HW 1 Mathematics 503, Mathematical Modeling, CSUF June 2 2007

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Contents

1 Problem 1 (section 1.1,#3, page 7)

problem:

3. In the blast wave problem take $C = 1$ (a v
and use $\rho = 1.25\ \mathrm{kg/m^3}$. Some of the radiu
data for the Trinity explosion is given in the

t	0.10	0.52	1.08	1.5	1.93
r	11.1	28.2	38.9	44.4	48.7

Using these data, estimate the yield of the T
kiloton equals $4.186(10)^{12}$ joules). Compare y
of approximately 21 kilotons.

solution:

The blast equation is given by

$$r = C\left(\frac{Et^2}{\rho}\right)^{\frac{1}{5}}$$

Where r is radius of blast in meters, t is time since explosion in seconds, E energy of explosion in Joules, ρ is air density in Kg/m^3

We are given $\rho = 1.25\ kg/m^3$ and $C = 1$, hence the above can be rewritten as

$$r^5 = 0.8Et^2$$

Let $x = E, A = 0.8t^2$, $b = r^5$, then we have

$$b = Ax$$

And now x is found by least squares solution. Once x is found, then E is found since $E = x$. The code is shown on the next page. The least squares solutions shows that

$$\boxed{E = 16.6175 \text{ kilotons}}$$

Problem 1 (#3, section 1.1, page 7) Code implementation by Nasser Abb
Modeling. CSUF Summer 2007. Use LeastSquare to fit data to find Energ

```
Remove["Global`*"];

eq = r == C ((energy * t^2)/ρ)^(1/5);
```

Formulate the data points

```
timePtsInMs = {0.1, 0.52, 1.08, 1.5, 1.93, 4.07, 15.0, 34.0};
timePtsInSec = timePtsInMs * 0.001;
A = 0.8 * timePtsInSec^2;

rPtsInMeter = {11.1, 28.2, 38.9, 44.4, 48.7, 64.3, 106.5, 145};
b = rPtsInMeter^5;
```

Now we have b = Ax, where x = energy. Solve for x using LeastSquares

```
x = Flatten[LeastSquares[Map[List, A], Map[List, b]]];
energyInJoules = x;
energyInKTon = energyInJoules / (4.186 * 10^12)
```

```
{16.6175}
```

Now plot the original data

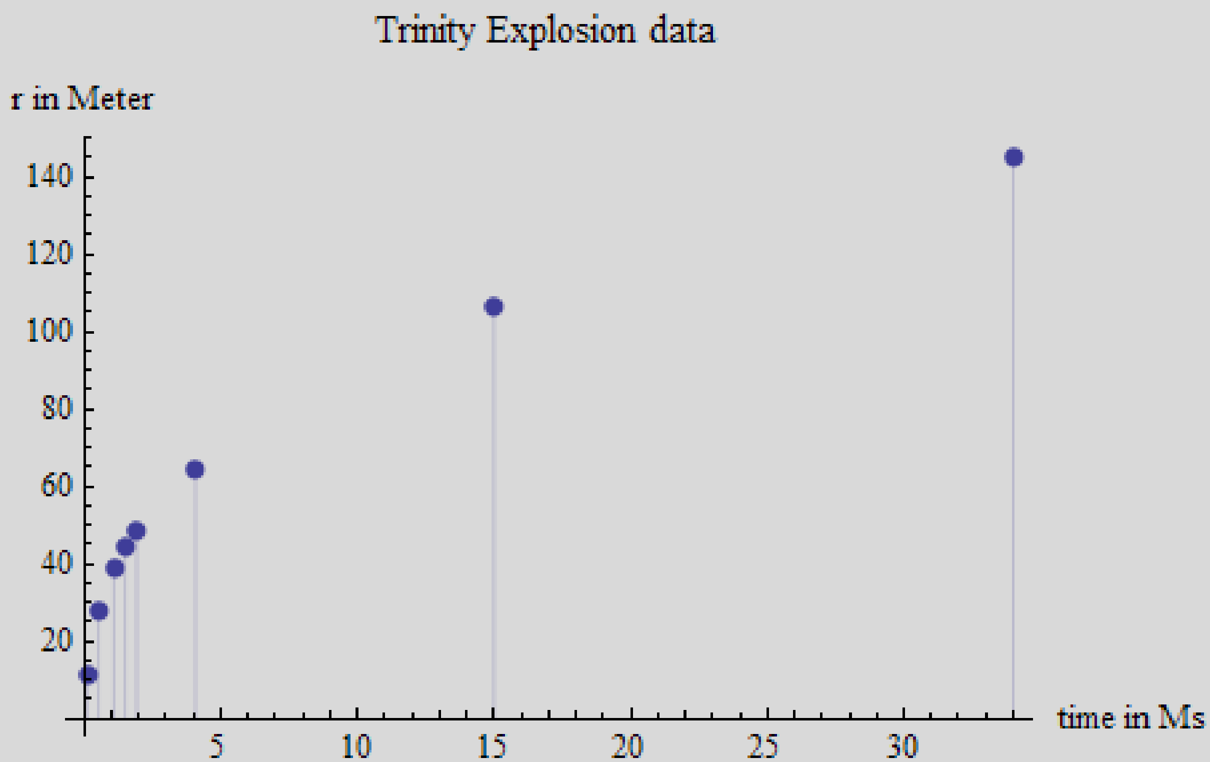
```
pts = Table[{timePtsInMs[[i]], rPtsInMeter[[i]]}, {i, 1, Length[tir
```

Display data in table format

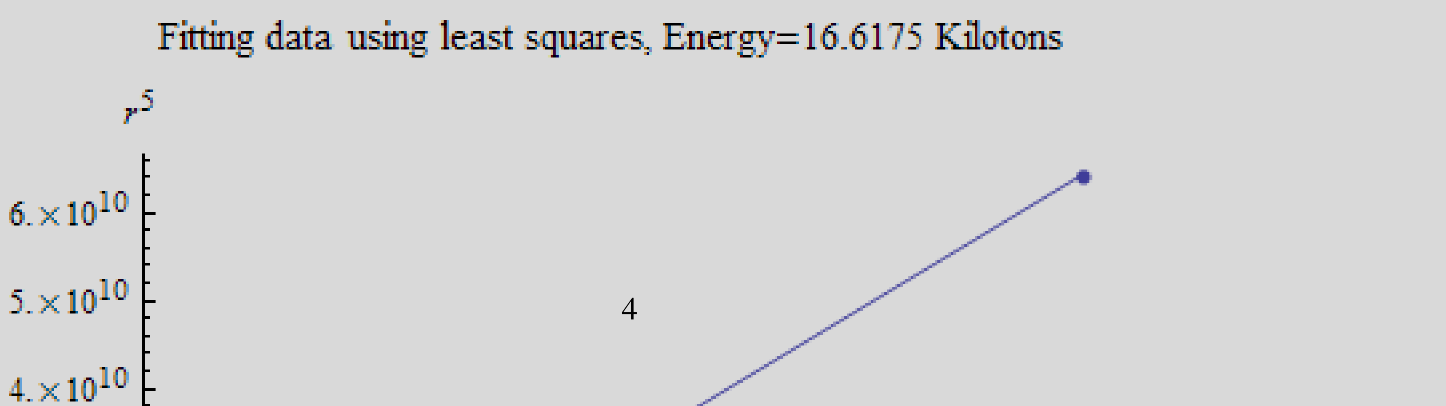
```
Grid[{Join[{"t"}, timePtsInMs], Join[{"r"}, rPtsInMeter] }, Fr
(*TableForm[pts,TableDirections→Column,TableHeadings→{None,
```


t	0.1	0.52	1.08	1.5	1.93	4.07	15.	34.

```
p1 = ListPlot[pts, PlotStyle → PointSize[.02], PlotRange → All,
  Frame → False, Filling → Axis, AxesLabel → {"time in Ms", "r
  PlotLabel -> "Trinity Explosion data"]
```




```
pts = Table[{A⟦i⟧, b⟦i⟧}, {i, 1, Length[timePtsInMs]}];
p1 = ListPlot[pts, PlotStyle → PointSize[.015], PlotRange → All
p2 = Plot[ energyInJoules * t, {t, 0, Last[A]}];
Show[{p1, p2}, PlotRange → All,
 PlotLabel → "Fitting data using least squares, Energy=16.617
 AxesLabel → {"0.8 t^2", "r^5"}]
```



2 Problem 2 (section 1.1, #11, page 18)

problem:

11. The problem is to determine the power P
ship of length l moving at a constant spe
reasonable, that P depends on the density
to gravity g, and the viscosity of water ν
well as l and V, then show that

$$\frac{P}{\rho l^2 V^3} = f(\mathrm{Fr},$$

where Fr is the Froude number and Re is t

$$\mathrm{Fr} \equiv \frac{V}{\sqrt{lg}}, \quad \mathrm{Re}$$

solution:

First make a list of all the physical variables and the corresponding dimensions.

Variable	P (power)	l (ship length)	V	ρ	ν	g
meaning	work rate(F*d/t)	in meters	ship speed	water density	water Viscosity	gravity
Dimension	$\frac{ML^2}{T^3}$	L	$\frac{L}{T}$	$\frac{M}{L^3}$	$\frac{L^2}{T}$	$\frac{L}{T^2}$

Hence we seek a physical law of the form

$$f(P, l, V, \rho, \nu, g) = 0$$

The function f is a combination of all the physical variables. Hence we write

$$\begin{aligned}
1 = [\pi] &= \left[P^a l^b V^c \rho^d \nu^e g^f\right] \\
&= \left(\frac{ML^2}{T^3}\right)^a (L)^b \left(\frac{L}{T}\right)^c \left(\frac{M}{L^3}\right)^d \left(\frac{L^2}{T}\right)^e \left(\frac{L}{T^2}\right)^f \\
&= \left(M^a L^{2a} T^{-3a}\right)\left(L^b\right)\left(L^c T^{-c}\right)\left(M^d L^{-3d}\right)\left(L^{2e} T^{-e}\right)\left(L^f T^{-2f}\right) \\
&= \left(M^{a+d}\right)\left(L^{2a+b+c-3d+2e+f}\right)\left(T^{-3a-c-e-2f}\right)
\end{aligned}$$

For the above combination to be dimensionless, we must have each exponent term equal to zero. Hence we obtain the following 3 equations

$$\begin{aligned}
a+d &= 0 \\
2a+b+c-3d+2e+f &= 0 \\
-3a-c-e-2f &= 0
\end{aligned}$$

Writing the above in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & -3 & 2 & 1 \\ -3 & 0 & -1 & 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

We see that the dimension matrix 3×6, hence its row space has dimension 6 and its column space has dimension 3. We need now to find the basis for the Null Space of the dimension matrix. Now reduce A to its reduce row echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & -3 & 2 & 1 \\ -3 & 0 & -1 & 0 & -1 & -2 \end{bmatrix} \overset{l_{21}=2}{\rightarrow} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -5 & 2 & 1 \\ -3 & 0 & -1 & 0 & -1 & -2 \end{bmatrix} \overset{l_{31}=-3}{\rightarrow} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -5 & 2 & 1 \\ 0 & 0 & -1 & 3 & -1 & -2 \end{bmatrix}$$

Now multiply last row by $-1 \to \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -5 & 2 & 1 \\ 0 & 0 & 1 & -3 & 1 & 2 \end{bmatrix}$, and now subtract last row from second row

$\to \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -1 \\ 0 & 0 & 1 & -3 & 1 & 2 \end{bmatrix}$ and this is the final reduce row echelon form

we have rank=3 (the first 3 columns are Linearly independent). Therefor, we use the first 3 variables as the pivot variables, which are a, b, c and use as the free variables those which correspond to the last 3 columns, which are d, e, f

Now since

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -1 \\ 0 & 0 & 1 & -3 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By back substitution, from the 3rd row we obtain $c - 3d + e + 2f = 0$, or $\boxed{c = 3d - e - 2f}$ and from the second row we obtain $b - 2d + e - f = 0$, or $\boxed{b = -e + f + 2d}$, and from the first row we obtain $a + d = 0$ or $a = -d$

Hence the solution now can be written in terms of the free variables as

$$x_{null} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} -d \\ -e+f+2d \\ 3d-e-2f \\ d \\ e \\ f \end{bmatrix}$$

$$= -d \begin{bmatrix} 1 \\ -2 \\ -3 \\ -1 \\ 0 \\ 0 \end{bmatrix} - e \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} - f \begin{bmatrix} 0 \\ -1 \\ 2 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

Therefor the basis for the null space of A are

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \\ 0 \\ 0 \\ -1 \end{bmatrix} \right\}$$

Hence

$$\begin{aligned}
\pi_1 &= P^1 l^{-2} V^{-3} \rho^{-1} \nu^0 g^0 = \frac{P}{l^2 V^3 \rho} \\
\pi_2 &= P^0 l^1 V^1 \rho^0 \nu^{-1} g^0 = \frac{V l}{\nu} \\
\pi_3 &= P^0 l^{-1} V^2 \rho^0 \nu^0 g^{-1} = \frac{V^2}{l\, g}
\end{aligned}$$

Hence the complete set is

$$\left\{ \pi_1 = \frac{P}{l^2 V^3 \rho}, \pi_2 = \frac{V l}{\nu}, \pi_3 = \frac{V^2}{l\, g} \right\}$$

Hence the general solution is $\pi = \pi_1^{\alpha} \pi_2^{\beta} \pi_3^{\gamma}$

The Pi theorem says that there is a physical law expressed in terms of the dimensionless quantities called $F(\pi_1, \pi_2, \pi_3) = 0$ corresponding to the physical law $f(P, l, V, \rho, \nu, g) = 0$

Now, we need to solve for P, hence we write

$$\begin{aligned}
\pi_1 &= g(\pi_2, \pi_3) \\
\frac{P}{l^2 V^3 \rho} &= g\left(\frac{V l}{\nu}, \frac{V^2}{l\, g} \right)
\end{aligned}$$

Hence if we let $g\left(\frac{Vl}{\nu}, \frac{V^2}{l\, g}\right) = \pi_2 \times \sqrt{\pi_3}$, then we see that

$$\boxed{\frac{P}{l^2 V^3 \rho} = g(F_r, R_e)}$$

where

$$F_r = \frac{V l}{\nu}, R_e = \frac{V}{\sqrt{l\, g}}$$

Let k be some constant, then the power needed is given by

$$\begin{aligned}
P &= k l^2 V^3 \rho \frac{V l}{\nu} \frac{V}{\sqrt{l\, g}} \\
&= k \frac{l^3 V^5 \rho}{\nu \sqrt{l\, g}}
\end{aligned}$$