

MAE91 HW#7

Required (due: 08/17/04):

Problem sets:

- 8.75 ✓
- 8.85 ✓
- 8.89 ✓
- 8.94 ✓
- 8.111 ✓
- 8.114 ✓
- 8.126 ✓
- 8.131 ✓

Bonus problems:

- 8.103 ✓
- 8.117 ✓
- 8.134 ✓

Discussion problems: (08/11/04):

- 8.95
- 8.99
- 8.108

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Course: MAE 91

HW set: # 7

Due: August 17, 2004

79/80

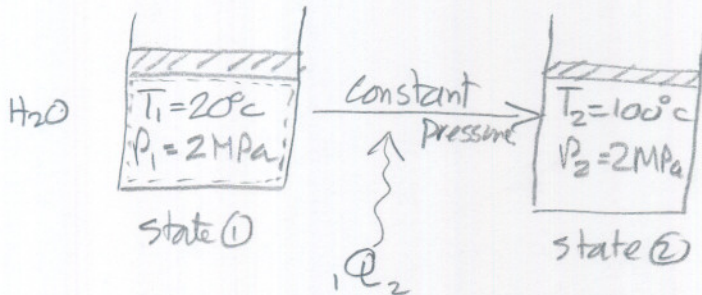
Bonus (15/15)

Problem 8.75

10/10

Statement

Piston/cylinder has constant pressure of 2000 kPa with water @ 20°C. It is heated to 100°C. Find heat transfer and entropy change using steam tables. Repeat calculation using constant heat capacity and incompressibility.



Find Q_2 and ΔS .

Assumptions

- C_p for water from table A.5 can be used for T different from 25°C.
- water assumed incompressible, to use $\Delta S = C_p \ln\left(\frac{T_2}{T_1}\right)$.

Laws

$${}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$$

$$h = u + Pv$$

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) \quad \text{For incompressible liquids.}$$

$$q_{12} = C_p (T_2 - T_1)$$

$${}_1w_2 = P(v_2 - v_1) \quad \text{for constant pressure.}$$

Using 1st Law for control mass

$$\boxed{1Q_2 - 1W_2 = m(u_2 - u_1)}$$

since not given mass m , using specific values.

$$\text{so } 1q_2 - 1w_2 = u_2 - u_1$$

$$1q_2 = (u_2 - u_1) + 1w_2$$

since constant pressure, then

$$\boxed{1w_2 = P(v_2 - v_1)}$$

$$\text{so } 1q_2 = (u_2 - u_1) + P(v_2 - v_1)$$

$$1q_2 = u_2 + Pv_2 - (u_1 + Pv_1)$$

$$\text{but } h = u + Pv$$

$$\text{so } \boxed{1q_2 = h_2 - h_1}$$

since in each state we are given 2 properties (T, P) then h_1, h_2 can be found from tables.

entropy change:

$$\text{using } m(s_2 - s_1) = \frac{1Q_2}{T} + S_{\text{gen}}$$

specific entropy change is $\boxed{(s_2 - s_1)}$. since we know 2 independent properties in each state, we can find s_2 and s_1 from tables.

now, to repeat calculations using constant heat capacity and incompressibility:

Since constant pressure, lookup C_p for water from table A.4.

$$\text{to find } 1Q_2, \text{ use } \boxed{1q_2 = h_2 - h_1 = C_p(T_2 - T_1)}$$

To find ΔS , use

$$\boxed{s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right)} \text{ assuming incompressible liquid.} \rightarrow$$

Numerical

From Table B.1.4 (compressed water):

$$(T=20^{\circ}\text{C}, P=2\text{MPa}) \longrightarrow \begin{aligned} h_1 &= 85.82 \text{ kJ/kg} \\ s_1 &= 0.2962 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

$$(T=100^{\circ}\text{C}, P=2\text{MPa}) \longrightarrow \begin{aligned} h_2 &= 420.45 \text{ kJ/kg} \\ s_2 &= 1.3053 \text{ kJ/kg}\cdot\text{K} \end{aligned}$$

$$\text{hence } q_{12} = h_2 - h_1 = 420.45 - 85.82 = \boxed{334.63 \text{ kJ/kg}}$$

$$s_2 - s_1 = 1.3053 - 0.2962 = \boxed{1.0091 \text{ kJ/kg}\cdot\text{K}}$$

using constant heat coefficients:

$$q_{12} = C_p (T_2 - T_1) = (4.18 \text{ kJ/kg}\cdot\text{K}) (100 - 20) = \boxed{334.4 \text{ kJ/kg}}$$

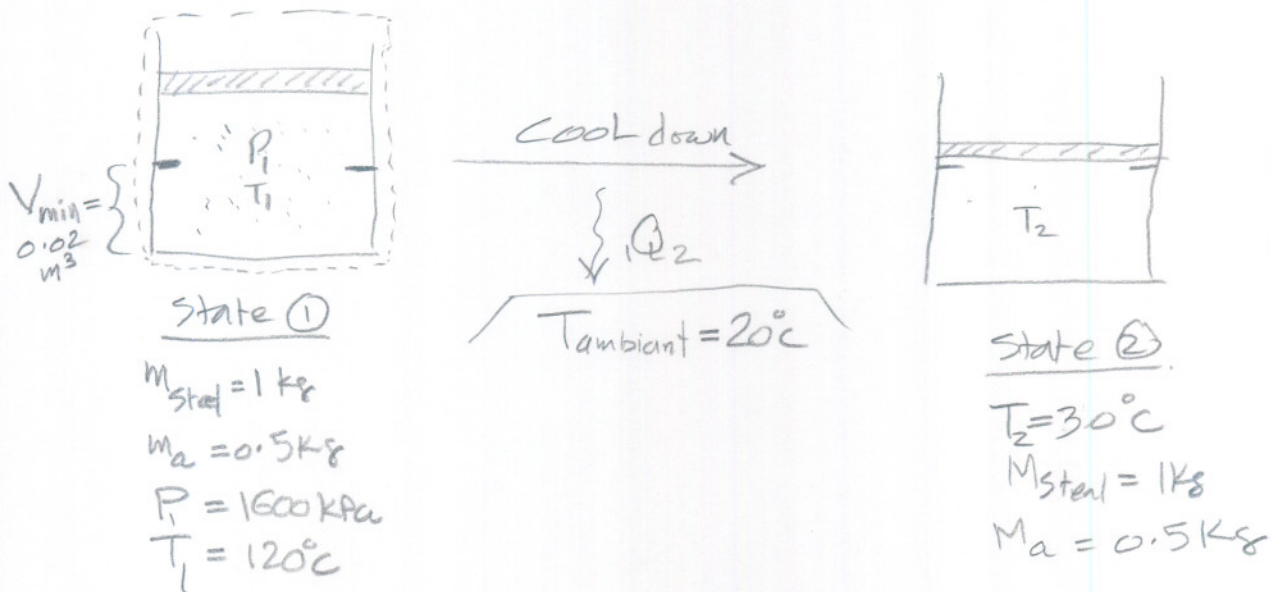
$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) = 4.18 \ln\left(\frac{100 + 273}{20 + 273}\right) = \boxed{1.00907 \text{ kJ/kg}\cdot\text{K}}$$

good approx.
good approximation

Problem 885 10/10

statement

Piston/cylinder of total 1 kg steel contains 0.5 kg ammonia @ 1600 kPa with both masses @ 120°C. Stop is placed so that minimum volume is 0.02 m³. Now whole system is cooled to 20°C by heat transfer to Ambient @ 20°C and during the process the steel keeps the same temp as the ammonia. Find work, heat transfer, total net entropy generated in process.



Find work done by process, Q_2 , S_{net} .

Assumptions

Control Mass.

C_p for steel constant at different T than 25°C (to use table A-3)

Laws

For solids, $u_2 - u_1 = C_p (T_2 - T_1)$

$$s_2 - s_1 = C_p \ln \left(\frac{T_2}{T_1} \right)$$

1st Law Control mass ${}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$

$${}_1Q_2 - {}_1W_2 = m_a(u_2 - u_1) + m_{\text{steel}}(u_2 - u_1)$$

$$W = \int P dV$$

steps $m(s_2 - s_1) = \int \frac{\delta Q}{T} + S_{\text{gen}}$

First thing to do is to find if piston hits stop or not. To do this, we know that since no spring is attached to piston, then piston will fall down at constant P . Call it $\boxed{P_{\text{float}} = P_1}$.

Then, just when it reaches stop, $P = P_{\text{float}}$ and

$$z_{\text{stop}} = \frac{V_{\text{stop}}}{\text{Mass}_{\text{amonia}}}. \quad \text{now from tables, find } T_{\text{stop}} @ (P_{\text{float}})_{\text{stop}}$$

if $T_{\text{stop}} > T_2$, then it will hit stop before find state.

and in this case, we will use P_{stop} for purpose of finding work done. i.e. constant P problem for work determination.

now assuming it hits stop, then we continue as:

$$Q_2 - \dots = \dots$$



$$\begin{aligned}
 {}_1Q_2 - {}_1W_2 &= m_a (u_2 - u_1) + m_{\text{Steel}} (u_2 - u_1) \\
 &\quad \downarrow \int_1^2 P dv \quad \downarrow \text{From table @ } (T_1, P_1) \quad \downarrow C_p (T_2 - T_1) \\
 &= P_{\text{float}} m_a (v_2 - v_1) \quad \downarrow \text{From Table @ } (T_2, P_{\text{stop}} = P_1) \\
 &\quad \downarrow \text{From table @ } (T_1, P_1) \quad \downarrow \text{From table @ } (T_2, P_{\text{stop}} = P_1)
 \end{aligned}$$

hence ${}_1Q_2$ is now found.

now apply entropy equation

From above calculation

$$\begin{aligned}
 m_a (s_2 - s_1) + m_{\text{Steel}} (s_2 - s_1) &= \frac{{}_1Q_2}{T_{\text{amb}}} + S_{\text{gen}} \\
 &\quad \downarrow \text{Table @ } (T_1, P_1) \quad \downarrow C_p \ln \left(\frac{T_2}{T_1} \right) \quad \downarrow \text{given} \\
 &\quad \downarrow \text{Table @ } (T_2, P_{\text{stop}})
 \end{aligned}$$

hence S_{gen} can now be found.

Numerical

$$v_{\text{stop}} = \frac{V_{\text{stop}}}{m} = \frac{0.02}{0.5} = 0.04 \text{ m}^3/\text{kg}$$

Just before hitting stop, $P = P_{\text{float}} = P_1 = 1600 \text{ kPa}$.

From table B.2.2

$T = 41.03$	$v = 0.08079$
$T = 50$	$v = 0.08506$
$T = ?$	$v = 0.04$

$$\text{so } \frac{50 - 41.03}{0.08506 - 0.08079} = \frac{50 - T}{0.08506 - 0.04} \rightarrow T = 44.65^\circ \text{C}$$

since $T > T_2$, then it will hit stop \rightarrow

now I used CD in back of book.

From program, Found $T = 41.02^\circ\text{C}$ @ $v = 0.04$.

Since $41.02 > 30^\circ\text{C} \Rightarrow$ it must hit the stop before cooling down all the way to 30°C .

now, From CD, I find rest of data:

$$u_1 = 1517 \text{ kJ/kg} \quad @ (T_1 = 120^\circ\text{C}, P_1 = 1.6 \text{ MPa}) \quad (\text{superheated})$$

$$u_2 = 678.6 \text{ kJ/kg} \quad @ (T_2 = 30^\circ\text{C}, v_2 = 0.04 \text{ m}^3/\text{kg})$$

$$s_1 = 5.502 \text{ kJ/kg}\cdot\text{K} \quad @ (T_1 = 120^\circ\text{C}, P_1 = 1.6 \text{ MPa})$$

$$v_1 = 0.1127 \text{ m}^3/\text{kg} \quad @ (T_1, P_1)$$

$$v_2 = 0.04 \text{ m}^3/\text{kg}$$

$$s_2 = 2.529 \text{ kJ/kg}\cdot\text{K} \quad @ (T_2 = 30^\circ\text{C}, v_2 = 0.04 \text{ m}^3/\text{kg})$$

$$\begin{aligned} {}_1W_2 &= \frac{P_m}{\text{flow}} (v_2 - v_1) = (1600) (0.5) (0.04 - 0.1127) \\ &= \boxed{-58.16 \text{ kJ}} \end{aligned}$$

$$\text{so } Q_2 - {}_1W_2 = m_a(u_2 - u_1) + m_{\text{steel}}(u_2 - u_1)$$

$$Q_2 = {}_1W_2 + m_a(u_2 - u_1) + m_{\text{steel}} c_p(T_2 - T_1)$$

$$\boxed{c_p = 0.46} \text{ kJ/kg}\cdot\text{K} \text{ from A.3.}$$

$$\begin{aligned} \text{so } Q_2 &= -58.16 + 0.5(678.6 - 1517) + 1(0.46)(30 - 120) \\ &= \boxed{-518.76 \text{ kJ}} \end{aligned}$$

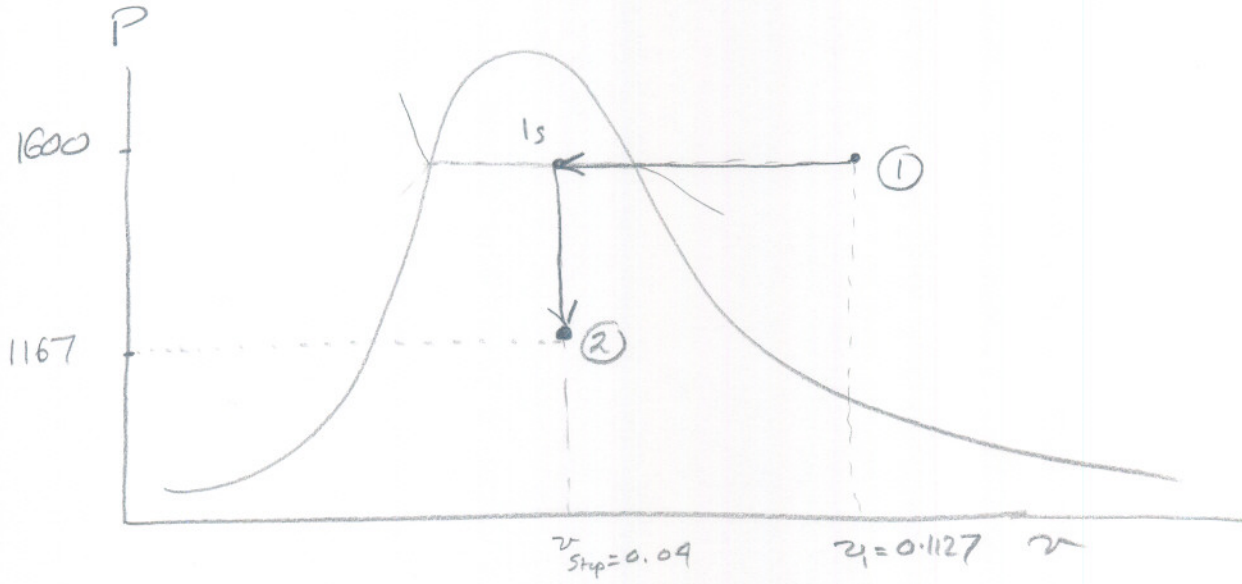
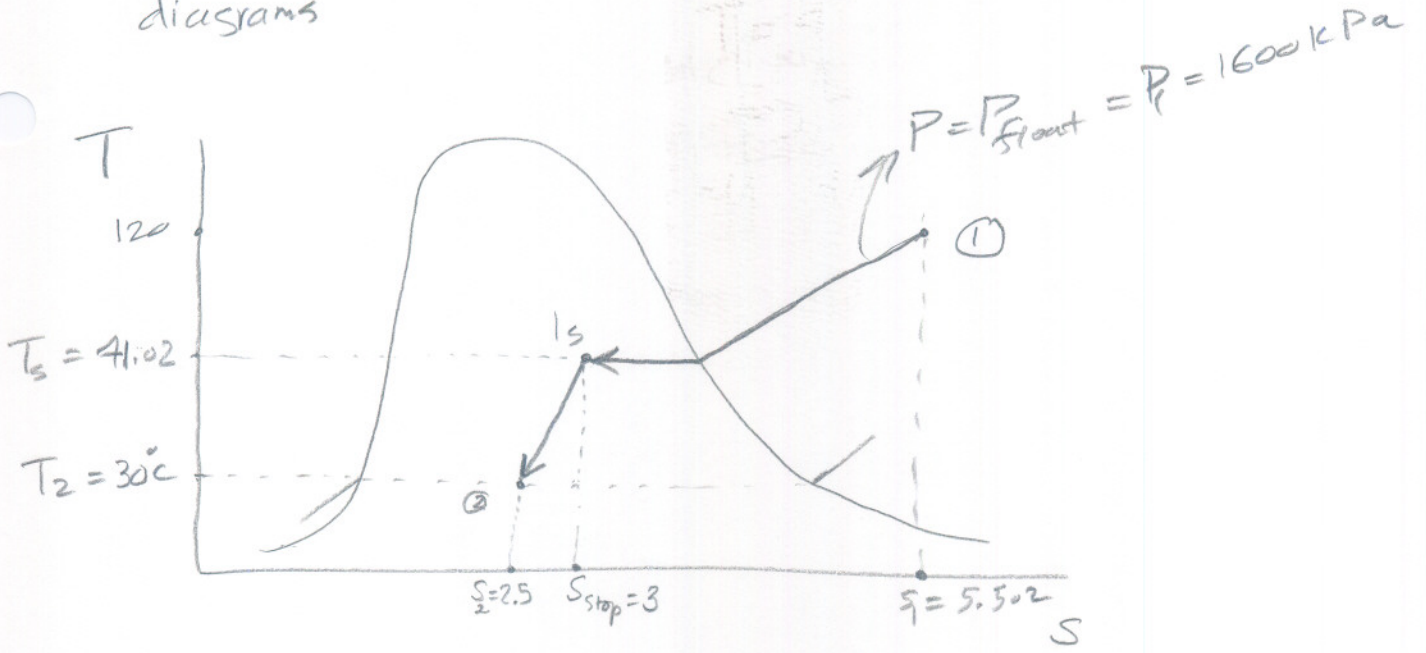
now from entropy equation

$$m_a(s_2 - s_1) + m_{\text{steel}}(s_2 - s_1) = \frac{Q_2}{T_{\text{amb}}} + S_{\text{gen}}$$

\downarrow
 $c_p \ln \frac{T_2}{T_1}$

$$\begin{aligned} \text{so } S_{\text{gen}} &= 0.5(2.529 - 5.502) + (1)(0.46) \ln \frac{30+273}{120+273} + \frac{518.76}{20+273} \\ &= -1.4865 + (-1.1963) + 1.7705 = \boxed{0.1673 \text{ kJ/K}} \rightarrow \end{aligned}$$

diagrams



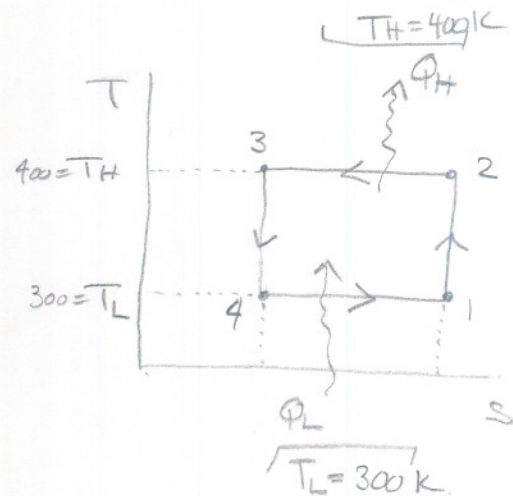
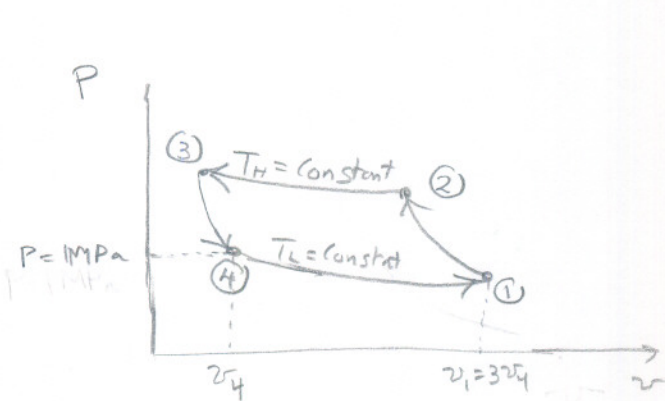
Problem 8.89

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Statement

Consider Carnot-cycle heat pump having 1 kg of nitrogen gas in cylinder/piston arrangement. This heat pump operates between reservoirs @ 300K and 400K. At the beginning of the low temp. heat addition, the pressure is 1 MPa. During this process the volume triples. Analyze each of the four processes in the cycle and determine

- the pressure, volume and temp at each point.
- the work and heat transfer for each process.



given: at point 4, $P = 1 \text{ MPa}$
at point 1, $V_1 = 3V_4$

Assumptions

ideal gas.

Laws.

$PV = \text{constant}$, during a reversible isothermal process.

$PV^{\gamma} = \text{constant}$, where $\gamma = \frac{C_p}{C_v}$, during an adiabatic reversible process when γ is constant.
(isentropic process).

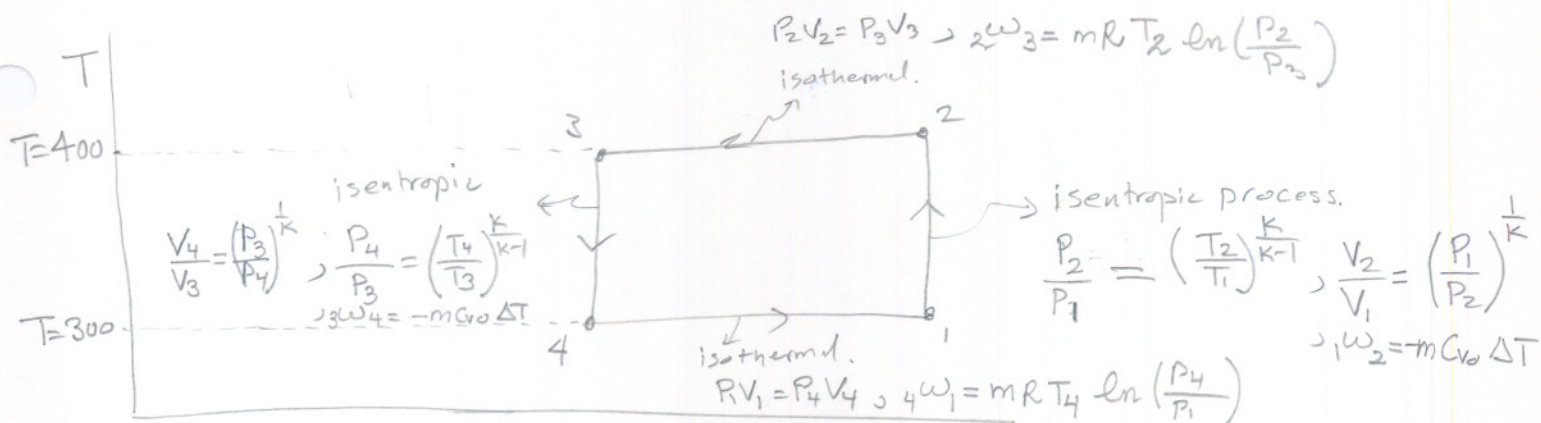
For isentropic

$$\begin{cases} \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \\ \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \end{cases}$$

$PV = mRT$, ideal gas law.

steps

looking at T-S diagram, each process is one of the above 2 types. it is either an isothermal process, where we can use $PV = \text{constant}$, or it is an isentropic process where we can use $PV^{\gamma} = \text{constant}$.



S

using these equations we can find P, V, T at each point.

we are given $P_4 = 1 \text{ MPa}$, and given that $V_1 = 3V_4$.

so from $P_1 V_1 = P_4 V_4 \Rightarrow P_1 = P_4 \frac{V_4}{V_1}$

$$P_1 = P_4 \frac{1}{3} = \boxed{\frac{P_4}{3}}$$

$$P_4 V_4 = mRT_4 \Rightarrow \boxed{V_4 = \frac{mRT_4}{P_4}} \Rightarrow \boxed{V_1 = 3V_4}$$

so now we know (P_4, V_4, T_4) , (P_1, V_1, T_1) . now we find (P, V, T) for points 2 and 3 \longrightarrow

a) we go from ① → ②, but now we use isentropic process.

$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{k}{k-1}}$$

k is found from table A.5 for N_2 .

$T_1 = T_L$, $T_2 = T_H$. P_1 was found already.

now to find V_2 .

From:
$$V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{k}}$$

now we know (P_2, V_2, T_2) .

now for point ③.

we can find this from 2 → 3 process using isothermal,
or from 3 → 4 process using isentropic.

using isentropic:

$$\frac{P_4}{P_3} = \left(\frac{T_4}{T_3} \right)^{\frac{k}{k-1}} \Rightarrow P_3 = P_4 \left(\frac{T_4}{T_3} \right)^{\frac{k-1}{k}}$$

Finally to find V_3 , $P_3 V_3 = P_2 V_2 \Rightarrow V_3 = \frac{P_2 V_2}{P_3}$

now we know (P_3, V_3, T_3) .

b) now to find work for each process:

$$1W_2 = -m C_{v0} (T_2 - T_1), \quad 1Q_2 = 0 \text{ (adiabatic)}$$

$$2W_3 = m R T_2 \ln \left(\frac{P_2}{P_3} \right), \quad 2Q_3 = 2W_3 \text{ (since } du=0)$$

$$3W_4 = -m C_{v0} (T_4 - T_3), \quad 3Q_4 = 0 \text{ (adiabatic)}$$

$$4W_1 = m R T_4 \ln \left(\frac{P_4}{P_1} \right), \quad 4Q_1 = 4W_1 \text{ (since } du=0)$$

numerical:



$$a) P_1 = \frac{P_4}{3} = \boxed{\frac{1}{3} \text{ MPa}}$$

$$V_4 = \frac{mRT_4}{P_4} = \frac{(1)(0.2968)(300)}{1 \times 10^3} = \boxed{0.08904 \text{ m}^3}$$

$$\text{so } V_1 = 3V_4 = \boxed{0.26712 \text{ m}^3}$$

so for point ①: $(P, V, T) = (\frac{1}{3} \text{ MPa}, 0.26712, 300)$

For point ④: $(P, V, T) = (1 \text{ MPa}, 0.08904, 300)$

$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{k}{k-1}}$$

$k = 1.4$ From table for N_2 .

$$P_2 = \left(\frac{1}{3} \times 10^3 \right) \left(\frac{400}{300} \right)^{\frac{1.4}{.4}} = \boxed{912.3559 \text{ kPa}}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{k}} = (0.26712) \left(\frac{\frac{1}{3} \times 10^3}{912.3559} \right)^{\frac{1}{1.4}} = \boxed{0.13012 \text{ m}^3}$$

so for point ②: $(P, V, T) = (912.35, 0.1302, 400)$

$$\frac{P_4}{P_3} = \left(\frac{T_4}{T_3} \right)^{\frac{k}{k-1}} \Rightarrow P_3 = (10^3) \left(\frac{300}{400} \right)^{\frac{1.4}{.4}} = 2.73706 \text{ MPa}$$

$$V_3 = \frac{P_2}{P_3} V_2 \Rightarrow V_3 = \frac{912.35 \times 10^{-3} \text{ (MPa)}}{2.73706 \text{ (MPa)}} \cdot (0.13012) = 0.0433 \text{ m}^3$$

so for point ③: $(P, V, T) = (2.737 \text{ MPa}, 0.0433, 400)$.

b).

$$1W_2 = -m c_{v0} (T_2 - T_1) = -0.7448 (400 - 300) = \boxed{-74.48 \text{ kJ}}$$

$$2W_3 = mRT_2 \ln\left(\frac{P_2}{P_3}\right) = (0.2968) 400 \ln\left(\frac{912.35 \times 10^{-3}}{2.73706}\right) = \boxed{-130.427 \text{ kJ}}$$

$$3W_4 = -m c_{v0} (T_4 - T_3) = -0.7448 (300 - 400) = \boxed{74.48 \text{ kJ}}$$

$$4W_1 = mRT_4 \ln\left(\frac{P_4}{P_1}\right) = 0.2968 (300) \ln\left(\frac{1}{\frac{1}{3}}\right) = \boxed{97.8204 \text{ kJ}}$$



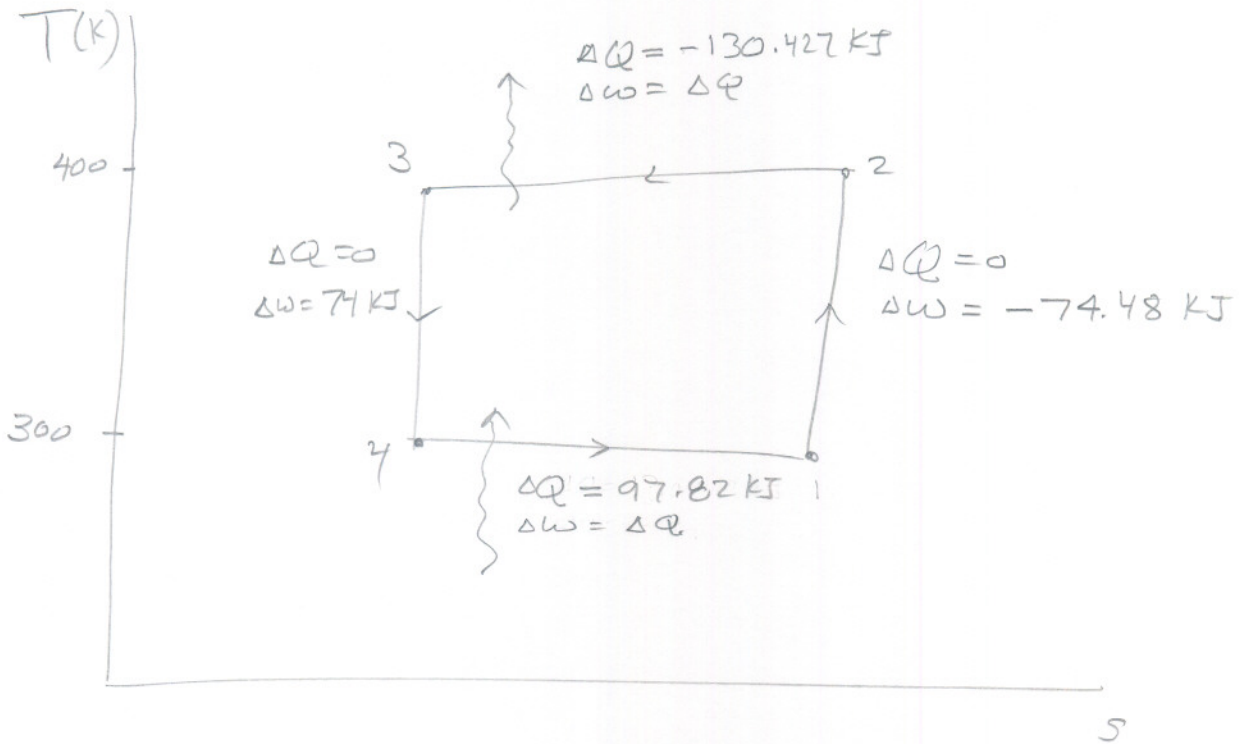
To find energy transfer.

$$1Q_2 = 0$$

$$2Q_3 = 2W_3 = -130.427 \text{ KJ}$$

$$3Q_4 = 0$$

$$4Q_1 = 97.82 \text{ KJ}$$



so Net Work = -65.2 KJ.

i.e. work into system.

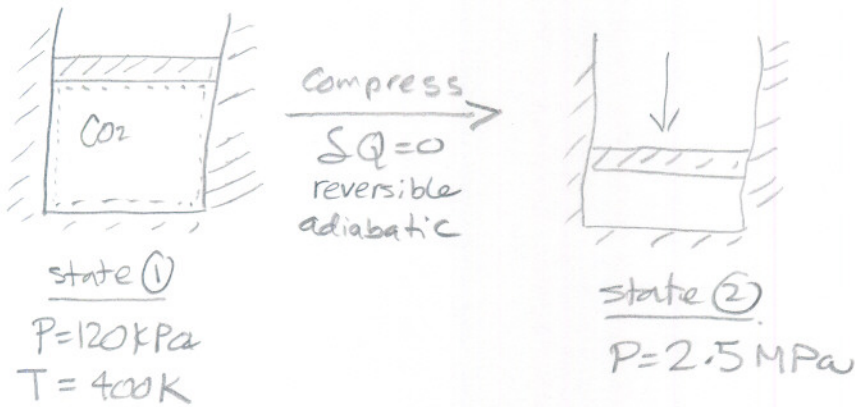
Problem 8.94

10/10

Statement.

Insulated cylinder/piston contains Carbon dioxide gas @ 120 kPa and 400 K. The gas is compressed to 2.5 MPa in a reversible adiabatic process. Find final Temp. and work per unit mass assuming

- variable specific heat (Table A.8)
- constant specific heat (Table A.5)
- constant specific heat (Table A.6)



Find T_2 .

Assumptions

ideal gas

Laws

$$s_2 - s_1 = (s_{T_2}^o - s_{T_1}^o) - R \ln \frac{P_2}{P_1}$$

$$P_2^{-1} u_2 = (u_2 - u_1) P^{-1} R T$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \quad \text{for ideal gas in isentropic process}$$

$$u_2 = -C_{v0} (T_2 - T_1)$$

Steps

a) since $\int m ds = \frac{\delta Q}{T} + S_{gen}$

but $S_{gen} = 0$ since reversible.

then $m ds = \frac{\delta Q}{T}$

but $\delta Q = 0$ since adiabatic.

This means $ds = 0$, i.e. $s_2 - s_1 = 0$.

to use table A.8, need to find standard entropy s_T° .

From eq. 8.28

$$s_2 - s_1 = (s_{T_2}^\circ - s_{T_1}^\circ) - R \ln \frac{P_2}{P_1}$$

we know $P_2, P_1, s_{T_1}^\circ$ From table A.8 @ $T = 400\text{K}$.

so $s_{T_2}^\circ = s_{T_1}^\circ + R \ln \frac{P_2}{P_1}$ → given
→ given

now using table A.8 we find T_2 for this $s_{T_2}^\circ$.

to find work:

From energy equation $\cancel{q_2} - w_2 = m(u_2 - u_1)$

so $w_2 = -(u_2 - u_1)$

→ table A.8 @ $T = T_1$
→ table A.8 @ $T = T_2$.



b) using table A.5. Constant specific heat.

since reversible adiabatic (i.e. isentropic), then

I can use $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$ to find T_2 .

k is found for CO_2 from table A.5

now to find work:

$$\begin{aligned} T_2 - {}_1w_2 &= u_2 - u_1 \\ -{}_1w_2 &= C_{v0}(T_2 - T_1) \end{aligned}$$

so ${}_1w_2 = -C_{v0}(T_2 - T_1)$
↳ From table A.5

c) using table A.6

using $S_{T_2}^{\circ}$ found in (a), $T_2 =$ From A.8.

now to find C_{v0} , use eq 8.30

$R = C_{p0} - C_{v0}$

i.e. $C_{v0} = C_{p0} - R$ ↳ For CO_2 from table A.5

↳ From table A.6 as follows

$$C_{p0} = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3$$

where $\theta = \frac{(T_2 + T_1)}{2} \left(\text{average } T \right)$
 $\frac{1000}{\text{K}}$

now that C_{v0} is found

use ${}_1w_2 = C_{v0}(T_2 - T_1)$ to find work.



Numerical

a) From table A.8 @ $T=400\text{K}$, $s_{T_1}^{\circ} = 5.1196 \text{ kJ/Ks-K}$

$$s_{T_2}^{\circ} = s_{T_1}^{\circ} + R \ln \frac{P_2}{P_1}$$

↳ from table A.5 $R = 0.1889 \text{ kJ/Ks-K}$.

$$s_{T_2}^{\circ} = 5.1196 + 0.1889 \ln \left(\frac{2.5}{120 \times 10^{-3}} \right) = 5.693 \text{ kJ/Ks-K}$$

from table A.8, $T_2 \approx 700\text{K}$

$$s_{T_2}^{\circ} = - (u_2 - u_1)$$

$$u_1 = 228.19 \text{ kJ/Ks} @ T=400\text{K} \text{ table A.8}$$

$$u_2 = 483.97 \text{ kJ/Ks} @ T=700\text{K} \text{ table A.8}$$

$$s_{T_2}^{\circ} = - (483.97 - 228.19) = -255.78 \text{ kJ}$$

$$b) T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$$

$$k = 1.289 \text{ From A.5}$$

$$P_1 = 1200 \text{ kPa}$$

$$P_2 = 2.5 \text{ MPa}$$

$$T_1 = 400 \text{ K}$$

$$s_{T_2}^{\circ} = 400 \left(\frac{2.5}{120 \times 10^{-3}} \right)^{\frac{1.289}{1.289}} = 790.19 \text{ K}$$

$$s_{T_2}^{\circ} = - C_{v_0} (T_2 - T_1)$$

$$\rightarrow = 0.653 \text{ kJ/Ks-K from A.5}$$

$$= -0.653 (790.19 - 400) = -254.794 \text{ kJ}$$



c) $T_2 \approx 700 \text{ K}$ using $S_{T_2}^0 = 0.1889 \text{ kJ/Ks-K}$ found in part (a).

$$C_{v0} = C_{p0} - R \\ = (C_0 + C_1 \theta + C_2 \theta^2 + C_3 \theta^3) - R$$

$$1000 \theta = \left(\frac{700 + 400}{2} \right) / 1000 = \boxed{0.55}$$

from table A.6, for CO_2

$$C_0 = .45$$

$$C_1 = 1.67$$

$$C_2 = -1.27$$

$$C_3 = .39$$

$$\text{so } C_{v0} = (.45 + 1.67(0.55) - 1.27(0.55)^2 + .39(0.55)^3) - 0.1889$$

↳ R for CO_2
From A.5

$$\boxed{C_{v0} = 0.86031 \text{ kJ/Ks-K}}$$

$$\text{so } q_2 = -C_{v0} (T_2 - T_1)$$

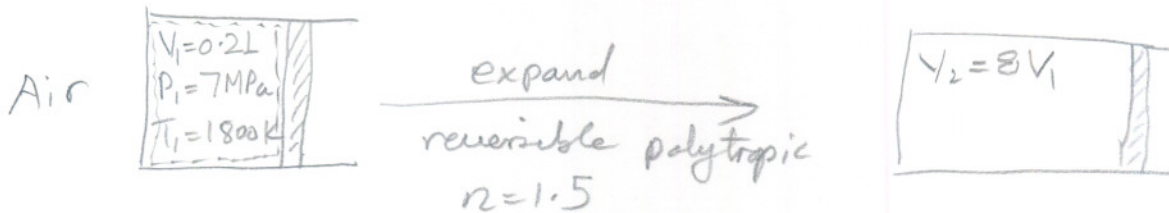
$$= -0.86031 (200 - 400) = \boxed{-258.093 \text{ kJ}}$$

Problem 8.111

9/10

statement

power stroke in an internal combustion engine can be approximated with a polytropic expansion. Consider air in a cylinder volume of 0.2L @ 7MPa and 1800K. It is now expanded in a reversible polytropic process with exponent $n=1.5$ through a volume ratio of 8:1. Show process on P-v and T-S diagrams and calculate work and heat transfer.



Find ${}_1W_2$ and ${}_1Q_2$.

Assumptions

- ideal gas.
- constant mass

Laws

$$PV^n = \text{constant}, \quad n = 1.5$$

$${}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$$

$$PV = mRT, \quad T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{n-1}$$

$${}_1W_2 = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$

steps

$${}_1W_2 = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$

can find m from $PV = mRT$

$$\text{so } m = \frac{P_1V_1}{RT_1}$$

need to find T_2 .

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = T_1 \left(\frac{V_1}{8V_1} \right)^{\gamma-1} = T_1 \left(\frac{1}{8} \right)^{\gamma-1}$$

$$\text{so } {}_1W_2 = \frac{P_1V_1}{RT_1} \frac{R \left(T_1 \left(\frac{1}{8} \right)^{\gamma-1} - T_1 \right)}{(1-n)}$$

$${}_1W_2 = \frac{P_1V_1 \left(\left(\frac{1}{8} \right)^{\gamma-1} - 1 \right)}{1-n} \quad \text{--- (1)}$$

but ${}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$

can find u_1, u_2 for air for T_1, T_2 using table A.7.1.

$$\text{so } {}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

but for ideal gas Table A.7.1 $u_2 - u_1 = C_v(T_2 - T_1)$

$$\text{so } {}_1Q_2 = m C_v (T_2 - T_1) + {}_1W_2 \quad \text{--- (2) } \rightarrow$$

Numerical

eq ①

$${}_1W_2 = \frac{P_1 V_1 \left(\left(\frac{1}{8} \right)^{n-1} - 1 \right)}{1-n}$$
$$= \frac{(7 \times 10^3) (0.2 \times 10^{-3}) \left(\left(\frac{1}{8} \right)^{1.5-1} - 1 \right)}{1-1.5}$$

$$\boxed{{}_1W_2 = 1.81 \text{ kJ}}$$

$$T_2 = T_1 \left(\frac{1}{8} \right)^{n-1} = 1800 \left(\frac{1}{8} \right)^{1.5-1} = 636.396 \text{ K}$$

from A.5

$C_{v_2} = 0.717$ for air.

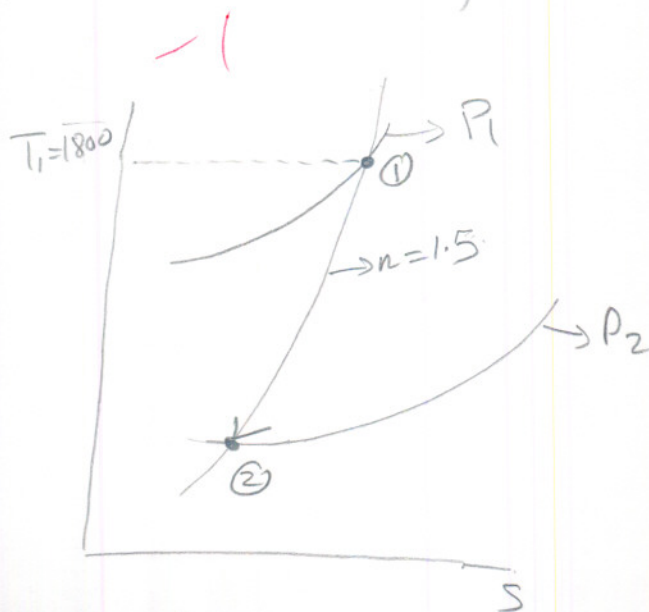
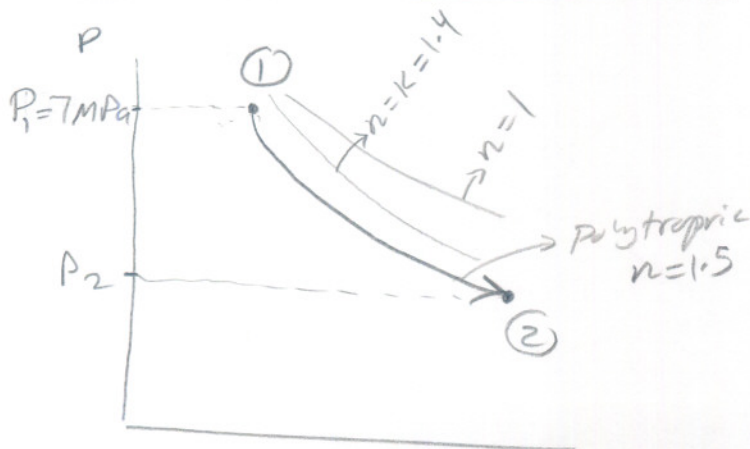
$$m = \frac{P_1 V_1}{R T_1} = \frac{(7 \times 10^3) (0.2 \times 10^{-3})}{(0.287) (1800)} = \boxed{2.71 \times 10^{-3} \text{ kg}}$$

eq ②

$${}_1Q_2 = m C_{v_2} (T_2 - T_1) + {}_1W_2$$

$$= (2.71 \times 10^{-3}) (0.717) (636.396 - 1800) + 1.81$$

$$\boxed{{}_1Q_2 = -0.459 \text{ kJ}}$$



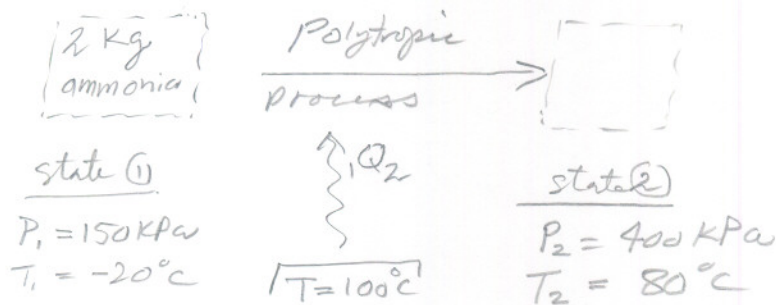
(using page 280, fig 8.18 as Example)

Problem 8.114

10/10

Statement

A device brings 2 kg of ammonia from 150 kPa and -20°C to 400 kPa and 80°C in a polytropic process. Find the polytropic exponent n , the work and heat transfer. Find total entropy generated assuming source at 100°C .



Find ${}_1W_2$, ${}_1Q_2$ and S_{gen} . Find n for $PV^n = \text{constant}$.

Assumptions

ideal gas for ammonia.

Laws

1st Law for control mass

$${}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$$

Entropy equation

$$m(s_2 - s_1) = \frac{{}_1Q_2}{T_{\text{source}}} + S_{\text{gen}}$$

$PV^n = \text{constant}$ for polytropic process.

$${}_1W_2 = \frac{mR(T_2 - T_1)}{1 - n} \text{ for ideal gas, } n \neq 1.$$

Steps

$PV^n = \text{constant}$. need to find n .

$$P_1 V_1^n = P_2 V_2^n$$

so $\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n \Rightarrow \log\left(\frac{P_1}{P_2}\right) = n \log\left(\frac{V_2}{V_1}\right)$

so $n = \frac{\log\left(\frac{P_1}{P_2}\right)}{\log\left(\frac{V_2}{V_1}\right)}$

since a control mass problem, then $V = 25 \text{ m}^3$
 v_1 and v_2 can be found at state ① and ② since
 (P_1, T_1) and (P_2, T_2) are known.

so $n = \frac{\log\left(\frac{P_1}{P_2}\right)}{\log\left(\frac{v_2}{v_1}\right)} = \frac{\log\left(\frac{P_1}{P_2}\right)}{\log\left(\frac{v_2}{v_1}\right)}$

Now, from Energy equation:

$${}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$$

but ${}_1W_2 = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$

Found from above.

so ${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$

Table ammonia using (P, T)

$$m(s_2 - s_1) = \frac{{}_1Q_2}{T_{\text{source}}} + S_{\text{gen}} \Rightarrow$$

$$S_{\text{gen}} = m(s_2 - s_1) - \frac{{}_1Q_2}{T}$$

ammonia tables using
 (P, T) at each state.



Numerical

state 1

$$P = 150 \text{ kPa}, \quad T = -20^\circ\text{C}$$

table B.2.2. (superheated)

$$u_1 = 1303.3 \text{ kJ/kg}, \quad s_1 = 5.7465 \text{ kJ/kg}\cdot\text{K}$$

$$v_1 = 0.7977 \text{ m}^3/\text{kg}$$

state 2

$$P = 400 \text{ kPa}, \quad T = 80^\circ\text{C}$$

table B.2.2

$$u_2 = 1468.1 \text{ kJ/kg}, \quad s_2 = 5.9907 \text{ kJ/kg}\cdot\text{K}, \quad v_2 = 0.4216 \text{ m}^3/\text{kg}$$

$$n = \frac{\log\left(\frac{P_1}{P_2}\right)}{\log\left(\frac{v_2}{v_1}\right)} = \frac{\log\left(\frac{150}{400}\right)}{\log\left(\frac{0.4216}{0.7977}\right)} = \boxed{1.538} \quad \left(\begin{array}{l} K=1.297 \\ \text{so } n > K \end{array}\right)$$

$$\begin{aligned} {}_1W_2 &= \frac{mR(T_2 - T_1)}{1-n} = \frac{(2)(0.4882)(80 - (-20))}{1 - (1.538)} \\ &= \boxed{-181.486 \text{ kJ}} \end{aligned}$$

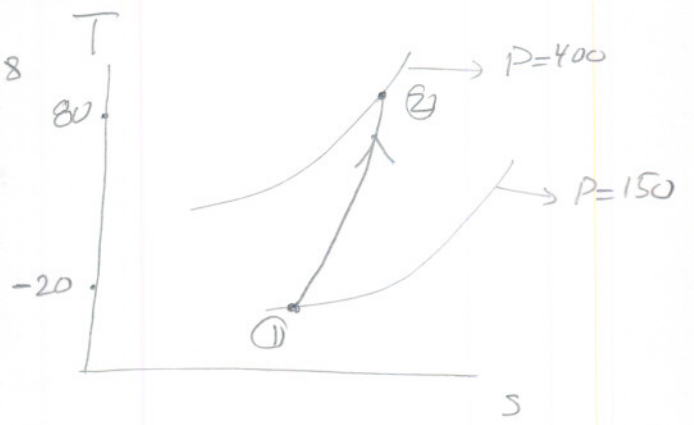
another way to find ${}_1W_2 = \frac{P_2 v_2 - P_1 v_1}{1-n} = \frac{m(P_2 v_2 - P_1 v_1)}{1-n}$

$$= \frac{(2)(400 \times 0.4216 - 150 \times 0.7977)}{1 - 1.538} = \boxed{-182.1 \text{ kJ}}$$

The second method is probably more accurate since using 'R' in the first method is for ideal gas @ 25°C, 100 kPa.

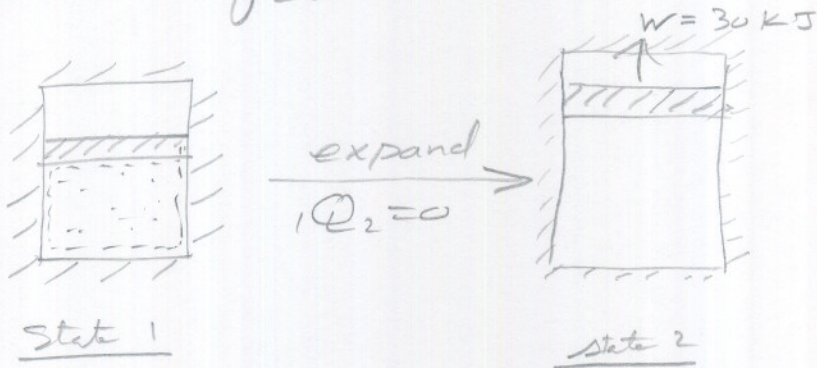
$$\begin{aligned} {}_1Q_2 &= m(u_2 - u_1) + {}_1W_2 = 2(1468 - 1303.3) - 182.1 \\ &= \boxed{147.3 \text{ kJ}} \end{aligned}$$

$$\begin{aligned} \text{So } S_{\text{gen}} &= m(s_2 - s_1) - \frac{{}_1Q_2}{T} \\ &= (2)(5.9907 - 5.7465) - \frac{147.3}{100 + 273} \\ &= \boxed{0.093494 \text{ kJ/K}} \end{aligned}$$



Statement

insulated cylinder/piston arrangement has an initial volume of 0.15 m^3 and contains steam @ 400 kPa and 200°C . The steam is expanded adiabatically and the work output is measured very carefully to be 30 kJ . It is claimed that final state of the water is in 2-phase region. What is your evaluation of this claim.



State 1
 $V_1 = 0.15 \text{ m}^3$
 $P_1 = 400 \text{ kPa}$
 $T_1 = 200^\circ\text{C}$

State 2

Find what phase substance is in.

Assumptions

- pure substance
- frictionless piston (reversible)

Laws

$$v = \frac{V}{m}$$

$${}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$$

$$S_2 = S_1 \text{ for reversible adiabatic process.}$$

$$\downarrow$$

$$S_{gen} = 0$$

$$\downarrow$$

$$S_Q = 0$$

Steps

$$Q_2 - W_2 = m(u_2 - u_1)$$

$$\text{so } -W_2 = m u_2 - m u_1$$

$$\text{but } v = \frac{V}{m}, \text{ so } \boxed{m = \frac{V_1}{v_1}}$$

$$\text{so } u_2 = \frac{-W_2 + m u_1}{m} = \frac{-W_2 + \frac{V_1}{v_1} u_1}{\frac{V_1}{v_1}}$$

$$\boxed{u_2 = \frac{-v_1 W_2 + V_1 u_1}{V_1}}$$

↖ Table @ (T₁, P₁) ↗ table @ (T₁, P₁)

so u_2 can be found.

but need another property to determine phase.
have to assume reversible process (i.e. frictionless piston). now I can write

$$\boxed{S_2 = S_1}$$

since a reversible adiabatic process.
↖ Table @ (T₁, P₁)

so now I know (u_2, S_2), can determine phase.

Numerical

from B.1.3 (superheated)

$$v_1 = 0.53422 \text{ m}^3/\text{kg}, \quad u_1 = 2646.83 \text{ kJ/kg}, \quad S_1 = 7.1706 \text{ kJ/kg}\cdot\text{K}$$

$$u_2 = \frac{-v_1 W_2 + V_1 u_1}{V_1} = \frac{(-0.53422)(30) + (0.15)(2646.83)}{0.15}$$
$$= \boxed{2539.986 \text{ kJ/kg}}$$

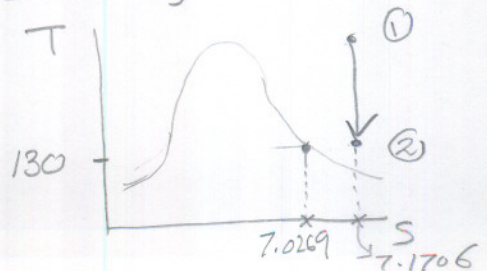
so @ state 2 we have ($u_2 = 2539.986, S_2 = 7.1706$).

From B.1.1, @ $u_2 = 2539.9, S_2 = 7.0269$

@ $T = 130^\circ\text{C}$

↘ ug

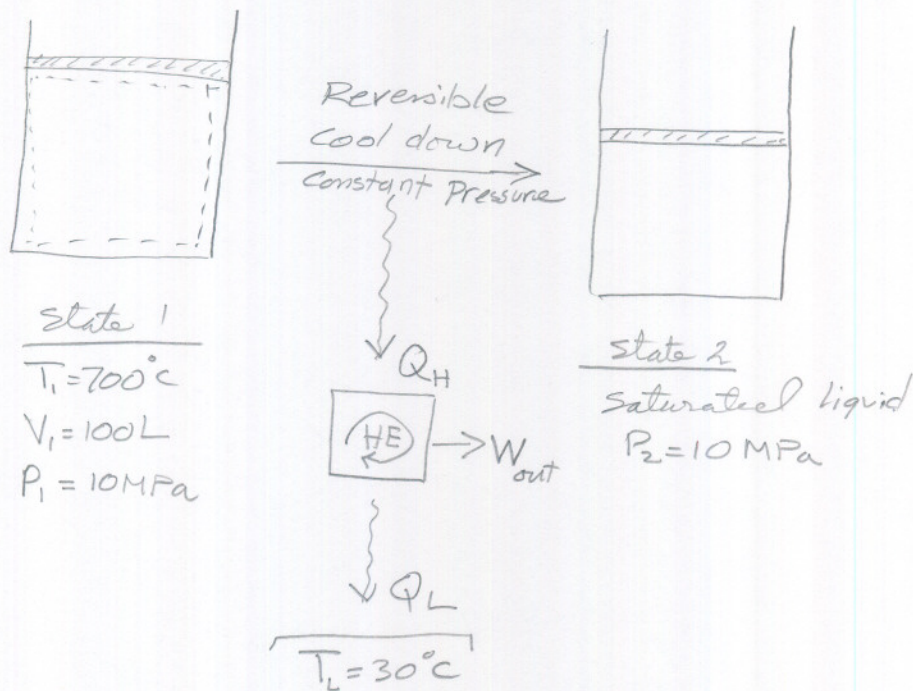
so since $S_2 > S @ u = 2539.9$, then in superheated
i.e. claim is WRONG.



10/10

Problem 8.131

A cylinder fitted with frictionless piston contains water. A constant hydraulic pressure on the back face of the piston maintains a cylinder pressure of 10 MPa. Initially the water is @ 700°C and the volume is 100L. The water is now cooled and condensed to saturated liquid. The heat released during this process is the Q supply to a cyclic heat engine that in turn rejects heat to the ambient at 30°C. If the overall process is reversible, what is the net work output of the heat engine?



Find W_{out} from heat engine

Assumptions

Control mass for cylinder/piston

Ignoring mass in H.E. for water.

Laws

$$v = \frac{V}{m}$$

$${}_1Q_2 - {}_1W_2 = m(u_2 - u_1) \quad 1^{st} \text{ Law for control mass}$$

$$m(s_2 - s_1) = \frac{\delta Q}{T} - S_{gen} \quad 2^{nd} \text{ Law.}$$

$$W_{out} = Q_H - Q_L \quad \text{for heat engine.}$$

$$S_{gen} = 0 \quad \text{for reversible process}$$

Steps

$$\text{For a heat engine, } W_{out} = Q_H - Q_L$$

Q_H can be found from 1st Law applied to cylinder/piston state change.

$$\text{Now } {}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$$

$${}_1Q_2 = \frac{W_{out}}{m} + m(u_2 - u_1)$$

$$\text{but constant } P, \text{ hence } {}_1W_2 = \int_1^2 P dV = P(V_2 - V_1)$$

V_1 is given.

we can find V_2 since $v = \frac{V}{m}$, $V = vm$, and mass is constant.

$$\boxed{m = \frac{V_1}{v_1}} \rightarrow \text{given}$$
$$\rightarrow \text{table @ } (T_1, P_1)$$

So $\boxed{V_2 = m v_2}$, v_2 can be found since in state 2 we know P and $x=0$ since saturated liquid.

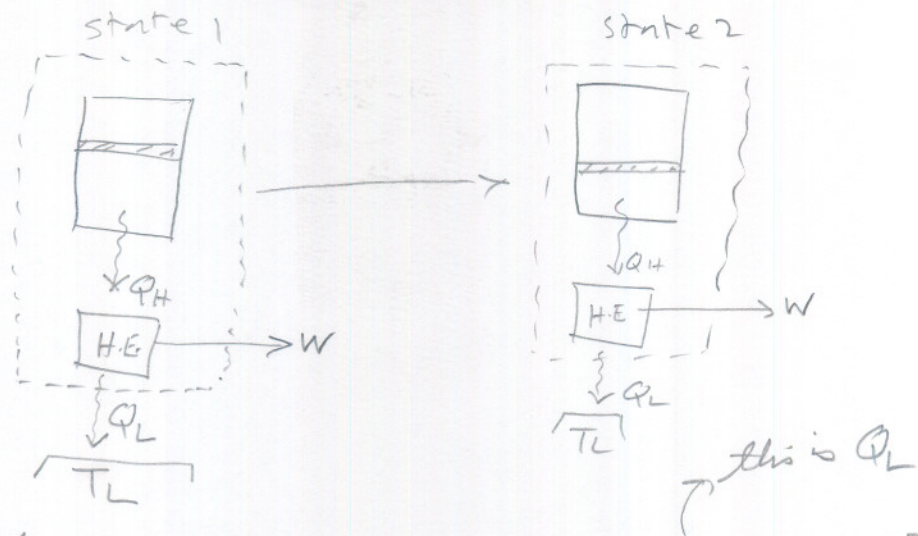
\rightarrow table @ (P_2, x_2) i.e. use u_f @ P_2

now u_1, u_2 can also be found from tables.

Table @ (P_1, T_1)

hence ${}_1Q_2$ is now found. this is the same as Q_H for the heat engine.

Now need to find Q_L for H.E. to do this, consider the cylinder/piston and H.E. as new control volume \rightarrow



apply 2nd Law: $m(s_2 - s_1) = \frac{\delta Q}{T_{amb.}} + S_{gen} = 0$; reversible.

so $Q_L = T_L m(s_2 - s_1)$
this is T_L

from table @ (P_1, T_1)
 from table @ $(P_2 = P_1, x=0)$ use S_f @ P_2

now that Q_L, Q_H are found, W_{out} is found from

$$W_{out} = Q_H - Q_L$$

Numerical

State 1

$T_1 = 700^\circ\text{C}$, $P_1 = 10 \text{ MPa}$, Table B.1.3 (superheated)

$v_1 = 0.04358$, $u_1 = 3434.72 \text{ kJ/kg}$, $h_1 = 3870.52 \text{ kJ/kg}$, $s_1 = 7.1687 \text{ kJ/kg}\cdot\text{K}$

$$m = \frac{V_1}{v_1} = \frac{100 \times 10^{-3}}{0.04358} = \boxed{2.2946 \text{ kg}}$$

Table B.1.2, $P = 10 \text{ MPa} \Rightarrow v_{f_2} = 0.001452 \text{ m}^3/\text{kg}$, $u_{f_2} = 1393.00$, $s_{f_2} = 3.3595 \text{ kJ/kg}\cdot\text{K}$

$$\text{so } V_2 = m v_{f_2} = (2.2946)(0.001452) = 3.3317 \times 10^{-3} \text{ m}^3$$

$$\text{so } W_2 = P(V_2 - V_1) = 10 \times 10^3 (3.3317 \times 10^{-3} - 100 \times 10^{-3}) = \boxed{-966.68 \text{ kJ}}$$

$$\text{so } Q_2 = m(u_2 - u_1) + W_2 = 2.2946(1393 - 3434.72) - 966.68 = \boxed{-5651.6 \text{ kJ}}$$

now Q_2 is Q_H for Heat engine \rightarrow

$$Q_L = T_L m (s_2 - s_1)$$

$$= (30 + 273) (2.2946) (3.3595 - 7.1687) = \boxed{-2648.39 \text{ KJ}}$$

So $W_{out} = Q_H - Q_L$

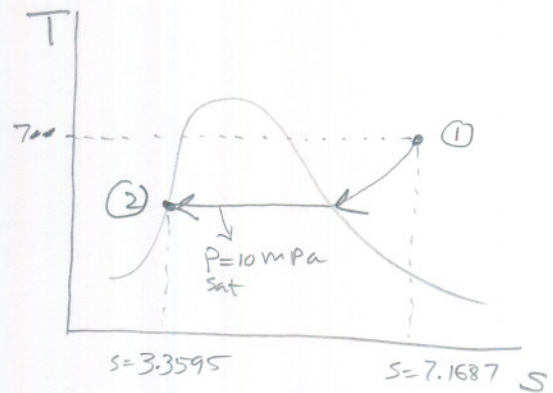
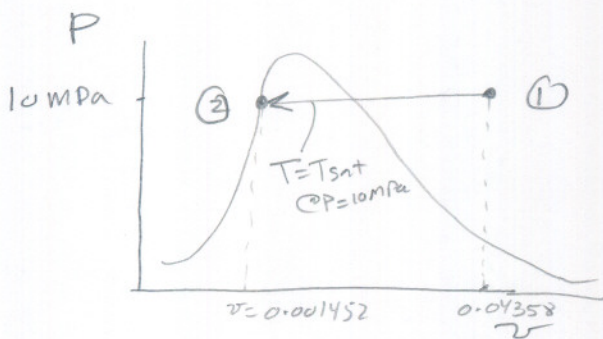
Now, for H.E, Q_H is an input heat, hence must be +ve.

for H.E, Q_L is an output heat, hence must be -ve.

So $W_{out} = 5651.6 - 2648.39$

$$= \boxed{3003.21 \text{ KJ}}$$

diagrams



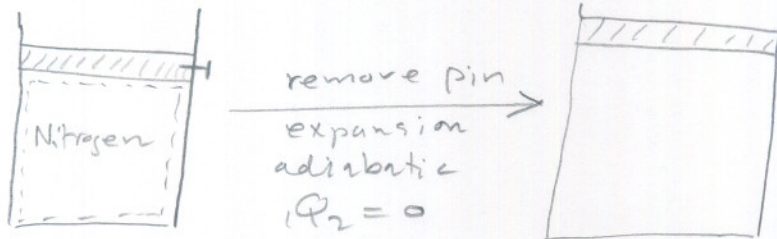
Problem 8.103 (Bonus)

5/5

Statement

Nitrogen at 200°C and 300 kPa in piston/cylinder of volume 5L. piston locked in with pin.

Force on piston require pressure inside of 200 kPa to balance it without pin. pin is removed and piston quickly comes to its equilibrium position without any heat transfer. Find final P_2 , T_2 and V_2 and S_{gen} due to this unrestrained expansion.



state 1
 $T_1 = 200^\circ\text{C}$
 $P_1 = 300 \text{ kPa}$
 $V_1 = 5 \text{ L}$

state 2
 $P_2 = ?$
 $T_2 = ?$
 $V_2 = ?$

$P_{float} = 200 \text{ kPa}$

Find P_2 , T_2 , V_2 , S_{gen} .

Assumptions

- irreversible process since sudden expansion.
- control mass.
- as soon as pin is removed, pressure is P_{float} .
- ideal gas.

Laws

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (\text{eq. 8.25 book})$$

$$PV = mRT$$

$$du = C_v dT \Rightarrow u_2 - u_1 = C_v (T_2 - T_1)$$

$$Q_2 - W_2 = m(u_2 - u_1)$$

$$W_2 = P_2 V_2 - P_1 V_1$$

apply 1st Law for control mass.
= 0 ∵ adiabatic

$$P_2 - W_2 = m(u_2 - u_1)$$

work done is $P_2 V_2 - P_1 V_1$.

but as soon as pin is removed we assume $P = P_{\text{float}}$.

$$\text{so } W_2 = P_{\text{float}} V_2 - P_{\text{float}} V_1$$

$$\text{so } m(u_2 - u_1) = -W_2 = -P_{\text{float}} (V_2 - V_1)$$

$$m(u_2 - u_1) = P_{\text{float}} V_1 - P_{\text{float}} V_2$$

let $P_2 = P_{\text{float}}$.

$$\text{so } m(u_2 - u_1) = P_2 V_1 - P_2 V_2$$

$$m(u_2 - u_1) = P_2 (V_1 - V_2)$$

but $m = \frac{P_1 V_1}{R T_1}$ so above becomes $\frac{P_1 V_1}{R T_1} (u_2 - u_1) = P_2 (V_1 - V_2)$

but $du = C_v dT$ for ideal gas.

$$\text{so } \frac{P_1 V_1}{R T_1} C_v (T_2 - T_1) = P_2 (V_1 - V_2) \quad \text{--- (1)}$$

$$\text{but } V_2 = m R \frac{T_2}{P_2} = \frac{P_1 V_1}{R T_1} R \frac{T_2}{P_2} = \frac{P_1 V_1 T_2}{T_1 P_2}$$

$$\text{so (1) becomes } \frac{P_1 V_1}{R T_1} C_v (T_2 - T_1) = P_2 \left(V_1 - \frac{P_1 V_1 T_2}{T_1 P_2} \right)$$

$$\text{ie } \frac{P_1 V_1}{R T_1} C_v (T_2 - T_1) = P_2 V_1 - \frac{P_1 V_1 T_2}{T_1}$$

$$P_1 V_1 C_v (T_2 - T_1) = R P_2 V_1 T_1 - P_1 V_1 T_2 R$$

$$P_1 V_1 C_v T_2 - P_1 V_1 C_v T_1 = R P_2 V_1 T_1 - P_1 V_1 T_2 R$$

$$T_2 (P_1 V_1 C_v + P_1 V_1 R) = T_1 (R P_2 V_1 + P_1 V_1 C_v)$$

$$T_2 = T_1 \frac{(R P_2 V_1 + P_1 V_1 C_v)}{P_1 V_1 C_v + P_1 V_1 R} = T_1 \frac{(R P_2 + P_1 C_v)}{P_1 (C_v + R)} \quad (1)$$

now T_2 is found.

need to find V_2 .

$$\text{From } P_2 V_2 = m R T_2$$

$$V_2 = \frac{m R T_2}{P_2} = \frac{P_1 V_1}{R T_1} \frac{R T_2}{P_2} = \frac{P_1 V_1 T_2}{T_1 P_2} \quad (2)$$

Numerical

$C_v = 0.745$ KJ/Kg-K From table A.5 for Nitrogen.

$R = 0.2968$ KJ/Kg-K From table A.5

$P_1 = 300$ KPa, $P_2 = 200$ KPa, $V_1 = 5$ L
From eq (1)

$$T_2 = (200 + 273.15) \frac{(0.2968)(200) + (300)(.745)}{300 (.745 + 0.2968)} = 428.2 \text{ K}$$

From equation (2)

$$V_2 = \frac{(300)(5 \times 10^{-3})(428.2)}{(473.15)(200)} = 0.006787 \text{ m}^3$$

need to find entropy generation next

$$m(s_2 - s_1) = \frac{\cancel{SQ}}{T} + S_{gen} \quad \text{since adiabatic}$$

$$\text{so } S_{gen} = m(s_2 - s_1)$$

for ideal gas, use equation 8.25

$$s_2 - s_1 = C_{p0} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Numerical

$$C_{p0} = 1.042 \text{ kJ/Ks-K} \quad \text{From table A.5}$$

$$\text{so } S_{gen} = \frac{P_1 V_1}{RT_1} \left(C_{p0} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right)$$

$$= \frac{(300)(5 \times 10^{-3})}{(.2968)(473.15)} \left(1.042 \ln \frac{728.2}{473.15} - .2968 \ln \frac{200}{300} \right)$$

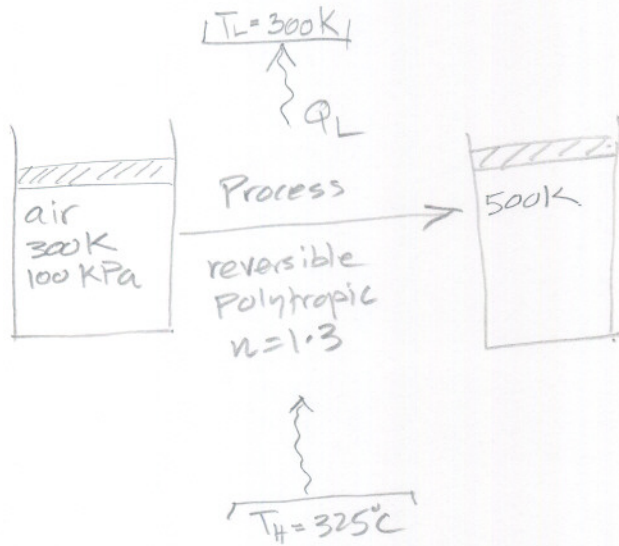
$$= \boxed{0.000174399 \text{ kJ/K}}$$

Problem 8.117 (Bonus)

5/5

Statement

A piston/cylinder contains air @ 300K, 100 kPa. A reversible polytropic process with $n=1.3$ brings the air to 500K. Any heat transfer if it comes in is from a 325°C reservoir, and if it goes out it is to the ambient @ 300K. Sketch the process in a P-v and T-s diagram. Find the specific work and specific heat transfer in the process. Find specific entropy generation (external to the air) in the process.



Sketch process on P-v and T-s.

Find work, heat transfer.

Find S_{gen}.

Assumptions

ideal gas for air
constant C_p, C_v

→

Laws

$${}_1W_2 = \frac{mR(T_2 - T_1)}{1-n} \quad (\text{eq 8.38}) \quad \text{For } n \neq 1 \\ \text{For reversible. Polytropic.}$$

$$u_2 - u_1 = C_v(T_2 - T_1) \quad \text{for ideal gas.}$$

$${}_1Q_2 - {}_1W_2 = m(u_2 - u_1) \quad \text{1st Law for control mass.}$$

$$m(s_2 - s_1) = \int_1^2 \frac{\delta Q}{T_{\text{source}}} + S_{\text{gen}} \quad \text{entropy equation.}$$

Steps

$${}_1Q_2 - {}_1W_2 = m(u_2 - u_1)$$

$${}_1q_2 - {}_1w_2 = (u_2 - u_1)$$

$$\text{but } \boxed{{}_1w_2 = \frac{R(T_2 - T_1)}{1-n}} \quad \text{--- (1)}$$

$$\text{so } {}_1q_2 = (u_2 - u_1) + {}_1w_2$$

$$\text{but } (u_2 - u_1) = C_v(T_2 - T_1)$$

$$\text{so } \boxed{{}_1q_2 = C_v(T_2 - T_1) + \frac{R(T_2 - T_1)}{1-n}} \quad \text{--- (2)}$$

now if ${}_1q_2$ is +ve, then it is going into.

if ${}_1q_2$ is -ve, then it is leaving.

so use the corresponding source T.

$$(s_2 - s_1) = \frac{{}_1q_2}{T_{\text{source}}} + S_{\text{gen}}$$

$$S_{\text{gen}} = (s_2 - s_1) - \frac{{}_1q_2}{T_{\text{source}}}$$

$$\text{page 284 shows } s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (\text{constant } C_p, C_v)$$

$$\text{so } S_{\text{gen}} = \left(C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right) - \frac{{}_1q_2}{T_{\text{source}}}$$

$$\text{but } P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{n}{n-1}} \quad \longrightarrow$$

$$s_u \quad S_{gen} = C_p \ln \frac{T_2}{T_1} - R \ln \left(\frac{P(T_2)^{\frac{n}{n-1}}}{P} \right) - \frac{q_2}{T_{source}} \quad (3)$$

Numerical

$$w_2 = \frac{R(T_2 - T_1)}{1 - n}$$

$R = 0.287 \text{ kJ/kg-K}$ For air, table A.5

$$s_u \quad w_2 = \frac{0.287(500 - 300)}{1 - 1.3} = \boxed{-191.33 \text{ kJ/kg}}$$

$C_{p0} = 0.717 \text{ kJ/kg-K}$ From table A.5

$$s_u \quad q_2 = C_{p0}(T_2 - T_1) + w_2$$

$$= 0.717(500 - 300) - 191.33 = \boxed{-47.93 \text{ kJ/kg}}$$

Since q_2 is negative, then it is leaving system.

so for T_{source} use 300 K

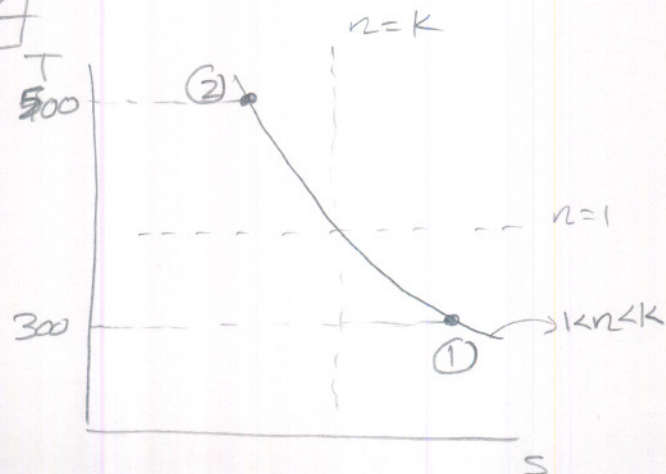
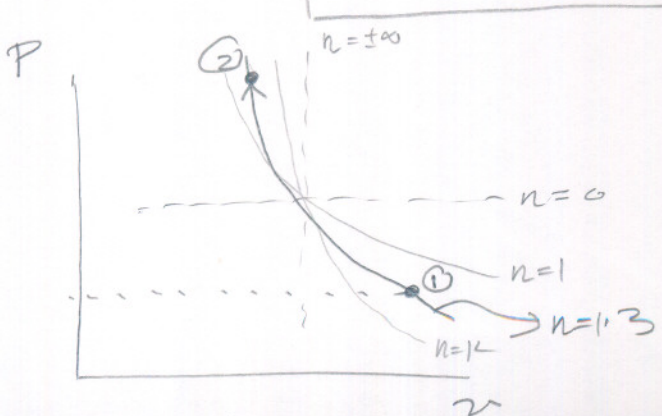
From equation (3)

$$S_{gen} = C_p \ln \frac{T_2}{T_1} - R \ln \left(\frac{P(T_2)^{\frac{n}{n-1}}}{P} \right) - \frac{q_2}{T_{source}}$$

$C_p = 1.004 \text{ kJ/kg-K}$, table A.5

$$S_{gen} = 1.004 \ln \frac{500}{300} - 0.287 \ln \left(\frac{500}{300} \right)^{\frac{1.3}{0.3}} + \frac{47.93}{300}$$

$$= \boxed{0.03736 \text{ kJ/kg-K}}$$

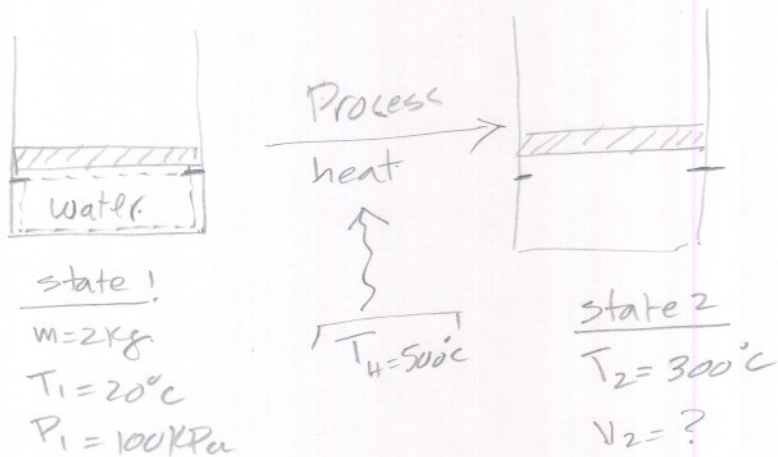


$k = 1.4$ for air, so $n < k$

Problem 8.134 (Bonus)

9/5

20°C , 100 kPa , and it is now heated to 300°C by a source at 500°C . A pressure of 1000 kPa will lift the piston off the lower stops. Find the Final Volume, Work, heat transfer, total entropy generation.



Find V_2 , W_2 , Q_2 , S_{gen} .

Assumptions

Control mass

Laws

1st Law for control mass $Q_2 - W_2 = m(u_2 - u_1)$

entropy equation $m(s_2 - s_1) = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$

$v = \frac{V}{m}$

steps

we need to find if $V_2 = V_{stop}$ or $> V_{stop}$.

$$V_{stop} = v_1 m.$$

we can find v_1 , since we know (T_1, P_1) .

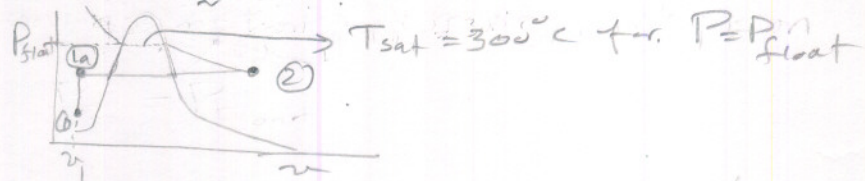
we are told we start in liquid phase.

as water is heated, as long as pressure remain smaller than $P_{float} = 1 \text{ MPa}$, then v will not change.

assume piston did not move, so $v_2 = v_1$.

now find T_{sat} for P_{float} from compressed liquid table. IF $T_{sat} < T_2$, then

v_2 must be $> v_1$.



if $T_{sat} > T_2$, then piston will move, and $v_2 = v_1$ and $P_2 = P_1$.

now, assuming piston did move, so we know (P_2, T_2) . from this we find v_2 from tables. and since mass is constant, then we can find V_2 .

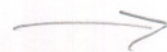
now that we know V_1, V_2 we can find work.

$${}_1W_2 = P_2 V_2 - P_1 V_1$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

From table
@ (P_2, T_2)

From
table @
 (P_1, T_1)



$$\text{now, } m(s_2 - s_1) = \frac{1\dot{Q}_2}{T_{\text{source}}} + S_{\text{gen}}$$

$$\text{so } S_{\text{gen}} = m(s_2 - s_1) - \frac{1\dot{Q}_2}{T_{\text{source}}}$$

↙
↘
→ calculated above

table @ (P₂, T₂)
table @ (P₁, T₁)
↓ given.

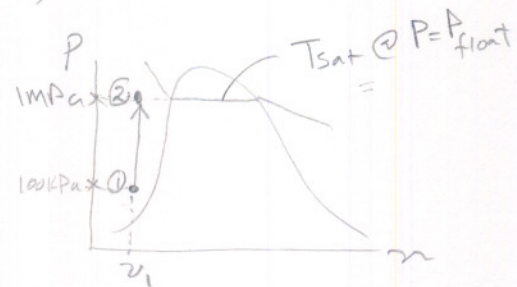
Numerical

From table @ (P₁, T₁) = (100 kPa, 20°C) B.1.1:

$$v_1 = v_g = 0.001002 \text{ m}^3/\text{kg}$$

$$s_1 = s_g = 0.2966 \text{ kJ/kg-K}$$

$$u_1 = u_g = 83.94 \text{ kJ/kg}$$



From table B.1.1, using $v_2 = v_1$ (assuming piston do not move)

Find Tsat for $P = P_{\text{float}} = 1 \text{ MPa}$, from compressed liquid table B.1.4.

From page 688 in book, I see that Tsat for $P = 1 \text{ MPa}$ must be $< 300^\circ\text{C}$, since Tsat for $P = 2 \text{ MPa} = 212.42^\circ\text{C}$.

This means that $v_2 > v_1$, i.e. piston moved.

$$\text{so } \boxed{P_2 = P_{\text{float}} = 1 \text{ MPa}}$$

$$v_2 = \frac{V_2}{m} \Rightarrow V_2 = v_2 m$$

↳ From superheated table @ $T_2 = 300^\circ\text{C}$
 $P_2 = 1 \text{ MPa}$

$$v_2 = 0.25794$$

$$u_2 = 2793.21$$

$$s_2 = 7.1228$$



$$\begin{aligned} \text{so } W_2 &= P_2 V_2 - P_1 V_1 \\ &= P_{\text{float}} (V_2 - V_1) \end{aligned}$$

but For Work,
 $P_2 = P_{\text{float}}$
 and $P_1 = P_{\text{float}}$ as well.

$$V_2 = v_2 m = (0.25794) (2)$$

$$V_1 = v_1 m = (0.001002) (2)$$

$$\text{so } W_2 = (1 \times 10^3) (2) (0.25794 - 0.001002)$$

$$\boxed{W_2 = 513.876 \text{ KJ}}$$

$$\begin{aligned} \dot{Q}_2 &= m(u_2 - u_1) + W_2 \\ &= (2) (2793.21 - 83.94) + 513.876 \\ &= \boxed{5932.416 \text{ KJ}} \end{aligned}$$

$$\begin{aligned} S_{\text{gen}} &= m(s_2 - s_1) - \frac{\dot{Q}_2}{T_{\text{surra}}} \\ &= (2) (7.1228 - 0.2966) - \frac{5932.416}{500 + 273} \\ &= \boxed{5.977 \text{ KJ/K}} \end{aligned}$$

diagrams

