HW#5. Problem 25.2
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Problem:

Use Euler's method with $h = 0.5$ and $h = 0.25$ to solve $\frac{dy}{dx} = yx^2 - 1.2y$ over the interval from $x = 0$ to 2.where $y(0) = 1$.Plot the results over the same graph as the analytical solution to visually compare the accuracy of the two step sizes.

Answer:

$h$ is the step size.

In Euler method, given $y' = f(x, y)$, future values of the solution $y$ are found using $y_{i+1} = y_i + h\, y_i'$ (Taylor series with truncation of terms). Here, we are given that $y' = y\left(x^2 - 1.2\right)$

## $\mathbf{h = 0.5}$

$i = 0$
$x_0 = 0$
$y_0 = y(x_0) = y(0) = 1$ From initial conditions
$y_0' = y_0\left((x_0)^2 - 1.2\right) = (1)\left(0^2 - 1.2\right) = -1.2$
hence, now ready to generate the next $y$
$y_1 = y_0 + h\, y_0' = 1 + 0.5(-1.2) = 0.4$
$i = 1$
$x_1 = x_0 + h = 0.5$
$y_1 = 0.4$ from previous step solution above
$y_1' = y_1\left((x_1)^2 - 1.2\right) = (4)\left(0.5^2 - 1.2\right) = -3.8$
hence, now ready to generate the next $y$
$y_2 = y_1 + h\, y_{i=1}' = 0.4 + 0.5\ (0.4)(-3.8) = 0.21$
$i = 2$
$x_2 = x_1 + h = 0.5 + 0.5 = 1.0$
$y_2 = 0.21$ from previous step solution above
$y_2' = y_2\left((x_2)^2 - 1.2\right) = (0.21)\left(1.0^2 - 1.2\right) = -0.042$
hence, now ready to generate the next $y$
$y_3 = y_2 + h\, y_2' = 0.21 + 0.5\ (-.042) = 0.189$
$i = 3$
$x_3 = x_2 + h = 1.0 + 0.5 = 1.5$
$y_3 = 0.189$ from previous step solution above
$y_3' = y_3\left((x_3)^2 - 1.2\right) = (0.189)\left(1.5^2 - 1.2\right) = 0.19845$
hence, now ready to generate the next $y$
$y_4 = y_3 + h\, y_3' = 0.189 + 0.5\ (0.19845) = 0.288225$

Hence, the $y$ solution is $[1, 0.4, 0.21, 0.189, 0.288225]$ at $x = [0, 0.5, 1.0, 1.5, 2.0]$

## $\mathbf{h = 0.25}$

$i = 0$
$x_0 = 0$
$y_0 = y(x_0) = y(0) = 1$
$y_0^{'} = y_0\left((x_0)^2 - 1.2\right) = (1)\left(0^2 - 1.2\right) = -1.2$
hence, now ready to generate the next $y$
$y_1 = y_0 + h\, y_0^{'} = 1 + 0.25\,(-1.2) = 0.7$
$i = 1$
$x_1 = x_0 + h = 0 + 0.25 = 0.25$
$y_1 = 0.7$ from previous step solution
$y_1^{'} = y_1\left((x_1)^2 - 1.2\right) = (0.7)\left(0.25^2 - 1.2\right) = -0.79625$
hence, now ready to generate the next $y$
$y_2 = y_1 + h\, y_1^{'} = 0.7 + 0.25\,(-0.79625) = 0.5009375$
$i = 2$
$x_2 = x_1 + h = 0.25 + 0.25 = 0.5$
$y_2 = 0.5009375$ from previous step solution
$y_2^{'} = y_2\left((x_2)^2 - 1.2\right) = (0.5009375)\left(0.5^2 - 1.2\right) = -0.475890625$
hence, now ready to generate the next $y$
$y_3 = y_2 + h\, y_2^{'} = 0.5009375 + 0.25\,(-0.475890625) = 0.38196484375$
$i = 3$
$x_3 = x_2 + h = 0.5 + 0.25 = 0.75$
$y_3 = 0.38196484375$ from previous step solution
$y_3^{'} = y_3\left((x_3)^2 - 1.2\right) = (0.38196484375)\left(0.75^2 - 1.2\right) = -0.243502587890625$
hence, now ready to generate the next $y$
$y_4 = y_3 + h\, y_3^{'} = 0.38196484375 + 0.25\,(-0.243502587890625) = 0.321089196777344$
$i = 4$
$x_4 = x_3 + h = 0.75 + 0.25 = 1.0$
$y_4 = 0.321089196777344$ from previous step solution
$y_4^{'} = y_4\left((x_4)^2 - 1.2\right) = (0.321089196777344)\left(1^2 - 1.2\right) = -0.0642178393554688$
hence, now ready to generate the next $y$
$y_5 = y_4 + h\, y_4^{'} = 0.321089196777344 + 0.25\,(-0.0642178393554688) = 0.305034736938477$
$i = 5$
$x_5 = x_4 + h = 1.0 + 0.25 = 1.25$
$y_5 = 0.305034736938477$ from previous step solution
$y_5^{'} = y_5\left((x_5)^2 - 1.2\right) = (0.305034736938477)\left(1.25^2 - 1.2\right) = 0.110575092140198$
hence, now ready to generate the next $y$
$y_6 = y_5 + h\, y_5^{'} = 0.305034736938477 + 0.25\,(0.110575092140198) = 0.332678509973526$
$i = 6$
$x_6 = x_5 + h = 1.25 + 0.25 = 1.5$
$y_6 = 0.332678509973526$ from previous step solution
$y_6^{'} = y_6\left((x_6)^2 - 1.2\right) = (0.332678509973526)\left(1.5^2 - 1.2\right) = 0.349312435472202$
hence, now ready to generate the next $y$
$y_7 = y_6 + h\, y_6^{'} = 0.305034736938477 + 0.25\,(0.110575092140198) = 0.332678509973526$
$i = 7$

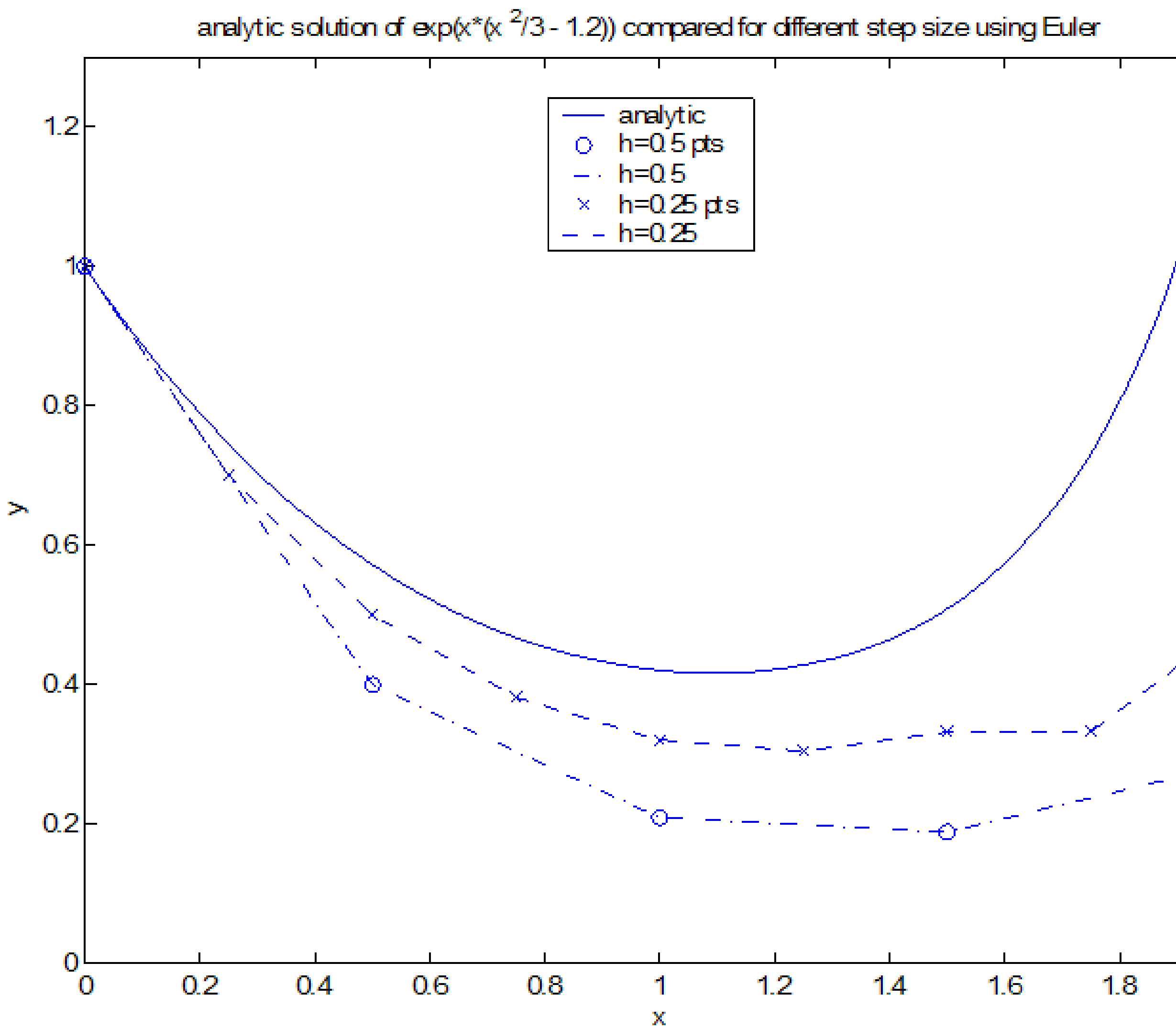
$x_7 = x_6 + h = 1.5 + 0.25 = 1.75$
$y_7 = 0.332678509973526$ from previous step solution
$y_7^{'} = y_7 \left( (x_7)^2 - 1.2 \right) = (0.332678509973526) \left(1.75^2 - 1.2\right) = 0.619613724825692$
hence, now ready to generate the next $y$
$y_8 = y_7 + h\, y_7^{'} = 0.332678509973526 + 0.25\,(0.619613724825692) = 0.487581941179949$

hence, the solution
$y = [1, 0.7, 0.5009375, 0.38196484375, 0.321089196777344, 0.305034736938477$
$, 0.332678509973526, 0.332678509973526, 0.487581941179949]$
at $x = [0, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0]$;

Now, use matlab to plot the analytical solution, and on top of the plot, the Euler solution for $h = 0.5$ and $h = 0.25$

```
>> x_5=[0 0.5 1 1.5 2];
 y_5=[1 0.4 0.21 0.189 0.288225];
 x_25=[0 0.25 0.5 0.75 1 1.25 1.5 1.75 2];
 y_25=[1  0.7 0.5009375  0.38196484375     0.321089196777344 ...
0.305034736938477  0.332678509973526  0.332678509973526 0.487581941179949];
>>
>> x_5'

                         0
                       0.5
                         1
                       1.5
                         2

>> y_5'

                         1
                       0.4
                      0.21
                     0.189
                  0.288225

>> x_25'
                         0
                      0.25
                       0.5
                      0.75
                         1
                      1.25
                       1.5
                      1.75
                         2

>> y_25'
                         1
                       0.7
                 0.5009375
             0.38196484375
         0.321089196777344
         0.305034736938477
         0.332678509973526
         0.332678509973526
         0.487581941179949
```

```
>>

>> y_analytic='exp(x*(x^2/3 - 1.2))';
ezplot(y_analytic,0,2);
hold on;
plot(x_5,y_5,'o')
plot(x_5,y_5,'-.')
plot(x_25,y_25,'x')
plot(x_25,y_25,'--')
ylim([0 1.3])
legend('analytic','h=0.5 pts','h=0.5','h=0.25 pts','h=0.25');
ylabel('y');
title(sprintf('analytic solution of %s compared for different step size using Eu
>>
```




Notice that as step size $h$ is decreased, the numerical solution is becoming closer to the analytical solution.