n is input to program July 19, 2002
2-3 signif digits for n=100 FEM

Math 503 Computing Assignment 1

1. Let V be the space of continuously differentiable functions y on the interval $[0,1]$ such that $y(0) = 0$ and $y(1) = 0$. On this space, consider the functional

J(y+h) approach

$$J[y] = \int_0^1 \left([y'(x)]^2 + q[y(x)]^2 - 2fy(x) \right) dx$$

where $q > 0$ and f are given constants. (a) Show that J achieves a minimum at y if and only if

$$\int_0^1 [(y'(x)\phi'(x) + qy(x)\phi(x) - f\phi(x)]\, dx = 0 \,,$$

for all $\phi \in V$. (b) Show that if $y \in V$ is twice continuously differentiable, and satisfies this optimality condition, then y satisfies the differential equation $-y''(x)+qy(x) = f$, $u(0) = 0$, $y(1) = 0$. (c) Conversely, show that if $y \in V$ is twice continuously differentiable, and satisfies the differential equation above, then y satisfies the optimality condition of part (a).

like problem #3 for membrane

due Thursday

2. (a) Let $\phi_1, \phi_2, \cdots \phi_n$ be a set of linearly independent functions in V. Suppose y is approximated by

$$v = \sum_{j=1}^{n} c_j \phi_j(x) \,, \quad \text{for some coefficients } \; c_1, c_2, \cdots, c_n \,.$$

Use the result of the previous problem, part (a), to find a linear system of equations, say $Ac = b$, that determines the vector c of coefficients $c_1, c_2, \cdots, c_n$. (b) Use this result to estimate the solution of the differential equation

$$-u''(x) + qu(x) = f \,, \quad u(0) = 0 \,, \quad u(1) = 0 \,,$$

on the interval $[0,1]$. For integer $n \geq 0$, discretize the interval with points $x_i = ih$, for $i = 0, 1, 2, \cdots, n+1$, where $h = 1/(n+1)$. For basis functions use the "tent" functions, defined by

$$\phi_i(x) = \phi\left(\frac{x}{h} - i\right) \quad \text{for} \quad i = 1, 2, \cdots, n \,,$$

where the function ϕ is defined by $\phi(x) = 1 - x$, for $0 < x < 1$, $\phi(x) = 0$, for $x > 1$, and $\phi(x) = \phi(-x)$. Take $q = 4$ and $f = 4$. (c) Compare the accuracy of the estimated solution with the estimated solution obtained by discretization of the differential equation using the central divided difference for the second derivative term.

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Computing Projects

A. Format of reports Submit a typed or legibly written report, outlined as follows.

a. Purpose and design of project Describe briefly the nature of your project and state precisely the questions you are investigating. Explain how your experiments and analysis are designed to address these questions. A few words about your computer program might be appropriate, but typically the annotated listing in the appendix will be enough.

b. Summary of numerical results Usually tables will do, with possibly additional observations about the numerical methods. However, be sure to present the summary of your results so that the reader is able to see clearly how they support your conclusions.

Table
No comments
comments

c. Discussion of numerical results Summarize in words your main results and conclusions. When appropriate, and if possible, provide explanations for your results.

d. Program listing In an appendix, provide a listing of your computer program. Include annotations which help to show clearly the content and logic of your program.

B. Grading The grade will be based on the following.

a. Technical content and completeness Have a clearly defined purpose, and design your experiments so that you can answer the questions you intended to study. The analysis and discussion of the results should be accurate and complete. Cover all topics requested in the assignment. Your program should be correct.

b. Organization of the report Organize the report so the reader can easily get a summary of the main results of your experiments, and thus see how you came to your conclusions.

c. Clarity and style of exposition Write to communicate. Get to the point and say what you want to say. Use complete sentences and acceptable grammar. BE BRIEF.