University Course

EE 152A
Digital Signal Processing

University Of California, Irvine
Fall 2004

My Class Notes

Nasser M. Abbasi

Fall 2004

Contents

Chapter 1

Introduction

1.1 syllabus

EECS152A Digital Signal Processing

Description: The study of the basic principles and techniques underlying the analysis and design of digital signal processing and digital filtering systems.

Prerequisites: ECE 120B (Signals and Systems II) and ECE 186 (Probability)

Text:	*Digital Signal Processing, 3rd Ed.* by John Proakis and Dimitris Manolakis Prentice-Hall	
Instructor:	Glenn Healey	
Office:	ET 616D	
Phone:	824-7104	
Email:	healey@ece.uci.edu	
Office Hours:	TBA	
Teaching Assistant:	Mohit Singh	
Email:	msingh@ea.nacs.uci.edu	
Office Hours:	TBA	
Lectures:	Tu,Th 5:00-6:20	
Grading:	Homework	30%
	Midterm Exam	30%
	Final Exam	40%

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EECS152A Outline

Tu	September 28	Chapter 1	
Th	September 30	Chapter 1	
Tu	October 5	Chapter 2	Homework #1
Th	October 7	Chapter 2	
Tu	October 12	Chapter 2	Homework #2
Th	October 14	Chapter 3	
Tu	October 19	Chapter 3	Homework #3
Th	October 21	Chapter 3	
Tu	October 26	MIDTERM EXAM	
Th	October 28	Chapter 4	
Tu	November 2	Chapter 4	
Th	November 4	Chapter 4	
Tu	November 9	Chapter 4	Homework #4
Th	November 11	Chapter 5	
Tu	November 16	Chapter 5	Homework #5
Th	November 18	Chapter 5	
Tu	November 23	Chapter 8	
Th	November 25	THANKSGIVING	
Tu	November 30	Chapter 8	Homework #6
Th	December 2	Advanced Topics	

Final Exam: Thursday December 9, 4-6pm

1.2 Text Book

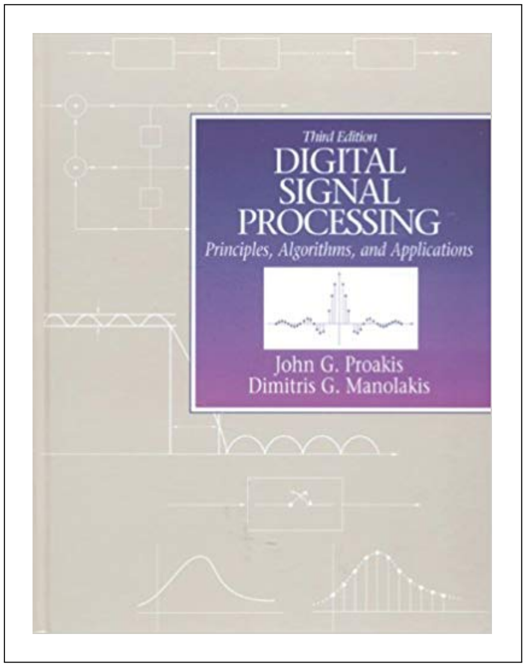
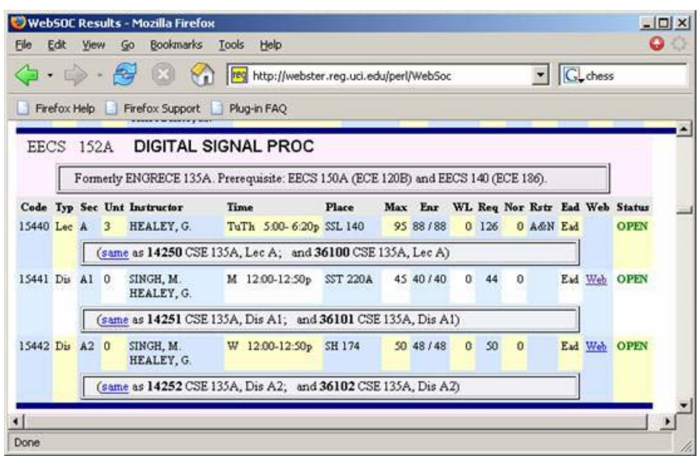


Figure 1.1: Official text book

1.3 Class information



EECS 152A DIGITAL SIGNAL PROC

Formerly ENGRECE 135A. Prerequisite: EECS 150A (ECE 120B) and EECS 140 (ECE 186).

Code	Typ	Sec	Unt	Instructor	Time	Place	Max	Enr	WL	Req	Nor	Rstr	Ead	Web	Status
15440	Lec	A	3	HEALEY, G.	TuTh 5:00- 6:20p	SSL 140	95	88 / 88	0	126	0	A&N	Ead		OPEN
(same as 14250 CSE 135A, Lec A; and 36100 CSE 135A, Lec A)															
15441	Dis	A1	0	SINGH, M. HEALEY, G.	M 12:00-12:50p	SST 220A	45	40 / 40	0	44	0		Ead	Web	OPEN
(same as 14251 CSE 135A, Dis A1; and 36101 CSE 135A, Dis A1)															
15442	Dis	A2	0	SINGH, M. HEALEY, G.	W 12:00-12:50p	SH 174	50	48 / 48	0	50	0		Ead	Web	OPEN
(same as 14252 CSE 135A, Dis A2; and 36102 CSE 135A, Dis A2)															

Figure 1.2: Course meeting time

Chapter 2

Study notes, cheat sheets

Local contents

2.1 some study notes

2.1.1 questions

1. To find $|H(\omega)|$ it might easier to use the relation that $|H(\omega)|^2 = H(\omega) H^*(\omega)$ this is true when $h(n)$ is real. can I also use it when input is real? see book page 320

2. need to learn better how to find fourier transform for periodic discrete signal. for example, if $f(x) = x^2$ then we get $F(u) = \sum_{n=0}^{N-1} x^2 e^{-j2\pi \frac{u}{N} x}$ how to evaluate this?

   only tricks I know are to use geometric series sum, $\sum_{n=0}^{N-1} a^n = \begin{cases} \frac{1-a^N}{1-a} & a \neq 1 \\ N & a = 1 \end{cases}$ this is if I can get the terms inside the sum to have the form a^n and the other trick is if I can express $f(x)$ itself in terms of $e^{j2\pi}$, this happens if $f(x)$ is a trignomtric function.

3. Should we used normalized $H(z)$ or leave it unnormalized? see question 4.51, HW 5 for example. if we do not normalize it, we are left with b_0 term?

4. Ask how did the book find the phase of $H(\omega) = \frac{1}{3}(e^{j\omega} + 1 + e^{-j\omega})$ to be 0 for $0 \leq \omega \leq \frac{2\pi}{3}$ and π for $\frac{2\pi}{3} \leq \omega < \pi$

2.1.2 notes

an analog signal is written as $A\cos(2\pi F t + \theta)$ where F is cycles per second.

A discrete signal is written as $A\cos(2\pi f n + \theta)$ where f is in samples per second.

(this should be cycle per second?, check)

$f = \frac{F}{F_s}$ where F_s is the sample rate in samples per second and F is the frequency of the discrete signal in cycles per sample.

To avoid aliasing we must have $f < |1/2|$ cycles per sample. And since $f = \frac{F}{F_s}$ then this means $F_s > 2F$ to avoid aliasing. To determine if aliasing exist given an analog signal and a sample rate, find f and see if it is $< 1/2$. example:

Given $x_a(t) = \cos(2\pi 10 t)$ and $F_s = 40$ samples/sec, then convert to discrete signal and find f. $x(n) = \cos(2\pi 10(nT)) = \cos(2\pi 10(n\frac{1}{F_s})) = \cos(2\pi 10(n\frac{1}{40})) = \cos(2\pi \frac{1}{4} n)$ hence $f = \frac{1}{4}$ cycles per sample. and since this is $< 1/2$, then no aliasing exist.

An analog sinisoidal signal is always periodic, but a discrete sinosoidal signal may not be. To detrermine, find f of the discrete signal and if f is rational number, then periodic. To find fundemental period, bring f to lowest terms (relative primes) and this will be the fundemental period.

A signal can be multi-dimensional and multi-channel. $f(x, y)$ multi-dimension, and $f(x) =$

$\begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix}$ is multi-channel, one dimension.

Learned linearity tests. $L[a_1x_1(n) + a_2x_2(n)] = L[a_1x_1(n)] + L[a_2x_2(n)]$ if these are the same, then system is linear.

$$\delta(n) = \begin{cases} 1 & n = 0 \\ \\ 0 & otherwise \end{cases} \quad \text{this is called a unit SAMPLE}$$

$$u(n) = \begin{cases} 1 & n \geq 0 \\ \\ 0 & otherwise \end{cases} \quad \text{this is called a unit STEP}$$

$$u(n) = \sum_{k=-\infty}^{k=\infty} \delta(n)$$

$$\delta(n) = u(n) - u(n-1)$$

any signal $x(n)$ can be written as $x(n) = \sum_{k=-\infty}^{k=\infty} x(k)\ \delta(n-k)$

To find $|H(\omega)|$ it might easier to use the relation that $|H(\omega)|^2 = H(\omega)\,H^*(\omega)$ this is true when $h(n)$ is real. can I also use it when input is real? Also, if I have the Z transform, I can use $|H(\omega)|^2 = H(z)\,H(z^{-1})\,|_{z=e^{jw}}$ i.e. multiply the z transforms as shown, and do everything in terms of z (easier) then at the end replace z by e^{jw}

2.1.3 agenda

1. sunday nov 28, 2004. working on last HW, HW6. Very long...
2. saturday nov 20, 2004. 8:50 PM. currently working on last HW for 152A and 203A
3. sunday nov 14, 2004. 9:40 AM. finsihed problem 1 for HW 5 for ECE 203A. coding mostly.
4. Sunday Nov 14, 2004. 4 AM. finsihed HW 5 for DSP. make a questions section to collect questions I needed answered. Use Geometrical argument for location of poles and zeros and coming up with $H(z)$ since it seems more natural.
5. Nov 12, 2004. Working on HW5. Forgot how to find the phase of transfer function.
6. No lecture on thursday Nov 11, 2004. Veternes day. I have been using Mathematica more now. it is worth learning it.
7. Oct5,2004. Lecture day. Talk about Nyquist and sampling theorm. How to convert discrete signal back to analog using the sinc function.

8. Oct 4, 2004. Monday. 10 PM. Finished HW1, started this note file to record notes on each chapter as I go so I use them to study from the exam.

2.2 Cheat sheet

$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$; $W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}-\frac{i\sqrt{3}}{2} & -\frac{1}{2}+\frac{i\sqrt{3}}{2} \\ 1 & -\frac{1}{2}+\frac{i\sqrt{3}}{2} & -\frac{1}{2}-\frac{i\sqrt{3}}{2} \end{bmatrix}$; $W_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

So if $x[n] = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$, $X(k) = x[n] W_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$

To find IDFT, do $x[n] = \frac{W^*}{4} X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} \downarrow \end{bmatrix}$

$\boxed{W_N^k} = e^{-j\frac{2\pi k}{N}}$

Properties of DFT: $X(k)$ periodic, period $= N$, Linear: $a_1 x_1 + a_2 x_2 \overset{DFT}{\longleftrightarrow} a_1 X_1 + a_2 X_2$

$x_p(n) = \sum_{\ell=-\infty}^{\infty} x[n-\ell N]$. $\boxed{X_1 X_2 = \text{circular Convolution of } x_1, x_2}$

ideal low pass $\Rightarrow h(n) = \begin{cases} \frac{\omega_c}{\pi} & n=0 \\ \frac{\omega_c}{\pi} \frac{\sin \omega_c n}{\omega_c n} & n \neq 0 \end{cases}$

LTI is Causal if specified by difference equation $y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$

$H(z) = \frac{\sum_{k=0}^{N} b_k z^{k}}{1 + \sum_{k=1}^{M} a_k z^{-k}}$. To do Circular Convolution. do same as Linear Convolution, just shift right!

$\Omega = 2\pi F$, $F \to$ cycle/sec. input signal $\omega = 2\pi f n$

Continuous $x(t)$ | discrete $x[n]$

energy $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int |X(F)|^2 dF$

Power: periodic (T) — CTFS (Fourier Series)

$c(k) = \frac{1}{T} \int_T x(t) e^{-j\omega_0 k t} dt$

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\omega_0 k t}$

$\boxed{P_x = \frac{1}{T} \int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k|^2}$

aperiodic (Transform): $X(\Omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\Omega t} dt$

$x(t) = \frac{1}{2\pi} \int_{\Omega=-\infty}^{\infty} X(\Omega) e^{j\Omega t} d\Omega$

$\boxed{\text{if } x(t) \text{ real then } c(-k) = c^*(k)}$

periodic T. DTFS: $C[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N} n}$

$x[n] = \sum_{k=0}^{N-1} c[k] e^{j\frac{2\pi k}{N} n}$

$E_N = \sum_{n=0}^{N-1} |x[n]|^2 = N \sum_{k=0}^{N-1} |c_k|^2$

aperiodic: $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} d\omega$

$E_x = \sum |x(n)|^2 = \frac{1}{2\pi} \int |X(\omega)|^2 d\omega$

$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}}$

DFT: $\boxed{X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{N} n} \quad k = 0,1,\ldots N-1 \\ x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\left(\frac{2\pi k}{N}\right) n} \quad n = 0,1,\ldots N-1}$

properties of Z transform

$u(n) \overset{z}{\longleftrightarrow} \frac{1}{1-z^{-1}}$ Roc $|z|>1$

$u(-n) \overset{z}{\longleftrightarrow} \frac{1}{1-z}$ Roc $|z|<1$

$n x(n) \overset{z}{\longleftrightarrow} -z \frac{dX(z)}{dz}$

$a^n u(n) \overset{z}{\longleftrightarrow} \frac{1}{1-a z^{-1}}$

$a^n x(n) \overset{z}{\longleftrightarrow} X(a^{-1} z)$

$\boxed{x(-n) = X(z^{-1})}$ so find $X(z)$ for $x(n)$ and replace z by z^{-1}

$x^*(n) \overset{z}{\longleftrightarrow} X^*(z^*)$

If $y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$

Then $H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$

Correlation

$r_{x_1 x_2}(\ell) = \sum_{n=-\infty}^{\infty} x_1(n) x_2(n-\ell) \overset{z}{\longleftrightarrow} X_1(z) X_2(z^{-1})$

$r_{x_1 x_2}(\ell) = x_1(\ell) * x_2(-\ell)$ (so Correlation is Convolution but without flipping)

Time		Frequency
Real, even	$\longleftrightarrow$	Real, even
Real, odd	$\longleftrightarrow$	Imaginary, odd
Im, even	$\longleftrightarrow$	Imaginary, even
Im, odd	$\longleftrightarrow$	Real, odd

$\delta(n) \overset{z}{\longleftrightarrow} 1$ All z

$u(n) \overset{z}{\longleftrightarrow} \frac{1}{1-z^{-1}}$ $|z|>1$

$a^n u(n) \overset{z}{\longleftrightarrow} \frac{1}{1-a z^{-1}}$ $|z|>a$

$\sum_{n=0}^{\infty} (a)^n = \frac{1}{1-a}$ $|a|<1$

$\sum_{n=0}^{N} (a)^n = \frac{1-a^{N+1}}{1-a}$ or $\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$ $a \neq 1$

Symmetry relationships for $X(\omega)$

$x^*(n) \longleftrightarrow X^*(-\omega)$

$x^*(-n) \longleftrightarrow X^*(\omega)$

IF $x[n]$ is real Then: $X(\omega) = X^*(-\omega)$, $|X(\omega)| = |X(-\omega)|$

Convolution

$y(n) = \sum_{k=-\infty}^{k=\infty} x(k) h(n-k)$

IIR $\Rightarrow y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$

$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$

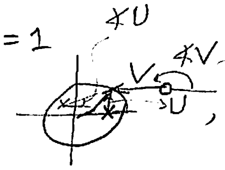
FIR = all zero $\Rightarrow y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$

$H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$

$$H(z)=\frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = b_0\frac{\prod_{k=1}^{M}(1-z_k z^{-1})}{\prod_{k=1}^{N}(1-p_k z^{-1})}=\frac{(z-z_1)(z-z_2)\cdots}{(z-p_1)(z-p_2)\cdots}$$

b_0 selected such that $|H(\omega_0)|=1$

$$|H(\omega)|=|b_0|\frac{|V_1(\omega)||V_2(\omega)|\cdots|V_m(\omega)|}{|U_1(\omega)||U_2(\omega)|\cdots|U_N(\omega)|}$$



$$\boxed{\measuredangle H(\omega)=\measuredangle b_0+\omega(N-M)+\measuredangle V_1+\measuredangle V_2+\cdots-[\measuredangle U_1+\measuredangle U_2+\cdots]}$$

$$|H(\omega)|^2 = H(\omega)H^*(\omega) = H(\omega)H(-\omega) = H(z)H(z^{-1})\Big|_{z=e^{j\omega}}$$

Real a_k, b_k.

Properties of DFT $X(N-k)=X^*(k)=X(-k)$ for real $x[n]$, $|X[N-k]|=|X[k]|$, $\measuredangle X[N-k]=-\measuredangle X[k]$

if $x[n]$ real & even, Then $X[k]=\sum_0^{N-1} x[n]\cos\frac{2\pi k}{N}n$, $x[n]=\frac{1}{N}\sum X(k)\cos\frac{2\pi k}{N}n$

if $x[n]$ is real & odd, Then $X[k]=\sum_0^{N-1}\left(-j\sin\frac{2\pi k}{N}n\right)$, $x[n]=j\frac{1}{N}\sum X(k)\sin\frac{2\pi k}{N}n$.

$x[N-n]=X[N-k]$, $x((n-\ell))_N \longleftrightarrow X(k)e^{-j\frac{2\pi k}{N}\ell}$; $x[n]e^{j\frac{2\pi\ell}{N}n}\longleftrightarrow X((k-\ell))_N$

$x^*[n]=X^*[N-k]$

$\tilde{r}_{xy}(\ell)=x(\ell)\,(N)\,y^*(-\ell)$ ← circular correlation $\quad x_1x_2\longleftrightarrow\frac{1}{N}X_1 (N) X_2$

given $x[n]$ of length L, $h[n]$ of length M, How to know Length of DFT? $N=L+M-1$

so if x has $L=4$, length $h[n]=3$, then we need $N=6$

note we can X_1, X_2 to find X_3, Then IDFT to find response of system. like Linear convolution, as long as we padd sequences.

$$\int_0^\infty e^{-t(1+j2\pi F)}dt=\frac{1}{1+j2\pi F}$$

if we are given some points of DFT, we can find rest using properties $\boxed{X(N-k)=X^*(k)=X(-k)}$

$N=8$. $X(6)=X(8-2)=X^*(2)$

for DFT when input is sin/cos, use

$$\sum_{n=0}^{N-1}e^{-j\frac{2\pi n}{N}(k-1)}=N\delta(k-1)$$

$$\sum_{n=0}^{N-1}e^{-j\frac{2\pi n}{N}(k+1)}=N\delta(k+1)$$

$$\cos\alpha=\frac{e^{j\alpha}+e^{-j\alpha}}{2}$$

$$\sin\alpha=\frac{e^{j\alpha}-e^{-j\alpha}}{2j}$$

so if $x(n)=\cos\frac{2\pi}{N}n\rightarrow DFT=\frac{N}{2}[\delta(k-1)+\delta(k+1)]$

$x(n)=\sin\frac{2\pi}{N}n\rightarrow DFT=\frac{N}{2j}[\delta(k-1)+\delta(k+1)]$

if $X(k)=\frac{N^2}{4j}[\delta(k-1)+\delta(k+1)]\Rightarrow x(n)=\frac{N}{2}\sin\left(\frac{2\pi}{N}n\right)$

DFT if given sequence $x(n)$ and asked to find its Energy do

$E=\sum_{n=0}^{N-1}x(n)x^*(n)$. Example. $x(n)=\cos\frac{2\pi k}{N}n \Rightarrow x(n)x^*(n)=\frac{1}{4}\left(2+e^{+j\frac{4\pi kn}{N}}+e^{-j\frac{4\pi kn}{N}}\right)$

$\Rightarrow E=\sum_{n=0}^{N-1}x(n)x^*(n)=\frac{1}{4}2N=\frac{N}{2}$.

x_1 = 0 1 2 3 4 5 6 7

x_2 = 0 1 2 3 4 5 6 7

$x_2=x_1((n-5))_8$ circnt space between. $\Rightarrow X_2=X_1e^{-j\frac{2\pi 5k}{N}}$ delay

Least square method for design IIR e FIR

2.3 some code

2.3.1 file 1

```
%Script to illustrate aliasing
% by Nasser M. Abbasi
% Oct 14, 2004.
%
%

clear all;
close all;

nSamples=10;
startTime=0;
Fs=input('Enter sampling frequency (Hz) > ');
F1=input('Enter first signal frequnecy (Hz) [1/8] >');
F2=input('Enter second signal frequnecy (Hz) [-7/8] >');
nSamples=input('Enter number of samples? >');


T=1/Fs;
endTime=nSamples*T;
ap=T/10;

t=[startTime:ap:endTime];

xa1= sin(2*pi*F1*t);
xa2= sin(2*pi*F2*t);
n=[1:10:length(t)];

xn1= xa1(n);
plot(t,xa1);
hold on;
plot(t(n),xn1,'o');

xn2= xa2(n);
plot(t,xa2,'r');
plot(t(n),xn2,'*');
```

2.3.2 file 2

```
function [x,n]=impseq(n0,n1,n2)
n=[n1:n2];
x=[(n-n0)==0];
```

Chapter 3

HWs

Local contents

3.1 HW 1

HW#1

EECS 152A, Digital Signal processing

UCI. Fall 2004

By Nasser Abbasi

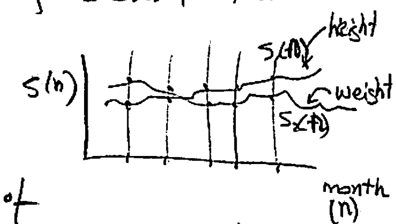
EECS 152 DSP
HW#1

9/10

Problem 1.1 (e)

Statment Classify the following signal according to whether they are (1) one or multidimensional (2) single or multichannel, (3) continous time or discrete time, (4) analog or digital (in amplitude). Give breif explanation.

1e) weight and height measurments of a child taken every month.



Answer

(1) this is $\boxed{\text{one dimension}}$ since function of one independent variable (the month in this case).

(2) ~~this~~ is $\boxed{\text{multichannal}}$. one channel is the height signal, and the second channel is the weight.

1

(3) This is $\boxed{\text{discrete time}}$. since time here is month number.

(4) This is $\boxed{\text{analog}}$. because weight and height are continous quantities.

$$\boxed{s(n) = \begin{bmatrix} s_1(n) \\ s_2(n) \end{bmatrix}}$$

EECS 152 DSP

HW#1

Problem 1.2

Statment

Deteemine which of following sinusoids are periodic and compute Fundemental period

(a) $\cos 0.01\pi n$ (b) $\cos\left(\pi \frac{30n}{105}\right)$ (c) $\cos 3\pi n$ (d) $\sin 3n$

(e) $\sin\left(\pi \frac{62n}{10}\right)$

Answer

General Form of discrete sinusoidal signal is

$$A \sin(2\pi f n + \theta) \quad \text{or} \quad A \cos(2\pi f n + \theta).$$

where A is amplitude, f is cycles per sample, θ is phase in radians, n is sample number.

discrete-time sinusoid is periodic only if f is rational.

(a) $\cos 0.01\pi n = \cos(2\pi f n + \theta)$

$\Rightarrow 0.1\pi n = 2\pi f n \Rightarrow f = \frac{0.1}{2}$ cycles per sample.

so f is rational $\Rightarrow$ $\boxed{\text{periodic}}$ ✓

(4) To Find Fundemental period. write $f = \frac{K}{N}$ where K, N are relative primes. then N is the Fundemental period.

so $f = \frac{1}{20} \Longrightarrow$ $\boxed{\text{F. Period} = 20}$ samples.

$\longrightarrow$

(b) $\cos\left(\pi \frac{30n}{105}\right)$.

write $\cos\left(\pi\frac{30n}{105}\right) = \cos(2\pi f n + \theta)$

so $\pi\frac{30n}{105} = 2\pi f n \Rightarrow f = \frac{15}{105} \Rightarrow \boxed{\text{periodic}}$ since rational

$\frac{15}{105} = \frac{3}{21} = \frac{1}{7} \Rightarrow \boxed{\text{Fund. periode} = 7}$ samples.

(c) $\cos 3\pi n$

write as $\cos 3\pi n = \cos(2\pi f n + \theta)$

so $3\pi n = 2\pi f n \Rightarrow f = \frac{2}{3}$. $3/2$ $\Rightarrow \boxed{\text{periodic}}$ since rational.

since $\frac{2}{3}$ allready realtivly prime $\Rightarrow \boxed{\text{Fund. period} = 3 \text{ samples}}$

(d) $\sin 3n$

$\sin 3n = \sin(2\pi f n + \theta) \Rightarrow 3n = 2\pi f n \Rightarrow f = \frac{3}{2\pi}$

$\Rightarrow \boxed{\text{NOT periodic}}$ since not rational.

(e) $\sin\left(\pi \frac{62n}{10}\right)$

$\sin\left(\pi\frac{62n}{10}\right) = \sin(2\pi f n + \theta) \Rightarrow 2\pi f n = \frac{62 n\pi}{10}$

$f = \frac{62}{20} \Rightarrow \boxed{\text{periodic}}$

$\frac{62}{20} = \frac{31}{10} \Rightarrow$ Fundemental period $= \boxed{10 \text{ samples}}$

1.5

Problem statment

Consider signal $x_a(t) = 3\sin(100\pi t)$

(a) sketch signal $x_a(t)$ for $0 \le t \le 30$ ms

(b) signal $x_a(t)$ is sampled with $F_s = 300$ samples/s. Determine the frequency of the discrete-time signal $x(n) = x_a(nT)$, $T = \frac{1}{F_s}$ and show that it is periodic.

(c) Compute sample values in one period of $x(n)$. sketch $x(n)$ on same diagram with $x_a(t)$. what is period of the discrete-time signal in milliseconds?

(d) Can you find sampling rate F_s such that signal $x(n)$ reaches its peak value of 3? what is the minmum F_s suitable for this task?

Solution

(a) $1\text{ ms} = 10^{-3}$ seconds

Calculate few values.

$t=0 \Rightarrow x_a(t) = 0$

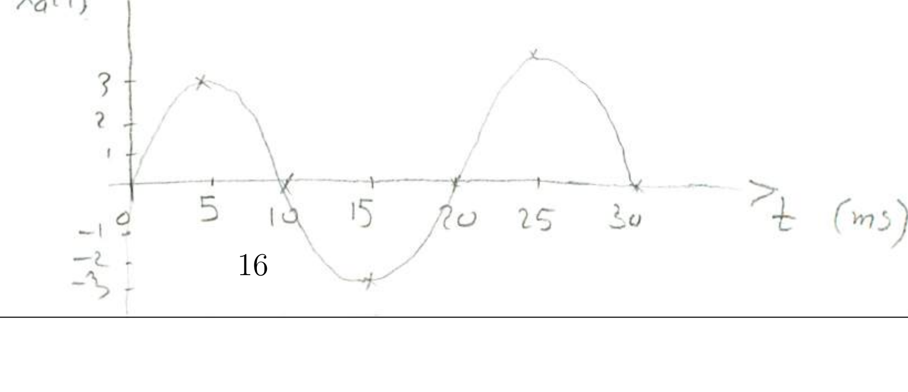
$t = 5\text{ms} \Rightarrow x_a(t) = 3\sin(100\pi \; 5\times10^{-3}) = 3\sin(.5\pi) = 3$

$t = 10\text{ ms} \Rightarrow x_a(t) = 3\sin(100\pi \; 10\times10^{-3}) = 3\sin(\pi) = 0$

$t = 15\text{ ms} \Rightarrow x_a(t) = 3\sin(100\pi \; 15\times10^{-3}) = 3\sin(1.5\pi) = -3$

$t = 20\text{ ms} \Rightarrow x_a(t) = 3\sin(100\pi \; 20\times10^{-3}) = 0$

⋮



(b) $x_a(t) = A \sin(2\pi F t + \theta)$ —— ⓪

so $x_a(nT) = A \sin(2\pi F nT + \theta) = A \sin(2\pi \frac{F}{F_s} n + \theta)$ —— ①

but $x(n) = A \sin(2\pi f n + \theta)$ —— ②

hence by comparing ①, ② $\Longrightarrow$ $\boxed{f = \frac{F}{F_s}}$ → cycles/sec, → samples/sec.

now Find F and F_s to find f.

since $x_a(t) = 3 \sin(100\pi t)$

Then by comparing to ⓪ $\Longrightarrow$ $2\pi F t = 100\pi t \Longrightarrow \boxed{F = 50 \text{ cycles/sec}}$

$F_s = 300$ samples/sec.

hence $f = \frac{50}{300} = \boxed{\frac{1}{6} \text{ cycles/sample.}}$

since f is a rational number $\Longrightarrow$ $\boxed{\text{periodic}}$

and Fundemental period is $\boxed{6 \text{ samples}}$

(c) $x(n) = A \sin(2\pi f n + \theta)$. but $A = 3$, $\theta = 0$, $f = \frac{1}{6}$.

so $x(n) = 3 \sin(2\pi \frac{1}{6} n) = \boxed{3 \sin(\frac{\pi}{3} n)}$

$n=1 \Rightarrow x(1) = 3 \sin(\frac{\pi}{3}) = 2.598$

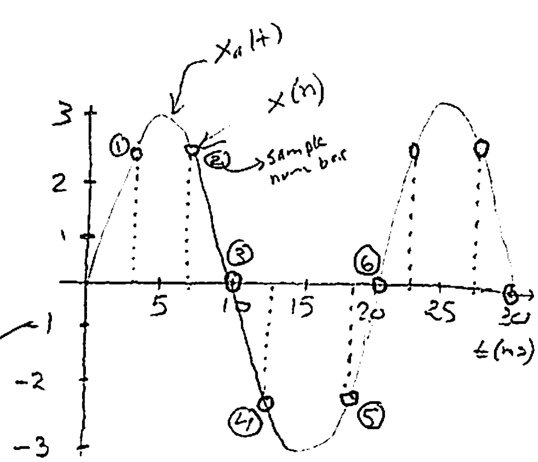
$n=2 \Rightarrow x(2) = 3 \sin(\frac{2\pi}{3}) = 2.598$

$n=3 \Rightarrow x(3) = 3 \sin(\pi) = 0$

$n=4 \Rightarrow x(4) = 3 \sin(\frac{4}{3}\pi) = -2.598$

$n=5 \Rightarrow x(5) = 3 \sin(\frac{5}{3}\pi) = -2.598$

$n=6 \Rightarrow x(6) = 3 \sin(2\pi) = 0$



since period of $x(n) = 6$ samples.

Then it takes $6 \times T$ seconds

or $6 \times \frac{1}{F_s} = 6 \frac{1}{300} = 0.02$ seconds $= \boxed{20 \text{ ms}}$ For period of $x(n)$ in ms.

(d)

Since $x_a(t)$ has a peak of 3 at $t=5$ ms.

then need to solve

$$3\sin(2\pi F t) = 3\sin\left(2\pi \frac{F}{F_s} n\right)$$

let $t = 5\times10^{-3}$, and given $F = 50$ cycles/seconds. (from part b)

so $\sin\left(2\pi(50)5\times10^{-3}\right) = \sin\left(2\pi \frac{50}{F_s} n\right)$

so

$$\boxed{1 = \sin\left(2\pi \frac{50}{F_s} n\right)}$$

so depending on n, we solve for F_s.

$$2\pi \frac{50}{F_s} n = \text{Arcsin}(1)$$

$$2\pi \frac{50}{F_s} n = m\frac{\pi}{2} \qquad \text{For } m = 1, 5, 9, 13, \ldots$$

4

so $\boxed{F_s = 200 \frac{n}{m}}$

choose $m=1 \implies \boxed{F_s = 200\, n}$

The minimum is when $n=1 \implies \boxed{F_s = 200 \text{ samples/second.}}$

at This sampling rate, $x(n) = 3$ at sample number 1 after $T=5$ ms time. (sample period).

3.2 HW 2

EECS152A: HOMEWORK #2

Due: October 12, 2004

Problems from the textbook: 1.7, 1.8, 1.9, 1.10(a)(b)(c)

HW#2

EECS 152A, Digital Signal processing

UCI. Fall 2004

By Nasser Abbasi

HW#2

Problem 1.7

12/15

Statment An analog Signal Contains frequencies up to 10 KHZ.

(a) what range of sampling Frequencies allow exact reconstruction of this signal From its Samples?

(b) suppose we sample with $F_s = 8$ KHZ. explain what happens to frequency $F_1 = 5$ KHZ.

(c) repeat (b) For frequency $F_2 = 9$ KHZ.

Answer

(a) $F_s > 2F_{max}$. where $F_{max} = 10$ KHZ. (given).

so $\boxed{F_s > 20 \text{KHZ}}$ allow exact reconstruction. ✓

(b) with $F_s = 8$ KHZ, Folding Frequency is $\frac{F_s}{2} = 4$ KHZ.

since $F_1 > \frac{F_s}{2}$ then F_1 will not be recovered but will alias to a frequency < 4 KHZ. F_1 will alias to a Frequency $\boxed{F_1 + K F_s}$ where K is ±1 or ±2 or ±3 etc....

Folding Frequency = 4 KHZ

so need to find K such that $F_1 + KF_s < |4 \text{KHZ}|$.

3

so with $\boxed{K = -1}$ we set alias Frequency $= 5 + (-1)8 = \boxed{-3 \text{KHZ}}$

hence $\boxed{F_1 \text{ will alias to } -3 \text{KHZ}}$ ✓

(c) here $F_2 = 9$ KHZ. so need to Find K: $9 + K(8) < |4|$

so $k = -1$.

so $\boxed{F_2 \text{ will alias to } 9 - 8 = 1 \text{ KHZ}}$ ✓

HW#2

Problem 1.8

Statment

An Analog ECG signal contains useful frequencies up to 100HZ.

(a) what is the Nyquist rate for this signal ?

(b) Suppose we sample this signal at rate 250 samples/sec, what is highest frequency that can be represented uniquely at this sampling rate ?

Answer

(a) Nyquist rate $= 2\ F_{max}$

$= 2\ (100\ \text{Hz}) = \boxed{200\ \text{Hz}}$ ✓

(b) $F_s = 250$ sample/sec. (or 250 Hz)

so highest Frequency that can be sampled uniquely is $\boxed{\frac{F_s}{2}} = \boxed{125\ \text{Hz}}$ ✓

Folding Frequency

2

HW#2

Problem 1.9

statment

An analog signal $x_a(t) = \sin(480\pi t) + 3\sin(720\pi t)$ is sampled at rate 600 times per second.

(a) Find Nyquist sampling rate for $x_a(t)$.

(b) Find Folding Frequency.

(c) what are the frequencies in radians in resulting discrete time signal $x(n)$?

(d) if $x(n)$ is passed through an ideal D/A convertor, what is the reconstructed signal $y_a(t)$?

Answer

(a) F_{max} : $2\pi F_{max} t \equiv 720\pi t$

2

so $F_{max} = \frac{720}{2} = 360$ Hz.

So Nyquist rate $= 2F_{max} = \boxed{720 \text{ Hz}}$

(b) Folding freqency $= \frac{F_s}{2} = \frac{600}{2} = \boxed{300 \text{ Hz}}$

(c)
$$x(n) = \sin\left(480\pi(nT)\right) + 3\sin\left(720\pi(nT)\right)$$
$$= \sin\left(480\pi\frac{n}{F_s}\right) + 3\sin\left(720\pi\frac{n}{F_s}\right)$$
$$= \sin\left(\frac{480}{600}\pi n\right) + 3\sin\left(\frac{720}{600}\pi n\right)$$

So $2\pi f_1 n \equiv \frac{480}{600}\pi n \Rightarrow f_1 = \frac{240}{600} = \frac{6}{15} = \frac{2}{5}$ sample/sec (ok $< \frac{1}{2}$)

but $\omega_1 = 2\pi f_1 \Rightarrow \omega_1 = 2\pi\frac{2}{5} = \boxed{\frac{4}{5}\pi}$

to find ω_2:

$$2\pi f_2 n = \frac{720}{600}\pi n \Rightarrow f_2 = \frac{360}{600} = \frac{18}{30} = \frac{9}{15} = \frac{3}{5} \text{ Samp/se}$$

2

$\frac{3}{5} > \frac{1}{2} \Rightarrow \boxed{f_2 \Rightarrow -\frac{2}{5}}$ so $\omega_2 = 2\pi\frac{-2}{5} = \boxed{\frac{-4}{5}\pi}$

HW# 2
Problem 1.9 (cont.)

(d) $$y_a(t) = \sum_{n=-\infty}^{n=+\infty} x_a\left(\frac{n}{F_s}\right) \text{sinc}\left(2\pi F_{max}\left(t - \frac{n}{F_s}\right)\right)$$

where $F_s = 600$ Hz.
$F_{max} = 360$ Hz.

so $$\boxed{y_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{600}\right) \text{sinc}\left(2\pi(360)\left(t - \frac{n}{600}\right)\right)}$$

this is called sinc interpolation.

2

Refer to the soln

Here, $f = 2/5$.
$F = fF_s = (2/5)600 = 240$.

$y_a(t) = -2\sin\left[2\pi(F)f\right]t$.

$= -2\sin\left[2\pi\,(240)\,2/5\right]t$.

$= -2\sin(480\pi)\,t$ //.

HW #2

Problem 1.10

statment

A digital communication link carries binary-coded words representing samples of an input signal

$$x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$$

the link is operated at 10,000 bits/s and each input sample is quantized into 1024 different voltage levels.

(a) what is the sampling frequency and the folding frequency?

(b) what is the Nyquist rate for $x_a(t)$?

(c) what are the frequencies in resulting discrete signal $x(n)$?

Answer. Each i/p sample = 10 bits, since $1024 = 2^{10}$.

$f_s = 1000$

(a) Consider each bit as sample. then $\boxed{F_s = 10{,}000 \text{ sample/sec}}$

so Folding frequency $= \frac{F_s}{2} = \boxed{5{,}000\ Hz}$ $f_{fold} = 500$

(b) Find F_{max}

(1) $1800\pi t \equiv 2\pi F_{max} t \Rightarrow F_{max} = \frac{1800}{2} = \boxed{900\ Hz}$

so Nyquist frequency $= 2 F_{max} = \boxed{1800\ Hz}$ ✓

(c) $F_1 = 300\ Hz$, $F_2 = 900\ Hz$.

Folding frequency = 5000 Hz. so no aliasing since Folding frequency > F_{max}.

$$x(n) = 3\cos 600\pi\left(\frac{n}{F_s}\right) + 2\cos 1800\pi\left(\frac{n}{F_s}\right)$$

$$= 3\cos 600\pi\left(\frac{n}{10000}\right) + 2\cos 1800\pi\left(\frac{n}{10000}\right)$$

$$= 3\cos\frac{3}{50}n\pi + 2\cos\frac{9}{50}n\pi$$

$\Rightarrow \boxed{f_1 = \left(\frac{3}{100}\right) \text{ samples/sec}}$, $\boxed{f_2 = \left(\frac{9}{100}\right) \text{ sample/sec}}$ $f_1 = \frac{3}{10}$; $f_2 = \frac{9}{10}$ or $\frac{1}{10}$

3.3 HW 3

HW#3

EECS 152A, Digital Signal processing

UCI. Fall 2004

By Nasser Abbasi

HW # 3, EECS 152A.
Problem 1.11

74/80

Statment Consider DSP system

Sampling period of A/D and D/A are $T = 5$ ms, $T' = 1$ ms. Determine output $y_a(t)$ of system. input is

$$x_a(t) = 3\cos 100\pi t + 2\sin 250\pi t \quad (t \text{ in sec})$$

Postfilter removes any frequency above $F_s/2$.

Solution

$$x(n) = x_a(nT) = 3\cos(100\pi)(nT) + 2\sin 250\pi(nT)$$

but $T = 5\text{ ms} = 5\times10^{-3}$ sec

so $x(n) = 3\cos\left(100\pi\right)\left(\frac{5}{1000}n\right) + 2\sin\left(250\pi\right)\left(\frac{5}{1000}n\right)$

$f_1 \Rightarrow \frac{500\pi n}{1000} = 2\pi f_1 n \Rightarrow f_1 = \frac{500}{2000} = \boxed{\frac{1}{4}}$ sample/sec

$f_2 \Rightarrow \frac{1250\pi n}{1000} = 2\pi f_2 n \Rightarrow f_2 = \frac{1250}{2000} = 0.625$ sample/sec.

$f_2 > |\frac{1}{2}| \Rightarrow$ alias, so $f_2 = 0.625 - 1 = \boxed{-0.375 \text{ Sample/sec}}$

hence $x(n) = 3\cos(2\pi f_1 n) + 2\sin(2\pi f_2 n)$

$$= 3\cos\left(2\pi \frac{n}{4}\right) + 2\sin\left(2\pi(-0.375)n\right)$$

$$= 3\cos\left(\frac{\pi}{2}n\right) - 2\sin\left(0.75\pi n\right)$$

$$\boxed{x(n) = 3\cos\left(\frac{\pi}{2}n\right) - 2\sin\left(\frac{3}{4}\pi n\right)}$$

or $\boxed{x(n) = 3\cos\left(2\pi\left(\frac{1}{4}\right)n\right) - 2\sin\left(2\pi\left(\frac{3}{8}\right)n\right)}$

For D/A

$T' = 1\ ms \quad = \boxed{F_s' = 1000\ Hz}$

so $\frac{F_1'}{F_s'} = f_1 \Rightarrow F_1' = f_1 F_s' = \left(\frac{1}{4}\right) 1000 = 250\ Hz.$

$\frac{F_2'}{F_s'} = f_2 \Rightarrow F_2' = \left(\frac{3}{8}\right) 1000 = 375\ Hz.$

So reconstructed signal is

$$\boxed{x_a'(t) = 3\cos(2\pi \overset{F_1'}{(250)} t) - 2\sin(2\pi \overset{F_2'}{(375)} t)}$$

which is different from input $x_a(t)$, due to aliasing.

5

Postfilter

remove frequency above $F_s/2$.

$F_s = \frac{1}{T} = \frac{1}{5\ ms} = 200\ Hz. \qquad$ so $\frac{F_s}{2} = 100\ Hz.$

looking at $x_a'(t)$, I see both signal components have frequencies > 100 Hz.

so $\boxed{y_a(t) = 0}$

HW3, problem 1.12 (b)

statment What is the analog signal we can obtain from $x(n)$ if in the reconstruction process we assume $F_s = 10kHz$?

use Example 1.4.2

$$x_a(t) = 3\cos 100\pi t.$$

Solution

For sampling, use $F_s = 200\ HZ$ as per example 1.4.2, part (b).

so $x(n) = 3\cos 100\pi \left(n \frac{1}{F_s}\right) = 3\cos\left(100\pi \frac{n}{200}\right)$

$$\boxed{x(n) = 3\cos\left(2\pi\left(\frac{1}{4}\right)n\right)}$$

For reconstruction, assuming $F_s = 10\,000$, we set

$\frac{F'}{F_s} = f_1$ but $f_1 = \frac{1}{4}$, $F_s = 10000$

so $F' = 40000$

so $y(a) = 3\cos\left(2\pi\,(40000)\,t\right)$

$$\boxed{y(a) = 3\cos(8000\,\pi t)}$$

5

HW 3, problem 2.5 (a,b)

Statment

Consider system $y(n) = T[x(n)] = x(n^2)$.

(a) determine if system is time invariant

(b) to clarify the result of part (a), assume signal

$$x(n) = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

is applied to the system.

(1) sketch $x(n)$.

(2) Determine and sketch $y(n) = T[x(n)]$

(3) sketch $y_2'(n) = y(n-2)$

(4) Determine and sketch $x_2(n) = x(n-2)$

(5) Determine and sketch $y_2(n) = T[x_2(n)]$

(6) Compare $y_2(n)$ and $y(n-2)$. what is your conclusion?

Solution

(a) a system is time invariant if $x(n) \xrightarrow{T} y(n)$ implies $x(n-k) \xrightarrow{T} y(n-k)$ for every input $x(n)$ and every delay k.

Given system, $y(n) = x(n^2)$ ——— ①

When we delay input by k, we get output

$\boxed{x(n^2 - k)}$ — ②

Now, if we delay output by k, then from ①, we get

$y(n-k) = x((n-k)^2) = \boxed{x(n^2 + k^2 - 2nk)}$ — ③

② ≠ ③ For every k ⟹

so $\boxed{\text{NOT Time invariant}}$

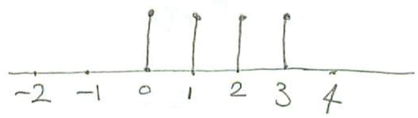
✓ 2

⟶

HW 3 problem 2.6 cont

(b)

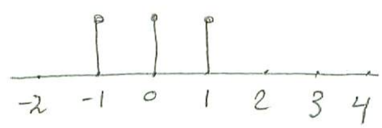
(1) $x(n)$



(2) $y(n) = \mathcal{T}[x(n)]$

n	-2	-1	0	1	2	3	4	5
$x(n)$	0	0	1	1	1	1	0	0
n^2	4	1	0	1	4	9	16	25
$x(n^2)$	0	1	1	1	0	0	0	0

$y(n)$



(3) sketch $y_2'(n) = y(n-2)$

n	-2	-1	0	1	2	3	4	5	6
$y(n)$	0	1	1	1	0	0	0	0	0
$n-2$	-4	-3	-2	-1	0	1	2	3	4
$y(n-2)$	0	0	0	1	1	1	0	0	0

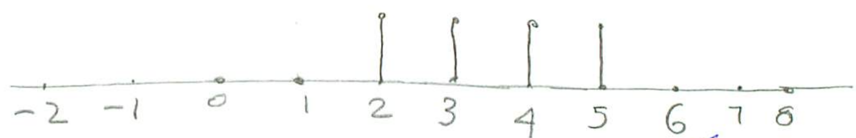
$y_2'(n) = y(n-2)$

HW problem 2.6 cont

(4) sketch $x_2(n) = x(n-2)$

n	-1	0	1	2	3	4	5	6	7	8
$x(n)$	0	1	1	1	1	0	0	0	0	0
$n-2$	-3	-2	-1	0	1	2	3	4	5	6
$x(n-2)$	0	0	0	1	1	1	1	0	0	0

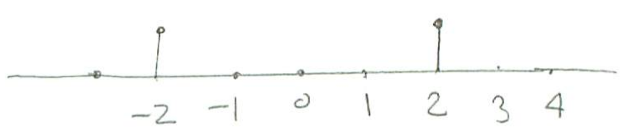
$x_2 = x(n-2)$



(5) Determin, sketch $y_2(n) = \mathcal{T}[x_2(n)]$
$\mathcal{T}[x_2(n)] = x_2(n^2)$

n	-2	-1	0	1	2	3	4	5	6
$x_2(n)$	0	0	0	0	1	1	1	1	0
n^2	4	1	0	1	4	9	16	25	36
$x_2(n^2)$	1	0	0	0	1	0	0	0	0

$y_2(n) = \mathcal{T}[x_2(n)]$



10

(6) $y_2(n)$ is output due to $x_2(n)$ which is a delayed input $x(n)$ by $K=2$
$y(n-2)$ is delayed output by $K=2$ due to same input $x(n)$.
since looking at both signals shows that they are different (part (5) $\neq$ part (3)) $\Longrightarrow$ system is $\boxed{\text{NOT time invariant}}$

HW#3, problem 2.7 a,b,c,d.

Problem

A discrete system can be
(1) static or dynamic
(2) Linear or non linear
(3) Time variant or invariant
(4) Causal or non causal
(5) stable or non stable.

determine the above for the following systems

(a) $y(n) = \cos(x(n))$

(b) $y(n) = \sum_{k=-\infty}^{\infty} x(k)$

(c) $y(n) = x(n)\cos(\omega_0 n)$

(d) $y(n) = x(-n+2)$

Answer

(a)

(1) $\boxed{\text{static}}$ since memorless (does not depend on past or future)

(2) $T[a x_1(n) + b x_2(n)] = \cos(a x_1(n) + b x_2(n)) = \cos(a x_1(n))\cos(b x_2(n)) - \sin(a x_1(n))\sin(b x_2(n))$

$a T[x_1(n)] + b T[x_2(n)] = a\cos(x_1(n)) + b\cos(x_2(n)) \Rightarrow$ $\boxed{\text{NOT Linear}}$

(3) delayed input gives output $= \cos(x(n-k))$
but delayed output $y(n-k) = \cos(x(n-k))$ $\Rightarrow$ $\boxed{\text{Time invariant}}$

(4) $\boxed{\text{Causal}}$ since output does not depend on future input.

(5) Select bounded input signal $x(n) = C\,\delta(n)$
where C is constant. then output

$y(0) = \cos(x(0)) = \cos(C)$
$y(1) = \cos(C\delta(1)) = \cos(0) = 1$
$y(2) = \cos(C\delta(2)) = \cos(0) = 1$
....
since $\cos \le 1 \Rightarrow$ bounded output. $\Rightarrow$ $\boxed{\text{stable}}$ BIBO

5

(b)

(1) $\boxed{\text{dynamic}}$ since requires memory. output does not only depend on current input.

(2) $\boxed{\text{Linear}}$

(3) $\boxed{\text{time Invariant}}$ since for each output at any time depend on all past and all future input.

(4) $\boxed{\text{not Causal}}$ since output depends on future values.

(5) for bounded input $x(n) = C\delta(n)$, where C is constant, (unit sample seq.)

$$y(n) = \cdots + C\delta(-2) + C\delta(-1) + C\delta(0) + C\delta(1) + C\delta(2) + \cdots$$

$$= \cdot 0 + 0 \quad 0 + C(1) + 0 + 0 + \cdots$$

$$= C$$

so $y(n) = C$. so $y(n)$ is bounded since C is some constant $< \infty$.

hence $\boxed{\text{Stable}}$ ~~BIBO~~

4

(c)

$$y(n) = x(n)\cos(\omega_0 n)$$

(1) $\boxed{\text{Static}}$ since depend on on current input.

(2) $T[a\,x_1(n) + b\,x_2(n)] = [a_1 x_1(n) + b\,x_2(n)]\cos(\omega_0 n)$ —— ①

$a\,T[x_1(n)] + b\,T[x_2(n)] = a\,x_1(n)\cos(\omega_0 n) + b\,x_2(n)\cos(\omega_0 n)$ —— ②

so ① = ② $\Longrightarrow$ $\boxed{\text{Linear}}$

(3) delayed input gives $y(n,k) = x(n-k)\cos(\omega_0 n)$

delayed output gives $y(n-k) = x(n-k)\cos(\omega_0(n-k))$

so $y(n,k) \neq y(n-k)$ for all k.

hence $\boxed{\text{NOT time invariant}}$

5

(4) $\boxed{\text{Causal}}$ since do not depend on future

(5) apply $C\delta(n)$ as input $\Longrightarrow y(n) = C\delta(n)\cos(\omega_0 n)$

$\Longrightarrow$ bounded output since $\cos \le |1|$. C constant $\Longrightarrow$ $\boxed{\text{Stable}}$ BIBO

HW #3, problem 2.7 Cont

(d) $y(n) = x(-n+2)$

(1) Static. since depend only on current input. X

(2) $T[a x_1(n) + b x_2(n)] = a x_1(-n+2) + b x_2(-n+2)$
$aT[x_1(n)] + b T[x_2(n)] = a x_1(-n+2) + b x_2(-n+2)$ $\Rightarrow$ $\boxed{\text{Linear}}$ ✓

(3) a delayed input gives

$$y(n,k) = x(-n+2-k)$$

a delayed output is $y(n-k) = x(-(n-k)+2) = x(-n+2+k)$

so $y(n,k) \neq y(n-k) \Longrightarrow$ $\boxed{\text{NOT time invariant}}$ ✓

(4) for $n=0$ we set

$$y(0) = x(2).$$

hence $y(0)$ depends on future input. $\Rightarrow$ $\boxed{\text{NOT Causal}}$ ✓

(5) supply input $x(n) = C\,\delta(n)$.

So output $y(n) = C\,\delta(-n+2)$

so
$$\begin{aligned}
&\vdots\\
y(-1) &= 0\\
y(0) &= 0\\
y(1) &= 0\\
y(2) &= C\delta(0) = C\\
y(3) &= 0\\
&\vdots
\end{aligned}$$

So BIBO $\boxed{\text{stable}}$ since C is constant.

✓

4

HW # 3 problem 2.10

problem The following input-output pairs have been observed during the operation of a time invariant system:

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{T} y_1(n) = \{\underset{\uparrow}{0}, 1, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$$

Can you draw any conclusions regarding the Linearity of the system? what is the impulse response of the system?

Answer

a system is Linear if any input $x(n)$ convolve with $h(n)$ will give the output from $x(n)$.

looking at $x_3(n)$, we see it is $\delta(n-3)$.

since this is time invariant, then $y_3(n)$ is the same as $h(n-3)$.

i.e a delayed input gives a delayed output for time invariant.

so, to find $h(n)$, we shift $x_3(n)$ to left by 3, and this gives $\delta(n)$. so $h(n)$ is shifted $y_3(n)$ to left by 3 as well. so $\boxed{h(n) = \{1\ 2\ 1\ 0\ \underset{\uparrow}{0}\}}$

so $H(z) = z^2 + 2z^3 + z^4$

now, looking at $x_1(n)$, $\Rightarrow X_1(z) = 1 + 2z^{-2}$.

so $Y_1(z) = H(z)X_1(z) = z^4 + 2z^3 + 3z^2 + 4z + 2$

so $y_1(n) = \{1, 2, 3, 4, \underset{\uparrow}{2}\}$ which is not $y_1(n)$.

$\rightarrow$ system not Linear

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HW# 3 problem 2.16

statment

(a) If $y(n) = x(n) * h(n)$, show that $\Sigma_y = \Sigma_x \Sigma_h$ where $\Sigma_x = \sum_{n=-\infty}^{\infty} x(n)$.

(b) compute convolution $y(n) = x(n) * h(n)$ of the following and check correctness by using test in (a)

(1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

(2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

(3) $x(n) = \{0, 1, -2, 3, -4\}$ $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

(4) $x(n) = \{1, 2, 3, 4, 5\}$ $h(n) = \{1\}$

(5) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$ $h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

Solution

(a)

$$\Sigma_y = \sum_{n=-\infty}^{\infty} y(n)$$

$$\Sigma_x = \sum_{n=-\infty}^{\infty} x(n)$$

$$\Sigma_h = \sum_{n=-\infty}^{\infty} h(n)$$

so $$\Sigma_x \Sigma_h = \left[\sum_{n=-\infty}^{\infty} x(n)\right]\left[\sum_{n=-\infty}^{\infty} h(n)\right].$$

$$= \left[\cdots + x(-1) + x(0) + x(1) + \cdots\right]\left[\cdots + h(-1) + h(0) + h(1) + \cdots\right]$$

$$\begin{aligned} = \cdots &+ x(-1)\left[\cdots + h(-1) + h(0) + h(1) + \cdots\right] \\ &+ x(0)\left[\cdots + h(-1) + h(0) + h(1) + \cdots\right] \\ &+ x(1)\left[\cdots + h(-1) + h(0) + h(1) + \cdots\right] + \\ &\cdots \end{aligned}$$

$$\begin{aligned} = \cdots &+ \left[\cdots + x(-1)h(-1) + x(-1)h(0) + x(-1)h(1) + \cdots\right] \\ &+ \left[\cdots + x(0)h(-1) + x(0)h(0) + x(0)h(1) + \cdots\right] \qquad (1) \\ &+ \left[\cdots + x(1)h(-1) + x(1)h(0) + x(1)h(1) + \cdots\right] \\ &\cdots \end{aligned}$$

but since $y(n) = x(n) * h(n)$

then $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

so
$$\left.\begin{aligned} y(0) &= \cdots + x(-1)\,h(1) + x(0)\,h(0) + x(1)\,h(-1) + \cdots \\ y(1) &= \cdots + x(-1)\,h(2) + x(0)\,h(1) + x(1)\,h(0) + \cdots \\ y(2) &= \cdots + x(-1)\,h(3) + x(0)\,h(2) + x(1)\,h(1) + \cdots \end{aligned}\right\} \text{—} (2)$$

Looking at (1), we see it is the same as (2)

for example $\boxed{y(0)}$ inside (2) can be seen inside (1) as follows

(1): $$\begin{aligned} \cdots &\left[\cdots + x(-1)h(-1) + x(-1)h(0) + \boxed{x(-1)h(1)} + \cdots\right] \\ + &\left[\cdots + x(0)h(-1) + \boxed{x(0)h(0)} + x(0)h(1) + \cdots\right] \\ + &\left[\cdots + \boxed{x(1)h(-1)} + x(1)h(0) + x(1)h(1) + \cdots\right] \\ &\cdots \end{aligned}$$

similarly $y(1)$ is the diagonal to the right of the above diagonal., and $y(2)$, is the diagonal to the right of that, etc...

so $$\boxed{\sum_x \sum_h = \sum_y}$$

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(b) (1) find convolution $x(n) = \{\underset{\uparrow}{1}, 2, 4\}$, $h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

n	k	-1	0	1	2	3	4	
	$x(k)$	0	1	2	4	0	0	$y(n)$
0	$h(0-k)$	$h(1)$ 1	$h(0)$ 1	$h(-1)$ 0	$h(-2)$ 0	$h(-3)$ 0	$h(-4)$ 0	1
1	$h(1-k)$	$h(2)$ 1	$h(1)$ 1	$h(0)$ 1	$h(-1)$ 0	$h(-2)$ 0	$h(-3)$ 0	3
2	$h(2-k)$	$h(3)$ 1	$h(2)$ 1	$h(1)$ 1	$h(0)$ 1	$h(-1)$ 0	$h(-2)$ 0	7
3	$h(3-k)$	$h(4)$ 1	$h(3)$ 1	$h(2)$ 1	$h(1)$ 1	$h(0)$ 1	$h(-1)$ 0	7
4	$h(4-k)$	$h(5)$ 0	$h(4)$ 1	$h(3)$ 1	$h(2)$ 1	$h(1)$ 1	$h(0)$ 1	7
5	$h(5-k)$	$h(6)$ 0	$h(5)$ 0	$h(4)$ 1	$h(3)$ 1	$h(2)$ 1	$h(1)$ 1	6

$x(k)$		0	1	2	4	0	0	0	0	0	
	$h(6-k)$	$h(7)$	$h(6)$	$h(5)$	$h(4)$	$h(3)$	$h(2)$	$h(1)$	$h(0)$	$h(-1)$	$y(n)$
6		0	0	0	1	1	1	1	1	0	4
7		0	0	0	0	1	1	1	1	1	0
8		0	0	0	0	1	1	1	1	1	0

so $\boxed{y(n) = \{\cdots, 0, 0, \underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4, 0, 0, \cdots\}}$

$\sum_y = 1+3+7+7+7+6+4 = 35$ same. so test (a) verified.

$\sum_x = 1+2+4 = 7$

$\sum_h = 1+1+1+1+1 = 5$

$\Rightarrow$ multiply $\Rightarrow$ 35 ✓

(2) $\rightarrow$

(2) $x(n) = \{\underset{\uparrow}{1}, 2, -1\}$, $h(n) = \{\underset{\uparrow}{1}, 2, -1\}$

n	k	-1	0	1	2	3	4	y(n)
	x(k)	0	1	2	-1	0	0	
-1	h(-1-k)	h(0) 1	h(-1) 0	h(-2) 0	h(-3) 0	h(-4) 0	h(-5) 0	0
0	h(0-k)	2	1	0	0	0	0	1
1	h(1-k)	-1	2	1	0	0	0	4
2	h(2-k)	0	-1	2	1	0	0	2
3	h(3-k)	0	0	-1	2	1	0	-4
4	h(4-k)	0	0	0	-1	2	1	2
5	h(5-k)	0	0	0	0	-1	2	0
6	h(6-k)	0	0	0	0	0	-1	0

so $\boxed{y(n) = \{0, 0, \underset{\uparrow}{1}, 4, 2, -4, 1, 0, 0, \dots\}}$

$\sum_y = 1+4+2-4+1 = 4$ same. test (a) verified.

$\sum_x = 1+2-1 = 2$

$\sum_h = 1+2-1 = 2$

$\left.\begin{array}{l}\sum_x = 2\\ \sum_h = 2\end{array}\right\}$ multiply $= 4$

$\rightarrow$

(3) $x(n) = \{ \underset{\uparrow}{0}, 1, -2, 3, -4, 0 \}$ $h(n) = \{ \underset{\uparrow}{\tfrac{1}{2}}, \tfrac{1}{2}, 1, \tfrac{1}{2} \}$

n	K	0	1	2	3	4	5	$y(n)$
	$x(k)$	0	1	-2	3	-4	0	
0	$x(0-k)$	$h(0)$ $\frac{1}{2}$	$h(-1)$ 0	0	0	0	0	0
1	$x(1-k)$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$
2	$x(2-k)$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}-\frac{1}{2}2 = -\frac{1}{2}$
3	$x(3-k)$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$1-2\frac{1}{2}+\frac{3}{2} = 1-1+\frac{3}{2} = \frac{3}{2}$
4	$x(4-k)$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}-2+\frac{3}{2}-\frac{4}{2} = -2$
5	$x(5-k)$	0	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}2+3-\frac{4}{2} = 2-2 = 0$
6	$x(6-k)$	0	0	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}-4 = -2\frac{1}{2}$
7		0	0	0	0	$\frac{1}{2}$	1	$-\frac{1}{2}\times 4 = -2$

so $\boxed{y(n) = \{\cdots, 0, \underset{\uparrow}{0}, \tfrac{1}{2}, -\tfrac{1}{2}, 1\tfrac{1}{2}, -2, 0, -2\tfrac{1}{2}, -2, 0, 0, \cdots\}}$

$\sum_y = -5$ same. verified using test (a)

$\sum_x = -2$

$\sum_h = 2\frac{1}{2}$ multiply $= -2 \times \frac{5}{2} = -5$

(2)

(4) $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$ $h(n) = \{\underset{\uparrow}{1}\}$

n	k	0	1	2	3	4	5	6
	$x(k)$	1	2	3	4	5	0	$y(n)$
0	$h(0-k)$	1	0	0	0	0	0	1
1	$h(1-k)$	0	1	0	0	0	0	2
2	$h(2-k)$	0	0	1	0	0	0	3
3	$h(3-k)$	0	0	0	1	0	0	4
4	$h(4-k)$	0	0	0	0	1	0	5
5	$h(5-k)$	0	0	0	0	0	1	0

$$\Rightarrow \boxed{y(n) = \{\dots, 0, 0, \underset{\uparrow}{1}, 2, 3, 4, 5, 0, 0 \dots\}}$$

$\sum_y = 15$ verified same.

$\left.\begin{array}{l}\sum_x = 15 \\ \sum_h = 1\end{array}\right\}$ $15 \times 1 = 15$

2

$\longrightarrow$

(5) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$ $h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

n	k	0	1	2	3	4	5	y(n)
	$x(k)$	1	-2	3	0	0	0	
0	$h(0-k)$	0	0	0	0	0	0	0
1	$h(1-k)$	0	0	0	0	0	0	0
2	$h(2-k)$	1	0	0	0	0	0	1
3	$h(3-k)$	1	1	0	0	0	0	-1
4	$h(4-k)$	1	1	1	0	0	0	2
5	$h(5-k)$	1	1	1	1	0	0	2
6	$h(6-k)$	0	1	1	1	1	0	1
7	$h(7-k)$	0	0	1	1	1	1	3

$$\Rightarrow \boxed{y(n) = \{\cdots 0, 0, 1, -1, 2, 2, 1, 3, 0, \cdots\}}$$

$\sum_y = 8$

verified same

$\sum_x = 2$

$\sum_h = 4$

$\left.\begin{matrix}\sum_x = 2 \\ \sum_h = 4\end{matrix}\right\} 2 \times 4 = 8$

2

HW #3 , problem 2.23

discrete time system $y(n) = n\,y(n-1) + x(n)$ $n \geq 0$ is at rest $(y(-1)=0)$. Check if system is Liner time invariant and BIBO

Solution

Since system is relaxed, need only to consider Linearity for relaxed system.

$$y(0) = 0 + x(0) = x(0)$$
$$y(1) = 1\cdot y(0) + x(1) = 1\cdot x(0) + x(1)$$
$$y(2) = 2\cdot y(1) + x(2) = 2\cdot[x(0)+x(1)] + x(2)$$
$$y(3) = 3\cdot y(2) + x(3) = 3\left[2\left[x(0)+x(1)\right] + x(2)\right] + x(3)$$
$$= 3\cdot 2\ [x(0) + x(1)] + 3\cdot x(2) + x(3)$$
$$y(4) = 4\cdot y(3) + x(4) = 4\left[3\cdot 2\,[x(0)+x(1)] + 3\cdot x(2) + x(3)\right] + x(4)$$
$$= 4.3.2\,[x(0)+x(1)] + 4.3\ x(2) + 4\,x(3) + x(4)$$

so $$y(n) = n!\,x(0) + n!\,x(1) + \frac{n!}{2!}x(2) + \frac{n!}{3!}x(3) + \frac{n!}{4!}x(4) + \cdots$$

$$= n!\left[\frac{x(0)}{0!} + \frac{x(1)}{1!} + \frac{1}{2!}x(2) + \frac{1}{3!}x(3) + \cdots\right]$$

$$\boxed{y(n) = n!\sum_{m=0}^{n}\frac{x(m)}{m!}} \longrightarrow$$

$$T\left[a x_1(n) + b x_2(n)\right] = n! \sum_{m=0}^{n} \frac{a x_1(m) + b x_2(m)}{m!}$$

$$= n!\left(\sum_{m=0}^{n} \frac{a x_1(m)}{m!} + \frac{b x_2(m)}{m!}\right)$$

$$= n!\left(\sum_{m=0}^{n} \frac{a x_1(m)}{m!} + \sum_{m=0}^{n} \frac{b x_2(m)}{m!}\right)$$

$$= n! \sum_{m=0}^{n} \frac{a x_1(m)}{m!} + n! \sum_{m=0}^{n} \frac{b x_2(m)}{m!}$$

$$= a T\left[x_1(n)\right] + b T\left[x_2(n)\right]$$

$$\Longrightarrow \boxed{\text{Linear}}$$

To check for time invariant.

a delayed input produces output

$$y(x,k) = n! \sum_{m=0}^{n} \frac{x(m-k)}{m!}$$

a delayed output is $y(x-k) = (n-k)! \displaystyle\sum_{m=0}^{n-k} \frac{x(m)}{m!}$

to see if $y(x,k) = y(x-k)$, $\boxed{\text{try } n=3,\ k=1}$

$$y(x,k) = 3! \sum_{m=0}^{3} \frac{x(m-1)}{m!} = 3!\left[\frac{\cancel{x(-1)}^{=0}}{1} + \frac{x(0)}{1} + \frac{x(1)}{2!} + \frac{x(2)}{3!}\right]$$

$$= \boxed{3!\left[x(0) + \frac{x(1)}{2!} + \frac{x(2)}{3!}\right]} \text{ —— } (1)$$

$$y(x-k) = 2! \sum_{m=0}^{2} \frac{x(m)}{m!} = 2!\left[x(0) + \frac{x(1)}{1} + \frac{x(2)}{2!}\right] \text{ —— } (2)$$

we see that $(1) \neq (2) \Rightarrow$ $\boxed{\text{NOT time invariant}}$ $\longrightarrow$

to check for BIBO stable;

give the system input $c\,\delta(n)$ and see if output $y(n)$ is bounded.

$$y(n) = n! \sum_{m=0}^{n} \frac{c\,\delta(m)}{m!}$$

$$= n! \left[c\,\delta(0) + c\,\delta(1) + c\,\frac{\delta(2)}{2!} + \cdots \right]$$

since $\delta(n) = 0$ for all $n \neq 0$, then

$$y(n) = n!\,[c]$$

$$\boxed{y(n) = c\,n!}$$ where c is a constant.

since $n!$ grows with no limit as n grows, so for a bounded input $\delta(n)$ we obtain unbounded output.

so $\boxed{\text{NOT stable}}$ ✓

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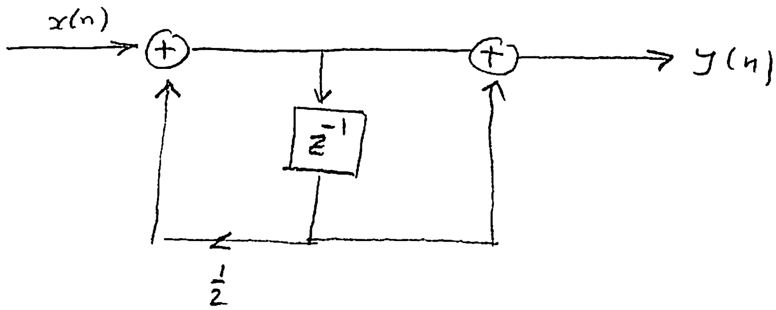
HW#3 problem 2.44

Consider system



(a) Compute first 10 samples of its impulse response.

(b) Find input-output relation.

(c) Apply input $x(n)=\{\underset{\uparrow}{1},1,1,\dots\}$ and compute the first 10 samples of the output.

(d) Compute the first 10 samples of output for input in part (c) by using convolution.

(e) Is system causal? Stable?

Solution

(a) $\boxed{y(n)=\frac{1}{2}y(n-1)+x(n)}$

$y(n)= x(n)+x(n-1)+\frac{1}{2}y(n-1)$

Assume relaxed system. so $y(-1)=0$

1

$$y(0)=0+x(0)=x(0)$$

$$y(1)=\frac{1}{2}y(0)+x(1)=\frac{1}{2}x(0)+x(1)$$

$$y(2)=\frac{1}{2}y(1)+x(2)=\frac{1}{2}\left(\frac{1}{2}x(0)+x(1)\right)+x(2)$$

$$=\frac{1}{4}x(0)+\frac{1}{2}x(1)+x(2)$$

$$y(3)=\frac{1}{2}y(2)+x(3)=\frac{1}{2}\left\{\frac{1}{4}x(0)+\frac{1}{2}x(1)+x(2)\right]+x(3)$$

$$=\frac{1}{8}x(0)+\frac{1}{4}x(1)+\frac{1}{2}x(2)+x(3)$$

so $\boxed{y(n)=\sum_{k=0}^{n}\frac{1}{2^k}x(n-k)}$

when $x(n) = \delta(n) = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$

then $y(0) = \delta(0) = 1$

$y(1) = \frac{1}{1}x(1-0) + \frac{1}{2}x(1-1) = x(1) + \frac{1}{2}x(0) = \frac{1}{2}$

$y(2) = \frac{1}{1}x(2-0) + \frac{1}{2}x(2-1) + \frac{1}{4}x(2-2) = x(2) + \frac{1}{2}x(1) + \frac{1}{4}x(0) = \frac{1}{4}$

$y(3) = \frac{1}{8}$, $y(4) = \frac{1}{16}$, $y(5) = \frac{1}{32}$, $y(6) = \frac{1}{64}$

$y(7) = \frac{1}{128}$, $y(8) = \frac{1}{256}$, $y(9) = \frac{1}{512}$.

(1)

so $\boxed{y = \{ \underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512} \}}$

is the response to impulse. i.e $\boxed{y(n) = \left(\frac{1}{2}\right)^n}$ $n \geq 0$

(b) h is given by part (a). since $h(n)$ is the impulse response.

So $\boxed{h(n) = \left(\frac{1}{2}\right)^n u(n)}$

(c) using difference equation

$y(0) = 0 + x(0) = x(0) = 1$

$y(1) = \frac{1}{2}y(0) + x(1) = \frac{1}{2} + 1 = \frac{3}{2}$

(2)

$y(2) = \frac{1}{2}\left(\frac{3}{2}\right) + x(2) = \frac{1}{2}\left(\frac{3}{2}\right) + 1 = \frac{3}{4} + 1 = \frac{7}{4}$

$y(3) = \frac{1}{2}y(2) + x(3) = \frac{1}{2}\left[\frac{7}{4}\right] + 1 = \frac{7}{8} + 1 = \frac{15}{8}$

$y(4) = \frac{1}{2}\left(\frac{15}{8}\right) + 1 = \frac{15}{16} + 1 = \frac{31}{16}$

$y(5) = \frac{1}{2}\left(\frac{31}{16}\right) + 1 = \frac{31}{32} + 1 = \frac{63}{32}$

$y(6) = \frac{1}{2}\frac{63}{32} + 1 = \frac{63}{64} + 1 = \frac{127}{64}$

$y(7) = \frac{255}{128}$, $y(8) = \frac{511}{256}$, $y(9) = \frac{1023}{512}$

so $y = \{1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \frac{63}{32}, \frac{127}{64}, \frac{255}{128}, \frac{511}{256}, \frac{1023}{512}, \ldots\} \longrightarrow$

so $y(n) = \dfrac{(2\cdot 2^n)-1}{2^n} = \boxed{\dfrac{2^{n+1}-1}{2^n}}$

(d) Now find $y(n)$ again using convolution

$x(n)=\{1,1,1,\cdots\}$ $h(n)=(\frac{1}{2})^n u(n) = \{1,\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},\cdots\}$

n	K	0	1	2	3	4	5	6	7	8	9	$y(n)$
	$x(k)$	1	1	1	1	1	1	1	1	1	1	
0	$h(0-k)$	$h(0)$ 1	$h(-1)$ 0	0	0	0	0	0	0	0	0	1
1	$h(1-k)$	$\frac{1}{2}$	1	0	0	0	0	0	0	0	0	$\frac{1}{2}+1=\frac{3}{2}$
2	$h(2-k)$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	0	0	0	0	$\frac{1}{4}+\frac{1}{2}+1=\frac{7}{4}$
3	$h(3-k)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	0	0	0	$\frac{1}{8}+\frac{1}{4}+\frac{1}{2}+1=\frac{15}{8}$
4	$h(4-k)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	0	0	$\frac{31}{16}$
5	$h(5-k)$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	0	0	0	0	0	$\frac{63}{32}$
6	$h(6-k)$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	$\frac{127}{64}$
7	$h(7-k)$	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	$\frac{255}{128}$
8	$h(8-k)$	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	$\frac{511}{256}$
9	$h(9-k)$	$\frac{1}{512}$	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{1023}{512}$

hence $y(n) = \{1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \cdots\}$ same as found in part (c)

(e) system is stable by ratio test $y(n)$ converges.. i.e. $\left|\dfrac{(\frac{1}{2})^{n+1}}{(\frac{1}{2})^n}\right| \to \frac{1}{2}$

as response to impulse.

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So $\boxed{\text{BIBO stable}}$

Since $y(n)=\frac{1}{2}y(n-1)+x(n)$, we see that $y(n)$ do not depend on Future $\Longrightarrow$ $\boxed{\text{Causal}}$

3.4 HW 4

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HW#4

EECS 152A, Digital Signal processing

UCI. Fall 2004

By Nasser Abbasi

HW 4, EECS 152A
Problem 4.5, Nasser Abbasi

Question
consider signal $x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{4} + \frac{1}{2}\cos\frac{3\pi n}{4}$
(a)Determine and sketch its power density spectrum
(b)Evaluate the power of the signal.

Solution
(a) I will use these relations for this problem: $\cos\frac{\pi}{3} = \frac{1}{2}, \cos\frac{2\pi}{3} = -\frac{1}{2}, \sin\frac{\pi}{3} = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos x = \frac{e^{jx}+e^{-jx}}{2}$

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{4\ \ 2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

First find the period. For the second term $2\cos\frac{\pi n}{4}$, we get $\frac{\pi n}{4} \equiv 2\pi f n$ hence $f = \frac{1}{8}$ hence periodic (since rational) and period is 8.
For the third term $\cos\frac{\pi n}{4}$, same period.
For the 4th term $\cos\frac{3\pi n}{4}$,we get $\frac{3\pi n}{4} \equiv 2\pi f n$ hence $f = \frac{3}{8}$, hence rational, hence periodic. Since lowest common multiplier allready, then period is 8.

$\boxed{\text{Hence the period of } x(n) \text{ is 8.}}$ ✓

Hence $x(n)$ can be written as $\boxed{x(n) = 2 + 2\cos\frac{2\pi}{8}n + \cos\frac{2\pi}{8}n + \frac{1}{2}\cos\frac{2\pi}{8}3n}$ ✓
Expand in complex exponentials we get

$$\begin{aligned} x(n) &= 2 + 2\left(\frac{e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}n}}{2}\right) + \left(\frac{e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}n}}{2}\right) + \frac{1}{2}\left(\frac{e^{j\frac{2\pi}{8}3n} + e^{-j\frac{2\pi}{8}3n}}{2}\right) \\ &= 2 + e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}n} + \frac{1}{2}e^{j\frac{2\pi}{8}n} + \frac{1}{2}e^{-j\frac{2\pi}{8}n} + \frac{1}{4}e^{j\frac{2\pi}{8}3n} + \frac{1}{4}e^{-j\frac{2\pi}{8}3n} \end{aligned}$$

Now convert all exponentials to the 'positive' side, so I can compare later with the IDFT. Using the periodicty of complex exponential, we know that

$$e^{-j\frac{2\pi}{8}n} = -e^{j\frac{2\pi}{8}3n}$$
$$e^{-j\frac{2\pi}{8}3n} = -e^{j\frac{2\pi}{8}n}$$
X

$$e^{-j\frac{2\pi}{8}n} = e^{j[-\frac{2\pi}{8}n + 2\pi n]} = e^{j2\pi(-1/8+1)n} = e^{j2\pi(7/8)n}$$

Hence

$$\begin{aligned} x(n) &= 2 + e^{j\frac{2\pi}{8}n} - e^{j\frac{2\pi}{8}3n} + \frac{1}{2}e^{j\frac{2\pi}{8}n} - \frac{1}{2}e^{j\frac{2\pi}{8}3n} + \frac{1}{4}e^{j\frac{2\pi}{8}3n} - \frac{1}{4}e^{j\frac{2\pi}{8}n} \\ &= \cancel{2 + \frac{5}{4}e^{j\frac{2\pi}{8}n} - \frac{7}{4}e^{j\frac{2\pi}{8}3n}} \quad X \end{aligned}$$

Now we know that IDFT is of the form

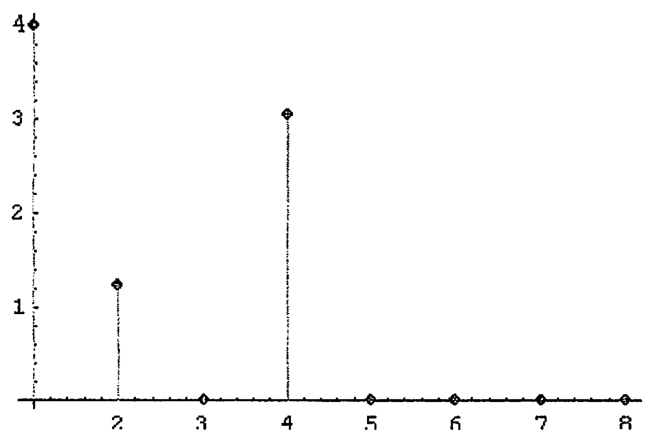
$$x(n) = \sum_{k=0}^{N-1} c(k) e^{j2\pi n\frac{k}{N}}$$

Hence by comparing term by term we see by inspection that
$c(0) = 2$
$c(1) = \frac{5}{4}$
$c(3) = -\frac{7}{4}$
And since $c(k)$ will have the same period as $x(n)$ we then write

$$\boxed{c(k) = \{2, \tfrac{5}{4}, 0, -\tfrac{7}{4}, 0, 0, 0, 0\}}$$ X

$C_K = \{2, 1, 1/2, 0, 0, 0, \frac{1}{2}, 1\}$

So power density spectrum is
$|c(k)|^2 = \{4, \frac{20}{16}, 0, \frac{49}{16}, 0, 0, 0, 0\}$ X X
This is a sketch of the power spectrum. y-axes is $|c(k)|^2$, and x-axis is k.



(b) Power of signal is given by $\sum_{k=0}^{N-1} |c(k)|^2 = 4 + \frac{20}{16} + 0 + \frac{49}{16} = \boxed{8.3125}$

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HW 4, EECS 152A
Problem 4.7 part (a), Nasser Abbasi

Question
Determine the periodic signal $x(n)$ with fundamental period $N = 8$ if their fourier coefficients are given by
(a) $c(k) = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$

Solution
(a) I will use these relations for this problem
$\cos x = \frac{e^{jx}+e^{-jx}}{2}, \sin x = \frac{e^{jx}-e^{-jx}}{2j}$

$$c(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi\frac{n}{N}k} \tag{1}$$

Expand given $c(k)$ in terms of complex exponentials, and compare terms to find $x(n)$

$$\begin{aligned} c(k) &= \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4} \\ &= \cos\frac{2\pi}{8}k + \sin\frac{2\pi}{8}3k \\ &= \frac{e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k}}{2} + \frac{e^{j\frac{2\pi}{8}3k} - e^{-j\frac{2\pi}{8}3k}}{2j} \\ &= \frac{1}{2}e^{j\frac{2\pi}{8}k} + \frac{1}{2}e^{-j\frac{2\pi}{8}k} + \frac{1}{2j}e^{j\frac{2\pi}{8}3k} - \frac{1}{2j}e^{-j\frac{2\pi}{8}3k} \quad ✓ \end{aligned} \tag{2}$$

Now write all the exponentials in 'negative' terms, so I can compare with (1).
Using periodicty property, $e^{j\frac{2\pi}{8}k} = -e^{-j\frac{6\pi}{8}k} = -e^{-j\frac{2\pi}{8}3k}$
and $e^{j\frac{2\pi}{8}3k} = e^{j\frac{6\pi}{8}k} = -e^{-j\frac{2\pi}{8}k}$
Hence (2) can be rewritten as

$$\begin{aligned} c(k) &= -\frac{1}{2}e^{-j\frac{2\pi}{8}3k} + \frac{1}{2}e^{-j\frac{2\pi}{8}k} - \frac{1}{2j}e^{-j\frac{2\pi}{8}k} - \frac{1}{2j}e^{-j\frac{2\pi}{8}3k} \\ &= \boxed{e^{-j\frac{2\pi}{8}3k}\left(-\frac{1}{2} - \frac{1}{2j}\right) + e^{-j\frac{2\pi}{8}}\left(\frac{1}{2} - \frac{1}{2j}\right)} \quad ✓ \end{aligned}$$

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Hence we see that $x(1) = 8\left(\frac{1}{2} - \frac{1}{2j}\right)$ and $x(3) = 8\left(-\frac{1}{2} - \frac{1}{2j}\right)$
Can also be written as $\boxed{x(1) = (4+4j) \text{ and } x(3) = (-4+4j)}$
or
Hence

$$\boxed{x(n) = \{0,\ (4+4j)\ ,\ 0\ ,\ (-4+4j)\ ,\ 0,\ 0,\ 0,\ 0\}}$$

HW 4, EECS 152A
Problem 4.9 part(a,b,c), Nasser Abbasi

Question
Compute Fourier transform for the following
(a)$x(n) = u(n) - u(n-6)$
(b)$x(n) = 2^n u(-n)$
(c)$x(n) = \frac{1}{4}^n u(n+4)$
Solution
using the Fourier transform for a discrete aperiodic sequence given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

(a) here $x(n) = \{\hat{1}, 1, 1, 1, 1, 0, 0, \cdots\}$
Hence

$$X(\omega) = \sum_{n=0}^{5} e^{-j\omega n} = \boxed{1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega} + e^{-j5\omega}}$$

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(b)

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-\infty}^{0} 2^n\, e^{-j\omega n} = \sum_{0}^{\infty} 2^{-n}\, e^{j\omega n} = \sum_{0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^n = \boxed{\frac{1}{1-\frac{e^{j\omega}}{2}}}$$

(c) $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-4}^{\infty} \frac{1}{4}^n\, e^{-j\omega n} = \sum_{n=-4}^{-1} \frac{1}{4}^n\, e^{-j\omega n} + \sum_{0}^{\infty} 2^{-n}\, e^{j\omega n}$

$$\begin{aligned} X(\omega) &= \left(\frac{1}{4}^{-4} e^{4\omega j} + \frac{1}{4}^{-3} e^{3\omega j} + \frac{1}{4}^{-2} e^{2\omega j} + \frac{1}{4}^{-1} e^{\omega j} \right) + \sum_{0}^{\infty} \left(\frac{e^{j\omega}}{4}\right)^n \\ &= \boxed{\left(64\, e^{4\omega j} + 32\, e^{3\omega j} + 16\, e^{2\omega j} + 4\, e^{\omega j}\right) + \frac{1}{1-\frac{e^{j\omega}}{4}}} \end{aligned}$$

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3.5 HW 5

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HW#5
EECS 152A
NASSER ABBASI

HW ~~4~~5, EECS 152A DSP.
Problem 4.27 Nasser Abbasi
UCI, Fall 2004.

Question
Determine and sketch the magnitude and phase diagrams for the following systems
(a)$y(n) = \frac{1}{2}[x(n) + x(n-1)]$
(b)$y(n) = \frac{1}{2}[x(n) - x(n-1)]$
(c)$y(n) = \frac{1}{2}[x(n+1) - x(n-1)]$
(d)$y(n) = \frac{1}{2}[x(n+1) + x(n-1)]$
(e)$y(n) = \frac{1}{2}[x(n) + x(n-2)]$
(f)$y(n) = \frac{1}{2}[x(n) - x(n-2)]$
(g)$y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)]$
(h)$y(n) = x(n) - x(n-8)$

Solution

part(a)

$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$

Hence, replace $x(n)$ with $\delta(n)$ in the above, we get

$h(n) = \frac{1}{2}[\delta(n) + \delta(n-1)]$

So, we get values only for $n = 0, 1$ i.e. $h = \left\{\boxed{\frac{1}{2}}, \frac{1}{2}\right\}$

using the Fourier transform for a discrete aperiodic sequence given by

$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ we get $H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{1} h(n) e^{-j\omega n} = \boxed{\frac{1}{2} + \frac{1}{2} e^{-j\omega}}$

i.e. $\boxed{H(\omega) = \frac{1}{2} + \frac{1}{2} e^{-j\omega} = \frac{1}{2} + \frac{1}{2}(\cos\omega - j\sin\omega) = \left(\frac{1}{2} + \frac{1}{2}\cos\omega\right) + j\left(-\frac{1}{2}\sin\omega\right)}$

$|H(\omega)| = \sqrt{\left(\frac{1}{2} + \frac{1}{2}\cos\omega\right)^2 + \left(-\frac{1}{2}\sin\omega\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}\cos^2\omega + \frac{1}{4}\cos^2\omega + \frac{1}{4}\sin^2\omega}$

$= \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{2}\cos\omega} = \boxed{\sqrt{\frac{1}{2}(1 + \cos\omega)}}$

So at $\omega = 0$, $|H(0)| = \sqrt{\frac{1}{2}(1 + \cos 0)} = 1$

I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.

at $\omega = \pi$, $|H(\pi)| = \sqrt{\frac{1}{2}(1 + \cos\pi)} = 0$

at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{\frac{1}{2}\left(1 + \cos\frac{\pi}{2}\right)} = \sqrt{\frac{1}{2}} = 0.707\,11$

For the phase, we have

$\Theta(\omega) = \tan^{-1}\frac{\operatorname{Im}(H(\omega))}{\operatorname{Re}(H(\omega))} = \tan^{-1}\left(\frac{-\frac{1}{2}\sin\omega}{\frac{1}{2}+\frac{1}{2}\cos\omega}\right) = \boxed{\tan^{-1}\left(\frac{-\sin\omega}{1+\cos\omega}\right)}$

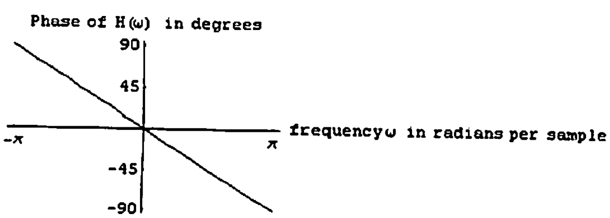
Phase diagram is an odd function and symmetrical across the y-axis. Look at few values, then I show the plot:

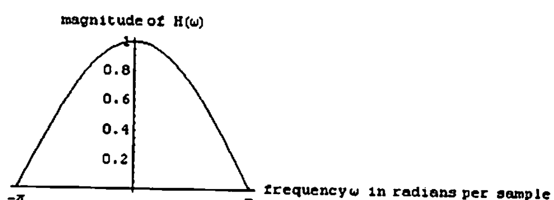
When $\omega = 0$,$\Theta(0) = \tan^{-1}\left(\frac{0}{2}\right) = 0^0$

at $\omega = \pi, \Theta(\pi) = \tan^{-1}\left(\frac{-\sin\pi}{1+\cos\pi}\right) = \tan^{-1}\left(\frac{0}{0}\right) =$ undefined, so point of discontinuity

at $\omega = \frac{\pi}{2}, \Theta\left(\frac{\pi}{2}\right) = \tan^{-1}\left(\frac{-\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}}\right) = \tan^{-1}\left(\frac{-1}{1}\right) = \frac{\pi}{4} = 45^0$

A plot of phase and $|H(\omega)|$ is below





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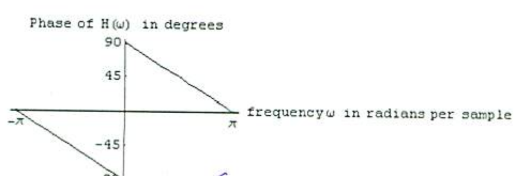
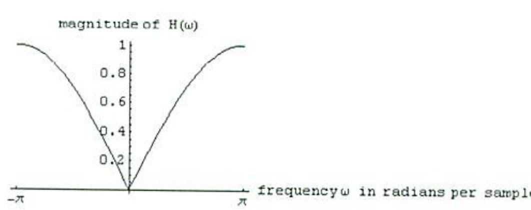
part(b)
$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$
Hence, replace $x(n)$ with $\delta(n)$ in the above, we get
$h(n) = \frac{1}{2}[\delta(n) - \delta(n-1)]$
So, we get values only for $n = 0, 1$ i.e. $h = \left\{\boxed{\frac{1}{2}}, -\frac{1}{2}\right\}$
using the Fourier transform for a discrete aperiodic sequence given by
$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$ we get $H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n} = \sum_{n=0}^{1} h(n)\, e^{-j\omega n} = \boxed{\frac{1}{2} - \frac{1}{2}e^{-j\omega}}$
i.e. $H(\omega) = \frac{1}{2} - \frac{1}{2}e^{-j\omega} = \frac{1}{2} - \frac{1}{2}(\cos\omega - j\sin\omega)$
$= \boxed{\left(\frac{1}{2} - \frac{1}{2}\cos\omega\right) + j\left(\frac{1}{2}\sin\omega\right)}$
$|H(\omega)| = \sqrt{\left(\frac{1}{2} - \frac{1}{2}\cos\omega\right)^2 + \left(\frac{1}{2}\sin\omega\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}\cos^2\omega - \frac{1}{2}\cos\omega + \frac{1}{4}\sin^2\omega}$
$= \sqrt{\frac{1}{2} - \frac{1}{2}\cos\omega} = \boxed{\sqrt{\frac{1}{2}(1 - \cos\omega)}}$
So at $\omega = 0$, $|H(\omega)| = 0$
I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.
at $\omega = \pi$, $|H(\pi)| = \sqrt{\frac{1}{2}(1 - \cos\pi)} = 1$
at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{\frac{1}{2}\left(1 - \cos\frac{\pi}{2}\right)} = \sqrt{\frac{1}{2}} = 0.70711$
A plot of $|H(\omega)|$ is shown below
For the phase, we have
$\Theta(\omega) = \tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))} = \tan^{-1}\left(\frac{\frac{1}{2}\sin\omega}{\frac{1}{2} - \frac{1}{2}\cos\omega}\right) = \boxed{\tan^{-1}\left(\frac{\sin\omega}{1-\cos\omega}\right)}$
Phase diagram is an odd function and symmetrical across the y-axis. Look at few values, then I show the plot:
When $\omega = 0$, $\Theta(0) = \tan^{-1}\left(\frac{0}{0}\right)$ undefined, so discontinuity point
at $\omega = \pi$, $\Theta(\pi) = \tan^{-1}\left(\frac{\sin\pi}{1-\cos\pi}\right) = \tan^{-1}\left(\frac{0}{2}\right) = 0$
at $\omega = \frac{\pi}{2}$, $\Theta\left(\frac{\pi}{2}\right) = \tan^{-1}\left(\frac{\sin\frac{\pi}{2}}{1-\cos\frac{\pi}{2}}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4} = 45^0$
A plot of the phase is shown below





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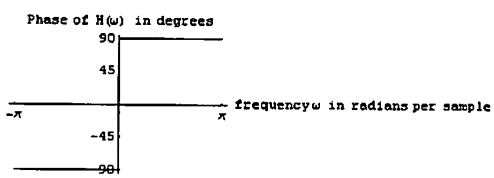
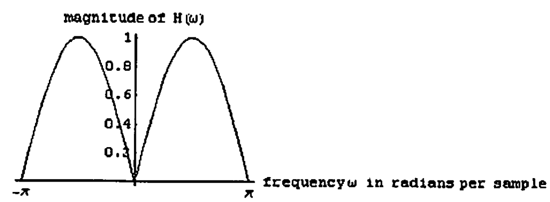
Part(c)
$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$
Hence, replace $x(n)$ with $\delta(n)$ in the above, we get
$h(n) = \frac{1}{2}[\delta(n+1) - \delta(n-1)]$
So, we get values only for $n = -1, 1$ i.e. $h = \left\{\frac{1}{2}, \boxed{0}, -\frac{1}{2}\right\}$
using the Fourier transform for a discrete aperiodic sequence given by
$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$ we get $H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n} = \sum_{n=-1}^{1} h(n)\, e^{-j\omega n} = \boxed{\frac{1}{2}e^{j\omega} - \frac{1}{2}e^{-j\omega}} = \boxed{j\sin(\omega)}$
$\boxed{|H(\omega)| = |\sin(\omega)|}$ This is just the magnitude of the sin function, which we know how it looks.
A plot of $|H(\omega)|$ is shown below
For the phase, since the complex number has only an imaginary part, its phase can only be $\pm 90^0$
When $0 < \omega < \pi$, $\sin(\omega)$ is positive, so $H(\omega)$ on the positive imaginary axis, i.e. phase is $+90^0$
When $-\pi < \omega < 0$, $\sin(\omega)$ is negative, so $H(\omega)$ on the negative imaginary axis, i.e. phase is -90^0
A plot of the phase is shown below





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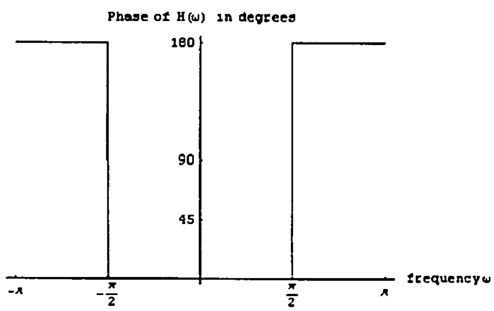
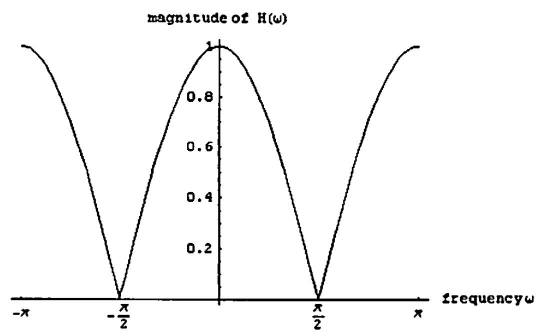
Part(d)
$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$
Hence, replace $x(n)$ with $\delta(n)$ in the above, we get
$h(n) = \frac{1}{2}[\delta(n+1) + \delta(n-1)]$
So, we get values only for $n = -1, 1$ i.e. $h = \left\{\frac{1}{2}, \boxed{0}, \frac{1}{2}\right\}$
using the Fourier transform for a discrete aperiodic sequence given by
$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ we get $H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=-1}^{1} h(n) e^{-j\omega n} = \boxed{\frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}} = \boxed{\cos(\omega)}$
This is just the cos function. $|H(\omega)| = |\cos(\omega)| = 1$ at $\omega = 0, \pm\pi$ and 0 at $\pm\frac{\pi}{2}$
This complex number has only real part, so its phase can be either a zero or 180^0 depending if the real part is positive or negatibe. when $0 < \omega < \frac{\pi}{2}$, $\cos(\omega)$ is positive, so $H(\omega)$ phase is zero. When $\frac{\pi}{2} < \omega < \pi$, then $\cos(\omega)$ is negative, so $H(\omega)$ phase is 180^0
when $-\frac{\pi}{2} < \omega < 0$, $\cos(\omega)$ is positive so phase is zero, when $-\pi < \omega < -\frac{\pi}{2}$, $\cos(\omega)$ is negative so phase is 180
A plot of the phase and magnitude is shown below





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Part(e)
$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n=0$
Hence, replace $x(n)$ with $\delta(n)$ in the above, we get
$h(n) = \frac{1}{2}[\delta(n) + \delta(n-2)]$
So, we get values only for $n = 0, 2$ i.e. $h = \left\{\boxed{\frac{1}{2}}, 0, \frac{1}{2}\right\}$
using the Fourier transform for a discrete aperiodic sequence given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \text{ we get } H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{2} h(n) e^{-j\omega n} = \boxed{\frac{1}{2} + \frac{1}{2}e^{-2j\omega}}$$

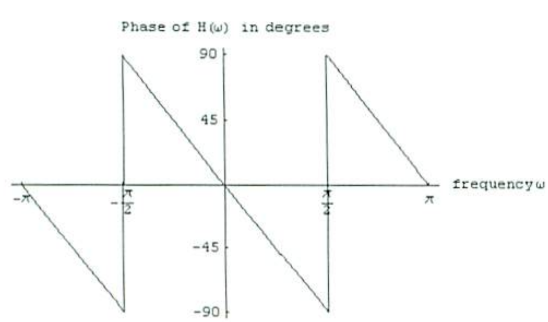
$$= \frac{1}{2} + \frac{1}{2}(\cos 2\omega - j \sin 2\omega) = \boxed{\left(\frac{1}{2} + \frac{1}{2}\cos 2\omega\right) + j\left(-\frac{1}{2}\sin 2\omega\right)}$$

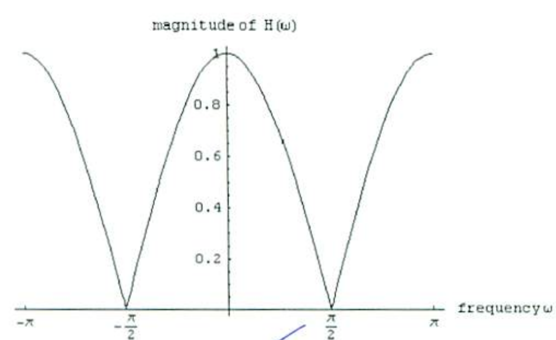
$$|H(\omega)| = \sqrt{\left(\frac{1}{2} + \frac{1}{2}\cos 2\omega\right)^2 + \left(-\frac{1}{2}\sin 2\omega\right)^2} = \sqrt{\left(\frac{1}{4} + \frac{1}{4}\cos^2 2\omega + \frac{1}{2}\cos 2\omega\right) + \frac{1}{4}\sin^2 2\omega}$$

$$= \boxed{\sqrt{\frac{1}{2} + \frac{1}{2}\cos 2\omega}}$$

I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.
at $\omega = 0$, $|H(0)| = \sqrt{\frac{1}{2} + \frac{1}{2}\cos 0} = 1$
at $\omega = \pi$, $|H(\pi)| = \sqrt{\frac{1}{2} + \frac{1}{2}\cos 2\pi} = 1$
at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{\frac{1}{2} + \frac{1}{2}\cos 2\frac{\pi}{2}} = 0$
at $\omega = \frac{\pi}{4}$, $\left|H\left(\frac{\pi}{4}\right)\right| = \sqrt{\frac{1}{2} + \frac{1}{2}\cos 2\frac{\pi}{4}} = \sqrt{\frac{1}{2}} = 0.70711$
A plot of $|H(\omega)|$ is shown below

For the phase, we have
$\Theta(\omega) = \tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))} = \tan^{-1}\left(\frac{-\frac{1}{2}\sin 2\omega}{\frac{1}{2}+\frac{1}{2}\cos 2\omega}\right) = \boxed{\tan^{-1}\left(\frac{-\sin 2\omega}{1+\cos 2\omega}\right)}$
at $\omega = 0$, $\Theta(\omega) = \tan^{-1}\left(\frac{-\sin 0}{1+\cos 0}\right) = \tan^{-1}\left(\frac{0}{2}\right) = 0$
at $\omega = \pi$, $\Theta(\omega) = \tan^{-1}\left(\frac{-\sin 2\pi}{1+\cos 2\pi}\right) = \tan^{-1}\left(\frac{0}{2}\right) = 0$
at $\omega = \frac{\pi}{2}$, $\Theta(\omega) = \tan^{-1}\left(\frac{-\sin \pi}{1+\cos \pi}\right) = \tan^{-1}\left(\frac{0}{0}\right) =$ undefined, discontinuity
A plot of the magnitude and phase are below





5

Part(f)

$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$

Hence, replace $x(n)$ with $\delta(n)$ in the above, we get

$h(n) = \frac{1}{2}[\delta(n) - \delta(n-2)]$

So, we get values only for $n = 0, 2$ i.e. $h = \left\{\boxed{\frac{1}{2}}, 0, -\frac{1}{2}\right\}$

using the Fourier transform for a discrete aperiodic sequence given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \quad \text{we get } H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n} = \sum_{n=0}^{2} h(n)\, e^{-j\omega n} = \boxed{\frac{1}{2} - \frac{1}{2}e^{-2j\omega}}$$

$$\text{so } H(\omega) = \frac{1}{2} - \frac{1}{2}(\cos 2\omega - j\sin 2\omega) = \boxed{\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega\right) + j\left(\frac{1}{2}\sin 2\omega\right)}$$

$$|H(\omega)| = \sqrt{\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega\right)^2 + \left(\frac{1}{2}\sin 2\omega\right)^2} = \sqrt{\left(\frac{1}{4} + \frac{1}{4}\cos^2 2\omega - \frac{1}{2}\cos 2\omega\right) + \left(\frac{1}{4}\sin^2 2\omega\right)}$$

$$= \boxed{\sqrt{\frac{1}{2} - \frac{1}{2}\cos 2\omega}}$$

I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.

at $\omega = 0$, $|H(0)| = \sqrt{\frac{1}{2} - \frac{1}{2}\cos 0} = 0$

at $\omega = \pi$, $|H(\pi)| = \sqrt{\frac{1}{2} - \frac{1}{2}\cos 2\pi} = 0$

at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{\frac{1}{2} - \frac{1}{2}\cos 2\frac{\pi}{2}} = 1$

at $\omega = \frac{\pi}{4}$, $\left|H\left(\frac{\pi}{4}\right)\right| = \sqrt{\frac{1}{2} - \frac{1}{2}\cos 2\frac{\pi}{4}} = \sqrt{\frac{1}{2}} = 0.707\,11$

A plot of $|H(\omega)|$ is shown below

For the phase, we have

$$\Theta(\omega) = \tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))} = \tan^{-1}\left(\frac{\frac{1}{2}\sin 2\omega}{\frac{1}{2} - \frac{1}{2}\cos 2\omega}\right) = \boxed{\tan^{-1}\left(\frac{\sin 2\omega}{1-\cos 2\omega}\right)}$$

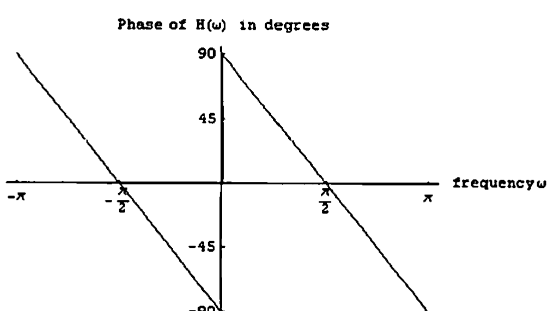
at $\omega = 0$, $\Theta(\omega) = \tan^{-1}\left(\frac{\sin 0}{1-\cos 0}\right) = \tan^{-1}\left(\frac{0}{0}\right)$ =undefined, discontinuity point

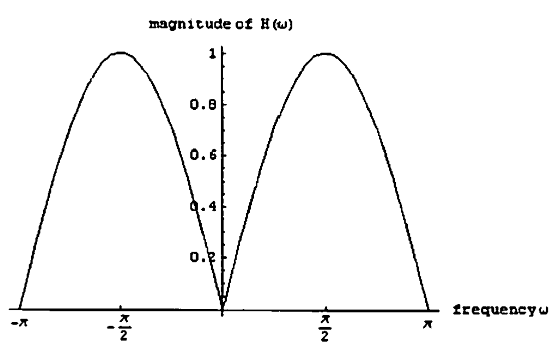
at $\omega = \pi$, $\Theta(\omega) = \tan^{-1}\left(\frac{\sin 2\pi}{1-\cos 2\pi}\right) = \tan^{-1}\left(\frac{0}{0}\right)$ =undefined, discontinuity point

at $\omega = \frac{\pi}{2}$, $\Theta(\omega) = \tan^{-1}\left(\frac{\sin \pi}{1-\cos \pi}\right) = \tan^{-1}\left(\frac{0}{2}\right) = 0$

at $\omega = \frac{\pi}{4}$, $\Theta(\omega) = \tan^{-1}\left(\frac{\sin\frac{\pi}{2}}{1-\cos\frac{\pi}{2}}\right) = \tan^{-1}\left(\frac{1}{1}\right) = 45^0$

A plot of the magnitude and phase are below

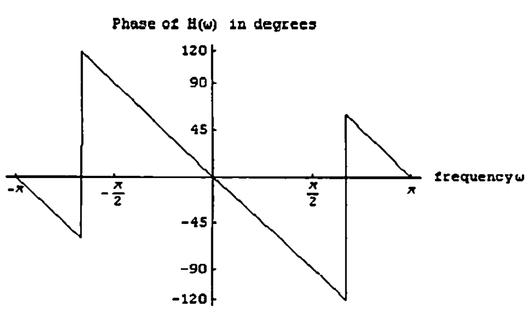


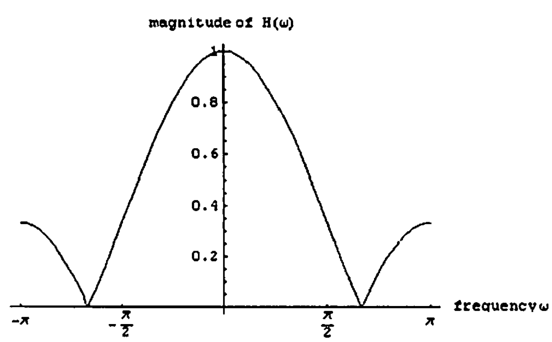


5

Part(g)
$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n=0$
Hence, replace $x(n)$ with $\delta(n)$ in the above, we get
$h(n) = \frac{1}{3}[\delta(n) + \delta(n-1) + \delta(n-2)]$
So, we get values only for $n = 0, 1, 2$ i.e. $h = \left\{\boxed{\frac{1}{3}}, \frac{1}{3}, \frac{1}{3}\right\}$
using the Fourier transform for a discrete aperiodic sequence given by
$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ we get $H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{2} h(n) e^{-j\omega n} = \boxed{\frac{1}{3} + \frac{1}{3}e^{-j\omega} + \frac{1}{3}e^{-2j\omega}}$
so $H(\omega) = \frac{1}{3} + \frac{1}{3}(\cos\omega - j\sin\omega) + \frac{1}{3}(\cos 2\omega - j\sin 2\omega) = \frac{1}{3} + \frac{1}{3}\cos\omega - \frac{1}{3}j\sin\omega + \frac{1}{3}\cos 2\omega - \frac{1}{3}j\sin 2\omega$
$= \boxed{\left(\frac{1}{3} + \frac{1}{3}\cos\omega + \frac{1}{3}\cos 2\omega\right) + j\left(-\frac{1}{3}\sin\omega - \frac{1}{3}\sin 2\omega\right)}$
$|H(\omega)| = \sqrt{\left(\frac{1}{3} + \frac{1}{3}\cos\omega + \frac{1}{3}\cos 2\omega\right)^2 + \left(-\frac{1}{3}\sin\omega - \frac{1}{3}\sin 2\omega\right)^2} = \boxed{\frac{1}{3}\sqrt{(1 + 2\cos\omega)^2}}$
I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.
at $\omega = 0$, $|H(0)| = \frac{1}{3}\sqrt{(1 + 2\cos 0)^2} = 1$
at $\omega = \pi$, $|H(\pi)| = \frac{1}{3}\sqrt{(1 + 2\cos\pi)^2} = \frac{1}{3}$
at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \frac{1}{3}\sqrt{\left(1 + 2\cos\frac{\pi}{2}\right)^2} = \frac{1}{3}$
at $\omega = \frac{\pi}{4}$, $\left|H\left(\frac{\pi}{4}\right)\right| = \frac{1}{3}\sqrt{\left(1 + 2\cos\frac{\pi}{4}\right)^2} = \frac{1}{3}\sqrt{\left(1 + 2\cos\frac{\pi}{4}\right)^2} = 0.804738$
A complete plot of $|H(\omega)|$ is shown below

For the phase, we have
$\Theta(\omega) = \tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))} = \tan^{-1}\left(\frac{-\frac{1}{3}\sin\omega - \frac{1}{3}\sin 2\omega}{\frac{1}{3} + \frac{1}{3}\cos\omega + \frac{1}{3}\cos 2\omega}\right) = \boxed{\tan^{-1}\left(\frac{-\sin\omega - \sin 2\omega}{1 + \cos\omega + \cos 2\omega}\right)}$
at $\omega = 0$, $\Theta(0) = \tan^{-1}\left(\frac{-\sin 0 - \sin 0}{1 + \cos 0 + \cos 0}\right) = \tan^{-1}\left(\frac{0}{3}\right) = 0$
at $\omega = \pi$, $\Theta(\pi) = \tan^{-1}\left(\frac{-\sin\pi - \sin 2\pi}{1 + \cos\pi + \cos 2\pi}\right) = \tan^{-1}\left(\frac{0}{1}\right) = 0$
at $\omega = \frac{\pi}{2}$, $\Theta\left(\frac{\pi}{2}\right) = \tan^{-1}\left(\frac{-\sin\frac{\pi}{2} - \sin 2\frac{\pi}{2}}{1 + \cos\frac{\pi}{2} + \cos 2\frac{\pi}{2}}\right) = \tan^{-1}\left(\frac{-1}{1-1}\right) = -90^0$
A plot of the magnitude and phase are below

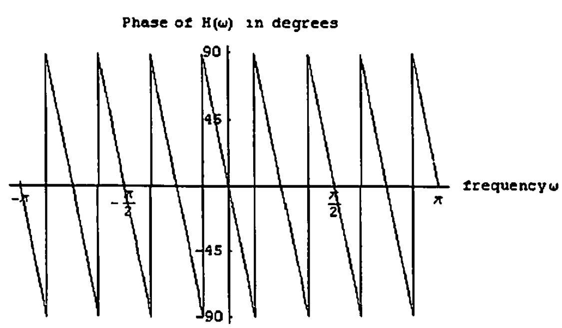


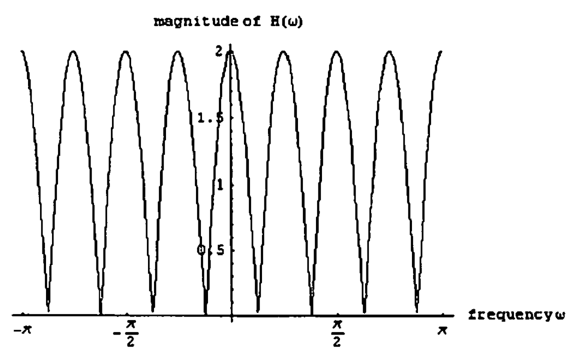


5

Part(h)
$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n=0$
Hence, replace $x(n)$ with $\delta(n)$ in the above, we get
$h(n) = \delta(n) - \delta(n-8)$
So, we get values only for $n = 0, 8$ i.e. $h = \left\{\boxed{1}, 0, 0, 0, 0, 0, 0, 0, 1\right\}$
using the Fourier transform for a discrete aperiodic sequence given by
$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ we get $H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{8} h(n) e^{-j\omega n} = \boxed{1 + e^{-8j\omega}} =$
$1 + (\cos 8\omega - j\sin 8\omega) = \boxed{(1+\cos 8\omega) + j(-\sin 8\omega)}$
$|H(\omega)| = \sqrt{(1+\cos 8\omega)^2 + (\sin 8\omega)^2} = \sqrt{(1 + \cos^2 8\omega + 2\cos 8\omega) + (\sin^2 8\omega)} = \boxed{\sqrt{2 + 2\cos 8\omega}}$
I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.
at $\omega = 0$, $|H(0)| = \sqrt{2 + 2\cos 0} = 2$
at $\omega = \pi$, $|H(\pi)| = \sqrt{2 + 2\cos 8\pi} = 2$
at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{2 + 2\cos 8\frac{\pi}{2}} = 2$
at $\omega = \frac{\pi}{4}$, $\left|H\left(\frac{\pi}{4}\right)\right| = \sqrt{2 + 2\cos 8\frac{\pi}{4}} = 2$
at $\omega = \frac{\pi}{3}$, $\left|H\left(\frac{\pi}{3}\right)\right| = \sqrt{2 + 2\cos 8\frac{\pi}{3}} = 1$
A plot of $|H(\omega)|$ is shown below

For the phase, we have
$\Theta(\omega) = \tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))} = \tan^{-1}\left(\frac{-\sin 8\omega}{1+\cos 8\omega}\right)$
at $\omega = 0$, $\Theta(0) = \tan^{-1}\left(\frac{0}{2}\right) = 0$
A plot of the magnitude and phase are below





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HW 5, EECS 152A DSP.
Problem 4.28 Nasser Abbasi
UCI, Fall 2004.

Question

An FIR system described by the difference equation $y(n) = x(n) + x(n-10)$
(a)Computer and sketch its magnitude and phase response
(b)Determine its response to the inputs
(1)$x(n) = \cos\left(\frac{\pi n}{10}\right) + 3\sin\left(\frac{\pi n}{3} + \frac{\pi}{10}\right) \quad -\infty < n < \infty$
(2)$x(n) = 10 + 5\cos\left(\frac{2\pi n}{5} + \frac{\pi}{2}\right) \quad -\infty < n < \infty$

Solution

part(a) Take the Z transform of both sides, we get $Y(z) = X(z) + z^{-10}X(Z)$
So, $Y(Z) = X(Z)\left(1 + z^{-10}\right)$ hence $H(Z) = \frac{Y(Z)}{X(Z)} = \left(1 + z^{-10}\right)$
This has a pole at z=0 or order 10, since pole inside unit circle , then stable. Also the Fourier transform exist since ROC defined on the unit circle. To find the Fourier transform, let $z = e^{j\omega}$ hence $1 + z^{-10} = 1 + \left(e^{jw}\right)^{-10}$

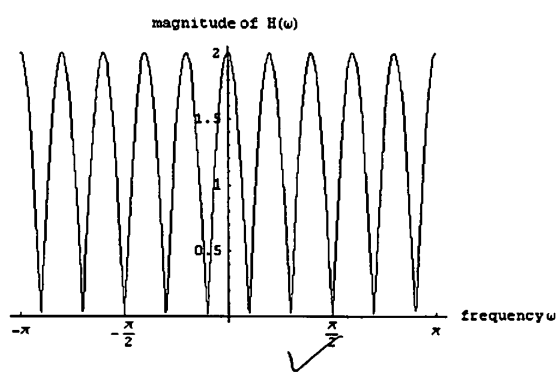
Hence $H(\omega) = \boxed{1 + e^{-10j\omega}} = 1 + \cos 10\,\omega - j\sin 10\omega = \boxed{(1 + \cos 10\omega) + j(\sin 10\omega)}$

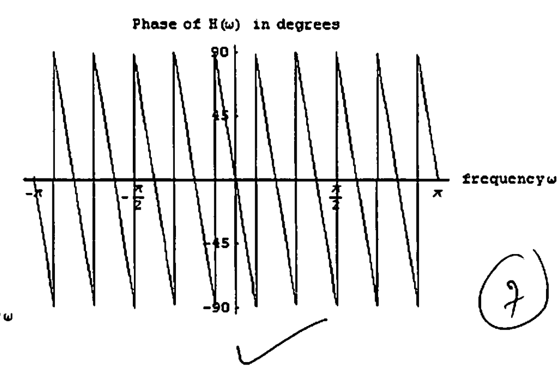
$|H(\omega)| = \sqrt{(1 + \cos 10\omega)^2 + \sin^2 10\omega} = \sqrt{1 + \cos^2 10\omega + 2\cos 10\omega + \sin^2 10\omega}$
$= \sqrt{2 + 2\cos 10\omega} = \boxed{\sqrt{2(1 + \cos 10\omega)}}$

Try few values: For $\omega = 0, |H(\omega)| = \sqrt{4} = 2$
For $\omega = \frac{\pi}{2}, |H(\omega)| = \sqrt{2(1 + \cos 5\pi)} = \sqrt{2(1-1)} = 0$
A plots for all ω values from $-\pi..\pi$ is below.

The phase, is given by $\Theta(\omega) = \boxed{\tan^{-1}\frac{\sin 10\omega}{1+\cos 10\omega}}$

try few values: $\omega = 0, \Theta(\omega) = \tan^{-1}\frac{0}{2} = 0$
$\omega = \pi, \Theta(\pi) = \tan^{-1}\frac{\sin\pi}{1+\cos 10\pi} = 0$
$\omega = \frac{\pi}{2}, \Theta\left(\frac{\pi}{2}\right) = \tan^{-1}\frac{\sin 10\frac{\pi}{2}}{1+\cos 10\frac{\pi}{2}}$ =undefined, discontinuity point
A plot for more points is shown below





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part(b)
To find response to $x(n) = \cos\left(\frac{\pi n}{10}\right) + 3\sin\left(\frac{\pi n}{3} + \frac{\pi}{10}\right)$, we note that the input is a combination of complex exponential, hence the output will not modify the frequencies of the input, but will scale the input, and shift the phase. i.e. the input is an eigenfunctions
i.e. if the input is $Ae^{j\omega_1 n}$ then the output is $\boxed{A\ |H(\omega)|\ e^{j(\omega_1 n + \Theta(\omega))}}$ evaluated at $\omega = \omega_1$

From part(a), we have $H(\omega) = 1 + e^{-10j\omega}, |H(\omega)| = \sqrt{2(1+\cos 10\omega)}, \Theta(\omega) = \tan^{-1} \frac{\sin 10\omega}{1+\cos 10\omega}$
First find the input frequencies and phase.
For $\cos\left(\frac{\pi n}{10}\right), \Rightarrow \omega_1 = \frac{\pi}{10}$
so response to this input is

$$\begin{aligned}
y_1(n) &= \sqrt{2(1+\cos 10\omega_1)} \quad \cos\left(\frac{\pi n}{10} + \tan^{-1}\frac{\sin 10\omega_1}{1+\cos 10\omega_1}\right) \\
&= \sqrt{2\left(1+\cos 10\frac{\pi}{10}\right)} \quad \cos\left(\frac{\pi n}{10} + \tan^{-1}\frac{\sin 10\frac{\pi}{10}}{1+\cos 10\frac{\pi}{10}}\right) \\
&= \sqrt{2(1-1)} \ \cos\left(\frac{\pi n}{10} + \tan^{-1}\frac{\sin 10\frac{\pi}{10}}{1+\cos 10\frac{\pi}{10}}\right) \\
&= 0
\end{aligned}$$

So response for $\boxed{\cos\left(\frac{\pi n}{10}\right) \text{ is zero.}}$
Now find the response for $3\sin\left(\frac{\pi n}{3} + \frac{\pi}{10}\right)$, here $\omega_2 = \frac{\pi}{3}$

$$\begin{aligned}
y_2(n) &= 3\sqrt{2(1+\cos 10\omega_2)} \quad \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\frac{\sin 10\omega_2}{1+\cos 10\omega_2}\right) \\
&= 3\sqrt{2\left(1+\cos 10\frac{\pi}{3}\right)} \quad \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\frac{\sin 10\frac{\pi}{3}}{1+\cos 10\frac{\pi}{3}}\right) \\
&= 3\sqrt{2(1+\cos(240^0))} \quad \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\frac{\sin(240^0)}{1+\cos(240^0)}\right) \\
&= 3\sqrt{2\left(1-\frac{1}{2}\right)} \quad \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{1-\frac{1}{2}}\right)\right) \\
&= 3 \ \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\left(-\sqrt{3}\right)\right) \\
&= 3 \ \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} - \frac{\pi}{3}\right) \\
&= 3 \ \sin\left(\frac{\pi n}{3} + \left(\frac{3-10}{30}\pi\right)\right) \\
&= 3 \ \sin\left(\frac{\pi n}{3} - \frac{7}{30}\pi\right)
\end{aligned}$$

(3) $\Rightarrow 6\cos\left(\frac{5\pi}{3}\right).\sin\left(\frac{\pi n}{3} - \frac{47\pi}{30}\right)$.

Hence the response of the system $\boxed{y(n) = 3\ \sin\left(\frac{\pi n}{3} - \frac{7}{30}\pi\right)}$
(2) $x(n) = 10 + 5\cos\left(\frac{2\pi n}{5} + \frac{\pi}{2}\right)$
The response of the system to the input 10 is simply $|H(\omega)| \times 10$
but $|H(\omega)|$ at $\omega = 0$ is 2, then $y_1(n) = 20$
To find response to $5\cos\left(\frac{2\pi n}{5} + \frac{\pi}{2}\right)$

$$y_2(n) = 5\sqrt{2(1+\cos 10\omega_2)} \quad \cos\left(\frac{2\pi n}{5} + \frac{\pi}{2} + \tan^{-1}\frac{\sin 10\omega_2}{1+\cos 10\omega_2}\right)$$

but $\omega_2 = \frac{2\pi}{5}$

$$
\begin{aligned}
y_2(n) &= 5\sqrt{2\left(1+\cos 10\frac{2\pi}{5}\right)} \quad \cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}+\tan^{-1}\frac{\sin 10\frac{2\pi}{5}}{1+\cos 10\frac{2\pi}{5}}\right) \\
&= 10 \quad \cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}+\tan^{-1}0\right) \\
&= 10 \quad \cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}\right)
\end{aligned}
$$

so response of system to $10+5\cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}\right)$ is $\boxed{20+10\ \cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}\right)}$ ✓ 4

HW5, EECS 152A DSP.
Problem 4.35 Nasser Abbasi
UCI, Fall 2004.

Question
Consider the filter $y(n) = 0.9\ y(n-1) + b\ x(n)$
(a)determine b such that $|H(0)| = 1$
(b)Determine the frequency at which $|H(\omega)| = \frac{1}{\sqrt{2}}$
(c)Is this filter a low pass or high pass
(d)Repeat parts (b),(c) for filter $y(n) = 0.9\ y(n-1) + b\ x(n)$

Solution
part(a) Take the Z transform of both sides, we get

$$\begin{aligned} Y(z) &= 0.9\ z^{-1}Y(z) + bX(z) \\ Y(z) - 0.9\ z^{-1}Y(z) &= bX(Z) \\ Y(z)\left(1 - 0.9\ z^{-1}\right) &= bX(Z) \\ H(z) &= \frac{Y(z)}{X(Z)} \end{aligned}$$

Hence

$$\boxed{H(z) = \frac{b}{(1-0.9\ z^{-1})}}$$

ROC: $0.9z^{-1} < |1|$ or $z > |0.9|$
Hence defined on the unit circle, and Fourier transform exist.
To find the Fourier transform, let $z = e^{j\omega}$ hence

$$\begin{aligned} H(\omega) &= \frac{b}{(1 - 0.9\ e^{-j\omega})} \\ &= \frac{b}{(1 - 0.9\ (\cos\omega - j\sin\omega))} \\ &= \frac{b}{(1-0.9\ \cos\omega) + 0.9j\sin\omega}\frac{(1-0.9\ \cos\omega) - 0.9j\sin\omega}{(1-0.9\ \cos\omega) - 0.9j\sin\omega} \\ &= \frac{(b - 0.9b\ \cos\omega) - 0.9bj\sin\omega}{(1-0.9\ \cos\omega)^2 - (0.9j\sin\omega)^2} \\ &= \frac{(b - 0.9b\ \cos\omega) - 0.9bj\sin\omega}{1.81 - 1.8\cos\omega} \end{aligned}$$

So $\operatorname{Re}(H) = \frac{(b-0.9b\ \cos\omega)}{1.81-1.8\cos\omega}$ and $\operatorname{Im}(H) = \frac{-0.9b\sin\omega}{1.81-1.8\cos\omega}$
Hence

$$\begin{aligned} |H(\omega)| &= \sqrt{\left(\frac{(b-0.9b\ \cos\omega)}{1.81-1.8\cos\omega}\right)^2 + \left(\frac{0.9b\sin\omega}{1.81-1.8\cos\omega}\right)^2} \\ &= \sqrt{\frac{(b-0.9b\ \cos\omega)^2 + (0.9b\sin\omega)^2}{(1.81-1.8\cos\omega)^2}} \end{aligned}$$

Let $\omega = 0$ and solve for b

$$|H(0)| = \sqrt{\frac{(b-0.9b\ )^2}{(1.81-1.8)^2}} = \sqrt{\frac{0.01b^2}{0.0001}} = \sqrt{\frac{b^2}{0.01}} = 10\ b$$

so

$$10\ b = 1$$

then

$$\boxed{b = \frac{1}{10}}$$

(5)

part(b)

solve for ω

$$\begin{aligned}
\frac{1}{\sqrt{2}} &= |H(\omega)| == \sqrt{\frac{(b-0.9b\ \cos\omega)^2 + (0.9b\sin\omega)^2}{(1.81-1.8\cos\omega)^2}} \\
\frac{1}{2} &= \frac{(b-0.9b\ \cos\omega)^2 + (0.9b\sin\omega)^2}{(1.81-1.8\cos\omega)^2} \\
(1.81-1.8\cos\omega)^2 &= 2(b-0.9b\ \cos\omega)^2 + 2(0.9b\sin\omega)^2 \\
3.24\cos^2\omega - 6.516\cos\omega + 3.2761 &= 2b^2 - 3.6b^2\cos\omega + 1.62b^2\cos^2\omega + 1.62b^2\sin^2\omega \\
0 &= 2b^2 - 3.6b^2\cos\omega + 3.24b^2\ - 3.24\cos^2\omega + 6.516\cos\omega - 3.2761
\end{aligned}$$

Let $\cos\omega = x$

$$0 = 2b^2 - 3.6b^2x + 3.24b^2\ - 3.24x^2 + 6.516x - 3.2761$$

with help of computer, Solution is:
$1.0056 - 0.15432\sqrt{20.995b^2 + 12.96b^4} - 0.55556b^2$,
$0.15432\sqrt{20.995b^2 + 12.96b^4} - 0.55556b^2 + 1.0056$
i.e.

$\boxed{\omega = \arccos\left(1.0056 - 0.15432\sqrt{20.995b^2 + 12.96b^4} - 0.55556b^2\right)}$

$\omega = \arccos\left(0.15432\sqrt{20.995b^2 + 12.96b^4} - 0.55556b^2 + 1.0056\right)$

for example, at $b = .1$ we get

$\omega = \arccos\left(1.0056 - 0.15432\sqrt{20.995(.1)^2 + 12.96(.1)^4} - 0.55556(.1)^2\right) = 0.37878$

$\omega = 0.105$

(4)

part(c)

$|H(\omega)| = \sqrt{\frac{(b-0.9b\ \cos\omega)^2+(0.9b\sin\omega)^2}{(1.81-1.8\cos\omega)^2}}$

at $\omega = \pi$ we have, when $b = (0.1)$, $|H(\pi)| = \sqrt{\frac{((0.1)-0.9(0.1)\ \cos\pi)^2+(0.9(0.1)\sin\pi)^2}{(1.81-1.8\cos\pi)^2}} = 5.2632\times 10^{-2}$

Hence we see that $|H(\omega)|$ is much smaller at high frequency than at DC, hence this is $\boxed{\text{low pass}}$

(5)

part(d)

Take the Z transform of both sides, we get

$$\begin{aligned}
Y(z) &= -0.9\ z^{-1}Y(z) + 0.1X(z) \\
Y(z) + 0.9\ z^{-1}Y(z) &= 0.1X(Z) \\
Y(z)\left(1+0.9\ z^{-1}\right) &= 0.1X(Z) \\
H(z) &= \frac{Y(z)}{X(Z)}
\end{aligned}$$

Hence

$$\boxed{H(z) = \frac{0.1}{(1+0.9\ z^{-1})}}$$

To find the Fourier transform, let $z = e^{j\omega}$ hence

$$\begin{aligned}
H(\omega) &= \frac{.1}{(1+0.9\ e^{-j\omega})} \\
&= \frac{.1}{(1+0.9\ (\cos\omega - j\sin\omega))} \\
&= \frac{.1}{(1+0.9\ \cos\omega) - 0.9j\sin\omega}\frac{(1+0.9\ \cos\omega)+0.9j\sin\omega}{(1+0.9\ \cos\omega)+0.9j\sin\omega} \\
&= \frac{(.1-0.09\ \ \cos\omega)+0.09j\sin\omega}{(1+0.9\ \cos\omega)^2-(0.9j\sin\omega)^2} \\
&= \frac{(.1-0.09\ \ \cos\omega)+0.09j\sin\omega}{1.8\cos\omega+1.81}
\end{aligned}$$

so

So $\mathrm{Re}(H) = \frac{.1-0.09\ \ \cos\omega}{1.81+1.8\cos\omega}$ and $\mathrm{Im}(H) = \frac{0.09\sin\omega}{1.81+1.8\cos\omega}$

$$\begin{aligned}
|H(\omega)| &= \sqrt{\left(\frac{.1-0.09\ \ \cos\omega}{1.81+1.8\cos\omega}\right)^2+\left(\frac{0.09\sin\omega}{1.81+1.8\cos\omega}\right)^2} \\
\frac{1}{2} &= \left(\frac{.1-0.09\ \ \cos\omega}{1.81+1.8\cos\omega}\right)^2+\left(\frac{0.09\sin\omega}{1.81+1.8\cos\omega}\right)^2 \\
\frac{1}{2} &= \frac{(.1-0.09\ \ \cos\omega)^2+(0.09\sin\omega)^2}{(1.81+1.8\cos\omega)^2} \\
\frac{1}{2} &= \frac{0.0181-0.018\cos\omega}{6.516\cos\omega+3.24\cos^2\omega+3.2761} \\
6.516\cos\omega+3.24\cos^2\omega+3.2761 &= 2(0.0181-0.018\ \cos\omega) \\
6.516\cos\omega+3.24\cos^2\omega+3.2761 &= 0.0362-0.036\cos\omega \\
0 &= -6.552\cos\omega-3.24\cos^2\omega-3.2399
\end{aligned}$$

Let $\cos\omega = x$

$0 = -6.552x - 3.24x^2 - 3.2399$, Solution is: $x = -1.1607, x = -0.86152$

i.e. $\boxed{\omega = \arccos(-0.86152) = 2.6091}$ the other root is not used as imaginary

To find if low or high filter, let $\omega = 0$ then

$|H(0)| = \sqrt{\left(\frac{.1-0.09\ \ \cos 0}{1.81+1.8\cos 0}\right)^2+\left(\frac{0.09\sin 0}{1.81+1.8\cos 0}\right)^2} = \sqrt{\left(\frac{.1-0.09}{1.81+1.8}\right)^2} = 2.7701\times 10^{-3}$

at $\omega = \pi$, $|H(\pi)| = \sqrt{\left(\frac{.1-0.09\ \ \cos\pi}{1.81+1.8\cos\pi}\right)^2+\left(\frac{0.09\sin\pi}{1.81+1.8\cos\pi}\right)^2} = \sqrt{\left(\frac{.1-0.09\ \ \cos\pi}{1.81+1.8\cos\pi}\right)^2} = 19.0$

Since $|H(\omega)|$ is much larger at large frequency, than at DC, then this is a $\boxed{\text{high pass filter}}$

5

HW#5
Problem 4.51
EECS 152A. Nasser Abbasi

Solve using geometrical argument.

(a) since we want $|H(\omega)|$ to be zero when $\omega=0$, then since $|H(\omega)| = \frac{|ZZ_k|\cdots}{|PP_k|\cdots}$, then we want $|ZZ_k|$ to be zero at $\omega=0$.

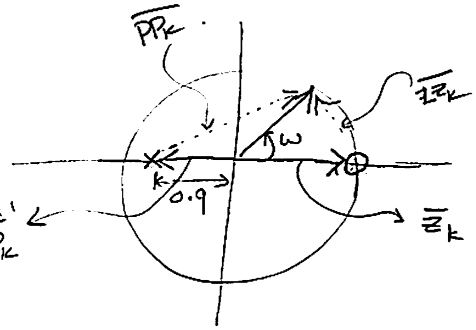
also since we want max response when $\omega=\pm\pi$ (max freq in discrete systems), then we want $|PP_k|$ to be smallest at $\omega=\pm\pi$. hence we set

$$H(\omega)=b_0\,\frac{1-z_k e^{-j\omega}}{1-P_k e^{-j\omega}}$$

$$\boxed{H(\omega)=b_0\,\frac{1}{1-0.9e^{j\pi}e^{-j\omega}}}$$

$$\boxed{H(z)=\frac{(b_0)}{1+0.9z^{-1}}}$$

there is a zero at P_k

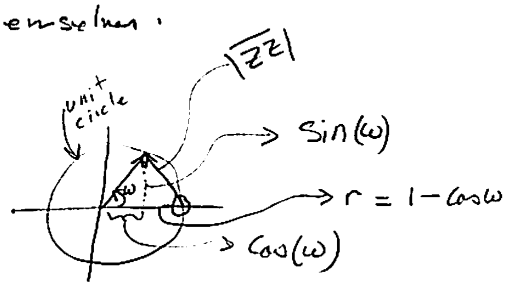


3

$$\boxed{H(z)=\frac{z-1}{z+0.9}}$$

I call vectors from zero location to tip of $e^{j\omega}$ as $\overline{ZZ_k}$ and vector from pole location to tip of $e^{j\omega}$ as $\overline{PP_k}$. to make it easier to differentiate from the actual vector $\overline{Z}_k$ and $\overline{P}_k$ themselves.

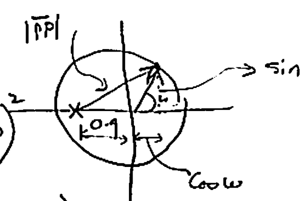
(b) $$|H(\omega)| = |b_0|\,\frac{|\overline{ZZ_k}|\cdots}{|\overline{PP_k}|\cdots}$$



to find $|\overline{ZZ}|$, use Phythagoras theorm

$$|\overline{ZZ}|^2 = \sin^2(\omega) + r^2$$, but $r = 1-\cos(\omega)$ from diagram.

so $$|\overline{ZZ}| = \sqrt{(1-\cos(\omega))^2+\sin^2\omega}$$



For $|\overline{PP}|$, we see that $|\overline{PP}|^2 = \sin^2\omega + (0.9+\cos\omega)^2$

so $$|\overline{PP}| = \sqrt{(0.9+\cos\omega)^2+\sin^2\omega}$$ $\Longrightarrow$

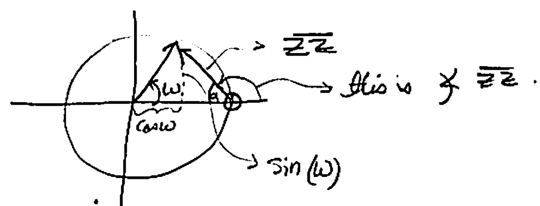
so $$\boxed{|H(\omega)| = |b_0| \frac{\sqrt{(1-\cos\omega)^2+\sin^2\omega}}{\sqrt{(0.9+\cos\omega)^2+\sin^2\omega}}}$$

for phase of $H(\omega)$,

$$\angle H(\omega) = \angle b_0 + \omega(\overset{\#\text{ poles}}{N}-\overset{\#\text{ zeros}}{M}) + \angle \overline{ZZ}_k \cdots - \left(\angle \overline{PP}_k + \cdots\right)$$

so $$\angle H(\omega) = \angle b_0 + \angle \overline{ZZ} - \angle \overline{PP}$$

$\angle \overline{ZZ}$:

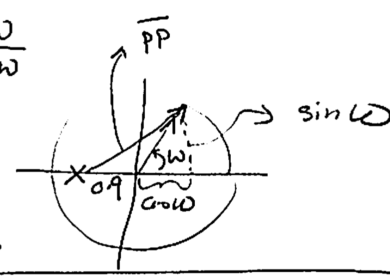


we see that $\angle \overline{ZZ} = 180^\circ - \alpha$

but $\alpha = \tan^{-1} \dfrac{\sin\omega}{1-\cos\omega}$.

so $\angle \overline{ZZ} = \pi - \tan^{-1} \dfrac{\sin\omega}{1-\cos\omega}$

$\angle \overline{PP}$:



we see that $\angle \overline{PP} = \tan^{-1} \dfrac{\sin\omega}{0.9+\cos\omega}$

so $$\boxed{\angle H(\omega) = \angle b_0 + \left(\pi - \tan^{-1}\frac{\sin\omega}{1-\cos\omega}\right) - \left(\tan^{-1}\frac{\sin\omega}{0.9+\cos\omega}\right)}$$

5

(c) need to find $|b_0|$. so that $|H(\omega)| = 1$ when $w=\pi$.

$$\left|H(\omega)\right|_{w=\pi} = |b_0| \frac{\sqrt{(1-\cos\pi)^2+\sin^2\pi}}{\sqrt{(0.9+\cos\pi)^2+\sin^2\pi}} = 1.$$

so $$|b_0| \frac{\sqrt{(1-(-1))^2}}{\sqrt{(0.9-1)^2}} = 1$$

$$|b_0| \frac{2}{0.1} = 1 \Rightarrow |b_0| = \frac{0.1}{2} = \boxed{0.05}$$

so $$\boxed{H(\omega) = \frac{0.05}{1+0.9e^{-j\omega}}}$$

(5)

(d) since $$H(z) = \frac{0.05}{1+0.9z^{-1}} = 0.05\left(\frac{1}{1+0.9z^{-1}}\right)$$

$$H(z) = 0.05 \frac{1}{1-(-0.9z^{-1})}$$

from table, we see that $a^n u(n) \xrightarrow{Z} \frac{1}{1-az^{-1}}$

so let $a=-0.9$, we get

$$h(n) = 0.05\,(-0.9)^n u(n)$$

$$= \boxed{(-1)^n\, 0.05\, (0.9)^n u(n)}$$

$$h = \{\underset{\uparrow}{0.05}, -0.045, 0.0405, -0.03645, \ldots\}$$

did not need to do this actually.

we see from $H(z) = \frac{Y(z)}{X(z)}$ that:

(5)

$$\frac{0.05}{1+0.9z^{-1}} = \frac{Y(z)}{X(z)} \rightarrow 0.05X(z) = Y(z) + 0.9Y(z)z^{-1}$$

$$\Rightarrow \boxed{\begin{aligned} &0.05x(n) = y(n) + 0.9\,y(n-1) \\ &y(n) = -0.9\,y(n-1) + 0.05\,x(n) \end{aligned}}$$

(e) need to find output if input is

$$x(n) = 2\cos\left(\frac{\pi}{6}n + 45^\circ\right)$$

we see here that $\omega = \frac{\pi}{6}$.

so $y(n) = 2\left|H(\omega=\frac{\pi}{6})\right| \cos\left(\frac{\pi}{6}n + 45^\circ + \measuredangle H(\omega=\frac{\pi}{6})\right)$

when $\omega = \frac{\pi}{6}$, $|H(\omega)| = 0.05 \dfrac{\sqrt{\left(1-\cos\frac{\pi}{6}\right)^2 + \sin^2\frac{\pi}{6}}}{\sqrt{\left(0.9+\cos\frac{\pi}{6}\right)^2 + \sin^2\frac{\pi}{6}}} =$ $\left(\frac{\pi}{6} = 30^\circ\right)$

$$= \frac{0.05\ (0.517638)}{1.8354} = \boxed{0.014101}$$

when $\omega = \frac{\pi}{6}$, $\measuredangle H(\omega) = \measuredangle 0.05 + \left(\pi - \tan^{-1}\frac{\sin 30^\circ}{1-\cos 30^\circ}\right) - \left(\tan^{-1}\frac{\sin 30^\circ}{0.9+\cos 30^\circ}\right)$

$= 0$ or π
but $+0.05 \Rightarrow \measuredangle = 0$

so $\measuredangle H(\omega) = 0 + \left(\pi - 75^\circ\right) - \left(15.807^\circ\right)$

$$= \boxed{89.19^\circ}$$

so $y(n) = 2\,(0.0141)\cos\left(\frac{\pi}{6}n + 45^\circ + 89.19^\circ\right)$

$$\boxed{y(n) = (0.0282)\cos\left(\frac{\pi}{6}n + 134.19^\circ\right)}$$

5

3.6 HW 6

94/100

HW#6

EECS 152A, Digital Signal processing

UCI. Fall 2004

By Nasser Abbasi

HW 4, EECS 152A DSP.
Problem 4.53 , Digital Signal Processing, 3rd edition, Proakis, anolakis
by Nasser Abbasi
UCI, Fall 2004.

Question
Derive the expression for the resonant frequency of a 2 pole filter with the poles at $p_1 = re^{j\theta}$ and $p_2 = p_1^*$ given by 4.5.25
Solution

Here, $\omega_0 = \theta$
Hence 4.5.25 is

$$\omega_r = \cos^{-1}\left(\frac{1+r^2}{2r}\cos\theta\right)$$

We know that

$$U_1(\omega) = \sqrt{1+r^2-2r\cos(\theta-\omega)}$$

and

$$U_2(\omega) = \sqrt{1+r^2-2r\cos(\theta+\omega)}$$

Take the product of U_1U_2 and minimize the result and solve for $\omega = \omega_r$

$$\begin{aligned} U_1U_2 &= \sqrt{1+r^2-2r\cos(\theta-\omega)}\sqrt{1+r^2-2r\cos(\theta-\omega)} \\ \frac{d}{d\omega}(U_1U_2) &= U_1\frac{d}{d\omega}(U_2) + U_2\frac{d}{d\omega}(U_1) \end{aligned}$$

But $\frac{d}{d\omega}(U_1) = \frac{1}{2}\frac{1}{\sqrt{1+r^2-2r\cos(\theta-\omega)}}(2r\sin(\theta-\omega)(-1)) = -\frac{r\sin(\theta-\omega)}{\sqrt{1+r^2-2r\cos(\theta-\omega)}}$
and $\frac{d}{d\omega}(U_2) = \frac{1}{2}\frac{1}{\sqrt{1+r^2-2r\cos(\theta+\omega)}}(2r\sin(\theta+\omega)(1)) = \frac{r\sin(\theta+\omega)}{\sqrt{1+r^2-2r\cos(\theta+\omega)}}$
Hence

$$\begin{aligned} \frac{d}{d\omega}(U_1U_2) &= U_1\frac{d}{d\omega}(U_2) + U_2\frac{d}{d\omega}(U_1) \\ &= \frac{\sqrt{1+r^2-2r\cos(\theta-\omega)}r\sin(\theta+\omega)}{\sqrt{1+r^2-2r\cos(\theta+\omega)}} - \frac{\sqrt{1+r^2-2r\cos(\theta+\omega)}r\sin(\theta-\omega)}{\sqrt{1+r^2-2r\cos(\theta-\omega)}} \end{aligned}$$

Take common denomenator

$$\frac{d}{d\omega}(U_1U_2) = \frac{\left(1+r^2-2r\cos(\theta-\omega)\right)\ r\sin(\theta+\omega) - \left(1+r^2-2r\cos(\theta+\omega)\right)\ r\sin(\theta-\omega)}{\sqrt{1+r^2-2r\cos(\theta+\omega)}\sqrt{1+r^2-2r\cos(\theta-\omega)}}$$

This derivative is mimumum when the numerator is zero.
Hence

$$0 = \left(1+r^2-2r\cos(\theta-\omega)\right)\ r\sin(\theta+\omega) - \left(1+r^2-2r\cos(\theta+\omega)\right)\ r\sin(\theta-\omega)$$

But for $r \neq 0$, divide the above by r to simplify, we get

$$0 = \left(1+r^2-2r\cos(\theta-\omega)\right)\ \sin(\theta+\omega) - \left(1+r^2-2r\cos(\theta+\omega)\right)\ \sin(\theta-\omega) \tag{1}$$

Expand (1) and use the following relations to simplify
$\sin(\theta+\omega) = \cos\theta\sin\omega + \sin\theta\cos\omega$
$\sin(\theta-\omega) = \sin\theta\cos\omega - \cos\theta\sin\omega$
$\cos(\theta-\omega) = \cos\theta\cos\omega + \sin\theta\sin\omega$

$\cos(\theta+\omega)=\cos\theta\cos\omega-\sin\theta\sin\omega$
Hence (1) becomes:

$$
\begin{aligned}
0 &= \left(1+r^2-2r\cos(\theta-\omega)\right)\ \ (\cos\theta\sin\omega+\sin\theta\cos\omega)\\
&\quad -\left(1+r^2-2r\cos(\theta+\omega)\right)\ \ (\sin\theta\cos\omega-\cos\theta\sin\omega)\\
&= (\cos\theta\sin\omega+\sin\theta\cos\omega)+r^2(\cos\theta\sin\omega+\sin\theta\cos\omega)-2r\cos(\theta-\omega)(\cos\theta\sin\omega+\sin\theta\cos\omega)\\
&\quad -\ (\sin\theta\cos\omega-\cos\theta\sin\omega)-r^2\ (\sin\theta\cos\omega-\cos\theta\sin\omega)+2r\cos(\theta+\omega)(\sin\theta\cos\omega-\cos\theta\sin\omega)\\
&= \cos\theta\sin\omega+\overbrace{\sin\theta\cos\omega}+r^2\cos\theta\sin\omega+\overbrace{r^2\sin\theta\cos\omega}-2r\cos(\theta-\omega)\cos\theta\sin\omega-2r\cos(\theta-\omega)\sin\theta\cos\omega\\
&\quad \overbrace{-\sin\theta\cos\omega}+\cos\theta\sin\omega\overbrace{-r^2\ \sin\theta\cos\omega}+r^2\cos\theta\sin\omega+2r\cos(\theta+\omega)\sin\theta\cos\omega-2r\cos(\theta+\omega)\cos\theta\sin\omega\\
&= 2\cos\theta\sin\omega+2r^2\cos\theta\sin\omega-2r\cos\theta\sin\omega\left(\cos(\theta-\omega)+\cos(\theta+\omega)\right)-2r\sin\theta\cos\omega\left(\cos(\theta-\omega)-\cos(\theta+\omega)\right)\\
&= 2\cos\theta\sin\omega+2r^2\cos\theta\sin\omega-2r\cos\theta\sin\omega(\cos\theta\cos\omega+\sin\theta\sin\omega+\cos\theta\cos\omega-\sin\theta\sin\omega)\\
&\quad -2r\sin\theta\cos\omega(\cos\theta\cos\omega+\sin\theta\sin\omega-\cos\theta\cos\omega+\sin\theta\sin\omega)\\
&=\\
&= 2\cos\theta\sin\omega+2r^2\cos\theta\sin\omega-2r\cos\theta\sin\omega(2\cos\theta\cos\omega)-2r\sin\theta\cos\omega(2\sin\theta\sin\omega)\\
&= 2\cos\theta\sin\omega+2r^2\cos\theta\sin\omega-4r\cos^2\theta\sin\omega\cos\omega-4r\sin^2\theta\cos\omega\sin\omega)\\
&= 2\cos\theta\sin\omega+2r^2\cos\theta\sin\omega-4r\sin\omega\cos\omega\left(\cos^2\theta+\sin^2\theta\right)\\
&= 2\cos\theta\sin\omega+2r^2\cos\theta\sin\omega-4r\sin\omega\cos\omega
\end{aligned}
$$

So the solution to $2\cos\theta\sin\omega+2r^2\cos\theta\sin\omega-4r\sin\omega\cos\omega=0$ will give us ω_r

$$
\begin{aligned}
2\cos\theta\sin\omega+2r^2\cos\theta\sin\omega-4r\sin\omega\cos\omega &= 0\\
\sin\omega\left[2\cos\theta+2r^2\cos\theta-4r\cos\omega\right] &= 0
\end{aligned}
$$

Hence, we get first solution as $\sin\omega=0$ or $\boxed{\omega=0}$
and we get the second solution when

$$
\begin{aligned}
2\cos\theta+2r^2\cos\theta-4r\cos\omega &= 0\\
\left(2+2r^2\right)\cos\theta-4r\cos\omega &= 0\\
4r\cos\omega &=\\
\cos\omega &= \frac{\left(2+2r^2\right)}{4r}\cos\theta\\
\cos\omega &= \frac{\left(1+r^2\right)}{2r}\cos\theta\\
&\boxed{\omega_r=\cos^{-1}\left(\frac{(1+r^2)}{2r}\cos\theta\right)}
\end{aligned}
$$

✓ 10

HW 6

Problem 4.57

(a) $y(n) = \frac{1}{2M+1} \sum_{k=-M}^{M} x(n-k)$.

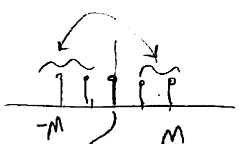
$$Y(z) = \frac{1}{2M+1} \sum_{k=-M}^{M} z^{-k} X(z) = \frac{1}{2M+1} X(z) \sum_{k=-M}^{M} z^{-k}$$

so $H(z) = \frac{1}{2M+1} \sum_{k=-M}^{M} z^{-k}$

so $\boxed{H(\omega) = \frac{1}{2M+1} \sum_{-M}^{M} e^{-j\omega k}}$ but $e^{-j\omega k} = \cos\omega k - j\sin\omega k$

so $H(\omega) = \frac{1}{2M+1} \sum_{-M}^{M} (\cos k\omega - j\sin k\omega)$

$= \frac{1}{2M+1} \underbrace{\sum_{-M}^{M} \cos k\omega}_{\Downarrow} \; \cancel{- j\sum_{-M}^{M} \sin k\omega} = 0$ since sin is an odd function.



$= 1$

$\because \cos\phi = 1$

$1 + 2\sum_{1}^{M} \cos k\omega \Rightarrow$ since cos is an even function.

So $\boxed{H(\omega) = \frac{1}{2M+1}\left(1 + 2\sum_{1}^{M} \cos k\omega\right)}$ ✓ S.

(b) $y(n) = \frac{1}{4M} x(n+M) + \frac{1}{2M} \sum_{-M+1}^{M-1} x(n-k) + \frac{1}{4M} x(n-M)$

$$Y(z) = \frac{1}{4M} z^{M} X(z) + \frac{1}{2M} \sum_{-M+1}^{M-1} z^{k} X(z) + \frac{1}{4M} X(z) z^{-M}$$

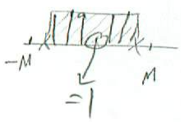
$$= \frac{1}{4M} z^{M} X(z) + \frac{1}{2M} X(z) \sum_{-M+1}^{M-1} z^{k} + \frac{1}{4M} X(z) z^{-M}$$

$$= X(z)\left[\frac{1}{4M} z^{M} + \frac{1}{2M}\sum_{-M+1}^{M-1} z^{k} + \frac{z^{-M}}{4M}\right]$$

so $H(z) = \dfrac{z^{M}}{4M} + \dfrac{1}{2M}\sum_{-M+1}^{M-1} z^{k} + \dfrac{z^{-M}}{4M}$

so $H(\omega) = \dfrac{e^{j\omega M}}{4M} + \dfrac{1}{2M}\left[\sum_{-M+1}^{M-1} e^{j\omega k}\right] + \dfrac{e^{-j\omega M}}{4M}$

$= \dfrac{1}{4M}\Big[\underbrace{e^{j\omega M} + e^{-j\omega M}}_{2\cos\omega M}\Big] + \dfrac{1}{2M}\Big[\underbrace{\sum_{-M+1}^{M-1}\cos\omega k}_{\Downarrow \; \left(1+2\sum_{1}^{M-1}\cos\omega k\right)} + \cancel{j\sum_{-M+1}^{M+1}\sin\omega k}^{=0}\Big]$



$= \dfrac{1}{4M}\,2\cos\omega M + \dfrac{1}{2M}\left(1+2\sum_{0}^{M-1}\cos\omega k\right)$

$$\boxed{H(\omega) = \frac{\cos\omega M}{2M} + \frac{1}{2M} + \frac{1}{M}\sum_{0}^{M-1}\cos\omega k} = \frac{\cos\omega M}{2M} + \frac{1}{2M}\left(1+2\sum_{0}^{M-1}\cos\omega k\right)$$

✓ 5

To find which provides better smoothing:

it is the filter which suppress high ω more.

for a fixed M, we see that $H_b(\omega)$ have one term less in the sum (it only goes up to $M-1$, but $H_a(\omega)$ goes to M.) so $H_b(\omega)$ will have smaller response.

so $\boxed{H(b) \text{ is better for smoothing}}$ ✓ 5

HW 4, EECS 152A DSP.
Problem 4.69 , Digital Signal Processing, 3rd edition, Proakis, anolakis
by Nasser Abbasi
UCI, Fall 2004.

Question

Determine the gain b_0 for the digital resonator described by 4.5.28 so that $|H(\omega_0)| = 1$

Solution

From page 342, equation 4.5.28 is

$$H(\omega) = b_0 \frac{1 - e^{-j2\omega}}{\left(1 - re^{j(\omega_0 - \omega)}\right)\left(1 - re^{-j(\omega_0 + \omega)}\right)} \tag{4.5.28}$$

$$H(\omega) = b_0 \frac{1 - (\cos 2\omega - j \sin 2\omega)}{(1 - r(\cos(\omega_0 - \omega) + j \sin(\omega_0 - \omega)))(1 - r\cos(\omega_0 + \omega) + j \sin(\omega_0 + \omega))}$$

Set $\omega = \omega_0$

$$\begin{aligned}
H(\omega) &= b_0 \frac{1 - (\cos 2\omega_0 - j \sin 2\omega_0)}{(1 - r(\cos(\omega_0 - \omega_0) + j \sin(\omega_0 - \omega_0)))(1 - r\cos(\omega_0 + \omega_0) + j \sin(\omega_0 + \omega_0))} \\
&= b_0 \frac{1 - \cos 2\omega_0 + j \sin 2\omega_0}{(1 - r)(1 - r\cos 2\omega_0 + j \sin 2\omega_0)} \\
&= b_0 \frac{1 - \cos 2\omega_0 + j \sin 2\omega_0}{1 - r\cos(2\omega_0) + j \sin 2\omega_0 - r + r^2 \cos 2\omega_0 - jr \sin 2\omega_0} \\
&= b_0 \frac{(1 - \cos 2\omega_0) + j \sin 2\omega_0}{(1 - r\cos 2\omega_0 - r + r^2 \cos 2\omega_0) + j(\sin 2\omega_0 - r \sin 2\omega_0)}
\end{aligned}$$

Hence

$$\begin{aligned}
|H(\omega)| &= b_0 \frac{\sqrt{(1 - \cos 2\omega_0)^2 + \sin^2 2\omega_0}}{\sqrt{(1 - r\cos 2\omega_0 - r + r^2 \cos 2\omega_0)^2 + (\sin 2\omega_0 - r \sin 2\omega_0)^2}} \\
&= b_0 \frac{\sqrt{1 + \cos^2 2\omega_0 - 2\cos 2\omega_0 + \sin^2 2\omega_0}}{\sqrt{(1 + r^2 - 2r)(1 + r^2 - 2r\cos 2\omega_0)}} \\
&= b_0 \frac{\sqrt{2(1 - \cos 2\omega_0)}}{\sqrt{(1 + r^2 - 2r)(1 + r^2 - 2r\cos 2\omega_0)}}
\end{aligned}$$

Set $|H(\omega)| = 1$ and solve for b_0

$$\begin{aligned}
1 &= b_0 \frac{\sqrt{2(1 - \cos 2\omega_0)}}{\sqrt{(1 + r^2 - 2r)(1 + r^2 - 2r\cos 2\omega_0)}} \\
b_0 &= \frac{\sqrt{(1 + r^2 - 2r)(1 + r^2 - 2r\cos 2\omega_0)}}{\sqrt{2(1 - \cos 2\omega_0)}} \\
&= \frac{\sqrt{(1 - r)^2 (1 + r^2 - 2r\cos 2\omega_0)}}{\sqrt{2(1 - \cos 2\omega_0)}}
\end{aligned}$$

10

$$b_0 = (1 - r)\frac{\sqrt{1 + r^2 - 2r\cos 2\omega_0}}{\sqrt{2 - 2\cos 2\omega_0}}$$

HW 6
4.79

(a) $P_1 = 0.8e^{j\frac{2\pi}{9}}$, $P_2 = 0.8e^{-j\frac{2\pi}{9}}$, $P_3 = 0.8e^{j4\frac{\pi}{9}}$, $P_4 = 0.8e^{-j4\frac{\pi}{9}}$

$z_1 = 1$, $z_2 = -1$, $z_3 = e^{3j\frac{\pi}{4}}$, $z_4 = e^{-3j\frac{\pi}{4}}$.

$$H(z) = b_0 \frac{(z-z_1)(z-z_2)(z-z_3)(z-z_4)}{(z-P_1)(z-P_2)(z-P_3)(z-P_4)}$$

$$= b_0 \frac{(z-1)(z+1)(z-e^{3j\frac{\pi}{4}})(z-e^{-j3\frac{\pi}{4}})}{(z-.8e^{j\frac{2\pi}{9}})(z-0.8e^{-j\frac{2\pi}{9}})(z-.8e^{j4\frac{\pi}{9}})(z-.8e^{-j4\frac{\pi}{9}})}$$

$$= \frac{(z^2-1)(e^{jw}-e^{3j\frac{\pi}{4}})(e^{jw}-e^{-j3\frac{\pi}{4}})}{(z^2-0.8z^{-j2\frac{\pi}{9}}-.8ze^{j\frac{2\pi}{9}}+.64)(z^2-0.8ze^{-j4\frac{\pi}{9}}-.8ze^{j4\frac{\pi}{9}}+.64)}$$

$$= \frac{(e^{2jw}-1)(e^{2jw}-e^{jw-j3\frac{\pi}{4}}-e^{3j\frac{\pi}{4}+jw}+1)}{(e^{2jw}-.8(e^{-j2\frac{\pi}{9}+jw}-e^{j\frac{2\pi}{9}+jw})+.64)(e^{2jw}-.8(e^{-j4\frac{\pi}{9}+jw}+e^{jw+j4\frac{\pi}{9}})+.64)}$$

$$= \frac{e^{4jw}-e^{3jw-j3\frac{\pi}{4}}-e^{3jw+3j\frac{\pi}{4}}+\cancel{e^{2jw}}-\cancel{e^{2jw}}+e^{j(w-\frac{3\pi}{4})}+e^{j(w+\frac{3\pi}{4})}-1}{(e^{2jw}-.8(e^{j(-\frac{2\pi}{9}+w)}-e^{j(\frac{2\pi}{9}+w)})+.64)(e^{2jw}-.8(e^{j(w-\frac{4\pi}{9})}+e^{j(w+\frac{4\pi}{9})})+.64)}$$

$$= \frac{e^{4jw}-e^{3jw}(e^{-j3\frac{\pi}{4}}-e^{3j\frac{\pi}{4}})+e^{jw}(e^{-j\frac{3\pi}{4}}+e^{j\frac{3\pi}{4}})-1}{(e^{2jw}-.8e^{jw}(e^{-j\frac{2\pi}{9}}-e^{j\frac{2\pi}{9}})+.64)(e^{2jw}-.8e^{jw}(e^{-j\frac{4\pi}{9}}+e^{j\frac{4\pi}{9}})+.64)}$$

$\rightarrow$

HW6_EECS_152.nb 1

In[118]:=

```
(*To TA, I could not simplify this any more, so I solved it
  using Mathematica, please see solution for b0 below *)

(*solution by Nasser Abbasi for HW 6, 152 *)

p1 = 0.8 Exp[I * 2 Pi / 9];
p2 = 0.8 Exp[- I * 2 Pi / 9];
p3 = 0.8 Exp[I * 4 Pi / 9];
p4 = 0.8 Exp[-I * 4 Pi / 9];
z1 = 1;
z2 = -1;
z3 = Exp[3 I Pi / 4];
z4 = Exp[- 3 I Pi / 4];
H[z_] := b0 (z - z1) (z - z2) (z - z3) (z - 4) / ((z - p1) (z - p2) (z - p3) (z - p4))
result = H[Exp[I w]];
Print["Result before substitution for w is=  ", result];

result = result /. w -> 5 Pi / 12;
Print["Result before substitution for w is=  ", result];

gain = Solve[Abs[result] == 1, b0];

Print["Gain is = ", gain];
```

Result before substitution for w is=

$$(b0\,(-4+e^{i w})\,(-1+e^{i w})\,(1+e^{i w})\,(-e^{\frac{3 i \pi}{4}}+e^{i w}))\,/\,(((-0.612836-0.51423\,i)+e^{i w})\,((-0.612836+0.51423\,i)+e^{i w})\,((-0.138919-0.787846\,i)+e^{i w})\,((-0.138919+0.787846\,i)+e^{i w}))$$

Result before substitution for w is= (19.967 - 10.6848 i) b0

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. More...

Gain is = {{b0 → -0.0441577}, {b0 → 0.0441577}}

So use $\boxed{b_0 = 0.044}$ X

4

so $H(z) = 0.044 \dfrac{(z-z_1)(z-z_2)(z-z_3)(z-z_4)}{(z-p_1)(z-p_2)(z-p_3)(z-p_4)}$ ✓ 5

0

Printed by Mathematica for Students

HW 6
4.84

$$y(n) = 0.9\,y(n-1) + 0.1\,x(n)$$

(a)

$$Y(z) = 0.9\,Y(z)\,z^{-1} + 0.1\,X(z)$$

$$Y(z)\left(1 - 0.9z^{-1}\right) = 0.1\,X(z)$$

$$\boxed{H(z) = \frac{0.1}{1 - 0.9z^{-1}}}$$

$$\boxed{H(\omega) = \frac{0.1}{1 - 0.9e^{-j\omega}}}$$

translation of frequency by $\frac{\pi}{2}$ is equivalent to multiplying the impulse response $h(n)$ by $e^{j\frac{\pi}{2}n}$.

so $h_{bp}(n) = e^{j\frac{\pi}{2}n}\, h_{lp}(n)$

but $e^{j\frac{\pi}{2}n} = \cos\frac{\pi}{2}n + j\sin\frac{\pi}{2}n$

$n=0 \Rightarrow 1$

$n=1 \Rightarrow J$

$n=2 \Rightarrow -1$

$n=3 \Rightarrow -J$

$n=4 \Rightarrow 1$

$n=5 \Rightarrow J$

$n=6 \Rightarrow -1$

$n=7 \Rightarrow -J$

$\vdots$

so multiply 2, 6, 10, 14, ... indexes by -1
multiply 1, 5, 9, 13, ... by J
multiply 3, 7, 11, 15, ... by $-J$

so $y(n) = 0.9\,y(n-1) + 0.1\,x(n)$

$k=1$ $\quad$ $k=0$

so $\boxed{y_{bp}(n) = J\,0.9\,y(n-1) + 0.1\,x(n)}$

3

(b) To find impulse response.

let $x(n) = \delta(n)$.

so $y_{bp}(n) = J\ 0.9\, y(n-1) + 0.1\,\delta(n)$.

at $n=0$, $y(0) = J\ 0.9\, y(-1) + 0.1$, assume $y(-1) = 0$

$\boxed{y(0) = 0.1}$

$n=1$

$y(1) = (0.1)(0.9)J = \boxed{(0.1)(0.9)J}$

$y(2) = J\ 0.9\,(0.09J) = \boxed{(-1)(0.1)(0.9)(0.9)}$

$y(3) = (-J)(0.1)(0.9)(0.9)(0.9)$

$y(4) = (1)(0.1)(0.9)(0.9)(0.9)(0.9)$

$y(5) = J(0.1)(0.9)\cdots$

so $h(n) = (0.1)(0.9)^n\, g(n)$.

where $g(n) = \begin{cases} J & \text{for } n = 1, 5, 9, \cdots \\ -J & \text{for } n = 3, 7, 11, \cdots \end{cases}$

Can also write it as $\boxed{h(n) = (0.1)\left(e^{j\frac{\pi}{2}}\right)^n (0.9)^n \quad \text{for } n \geq 0}$

(c) One problem I see is that the output of the filter is real only for even numbered samples.

$J, -1, -J, 1, J, -1, -J, 1, \ldots$
1 2 3 4 5 6 7 8

√ 5

HW 6

5.1

using property of DFT that for real valued sequence 5.2.24.

$$X(N-k) = X^*(k) = X(-k) .$$

here $N=8$. $X = \{0.25, 0.125 - j0.3018, 0, 0.125 - j0.0518, 0, ?, ?, ?\}$

so $X(5) = X(8-3) = X^*(3) = \boxed{0.125 + j0.0518}$

$X(6) = X(8-2) = X^*(2) = \boxed{0}$

$X(7) = X(8-1) = X^*(1) = \boxed{0.125 + j0.3018}$

✓ 10

HW 6

5-7

$$x_c(n) = x(n) \cos \frac{2\pi k_o n}{N}$$

$$x_s(n) = x(n) \sin \frac{2\pi k_o n}{N}.$$

$$x_c(n) = x(n) \left(\frac{e^{j\frac{2\pi k_o n}{N}} + e^{-j\frac{2\pi k_o n}{N}}}{2} \right).$$

so
$$X_c(k) = \frac{1}{2}\sum_{n=0}^{N-1} x(n) \left(e^{j\frac{2\pi k_o n}{N}} + e^{-j\frac{2\pi k_o n}{N}} \right) e^{-j\frac{2\pi k n}{N}}$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} x(n) \left(e^{j\frac{2\pi k_o n}{N} - j\frac{2\pi k n}{N}} + e^{-j\frac{2\pi k_o n}{N} - j\frac{2\pi k n}{N}} \right)$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} x(n) \left(e^{-j\frac{2\pi n}{N}(k_o - k)} + e^{-j\frac{2\pi n}{N}(k_o + k)} \right)$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi n}{N}(k_o - k)} + \frac{1}{2}\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi n}{N}(k_o + k)}$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} \underbrace{x(n) e^{j\frac{2\pi n}{N}k_o}} e^{-j\frac{2\pi k n}{N}} + \frac{1}{2}\sum_{n=0}^{N-1} \underbrace{x(n) e^{-j\frac{2\pi n}{N}k_o}} e^{-j\frac{2\pi n k}{N}}$$

$$= \frac{1}{2} \operatorname{DFT}\left(x(n) e^{j\frac{2\pi n}{N}k_o} \right) + \frac{1}{2} \operatorname{DFT}\left(x(n) e^{-j\frac{2\pi n}{N}k_o} \right)$$

but from Table 5.2, we see that

$$\operatorname{DFT}\left(x(n) e^{j\frac{2\pi n}{N}k_o} \right) = X((k - k_o))_N$$

and
$$\operatorname{DFT}\left(x(n) e^{-j\frac{2\pi n}{N}k_o} \right) = X((k + k_o))_N$$

So
$$\boxed{X_c(k) = \frac{1}{2} X((k-k_o))_N + \frac{1}{2} X((k+k_o))_N}$$

$$X_s(n) = x(n) \sin \frac{2\pi k_0 n}{N}$$

$$= x(n)\left(\frac{e^{j\frac{2\pi k_0 n}{N}} - e^{-j\frac{2\pi k_0 n}{N}}}{2j}\right)$$

$$x_s(n) = \frac{1}{2j}\, x(n)\, e^{j\frac{2\pi k_0 n}{N}} - \frac{1}{2j}\, x(n)\, e^{-j\frac{2\pi k_0 n}{N}}$$

$$\mathcal{X}(x_s(n)) = \frac{1}{2j}\left(\sum_{n=0}^{N-1} x(n)\, \underbrace{e^{j\frac{2\pi k_0 n}{N}} e^{-j\frac{2\pi k n}{N}}}\right) - \frac{1}{2j}\sum_{n=0}^{N-1} \underbrace{x(n)\, e^{-j\frac{2\pi k_0 n}{N}} e^{-j\frac{2\pi k n}{N}}}$$

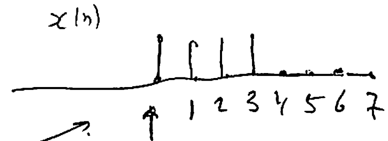
$$\boxed{\mathcal{X}(x_s(n)) = \frac{1}{2j}\, \mathcal{X}((k-k_0))_N - \frac{1}{2j}\, \mathcal{X}((k+k_0))_N}$$

✓ 10

HW 6

5.11

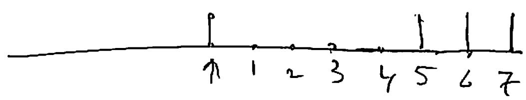
$$x(n) = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & 4 \le n \le 7 \end{cases}$$



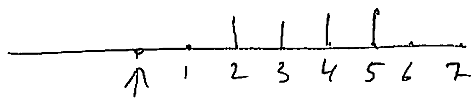
has $X(k)$ as its DFT

compute DFT of

(a) $x_1(n)$



(b) $x_2(n)$



(a) use the circular shift property in time.

we see that $x_1(n)$ is same as $x(n)$, when $x(n)$ is circularly shifted to right by 5 units.

so $x_1(n) = x((n-5))_8$

amount of circular shift

but circular shift in time means multiply X by $e^{-j\frac{2\pi k}{N}\ell}$

so $$\boxed{X_1(k) = X(k)\, e^{-j\frac{2\pi k}{8}5}}$$

(b) we see that $x_2(n)$ is same as $x(n)$ when $x(n)$ is circularly shifted to right by 2 units. so

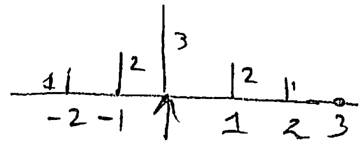
$$\boxed{X_2(k) = X(k)\, e^{-j\frac{2\pi k}{8}2}}$$

10

HW6

5.25 (a)

a) Find Fourier Transform $X(\omega)$ of $x(n)$



b) Compute 6 points DFT $V(k)$ of $v(n) = \{3, 2, 1, 0, 1, 2\}$.

(c) any relation between $X(\omega)$ and $V(k)$?

(a) from definition of $X(\omega)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}.$$

$$= x(-2)\,e^{-j\omega(-2)} + x(-1)\,e^{-j\omega(-1)} + x(0)\,e^{0} + x(1)\,e^{-j\omega} + x(2)\,e^{-j\omega 2} + x(3)\,e^{-j\omega 3}$$

$$= e^{2j\omega} + 2e^{j\omega} + 3 + 2e^{-j\omega} + 1\,e^{-2j\omega}$$

$$\boxed{X(\omega) = 2\cos 2\omega + 4\cos\omega + 3}$$

(b) $$\text{DFT} = \sum_{n=0}^{N-1} v(n)\, e^{-j2\pi\frac{k}{N}n}$$

$$= \sum_{0}^{5} v(n)\, e^{-j2\pi\frac{k}{N}n} = v(0)\,e^{0} + v(1)\,e^{-j2\pi\frac{k}{6}} + v(2)\,e^{-j\frac{2\pi k 2}{N}} + v(3)\,e^{-j\frac{2\pi k}{N}3} + v(4)\,e^{-j\frac{2\pi k}{N}4} + v(5)\,e^{-j\frac{2\pi k}{N}5}$$

$$= 3 + 2e^{-j\frac{\pi k}{3}} + 1\,e^{-j\frac{4\pi k}{N}} + 0 + 1\,e^{-j\frac{2\pi}{N}4} + 2e^{-j\frac{2\pi k 5}{N}}$$

$$= 3 + 2e^{-j\frac{\pi k}{3}} + e^{-j\frac{2}{3}\pi k} + e^{-j\frac{4}{3}\pi} + 2e^{-j\frac{5}{3}\pi k} \longrightarrow$$

but $e^{-j\frac{\pi}{3}} = e^{-j\frac{5\pi}{3}}$, $e^{-\frac{2}{3}\pi} = e^{-\frac{4}{3}}$ $\Rightarrow \boxed{3 + 4\cos\frac{\pi k}{3} + 2\cos\frac{2}{3}\pi k}$

compare $X(w) = 2\cos 2w + 4\cos w + 3$

with $DFT(v(n)) = 2\cos\frac{2\pi k}{3} + 4\cos\frac{\pi k}{3} + 3$

$\Rightarrow$ when $\boxed{w = \frac{\pi k}{3}}$ they are the same.

10

Chapter 4

Exams

Local contents

4.1 Exam 1

1

EECS152A Exam #1 October 26, 2004

Name: NASSER ABBASI

I.D.:

This is an 80 minute, CLOSED BOOK exam. If you have any questions, please ask. GOOD LUCK!

Question 1: 10

Question 2: 20

Question 3: 15

Question 4: 15

Question 5: 10

Question 6: 7

Question 7: 17

TOTAL: 94

Question 1 (10 points) Consider the continuous-time signal

$2\pi F_1 t = 80\pi t$
$\Rightarrow F_1 = 40$ Hz.

$$x(t) = 5\sin 80\pi t + 3\cos 25\pi t - \cos 70\pi t$$

a) What is the Nyquist rate for this signal?

$F_1 = 40$ Hz , $F_2 = 12.5$ Hz $F_3 = 35$ Hz.

so $\boxed{F_{max} = 40 \text{ Hz}}$ $\rightarrow$ Nyquist freq $= \boxed{80 \text{ Hz}}$

i.e $2F_{max}$.

5

b) For what values of the sampling rate F_s will sinc interpolation allow us to recover $x(t)$ exactly from its samples?

for values of $F_s >$ Nyquist frequency.

5

3

Question 2 (20 points) Consider the continuous-time signal

$F = 25\ Hz$

$$x_1(t) = 10\cos(50\pi t + \pi)$$

Suppose that we sample $x_1(t)$ at a rate $F_s = 20$ Hz to generate the sampled signal $x_1(n)$.
a) Determine the sampled signal $x_1(n)$.

$x_1(n) = 10\cos(50\pi(nT) + \pi)$

$= 10\cos(50\pi(\frac{n}{20}) + \pi)$

$x_1(n) = 10\cos(\frac{5}{2}\pi n + \pi)$

but $f = \frac{5}{4} > |\frac{1}{2}| \Rightarrow f_1 = \frac{1}{4} \Rightarrow \boxed{x_1(n) = 10\cos(2\pi(\frac{1}{4})n + \pi)}$

5

$\frac{1}{10000}$

b) If we apply sinc interpolation to $x_1(n)$, determine the recovered continuous-time signal $x_1'(t)$.

$2\pi f n = \frac{5}{2}\pi n \Rightarrow f = \frac{5}{4} > |\frac{1}{2}|$ so aliasing.

hence $\boxed{f \Rightarrow \frac{1}{4}}$

so $\boxed{x_1(n) = 10\cos(2\pi(\frac{1}{4})n + \pi)}$. now, note that $T = \frac{1}{20}$ sec.

So $x_1'(t) = 10\cos(2\pi(\frac{1}{4})\frac{t}{\frac{1}{20}} + \pi)$

$= 10\cos(\frac{40}{4}\pi t + \pi) = \boxed{10\cos(10\pi t + \pi)}$

8

c) Consider the continuous-time signal

$$x_2(t) = 10\cos(2\pi Ft + \pi)$$

Suppose that we sample $x_2(t)$ at $F_s = 20$ Hz to generate the sampled signal $x_2(n)$. Determine all values of F for which the signals $x_2(n)$ and $x_1(n)$ are equal for all samples n.

$x_2(n) = 10\cos(2\pi F(nT) + \pi)$

$= 10\cos(2\pi\frac{F}{F_s}n + \pi)$

$\boxed{x_2(n) = 10\cos(2\pi\frac{F}{20}n + \pi)}$ but $x_1(n) = 10\cos(2\pi\frac{1}{4}n + \pi)$.

So solve $2\pi\frac{F}{20}n = 2\pi\frac{1}{4}n$

7

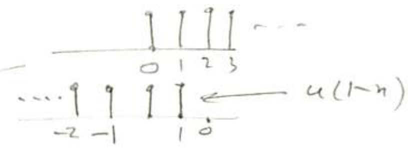
so $\boxed{F_0 = 5\ Hz}$ so $F = F_0 + KF_s$ $K = 0, \pm1, \pm2 \ldots$

$\boxed{F = 5, 25, 45, 65, \ldots, -15, -35, -55, \ldots}$

4

Question 3 (15 points) a) Compute the z-transform of

$$x(n) = \left(\frac{3}{2}\right)^n u(1-n)$$



$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{\infty} \left(\frac{3}{2}\right)^n u(1-n)\, z^{-n}$$

for n=−1 case

$$X(z) = \sum_{n=-\infty}^{n=1} \left(\frac{3}{2}\right)^n z^{-n}$$

$$= \sum_{-1}^{\infty} \left(\frac{3}{2}\right)^{-n} z^{n} = \left[\sum_{0}^{\infty} \left(\frac{3}{2}\right)^{-n} z^{n}\right] + \left(\frac{3}{2}\frac{1}{z}\right)$$

$$= \sum_{0}^{\infty} \left(\frac{2z}{3}\right)^{n} + \left(\frac{3}{2}\frac{1}{z}\right) = \frac{1}{1-\frac{2}{3}z} + \frac{3}{2}\frac{1}{z}$$

$\left|\frac{2}{3}z\right| < 1$, $z \neq 0$

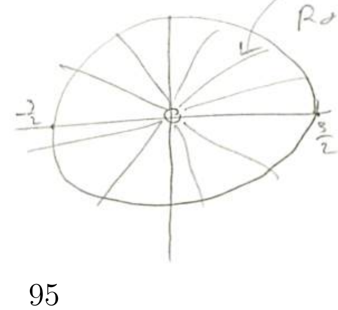
$$= \frac{2z + 3\left(1-\frac{2}{3}z\right)}{2z - \frac{4}{3}z^2} = \frac{2z+3-2z}{2z\left(1-\frac{1}{3}z\right)} = \boxed{\frac{3}{2z\left(1-\frac{1}{3}z\right)}}$$

10

b) What is the region of convergence?

$\left|\frac{2}{3}z\right| < 1$ i.e. $|z| < \frac{3}{2}$ and $z \neq 0$.

ROC . exclude origin



5

5

Question 4 (15 points) Consider the system

$$y(n) = 2x(n) + 4$$

a) Is this system linear?

NO. ✓ 5

$T[a\,x_1(n) + b\,x_2(n)] \stackrel{?}{=} a\,T[x_1(n)] + b\,T[x_2(n)]$

$2[a\,x_1(n) + b\,x_2(n)] + 4 \stackrel{?}{=} a[2x_1(n)+4] + b[2x_2(n)+4]$

$2a\,x_1(n) + 2b\,x_2(n) + 4 \stackrel{?}{=} 2a\,x_1(n) + 4a + 2b\,x_2(n) + 4b$ ✓

b) Write down the definition of a linear system. Use the definition of linear system to justify your answer to part a).

definition is: if $T[a\,x_1(n) + b\,x_2(n)] = a\,T[x_1(n)] + b\,T[x_2(n)]$

for any a, b, and for any $x_1(n)$, $x_2(n)$ 5

by applying this definition, we see that LHS $\neq$ RHS

for all a, b. it is only true for $a = \frac{1}{2}$, $b = \frac{1}{2}$, but

not for say $a = 1$, $b = 1$.

Hence NOT Linear 5

6

Question 5 (10 points) Consider the system

$$y(n) = \sum_{k=0}^{n} x(k) \quad = x(0) + x(1) + x(2) + \cdots + x(n)$$

a) Is this system time-invariant?

NO ✓. 5.

b) Justify your answer to part a).

a system is time invariant if delayed input produces the delayed output of the input when the input was not delayed.

i.e $y(n, L) = y(n-L)$ for all delay L.

$$\boxed{\begin{aligned} y(n, L) &= \sum_{k=0}^{n} x(k-L) \\ y(n-L) &= \sum_{k=0}^{n-L} x(k) \end{aligned}}$$

$= x(-L) + x(1-L) + x(2-L) + \cdots + x(n-L)$

$= x(0) + x(1) + x(2) + \cdots + x(n-L)$

we see $y(n, L) = y(n-L)$ only for $L = 0$.

$\Rightarrow$ NOT time-invariant

5.

for example, for $n=3$, $L=1$ we get

$y(n, L) = x(-1) + x(0) + x(1) + x(2)$

$y(n-L) = x(0) + x(1) + x(2)$

7

Question 6 (10 points) Let $x(n) = \delta(n)$ and let $y(n) = \left(\frac{1}{2}\right)^n u(n)$. Compute the crosscorrelation $r_{xy}(l)$ of $x(n)$ and $y(n)$.

$$r_{xy}(\ell) = \sum_{n=-\infty}^{\infty} x(n)\, y(n-\ell)$$

$$r_{xy}(\ell) = \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n \delta(n-\ell)$$

but $\delta(n-\ell) = 0$ for all $n-\ell \neq 0$.

i.e when $n \neq \ell \Rightarrow \delta(n-\ell) = 1$

so when $\ell = n \Rightarrow \left(\tfrac{1}{2}\right)^{\ell}$

so $\boxed{r_{xy}(\ell) = \left(\tfrac{1}{2}\right)^{\ell}}$

7

8

Question 7 (20 points) The following input-output pairs are observed during the operation of a linear time-invariant system

$$x_1(n) = \{0, 0, \underset{\uparrow}{4}\} \longleftrightarrow y_1(n) = \{0, 4, \underset{\uparrow}{6}, -8\}$$

$$x_2(n) = \{\underset{\uparrow}{2}, 4\} \longleftrightarrow y_2(n) = \{2, 7, \underset{\uparrow}{2}, -8\}$$

a) From this information, is it possible to determine the output $y_3(n)$ of the system for the input

YES

$$x_3(0) = 1 \quad x_3(1) = -1$$

Assume that $x_3(n)$ is zero for all other values of n.

note: if input is delta, then output is impulse response $h(n)$

5

b) If you answered no to part a), explain why not. If you answered yes to part a), determine the output sequence $y_3(n)$. I only need to use $x_1(n)$, $y_1(n)$.

Find $H(z)$ from $\boxed{x_1(n), y_1(n)} \Rightarrow X_1(z) = 4z^{-2}$

$Y_1(z) = 4z^{-1} + 6z^{-2} - 8z^{-3}$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{4z^{-1} + 6z^{-2} - 8z^{-3}}{4z^{-2}} = \boxed{z + \frac{3}{2} - 2z^{-1}}$$

7 +1

so $\boxed{h(n) = \{1, \underset{\uparrow}{\tfrac{3}{2}}, -2, 0, \cdots\}}$ now can use $H(z)$ to find $y_3(n)$ for other inputs:

$$Y_3(z) = H(z)X_3(z) = \left(z + \frac{3}{2} - 2z^{-1}\right)(1) = z + \frac{3}{2} - 2z^{-1}$$

$(1 - z^{-1})$

$$\Rightarrow \boxed{y_3(n) = \{1, \underset{\uparrow}{\tfrac{3}{2}}, -2\}}$$

for $x_3(1) = -1$, we get

$$Y_3(z) = H(z)X_3(z) = \left(z + \frac{3}{2} - 2z^{-1}\right)(-1) \Rightarrow \boxed{y_3(n) = \{-1, \underset{\uparrow}{-\tfrac{3}{2}}, 2\}}$$

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