Solutions to HW #2 Math 307 Spring 2007

Section 1.5

• #2 $A = LU = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix}}_{U}$

a) $\boxed{\ell_{32} = 4}$

b) Pivots = 5, 2, 6

c) Since we could factor the matrix into LU, then no row exchanges are required. Note that the pivots are nonzero.

\# 5 $Ax = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$ to factor into LU, we carry out Gaussian elimination.

$$\left[\begin{array}{ccc|c} 2 & 3 & 3 & 2 \\ 0 & 5 & 7 & 2 \\ 6 & 9 & 8 & 5 \end{array}\right] \xrightarrow[R3-3R1]{} \left[\begin{array}{ccc|c} 2 & 3 & 3 & 2 \\ 0 & 5 & 7 & 2 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

U

, hence $E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ $L_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$,

therefore $A = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix}}_{U}$, $Ux = c \Rightarrow \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$.

• # 6 $E = \begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix}$ $E^2 = \begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 6 \cdot [1\ 0] + 1 \cdot [6\ 1] \end{bmatrix}$ writing the second row using the row view, in general the second row will be $E^n \to [n6 \quad 1]$

$= \begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$

$E^8 = \begin{bmatrix} 1 & 0 \\ 6 \cdot 8 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 48 & 1 \end{bmatrix}$ $E^{-1} = \begin{bmatrix} 1 & 0 \\ -6 & 1 \end{bmatrix}$ be careful, E^{-1} is not given by taking the reciprocal of the elements.

• # 9 $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} d_1 & & \\ & d_2 & \\ & & d_3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

a) The diagonal matrix contains the pivots, so the matrix A is nonsingular when $d_1 \neq 0$ and $d_2 \neq 0$ and $d_3 \neq 0$ or equivalently $d_1 d_2 d_3 \neq 0$.

b) $\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{\vec{c}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\vec{b}}$

$c_1 = 0$
$-1c_1 + c_2 = 0 \Rightarrow c_2 = 0$
$-1c_2 + c_3 = 1 \Rightarrow c_3 = 1$

and then we solve $U\vec{x} = \vec{c}$

$d_1x_1 - d_2x_2 = 0 \quad x_1 = 1/d_3$
$d_2x_2 - d_3x_3 = 0 \Rightarrow x_2 = 1/d_3$
$d_3x_3 = 1 \Rightarrow x_3 = 1/d_3$

Note that b) refers to the same matrix as A. The key is to solve $Ax = b$ by first decomposing A into LU: $LU\vec{x} = \vec{b}$. [let $c = Ux$], and then solve $L\vec{c} = b$ for $\vec{c}$, and then $U\vec{x} = \vec{c}$ for $\vec{x}$.

• #24

$$A=\begin{bmatrix}1&0&1\\2&2&2\\3&4&5\end{bmatrix}\xrightarrow[E_{21},\,E_{31}]{}\begin{bmatrix}1&0&1\\0&2&0\\0&4&2\end{bmatrix}\xrightarrow[E_{32}]{}\underbrace{\begin{bmatrix}1&0&1\\0&2&0\\0&0&2\end{bmatrix}}_{U}, \quad E_{21}=\begin{bmatrix}1&0&0\\-2&1&0\\0&0&0\end{bmatrix},\ E_{31}=\begin{bmatrix}1&0&0\\0&1&0\\-3&0&1\end{bmatrix},\ E_{32}=\begin{bmatrix}1&0&0\\0&1&0\\0&-2&0\end{bmatrix}$$

$$L=\begin{bmatrix}1&0&0\\2&1&0\\3&2&0\end{bmatrix}$$ ← found by placing the multipliers in the appropriate locations. Check $LU=A$.

#42 To get two matrices that commute, it is simple to use $P=I$ and it will commute with anything.

$$\underset{P_1}{\begin{bmatrix}1&0\\0&1\end{bmatrix}}\underset{P_2}{\begin{bmatrix}0&1\\1&0\end{bmatrix}}=\begin{bmatrix}0&1\\1&0\end{bmatrix}=\underset{P_2}{\begin{bmatrix}0&1\\1&0\end{bmatrix}}\underset{P_1}{\begin{bmatrix}1&0\\0&1\end{bmatrix}} \ \checkmark$$

You could also have chosen other permutation matrices that would work.

$$P_1P_2=\underset{P_1}{\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}}\underset{P_2}{\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}}=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix} \qquad P_2P_1=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$$

$P_1P_2 \neq P_2P_1$! Most of them don't commute.

Nice & short HW, eh?