B^2}$

Note the time constant when the shaft is free is proportional to J $\Rightarrow 2 \times J \Rightarrow 2 \times \tau_c^{free}$

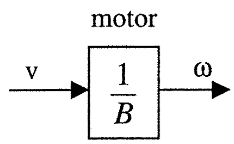
$\boxed{\text{Twice as Long (i.e. 10 sec)}}$

4 d) Why does a DC brushed motor have brushes? (Explain in 15 words or less).

To reverse the direction of current through coils, so motor keeps spinning (commutation)

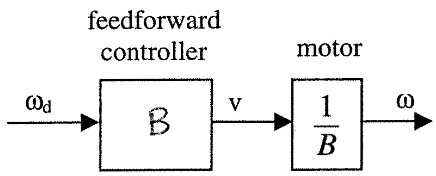
Problem 3: Control Theory (25 pts)

1) Consider the problem of controlling the velocity of a motor. A simple model of the motor is given by the following block diagram:



where v is the voltage input to the motor and ω is the angular velocity of the shaft.

4 a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What gain value should the controller box have to make the output equal the desired output? Write the gain in the controller box.



Assume gain is an unknown constant G

then $V = G\omega_d$

$\omega = \frac{1}{B} V = \frac{G}{B}\omega_d$

To make $\omega = \omega_d$, we want $G = B$

6 b) Name *two* disadvantages and *two* advantages of an open-loop controller like this one.

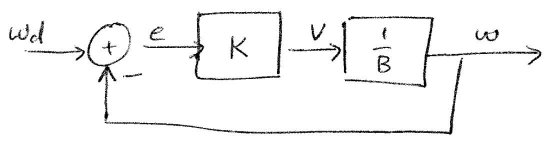
Disadvantages

1. can't respond to unexpected disturbances
2. requires a good model of plant
3. difficult to make a good model if plant is complex

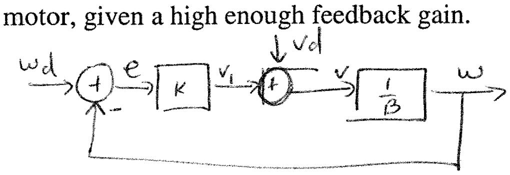
Advantages

1. Can theoretically produce a perfect output (doesn't need an error to generate a control input)
2. Doesn't require a sensor
3. Can perform control calculations "off-line," rather than in real time.

7 c) Draw a block diagram of a feedback controller for the motor, label all arrows, including the error signal.



8 d) Prove that your feedback controller cancels an additive disturbance V_d to the voltage input to the motor, given a high enough feedback gain.



$V_1 = Ke = K(\omega_d - \omega)$

$V = V_1 + V_d = K(\omega_d - \omega) + V_d$

$\omega = \frac{1}{B} V = \frac{1}{B}(V_1 + V_d)$

$\omega = \frac{1}{B}(K(\omega_d - \omega) + V_d)$

$\omega = \frac{K}{B}(\omega_d - \omega) + \frac{1}{B} V_d$

$\omega\left(\frac{B+K}{B}\right) = \frac{K}{B}\omega_d + \frac{V_d}{B} \Rightarrow \omega = \frac{K}{K+B}\omega_d + \frac{1}{K+B} V_d$

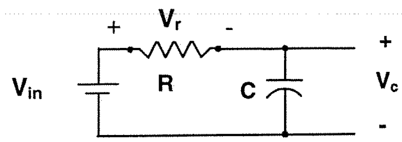
(1 as $K \to \infty$; 0)

$\omega = \omega_d$ as $K \to \infty$

V_d cancelled

Problem 4: Signal Processing, Differential Equations, and Frequency Analysis (25 pts)

5 a) Describe a practical situation in which the following circuit would be useful:



Low Pass Filter (Passive)

Could be used to filter out high frequency radio noise, or noise from an input signal to a system (eg. potentiometer)

5 b) Assume V_{in} is a step input at time zero and $V_o(0) = 0$. Find $V_o(t)$.

$V_{in} = V_c + V_r$

$V_{in} = C\frac{dV_c}{dt} + \frac{V_c}{R}$

Homogeneous: $\frac{V_c}{R} = -C\frac{dV_c}{dt}$

$-\frac{1}{RC}V_c = \frac{dV_c}{dt}$

$V_c(t) = Ae^{-\frac{1}{RC}t}$

Particular: let $V_c = V = \text{const.}$

$V_c = V_{in} \rightarrow V_c(t) = Ae^{-\frac{1}{RC}t} + V_{in}$

& $V_c(0) = 0$ so $A + V_{in} = 0 \rightarrow A = -V_{in}$

$$V_c(t) = V_{in}\left(1 - e^{-\frac{t}{RC}}\right)$$

5 c) Find the transfer function for the above circuit.

$$\frac{V_c}{V_{in}} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} = \frac{1}{1 + RCs} = H(s)$$

5 d) Find the frequency response of the above circuit. Be sure to provide equations for how the circuit scales and phase shifts a sinusoidal input.

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (RC\omega)^2}}$$

$$\phi(j\omega) = \tan^{-1}\frac{0}{1} - \tan^{-1}\frac{RC\omega}{1} = -\tan^{-1} RC\omega$$

5 e) Assume R = 100 ohm, C = .01 F. How much more attenuated will a 1000 Hz sinusoidal input signal be than a 100 Hz input signal? (Provide proof).

$\tau = RC = (100\Omega)(.01F) = 1\text{ sec}$ $\omega = 2\pi f$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(RC\omega)^2}} = \frac{1}{\sqrt{1+\omega^2}}$$

$$|H(2\pi 100 j)| = \frac{1}{\sqrt{1+(2\pi 100)^2}} \approx \frac{1}{2\pi 100} = A$$

$$|H(2\pi 1000 j)| = \frac{1}{\sqrt{1+(2\pi 1000)^2}} \approx \frac{1}{2\pi 1000} = B$$

$B = \frac{1}{10}A \longrightarrow$ 1000 Hz will be attenuated by a factor of $\frac{1}{10}$

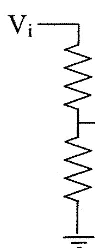
Mean 63.5
σ 12.6

MAE 106 Midterm Exam
Winter 2003

University of California, Irvine
Department of Mechanical and Aerospace Engineering

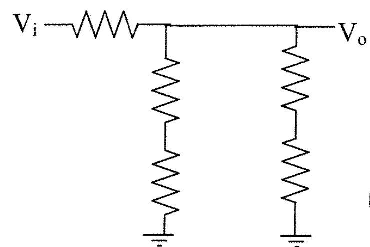
Problem 1 Circuits (25 pts)

4 a) Find V_o for the following two circuits. Assume all resistors values = R.



Answer:
$V_o = \frac{1}{2} V_i$

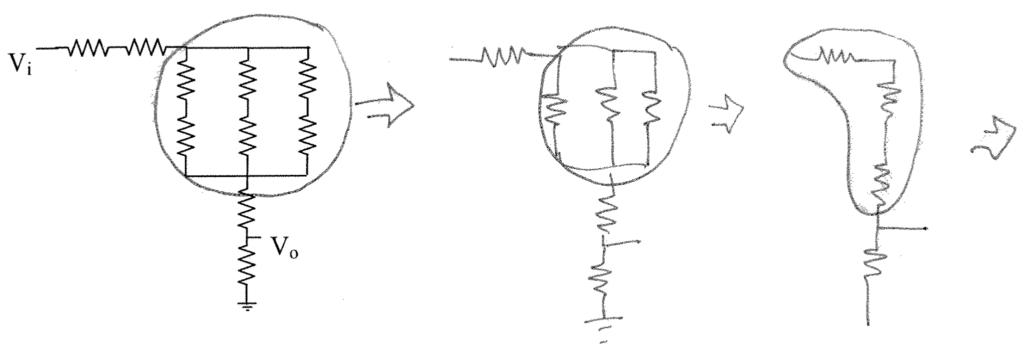
$V_o = \frac{R}{R+R} V_i$

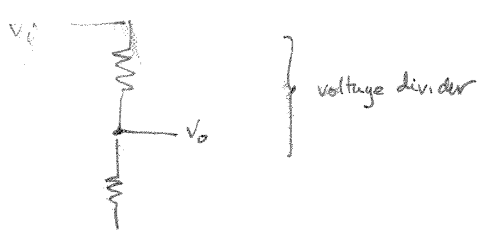


Answer:
$V_o = \frac{1}{2} V_i$

$R_{eq} = \frac{(2R)(2R)}{2R+2R} = \frac{4R^2}{4R} = R$

4 b) For the following circuit, draw conceptual sketches to show how you would simplify the network to solve for V_o. You do not need to find V_o, just illustrate the steps.





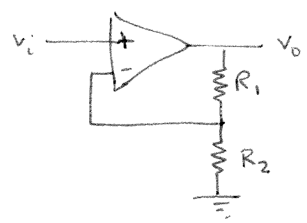
4 c) What are the two “golden rules” of op-amp analysis?

1. $V_+ = V_-$
2. $i_+ = i_- = 0$

4 d) What two conditions must be true for these golden rules to apply?

1. negative feedback
2. not saturated

5 e) Using an op-amp and resistors, design a circuit to amplify an input signal by a factor of 20. Show mathematically that your design works.



$$\frac{V_o - V_i}{R_1} = \frac{V_i}{R_2}$$

$$V_o - V_i = \frac{R_1}{R_2} V_i$$

$$V_o = \left(1 + \frac{R_1}{R_2}\right) V_i$$

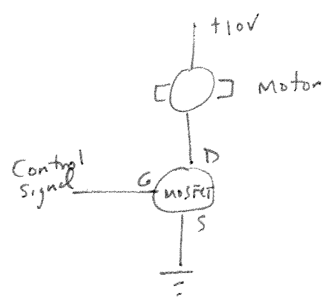
Desire

$$1 + \frac{R_1}{R_2} = 20$$

$$\frac{R_1}{R_2} = 19$$

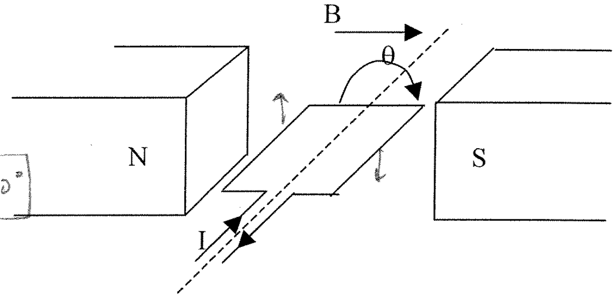
$$\boxed{R_1 = 19 R_2}$$

4 f) Assume that you have a low-power control signal from a computer, and that you would like to make a motor spin when the control signal is +5 v, and to stop spinning when the control signal is 0 V. Design a circuit using a MOSFET to achieve this control.



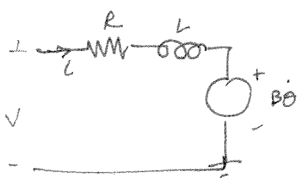
Problem 2: Motors (25 pts)

3 a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at $\theta = 0°$ as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.



$\theta = -90°$ or $270°$

4 b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:



$V = iR + L\frac{di}{dt} + B\dot{\theta}$

6 b. Solve this differential equation for the current through the motor as a function of time when:

- the shaft of the motor is held fixed
- a constant voltage v is applied across the motor at time = 0
- the initial current i(t = 0) through the inductor is zero

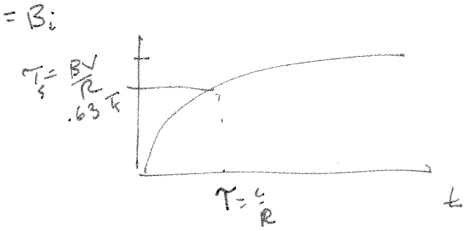
shaft fixed $\Rightarrow B\dot{\theta} = 0$

$V = L\frac{di}{dt} + iR$

$i(t) = \frac{v}{R}\left(1 - e^{-t/\tau}\right) \quad \tau = \frac{L}{R}$

4 3 c. Plot the torque that the motor generates as a function of time for the conditions described in part b. Label the axes, the final value of the torque, and the time at which the torque has reached 63% of its final value. Assume the motor's torque constant is some constant B, equal to the back EMF constant.

$T = Bi$



1 d. What is the term for the maximum torque a motor can produce when its shaft is held constant?

Stall torque

1 e. What is the term for the maximum speed that a motor free to spin will reach?

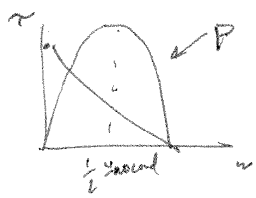
no load speed

2 f. What happens to an unloaded motor spinning at steady-state speed ω when you double its input voltage?

spins at 2ω

4 g. At what speed does an ideal DC brushed motor produce maximum power and why?

$\frac{1}{2}\omega_{no\ load}$



Torque-speed curve is a line

$\tau = m\omega + \tau_s$ but $\tau = 0 \Rightarrow \omega = \omega_{max}$ $\quad 0 = m\omega_m + \tau_s$

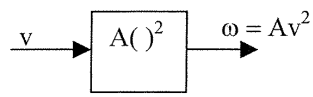
Power: $\tau \cdot \omega = m\omega^2 + \tau_s\omega$ $\quad m = -\frac{\tau_s}{\omega_m}$

$\frac{dP}{d\omega} = 2m\omega + \tau_s = 0$

$\omega = \frac{-\tau_s}{2m} = \frac{-\tau_s}{2(-\frac{\tau_s}{\omega_m})} = \boxed{\frac{\omega_m}{2}} = \frac{1}{2}\omega_{max} = \frac{1}{2}\omega_{no\ load}$

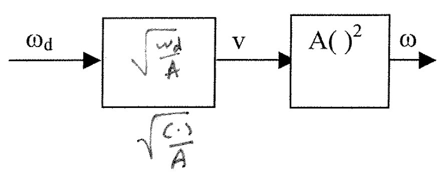
Problem 3: Control Theory (25 pts)

1) You design a new type of motor for which the speed is proportional to the input voltage squared:



where v is the voltage input to the motor and ω is the angular velocity of the shaft and A is a constant.

6 a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What function should the controller box compute to make the output equal the desired output? Write this function controller box.



Assume $\omega = \omega_d$, then

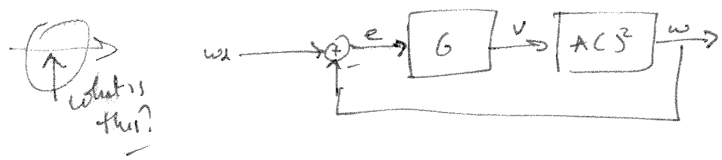
$\omega = Av^2 = \omega_d$

$v = \sqrt{\frac{\omega_d}{A}}$

6 b) If the estimate of A used by the feedforward controller is too small by 10%, how much faster will the actual speed be than the desired speed?

$v = \sqrt{\frac{\omega_d}{.9A}}$ $\quad \omega = Av^2 = \frac{A}{.9A}\omega_d = \frac{\omega_d}{.9} = 1.11\,\omega_d$ 11% faster

7 c) Draw a block diagram of a feedback controller for the motor, label all arrows, including the error signal.



6 d) Prove that, the actual velocity equals the desired velocity as the feedback gain gets large.

$v = Ge = G(\omega_d - \omega)$

$\omega = Av^2 = A\,G^2(\omega_d - \omega)^2$

$(\omega_d - \omega)^2 = \frac{\omega}{AG^2}$

as $G \to \infty$ $\quad (\omega_d - \omega)^2 \to 0 \Rightarrow \omega_d = \omega$

2

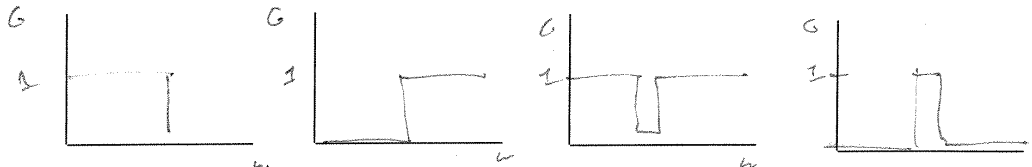
i) What two pieces of information do you need to describe the frequency response of a system?

magnitude / scaling
phasing

4

j) Draw an example of the magnitude response of the following types of filters. Make sure to label the axes.

G = scaling factor



Low Pass Filter High Pass Filter Notch Filter Band-pass filter

3

k) Prove that a mass M acts like a low-pass filter, if force f is considered its input and position x its output

$f = m\ddot{x}$

$f = m s^2 x$

$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2}$

$G(j\omega) = \frac{1}{-m\omega^2}$

$|G(j\omega)| = \frac{1}{m\omega^2}$

as $\omega \to \infty$ $|G(j\omega)| \to 0$

Problem 4: Frequency Analysis: Motivation, Theory, and Practice (25 pts)

4 a) Give two reasons for studying frequency domain analysis:

1. intuition – systems behave like filters
2. ease – solving for freq. response solving for time dynamics

4 g) If you input a sinusoidal input u(t) = asin(ωt) into a linear, time-invariant system, what is the output x(t)? Express your answer in terms of an equation and in words.

$x(t) = aG\sin(\omega t + \phi)$

sinusoidal output at same frequency but ~~at shift~~ w/ scaling G & phase shift ϕ

8 h) Below is a proof of the correct answer to part b). On the left are the equations for each step of the proof. On the right is a description of what is happening in the proof. Fill in all blanks to complete the proof.

Step	Description	Equation
1	An n^{th} order linear system with output x(t) and input u(t) can be described by a differential equation like this:	$\frac{d^n x}{dt^n} + \frac{d^{n-1} x}{dt^{n-1}} + \ldots x = u$
2	taking the L.T.	X(s)=B(s)/A(s)U(s)+IC(s)/A(s)
3	Assume the system is stable, then in the steady state Equation 2 can be simplified to:	$X(s) = \frac{B(s)}{A(s)} U(s)$
4	The Laplace transform of the sinusoidal input is:	$\frac{\omega}{s^2+\omega^2}$
5	Thus, the steady state output of the system in the frequency domain is:	$X(s) = \frac{B(s)}{A(s)} \frac{\omega}{s^2+\omega^2}$
6	PFE (Partial Fraction Expansion)	$K_1/(s+jw)+K_2/(s-jw)+K_3/(s-s1)+K_4(s-s2)+\ldots$
7	But the system is stable, so the K3 and K4 terms go to zero with time, and the output in the time domain is thus Equation 7, with K_1=-a/(2j)G(-jw), and K_2 = a/(2j)G(jw), G(s)=B(s)/A(s)	$K_1 e^{-j\omega t} + K_2 e^{+j\omega t}$
8	Simplifying Equation 7 using the fact $\sin(\theta) = 1/2j(e^{j\theta}-e^{-j\theta})$ gives:	$x(t) = \lvert G(j\omega)\rvert \sin(\omega t + \phi_G)$
	PROOF COMPLETE	PROOF COMPLETE