Astronautics, Spring quarter 2003, HW 6, UCI

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1 problem 1, chapter 7. Weisle book

Problem: Given that total $\Delta V = 4.29$ km/sec to transfer from LEO at inclination 28^0 to GEO. $I_{sp} = 453$ sec, $m_p = 16,000$ kg, $m_s = 1300$ kg. How much payload can tug deliver to GEO? can tug make a round trip without payload? if it can, how much payload could it carry to GEO and still return to LEO?

Assumptions

m_p given is that starting from LEO, and not from surface of earth.

roundtrip is back to LEO, not earth.

Method

We are given ΔV and asked to find payload that could be carried given the physical properties of the spacecraft.

Solve from total $\Delta V = V_e \ln Z$

For the other parts of the problem, use the rocket equation again to solve for different variables as shown in the analysis.

Analysis

$g = 9.8$ m/s.

so $V_e = I_{sp} g = 453(9.8) = 4439.4$ m/s

$\Delta V = V_e \ln Z$

hence $4290 = 4439.4 \ln Z$

hence $\ln Z = \frac{4290}{4439.4} = 0.9663468036$

hence $Z = e^{0.9663468036} = 2.628325112$

so $\frac{1+\lambda}{\epsilon+\lambda} = 2.628325112$

λ is the payload ratio

ϵ is the structural ratio

$\lambda = \frac{m_L}{m_s + m_p}$

$\epsilon = \frac{m_s}{m_s + m_p}$

so $\frac{1+\lambda}{\epsilon+\lambda} = \frac{1 + \frac{m_L}{m_s+m_p}}{\frac{m_s}{m_s+m_p} + \frac{m_L}{m_s+m_p}} = \frac{\frac{m_s+m_p+m_L}{m_s+m_p}}{\frac{m_s+m_L}{m_s+m_p}} = \frac{m_s+m_p+m_L}{m_s+m_L} = \frac{1300+16000+m_L}{1300+m_L}$

$\frac{1300+16000+m_L}{1300+m_L} = 2.628325112$

solve for m_L

$1300 + 16000 + m_L = 2.628325112\ (1300) + 2.628325112\ m_L$

$1.628325112\ m_L = 1300 + 16000 - 2.628325112\ (1300) = 13883.17735$

$m_L = \frac{13883.17735}{1.628325112} = 8526.047561 \approx 8526$ kg

To find if tug can make a round-trip to LEO without payload, make m_{p_new} as the unknow, solve for it, and compare it to the given m_p.

Now we have an aditional ΔV which is that needed to go back from GEO to LEO.

So, our ΔV now is $4290+4290=8580$ km/s

$\Delta V=V_e\ln Z$

$8580=4439.4\ln Z$

hence $\ln Z=\frac{8580}{4439.4}=1.932693607$

hence $Z=e^{1.932693607}=6.908092891$

$Z=\frac{m_0}{m_f}$

Here, m_0 is the initial mass at start of the trip, which is $m_s+m_{p\_new}$, and m_f is the final mass at the end of the trip, which now is $m_f=m_s$

So, $Z=\frac{m_0}{m_f}=\frac{m_s+m_{p\_new}}{m_s}$ solve for m_{p_new} and compared to give m_p to see if less than.

$6.908092891=\frac{1300+m_{p\_new}}{1300}$

$m_{p\_new}=6.908092891\,(1300)-1300=7680.520758\text{kg}\approx 7680.5$ kg

Compare this to the m_p that the tug actually has which is $16,000$ kg, so the answer is Yes, it can make a round trip back to LEO with no payload.

To find how much payload it can carry and still make a round trip to LEO. Since the m_p needed to make a round trip with NO payload was found above to be 7680.5 kg, then the m_p that we can use to make one second half of the round trip with no payload is $\frac{7680.5}{2}=3840.25$ kg

So, given that we started with $m_p=16000$ kg, then the m_p that we have at our disposal in the first half of the trip is the difference $16000-3840.25=12159.75$ kg. This is the m_p we can use for the one way trip from LEO to GEO with a payload. We know find this payload.

$\Delta V=V_e\ln Z$

$4290=4439.4\ln Z$

hence $\ln Z=\frac{4290}{4439.4}=0.9663468036$

hence $Z=e^{0.9663468036}=2.628325112$

$Z=\frac{m_s+m_L+m_p}{m_{s+m_L}}=\frac{1300+m_L+12159.75}{1300+m_L}$ solve for m_L

$2.628325112=\frac{1300+m_L+12159.75}{1300+m_L}$

$2.628325112\,(1300)+2.628325112\ m_L=1300+m_L+12159.75$

$1.628325112\ m_L=1300+12159.75-2.628325112\,(1300)$

$m_L=6167.642613\approx 6167.6$ kg

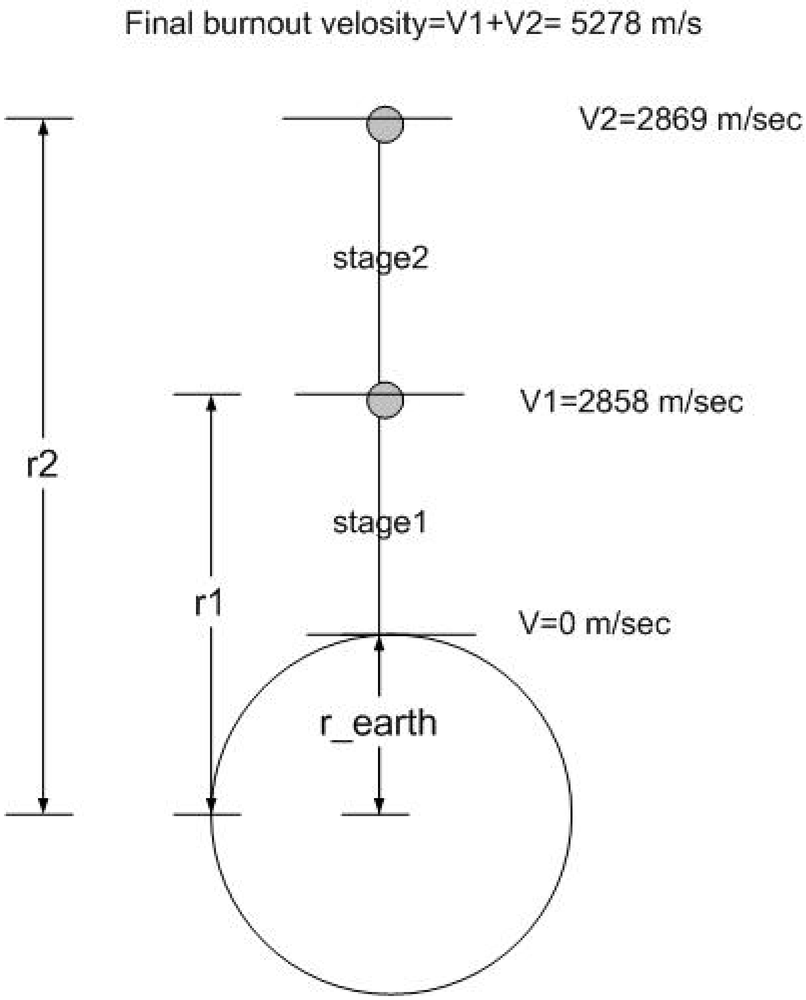
2 problem 7.4

see problem on page 226, Weisel book.

Assumptions: burn-time is zero long. g (earth accelaration) does not change during the flight of the spacecraft.

Method: Use the rocket equation

Analysis



for the overall system

$m_0 = m_{p1} + m_{s1} + m_{p2} + m_{s2} + m_L = 1167 + 113 + 415 + 41 + 250 = 1986$ kg

$m_f = m_L + m_{s2} = 250 + 41 = 291$ kg

burn out for end of first stage:

$V_e = I_{sp}\ g = 282\,(9.8) = 2763.6$ m/sec

First stage:

$V_{bo1} = V_e \ln Z$

$m_{10} = m_{p1} + m_{s1} + m_{p2} + m_{s2} + m_L = 1167 + 113 + 415 + 41 + 250 = 1986$ kg

$m_{1f} = m_{s1} + m_{p2} + m_{s2} + m_L = 41 + 415 + 41 + 250 = 747$ kg

$V_{bo1} = 2763.6\ \ln\left(\frac{m_{10}}{m_{1f}}\right)$

$V_{bo1} = 2763.6 \ln\left(\frac{1986}{747}\right) = 2763.6 \ln 2.6586 = 2702\text{m/sec} \approx 2.7\text{km/sec}$

second stage:

$V_{bo2} = V_e \ln Z$

$m_{20} = m_{p2} + m_{s2} + m_L = 415 + 41 + 250 = 706$ kg

$m_{2f} = m_L = 250$ kg

$V_{bo2} = 2763.6\ \ln\left(\frac{m_{20}}{m_{2f}}\right) = 2763.6\ \ln\left(\frac{706}{250}\right)$

$V_{bo2} = 2763.6 \ln 2.824 = 2869.043 \approx 2869\text{m/sec}$

so, final burnout velosity is the sum of the above 2 velosities:

$2858.29 + 2869 = 5727.29\text{m/sec}$

To find max altitude with 250 kg.

Find the mechanical energy E at surface of earth and at end of last stage, and use to solve for the unknowns $r_{\max}$ since E does not change over the path.

Convert μ_{earth} to m/sec, which is

$3.986012 \times 10^5\,(\text{km/sec})^3 \Longrightarrow 3.986012 \times 10^5 \times 10^9\,(\text{m/sec})^3 = 3.986012 \times 10^{14}\,(\text{m/sec})^3$

At surface of earth, and noting that the velosity of the rocket is zero at that point, we get $E = \frac{V_{earth}^2}{2} - \frac{\mu}{r_{earth}} = \frac{0}{2} - \frac{\mu}{r_{earth}}$

$E = -\frac{\mu}{r_{earth}} = -\frac{3.986012 \times 10^{14}}{6378.145 \times 10^3} = -62494847\ \text{m}^2/\text{s}^2$

now, final velosity is 5727.29 m/sec, so

$E = \frac{V_{\max}^2}{2} - \frac{\mu}{r_{\max}}$

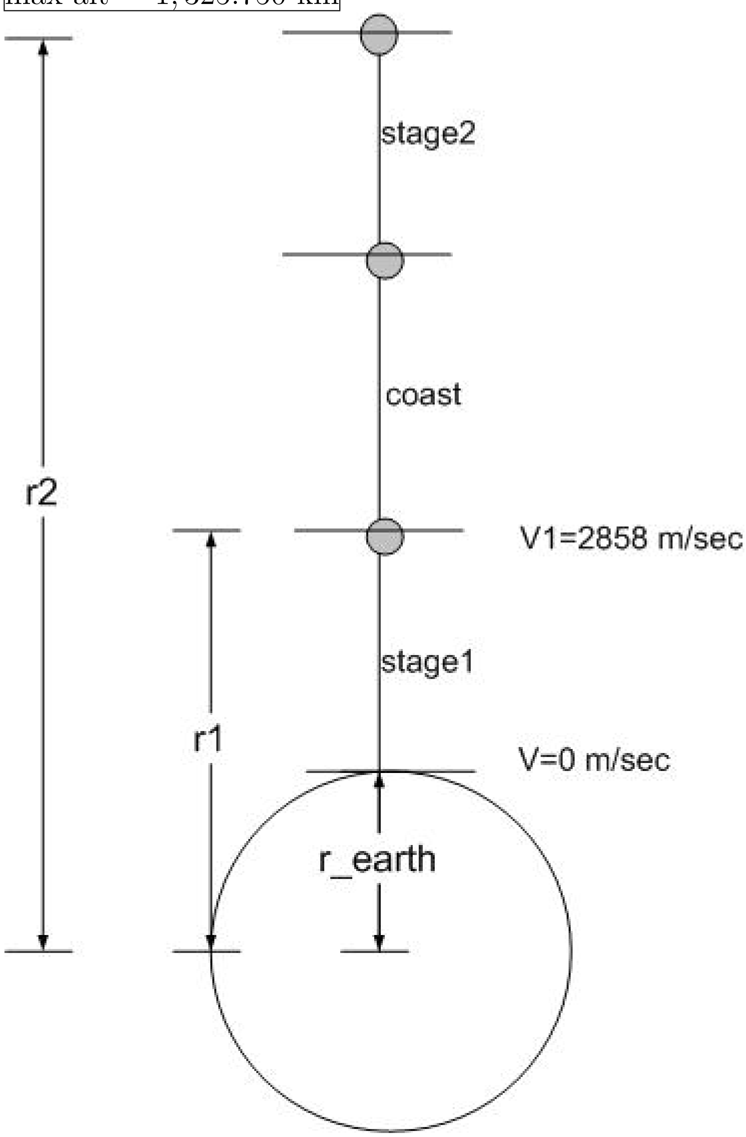
$\frac{\mu}{r_{\max}} = \left(\frac{V_{\max}^2}{2} - E\right) = \frac{5727.29^2}{2} - (-62494847) = 78895772$

$r_{\max} = \frac{\mu}{78895772} = \frac{3.986012 \times 10^{14}}{78895772} = 5052250\ \text{m}$

So, max altitude $5052250 - 6378000 = -1325750\ \text{m} = -1,325.750\ \text{km}$

Not sure why I get negative ALT. I think this is because zero potential energy reference is usually taken at ∞. This should then be

$\boxed{\text{max alt} = 1,325.750\ \text{km}}$



At end of stage 1 we know the velocity. Let the spacecraft coast from that point untill its velosity becomes zero. Then start stage2.

At end of stage one, the mass of spacecraft is $m_{1f} = 706$ kg.

The K.E. the spacecraft have at this point is $0.5mV^2$, then at end of coast, this K.E. will all be exchanged by P.E. gained in going up, so solve for distance travelled

$\frac{1}{2}mV^2 = mgh$

$\frac{1}{2}(706)(2858.29^2) = 706(9.8)h$

$h = \frac{\frac{1}{2}(706)\left(2858.29^2\right)}{706(9.8)} = 416827\text{m} = 416.8\text{km}$

Now, the spacecraft fires its second stage rocket, at end of the second stage it will have gained a velosity of 2869 m/sec (found from above). Mass of spacecraft at end of stage 2 is $m_L = 250\text{kg}$

From $\frac{1}{2}mV^2 = mgh$

$\frac{1}{2}V^2 = gh$

$h = \frac{\frac{1}{2}V^2}{g} = \frac{0.5(2869)^2}{9.8} = 419957\text{m} = 419.957\text{km}$