Study Notes

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Notes on Linear algebra

- **1. If M is a symmetric matrix then $C\,M\,C^T$ is a diagonal matrix, where C is a matrix whose rows are the eigen vectors of M. example**

```
Remove["Global`*"]
```

$$m = \begin{pmatrix} 1 & 4 & 5 \\ 4 & 8 & 9 \\ 5 & 9 & 4 \end{pmatrix};$$

```
v = Eigenvectors[m];

Chop[N[v.m.Transpose[v]]] // MatrixForm
```

$$\begin{pmatrix} 48.2835 & 0 & 0 \\ 0 & -6.37096 & 0 \\ 0 & 0 & -29.445 \end{pmatrix}$$

■ **2 If M is a symmetric matrix then** $e^M = C^T \begin{pmatrix} e^{\lambda_1 t} & 0 & 0 \\ 0 & e^{\lambda_2 t} & 0 \\ 0 & 0 & e^{\lambda_n t} \end{pmatrix} C$ **where** λ_i **are the eigenvalues of M and** C **is a matrix whose rows are the eigen vectors of** M**. example**

```
Remove["Global`*"]
```

$$\text{badMatrix} = \begin{pmatrix} 1 & 4 & 5 \\ 4 & 8 & 7 \\ 5 & 7 & 4 \end{pmatrix};$$

$$\text{goodMatrix} = \begin{pmatrix} 0.624961238712114` & 0.9156593589568051` & 0.9594314541 \\ 0.9156593589568051` & 0.9588324892854148` & 0.6159355337 \\ 0.9594314541903537` & 0.6159355337104255` & 1.3130013229 \end{pmatrix}$$

```
m = badMatrix;
{λ, v} = Eigensystem[m];
Chop[N[Transpose[v].DiagonalMatrix[e^λ].v]] // MatrixForm
```

$$\begin{pmatrix} 3.51857 \times 10^6 & 6.48044 \times 10^6 & 5.29451 \times 10^6 \\ 6.48044 \times 10^6 & 1.19356 \times 10^7 & 9.75134 \times 10^6 \\ 5.29451 \times 10^6 & 9.75134 \times 10^6 & 7.96683 \times 10^6 \end{pmatrix}$$

Now evaluate e^M using Mathematica function

```
(N[MatrixExp[m]]) // MatrixForm
```

$$\begin{pmatrix} 1.19687 \times 10^6 & 2.20437 \times 10^6 & 1.80097 \times 10^6 \\ 2.20437 \times 10^6 & 4.05998 \times 10^6 & 3.317 \times 10^6 \\ 1.80097 \times 10^6 & 3.317 \times 10^6 & 2.70998 \times 10^6 \end{pmatrix}$$