# My Mathematics 597 page
# Summer 2008 course at California State University, Fullerton

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## Contents

# 1 Introduction

This course part of my Masters degree in Applied Mathematics at California State University, Fullerton.

| Status | Sec | Sched # | GE | Site | FootNotes | Units | Type | Days | Time | BldgRoom |
|---|---|---|---|---|---|---|---|---|---|---|
| CLOSED | 01 | 10646 | | | AA | 3.0 | Supv | MWR | 0600PM-0820PM | MH 484 |
| | 02 | 10647 | | | BB | 3.0 | Supv | MWR | 0600PM-0850PM | MH 380 |

Figure 1: class schedule

## 1.1 Class description

Keep the project within scope of what GE asks for.

Math 597 Summer 2008

**Applied Mathematics Graduate Project**

*Class Time*: MWTH 5:30-7:50 P.M. *Class Room*: MH 484 (Session A), MH 380 (Session B)

*Instructors*:
William Gearhart, Ph.D. *Office*: MH 182F *Phone*: 714-278-3184
*Email*: wgearhart@fullerton.edu

Angel R. Pineda, Ph.D. *Office*: MH 182L *Phone*: 714-278-8478
*Email*: apineda@fullerton.edu *Homepage*: http://math.fullerton.edu/apineda/

*Office Hours*: Monday, Wednesday 8-9 pm, Thursday 4:30 – 5:30 pm or by appointment.

Dr Gearhart will be here 4:30–5:30 MWTh

*Course Description:*
This course is the capstone experience of the masters in applied mathematics. We will serve as a team of consultants for GE Healthcare with two tasks dealing with time-resolved angiography in magnetic resonance imaging (MRI):

1. Understanding the mathematics of the **H**ighly Constrained Back**pr**ojection (HYPR). We are interested in placing this and related algorithms (I-HYPR, HYPR-LR) in a mathematical framework to study their relation to other iterative reconstruction algorithms. We want to understand their resolution, noise amplification and artifacts. This analysis will help GE make a short term decision as to whether they will include this method in the MRI scanners.
2. Understanding how prior information can be incorporated in a mathematical description of the blood vessels being imaged using level-set techniques. We will compare the HYPR technique to a method that evolves the surfaces of vessels until they best match the data. This comparison is a long term goal of GE and Dr. Pineda in terms of finding the optimal way of using prior information for reconstruction with limited data.

7 weeks time frame

*Course Homepage (Blackboard):*
- *Email:* make sure that your email on Blackboard is one that you check regularly. Homework assignments, announcements and other class related information will be sent via email.
- *Course Documents:* documents related to the course will be posted here.
- *Discussion Board (under communication tool):* this online forum allows for students and faculty to communicate about the course (anonymously if desired).

*Grading:*
Major Project Reports to GE (Presentation and Written)

| Midterm Report | Final Report |
|---|---|
| Monday June 23 | Friday August 8 |

Reports include presentation. & weekly written presentation

Internal Weekly Presentations of Group Progress

There will be a final presentation on Thursday August 7, 2008 by the students to the instructors. The deliverables to GE will be a written report, a presentation and all of the code we used to generate the results.

Letter Grades for each students will be assigned after the Final Report (presentation and written) have been submitted. One letter grade will be based on the degree of satisfaction from our client (GE Healthcare) and this grade will be shared by all students. The second letter grade will be based on class participation, attendance, collaboration, and contribution to the project. The research notebooks and a 1 page description written by the student of their contributions will also be used to assign the final grade. In the case that the final report is not completed, a grade of incomplete will be assigned.

GE matlab code

one Grade based on Group.
Individual Grade based on note book.
also presentations & attendance
each will write description of my contribution to project. type it

*Suggestion and comments*

- This project will consist of a real consulting experience which by its nature is subject to change. We will be responsive to our clients needs and will be flexible. As opposed to other courses where the emphasis is on giving the right answers, this course is about asking the right questions.
- Unlike other courses where your instructors have the answers and are testing you, in this course, the instructors are simply more experienced members of the team who will help guide the work. We don't have all the answers!
- The course will involve reading papers, asking questions, writing code, and sharing your results with the team and with our clients.
- Make sure to ask questions and offer comments as this will make the team stronger.
- Write all your computations in your research notebook and date them. This will make it easier later when writing the reports. Try writing sections of the report as you go if possible.
- We are a team. Our success depends on all of us working well together.

*Academic Integrity*

Students who violate university standards of academic integrity are subject to disciplinary sanctions, including failure in the course and suspension from the university. Since dishonesty in any form harms the individual, other students and the university, policies on academic integrity are strictly enforced. I expect that you will familiarize yourself with the academic integrity guidelines found in the current student handbook:
http://www.fullerton.edu/deanofstudents/judicial/policies.htm

Examples of actions that constitute academic dishonesty include, but are not limited to:

- Unacceptable examination behavior – communicating with fellow students, copying material from another student's exam or allowing another student to copy from an exam, possessing or using unauthorized materials, or any behavior that defeats the intent of an exam.
- Plagiarism – taking the work of another and offering it as one's own without giving credit to that source, whether that material is paraphrased or copied in verbatim or near-verbatim form.
- Unauthorized collaboration on a project, homework or other assignment where an instructor expressly forbids such collaboration.

*Emergency Evacuation*

In the event of an emergency such as an earthquake or a fire:

- Take all your personal belongings and leave the classroom. Use the stairways located at the east, west or center of the building.
- Do not use the elevator. They may not be working once the alarm sounds.
- Go to the lawn area towards Nutwood Avenue. This provides a safe distance from falling debris from buildings. Stay with class members.
- For additional information on exits, fire alarms and telephones, building evacuation maps are located near each elevator.
- Anyone who may have difficulty evacuating the building, please see instructor.

***The material in this syllabus may be changed at the instructors' discretion***

# 2 Reports

## 2.1 HYPR simulator software

Matlab program I wrote for the project is here

## 2.2 Midterm group presenations

### 2.2.1 HYPR presentation

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# Initial Goals:

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- Understand the mathematics of HYPR and related algorithms (Wright HYPR, First Iteration of I-HYPR).
- Study their mathematical relation to the MLEM algorithm.
- Develop and validate a MATLAB tool to explore the properties of the reconstruction algorithms.
- Explore illustrative examples of where these algorithms work well and where they fail with a time-resolved angiography application in mind.

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# HYPR Algorithm:

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Data Acquisition:

$$s_t = R_{f_t}[I_t]$$

$t$: Time
$\phi_t$: Angle of projection at time t
$s_t$: Projection at time t
$I_t$: True image at time t
$R_{\phi_t}$: Radon iransform for angle $\phi_t$

Image Reconstruction:

$$J_t = C\left[\frac{1}{N_p}\sum_{i=1}^{N_p}\frac{R^u_{\phi_{t_i}}[s_{t_i}]}{R^u_{\phi_{t_i}}[s_{c_{t_i}}]}\right]$$

$C$: Composite image
$N_p$: Number of projections per time frame
$S_{t_i}$: Projection data at time $t_i$
$S_{c_{t_i}}$: Projection of composite image at time $t_i$
$R^u_\phi$: Transpose of Radon transform (unfiltered backprojection)
$J_t$: Reconstructed image

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## MLEM Algorithm:

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Let $g = Hf + \varepsilon$

$H$ : Matrix Projection Operator

$f$: Discrete Object

$g$: Projection

$\varepsilon$: Poisson Noise

MLEM maximizes the likelihood that g came from f.

MLEM Algorithm: $f_n^{(k+1)} = \left(\frac{f_n^{(k)}}{s_n}\right)\sum_m \left(\frac{g_m}{\left(Hf^{(k)}\right)_m} H_{mn}\right)$ *where* $s_n = \sum_m H_{mn}$

Using Matrix Notation Unfiltered Backprojection is $H^T$ :

$$\Rightarrow \quad f_n^{(k+1)} = f_n^{(k)} \frac{1}{s_n}\left(H^T\left[\frac{g}{Hf^{(k)}}\right]\right)_n$$

**Notation adopted from* *__Foundations of Image Science,__* *by Barrett and Myers*



## Comparison of MLEM & HYPR

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| MLEM-1 Algorithm | HYPR |
|---|---|
| $g = Hf + \varepsilon$ | $s_t = R_{\phi_t}\left[I_t\right]$ |
| $f_n^{(1)} = f_n^{(0)} \frac{1}{z_n}\left[H^T\left[\frac{g}{(Hf^{(0)})}\right]\right]_n$ | $J_n = \frac{1}{N_p} C_n \cdot R_\phi^u \left(\frac{s}{R_\phi(C)}\right)_n$ |
| $H$ | $R_\phi$ – *Radon Transform* |
| $H^T$ | $R_\phi^u$ – *Unfiltered backprojection* |
| $z_n$ | $N_p$ – *Projections per time frame* |
| $g$ | $s$ – *Original projection* |
| $f^{(0)}$ | $C$ – *Composite image* |

For this method to match the original HYPR in the first iteration we need that

$$\frac{R_\phi^u(s)}{R_\phi^u(s_c)} = R_\phi^u\left(\frac{s}{s_c}\right)$$

The ratio of unfiltered backprojections is the unfiltered backprojection of the ratio.

## Relevant Properties of MLEM



- Multiplicative update on each iteration, so if the initial estimate is zero, subsequent estimates remain zero. This property reduces streaking artifacts by using the composite image as the initial guess.

- Enforces non-negativity constraint. If initial estimate is positive and H has non-negative entries, future updates remain non-negative.

- Non-linear and iterative: while hugely popular in the research community, adoption in clinical nuclear medicine was slow because of unpredictable nature of artifacts. This may be something to discuss with clinical collaborators.

- Noise properties for time resolved MRA very different than in nuclear medicine where the major source of noise is the Poisson noise in the projections.

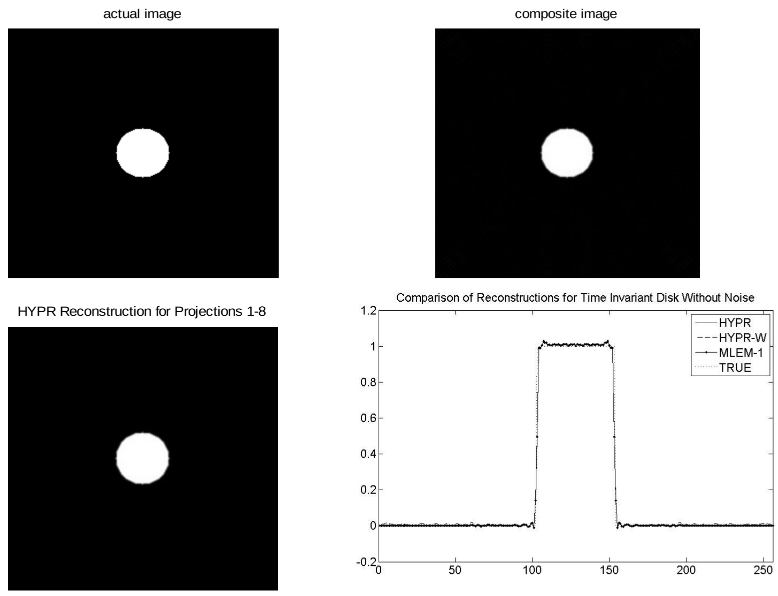




## Computational Comparison of HYPR, HYPR-W & MLEM-1



- In the following slide we compare Original HYPR to 1-step MLEM algorithm
- Time-invariant disk used
- 128 projection angles used (bit-reversed ordering)
- Window size: 8 projections
- Also implemented HYPR-W (Huang and Wright)
- For a stationary disk, all methods give a similar result, but the MLEM implementation is slightly better.




For a stationary disk, with no noise, all methods are similar.





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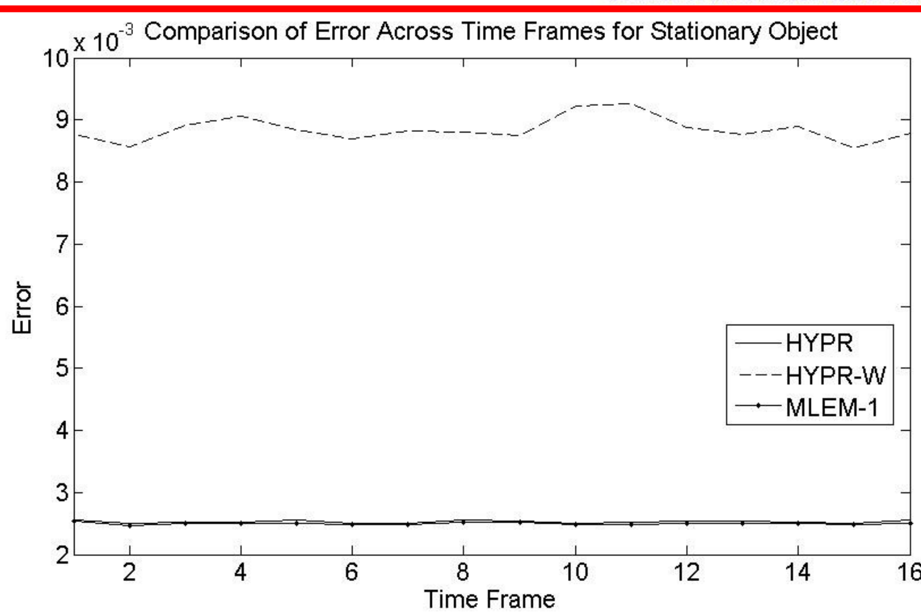


MLEM-1 and HYPR are the same method with different implementation.

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# MATLAB Computational Workbench

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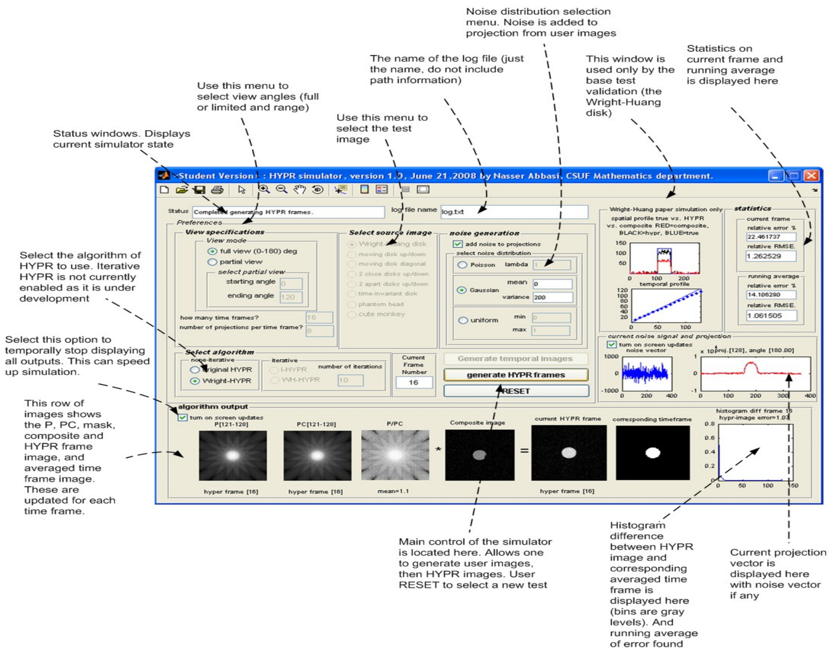






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# Time Dependent Intensity with Stationary Boundary

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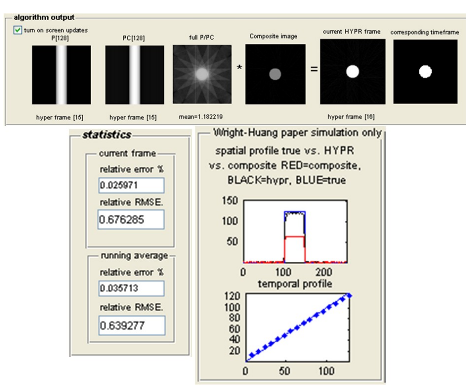
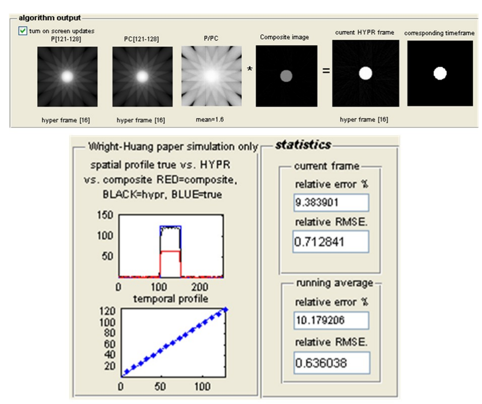


```
generated data for simulation [wrightPaperDiskTag]
Completed image generation...
number of time frames = [16]
number of projections per time frame = [8]
[22-Jun-2008 13:56:42]Enter generate HYPR ...

Enter original HYPR ...
No NOISE is being added
frame rmse     relativeError    rmse_averaged    relativeError_averaged  averageMask
1     1.026815    0.124615    1.026815    0.124615    0.810999
2     0.789398    0.080273    0.908107    0.102444    0.834959
3     0.658115    0.049115    0.824776    0.084668    0.856301
4     0.579282    0.057853    0.763403    0.077964    0.888973
5     0.551421    0.015053    0.721006    0.065382    0.905486
6     0.539332    0.016687    0.690727    0.057266    0.934073
7     0.544661    0.011765    0.669861    0.050766    0.957154
8     0.555759    0.021502    0.655598    0.047108    0.988724
9     0.565009    0.019138    0.645533    0.044000    1.005649
10    0.582615    0.021568    0.639241    0.041757    1.030899
11    0.600318    0.024314    0.635702    0.040171    1.053254
12    0.618896    0.016745    0.634302    0.038219    1.085530
13    0.629853    0.033764    0.633959    0.037876    1.101912
14    0.649074    0.025293    0.635039    0.036977    1.132088
15    0.661592    0.027750    0.636809    0.036362    1.153034
16    0.676285    0.025971    0.639277    0.035713    1.182219
Done,  totalHistError =1.33 ...
```

Running the Wright-Huang test case using the original HYPR algorithm



```
Enter Wright-Huang HYPR ...
No NOISE is being added
frame rmse       relErr      MeanRmse    MeanRelErr  MeanMask
1     0.982878   38.035191   0.982878    38.035191   0.396131
2     0.734453   26.583094   0.858665    32.309142   0.483218
3     0.604836   19.085338   0.774055    27.901208   0.557789
4     0.532656   14.054107   0.713705    24.439433   0.639344
5     0.520571   9.217883    0.675079    21.395123   0.715395
6     0.519005   5.920670    0.649066    18.816047   0.798848
7     0.535644   3.097390    0.632863    16.570525   0.877456
8     0.556542   0.955617    0.623323    14.618661   0.958114
9     0.574512   1.083784    0.617900    13.114786   1.035286
10    0.599367   2.818749    0.616046    12.085182   1.114582
11    0.621385   4.294862    0.616532    11.376971   1.193476
12    0.645831   5.560250    0.618973    10.892245   1.277558
13    0.658385   6.633323    0.622005    10.564635   1.353904
14    0.682260   7.683917    0.626309    10.358870   1.434595
15    0.695441   8.459228    0.630918    10.232227   1.508504
16    0.712841   9.383901    0.636038    10.179206   1.596524
Done, totalHistError =1.35 ...
```

Test 1b results

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## Time Dependent Boundary with Constant Intensity

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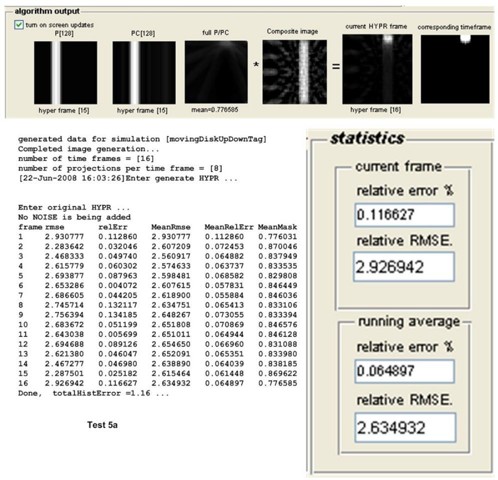


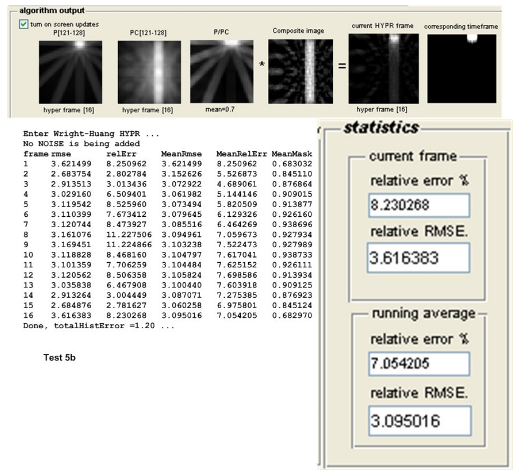




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## Two Vessels

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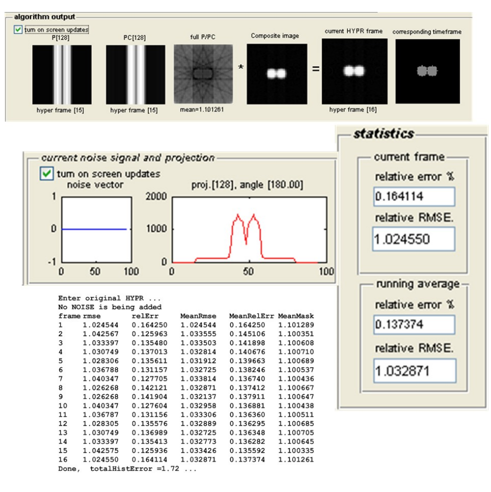


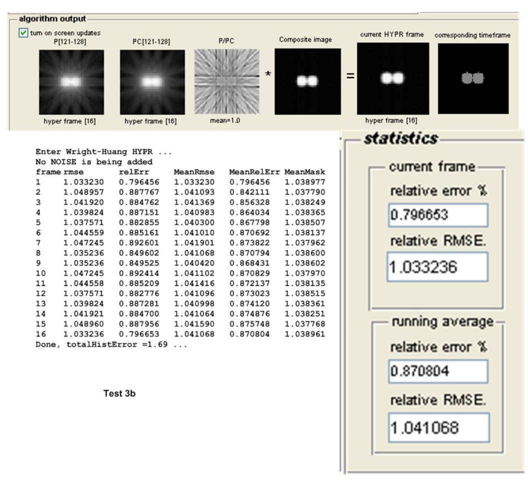

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## Two Moving Vessels

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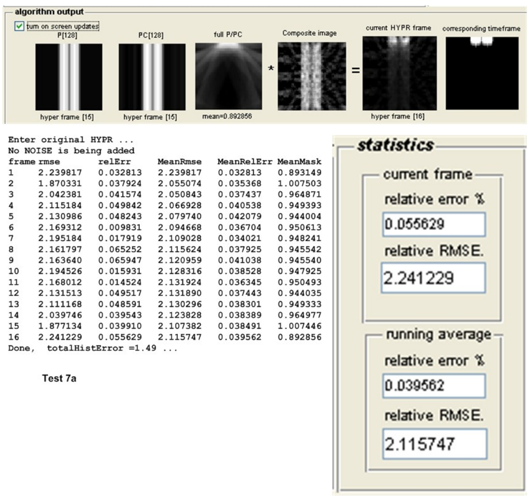


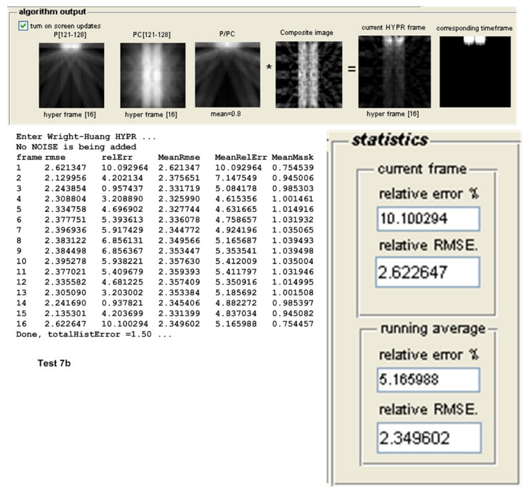






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## Preliminary Results

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- Identification of the mathematical structure of HYPR
- Verification of HYPR as the first step of MLEM
- Validation of HYPR Computational Workbench
- HYPR surprisingly robust to vessel motion
- Comparison with HYPR-W

## Our Intended Direction

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- Validation of Computational Tools.
- Characterize the noise amplification and resolution of the HYPR algorithm through simulations and analytical approximations.
- Test on clinically relevant objects (models of occluded arteries, vessels with different time uptake characteristics).
- Comparisons to alternative algorithms (I-HYPR, HYPR-LR, positively constrained least-squares, Level Set Methods).

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### 2.2.2 Level set report

## Level Set Method - Introduction

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Research Plan: Develop a level set method for image reconstruction that will be:

- useful when there is sparse angular data, and filtered backprojection reconstruction is inadequate.

- improves image quality by using the prior information that the object can be represented by a piecewise constant function with a few number of intensities.

The main idea of a level set method is to represent a closed curve as the level set of a function $\phi(x, y)$

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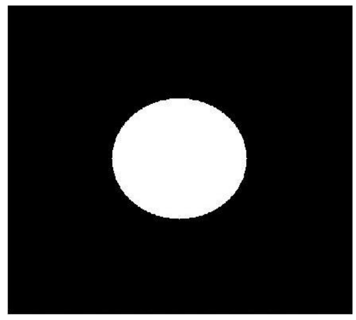
To begin our research, we will apply level set methods to simple images, such as the one on the right.



This image can be approximated by a piecewise constant function:

$$u(\varphi, c_1, c_2) = \sum_{j=1}^{2} c_j \psi_j = c_1 H(\varphi) + c_2 (1 - H(\varphi))$$

where $c_1$ represents the material in one region, $c_2$ represents the material in the other the region, $\phi$ is the level set function

and $H(z) = ¿\{1 \text{ if } z ³ 0 ¿¿¿¿$ the Heaviside step function, and

$$\psi_1 = H(\phi) \qquad \psi_2 = 1 - H(\phi)$$



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Level Set - The Problem
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Our approach is to minimize the functional:

$$F(\phi, c_1, c_2) = \frac{1}{2}\|\mathbf{P}u - g\|^2 + \beta \int_{\Omega} |\nabla H(\phi)| dxdy$$

Data agreement between the model prediction $\mathbf{P}u$ (**P** is the projection operator) and the data, *g*.

Regularization term, where β is a weight factor, Ω is a bounded region containing the object, and the integral represents the length of the boundary.

## Level Set - The Problem



F is a function of three variables. To minimize F, set the partial derivatives, with respect to each variable, equal to zero. This yields the three equations:

1.
$$\begin{pmatrix} \int_{\Omega_1} P * P y_1 \, dxdy & \int_{\Omega_1} P * P y_2 \, dxdy \\ \int_{\Omega_2} P * P y_1 \, dxdy & \int_{\Omega_2} P * P y_2 \, dxdy \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \int_{\Omega_1} P * g \, dxdy \\ \int_{\Omega_2} P * g \, dxdy \end{pmatrix}$$

2.
$$\frac{\partial F}{\partial \phi} = P * (Pu - g)\frac{\partial u}{\partial \phi} - \beta \nabla \cdot \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \delta(\phi) = 0 \quad \text{where,} \quad \frac{\partial u}{\partial \phi} = (c_1 - c_2)\,\delta(\phi)$$



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## Level Set - The Problem

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To solve for $\phi$, we introduce an artificial time variable $t$ and solve numerically the following partial differential equation:

$$\frac{\partial \varphi}{\partial t} = -\left[ P * (Pu - g)\frac{\partial u}{\partial \varphi} - \beta \nabla \cdot \left( \frac{\nabla \varphi}{|\nabla \varphi|} \right) \delta(\varphi) \right]$$

- This equation will be discretized with respect to the time and the spatial variables.

- Use will be made of the MATLAB Image Processing Toolbox and the Level Set Toolbox developed for use with MATLAB.

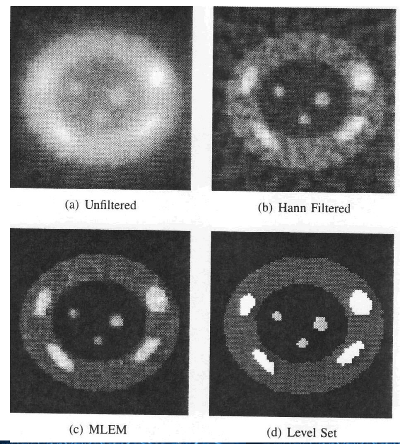
**1. "Level Set Reconstruction for Sparse Angularly Sampled Data" by S. Yoon, et al 2004.**

- Paper presents an iterative algorithm for a sparse set of projections of a time invariant object

- Assumes a piecewise constant function to represent the underlying image

- Incorporates a multiphase level set framework.

- Method provides better image contrast than the Hann filtered backprojection method, and the Maximum Likelihood Expectation Maximization (MLEM) algorithm.



(a) Unfiltered (b) Hann Filtered (c) MLEM (d) Level Set



Level Set - Literature Search

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**2. "Level Set Methods for Dynamic Tomography", by Y. Shi and W.C. Karl, 2003**

- Uses a variational method for the reconstruction of dynamic objects from noisy, sparse projection data.

- Simultaneously reconstructs multiple dynamic objects using this level set method for boundary representation.

**3. "3D Tomographic Reconstruction of Binary Images From Cone Beam Projections", by B. Jean-Pierre, P. Francoise, D. Jean-Marc, B. Michel, 2002**

- Shows that regularization through 3-D curvature can be introduced to manage lack of data and noise.

## Level Set - Status of Work

- Identified the functional $F(f, c_1, c_2) = \frac{1}{2}\|Pu - g\|^2 + \beta \int_\Omega |\tilde{N}H(f)| \, dxdy$ to be minimized.

- Used variational analysis to show that F will be minimized by solving the PDE:

$$\frac{\partial \varphi}{\partial t} = -\left[ P*(Pu-g)\frac{\partial u}{\partial \varphi} - \beta\kappa(\varphi)\delta(\varphi) \right] \quad \text{where} \quad k(f) = \tilde{N}\cdot\left(\frac{\tilde{N}f}{|\tilde{N}f|}\right)$$

- Numerically implemented the term $P^*(Pu-g)$ in MATLAB

- Working now to discretize the PDE, and in particular to effectively approximate the curvature term and the delta function.





## Level Set - Challenges

**Computational Challenges:**

- Properly estimating the non-trivial terms $\kappa(\phi)$ and $\delta(\phi)$

- Stability issues may arise in solving the PDE

- Finding values of $\beta$ that produce accurate images with out compromising contours

## Level Set - Evaluation of Results

Comparisons will be made with:

- HYPR
- Filtered Back-Projection
- I-HYPR



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## Level Set - Future Research

- Effects of noise
- Sparse data
- Nature of artifacts
- Relationships to other level set methods
- Test robustness for images that do not satisfy initial assumptions (such as the piecewise constant assumption)
- Test the level set method in cases that have clinical relevance

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References

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[1] Jean-Pierre, B., P. Francoise, D. Jean-Marc, and B. Michel. "3D Tomographic Reconstruction of Binary Images From Cone Beam Projections", Proceedings, IEEE International Symposium on Biomedical Imaging, 2002, 677-680

[2] Y. Shi and W. C. Karl, "Level Set Methods for Dynamic Tomography", Proceedings, IEEE International Symposium on Biomedical Imaging, Nano to Macro, Vol. 1, April 2004, 620-623.

[3] Yoon, S, et al, "Level Set Reconstruction for Sparse Angularly Sampled Data", Proceedings, IEEE Nuclear Science Symposium, Vol. 6, 2006, 3420-3423.

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## 2.3 Final group presenations

### 2.3.1 Final HYPR presentation

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Evaluation of Temporal and Spatial Characteristics of 2D HYPR Processing Using Simulations

By Y. Wu, O. Wieben, C. Mistretta, F Korosec

Summarized By Kacie Jacklin

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Department of Mathematics Department of Mathematics Department of Mathematics Department of Mathematics

## Purpose of Study



- Evaluate the temporal and spatial characteristics of images produces using the HYPR algorithm.
- Matlab was used to evaluate the properties of HYPR.
  - Bit-reversed ordering was used in obtaining the projections.



## HYPR Algorithm

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- Spatial information comes from a nearly fully sampled, high spatial resolution, high-quality reference image.
- Temporal information comes from a more sparsely samples temporal weighting image.
- Multiplication of temporal weighting images by spatial-reference composite images yields
  - high signal-to-noise ratio (SNR),
  - low artifact images,
  - good spatial and temporal resolution.

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# HYPR continued…

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$$HYPR(x,y,z) = \frac{1}{N_p} C(x,y,z) \sum \frac{P(r,\theta,\varphi)}{P_c(r,\theta,\varphi)}$$

To prevent the ratio from going to infinity as $P_c$ approaches zero, all values of $P_c$ between zero and a certain threshold, 5% of the maximum value of all the points along all profile $P_c$, are set to equal this threshold.

The equation to quantify the accuracy of the signal in a HYPR image:

$$D = \sqrt{\frac{\sum (HYPR(x,y,z) - INPUT(x,y,z))^2}{\sum INPUT(x,y,z)^2}}$$

To calculate the temporal accuracy, The cross-correlation between the temporal waveforms of the HYPR image and the waveforms of the input image. Cross-correlation is the covariance or the signal similarity between two intensities.



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# SNR

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- SNR - Signal to Noise Ratio
- Signal - measured as the mean intensity of all pixels within the object.
- Noise - measured as the standard deviation of intensities of all pixels within a large region of interest outside the object.
- SNR equals the ratio of then two quantities.
- The SNR of a HYPR image is dominated by the SNR of the composite image.

# Composite & Weighting Images



- Filtered backprojection to a large number of profiles yields a composite image that is free of artifacts and has a relatively high SNR.
- Weighting images provide temporal information into the time series of HYPR images. Interference between signals occur when the objects overlap in projections.
- When an image has sparse signal intensity, the weighting images using as few as 8 to 16 projections provide relatively accurate results.
- A sliding window approach results in more accurate intensities in the composite image.



# Figure 3

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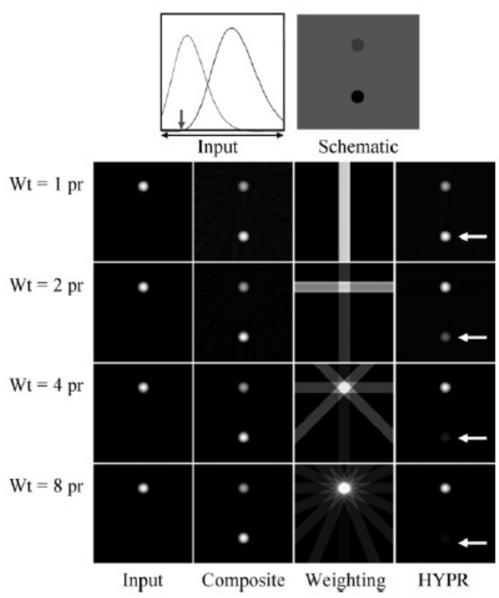


- The composite image is taken over the whole timeframe.
- The objects overlap in the projection.
- With one projection, the HYPR image shows both images.
- This is an early timeframe and should only show the top image.
- As the number of projections is increased, a more accurate depiction is achieved in the HYPR image.

# Figure 2 & 4

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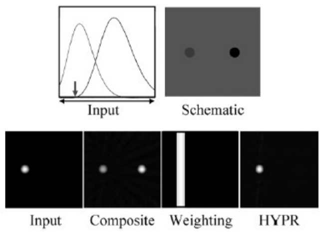
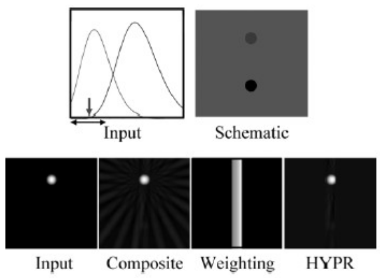



•When the objects don’t overlap in the projection, the reconstruction is more accurate.




•A sliding window approach for the composite image is used in this case.
•25 projections were used to create the composite image.
•We obtain better temporal accuracy.



# Figure 5

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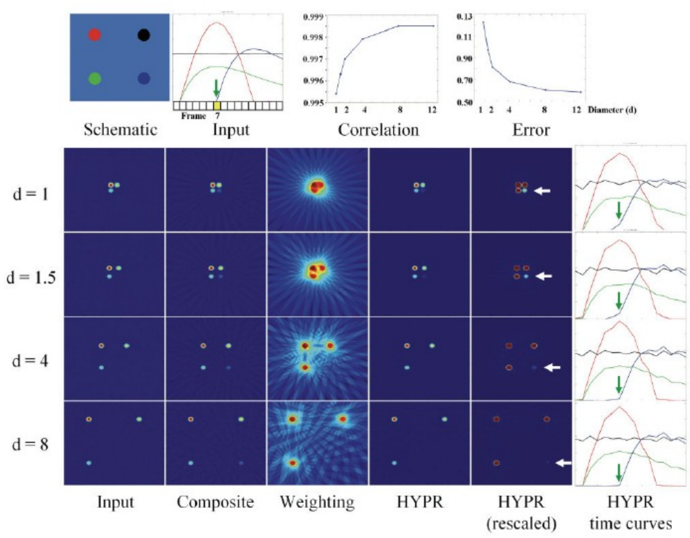
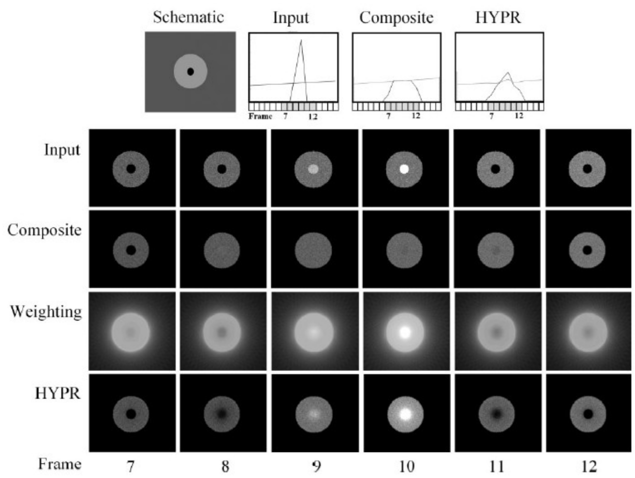



•4 objects with varying intensities.
•This causes the HYPR image to be less accurate.
•The arrow depicts timeframe 7.
•At this time, the blue object has zero intensity. The HYPR image shows otherwise.

# Figure 6





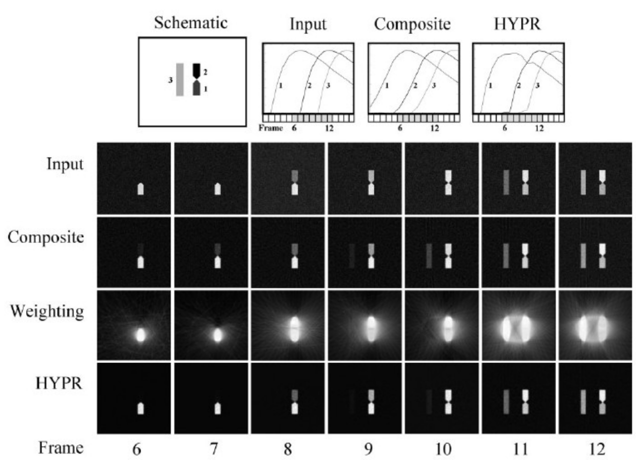

- •A circular object within an annular object.
- •The circular object’s intensity increases and decreases rapidly.
- •The annular object’s intensity increases at a slow steady rate.
- •The objects overlap in every projection.
- •This degrades the HYPR image.
- •The composite image is a lot worse than the HYPR image.





# Figure 7






- •This figure depicts vascular stenosis.
- •3 signal varying objects that are very close together.
- •Sliding window is used for the composite image with a width of 5 timeframes.
- •Noise was also added to the image.
- •The temporal waveforms for the HYPR image are distorted, as well as the waveforms of the composite.

## Conclusion

- Scenarios when HYPR can produce a less accurate image:
  - Objects are close to each other,
  - Signal intensities change dramatically,
  - Low temporal correlation,
  - A low number of projections is taken.
- Even when there are scenarios that are ill-suited for HYPR, it still performs relatively well.
- HYPR images demonstrate better temporal variations than the sliding window composite image.
- Composite window width can play a part in the quality of the HYPR image produced.



### 2.3.2 W-Hypr presenations



“Time Resolved MR Angiography With Limited Projections”

Yuexi Huang and Graham A. Wright

By Kacie Jacklin

## Key Points to HYPR

- Data Sparsity/Undersampling - Limited Projections
- Uniformity of Signal Dynamics - This assumption yields the property that the artifacts are proportional between the limited-projection image and the corresponding limited-projection image calculated from the composite and cancel each other out after normalization.
- Bit-reversed ordering of acquiring projections is used.
- Unfiltered backprojection can help limit the artifacts.





## Original HYPR

$$HYPRimage(x,y,z) = \frac{1}{N_{pr}} \times C(x,y,z) \times \sum \frac{P(r,\theta,\varphi)}{P_c(r,\theta,\varphi)}$$

$N_{pr}$ - Number of limited projections in the time frame

$C(x,y,z)$ - Time-averaged composite image

$P(r,\theta,\varphi)$ - Unfiltered backprojection of a certain raw projection

$P_c(r,\theta,\varphi)$ - Unfiltered backprojection of the corresponding projection from the composite image

*As the number of limited projections increases to equal the total number of projections,
the HYPR image is equivalent to the composite image.
*This equation leads to constraints in the denominator. If there are pixels with a value of
zero, or near zero, it can lead to artifacts in the HYPR image.

## Wright HYPR

$$HYPRimage(x,y,z) = C(x,y,z) \times \frac{\sum P(r,\theta,\varphi)}{\sum P_c(r,\theta,\varphi)}$$

$C(x,y,z)$ - Time - averaged composite image
$P(r,\theta,\varphi)$ - Unfiltered backprojection of a certain raw projection
$P_c(r,\theta,\varphi)$ - Unfiltered backprojection of the corresponding projection from the composite image

As the number of limited projections increases to equal the total number of projections,

$$\frac{\sum P(r,\theta,\varphi)}{\sum P_c(r,\theta,\varphi)} = 1$$

Then $HYPRimage = C(x,y,z)$

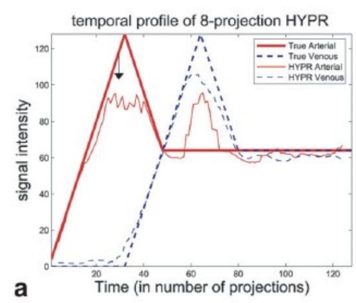
*In other words, as the number of limited projections increases to the number of projections of the composite image, the ratio of the sums is one and the HYPR image is equivalent to the composite image.

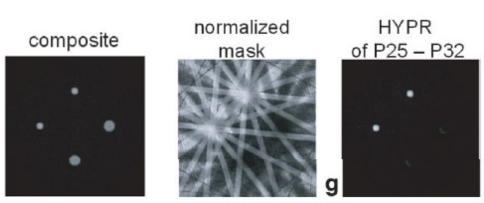


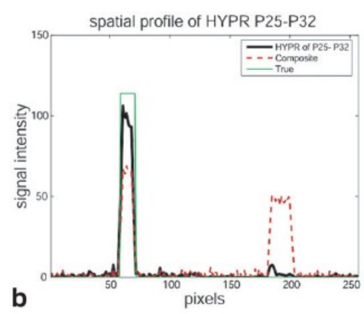


## Simulation

- 4 object computer model
- Two arteries and two veins a certain distance apart, veins larger than arteries
- Arterial signals increased earlier than venous signals
- HYPR picks up venous intensity early.
- HYPR detects venous intensity for the arterial image, this is called "cross-talk".



## Simulation

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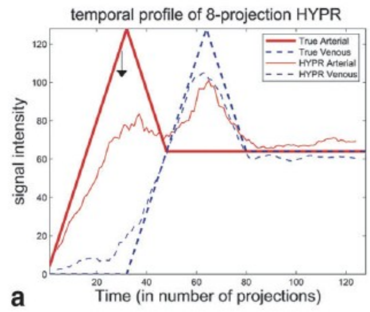
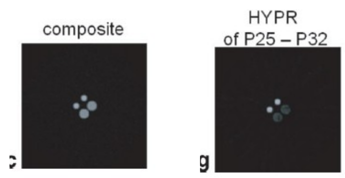
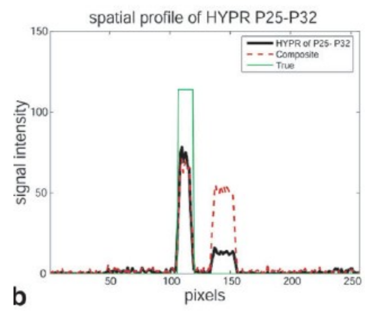
- Less sparse than previous simulation.
- Two arteries and two veins closer together than previous simulation, veins larger than arteries.
- Arterial signals increased earlier than venous signals
- HYPR picks up venous intensity early and has a lower arterial intensity.
- HYPR detects venous intensity for the arterial image.

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## Simulation

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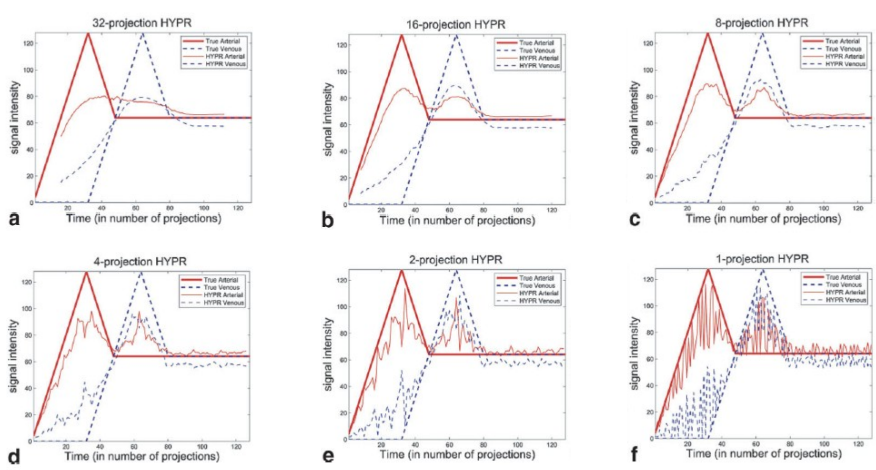
- Same dynamics as previous simulation.
- HYPR detects venous intensity for the arterial image.
- Sliding window reconstruction is applied for updating one projection for each HYPR reconstruction.
- As the number of projections is reduces, the greater the fluctuations in intensity.

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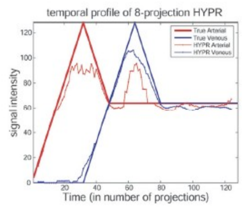
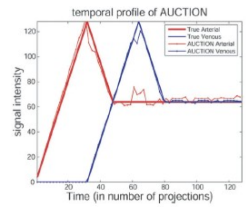
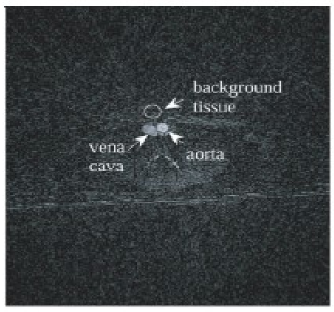
## Background Tissue Signals



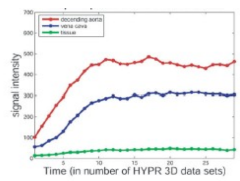
- Comparison of HYPR and AUCTION
- Arterial signal is less intense
- Background tissue is reconstructed more accurately than other signals.

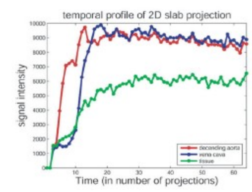


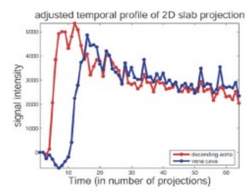




temporal profile of HYPR 3D data sets






## Signal-to-Noise Ratio



- Filtered backprojection applied to limited-projection images produces a SNR that is significantly lower than that of the composite image.
- Unfiltered backprojection produces a higher SNR than filtered backprojection.
- The SNR of a HYPR image is dominated by the low SNR of the limited projection image.

Ex) Assume we have a circular shaped object that we are projecting,
$SNR_c$ − SNR of the composite image
$N_v$ − diameter of the object in pixels=5
$N_{pix}$ −matrix size of the composite image in pixels=256
$N_p$ − number of projections per HYPR group=16

$$SNR = SNR_c \frac{N_v}{\sqrt{N_{pix}}} \sqrt{N_p} = SNR_c \frac{5}{\sqrt{256}} \sqrt{16} = SNR_c (1.25)$$

## Findings



- In Original HYPR, there is need to avoid the pixels that are zero (or near zero), these cause artifacts (spikes) in the HYPR image when the projections are normalized.
- In Wright HYPR, this is avoided since the denominator is the sum of a number of projections. The likelihood of zeros in the denominator is reduced.
- The number of acquisitions taken using bit-reversed ordering must be a power of 2.
- Large vessels cause signal interference to small vessels in HYPR, especially when the vessels are close to each other.
- Since this is a sparse data set, the interference of the nonuniform dynamics is relatively minor in terms of the overall image contrast.

## 2.4 Different team members writings

### 2.4.1 Current Status of HYPR Computational Investigation By Doug

# Current Status of HYPR Computational Investigation

The original HYPR, Wright HYPR, and MLEM reconstruction methods have been implemented in MATLAB, using the built-in radon and iradon functions for projection and backprojection. Three exploratory simulations are presented below. Going forward, the computational team will be further investigating these algorithms and working with the theoretical team to test the efficacy of any new algorithms that are developed. These tasks support the overall goals of understanding the mathematical justification of the HYPR method and, if possible, deriving a superior method that may be an enhancement and/or combination of current methods.

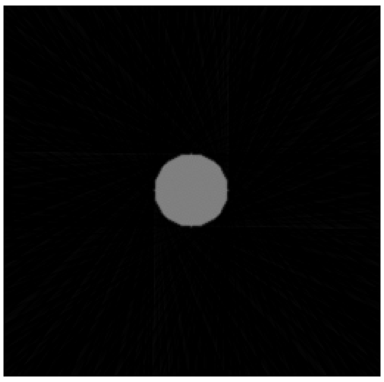
**FIRST SIMULATION**

The configuration of the first simulation was similar to that used in *Time-Resolved MR Angiography With Limited Projections*, by Huang and Wright. Specifically, a disk 50 pixels was centered in a 256x256 background. The intensity of the disk varied linearly from 0 to 127, resulting in 128 images. The size of the HYPR time frame was set to 8 projections, so that 16 time frames were used. The 128 projection angles varied linearly from 0 to 179, although the order of the angles was bit-reversed. The simulation was run using original and Wright HYPR, and the results of both were largely consistent with those in Huang and Wright. Small differences could be the result of inexact replication of their simulation or minor errors in our code. Any minor errors will likely be discovered once this (Stang) simulation is compared to the Abbasi simulation.

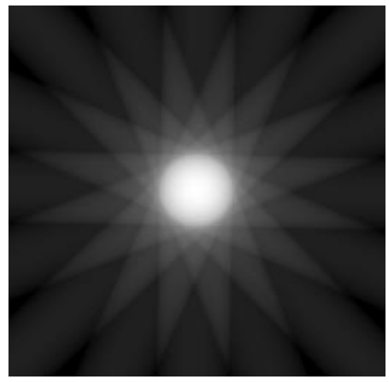
Error was assessed by computing the mean absolute difference of each reconstructed time frame (there is one reconstruction for each 8 projections) with the mean of the corresponding 8 actual images. By this measure, the errors for original HYPR were smaller than for the Wright method. Again, this result will be compared with the equivalent result from the Abbasi simulation.

The figures shown below are the composite image, the sum of the unfiltered backprojections of raw projections 121-128, the sum of the unfiltered backprojections of composite projections 121-128, the product of the ratio of these backprojections with the composite image, and finally the mean of the 8 actual images corresponding to this time frame, respectively. The Wright reconstruction can be seen to be close to the actual images.

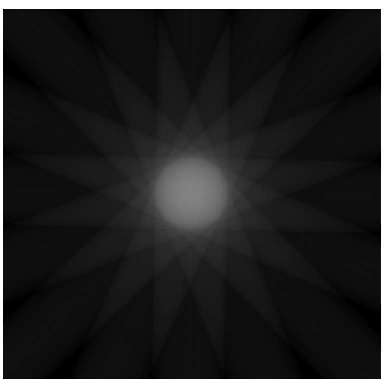
Composite Image



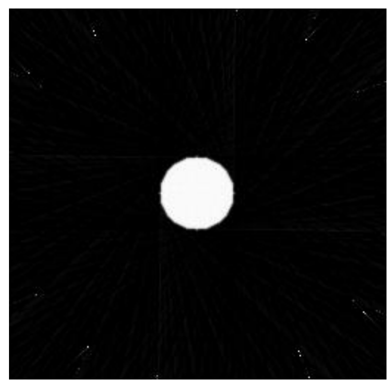
Unf. BP of Original Proj. 121-128



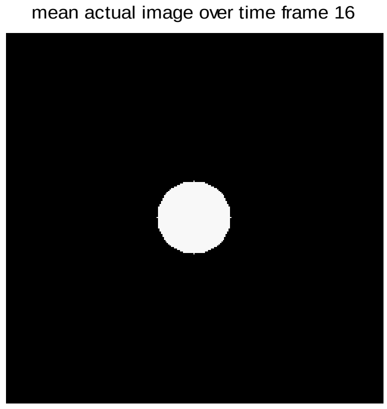
Unf. BP of Composite Proj. 121-128



Wright reconstruction for time frame 16 (proj. 121-128)



mean actual image over time frame 16

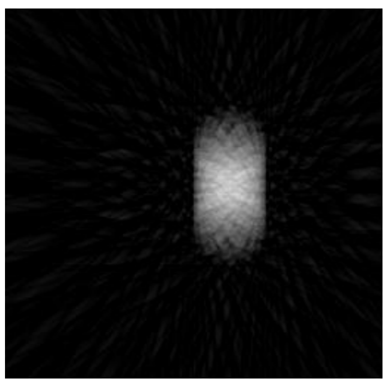


**SECOND SIMULATION**

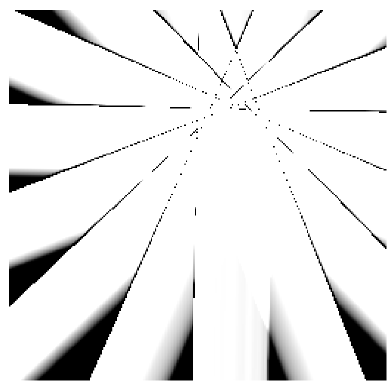
The second simulation examined the effect on the original HYPR method results when the disk moves over time, with a view to investigating the effect of blood flow on the reconstruction. Specifically, a disk with radius 25 pixels was centered at off-center coordinates (25,-25) in a 256x256 background. 128 different projection angles were used, again in bit-reversed order. Every eight projections, the disk moved 4 pixels.

The figures shown below are the composite image, the sum of the ratios of the unfiltered backprojections for time frame 16, the sum of the unfiltered backprojections of composite projections 121-128, the reconstructed image, and finally the actual image of at this time frame, respectively. The reconstruction and the composite image are clearly corrupted by the movement of the disk.

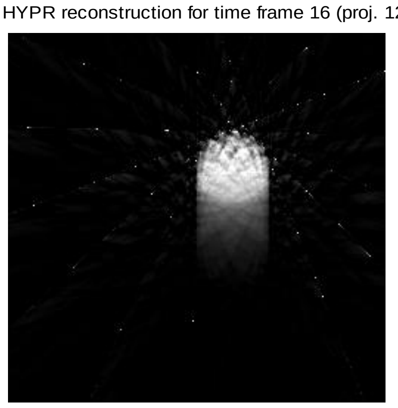
composite image



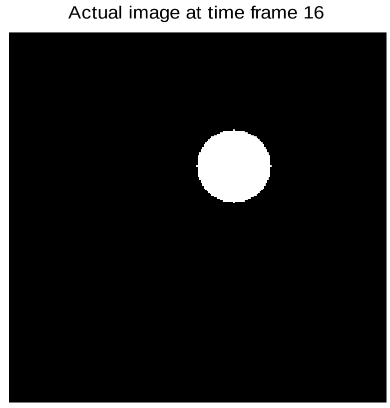
Sum of ratios of Unf. BP for time frame 16 (proj. 121-128)



Original HYPR reconstruction for time frame 16 (proj. 121-128)



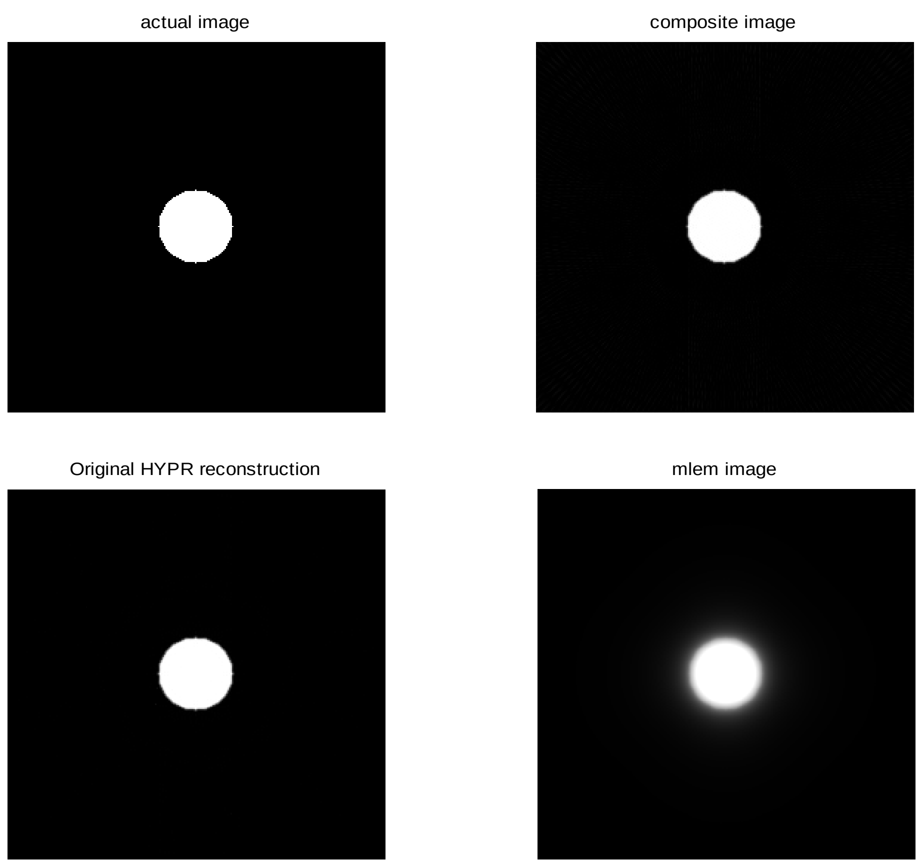
Actual image at time frame 16



## THIRD SIMULATION

The third simulation compared original HYPR and a 1-step MLEM. A disk with radius 25 pixels was centered in a 256x256 background. 128 different projection angles were used, and the size of the time frame was set to 128 projections for simplicity.

The figures shown below are the actual image, the composite image, the HYPR reconstruction, and finally the MLEM image. The HYPR image is clearly more accurate than the MLEM image. Investigation into this discrepancy will be ongoing. More MLEM iterations may be required, although even at 3 iterations, the mean absolute error is higher for MLEM than for HYPR.

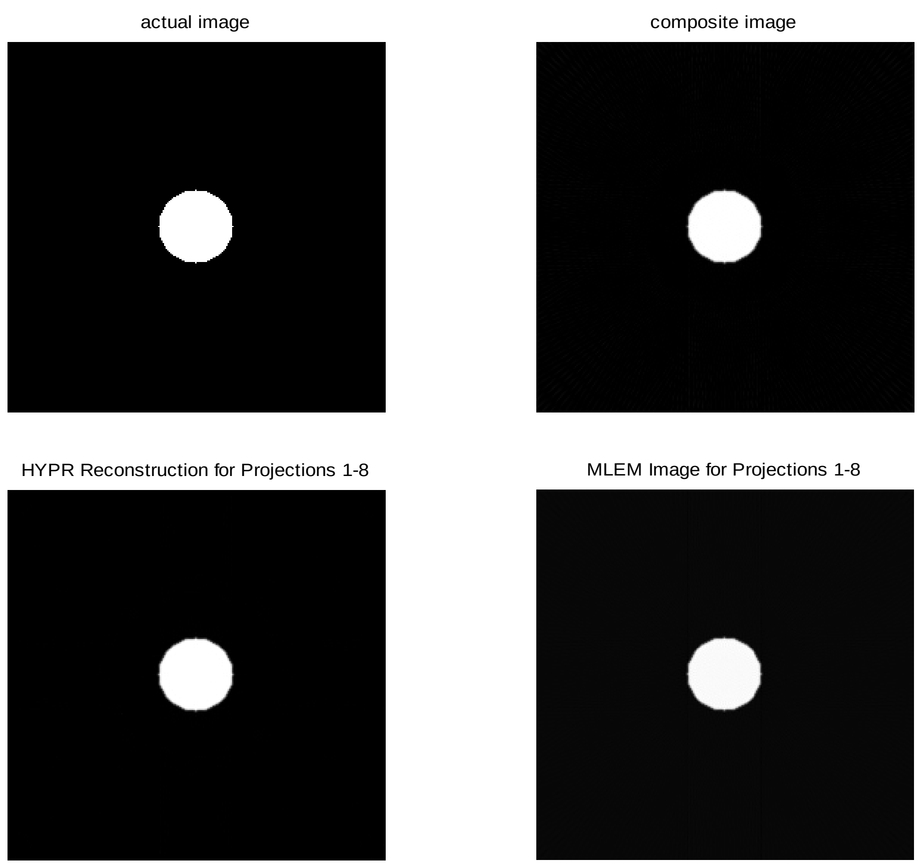


### 2.4.2 MLEM vs. HYPR by Doug

**MLEM vs. HYPR**

Original HYPR was compared a 1-step MLEM algorithm. A time-invariant disk with radius 25 pixels was centered in a 256x256 background. 128 different projection angles were used (ordered using bit-reversed ordering), and the size of the window was set to 8 projections.

The figures shown below are the actual image, the composite image, the HYPR reconstruction for the first HYPR frame, and the corresponding MLEM image. The HYPR and the MLEM images are indistinguishable, although the mean absolute error is slightly higher for HYPR than for MLEM. More detailed comparisons of MLEM and HYPR are planned.



### 2.4.3 MLEM vs HYPR By Theory group

The Mathematics that connects the MLEM algorithm to HYPR image reconstruction:

According to O'Halloran's paper entitled *Iterative Projection Reconstruction of Time-Resolved Images Using Highly-Constrained Back-Projection (HYPR),* the MLEM algorithm is mathematically equivalent to HYPR. MLEM stands for Maximum-Likelihood Expectation-Maximization. The MLEM algorithm can be used in image reconstruction for medical purposes. Positron Emission Tomography (PET) and Single-Photon Emission Computed Tomography (SPECT) are two types of image reconstruction processes where the MLEM algorithm is used. The purpose here is to show that the MLEM algorithm will work for HYPR reconstructions.

The MLEM algorithm is a process that approximates the solution to

$$g = H\theta$$

where we can look at H as a forward projection matrix, $\theta$ as the original image being projected, and g as the projection produced in order to link the two processes together. The goal is to tie this to the equation

$$s_t = R_{\phi t}[I_t]$$

from the HYPR process where $R_{\phi t}$ is the Radon transform over the sets of angles $\phi t$, $I_t$ is the image being projected, and $s_t$ is the sinogram produced from the projection.

We will look at the first iteration of the MLEM algorithm and see how it can be translated into the HYPR process of image reconstruction. The first step of MLEM is as follows:

$$\hat{\theta}_n^{(1)} = \hat{\theta}_n^{(0)} \frac{1}{z_n} \sum_{m=0}^{M} H_{mn} \frac{g_m}{(H\hat{\theta}^{(0)})_m} \tag{1}$$

This can be rewritten in matrix form.

$$\hat{\theta}_n^{(1)} = \hat{\theta}_n^{(0)} \frac{1}{z_n} \left[ H^{\mathrm{T}} \left[ \frac{g}{(H\hat{\theta}^{(0)})} \right] \right]_n \tag{2}$$

The portion of the equation

$$H^{\mathrm{T}} \left[ \frac{g}{H\hat{\theta}^{(0)}} \right]$$

can be looked as the vector that is produced from unfiltered back projection on the image produced by the ratio

$$\frac{g}{H\hat{\theta}^{(0)}} .$$

Here the division is done in an element-by-element fashion to produce the vector whose elements are the ratios of the respective elements of g and $H\hat{\theta}^{(0)}$. The difference here is that $H^T$ is applied to the ratio where in HYPR the back projection is done then the ratio is created.

In HYPR the equation we want to tie to equation (1) above is as follows

$$J = C \cdot R_{\phi}^{T} \left( \frac{s}{R_{\phi}(C)} \right) \tag{3}$$

where we have that C is the composite image, s is the vector of image space projections, and $R_{\phi}$ is the radon transform. $R_{\phi}^{T}$ is unfiltered back projection.

The only thing left to tie together is C and

$$\hat{\theta}_n^{(0)} \frac{1}{z_n} = \hat{\theta}_n^{(0)} \frac{1}{\left[ H^T [1] \right]_n}$$

### 2.4.4 Computational results By Theory group

# Computational Results

- Original HYPR, Wright HYPR implemented in MATLAB
- Simulations configured similarly to those in "Time-Resolved MR Angiography with Limited Projections" by Huang and Wright
- Results largely consistent with Huang and Wright
  - this statement applies only to Wright method, since Huang and Wright only simulate Wright method

# Computational Results

- Error measured using mean absolute error of reconstruction compared to mean of actual images over time frame
- By this measure, original HYPR appears to be more accurate than Wright method
- No noise considered yet
- Preliminary MLEM method implemented
  - Will be used to test various hypotheses, such as equality between HYPR and first iteration of MLEM
- Goals of computational tasks
  - Validation of theories about mathematical justification of HYPR
  - Exploration of any new algorithms formulated by team

### 2.4.5 GP597 report By Hassan

**Goals of the Project:**

***Use Applied Mathematics to optimize performance of Medical imaging system.**

***Mathematics of HYPER and related algorithms (Wright HYPER,1 HYPER),Study**

**Their relations to ML-EM algorithm and understand their resolution, noise amplification and artifacts.**

***Implement HYPER and related algorithms**

**(Wright HYPER,1 HYPER) and ML-EM using MATLAB, study their comparison.**

***Mathematics formulation and simulation of projection of a dynamic disk with radius r that moves in different configurations with respect to time.**

### 2.4.6 HYPER report By Hassan

**What's the goal of project?**

**Understanding the mathematics of Highly-Constrained Backprojection (HYPER) is part of the work of graduate students in the Applied Mathematics Project from GE Health care Technologies.**

**We are in a search to optimize performance of medical MRI imaging system through applied mathematics, by analyzing the original HYPER algorithm and related HYPER algorithms(Wright HYPER,1 HYPER), also we study the Expectation-Maximization (EM) algorithm which is an important tool for maximum likelihood (ML) estimation and theoretical formulation for estimating statistical properties of medical image reconstructed . Since the noise and its potential adverse effects on medical image quality, it requires a detail understanding of the statistical properties of the image. We use MATLAB program to run simulations of a simple circular dynamic models such as a disk with radius r that moves in different configurations with respect to time.**

**We form a projection of a disk which is represented by a two dimensional functions f(x,y) by combining a set of line integrals that's parameterized by ($\theta$,p) and satisfy the equation, x cos($\theta$)+y sin($\theta$)=p. The line integral g(t,$\theta$,p) which is known as the Radon transform of the function f(x,y) can be written as**

**$g(t,\theta,p)=\iint f(x,y)((x\cos(\theta)+y\sin(\theta)-p)\,dx\,dy$, Where time (t) is fixed.**

**Next, in order to reconstruct the image f(x,y) we use Filtered Backprojection and The Central-Slice Theorem.**

**Thank you for your time**

# 3 My project notebook

UP

**Project notebook for Nasser Abbasi**



## Monday June 5, 2008

### *Some notes on HYPR and related*

This section will contain useful notes I found related to this project

1. From paper "Multidimensional MRI of Cardiac Motion Acquisition, Reconstruction and Visualization" By Andreas Sigfridsson

   "HYPR: Projection imaging has gained much interest, because of the forgiving appearance when using large undersampling factors and thus rapid image acquisition. HighlY constrained backPRojection (HYPR) [28] has demonstrated an impressive reduction factor of 225 for time resolved imaging. Temporal averaging is used to reconstruct a composite image, which is then used to constrain backprojections of individual radial read-outs, depositing the projection data only in the objects being imaged. This requires, however, that the objects in the imaging volume do not change position over time. Thus, while it might be useful for contrast enhanced vessel angiography, it is not directly applicable for imaging of cardiac motion."

my comment: Note that HYPR is useful for object that do not move. I also read somewhere else, that within the object, the blood flow should be changing at fixed rate (do HYPR might not work for using on places where one part of flow is higher. We then just need to assume that these conditions are met, and we do not need to worry about what if they are not for this project.

2. The term "gridding" used in the Mistretta paper seems to mean as follows I saw on this link http://adsabs.harvard.edu/abs/2004JOSAA..21..499P O'Sullivan JD. A fast sinc function gridding algorithm for Fourier inversion in computer tomography. IEEE Trans Med Imaging 1985; MI-4(4):200{207.

   : "…by the use of gridding techniques that provide an efficient means to compute a uniformly sampled version of a function g from a nonuniformly sampled version of Fg, the Fourier transform of g, or vice versa…."

   I am not sure what nonuniform sampled version of the spectrum means, I am guessing it means those slices that are taken from the k-space projection (first row in Mistretta paper) are not taken at uniform angles and at some time more slices are sampled than at other times.

3. I really need to try to implement HYPR to understand how it works more. But need to find how to obtain the k-space projection data and how to read it to start the process. But first need to write the full algorithm. There is Matlab code to do HYPR simulation from the paper, see if we can get that.

## Tuesday June 3, 2008

6/4/08 made a more detailed diagram of HYPR algorithm, to review with group at class tonight.

## Thursday June 5, 2008

Made a visio diagram of HYPR hypr.png

## Friday June 6, 2008

Working on the backprojection formulation using matrix based. The algorithm for backprojection is I currently do it in the simulation uses radon/iradon. However, this is FourierTransform based (i.e. to do backprojection, iradon uses the central slice theorem). We need to do it using as in first assignment, using matrices and transpose and all that.

The problem is how to formulate this with many projection to construct the composit image. I think it should be simply SUM over I of A'*g(i) as in my note above. Instructors said to stage the g(i) vectors (the projections) to make one large vector and then use A' on that. But the dimensions do not work out. Even if I make A to back a bunch of A's stacked next to each other, I get the same as if I did a SUM. So I am not sure why they said that. Need to sort this out.

Reading the PPT file that Dr Pineda send to us today to see if it will help me.
Spend more time reading the Kak book. Very useful stuff.

## Saturday June 7, 2008

Cleaning up my notes on derivation of HYPR.

## Monday June 9, 2008

Updated my notes on HYPR. PDF HTML
Few things needs to understand:
What does this mean? "angular undersampling factors of 100 may be possible" from the main HYPR paper (A5). I am still now sure I understand how HYPR allows undersampling? Need to think more about this.
Why does appendix A talks about single projection then uses a sum over all projections? (part about SNR)

## Tuesday June 10, 2008

Reading the Wright-HYPR paper.
Questions on it:

1. It says the the composite image C is "time average". Does this mean when making the composite image we need to average the resulting of the filtered backprojections?
2. What does this mean? "Since the profiles of the projection lines are normalized (divided) before they are summed, this is a nonlinear process."

From Wright-HYPR
"Unfiltered backprojection has a significantly higher SNR than filtered backprojection due to the over-weighting of the low frequency data (data at the center of the *k*-space)."

Some definitions from
"Projection lines: Projection lines are thin continuous parallel lines that project out from a drawing to help describe a component. They are drawn two at a time with a dimension and a dimension line between them."

Wrote matlab function to generate disk image of different sizes and centered and simulate for different loci see my main project page for table

## Wed June 11, 2008

Worked on my HYPR report, read Wright paper and I-HYPR

## Thursday June 12, 2008

Corrected my HYPR report. There was a mistake in the GE PPT.

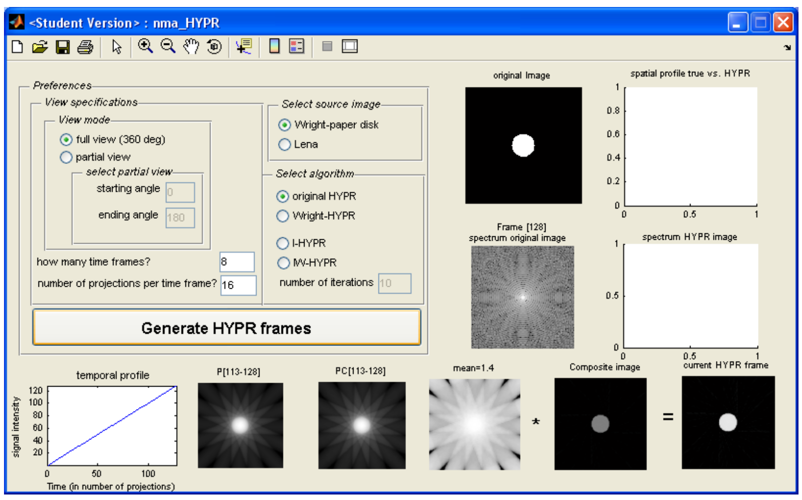
Added algorithm psudo-code as well.

## Friday June 13, 2008

Staring work on HYPR implementation

## Saturday June 14, 2008

Working on HYPR implementation. I can now reproduce the plots in Wright Huang Paper using disk. I think I found an error in the paper. It is 16 projections per frame, not 8. Send email to the author Dr Huang.
This is how the UI look like now




Still need to implement W-HYPR and I-HYPR and make it more robust.

## Sunday June 15, 2008

W.H. paper is correct, it is 8 projections, but they count projection differently from what we do. So their 8 projections is what I call 16 projections. So all is ok.

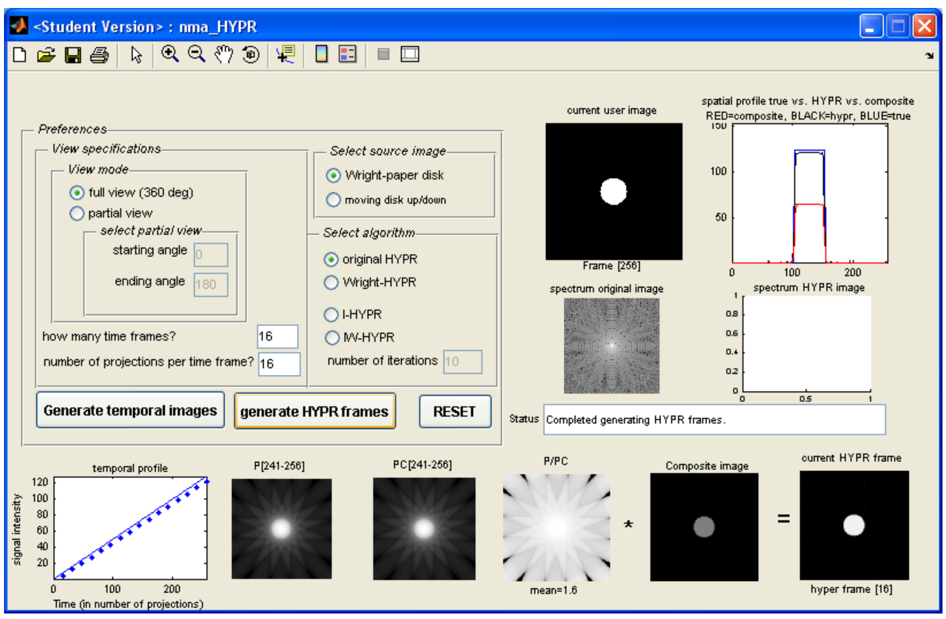
9AM: Things needs to do for today

1. if spatial images already created for current image, do not redo it. This needs for me to use UserData and keep track of this.
2. generate the profile for intensity
3. check for no power of 2 number of projections, and if so, do not use bitordering since that works only for power of 2.
4. Try to do the HYPR for moving disk as well.
5. add plot/result of error between current HYPR frame and average of real frames used to generate the HYPR frame.

6. Updated my document on HYPR projections and clarified it.
7. 11:15 pm: Need to add error, and 3D view of spectrum. Finished HYPR and WH-HYPR. Tommorrow I can do I-HYPR

Now it is all complete for HYPR, I get the same results for all the plots of the paper. Here is what the UI looks like now.

Next, I need to implement WH hyper and I-Hyper. Should be easy to do. Next, add 2 small objects (disks) next to each other and see the effect of small objects, and compare to 2 disks further apart. HYPR should do better with objects with more space between them I think




Some observations:
To obtain a good HYPR frame reconstruction, projections per frame must be taken at angles that are uniformly distributed around 360. If one takes a time frame projections at angles such as 1,2,3,4…,20 degress say, then HYPR frames reconstruction will not resemble the original images well. Hence use bitordering, and for this user must supply a power of 2 total number of projections.

## Monday June 16, 2008

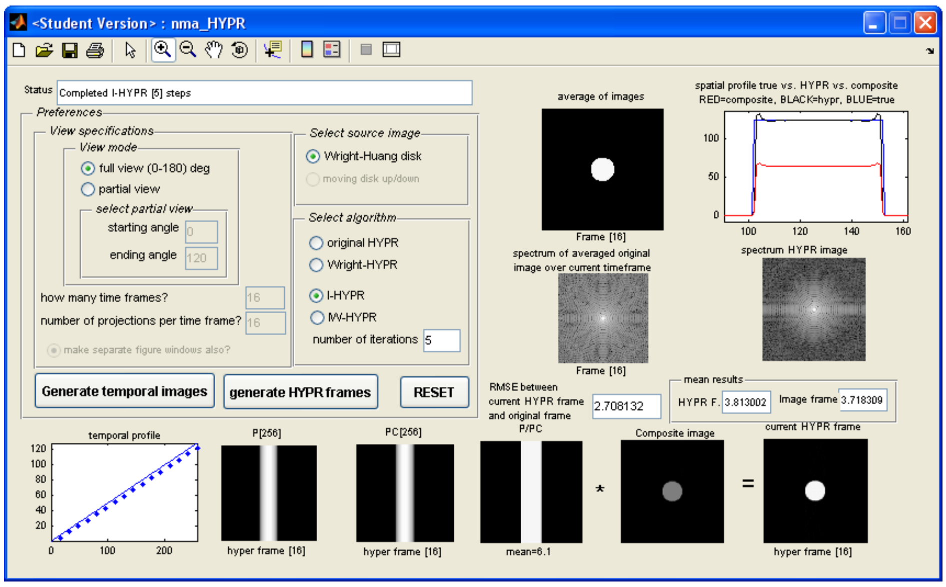
Adding more stats
Original, RMSE 2.937
WH hypr RMSE 3.064

HYPR frame in W-HYPR is more than image frame (averaged) than in the case with original HYPR. So W-HYPR for somereason generates HYPR frames with more intensity?

I-HYPR is working. RMSE after 2 steps went down to 2.754

3 PM. Moving sotware to laptop. Completed initial report with results. See my updated HYPR report.

Current UI




## Tuesday June 17, 2008

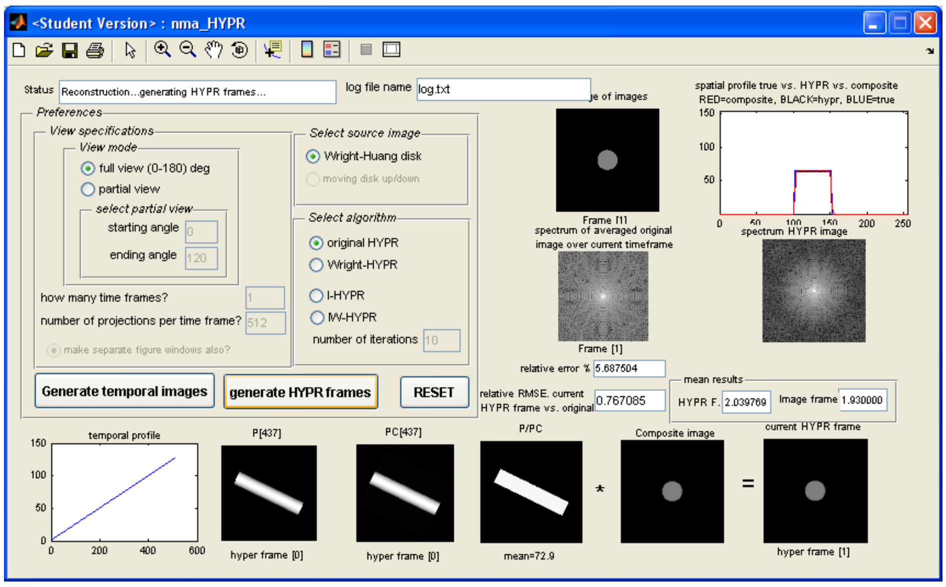
From Wright-Huang paper

"A uniform acquisition order, such as the bit-reversed order, is required to reduce imaging artifacts."

"Spatial resolution, temporal resolution, signal-to-noise ratio (SNR), field of view (FOV), and the extent of artifacts are common tradeoffs in magnetic resonance imaging (MRI)."

Send email to Dr Huang. With this one simple test (disk, change intensity) original HYPR gives less relative error and less RMSE. Should I be trying different configurations?

Current UI. Added log file, more statistics




Send email to Dr Huang with result of simulation in the hope to get his input on why WH-HYPR produces larger relative error in the HYPR image with the above simple simulation. May be the disk simulation does not reflect or show the main strength of WH-HYPR ?

Here is the PDF file with results of a test described in the pdf file.

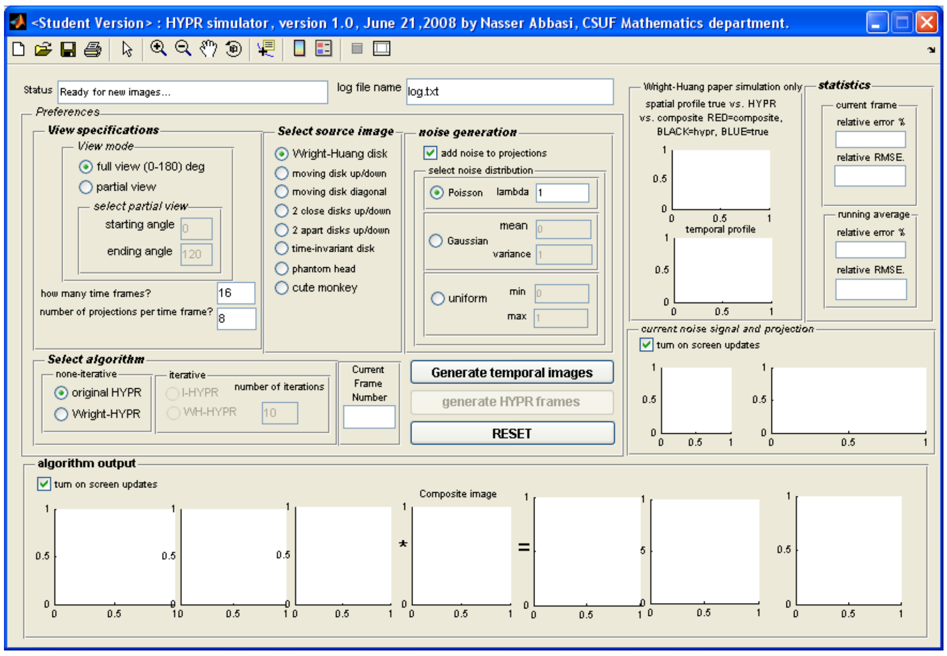
## Wednesday to Saturday June 21, 2008

Been working on running experiments on HYPR and developing a HYPR simulator to help me with this.

## Thursday to Monday June 24, 2008

Completed version 1.0 of the HYPR simulator and also completed the midterm report. Helped with editing for the PPT slides.

This is how the UI looks now




## Tuesday to Thursday 6/26/08

Made version 1.1 of the HYPR simulator. See HERE for web page and more information.

Class.

At time, worked for few more hours to add support to dynamics phantom clip and another image from Dr Pineda he send.

## Friday 6/27/08

Read a little from the book the mathematics of medical imaging on radon tansform and filter theory (which is really nothing but linear system stuff studied in my mechanical eng.).

Working on splitting the 2 windows. One is a configuration only UI (where preferences are entered) and a separate window for all the plots. This allow more real estate for displaying the images and it also allows me to improve the preferences entry and add more options as I am running out of space already.

## Saturday 6/28/08

Work on simulator. Looking at adding noise

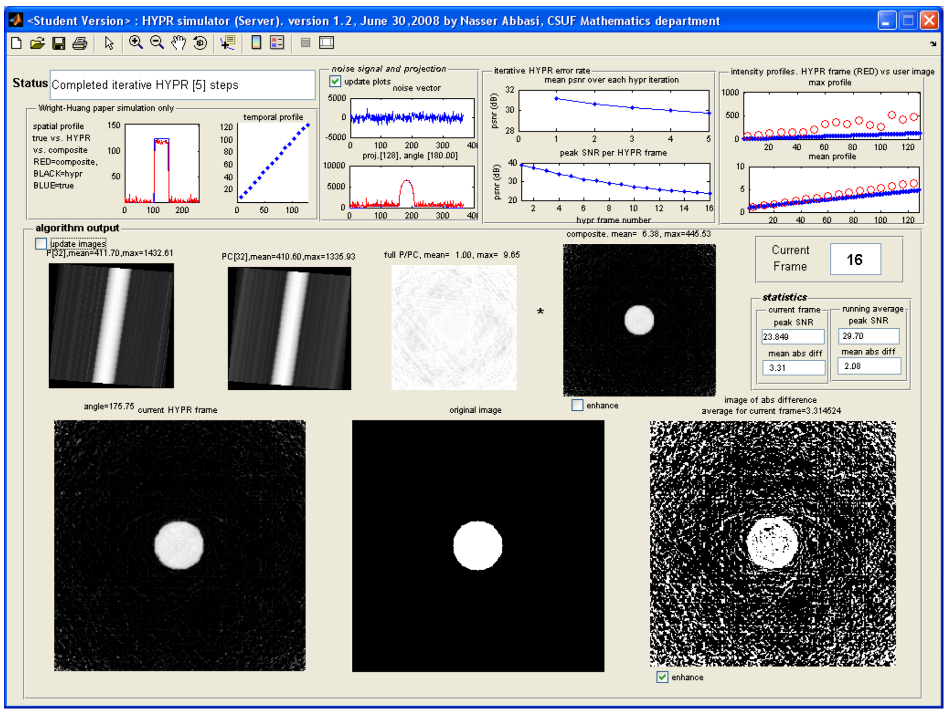
## Sunday 6/29/08

Work on HYPR, read papers

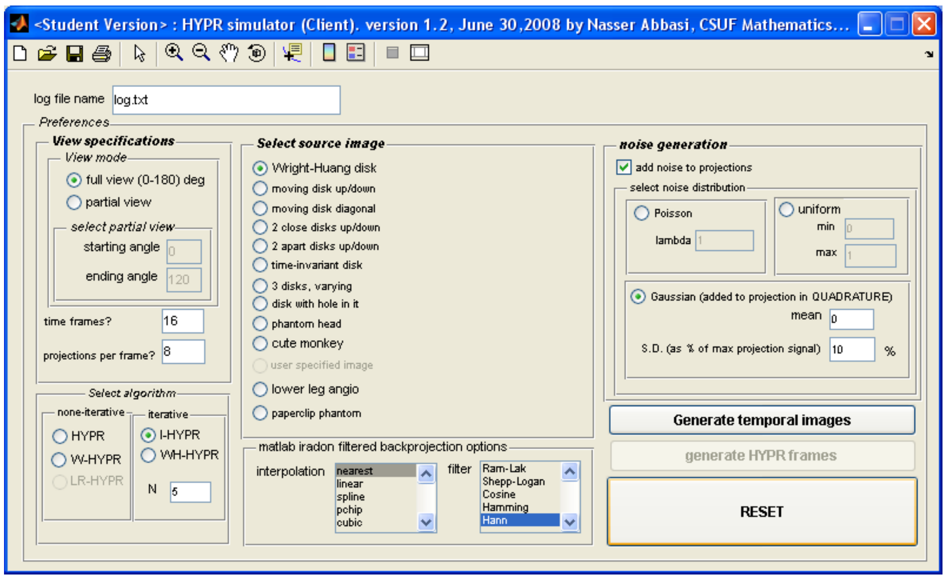
## Monday 6/30/08

Work on HYPR, class

Server UI




Client UI




## Tuesday to Thursday 7/03/08

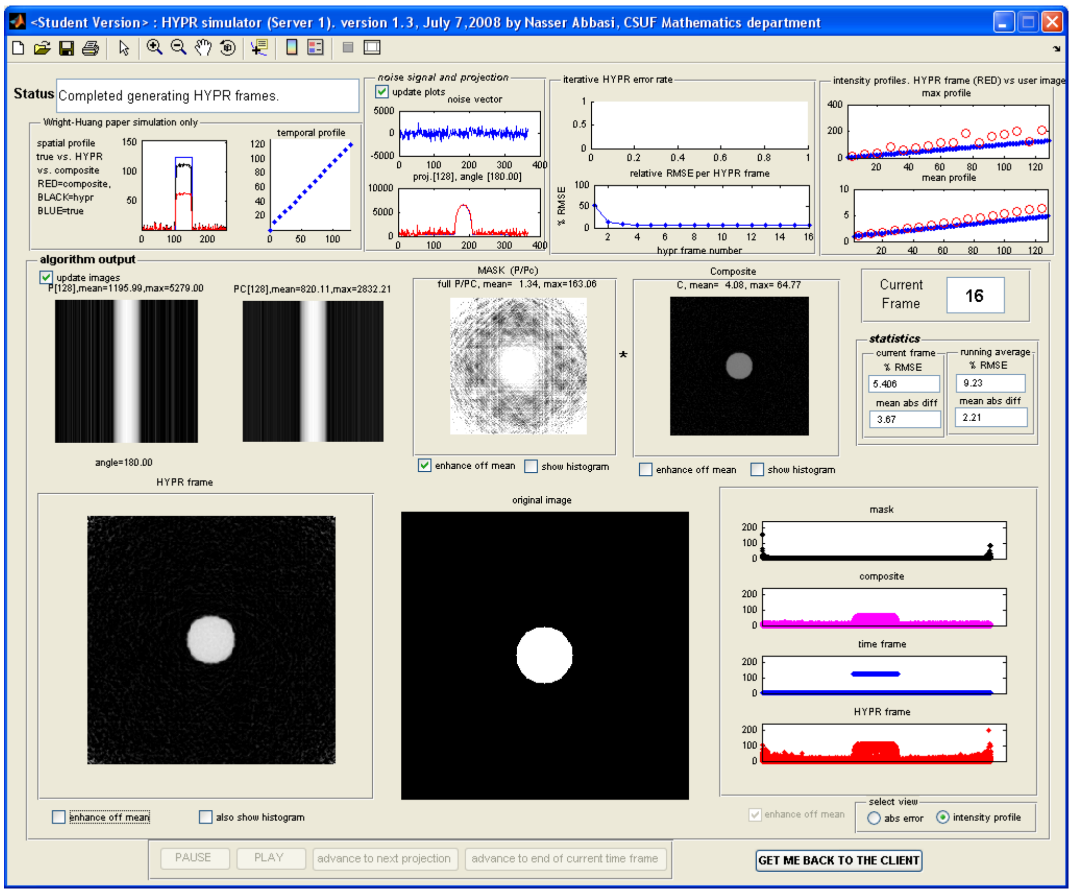
Worked on “initial findings and animation” report.
Updated HYPR simulator to 1.2.1 (fixed 2 small boundary conditions problems and changed RMSE to become normalized).
Read papers, learn about SNR, Contrast, and CNR.
Update my HYPR report, concentrate on I-HYPR for class talk

## Friday, Saturday and Sunday 7/6/08

Working on adding more analysis features to simulator.
Need to know the following on HYPR

How is a time frame determined? i.e. what are the basic of it? It must have something to do when acquisition occurs.

5:00 AM Sunday. Ok, go to sleep. All what is left now is to clean up stuff, and synch things up. Should be done by Thursday. Here is the UI now. Added intensity profile plot also.




## Monday 7/7/08 to Thursday 7/17/08

Went to SIAM on Monday 7/708. Then spend all the next week working on v 1.3 of simulator. Many things added. Plane to finish it by next Monday so we can start using it to analyze the algorithms in detail and write the final report.
Wrote a small report on matlab iradon and why the all-at-once does not give the same result as the one-at-time method.

## Friday 7/18/08 to Monday 7/28/08

Completed HYPR simulator. Final version is 1.4.1
Made report on HYPR-LR and reviewed finding in class
Working now on final report (4 pages) and power points (4 slides) for summary of work done.

## Tuesday 7/29/08 to Friday 8/1/08

Worked on power points and report. Did review in class and handed out my reports.
Applied changes to power point slides and emailed updated copy.
Working on documentation for HYPR.

# 4 My report on effect of using sliding window for HYPR

## Findings Related to the Effect of Using Sliding Window Composite of varying sizes on the Accuracy of Original HYPR, Wright-Huang HYPR and HYPR-LR Using HYPR Simulator Software applied to GE phantom clip and to Crosstalk test case

by Nasser M. Abbasi
August 6, 2008

Notice: This whole report with all supporting documentations and images are contained in this one ZIP file (8 MB)

## Introduction

This report contains results obtained using simulation to compare the accuracy of HYPR image reconstruction using the original HYPR, Wright-Huang HYPR and HYPR-LR algorithms applied to two different input data: The first using the GE phantom clip (images in this clip exhibit large spatial and temporal dynamic), and the second input data using a test case which exhibits cross talk problem (2 objects close to each others with different temporal dynamics). This second case was obtained from the I-HYPR paper[(4)] and shown under figure 4 in that paper. This paper if available to download from my project web page in the Papers table under item #2.

In this simulation (version 1.5 of HYPR simulator was used, which now supports composite sliding window) we used a sliding window composite algorithm to generate a new composite image when a new HYPR image is being reconstructed.

The sliding window algorithm for generating the composite image is a known method which attempts to improve the result of the final HYPR images by reducing cross talk effects, but can increase streak artifacts. See LR-HYPR paper[(1)] for more discussion on this topic. This paper can be downloaded from the above mentioned table as well at item #9.

We have modified the original HYPR[(2)], Wright-HYPR[(3)] and HYPR-LR[(1)] algorithms to be able to support a sliding window composite in the HYPR simulation software.

In this small study, our goal was to determine how each algorithm's accuracy changes with window size.

We used windows of varying sizes and in each case, we ran simulation using noise and without noise. We also run the algorithm without the use of sliding window. Two different tests were done.

## Simulation results

### *First test case: GE phantom clip*

In this test, we used as input to HYPR algorithm the GE phantom clip which exhibits large spatial and temporal dynamics.

This set of data we broken into 8 time frames with 8 projections per time frame. Then we ran the modified O-HYPR and W-HYPR which now supports sliding window and compared the accuracy as the window size is changed. This is the result.

**GE Phantom Clip, No Noise case**

| Algorithm | Window Size | | | No sliding window |
|---|---|---|---|---|
| | 3 | 5 | 7 | |
| O-HYPR | 8.32 | 6.76 | 6.65 | 6.83 |
| W-HYPR | 8.45 | 6.69 | 6.54 | 6.77 |
| LR-HYPR[1] | 18.49 | 9.79 | 7.34 | 6.70 |

**GE Phantom Clip, noise is zero mean and 5% S.D.**

| Algorithm | Window Size | | | No sliding window |
|---|---|---|---|---|
| | 3 | 5 | 7 | |
| O-HYPR | 12.44 | 12.20 | 11.08 | 10.73 |
| W-HYPR | 10.86 | 9.65 | 9.55 | 9.60 |
| LR-HYPR[1] | 20.67 | 17.23 | 14.50 | 13.87 |

**RMSE results (Lower values means more accurate reconstruction)**
**Data contains total of 8 time frames**

(1) LR-HYPR was run using circular filter with size 20

### Observations on the above test results

We first notice that W-HYPR had the best results with and without noise. We also observe that the most accurate results was obtained using the sliding window method by limiting the composite size to smaller size than the case would be without the use of sliding window. W-HYPR with sliding window of 7 was more accurate than when using all the available time frames.

### *Second test case: Cross talk*

In this test case, we used the test case as described in the I-HYPR paper[(4)] under figure 4.

**Fig-4 I-HYPR paper, noise is zero mean and 5% S.D.**

| Algorithm | Window Size | | | | | | | No sliding window |
|---|---|---|---|---|---|---|---|---|
| | 3 | 5 | 7 | 9 | 11 | 13 | 15 | |
| O-HYPR | 57.76 | 204.54 | 40.76 | 56.77 | 44.33 | 37.74 | 39.78 | 33.45 |
| W-HYPR | 31.22 | 24.79 | 22.02 | 22.01 | 20.80 | 21.07 | 21.87 | 22.06 |
| LR-HYPR[1] | 124.39 | 93.89 | 87.17 | 84 | 85.91 | 86.11 | 84.12 | 75.65 |

**Fig-4 I-HYPR paper, No Noise added**

| Algorithm | Window Size | | | | | | | No sliding window |
|---|---|---|---|---|---|---|---|---|
| | 3 | 5 | 7 | 9 | 11 | 13 | 15 | |
| O-HYPR | 33.95 | 13.25 | 9.88 | 8.66 | 7.22 | 5.97 | 5.79 | 5.85 |
| W-HYPR | 21.88 | 12.40 | 10.22 | 9.16 | 7.75 | 6.54 | 6.54 | 6.63 |
| LR-HYPR[1] | 86.06 | 18.46 | 10.67 | 8.52 | 6.97 | 5.96 | 5.58 | 5.59 |

**RMSE results (Lower values means more accurate reconstruction)**
**Data contains total of 16 time frames, 8 projections per frame**

**(1) LR-HYPR was run using circular filter with size 20**

**Observations on the above test results**

In this test case, we wanted to determine the effect of sliding window on cross talk. There were 16 time frames with 8 projections per time frame.

When noise was present, W-HYPR was the most accurate. The accuracy of W-HYPR was improved more with the use of sliding window where we see that the most accurate result was obtained with window of size 11.

With no noise present, LR-HYPR was the most accurate. The use of sliding window with LR-HYPR did not result in improvement of accuracy compared to the case when no sliding window was used (5.58 with window of size 15 vs. 5.59 with no sliding window). By the nature of LR-HYPR, it works best with objects that are close to each others and exhibit large temporal dynamics.

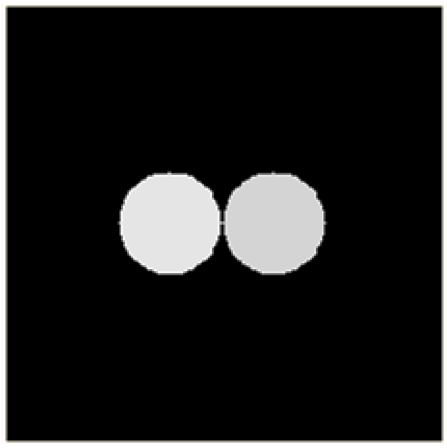
## Conclusions

1. The use of sliding window with Original HYPR and Wright-Huang HYPR results in more accurate HYPR reconstruction.
2. In both test cases, O-HYPR and W-HYPR did better with sliding window than without sliding window. However, the size of the sliding composite window is difficult to predict. Doing some earlier simulations on typical images that are expected to be acquired could help in determining the size.
3. With smaller sliding composite window, cross talk was reduced; however, in place of it streak artifacts showed up (see images below in appendix). LR-HYPR had the least amount of streaks show up at small window sizes.
4. It is recommended that O-HYPR and W-HYPR be implemented with sliding window algorithm, however, since the wrong size of the sliding window could result in worst reconstruction, the determination of the correct size for each different conditions can be difficult to predict. More research is required to study the affect of sliding window composite on accuracy of reconstruction as it can depend on the nature of the images being reconstructed.
5. The more parameters are available to adjust (we have now introduced a new parameter which is the sliding window size), the more combinations that are available to adjust and this can make it more difficult to determine the optimal set of parameters. However, the advantage comes from when we are able to determine the most optimal set of parameters for a given input, as this can result in a more accurate HYPE reconstruction as was demonstrated above.

## Appendix

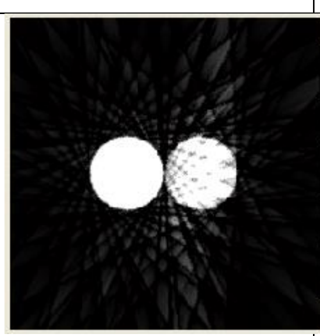
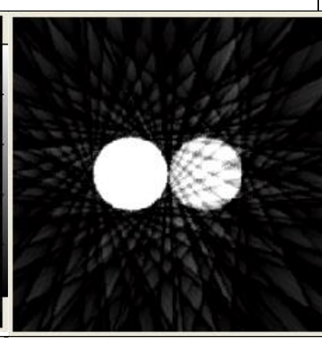
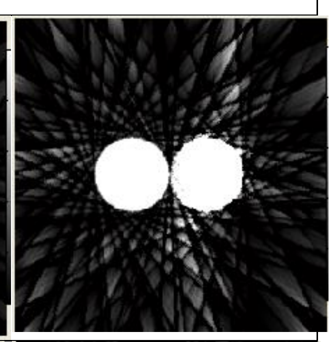
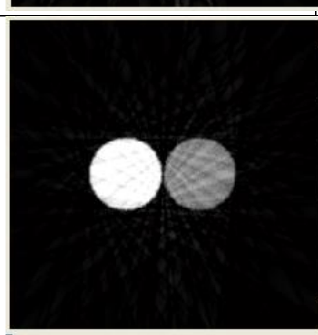
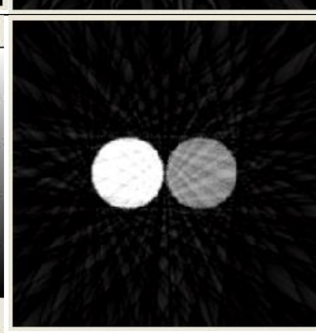
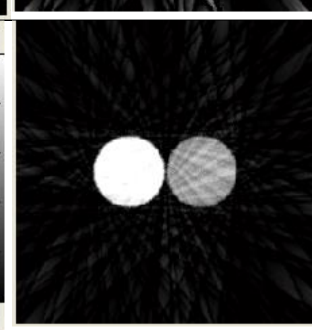
This appendix contains a detailed look at how the different window size affected the cross talk problem. We show the HYPR image reconstructed at the end of time frame 4 for sliding windows of size 3, 5,7,9,11,13, and 15. We do this for O-HYPR, W-HYPR and LR-HYPR. And compare each to the original image at the same time frame.

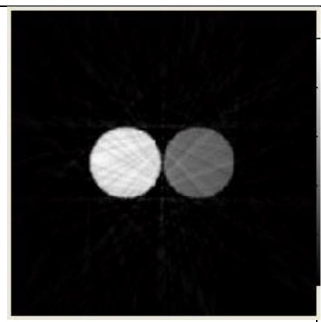
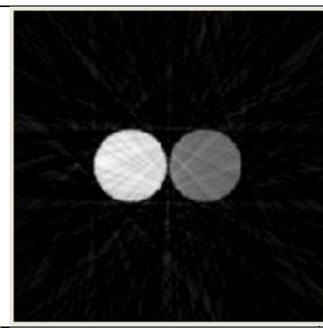
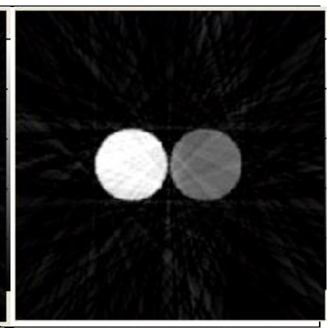
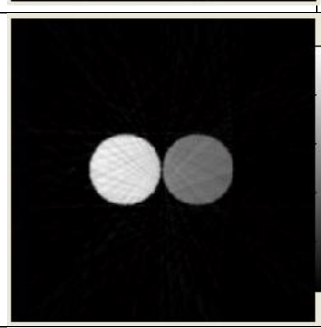
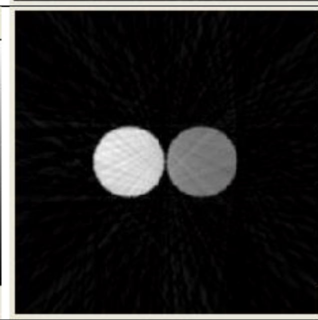
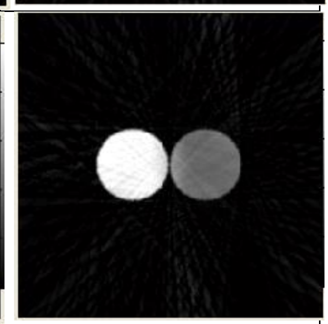
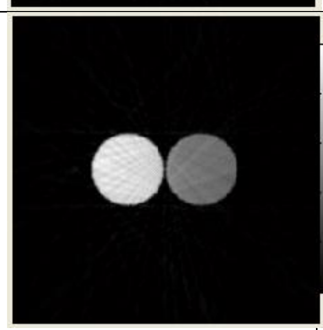
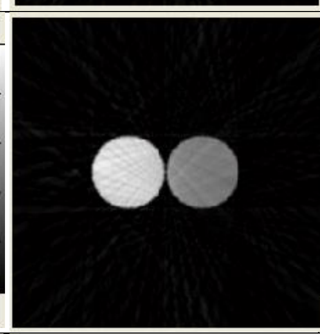
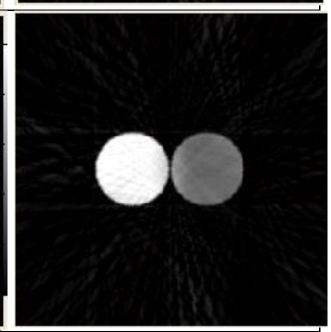
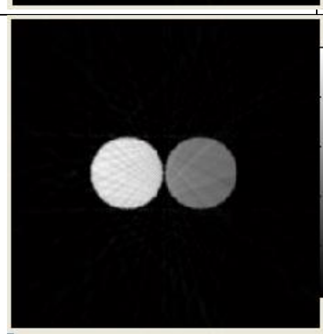
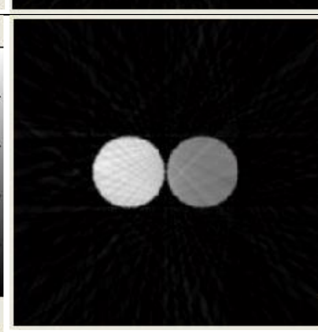
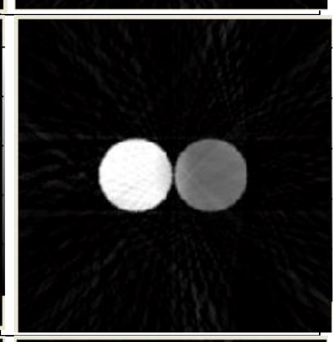
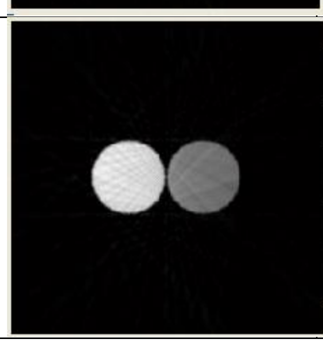
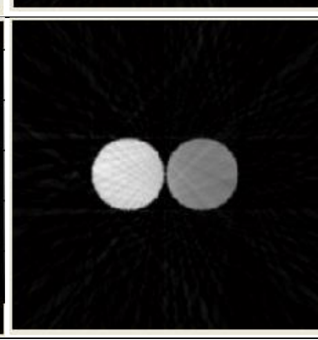
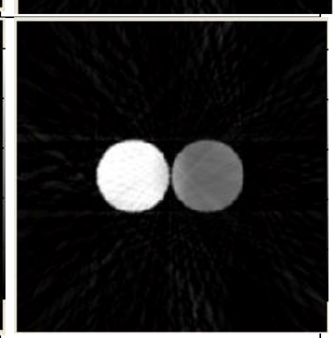
At the end of the time frame 4, the following is the actual image at input and how it looked like



Cross talk test. 2 images close to each other with varying intensities.

***NO NOISE. Showing cross talk at time frame 4 as window size changes***

| Window size | O-HYPR | W-HYPR | LR-HYPR |
|---|---|---|---|
| **3** |  |  |  |
| **5** |  |  |  |

| | | | |
|---|---|---|---|
| **7** |  |  |  |
| **9** |  |  |  |
| **11** |  |  |  |
| **13** |  |  |  |
| **15** |  |  |  |

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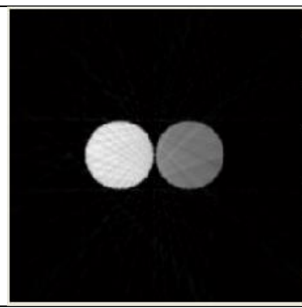
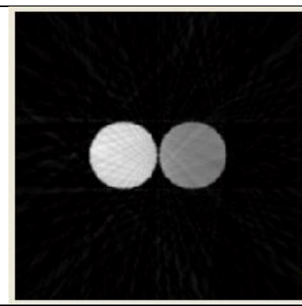
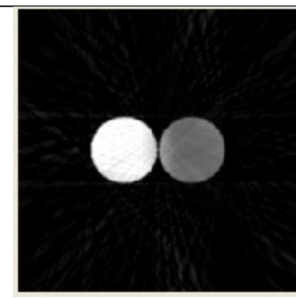
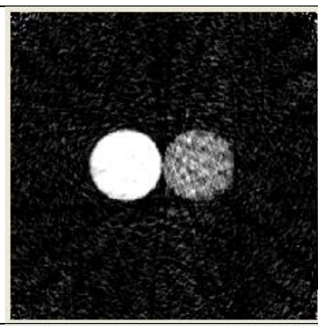
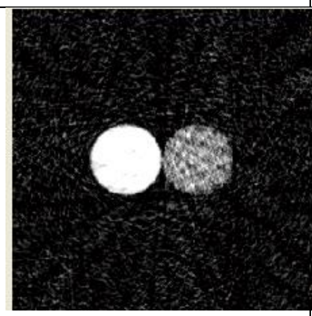
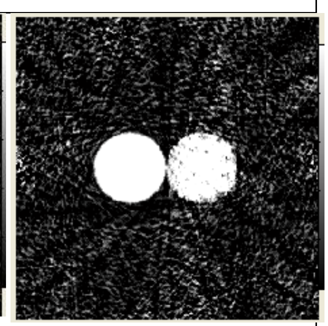
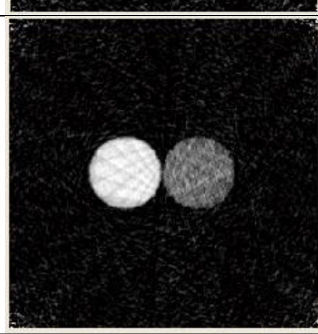
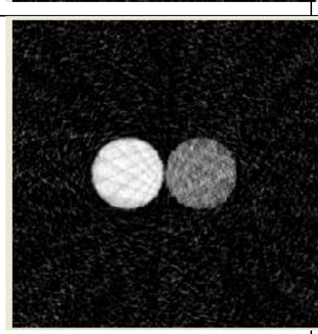
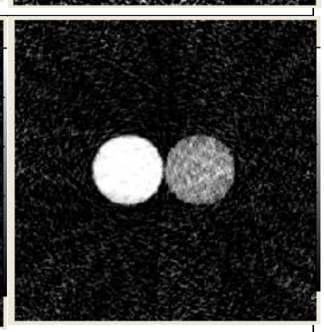
| NONE |  |  |  |

***NOISE ADDED. Showing cross talk at time frame 4 as the window size was changed. Noise is Gaussian with zero mean and 5% S.D. of maximum projection signal.***

| Window size | O-HYPR | W-HYPR | LR-HYPR |
|---|---|---|---|
| 3 |  |  |  |
| 5 |  |  |  |

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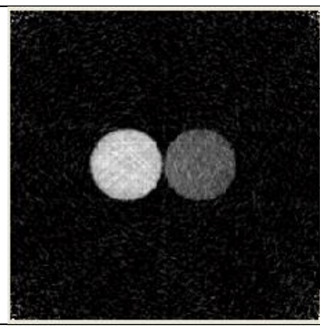
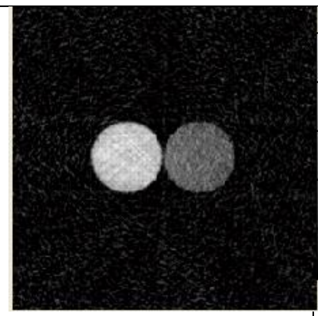
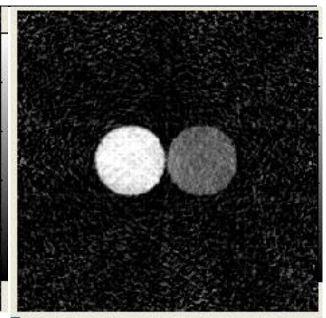
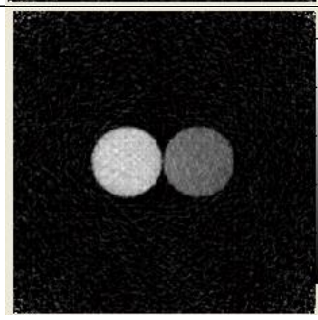
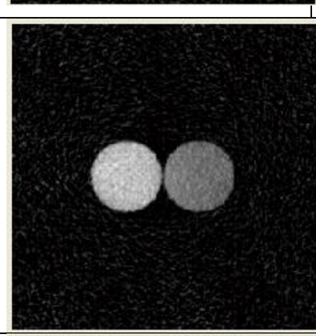
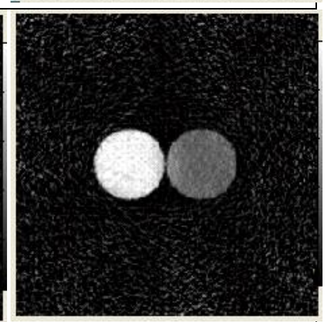
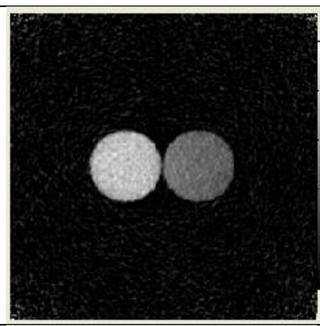
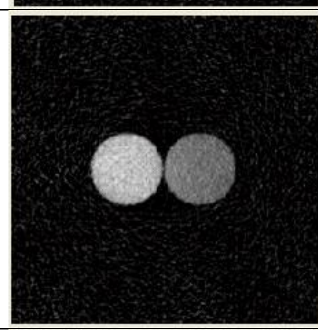
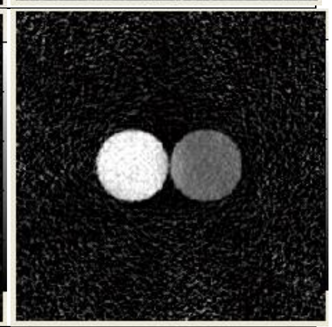
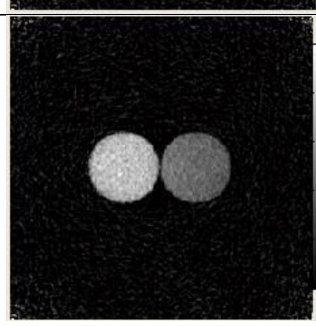
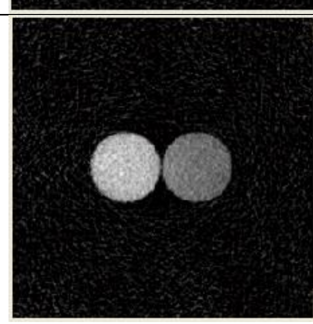
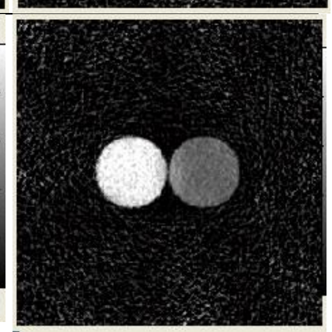
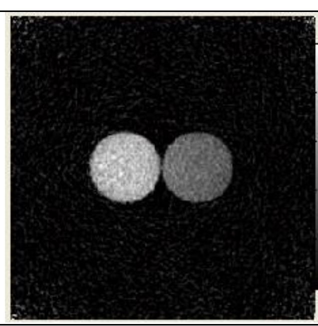
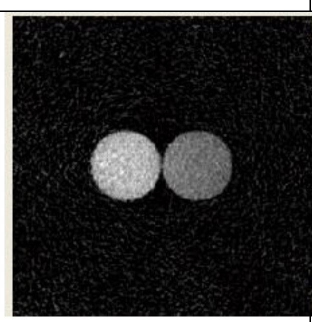
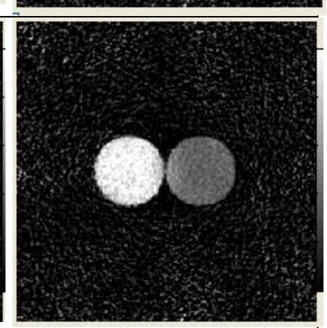
| **7** |  |  |  |
| **9** |  |  |  |
| **11** |  |  |  |
| **13** |  |  |  |
| **15** |  |  |  |

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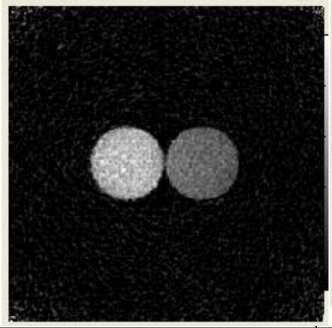
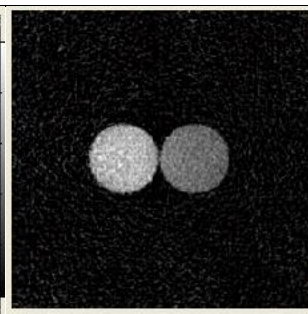
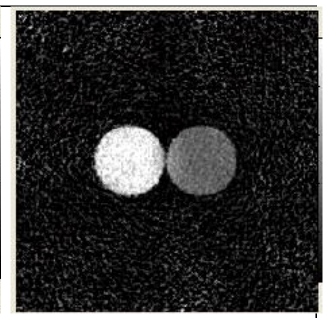
| **NONE** |  |  |  |

## References

(1)Improved Waveform Fidelity Using Local HYPR Reconstruction (HYPR LR) Kevin M. Johnson, Julia Velikina,Yijing Wu, Steve Kecskemeti,, Oliver Wieben, and Charles A. Mistretta
(2) Highly Constrained Back projection for Time-Resolved MRI by C. A. Mistretta, O. Wieben, J. Velikina, W. Block, J. Perry, Y. Wu, K. Johnson, and Y. Wu
(3) Time-Resolved MR Angiography With Limited Projections by Yuexi Huang1,and Graham A. Wright
(4) Iterative projection reconstruction of time-resolved images using HYPR by O'Halloran et.all
(5) Various reports on HYPR from the Mathematics 597 project at CSUF Fullerton, summer 2008 http://12000.org/my_courses/FULLERTON_COURSES/summer_2008/project/

# 5 Matlab functions and simulation

This section will contain collection of functions and simulation I made during work on this project.

1. M file to generate a disk of some radius and center. added June 9, 2008. This function

returns a 2D matrix of a disk (white=1,black=0) nma_makeDisk.m

2. This file is a driver for the above function. Shows examples of how to call the function nma_driver_makeDisk.m

# 6 Class handouts and reference papers

| # | date | handout description | link |
|---|---|---|---|
| 1 | Tuesday 5/27/2008 | Paper: Highly Constrained Backprojection for Time-Resolved MRI by C. A. Mistretta, O. Wieben, J. Velikina, W. Block, J. Perry, Y. Wu, K. Johnson, and Y. Wu1 | link |
| 2 | Tuesday 5/27/2008 | Paper: Iterative projection reconstruction of time-resolved images using highly-constrained back-projection (HYPR) by Rafael L. O'Halloran, Zhifei Wen, James H. Holmes, Sean B. Fain | link |
| 3 | Tuesday 5/27/2008 | Paper: Level Set Reconstruction for Sparse Angularly Sampled Data by Sungwon Yoon; Pineda, R.; Fahrig, R. | link |
| 4 | Tuesday 5/27/2008 | Paper: Reconstructing absorption and diffusion shape profiles in optical tomography by a level set technique by M. Schweiger, S. R. Arridge, O. Dorn, A. Zacharopoulos, and V. Kolehmainen | link |
| 5 | Tuesday 5/27/2008 | 3 pages from book, on discretization delimma | PDF |
| 6 | Tuesday 5/27/2008 | 3 pages from book Foundations of Image Science on MLEM algorithm | PDF |
| 7 | Thursday 5/29/2008 | Tomographic Image Reconstruction Derivation of the central slice theoreom | link |
| 8 | Thursday 6/5/08 | Professor's Gearhart Derivation of Equation (7) in the paper by Sungwon Yoon, A Pineda, and R. Fahrig | PDF |
| 9 | Monday 6/9/08 | Paper (Wright- Huang -HYPR) Time-Resolved MR Angiography With Limited Projections Yuexi Huang and Graham A. Wright | PDF |
| 10 | Monday 6/9/08 | PPT presentation of HYPR by GE | PDF |
| 11 | Wed 6/12/08 | Scan of page from Kak/Stany showing analytical solution to projection of ellipes | image |
| 12 | Monday 6/16/08 | Paper: Improved Waveform Fidelity Using Local HYPR Reconstruction (HYPR LR) by Kevin M. Johnson, Julia Velikina,Yijing Wu,Steve Kecskemeti,Oliver Wieben, and Charles A. Mistretta | PDF |
| 13 | Thursday 6/19/08 | The EM algorithm handout given to us by Dr Gearhart | PDF |
| 14 | Thursday 6/19/08 | Handout from Dr Pineda, the goals of the HYPR project | PDF |
| 15 | Thursday 6/26/08 | Siavash Jalal write up on EM | PDF |
| 16 | Wed 7/02/08 | Paper: Projection Reconstruction MR Imaging Using FOCUSS Jong Chul Ye, Sungho Tak, Yeji Han, and Hyun Wook Park | PDF |
| 17 | Wed 7/02/08 | Paper: An Application of Highly Constrained Back-projection (HYPR) to Time-Resolved VIPR Acquisition J. V. Velikina1, C. A. Mistretta1, K. M. Johnson1, O. Wieben1 | PDF |
| 18 | Tuesday 7/8/08 | Talk by Jeff Fessler at SIAM 2008 in San Diego on MRI | PDF |
| 19 | Thursday 7/10/08 | Send to us by Dr Pineda: Paper Evaluation of Temporal and Spatial Characteristics of 2D HYPR Processing Using Simulations by Yan Wu, Oliver Wieben, Charles A. Mistretta, and Frank R. Korosec | PDF |

| | | | |
|---|---|---|---|
| 20 | Sunday 7/12/08 | Paper: Time-Resolved Contrast-Enhanced 3D MR Angiography by Frank R. Korosec, Richard Frayne, Thomas M. Grist, Charles A. Mistretta | PDF |
| 21 | Sunday 7/12/08 | Paper: MAGNETIC RESONANCE IMAGING (MRI) SIMULATION ON A GRID COMPUTING ARCHITECTURE H. BENOIT-CATTIN, F. BELLET, J. MONTAGNAT, C. ODET | PDF |
| 22 | Sunday 7/12/08 | Thesis: Multidimensional MRI of Cardiac Motion Acquisition, Reconstruction and Visualization Andreas Sigfridsson LIU-TEK- | PDF |
| 23 | Monday 7/12/98 | Siavash derivation of SNR for HYPR | PDF |
| 24 | Sunday 7/19/08 | from Doug, MLEM related power points. Mathematical relation of MLEM to HYPR | PDF |
| 25 | Tuesday 7/22/08 | Paper A self referencing level set method for image reconstruction 2002 | PDF |
| 26 | Saturday 7/26/08 | Paper: 3D Time-Resolved Contrast-Enhanced Cerebrovascular MR Angiography with Subsecond Frame Update Times Using Radial k-Space Trajectories and Highly Constrained Projection Reconstruction Y. Wu, N. Kim, F.R. | PDF |
| 27 | Sunday 7/27/08 | Paper: Undersampled Radial MRI with Multiple Coils. Iterative Image Reconstruction Using a Total Variation Constraint by Kai Tobias Block, Martin Uecker, and Jens Frahm | PDF |
| 28 | Sunday 7/27/08 | Paper: Radial Single-Shot STEAM MRI By Kai Tobias Block and Jens Frahm | PDF |
| 29 | Sunday 7/27/08 | Paper: Novel Radial MRI Technique for Obtaining High Resolution Black Blood Images of the Heart with and without Fat Suppression from a Single k-space Data Set by Zhiqiang Li, Ali Bilgin, Arthur F. Gmitro, and Maria . Altbach1 | PDF |
| 30 | Friday 8/15/08 | Paper: HYPRIT: Generalized HYPR Reconstruction by Iterative Estimation Samsonov AA, Wieben O, Block WF. | PDF |
| 31 | Friday 8/15/08 | Paper: More Optimal HYPR Reconstructions Using a Combination of HYPR and Conjugate-Gradient Minimization by M. A. Griswold1, K. Barkauskas, M. Blaimer, J. L. Sunshine, and J. L. Duerk | PDF |

## 7 Link

1. urlhttp://scien.stanford.edu/class/psych221/projects/02/insomnia/ NOISE measurements in MRI (SNR) and matlab code
2. `http://visielab.ua.ac.be/staff/sijbers/snr_ref.html` web page of references on SNR in MRI
3. `http://www.eng.warwick.ac.uk/oel/courses/undergrad/lec13/applications.htm` 3. http://www.eng.warwick.ac.uk/oel/courses/undergrad/lec13/applications.htm good notes on MRI andbackprojection in general. Warick univ. England.
4. `http://www.patentstorm.us/patents/7277597/description.html` talk about radial acquiztion.
5. `http://www.impactscan.org/slides/eanm2002/sld001.htm` on Filtered backprojection and CT
6. `http://www.eecs.umich.edu/~fessler/` Dr John Fessler web page. He does MRI and this page contains software and papers. The link below is MRI data found

on this page.

7. http://www.nap.edu/readingroom/books/biomedical/ on-line medical imaging book Mathematics and Physics of Emerging Biomedical Imaging
8. http://www.eecs.umich.edu/~fessler/course/516/a/books.txt Dr Fessler recommended books on medical imaging.
9. http://www.eecs.umich.edu/~fessler/result/mr/angio/ MRI data. This is from Fessler group.
10. http://www.radiology.mcg.edu/radiologyphysics/mri/MR%20cha8%20SNR.ppt very good PPT on MRI
11. http://www.radiology.mcg.edu/radiologyphysics/ Where the above was taken. (the chp4 one is GOOD) also the k-space one
12. http://www-cellbio.med.unc.edu/henson_mrm/ looks like have MRI data here. Check it out
13. http://www.ehealthmd.com/library/mri/MRI_whatis.html good description of how MRI works, but no pictures.
14. http://www.mabot.com/brain/ some brain MRI images
15. http://www.hull.ac.uk/mri/lectures/gpl_page.html intro to MRI
16. http://www.sph.sc.edu/comd/rorden/mritut.html MRI tutorial and nice software. Download software to my references folder for math 597 csuf and tried it. No more time.
17. http://www.fonar.com/glossary.htm MRI glossary
18. http://dnl.ucsf.edu/users/dweber/dweber_docs/mri_quality.html good page on MRI quality
19. http://www.dimag.com/cardiovascular/journal/showArticle.jhtml?articleID=201202400 good discussion on sampling for imaging
20. http://airto.bmap.ucla.edu/BMCweb/SharedCode/SpeedLimit/SpeedLimit html good article on MRI
21. http://www.bic.mni.mcgill.ca/cgi/bw/submit_request MRI simulation data request. I did it, but no reply.
22. http://www.nitrc.org/projects/pediatric_mri/ MRI data
23. http://www.bic.mni.mcgill.ca/nihpd/info/data_access.html data access
24. http://mipav.cit.nih.gov/download.php some MRI application, requires Java stuff.
25. http://www.e-mri.org/quality-artifacts/signal-to-noise-ratio.html SNR for MRI
26. http://dynamo.ecn.purdue.edu/~bouman/software/tomography/ MRI matlab data
27. http://en.wikipedia.org/wiki/Projection-slice_theorem Central slice theorem
28. http://www.slaney.org/pct/ Book Principles of Computerized Tomographic Imaging. See chapter 7. Here is the web page of the book which can be downloaded for free or buy from amazon by Avinash C. Kak and Malcolm Slaney
29. http://www.archive.org/details/Lectures_on_Image_Processing on-line lectures on digital image processing.

30. http://www.ismrm.org/ The International Society for Magnetic Resonance in Medicine

31. http://www.ismrm.org/07/Session53.htm This page contains papers on Compressed Sensing and HYPR (It is part of workshop by ISMRM held in 2007)

32. http://www.cs.ubc.ca/~mitchell/ToolboxLS/ Level set Matlab toolbox (thanks to Dr Pineda for the link)

33. http://focus.ti.com/docs/solution/folders/print/275.html contains a detailed block diagram of MRI