

INSTRUCTION: Show all the necessary work. Write your answer on a separate sheet preferably hand written clear and legible. Post your answer sheet on D2L by Sunday **June 20**.

1. If $x = (-3, 9, 9)$ and $y = (3, 0, -5)$, find a vector z in R^3 such that $4x - y + 2z = 0$ and its additive inverse.
2. Determine whether the given set S of vectors is closed under addition and is closed under scalar multiplication. The set of scalars is the set of all real numbers. Justify your answer.

a) The set $S = Q$, the set of all rational numbers.

b) The set S of all solutions to the differential equation

$$y' + 3y = 0 \quad (\text{do not solve the differential equation})$$

3. Let $S = \{(x, y) \in R^2 : x \geq 0, y \geq 0\}$. Is S a subspace of R^2 . Justify your answer.
4. Let $V = C^2(I)$ and S is a subset of V consisting of those functions satisfying the differential equation

$$y'' + 2y' - y = 0,$$

On I . Determine if S is a subspace of V .

5. a) Determine the null space of the given matrix A , nullspace(A).

$$A = \begin{bmatrix} 2 & 6 & 4 \\ -3 & 2 & 5 \\ -5 & -4 & 1 \end{bmatrix}$$

- b) Determine if $w = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is in the nullspace(A).

6. Let $v_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ vectors in R^2 . Express the vector $v = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$ as a linear combination of v_1, v_2 .

7. Let $v = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$, $v_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ be in R^3 . Let $W = \text{span}(v_1, v_2)$. Determine if v is in W .

8. Determine whether the given set $\{(1, -1, 0), (0, 1, -1), (1, 1, 1)\}$ in R^3 is linearly independent or linearly dependent.

9. Use the Wronskian to show that the given functions are linearly independent on the given interval I .

$$f_1(x) = 1, f_2(x) = 3x, f_3(x) = x^2 - 1, I = (-\infty, \infty)$$

10. Determine whether the set of vectors,

$$S = \{(1, 1, 0, 2), (2, 1, 3, -1), (-1, 1, 1, -2), (2, -1, 1, 2)\}$$

is a basis for R^4 .

11. Determine whether the set $S = \{1 - 3x^2, 2x + 5x^2, 1 - x + 3x^2\}$ is basis for $P_2(R)$.

12. Find the dimension of the null space of the given matrix A .

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 2 & 3 & -2 \\ 1 & 2 & -2 \end{bmatrix}$$

13. Determine the component vector of the given vector space V relative to the given ordered basis B .

$$V = R^2; B = \{(2, -2), (1, 4)\}; v = (5, -10).$$

14. a) find n such that $\text{rowspace}(A)$ is a subspace of R^n and determine the basis for $\text{rowspace}(A)$.
- a) find m such that $\text{colspace}(A)$ is a subspace of R^m , and determine a basis for $\text{colspace}(A)$.

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 1 & 1 & -2 & 6 \\ 3 & 1 & 4 & 2 \end{bmatrix}$$

- Note:**
1. You can use a theorem whenever applicable.
 2. Check the video clips posted on D2L related to this chapter.