HW 4. CEE 247. Structural Dynamics. UCI. Fall 2006.

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November 12, 2006

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Contents

1 Problem

Determine and plot the deformation response $u(t)$ from a SDOF system with natural period of 2 sec and $\zeta = 5\%$ to the El Centro ground motion N-S component. Implement one of the numerical time-stepping algorithms.

2 Solution

Derivation used was based on TextBook 'Structural Dynamics' 5th edition by Paz and Leigh. Page 185.

First, The El Centro data needed was downloaded from this web site

`http://nisee.berkeley.edu/data/`

```
Here is listing of the first few lines of the file:

Data for El Centro 1940 North South Component (Peknold Version)
1559 points at equal spacing of 0.02 sec
Points are listed in the format of 8F10.5, i.e., 8 points across in
a row with 5 decimal places
The units are (g)
*** Begin data ***
   0.00630   0.00364   0.00099   0.00428   0.00758   0.01087   0.00682   0.00277
  -0.00128   0.00368   0.00864   0.01360   0.00727   0.00094   0.00420   0.00221
   0.00021   0.00444   0.00867   0.01290   0.01713  -0.00343  -0.02400  -0.00992
   0.00416   0.00528   0.01653   0.02779   0.03904   0.02449   0.00995   0.00961
   0.00926   0.00892  -0.00486  -0.01864  -0.03242  -0.03365  -0.05723  -0.04534
```

We start by the equation of motion for SDOF system, using relative motion to support subjected to suppose accelaration of $\ddot{u}_g$

$$m\ddot{u}_r + c\dot{u}_r + ku_r = \overbrace{-m\ddot{u}_g}^{\text{effective force}}$$

The ground accelaration $\ddot{u}_g$ is given from the El-centro earthquake measurements.

Solve using the method of linear acceleration. We start by writing the above equation as

$$m\Delta\ddot{u}_i + c\Delta\dot{u}_i + k\Delta u_i = -m\Delta\ddot{u}_{g_i}$$

Where in the above all the mass displacement, velocity and acceleration are relative to the support and are not the absolute values.

Rewrite the above, to remove the unknown mass m as follows

$$\boxed{\Delta\ddot{u}_i + 2\zeta\omega_n\ \Delta\dot{u}_i + \omega_n^2\ \Delta u_i = -\Delta\ddot{u}_{g_i}} \tag{1}$$

Following the analysis of the text book, from equation 6.36 on page 186, and using $\Delta\ddot{u}_{g_i} = \ddot{u}_{g_{i+1}} - \ddot{u}_{g_i}$ we have the following expression for the change of displacement at time step i

$$\boxed{\Delta u_i = \frac{-\overbrace{\left(\ddot{u}_{g_{i+1}} - \ddot{u}_{g_i}\right)}^{\Delta\ddot{u}_{g_i}} + \frac{6}{\Delta t}\dot{u}_i + 3\ddot{u}_i + 2\zeta\omega_n\left(3\dot{u}_i + \frac{\Delta t}{2}\ddot{u}_i\right)}{\omega_n^2 + \frac{6}{\Delta t^2} + \frac{3\times 2\zeta\omega_n}{\Delta t}}} \tag{2}$$

and hence we can now find u_{i+1} using

$$u_{i+1} = u_i + \Delta u_i \tag{3}$$

Now, we can find $\Delta\dot{u}_i$ from equation 6.31 in the book

$$\boxed{\Delta\dot{u}_i = \frac{3}{\Delta t}\Delta u_i - 3\dot{u}_i - \frac{\Delta t}{2}\ddot{u}_i} \tag{4}$$

Hence

$$\dot{u}_{i+1} = \dot{u}_i + \Delta\dot{u}_i \tag{5}$$

And $\Delta\ddot{u}_i$ is obtained directly from equation (1) above

$$\boxed{\Delta\ddot{u}_i = -\Delta\ddot{u}_{g_i} - 2\zeta\omega_n\ \Delta\dot{u}_i - \omega_n^2\ \Delta u_i} \tag{6}$$

Hence

$$\ddot{u}_{i+1} = \ddot{u}_i + \Delta\ddot{u}_i \tag{7}$$

Now that $u_{i+1}, \dot{u}_{i+1}, \ddot{u}_{i+1}$ have been obtained, the process is repeated for the next step.

We start the process by using the initial conditions of

$$u_0 = 0$$
$$\dot{u}_0 = 0$$

The El-centro data file gives the values of $\ddot{u}_{g_i}$, which we will use to solve this problem. $\Delta t = 2\,\text{sec}$ for this data.

Hence the algorithm is as follows

1. $i = 1$, $u_i = \dot{u}_i = 0$

2. read $\ddot{u}_{g_i}$ and $\ddot{u}_{g_{i+1}}$ from El-centro file, and find $\Delta\ddot{u}_{g_i} = \ddot{u}_{g_{i+1}} - \ddot{u}_{g_i}$
3. Find Δu_i from eq (2). Find u_{i+1} from eq (3)
4. Find $\Delta\dot{u}_i$ from eq (4). Find $\dot{u}_{i+1}$ from eq (5)
5. Find $\Delta\ddot{u}_i$ from eq (6). Find $\ddot{u}_{i+1}$ from eq (7)
6. $i = i + 1$ and go to step 2. Stop when i is the length of the el-centro data less than 1

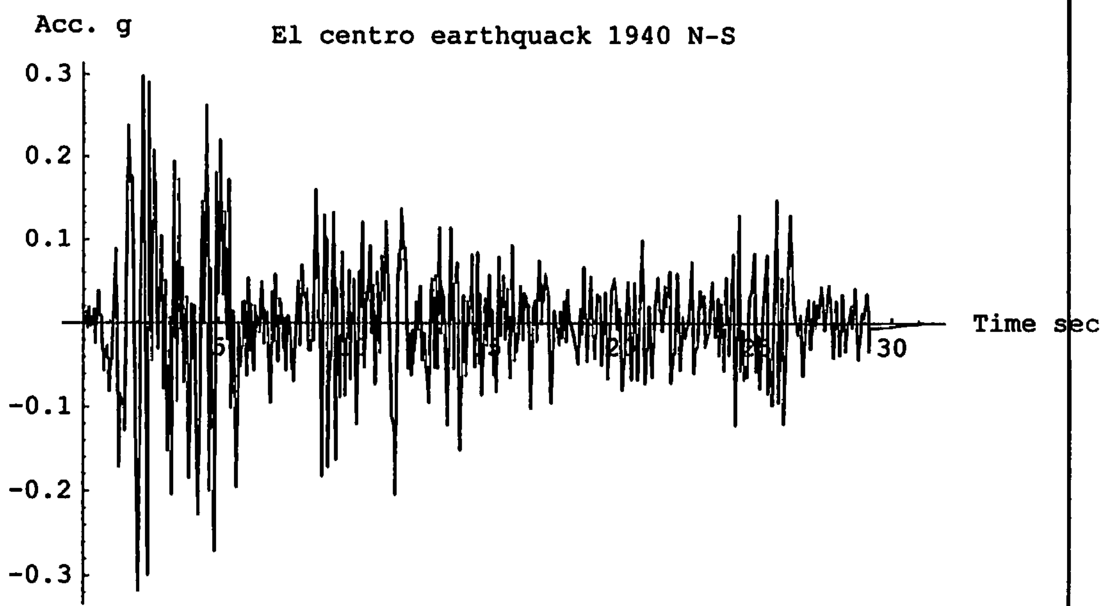
The following is a listing of the program and the output

EL-Centro Earthquake 1940 N-S response analysis
by Nasser Abbasi

```
Remove["Global`*"];
SetDirectory["E:/nabbasi/data/nabbasi_web_Page/my_courses/
    UCI_COURSES/CREDIT_COURSES/fall_2006/CEE_247/HWs/HW4"];
```

- **Load El-Centro data and plot the earthquack recorded accelration**

```
data = Import["el_centro.txt", "Table"];
dataLength = 8 Length[data];

g = Table[{0, 0}, {dataLength}];
t = Table[0, {dataLength}];
u = Table[{0, 0}, {dataLength}];
v = Table[{0, 0}, {dataLength}];
acc = Table[{0, 0}, {dataLength}];

delT = 0.02; (*sec*)
k = 0;
currentTime = 0;
For[i = 1, i ≤ Length[data], i = i + 1,
  For[j = 1, j ≤ 8, j = j + 1,
   {
    k = k + 1;

    g[[k, 1]] = currentTime;
    u[[k, 1]] = currentTime;
    v[[k, 1]] = currentTime;
    acc[[k, 1]] = currentTime;

    g[[k, 2]] = data[[i, j]];

    currentTime = currentTime + delT;
   }
  ]
 ];

ListPlot[g, PlotJoined → True,
  PlotRange → All, AxesLabel → {"Time sec", "Acc. g"},
  PlotLabel -> "El centro earthquack 1940 N-S"];
```


Acc. g
El centro earthquack 1940 N-S
0.3
0.2
0.1
-0.1
-0.2
-0.3
5
30
Time sec

■ Implementation of Linear Accelration step-by-step algorithm

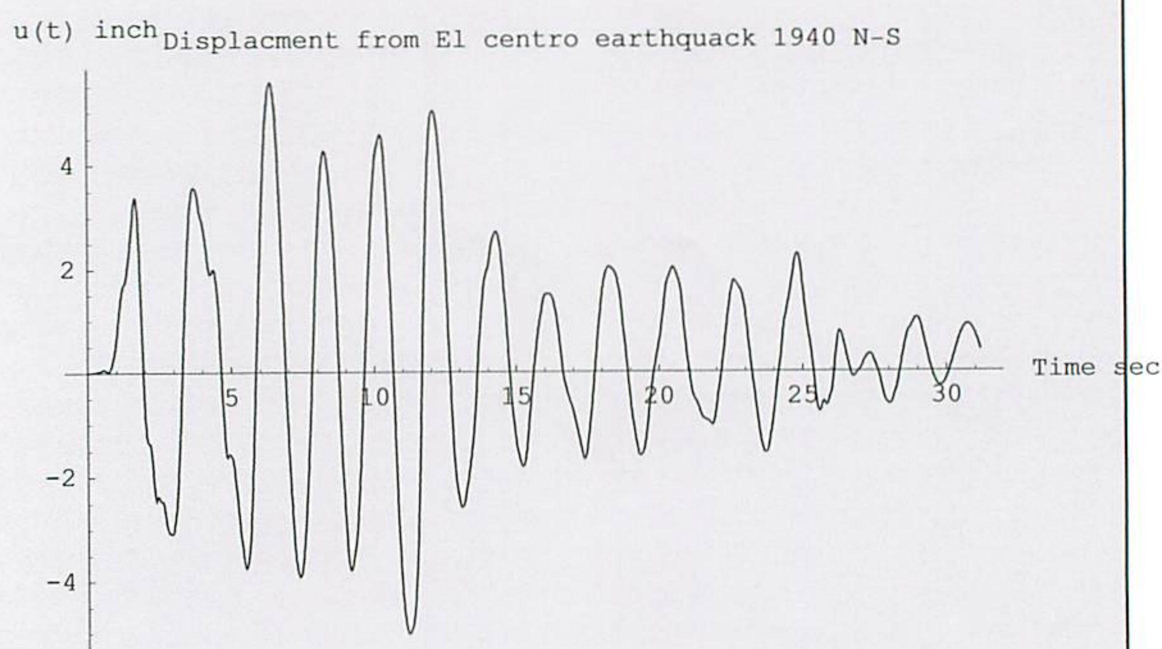
```
u[[1, 2]] = 0.;
v[[1, 2]] = 0.;
acc[[1, 2]] = 0.;

ξ = 0.05;
ω = π;

gConversionFactor = 32.174 * 12;

For[i = 1, i < dataLength, i = i + 1,
  {
   delg = (g[[i + 1, 2]] - g[[i, 2]]) gConversionFactor;
   delU = 1/(ω^2 + 6/delT^2 + (3*2 ξ ω)/delT) (-delg + 6/delT v[[i, 2]] +
        3 acc[[i, 2]] + 2 ξ ω (3 v[[i, 2]] + delT/2 acc[[i, 2]]));

   u[[i + 1, 2]] = u[[i, 2]] + delU;

   delV = 3/delT delU - 3 v[[i, 2]] - delT/2 acc[[i, 2]];

   v[[i + 1, 2]] = v[[i, 2]] + delV;
   delAcc = -delg - 2 ξ ω delV - ω^2 delU;
   acc[[i + 1, 2]] = acc[[i, 2]] + delAcc;
  }
 ];
```

$$\text{delU} = \frac{1}{\omega^2 + \frac{6}{\text{delT}^2} + \frac{3*2\,\xi\,\omega}{\text{delT}}}\left(-\text{delg} + \frac{6}{\text{delT}}\,\text{v[[i, 2]]} + 3\,\text{acc[[i, 2]]} + 2\,\xi\,\omega\left(3\,\text{v[[i, 2]]} + \frac{\text{delT}}{2}\,\text{acc[[i, 2]]}\right)\right)$$

$$\text{delV} = \frac{3}{\text{delT}}\,\text{delU} - 3\,\text{v[[i, 2]]} - \frac{\text{delT}}{2}\,\text{acc[[i, 2]]}$$

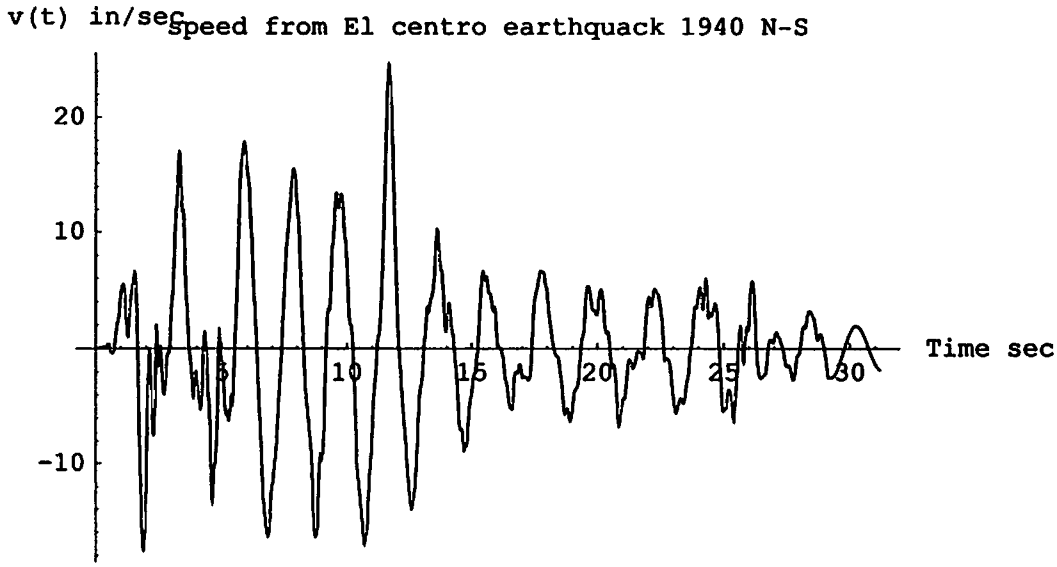
■ Plot results

```
ListPlot[u, PlotJoined → True,
  PlotRange → All, AxesLabel → {"Time sec", "u(t) inch"},
  PlotLabel -> "Displacment from El centro earthquack 1940 N-S"];
```



Super!

```
ListPlot[v, PlotJoined → True,
  PlotRange → All, AxesLabel → {"Time sec", "v(t) in/sec"},
  PlotLabel -> "speed from El centro earthquack 1940 N-S"];
```



```
ListPlot[acc / (32 * 12), PlotJoined → True,
  PlotRange → All, AxesLabel → {"Time sec", "acc(t) g"},
  PlotLabel -> "accelaration from El centro earthquack 1940 N-S"];
```