HW5, Problem 25.4.

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Problem: Use midpoint method with h=0.5 and h=0.25 to solve $\frac{dy}{dx} = yx^2 - 1.2y$ over the interval $x = 0$ to 2 with $y(0) = 1$.

Solution:

I illustrate the method, then write a matlab function to implement it.

Recall that in Euler method, we obtain an estimate to future value of y (at next step) by using the value for the rate of change of y at the current step. i.e. $y_{i+1} = y_i + h\ y_i^{'}$. i.e. we assume that the rate of change of y has remained the same over the duration of the step size.

In midpoint method the rate of change of y (which is used to make an h Euler step to estimate the future value of y) is taken to be the rate of change of y at the mid-point between the start and end of the x-step.

To find the rate at the mid-point, an $\frac{h}{2}$ Euler step is first used to estimate y at the mid-point.

Using this midpoint value for y', we now obtain the future value of y.

In otherwords, $y_{i+1} = y_i + h\ f\left(x_{i+\frac{h}{2}}, y_{i+\frac{h}{2}}\right)$

Algorithm

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Find derivative of y at start of current step. Use initial
     conditions for y at start of first step.

Use this derivative to make a h/2 Euler step to find y at half-way
Find derivative of y at this half-way y value, and at x=x_start_step+h/2
Use this new mid-point derivative to make a full h Euler step from the
    start of the step to find y at end of current step.

repeat the above for each step.
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I show how this is done for one step, then run the matlab function I wrote to fully solve the mid-point method.

Let $h = 0.5$

First step, i=0:

First find the rate at start of the step:

$y_0' = y_0\left(x_0^2 - 1.2\right) = 1\,(0 - 1.2) = -1.2$

Now, find the mid-point:

$y_{mid-point} = y_0 + \frac{h}{2}\ y_0^{'} = 1 + \frac{0.5}{2}\,(-1.2) = 0.7$

Now Use $y_{mid-point}$ to find the rate of change at the mid-point of the step:

$y'_{mid-point} = y_{mid-point}\left(\left(x_0 + \frac{h}{2}\right)^2 - 1.2\right) = 0.7\left(\left(0 + \frac{0.5}{2}\right)^2 - 1.2\right) = -0.7962$

Now, use this rate at the mid-point to make a full Euler step to find y at the end of the step

$y_1 = y_0 + h\ y_{mid-point}^{'} = 1 + 0.5\,(\ -0.7962) = 0.6019$

Continue the above process for all the steps.