HW 6

Math 2520
Differential Equations and Linear Algebra

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Contents

1 Problem 1

$$
\begin{aligned}
x_1' &= -k_1 x_1 \\
x_2' &= k_1 x_1 - k_2 x_2
\end{aligned}
$$

Where

$$
\begin{aligned}
k_1 &= \frac{10}{V_1(t)} = \frac{10}{25} = \frac{2}{5} \\
k_2 &= \frac{10}{V_2(t)} = \frac{10}{50} = \frac{1}{5} \\
x_1(0) &= 15 \\
x_2(0) &= 0
\end{aligned}
$$

a) Find the amount of salt in each tank at time $t \geq 0$. b) Find the maximum amount of salt ever in tank 2.

Solution

1.1 Part a

The system in matrix form is

$$
\begin{aligned}
x' &= Ax \\
\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} &= \begin{bmatrix} -k_1 & 0 \\ k_1 & -k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\
&= \begin{bmatrix} -\frac{2}{5} & 0 \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\end{aligned}
$$

The eigenvalues of A are found from solving $\det(A - \lambda I) = 0$ or

$$
\begin{aligned}
\begin{vmatrix} -\frac{2}{5} - \lambda & 0 \\ \frac{2}{5} & -\frac{1}{5} - \lambda \end{vmatrix} &= 0 \\
\left(-\frac{2}{5} - \lambda\right)\left(-\frac{1}{5} - \lambda\right) &= 0
\end{aligned}
$$

Hence $\lambda_1 = -\frac{2}{5}, \lambda_2 = -\frac{1}{5}$. Now the eigenvector for each eigenvalue is found.

$\lambda_1 = -\frac{2}{5}$

$$\begin{bmatrix} -\frac{2}{5}-\lambda_1 & 0 \\ \frac{2}{5} & -\frac{1}{5}-\lambda_1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{2}{5}-\left(-\frac{2}{5}\right) & 0 \\ \frac{2}{5} & -\frac{1}{5}-\left(-\frac{2}{5}\right) \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Hence $v_1 = t$ is free variable and v_2 is base variable. From second row $\frac{2}{5}t + \frac{1}{5}v_2 = 0$ or $2t + v_2 = 0$ or $v_2 = -2t$. The solution is

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} t \\ -2t \end{bmatrix} = t\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Choosing $t = 1$ this gives the first eigenvector

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$\underline{\lambda_2 = -\frac{1}{5}}$

$$\begin{bmatrix} -\frac{2}{5}-\lambda_2 & 0 \\ \frac{2}{5} & -\frac{1}{5}-\lambda_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{2}{5}-\left(-\frac{1}{5}\right) & 0 \\ \frac{2}{5} & -\frac{1}{5}-\left(-\frac{1}{5}\right) \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{5} & 0 \\ \frac{2}{5} & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 - 2R_2$

$$\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Hence v_1 is base variable and $v_2 = t$ is free variable. Therefore $\frac{1}{5}v_1 = 0$ or $v_1 = 0$. The eigenvector is

$$\vec{v}_2 = \begin{bmatrix} 0 \\ t \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Choosing $t = 1$ this gives the second eigenvector is

$$\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Therefore the solution basis are

$$\vec{x}_1(t) = e^{\lambda_1 t}\vec{v}_1 = e^{-\frac{2}{5}t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\vec{x}_2(t) = e^{\lambda_2 t}\vec{v}_2 = e^{\frac{-1}{5}t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

And the solution is linear combination of the above basis, which gives

$$\vec{x}(t) = c_1 \begin{bmatrix} e^{-\frac{2}{5}t} \\ -2e^{-\frac{2}{5}t} \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ e^{\frac{-1}{5}t} \end{bmatrix} \tag{1}$$

The scalar solutions are therefore

$$x_1(t) = c_1 e^{-\frac{2}{5}t}$$

$$x_2(t) = -2c_1 e^{-\frac{2}{5}t} + c_2 e^{\frac{-1}{5}t}$$

Now c_1, c_2 are found from initial conditions. At $t = 0, x_1(0) = 15, x_2(0) = 0$. Hence (1) becomes

$$\begin{bmatrix} 15 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Or

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix} \tag{2}$$

The augmented matrix is

$$\begin{bmatrix} 1 & 0 & 15 \\ -2 & 1 & 0 \end{bmatrix}$$

$R_2 = R_2 + 2R_1$ gives

$$\begin{bmatrix} 1 & 0 & 15 \\ 0 & 1 & 30 \end{bmatrix}$$

Hence (2) becomes

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 30 \end{bmatrix}$$

Second row gives $c_2 = 30$ and first row gives $c_1 = 15$. Hence

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 30 \end{bmatrix}$$

And the solution (1) becomes

$$\begin{aligned} x_1(t) &= 15e^{-\frac{2}{5}t} \\ x_2(t) &= -2\,(15)\,e^{-\frac{2}{5}t} + 30e^{\frac{1}{5}t} \end{aligned}$$

Or

$$\begin{aligned} x_1(t) &= 15e^{-\frac{2t}{5}} \\ x_2(t) &= -30e^{-\frac{2t}{5}} + 30e^{\frac{-t}{5}} \end{aligned} \tag{3}$$

The above is the amount of salt in each tank for $t \geq 0$.

1.2 Part b

The solution in (3) above shows that at $t \to \infty$ then $x_2(t) \to 0$ because both exponential are raised to negative power of t. This is as expected, as with time, and with more fresh water coming in and mixture discharges, we expect the initial salt in the tank to eventually vanish leaving only pure water in the tank. The following plot shows how the amount of salt changes in each tank as function of time

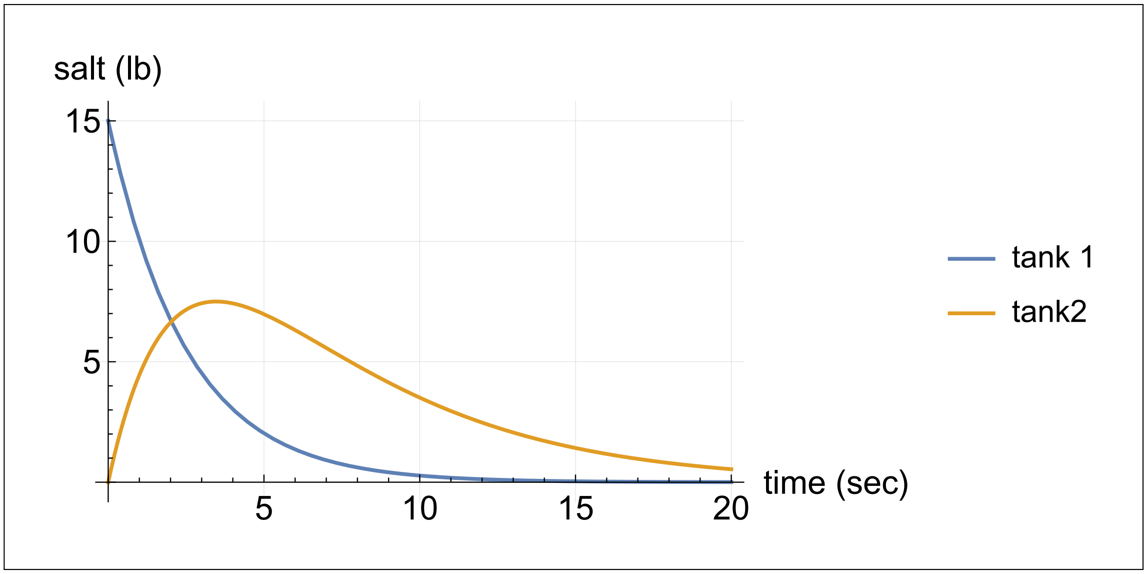

Figure 1: salt amount vs. time for each tank

The above shows that salt starts in tank 1 at amount $x_1(0) = 15$ which is the initial value, and continues to decreases exponentially where is becomes close to zero after about 15 seconds. While for tank 2, which initially has no salt, its salt content initially increases to a maximum value after about 3 seconds and then starts to decrease where it will eventually becomes zero. The initial increase in tank 2 is because of the salt coming from tank 1 in the mixture. But as salt decrease in tank 1 with time, so will the salt in tank 2 as well.

Code used for the plot above is

```
ClearAll[x1,x2,t];
x1[t_] := 15*Exp[-2/5*t]
x2[t_] := -30*Exp[-2/5*t]+30*Exp[-t/5]

Plot[{x1[t],x2[t]},{t,0,20},
   AxesLabel->{"time (sec)","salt (lb)"},
   BaseStyle->14,
   GridLines->Automatic,GridLinesStyle->LightGray,
   PlotLegends->{"tank 1","tank2"}
]
```

2 Problem 2

Determine all the equilibrium points of the given system.

$$x' = x - x^2 - xy$$
$$y' = 3y - xy - 2y^2$$

Solution

The equilibrium points are the solutions x, y to

$$x - x^2 - xy = 0 \tag{1}$$
$$3y - xy - 2y^2 = 0 \tag{2}$$

We can start with either equation, find one unknown from it, and use that to solve for the second unknown using the second equation. Starting with (1) and solving for x. Writing (1) as

$$x^2 + x\left(y-1\right) = 0$$
$$x\left(x + \left(y-1\right)\right) = 0$$

Then the solutions are

$$x = 0$$
$$x + \left(y-1\right) = 0$$

Or

$$x = 0 \tag{3}$$
$$x = 1 - y \tag{4}$$

For each one of the above solutions, we go back to (2) and solve for y now. When $x = 0$, then (2) gives

$$3y - 2y^2 = 0$$
$$y\left(3 - 2y\right) = 0$$

Hence $y = 0, y = \frac{3}{2}$ are the solutions. So now we have the following solutions found for the case when $x = 0$

$$\left\{(0,0), \left(0, \frac{3}{2}\right)\right\} \tag{5}$$

And when $x = 1 - y$ then (2) gives

$$3y - \left(1-y\right)y - 2y^2 = 0$$
$$2y - y^2 = 0$$
$$y\left(2-y\right) = 0$$

Hence $y = 0, y = 2$ are the solutions. When $y = 0$, the corresponding x from $x = 1 - y$ is 1. And when $y = 2$, the corresponding x is -1, So now we have the following solutions found

$$\{(1,0),(-1,2)\} \tag{6}$$

Adding (6,5) gives the list of equilibrium points as

$$\left\{(0,0),\left(0,\frac{3}{2}\right),(1,0),(-1,2)\right\}$$

3 Problem 3

Using the definition of Laplace transform, determine Laplace transform of

$$f(t) = te^t$$

Solution

By definition

$$\mathscr{L}\left(f(t)\right) = \lim_{N\to\infty}\int_0^N f(t)e^{-st}dt$$

Therefore

$$\begin{aligned}\mathscr{L}\left(f(t)\right) &= \lim_{N\to\infty}\int_0^N te^te^{-st}dt\\ &= \lim_{N\to\infty}\int_0^N te^{-st+t}dt\\ &= \lim_{N\to\infty}\int_0^N te^{t(1-s)}dt\end{aligned}$$

Integration by parts. $\int udv = uv - \int vdu$. Let $u = t, dv = e^{t(1-s)}$, therefore $du = dt, v = \frac{e^{t(1-s)}}{1-s}$. Hence the above becomes

$$\begin{aligned}\mathscr{L}\left(f(t)\right) &= \frac{1}{1-s}\lim_{N\to\infty}\left[te^{t(1-s)}\right]_0^N - \lim_{N\to\infty}\int_0^N \frac{e^{t(1-s)}}{1-s}dt \qquad (1)\\ &= \frac{1}{1-s}\lim_{N\to\infty}\left[te^{t(1-s)}\right]_0^N - \frac{1}{1-s}\lim_{N\to\infty}\int_0^N e^{t(1-s)}dt\end{aligned}$$

But

$$\begin{aligned}\lim_{N\to\infty}\left[te^{t(1-s)}\right]_0^N &= \lim_{N\to\infty}\left(Ne^{N(1-s)}\right) - 0\\ &= \lim_{N\to\infty}\left(Ne^{N(1-s)}\right)\end{aligned}$$

But

$$\begin{aligned}\lim_{N\to\infty}\left(Ne^{N(1-s)}\right) &= \lim_{N\to\infty}N\lim_{N\to\infty}e^{N(1-s)}\\ &= (\infty)\left(\lim_{N\to\infty}e^{N(1-s)}\right)\end{aligned}$$

For $s > 1$, $\lim_{N\to\infty}e^{N(1-s)} = e^{-\infty} = 0$ since $1 - s < 0$, and therefore the exponential is raised to negative infinity. Hence the above becomes

$$\begin{aligned}\lim_{N\to\infty}\left(Ne^{N(1-s)}\right) &= (\infty)(0)\\ &= 0\end{aligned}$$

Therefore (1) simplifies to

$$\begin{aligned}\mathscr{L}\big(f(t)\big) &= \frac{1}{1-s}\lim_{N\to\infty}\int_0^N e^{t(1-s)}dt\\ &= \frac{1}{s-1}\frac{1}{1-s}\lim_{N\to\infty}\left[e^{t(1-s)}\right]_0^\infty\\ &= \frac{1}{(s-1)(1-s)}\lim_{N\to\infty}\left[e^{t(1-s)}\right]_0^\infty\end{aligned}$$

But for $s > 1$ then $\lim_{N\to\infty}\left[e^{t(1-s)}\right]_0^\infty = \lim_{N\to\infty} e^{N(1-s)} - 1 = 0 - 1 = -1$. The above then becomes

$$\begin{aligned}\mathscr{L}\big(f(t)\big) &= \frac{-1}{(s-1)(1-s)}\\ &= \frac{1}{(s-1)(s-1)}\\ &= \frac{1}{(s-1)^2}\end{aligned}$$

For $s > 1$.

4 Problem 4

Find the inverse Laplace transform of the given functions

a $F(s) = \frac{2}{s(s-2)}$

b $F(s) = \frac{2s+2}{s^2+2s+5}$

Solution

4.1 Part a

Using partial fractions, let

$$\frac{2}{s(s-2)} = \frac{A}{s} + \frac{B}{s-2}$$

Therefore

$$\begin{aligned} A &= \left. \frac{2}{(s-2)} \right|_{s=0} = -1 \\ B &= \left. \frac{2}{s} \right|_{s=2} = 1 \end{aligned}$$

Hence

$$\frac{2}{s(s-2)} = -\frac{1}{s} + \frac{1}{s-2}$$

Using Table 10.2.1 in textbook,

$$\begin{aligned} \mathscr{L}^{-1}\left(\frac{1}{s}\right) &= 1 \qquad s > 0 \\ \mathscr{L}^{-1}\left(\frac{1}{s-2}\right) &= e^{2t} \qquad s > 2 \end{aligned}$$

Therefore, by linearity of $\mathscr{L}^{-1}$

$$\begin{aligned} \mathscr{L}^{-1}\left(\frac{2}{s(s-2)}\right) &= -\mathscr{L}^{-1}\left(\frac{1}{s}\right) + \mathscr{L}^{-1}\left(\frac{1}{s-2}\right) \\ &= -1 + e^{2t} \qquad s > 2 \end{aligned}$$

4.2 Part b

$$F(s) = \frac{2s+2}{s^2+2s+5}$$

Completing the squares in the denominator

$$\begin{aligned} s^2 + 2s + 5 &= (s+A)^2 + B \\ &= s^2 + 2sA + A^2 + B \end{aligned}$$

Comparing coefficients of s shows that

$$\begin{aligned} 2A &= 2 \\ A^2 + B &= 5 \end{aligned}$$

Hence $A = 1$ and $B = 4$. Therefore

$$\begin{aligned} \frac{2s+2}{s^2+2s+5} &= \frac{2s+2}{(s+1)^2+4} \\ &= \frac{2(s+1)}{(s+1)^2+4} \end{aligned}$$

Using the first shifting property (theorem 10.5.1 in book, which says)

$$\mathscr{L}\left(e^{at} f(t)\right) = F(s-a)$$

Then for $a = -1$, we see that

$$\mathscr{L}\left(e^{-t} f(t)\right) = F(s+1) \tag{1}$$

Where $f(t) = \mathscr{L}^{-1}(F(s))$. Therefore we just now need to find $f(t)$ using

$$f(t) = \mathscr{L}^{-1}\left(\frac{2s}{s^2+4}\right)$$

To complete the solution. But $\mathscr{L}^{-1}\left(\frac{s}{s^2+4}\right) = \cos 2t$ for $s > 0$ from Table 10.2.1. Hence

$$\mathscr{L}^{-1}\left(\frac{2s}{s^2+4}\right) = 2\cos(2t) \qquad s > 0$$

Therefore using (1) the final result is given by

$$\begin{aligned} \mathscr{L}^{-1}\left(\frac{2s+2}{s^2+2s+5}\right) &= e^{-t}\mathscr{L}^{-1}\left(\frac{2s}{s^2+4}\right) \\ &= e^{-t}(2\cos(2t)) \\ &= 2e^{-t}\cos s(2t) \qquad s > 0 \end{aligned}$$

5 Problem 5

Use the Laplace transform to solve the following given initial-value problems

a $y' + y = 8e^{3t}, y(0) = 2$

b $y'' + y' - 2y = 10e^{-t}, y(0) = 0, y'(0) = 1$

Solution

5.1 Part a

Taking the Laplace transform of both sides of $y' + y = 8e^{3t}$ gives (using linearity)

$$\mathscr{L}\left(y'\right) + \mathscr{L}(y) = 8\mathscr{L}\left(e^{3t}\right)$$

Assuming $\mathscr{L}(y) = Y(s)$, and using the property that $\mathscr{L}\left(y'\right) = s\mathscr{L}(y) - y(0)$ and from table 10.2.1 $\mathscr{L}\left(e^{3t}\right) = \frac{1}{s-3}, s > 3$, then the above becomes

$$sY(s) - y(0) + Y(s) = \frac{8}{s-3}$$

but $y(0) = 2$, hence the above simplifies to

$$\begin{aligned} sY(s) - 2 + Y(s) &= \frac{8}{s-3} \\ Y(s)(s+1) - 2 &= \frac{8}{s-3} \\ Y(s)(s+1) &= \frac{8}{s-3} + 2 \\ Y(s) &= \frac{8}{(s-3)(s+1)} + \frac{2}{(s+1)} \qquad (1) \end{aligned}$$

Looking at first term above, and using partial fractions

$$\frac{8}{(s-3)(s+1)} = \frac{A}{(s-3)} + \frac{B}{(s+1)}$$

Therefore

$$A = \left.\frac{8}{(s+1)}\right|_{s=3} = \frac{8}{4} = 2$$

And

$$B = \left.\frac{8}{(s-3)}\right|_{s=-1} = \frac{8}{-4} = -2$$

Therefore (1) becomes

$$\begin{aligned} Y(s) &= \frac{2}{(s-3)} - \frac{2}{(s+1)} + \frac{2}{(s+1)} \\ &= \frac{2}{(s-3)} \end{aligned}$$

From table 10.2.1 $\mathscr{L}\left(e^{3t}\right) = \frac{1}{s-3}, s > 3$. Hence

$$\begin{aligned} y(t) &= \mathscr{L}^{-1}\left(\frac{2}{(s-3)}\right) \\ &= 2\mathscr{L}^{-1}\left(\frac{1}{(s-3)}\right) \\ &= 2e^{3t} \end{aligned}$$

5.2 Part b

Taking the Laplace transform of both sides of $y'' + y' - 2y = 10e^{-t}$ gives (using linearity)

$$\mathscr{L}\left(y''\right) + \mathscr{L}\left(y'\right) - 2\mathscr{L}(y) = 10\mathscr{L}\left(e^{-t}\right) \tag{1}$$

Assuming $\mathscr{L}(y) = Y(s)$, and using the property that

$$\mathscr{L}\left(y'\right) = s\mathscr{L}(y) - y(0)$$

And

$$\mathscr{L}\left(y''\right) = s^2\mathscr{L}(y) - sy(0) - y'(0)$$

And from table 10.2.1 $\mathscr{L}\left(e^{-t}\right) = \frac{1}{s+1}, s > -1$, then (1) becomes

$$\begin{aligned} \left(s^2\mathscr{L}(y) - sy(0) - y'(0)\right) + \left(s\mathscr{L}(y) - y(0)\right) - 2\mathscr{L}(y) &= 10\left(\frac{1}{s+1}\right) \\ \left(s^2Y - s(0) - 1\right) + (sY - 0) - 2Y &= 10\left(\frac{1}{s+1}\right) \\ s^2Y - 1 + sY - 2Y &= 10\left(\frac{1}{s+1}\right) \\ Y\left(s^2 + s - 2\right) &= 10\left(\frac{1}{s+1}\right) + 1 \\ Y &= \frac{10}{(s+1)\left(s^2 + s - 2\right)} + \frac{1}{\left(s^2 + s - 2\right)} \end{aligned}$$

But $\left(s^2 + s - 2\right) = (s+2)(s-1)$. The above becomes

$$Y = \frac{10}{(s+1)(s+2)(s-1)} + \frac{1}{(s+2)(s-1)} \tag{2}$$

Using partial fractions to simplify the above, the first term becomes

$$\frac{10}{(s+1)(s+2)(s-1)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-1}$$

Hence

$$
\begin{aligned}
A &= \left.\frac{10}{(s+2)(s-1)}\right|_{s=-1} = \frac{10}{(-1+2)(-1-1)} = -5\\
B &= \left.\frac{10}{(s+1)(s-1)}\right|_{s=-2} = \frac{10}{(-2+1)(-2-1)} = \frac{10}{3}\\
C &= \left.\frac{10}{(s+1)(s+2)}\right|_{s=1} = \frac{10}{(1+1)(1+2)} = \frac{5}{3}
\end{aligned}
$$

And for the second term in (2)

$$
\frac{1}{(s+2)(s-1)} = \frac{D}{(s+2)} + \frac{E}{(s-1)}
$$

Hence

$$
\begin{aligned}
D &= \left.\frac{1}{(s-1)}\right|_{s=-2} = \frac{1}{(-2-1)} = -\frac{1}{3}\\
E &= \left.\frac{1}{(s+2)}\right|_{s=1} = \frac{1}{(1+2)} = \frac{1}{3}
\end{aligned}
$$

Using all the above in (2) gives

$$
\begin{aligned}
Y &= \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-1} + \frac{D}{(s+2)} + \frac{E}{(s-1)}\\
&= -5\frac{1}{s+1} + \frac{10}{3}\frac{1}{s+2} + \frac{5}{3}\frac{1}{s-1} - \frac{1}{3}\frac{1}{(s+2)} + \frac{1}{3}\frac{1}{(s-1)}\\
&= -5\frac{1}{s+1} + 3\frac{1}{s+2} + 2\frac{1}{s-1} \qquad (3)
\end{aligned}
$$

But from table 10.2.1

$$
\begin{aligned}
\mathscr{L}^{-1}\left(\frac{1}{s+1}\right) &= e^{-t} \qquad s > -1\\
\mathscr{L}^{-1}\left(\frac{1}{s+2}\right) &= e^{-2t} \qquad s > -2\\
\mathscr{L}^{-1}\left(\frac{1}{s-1}\right) &= e^{t} \qquad s > 1
\end{aligned}
$$

Using these results in (3) gives the final solution as

$$
\begin{aligned}
y(t) &= \mathscr{L}^{-1}(Y(s))\\
&= -5\mathscr{L}^{-1}\left(\frac{1}{s+1}\right) + 3\mathscr{L}^{-1}\left(\frac{1}{s+2}\right) + 2\mathscr{L}^{-1}\left(\frac{1}{s-1}\right)\\
&= -5e^{-t} + 3e^{-2t} + 2e^{t}
\end{aligned}
$$