HW1. CEE 247. Structural Dynamics. UCI. Fall 2006

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Contents

no force no damping

$m\ddot{u} + ku = 0$

$m\ddot{u} + c\dot{u} + ku = 0$

$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = 0$

$u(t) = A\cos\omega_n t + B\sin\omega_n t$

$u(t) = C\sin(\omega_n t + \phi)$

$A = u_0 \qquad B = \frac{v_0}{\omega_n} \qquad C = \sqrt{A^2 + B^2}$

$\phi = \tan^{-1}\left(\frac{A}{B}\right)$

damped:

$u(t) = e^{-\zeta\omega_n t}\left(A\cos\omega_d t + B\sin\omega_d t\right)$

$u(t) = Ce^{-\zeta\omega_n t}\cos(\omega_d t - \alpha)$

$A = u_0 \qquad B = \frac{v_0}{\omega_d} + \frac{u_0\zeta\omega_n}{\omega_d} \qquad C = \sqrt{A^2 + B^2}$

$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$

$\omega_d = \omega_n\sqrt{1 - \zeta^2}$

To determine ζ

$\delta = \ln\left(\frac{u_n}{u_{n+1}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$

$\ln\left(\frac{u_n}{u_{n+k}}\right) = k(2\pi\zeta)$

misc. relations

$\omega_n = \sqrt{\frac{k}{m}}$

$\zeta = \frac{c}{c_{cr}}$

$C_{cr} = 2\sqrt{km} = 2\ \omega_n\ m$

stiffness in series: $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

stiffness in parallel: $k = k_1 + k_2$

e

Stiffness $P\ = \left(3\frac{EI}{L^3}\right)\delta$

$P = 2 \times \left(k = 3\frac{EI}{L^3}\right)\delta$

$P = 2 \times \left(k = \frac{48\ EI}{L^3}\right)\delta$

$a_t = e\dot{\omega}$

$a_n = e\omega^2$

$$\begin{aligned} F_{centric} &= m_e a_n \\ &= m_e\ e\omega^2 \\ F_{vertical} &= \overbrace{m_e\ e\omega^2}^{F_0} \sin\omega t \end{aligned}$$

$m\ddot{u} + c\dot{u} + ku = \overbrace{m_e\ e\omega^2}^{F_0} \sin\omega t$

$m\ddot{u} + ku = F_0 \sin\bar{\omega}t$

$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{F_0}{k}\frac{1}{1-r^2}\sin\bar{\omega}t$

$A = u(0)$

$B = \frac{v(0)}{\omega_n} - \frac{F_0}{k}\frac{r}{1-r^2}$

if all initial conditions are zero, the above solution can as

$u(t) = \overbrace{\frac{F_0}{k}}^{u_{st}}\overbrace{\frac{1}{1-r^2}}^{R_d}(\sin\bar{\omega}t - r\sin\omega_n t)$

DAMPED

$m\ddot{u} + c\dot{u} + ku = F_0 \sin\bar{\omega}t$

$u(t) = \overbrace{e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)}^{\text{transient}} + \overbrace{\frac{F_0}{k}\frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}\sin(\bar{\omega}t - \theta)}^{\text{steady state}}$

$u_{ss}(t) = \overbrace{\frac{F_0}{k}}^{u_{st}}\overbrace{\frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}}^{R_d}\sin(\bar{\omega}t - \theta)$

$\theta = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$

$r = \frac{\bar{\omega}}{\omega_n}$

$A = u(0)$

$B = \frac{v(0)}{\omega_n} - \frac{F_0}{k}\frac{r}{1-r^2}$

if all initial conditions are zero, the above solution can as

$$u(t) = \overbrace{\frac{F_0}{k}}^{u_{st}} \overbrace{\frac{1}{1-r^2}}^{R_d} (\sin\bar{\omega}t - r\sin\omega_n t)$$

max shear in column

$V_{\max} = k\ u_{\max}$

max moment in column

$M_{\max} = V_{\max} L$

logarithimc decrement

Measure u_{ss} at 2 successive periods

$\ln\frac{u_1}{u_2} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \approx 2\pi\zeta$

Harmoice vibration test

excite system with frequencies

near resonance.. Measure u_{ss} near resonance.

Hence $R_d = \frac{u_0}{u_{st}} = \frac{1}{2\zeta}$

where $u_{st} = \frac{F_0}{k} = \frac{m_e\ e\ \bar{\omega}^2}{k}$

half-power

excite structure with range of frequencies near ω_n

plot response of system. Take max $u_{\max}$

take $\frac{u_{\max}}{\sqrt{2}}$ line

Determine $\bar{\omega}_1$ and $\bar{\omega}_2$ at each x-axis

$\zeta = \frac{\bar{\omega}_2-\bar{\omega}_1}{\bar{\omega}_2+\bar{\omega}_1}$

ground displaceement

$u_s(t) = u_{s0}\sin\bar{\omega}t$

equation of motion

$m\ddot{u} + c(\dot{u} - \dot{u}_s) + k(u - u_s) = 0$

solution

$$\begin{aligned} m\ddot{u} + c(\dot{u} - \dot{u}_s) + k(u - u_s) &= 0 \\ m\ddot{u} + c\dot{u} + ku &= ku_{s0}\sin\bar{\omega}t + c\bar{\omega}u_{s0}\sin\bar{\omega}t \\ &= F_0\sin(\bar{\omega}t + \beta) \end{aligned}$$

where

$F_0 \equiv u_{s0} k \sqrt{1 + (2\zeta r)^2} \qquad \tan^{-1}\beta = 2\zeta r$

steady state solution is

$$\begin{aligned} u_{ss}(t) &= \frac{F_0}{k} R_d \sin(\bar{\omega} t + \beta - \theta) \\ &= u_{s0} \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1-r)^2 + (2\zeta r)^2}} \sin(\bar{\omega} t + \beta - \theta) \end{aligned}$$

Hence Transmissibility (ratio of abs. motion of structure to abs. motion of ground) is

$T_r = \frac{|u_{ss}|}{u_{s0}} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$

using relative displacement

$m\ddot{u}_r + c\dot{u}_r + ku_r = F_{eff} \sin\bar{\omega} t$

$F_{eff} = -m\ddot{u}_s$

Hence equation of motion becomes

$m\ddot{u}_r + c\dot{u}_r + ku_r = \overbrace{m\ u_{s0}\ \bar{\omega}^2}^{F_{0,effective}} \sin\bar{\omega} t$

steady state solution

$$\begin{aligned} u_r(t) &= \frac{u_{s0} m \bar{\omega}^2}{k} R_d \sin(\bar{\omega} t - \theta) \\ &= u_{s0} r^2 R_d \sin(\bar{\omega} t - \theta) \end{aligned}$$

Hence steady state max amplitude is

$$U_0 = u_{s0} r^2 R_d = u_{s0} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Vibration isolation. Assuming applied force to struct is $F_0 \sin\bar{\omega} t$ then max force transmitted to support is

$A_t = F_0 T_r = F_0 \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r)^2+(2\zeta r)^2}}$