Math2520-01 **Assignment 5**

<u>INSTRUCTION</u>: ***Show all the necessary work***. Write your answer on a separate sheet preferably hand written clear and legible. Post your answer sheet on D2L by Monday July 5.

1. Solve the following Differential Equations.

   a) $y'' - y' - 2y = 5e^{2x}$

   b) $y'' + 16y = 4\cos x$

   c) $y'' - 4y' + 3y = 9x^2 + 4,\ y(0) = 6,\ y'(0) = 8$

2. Use the variation of parameters method to find the general solution to the given differential equation.

   $y'' + y = \tan^2(x)$

3. Show that the given vector functions are linearly independent on $(-\infty, \infty)$.

$$x_1(t) = \begin{bmatrix} t \\ t \end{bmatrix}, x_2(t) = \begin{bmatrix} t \\ t^2 \end{bmatrix}$$

4. Show that the given vector functions are linearly dependent on $(-\infty, \infty)$.

$$x_1(t) = \begin{bmatrix} e^t \\ 2e^{2t} \end{bmatrix}, x_2(t) = \begin{bmatrix} 4e^t \\ 8e^{2t} \end{bmatrix}$$

5. Show that the given functions are solutions of the system $x'(t) = A(x)x(t)$ for the given matrix A and hence find the general solution to the system (remember to check linear independence). Then find the particular solution for the given auxiliary conditions.

$$x_1(t) = \begin{bmatrix} e^{4t} \\ 2e^{4t} \end{bmatrix}, x_2(t) = \begin{bmatrix} 3e^{-t} \\ e^{-t} \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 3 \\ -2 & 5 \end{bmatrix}, x(0) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

6. Solve the initial-value problem $x' = Ax,\ x(0) = x_0$.

$$A = \begin{bmatrix} -1 & 4 \\ 2 & -3 \end{bmatrix},\quad x(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

7. Use the variation of parameters technique to find a particular solution x_p to $x' = Ax + b$ for the given A and b. Also obtain the general solution to the system of differential equations.

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix},\quad b = \begin{bmatrix} 0 \\ 4e^t \end{bmatrix}$$