Astronautics, Spring quarter 2003, HW 4, UCI

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1 problem 3.5 from BMW book

Analysis:

Since V_{c2} is given as $0.5\frac{DU}{TU}$, and $V_{c1} = 1DU/TU$, then service orbit must be the larger orbit (outside orbit), since the smaller the distance from the attracting body the larger the velosity of the orbiting body).

First find r_1 and r_2

Since $V_{c1} = \sqrt{\frac{\mu}{r_1}} \Longrightarrow r_1 = \frac{\mu}{V_{c1}^2} \Longrightarrow r_1 = \frac{1}{1^2} = 1DU$

Similarly $r_2 = \frac{\mu}{V_{c2}^2} \Longrightarrow r_2 = \frac{1}{0.5^2} = 4DU$

From geometry, for the transfer orbit, $2a_t = r_1 + r_2 \Longrightarrow a_t = 5DU$

But $\xi_t = -\frac{\mu}{a_t} \Longrightarrow \xi_t = -0.2\left(\frac{DU}{TU}\right)^2$

Now, velocity in the transfer orbit is given by $V_t = \sqrt{2\left(\frac{\mu}{r} + \xi_t\right)}$ hence at point 1, $r = r_1 = 1$ DU, so we get $V_{t1} = \sqrt{2\left(\frac{1}{1} - 0.2\right)} \Longrightarrow V_{t1} = 1.264911DU/TU$

Similarly, $V_{t2} = \sqrt{2\left(\frac{\mu}{r_2} + \xi_t\right)}$ hence at point 2, $r_2 = 4$ DU, so we get $V_{t2} = \sqrt{2\left(\frac{1}{4} - 0.2\right)} \Longrightarrow V_{t2} = 0.316227\frac{DU}{TU}$

So, $\Delta V_1 = |V_{c1} - V_{t1}| = |1 - 1.264911| = 0.264911$ DU/TU

$\Delta V_2 = |V_{c2} - V_{t2}| = |0.5 - 0.316227| = 0.183773$ DU/TU

hence, minumum $\Delta V = \Delta V_1 + \Delta V_2 = \boxed{0.448684 \text{ DU/TU}}$

2 problem 3.8 from BMW book

Compute the minimum ΔV required to transfer between 2 coplaner elliptical orbits which have their major axes aligned. The parameters are:

$r_{p1} = 1.1$ DU. $r_{p2} = 5$ DU

$e_1 = 0.290$ DU. $e_2 = 0.412$ DU

Assume both preigrees lie on the same side of the earth.

Assumptions:

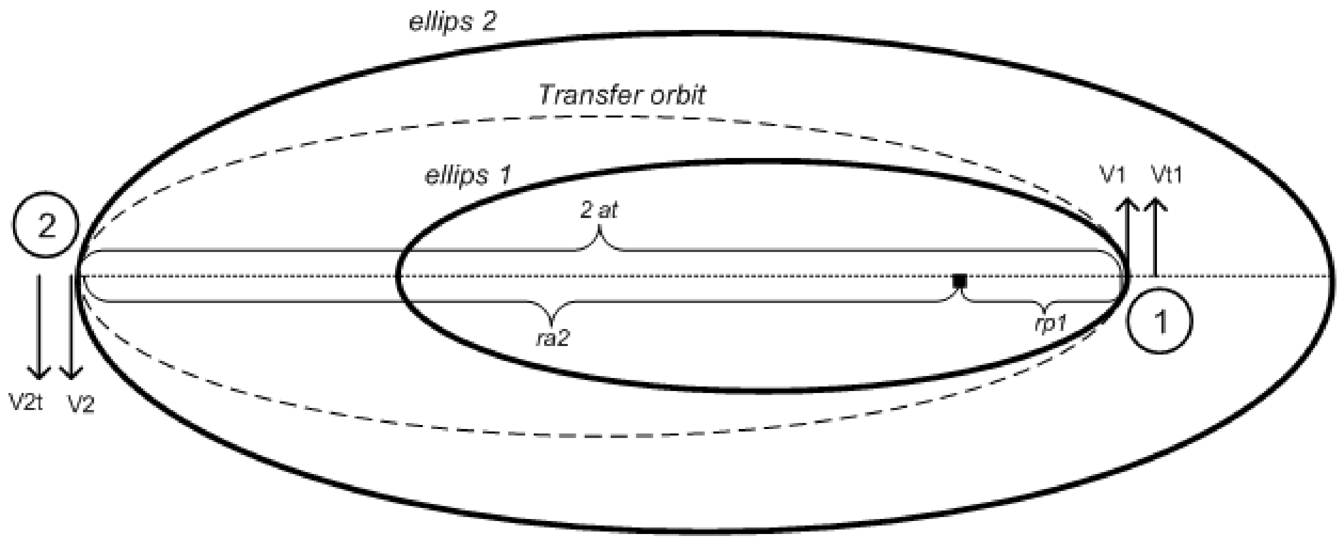
Method: First find $V_1$and $V2$, the velositites for ellips 1 at its perigree and for ellips 2 at its apegee.

Next find ξ_t, the energy for the transfer orbit. From this, find V_{1t} and V_{2t}, the velosities in the transfer orbit at point 1 and point 2 respectively.

Finally, final ΔV follows as from the sum of the ΔV at point 1 and point 2.

This is the minumum, since the transfer orbit is a Homann orbit.

Analysis:



since $a = rp + ae$ from ellips geometry.
$rp = a(1-e)$

$a = \frac{rp}{1-e}$

For ellips 1:

$a_1 = \frac{r_{p1}}{1-e_1} \Rightarrow a_1 = \frac{1.1}{1-0.29} = 1.5492957$ DU.

$\xi_1 = -\frac{\mu}{2a_1} = -\frac{1}{(2)1.5492957} = -0.322727 \quad DU^2/TU^2$

But $\xi = \frac{V^2}{2} - \frac{\mu}{r}$, hence, since ξ is constant over the orbit, we can use this relationship to solve for V for different r.

At point 1, for first ellips, $r = r_{p1}$, hence

$-0.322727 = \frac{V_1^2}{2} - \frac{1}{1.1} \Rightarrow \boxed{V_1 = 1.082925 \text{ DU/TU.}}$

For ellips 2:

Here we want to find the velosity V_2, the velosity at the apegee for ellips 2. So, need to find r_{A2} for ellips 2.

Since $r_{p2} = 5$ DU, and $e_2 = 0.412$, we get

$a = \frac{rp}{1-e} \Rightarrow a_2 = \frac{5}{1-0.412} = 8.5034$ DU.

Hence, since $r_A = a + ae = a(1+e) \Rightarrow r_{A2} = 8.50034(1+0.412) = 12.0068$ DU.

Now find ξ_2 the energy for ellips 2

$\xi_2 = -\frac{\mu}{2a} = -\frac{1}{(2)8.5034} = -0.0588 \quad DU^2/TU^2$

but $\xi_2 = \frac{V_2^2}{2} - \frac{\mu}{r_A}$ then

$-0.0588 = \frac{V_2^2}{2} - \frac{1}{12.0068} \Rightarrow \boxed{V_2 = 0.2212968 \text{ DU/TU}}$

For the transfer orbit:

From geometry, $2a = r_{p1} + r_{A2} = 1.1 + 12.0068 \Longrightarrow a_t = 6.5534$ DU

so $\xi_t = -\frac{\mu}{(2)6.5534} = -0.0762963 \; DU^2/TU^2$

So, $V_{1t} = \sqrt{2\left(\frac{\mu}{r_{p1}} + \xi_t\right)} = \sqrt{2\left(\frac{1}{1.1} - 0.0762963\right)} = 1.2905$ DU/TU

So, $V_{2t} = \sqrt{2\left(\frac{\mu}{r_{A2}} + \xi_t\right)} = \sqrt{2\left(\frac{1}{12.0068} - 0.0762963\right)} = 0.11823567$ DU/TU

so ΔV at point 1 $= |1.082925 - 1.2905| = 0.207575$ DU/TU
so ΔV at point 2 $= |0.2212968 - 0.11823| = 0.1030668$ DU/TU
hence minumum $\Delta V = 0.207575 + 0.1030668 = 0.3106418 DU/TU$