HW2. CEE 247. Structural Dynamics. UCI. Fall 2006

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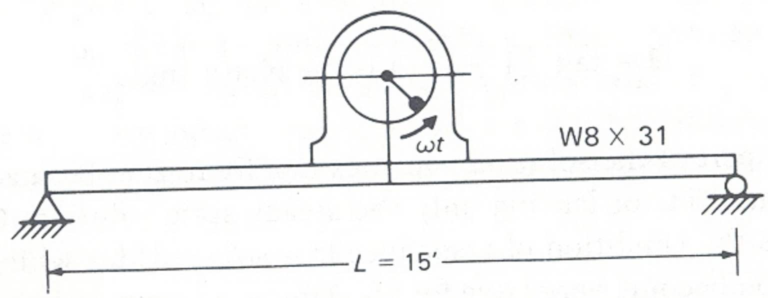
Contents

1 Problem 3.2

3.1 An electric motor of total weight $W = 1000$ lb is mounted at the center of a simply supported beam as shown in Fig. 3.15. The unbalance in the rotor is $W'e = 1$ lb · in. Determine the steady-state amplitude of vertical motion of the motor for a speed of 900 rpm. Assume that the damping in the system is 10% of the critical damping. Neglect the mass of the supporting beam.



Solution

The rotating mass generates a force of $\frac{W'}{g} e\bar{\omega}^2$ where $\bar{\omega}$ is the angular speed of the rotating weight and e is the distance of the mass from the center of the motor.

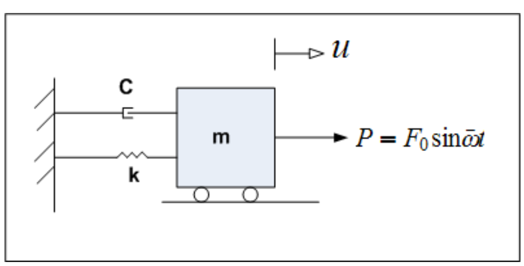
Hence the vertical load is

$$P = m'e\bar{\omega}^2 \sin\theta\left(t\right)$$

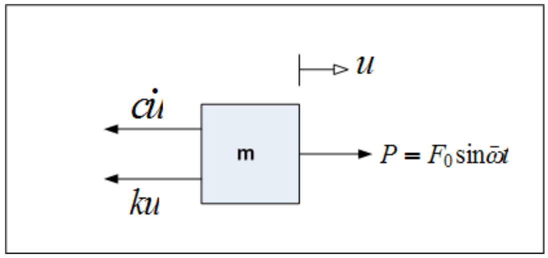
But $\theta\left(t\right) = \bar{\omega}t$, hence the vertical force is

$$\boxed{P = \overbrace{m'e\bar{\omega}^2}^{F_0} \sin\bar{\omega}t}$$

The physical idealized model is



Hence the equation of motion can now be written from the free body diagram as (where M is the mass of the electric motor) and assuming the mass is moving to the right, and taking u relative to the static equilibrium position.



$$F = Ma$$
$$F_0 \sin \bar{\omega} t - c\dot{u} - ku = M\ddot{u}$$
$$M\ddot{u} + c\dot{u} + ku = F_0 \sin \bar{\omega} t$$

The above ODE has the solution

$$\boxed{u(t) = e^{-\zeta \omega_n t}\left(A \cos \omega_d t + B \sin \omega_d t\right) + u_0 \sin\left(\bar{\omega} t - \phi\right)}$$

Where $u_0 = u_{st}\ R_d$, where $u_{st} = \frac{F_0}{k}$ and $R_d = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$ and $\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$ and $r = \frac{\bar{\omega}}{\omega_n}$ and $\omega_n = \sqrt{\frac{k}{M}}$

At steady state, the transient solution decays to zero thanks to the negative exponential term in it, and the solution becomes

$$\boxed{u(t) = u_0 \sin\left(\bar{\omega} t - \phi\right)}$$

Which has amplitude of u_0. Hence we need to evaluate u_0

$$\begin{aligned} u_0 &= u_{st}\ R_d \\ &= \frac{F_0}{k} \frac{1}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}} \end{aligned}$$

But motor runs at 900 rpm, hence

$$\begin{aligned} \bar{\omega} &= \frac{2\pi (900)}{60} = 30\pi \\ &= \boxed{94.248\ rad/sec} \end{aligned}$$

We are given that $W'e = 1$ lb.in, Hence

$$\begin{aligned} F_0 &= \frac{W'}{g} e\bar{\omega}^2 \\ &= \frac{1}{386} (30\pi)^2 \\ &= \boxed{23.012 lb} \end{aligned}$$

Now we need to find k, the stiffness of the beam against bending. For this geometry,

$$k = \frac{48EI}{L^3}$$

But $E = 30,000$ ksi for steel, and for $W8 \times 31$ from tables we find $I_{xx} = 110$ in^4, hence

$$\begin{aligned} k &= \frac{48 \times 30 \times 10^6 \times 110}{(15 \times 12)^3} \\ &= \boxed{27,160 \ \ lb/in} \end{aligned}$$

Now to find ω_n.Recall that that $W = 1000$ lb, hence

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{M}} \\ &= \sqrt{\frac{27160 \times 386}{1000}} \\ &= \boxed{102.39 \quad rad/sec} \end{aligned}$$

Hence

$$\begin{aligned} r &= \frac{\bar{\omega}}{\omega_n} \\ &= \frac{30\pi}{102.39} \\ &= \boxed{0.920\,48} \end{aligned}$$

Putting all these together we obtain

$$\begin{aligned} u_0 &= \frac{F_0}{k} \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \\ &= \frac{23.012}{27160} \times \frac{1}{\sqrt{(1-0.920\,48^2)^2 + (2 \times 0.1 \times 0.920\,48)^2}} \end{aligned}$$

Hence

$$\boxed{u_0 = 0.0035422 \quad inch}$$

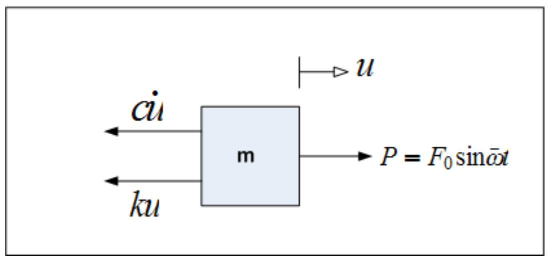
(Note: The back of the book gives $u_0 = 0.0037$, I think the book used a slightly different steel table to obtain I_{xx} which could have been slightly different than the one I used.)

2 Problem 3.3

Determine the maximum force transmitted to the supports of the beam in Problem 3.1.

solution

From the free body diagram:



We see that the forces transmitted to the support are

$$f_{tr} = c\dot{u} + ku$$

Where u here is taken as the steady state solution from problem 3.2, which is

$$u(t) = u_0 \sin(\bar{\omega}t - \phi)$$

Where $u_0 = u_{st}\ R_d$, and $\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$. Differentiate the above equation and substitute the results back into the f_{tr} equation we obtain

$$\begin{aligned} f_{tr} &= c\bar{\omega}u_0 \cos(\bar{\omega}t - \phi) + ku_0 \sin(\bar{\omega}t - \phi) \\ &= \sqrt{(c\bar{\omega}u_0)^2 + (ku_0)^2} \sin(\bar{\omega}t - \phi + \beta) \end{aligned}$$

Where $\beta = \tan^{-1}\left(\frac{k}{c\bar{\omega}}\right)$

Hence we see that the maximum force transmitted to the supports are given by

$$\boxed{f_{tr_0} = u_0\sqrt{(c\bar{\omega})^2 + k^2}} \tag{1}$$

We now plug into the above equation the results we obtain from problem 3.2 to determine f_{tr_0}. All the variables in the above expression are known, which are repeated here

$$\begin{aligned}
\omega_n &= 102.39 \ \ \text{rad/sec} \\
M &= \frac{1000}{386} \text{ lb} \\
\zeta &= 0.1 \\
u_0 &= 0.0037 \text{ in} \\
k &= 27160 \text{ lb/in} \\
\bar{\omega} &= 30\pi \text{ rad/ sec}
\end{aligned}$$

We just need to find the damping c. Since $\zeta = \frac{c}{c_{cr}}$ and $c_{cr} = 2\omega_n M$, hence

$$\begin{aligned}
c_{cr} &= 2 \times 102.39 \times \frac{1000}{386} \\
&= 530.52 \text{ lb-sec/in}
\end{aligned}$$

Hence

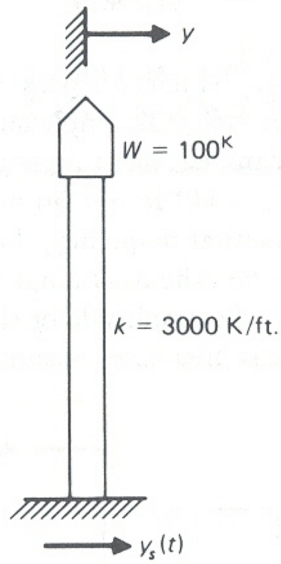
$$\begin{aligned}
c &= \zeta c_{cr} \\
&= 0.1 \times 530.52 \\
&= \boxed{53.052 \ \ \text{lb-sec/in}}
\end{aligned}$$

Now substitute all the above values into equation (1) we obtain

$$\begin{aligned}
f_{tr_0} &= 0.0037\sqrt{(53.052 \times 30\pi)^2 + 27160^2} \\
&= \boxed{102.18 \ lb}
\end{aligned}$$

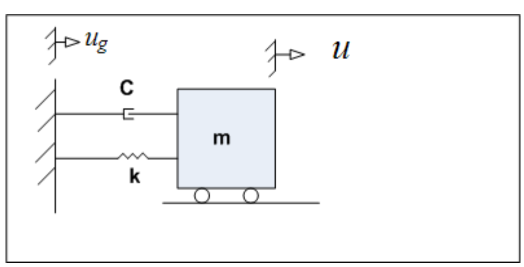
3 Problem 3.8

Consider the water tower shown in Fig. 3.17 which is subjected to ground motion produced by a passing train in the vicinity of the tower. The ground motion is idealized as a harmonic acceleration of the foundation of the tower with an amplitude of 0.1 g at a frequency of 10 cps. Determine the motion of the tower relative to the motion of its foundation. Assume an effective damping coefficient of 10% of the critical damping in the system.

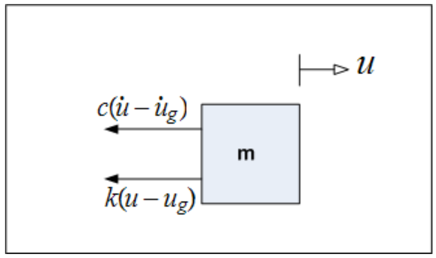


solution

The physical idealized system is the following



Where in the above, u is the absolute displacement of the tower, and u_g is the absolute displacement of the ground. The free body diagram is



Applying newton's second law we obtain

$$
\begin{aligned}
F &= ma \\
-k(u-u_g) - c\left(\dot{u}-\dot{u}_g\right) &= m\ddot{u}
\end{aligned}
$$

Expand and rearrange

$$
m\ddot{u} + c\dot{u} + ku = ku_g + c\dot{u}_g \tag{1}
$$

Let the relative motion between the mass and the ground be u_r, hence $u_r = u - u_g$ or $u = u_r + u_g$, similarly

$$
\begin{aligned}
\dot{u}_r &= \dot{u} - \dot{u}_g \\
\dot{u} &= \dot{u}_r + \dot{u}_g
\end{aligned}
$$

And

$$
\begin{aligned}
\ddot{u}_r &= \ddot{u} - \ddot{u}_g \\
\ddot{u} &= \ddot{u}_r + \ddot{u}_g
\end{aligned}
$$

Using the above expressions for $u, \dot{u}, \ddot{u}$, we can now rewrite (1) as

$$
m\left(\ddot{u}_r + \ddot{u}_g\right) + c\left(\dot{u}_r + \dot{u}_g\right) + k\left(u_r + u_g\right) = ku_g + c\dot{u}_g \tag{2}
$$

Expand (2) and cancel terms we obtain

$$
\begin{aligned}
m\ddot{u}_r + m\ddot{u}_g + c\dot{u}_r + c\dot{u}_g + ku_r + ku_g &= ku_g + c\dot{u}_g \\
m\ddot{u}_r + c\dot{u}_r + ku_r + m\ddot{u}_g &= 0
\end{aligned}
$$

Or

$$\boxed{m\ddot{u}_r + c\dot{u}_r + ku_r = -m\ddot{u}_g} \tag{3}$$

The above is the equation of motion of the tower using relative displacement. Hence we can view the term $m\ddot{u}_g$ as the effective force acting on the tower due to the acceleration of the ground.

Now using the fact that the ground motion is harmonic, we can write

$$u_g = u_{g_0} \sin \bar{\omega} t$$

Where u_{g_0} is the maximum amplitude of the ground displacement, and $\bar{\omega}$ is the ground motion frequency. Hence from the above we obtain that

$$\ddot{u}_g = -\overbrace{u_{g_0}\bar{\omega}^2}^{0.1g} \sin \bar{\omega} t$$

Plug the above into (3) we obtain

$$\boxed{m\ddot{u}_r + c\dot{u}_r + ku_r = \overbrace{mu_{g_0}\bar{\omega}^2}^{F_0} \sin \bar{\omega} t}$$

The above is now in standard 2nd order linear system, the steady state solution for u_r is

$$u_r(t) = u_0 \sin(\bar{\omega} t - \phi)$$

Where $\tan\phi = \frac{2\zeta r}{2-r^2}$ and $u_0 = u_{st} R_d$, where $u_{st} = \frac{F_0}{k} = \frac{mu_{g_0}\bar{\omega}^2}{k}$, and $R_d = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

$$\begin{aligned} u_0 &= u_{st} R_d \\ &= \frac{F_0}{k} \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \\ &= \frac{mu_{g_0}\bar{\omega}^2}{k} \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \end{aligned}$$

But we are told that $u_{g_0}\bar{\omega}^2 = 0.1g$, Hence the above becomes

$$u_0 = \frac{m \times (0.1 \times g)}{k} \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

We are given that

$$\begin{aligned} \bar{\omega} &= 2\pi f \\ &= 2\pi (10) \\ &= 20\pi \quad \text{rad/sec} \end{aligned}$$

and

$$
\begin{aligned}
k &= \frac{3000 \times 10^3}{12} \\
&= \boxed{2.5 \times 10^5 \quad lb/in}
\end{aligned}
$$

Then

$$
\begin{aligned}
\omega_n &= \sqrt{\frac{k}{m}} \\
&= \sqrt{\frac{2.5 \times 10^5 \quad \times 386}{100000}} \\
&= \boxed{31.064 \quad \text{rad/sec}}
\end{aligned}
$$

Then

$$
\begin{aligned}
r &= \frac{\bar{\omega}}{\omega_n} \\
&= \frac{20\pi}{31.064} \\
&= \boxed{2.0227}
\end{aligned}
$$

and since $\zeta = 0.1$ we obtain

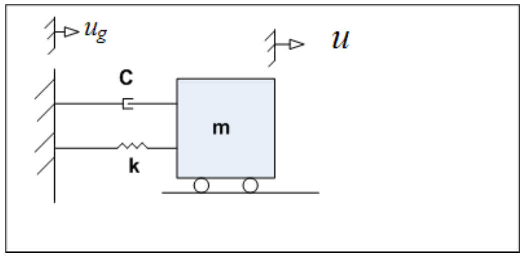
$$
\begin{aligned}
u_0 &= \frac{0.1m \times g}{k} \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \\
&= \frac{0.1 \times \frac{100000}{g} \times g}{2.5 \times 10^5} \frac{1}{\sqrt{(1 - 2.0227^2)^2 + (2 \times 0.1 \times 2.0227)^2}} \\
&= \frac{0.1 \times 100000}{2.5 \times 10^5} \times 0.32075 \\
&= 0.01283 \ in \\
&\approx \boxed{0.013 \ in}
\end{aligned}
$$

4 Problem 3.9

Determine the transmissibility of the above problem

Solution

We need to determine first the expression that represents the force that is transmitted to the ground. From the idealized system diagram



We see that the force transmitted to the support is

$$f_{tr} = c\dot{u}_r + ku_r$$

But

$$u_r = u_0 \sin(\bar{\omega}t - \phi)$$

Where $u_0 = \frac{F_0}{k}$ where F_0 here is the effective force.

Hence

$$\begin{aligned}
f_{tr} &= c\left(u_0\bar{\omega}\cos(\bar{\omega}t - \phi) + ku_0\sin(\bar{\omega}t - \phi)\right) \\
&= u_0\left[c\bar{\omega}\cos(\bar{\omega}t - \phi) + k\sin(\bar{\omega}t - \phi)\right] \\
&= u_0\sqrt{(c\bar{\omega})^2 + k^2}\sin(\bar{\omega}t - \phi + \beta)
\end{aligned}$$

Where $\tan\beta = \frac{c\bar{\omega}}{k}$

Hence Max force transmitted is A_{tr}

$$\begin{aligned}
A_{tr} &= u_0\sqrt{(c\bar{\omega})^2 + k^2} \\
&= u_{st}R_d\sqrt{(c\bar{\omega})^2 + k^2} \\
&= \frac{F_0}{k}R_d\sqrt{(c\bar{\omega})^2 + k^2}
\end{aligned}$$

But $\sqrt{(c\bar{\omega})^2 + k^2} = k\sqrt{1 + (2r\zeta)^2}$ hence

$$\begin{aligned} A_{tr} &= F_0 R_d \sqrt{1+(2r\zeta)^2} \\ &= F_0 \frac{\sqrt{1+(2r\zeta)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \end{aligned}$$

But $T_r = \frac{A_{tr}}{F_0}$ hence

$$T_r = \frac{\sqrt{1+(2r\zeta)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

Now, since from the earlier problem we found that $r = 2.\,022\,7$, and given that $\zeta = 0.1$, then

$$\begin{aligned} T_r &= \frac{\sqrt{1+(2\times 2.\,022\,7\times .1)^2}}{\sqrt{(1-2.\,022\,7^2)^2+(2\times 2.\,022\,7\times .1)^2}} \\ &= \boxed{0.346} \end{aligned}$$

5 Problem 3.12

Problem 3.12

Determine the damping in a system in which during a vibration test under a harmonic force it was observed that at a frequency 10% higher than the resonant frequency, the displacement amplitude was exactly one-half of the resonant amplitude.

Solution

We start with the expression for the maximum amplitude steady state displacement given by

$$\begin{aligned}
u_0 &= u_{st} R_d \\
&= u_{st} \frac{1}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}} \\
&= u_{st} \frac{1}{\sqrt{\left(1-\left(\frac{\bar{\omega}}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\bar{\omega}}{\omega_n}\right)^2}}
\end{aligned}$$

We have 2 cases, case 1 is when $\bar{\omega} = \omega_n$ (resonance), and the second case is when $\bar{\omega} = 1.1\omega_n$. Hence we obtain the following equation

$$\begin{aligned}
\frac{u_{st}}{\sqrt{\left(1-\left(\frac{1.1\omega_n}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{1.1\omega_n}{\omega_n}\right)^2}} &= \frac{1}{2}\frac{u_{st}}{\sqrt{\left(1-\left(\frac{\omega_n}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega_n}{\omega_n}\right)^2}} \\
2\sqrt{\left(1-\left(\frac{\omega_n}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega_n}{\omega_n}\right)^2} &= \sqrt{\left(1-\left(\frac{1.1\omega_n}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{1.1\omega_n}{\omega_n}\right)^2} \\
2\sqrt{\left(2\zeta\right)^2} &= \sqrt{\left(1-\left(1.1\right)^2\right)^2 + \left(2\zeta \times 1.1\right)^2} \\
4\left(2\zeta\right)^2 &= \left(1-\left(1.1\right)^2\right)^2 + \left(2\zeta \times 1.1\right)^2 \\
16\zeta^2 &= 0.0441 + 4.84\zeta^2 \\
11.16\zeta^2 &= 0.0441 \\
\zeta &= \sqrt{\frac{0.0441}{11.16}} \\
&= 0.062862
\end{aligned}$$

Hence

$$\boxed{\zeta \approx 6.3\ \%}$$

6 Problem 3.17

A structural system modeled as a damped oscillator is subjected to the harmonic excitation produced by an eccentric rotor. The spring constant k and the mass m are known but not the damping and the amount of unbalance in the rotor, From measured amplitudes U_r at resonance and U_1 at a frequency ratio $r_1 \neq 1$, determine expressions to calculate the damping ratio ξ and the amplitude of the exciting force F_r at resonance.

Solution

Considering maximum amplitude of steady state is given by

$$\begin{aligned} u_0 &= u_{st} R_d \\ &= \frac{u_{st}}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}} \end{aligned}$$

But $u_{st} = \frac{F_0}{k} = \frac{me\omega^2}{k}$

We are given one case where u_r (resonance) and another case where $u_{r\neq 1}$.When $r = 1$, we obtain

$$u_r = \frac{1}{2\zeta} \frac{me\omega_n^2}{k} \tag{1}$$

When $r \neq 1$ we write (where we call u_0 when $r = r_1$ as u_1)

$$u_1 = \frac{me\omega_1^2}{k} \frac{1}{\sqrt{\left(1-r_1^2\right)^2 + \left(2\zeta r_1\right)^2}} \tag{2}$$

Square equation (1) and (2) and divide by each others we obtain

$$
\begin{aligned}
\frac{u_r^2}{u_1^2} &= \left(\frac{\omega_n^2}{\omega_1^2}\right)^2 \left(\frac{\left(1-r_1^2\right)^2+\left(2\zeta r_1\right)^2}{\left(2\zeta\right)^2}\right)\\
\frac{u_r^2}{u_1^2} &= \left(\frac{1}{r_1}\right)^4 \left(\frac{\left(1-r_1^2\right)^2+\left(2\zeta r_1\right)^2}{\left(2\zeta\right)^2}\right)\\
4\zeta^2 u_r^2 r_1^4 &= u_1^2\left(1-r_1^2\right)^2+4u_1^2\zeta^2 r_1^2\\
4\zeta^2 r_1^2\left(u_r^2 r_1^2-u_1^2\right) &= u_1^2\left(1-r_1^2\right)^2\\
\zeta^2 &= \frac{u_1^2\left(1-r_1^2\right)^2}{4r_1^2\left(u_r^2 r_1^2-u_1^2\right)}
\end{aligned}
$$

Hence

$$
\boxed{\zeta = \frac{u_1\left(1-r_1^2\right)}{2r_1\sqrt{u_r^2 r_1^2-u_1^2}}}
$$

Since

$$
\begin{aligned}
u_r &= u_{st}R_d\\
&= \frac{F_r}{k}R_d
\end{aligned}
$$

Hence

$$
F_r = \frac{u_r k}{R_d}
$$

But $R_d = \frac{1}{2\zeta}$ at resonance, hence

$$
F_r = 2u_r k\zeta
$$

But from first part, we found expression for ζ, which we plug into the above to obtain

$$
F_r = 2ku_r\frac{u_1\left(1-r_1^2\right)}{2r_1\sqrt{u_r^2 r_1^2-u_1^2}}
$$

Hence

$$
\boxed{F_r = \frac{u_r u_1\left(1-r_1^2\right)k}{r_1\sqrt{u_r^2 r_1^2-u_1^2}}}
$$