H.W #1

MAE 146

Astronautics

UCI

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April 16, 2003

30/30

problem 1.1 page 44 BMW Book

problem statment: given $\bar{r} = 2\bar{i} + 2\bar{j} + 2\bar{k}$, $\bar{v} = -.4\bar{j} + .2\bar{j} + .4\bar{k}$

Find $\bar{h}$ and $\mathcal{E}$ for the satellite.

Assumptions: an earth satellite. so use $\mu_{\oplus} = 1\ DU^3/TU^2$

Method: to find $\bar{h} = \bar{r} \times \bar{v}$, To Find $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$

Analysis:

$$\bar{h} = \bar{r} \times \bar{v} = \begin{vmatrix} i & 2 & -.4 \\ j & 2 & .2 \\ K & 2 & .4 \end{vmatrix} = +\bar{i}(2\times.4 - .2\times2) - \bar{j}(2\times.4 + .4\times2) + \bar{k}(2\times.2 + .4\times2)$$

$$= \bar{i}(.8 - .4) - \bar{j}(.8 + .8) + \bar{k}(.4 + .8)$$

so specific Angular Momentum = $\boxed{\bar{h} = .4\bar{i} - 1.6\bar{j} + 1.2\bar{k}}$ in DU^2/TU

$|\bar{h}| = \sqrt{.4^2 + (-1.6)^2 + 1.2^2} = 2.0396\ DU^2/TU$

To find $\mathcal{E}$, the specific Mechanical Energy for the satellite.

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v = |\bar{v}| = \sqrt{(-.4)^2 + (.2)^2 + (.4)^2} = 0.6$$

$$r = |\bar{r}| = \sqrt{2^2 + 2^2 + 2^2} = 3.46412$$

assuming an earth satellite, $\mu_{\oplus} = 1.407647 \times 10^{16}\ ft^3/sec^2$ or $1\ \frac{DU^3}{TU^2}$

$$\text{so}\quad \mathcal{E} = \frac{0.6^2}{2} - \frac{1}{3.46412} = 0.18 - 0.288673 =$$

$$\boxed{\mathcal{E} = -0.108673}\ DU^2/TU^2$$

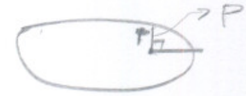
problem (1.2)

problem: given $\nu = 90°$, $v = 45{,}000$ ft/sec ; $r = 4{,}000$ nmi. Find e

Assumption: earth satellite.

method: convert all units to Canonical.

find h from $h = \sqrt{P\mu}$ since $\nu = 90° \rightarrow r \equiv P$



find $\mathcal{E}$ from $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$

find e from $e = \sqrt{1 + \frac{2\mathcal{E}h^2}{\mu^2}}$

Analysis

$r = 4{,}000 \times 2.903665564 \times 10^{-4} = 1.16146626$ $DU_\oplus$

$V = 45000 \times 3.85560785 \times 10^{-5} = 1.735023533$ $DU_\oplus / TU_\oplus$

$\mu = 1 \frac{DU_\oplus^3}{TU_\oplus^2}$

$h = \sqrt{P\mu} = \sqrt{1.16146626 \times 1} = 1.077713441$

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = \frac{(1.735023533)^2}{2} - \frac{1}{1.16146626} = 0.644172658 \; DU^2/TU^2$$

$$e = \sqrt{1 + \frac{2(0.644172658)(1.077713441)^2}{1^2}}$$

$$e = \sqrt{1 + 1.496369617} = 1.57999 \simeq \boxed{1.58}$$

✓

$\boxed{\text{what is happen when } e > 1}$

You have to mention is Hyperbola

problem 1.3

problem: given r_A and r_p, determin period of orbit.

Assumptions: earth satellite. use $\mu_\oplus = 1$

method: since r_A, r_p are given, find a from $2a = r_A + r_p$. Then use $T = 2\pi\sqrt{\frac{a^3}{\mu}}$ (this is kepler equation)

Analysis:

$r_\oplus$ in canonical

$$r_A = 600\,(2.903665564 \times 10^{-4}) + 1 = 1.174219934$$

$$r_p = 100\,(2.903665564 \times 10^{-4}) + 1 = 1.029036656$$

but $2a = r_A + r_p \Rightarrow a = \frac{r_A + r_p}{2} = 1.101628292$

$$T = 2\pi\sqrt{\frac{a^3}{1}}$$

$= 7.26494766\ TU_\oplus$

convert to seconds $\Rightarrow \frac{7.2649466}{1.239446309E-3} = 5861.4$ sec

$T\ \boxed{= 1.628\ hrs}$

1.4 (1.4)

problem: Six constants of integration (or effectively 6 orbital Elements) are required for a complete solution to the 2 body problem. Why, in general, is a completely determined closed solution of the N-body problem an impossibility if $N \geq 3$?

Assumptions: the 2 body problem mentioned above is where one body is moving <u>relative</u> to the second body.

method: solution of set of differential Equations, constants of integration

Analysis:

The 2 body problem solution is to find the position of one body relative to another as time changes. i.e to find $\underline{r}(t)$

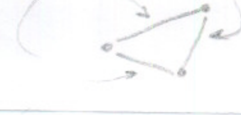
we solve $\ddot{\underline{r}} + \frac{\mu}{r^3}\underline{r} = 0$ to obtain $\underline{r}(t)$

the above, in cartesian coordinate, is a set of 3 D.E's. (one each for x, y, z)

since we have 3 second order D.E.'s, we need to integrate twice to solve each. hence we will have a total of 6 constants of integrations that we need to determine in order to solve the DE's.

for 2 body problem, we can find 6 independent quantities that fully describe the orbit of one body relative to the second (these are called the classical orbital Elements, see page 58 BMW)

but for N=3, we have a set of 3 DE's, one for each body relative to the other:



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so we have a total of 9 DE's. (since each D.E. of the original 3 is itself is a set of 3DE's. one for x, y, z) hence we need now 18 constants of integration to solve.

for $N \geq 3$, it is not possible to find enough independent quantities to satisfy solving (or finding) all the constants of integrations. hence no closed form solution exist for $N \geq 3$.

In particular, for $N=3$, the center of mass of the 3 bodies can provide 6 quantities (3 for V_x, V_y, V_z and 3 from r_x, r_y, r_z) at $t=0$. this will provide 6 of the 18 constants. we can find another 6 constants (3 from angular momentum, one from energy, one from elimination of time, one from elimination of something called ascending node) This Leaves 6 unknowns.

so in general, for n bodies we need 6n independent quantities to fully solve it, we can find 12, hence (6n-12) is what remains.

references read while working on this:

1. scienceworld.wolfram.com
2. N-body simulation www.chara.gsu.edu/~harvin/Thesis
3. "The Three body problem" by Eugenii V. Donev
4. section 2.2 from Wiesel Book
5. N-body algorithms by Tancred Lindholm

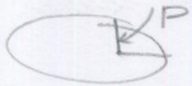
1.5 (1.5)

Problem: given a earth satellite with semi-major axis $a = 30\times10^6$ ft, $e = 0.2$ Find:

a) r_A, r_P

b) $\mathcal{E}$ (specific mechanical Energy)

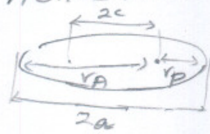
c) P parameter



d) $|\vec{r}|$ at $\nu = 135°$

Assumptions:

Method: a) use properties of ellipse: $e = \frac{c}{a}$ to find c, hence this finds r_A, r_P



b) use $\mathcal{E} = -\frac{\mu}{2a}$ to find $\mathcal{E}$

c) use $p = a(1-e^2)$

d) use $r = \frac{p}{1+e\cos\nu}$ (the general conic locus equation)

Analysis: a) $e = \frac{c}{a} \Rightarrow c = ea = 30\times10^6 \times 0.2 = 6\times10^6$ ft.

$r_P = \frac{2a-2c}{2} = a - c = 30\times10^6 - 6\times10^6 = 24\times10^6$ ft.

so $\boxed{r_P = 24\times10^6 \text{ ft}}$ ✓

so $r_A = 2c + r_P = 2\times6\times10^6 + 24\times10^6 = 36\times10^6$ ft

$\boxed{r_A = 36\times10^6 \text{ ft}}$ ✓

b) $\mathcal{E} = -\frac{\mu}{2a} = -\frac{1.407646882\times10^{16}}{2\times30\times10^6} = \boxed{-234607813 \text{ ft}^2/\text{sec}^2}$

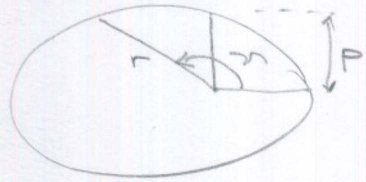
or $\boxed{\mathcal{E} = -0.34876 \frac{DU^2}{TU^2}}$

c) $P = a(1-e^2) = 30\times10^6(1-0.2^2)$ ✓

$= 28{,}800{,}000 = \boxed{28.8\times10^6 \text{ ft}}$

→

d) $r = \frac{p}{1+e\cos\nu}$



$$r = \frac{28.8\times10^{6}}{1+0.2\cos135^\circ} = 33{,}543{,}811.29 \text{ ft.}$$

$$\boxed{r = 3.354\times10^{7}\ \text{ft}}$$

✓

1.6

problem: Find equation for the velosity of satellite as a function of the total specific mechanical Energy and distance from the center of the earth.

Assumptions: same assumptions used to derive $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$.

Method: this problem is confusing to me. as the answer is allready given by $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$ and was derived in class and text book. (BMW, page 15 section 1.4.1)

Unless I am missing something. $V(\mathcal{E}, r)$ follows from above by simple rearrangment:

$$\frac{v^2}{2} = \mathcal{E} + \frac{\mu}{r} \Rightarrow \boxed{V = \sqrt{2\left(\mathcal{E} + \frac{\mu}{r}\right)}}$$

this expresses V as function of $\mathcal{E}$ and r which is what is asked to do.

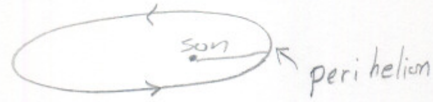
Dr the problem talks about "total specific mechanical Energy", I have assumed here that the word "total" is extra, since I have only seen references to the phrase "specific Mechanical Energy" which is $\mathcal{E}$, i.e without the word "total" in front off it.

if total specific Mechanical Energy is not meant to be $\mathcal{E}$, then my solution is obviously wrong.

(Name Abbasi

problem 2.3 Weisel

problem: Halley comet last passed perihelion on feb 9, 1986. it has a semimajor axis $a = 17.9564$ AU and $e = 0.967298$ (one AU = distance from sun to earth). calculate the period of Halley's comet and predict the date of next return. solve kepler's equation and calculate E, ν and the scalar radius vector r for your current date.



Assumptions:

Method: to find period use $\mathbb{P} = 2\pi\sqrt{\dfrac{a^3}{\mu_\odot}}$

to find ν and r on April 23, 2003:
find duration t = April 23, 2003 — feb 9, 1986. Then find M.
Then from $M = E - e\sin E$ solve for E using Newton method.
Then from $\tan\frac{\nu}{2} = \tan\left(\frac{E}{2}\right)\sqrt{\frac{1+e}{1-e}}$ solve for ν, and find r from $r = a(1-e\cos E)$

50/50

Analysis: $\mathbb{P} = 2\pi\sqrt{\dfrac{a^3}{\mu_\odot}} = 2\pi\sqrt{\dfrac{17.9564^3}{1}} = 478.08\ TU_\odot$

$= 27792.66$ days

$\boxed{\mathbb{P} \triangleq 76.352 \text{ years}}$ mean orbit Period.

so date of next return = $\boxed{2062}$ approx.

to find exact date: From feb 9 to Dec 31 = $365 - (9+31) = 325$ days

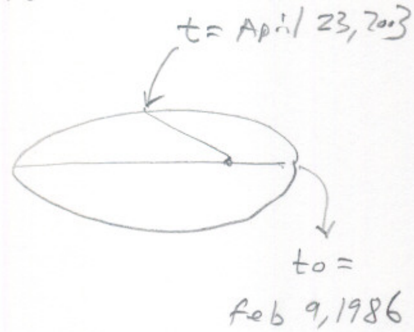
so $27792 - 325 = 27467$ days from start of 1987.
$= 75$ years and 92 days. (using 365 days per year)

$\Rightarrow 1987 + 75 \Rightarrow$ year 2062 + 92 days.

but there are one leap year each 4 years. (366 days is leap years).
so There are 18 leap years from 1987 to 2062. so add 18 days to 92 days $\Rightarrow$ 110 days. 110 days from start of year is

$\underbrace{31 \quad 28 \quad 31}_{90 \text{ days}} \quad \underbrace{20}_{20 \text{ days}} \Rightarrow$ April 20. so final date is $\boxed{2062, \text{April } 20}$

time in orbit = April 23, 2003 — Feb 9, 1986



From Feb 19, 1986 to 1/1/1987 = 325 days.

From 1/1/2003 to 4/23/2003 = 31+28+31+23
= 113 days.

	leap				leap				leap				leap		
1987	88	89	90	91	92	93	94	95	96	97	98	99	2000	2001	2002
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

16 years. = 5840 days. add 4 days for leap years = 5844 days.

so total duration = 325 + 113 + 5844 = $\boxed{6282 \text{ days}}$

$$t = \frac{6282}{58.132821} = 108.062879 \quad TU_{\odot}$$

now $$t = (E - e\sin E)\sqrt{\frac{a^3}{\mu}}$$

so $$t\sqrt{\frac{\mu}{a^3}} = (E - e\sin E)$$

so $$108.062879\sqrt{\frac{1}{17.95643}} = (E - e\sin E)$$

$$1.420193848 = E - e\sin E$$

solve for E using Newton Method:

$$F(E) = E - e\sin E - 1.420193848 = 0$$

$$F'(E) = 1 - e\cos E$$

$$E_{i+1} = E_i - \frac{F(E_i)}{F'(E_i)}$$ choose $E_0 = 1.420193848$

→

$e = 0$

$$E_1 = E_0 - \frac{F(E_0)}{F'(E_0)} = 1.420193848 - \frac{(1.420193848 - 0.967298 \sin 1.420193848) - 1.420193848}{1 - 0.967298 \cos 1.420193848}$$

$$= 2.2214698 \qquad E_1 \neq E_0 \quad \text{continue.}$$

$$E_2 = E_1 - \frac{F(E_1)}{F'(E_1)} = 2.2214698 - \frac{2.22146 - e \sin 2.2214 - 1.4219}{1 - .967298 \cos 2.221498}$$

$$= 2.20153 \qquad E_2 \neq E_1$$

$$E_3 = E_2 - \frac{F(E_2)}{F'(E_2)} = 2.20153 - \frac{(2.20153 - 0.967298 \sin 2.20153 - 1.42019)}{1 - 0.967298 \cos 2.20153}$$

$$= 2.20143 \qquad E_3 \neq E_2$$

$$E_4 = E_3 - \frac{F(E_3)}{F'(E_3)} = 2.201433 \qquad E_4 = E_3 \quad \text{stop.}$$

so $\boxed{E = 2.201433}$ 2.983

$$\tan\left(\frac{\nu}{2}\right) = \tan\left(\frac{E}{2}\right)\sqrt{\frac{1+e}{1-e}}$$

so $\tan\left(\frac{\nu}{2}\right) = \tan\left(\frac{2.201433}{2}\right)\sqrt{\frac{1+0.967298}{1-0.967298}} = 15.26609$ ✓

so $\frac{\nu}{2} = 1.505385$ radians $\Rightarrow \boxed{\nu = 86.25^\circ}$

$$r = a(1 - e\cos E) = 17.9564(1 - 0.967298 \cos 2.201433) = \boxed{28.1983183\ DU_\odot}$$

25.76

$$r = \frac{a(1-e^2)}{1+e\cos\nu}$$

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Problem 1.20 BMW

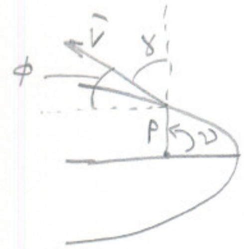
1. problem prove that flight path angle $\phi = 45^\circ$ when $\nu = 90^\circ$ on all parabolic trajectories.

2. assumptions:

3. Method: use fact that Energy $= \phi$ for parabola orbit.

4. Analysis:

since parabola then



$$\mathcal{E} = 0 = \frac{V^2}{2} - \frac{\mu}{r} \quad \text{———} \quad (0)$$

since $\nu = 90^\circ$, then $P = r$ also.

from (0) $\quad V^2 = \frac{2\mu}{P}$ ——— (1)

now, $h = PV \sin\gamma$ ——— (2) where I replaced r by P

also $h^2 = r\mu = P\mu$ ——— (3) true for any orbit

from (2) and (3) $\Rightarrow \sqrt{P\mu} = PV\sin\gamma$ ——— (4)

eliminate V from (4) by using (1)

$$\Rightarrow \sqrt{P\mu} = P\sqrt{\frac{2\mu}{P}}\,\sin\gamma$$

square $\Rightarrow$

$$P\mu = P^2\,\frac{2\mu}{P}\,\sin^2\gamma$$

$$1 = 2\sin^2\gamma$$

$$\sin^2\gamma = \frac{1}{2}$$

$$\sin\gamma = \sqrt{\frac{1}{2}}$$

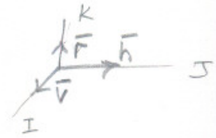
$\gamma = 45^\circ$ (since first quadrant)

but $\phi = 90 - \gamma = 45^\circ$ QED

(Nasser Abbasi)

problem 2.1 BMW

problem: Determine orbital elements for an earth orbit which has the following position and velocity vectors:



$$\bar{r} = 1\hat{K} \quad DU$$
$$\bar{V} = 1\hat{I} \quad DU/TU.$$

Assumptions: use geocentric-equatorial coordinate system.

Method: use method of orbital elements determination for $\bar{r}, \bar{v}$ as described in 2.4 section, book BMW

Analysis:

$$\bar{h} = \bar{r}\times\bar{v} = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = +i(0-0) - j(0-1) + k(0-0)$$
$$= \hat{J}$$

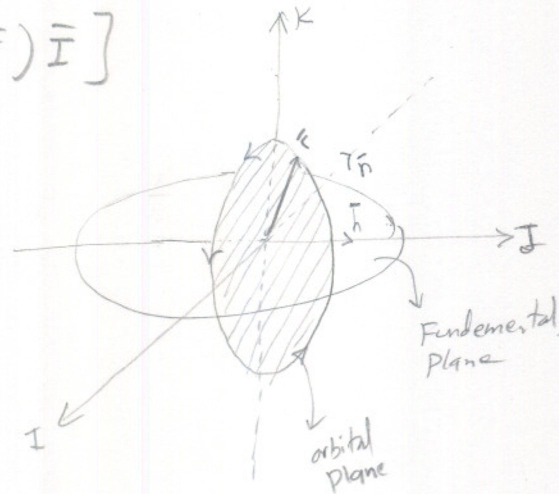
$$|\bar{h}| = 1$$

$$\bar{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\bar{r} - (\bar{r}\cdot\bar{v})\bar{v}\right] \qquad \text{from eq 2.4-5 BMW}$$

but $v = 1$, $r = 1$

$$\bar{e} = \frac{1}{\mu}\left[\left(1-\frac{1}{1}\right)1\bar{K} - (1\bar{K}\cdot 1\bar{I})\bar{I}\right]$$

$$= \frac{1}{\mu}[0 - 0] = \bar{0}$$



$\boxed{e = 0 \quad \text{hence} \quad \text{circle orbit}}$

hence $\boxed{a = r = 1 \quad DU}$

$$\cos i = \frac{\bar{h}\cdot\bar{K}}{h} = \frac{\hat{J}\cdot\hat{K}}{1} = 0$$

since $i < 180°$ $\Longrightarrow$ $\boxed{i = 90°}$

$$\bar{n} \equiv \bar{K}\times\bar{h} = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = +i(0-1) - j(0-0) + k(0-0) = -\bar{I}$$

$$\Omega = \cos^{-1}\left(\frac{\bar{n}\cdot\bar{I}}{n}\right) = \cos^{-1}\left(-\frac{\bar{I}\cdot\bar{I}}{1}\right) = \cos^{-1}(-1)$$

$\boxed{\Omega = 180°}$

→

$\omega = \cos^{-1} \frac{\bar{n} \cdot \bar{e}}{}$

also ν_0 (angle between $\bar{e}, \bar{r}_0$) undefined.

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problem 1.22 BMW

problem: A space vehicle destined for Mars was first launched into a 100 n mi circular parking orbit

a. What was speed of vehicle at injection into parking orbit?

The vehicle increased its velocity to 37,000 ft/sec which placed it on an interplanetary trajectory towards mars.

b. find $e, h, \mathcal{E}$ relative to the earth for the escape orbit. What kind of orbit is it?

c. compare the velocity at 1,000,000 n mi from the earth with the hyperbolic excess velocity V_∞. Why are the two so nearly alike?

Assumptions: speed increase to 37,000 ft/sec is instantanous.

Method:

<u>a</u> find $\mathcal{E}$ from $\mathcal{E} = -\frac{\mu}{2a}$, then find v from $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$

<u>b</u> find e from $r_p = a(1-e)$. we can find a by using

$$\mathcal{E} = \frac{v_p^2}{2} - \frac{\mu}{r_p} = \frac{-\mu}{2a}$$

we know V_p and r_p, hence a is found.

<u>c</u> find V at 1,000,000 n mi from equating $\mathcal{E}$ at r_p and $\mathcal{E}$ at 1,000,000 n mi. then find V_∞ at $r = \infty$, and compare the 2 velosities.

→

Analysis:

<u>a</u> $\mathcal{E} = -\frac{\mu}{2a}$ which is valid for all conic orbits.

converts to Geocentric Canonical units

$Alt = (100)(2.903665564\,E{-4})$

$= 0.029036655 \quad DU_\oplus$

so orbit is at 1+ALT from center of earth.

since circular orbit, then $a = 1.0 + 0.029036655\ DU_\oplus$

i.e. $r = a$

$\boxed{a = 1.029036655}\ DU_\oplus$

so $\mathcal{E} = -\frac{\mu}{2a} = -\frac{1}{2.058073311}$

then $\mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r} \Rightarrow V^2 = 2\mathcal{E} + \frac{2\mu}{r}$

or $V = \sqrt{2\mathcal{E} + \frac{2\mu}{r}} = \sqrt{\frac{-2}{2.058073311} + \frac{2}{1.029036655}}$

$V_{cs} = 0.985790384 \quad DU_\oplus/TU_\oplus$

$V_{cs} = 25{,}567.7$ ft/sec.

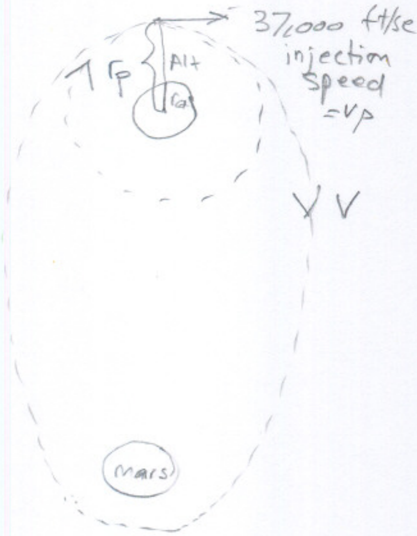
all this is the same as saying

$V_{cs} = \sqrt{\frac{\mu}{r_{cs}}}$

→

b. $r_p = 1 + r_\oplus$

V = speed in the new orbit to mars.



$r_p = a(1-e)$

I know r_p above. so I need to find a only to determin e.

from $\mathcal{E} = -\frac{\mu}{2a}$ we find a. since

$\mathcal{E} = \frac{V_p^2}{2} - \frac{\mu}{r_p}$ is known.

first convert to canonical: $V_p = 37{,}000 \times 3.85560785 \times 10^{-5}$

$= 1.426574905$ DU$_\oplus$/TU$_\oplus$

and $r_p = 1.029036655$ from part (a).

so $\mathcal{E} = \frac{1.426574905^2}{2} - \frac{1}{1.029036655} = 0.045775298$ DU²/TU²

so from $\mathcal{E} = -\frac{\mu}{2a} \Rightarrow a = \frac{-\mu}{2\mathcal{E}} = \frac{-1}{2 \times 0.045775298}$

$\boxed{a = -10.92292179 \text{ DU}_\oplus \quad a = -37{,}617.6 \text{ n mi}}$

Since $\mathcal{E}$ is +ve, this is a hyperbola also a in $-$ve as expected.

so since $r_p = a(1-e) \Rightarrow r_p = a - ae$

so $e = -\left(\frac{r_p - a}{a}\right) = -\left(\frac{1.029036655 - (-10.92292179)}{-10.92292179}\right)$

$\boxed{e = 1.0942089}$ L.Lb

$\rightarrow$

$h = r_p v_p =$

$= 1.029036655 \times 1.426574905$

$\boxed{h = 1.4679978 \quad DU^2/TU}$

the orbit is a hyperabola since $e > 1$.

<u>C</u> now we are asked to find V at ALT = 1,000,000 nmi from earth to V_∞.

$r = ALT + r_\oplus = 1\times10^6 \times 2.903665564\times10^{-4} + 1$

$= 290.3665564 + 1$

$= 291.3665564 \quad DU_\oplus$

Since $\mathcal{E}$ is constant over the orbit, then

$$\mathcal{E} = \frac{V_p^2}{2} - \frac{\mu}{r_p} = \frac{V^2}{2} - \frac{\mu}{r}$$

Mars

where V_p is the burn out velocity. it is 37,000 ft/sec in this example. and $r_p = Alt + r_\oplus$ at time of burn out.

so $$\frac{V^2}{2} = \frac{V_p^2}{2} - \frac{\mu}{r_p} + \frac{\mu}{r} = \frac{1.426574905^2}{2} - \frac{1}{1.029036655} + \frac{1}{291.3665564}$$

so $\frac{V^2}{2} = 0.049207401 \Rightarrow V = 0.313711335 \; DU/TU = \boxed{1 \quad n\,mi/sec.}$

$\longrightarrow$

now find the hyperbolic excess velosity V_∞

$$\mathcal{E} = \frac{V_{bo}^2}{2} - \frac{\mu}{r_{bo}} = \frac{V_\infty^2}{2} - \cancel{\frac{\mu}{r_\infty}}^{=0}$$

so $$\frac{V_\infty^2}{2} = \frac{V_{bo}^2}{2} - \frac{\mu}{r_{bo}}$$

$$= \frac{1.426574905^2}{2} - \frac{1}{1.029036655}$$

$$= 0.045775298$$

so $V_\infty = 0.30257327$ DU/TU

$$\boxed{V_\infty = 1.291 \quad nmi/sec.}$$

so

V at 1,000,000 nmi from earth	= 1 n mi/se
V_∞	= 1.291 n mi/sec

the escape speed is $\sqrt{\frac{2\mu}{r_p}} = \sqrt{\frac{2}{1.029036655}} = 1.394118131$ DU/TU

$= 36,158$ ft/sec.

the above is the escape speed that will cause the satallite to escape the circular orbit such that at $r \gg 1$, $V \to 0$. i.e no excess velosity at $r=\infty$.

notice that the burn out speed is 37,000 ft/sec. which is is almost like the Vescap. the difference, which is small, explain why the difference in V at $r=1,000,000$ and $r=\infty$ is small also.

problem 1.14 BMW (Abbasi)

problem: given equation $r = \frac{P}{1+e\cos\theta}$ plot at least 4 points, sketch and identify the locus and label the major dimensions for the following sections:

a. $P=2$ $e=0$

b. $P=6$ $e=.2$

c. $P=6$ $e=.6$

d. $P=3$ $e=1$

e. $P=2$ $e=2$

Assumptions:

method: plug in $\theta=0$ (to find r_P), $\theta=90$ (to find P), $\theta=180$ (to find r_A) $\theta=135°$ ($45°$ off P). This gives 4 points. by symmetry plot the bottom half of the section.

Analysis:

a since $e=\frac{c}{a}$ and $e=0$, hence $c=0$ i.e. this is a circle.

hence $r=2$.

b $\theta=0°$ $r_P = \frac{6}{1+.2\cos 0°} = \frac{6}{1.2} = 5$

$\theta=90°$ $P = \frac{6}{1} = 6$

$\theta=135°$ $r = \frac{6}{1+.2\cos 135°} = 6.988$

$\theta=180°$ $r_A = \frac{6}{1+.2\cos 180} = \frac{6}{1-.2} = 7.5$

See plot next page.

c $\theta=0°$ $r_P = \frac{6}{1+.6\cos 0} = \frac{6}{1.6} = 3.75$

$\theta=90°$ $P = 6$

$\theta=135°$ $r = \frac{6}{1+.6\cos 135} = 10.4214$

$\theta=180°$ $r_A = \frac{6}{1-.6} = 15$

see plot next page.

$\longrightarrow$

d $\nu = 0°$ $\quad r_p = \dfrac{3}{1+\cos 0°} = \dfrac{3}{2}$ (since $e=1$, this is a parabola).

$\nu = 90°$ $\quad P = \dfrac{3}{1} = 3$

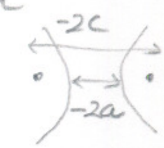
$\nu = 135°$ $\quad r = \dfrac{3}{1+\cos 135°} = 10.242$

plot is next page.

e since $e>1$ this is a hyperbola.

magnitude of distance between foci is $2c$

magnitude of distance between edges is $2a$



$r_A = \dfrac{2}{1+2\cos 180} = -2$

$r_p = \dfrac{2}{3}$ (since $\nu = \phi$)

so $2a = -2 + \frac{2}{3} = \dfrac{-6+2}{3} = \boxed{-\frac{4}{3} = 2a}$

$2c = 2a - 2(r_p) = 2(a - r_p) = 2\left(-\frac{4}{3} - \frac{2}{3}\right)$

$= 2\left(\dfrac{-4-2}{3}\right) = 2\left(\dfrac{-6}{3}\right)$

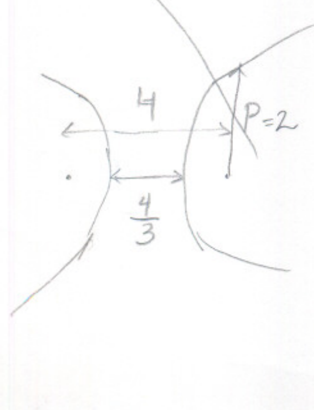
$= \dfrac{-12}{3} = \boxed{-4 = 2c}$

$\nu = 90°$ $\quad P = \dfrac{2}{1+2\cos 90} = 2$

plot is next page.

Please see different solution next page without needing to find r_A, r_p

i.e



e) $e=2 \qquad p=2$

I'll use general representation (not polar) for hyperabola.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{———} \; (1)$$

where $p = \frac{b^2}{a}$, a the semimajor axis, b the semiminor axis.

now $p = a(1-e^2)$

hence $a = \frac{2}{1-4} = \boxed{-\frac{2}{3}}$ (-0.666) ✓ $\Rightarrow \boxed{2a = -\frac{4}{3}}$ ✓

hence (1) now is $\frac{x^2}{(-2/3)^2} + \frac{y^2}{2(-2/3)} = 1$

i.e $\boxed{9x^2 - 3y^2 = 4}$ ✓

but $e = \frac{c}{a} \Rightarrow C = -\frac{4}{3}$ (-1.333) $\Rightarrow \boxed{2c = -\frac{8}{3}}$

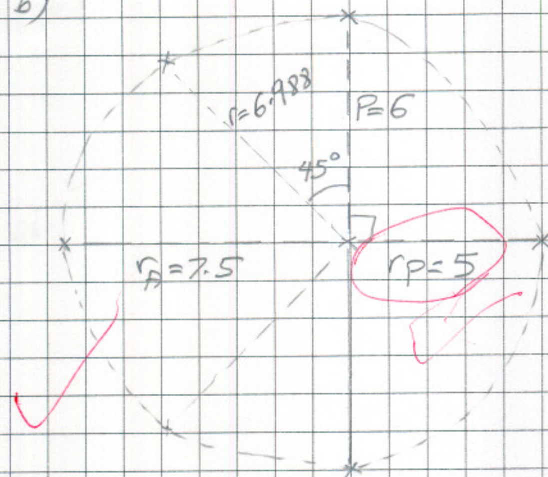
let me find few more points

x	y
-3	~ 5.066
-4	~ 6.83
-5	~ 8.582
-0.6666	0
-1.3333	2

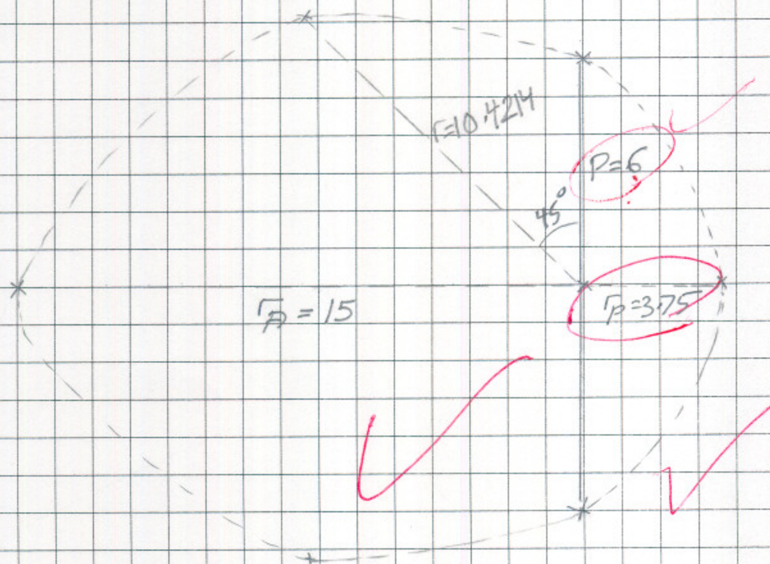
Plot next page $\rightarrow$

Part b)

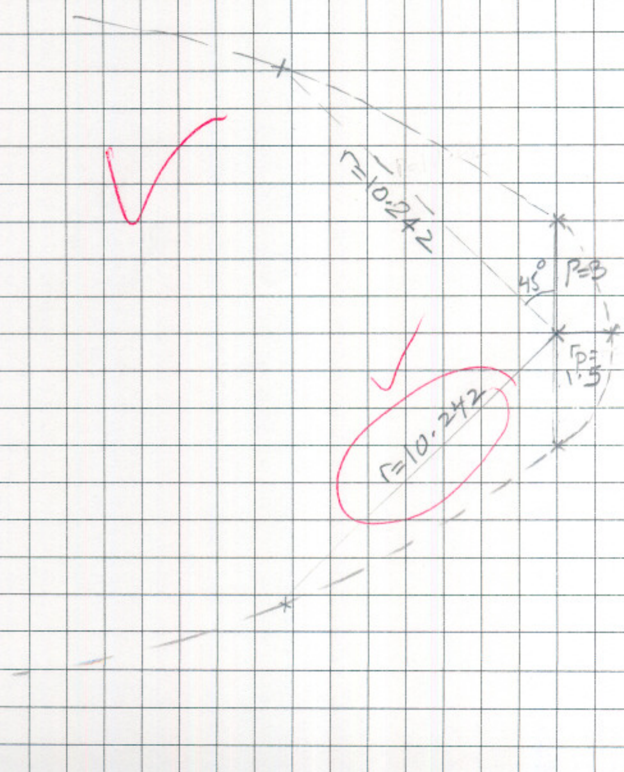
Is it ellipse?



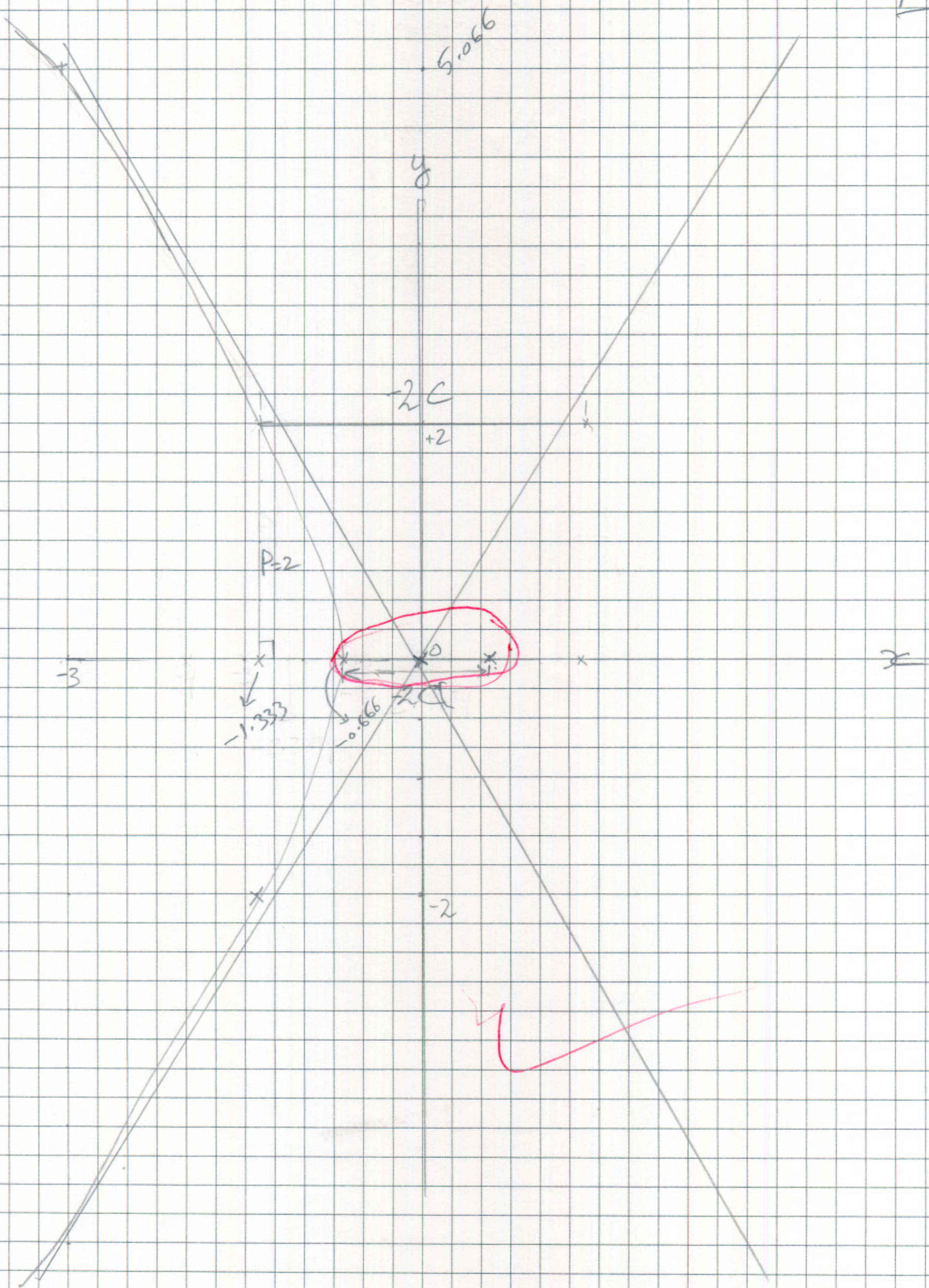
Part c)



part d)



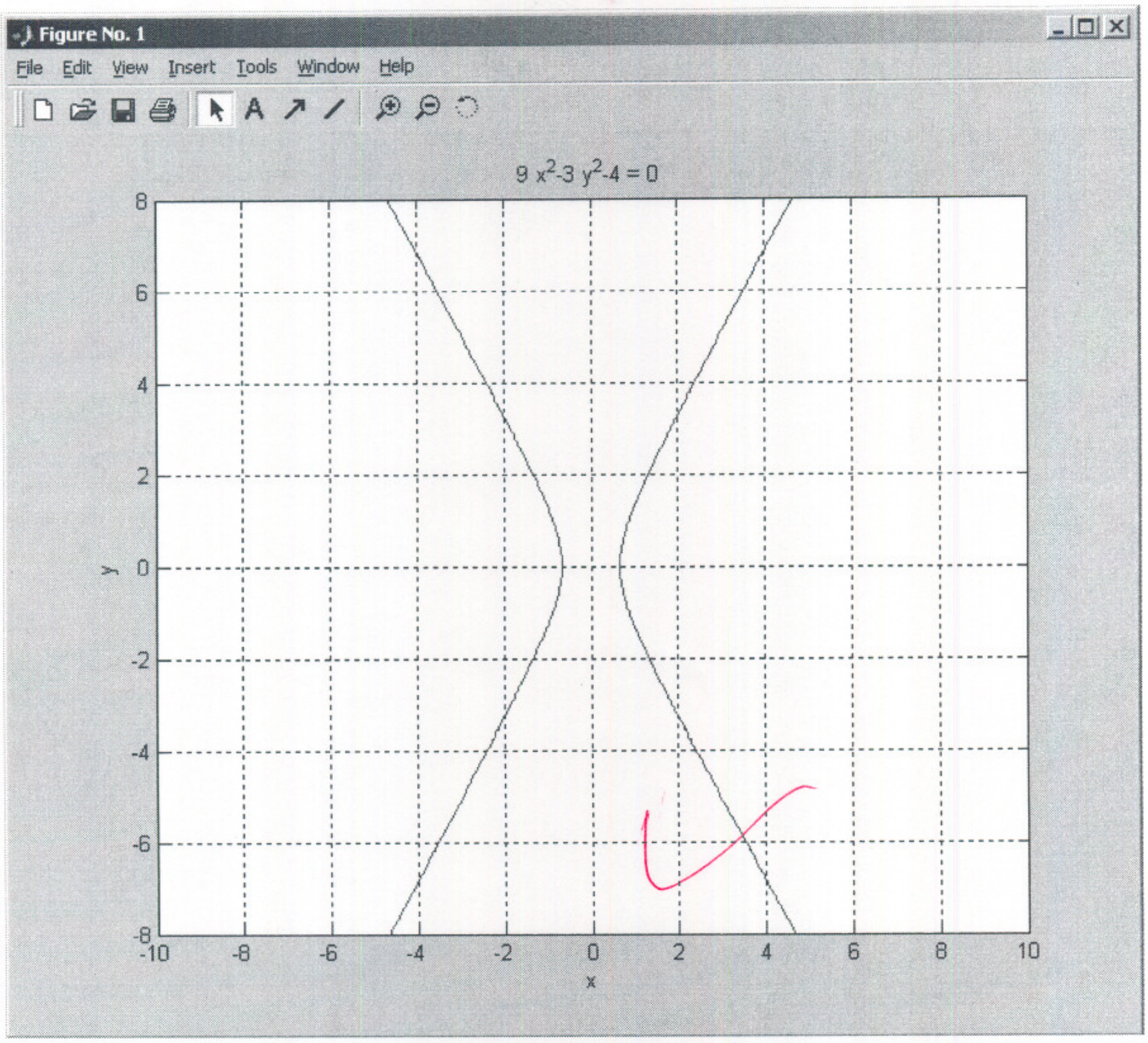
Part e



(need to set polar paper)

this is hard to draw by hand! next I draw again using matlab →

```
>>
>> ezplot('9*x^2-3*y^2-4',[-8 8]);
```



Name Abbasi

problem 2.2 BMW

problem $\bar{r} = -.707\bar{I} + .707\bar{J}$ DU

$\bar{v} = \frac{1}{2}\bar{J}$ DU/TU

Determine orbital elements and sketch the orbit.

Assumptions: use geocentric-equatorial coordinate system.

Method: per section 2.4 BMW.

Analysis:

$$\bar{h} = \bar{r}\times\bar{v} = \begin{vmatrix} i & j & k \\ -.707 & .707 & 0 \\ 0 & .5 & 0 \end{vmatrix} = \bar{i}(0-0) - \bar{j}(0-0) + \bar{k}(-.707\times.5-0)$$

$$= -0.3535\ \bar{k}$$

$h = 0.3535$ DU²/TU

$$\bar{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\bar{r} - (\bar{r}\cdot\bar{v})\bar{v}\right]$$ from eq 2.4-5 BMW

$v = \frac{1}{2}$, $r = \sqrt{(-.707)^2 + (.707)^2} = 0.999848988$

$$\bar{e} = \frac{1}{\mu}\left[\left(.5^2 - \frac{1}{0.999848988}\right)(-.707\bar{I} + .707\bar{J}) - (.707\times.5)\frac{1}{2}\bar{J}\right]$$

$$= \frac{1}{\mu}\left[+.530356781\,\bar{I} - .530356781\,\bar{J} - .707\,\bar{J}\right]$$

$$= \frac{1}{\mu}\left[.530356781\,\bar{I} - 1.237356781\,\bar{J}\right]$$

$e = 1.346228108$

$$\bar{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\bar{r} - (\bar{r}\cdot\bar{v})\,\bar{v}\right] = \frac{1}{\mu}\left[\left(\left(\frac{1}{2}\right)^2 - \frac{1}{\sqrt{\left(\frac{-707}{1000}\right)^2 + \left(\frac{707}{1000}\right)^2}}\right)\bar{r}\right.$$

$$\left. - \left(\left(\frac{1}{2}\right)\frac{707}{1000}\right)\frac{1}{2}\bar{J}\right]$$

$$\bar{e} = \left(\frac{1}{4} - \frac{1}{\sqrt{2\left(\frac{707}{1000}\right)^2}}\right)\left(-\frac{707}{1000}\bar{I} + \frac{707}{1000}\bar{J}\right) - \frac{1}{4}\,\frac{707}{1000}\bar{J}$$

$$\bar{e} = \left(\frac{1}{4} - \frac{1}{\sqrt{2}}\,\frac{1000}{707}\right)\left(-\frac{707}{1000}\bar{I} + \frac{707}{1000}\bar{J}\right) - \frac{1}{4}\,\frac{707}{1000}\bar{J}$$

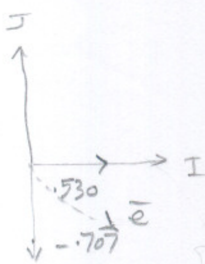
$$\bar{e} = \left(\frac{\sqrt{2}\;707 - 4000}{4\sqrt{2}\;707}\right)\left(-\frac{707}{1000}\bar{I} + \frac{707}{1000}\bar{J}\right) - \frac{1}{4}\,\frac{707}{1000}\bar{J}$$

$$\bar{e} = -\frac{707\sqrt{2} - 4000}{4\sqrt{2}\;1000}\bar{I} + \frac{\sqrt{2}\;707 - 4000}{4\sqrt{2}\;1000}\bar{J} - \frac{1}{4}\,\frac{707}{1000}\bar{J}$$

$$\bar{e} = \frac{4000 - 707\sqrt{2}}{4\sqrt{2}\;1000}\bar{I} + \frac{707\sqrt{2} - 4000 - \sqrt{2}\;707}{4\sqrt{2}\;1000}\bar{J}$$

$$\bar{e} = \frac{4000 - 707\sqrt{2}}{4\sqrt{2}\;1000}\bar{I} + \frac{-4000}{4000\sqrt{2}}\bar{J}$$

$$\bar{e} = \frac{4000 - 707\sqrt{2}}{4\sqrt{2}\;1000}\bar{I} - \frac{1}{\sqrt{2}}\bar{J} \approx (.530\,I - .707\,J)$$



$$|\bar{e}| = \sqrt{\left(\frac{4000 - 707\sqrt{2}}{4\sqrt{2}\;1000}\right)^2 + \frac{1}{2}}$$

$$= \sqrt{\frac{4000^2 - 8000 \times 707\sqrt{2} + 707^2 \times 2}{16 \times 2 \times 1000 \times 1000} + \frac{1}{2}} = \sqrt{0.28127 + 0.5} = \sqrt{0.7812}$$

$$\boxed{e = 0.8839}$$

$$\bar{h} = -\left(\frac{707}{1000} \times \frac{5}{10}\right) \bar{k}$$

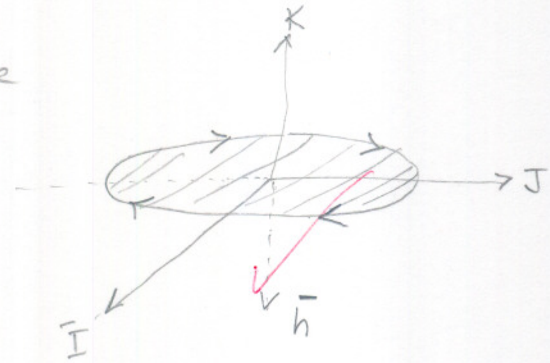
so $h = \dfrac{707}{2000}$

so $P = \dfrac{h^2}{\mu} = \left(\dfrac{707}{2000}\right)^2 = \dfrac{499849}{4000000} \simeq \dfrac{5 \times 10^5}{4 \times 10^6} \simeq \dfrac{5}{40} \cong \boxed{\dfrac{1}{8}\ DU}$

<u>inclination i</u>

$$i = \cos^{-1} \frac{h \cdot K}{h} = \cos^{-1} \frac{-\frac{707}{2000}}{\frac{707}{2000}} = \cos^{-1} -1$$

$\boxed{i = 180^\circ}$ so orbit is retrograde



$$\bar{n} = \bar{K} \times \bar{h}$$

$$\bar{n} = \begin{vmatrix} i & j & K \\ 0 & 0 & 1 \\ 0 & 0 & \frac{-707}{2000} \end{vmatrix} = \bar{0}$$

undefined. makes sense as orbit plane is on the fundental plane, i.e it does not cross it any where.

so $\boxed{\Omega \text{ undefined}}$

also $\boxed{\omega \text{ undefined}}$ since no $\bar{n}$ exist.

$$\cos \nu_0 = \frac{e \cdot r}{er} = \frac{\frac{-707}{1000}\left(\frac{4000 - 707\sqrt{2}}{4\sqrt{2}\ 1000}\right) - \frac{1}{\sqrt{2}} \frac{707}{1000}}{(0.8839)(1)}$$

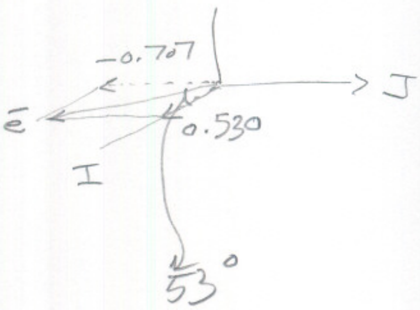
$= -0.9898 \Rightarrow \boxed{\nu_0 = 171.8^\circ}$ since $r \cdot v > 0$

$\Pi = \Omega + \omega$. undefined since both Ω and ω are undefined.

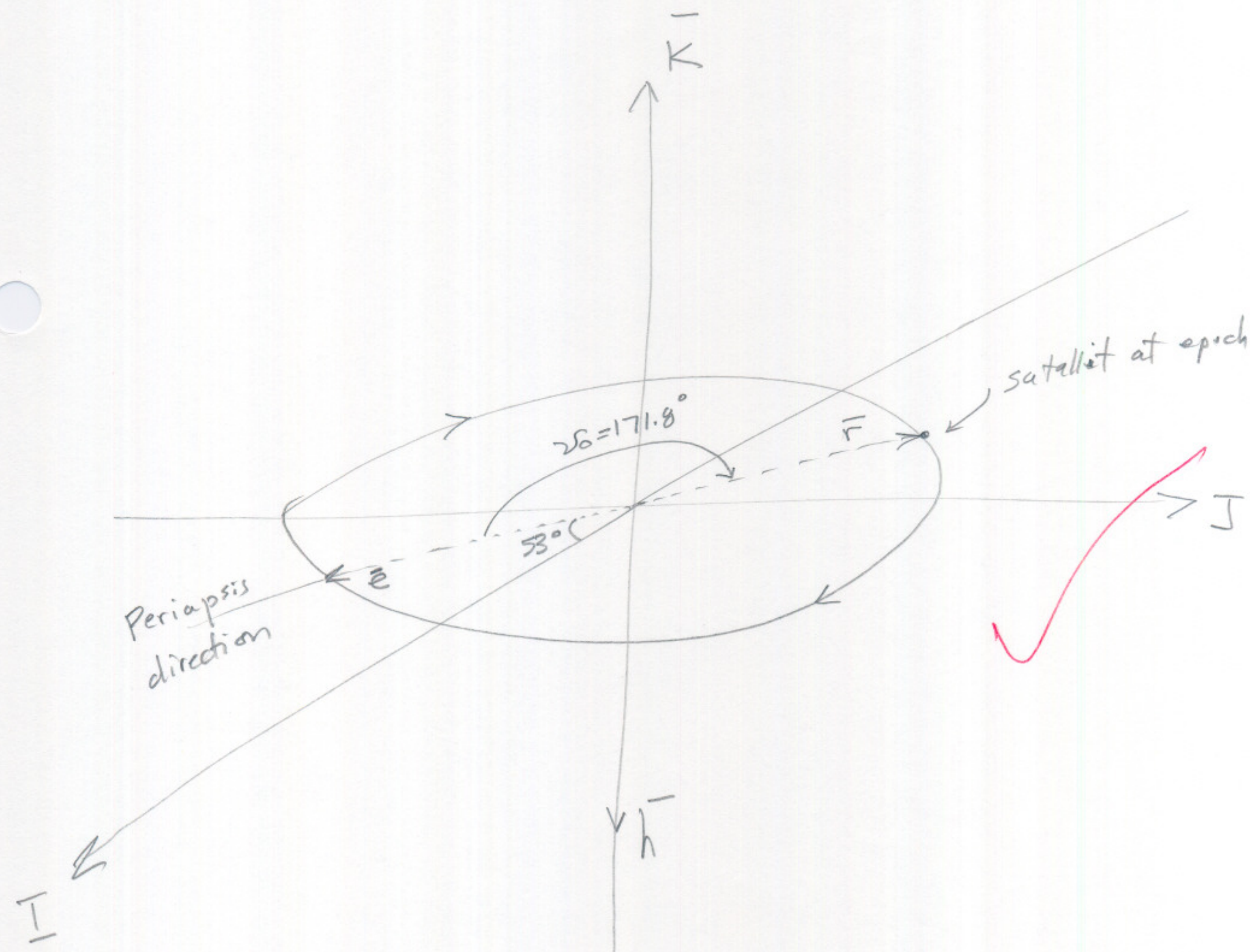
$\bar{e}$ is on the Fundamental plane. $(0.530\bar{I} - 0.707\bar{J})$

so angle $\bar{e}$ makes with I is

$\tan^{-1} \frac{.707}{.53} = 53^\circ$



so sketch of orbit is



problem 2.7 BMW

Problem: given $r_0 = -I - J - K$ $DU_\oplus$

$V_0 = \frac{1}{3}(I - J + K)$ $DU_\oplus / TU_\oplus$

in geocentric equatorial coordinate system. Determine the orbital elements.

Assumptions:

Method: per section 2.4 BMW.

Analysis:

$$\bar{h} = \bar{r} \times \bar{v} = \begin{vmatrix} i & j & k \\ -1 & -1 & -1 \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{vmatrix} = \bar{i}\left(-\frac{1}{3} - \frac{1}{3}\right) - \bar{j}\left(-\frac{1}{3} + \frac{1}{3}\right) + \bar{k}\left(\frac{1}{3} + \frac{1}{3}\right)$$

$$= -\frac{2}{3}\bar{I} + 0\bar{J} + \frac{2}{3}\bar{K}$$

$$\text{so } h = \sqrt{\left(-\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \sqrt{2}\,\frac{2}{3}$$

$$\text{so } p = \frac{h^2}{\mu} = 2 \cdot \frac{4}{9} = \boxed{\frac{8}{9}}$$

$$\bar{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\bar{r} - (\bar{r} \cdot \bar{v})\bar{v}\right]$$

$$v = \sqrt{\left(\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}} = \sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{3}$$

$$r = \sqrt{1 + 1 + 1} = \sqrt{3}$$

$$\text{so } \bar{e} = \left(\frac{3}{9} - \frac{1}{\sqrt{3}}\right)(-I - J - K) - \left(-\frac{1}{3} + \frac{1}{3} - \frac{1}{3}\right)\left(\frac{1}{3}I - \frac{1}{3}J + \frac{1}{3}K\right)$$

$$= \frac{3\sqrt{3} - 9}{9\sqrt{3}}(-I - J - K) + \frac{1}{9}I - \frac{1}{9}J + \frac{1}{9}K$$

$$= \frac{9 - 3\sqrt{3}}{9\sqrt{3}}I + \frac{9 - 3\sqrt{3}}{9\sqrt{3}}J + \frac{9 - 3\sqrt{3}}{9\sqrt{3}}K + \frac{1}{9}I - \frac{1}{9}J + \frac{1}{9}K$$

$$= \frac{9 - 3\sqrt{3} + \sqrt{3}}{9\sqrt{3}}\bar{I} + \frac{9 - 3\sqrt{3} - \sqrt{3}}{9\sqrt{3}}\bar{J} + \frac{9 - 3\sqrt{3} + \sqrt{3}}{9\sqrt{3}}\bar{K} \rightarrow$$

so $|\bar{e}| = \sqrt{2\left(\frac{9-3\sqrt{3}+\sqrt{3}}{9\sqrt{3}}\right)^2 + \left(\frac{9-3\sqrt{3}-\sqrt{3}}{9\sqrt{3}}\right)^2}$

$$= \sqrt{2\left(\frac{9-2\sqrt{3}}{9\sqrt{3}}\right)^2 + \left(\frac{9-4\sqrt{3}}{9\sqrt{3}}\right)^2} = \sqrt{2\,\frac{(9-2\sqrt{3})^2 + (9-4\sqrt{3})^2}{(9\sqrt{3})^2}}$$

$$= \frac{\sqrt{2}}{9\sqrt{3}}\sqrt{(9-2\sqrt{3})^2 + (9-4\sqrt{3})^2} = \frac{\sqrt{2}}{9\sqrt{3}}\sqrt{9^2+(4)(3)-4\times9\sqrt{3}+9^2+16(3)-8\times9\sqrt{3}}$$

$$= \frac{\sqrt{2}}{9\sqrt{3}}\sqrt{2\times9^2+12+48-12\times9\sqrt{3}} = \frac{\sqrt{2}}{9\sqrt{3}}\sqrt{162+60-108\sqrt{3}}$$

$$= \frac{\sqrt{2}}{9\sqrt{3}}\sqrt{222-108\sqrt{3}} = \frac{\sqrt{2}}{9\sqrt{3}}\sqrt{2(111-54\sqrt{3})}$$

$$= \frac{2}{9\sqrt{3}}\sqrt{111-54\sqrt{3}} = \boxed{0.536246}$$

<u>inclination i</u>

$$i = \cos^{-1}\frac{\bar{h}\cdot\bar{K}}{h} = \cos^{-1}\frac{\frac{2}{3}}{\sqrt{2}\ \frac{2}{3}} = \cos^{-1}\frac{1}{\sqrt{2}} = 45^\circ$$

$$\bar{n} = \bar{K}\times\bar{h}$$

$$= \begin{vmatrix} I & J & K \\ 0 & 0 & 1 \\ -\frac{2}{3} & 0 & \frac{2}{3} \end{vmatrix} = I(0-0) - J\left(0+\frac{2}{3}\right) + K(0) = -\frac{2}{3}\bar{J}$$

so $\Omega = \cos^{-1}\frac{\bar{n}\cdot\bar{I}}{n} = \cos^{-1}\frac{0}{2/3} = \cos^{-1}0 = \boxed{90^\circ}$ but $\bar{n}\cdot\bar{j} < 0$ so $\boxed{\Omega = 270^\circ}$

$$\omega = \cos^{-1}\left(\frac{\bar{n}\cdot\bar{e}}{ne}\right) = \frac{\left(-\frac{2}{3}\right)\left(\frac{9-4\sqrt{3}}{9\sqrt{3}}\right)}{\left(\frac{2}{3}\right)(0.536246)} = \cos^{-1}-0.2538 \cong 104.702^\circ$$

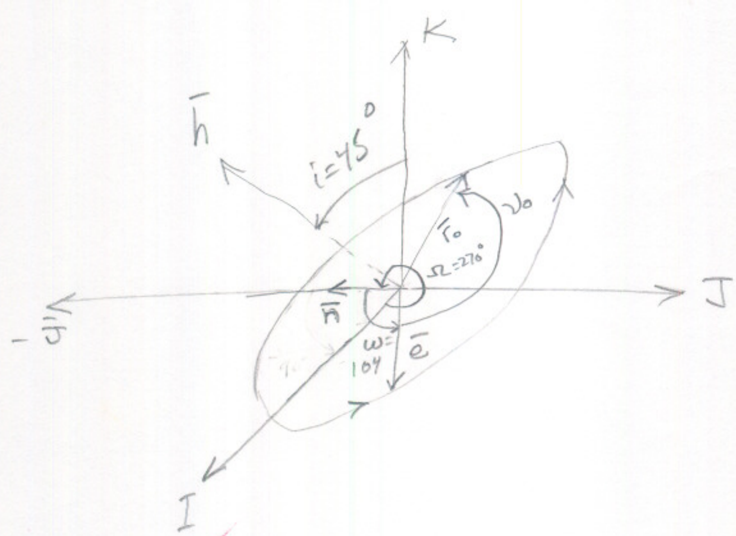
$$\boxed{\cong 104^\circ\ 42'}$$

$$\nu_0 = \cos^{-1} \frac{\bar{e}\cdot\bar{r}}{er} = \frac{-\left(\frac{9-2\sqrt{3}}{9\sqrt{3}}\right) - \left(\frac{9-4\sqrt{3}}{9\sqrt{3}}\right) - \left(\frac{9-2\sqrt{3}}{9\sqrt{3}}\right)}{(0.536246)\sqrt{3}}$$

$$= \cos^{-1} \frac{-9+2\sqrt{3} - 9+4\sqrt{3} - 9+2\sqrt{3}}{9\,(0.536246)\,3} = \cos^{-1} \frac{-27+8\sqrt{3}}{9\,(0.536246)\,3}$$

$$= \cos^{-1} -0.9077918 = 155.20^\circ$$

$$\boxed{\nu_0 \simeq 155^\circ \ 12'}$$



$$\Pi = \Omega + \omega = 270 + 104.7 = 374.7^\circ$$

$$u_0 = \omega + \nu_0 = 104.702 + 155.20 = 259.9^\circ$$

104 +

Nasser Abbasi

problem 2.6 BMW

problem: given $\bar{r} = 1.2\bar{K}$ DU, $\bar{V} = .4\bar{I} - 0.3\bar{K}$ DU/TU,
find $p, e, i, u_0, \Omega, \omega, \nu_0, l_0$ and the latitude of impact.

Assumptions:

Method: per section 2.4 in BMW book.

$$\bar{r} = 1.2\bar{K}$$

$$\bar{v} = 0.4\bar{I} - 0.3\bar{K}$$

$$\bar{h} = \bar{r}\times\bar{v} = \begin{vmatrix} I & J & K \\ 0 & 0 & 1.2 \\ 0.4 & 0 & -0.3 \end{vmatrix} = \bar{I}(0) - J\left(-1.2\times 0.4\right) + K(0)$$

$$= 0.48\,\bar{J} \quad DU^2/TU$$

so $h = 0.48 \quad DU^2/TU$

$$p = \frac{h^2}{\mu} \implies \boxed{p = 0.2304 \quad DU}$$

$$v = \sqrt{0.4^2 + 0.3^2} = 0.5$$

$$r = 1.2$$

so $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = \frac{0.5^2}{2} - \frac{1}{1.2} = -0.7083333$

$$e = \sqrt{1 + \frac{2\mathcal{E}h^2}{\mu^2}} = \sqrt{1 + \frac{2(-0.7083333)(0.48)^2}{1}} = \boxed{0.82073}$$

$$\bar{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\bar{r} - (\bar{r}\cdot\bar{v})\bar{v}\right]$$

$$= \left(0.5^2 - \frac{1}{1.2}\right)1.2\bar{k} - (-1.2\times .3)(.4\bar{I} - .3\bar{k})$$

$$= -0.7\bar{k} + 0.144\bar{I} - 0.108\bar{k}$$

$$\boxed{\bar{e} = 0.144\bar{I} - 0.808\bar{k}}$$

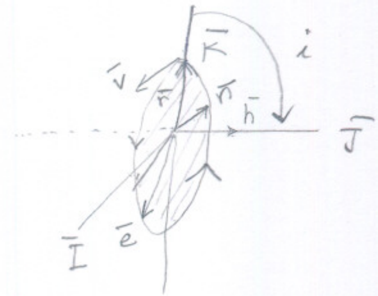
$\longrightarrow$

inclination i

$$i = \cos^{-1} \frac{\bar{h} \cdot \bar{K}}{h} = \cos^{-1} \frac{0.48\bar{J} \cdot \bar{K}}{0.48} = \cos^{-1} 0 = 90^\circ$$

(i always $< 180^\circ$)

(should i be -90° since using R.H.R ?)



$$\bar{n} = \bar{K} \times \bar{h} = \begin{vmatrix} I & J & K \\ 0 & 0 & 1 \\ 0 & 0.48 & 0 \end{vmatrix} = I(-.48) - J(0) + K(0) = -.48\bar{I}$$

$$\Omega = \cos^{-1} \frac{\bar{n} \cdot \bar{I}}{n} = \frac{-.48}{.48} = -1$$

so $\boxed{\Omega = +180^\circ}$

$$\omega = \cos^{-1} \frac{\bar{n} \cdot \bar{e}}{ne} = \cos^{-1} \frac{(-.48\bar{I}) \cdot (.144\bar{I} - .808\bar{K})}{(.48)(0.82073)}$$

$$\omega = \cos^{-1} -.1754535 = 1.7471627 \text{ radians}$$

$$\boxed{\omega = 100.105^\circ}$$

$$\nu_0 = \cos^{-1} \frac{\bar{e} \cdot \bar{r}}{er} = \frac{(.144\bar{I} - 0.808\bar{K}) \cdot (1.2\bar{K})}{0.82073 \times 1.2}$$

$$= \cos^{-1} -0.984489$$

$$\boxed{\nu_0 = 169.895^\circ}$$

$$u_0 = \cos^{-1} \frac{\bar{n} \cdot \bar{r}}{nr} = \frac{-.48\bar{I} \cdot 1.2\bar{K}}{.48 \times 1.2} = 0 \Rightarrow u_0 = 90^\circ$$

$$l_0 = \Omega + u_0 = 180 + 90 = \boxed{270^\circ}$$

(Abbasi

problem 3.3 expensive book.

problem; see page 93.

40/40

Assumptions:

Method:

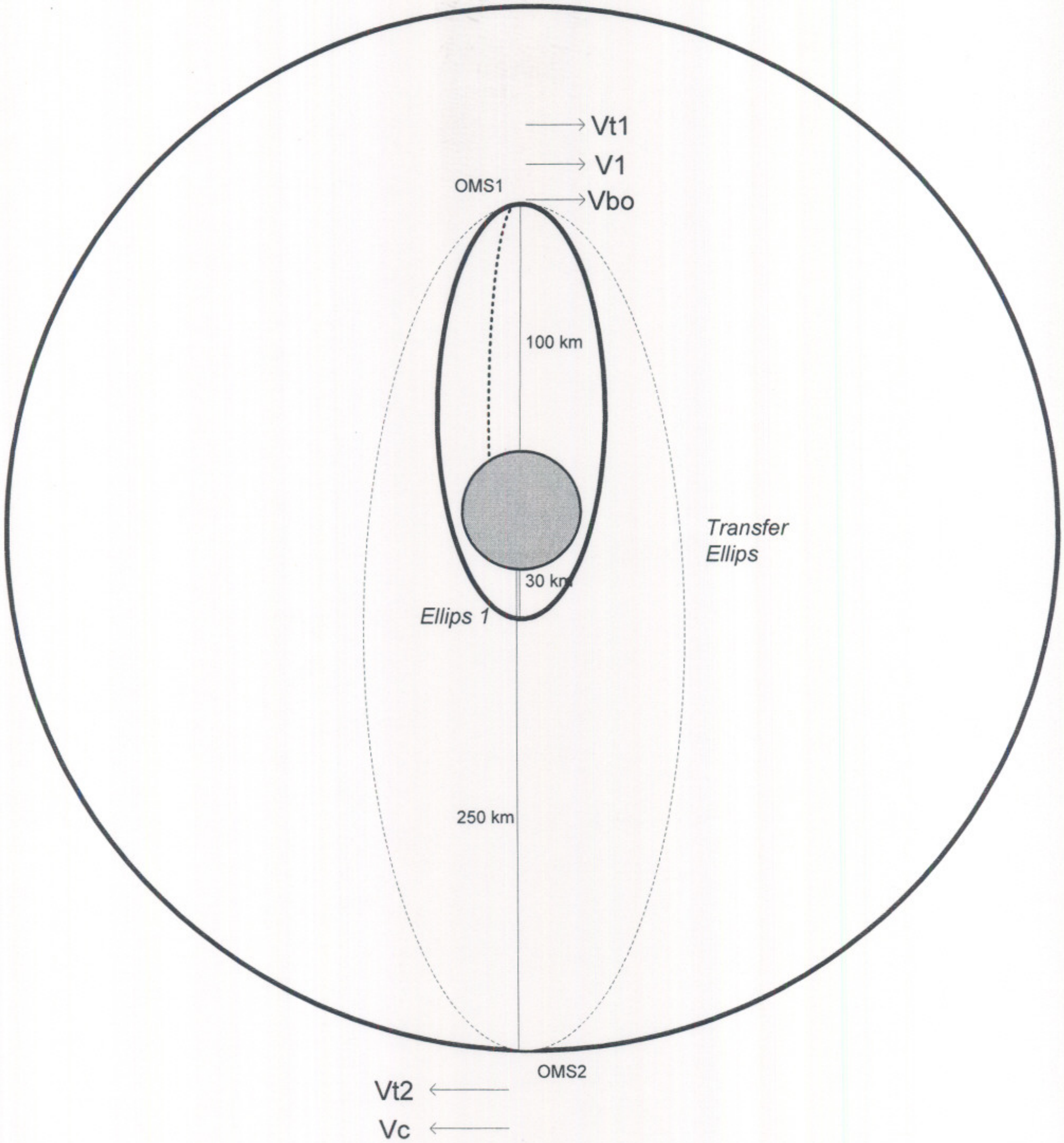
See diagram next page.

Need to find Vbo at OMS1 point. V_{bo} is the Same as V_1 (the Velosity for ellips 1 at the apogee). so find $\mathcal{E}_t$ for ellips 1, and from that find V_1.

to solve part (b); find V_{t1}, which is speed in the transfer ellips at OMS1 point. $V_1 - V_{t1}$ gives the answer.

for part (c): find V_c, which is the circular speed for final orbit, find V_{t2}, which is transfer orbit speed at OMS2. difference gives the answer.

→



Vt1
V1
OMS1
Vbo
100 km
Transfer
Ellips
30 km
Ellips 1
250 km
OMS2
Vt2
Vc

Analysis:

First convert to canonical units.

$$r_{1A} = 100\ km + r_\oplus = (100)\cdot\frac{1}{6378.145}+1 = 1.015678539\ \text{DU}$$

$$r_{1P} = 30\ Km + r_\oplus = (30)\ \frac{1}{6378.145}+1 = 1.00470356193\ \text{DU}$$

$$r_{2P} = r_{1A} = 1.015678539\ \text{DU}$$

$$r_{2A} = 250\ Km + r_\oplus = (250)\ \frac{1}{6378.145}+1 = 1.03919635\ \text{DU}$$

(a)

<u>For ellips 1</u>

$$\mathcal{E}_1 = -\frac{\mu}{2a_1}$$

but $2a_1 = r_{1A} + r_{1P} = 1.015678539 + 1.00470356193$

$$= 2.0203821\ \text{DU}$$

so $\mathcal{E}_1 = -\dfrac{1}{2.0203821} = -0.49495588\ \text{DU}^2/\text{TU}^2$

but $V_1 = \sqrt{2\left(\frac{\mu}{r_{1A}} + \mathcal{E}_1\right)} = \sqrt{2\left(\frac{1}{1.015678539} - 0.49495588\right)}$

$$V_1 = 0.989553\ \text{DU/TU}$$

hence $\boxed{V_{bo} = 0.989553\ \text{DU/TU} = 7.82278\ \text{KM/sec}}$

→

(b) To find ΔV_1 need to find V_{t1}, the speed for transfer orbit at OMS1.

for transfer ellipse,

$$2a_t = r_{tP} + r_{tA} = 1.015678539 + 1.03919635$$

$$= 2.054874889 \text{ DU}.$$

hence $\mathcal{E}_t = -\frac{\mu}{2a_t} = -\frac{1}{2.054874889} = -0.4866476 \text{ DU}^2/\text{TU}^2$

So $V_{t1} = \sqrt{2\left(\frac{\mu}{r_{tP}} + \mathcal{E}_t\right)} = \sqrt{2\left(\frac{1}{1.015678539} - 0.4866476\right)}$

$$= 0.997913674 \text{ DU/TU}$$

hence $\Delta V_1 = |V_{t1} - V_1| =$

$$= |0.997913674 - 0.989553| = 0.008360674 \text{ DU/TU}$$

$$\boxed{= 0.0660942 \text{ KM/sec}}$$

ΔV_1 is speed increase since $V_{t1} > V_1$

→

part c   to find ΔV_2, need to find V_{t2} and V_C.

$$V_{t2} = \sqrt{2\left(\frac{\mu}{r_{tA}} + \mathcal{E}_t\right)} = \sqrt{2\left(\frac{1}{1.03919635} + 0.49495588\right)}$$

$= 0.966774198$ DU/TU

$= 7.64271$ km/sec.

$$V_C = \sqrt{\frac{\mu}{r_c}}$$ but $r_c = 250$ km $= 1.03919635$ DU

so $V_c = \sqrt{\frac{1}{1.03919635}} = 0.980959762$ DU/TU

hence $\Delta V_2 = \left| V_{t2} - V_c \right| = 0.01418556Y$ DU/TU

$\boxed{\Delta V_2 = 0.1121421 \text{ km/se}}$

since $V_{t2} < V_C$, then ΔV_2 is a speed increase.

Aybosi

Problem 3.6 BMW

Problem: determine which of the following orbits could be used to transfer between two circular co-planar orbits with radii 1.2 DU and 4 DU.

a. $r_p = 1\,DU$, $e = .5$

b. $a = 2.5\,DU$, $e = 0.56$

c. $\mathcal{E} = -0.1\,DU^2/TU^2$, $h = 1.34\,DU^2/TU$

d. $P = 1.95\,DU$, $e = 0.5$

Assumptions: earth orbit, i.e. $\mu = 1$

Method: for each case, find r_p, r_a for the transfer orbit and apply these rules; (see page 167 BMW)

① IF $(r_p > r_1)$ Then not possible.

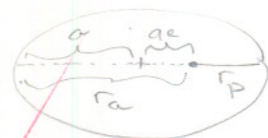
② IF $(r_p \le r_1$ AND $r_a < r_2)$ Then not possible

③ IF $(r_p \le r_1$ AND $r_a \ge r_2)$ Then possible

Analysis:

a since $r_p \le r_1$, need to find r_a to decide.

from ellips geometry



$$r_p = a - ae$$

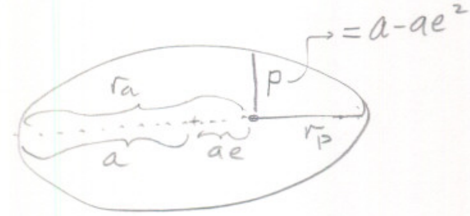
$$\therefore a = \frac{r_p}{1-e} = \frac{1}{.5} = 2\ DU$$

$$\therefore r_a = a + ae = a(1+e) = 2(1.5) = 3\ DU$$

hence $r_a < r_2$.

This is case ②. $\Longrightarrow$ $\boxed{\text{NOT Possible}}$

b. From ellips geometry



when $\vartheta = 0°$

$$r_p = \frac{P}{1+e}$$

but $P = a - ae^2$, then $r_p = \dfrac{a(1-e^2)}{1+e} = \dfrac{2.5(1-0.56^2)}{1+0.56}$

$$= \frac{1.716}{1.56} = 1.1 \text{ DU}$$

since $r_p \le r_1$ then this is case ① or ②. need to find r_A to decide.

$$r_A = a + ae = a(1+e) = 2.5(1+0.56) = 3.9 \text{ DU}$$

hence $r_A < r_2$. this is case ② $\Longrightarrow$ $\boxed{\text{NOT Possible}}$

c.

Since $P = \dfrac{h^2}{\mu} \Rightarrow P = \dfrac{1.34^2}{1} = 1.7956$ DU/TU

since $\mathcal{E} = -\dfrac{\mu}{2a} \Rightarrow a = -\dfrac{\mu}{2\mathcal{E}} = \dfrac{-1}{(2)(-0.1)} = 5$ DU

but $P = a - ae^2 \Rightarrow e^2 = \dfrac{a-P}{a} = \dfrac{5-1.7956}{5} = 0.64088$

$$\therefore e = \sqrt{0.64088} = 0.8005498$$

Then $r_p = a - ae = a(1-e) = 5(1-0.8005498)$

$$= 0.99725 \text{ DU}$$

since $r_p \le r_1$ Then need to find r_A to decide.

$$r_A = a + ae = a(1+e) = 5(1+0.80054998) = 9 \text{ DU}$$

since $r_A \geq r_2$ $\Longrightarrow$ $\boxed{\text{POSSIBLE}}$

d

since $P = a - ae^2 = a(1-e^2)$

Then $a = \frac{P}{1-e^2} = \frac{1.95}{1-0.5^2} = 1.096875$ DU

Then $r_P = a - ae = a(1-e) = 1.096875(1-.5)$

$= 0.5484$ DU

Since $r_p \le r_1$ then can be case ② or ③. Find r_A to decide.

$r_A = a + ae = a(1+e) = 1.096875(1.5)$

$= 1.6453$ DU

since $r_A < r_2 \Rightarrow \boxed{\text{NOT POSSIBLE}}$

Answer:

a $\rightarrow$ NOT possible

b $\rightarrow$ NOT possible

c $\rightarrow$ possible

d $\rightarrow$ NOT possible

Problem 3.2 Expensive book

problem: see page 93.

Assumptions:

Method:

set up the transfer ellips orbit. find $\mathcal{E}_t$. from this find V_{t_1} which is the same as V_{bo}.

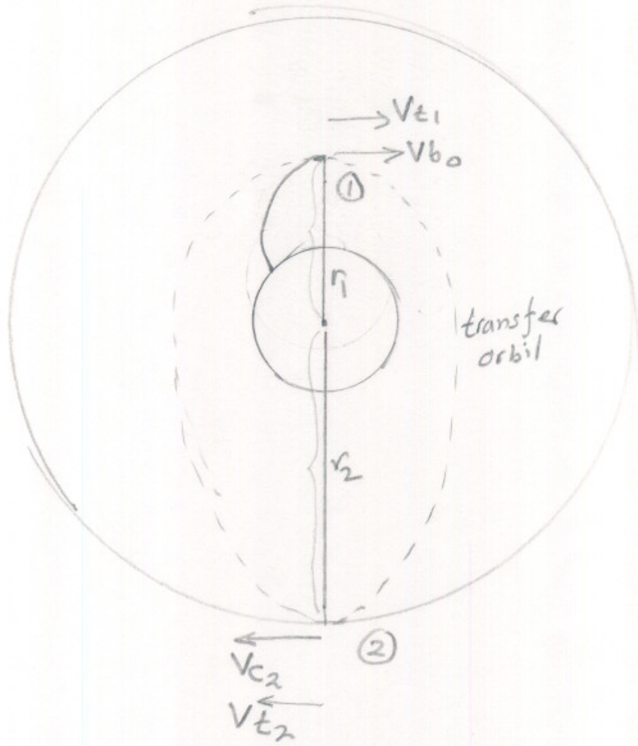
find V_{t_2}, V_{c_2} and from these find ΔV_2 which is the maneuver required to rendezvous.

Coast time is $\frac{\mathbb{P}}{2}$. $\mathbb{P}$ is known from $\mathbb{P} = \frac{2\pi}{\sqrt{\mu}}\sqrt{a_t^3}$

for part (c). since coast time is now known, we find where CM must be by geometry.

→

Analysis



since $\mu \equiv GM$

Then $\left.\begin{aligned} \mu_\oplus &= GM_\oplus \\ \mu_{moon} &= GM_{moon} \end{aligned}\right\} \Rightarrow \dfrac{\mu_{moon}}{\mu_\oplus} = \dfrac{\cancel{G} M_{moon}}{\cancel{G} M_\oplus}$

$= \dfrac{M_{moon}}{M_\oplus} = \dfrac{0.01213 \cancel{M_\oplus}}{\cancel{M_\oplus}} = 0.01213$

So $\dfrac{\mu_{moon}}{\mu_\oplus} = 0.01213$. but $\mu_\oplus = 1$ using Canonical units

hence $\mu_{moon} = 0.01213 \ \dfrac{DU^3}{TU^2}$

part a. $V_{bo} = V_{t1}$ since V_{bo} is parallel to r_1, this must be the same as V_{t1} at that point

Now $2a_t = r_1 + r_2$

but $r_1 = 30 + 1740 = 1770$ KM, $r_2 = 250 + 1740 = 1990$ KM →

Convert to Canonical Units

$r_1 = 0.27751 \quad DU$

$r_2 = 0.312 \quad DU$

so $2a_t = r_1 + r_2 = 0.58951 \quad DU$

so $\mathcal{E}_t = \dfrac{-\mu}{2a_t} = -\dfrac{0.01213}{0.58951} = -0.02057641 \quad \dfrac{DU^2}{TU^2}$

but $\mathcal{E}_t = \dfrac{V^2}{2} - \dfrac{\mu}{r}$

so $\mathcal{E}_t = \dfrac{V_{t_1}^2}{2} - \dfrac{\mu}{r_1} \Rightarrow V_{t_1} = \sqrt{2\left(\frac{\mu}{r_1} + \mathcal{E}_t\right)}$

so $V_{t_1} = \sqrt{2\left(\frac{0.01213}{0.27751} - 0.02057641\right)} = 0.2150987 \quad DU/TU$

but $V_{bo} = V_{t_1}$ hence $\boxed{\begin{aligned} V_{bo} &= 0.2150987 \quad DU/TU \\ &= 1.70173 \quad KM/sec \end{aligned}}$

Part b

need to find ΔV at point 2.

First find V_{t2}

$V_{t2} = \sqrt{2\left(\frac{\mu}{r_2} + \mathcal{E}_t\right)} = \sqrt{2\left(\frac{0.01213}{0.312} - 0.02057641\right)}$

$V_{t2} = 0.1913206 \quad DU/TU$

$\longrightarrow$

but V_c (speed in orbit circular) $= \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{0.01213}{0.312}} = 0.19717557$ DU/TU

hence magnitude of maneuver $= |V_c - V_{t_2}|$

$= |0.19717557 - 0.1913206| = 5.854969299 \times 10^{-3}$ DU/TU

$$\boxed{= 0.0462857 \text{ KM/s}}$$

Part C

From geometry, Coast time $= \frac{P}{2}$

so Coast time $= \frac{1}{2}\left(2\pi\sqrt{\frac{a_t^3}{\mu}}\right)$, but $a_t = 0.294755$ DU

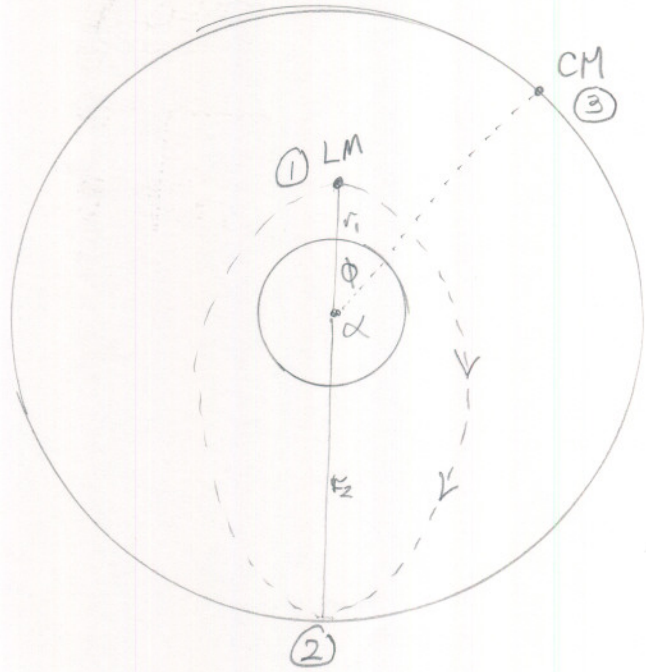
$= \frac{1}{2}\left(2\pi\sqrt{\frac{0.294755^3}{0.01213}}\right)$

$= 4.56468$ TU

$= 3682.84$ sec $= 61.3807$ mins.

$$\boxed{= 1 \text{ hr and } 1.3807 \text{ min}}$$

part d



Need to find Φ.

we know it takes 4.56468 TU for LM to go from point 1 to point 2. hence CM must take the same time from 3 to 2.

Find $\mathbb{P}$ of CM. $= 2\pi\sqrt{\dfrac{a_{cm}^3}{\mu}}$

but a for CM $= r_2$ since circle.

Then $\mathbb{P}$ for CM $= 2\pi\sqrt{\dfrac{0.312^3}{0.01213}} = 9.9421739$ TU.

hence 

by symmetry, $\dfrac{2\pi}{\alpha} \equiv \dfrac{9.9421739}{4.56468}$

solve for $\alpha \Rightarrow \quad \alpha = \dfrac{(2\pi)(4.56468)}{9.9421739} = 2.884754$ radians

$= \dfrac{180}{\pi}(2.884754) = 165.28^\circ$

so $\phi = \pi - \alpha$

hence $\Phi = 0.256838$ radians

$= (0.256838)\dfrac{180}{\pi} = \boxed{14.7157^\circ}$

so CM must be ahead of LM by this amount.

time differential follows easily from $\dfrac{\mathbb{P}}{\Delta T} \equiv \dfrac{2\pi}{0.2568} \Rightarrow \boxed{\Delta T = 0.406\,TU}$

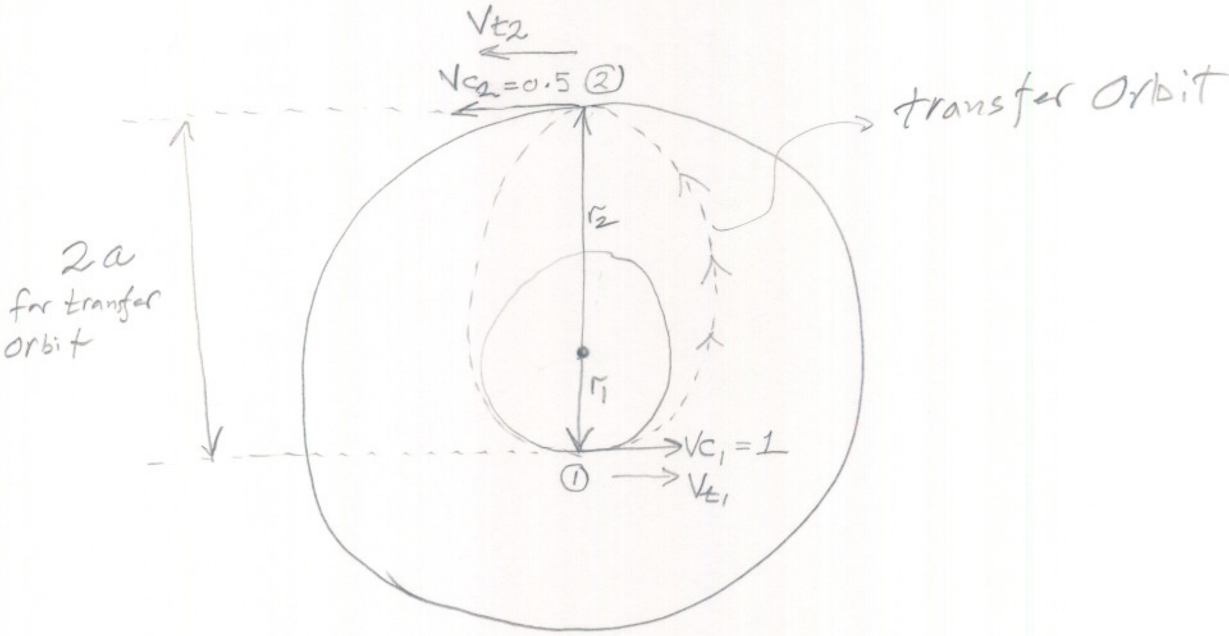
Abbasi

problem 3.5 BMW

problem: you are in a circular orbit with velosity 1 DU/TU. your service module is in another circular orbit with velosity .5 DU/TU. what is the minumum ΔV needed to transfer to the service Module's orbit?

Assumptions:

Method: Apply Hohmann transfer orbit (Known to require least Energy). Find V_{t1}, Find ΔV_1 as the difference between V_{c1} and V_{t1}. do the same to find ΔV_2. Add the ΔV's.



Analysis:

Since $V_{c2}$is given as 0.5 DU/TU, and $V_{c1} = 1$ DU/TU, then service orbit must be the larger orbit (outside orbit), since the smaller the distance from the attracting body the larger the velosity of the orbiting body).

First find r_1 and r_2

Since $V_{c1} = \sqrt{\frac{\mu}{r_1}} \Rightarrow r_1 = \frac{\mu}{V_{c1}^2} \Rightarrow \boxed{r_1 = \frac{1}{1^2} = 1 \text{ DU}}$

Similarly $r_2 = \frac{\mu}{V_{c2}^2} \Rightarrow \boxed{r_2 = \frac{1}{0.5^2} = 4 \text{ DU}}$

From geometry, for the transfer orbit, $2a_t = r_1 + r_2 \Rightarrow \boxed{a_t = 5 \text{ DU}}$

But $\xi_t = -\frac{\mu}{a_t} \Rightarrow \boxed{\xi_t = -0.2 \ DU^2/TU^2}$

Now, velosity in the transfer orbit is given by $V_t = \sqrt{2(\frac{\mu}{r} + \xi_t)}$ hence at point 1, $r = r_1 = 1$ DU, so we get $V_{t1} = \sqrt{2\left(\frac{1}{1} - 0.2\right)} \Rightarrow \boxed{V_{t1} = 1.264911 \ \text{DU/TU}}$

Similarly, $V_{t2} = \sqrt{2(\frac{\mu}{r_2} + \xi_t)}$ hence at point 2, $r_2 = 4$ DU, so we get

$V_{t2} = \sqrt{2\left(\frac{1}{4} - 0.2\right)} \Rightarrow \boxed{V_{t2} = 0.316227 \ \text{DU/TU}}$

So, $\Delta V_1 = |V_{c1} - V_{t1}| = |1 - 1.264911| = 0.264911$ DU/TU

$\Delta V_2 = |V_{c2} - V_{t2}| = |0.5 - 0.316227| = 0.183773$ DU/TU

hence, minumum $\Delta V = \Delta V_1 + \Delta V_2 = \boxed{0.448684 \ \text{DU/TU}}$