SIGNAL TO NOISE RATIO CONSIDERATION IN HYPR AND WHYPR

What is SNR?

SNR or signal to noise ratio compares the level of signal to the level of background noise.

Signal to noise ratio in imaging is defined as ratio of the mean pixel value to the standard deviation of pixel value.

$$SNR = \frac{mean(f)}{std(f)}$$

Notifications and assumptions

N_p	Number of projection in each time frame
N_v	Number of vascular pixels
N_r	Number of read out point
N_f	Number of time frames
N_{pix}	Number of pixels

$$\mu_C = mean(C)$$

$$mean(p) = \mu_p \approx N_v \times \mu_C \approx \mu_{p_c} = mean(p_c)$$

$$Var(p) = \frac{\sigma^2}{N_r} \quad , \quad Var(C) = \frac{\sigma^2}{N_p N_r N_f} \quad , \quad Var(p_c) = \frac{N_{pix}\sigma^2}{N_p N_r N_f}$$

$$Cov(C, p) = Cov(C, p_c) = Cov(p, p_c) = 0$$

Original HYPR

In Original HYPR we have:

$$H = \frac{1}{N_p} C \sum_{i=1}^{N_p} \frac{p}{p_c}$$

By using Delta method which is uses Taylor expansion for approximate variance g when g is a function of random variables x_i as:

$$Var(g) \approx \sum_i \sum_j \frac{\partial g}{\partial x_i} \times \frac{\partial g}{\partial x_j} \times Cov(x_i, x_j)$$

Then we have:

$$Var(H) \approx \left(\frac{1}{N_p}\right)^2 \times \left(Var(c) \times \left(\sum_{i=1}^{N_p} \frac{\mu_p}{\mu_{p_c}} \right)^2 + Var(p) \times \sum_{i=1}^{N_p} \left(\frac{\mu_c}{\mu_{p_c}} \right)^2 + Var(p_c) \sum_{i=1}^{N_p} \left(\frac{\mu_c \mu_p}{\mu_{p_c^2}} \right)^2 \right)$$

Replacing properties in assumption section we have:

$$Var(H) \approx \left(\frac{1}{N_p}\right)^2 \times \left(\frac{\sigma^2}{N_p N_r N_f} \times (N_p)^2 + \frac{\sigma^2}{N_r} \times \sum_{i=1}^{N_p} \left(\frac{\mu_c}{N_v \times \mu_c}\right)^2 + \frac{N_{pix}\sigma^2}{N_p N_r N_f} \sum_{i=1}^{N_p} \left(\frac{\mu_c \times N_v \times \mu_c}{(N_v \times \mu_c)^2}\right)^2\right)$$

$$Var(H) \approx \frac{\sigma^2}{N_p N_r N_f}\left(1 + \frac{N_f}{N_{v^2}} + \frac{N_{pix}}{N_p N_{v^2}}\right)$$

$$SNR = \frac{mean(H)}{\frac{\sigma}{(N_p N_r N_f)^{1/2}}\left(1 + \frac{N_f}{N_{v^2}} + \frac{N_{pix}}{N_p N_{v^2}}\right)^{1/2}}$$

but $mean(H) = mean(c)$

So we have :

$$SNR_H = \frac{SNR_C}{\left(1 + \frac{N_f}{N_{v^2}} + \frac{N_{pix}}{N_p N_{v^2}}\right)^{1/2}}$$

Wright HYPR

In Wright HYPR we have:

$$H_W = C \frac{\sum_{i=1}^{N_p} p}{\sum_{i=1}^{N_p} p_c}$$

By using Delta method we have:

$$Var(H_W) \approx Var(c) \times \left(\frac{\sum_{i=1}^{N_p} \mu_p}{\sum_{i=1}^{N_p} \mu_{p_c}} \right)^2 + \sum_{i=1}^{N_p} Var(p) \left(\frac{\mu_c}{\sum_{i=1}^{N_p} \mu_{p_c}} \right)^2 + \sum_{i=1}^{N_p} Var(p_c) \left(\frac{\mu_c \sum_{i=1}^{N_p} \mu_p}{\left(\sum_{i=1}^{N_p} \mu_{p_c} \right)^2} \right)^2$$

$$Var\left(H_W \right) \approx \frac{\sigma^2}{N_p N_r N_f} + \sum_{i=1}^{N_p} \frac{\sigma^2}{N_r} \left(\frac{\mu_c}{\sum_{i=1}^{N_p} N_v \times \mu_c} \right)^2 + \sum_{i=1}^{N_p} \frac{N_{pix} \sigma^2}{N_p N_r N_f} \left(\frac{\mu_c \sum_{i=1}^{N_p} N_v \text{¿} \mu_c}{\left(\sum_{i=1}^{N_p} N_v \text{¿} \mu_c \right)^2} \right)^2$$

$$= \frac{\sigma^2}{N_p N_r N_f} + \frac{\sigma^2}{N_p N_{v^2} N_r} + \frac{N_{pix} \sigma^2}{N_{p^2} N_r N_f N_{v^2}} = \frac{\sigma^2}{N_p N_r N_f} \left(1 + \frac{N_f}{N_{v^2}} + \frac{N_{pix}}{N_p N_{v^2}} \right)$$

So we have :

$$SNR_{H_W} = \frac{SNR_C}{\left(1 + \frac{N_f}{N_{v^2}} + \frac{N_{pix}}{N_p N_{v^2}} \right)^{1/2}}$$

- SNR in WHYPR is equal to SNR in original HYPR
- We expect WHYPR yield similar results as Original HYPR

References

1. Mistretta CA, Wieben O, Velikina J, Block W, Perry J, Wu Y, Johnson K, Wu Y. Highly constrained backprojection for time-resolved MRI. Magn Reson Med 2006;55:30–40.

2. Yuexi Huang GAW. Time-resolved MR angiography with limited projections. Magn Reson Med 2007;58:316-325.

3.John A. Rice, Mathematical Statistics and Data Analysis 3th Edition.