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HW1. CEE 247. Structural Dynamics. UCI. Fall 2006

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October 4, 2006

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1 Problem 1.3

Problem 1.3
Determine the natural frequency for horizontal motion of the steel frame in Fig. P1.3. Assume the horizontal girder to be infinitely rigid and neglect the mass of the columns.

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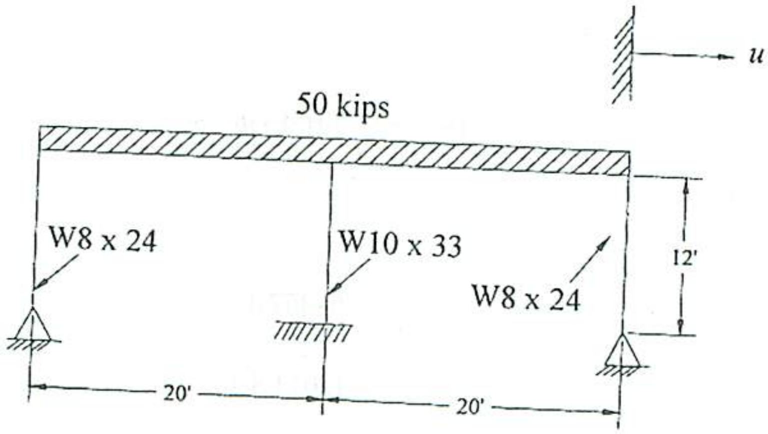

Fig. P1.3

Solution

From steel manual, we obtain the following values for I_{xx} for the different W sections:

$W8 \times 24 \rightarrow I_{xx} = 82.5\ in^4$, $W10 \times 33 \rightarrow I_{xx} = 170\ in^4$, and $E = 30 \times 10^6\ psi.$

Weight of girder$W = 50\ kips$ or 50,000 lbs.

Gravity Acceleration $g = 386\ in/\sec^2$

$L = 12 \times 12 = 244''$

We start by finding the effective stiffness k_e

$$
\begin{aligned}
k_e &= 2\left(\frac{3EI_{8\times24}}{L^3}\right) + \frac{12EI_{10\times33}}{L^3} \\
&= 2\left(\frac{3 \times 30 \times 10^6 \times 82.5}{244^3}\right) + \frac{12 \times 30 \times 10^6 \times 170}{244^3} \\
&= 1022.2 + 4212.9
\end{aligned}
$$

?

Hence

−1

$$\boxed{k_e = 5235.2\ lb/in}$$

$$\begin{aligned}\omega_n &= \sqrt{\frac{k_e}{m}}\\ &= \sqrt{\frac{k_e \times g}{W}}\\ &= \sqrt{\frac{5235.2 \times 386}{50000}}\end{aligned}$$

Hence

$$\boxed{\omega_n = 6.3573 \ rad/sec}$$

But $\omega_n = 2\pi f$ hence

$$\begin{aligned}f &= \frac{6.3573}{2\pi}\\ &= 1.0118 \text{ hz}\end{aligned}$$

2 Problem 1.6

Problem 1.7

Consider the simple pendulum of weight W illustrated in Fig. P1.7. If the cord length is L, determine the motion of the pendulum. The initial angular displacement and initial angular velocity are θ_0 and $\dot{\theta}_0$, respectively. (Assume the angle θ is small)

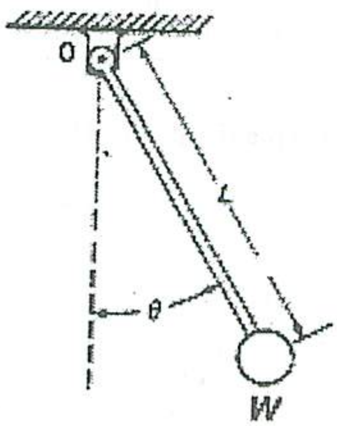

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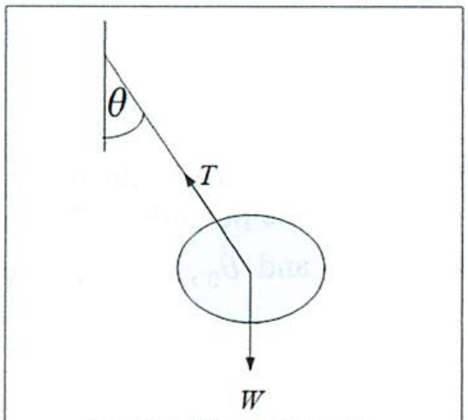
Fig. P1.7

Solution

The equation of motion is derived in 2 methods. One based on the force method and the second is based on energy method. In both method we assume there is no damping in the system and no friction nor air resistance.

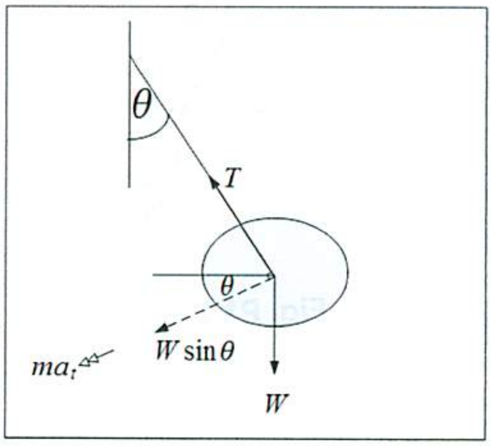
First method

First draw a free body diagram showing forces acting on pendulum mass, which are the weight W and the tension T in the cord.



Free Body Diagram of Pendulum

Next resolve the forces parallel and perpendicular to the direction of motion as follows



Where a_t is the tangential acceleration and m is the mass of the pendulum bob. Since $a_t = \frac{d^2(s)}{dt^2}$ where s is an arc length, and $s = L\theta$ when θ is in radians, hence $a_t = L\frac{d^2\theta}{dt^2} = L\ddot{\theta}$

So applying Newton second law of motion along the tangential direction we obtain

$$\sum F_t = ma_t$$
$$-W\sin\theta = mL\ddot{\theta}$$

Where the minus sign is due to the fact that force acts in the direction opposite to increasing θ.
But $W = mg$ hence

$$-g\sin\theta = l\ddot{\theta}$$
$$\ddot{\theta} + \frac{g}{L}\sin\theta = 0$$

This is the second order differential equation of motion we need to solve. This ODE is non-linear in θ. Assuming θ is small, and since $\sin\theta = \theta - \frac{\theta^3}{3!} + \cdots$, hence we see that for small θ, $\sin\theta \approx \theta$, we the ODE becomes

$$\ddot{\theta} + \frac{g}{L}\theta = 0$$

This is free vibration undamped motion. Assume the solution is $\theta(t) = Ae^{\lambda t}$, we obtain the characteristic equation

$$\lambda^2 + \frac{g}{L} = 0$$

which has solution $\lambda = \pm i\sqrt{\frac{g}{L}}$, Hence the solution is

This has the solution

$$\theta(t) = A\cos\lambda t + B\sin\lambda t$$

Now we find A, B from initial conditions. at $t = 0$, $\boxed{\theta_0 = A}$, and at $t = 0$, $\dot{\theta}_0 = B\lambda \Rightarrow \boxed{B = \frac{\dot{\theta}_0}{\lambda}}$

Hence the natural frequency

$$\boxed{\omega_n = \sqrt{\frac{g}{L}}}$$

Hence the general solution is

$$\boxed{\theta(t) = \theta_0\cos\omega_n t + \frac{\dot{\theta}_0}{\omega_n}\sin\omega_n t}$$

Let $C = \sqrt{A^2 + B^2}$, and $\phi = \arctan\frac{A}{B}$, the solution can also be written as

$$\theta(t) = C\sin(\omega_n t + \phi)$$

We see that the natural frequency ω_n of the bob is

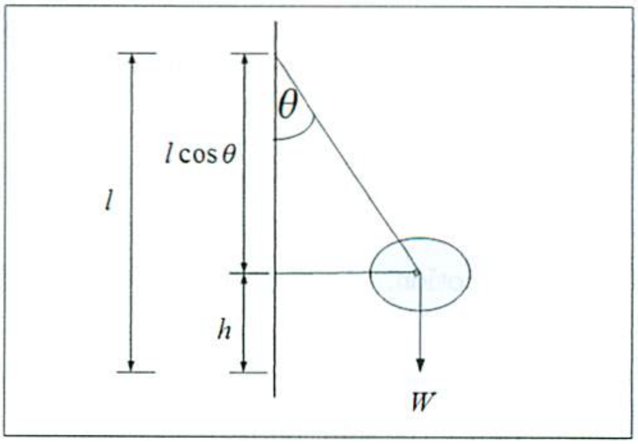
$$\omega_n = \sqrt{\frac{g}{L}}$$

And it does not depend on the mass of the bob.

Second method

Here is another derivation. Since there is no damping in the system then the energy of the system $E = PE + KE$ is constant.

But $PE = mgh$, where h is the height above the reference level. Taking the reference level when the bob is at the lowest point, we see that at any instance of time $h = L - L\cos\theta = L(1 - \cos\theta)$



And the KE any that moment is given by $\frac{1}{2}mv^2$, but $v = L\dot{\theta}$, hence $KE = \frac{1}{2}m\left(l\dot{\theta}\right)^2$, Hence we obtain the energy as

$$E = mgL\left(1 - \cos\theta\right) + \frac{1}{2}m\left(L\dot{\theta}\right)^2$$

Since E is constant, then $\frac{dE}{dt} = 0$, hence

$$g\dot{\theta}\sin\theta + L\dot{\theta}\ddot{\theta} = 0$$
$$\dot{\theta}\left(\frac{g}{L}\sin\theta + \ddot{\theta}\right) = 0$$

We have 2 solution. Ignoring the solution that $\dot{\theta} = 0$ since this is trivial. We obtain the same ODE as above which is

$$\ddot{\theta} + \frac{g}{L}\sin\theta = 0$$

The advantage of the derivation based on energy is that one is working with scalar quantities. hence one does not need to worry about sign of forces and direction of motion as one would with the force method.

Excellent!

3 Problem 2.4

Problem 2.4
It is observed experimentally that the amplitude of free vibration of a certain structure, modeled as a single degree-of-freedom system, decreases in 10 cycles from 1 in to 0.4 in. What is the percentage of critical damping?

$\frac{10}{10}$

Solution

Since

$$\ln\left(\frac{u_i}{u_{i+j}}\right) = j2\pi\xi$$

And since $j = 10$ in this case (10 cycles), hence

$$\xi = \frac{\ln\left(\frac{u_i}{u_{i+10}}\right)}{20\pi}$$

But $u_i = 1''$ and $u_{i+10} = .4''$, hence

$$\begin{aligned}\xi &= \frac{\ln\left(\frac{1}{.4}\right)}{20\pi} \\ &= 1.4583 \times 10^{-2}\end{aligned}$$

Therefore

$$\boxed{\xi \approx 1.5\%}$$

4 Problem 2.6

Problem 2.6

A structure is modeled as a damped oscillator having a spring constant $k = 30$ kip/in and undamped natural frequency $\omega = 25$ rad/sec. Experimentally it was found that a force of 1 kip produced a relative velocity of 1.0 in/sec in the damping element. Determine:

a) The damping ratio ξ.
b) The damped period T_D.
c) The logarithmic decrement δ.
d) The ratio between two consecutive amplitudes.

$k = 30,000 \;\; lb/in$

$\omega_n = 25$ rad/sec

Since viscous damping force is proportional to speed, hence $\frac{F_d}{v} = c$, then

$$\begin{aligned} c &= \frac{1000\,(lb)}{1\,(in/\text{sec})} \\ &= 1000 \;\; \text{lb-sec/in} \end{aligned}$$

a) $\xi = \frac{c}{c_{cr}}$, but

$$\begin{aligned} c_{cr} &= \frac{2k}{\omega_n} \\ &= \frac{2 \times 30000}{25} \\ &= 2400 \text{ lb-sec/in} \end{aligned}$$

Hence

$$\begin{aligned} \xi &= \frac{c}{c_{cr}} \\ &= \frac{1000}{2400} \end{aligned}$$

Hence

$$\boxed{\xi = 0.416\,67}$$

b) Since

$$T_D = \frac{T}{\sqrt{1-\xi^2}}$$

But $T = \frac{2\pi}{\omega_n} = \frac{2\pi}{25} = 0.251\,33\,\text{sec}$

Hence

$$T_D = \frac{0.251\,33}{\sqrt{1-0.416\,67^2}}$$

Then

$$\boxed{T_D = 0.276\,47\,\text{sec}}$$

c)The logarithmic decrement $\delta = \ln \frac{u_i}{u_{i+1}}$, But $\ln \frac{u_i}{u_{i+1}} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$, hence

$$\delta = \frac{2\pi \times 0.416\,67}{\sqrt{1-0.416\,67^2}}$$

Hence

$$\boxed{\delta = 2.\,879\,9}$$

c)Since

$$\ln \frac{u_i}{u_{i+1}} = \delta$$

Then

$$\begin{aligned} \frac{u_i}{u_{i+1}} &= e^{\delta} \\ &= e^{2.\,879\,9} \end{aligned}$$

Hence

$$\frac{u_i}{u_{i+1}} = 17.\,812$$