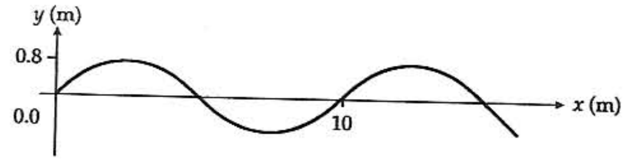
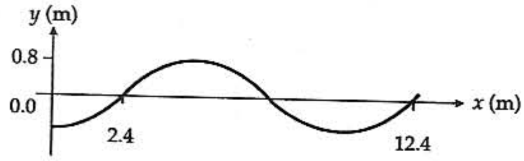
16.4





*16.6 The distance the waves have traveled is

$$d = (7.80 \text{ km/s})t = (4.50 \text{ km/s})(t + 17.3 \text{ s})$$

where t is the travel time for the faster wave.

Then, $(7.80 - 4.50)(\text{km/s})t = (4.50 \text{ km/s})(17.3 \text{ s})$

or $t = \dfrac{(4.50 \text{ km/s})(17.3 \text{ s})}{(7.80 - 4.50)(\text{km/s})} = 23.6 \text{ s}$, and

the distance is $d = (7.80 \text{ km/s})(23.6 \text{ s}) = \boxed{184 \text{ km}}$

16.7 (a) $\phi_1 = (20.0 \text{ rad/cm})(5.00 \text{ cm}) - (32.0 \text{ rad/s})(2.00 \text{ s}) = 36.0 \text{ rad}$

$\phi_1 = (25.0 \text{ rad/cm})(5.00 \text{ cm}) - (40.0 \text{ rad/s})(2.00 \text{ s}) = 45.0 \text{ rad}$

$\Delta\phi = 9.00 \text{ radians} = 516° = \boxed{156°}$

(b) $\Delta\phi = \left|20.0x - 32.0t - [25.0x - 40.0t]\right| = \left|-5.00x + 8.00t\right|$

At $t = 2.00$ s, the requirement is

$$\Delta\phi = \left|-5.00x + 8.00\,(2.00)\right| = (2n+1)\pi \text{ for any integer } n.$$

For $x < 3.20$, $-5.00x + 16.0$ is positive, so we have

$$-5.00x + 16.0 = (2n+1)\pi, \text{ or}$$

$$x = 3.20 - \frac{(2n+1)\pi}{5.00}$$

The smallest positive value of x occurs for $n = 2$ and is

$$x = 3.20 - \frac{(4+1)\pi}{5.00} = 3.20 - \pi = \boxed{0.0584 \text{ cm}}$$

16.12 $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{1350 \text{ kg}\cdot\text{m/s}^2}{5.00 \times 10^{-3} \text{ kg/m}}} = \boxed{520 \text{ m/s}}$

16.17 If the tension in the wire is T, the tensile stress is

$$\text{Stress} = T/A \quad \text{so} \quad T = A(\text{stress})$$

The speed of transverse waves in the wire is

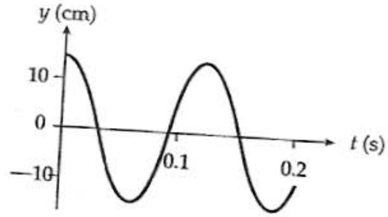
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{A(\text{Stress})}{m/L}} = \sqrt{\frac{\text{Stress}}{m/AL}} = \sqrt{\frac{\text{Stress}}{m/(\text{Volume})}} = \sqrt{\frac{\text{Stress}}{\rho}}$$

where ρ is the density. The maximum velocity occurs when the stress is a maximum:

$$v_{\max} = \sqrt{\frac{2.70 \times 10^9 \text{ Pa}}{7860 \text{ kg/m}^3}} = \boxed{586 \text{ m/s}}$$

16.23 (a) See figure at right.

(b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{50.3} = \boxed{0.125 \text{ s}}$



16.24 Using data from the observations, we have $\lambda = 1.20$ m and $f = \frac{8.00}{12.0 \text{ s}}$. Therefore,

$$v = \lambda f = (1.20 \text{ m})\left(\frac{8.00}{12.0 \text{ s}}\right) = \boxed{0.800 \text{ m/s}}$$

16.36 (a) $\omega = 2\pi f = 2\pi(500) = 3140$ rad/s, $k = \omega/v = (3140)/(196) = 16.0$ rad/m

$$\boxed{y = (2.00 \times 10^{-4} \text{ m}) \sin(16.0x - 3140t)}$$

(b) $v = 196 \text{ m/s} = \sqrt{\frac{T}{4.10 \times 10^{-3} \text{ kg/m}}}$

$T = \boxed{158 \text{ N}}$

16.39 Suppose that no energy is absorbed or carried down into the water. Then a fixed amount of power is spread thinner farther away from the source, spread over the circumference $2\pi r$ of a expanding circle. The power-per-width across the wave front

$$\frac{\wp}{2\pi r}$$

is proportional to amplitude squared so amplitude is proportional to

$$\sqrt{\frac{\wp}{2\pi r}}$$

$A = 5.00 \times 10^{-2}$ m $\quad \mu = 4.00 \times 10^{-2}$ kg/m $\quad \wp = 300$ W $\quad T = 100$ N

Therefore,

$$v = \sqrt{\frac{T}{\mu}} = 50.0 \text{ m/s}$$

$$\wp = \frac{1}{2}\mu\omega^2 A^2 v$$

$$\omega^2 = \frac{2\wp}{\mu A^2 v} = \frac{2(300)}{(4.00 \times 10^{-2})(5.00 \times 10^{-2})^2(50.0)}$$

$$\omega = 346 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \boxed{55.1 \text{ Hz}}$$

16.46 Equation 16.26, with $v = \sqrt{T/\mu}$ is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

If $y = e^{b(x - vt)}$,

then $\frac{\partial y}{\partial t} = -bve^{b(x - vt)}$ and $\frac{\partial y}{\partial x} = be^{b(x - vt)}$

$$\frac{\partial^2 y}{\partial t^2} = b^2v^2e^{b(x - vt)} \quad \text{and} \quad \frac{\partial^2 y}{\partial x^2} = b^2e^{b(x - vt)}$$

Therefore, $\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$, and $e^{b(x - vt)}$ is a solution.

(a) From $y = x^2 + v^2t^2$,

evaluate $\frac{\partial y}{\partial x} = 2x \qquad \frac{\partial^2 y}{\partial x^2} = 2$

$$\frac{\partial y}{\partial t} = v^2 2t \qquad \frac{\partial^2 y}{\partial t^2} = 2v^2$$

Does $\frac{\partial^2 y}{\partial t} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$?

By substitution: $2 = \frac{1}{v^2}2v^2$ and this is true, so the wave function does satisfy the wave equation.

(b) Note $\frac{1}{2}(x + vt)^2 + \frac{1}{2}(x - vt)^2$

$$= \frac{1}{2}x^2 + xvt + \frac{1}{2}v^2t^2 + \frac{1}{2}x^2 - xvt + \frac{1}{2}v^2t^2$$

$= x^2 + v^2t^2$ as required

So $\boxed{f(x + vt) = \frac{1}{2}(x + vt)^2}$ and $\boxed{g(x - vt) = \frac{1}{2}(x - vt)^2}$

(c) $y = \sin x \cos vt$ makes

$$\frac{\partial y}{\partial x} = \cos x \cos vt \qquad \frac{\partial^2 y}{\partial x^2} = -\sin x \cos vt$$

$$\frac{\partial y}{\partial t} = -v \sin x \sin vt \qquad \frac{\partial^2 y}{\partial t^2} = -v^2 \sin x \cos vt$$

Then $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

becomes $-\sin x \cos vt = \frac{-1}{v^2}v^2 \sin x \cos vt$ which is true as required.

Note $\sin(x + vt) = \sin x \cos vt + \cos x \sin vt$

$\sin(x - vt) = \sin x \cos vt - \cos x \sin vt$

So $\sin x \cos vt = f(x + vt) + g(x - vt)$ with

$\boxed{f(x + vt) = \frac{1}{2}\sin(x + vt)}$ and $\boxed{g(x - vt) = \frac{1}{2}\sin(x - vt)}$